RESEARCH PAPER P-370

ON THE ECONOMIC APPLICATION OF AIRLIFT TO RESUPPLY AND ITS IMPACT ON INVENTORY LEVELS

A. Soriano
D. Gross

October 1967

INSTITUTE FOR DEFENSE ANALYSES
SYSTEMS EVALUATION DIVISION

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ABSTRACT

This study demonstrates how, in a supply system, the inventory stock levels needed to maintain a specified degree of customer service vary as a function of the requisition leadtime and the demand pattern.[1] Customer service is measured in terms of the percentage of demands satisfied immediately from on-shelf inventory.

The study presents a practical method to determine the savings in inventory holding cost made possible by reducing requisition leadtime, in problems involving several million different line items. In most cases the reduction of requisition leadtime may involve some additional cost, as in the case of using a more rapid means of transportation (airlift instead of sealift). For such cases, it is essential to compare the potential savings in inventory holding cost with the added cost of reducing the requisition leadtime.

The approach developed has been applied successfully to the peacetime military overseas resupply problem involving over 200 supply distribution points with more than 2 million different line items. This study presents primarily the conceptual development behind the methodology used in WSEG Report 99.[2]

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[1] The paper, The Effect of Reducing Leadtime on Inventory Levels - Simulation Analysis, presented by D. Gross at the 31st National ORSA meeting, New York, May 31, 1967, is included as part of this document.

[2] WSEG Report 99, The Peacetime Value of Strategic Airlift, Weapons Systems Evaluation Group, Institute for Defense Analyses, June 1966, SECRET.

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CONTENTS

I. INTRODUCTION AND STATEMENT OF PROBLEM

A. INTRODUCTION

U.S. forces stationed overseas in peacetime require a continuous large flow of supplies. Most of the resupply is shipped to distribution points overseas from the military supply system in CONUS. During peacetime, emergency and routine resupply modes are used to supply the overseas military forces. Emergency resupply must be moved quickly, and frequently involves shipment by air. Routine resupply involves a longer requisition leadtime and is usually accomplished by sealift.

The shipment of supplies by air cuts down the requisition leadtime (pipeline time) and hence allows the reduction of inventory stocks without the sacrifice of customer service. A great increase in volume of air shipments will be possible when the C-5 and like aircraft become available for regular air cargo transportation by the mid 1970's. Such large aircraft will permit air cargo transportation charges well below current levels. One of the purposes of this study is to demonstrate that it will be economical to ship by air significantly greater amounts of cargo than are airlifted today. Theoretically, routine fast air shipment is justified on economic grounds for any commodity if the resultant savings in inventory holding cost are larger than the increase in transportation cost when airlift is used instead of sealift.

This paper presents a procedure for estimating net dollar savings the Department of Defense (DOD) might realize if inventory requirements for some air transportable items were decreased through substitution of airlift for sealift. The paper also presents estimates of such net DOD savings, but only under a wide variety of unclassified assumptions regarding airlift and sealift costs. In addition, relevant unclassified aspects of current Service logistics policies are discussed.

B. STATEMENT OF PROBLEM

The net dollar savings that the Department of Defense might realize if inventory requirements for some air transportable peacetime military resupply items were decreased through the substitution of airlift for sealift would be equal to the reduction in inventory costs, less the amount by which air transportation cost exceeded sea transportation cost.

To estimate savings it was necessary first to establish an economic criterion to determine which commodities should be eligible for airlift. From an economic consideration, high-cost, low-weight (and/or low-volume) line items should qualify for airlift movement, while low-cost, high-weight (and/or high-volume) line items should not. A method has been developed to determine the economic break-even point. This method facilitated the selection of the line items to be shipped by air on economic grounds. To estimate the annual savings resulting from the optimal use of airlift for peacetime resupply, two alternative courses of action have been considered: (1) sealift used only for routine resupply shipment, and (2) sealift used only for line items for which airlift could not be justified economically.

For the most part, practical considerations made it necessary to deal with whole classes of commodities rather than with individual items. The method developed is an approximate one that can be used effectively in problems involving a very large number of different line items where it could be reasonably well expected that the errors of low demand cases will either balance out or have a small effect, relatively speaking. The method yields poor approximations in case of items with very low demand.

It should be pointed out that only the long-range (steady-state) situation has been studied here, and no consideration has been given to the cost or savings resulting from the changes during the transition. Nevertheless, the importance of considering the transition (transient state) and its effect have been well recognized and it is expected that this aspect will form the theme of a future study.

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II. MATHEMATICAL DEVELOPMENT

A. PIPELINE INVENTORY ANALYSIS

Introduction

The part of the military supply system of concern in this study comprises more than 200 supply distribution points and over 2.5 million line items. Demand data accumulated in some of these supply distribution points indicate that there is no clear way of relating the demand patterns for the same line item in different supply distribution points. Hence, a line-item-by-line-item analysis of the problem could have involved a study of anywhere from 2.5×10^6 to 5×10^8 different cases. The need to project the line-item demand to the mid 1970 era complicates the problem even further. The magnitude of the effort required by such a detailed analysis could not be justified for the objectives set for this study. It was considered preferable to develop a methodology that will allow a first order estimation of the potential yearly savings that could be realized if airlift is allocated on economic grounds.

It should be noted that the totality of the military supply system is a multi-echelon inventory system. However, that portion of the supply system affected by the application of airlift movement to peacetime resupply of military overseas installations can be treated as a single echelon problem.

