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RESEARCH TRANSLATION

Dissipation of Stratus Clouds in a Turbulent Atmosphere

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Translation of

Dissipatsiia sloistoi oblachnosti v turbulizovannoi atmosfere

by

Iu. V. Shulepov and M. V. Buikov

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DISSIPATION OF STRATUS CLOUDS
IN A TURBULENT ATMOSPHERE

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Iu. V. Shulepov and M. V. Buikov

We examine the horizontal dissipation of a spatially homogeneous stratus cloud in a turbulent atmosphere, in which isothermy, an inversion in the supracloud layer, and downdrafts are present; the time required for total dissipation of the cloud is computed.

Stratus clouds can dissipate not only by gravitational fall-out of the droplets (descent and evaporation of droplets in the unsaturated layer of air beneath the cloud), but by the transport of cloud droplets out of the saturation zone by turbulent pulsations with subsequent evaporation of the droplets.

As in the absence of turbulence, evaporation of droplets outside the cloud leads to complete saturation of the air layers adjacent to the top and base of the cloud, i.e., in this case, too, allowance for the collective nature of the evaporation process indicates that the time required for evaporation of the entire cloud will be many times greater than that required for the evaporation of a single drop. Actually, cloud dissipation is caused simultaneously by turbulence and sedimentation mechanisms, and by other mechanisms, such as coagulation. However, one of these mechanisms may prove to be stronger, depending on the physical conditions (cloud droplet spectrum, turbulence level, extent of undersaturation below the cloud, etc.). Therefore, the simultaneous examination of both mechanisms involves quite serious mathematical difficulties.

The problem of cloud dissipation in a turbulent atmosphere has not been examined, essentially, in the manner in which we have formulated it. L. T. Matveev [1] noted that a stratus cloud is relatively stable colloidally. Milburn [2] took up the problem of the dissipation of a stratus cloud in a turbulent atmosphere. As in [3], which he devoted to the dissipation of a stratus cloud in a non-turbulent atmosphere, in [2] Milburn chose a physically false model of a cloud, considering that undersaturation takes place in it. In deriving the equation for the water content of the cloud, he made unfounded assumptions concerning the relation between factors of different order for the function of drop-size distribution, e.g., he states that $\bar{r} = (\overline{r^3})^{1/3}$. Furthermore, in [2] he does not pose the question of computing the time required for complete dissipation and he solves the derived system of equations for short time spans, which in general has no practical meaning.

The problem of stratus cloud dissipation in a nonturbulent atmosphere can be formulated within the formalism of the kinetic equation for describing cloud processes.

Let us examine two cases of dissipation: 1) n an isothermal atmosphere, and 2) for a linear lapse rate with an inversion above the cloud. Although isothermy is rarely encountered in nature, it is useful to examine it not only from the standpoint of methodology, but for estimating the dissipation time of the cloud if complete information is not available on the lapse rate; it may be maintained, approximately, that dissipation occurs in an atmosphere having some constant mean temperature. It should also be noted that we are not speaking of isothermy for the whole atmosphere, but only of isothermy in the relatively narrow layer where dissipation occurs.

Since the solution to the problem has certain properties that depend on the presence or absence of ordered downdrafts, we shall examine these two cases separately.

Let us turn to the mathematical formulation of the problem and begin with the kinetic equation for the function of the drop-size distribution f(r, z, t) which, with allowance for the condensation processes, convective and turbulent transfer and sedimentation of cloud droplets, has the form

$$\frac{\partial f}{\partial t} + \frac{D(q - q_s(T))}{\rho r}\left(\frac{\partial f}{\partial r} - \frac{f}{r}\right) + (v + ar^2)\frac{\partial f}{\partial z} = k\frac{\partial^2 f}{\partial z^2}. \qquad (1)$$

where D is the diffusivity of water vapor [in air], ρ is the density of water, $q(z,t)$ and $q_s(T)$ are the vapor density and saturated vapor density at temperature T, v is the velocity of the ordered downdrafts, k is the eddy diffusivity, and ar^2 is the fall speed of a cloud droplet in stationary air (in what follows we shall neglect sedimentation and omit that term).

