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NONLINEAR MICROWAVE TUBE STUDIES

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ABSTRACT

An analytical, nonlinear space charge wave theory is given for the velocity modulated, confined electron beam of finite radial extension. The present theory gives a more accurate nonlinear description of the electron velocities and the ac current amplitudes at the fundamental and harmonic frequencies than earlier space charge wave type theories do. Although the theory is not strictly valid after electron overtaking has occurred, a formulation is used which allows the electron velocities to be multi-valued functions of the space coordinates. Therefore reasonably good results should be expected also in the overtaking range. For very large signals the results transform to those of Webster's ballistic theory. The analytical results obtained agree well with computer experiments.

I. INTRODUCTION AND SUMMARY

Webster [1] developed, ignoring space charge forces, an elementary theory of the velocity modulated (klystron) electron beam. Thus, with the velocity modulation $v_1^o \sin\omega t_o$ imposed on a beam with the initial velocity v_o, Webster writes for the electron drift distance z, measured from the modulation grids,

$$z = (v_o + v_1^o \sin\omega t_o)(t - t_o) \tag{1}$$

For the infinitely wide klystron beam Olving [2], taking the space charge debunching forces into account, obtained

$$z = v_o(t-t_o) + \frac{v_1^o}{\omega_p}\sin[\omega_p(t-t_o)]\sin\omega t_o \tag{2a}$$

or

$$\beta_e z = \omega(t-t_o) + A_o \sin[\omega_p(t-t_o)]\sin\omega t_o \tag{2b}$$

where ω_p is the angular plasma frequency of the beam electrons,

$$\beta_e = \frac{\omega}{v_o} \text{ and } A_o = \frac{v_1^o}{v_o}\frac{\omega}{\omega_p},$$

Eq. (2) exactly accounts for the space charge effects, but, unlike Eq. (1), it is not valid after electron overtaking has occurred. By Fourier analysis Webster and Olving found for the ac beam current I_{ac} (dc current I_o)

$$\frac{I_{ac}}{I_o} = 2\sum_{\nu=1}^{\infty} J_\nu(\nu X)\cos\nu(\omega t - \beta_e z) \tag{3}$$

where J_ν is the Bessel function. In Webster's case the bunching parameter

$$X = \frac{v_1^o z\omega}{v_o^2} = A_o \frac{\omega_p z}{v_o}$$

while in Olving's case

$$X = A_o \cdot \sin(\frac{\omega_p z}{v_o}) .$$

Eq. (3) decribes the current nonlinearities and harmonic frequency components.

For the radially finite electron beam, as opposed to the infinite beam, the space charge debunching forces become nonlinear which is bound to complicate the analysis. Retaining terms only of the three lowest orders, instead of Eq. (2b), one now expects to obtain an equation of the form

$$\beta_e z = \omega(t-t_o) + A[\sin \omega_q(t-t_o) - A^2F_1] \sin \omega t_o - A^2F_2 \sin 2\omega t_o - A^3F_3 \sin 3\omega t_o \tag{4}$$

where ω_q is the reduced angular plasma frequency, the drive parameter

$$A = \frac{v_1^o \omega}{v_o \omega_q}$$

and F_1, F_2 and F_3 are some functions of $\omega_q(t-t_o)$. The first step of the present work is to express these functions in Lagrangian variables. By Fourier analysis an equation corresponding to Eq. (3) will then be derived for the ac beam currents. This equation should give the nonlinearities and high harmonic frequency components with good accuracy. The accuracy should also be reasonable in the range where overtaking has occurred.

Starting from the linear space charge wave theory of Hahn and Ramo [3], Paschke [4, 5] developed for the klystron case a nonlinear theory which describes the first nonlinearities at small signal levels. Similar methods were used by Olving and Wallander [6, 7] on the klystron and by Nilsson [8] on the TWT. For the ac beam currents in a klystron these methods give equations, containing terms to the third order, of the type

$$\frac{I_{ac}}{I_o} \simeq A\left[\sin\left(\frac{\omega_q z}{v_o}\right)+A^2\Gamma_1\right]\cos(\omega t-\beta_e z) + A^2\Gamma_2\cos 2(\omega t-\beta_e z) + A^3\Gamma_3\cos 3(\omega t-\beta_e z) \qquad (5)$$

where Γ_1, Γ_2 and Γ_3 are functions of $\omega_q z/v_o$. It is hardly feasable to calculate terms of higher order because of the amount of work required. Another drawback is that Eulerian formulation is used, that is electron velocities and other beam quantities are considered functions of present time and space coordinates. These functions are single-valued even when the velocity is actually many-valued, which means that the analysis deteriorates very fast when electron overtaking occurs. The theory of the present report is essentially Lagrangian which allows many-valued velocities to be taken into account.

In the present work the beam model used is the hydrodynamic disc-electron beam [7]. The beam is supposed to consist of an infinite number of rigid, charged, infinitesimal discs (Fig. 1).

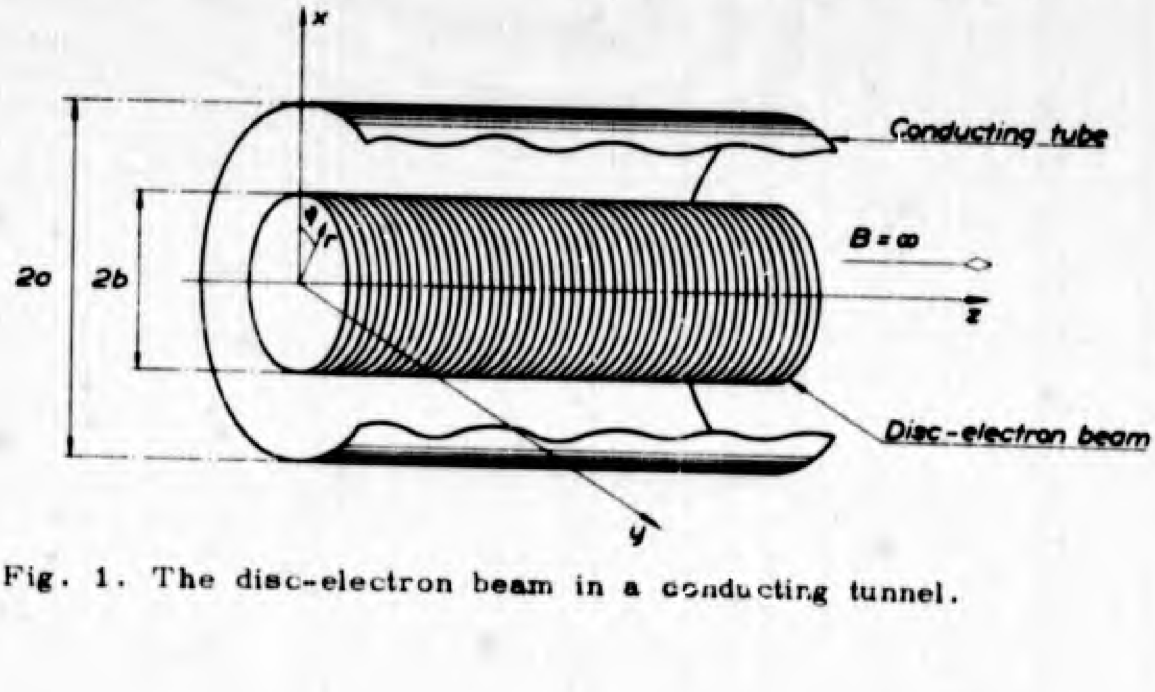

Fig. 1. The disc-electron beam in a conducting tunnel.

Other means for electron beam studies are offered by computer experiments [9, 10], in which Newton's equation of motion is solved on a digital computer for a finite number of discs. Thereby no restrictions are imposed by electron overtaking. The results of the present analytical theory are in good agreement with the numerical computer theories [10].

II. THE NONLINEAR SPACE CHARGE WAVE EQUATIONS

In this section nonlinear differential equations will be derived for longitudinal space charge waves on the disc-electron beam. An essentially Lagrangian formulation will be used. The equations are exactly valid provided no overtaking has occurred. The linear and nonlinear solutions will be given in Sections III and IV respectively.

The homogeneous, confined electron beam and the coordinate system are shown in Fig. 1. We introduce the following notations

t = time

$z = z_o + z_d$ position of an electron disc

$v = v_o + v_d$ velocity of an electron disc

$i = i_o + i_d$ convection current density

$\rho = \rho_o + \rho_d$ electron space charge density

Here z_o, v_o, i_o and ρ_o are the quantities in the absence of modulation, while v_d, i_d and ρ_d are corrections due to the disc displacements z_d which are caused by the modulation. Actually z_o can be taken as $v_o(t-t_o)$.

E_z, H_φ field components due to the modulation

$c = (\epsilon_o \mu_o)^{-1/2}$ velocity of light in vacuum

m, $-e$ electron mass and charge

All the modulation dependent quantities, including the electromagnetic fields, are considered as functions of the time t and the undisturbed electron position z_o. The field quantities are also functions of the radial coordinate r.

The modulation dependent beam quantities will now be expressed in terms of the displacement z_d.