Several books and numerous papers have been written on inventory control theory, and no attempt will be made here to cover the subject matter thoroughly. Instead, attention will be concentrated specifically on the problem on hand. The treatment to be included here is based on an expected value approach. The effect of the stochastic properties of the supply system are evaluated in Section B of this chapter.

Definitions of Terms

To facilitate the presentation of some of the ideas, it is necessary to make the following definitions of terms and symbols. These definitions are in general agreement with the definitions included in the U.S. Joint Chiefs of Staff Publication 1, Dictionary of United States Military Terms for Joint Usage, 1 December 1964.

- Operating Level - The quantities of materiel (number of units of each line item) required to sustain operations in the interval between the receipt of successive shipments.
- Safety Level - The quantity of materiel (number of units of each line item), in addition to the operating level, required to be on hand to permit continuous operations in the event of minor interruptions of normal replenishment or unpredicted fluctuations in demand.
- Pipeline Level or Pipeline Inventory - The quantity of materiel that will be in the channel of support by means of which materiel flows from sources of supply to the requisitioning activity.
- Reorder Level - The level of inventory at which, when the base or depot assets fall to or below it, a requisition for replenishment is issued.
- Pipeline Time or Order and Shipping Time or Requisition Leadtime - The elapsed time between the initiation of a requisition for replenishment and the receipt of the requested replenishment.
- Requisitioning Objective - The maximum quantities of materiel to be maintained on hand and on order to sustain current operation. It consists of the sum of stocks represented by the operating level, safety level, and the order and shipping time.
- Mobilization Reserve Level or Strategic Level - The quantity of materiel placed in particular geographic locations for strategic reasons or in anticipation of major interruptions in the supply distribution system. It is, in such cases, over and above requisition objective.

Supply Cycle Time - The elapsed time between the issuing of successive requisitions.

Order Quantity - The quantity of units of a given line item ordered each time that a requisition for that line is initiated.

Availability - The percentage of demands (or unit requests) satisfied immediately from on-shelf inventory. Availability is used here as a measure of the level of customer service.

Definitions of Symbols

A_{ij} = The dollar value of the yearly demand for the ith line item in the jth supply distribution activity expressed in terms of dollars per year. In the commodity class approach A_{ij} refers to the dollar value of the yearly demand of the ith commodity class in the jth supply distribution activity.

C^* = Yearly inventory investment.

$C(\bar{t}_s-\bar{t}_a;m_1,m_2)$ = Additional inventory investment required as a result of increasing the pipeline time by an amount equal to $\bar{t}_s-\bar{t}_a$, for a case wit' m_1 supply distribution activities and m_2 line items in each.

c_u = Unit cost expressed in terms of dollars per unit.

I_o = Maximum operating level expressed in terms of units.

$\bar{I}_o$ = Average operating level expressed in terms of units per unit of time.

I_p = Maximum pipeline level expressed in terms of units.

$\bar{I}_p$ = Average pipeline level expressed in terms of units per unit of time.

I_R = Maximum mobilization reserve level expressed in terms of units.

$\bar{I}_R$ = Average mobilization reserve level expressed in terms of units per unit of time.

I_s = Maximum safety level expressed in terms of units.

$\bar{I}_s$ = Average safety level expressed in terms of units per unit of time.

q = The order quantity expressed in terms of units.

$\bar{t}_r$ = The average time interval between successive replenishment requisitions, or the average supply cycle time.

$\bar{t}$ = The average pipeline time.

$\bar{t}_a$ = The average pipeline time when resupplies are airlifted.

$\bar{t}_s$ = The average pipeline time when resupplies are sealifted.

$\bar{\lambda}_d = \bar{\lambda}$ = The average daily demand for a given line item expressed in terms of units per day.

$\bar{\lambda}_y$ = The average yearly demand rate for a given line item expressed in terms of units per year.

The Analytic Approach

As specified above, an expected value model has been used for the analysis, while the effect of the stochastic properties of the system were estimated by the use of a simulation model. The expected value model will be presented here, the simulation model and corresponding results will be described in Section B of this chapter and the Appendix.

The character of the military inventory fluctuation can be easily illustrated graphically in terms of average behavior. In Figure 1, some of the various cases that may characterize the daily inventory depletion process of an item are shown. Computation of the average inventory on hand during the 14 days of depletion will result in different answers depending

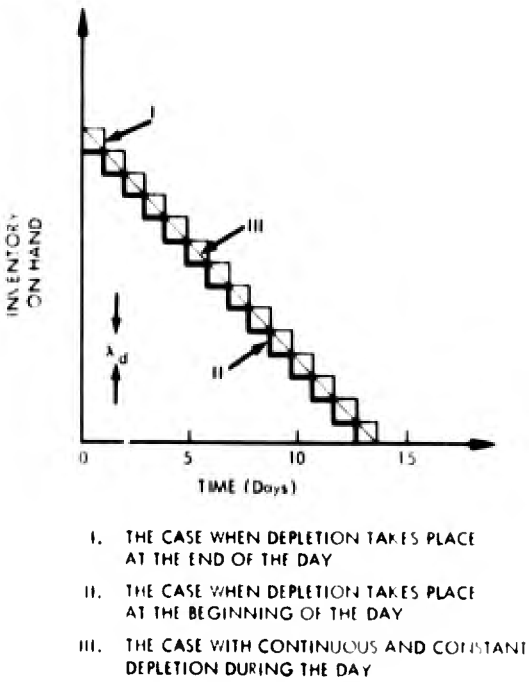

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FIGURE 1. Inventory Depletion for a Given Line Item

on the way the inventory is depleted each day. Cases I and II result in the upper and lower values for the average inventory on hand during this period. Case III is equivalent to the average of the upper and lower values, and hence will be assumed to characterize the inventory problem under consideration in this report.