Our method does not require finding the distribution function in explicit form, therefore we shall proceed from eq. (1) to the equation for liquid-water content, for which we multiply both sides of (1) by $\frac{4\pi}{3}\rho r^3 N_0$ and integrate over r from 0 to ∞. After simple transformation. for the liquid-water content

$$w(z, t) = \frac{4\pi N_0}{3} \int\limits_0^\infty dr\, r^3 f(r, z, t)$$

(N_0 is the number of droplets in a unit volume) and on the assumption that $f \rightarrow 0$ when $r \rightarrow \infty$, we get the following equations

$$\left.\begin{aligned} \frac{\partial w}{\partial t} + v\frac{\partial w}{\partial z} &= k\frac{\partial^2 w}{\partial z^2} + \varepsilon(z, t); \\ \varepsilon(z, t) &= 4\pi D N_0 (q - q_s(T)) \int\limits_0^\infty dr\, r\, f(r, z, t). \end{aligned}\right| \tag{2}$$

The equations for vapor density and temperature are:

$$\frac{\partial q}{\partial t} + v\frac{\partial q}{\partial z} = k\frac{\partial^2 q}{\partial z^2} - \varepsilon(z, t), \tag{3}$$

$$\frac{\partial T}{\partial t} + v\left(\frac{\partial T}{\partial z} - \gamma_a\right) = k\frac{\partial^2 T}{\partial z^2} + \alpha\varepsilon(z, t), \tag{4}$$

where $\alpha = L/c_p\rho_{air}$; L is the latent heat of vaporization, c_p is the specific heat of the air at constant pressure, ρ_{air} is the density of air, and $\gamma_a = g/c_p$ is the adiabatic lapse rate.

Let us assume that at the initial moment a cloud of infinite horizontal extent is 2H thick, has a constant liquid-water content w_0, and has known vapor density q and temperature T at t = 0. However, since these conditions are written differently for isothermy and linear lapse rate, we shall not give them here, but will formulate the basic premises of the approximation method of solving the dissipation problem.

Let us note that linear combinations of the functions w, q, T, $S = w + q$, and $\Phi = T - \alpha w$ satisfy the following equations, as can easily be seen from (2), (3), and (4):

$$\frac{\partial S}{\partial t} - v\frac{\partial S}{\partial z} - k\frac{\partial^2 S}{\partial z^2} = 0, \tag{5}$$

$$\frac{\partial \Phi}{\partial t} + v\frac{\partial \Phi}{\partial z} - k\frac{\partial^2 \Phi}{\partial z^2} = v\gamma_a. \tag{6}$$

The initial and boundary conditions for these equations follow from the corresponding conditions for the functions q, w, and T. Equations (5) and (6) are simpler than (2), (3), and (4), but they cannot provide a complete solution to the problem, because one cannot determine w, q, and T from S and Φ. To find them, one must solve one of the equations (2), (3), or (4), but this cannot be done if we limit ourselves to an approximate solution to the problem, considering only the physical essence of the phenomenon.

Let us note, first of all, that theoretically one may determine the cloud boundaries in two ways. They can be regarded as a surface where the liquid-water content becomes vanishingly small ($w \approx 0$), or they may be regarded as a surface where $q(z,t) = q_s[T(z,t)]$, since usually there is saturation within the cloud while there is a moisture deficit outside it. Naturally, cloud boundaries determined from the liquid-water content will differ from those determined from the vapor density, since cloud droplets may exist for a time in the presence of a moisture deficit, therefore cloud boundaries determined from the condition $w \approx 0$ will show a somewhat larger cloud thickness than those determined from the other condition. It follows from the physical basis of the problem that the difference between the cloud boundaries found by the two methods will be of the order of $\sqrt{kt_0}$, where $t = \rho r_0^2/2D(q_s(T)-q_0)$ is the evaporation time of a cloud droplet of mean statistical radius for a characteristic non-cloud undersaturation $q_s(T) - q_0$. This difference in determining the boundary may be ignored if it is much smaller than the cloud thickness. This condition is not applicable to thin clouds and undersaturation, but is fulfilled in most real cases. For example, for $r_0 = 6\mu$, $q_s(T) - q = 10^{-7}$ g/cm^3, $\sqrt{kt_0} \sim 20$ m, which is less than the actual cloud thicknesses.

