A DYNAMIC MODEL OF URBAN STRUCTURE

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A. History of T.O.M.M.

Versions of the Time-Oriented Metropolitan Model (T.O.M.M.) discussed in this paper have served as the primary spatial-location model for two, large-scale urban simulations. T.O.M.M. has served as a key element in the Pittsburgh urban simulation (City Planning Department's Community Renewal Program)[1] and is currently serving as the spatial-location device in the M.E.T.R.O. Project at the University of Michigan.[2] The model was partially validated for the Pittsburgh metropolitan area and is currently in the process of being modified and validated using data from the Lansing, Michigan metropolitan area. Like other land use, population, and commercial location models, the degree to which T.O.M.M. can be validated is a function of data availability.

Because of the incompleteness, incompatibility, and non-existence of most small-area data and the lack of time-series data, T.O.M.M. remains in the developmental stage. Although two versions have been calibrated for Pittsburgh and Lansing, the version presented here is untested and describes the latest model modifications.

B. Overview of T.O.M.M.

The model is one describing the interaction of variables in an urban system. Three classes of variables are included: an exogenous

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employment sector, an endogenous commercial employment sector, and a household or population sector.

The city is divided into a disjoint and exhaustive set of areal units. Variables are located, spatially, within particular areal units (e.g., census tracts). Superimposed on this system of variables is a transportation system providing accessibilities between various activities within the areal units. In the formal model of T.O.M.M., the number of areal units is variable and in practice would generally be determined by characteristics of the urban area involved and data availability.

Exogenous employment refers to those people employed in establishments or activities whose primary clientele, reason for being, or set of customers lies outside the urban study area. Exogenous employment variables can be influenced by factors external to the urban community or urban study area but cannot be influenced by factors internal to the community. In addition, the model hypothesizes a one-way interaction between the exogenous employment sector and the endogenous commercial employment and household sectors. That is, exogenous employment influences endogenous, commercial employment and household variables but household and endogenous employment do not influence exogenous employment. In short, the urban system is a partially decomposable subsystem of a larger (national) socio-economic system. The links between the urban subsystem (endogenous and household sectors) and the larger system are through exogenous employees and exogenous institutional and economic activities.[3] In the short run, the number of people employed at the Oldsmobile plant in Lansing, the number of State employees, and the number of Michigan State University employees cannot be said to be a function of what happens or what goes on in the Lansing metropolitan area. Consequently, T.O.M.M. considers the number, type (white-collar or bureaucratic, and blue-collar or industrial), and spatial location of exogenous employees as givens.

Endogenous commercial activity includes all people employed or engaged in activities serving the metropolitan region. Neighbohood grocery stores, regional shopping centers, secondary and elementary school teachers, librarians, doctors, lawyers, retail clerks, etc., all comprise elements of the endogenous employment sector. Provisions are made in T.O.M.M. for several categories or major classes of endogenous, commercial employees. Ideally, the sectoring or classification of endogenous employees would be based on the differential locational behavior of various classes. Employment classification has been operationalized in a very crude way in the Pittsburgh and M.E.T.R.O. projects by using the same three employment classes as did Lowry in his study of the Pittsburgh region:

> "Neighborhood facilities: Food stores; drug stores; gasoline service stations; personal services (part); elementary and secondary schools; domestic services.
>
> "Local facilities: Part of the following: Eating and drinking places; medical and health services; welfare and religious services; personal services; finance, insurance, and real estate services; automotive dealers and repair services; department, general merchandise, and variety stores; amusement and recreation facilities; public administration; miscellaneous retail and service trades not listed above.
>
> "Metropolitan facilities: Part of most groups listed under 'local facilities,' with large shares of department stores, financial services and public lodgings, business services, and public administration."[4]

These classifications reflect differential location behavior due to differences in type of consumer services and reflect (crudely) different returns to scale (establishment size) for various enterprises.

The household or population sector similarly is divided into a number of categories or classes of households. The number of categories is also a variable in the program. The classification of households would be on the basis of income, family size, race, employment, age of head of household, national origin, and the like.

Again the primary criterion for dividing households into categories should be differential locational behavior.

An overview of the dynamics of T.O.M.M. is as follows: numbers of exogenous employees, by type (bureaucratic or industrial), and by location are taken as given. Exogenous employees support (or create) a certain number of households in the region. Households locate spatially in the urban system based partly on accessibility to particular places of employment. These households in turn generate demands for services and endogenous commercial employment. The additional endogenous employees in turn support more households (which are then located), etc. The locational behavior of households is said to be a function of the composite cost surface for particular household types. The composite cost surface is in turn a function of cost surfaces for individual households based on access to exogenous employment; access to endogenous employment (as employees and as customers), and externalities associated with various areal units or sites. The locational behavior of endogenous commercial activities is based primarily on accessibility to customers where different types of households exert differential rates of attraction.

Once a change in the level or location of employees is affected, the urban system moves toward a new equilibrium between exogenous employees, households, and endogenous employees. The theoretical origin of this work lies in Ira Lowry's excellent Model of Metropolis. The primary differences between T.O.M.M. and the Lowry model is that variables in T.O.M.M. are of a much more disaggregate nature, the concept of site amenities is introduced, and zoning constraints are explicit. The other major difference concerns the way in which the concept of time is handled.

C. Theoretical Origins of T.O.M.M.

The model described by Ira Lowry in A Model of Metropolis provides the general orientation and many of the specifics for T.O.M.M. In particular, the causal hierarchy, discussed above, is the same as Lowry's.[5] (See Figure 1.)

T.O.M.M., as will be seen below, consists of a large system of equations and constraints. The partial equilibrium that characterizes a short-run "solution" to the urban location problem is really a solution to the system of equations and constraints. The iterative solution procedure used in T.O.M.M. (see Figure 1) parallels Lowry's[6] in its broad features but differs significantly in specific mechanisms.

Lowry Model

Lowry's model takes the urban area (divided into appropriate areal units) initially as empty. By assuming or locating exogenous employees on the urban space, households are generated. These households are considered as homogeneous. More realistically, Lowry generates population densities for each areal unit based on employment (exogenous and endogenous) locations. Densities are then subjected to a maximum residential density constraint (i.e., zoning restriction). The resulting densities are then converted to households. The existence of households creates an aggregate or city-wide demand for

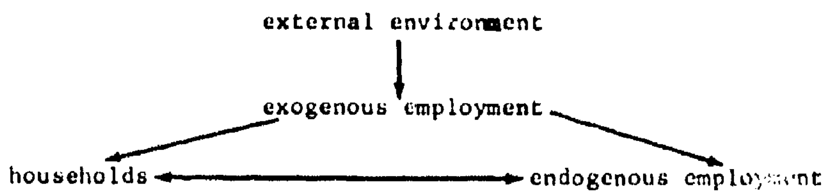

Figure 1

endogenous, commercial employees. Endogenous employees are then distributed to areal units on the basis of customer or client location, where customers are households and employees (only employees located within an areal unit were considered as potential customers for that unit).[7]

Disaggregation and Descriptive Power

T.O.M.M. is essentially a second phase of the Lowry effort. Changes and the addition of complexity were made because of a desire for a descriptive model with a policy orientation, containing more realistic (and, unfortunately, more complex) locational decision rules. These desires led to the following elaborations and structural changes in the Lowry model:

a. Introduction of the concept of short-run stability in urban land uses and location. The urban locational system cannot be considered as a self-correcting mechanism except in the very long run. Consequently, historical accidents, noise in the system, and "irrational" locational decisions tend to get perpetuated.

b. Inclusion of two types of exogenous employees, white collar or bureaucratic [EBUR(I)] and blue collar or industrial [EIND(I)], to partially capture the difference in locational (for household locations) and travel (as customers of endogenous employees) behavior for different kinds of employees. The Lowry model considers all employees (both endogenous and exogenous) as homogeneous with respect to household and endogenous commercial employment locations. T.O.M.M. also assumes that different kinds of endogenous commercial establishments (collections of employees) exert differential locational behaviors.

c. Inclusion of several household types, rather than assuming all households were homogeneous.

d. Inclusion of the effect of site amenities and economic externalities as factors in determining site valuation and hence locational behavior for the various household types.

e. Explicit consideration of the effect of a certain kind of market imperfection, density and land use (zoning) restrictions, on rents and, hence, on locational behavior.

Disaggregation of households (into categories) and exogenous employees, by reducing the level of model abstraction, increases the correspondence of the model to the "real world" and increases the descriptive power of the model. Greater disaggregation of variables should also make the model more amenable to policy studies and experiments. The impact of changes in the transportation system on specific segments of the population can be assessed. If race is used as a basis for categorizing households, it would be relatively easy to assign the locational parameters of white households to non-whites as a means to assess the impact of segregation, etc. The impact of funneling new, exogenous employment opportunities to the urban fringe rather than the central city area could be analyzed, etc.

The inclusion of site amenities and economic externalities as factors in the distribution of households within areal units allows the planner to assess the impact of public policy and investment decisions within particular areas.

Dynamic Properties of Urban Locational Behavior

The spatial distribution of activities in an urban area has many determinants, some of which are regular, habitual, and predictable and some of which are not. The unsystematic portion of urban locational behavior creates some special problems in the long-run analysis of that behavior. Unlike the "noise" or error terms in most empirical works in economics and social behavior, a "mistake" cannot be easily corrected during the next time period.[8] A shopping center built on an "uneconomic site" does not just disappear. The buildings remain and are almost always used during the next period. Deviations from the "most economic" locational decision are compensated by changes in prices, portion of household budget spent on transportation, and the like, and not, necessarily, by changes in the activity or its location. In short, effects of deviations from the "normal" behavior of firms and individuals can be cumulative and are not always self-correcting or canceling. A freeway built where there was no "demand" for one represents a change in supply, a change in transport price, and hence generates its own "demand." Similarly, shopping centers and subdivisions built in inaccessible places usually manage to generate a supply of surrounding households, streets and roads, and the like. As neighborhoods age and change character (mix of households, employment, etc.), large, Victorian houses, no longer economically feasible, are carved into apartments, not torn down. A new school tends to generate new subdivisions regardless of initial needs or correlation of plans. This two-way interaction of supplies and demands superimposed on the permanency of physical structures creates great difficulties in building models of an urban system.

The Lowry model is a static model in the sense that it attempts to regenerate the economic and population distributions of an urban area at a given point in time from an empty space, containing only exogenous employees (i.e., from the beginning of time). To the extent that imperfections in real developments are magnified rather than self-correcting, a static model is likely to provide an inadequte prediction or explanation of urban locational dynamics. Dziewonski makes a similar point --

> "Obviously some changes do take place; advantages and disadvantages of the location in the same place are not the same at all points in time (as static models assume); for both theoretical and practical purposes these changes should be incorporated into our conceptual framework and methods of analysis. But in spite of many and continuous efforts, the introduction of the time element into the theory of location is not yet satisfactorily tackled.[9]

Static Friction and Sliding Friction

The analysts or urban structure have long recognized the friction which impedes (the inverse of accessibility) travel between distant points as an important determinant of locational behavior. What T.O.M.M. attempts to do is explicitly recognize the existence of another kind of impediment to location associated with relocation. This difference in impediments is similar to the difference between static friction and sliding friction in the physical sciences. The present version of T.O.M.M. incorporates static friction by assuming that during any given time period, only a portion of existing households and land uses are free to move. Locational stability becomes an extremely important concept in both the long and short run, when one considers the decision to change location as containing considerations distinct from the decision to travel. Consider the plight of a low-income household. It may be that such a household has a more desired location that its current one. The costs of moving are not negligible however. Even if another neighborhood is more desirable on all other dimensions, moving from one social structure to another is usually costly. The cash position for a household may be such that moving costs cannot be absorbed. In the case of non-whites, artificial barriers may exist. Consider the firm.

> "...(I)n adapting its activities to the characteristics of the site and vice versa, an establishment makes an investment which is seldom recoverable on the market. The search for alternative sites is tedious, transaction costs are high, and a move itself can be expensive."[10]

Most existing location models assume that location is determined by a desire to minimize total travel costs (for all kinds of trips) and that differences in locational behavior for various segments of the population result from differences in trip purposes and differences in transportation budgets, rents at various locations being considered fixed by the market. T.O.M.M. attempts to modify the concept of locational behavior in three ways:

(1) By allowing only a certain portion of land uses and activities to re-locate during a given analysis period.

(2) By including site amenities along with travel and rent considerations in re-location decisions.

(3) By explicitly including effects of (government zoning) density and land use restrictions on rents in an area and, ultimately, on the desirability of that area.

Accessibility

Another difference between T.O.M.M. and the Lowry model concerns the concept of accessibility. Accessibility, as used here, is inversely porportional to distance. Lowry's model, as applied to the Pittsburgh region, used airline distance as the appropriate measure of separation of areal units. Airline distance was then fitted to various functions as the measure of accessibility.[11] T.O.M.M., in its current version, is flexible in its treatment of accessibility. A subroutine, ACCESS(I,J,MM), is used to calculate the measure of the degree of separation between areal units I and J, based on relationship MM. This allows use of airline distance from I to J, actual travel times, shortest-route distance, etc. It also allows the use of different functional relationships for expressing the effects of accessibility for different activities. There is no reason to believe that accessibility considerations for neighborhood grocery stores are the same as those for central business district department stores.