From the definition of velocity one has

$$v = \frac{dz}{dt}, \qquad v_o = \frac{dz_o}{dt}, \qquad v_d = \frac{dz_d}{dt} \tag{6 a, b, c}$$

where the time derivative is the linear operator

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v_o \frac{\partial}{\partial z_o}$$

Charge conservation yields

$$\rho = \rho_o + \rho_d = \frac{\rho_o}{\frac{\partial z}{\partial z_o}} \quad \text{or} \quad \rho_d = - \rho_o \frac{\frac{\partial z_d}{\partial z_o}}{1 + \frac{\partial z_d}{\partial z_o}} \tag{7}$$

For the current density correction i_d one has

$$i_d = i - i_o = \rho v - \rho_o v_o = \rho_o v_d + v_o \rho_d + \rho_d v_d \tag{8}$$

Introduction of Eqs. (6c) and (7) into Eq. (8) yields

$$i_d = \rho_o \frac{\frac{\partial z_d}{\partial t}}{1 + \frac{\partial z_d}{\partial z_o}} \tag{9}$$

The beam quantities are connected to the field quantities through Maxwell's equations and Newton's equation of motion. Maxwell's equations are usually formulated in the Eulerian variables t, z and r and contain partial derivatives. For these derivatives we use the notations $\frac{\delta}{\delta t}$, $\frac{\delta}{\delta z}$ and $\frac{\delta}{\delta r}$. These operators have to be expressed in the form of partial derivatives in the variables t, z_o and r. The following relations are derived in Appendix I.

Operator in independent variables t, z and r	Corresponding operator in independent variables t, z_o and r
$\frac{\delta}{\delta t}$	$\frac{\partial}{\partial t} - \frac{\frac{\partial z_d}{\partial t}}{1 + \frac{\partial z_d}{\partial z_o}} \cdot \frac{\partial}{\partial z_o}$
$\frac{\delta}{\delta z}$	$\frac{\frac{\partial}{\partial z_o}}{1 + \frac{\partial z_d}{\partial z_o}}$
$\frac{\delta}{\delta r}$	$\frac{\partial}{\partial r}$

(10)

With the quasistatic approximation and with no azimuthal variation, Maxwell's equations immediately yield the following equations in t, z and r variables

$$(P + \frac{\delta^2}{\delta z^2}) E_z = \frac{1}{\epsilon_o} \frac{\delta \rho_d}{\delta z} \tag{11}$$

where the Bessel operator

$$P = \frac{1}{r} \frac{\delta}{\delta r} r \frac{\delta}{\delta r} \left(= \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r}\right)$$

and

$$\epsilon_o \frac{\delta^2 E_z}{\delta r \delta t} + \frac{\delta^2 H_\varphi}{\delta z^2} = 0 \tag{12}$$

Since E_z, H_φ and ρ_d are functions of t, z_o and r the expressions (10) must be used for the derivatives. Simultaneous introduction of Eq. (7) for ρ_d into Eq. (11) yields $(\frac{\partial}{\partial r}\rho_o z_d = 0)$

$$(P-Q^2)(E_z + \frac{\rho_o}{\epsilon_o} z_d) = 0 \tag{13}$$

$$\epsilon_o \Omega \frac{\partial E_z}{\partial r} - Q^2 H_\varphi = 0 \tag{14}$$

where Q^2 and Ω are the operators

$$Q^2 = - \frac{\frac{\partial}{\partial z_o}}{1 + \frac{\partial z_d}{\partial z_o}} \cdot \frac{\frac{\partial}{\partial z_o}}{1 + \frac{\partial z_d}{\partial z_o}} = - \frac{\frac{\partial^2}{\partial z_o^2}}{(1 + \frac{\partial z_d}{\partial z_o})^2} + \frac{\frac{\partial^2 z_d}{\partial z_o^2} \frac{\partial}{\partial z_o}}{(1 + \frac{\partial z_d}{\partial z_o})^3}$$

and

$$\Omega = \frac{\partial}{\partial t} - \frac{\frac{\partial z_d}{\partial t}}{1 + \frac{\partial z_d}{\partial z_o}} \cdot \frac{\partial}{\partial z_o}$$

Since the discs must move under the influence of the integral of the longitudinal electric field E_z over the disc surface one has from Newton's equation of motion

$$\frac{d^2 z_d}{dt^2} = - \frac{e}{m} \int_0^b \frac{E_z 2\pi r \, dr}{\pi b^2} \tag{15}$$

where

$$\frac{d^2}{dt^2} = (\frac{\partial}{\partial t} + v_o \frac{\partial}{\partial z_o})^2$$

For longitudinal space charge waves Eqs. (13) and (15) have to be solved simultaneously with initial conditions and boundary conditions taken into account. Nonlinearities in the equations arise due to the nonlinearities in the operators Q^2 and Ω. The equations are exactly valid as long as no overtaking occurs, that is, as long as $\frac{\partial z_d}{\partial z_o} > -1$. After overtaking has occurred, Eq. (13) will give the wrong field E_z since Eqs. (7) and (10) are no longer valid.

III. THE LINEAR SOLUTION

We now consider the linearized displacement wave z_1 propagating on the beam together with the linearized longitudinal electric field E_1. The quantities z_1 and E_1 satisfy the linearized Eq. (13), viz.

$$(P-Q_1^2)(E_1+\frac{\rho_o}{\epsilon_o}z_1) = 0 \tag{16}$$

where Q_1^2 is the linearized operator Q^2

$$Q_1^2 = -\frac{\partial^2}{\partial z_o^2}$$

Let z_1 and E_1 be waves of the angular frequency ω. Only beams with $v_1^o << v_o$ and $\omega_q << \omega$ are dealt with in the present report. For such beams it is well known that the wave pattern moves very slowly relative to the electrons in the beam (velocity v_o) and that the amplitude does not change appreciably over a distance of one beam wavelength. Thus we can write

$$Q_1^2 = -\frac{\partial^2}{\partial z_o^2} \simeq -\frac{1}{v_o^2}\frac{\partial^2}{\partial t^2} = \frac{\omega^2}{v_o^2} = \beta_e^2$$

Then the solutions of Eq. (16) become

$$E_1 = B_1 I_o(\beta_e r) - \frac{\rho_o}{\epsilon_o} z_1 \qquad r < b \text{ (inside the beam)} \tag{17}$$

$$E_1 = B_2[I_o(\beta_e r) + \alpha K_o(\beta_e r)] \qquad b < r < a \text{ (outside the beam)} \tag{18}$$

and Eq. (14) gives for the magnetic field H_φ

$$Q^2 H_\varphi = \epsilon_o \Omega \beta_e B_1 I_1(\beta_e r) \qquad r < b \qquad (19)$$

$$Q^2 H_\varphi = \epsilon_o \Omega \beta_e B_2 [I_1(\beta_e r) - \alpha K_1(\beta_e r)] \qquad b < r < a \qquad (20)$$

where I_o, I_1, K_o and K_1 are modified Bessel functions and B_1, B_2 and α are integration coefficients. Now E_1 and H_φ must be continous across the beam surface $(r = b)$ and E_1 must disappear at the surface of the conducting tube $(r = a)$. These boundary conditions determine the coefficient B_1, viz.

$$B_1 = \beta_e b I_1(\beta_e b) \left[\frac{K_1(\beta_e b)}{I_1(\beta_e b)} + \frac{K_o(\beta_e a)}{I_o(\beta_e a)} \right] \frac{\rho_o}{\epsilon_o} z_1$$

For z_1 one then gets from Eq. (15)

$$\frac{d^2 z_1}{dt^2} = - \omega_p^2 R^2 z_1 = - \omega_q^2 z_1 \qquad (21)$$

where

$$\omega_p = \left(- \frac{e}{m} \frac{\rho_o}{\epsilon_o}\right)^{1/2} = \text{angular plasma frequency}$$

$$R(\beta_e b, \frac{a}{b}) = \left[1 - 2 I_1(\beta_e b) K_1(\beta_e b) - 2 \frac{K_o(\beta_e a)}{I_o(\beta_e a)} I_1^2(\beta_e b) \right]^{1/2} =$$

= plasma frequency reduction factor and

$\omega_q = \omega_p R$ = reduced angular plasma frequency

Assume now that the beam is given a pure velocity modulation $v_1^o \sin\omega t$ in the plane $z = 0$. Since the beam is undisplaced $(z_1 = 0)$ in the plane $z = 0$ the solution of Eq. (21) becomes

$$\beta_e z_1 = A \sin\left(\frac{\omega_q z_o}{v_o}\right) \sin(\omega t - \beta_e z_o) \tag{22}$$

The velocity caused by the modulation, Eq. (6c), is

$$v_d = v_1^o \cos\left(\frac{\omega_q z_o}{v_o}\right) \sin(\omega t - \beta_e z_o) \tag{23}$$

By Eq. (22) one immediately obtains the linear term of Eq. (4) since $z = z_o + z_1$ and $z_o = v_o(t - t_o)$.

As one should expect, the linearized solutions, Eqs. (22) and (23), are the same as those obtained by the Eulerian analysis [7] except for the fact that z has been replaced by z_o. With both analyses the same expression is obtained for the reduction factor R which is depicted in Fig. 2. This also shows the reduction factor, calculated by Branch and Mihran [11], for the fundamental mode of propagation in the usual "point-electron" beam.

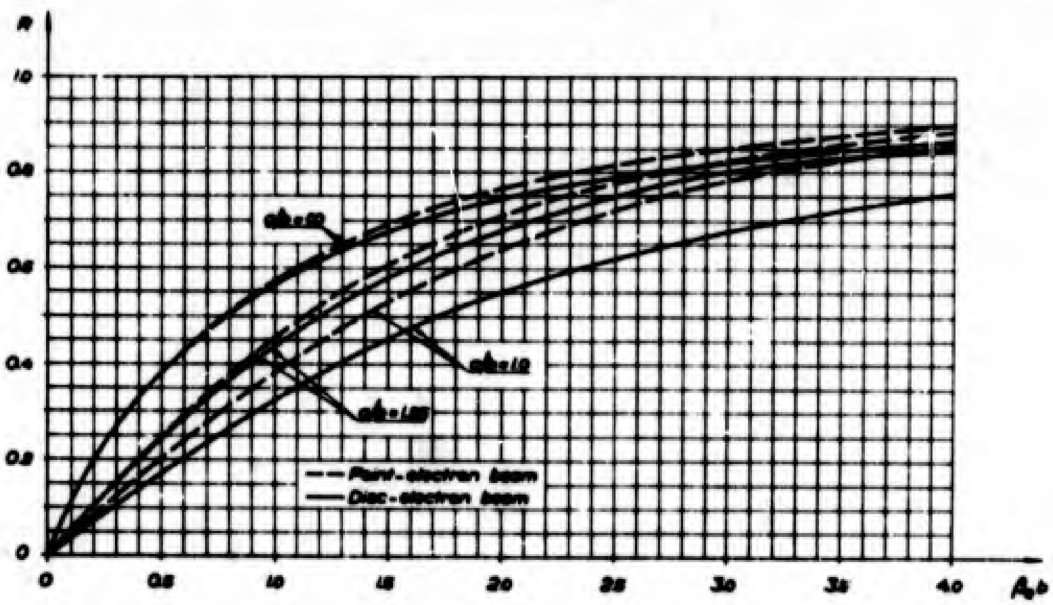

Fig. 2. The plasma frequency reduction factor $R(\beta_e b, \frac{a}{b})$ for the disc-electron beam and the point-electron beam.