However, if we consider that the cloud boundaries determined by the conditions $w = 0$ and $q = q_s(T)$ are equivalent, it follows from this equivalency that the following equality will be fulfilled at the boundary for our S function

$$S(z, t) = q_s\,[T(z, t)]. \qquad (7)$$

The temperature in the right-hand side of this equality may be found by the second auxiliary function Φ. Determining Φ for $w = 0$, we find $T(z,t) = \Phi$. The equation for determining the moving boundaries of a dissipating cloud has the form

$$S\,(z(t), t) = q_s[\Phi(z(t), t)]. \qquad (8)$$

Thus, the entire kinetics of cloud dissipation can be described on the basis of S and Φ. We shall now seek S and Φ for specific cloud models and solve eq. (8).

We consider the function $q_s(T)$ to be known and take it in the form

$$q_s(T) = \frac{q_s(T_0)\,T_0}{T} e^{\frac{E}{T_0} - \frac{E}{T}}, \qquad (9)$$

where $E = L\mu/R$ (μ is the molecular weight of water, R is the gas constant), T_0 is a characteristic temperature whose specific meaning will depend on the model.

The time of total evaporation of all the cloud droplets (t_d is the dissipation time) may be determined from (3) as follows. By the time all the cloud droplets have evaporated, the size of the saturation zone will have approached zero. This can easily be demonstrated by the opposite case. Since in the case of turbulent transfer, streams of droplets and vapor come from a region of saturation, it follows that the existence of a zone of saturation would infer the existence of a zone with unevaporated droplets. It is physically evident that eq. (8) has two solutions, $z_1(t)$ and $z_2(t)$, for the motion of the cloud top and base. The cloud thickness for each moment of time is $|z_1(t) - z_2(t)|$ and, in agreement with what has been said, the total dissipation time may be determined from the equation $z_1(t_d) = z_2(t_d)$. If there is symmetry with respect to the center of the cloud, $z_1(t) = -z_2(t)$, when determining the dissipation time one may assume that $z = 0$ in (8) (since $z_1(t_d) = 0$)

$$S(0, t_d) = q_s[\Phi(0, t_d)]. \tag{10}$$

In the further computations, it will be more convenient to use the moving system of coordinates $z \rightarrow z - vt$ in eqs. (5) and (6):

$$\frac{\partial S}{\partial t} - k\frac{\partial^2 S}{\partial z^2} = 0; \tag{11}$$

$$\frac{\partial \Phi}{\partial t} - k\frac{\partial^2 \Phi}{\partial z^2} = \gamma_a v. \tag{12}$$

Thus, the existence of downdrafts in the moving system of coordinates is equivalent to the existence of heat sources with an intensity of $v\gamma_a$.

1. Isothermy

Given constant atmospheric temperature at the initial moment of time, the following formulas obtain for the initial values of liquid-water content and vapor density

$$w = \begin{cases} w_0, & |z| < H; \\ 0, & |z| > H; \end{cases} \quad q(z,t) = \begin{cases} q_s(T_0), & |z| < H, \\ q_0, & |z| > H, \end{cases} \quad T(z,t) = T_0, \quad t = 0 \quad ,$$

whence for S and Φ we get the following initial conditions:

$$S=\begin{cases} w_0+q_s(T_0), & |z|<H; \\ q_0, & |z|>H; \end{cases} \qquad \Phi=\begin{cases} T_0-aw_0, & |z|<H; \\ T_0, & |z|>H; \end{cases} \qquad t=0.$$

It is evident that the solutions of (11) and (12) for these initial conditions will be

$$\left.\begin{aligned} S&=\frac{w_0+\Delta q}{2}\left[E\left(\frac{H-z}{2\sqrt{kt}}\right)+E\left(\frac{H+z}{2\sqrt{kt}}\right)\right]+q_0; \\ \Phi&=\gamma_a vt+T_0\frac{aw_0}{2}\left[E\left(\frac{H-z}{2\sqrt{kt}}\right)+E\left(\frac{H+z}{2\sqrt{kt}}\right)\right]. \end{aligned}\right\} \tag{13}$$

where $$E(\xi)=\frac{2}{\sqrt{\pi}}\int_0^{\xi} e^{-x^2}dx; \; \Delta q=q_s(T_0)-q_0.$$