The cost of travel between location I and location J is a function of their spatial separation.

$$\text{Travel Cost}_{ij} = f(\text{distance}_{ij})$$

$$\text{Travel Cost}_{ij} \propto \frac{1}{\text{ACCESS(I,J,MM)}}$$

Sample functional relationships that can be considered are:

MM = 1	d_{ij}^{-1}
MM = 2	$(d_{ij})^{-k}$
MM = 3	$[\ln(d_{ij})]^{-k}$
MM = 4	$e^{-kd_{ij}}$

where d_{ij} measure of distance, travel time, etc. areal unit i to j.

k empirically determined constant.

The different functional relationships (MM's) reflect different locational behaviors for different population and endogenous, commercial activities. The measure-of-distance, d_{ij}, permits T.O.M.M. to be applied in cities with different data bases. d_{ij} also becomes a policy variable reflecting changes in the transportation system or travel costs.[12] The use of a subroutine, ACCESS(I,J,MM), also allows for calculation of the impacts of modal splits -- different models, MM, reflecting different modal splits, autos, buses, rapid transit, etc.

D. Residential Locational Behavior

Consider the location decision for a single household.[13] For the moment assume the household is merely interested in choosing a particular site, on which to locate, from among a set with similar site improvements (buildings, landscaping, etc.). Observing that few households take such decisions lightly, it seems reasonable to assume a degree of rationality on the part of this behavioral unit. In particular, we are assuming that a household, in choosing an urban property site, behaves as if it is maximizing its net benefit.[14]

In the single-household, homogeneous site-improvement case, consider two relevant costs associated with household location as found in Figure 2: site re[illegible] or land and improvements, and travel costs to and from work.[15] Note the particular location of a household's place of employment (trip destination where home is the origin) is important in calculating travel costs, whereas general accessibility to employment locations may be the important factor in determining site rent. In the short run, site rent is fixed by the market. Many spatial location models do not explicitly consider individual household travel costs,[16] but rather consider them to be a function of the household site (i.e., general accessibility) with no reference to specific trip destinations for a household(s) on that site. In the aggregate, this is equivalent to hypothesizing that a household found on a given site is equally likely to be employed in any job in the urban area. If households and employment are considered homogeneous, this is reasonable.

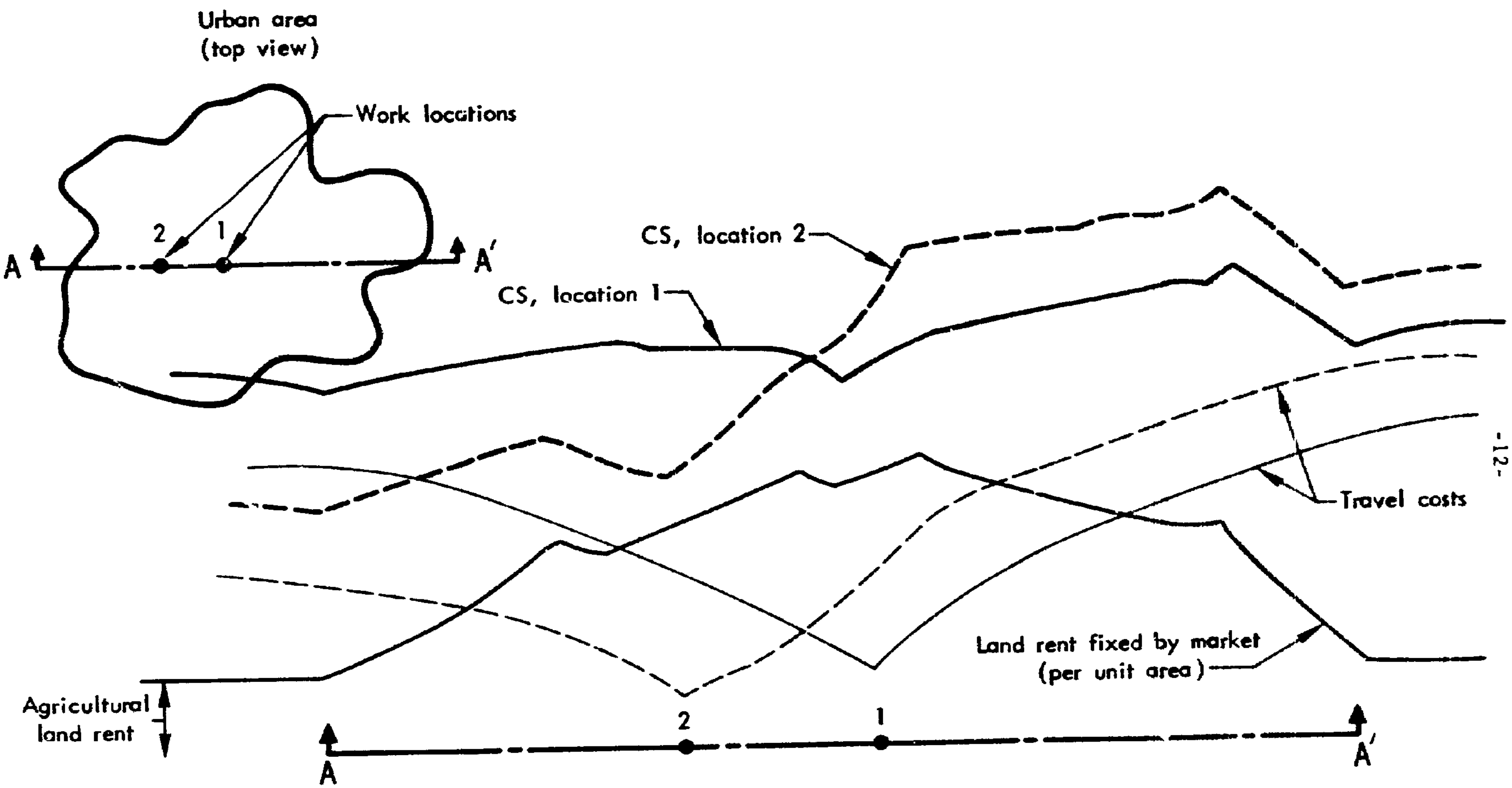

Fig. 2—Locational cost surface (CS) for cross section AA'
(land rent and work travel costs in dollars per
year at two alternative work locations)

As can be seen in Figure 2, our household, if employed at location 1 (near city center), would be fairly indifferent about its preferred location if it were attempting to minimize its net locational cost. Note that in adding the two costs, we are assuming the household is indifferent to locations having the same total cost -- i.e., it is indifferent to the particular mix of travel cost and site rent.[17] That is, the (top) solid line in Figure 2 describes the urban cost surface for all employees of establishments at location 1.

If employed at location 2, a rational household would restrict his locational search to the "A" side of town and avoid the A'-side altogether.

Consider the dynamics of this process. Assume the only changes in the urban area from time t to t+1 are the introducti n of additional employees at locations 1 and 2. Workers at location 1 (assuming they all have identical cost surfaces), other things being equal, will distribute themselves evenly (roughly) about the city (assuming all axes passing through location 1 have the same cost surfaces as AA'). On the other hand, the workers at 2 will all bid for sites on the "A" side of town. It may be that the number of households bidding on a particular set of sites exceeds the number of available sites. If that is the case, in a market with perfect information, the site rent will be bid upwards until:

(1) All households bidding on that set of sites are able to be accommodated on the set of sites, and

(2) No less expensive sites exist in the city for households employed at location 2.

Assuming travel costs are invariant with the supply of sites, the total cost surface will increase by the amount of increases in individual site rents. Everyone's rent surface increases at that set of sites (by an equal amount), regardless of employment location.

In the long run, it can be shown that both density and site rent, rather than being "fixed" by the market, vary with general accessbility. Hence, an urban area which historically (due to natural harbors, rail heads, certain kinds of economies of scale, etc.) had found most of its establishments and employees located in a particular (central) place, also tended to grow so that densities and site rents were highest in the center and diminished uniformly as one moved out from the center. In addition, the slopes of the travel-cost curves (see Figure 2) were much steeper, further increasing the degree of centrality in the urban area.

Consider the implications of forces leading to the decentralization of economic activity in an urban area: availability of several modes of <u>inter</u>city transportation -- interstate highways and airports on the outskirts of urban areas -- land area requirements of labor intensive and many consumer oriented firms for employee and customer parking, etc. This decentralization of economic activity leads to decentralization of residential units as well. The dramatic decrease in <u>intra</u>city transportation costs (slope of travel-cost curves) attributable to the automobile, as suggested above, further flattens the urban rent (cost) surface, representing another set of dispersing forces.

Independently, increased decentralization of economic activities and decreased intercity travel costs would lead to a sprawling urban area with multiple centers and islands of intensive site use. Together they accelerate it. A theory of urban spatial location must not only be able to explain or describe the forces of centralization and urban concentration, but if it hopes to be useful to policy makers, it must reflect the forces of dispersal. Because of relatively recent changes in intracity transportation costs (post-World War II and the automobile) and in intercity transportation alternatives, models whose parameter values are heavily dependent on historical values may be extremely inappropriate for describing or predicting future changes in spatial organization.[18] T.O.M.M. has as an explicit objective the explanation of changes. Hence the model structure and parameter values will be strongly influenced by the postwar experience.

We have, at this point, considered but two kinds of costs -- urban land rent, fixed by the market in the short run, and home-to-work travel costs. In completing the theoretical treatment of residential location decisions, the following items will be incorporated into the theory:

1. Land Rent Changes

The concept of changes in land rent for particular sites based on demand-supply imbalances will be introduced to give the theory some long run validity.

2. Site Amenities

Particular benefits and costs associated with particular sites will be introduced as crucial determinants of location decisions.

3. Multiple Trip Purposes

By positing different trip destinations for households, the role of trips other than work trips (shopping trips, etc.) in the location decision will be introduced.

4. Income and Budget Effects -- Household Stratification

By stratifying households, partially by income, we will be attempting to reflect differences among household types in valuations of the various costs (and their mix) associated with locational decisions. The introduction of different preference functions (valuations of costs and benefits) should also capture the effect of household budget constraints on locational decisions.[19]

5. Site Aggregation

Partly due to computer storage limitations and partly because of data availability, individual sites are aggregated into larger areal units (census tracts, for example). Property within an areal unit is assumed to be homogeneous for any particular household type (but not across household types).

Household Locational Valuation

The theory of residential location behavior as revealed at this stage of the analysis is that households (of a given type) locate in such a way as to minimize total costs (or maximize benefits) associated with their locational decision. In other words, households tend to pick the lowest point on their locational cost surface (see Figure 2). A household's costing of its locational surface at any given site in the urban area is said to consist of the sum of two costs:

(a) Rent for land and improvements at the site (set by the urban property market);

(b) Travel costs from the household's place of employment.

Define:

TLCV(L,I,J)	as the total locational cost valuation at location I for a type L household (principally) employed at location J.
ACCESS(I,J,MM)	as the relevant measure of accessibility of location I to location J (some function, MM, discussed above, of distance, travel time, etc.), where accessibility is directly related to distance.
MVAL(L,I)	as market value of land and improvements for a site at location I, for type L households. This assumes that improvements on particular sites are homogeneous over all sites found at location I, for households of type L (only).

Then,

(1) $$TLCV(L,I,J) = f_L[ACCESS(I,J,MM), MVAL(L,I)] .$$

In addition, certain characteristics peculiar to particular locations, I, are important elements of the total locational cost valuation (TLCV) for a household. Although, in some sense, the list of such site amenities is particular to the city,[20] let us identify a few that seem generally relevant:

Define:

DETR(I)	as portion of buildings deteriorating or dilapidated at location I
SCHOL(I)	as quality and/or quantity of (public?) school facilities at location I
PUBFC(I)	as quality and/or quantity of public facilities (parks, sewers, paved streets and sidewalks, etc.) at location I
$\frac{NHTT(L,I)}{NTT(I)}$	as portion of households (neighbors) of a type similar to the locating household, at location I

With these additions,

$$(2)\quad TLCV(L,I,J) = f'_L\left[\frac{1}{ACCESS(I,J,MM)}, MVAL(L,I), DETR(I), SCHOL(I), PUBFC(I), \frac{NHTT(L,I)}{NTT(I)}\right].$$

This relationship can be seen graphically in Figure 3 for a given household type, L, employed at location 1. Figure 3 demonstrates the effect of partitioning the urban area into locations or areal units (aggregates of individual sites -- set of I's).

Figure 3 gives the values of TLCV(L,I,1) over all I's on the axis AA'. Note that area (site) amenities can have either a positive or negative effect (cost).