IV. THE NONLINEAR SOLUTION

First we rewrite Eq. (13) in the form

$$\left[\left(1+\frac{\partial z_d}{\partial z_o}\right)^3 P+\left(1+\frac{\partial z_d}{\partial z_o}\right)\frac{\partial^2}{\partial z_o^2} - \frac{\partial^2 z_d}{\partial z_o^2}\frac{\partial}{\partial z_o}\right]\left(E_z+\frac{\rho_o}{\epsilon_o} z_d\right)= 0 \quad (24)$$

The method of successive approximations will be used to solve Eq. (24). The electric field E_z and the displacement z_d are expanded in power series which, retaining only the first three terms, can be written

$$E_z = E_1 + E_2 + E_3$$

$$z_d = z_1 + z_2 + z_3$$

where E_1 and z_1 are the linear solutions already calculated. The second order solutions E_2 and z_2 are proportional to A^2 and the third order solutions E_3 and z_3 are proportional to A^3.

From Eqs. (15) and (24) we get the following differential equations for the terms of the three lowest orders.

$$\left.\begin{aligned} &(P+\frac{\partial^2}{\partial z_o^2})(E_1+\frac{\rho_o}{\epsilon_o} z_1) = 0 && (25a)\\ &\frac{d^2 z_1}{dt^2} = -\frac{e}{m}\int_o^b \frac{E_1 2\pi r dr}{\pi b^2} && (25b)\end{aligned}\right\}$$

$$\left.\begin{aligned}
&\left(P+\frac{\partial^2}{\partial z_o^2}\right)\left(E_2+\frac{\rho_o}{\epsilon_o}z_2\right)= \\
&= -\left(3\frac{\partial z_1}{\partial z_o}P+\frac{\partial z_1}{\partial z_o}\frac{\partial^2}{\partial z_o^2}-\frac{\partial^2 z_1}{\partial z_o^2}\frac{\partial}{\partial z_o}\right)\left(E_1+\frac{\rho_o}{\epsilon_o}z_1\right) && (26a) \\
&\frac{d^2 z_2}{dt^2} = -\frac{e}{m}\int_o^b \frac{E_2 2\pi r dr}{\pi b^2} && (26b)
\end{aligned}\right\}$$

$$\left.\begin{aligned}
&\left(P+\frac{\partial^2}{\partial z_o^2}\right)\left[E_3(\omega)+\frac{\rho_o}{\epsilon_o}z_3(\omega)\right]+\left(P+\frac{\partial^2}{\partial z_o^2}\right)\left[E_3(3\omega)+\frac{\rho_o}{\epsilon_o}z_3(3\omega)\right]= \\
&= -\left(3\frac{\partial z_1}{\partial z_o}P+\frac{\partial z_1}{\partial z_o}\frac{\partial^2}{\partial z_o^2}-\frac{\partial^2 z_1}{\partial z_o^2}\frac{\partial}{\partial z_o}\right)\left(E_2+\frac{\rho_o}{\epsilon_o}z_2\right)+ && (27a) \\
&-\left\{\left[3\frac{\partial z_2}{\partial z_o}+3\left(\frac{\partial z_1}{\partial z_o}\right)^2\right]P+\frac{\partial z_2}{\partial z_o}\frac{\partial^2}{\partial z_o^2}-\frac{\partial^2 z_2}{\partial z_o^2}\frac{\partial}{\partial z_o}\right\}\left(E_1+\frac{\rho_o}{\epsilon_o}z_1\right) \\
&\frac{d^2 z_3(\omega)}{dt^2} = -\frac{e}{m}\int_o^b \frac{E_3(\omega) 2\pi r dr}{\pi b^2} && (27b) \\
&\frac{d^2 z_3(3\omega)}{dt^2} = -\frac{e}{m}\int_o^b \frac{E_3(3\omega) 2\pi r dr}{\pi b^2} && (27c)
\end{aligned}\right\}$$

It has been assumed that the second order quantities E_2 and z_2 in Eqs. (26) consist of terms with frequency 2ω (i.e. no dc terms). This is true for cases with $\omega_q << \omega$ and $v_1^o << v_o$.

The third order quantities in Eqs. (27) are assumed to consist of terms with frequencies ω and 3ω.

In a beam that is given the velocity modulation $v_1^o \sin\omega t$ in the plane $z = 0$ one gets the following initial conditions (for $z_o = 0$)

$$z_1 = 0, \quad z_2 = 0, \quad z_3 = 0 \tag{28 a, b, c}$$

and

$$\frac{dz_1}{dt} = v_1^o \sin\omega t, \quad \frac{dz_2}{dt} = 0, \quad \frac{dz_3}{dt} = 0 \tag{29 a, b, c}$$

The procedure to solve Eqs. (26) and (27) is as follows. The solutions [Eqs. (17), (18), (22)] of the first order equation [Eq. (25)] are inserted into the r.h.s. of Eq. (26a). This equation then becomes a linear differential equation with a drive term that is a known function of t, z_o and r. The simultaneous solution of Eq.s (26a) and (26b), with the initial conditions at $z_o = 0$ and the boundary conditions at the beam and tube surfaces taken into account, gives the second order displacement z_2 and field E_2. To find the third order quantities the first and second order solutions are used in the r.h.s. of Eq. (27a) and the procedure is repeated.

The quantities of interest in a klystron analysis are first the displacement z_d, from which the beam ac currents can be calculated, and second the velocity v_d, which gives the modulation dependent velocity spread in the beam. Using the notations

$$m = \frac{R(2\beta_e b, \frac{a}{b})}{R(\beta_e b, \frac{a}{b})}, \qquad n = \frac{R(3\beta_e b, \frac{a}{b})}{R(\beta_e b, \frac{a}{b})}, \qquad Z_o = \frac{\omega_q}{v_o} z_o$$

where $R(2\beta_e b, \frac{a}{b})$ and $R(3\beta_e b, \frac{a}{b})$ are the reduction factors for waves of the frequencies 2ω and 3ω, the results for z_d and v_d are as follows.

The nonlinear displacement

$$\beta_e z_d = \beta_e(z_1 + z_2 + z_3) = [A\sin Z_o - A^3F_1(Z_o)]\sin(\omega t-\beta_e z_o) +$$

$$- A^2F_2(Z_o)\sin 2(\omega t-\beta_e z_o) - A^3F_3(Z_o)\sin 3(\omega t-\beta_e z_o) \qquad (30)$$

where

$$F_1(Z_o) = \frac{1}{16}\cdot\frac{(1-m^2)}{(4m^2-m^4)}\left[3m^2\sin^3 Z_o - \frac{16+12m^2-m^4}{4-m^2}\sin Z_o +\right.$$

$$\left. + (8+m^2)Z_o\cos Z_o - 8\frac{1-m^2}{2m+m^2}\sin(1+m)Z_o + 8\frac{1-m^2}{2m-m^2}\sin(1-m)Z_o\right] \qquad (31)$$

$$F_2(Z_o) = \frac{1-m^2}{4m^2-m^4}\left[-1+\frac{m^2}{2}\sin^2 Z_o + \cos m Z_o\right] \qquad (32)$$

$$F_3(Z_o) = \frac{1}{8}\cdot\frac{1-m^2}{4m^2-m^4}\left\{\left[\frac{48-30m^2+9m^4}{4(1-m^2)} - \frac{8+m^2}{1-n^2}\right]\sin Z_o +\right.$$

$$+ \left[\frac{3(5m^2+m^4)}{(1-m^2)(9-n^2)} - \frac{3(2m^2+m^4)}{4(1-m^2)}\right]\sin 3Z_o +$$

$$+ \frac{2+4m^2-6n^2}{n^2-(1+m)^2}\sin(1+m)Z_o + \frac{2+4m^2-6n^2}{n^2-(1-m)^2}\sin(1-m)Z_o +$$

$$+ \frac{1}{n}\left[\frac{8+m^2}{1-n^2} - \frac{9(5m^2+m^4)}{(1-m^2)(9-n^2)} + \frac{4+12m+2m^2}{n^2-(1+m)^2}(1+m) +\right.$$

$$\left.\left. + \frac{4-12m+2m^2}{n^2-(1-m)^2}(1-m)\right]\sin nZ_o\right\} \qquad (33)$$

With Eq. (30) one immediately obtains Eq. (4) since $z = z_o + z_d$ and $z_o = v_o(t-t_o)$.