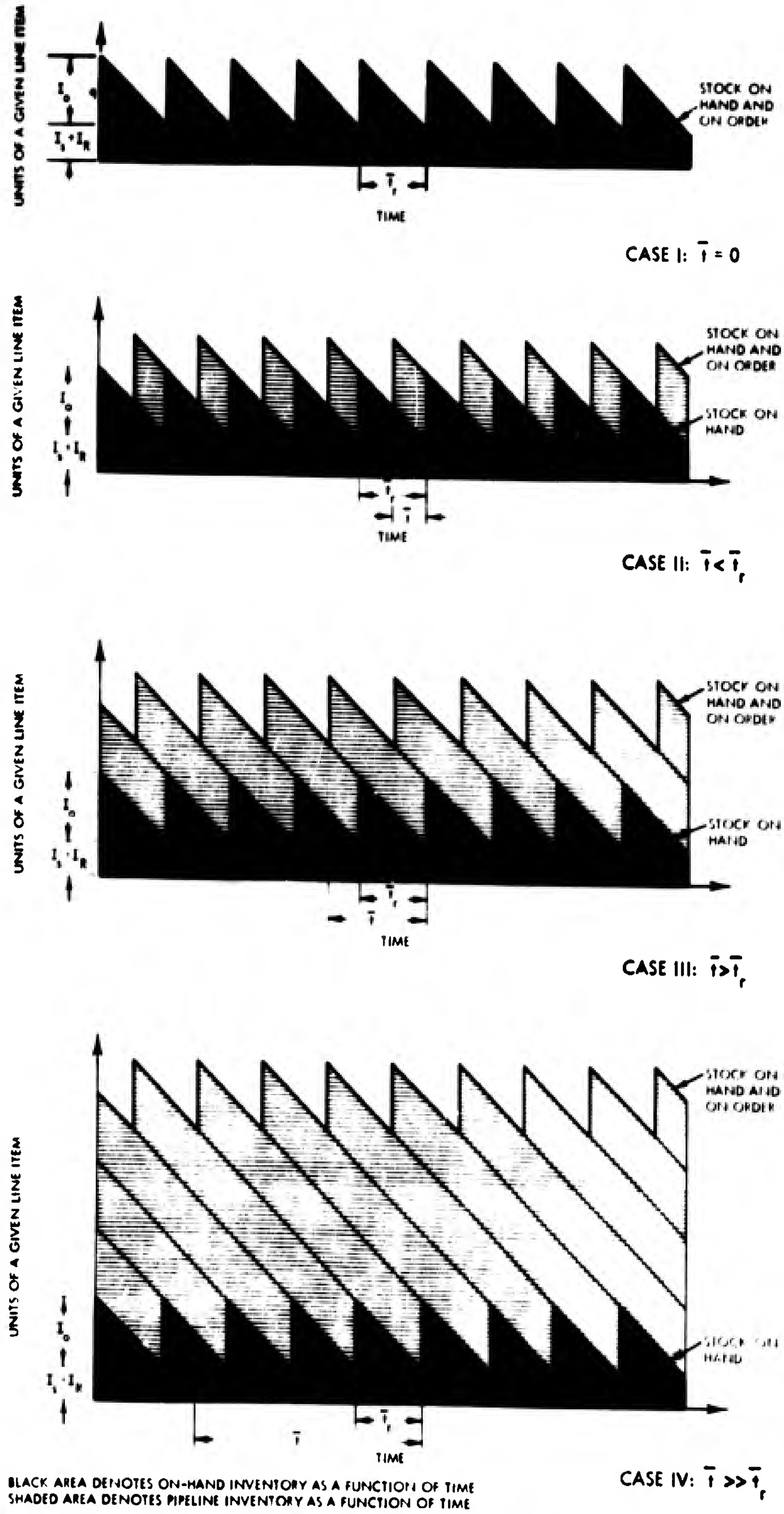
Figure 2 demonstrates the increase in pipeline inventory with the increase in pipeline time with respect to supply cycle time, $\bar{t}_r$; in all the cases shown, the level of inventory on hand as a function of time is identical. The inventory in the pipeline can be regarded as maintaining the level of inventory on hand independent of the pipeline time.

In this problem, the inventory in the pipeline is necessary because of the time that elapses between placement of a requisition and the receipt of the requested replenishment. This analysis is a comparison of two situations (airlift and sealift) each involving another set of pipeline times. The problem is to compare their inventory costs and transportation costs.

By definition, the average on-hand inventory is equal to

$$\bar{I}_S(i,j,\bar{t}(j)) + \bar{I}_O(i,j,\bar{t}(j)) + \bar{I}_R(i,j,\bar{t}(j)) . \qquad (1)$$

It should be noted that in this chapter, it will be assumed that the availability is fixed at some prescribed value. The inventory and inventory cost equations to be developed below will correspond to the specified availability value.