For the moment let us retain the assumptions that the value (cost) of site and improvements is fixed by the market and that work trips are the only travel costs associated with location. For the moment, let us also assume we know the number of household units of a given type, L, employed at J, EY(L,J).[21]

It seems clear that the number of households desiring to locate at I is a function of an aggregate of individual locational cost valuations of areal unit I. Define this aggregate valuation as:

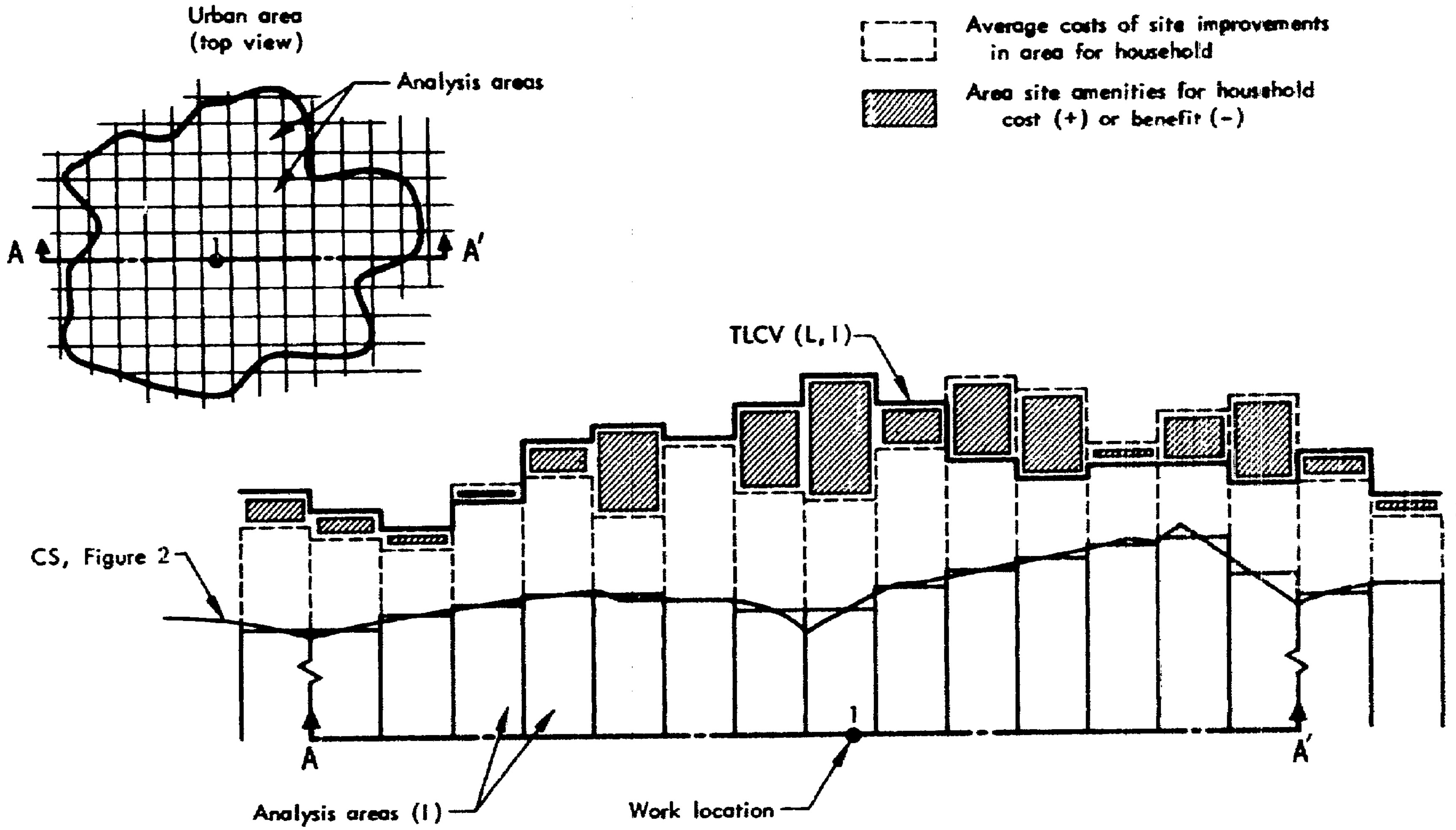

Fig. 3—Total locational cost valuation surface (TLCV) along cross section AA'
(for a given household, L, employed at location 1)

TCS(L,I) total rent surface at I, as "collectively valued" by households of type L.

$$(3) \qquad TCS(L,I) = g[\sum_{J=1}^{N} TLCV(L,I,J)]$$

$$(4) \qquad TCS(L,I) = g\left[\sum_{J=1}^{N} f_L[\frac{1}{ACCESS(I,J,MM)}, MVAL(L,I), DETR(I), SCHOL(I), PUBFC(I), \frac{NHTT(L,I)}{NTT(I)}]\right].$$

The functional, g, indicates the method of aggregating individual valuation functions. We are attempting to aggregate the cost valuation elements [MVAL(L,I), for example] in individual's preference (valuation) function to form a preference index for a bundle of goods [TLCV(L,I,J)], then attempting to use these indices to order different bundles of goods [to order all areas, I, in TLCV(L,I,J)] for a given individual, and finally aggregating over "all" individuals to form a collective preference for individual areas. To do this, we will assume a particular form for a individual's utility function. In particular, we assume TLCV(L,I,J) to be linear and homogeneous in the valuation elements:

$$(5) \qquad TLCV(L,I,J) = \alpha_1 * \frac{1}{ACCESS(I,J,MM)} + \alpha_2 * MVAL(L,I) + P(L) * DETR(I) + WS(L) * SCHOL(I) + WPF(L) * PUBFC(I) + Q(L) * [\frac{NHTT(L,I)}{NTT(I)}].$$

That the aggregation problem is crucial in the modeling of systems of social choice is not an unknown difficulty. A reasonably common way to cope with such problems is to restrict the applicability of aggregate relationships [TCS(L,I)'s] to those situations where aggregations are meaningful. This general consideration argues that when we aggregate TLCV(L,I,J)'s to get TCS(L,I)'s we must aggregate over groups of individuals who are likely to be homogeneous in their valuations and behavior. This means that valuations of locations within a particular aggregation of individuals must rise and fall together over the range of alternatives and not cancel one another. Thus, if any aggregation is to be meaningful,[72] it must be over "similar" individuals. This general consideration not only justifies disaggregating by household types, L, it requires it. And, incidently, it provides a guide for partitioning households into types, L.

If we assume that, over the range of possible locations, I, TLVC(L,I,J) is a cardinal (dis)utility measure with respect to location for a type L household employed at J, and the same measure (function) holds for all type L workers regardless of employment location, then it makes sense to aggregate TLCV(L,I,J) over all such individuals.

$$TCS'(L,I) = \sum_{J=1}^{N} [EY(L,J) * TLCV(L,I,J)]$$

where

EY(L,J) is defined as the number of type-L households employed at J.

If we modify the definition of TCS'(L,I) to mean the cost valuation or total rent surface of the "typical" type L household,

$$TCS(L,I) = \sum_{J=1}^{N} \left[\frac{EY(L,J)}{\sum_{K=1}^{N} EY(L,K)} * TLCV(L,I,J) \right]$$

or

(6)

$$TCS(L,I) = \alpha_{1L} * \sum_{J=1}^{N} \left[\frac{EY(L,J)}{\sum_{K=1}^{N} EY(L,K)} * \frac{1}{ACCESS(I,J,MM)} \right]$$

$$+ \alpha_{2L} * \sum_{J=1}^{N} \left[\frac{EY(L,J)}{\sum_{K=1}^{N} EY(L,K)} * MVAL(L,I) \right]$$

$$+ P(L) * \sum_{J=1}^{N} \left[\frac{EY(L,J)}{\sum_{K=1}^{N} EY(L,K)} * DETR(I) \right]$$

$$+ WS(L) * \sum_{J=1}^{N} \left[\frac{EY(L,J)}{\sum_{K=1}^{N} EY(L,K)} * SCHOL(I) \right]$$

$$+ WPF(L) * \sum_{J=1}^{N} \left[\frac{EY(L,J)}{\sum_{K=1}^{N} EY(L,K)} * PUBFC(I) \right]$$

$$+ Q(L) * \sum_{J=1}^{N} \left[\frac{EY(L,J)}{\sum_{K=1}^{N} EY(L,K)} * \frac{NHTT(L,I)}{NTT(I)} \right] .$$

Simplifying,

(7)

$$TCS(L,I) = \alpha_{1L} * \sum_{J=1}^{N} \left[\frac{EY(L,J)}{\sum_{K=1}^{N} EY(L,K)} * \frac{1}{ACCESS(I,J,MM)} \right]$$

$$+ \alpha_{2L} * MVAL(L,I) + P'(L) * DETR(I) + WS'(L) * SCHOL(I)$$

$$+ WPF'(L) * PUBFC(I) + Q'(L) * \frac{NHTT(L,I)}{NTT(I)} .$$

The formulation of the problem represented by (7) makes it clear how the model should be modified to include non-work trip considerations in household locational decisions. If we assume cost of access to commercial activity (shopping trips, etc.) is important and that an appropriate measure of commercial activity is number of employees, (7) becomes:

$$(8) \qquad TCS(L,I) = \alpha_1 * \sum_{J=1}^{N} \frac{EY(L,J)}{\sum_{K=1}^{N} EY(L,K)} * \frac{1}{ACCESS(I,J,MM)}$$

$$+ \alpha_{3L} * \sum_{J=1}^{N} \sum_{K=1}^{M} \frac{ERRM(K,J)}{ACCESS(I,J,MN)}$$

$$+ \alpha_{2L} * MVAL(L,I) + P'(L) * DETR(I)$$

$$+ WS'(L) * SCHOL(I) + WPF'(L) * PUBFC(L)$$

$$+ Q'(L) * \frac{NHTT(L,I)}{NTT(I)} .$$

where

ERRM(K,J) — is defined as the number of endogenous, commercial employees of type K, located in areal unit J. As described above, three types (K) of endogenous commercial employees are considered in the present version of T.O.M.M. (M = 3).

Income and budget effects have been introduced by stratifying households into types (L's), site aggregations have been dealt with (N areal units or site aggregations), as have site amenities, and non-work trips. These are all, in some sense, operational variables in that one can easily move from the variable definitions in the text to data sources, given appropriate areal unit definitions (such as census tracts). Many loose ends remain, however. We have not arrived at an appropriate definition of market value or specified how this component of the rent surface changes in response to demand-supply imbalances. While we have discussed specific exogenous [EBUR(I) and EIND(I)] and endogenous [ERRM(K,I)] employment types, we have not discussed how we move from that to "households employed at a given location," [EY(L,I)]. And, finally, the total cost surface [TCS(L,I)] may be defined in some sense, but we have yet to relate this to household location decisions and land use. To do so involves introduction of supply considerations (zoning and density considerations, etc.), land-use intensity factors (single family vs. multiple family units), economic competition for land (residential vs. commercial uses), and overall model solution procedures. The loose ends will be dealt with in the above order.

Land Rent and Changes - MVAL(L,I)

Perhaps the most intractable problem faced by T.O.M.M.[24] hinges on qualitative differences in the supply of housing. More specifically, one important component of the market value of land relates to the nature of improvements. How many rooms does a housing unit have? What is the quality of construction and landscaping? Is the unit a single-family residence or part of a multiple family structure? The problem is partially one of available data. In any event, we will "deal" with the problem of making heroic-but-reasonable assumptions.

We assume that the supply of housing within an areal unit is homogeneous within a particular household type, L, with regard to market value. By utilizing available census data fully[25] one can obtain a dollars/unit figure for households of a given type.[26] We assume that when a household estimates market value for an area, he only looks to that part of the housing supply "appropriate" for his type of household. Here again, we assume that variations about the average cost per unit for a given household type are random, with zero mean, and reflect taste differences within the household category (which are much smaller than taste differences between categories). This is equivalent to saying that the urban housing market consists of several (one for each type of household, L), non-overlapping markets designed to serve different kinds of households. A larger-than-average yard may increase the cost slightly, etc., but not significantly. If we assume the present value of a stream of expected current and future rents, discounted to reflect rates of return and risk, equals purchase price then it does not matter if rent/unit or purchase price/unit is used for MVAL(L,I).

We may now ask how the market value of a particular supply of housing units changes over time. It is clear that if fewer households are located in an area than there are housing units, prices will fall. Assuming structures do not disappear and are immobile, the drop in housing unit prices in an areal unit should be related to the decline from the previous period in households locating in that area. Conversely, if the number of households desiring to locate in a given area exceeded available housing units or led to residential densities which exceed zoning codes, the result would be an increase in market value. If an increase in housing units were necessary, and if the number of units involved could be constructed during the relevant time period (t-1 to t) which did not result in density excesses, it is assumed that any new units would be made available at the current market price.

(9) $$MVAL(L,I)_t = MVAL(L,I)_{t-1} - \beta_{1L} * \left[\frac{NH(L,I)_{t-1}}{NT(L,I)_{t-1}}\right] + \beta_{2L} * \left[\frac{\left(\sum_{L=1}^{LH} h_L[TCS(L,I)_{t-1}]\right) - NMAX(I)}{NMAX(I)}\right].$$

where

$$\begin{cases} \beta_{1L} = 0 \quad \text{if} \quad NH(L,I)_{t-1} \leq 0. \\ \beta_{2L} = 0 \quad \text{if} \quad NMAX(I) \geq \sum_{L=1}^{LH} h_L[TCS(L,I)_{t-1}] \end{cases}$$

Define:

$NT(L,I)_c$ as the number of households, type L, located in areal unit I at the end of the last analysis period (t-1).

NMAX(I) as the maximum number of households (housing units), that can be held in I, based on the total area and the residential zoning patterns in I (to be described more completely below) -- the holding capacity of I.