Substituting the expressions for S and Φ in eq. (8) to determine the moving cloud boundaries, we get the equation

$$\frac{1}{2}\left[w_0^*+\Delta q\right]\left[E\left(\frac{H-z}{2\sqrt{kt}}\right)+E\left(\frac{H+z}{2\sqrt{kt}}\right)\right]=\Delta q+q_s'(T_0)\gamma_a vt, \tag{14}$$

where $$w_0^*=w_0(1+aq_s'(T_0)).$$

In deriving this equation, we assumed that $q_s(T)$ could be expanded in a series. Inasmuch as this equation is invariant for the substitution of $-z$ for z, we shall consider the case $z>0$. Given $H/2\sqrt{kt}\ll 1$, from (14) we get the following equation for the motion of the cloud boundary

$$z(t)=2\sqrt{kt\ln\frac{(w_0^*+\Delta q)H}{(\Delta q+q_s'(T_0)\gamma_a vt)\sqrt{\pi kt}}}. \tag{15}$$

This formula implies that the motion decreases with time, in connection with the evaporation of the moisture stored in the dissipating cloud. From $z(t_d)=0$, we can find the equation for determining the total dissipation time of the cloud from (15)

$$1=\frac{t_d}{t_{d0}}\left(1+a\frac{t_d}{t_{d0}}\right)^2, \tag{16}$$

where
$$a = \frac{q_s'(T_0)\,\gamma_a\, v t_{d_0}}{\Delta q};$$

$t_{d_0} = H^2/\pi k(w_0^* + \Delta q/\Delta q)^2$ is the cloud dissipation t'me in the absence of downdrafts. As shown by determinations of the parameters (H, w_0, T_0, q_0) over a broad range of variations and for all $v \geq 1$ cm/sec, $a \gg 1$. Consequently, eq. (16) may be solved approximately

$$t_d = t_{d_0} a^{-2/3}. \tag{17}$$

Formula (15) for the motion of the boundary is asymptotically valid for $2\sqrt{kt} \gg H$. For this inequality to be satisfied, $2\sqrt{kt_d}$ must be greater than H. This inequality will be fulfilled if

$$1 \ll \left(\frac{w_0^* + \Delta q}{\Delta q}\right)^2 a^{-2/3},$$

which will be satisfied when $w_0 \gg \Delta q$, i.e., if the initial liquid-water content of the cloud is greater than the undersaturation below the cloud.

It follows from (15) that at certain moments the thickness of the dissipating cloud will exceed the initial thickness of the cloud, $z(t) > H$. For $\Delta q > w_0$, as qualitative analysis of eq. (14) shows, $z(t) < H$. Thus, upon dissipating the cloud will either expand at first and then contract and disappear ($\Delta q < w_0$) or it will begin to "melt," i.e., diminish from the very start of dissipation, depending on the relation between w_0 and Δq. It may also be shown that in the case of cloud "melting," the cloud boundaries move more rapidly than strictly by diffusion $2\sqrt{kt}$, because of the intensive removal of vapor from the cloud, i.e., by diffusion of the undersaturation into the cloud.

To determine the dissipation time, we set $z(t_d) = 0$ in eq. (14) and, allowing that $H \gg 2\sqrt{kt}$, for E we take the asympotic expression for large values of the argument

$$w_0^* - q_s'(T_0)\,\gamma_a\, v t_d = \frac{2}{\sqrt{\pi}} \frac{\sqrt{kt_d}}{H} e^{-\frac{H^2}{4kt_d}} (\Delta q + w_0^*). \tag{18}$$

The second term in the left-hand side of (18) describes the evaporation of cloud moisture resulting from the temperature increase during adiabatic compression. If this quantity of moisture is small ($w_0 \gg q_s'(T_0)\gamma_a v t_d$), from formula (18), taking its logarithm and omitting the terms of the next order of smallness, we get the following equation for t_d

$$t_d = \frac{H^2}{4k}\left[\ln \frac{2\Delta q}{\sqrt{\pi}\, w_0^*}\right]^{-1}. \tag{19}$$

This formula has rather limited application, since in deriving it we assumed that $H \gg 2\sqrt{kt_d}$, which can be ensured for a large value of the logarithm. Therefore, we shall also give the expression for the dissipation time when $\Delta q = w_0^*$. In particular, for $v = 0$, the dissipation time is determined from the equation

$$E\left(\frac{H}{2\sqrt{kt_d}}\right) = 0{,}5,$$

whence

$$t_d \approx \frac{1.1\,H^2}{k}.$$

The presence of downdrafts reduces the dissipation time in the last two cases; it is difficult to obtain the analytic formulas but the dissipation time t_d may be determined from (14) with $z = 0$ by numerical methods.

