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METHOD OF CALCULATING THE WIND TUNNEL INTERFERENCE FOR STEADY AND OSCILLATING WINGS IN TUNNELS OF ARBITRARY WALL CONFIGURATION

C. F. Lo

ARO, Inc.

March 1968

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FOREWORD

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This technical report has been reviewed and is approved.

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ABSTRACT

A method of calculating the wind tunnel interference induced by arbitrary wall configurations has been developed wherein a linearized equation of the perturbation velocity potential is used to describe the flow in the tunnel. A series solution of the interference velocity potential satisfies the differential equation exactly. The constant coefficients in the series solution are determined by the point matching technique along the boundary. Results are presented for slotted tunnel walls with homogeneous boundary conditions. Solutions presented show excellent agreement with the results obtained by other methods for a stationary as well as an oscillating wing. Finally, the method is applied to compute a set of zero interference configurations for a combination of slot openings of side walls, roof, and floor.

CONTENTS

APPENDIXES

I. ILLUSTRATIONS

NOMENCLATURE

a	Slot width
A_m; B_j, B_{1n}, B_{2n}; C_{1n}, C_{2n}, C_{3n}, C_{4n}	Series coefficient constants
$\{B\}$	Column matrix
b	Tunnel semiwidth
C_L	Lift coefficient, $\dfrac{L}{1/2\, \rho_d\, U^2 S}$
$\{D$	Column matrix
h	Tunnel semiheight
K	Geometric slot parameter, $\dfrac{\ell}{\pi}\ \ln \left[\csc \dfrac{\pi a}{2\ell}\right]$
L	Lift
ℓ	Slot spacing
	Coefficient matrix

$\bar{n}$	Normal at the wall
P	Nondimensional slot parameter, $(1 + K/h)^{-1}$
S	Wing area
s	Wing span
U	Free-stream velocity
ΔW	Upwash velocity
x, y, z	Cartesian coordinate system (see Fig. 1)
α	ω/U
$\Delta\alpha_i$	Induced angle
Γ	Circulation about a wing
δ	Lift interference factor
ϵ_B	Error on boundary
λ	Tunnel height-to-width ratio, h/b
ρ_d	Air density
ρ, θ	Polar coordinate system (see Fig. 2)
ϕ	Perturbation velocity potential, $\phi_i + \phi_m{}^*$
ϕ_i	Interference velocity potential
$\phi_m{}^*$	Velocity potential of wing
ω	Angular frequency

SUBSCRIPTS

B	Boundary
o	Plane of wing
r	Roof and floor
w	Side walls
∞	Downstream wake

SECTION I
INTRODUCTION

Several investigations have been made theoretically and experimentally to determine an optimum test section of a Vertical/Short Takeoff and Landing (V/STOL) wind tunnel (see references cited in Ref. 1). Although a zero interference configuration has not yet been determined, all results indicate that the wind tunnel with the mixed boundaries is a promising configuration to alleviate the interference effects.

As the first step to obtain a theoretical solution to the interference problem, a method is required to calculate the tunnel interference on the horizontal trailing wake which is the limiting case of a V/STOL model. Unfortunately, the available theoretical methods are inadequate for the mixed boundary problem. The classical image method has no proper image system satisfying the boundary conditions. In recent computer investigations (Refs. 2 and 3), a vortex lattice network has been utilized to represent the model and wind tunnel walls. However, the technique is difficult to use because the choice of the size of networks and the distribution of networks seem ambiguous.

In this report, the point matching technique (Ref. 4) is introduced to calculate the interference. The attractive feature of the method is that it is applicable to a tunnel with an arbitrary cross-section and wall configuration. Calculations were made for the case of a horizontal wake (parallel to the tunnel centerline) and compared to solutions obtained by other methods. In addition, the wall interference is calculated at a point in the downstream wake of an oscillating wing. Solutions are also obtained for the wall interference for the horizontal wake case in a wind tunnel with various slot configurations. The results indicate the proper combination of slotted tunnel walls that will result in zero lift interference for the case of a horizontal wing wake.

SECTION II
GENERAL ANALYSIS

The flow field, in terms of the perturbation velocity potential, is governed by Laplace's equation for the problem of the wall interference in a wind tunnel. In the present analysis a wing of span "s" is considered and is represented by a single horseshoe vortex. The perturbation potential is written as:

$$\phi = \phi_m{}^* + \phi_i \tag{1}$$

where

$\phi_m{}^*$ = the potential caused by the wing in the absence of tunnel walls,
ϕ_i = the interference potential induced by the walls.

The potential at the point (x, y, z), as shown in Fig. 1, Appendix I, because of the wing at the origin with the circulation Γ about the wing, is

$$\phi_m{}^* = \frac{\Gamma s}{4\pi} \frac{z}{y^2 + z^2} \left[1 + \frac{x}{(x^2 + y^2 + z^2)^{1/2}} \right] \tag{2}$$

The potential for the wake far downstream from the wing is

$$\phi_m = \lim_{x \to \infty} \phi_m{}^* = \frac{\Gamma s}{2\pi} \frac{z}{y^2 + z^2} \tag{3}$$

which is independent of x. This will be used as the basic flow far downstream when no walls are present.

2.1 DIFFERENTIAL EQUATION OF THE INTERFERENCE POTENTIAL

The governing equation for the velocity potential in incompressible flow is

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \tag{4}$$

If the tunnel geometry is independent of x, then it may be assumed by observing Eq. (3) that the potential of the far downstream wake for steady flow is

$$\phi = \phi\ (y,z) \tag{5}$$

Substituting Eq. (5) into Eq. (4), the governing equation results in

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

or in polar coordinates, $\rho = (y^2 + z^2)^{1/2}$ and $\theta = \tan^{-1} (z/y)$,

$$\frac{\partial^2 \phi}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \phi}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \phi}{\partial \theta^2} = 0 \tag{6}$$

The equation describing the interference potential is obtained by combining Eqs. (1), (3), and (6),

$$\frac{\partial^2 \phi_i}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \phi_i}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 \phi_i}{\partial \theta^2} = 0 \tag{7}$$

2.2 BOUNDARY CONDITIONS

It is assumed that along the tunnel wall there are longitudinal slots. The condition to be satisfied on the solid portion of the boundary is that there is no flow through the wall. This can be expressed as

$$\frac{\partial \phi}{\partial \overline{n}} = 0 \text{ at the solid wall portion} \tag{8}$$

where $\overline{n}$ is the normal at the wall. Along the slotted portion the pressure is constant which corresponds to

$$\phi = 0 \text{ at the slotted portion.} \tag{9}$$

Instead of separate conditions on the solid and slotted portions of the tunnel (discrete slots), an equivalent homogeneous boundary condition (Ref. 5) can be introduced to satisfy all points of a uniformly spaced slotted wall. The condition, based on a periodic slot of width a and spacing ℓ, is as follows:

$$\phi + K\frac{\partial \phi}{\partial \overline{n}} = 0 \text{ at the boundaries} \tag{10}$$

where K is the geometric slot parameter

$$K = \frac{\ell}{\pi} \ln\left[\csc \frac{\pi a}{2\ell}\right] \tag{11}$$

and a/ℓ is the open area ratio as shown in Fig. 2.