$h_L[TCS(L,I)]_{t-1}$ as the functional which converts the total cost surface for household type L to the number of such households desiring to locate in I during period (t-1) -- the household demand potential of I. T.O.M.M., during the solution procedure, calculates a household potential for each analysis area I and for each household type L, irrespective of locational constraints. Where constraints are violated, excess households are relocated. h_L calculates this potential. The exact form of h_L will be discussed at length below, (14).

$NH(L,I)_{t-1} \equiv NT(L,I)_t - NT(L,I)_{t-1}$

Note that there is a one-period time lag between the demand-supply imbalance (at t-1) and subsequent price adjustments (at t) in (9). This response lag represents the presence of contracts, leases, and construction lags in the urban property market (as well as a computational convenience).[27]

Households, by Employment Location

In the absence of much greater employment or occupation detail, we will assume a simple, linear relationship between household types and employee types.

$$(10) \qquad EY(L,I) = \sum_{K=1}^{M} [G'(K,L) * ERRM(K,I)] + G'(M+1,L) * EIND(I) + G'(M+2,L) * EBUR(I) .$$

where the G's are empirically estimated parameters.

(10) implies that each employee of a given type generates G(,L) households of type L. Observe that, summing over all areal units, (10) becomes:

$$(11)\qquad \sum_{I=1}^{N} EY(L,I) = \sum_{K=1}^{M} [G'(K,L) * \sum_{I=1}^{N} ERRM(K,I)]$$

$$+ G'(M+1,L) * \sum_{I=1}^{N} EIND(I)$$

$$+ G'(M+2,L) * \sum_{I=1}^{N} EBUR(I) .$$

If we assume that households with no employees and households with more than one employee are distributed spatially in exactly the same way as households with exactly one employee, then it is reasonable to use (11) to forecast or generate population as well. Because of this feature, it may be wise to use cross sectional data from a sample of similar cities to estimate the G's in (12).

$$(12)\qquad TNHH(L) \equiv \sum_{K=1}^{M} [G(K,L) * \sum_{I=1}^{N} ERRM(K,I)]$$

$$+G(M+1,L) * \sum_{I=1}^{N} EIND(I) + G(M+2,L) * \sum_{I=1}^{N} EBUR(I) .$$

where

TNHH(L) is defined as the total number of households, type L, in the urban area during the analysis period.

We are now in a position, using (12), (11), (9), and (8), to consolidate our definition of TCS(L,I). We shall do this in the next section.

Residential Location Decisions

The total cost surface, by household type [TCS(L,I)] is now well specified. Referring back to our previous discussion, only a portion of all existing households are considered mobile from the beginning of one period to the beginning of the next. Define NHS(L,I) as the number of stable or immobile households in an area. T.O.M.M. essentially consists of an iterative solution procedure which begins an analysis period with a city containing exogenous employees and stable (immobile) households. The stable households are the first trial total for the household distribution in the city, during the analysis period:

$$NHTT(K,I) = NHS(L,I) .$$

Endogenous commercial employment is calculated (see discussion below) based on the city's population characteristics [NHTT(L,I)'s]. Relationship (12) is then used to calculate a population total.

$$(12a) \quad TNHH(L) = \sum_{I=1}^{N} NHTT(L,I) + \sum_{I=1}^{N} NH(L,I) .$$

The modifications in trial totals, NH(L,I)'s, are then calculated using the total cost surface values [TCS(L,I)] to achieve a new set of trial totals, such that (12a) holds.

$$NHTT'(L,I) = NHTT(L,I) + NH(L,I) .$$

This procedure is repeated until a "solution" (to be defined below) is reached. The calculation of the locating and relocating households, NH(L,I), is what will now be described.

The precise definition of $h_L[TCS(1,I)_t]$ as referred to in (9) and (14) should be presented here. The household trial total calculated for an area in the previous analysis period, $NHTT'(L,I)_{t-1}$, before being subjected to density and immobile-household constraints, is used as the potential population for the area. The magnitude of subsequent price increases in that area is, in (9) and (14), related to the difference between these potentials and what the area can actually hold:

$$\sum_{L=1}^{LH} h_L[TCS(L,I)_{t-1}] - NMAX(I) = \sum_{L=1}^{LH} NHTT'(L,I)_{t-1} - NMAX(I).$$

At any given point in the solution procedure, a number of households (+ or -) must be located in areal units. TCS(L,I), representing the average total cost of housing in area I for households of a given type, forms the basis for the locational decisions. In particular, one expects the following:

$$NH(L,I) \qquad \frac{1}{TCS(L,I)} \qquad \text{or}$$

(13) $$NH(L,I) = K'_L * [CONST(L) - \gamma * TCS(L,I)]$$

where

CONST(L) can be interpreted as an area-wide average "total cost" of housing for type L households, and where K'_L is defined such that

$$\sum_{I=1}^{N} NH(L,I) = TNHH(L) - \sum_{I=1}^{N} NHTT(L,I) .$$

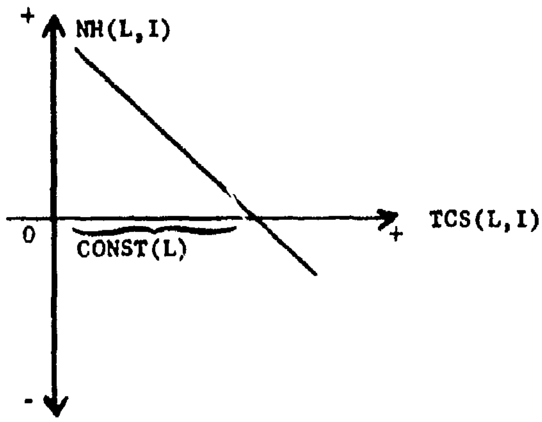

Figure 4

Consolidating our notions of TCS(L,I), using (8), (9), (11), (12), and (13), the following form seems appropriate for parameter estimation using linear regression techniques:[28]

$$(14)\quad NH(L,I) = WBUR(L) * \sum_{J=1}^{N} \frac{EBUR(J)}{TNHH(L)} * \frac{1}{ACCESS(I,J,MM)}$$

$$+ WIND(L) * \sum_{J=1}^{N} \frac{EIND(J)}{TNHH(L)} * \frac{1}{ACCESS(I,J,MM)}$$

$$+ \sum_{K=1}^{M} WCOM(K,I) * \sum_{J=1}^{N} \frac{ERRM(K,J)}{TNHH(L)} * \frac{1}{ACCESS(I,J,NN)}$$

$$V(L) * MVAL(L,I) + V1(L) * \left[\frac{NT(L,I)_{t-2} - NT(L,I)_{t-1}}{NT(L,I)_{t-1}}\right]$$

$$+ V2(L) * \left[\frac{\left[\sum_{L=1}^{LH} h_L \left(TCS(L,I)_{t-1}\right)\right] - NMAX(I)}{NMAX(I)}\right]$$

$$+ P(L) * DETR(I) + WS(L) * SCHOL(I) + WPF(L) * PUBFC(I)$$

$$+ Q(L) * \left[\frac{NT(L,I)}{\sum_{L=1}^{LH} NT(L,I)}\right] + CONST(L).$$

where: **Predicted values**

$WBUA(L) = \alpha_1 * G(M+2,L) * Y$	≤ 0
$WIND(L) = \alpha_1 * G(M+1,L) * Y$	≤ 0
$WCOM(K,L) = \alpha_{3L} * \alpha_1 * G(K,L) * Y$	≤ 0
$V(L) = \alpha_{2L} * Y$	≤ 0
$V1(L) = \beta_{1L} * Y$	≤ 0
$= 0, \text{ if } NT(L,I)_{t-1} \geq NT(L,I)_{t-2}$	
$V2(L) = \beta_{2L} * Y$	≥ 0
$= 0, \text{ if } NMAX \geq \sum_{L=1}^{LH} h_L \left[TCS(L,I)_{t-1}\right]$	
$P(L) = P'(L) * Y$	≤ 0
$WS(L) = WS'(L) * Y$	≥ 0 ?
$WPF(L) = WPF'(L) * Y$	≥ 0 ?
$Q(L) = Q'(L) * Y$	?
$CONST(L)$	> 0

In estimating parameters in (14), NH(L,I) refers to the <u>change</u> in households of type L (+ or -) in an area from one time period to the next where the length of time is identical to the analysis period to be used in applications of T.O.M.M.

On first glance, it would appear that substantial problems may exist with relationship (14). For instance, in the long run, market value of land and improvements adjusts to changes in travel costs. Indeed, it is precisely this adjustment process we are attempting to describe. If the location costs associated with a particular area get too far "out of line" [CONST(L) - TCS(L,I)], it is a price adjustment process which prevents everyone from moving in or from moving out of an area (9). In the long run (with all such adjustments made), it is clear that the average locational costs associated with each area would, through price adjustments caused by over-crowding or vacancies (9), reach a constant level --

TCS(L,I) = CONST(L), for all I

-- and there would be no motivation for anyone to move -- NH(L,I) = 0, for all I. On the other hand, we have argued that substantial imperfections exist in urban property markets. Not all households are free to move, in the short run. Contracts and leases, construction lags, imperfect information, and the like make price adjustments less than instantaneous. Changes in the exogenous employment sector bring about changes in the costs affecting location decisions, keeping the market in more-or-less constant disequilibrium.

Nevertheless, the long run tendency of TCS(L,I) to seek a constant value, over all I, undoubtedly will cause collinearity problems. The severity of these problems relate to the speed of the market adjustment process and, hence, the time span encompassed by the change variables [NH(L,I)] used in estimating the coefficients of (14). Unfortunately the problem may ultimately rest more with the collinear nature of the real world, and not so much with our model specification.

Residential Location Constraints

By allowing only a portion of residential activities (households) to (re)locate in a given analysis period, T.O.M.M. provides a set of minimum constraints on residential land use -- residential activity in a given area at the end of an analysis period consists at least of those households considered immobile during the period [NHS(L,I)].

(15) $NH(L,I) + NHTT(L,I) \geq NHS(L,I)$.

On the other hand, it is obvious other constraints, setting upper limits on the changes from one period to the next, exist. Some are market constraints. It takes time to plan and construct housing units, especially in large quantities. Construction lags limit growth in any one area. Important non-market constraints exist as well. Primarily these consist of municipal zoning regulations limiting the type and intensity of land use in an area. While not affecting existing uses (non-conforming land uses are provided for in most zoning ordinances through granted variances and the non-retroactive nature of the law), zoning laws constrain all changes in locational activity in a district.

A given areal unit I is partitioned into a disjoint and exhaustive set of zoning districts.

AT(I)	total area included in I.
AZR1(I)	area zoned for single family (detached) residential use
AZR2(I)	area zoned for duplex and row house residential use
AZR3(I)	area zoned for multiple family, hi-rise apartments, and special residences
AZC(I)	area zoned for commercial (endogenous) activity
AZSI(I)	area zoned for institutional, special, and industrial (for exogenous activity)
AZG(I)	area zoned for public and governmental uses (streets, parks, etc.)
AU(I)	unusable land in areal unit I (steep hill-sides, bodies of water, swamps, etc.)

$$AT(I) \equiv AU(I) + AZR1(I) + AZR2(I) + AZR3(I) + AZC(I) + AZSI(I) + AZG(I) .$$

It is possible to work directly from a municipal zoning map to calculate values for the above areas. The principal difficulty lies in subtracting unusable land from each of the land use categories.

Zoning ordinances are such that one moves from a highly restricted category (AZR1) towards increasing permissiveness. For example, area zoned commercial, AZC(I), permits all types of residential usage as well. Multiple-family-dwelling unit districts, AZR3, also permit single-family detached units. Single-family districts, AZR1, permit <u>only</u> single-family residences however.

In order to appropriately handle the progressively-restrictive features of zoning, it is necessary to assume a pecking order in land use based on the relative economic strengths of various activity categories. First, activities in AZSI and AZC are assumed given and are exogenous to the model, as is the unusable land area, AU. These activities all represent potential policy variables however and may be manipulated outside of the model. Commercial activities are assumed to be able to bid away land from any and all residential uses if necessary. Zoning laws are such that they put maximums on use intensity (building height limitations, minimum lot sizes, etc.). It is assumed commercial activity can go on in a district unhindered by residential uses and subject only to the commercial zoning constraint. The exact nature of this constraint will be discussed below.

While it would be ideal if there were a one-to-one correspondence between household types, L, and the type of housing unit desired, the necessary additional stratification of households would place undue strains on the data, the model, and computer storage capacity. Instead, we shall merely use the zoning constraints to estimate the "residential holding capacity" for a given area -- the number of households the area can accommodate under the zoning laws.

Most zoning ordinances contain density restrictions for the various zoning categories -- minimum lot sizes for single family (detached) units in AZR1 districts and maximum building heights, distance from lot lines, etc. for AZR3 districts. From this we should be able to calculate for each type of district, the "maximum" allowable density in households per unit area. The "residential holding capacity" (in households) of an area is then,

(16) NMAX(I) = AZR1(I) * ACR1 + AZR2(I) * ZCR2 + AZR3(I) * ZCR3

where

ZCR<u>K</u> is the maximum density, in housing units per unit area, for type <u>K</u> housing units.