Let us cite some numerical examples that illustrate the resulting formulas. For $w_0 = 10^{-6}$ g/cm^3, $\Delta q = 10^{-7}$ g/cm^3, $2H = 500$ m, $k = 10^5$ cm^2/sec, $q_s(T_0) = 10^{-6}$ g/cm^3 from formula (16), we get $t_{d0} = 10^5$ sec. For $v = 10$ cm/sec and these same values, from formula (17) we get $t_d \approx 10^4$ sec. For $\Delta q = 10^{-6}$ g/cm^3, $2H = 500$ m, $w_0 = 10^{-7}$ g/cm^3, from formula (19) we get $t_d \approx 3 \times 10^3$ sec. From this it is evident that the dissipation time decreases with increasing undersaturation below the cloud and increasing downdraft velocity.

2. Linear Lapse Rate below the Cloud and Inversion above it

Now let us examine the case where the temperature is constant within the cloud at the initial moment of time, but increases above the cloud and decreases linearly below the cloud, i.e., when t = 0 the cloud is bounded by the lines $q = q_s(T)$. The initial values for liquid-water content, vapor density, and temperature are the following (here, as before, it is assumed that the origin of the selected coordinate system ($z = 0$) is in the center of the moving cloud):

$$w(z,\ t) = \begin{cases} w_0, & |z| < H; \\ 0, & |z| > H; \end{cases}$$

$$T(z,\ t) = \begin{cases} T_0 + \gamma_1 (z - H), & z > H; \\ T_0, & |z| < H; \\ T_0 - \gamma_2 (z + H); & z < -H; \end{cases} \qquad q = q_s(T_0), \quad t = 0.$$

The initial conditions for S and Φ are:

$$S(z,\ t) = \begin{cases} w_0 + q_s(T_0), & |z| < H; \\ q_s(T_0), & |z| > H; \end{cases}$$

$$\Phi(z,\ t) = \begin{cases} T_0 + \gamma_1 (z - H), & z > H; \\ T_0 - \alpha w_0, & |z| < H; \\ T_0 - \gamma_2 (z + H), & z < -H; \end{cases} \qquad t = 0.$$

The solutions of eqs. (11) and (12) with these initial conditions are

$$\left.\begin{aligned} & S(z,\ t) = q_s(T_0) + \frac{w_0}{2}\left[E\left(\frac{H - z}{2\sqrt{kt}}\right) + E\left(\frac{H + z}{2\sqrt{kt}}\right)\right], \\ & \Phi(z,\ t) = \frac{(\gamma_1 - \gamma_2) z}{2} - \frac{(\gamma_1 + \gamma_2) H}{2} - E\left(\frac{H - z}{2\sqrt{kt}}\right) \times \\ & \times \left(\frac{\alpha w_0}{2} + \frac{\gamma_1 (z - H)}{2}\right) - E\left(\frac{H + z}{2\sqrt{kt}}\right)\left(\frac{\alpha w_0}{2} - \frac{\gamma_2 (z + H)}{2}\right) + \\ & + \frac{1}{\sqrt{\pi}}\left(\gamma_1 \sqrt{kt}\, e^{-\frac{(H - z)^2}{4kt}} + \gamma_2 \sqrt{kt}\, e^{-\frac{(H + z)^2}{4kt}} + \gamma_a v t\right). \end{aligned}\right\} \qquad (20)$$

We substitute these expressions for S and Φ into eq. (8) to determine the moving boundaries of the cloud:

$$\begin{aligned} & \frac{AH}{2\sqrt{kt}}\left[E\left(\frac{H - z}{2\sqrt{kt}}\right) + E\left(\frac{H + z}{2\sqrt{kt}}\right)\right] = -\frac{H}{2\sqrt{kt}} - \frac{z - H}{4\sqrt{kt}} E\left(\frac{H - z}{2\sqrt{kt}}\right) + \\ & \qquad + \frac{z + H}{4\sqrt{kt}} E\left(\frac{z + H}{2\sqrt{kt}}\right) + \frac{1}{2\sqrt{\pi}}\left(e^{-\frac{(H - z)^2}{4kt}} + e^{-\frac{(H + z)^2}{4kt}}\right) + \qquad (21) \\ & + \frac{\gamma_a v t}{2\gamma_1 \sqrt{kt}} + \frac{(\gamma_1 - \gamma_2)(z + H)}{4\gamma_1 \sqrt{kt}}\left(1 - E\left(\frac{z + H}{2\sqrt{kt}}\right)\right) + \frac{\gamma_2 - \gamma_1}{2\sqrt{\pi}\,\gamma_1} e^{-\frac{(z + H)^2}{4kt}}. \end{aligned}$$