The nonlinear velocity

$$v_d = \frac{dz_d}{dt} = \omega_q \frac{dz_d}{dZ_o} = \omega_q \frac{d(z_1+z_2+z_3)}{dZ_o} = v_1^o\left[\cos Z_o - A^2\frac{dF_1(Z_o)}{dZ_o}\right]\sin(\omega t-\beta_e z_o)$$

$$- v_1^o A \frac{dF_2(Z_o)}{dZ_o}\sin 2(\omega t-\beta_e z_o) - v_1^o A^2 \frac{dF_3(Z_o)}{dZ_o}\sin 3(\omega t-\beta_e z_o) \quad (34)$$

where

$$\frac{dF_1(Z_o)}{dZ_o} = \frac{1}{16}\,\frac{1-m^2}{4m^2-m^4}\left[\frac{16-7m^2-\frac{9}{4}m^4}{4-m^2}\cos Z_o - \frac{9}{4}m^2\cos 3Z_o + \right.$$

$$\left. - (8+m^2)Z_o \sin Z_o - \frac{8(1+m-m^2-m^3)}{2m+m^2}\cos(1+m)Z_o + \frac{8(1-m-m^2+m^3)}{2m-m^2}\cos(1-m)Z_o\right] \quad (35)$$

$$\frac{dF_2(Z_o)}{dZ_o} = \frac{1-m^2}{4m^2-m^4}\left[\frac{m^2}{2}\sin 2Z_o - m\sin mZ_o\right] \quad (36)$$

$$\frac{dF_3(Z_o)}{dZ_o} = \frac{1}{8}\,\frac{1-m^2}{4m^2-m^4}\left\{\left[\frac{48-30m^2+9m^4}{4(1-m^2)} - \frac{8+m^2}{1-n^2}\right]\cos Z_o + \right.$$

$$+ \left[\frac{9(5m^2+m^4)}{(1-m^2)(9-n^2)} - \frac{9(2m^2+m^4)}{4(1-m^2)}\right]\cos 3Z_o + \frac{2+4m^2-6n^2}{n^2-(1+m)^2}(1+m)\cos(1+m)Z_o +$$

$$+ \frac{2+4m^2-6n^2}{n^2-(1-m)^2}(1-m)\cos(1-m)Z_o + \left[\frac{8+m^2}{1-n^2} - \frac{9(5m^2+m^4)}{(1-m^2)(9-n^2)} + \right.$$

$$\left.\left. + \frac{4+12m+2m^2}{n^2-(1+m)^2}(1+m) + \frac{4-12+2m^2}{n^2-(1-m)^2}(1-m)\right]\cos nZ_o\right\} \quad (37)$$

The nonlinear displacement expressions can be transformed to those obtained from the Eulerian analysis [7] by replacing z_o with $z - z_d$ and then retaining all terms to the third order.

In Fig. 3 the ratios m and n are depicted for various beam geometries. The functions F_1, F_2 and F_3 and their derivatives are depicted in Figs. 4 and 5 versus normalized drift length Z_o.

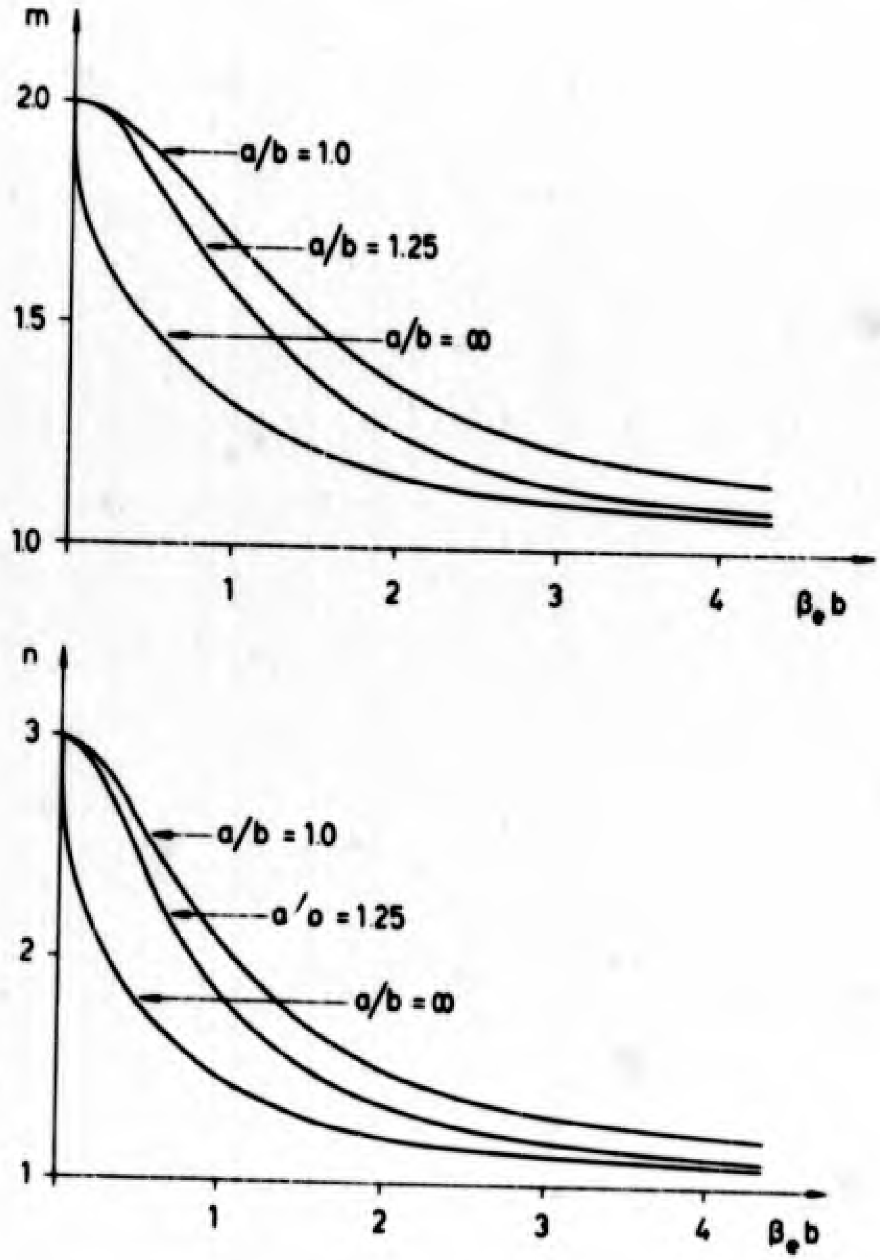

Fig. 3. The ratios $m = \dfrac{R(2\beta_e b, \frac{a}{b})}{R(\beta_e b, \frac{a}{b})}$ and $n = \dfrac{R(3\beta_e b, \frac{a}{b})}{R(\beta_e b, \frac{a}{b})}$ versus normalized beam radius for some values of $\frac{a}{b}$.

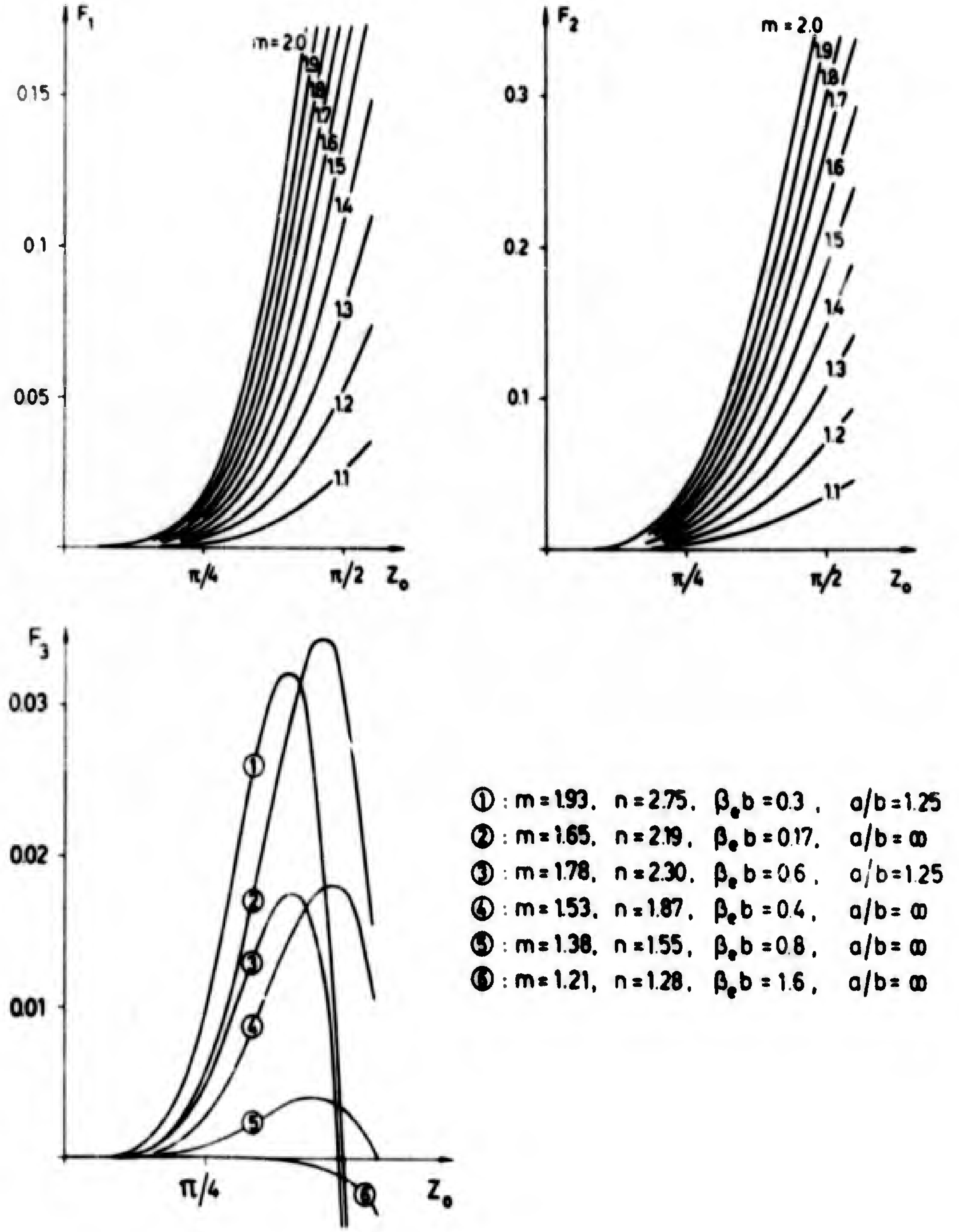

Fig. 4. The functions $F_1(Z_o)$, $F_2(Z_o)$ and $F_3(Z_o)$ in Eq. (30).