3-28-66-38 **FIGURE 2. Pipeline Inventory for Some Particular Cases**

Since

$$\bar{I}_R(i,j,\bar{t}(j)) = \bar{I}_R(i,j). \qquad (2)$$

$\bar{I}_R$, in other words, is independent of the pipeline time and hence will be omitted entirely from the analysis which follows. It can be easily verified by the geometry of Figure 2 that the average inventory in the pipeline is given by

$$\bar{I}_p(i,j,\bar{t}(j)) = q_{ij} \frac{\bar{t}(j)}{\bar{t}_r(j)} \qquad (3)$$

An arbitrary item i say, in the jth supply distribution activity will be considered now. Accordingly, the average yearly amount of capital that will be invested in on-hand and on-order inventories (excluding $\bar{I}_R$) as a function of the average pipeline time is given by

$$C^*(\bar{t}(j)) = \left[I_s(i,j,\bar{t}(j)) + \bar{I}_o(i,j,\bar{t}(j)) + q_{ij} \frac{\bar{t}(j)}{\bar{t}_r(j)} \right] c_u(i) \qquad (4)$$

where $c_u(i)$ is the unit cost. In a situation involving m_1 supply distribution activities with m_2 line items in each, the corresponding relationship can be given by

$$C^*(\bar{t}(1),\bar{t}(2),\ldots,\bar{t}(m_1);m_1,m_2) = \sum_{j=1}^{m_1} \sum_{i=1}^{m_2} C^*_{ij}(\bar{t}(j))$$

$$= \sum_{j=1}^{m_1} \sum_{i=1}^{m_2} \left[\bar{I}_s(i,j,\bar{t}(j)) + \bar{I}_o(i,j,\bar{t}(j)) + q_{ij} \frac{\bar{t}(j)}{\bar{t}_r(j)} \right] c_u(i) \qquad (5)$$

Assuming that pipeline times are independent of the supply distribution activity, Equation (5) becomes

$$C^*(\bar{t};\, m_1,m_2) = \sum_{j=1}^{m_1} \sum_{i=1}^{m_2} C^*_{ij}(\bar{t})$$

$$= \sum_{j=1}^{m_1} \sum_{i=1}^{m_2} [\bar{I}_s(i,j,\bar{t}) + \bar{I}_o(i,j,\bar{t}) + q_{ij} \frac{\bar{t}}{\bar{t}_r(j)}]\, c_u(i) \qquad (6)$$

The additional capital investment that will be required for a case defined by pipeline time $\bar{t}_s$, over that required for a case defined by pipeline time $\bar{t}_a$, is equal to

$$C(\bar{t}_s - \bar{t}_a;\, m_1,m_2) = \sum_{j=1}^{m_1} \sum_{i=1}^{m_2} C^*_{ij}(\bar{t}_s - \bar{t}_a)$$

$$= \sum_{j=1}^{m_1} \sum_{i=1}^{m_2} [\Delta(\bar{I}_{os}(i,j)) + q_{ij} \frac{\Delta\bar{t}}{\bar{t}_r(j)}]\, c_u(i) \qquad (7)$$

where $\Delta\bar{t} = \bar{t}_s - \bar{t}_a$

$$\Delta(\bar{I}_{os}(i,j)) = [\bar{I}_s(i,j,\bar{t}_s) + \bar{I}_o(i,j,\bar{t}_s)] - [\bar{I}_s(i,j,\bar{t}_a) + \bar{I}_o(i,j,\bar{t}_a)].$$

Since $\bar{t}_r(j) = \frac{q_{ij}}{\lambda_y(i,j)}$

where $\lambda_y(i,j)$ is the yearly demand for the <u>i</u>th item in the <u>j</u>th supply distribution activity, and q_{ij} is the corresponding reorder quantity, hence rewriting Equation (7) yields,

$$C(\Delta\bar{t};\, m_1,m_2) = \sum_{j=1}^{m_1} \sum_{i=1}^{m_2} [\Delta(\bar{I}_{os}(i,j)) + \lambda_y(i,j)\Delta\bar{t}]\, c_u(i)$$

$$= \sum_{j=1}^{m_1} \sum_{i=1}^{m_2} A_{ij}[\Delta\bar{t} + \Delta(\bar{I}_{os}(i,j))/\lambda_y(i,j)] \qquad (8)$$

A method to estimate $\Delta(\bar{I}_{os}(i,j))$ is presented in the following section. The method described above, together with the simulation results given in Section B of this chapter and the discussion given in Chapter III, formed the backbone of the analysis methodology of this study.

It may be of interest to note that the treatment given above applies to a general (s,S) inventory system. In terms of the notations used above,

$$s = I_p + I_s \qquad (9)$$

$$S = I_p + I_s + I_o. \qquad (10)$$

B. SAFETY LEVEL ANALYSIS

Introduction

The allowed reduction in safety stock level and accompanying reduction in average on-shelf inventory level are evaluated, for prescribed system performance requirements, as a function of mean leadtime. The general simulation technique developed for this purpose allows the treatment of the (s,S) periodic review inventory problem with both demands and leadtime being stochastic. Some results are presented for the performance of a military supply system, in which the reduction in leadtime is achieved by resorting to airlift rather than sealift. In addition, generalizations are given in the Appendix regarding the effects of certain parameters on the allowable reductions for general situations. Finally, a simplified means of predicting these effects by an empirical method is indicated.