In this equation, as before, it is assumed that $q_s(T)$ can be expanded in a series with respect to the difference $T - T_0$ (which places some limitations on t_d) and the equation reduced to dimensionless form, and the terms that are dependent on the difference $\gamma_1 - \gamma_2$ are selected. If we set $\gamma_1 = \gamma_2 = \gamma$, the resulting equation is invariant when $-z$ is substituted for z, the two boundaries move symmetrically, and the dissipation time is determined from the condition $z(t_d) = 0$. If $H \ll 2\sqrt{kt_d}$ and $|\gamma_1 - \gamma_2/2\gamma_1| \ll 1$, it is natural to expect that for times close to the dissipation time, both boundaries will be near the point $z = 0$, i.e., $z \ll 2\sqrt{kt}$. Then the equation of motion of the boundaries will have the following form (we discard the terms of the next order of smallness)·

$$ax^2 + bx + c = 0, \tag{22}$$

where

$$x = \frac{z}{2\sqrt{kt}}, \quad a = \left(\frac{H}{2\sqrt{kt}}\right)^2 \frac{2A}{\sqrt{\pi}} + \frac{1}{\sqrt{\pi}}.$$

$$b = \frac{\gamma_1 - \gamma_2}{2\gamma_1}, \quad c = \frac{1}{\sqrt{\pi}} + \frac{\gamma_a vt}{2\gamma_1\sqrt{kt}} - \frac{AH^2}{2\sqrt{\pi}\,kt}, \quad A = \frac{w_0\,(1 + aq_s'(T_0))}{q_s'(T_0)\,\gamma_1 H}.$$

As this equation shows, there will be two boundaries, and the criterion for satisfaction of inequality $z \ll 2\sqrt{kt}$ will be, as is to be expected, $|\gamma_1 - \gamma_2| \ll 2\gamma_1$. The moment the two boundaries fuse may be regarded as the dissipation time, i.e., the moment when the two roots of the equation are equal ($b^2 = 4ac$), which yields the following approximate equation for determining t_d:

$$\frac{1}{\sqrt{\pi}} + \frac{\gamma_a vt_d}{2\gamma\sqrt{kt_d}} = \frac{AH^2}{\sqrt{\pi}\,kt_d}. \tag{23}$$

For small vertical velocities ($v \ll 4\,\frac{\gamma}{\gamma_a}\,\frac{k}{\sqrt{AH}}$), the formula for the dissipation time becomes

$$t_d = \frac{w_0^* H}{4q_s'(T_0)\,\gamma k}. \ \ldots \tag{24a}$$

However, in this case the velocity cannot exceed 1 cm/sec for the characteristic values of the parameters. However, for $v \gtrsim 1$ cm/sec in eq. (23), we may neglect the first term and then we will get the following expression for the dissipation time

$$t_d = \left(\frac{w_0^* H}{q_s'(T_0)\, \gamma_a v\, 2\sqrt{\pi k}} \right)^2 . \tag{24}$$

This formula holds if $H \ll 2\sqrt{kt}$, which gives the upper limit of v (the lower limit is determined from the excess of the second term on the left-hand side of (23) over the first term), therefore the region of change is determined by the inequality

$$\frac{1}{4} \frac{\gamma k}{\gamma_a H} A^{-1} \ll v \ll 4 \frac{\gamma k}{\gamma_a H} A.$$

Obviously, this inequality is not contradictory when $A \gg 1$.

The examined case is completely analogous to that given earlier for $w_0 > \Delta q$; the smallness of the termperature gradient $(q'_s(T_0)\,\gamma H)$ is substituted here for the smallness of the subcloud undersaturations. When $A < 1$, by qualitative analysis of eq. (21) we can also show that the cloud dimensions begin to decrease from the very beginning of the dissipation process and that the cloud boundaries move at a greater speed than indicated by the purely diffusion law. To determine the dissipation time in this case, we should consider $H \gg 2\sqrt{kt}$ in (21) on the assumption that $|\gamma_1 - \gamma_2| \ll 2\gamma_1$, naturally again assuming that $z \ll 2\sqrt{kt}$. Expanding the integrals asymptotically and again omitting the terms of the next order of smallness, we get

$$w_0^* - q_s'(T_0)\, \gamma_a v t_d = \frac{2kt_d}{\sqrt{\pi} H} e^{-\frac{H^2}{4kt_d}} q_s'(T_0)\, \gamma. \tag{25}$$