2.3 METHOD OF SOLUTION

The general solution of the interference potential of Eq. (7) may be written in series form with undetermined constant coefficients, i. e.,

$$\phi_i = \sum_{n=0}^{\infty}\left[C_{1n}\sin n\theta + C_{2n}\cos n\theta\right]\left[C_{3n}\rho^n + C_{4n}\rho^{-n}\right] \tag{12}$$

Since ϕ_i must be finite at $\rho = 0$, C_{4n} must be zero and Eq. (12) becomes

$$\phi_i = \sum_{n=0}^{\infty}\left[B_{1n}\rho^n \sin n\theta + B_{2n}\rho^n \cos n\theta\right] \tag{13}$$

Equation (13) is an exact solution to the differential equation and it may be applied by the point matching method to approximate the boundary conditions so as to determine the constants. If the series, Eq. (13), is truncated at $n = N$, then there are $2N + 1$ unknown constants. For an arbitrary cross-sectional shape, $2N + 1$ points may be chosen along the boundary. Evaluation of either Eqs. (8) and (9) or Eq. (11) at these points results in $2N + 1$ simultaneous linear algebraic equations for the undetermined constants B_{1n} and B_{2n}. The inversion of this set of simultaneous equations completes the solution of the problem.

The mathematical expressions and method of solution proceeds as follows: Let the $2N + 1$ linear equations be written in matrix form

$$[M]\ \{B\} = \{D\} \tag{14}$$

where

$[M]$ is the coefficient matrix,

$\{B\}$ is the unknown vector and

$\{D\}$ is the right-hand side column matrix of this set of linear equations.

Since $[M]$ is a square matrix, the unknown constants B_n may be evaluated as

$$\{B\} = [M]^{-1}\ \{D\} \tag{15}$$

Alternately, a more accurate result may be obtained (Ref. 6) by the method of least squares and by choosing more matching points along the boundary than the number of undetermined constants. Let the error on the boundary be expressed as

$$\epsilon_B = \phi_{i_B} - \phi'_{i_B} \tag{16}$$

where ϕ'_{i_B} is the exact expression for interference potential on the boundary. The error, ϵ_B, is expressed in terms of the undetermined constants B_j, $j = 1, 2, \ldots 2N + 1$, where $(2N + 1)$ is less than the number of matching points, Q. The sum of the squares of the errors

$$S(B_j) = \sum_{i=1}^{Q} \epsilon_{B_i}^2 = \sum_{i=1}^{Q} \left(\sum_{j=1}^{2N+1} M_{ij} B_j - D_i \right)^2 \tag{17}$$

is then minimized through the $(2N + 1)$ relationships

$$\frac{\partial S}{\partial B_k} = 0,\ k = 1, 2, \ldots 2N + 1 \tag{18}$$

The resulting system of algebraic equations is

$$\sum_{i=1}^{Q} M_{ik}\left[\sum_{j=1}^{2N+1} M_{ij}B_j\right] = \sum_{i=1}^{Q} M_{ik}D_i$$

or

$$[M]^T [M] \{B\} = [M]^T\{D\} \tag{19}$$

The solution is then given by the inversion of Eq. (19).

The error in the domain of the solutions for the above two methods is bounded by the boundary error, ϵ_B. It is known from the maximum-minimum theory of harmonic functions that the absolute value of error has its maximum value on the boundary. Therefore, the smaller the boundary error, the smaller the error in the interior domain becomes. In other words, the greater the number of points, the more accurate is the solution.

2.4 LIFT INTERFERENCE FACTOR

Once the interference potential is obtained, the induced velocity caused by the walls can be calculated at any point in the tunnel. The wall interference on lift is expressed as an angle whose tangent is the vertical component of interference velocity divided by the free-stream speed, $\Delta W/U$. The classical interference factor δ is defined by

$$\delta S C_L/C = \Delta\alpha_i \approx \tan \Delta\alpha_i = \Delta W/U$$

where

S = wing area
C_L = wing lift coefficient
C = wind tunnel cross-sectional area

The factor may also be expressed in terms of wing circulation and wing span as

$$\delta = C\Delta W/2\Gamma s$$

The interference factor at the plane of wing δ_o is

$$\delta_o = C\Delta W_\infty/4\Gamma s \tag{20}$$

where

$$\Delta W_\infty = \left(\frac{\partial \phi_i}{\partial z}\right)_{\substack{y=0\\ z=0}}$$

ΔW_∞ is the interference velocity far downstream from the wing and by symmetry is twice the value at the plane of the wing.

SECTION III
RESULTS

3.1 RECTANGULAR TUNNEL WITH SOLID SIDE WALLS AND SLOTTED ROOF AND FLOOR

The first example chosen to verify the present method is the wind tunnel interference of a rectangular cross section with solid side walls and slotted roof and floor. This case has been investigated by analytical methods as well as an experimental method utilizing an electrical analogy to obtain theoretical solutions.

The coordinate system of the test section is shown in Fig. 2. For the equivalent homogeneous boundary conditions, the overall effect of the slotted roof or floor is given by Eq. (10),

$$\phi_i \pm K_r \frac{\partial \phi_i}{\partial z} = -\left(\phi_m \pm K_r \frac{\partial \phi_m}{\partial z}\right) \text{ at } z = \pm h \tag{21}$$

where K_r is the geometric slot parameter of the roof and floor defined in Eq. (11). For the solid side walls, the normal velocity at the boundary is zero.

$$\frac{\partial \phi_i}{\partial y} = -\frac{\partial \phi_m}{\partial y} \text{ at } y = \pm b \tag{22}$$

The wing is located at the center of the tunnel for convenience. However, it should be noted that the method is applicable for a wing located at any position in the tunnel. The potential of the wing from Eq. (3) is

$$\phi_m = \frac{\Gamma s}{2\pi} \frac{z}{y^2 + z^2} \tag{23}$$

Let $\theta = 0$ be in the plane of the wing. Then, because of the symmetrical location of the wing and the asymmetrical potential of the wing about $\theta = 0$, it is apparent that $\phi_i(\theta) = \phi_i(\pi - \theta)$ and $\phi_i(-\theta) = -\phi_i(\theta)$. Thus the solution, Eq. (13), may be written in the form

$$\phi_i = \sum_{m=1,3,5}^{\infty} A_m \rho^m \sin m\theta \tag{24}$$

By applying the boundary conditions of Eqs. (21) and (22) to Eqs. (23) and (24), the following sets of equations are obtained:

$$\sum_{m=1,3,5}^{\infty} A_m \left[\rho^m \sin m\theta_r + K_r m \rho_r^{m-1} \cos (m-1)\theta_r\right] = -\frac{\Gamma s}{2\pi}\left[\frac{h}{\rho_r^2} + K_r \frac{y^2 - h^2}{\rho_r^4}\right] \tag{25}$$

where

$$\rho_r = (y^2 + h^2)^{\frac{1}{2}} \qquad \theta_r = \tan^{-1}(h/y) \tag{26}$$

and

$$\sum_{m=1,3,5}^{\infty} A_m\, m\, \rho_w^{m-1} \sin(m-1)\theta_w = \frac{\Gamma_s}{2\pi}\, \frac{2zb}{\rho_w^4} \tag{27}$$

where

$$\rho_w = (b^2 + z^2)^{\frac{1}{2}} \qquad \theta_w = \tan^{-1}(z/b) \tag{28}$$

The constant coefficients A_m are determined by the point matching technique from Eqs. (25) and (27).