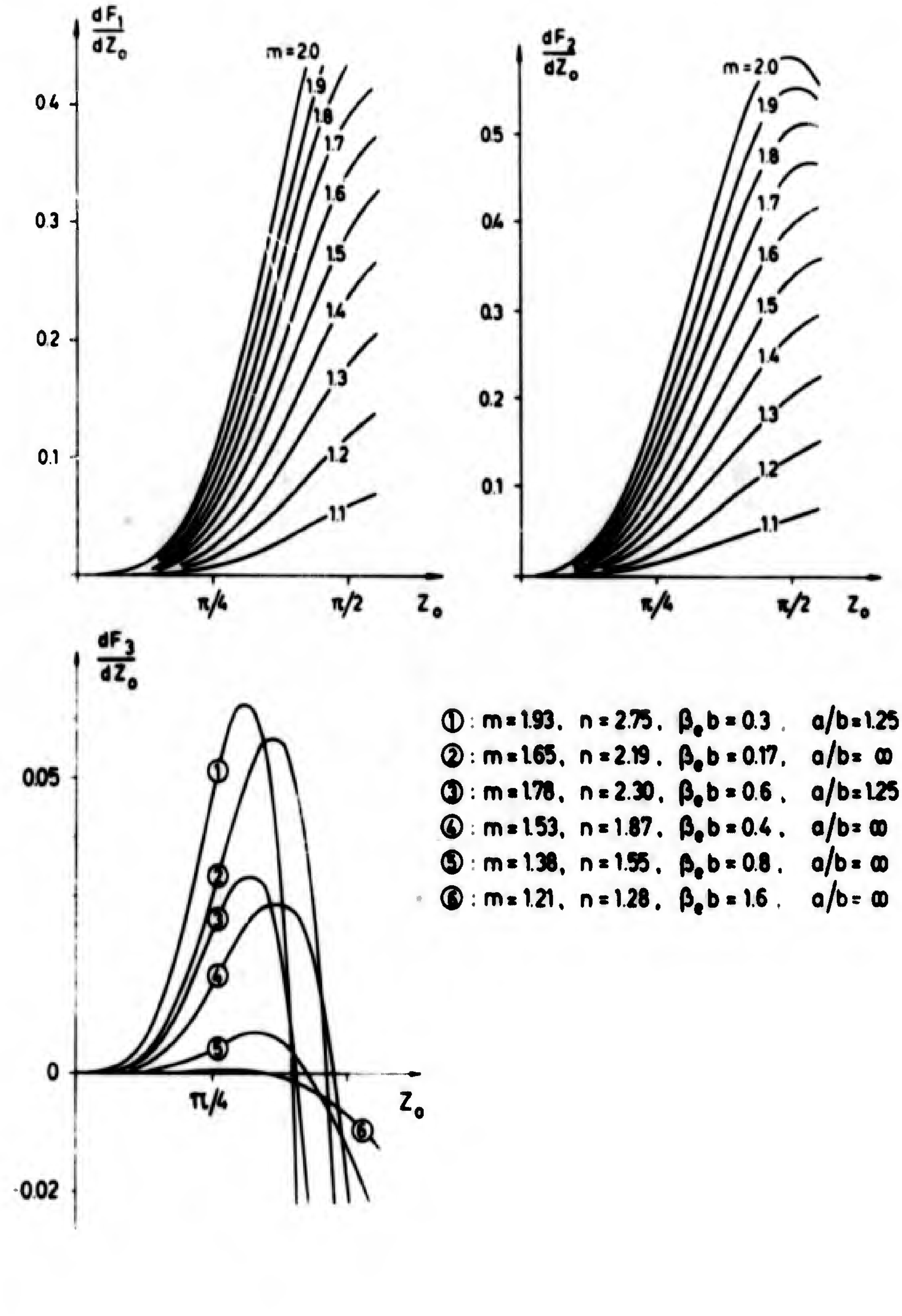

Fig. 5. The derivatives $\frac{dF_1}{dZ_o}$, $\frac{dF_2}{dZ_o}$ and $\frac{dF_3}{dZ_o}$ in Eq. (34)

For the infinitely wide beam ($m = 1$ and $n = 1$) F_1, F_2 and F_3 and their derivatives equal zero. In that case the linear terms of Eqs. (30) and (34) are the exact solutions [2].

V. THE DISC-ELECTRON DISPLACEMENT AND VELOCITY

The theory of the preceding chapters is not strictly valid after overtaking has occurred. To investigate the practical validity of the third order displacement expression, Eq. (30), after overtaking the electron phase position $\beta_e z-\omega t$ is depicted in Fig. 6 versus the undisturbed phase position $\beta_e z_o-\omega t$ that an electron would have in a case with zero modulation. The following relation is used

$$\beta_e z-\omega t = \beta_e z_o-\omega t+\beta_e z_d$$

Comparison is made with computer experiments [10] and the third order Eulerian theory [7]. The curves are for the infinitely wide beam and for a thin beam, $\beta_e b = 0.17$ and $\frac{a}{b} = \infty$. Overtaking has occurred in all the diagrams with the exception of 6d. For the cases in 6a and 6d the present theory and the computer experiments agree completely. The diagrams 6b and 6e depict the situation at the distances where, according to the computer experiments, the amplitude of the fundamental frequency current should have its first maximum.

Assuming that the computer experiments are accurate it is evident from Fig. 6 that the present third order theory gives a good description of the physical situation also somewhat beyond overtaking. Apparently this is not true for the Eulerian analysis. The difference between the two theories is due to the fact that the displacement z_d, after overtaking, is a multi-valued function of z while it is still a single-valued function of z_o. The Eulerian analysis always gives single-valued functions of z. Even our solution for z_d is incorrect after overtaking since Eq. (13) gives the wrong electric field E_z. However, the displacement is obtained upon integrating the field twice [Eq. (15)] which makes the error less critical.

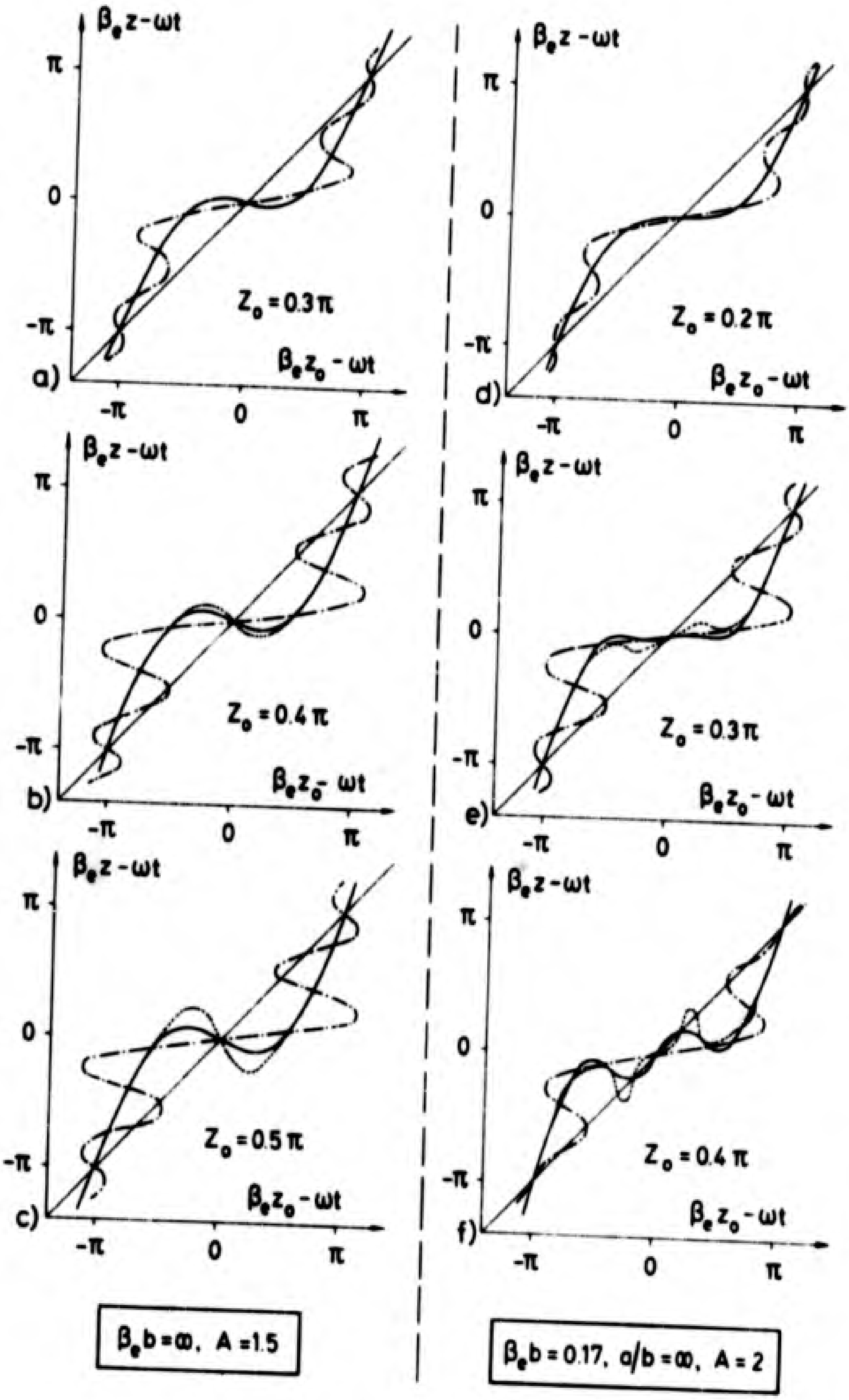

Fig. 6. Disc-electron phase position $\beta_e z-\omega t$ versus undisturbed phase $\beta_e z_o-\omega t$ for two klystron beams at three normalized drift lengths Z_o.