Residential Holding Capacity Constraint

From (13) we have calculated the number of households, by type, desiring to (re)locate in I. The "residential holding capacity constraint" is then applied to see if the area, I, has sufficient room.

$$(17) \qquad \sum_{L=1}^{LH} [NH(L,I) + NHTT(L,I)] \leq NMAX(I) \ .$$

If (17) is satisfied, the households are located in area I:

$$(18) \qquad NHTT'(L,I) = NH(L,I) + NHTT(L,I) \ .$$

If (17) is exceeded, excess households are drawn proportionately from the (re)locating households.

$$(19) \qquad NH'(L,I) = \alpha_I * NH(L,I)$$

where

$$\alpha_I = \frac{NMAX(I) - \sum_{L=1}^{LH} [NHTT(L,I)]}{\sum_{L=1}^{LH} NH(L,I)} \ .$$

The excess households, [NH(L,I) - NH'(L,I)], are first located in areas failing to satisfy the minimum residential activity constraint (15). After all areas are brought up to the minimum, remaining excess households are located, only in areas satisfying (17), on the basis of the relative attractiveness of areas calculated in (13). Thus, new trial totals are achieved for all household types in all areas, using (18). Note that households have been allocated in such a way that:

$$(20) \qquad \sum_{I=1}^{N} NHTT'(L,I) = \sum_{I=1}^{N} [NH(L,I) + NHTT(L,I)] = \Sigma NHH(L)$$

as calculated from (12).

E. Location of Endogenous Employment

Demand for Endogenous Activities

T.O.M.M. assumes that households generate certain market demands for goods and services in an urban area, which, in turn, generate demands for employees. T.O.M.M. further assumes that changes in "demand for employees" are met by increases in "output" (supply of employees) through in-migration or out-migration, [(12) and (21)], rather than through changes in price (wages) or unemployment levels. Further, it is assumed that different distributions of households generate different demands for goods, services and employees:

$$ERM(K) = \sum_{L=1}^{LH} A(K,L) * TNHH(L) \tag{21}$$

where

ERM(K) is defined as the total endogenous employment of type K in a region, and A(K,L) are empirically estimated parameters reflecting differential demands of household types.

Cross-section data from comparable cities can easily be used to estimate the A(K,L)'s in (21).

Distribution and Location of Endogenous Employment

Much like households, certain endogenous activity is considered immobile during any analysis period. Operationally, this means some portions of last period's employment totals must remain stable:

$$ER(K,I) + ERRM(K,I) \geq STABC\,[EM(K,I)] \tag{22}$$

where

ER(K,I) is defined as additional (+ or -) employees of type K to be located in areal unit I.

EM(K,I) is the number of employees of type K located in I at the end of the last analysis period.

T.O.M.M. hypothesizes that the sole objective in employee location (establishment formation, location, and growth) is to minimize travel costs of customers (maximizing accessibility) subject to minimum efficient establishment size constraints. Two classes of customers are assumed: those traveling from their household locations and exogenous employees traveling from their employment locations.[29] In addition, different household types value travel costs differently and have differential attractions for establishments. Reasoning, similar to that associated with the travel cost portion of the household cost surface, leads to the following customer travel cost function for establishment location decisions:

$$(23) \qquad Cost(K,I) = \sum_{J=1}^{N} \left[\frac{\sum_{L=1}^{LH} [C'(K,L) * NHTT(L,J)]}{ACCESS(I,J,MM)} + \frac{DI'(K) * EIND(J)}{ACCESS(I,J,MN)} + \frac{DB'(K) * EBUR(J)}{ACCESS(I,J,NN)} \right].$$

where

Cost(K,I) represents the total travel costs associated with customers for establishments with type-K employees, if the establishment were located at I.

It seems clear that employees (representing new establishments or growth in existing establishments) will tend to (re)locate such that

$$[ER(K,I) + ERRM(K,I)] \propto \frac{1}{Cost(K,I)} \qquad \text{or}$$

$$ER(K,I) + ERRM(K,I) = - \delta * Cost(K,I) + CNST(K)$$

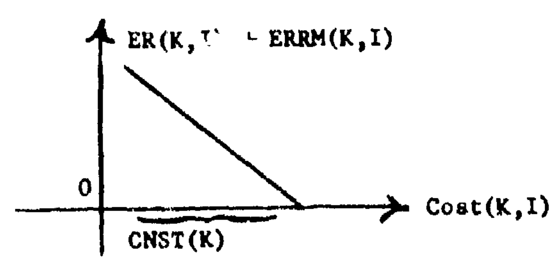

Figure 5

(24)
$$ER(K,I) + ERRM(K,I) = \sum_{J=1}^{N} \frac{\sum_{L=1}^{LH} [C(K,L) * NHTT(K,J)]}{ACCESS(I,J,MM)}$$
$$+ \frac{DI(K) * EIND(J)}{ACCESS(I,J,MN)} + \frac{DB(K) * EBUR(J)}{ACCESS(I,J,NN)} + CNST(K) .$$

where CNST(K) represents a maximum feasible separation of type K enterprises from its customers.

(24) is in a form allowing direct parameter estimation using linear regression techniques, if the constant of proportionality is assumed to be 1.0. The parameters in (24) -- C(K,L)'s, DI(L), and DB(L) -- embody the notion that different types of commercial activities look to different customers. Hence, the stratification of endogenous employment into the M categories ought to reflect this tendency.

Considerations somewhat different than those affecting household locations exist however. It is at least feasible for a one-member household to survive. Such latitude in size at the small end of the spectrum is not available to (endogenous) commercial enterprises however. Regardless of how socially desirable one-member (or zero-member) used car establishments might be, evidence suggests that economies of scale make survival of such firms highly unlikely. Using Lowry's notion of the minimum efficient establishment size,[30] to reflect such economies of scale considerations, endogenous employment of a type K is located in an areal unit only if total employment exceeds the minimum efficient establishment size for employment of that type:

(25)
$$[ER(K,I) + ERRM(K,I)] \geq Z(K)$$

An additional restriction exists as well. No endogenous commercial employees can be located in an area with no land area zoned commercial (where AZC(I) = 0). No maximum number of employees for an area is assumed.

Computationally T.O.M.M. uses (24) to generate a potential for net, additional employees of type K in area I, ER'(K,I), where the constant of proportionality in (24) is such that (21) holds -- the potentials adds up to the city-wide employment totals. The proposed total of employees in an area is then subjected to the constraints and conditions:

i. $[ER'(K,I) + ERRM(K,I)] \geq STABC(EM(K,I))$.

ii. $[ER'(K,I) + ERRM(K,I)] \geq Z(K)$.

iii. $[ER'(K,I) + ERRM(K,I)] = 0 \quad \text{if} \quad AZC(I) = 0$.

If constraints i. and ii. are satisfied, the proposed employment addition (+ or -) is accepted:

$$ER(K,I) = ER'(K,I).$$

If i. is not satisfied,

$$ER(K,I) \equiv STABC(EM(K,I)) - ERRM(K,I),$$

initially and if this revised employment total fails ii., total employment is set to zero in that areal unit. All areal units in which i., ii., or iii. were violated are removed from further consideration and any remaining employees [as defined by (21)] are located in those areal units satisfying i. and ii. (and having $AZC(I) \geq 0$), proportionally, based on potentials calculated in (24).

An explicit description of how the theory and computations embodied in the household and endogenous employment location discussions are integrated in T.O.M.M. follows.

F. Detailed Description of T.O.M.M.

Figure 6 gives an overview of the relationships and solution procedure imbedded in T.O.M.M. What follows in this section is a detailed description of the internal workings of each "box" in the flowchart. Numbers in the text (15.) reference items in the Figure 6 flowchart (15. Apply minimum constraints....).

(Insert Figure 6 here)

Information Inputs and Data Files

1. Empirically-deterimed parameter values and constants needed to calculate population and employment totals and area population and endogenous employment potentials are read in at this point and are available for use later in the program (13., 14., 19., 21., 22.). In addition, tolerance limits for an equilibrium solution are read in at this time (31.). The number of household types, LH, number of endogenous commercial categories, M, the number of areal units, N, and the number (years) of the initial analysis period, (TO, and the final analysis period, T, are also read in at the beginning of the study period. The entire study period consists of the period from TO to T, where each analysis period represents an increment of 1.

2. The patterns of zoning representing restrictions on land uses, by areal unit (I), are either read in at this point, if it is the beginning of a study period, or are updated (signifying a change in the zoning laws) if it is the beginning of a new analysis period within the study period.

AT(I)	total land area in areal unit I
AU(I)	unusable land (swamps, slopes, bodies of water, streets, etc.) in areal unit I
AZSI(I)	area zoned for and/or occupied by exogenous uses in area unit I
AZC(I)	area zoned commercial and light industrial (endogenous commercial activities), areal unit I

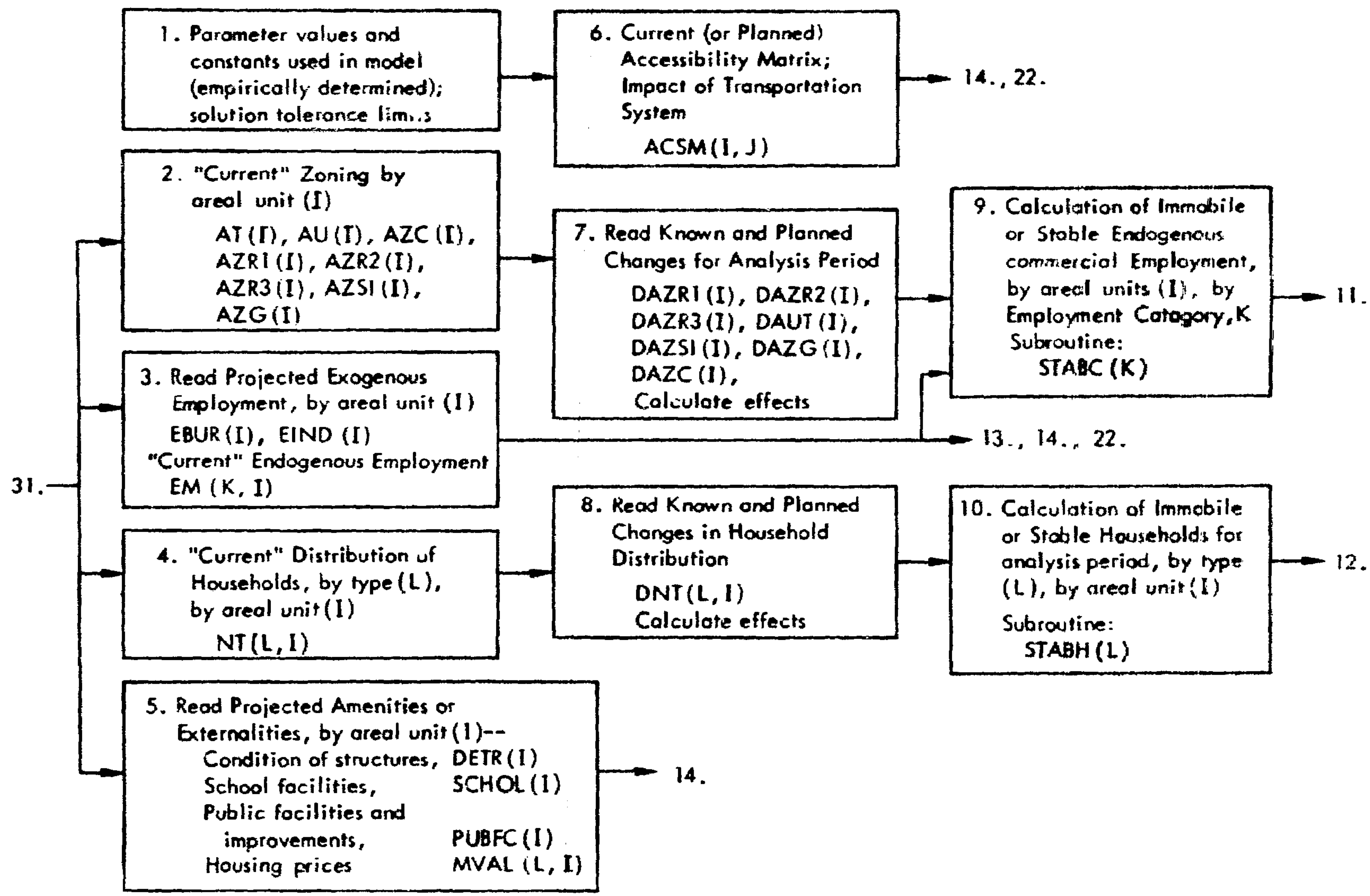

Fig. 6—T. O. M. M. solution procedure

9. → **11.** Trial Total for Endogenous Employment, by areal unit (I), by type (K), using STABC (K):

ERRM (K, I)

3. → 30. → **13.** Calculation of Total Households supported by city-wide employment totals, by type (L):

TNHH (L)

3., 6., 5., 18. →

10. → **12.** Trial Total for Households by type (L), by area (I) using STABH (L):

NHTT (L, I) → 20.