The lift interference factor at the plane of wing only involves the first constant A_1, as seen from Eqs. (20) and (24)

$$\delta_o = \frac{CA_1}{4\Gamma_s} \tag{29}$$

The interference factor is tabulated in Table I (Appendix II) corresponding to truncating the series at various numbers of terms to indicate the convergence of the series. It can be seen that there are only small differences in the interference factor as the number of terms increases. Table II gives the results of the series truncated after the ninth term for various numbers of matching points. The value of the interference factor is seen to converge as the number of matching points increases.

The convergence of the interference factor for various height-to-width ratios was also examined. It may be seen from Table III that the case of $\lambda = 1.0$ (square tunnel) converges most rapidly. It was pointed out in Ref. 6 that this may be partially because the ratio of maximum to minimum boundary radius (ρ_{max}/ρ_{min}) has the smallest value.

The interference factor is plotted as a function of the nondimensional slot parameter, $P_r = (1 + K_r/h)^{-1}$, in Fig. 3 for various height-to-width ratios. The value $P_r = 1.0$ corresponds to solid roof and floor, and $P_r = 0.0$ corresponds to open roof and floor. Results from Refs. 5, 7, and 8 are also shown in the figure. It can be seen that the agreement between the different methods is excellent.

3.2 RECTANGULAR TUNNEL WITH FOUR WALLS SLOTTED

The wind tunnel interference in the rectangular tunnel with four walls slotted is chosen as the second example (see Fig. 2). This particular configuration has not been solved analytically. The only available solution has been obtained by Rushton (Ref. 8) by an electrical analogy technique.

Again, the equivalent homogeneous boundary conditions will be used. For the slotted roof or floor Eq. (25) is used. The boundary condition for the slotted side walls is governed by

$$\phi_i \pm K_w \frac{\partial \phi_i}{\partial y} = -\left(\phi_m \pm K_w \frac{\partial \phi_m}{\partial y}\right) \text{at } y = \pm b \tag{30}$$

Combining Eqs. (23), (24), and (30), the following equation is obtained

$$\sum_{m=1,3,5}^{\infty} A_m \left[\rho_w^m \sin m\theta_w + K_w m \rho_w^{m-1} \sin (m-1)\theta_w\right] = -\frac{\Gamma s}{2\pi}\left[\frac{z}{\rho_w^2} - K_w \frac{2zb}{\rho_w^4}\right] \tag{31}$$

where ρ_w and θ_w are polar coordinates as defined in Eq. (28).

The constant coefficients A_m are determined by the point matching techniques from Eqs. (25) and (31) for $K_r = K_w$. The lift interference factors are plotted in Fig. 4 and show good agreement with Rushton's electrical analogy results.

3.3 OSCILLATING WING IN A SLOTTED WIND TUNNEL

This section is devoted to the case of the harmonically oscillating wing that is restricted to a rotatory oscillation about the center of the chord. The interference factor in the infinite wake can only be used as an indication of the amplitude, but not the phase, of the interference factor in the plane of the wing. For a wing of very small chord with uniform spanwise loading, the expression of the potential field as $x \to \infty$ of a small wing oscillating with an angular frequency ω has been derived by Rushton (Ref. 8) and Goodman (Ref. 9) and is given by

$$\phi_m(y,z) = \frac{USC_L\omega}{4\pi} \frac{z}{U} \frac{z}{\rho} K_1\left(\frac{\omega}{U}\rho\right) \tag{32}$$

where K_1 is the modified Bessel function of the second kind of order 1.

For a tunnel of constant cross section, it may be assumed that the total perturbation potential of the far downstream in a harmonically oscillating wake is given by

$$\phi = \text{Real Part of} \left[\phi(y,z)\, e^{i\omega\left(t - \frac{x}{U}\right)}\right] \tag{33}$$

Equations (4) and (33) lead to the differential equation for ϕ (y, z)

$$\frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = \left(\frac{\omega}{U}\right)^2 \phi \tag{34}$$

or in polar coordinates

$$\frac{\partial^2\phi}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\phi}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2\phi}{\partial\theta^2} = \left(\frac{\omega}{U}\right)^2 \phi \tag{35}$$

From Eqs. (1), (32), and (35), the differential equation for the interference potential is given as

$$\frac{\partial^2\phi_i}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\phi_i}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2\phi_i}{\partial\theta^2} = \left(\frac{\omega}{U}\right)^2 \phi_i \tag{36}$$

The general solution of Eq. (36) in series form becomes

$$\phi_i = \sum_{n=0}^{\infty}\left[C_{1n}\sin n\theta + C_{2n}\cos n\theta\right]\left[C_{3n} I_n(\alpha\rho) + C_{4n} K_n(\alpha\rho)\right] \tag{37}$$

where I_n and K_n are the modified Bessel function of the first and second kind of order n, respectively, and $\alpha \equiv \omega/U$.

For a rectangular wind tunnel with solid side walls and slotted roof and floor, the boundary conditions are the same as that of the nonoscillating wing case, Eqs. (21) and (22). Using the properties of symmetry about $\theta = \pi/2$, asymmetry about $\theta = 0$ and $C_{4n} = 0$, ϕ_i can be written as

$$\phi_i = \sum_{m=1,3,5}^{\infty} A_m I_m(\alpha\rho)\sin m\theta \tag{38}$$

From Eqs. (32) and (38) and the boundary conditions of Eqs. (21) and (22), the constants A_m may be determined by the following sets of equations:

$$\begin{aligned}\sum_{m=1,3,5}^{\infty} A_m \Big\{ & I_m(\beta_r)\sin m\theta_r + K_r\Big[m\, I_m(\beta_r)\cos(m-1)\theta_r \\ & + \beta_r\, I_{m+1}(\beta_r)\sin m\theta_r \sin\theta_r\Big]/\rho_r\Big\} \\ = -\frac{USC_L}{4\pi}\frac{\alpha}{\rho_r^3} & \Big\{h\, K_1(\beta_r)\rho_r^2 + K_r\Big[(y^2 - h^2)K_1(\beta_r) \\ & - \beta_r h^2 K_0(\beta_r)\Big]\Big\}\end{aligned} \tag{39}$$

where

$$\rho_r = (h^2 + y^2)^{1/2} \qquad \theta_r = \tan^{-1}(h/y) \tag{40}$$

$$\beta_r = a\rho_r$$

and

$$\sum_{m=1,3,5}^{\infty} A_m \Big[\beta_w I_{m+1}(\beta_w) \sin m\theta_w \cos \theta_w + m I_m(\beta_w) \sin(m-1)\theta_w\Big] \tag{41}$$

$$= \frac{USC_L}{4\pi} \frac{azb}{\rho_w^2} \Big[2K_1(\beta_w) + \beta_w K_0(\beta_w)\Big]$$

where

$$\rho_w = (b^2 + z^2)^{1/2} \qquad \theta_w = \tan^{-1}(z/b) \qquad \beta_w = a\rho_w \tag{42}$$

The constant coefficients are determined from this system of equations by the point matching technique along the boundaries as before.