System availability which has been used as the criterion for system performance is the percent (or fraction) of unit requests filled from on-shelf inventory. As the safety stock level is increased, so is the average availability since the larger the safety stock, the less chance there is for on-shelf inventory to be completely depleted when a request for items occurs. Increasing safety stock level, however, increases the average on-shelf inventory as well. When the mean leadtime is reduced, it is possible to reduce the safety stock levels (and correspondingly average on-shelf inventory) and still provide the previous system performance because the variation of the demand over the leadtime is also reduced. The purpose of safety stock is to guard against the variation, hence the allowable reductions without adversely affecting system performance. The amount of these reductions, for various availability specifications, can be determined by use of simulation programs.

A variety of factors influence the amount of permissible safety stock reduction. Among these are the means, variances, type of distributions of the demand and leadtimes, and the parameters of the inventory control policy. These parameters include such items as the quantity ordered, length of review period, and the decision rule used in determining if an order should be placed upon review. It is also of interest to determine which of the above factors are most significant and the magnitude of their effect.

Simulation Model

The simulation model developed is for a periodic review (s,S) inventory policy. The model is governed in the following manner: once every review period, r, the program calls for a review of the inventory position, to show items on-hand and on-order. If the inventory is below a level s, an order is placed for the amount required to bring the inventory position back to a level S. This procedure is most common in military supply systems (and, in fact, in a large number of nonmilitary systems as well).

The simulation model operates on a basic time period.

The user can decide whether the basic period is to be one day, week, month, or some other interval. He can also determine the value of r when this is expressed in terms of basic time periods. When r=1, inventory is reviewed once each period. The user also specifies s and S. Thus, the model is flexible enough to accommodate almost any situation of a periodic (s,S) inventory policy. The demand per period and the leadtime in number of periods are the data required to operate the simulator. These can be generated either by the use of a variety of probability distributions, or by use of actual historical data. For the study presented here, Monte Carlo techniques have been used to generate the required input data.

Required supplies to meet the demand were assumed to be withdrawn from inventory in a "lump sum" fashion at the end of each basic time period. Variations of the program for "lump sum" withdrawal at the beginning of a period and continuous withdrawal during a period also exist; however, the former was felt to be the most realistic for this study.

As output, the simulation program records a variety of system performance measures and characteristics. The two of interest in this study are the average on-shelf inventory and the percent of units not filled from on-shelf inventory. The latter is equal to one minus the availability. Since the data (demand and leadtime) are random variables, the output measures of interest (average on-shelf inventory and percent units not filled from on-shelf inventory) are also random variables. We are interested in estimating their means. In order to increase the precision of our estimates, we can take longer runs (that is, run the simulator for more periods) and/or take replications of each run (that is, using a different sample of the basic generating random variables repeat the run under the same condition of the previous run). We pay, of course, for increased precision by longer computer running time. The procedure used in the selection of run lengths and number of repetitions is discussed in detail below.

We introduce the following notation:

P_o' = average availability (expressed in terms of fraction or percent)

P_o = average percent of units not filled from on-shelf inventory = $1-P_o'$

OS = average on-shelf inventory level (units)

s = lower inventory control point (units)

S = upper inventory control point (units)

μ_x = mean demand per period (units per period)

μ_t = mean leadtime (periods)

I_o = operating inventory level = S-s (units)

I_p = average on-order inventory level (pipeline inventory) = $\mu_x\mu_t$ (units)

I_s = safety level (units)

r = review period (time).

The lower inventory control point, s, can be looked upon as being made up of two components; one to provide for average usage until the next order is received, and the other to guard against fluctuations in both demand and leadtime. Thus, we can write[1] $s = I_p + I_s$. Reducing μ_t reduces the fluctuations in demand over the leadtime, hence, we are able to reduce I_s while maintaining previous system performance. Since OS is a function of both I_o and I_s, it can also be reduced. Hence, we are interested in determining the allowed reduction in I_s (and resulting reduction in OS) that can be achieved by a reduction in μ_t, while keeping the system performance constant, as measured by P_o'. To accomplish this, the simulation model will

[1]It should be noted that since we have a periodic review policy, safety stock is also needed to provide for average usage and demand fluctuation for one period in addition to the leadtime. We include this in I_s.

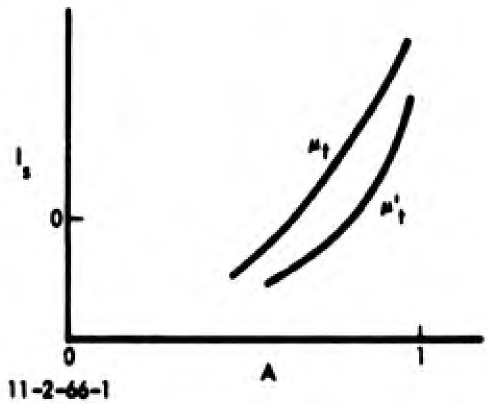


FIGURE 3. I_s Versus A

FIGURE 4. OS Versus A

be used to provide the curves shown in Figures 3 and 4.

For any fixed value of average availability, P_o', we can obtain (from Figure 3) the reduction in safe inventory level I_s when the mean leadtime is reduced from μ_t to μ_t' and from Figure 4 we can obtain the resulting reduction is OS. For scaling purposes, it was decided to plot I_s/μ_x and OS/μ_x versus P_o', that is, all I_s and OS values are normalized by mean demand.