——————— present analysis   - - - - - - - - - - - - - - computer exp. [10]

— · — · — · — · Eulerian theory [7]

The velocity deviations v_d maximize the ac voltage that can be applied over the output gap and thus set a limit on the output power from a klystron. It is important to minimize v_d in the gap. Mihran [12] has studied this problem with computer experiments on disc-electron beams. In Fig. 7 the velocity v_d, Eq. (34), is depicted versus normalized drift length Z_o for some of the discs in a period. The velocity deviations of the slowest and fastest electrons are shown in Fig. 8 for two cases. These velocity deviations pass through a distinct minimum, the velocity neck [12]. At least for the case $A = 1.7$ the present theory predicts the velocity neck in good agreement with Mihran's computer experiment despite the fact that overtaking already occurs in the vicinity of $Z_o = 0.2\pi$.

The location Z_{on} of the velocity neck is the location where the fundamental frequency component of the velocity deviation v_d, Eq. (34), is equal to zero, viz.

$$\cos Z_{on} - A^2\left(\frac{dF_1}{dZ_o}\right)_{Z_o = Z_{on}} = 0 \qquad (38)$$

Since the 3ω component in Eq. (34) generally is quite small (cf. Fig. 5) the maximum velocity deviation v_{dn} at the velocity neck obeys

$$v_{dn} \simeq v_1^o A\left(\frac{dF_2}{dZ_o}\right)_{Z_o = Z_{on}} \qquad (39)$$

Figs. 9 and 10 show Z_{on} and v_{dn}. The lines are broken in the regions with overtaking. Due to the assumptions $v_1^o << v_o$ and $\omega_q << \omega$ the curves are strictly valid only for cases with $v_{dn}/v_o >> (v_1^o/v_o)^2$ and $v_{dn}/v_o >> (v_1^o/v_o)(\omega_q/\omega)$. Fig. 10 also shows some results from Mihran's computer calculations. The discrepancies for large values of A are due to overtaking.

One concludes that the velocity deviation v_{dn} increases with the parameter m. This means (cf. Fig. 3) that the velocity spread at the velocity neck in general is larger in thin beams inside narrow conducting tubes than it is in wide beams with large spacing between the tube and the beam.

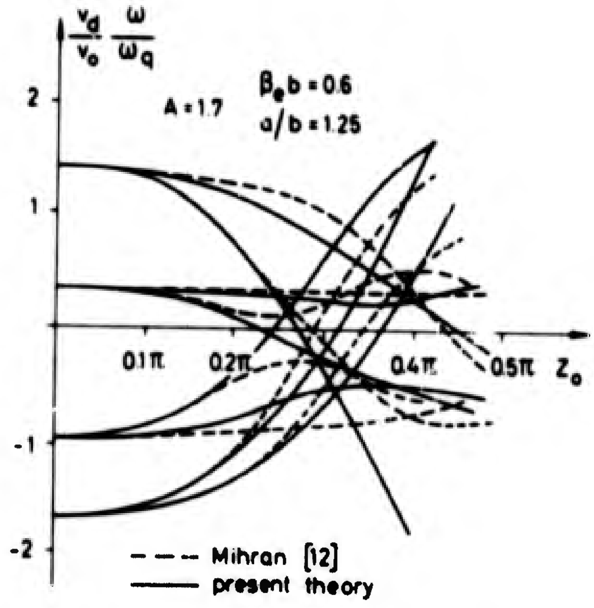

Fig. 7. Velocity deviations v_d versus normalized drift length Z_o for a few electrons. The picture is asymmetrical around the horisontal axis due to the fact that the positive and negative initial velocities chosen are unequal.

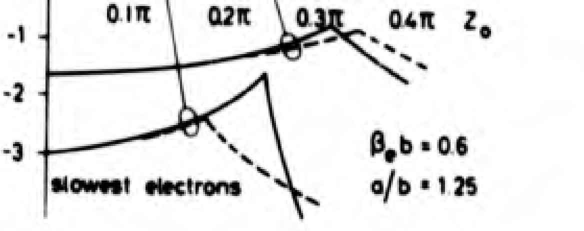

Fig. 8. Velocity deviation v_d of fastest and slowest electrons versus normalized drift length Z_o.

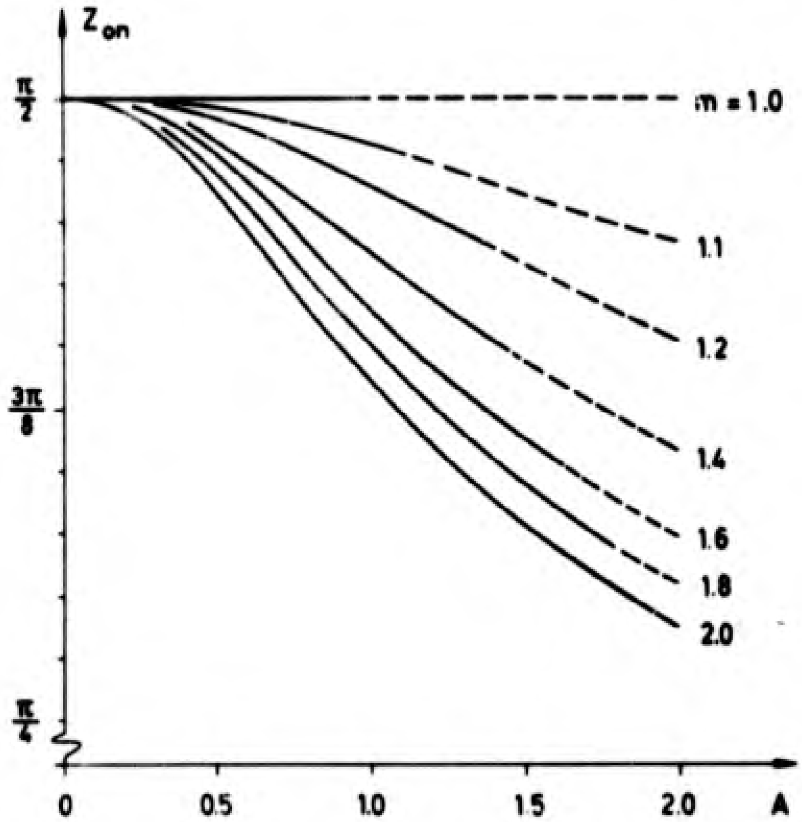

Fig. 9. The location Z_{on} of the velocity neck.

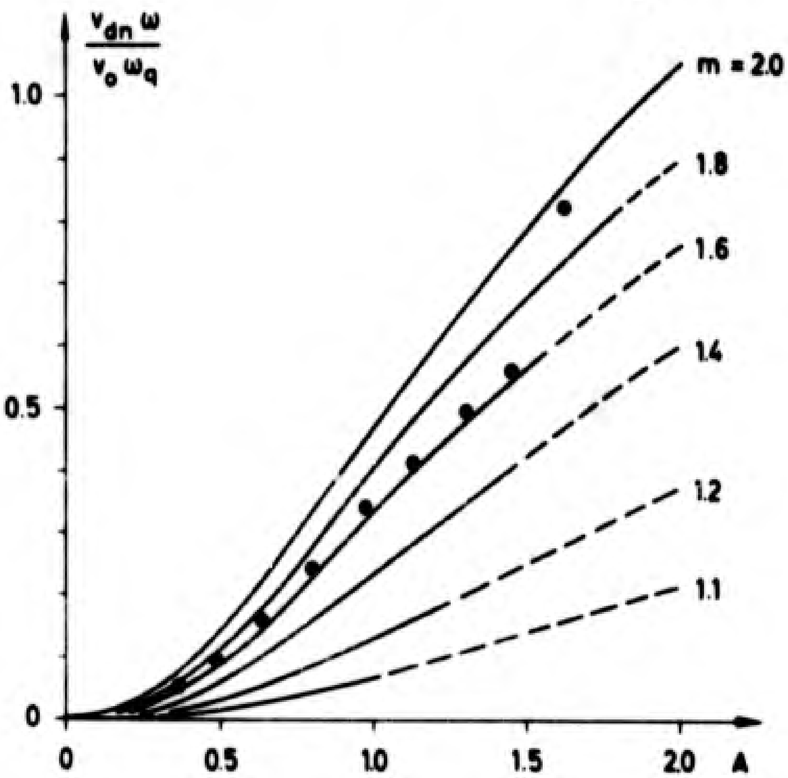

Fig. 10. The maximum velocity deviation v_{dn} at the velocity neck. The dots are from Mihran's [12] computer calculations on a beam with $\beta_e b = 0.71$, $a/b = 1.22$, $m = 1.7$ and $\omega_q/\omega = 0.123$.

VI. THE HIGH FREQUENCY CURRENT AMPLITUDES

In this chapter we will use Eqs. (30) through (33) to find analytical expressions for the a c currents of the fundamental frequency and of the harmonic frequencies.

One can expand the beam convection current $I(z, t)$ in the following Fourier series

$$I(z, t) = I_o + \sum_{\nu=1}^{\infty} I_\nu \cos \nu(\omega t - \beta_e z)$$

where I_o is the beam dc current and I_ν the amplitude of the νth harmonic frequency current. For beams with $v_1^o << v_o$ and $\omega_q << \omega$ the amplitudes I_ν are slowly varying functions of z.