14. Allocation of mobile households, by type, to analysis areas, I, based on current characteristics and ammenities of areal units (I) and access to different employment and economic opportunities:

$$NH(L, I) = WBUR(L) * \sum_{J=1}^{N} \frac{EBUR(J)}{TNHH(L) * ACCESS(I, J, MM)}$$

$$+ WIND(L) * \sum_{J=1}^{N} \frac{EIND(J)}{TNHH(L) * ACCESS(I, J, MN)}$$

$$+ \sum_{K=1}^{M} \left[WCOM(K, L) * \sum_{J=1}^{N} \frac{ERRM(K, J)}{TNHH(L) * ACCESS(I, J, NN)} \right]$$

$$+ V(L) * MVAL(L, I) - V1(L) * \left[\frac{NH(L, I)_{t-1}}{NT(L, I)_{t-1}} \right]$$

$$+ V2(L) * \left[\frac{\sum_{L=1}^{LH} \left[h_L(TRS(L, I)_{t-1}) \right] - NMAX(I)}{NMAX(I)} \right]$$

$$+ P(L) * DETR(I) + WS(L) * SCHOL(I) + WPF(L) * PuBFC(I)$$

$$+ Q(L) * \left[\frac{NT(L, I)}{\sum_{L=1}^{LH} NT(L, I)} \right] + CONST(L)$$

Note: V1 (L) = V2 (L) = 0 if first iteration in analysis period.

→ 15.

18. (yes) reallocate deviant areal units → 14.

Fig. 6—(cont)

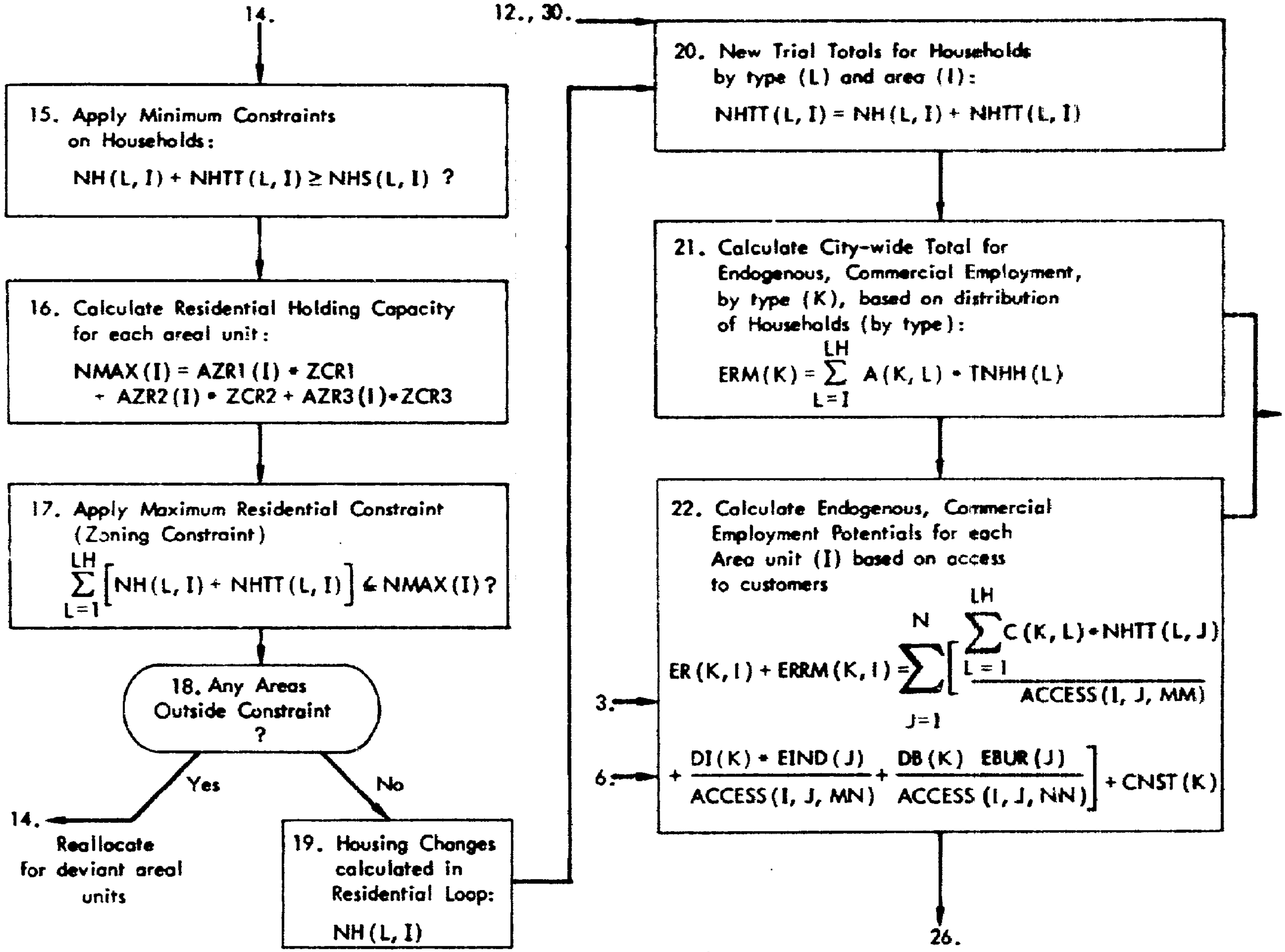
14.
15. Apply Minimum Constraints on Households:
NH(L, I) + NHTT(L, I) ≥ NHS(L, I) ?
16. Calculate Residential Holding Capacity for each areal unit:
NMAX(I) = AZR1(I) • ZCR1 + AZR2(I) • ZCR2 + AZR3(I) • ZCR3
17. Apply Maximum Residential Constraint (Zoning Constraint)
$\sum_{L=1}^{LH} [NH(L, I) + NHTT(L, I)] \leq NMAX(I)$?
18. Any Areas Outside Constraint ?
Yes
No
14.
Reallocate for deviant areal units
19. Housing Changes calculated in Residential Loop:
NH(L, I)
12., 30.
20. New Trial Totals for Households by type (L) and area (I):
NHTT(L, I) = NH(L, I) + NHTT(L, I)
21. Calculate City-wide Total for Endogenous, Commercial Employment, by type (K), based on distribution of Households (by type):
$ERM(K) = \sum_{L=1}^{LH} A(K, L) \cdot TNHH(L)$
22. Calculate Endogenous, Commercial Employment Potentials for each Area unit (I) based on access to customers
$ER(K, I) + ERRM(K, I) = \sum_{J=1}^{N} \left[\frac{\sum_{L=1}^{LH} C(K, L) \cdot NHTT(L, J)}{ACCESS(I, J, MM)} + \frac{DI(K) \cdot EIND(J)}{ACCESS(I, J, MN)} + \frac{DB(K) \quad EBUR(J)}{ACCESS(I, J, NN)} \right] + CNST(K)$
3.
6.
26.

Fig. 6—(cont)

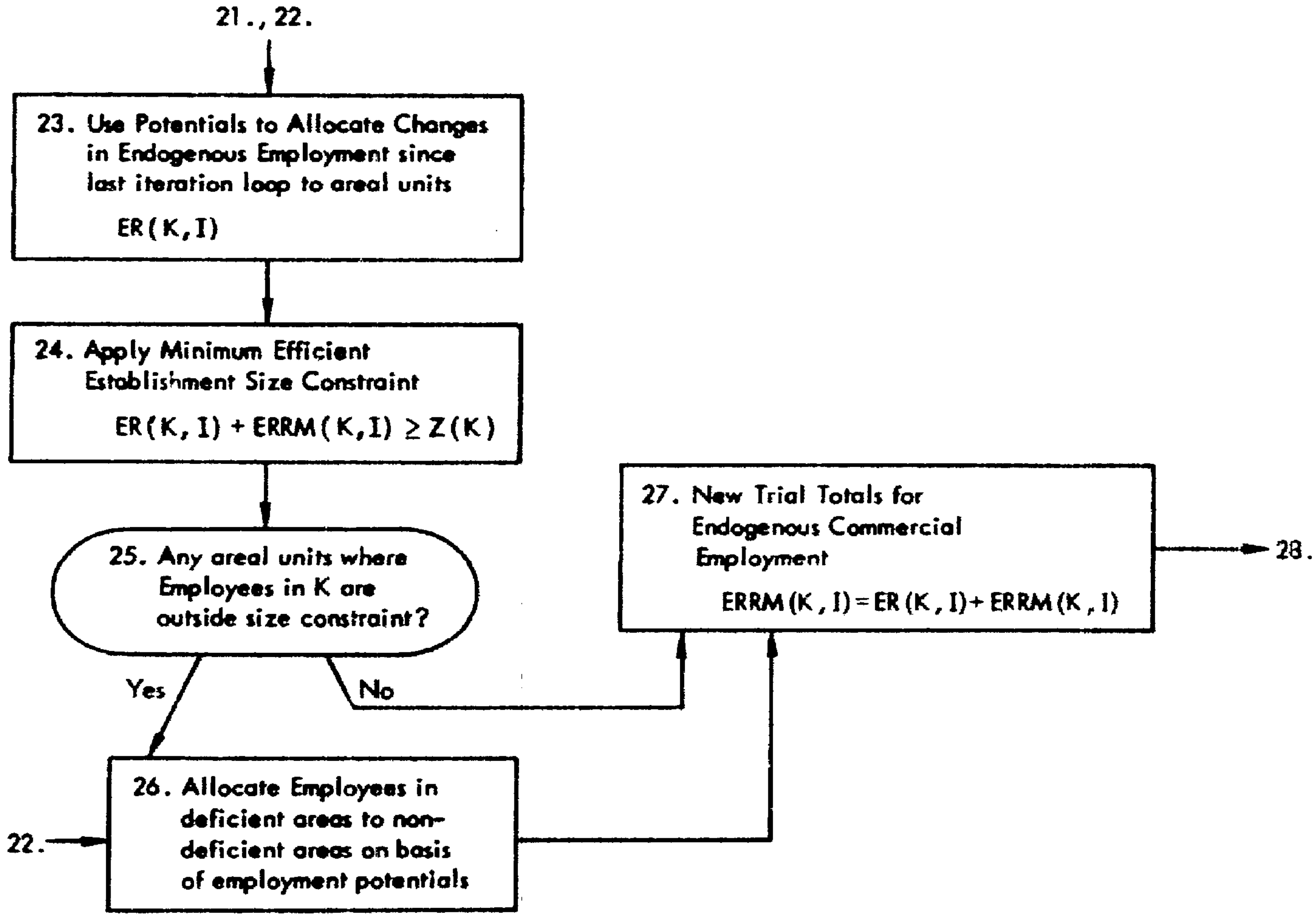
21., 22.
23. Use Potentials to Allocate Changes in Endogenous Employment since last iteration loop to areal units
ER(K,I)
24. Apply Minimum Efficient Establishment Size Constraint
ER(K,I) + ERRM(K,I) ≥ Z(K)
25. Any areal units where Employees in K are outside size constraint?
Yes
No
26. Allocate Employees in deficient areas to non-deficient areas on basis of employment potentials
22.
27. New Trial Totals for Endogenous Commercial Employment
ERRM(K,I) = ER(K,I) + ERRM(K,I)
28.

Fig. 6—(cont)

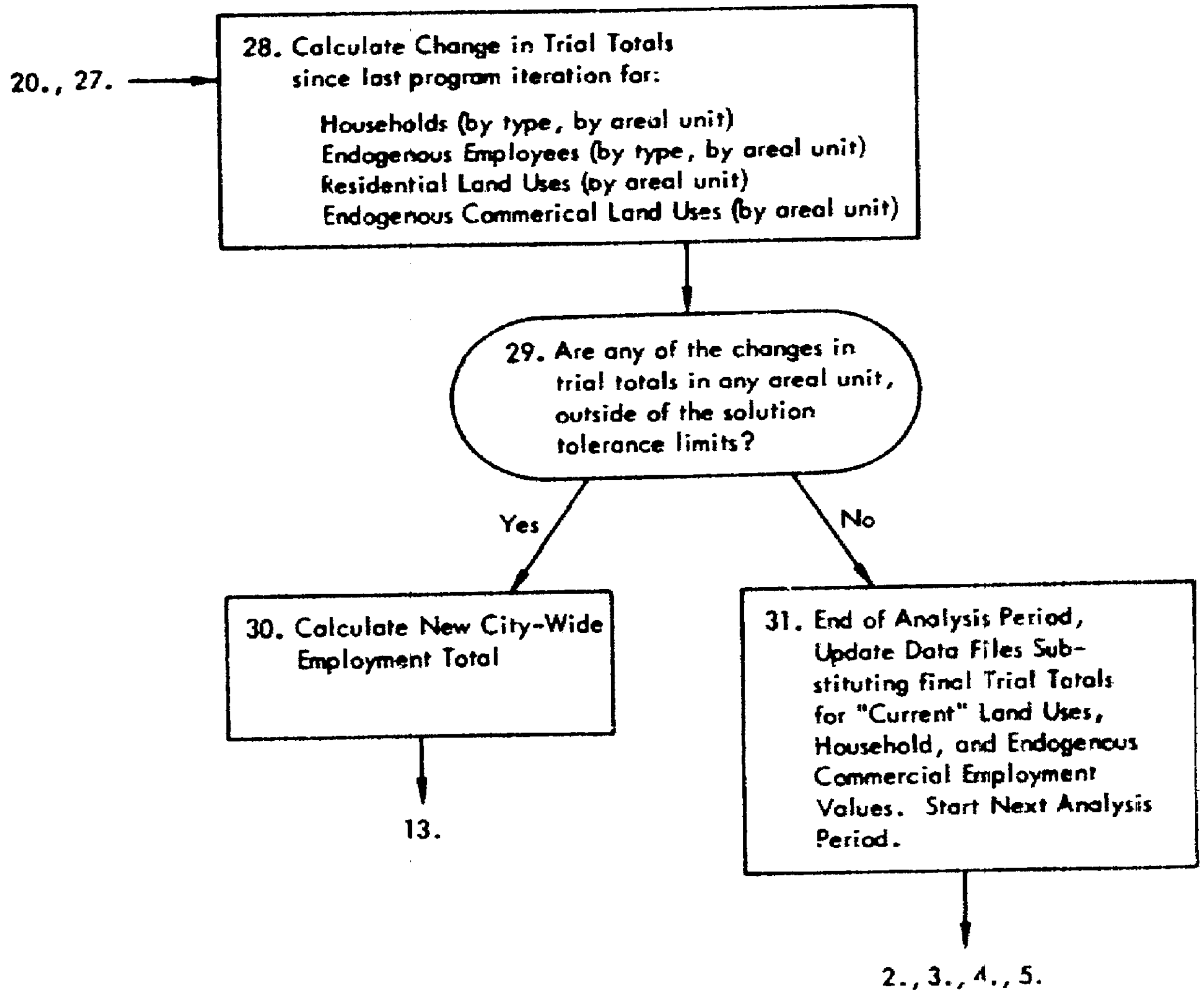
20., 27.
28. Calculate Change in Trial Totals since last program iteration for:
Households (by type, by areal unit)
Endogenous Employees (by type, by areal unit)
Residential Land Uses (by areal unit)
Endogenous Commerical Land Uses (by areal unit)
29. Are any of the changes in trial totals in any areal unit, outside of the solution tolerance limits?
Yes
No
30. Calculate New City-Wide Employment Total
13.
31. End of Analysis Period, Update Data Files Substituting final Trial Totals for "Current" Land Uses, Household, and Endogenous Commercial Employment Values. Start Next Analysis Period.
2., 3., 4., 5.