From Figures 3 and 4, tables showing allowable reductions for various specified values of P_o' can be obtained. Plotting such sets of curves for a variety of demand and leadtime distributions will enable us to estimate reductions possible in a large number of different cases. Of course we do not know the exact characteristics of the demand and leadtime distributions associated with a particular military system. By studying a variety of cases it is possible to get a range of values, on which to base an estimate of the total effect on the military supply system in reducing the leadtime. In addition, this will allow us to make some general conclusions regarding the sensitivity of the demand and leadtime distributions. Aside from the theoretical interest in sensitivity, it is felt that the sensitivity analysis could help in providing a more reasonable estimate of the total effect; it could also indicate the extent to which future data should be collected. Furthermore, by varying certain policy conditions such as review period length, r, and size of I_o, we can study the effect on inventory levels of these parameters.

A case is defined as a specific demand distribution, a specific leadtime distribution, and fixed values of review period r and operating inventory level I_o. To generate the curve sets described in Figures 3 and 4, for a particular case it is necessary to vary safety level, I_s, as a particular setting for I_s will yield a single point on a curve in each figure. Changing I_s requires changing the lower inventory control point, s, one of the model inputs. Assuming the mean value of the leadtime is μ_t, by varying s we can generate points from which to plot the μ_t curves shown in Figures 3 and 4. Then, by reducing μ_t to μ_t' and again varying s, we can generate the points needed to plot the μ_t' curves.

The results of one s setting for a particular case is referred to as a run. The number of basic time periods simulated in a run is referred to as the run length. In order to gain knowledge concerning the confidence of our estimates of OS and P_o', it was necessary to replicate the runs, that is, different samples of the basic generating random variables were used when all input values were kept fixed. The resulting estimates of OS and P_o' from each run were averaged and these average values were used to plot a point. The procedure followed in deciding on the run length and number of replications to be used is given below.

Precision of Simulation Results

The basic time period was taken to represent one week, since this was the smallest period for which any real data were available. The simulation model had a run length limit of 2,000 periods, thus a maximum run length of 2,000 weeks is possible. Measurements of computer timing showed that running time versus run length was highly nonlinear and that the time differential to run 2,000 rather than 1,000 was quite small; thus, it was decided to utilize the maximum possible run length. It should be noted that the maximum length of a run was imposed by computer memory constraints, and that this limitation could have been overcome in several different ways at the cost of some computer time.

To determine the number of replications we introduce the following notations:

N = number of replications

K_{OS} = desired precision in estimating OS (units)

K_{P_o} = desired precision in estimating P_o (percent)

$\overline{OS}$ = estimate of OS from a single run

$\overline{P}_o$ = estimate of P_o from a single run

$\overline{\overline{OS}}$ = estimate of OS from N runs = $\Sigma\ \overline{OS}/N$

$\overline{\overline{P}}_o$ = estimate of P_o from N runs = $\Sigma\ \overline{P}_o/N$

$\sigma_{\overline{OS}}$ = standard deviation of $\overline{OS}$

$\sigma_{\overline{P}_o}$ = standard deviation of $\overline{P}_o$.

If N is sufficiently large so that the central limit theorem applies, then for a confidence of 0.975 it can be shown that the following must be true:

$$N \geq (2.24/K_{OS})^2\ \sigma^2_{\overline{OS}} \tag{11}$$

$$N \geq (2.24/K_{P_o})^2\ \sigma^2_{\overline{P}_o}\ . \tag{12}$$

However, we do not know $\sigma^2_{\overline{OS}}$ nor $\sigma^2_{\overline{P}_o}$. In order to estimate these, a case with relatively large variation in demand and leadtime was run thirty times and sample variations $s^2_{\overline{OS}}$ and $s^2_{\overline{P}_O}$ were calculated. It is expected that the larger the variation in the input, the larger the variation in the outputs; thus to be conservative, a case with large variations was chosen from which to calculate the s^2's. Furthermore, using the χ^2 distribution, upper tail 0.975 confidence interval estimates were obtained. These are denoted by $\hat{s}^2_{\overline{OS}}$ and $\hat{s}^2_{\overline{P}_O}$. These, then, were used in Equations (11) and (12) to provide an overall confidence of at least 0.975^2. Equations (11) and (12) then yield:

$$N \geq 30.4/K^2_{OS} \quad (13)$$

$$N \geq 0.004/K^2_{P_O} . \quad (14)$$

Equations (13) and (14) allow us to plot trade-off curves of desired precision K versus number of replications N, as shown in Figures 5 and 6.