The amplitudes are found from the Fourier integral

$$I_\nu = \frac{1}{\pi} \int_{-\pi}^{\pi} I \cos \nu(\omega t - \beta_e z)\, d\omega t \tag{40}$$

Charge conservation implies

$$I\, dt = \sum_r I_o \left| dt_o \right|_r \tag{41}$$

The relation $z_o = v_o(t - t_o)$ and Eq. (30) yield

$$\omega t - \beta_e z = \omega t_o - X_1(Z_o) \sin \omega t_o + X_2(Z_o) \sin 2\omega t_o + $$
$$+ X_3(Z_o) \sin 3\omega t_o \tag{42}$$

where

$$X_1(Z_o) = A[\sin Z_o - A^2 F_1(Z_o)]$$

$$X_2(Z_o) = A^2 F_2(Z_o)$$

$$X_3(Z_o) = A^3 F_3(Z_o)$$

By the use of Eqs. (41) and (42) one can write Eq. (40) in the form

$$I_\nu = \frac{1}{\pi} \int_{-\pi}^{\pi} I_o \cos \nu(\omega t_o - X_1 \sin \omega t_o + X_2 \sin 2\omega t_o + X_3 \sin 3\omega t_o)\, d\omega t_o \qquad (43)$$

This integral is to be performed for constant z. Using the assumptions $v_1^o \ll v_o$ and $\omega_q \ll \omega$ one has $Z_o \simeq Z = \frac{\omega_q}{v_o} z$ in the arguments of X_1, X_2 and X_3. Then the solution for the current amplitudes is (Appendix II)

$$\frac{I_\nu}{I_o} = 2 \sum_{\lambda=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} J_{\nu+2\lambda+3\mu}(\nu X_1)\, J_\lambda(\nu X_2)\, J_\mu(\nu X_3) \qquad (44)$$

where J_p denotes Bessel functions and X_1, X_2 and X_3 are the functions $X_1(Z)$, $X_2(Z)$ and $X_3(Z)$.

For the infinitely wide beam F_1, F_2 and $F_3 = 0$ and Eq. (44) reduces to

$$\frac{I_\nu}{I_o} = 2J_\nu(\nu A \sin Z) \qquad (45)$$

This is as expected from Eq. (3).

Eq. (45) also is the result for beams with finite transverse dimensions when the nonlinearities F_1, F_2 and F_3 are ignored. This equation is commonly used in klystron analysis [12, 13, 14].

When $\omega_q \to 0$ the space charge forces disappear and for X_1, X_2 and X_3 we get the following limit values

$$X_1 = AZ = \beta_e z \frac{v_1^o}{v_o}, \qquad X_2 = 0, \qquad X_3 = 0$$

For negligible space charge forces Eq. (44) therefore transforms to Webster's ballistic analysis [1], viz.

$$\frac{I_\nu}{I_o} = 2J_\nu(\nu AZ) \qquad (46)$$

First we will study the fundamental frequency current amplitude I_1 around its first maximum. By the use of Eq. (43) the ratio $\frac{I_1}{I_o}$ has been depicted in Fig. 11 versus the normalized drift length Z for a few beams with A as parameter. Comparison is made with results from computer experiments [10] and in Fig. 11b with the third order Eulerian analysis [7]. The present theory agrees very well with oomputer experiments for the thin beam, Fig. 11a, while the agreement is less for the infinitely wide beam, Fig. 11c. This is due to the fact that electron overtaking occurs at higher A-values for thinner beams. The overtaking points are shown in Fig. 11. For the beam in Fig. 11a overtaking occurs only for the curves with $A \geq 2$ while in Figs. 11b and 11c it already occurs for the curves with $A = \frac{4}{3}$. In fact for the infinitely wide beam [2] in Fig. 11c overtaking is just about to occur when $A = 1$ and $Z = \frac{\pi}{2}$.

From Fig. 11b it can be concluded that the present theory is significantly more accurate than the third order Eulerian theory. The reason is, of course, that the method used to calculate z_d is more accurate. However, for smaller values of A than those used in Fig. 11, say $A \lesssim 0.7$, the two theories agree.

In the regions of Fig. 11 where the present theory agrees with the computer experiments, X_2 is less than a few times 10^{-1} while X_3 is less than 10^{-1}. For these values of X_2 and X_3 most terms in the series, Eq. (44), for the fundamental frequency current amplitude are negligible and it is sufficient to retain only two terms, viz.

$$\frac{I_1}{I_o} = 2J_1(X_1)\,[J_o(X_2) + J_1(X_2)] \qquad (47)$$

For beams with finite transverse dimensions Eq. (47) contains more information about the nonlinearities than Eq. (45). Note

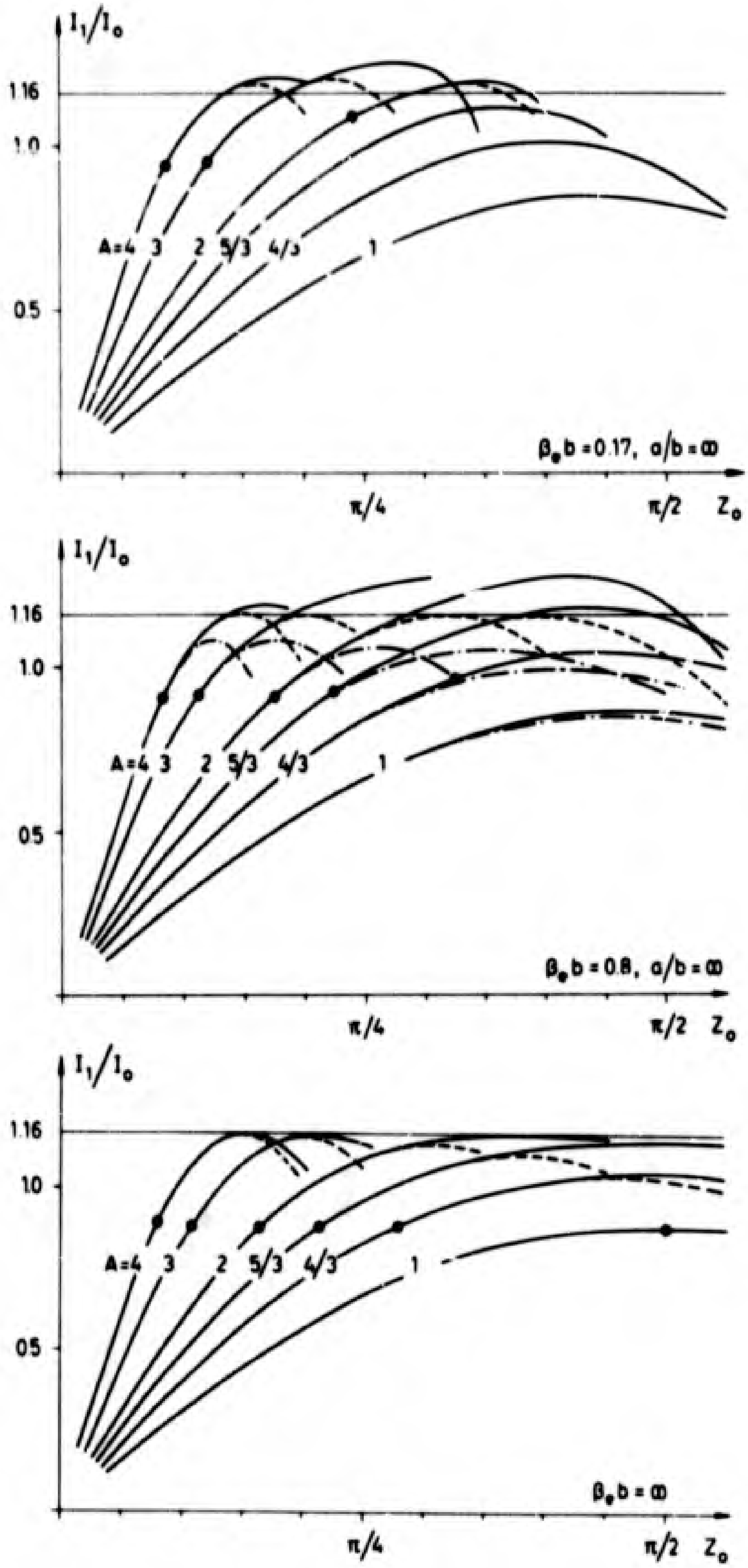

Fig. 11. Fundamental frequency current amplitude I_1 versus normalized drift length $Z(=\frac{\omega_q}{v_0}z)$ for three beams.

———————— present theory ‐ ‐ ‐ ‐ ‐ ‐ ‐ ‐ ‐ ‐ ‐ ‐ ‐ ‐ computer exp. [10]

— . — . — . — . Eulerian theory [7]

● denotes first overtaking location

that Eq. (47) contains only the quantities X_1 and X_2 and not the quantity X_3. This means that when using Eq. (47) one only has to know m, which is the ratio of the plasma frequency reduction factors for the fundamental frequency ω and the second harmonic frequency 2ω.

The amplitudes of the second, third and fifth harmonic currents are depicted in Fig. 12 for a beam with $\beta_e b = 0.8$ and $\frac{a}{b} = \infty$. The figures show close agreement between the present theory and the computer experiments for medium and very high values of the drive parameter A. The results of the third order Eulerian theory for the second and third harmonic currents are depicted for $A = 1$. The deviations are due to the fact that the third order Eulerian theory does not contain any information on the nonlinear depression which is an effect of higher order than the third. To give any information on the fifth harmonic, the Eulerian theory would have to be extended to the fifth order.

Though the displacement z_d has been calculated only to the third order the current expression, Eq. (44), apparently contains a considerable part of the higher order effects. This means that these effects are mainly due to the transformation from the Lagrangian independent variables t and t_o to the laboratory variables t and z. This transformation is done exactly in Eq. (43). The results of the third order Eulerian theory are obtained if effects only to the third order are retained in the transformation.