The upwash interference at the centerline in the infinite wake is

$$\Delta W_\infty = \lim_{\rho \to 0} \frac{\partial \phi_1}{\partial z} = a A_1/2 \tag{43}$$

The lift interference factor in the infinite wake becomes

$$\delta_\infty = \frac{a A_1 C}{2\ USC_L} \tag{44}$$

Figure 5 shows the interference factor δ_∞ as a function of the reduced frequency $\omega h/U$ for various values of the nondimensional slot parameter, $P_r = (1 + K_r/h)$, for $\lambda = 0.5$. It is seen that the interference factor is of the same order of magnitude as the nonoscillating case at low frequencies but is negligible at high frequencies.

It is also interesting to note the slot parameter as a function of frequency parameter for the zero interference condition as shown in Fig. 6. It can be seen that the higher the frequency the greater the percent opening required for zero interference. For values of the slot parameter less than 0.38, zero interference is never reached.

SECTION IV
APPLICATION TO OPTIMIZATION OF WALL CONFIGURATIONS

Since the point matching technique permits an interference evaluation of wall configurations with mixed boundaries, it becomes possible to optimize the wall configuration of a tunnel on the basis of a number of criteria. An example of applying the method in search of a set of zero interference wall configurations can proceed as follows.

The first consideration could be the tunnel height-to-width ratio. From the study of a finite span wing in a closed test section (Ref. 3), it was found that the tunnel with a height-to-width ratio, λ, from 0.667 to 0.8 gives a reasonably small rate of variation of the correction for angle of attack and velocity along the lateral and longitudinal axes. In the investigation of "recirculation limits," Rae (Ref. 10) concluded that a height-to-width ratio of the order of 0.8 is probably desirable for V/STOL testing in a closed tunnel. Therefore height-to-width ratios from 0.5 to 1.0 can be chosen in a study of optimum configurations.

The second consideration could be the distribution of slot openings in the tunnel walls. If it is assumed that the slot parameters of roof and floor have the same value, P_r, which may be different from the values for the walls, P_w, the properties of symmetry about $\theta = \pi/2$ and asymmetry about $\theta = 0$ for the interference potential are preserved. The solution of interference potential has the form of Eq. (24). The boundary condition of the slotted roof or floor is given in Eq. (25), and for the slotted side walls in Eq. (31).

The interference factor is plotted in Fig. 7 as a function of the roof and floor slot parameter P_r for $\lambda = 0.667$ and various values of slot parameter P_w. It can be seen that proper combinations of P_r and P_w give zero interference. Similarly, the combination of P_r to P_w for zero interference is obtained for other height-to-width ratios. The values of wall slot parameter that result in zero interference are shown in Fig. 8 for various values of λ. It is concluded that the interference factor is insensitive to the opening of side walls except for the case of a square tunnel. This gives the freedom on the side walls to satisfy other criteria.

SECTION V
CONCLUDING REMARKS

A new method has been proposed for calculating the wind tunnel interference of arbitrary wall configurations on a lifting wing. The total perturbation potential is split into two parts: the potential caused by the wing and the interference potential induced by the walls. The unknown constants of a truncated series solution for the interference potential are determined by the point matching technique along the mixed boundaries.

Calculations of the wall interference on steady and oscillating wings are made to evaluate the proposed method. The agreement is excellent between the results calculated by the proposed method, the available data from the classical solutions, and the solutions to the equations as obtained by electrical analogy.

Finally, a set of zero interference configurations are found for combinations of slot opening on the side walls and the roof and floor. The insensitivity of the interference to the opening on the side walls leaves one adjustable variable to fulfill some other optimizing criterion.

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APPENDIXES

I. ILLUSTRATIONS

II. TABLES

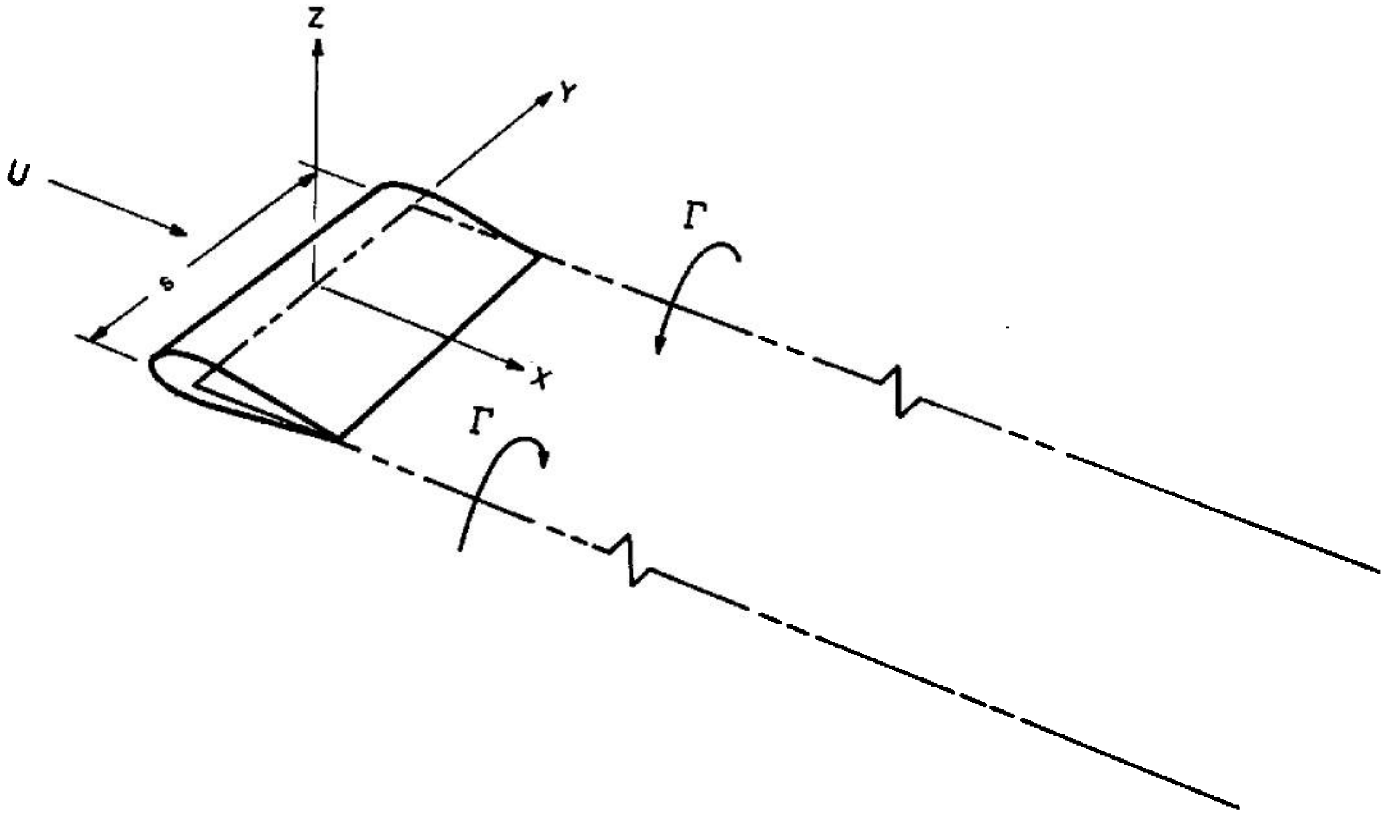
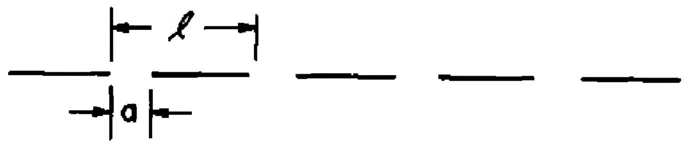