Neglecting the term with v, for t_d we get

$$t_d = \frac{H^2}{4k} \left[\ln \frac{q_s'(T_0)\, \gamma H}{2\sqrt{\pi}\, w_0^*} \right]^{-1} . \tag{26}$$

Let us compare this equation with (19). The remarks on the limitations of (19) also apply to (26). Let us give some numerical examples. For $w_0 = 10^{-8}$ g/cm^3, H = 200 m, $q_s(T_0) = 10^{-8}$ g/cm^3, v = 10 cm/sec, $k = 10^5$ cm^2/sec, from (24) we get $t_d = 3.5 \times 10^3$ sec $\approx$ 1.0 hr. For these same values of the parameters, but with $\gamma = 10^{-4}$ deg/cm and v = 0, from (24a) we get $t_d \approx 10^4$ sec, i.e., a value three times larger.

Thus, downdrafts also reduce the dissipation time.

3. Conclusion

The dissipation times for the most characteristic cases are given in table 1.

Table 1

Dissipation Time (hr) of a Cloud

Δq g/cm^3	γ deg/cm	v cm/sec		
		1×10^{-1}	1.0	10.0
		$w_0 = 10^{-7}$ g/cm^3		
10^{-8}		0.25	0.2	0.1
	10^{-4}	0.7	0.4	0.14
		$w_0 = 5 \times 10^{-8}$ g/cm^3		
10^{-8}		1.0	0.8	0.3
	10^{-4}	1.4	1.2	0.4

Owing to a number of mathematical difficulties, we have made a separate study of stratus-cloud dissipation due to sedimentation of the cloud droplets and that due to their removal from the cloud by turbulent pulsations, even though these mechanisms operate simultaneously in actual situations. It is interesting to compare the results obtained and to determine which of these mechanisms plays the decisive role in various situations.

It is natural to assume that the mechanism which causes dissipation in the shortest time is the dominant mechanism. First let us compare the formulas obtained for the dissipation time for the cases of turbulent and nonturbulent atmospheres. In a nonturbulent atmosphere, the dissipation time is expressed through the characteristics of the cloud and the atmosphere

$$t_{dc} = \frac{H}{ar_0^2}\left(1 + \frac{w_0}{\Delta q}\right) \quad \textbf{for} \quad v = 0. \tag{27}$$

Comparing (27) with (16) for the case $w_0 >> \Delta q$, we get

$$\frac{t_{dc}}{t_{dm}} = \frac{4\pi k\Delta q}{ar_0^2 H w_0},$$

from which it is evident that with small cloud thickness and large eddy diffusivity, the removal of droplets by turbulent pulsations will be the dominant factor (with $k = 10^5$ cm^2/sec, H = 500 m, $\Delta q/w_0 = 10^{-1}$, $r_0 = 6\mu$, $t_{dc}/t_{dm} = 6$).

The relationship for dissipation times resulting from the sedimentation and turbulence mechanism may be obtained in a similar manner for another limiting case, $w_0 << \Delta q$. Comparing (27) with (19) for this case, we get

$$\frac{t_{dc}}{t_{dm}} = \frac{16k \ln \frac{\Delta q}{w_0}}{ar_0^2 H}.$$

In contrast to the preceding formula, here $k/dr_0^2 H$ is multiplied by a number much larger than unity, therefore a situation may arise where turbulence will play a role given these same values of the parameters and this same limiting case.

The dissipation-time formulas for sedimentation and turbulence may also be compared for the case of a linear lapse rate, in which case the expression $q_s'(T_0)\gamma H/w_0$ should be substituted for $\Delta q/w_0$ in the relations obtained for the dissipation time. The quantitative conclusions regarding the relation between the sedimentation and turbulence mechanism remain as before.

Given downdraft velocities ($v \neq 0$) for the most characteristic cases in which turbulence is absent, the dissipation of a cloud is determined by evaporation, given a steady temperature increase due to adiabatic compression.

In the case of turbulence, the cloud will evaporate more rapidly, because the cloud droplets will be transported by turbulent pulsations into a region of greater undersaturations than those which exist inside the cloud.

The various mechanisms of stratus-cloud dissipation examined here show that a cloud is a stable colloidal system and that it may exist for several hours after its moisture supply has been curtailed.

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