Fig. 6—(cont)

AZR1(I) area zoned for single-family dwelling units, areal unit I

AZR2(I) area zoned for two-family and row dwelling units, areal unit I

AZR3(I) area zoned for multiple-family units and special residences, areal unit I

AZG(I) area zoned for institutional, governmental, and public uses

3. Read projected exogenous employment totals at the start of each analysis period in the total study period, for area unit, I.

EBUR(I) total bureaucratic or white-collar exogenous employment

EIND(I) total industrial or blue-collar exogenous employment

The numbers of current endogenous, commercial employees, by type (K), in each areal unit (I) are either read in at this point, if it is the beginning of a study period, or are the final trial totals from the previous analysis period (33.), EM(K,I)

4. The numbers of current households, by type (L), in each areal unit (I) are either read in at this point, if it is the beginning of a study period, or are calculated on the basis of firal trial totals (20., 33.) from the previous analysis period.

NT(L,I) households, type L, in areal unit I.

Initial Conditions at Start of Analysis Period

5. Read projected (and planned) amenities or externalities, by areal unit (I), at the beginning of each analysis period.

DETR(I) index of deterioration and dilapidation of structures

SCHOL(I) index of educational facilities

PUBFC(I) index of investment in public facilities and improvements (parks, street lights, sewers, utilities, etc.)

6. Read in current (or planned) accessibility matrix for urban area. Matrix entries can refer to travel time, I to J, shortest-route distance, airline distance, etc., depending on data availability.

   ACSM(I,J) time-distance, areal unit I to J

7. Read in known and planned changes (+ or -) in permissible land uses (zoning laws) during analysis period, for each areal unit (I).

   DAZSI(I) change in basic or exogenous land use

   DAUT(I) change in stock of unusable land

   DAZG(I) change in land devoted to public uses

   DAZG(I) change in land devoted to retail or exogenous, commercial uses

   DAZR1(I),
   DAZR2(I),
   DAZR3(I) change in land devoted to household or residential uses

   The effects of these known changes are then used to calculate new zoning regulations for each areal unit.

   AZSI(I) = DAZSI(I) + AZSI(I)

   AU(I) = DAUT(I) + AU(I)

   AZC(I) = DAZC(I) + AZC(I)

   AZR1(I) = DAZR1(I) + AZR1(I)

   AZR2(I) = DAZR2(I) + AZRs(I)

   AZR3(I) = DAZR3(I) + AZR3(I)

   These changes can result from public decisions (urban renewal treatments, etc.) or private (known plans of developers, etc.) actions.

8. Read in known and planned changes (+ or -) in population distribution during the analysis period, for each areal unit, I, for each household type, L.

   DNT(L,I) change in total households, type L, in areal unit I

The effects of these known changes are then used to calculate new "beginning totals":

NT(L,I) = '(L,I) + DNT(L,I)

These changes can result from public decisions (urban renewal treatment) or private (plans of real estate developer, etc.) actions.

9 and 10. Assuming an analysis period covers a reasonably short period of time (say one to five years), it is not realistic to assume that there can be a complete reshuffling of land use. Clearly, the factors present in the original locational decision for an activity continue to be important. Once a location or land use comes into being, however, additional forces are created that act to tie the activity to the existing location. Some investments (buildings, equipment, etc.) are relatively permanent and immovable, for example, which makes the cost of re-locating different from the original location cost. Although the calculation of the effects of those forces that tie activities to existing locations is a potentially complicated research area,[31] T.O.M.M. in its present form handles the problem in a straightforward, simplistic way. Two subroutines are used that assume that only a certain percentage of activitiesoor population are mobile during any analysis period. The "stationary" or stable employees and households form lower bounds for activities in all areal units (15., 28.).

NHS(L,I) = STABH(NT(L,I))   stable households in areal unit, I
ERRM(K,I) = STABC(EM(K,I))

11. Calculate first trial totals for endogenous commercial activities during analysis period.

ERRM(K,I) = STABC(EM(K,I))   trial total for endogenous, commercial land use.

12. Calculate first trial totals for households, by type (L), during analysis period.

NHTT(L,J) = STABH(NT(L,J)) = NHS(L,J)

Residential Loop

13. Calculate total households, by type L, supported by city-side employment. Calculation is based on labor-force participation rates and employment totals at start of present iteration.

$$TNHH(L) = \sum_{K=1}^{M} \left[G(K,L) * \sum_{I=1}^{M} ERRM(K,I) \right] + G(M+1,L) * \sum_{I=1}^{M} EIND(I) + G(M+2,L) * \sum_{I=1}^{M} EBUR(I) .$$

14. Allocation of households, by type, to areal units, with the number of additional households desiring to locate in a given area being inversely proportional to the total rent surface at I (evaluated by type L households) relative to all others.

$$\frac{1}{NH(L,I)} \propto TCS(L,I) \qquad \text{or}$$

$$NH(L,I) = K_L * [- \gamma * TCS(L,I) + CONST(L)]$$

where

$$K_L = \frac{TNHH(L,I) - \sum_{J=1}^{N} NHTT(L,J)}{\sum_{J=1}^{N} \frac{1}{TRS(L,J)}}$$

15. Apply minimum constraints -- are households in an areal unit greater than the assumed immobile ones?
Is $[NH(L,J) + NHTT(L,J)] \geq NHS(L,J)$?
If not, note that J was outside of constraints and is not to be considered in further residential loop calculations.
$NH(L,J) = NHS(L,J) - NHTT(L,J)$
If constraint satisfied, leave NH(L,J) unaltered.

16. Calculate Residential Holding Capacity for areal unit, based on zoning map and densities in zoning ordinance (ZCR1, ZCR2, ZCR3).

$$NMAX(I) = AZR1(I) * ZCR1 + AZR2(I) * ZCR2 + AZR3(I) * ZCR3.$$

17. Apply Residential Holding Capacity Constraint to see if more households were assigned to an areal unit than it can hold.

Is $\sum_{L=1}^{LH} [NH(L,I) + NHTT(L,I)] \geq NMAX(I)$?

If greater than the maximum allowable, note that I was outside of constraints and is not to be considered in further calculations in the residential loop. Reduce the additional households proportionately so that

$$\sum_{L=1}^{LH} NH(L,I) = NMAX(I) - \sum_{L=1}^{LH} NHTT(L,T)$$

If constraint was satisfied, leave NH(L,I) unaltered.

18. Check to see if either the minimum (stable households) or maximum (residential density) constraints were violated in any analysis area during the current residential loop. If one or more constraint violations were observed, accumulate the <u>net</u> households outside of the constraints (those over maximum density are + and those under minimum are -) to be reallocated. Go through residential loop again (go to 14.), reallocating households outside of the constraints only in those areal units in which constraints were not violated. Continue iteration through residential loop (14., 15., 16., 17., and 18.) until all households have been allocated and no constraints are violated. Then calculate additional (+ or -) households in each tract for current program iteration (19.).

19. Record change in total households in each analysis area since last iteration, NH(L,I).

It should be noted at this point that when constraints are violated in a given areal unit, those households involved (over or under the original total) are reassigned to other areal units that have not had a constraint violated. This reassignment or reallocation is made on the basis of the original calculations of potential residential density. This is equivalent to saying that households locate as near to their desired location (based on the total rent surface) as possible without violating constraints. If all the households that "want" to locate in the analysis area with the highest density potential are unable to do so, they locate in other areas inversely proportional to those areas' rent surfaces. A similar argument holds for those households "assigned" to areal units to bring the area up to its minimum complement of households [NHS(L,I)].

20. Calculate new trial totals for current program iteration based on output from residential loop (19.).

$$NHTT(L,J) = NHTT(L,J) + NH(L,J)$$

<u>Endogenous, Commercial Employment Loop</u>

21. Calculate city-wide totals for endogenous, commercial employment, by type, ERM(K), based on distribution of total households, TNHH(L).

$$TNHH(L) = \sum_{I=1}^{N} NHTT(L,I)$$

$$ERM(K) = \sum_{L=1}^{LH} A(K,L) * TNHH(L)$$

Different types of households exert differential support for different commercial activities. The A(K,L) coefficients are empirically determined and reflect this notion.

22. Calculate endogenous, commercial employment potentials for each areal unit, I, based on access to households and exogenous employment activities. Employment potentials are inversely proportional to customer travel costs for firms located at J.

$$ER(K,I) + ERRM(K,I) \propto \frac{1}{Cost(K,I)} \quad or$$

$$Cost(K,I) = \sum_{J=1}^{N} \left[\frac{\sum_{L=1}^{LH} [C(K,L) * NHTT(L,J)]}{ACCESS(I,J,MM)} + \frac{DI(K) * EIND(J)}{ACCESS(I,J,MN)} + \frac{DB(K) * EBUR(J)}{ACCESS(I,J,NN)} \right]$$

The parameters C(K,L), DI(K), and DB(K) are empirically determined and represent differential forces in the commercial employment location process. Different household types and different employee types exert differential attractions for commercial activities.

23. The potentials calculated for each areal unit (in 22.) are then used to allocate <u>changes</u>, ER(K,J), in commercial employment since the last program iteration to areal units. ERRM(K,J) is the current trial total for endogenous employment, K, in areal unit I.

$$[ER(K,I) + ERRM(K,I)] \propto \frac{1}{Cost(K,I)} = - \delta * Cost(K,I) + CNST(K).$$

24. The total number of (potential) employees, type K, in an areal unit is subjected to a minimum-efficient-establishment-size constraints, Z(K).

$$[ER(K,I) + ERRM(K,I] \geq Z(K)$$

"Controlling the distribution of retail employment was anticipated as a major problem in model design. The difficulty, essentially, is that the potential functions do not allow for those external economies of scale which, in the real world, encourage the clustering of retail establishments. The control device...chosen was a minimum-size constraint imposed on the distribution of employment for each kind of business. No tract would be allowed, for instance, to have a three-man department store."[32]

25. and 26. Check to see if any employees are assigned to areal units which will not support an establishment or has no land zoned commercial [AZC(I) = 0]. Check to see that employees are above minimum levels as well [STABC(EM(K,I))]. Take all employees located in deficient areal units or areal units not allowing commercial land use and relocate them in those areal units with sufficient employees to form an establishment. The reallocation is based on the relative potentials of the areal units with sufficient employees and AZC(I) > 0.

27. Calculate New Trial Totals for endogenous commercial employment based on changes since last program iteration.

ERRM(K,I) = ERRM(K,I) + ER(K,I)

28. <u>Changes</u> in trial totals since last program iteration:

NH(L,J) households, by type by areal unit

ER(K,J) endogenous commercial employment, by type, by areal unit

MVAL(L,J) market price of housing unit, by household type, by areal unit.

29. and 30. Because of the iterative solution procedure that the program represents, convergence to an equilibrium may require a great number of program iterations. The program may go through several iterations (31. to 13. and re-loop) as it nears convergence where only one or two households in one or two areal units may change from iteration to iteration. The tolerance limits specify how near an exact solution we require. Experiments with T.O.M.M. have shown that the program converges uniformly so that as changes from iteration to iteration diminish, the program approaches the "exact" equilibrium solution (see discussion on convergence, below). If all changes in all areal units for all variables (28.) are less than a tolerance limit, the program assumes it has found a solution for that analysis period (31.), otherwise, it returns to 13. and starts another iteration.