Fig. 1 Horseshoe Vortex Representation of a Wing



GEOMETRIC SLOT PARAMETER: $K = \frac{\ell}{\pi} \ln \left[\csc \frac{\pi a}{2\ell} \right]$

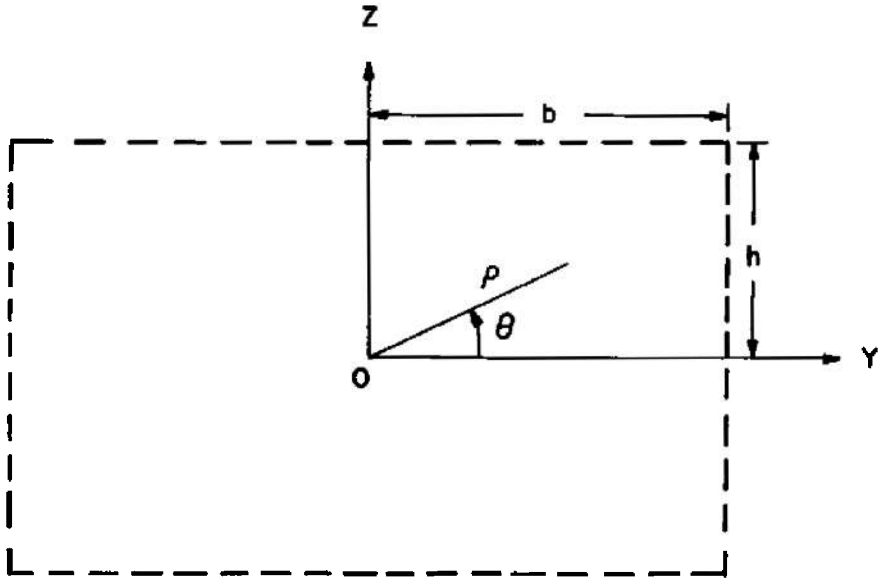


Fig. 2 Coordinate System and Geometry of Wind Tunnel Cross Section and Longitudinal Slots

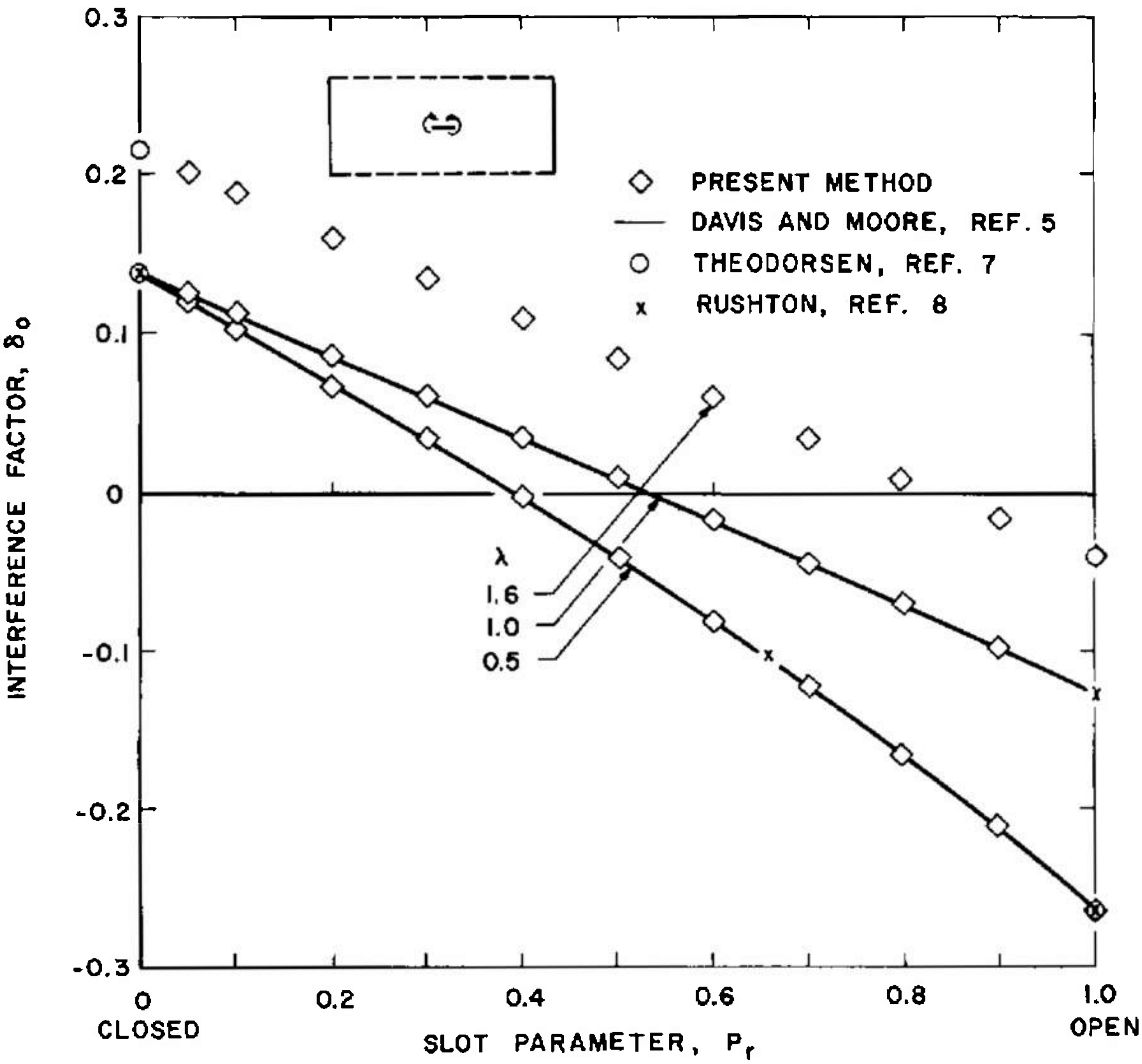


Fig. 3 Lift Interference Factor for Rectangular Tunnels with Slotted Roof and Floor