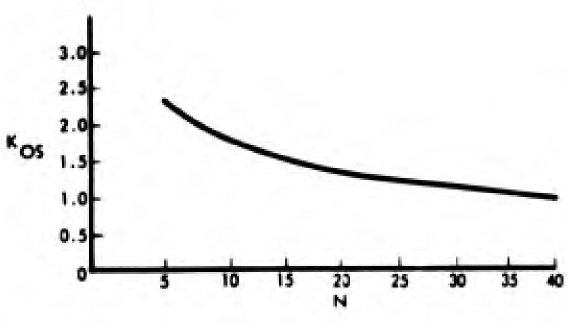


FIGURE 5. K_{OS} Versus N

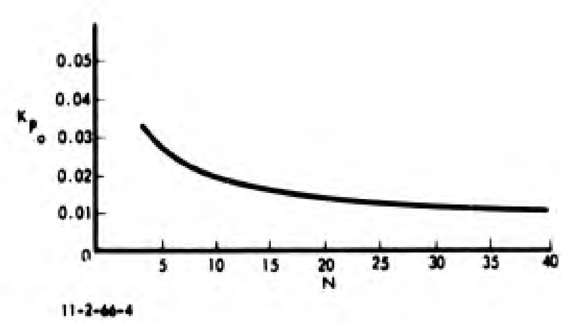


FIGURE 6. K_{P_O} Versus N

It should be kept in mind that for small N, the curves may not be accurate because of the need to make the normal distribution assumption. If an N of 15 were chosen, we would expect our estimates of OS to be within $\pm$ 1.42 units and our estimates of P_O to be within $\pm$ 0.0164 or $\pm$ 1.64 percent, with confidence of 0.95. This was considered to be satisfactory for the purposes of this study. In addition, considering all the cases desired to be run and deciding on a minimum of five points on which to base each curve, it was found that available computer time would permit determinations for as much as N=15. Therefore, it was decided to replicate each run fifteen times. During the runs, sample variances were computed for the $\overline{OS}$ and $\overline{P}$ estimates and, in the vast majority of cases, these were found to be well below the estimates of upper intervals, $\hat{s}^2_{\overline{OS}}$ and $\hat{s}^2_{\overline{P}_o}$. It is believed, therefore, that the precision obtained is certainly as good as stated and in most cases better. It also turns out that for high values of P'_o (low values of P_o), the actual sample variances of $\overline{P}_o$ are much smaller than $\hat{s}^2_{\overline{P}_o}$, indicating better than stated precision for high values of P'_o.

Current Operating Procedure for Military Systems Under Study

This study is concerned with the military supply system during a period in which most supplies have been sealifted overseas. The mean leadtime, including transportation, delays and processing times, was estimated to be about 13 weeks. Furthermore, it was estimated that shipping by air will reduce the mean leadtime to about 2 weeks. The inventory control policies, governing the supply system studied, set both safety levels I_s and operating inventory I_o at one month of supply.[1] Thus we have,

$$I_s = I_o = 4.3\mu_x \qquad (15)$$

where, μ_x is mean weekly demand. Then the policy called for requires that

[1]Since inventory levels are normalized by average demand, the terms one week of supply and one month of supply will represent the average demand over the corresponding period.

$$s = I_s + I_p$$
$$S = I_s + I_p + I_o \,, \quad (16)$$

where I_s and I_o are given by Equation (15), for a μ_t = 13 weeks. Inventory was reviewed weekly, thus r=1. There are, of course, some particular values of average availability, P_o', and average on-shelf inventory, OS, associated with this procedure. We are interested in obtaining, for a μ_t=2 weeks, I_o given by Equation (15) and an r=1, what the new I_s can be, still maintaining the same P_o'. We are also interested in the resulting reduction in average on-shelf inventory level OS. This can be obtained from the curves of Figures 5 and 6. In addition, the average availability, P_o', associated with current procedure can also be estimated. From available data and knowledge of supply personnel, the P_o' was expected to be about 0.95 or slightly better. This prior knowledge concerning P_o' will indicate whether our assumptions of demand and leadtime distribution shapes and variances are realistic, since under the input settings representing normal procedure, the resulting $\overline{\overline{P}}_o'$, should be in the neighborhood of 0.95.

Form of the Results

Twenty-two different cases were studied. For each case, sets of curves as described in Figures 3 and 4 were plotted. A sample set for one case, Figures A-3 and A-4, is given in the appendix. From these sets of curves, Tables A-5 to A-26 (presented in the last section of the appendix) were calculated; each table represented one of the twenty-two cases studied. The starred row of each table represents data for the military supply system for the time frame considered. The associated $\overline{\overline{P}}_o'$ and the I_s and $\overline{\overline{OS}}$ values required when μ_t is reduced from thirteen weeks are given. In addition, the reductions in I_s and $\overline{\overline{OS}}$ when μ_t is reduced are also calculated. The remaining rows of the table show, for other $\overline{\overline{P}}_o'$ the I_s, $\overline{\overline{OS}}$ values and reductions possible when μ_t is lowered. Note that in Tables A-5 through A-8 inclusive, two reductions in μ_t from thirteen to six and thirteen to two, respectively, are studied while in all others, only a reduction from thirteen to two weeks is considered. While thirteen and two are the μ_t's of interest, a middle value was included in some cases for two reasons. One, this middle value could represent a new faster transport ship, and two, it was of theoretical interest to get some idea of the rate of the reductions possible.

Directly to the right of each table number is a vector of descriptive information giving the particular details of the case as, for example, in Table A-5 the description (P,10,.316; N,μ_t,.3;1;4.3) is given. The first set of three characters gives demand information, the next set of three gives leadtime information, then the review period length followed by I_o/μ_x. Note that in addition to I_o, all values of I_s, $\overline{OS}$ and the reductions are also normalized by dividing by μ_x; that is, all values can be looked at as given in terms of weeks of supply. Case A1 in Table A-5, then, has a weekly Poisson demand with an average of 10 units per week (first and second descriptors respectively). The next value is the standard deviation to mean ratio (coefficient of variation). In the Poisson case this is redundant since the Poisson is a one parameter distribution; however, in other cases, this information is needed. The leadtime is normal with a coefficient of variation of 0.3. The μ_t's, the mean leadtimes, are given in the table itself. The one refers to a weekly review and the 4.3 indicates that I_o = S-s was set at one month of supply which for the particular case described above is equal to 43 units. Other letters used for distribution designations are U for uniform, E for exponential and C for constant (deterministic).

Tables A-5 through A-19 are for cases with r=1 and I_o/μ_x=4.3.