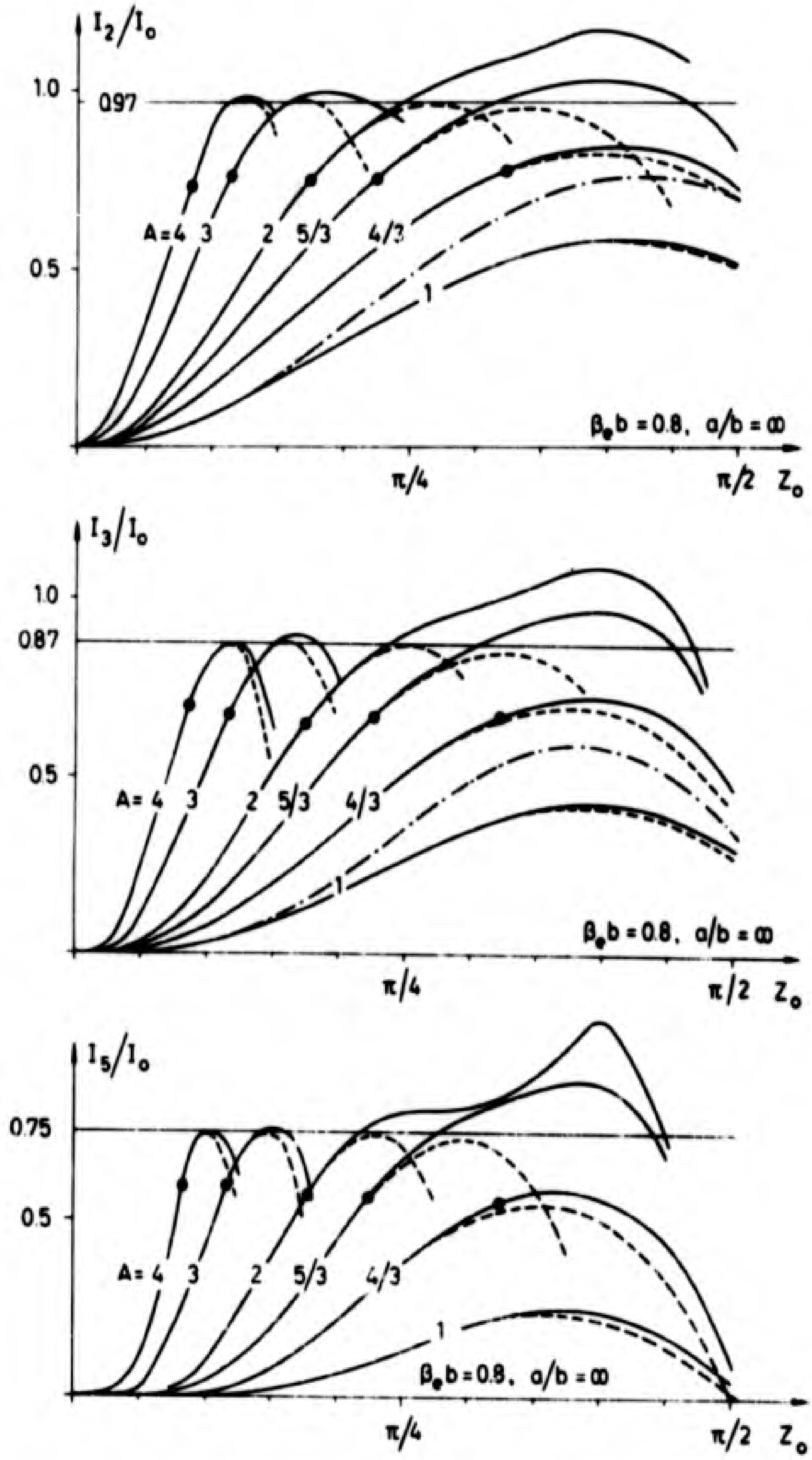

Fig. 12. Amplitude of second, third and fifth harmonic currents versus normalized drift length $Z(=\frac{\omega_q}{v_o} z)$.

——————— present theory - - - - - - - - - - - computer exp. [10]

— . — . — . — Eulerian theory [7]

• denotes first overtaking location

VII. CONCLUDING REMARKS

It is well known [2] that the electron displacement $z - v_o(t-t_o)$ for space charge waves in an infinitely wide electron beam klystron can be expressed simply and exactly in Lagrangian variables [Eq. (2)]. By the use of Fourier analysis one can, from this expression, calculate the ac current in Eulerian variables. The result is an infinite series which contains all harmonic frequencies [Eq. (3)].

The simplicity of the expression for the electron displacement in the above case suggests the use of Lagrangian variables also for the radially finite beam case. Although the electron displacement now, by necessity, becomes an infinite power series in modulation amplitude, one can expect the series to converge much more rapidly than the corresponding series in Eulerian formulation. In the present work the first three displacement terms are calculated in Lagrangian variables [Eqs. (4) and (30)]. Fourier analysis is then used to express the ac current in Eulerian variables [Eq. (44)]. The ac current series again contains all harmonics. The point is now that, in spite of the fact that a third order electron displacement forms the basis of the theory, the current harmonics are described to a higher order than the third. The fifth harmonic is shown in Fig. 12. Thus the present method gives rather sofisticated information about the nonlinearities within the frame of a third order theory.

Another basic advantage of the Lagrangian formulation is that electron overtaking is taken into account. Even though the theory still is strictly valid only before overtaking occurs we have found that it can be used with good accuracy up to the saturation level.

The general conclusion is that the present theory can be used for larger modulations and contains more information about the nonlinearities than the earlier nonlinear space charge wave theories in Eulerian formulation [4, 5, 6, 7] which involve about the same amount of labour.

APPENDIX I

Assume a function $X(t, z_o)$. Then we get

$$\frac{\delta X}{\delta t} = \frac{\partial X}{\partial t} + \frac{\partial X}{\partial z_o} \cdot \frac{\delta z_o}{\delta t} = \frac{\partial X}{\partial t} - \frac{\partial X}{\partial z_o} \cdot \frac{\delta z_1}{\delta t} \qquad (AI.1)$$

Let $X(t, z_o) = z_1(t, z_o)$ in Eq. (AI.1). Then

$$\frac{\delta z_1}{\delta t} = \frac{\dfrac{\partial z_1}{\partial t}}{1 + \dfrac{\partial z_1}{\partial z_o}}$$

which inserted into Eq. (AI.1) yields

$$\boxed{\frac{\delta X}{\delta t} = \frac{\partial X}{\partial t} - \frac{\dfrac{\partial z_1}{\partial t}}{1 + \dfrac{\partial z_1}{\partial z_o}} \frac{\partial X}{\partial z_o}} \qquad (AI.2)$$

For the derivative $\frac{\delta}{\delta z}$ one similarly gets

$$\frac{\delta X}{\delta z} = \frac{\partial X}{\partial z_o} \cdot \frac{\delta z_o}{\delta z} = \frac{\partial X}{\partial z_o} \cdot \frac{1}{\dfrac{\partial z}{\partial z_o}}$$

that is

$$\boxed{\frac{\delta X}{\delta z} = \frac{\dfrac{\partial X}{\partial z_o}}{1 + \dfrac{\partial z_1}{\partial z_o}}} \qquad (AI.3)$$

APPENDIX II

The solution of the integral, Eq. (43), can be found by the following procedure. By the use of simple trigonometric formulas the integrand is rewritten

$$\cos \nu(\omega t_o - X_1 \sin\omega t_o + X_2 \sin 2\omega t_o + X_3 \sin 3\omega t_o) =$$

$$= [\cos\nu\omega t_o \cdot \cos(\nu X_1 \sin\omega t_o) + \sin\nu\omega t_o \cdot \sin(\nu X_1 \sin\omega t_o)] \cdot$$

$$\cdot [\cos(\nu X_2 \sin 2\omega t_o) \cdot \cos(\nu X_3 \sin 3\omega t_o) - \sin(\nu X_2 \sin 2\omega t_o) \cdot \sin(\nu X_3 \sin 3\omega t_o)] +$$

$$+ [\cos\nu\omega t_o \cdot \sin(\nu X_1 \sin\omega t_o) - \sin\nu\omega t_o \cdot \cos(\nu X_1 \sin\omega t_o)] \cdot$$

$$\cdot [\cos(\nu X_2 \sin 2\omega t_o) \cdot \sin(\nu X_3 \sin 3\omega t_o) + \sin(\nu X_2 \sin 2\omega t_o) \cdot \cos(\nu X_3 \sin 3\omega t_o)]$$

(AII.1)

Now one has the following series [15]

$$\left.\begin{aligned} \cos(x\sin\theta) &= J_o(x) + 2\sum_{k=1}^{\infty} J_{2k}(x)\cos 2k\theta \\ \sin(x\sin\theta) &= 2\sum_{k=o}^{\infty} J_{2k+1}(x)\sin[(2k+1)\theta] \end{aligned}\right\} \qquad \text{(AII.2)}$$

The introduction of the series (AII.2) into the integrand AII.1 makes the Fourier integral, Eq. (43), trivial and the result, Eq. (44), follows after some rearranging.

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13. ABSTRACT

An analytical, nonlinear space charge wave theory is given for the velocity modulated, confined electron beam of finite radial extension. The present theory gives a more accurate nonlinear description of the electron velocities and the ac current amplitudes at the fundamental and harmonic frequencies than earlier space charge wave type theories do. Although the theory is not strictly valid after electron overtaking has occurred, a formulation is used which allows the electron velocities to be multi-valued functions of the space coordinates. Therefore reasonably good results should be expected also in the overtaking range. For very large signals the results transform to those of Webster's ballistic theory. The analytical results obtained agree well with computer experiments.

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