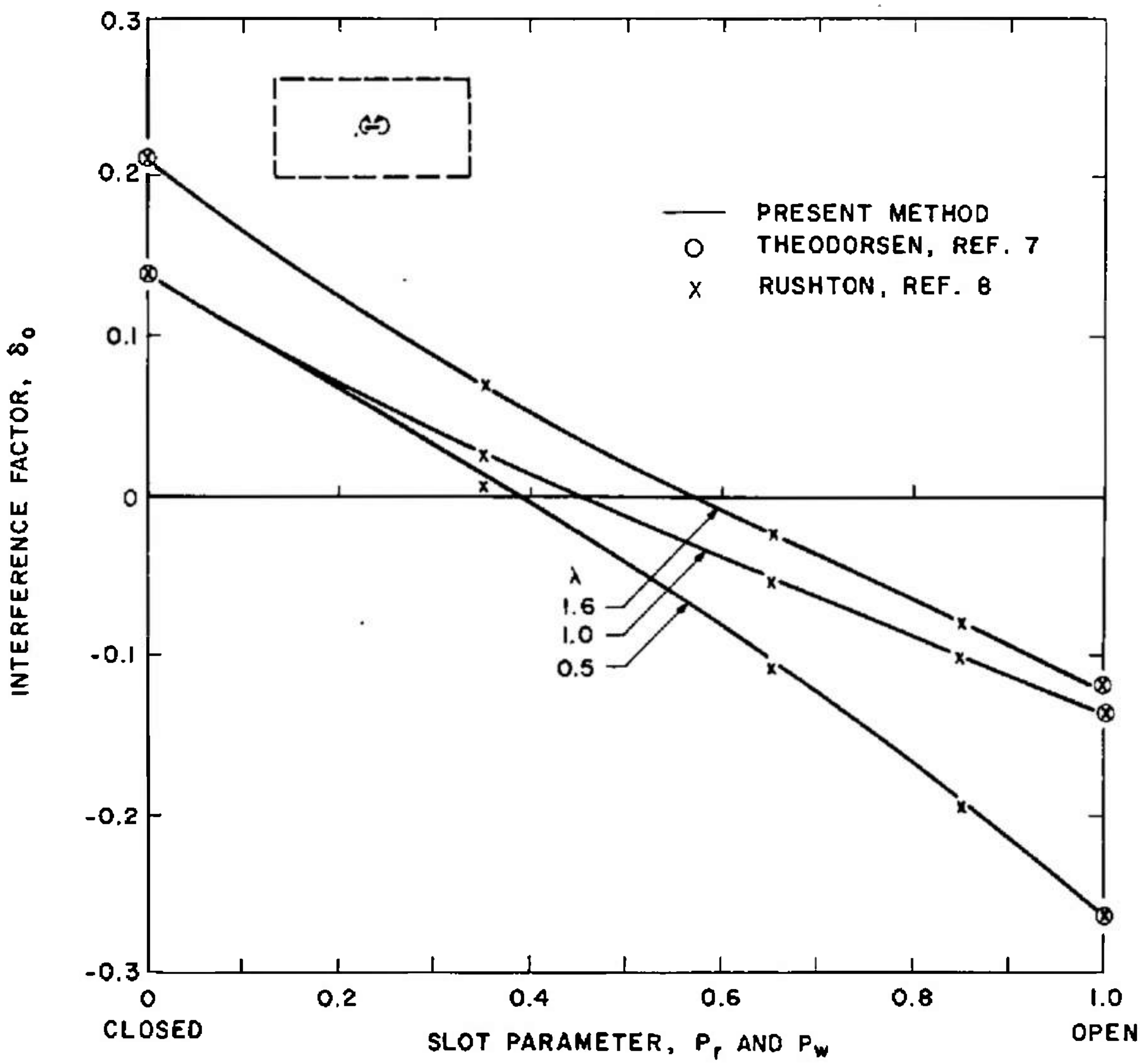


Fig. 4 Lift Interference Factor for Rectangular Tunnels with Four Walls Slotted, $P_r = P_w$

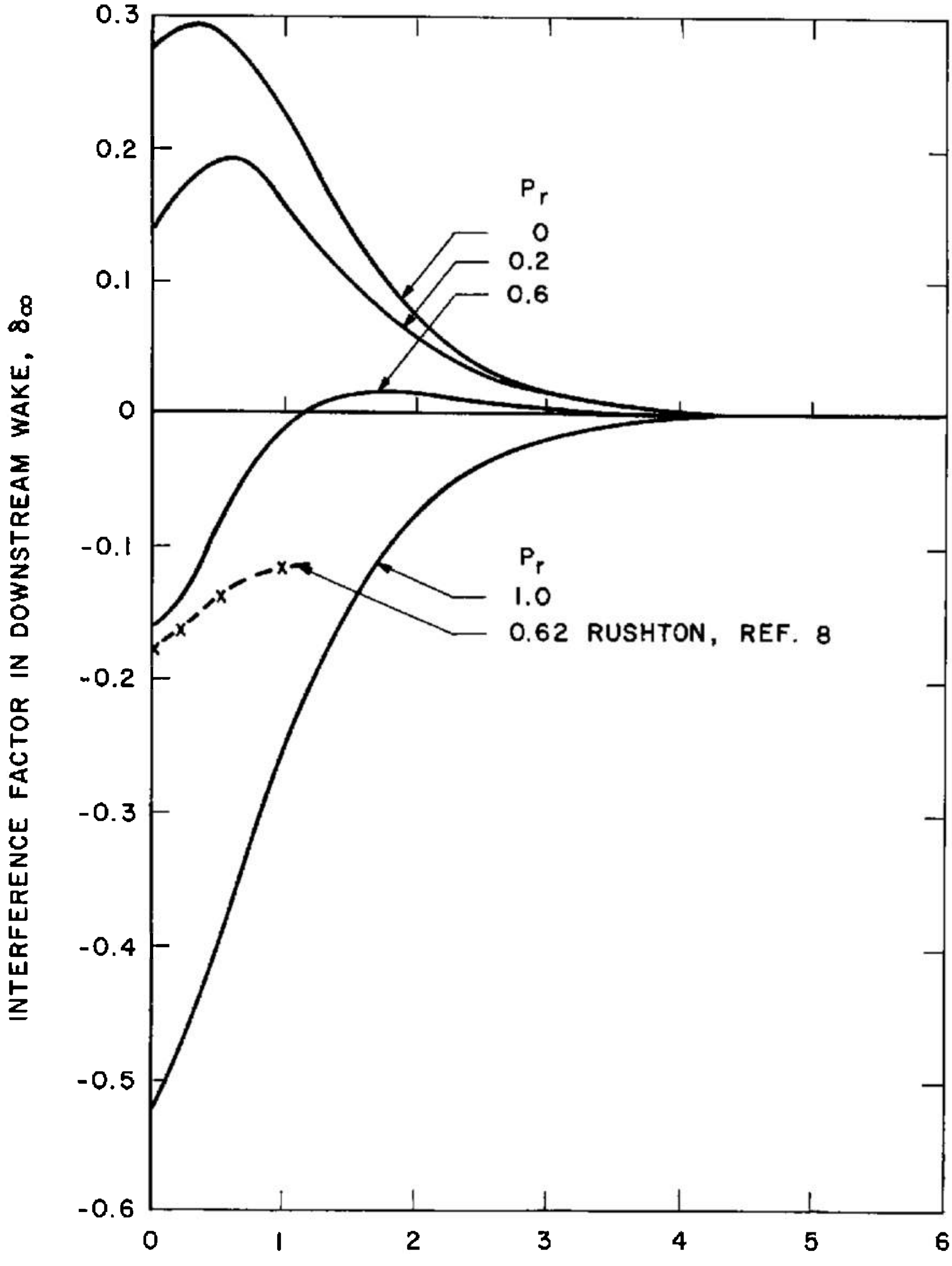


Fig. 5 Lift Interference Factor as a Function of Oscillation Frequency, $\lambda = 1/2$