The tolerance limits applied (read in at 1.) to changes in variables calculated in 28. for each areal unit are as follows:

TL1 Endogenous, commercial land use

TL2 Total employment

TL3 Residential land use

TL4 Households, by type

TL5 Totals of all land uses

31. End of analysis period. Print out final distributions of employment and population. If analysis period is last in the study period, end program. Otherwise update data files, making ending employment, population, and market value distributions the beginning ones for the next analysis period:

MVAL(L,I)

NT(L,J) = NHTT(L,J)

EM(K,J) = ERRM(K,J)

To to 2., 3., 4., and 5.

G. Parameter Estimation

Generally speaking, the values of parameters are being estimated using multiple regression techniques. In so doing, we quite clearly raise problems of identification and lagged varirbles.[33] These are minimized when we use changes in variables as the dependent variable (14.) but not eliminated. A far more troublesome feature of our regression equations relates to the problem of simulatneously estimating a regression coefficient, choosing from one of the several accessibility models, and estimating parameters for the accessibility model. Consider, for example, the potential function calculated in (14.):

$$NH(L,J) = WBUR(L) * \sum_{I=1}^{N} \frac{EBUR(I)}{ACCESS(I,J,MM)} * \frac{1}{TNHH(L)}$$

(other terms ignored, for convenience).

Assume that we are able to pick, a priori, the best measure of accessibility as MM = 2:

$$ACCESS(I,J,2) = d_{ij}^{-k}$$

We are faced with the problem of estimating both k and WBUR(L) simultaneously. In practice, we have been assuming various values of k, estimating WBUR(L) under the assumption and calculating goodness-of-fit. The values of k are changed in the direction yielding better goodness-of-fit measures.

The constraining effect of data availability becomes apparent when we attempt to estimate parameters. NH(L,J), above, refers to the change in households, type L, from one analysis period to the next. At least two observations on households are necessary to calculate this change. The time-period between observations determines the appropriate length of the analysis period. If forced to use only one set of observations (at one point in time) to estimate parameters, then we are assuming, in effect, that average change equals marginal change and that the forces affecting locational changes today are no different than those of 10, 50 or 100 years ago.

Perhaps the severest data limitation generally encountered in calibrating T.O.M.M. is the lack of data on employment, by location. Employment data, if available, is usually by place of residence or location of accounting office issuing the employee's payroll (data from tax records). Small-area data to be collected in the 1970 Census for resident-workplace pairs should help alleviate this critical problem.

H. Convergence Properties

The rate of convergence of T.O.M.M. is a function of the number of household types, number of endogenous employment types, number of areal units, and the solution tolerance limits. For "reasonable" values of these variables and "reasonable" parameter estimates:

Household types	(5-6)
Endogenous employment types	(3)
Areal units	(45-100)
Tolerance limits	(2-10 households, employees, etc., per analysis area)

Previous versions of T.O.M.M. converged in 3 to 6 program iterations or in a total of 2-3 minutes on an IBM 7090, for each analysis period.

The most important variables in terms of convergence rates and aggregate system behavior appear to be the labor-force participation rates [G(K,L)'s], or the average numbers of households to supply one worker. Within ranges of .6 and 1.0, the previous versions of the program behaved reasonably and converged rapidly. Below average values of .6 the city stagnated and above 1.0, it grew without bound. Because of their relations to population totals, this phenomena seems to be related to the concept of "the urban size ratchet" discussed by Thompson.

> "Perhaps some critical size exists, short of which growth is not inevitable and even the very existence of the place is not assured, but beyond which absolute contraction is highly unlikely, even though the growth rate may slacken, at times even to zero....
>
> In sum, if the growth of an urban area persists long enough to reach some critical size (a quarter of a million population?), structural characteristics such as industrial diversification, politicia power, huge fixed investments, a rich local market, and a steady supply of industrial leadership may almost endure its continued growth and fully ensure against absolute decline (> .6) -- may in fact effect irreversible aggregate growth (> 1.0)."[34]

The changes included in this version of T.O.M.M. should not affect convergence rates or aggregate system behavior in significant ways.

I. Summary and Future Directions

T.O.M.M. is still in the developmental stage. This version, however, approaches the limits of this particular approach to urban locational phenomena. Futhre efforts on T.O.M.M. should focus on developing an appropriate data base and on the considerable parameter estimation problems. Additional theoretical refinements would appear to have only marginal payoffs. Only careful empirical testing can determine whether the current version of T.O.M.M. represents a desirable addition to Lowry's pioneering work and achieves its dual objectives of utilizing only "readily" available data and incorporating policy variables in a meaningful way.

FOOTNOTES

1. Crecine, John P., Time Oriented Metropolitan Model, CRP Technical Bulletin No. 5, Pittsburgh City Planning Department, January 1964.

   Steger, Wilbur A., "Analytic Techniques to Determine the Needs and Resources for Urban Renewal Action," in Proceedings IBM Scientific Computing Symposium: Simulation Models and Gaming (IBM, White Plains, New York), pp. 85-93.

   Steger, Wilbur A., "The Pittsburgh Urban Renewal Simulation," Journal of the American Institute of Planners, May 1965, pp. 144-149.

   Crecine, John P. (with the assistance of Kenneth Hadden and Karen Wirth), "Accessibilities, Externalities, and Urban Structure: A Computer Simulation Model," University of Michigan, unpublished, May 1967.

2. Richard Duke, Director.

   Duke, Richard D., and Ray, Paul H., "The Environment of Decision-Makers in Urban Simulations," in Proceedings of Symposium on Models of the Decision-Maker's Environment (Markham Press, forthcoming).

   I wish to express my appreciation to Richard Duke, Paul Ray, Tom Borton, Don Kiel, and Roy Miller for their generous aid and assistance.

3. Isard, Walter, Methods of Regional Analysis: An Introduction to Regional Science (Wiley, New York), 1960, pp. 1-4.

   Thompson, W., A Preface to Urban Economics (John Hopkins Press, Baltimore), 1965, pp. 27-31.

   Lowry, I. S., A Model of Metropolis, RM-4035-RC, The RAND Corporation, Santa Monica, 1964, pp. 2-3.

   In the long run, it could be that the supply of labor in the urban area has a degree of influence over the decision of exogenous activities to locate within the urban area. T.O.M.M. takes the location and scale decisions of exogenous activities as givens.

4. Lowry, I. S., op. cit., p. 63.

5. Also consistent with Isard, op. cit., pp. 569-673, and Thompson, op. cit., pp. 11-37.

6. The solution procedure was developed by John F. Muth, now at Michigan State University, while at Carnegie-Mellon Univer[illegible]y.

7. Lowry, I. S., op. cit., pp. 8-18.

8. The stochastic disturbance terms can be serially correlated.

9. Dziewonski, Kasimierz, "A New Approach to Theory and Empirical Analysis of Location," Regional Science Association: Papers, XVI, Cracow Congress, 1965, p. 18.

10. Lowry, Ira S., Seven Models of Urban Development: A Structural Comparison, P. 3673, The RAND Corporation, Santa Monica, 1967.

11. These functions were generally of the following form:

$$c_1 \, d_{ij}^{-c_2}$$

where d is airline distance and c_1 and c_2 are empirically-determined constants reflecting differences in costs of social interaction for different forms of interaction.

12. Mills argues strongly for the use of airline distance from the center city as the only necessary measure of accessilibity. Mills, Edwin S., "The Value of Land," Johns Hopkins University (unpublished), 1967. The reason given was the high correlation between various measures of accessibility and distance from center city ($R^2 \approx .8$). For a model attempting to describe the dynamics of locational behavior, this would be an especially inappropriate assumption. First, and most importantly, individuals locate with reference to a particular employment location. This is why Mills is forced to assume centrality in his model of urban land values. The center city is assumed to be more accessible to the outside world than any other point. Hence, in Mills' model, export industries will locate around the center and other activities locate about the export industries. Mills, op. cit., pp. 19-20. While the presence of a "single point" in an urban area that is most preferred for intercity transportation (harbor, railhead, etc.) may have some historical validity, with limited access interstate highways bypassing center cities and airports outside of urban areas, the centrality assumption no longer seems reasonable. This is not to say such historical considerations are not important. The importance of an area's history is summarized in T.O.M.M. as the existing stock of land uses. The forces which brought about an extremely centralized urban area with dense population near the center city (after a railhead, harbor, or water-rail interchange point) and along the transportation lines (rail), falling off quickly, have largely been replaced with the dispersive forces of the automobile and limited access freeway. Locational parameters estimated on the basis of urban development to date (i.e., filling an empty plane -- city -- using cert[illegible] [illegible]ptions and location parame[illegible] in such a way to reproduce an existing distribution of population, commercial activity and land use), because of this historical bias, would be especially inappropriate for estimating future development.

13. A somewhat parallel argument holds for endogenous commercial activity -- see below.

14. To be defined more completely below.

15. In Figure 2, these costs are stated in comparable units; dollar/land area/unit time and dollars/unit time. Household travel for non-work purposes will be introduced below.

16. See Lowry, Model of Metropolis, op cit., pp. 31-32, for a cogent statement of this position.

17. By stratifying households, by type, we allow different classes of households to prefer different cost mixes -- through differential valuations of component costs.

18. For an example of such historical models, see Edwin S. Mills, op. cit., and his "An Aggregative Model of Resource Allocation in the Urban Area," American Economic Review, May 1967, pp. 197-210, and Richard Muth, "Economic Growth and Rural-Urban Land Conversions," Econometrica, January 1961, pp. 1-23.

19. In models that treat households as homogeneous, problems arising when, for example, a household's budget is less than the minimum point on the rent-surface (see Figure 2) are ignored. In T.O.M.M. it is assumed that budget problems are largely eliminated by different valuations of cost components for different (i.e., low income) households. For instance, by "assigning" zero cost to travel time, the rent surface may be lowered so some points fall below the household's budget constraint. This is equivalent to treating the travel cost as the residual of household budget less site and improvement rents.

20. Topography, ocean or lake frontage, view, smog count, etc. are examples of site amenities present in limited numbers of cities.

21. Immediately questions arise concerning households with no member in the labor force and those with more than one member in the labor force. These issues will be dealt with below.

22. Meaningful in the sense it does not require knowledge of all components (individuals) of the aggregation to determine system behavior.

23. This assumption could be interpreted as assuming all households making locational decisions have full information of site amenities in all areal units, not just those in the "neighborhood" of their employment location, etc.

24. I must add, however, merely because other modeling efforts with purposes similar to T.O.M.M.'s do not face the problem, does not mean the problem is non-existent.

25. Land use and property transactions data exist in many cities and, if properly coded by areal units, may be superior to census data. Prices in the real estate classified section of newspapers, while undesirable because they represent asking prices, may represent a reasonable source of data if the disparity between asking and selling price is small or predictable.

26. This means obtaining access to the original, raw census data, by individual households or estimating census tract market values from component census blocks containing a particular type of household, exclusively (or nearly so). If an area contains mixed housing types (single family and multiple units) it may be necessary to construct an index of market value based on a weighted average of various housing unit costs reflecting this mix in types.

27. The price adjustment phenomena outlined here closely parallels Kenneth Arrow's discussion on that subject, "Toward a Theory of Price Adjustments," in Abramovitz, Moses, <u>et al</u>., <u>The Allocation of Economic Resources</u>, 1959, pp. 41-51. In his analysis, Arrow points out that "...the difference between supply and demand is a major factor in explaining the movement of prices....However, the 'price' whose movements are explained...must be thought of as the average price," <u>op. cit</u>., pp. 47-48. This parallels our definition of MVAL(L,I). In addition, Arrow's analysis suggests:

    i. the speed of price adjustment will be greater during a period of full utilization of capacity than in a situation of excess capacity [$|V2(L)| > |V1(L)|$].

    ii. Price adjustment will be more rapid in industries where inventories play a significant role (as in housing markets).

    iii. Well-organized exchanges (real estate market) would display the greatest degree of price flexibility, because of the present of greater information.

    iv. Offsetting iii., for us, and leading to an absence of information <u>relevant</u> for price adjustments is the market where products (houses) are poorly standardized. The heterogenous nature of the housing market should lead to longer (price) response times and greater unsystematic behavior in price adjustments. <u>Op. cit</u>., p. 48.

28. Empirical work using market transactions data suggests that MVAL(L,I) is not particularly collinear with site amenities, at the neighborhood level. Crecine, John P., Davis, O. A., and Jackson, John E., "Urban Property Markets: Some Empirical Results and Their Implications for Municipal Zoning," Journal of Law and Economics, October 1967, pp. 79-100.

29. Other endogenous employees were not included as potential customers to avoid parameter estimation problems.

30. Lowry, I. S., op. cit., pp. 70-74.

31. Dziewonski, K., op. cit., pp. 19-20.

32. Lowry, I. S., op. cit., pp. 71-72.

33. Malinvaud, E., Statistical Methods of Econometrics (Rand McNally, Chicago), 1966, pp. 473-523, 559-614.

34. Thompson, W., op. cit., pp. 23-24.