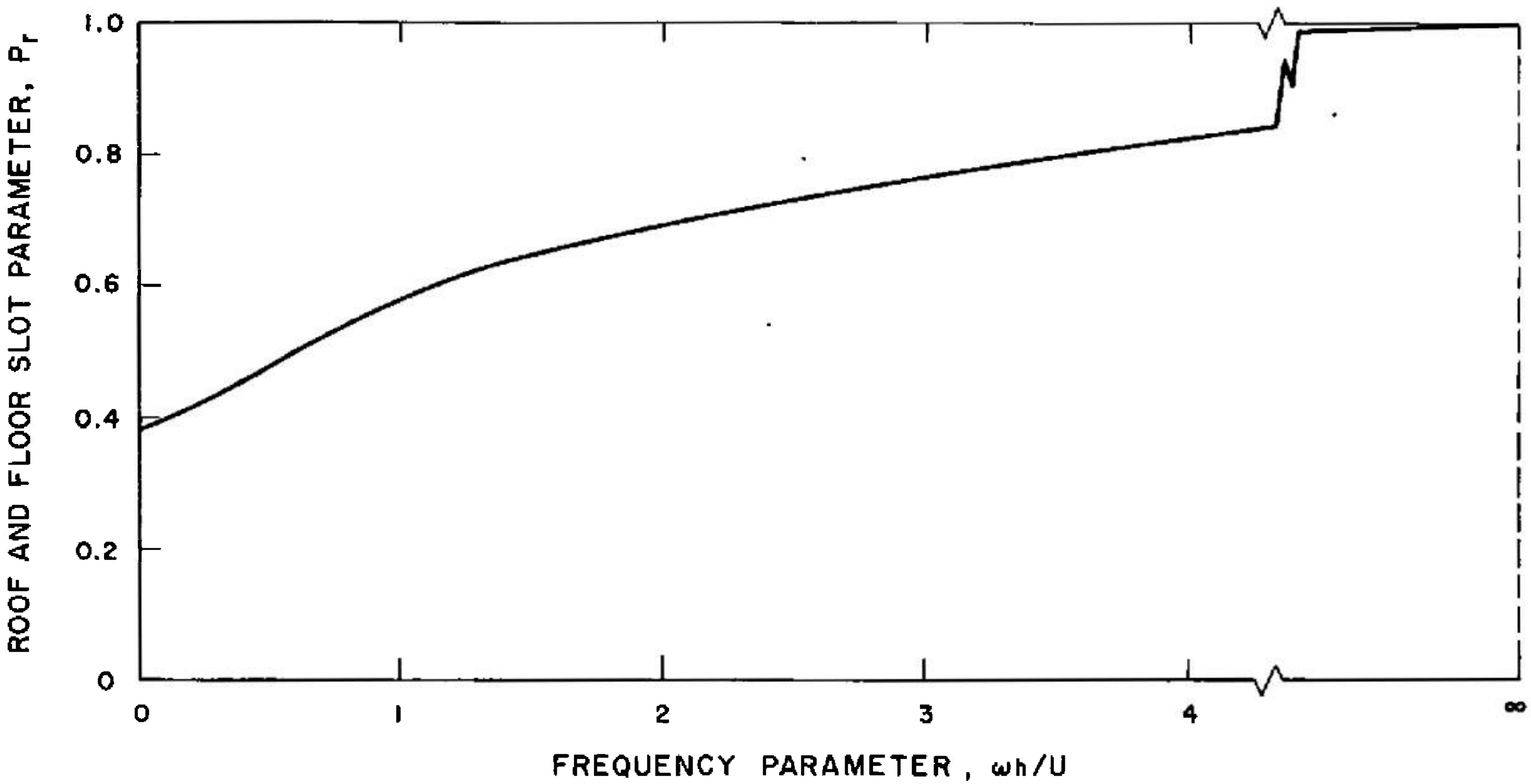


Fig. 6 Slot Parameter Required for Zero Interference Solid Side Walls, $\lambda = 1/2$

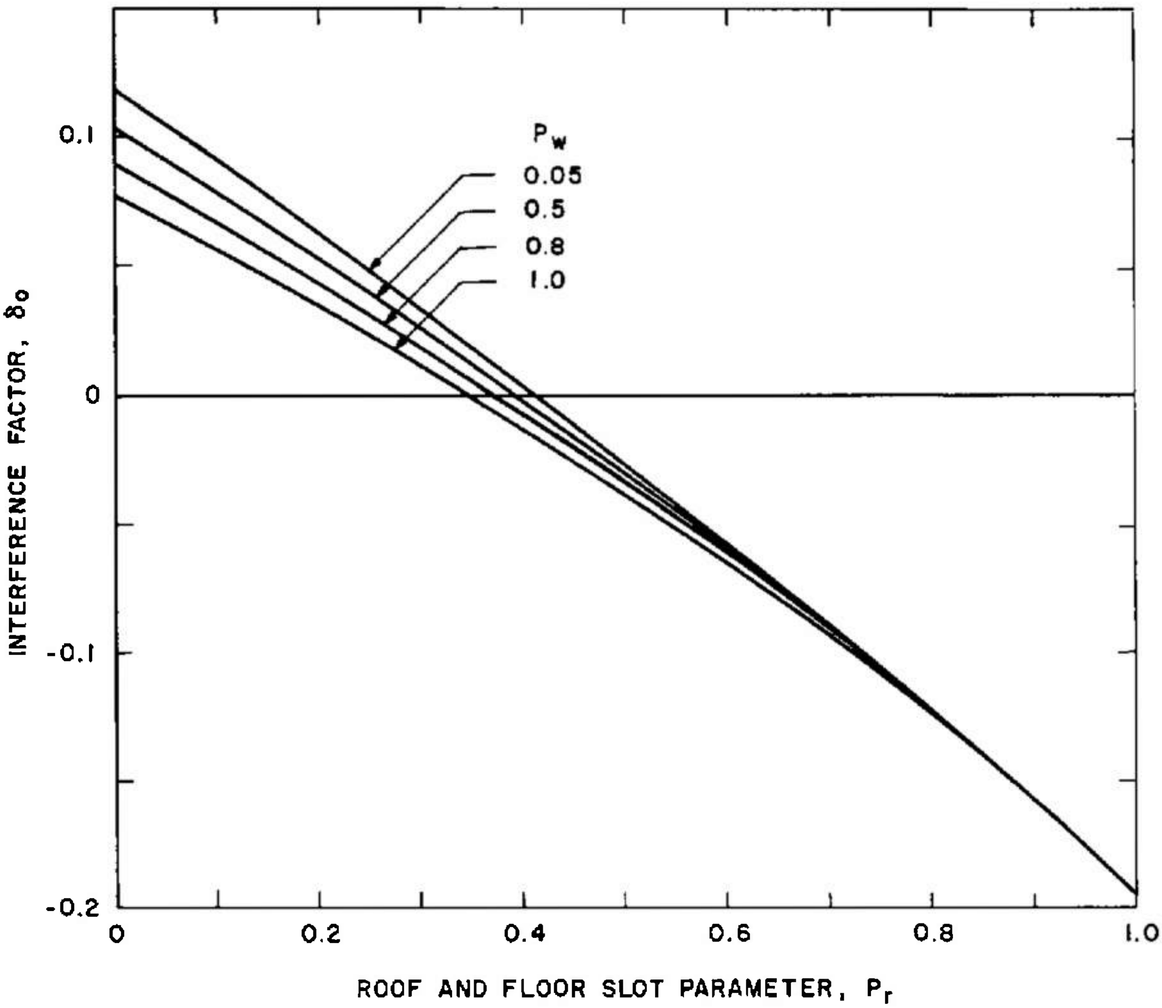


Fig. 7 Lift Interference Factor as a Function of the Roof and Floor Slot Parameter, $\lambda = 0.667$

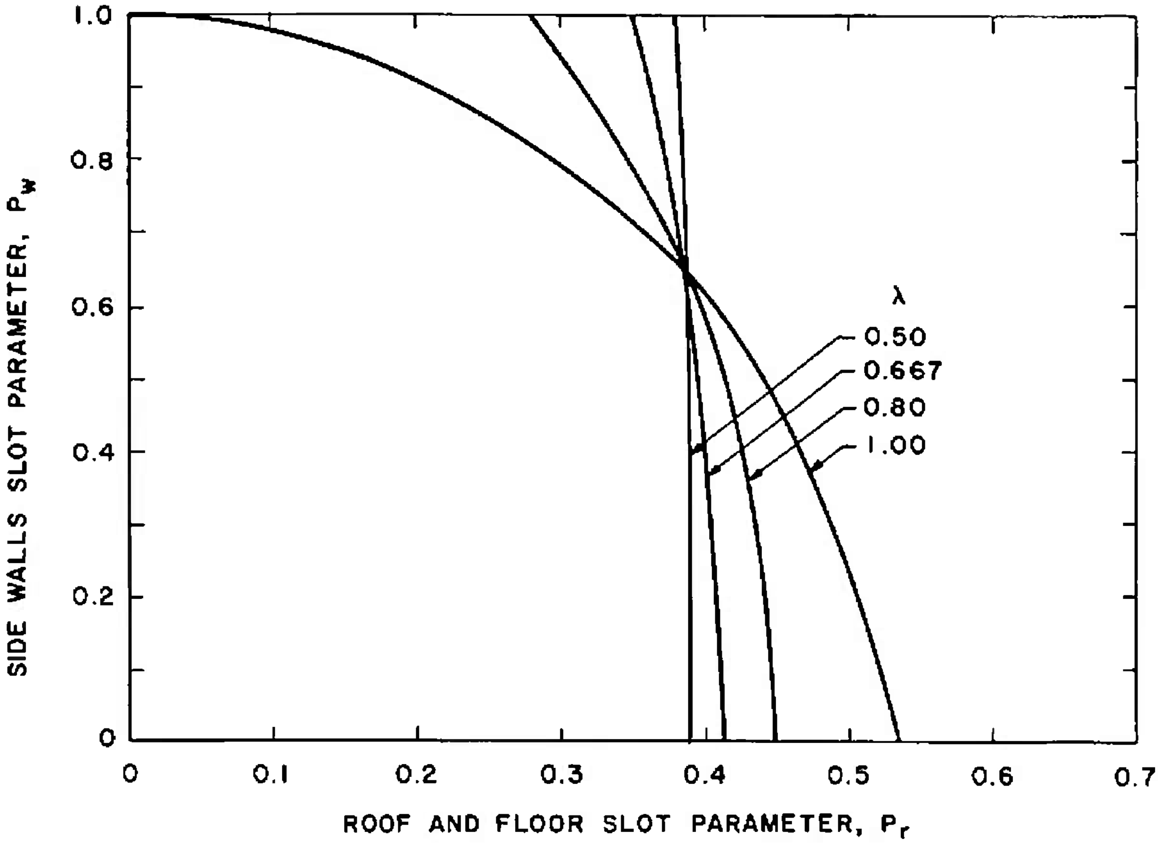


Fig. 8 Slot Parameters Required for Zero Interference

TABLE I
THE INTERFERENCE FACTOR FOR VARIOUS NUMBERS OF TERMS RETAINED IN THE SERIES SOLUTION SQUARE TUNNEL ($\lambda = 1$), SOLID SIDE WALLS, AND SLOTTED ROOF AND FLOOR

No. of Terms	No. of Matching Points	Slot Parameter P_r	Interference Factor δ_o
9	9	0.05 0.5 1.0	0.1241955 0.0899108 -0.1250009
11	11	0.05 0.5 1.0	0.1240588 0.0893592 -0.1249996
13	13	0.05 0.5 1.0	0.1240902 0.0894877 -0.1249995
15	15	0.05 0.5 1.0	0.1240832 0.0894588 -0.1249999
17	17	0.05 0.5 1.0	0.1240847 0.0894652 -0.1249999
19	19	0.05 0.5 1.0	0.1240844 0.0894639 -0.1249999

TABLE II
THE INTERFERENCE FACTOR FOR VARIOUS NUMBERS OF MATCHING POINTS, SERIES SOLUTION RETAINED TO NINTH TERMS SQUARE TUNNEL ($\lambda = 1$), SOLID SIDE WALLS, AND SLOTTED ROOF AND FLOOR

No. of Matching Points ($0 \leq \theta \leq \pi/2$)	Slot Parameter P_r	Interference Factor δ_o	$\Delta\delta_o$*
9	0.05	0.1241955	-
	0.5	0.0899108	-
	1.0	-0.1250009	-
11	0.05	0.1241300	.0000655
	0.5	0.0896548	.0002560
	1.0	-0.1249965	-.0000044
13	0.05	0.1241119	.0000181
	0.5	0.0895661	.0000887
	1.0	-0.1249984	.0000019
15	0.05	0.1241063	.0000056
	0.5	0.0895332	.0000329
	1.0	-0.1249994	.0000010
17	0.05	0.1241048	.0000015
	0.5	0.0895170	.0000162
	1.0	-0.1249999	.0000005
19	0.05	0.1241046	.0000002
	0.5	0.0895073	.0000097
	1.0	-0.1250002	.0000003

*$\Delta\delta_o$ increment of the interference factor due to increasing number of matching points.

TABLE III
THE INTERFERENCE FACTOR FOR VARIOUS HEIGHT-TO-WIDTH RATIOS

Roof and Floor Slot Parameter, $P_r = 0.5$

Solid Side Walls

Size of Coefficient Matrix*	Height-Width Ratio, λ	
	0.5	1.0
9 x 9	-.0398929	0.0899108
19 x 9	-.0410906	0.0895073
ρ_{max}/ρ_{min}	2.236	1.414

All Walls Slotted, $P_r = P_w = 0.3$

Size of Coefficient Matrix*	Height-Width Ratio, λ		
	0.5	1.0	1.6
9 x 9	0.5235	0.0430	0.1178
19 x 9	0.0320	0.0429	0.0853
29 x 9	0.0322	0.0429	0.0853
ρ_{max}/ρ_{min}	2.236	1.414	1.881

*The size of coefficient matrix Q x N represents Q matching points and N terms in the series.

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13 ABSTRACT

A method of calculating the wind tunnel interference induced by arbitrary wall configurations has been developed wherein a linearized equation of the perturbation velocity potential is used to describe the flow in the tunnel. A series solution of the interference velocity potential satisfies the differential equation exactly. The constant coefficients in the series solution are determined by the point matching technique along the boundary. Results are presented for slotted tunnel walls with homogeneous boundary conditions. Solutions presented show excellent agreement with the results obtained by other methods for a stationary as well as an oscillating wing. Finally, the method is applied to compute a set of zero interference configurations for a combination of slot openings of side walls, roof, and floor.

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