Tables A-20 and A-22 vary r from its setting of one while Tables A-23 and A-26 vary I_o. These tables will serve as the basis for the following discussion.

Analyses of Results

The results given in Tables A-5 through A-26 inclusive, and their corresponding $\overline{OS}/\mu_x$ vs. P'_o and I_s/μ_x vs. P'_o curves, will be used for the analysis below.

Inventory Reductions Possible for Military System Under Study

Observing the top rows of Tables A-5 through A-20 inclusive, we see that in the large majority of the cases we can expect a reduction in on-shelf inventory and I_s of slightly more than three weeks. Table 1 summarizes the top rows of T. ..es A-5 through A-19 (Tables A-14 and A-15, the constant leadtime cases are omitted).

Table 1. ACHIEVABLE INVENTORY REDUCTIONS

Assumed Reduction in Leadtime in Weeks From To	$\overline{P}'_o$			Safety Level Reduction (weeks of supply)			Reduction in On-hand Inv. (weeks of supply)		
	Avg	Max	Min	Avg	Max	Min	Avg	Max	Min
13-2	0.95	0.99	0.85	3.3	3.8	3.0	3.4	4.0	3.2
13-2[a]	0.97	0.99	0.96	3.2	3.4	3.0	3.3	3.4	3.2
13-6	0.98	0.99	0.97	1.9	1.9	1.8	1.9	2.0	1.7
6-2	0.98	0.99	0.97	1.4	1.5	1.2	1.4	1.6	1.2

[a]Cases A14, A15 (the exponential leadtime cases) excluded.

The second row of Table 1 probably includes the most realistic cases judging from the range of $\overline{P}'_o$ since it is believed that persons supplying military systems were trying to maintain an availability of about 0.95. Hence, a reduction in I_s and OS of approximately three weeks of supply ($3\mu_x$) can be expected if leadtime is cut from thirteen weeks to two weeks. Reducing leadtime from thirteen to six weeks will yield a reduction of approximately two weeks.

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III. THE AIRLIFT ECONOMIC CRITERION

The model developed in the first section of Chapter II is a deterministic one based on expected values. However, results obtained from the stochastic simulation analysis of the problem have been used in conjunction with the deterministic model in order to include the statistical effects in the treatment. The method developed could be used for a line-item-by-line-item analysis, although practical considerations precluded such analysis here.

With slight adjustment of some conceptual ideas, the model initially developed for a line-item-by-line-item analysis provided the methodology to be presented here. The adjustment required that each commodity class be regarded as being composed of homogeneous line items. This homogeneity is needed with respect to such relevant characteristics as unit weight, unit cost, and inventory control procedures. It is well understood that this assumption does not hold true for the cases on hand; on the other hand, the available aggregate data allow only such gross type analysis. The adjusted approach, referred to as the commodity class approach, is based on expected or average values. Using this approach the accuracy of the results obtained will increase as the degree of homogeneity increases; that is, the more data obtainable, the finer the commodity classes selected and the more accurate the results.

Taking into account that with the aggregate data obtainable only order-of-magnitude estimates were possible, the assumption of homogeneity can be made and the commodity classes methodology can be used. Accordingly, the additional inventory investment required, C_{ij}, for the supply system due to an increase in the pipeline time to distribution activity j for the ith commodity class, in order to maintain availability P'_o, can be given by

$$C_{ij} = C_{ij}\ (P'_o|\Delta\bar{t}(j)) = A_{ij}\left[\Delta\bar{t}(j) + \Delta\ (\bar{I}_{os}/y(i,j))\right] \quad (1)$$

where i and j denote the ith commodity class and jth supply distribution activity, and

$\Delta\bar{t}(j)$ = average pipeline time differential = $\bar{t}_s(j) - \bar{t}_a(j)$

$\bar{t}_s(j)$ = average order and shipping time (pipeline time) when corresponding supplies are transported from CONUS to overseas supply distribution activity j by surface means (sealift).

$\bar{t}_a(j)$ = average order and shipping time (pipeline time) when corresponding supplies are transported from CONUS to overseas supply distribution activity j by air (airlift).

P'_o = probability of not having a stockout to be referred to also as the availability.

$C_{ij}=C_{ij}(P'_o|\Delta\bar{t}(j))$ = the yearly average additional investment that will have to be tied up in inventory in the supply system for the ith commodity class in order to maintain the availability P'_o, given that the pipeline time to the jth supply distribution activity is increased by $\Delta\bar{t}(j)$.

$\Delta(\bar{I}_{os}/\lambda_y(i,j))$ = average on-shelf inventory differential, obtained from the increase in the safety level which is required in order to maintain the same P'_o, when pipeline time is increased by $\Delta\bar{t}(j)$. It is expressed in terms of $\bar{I}_{os}(i,j)/\lambda_y(i,j)$.

$\bar{I}_{os}(i,j)$ = average on-shelf inventory expressed in units.

$\lambda_y(i,j)$ = the yearly demand rate expressed in terms of units per year.

The distance that supplies are transported by surface means is greater than the corresponding aerial distance. Hence, let

$$k_2 = \frac{M_s(j)}{M_a(j)} \qquad (2)$$

where

$M_a(j)$ = the average distance in miles that supplies are moved when they are airlifted from CONUS.

$M_s(j)$ = the average distance in miles that supplies are moved when they are sealifted from CONUS.

In considering the possible savings that result from a reduction of inventory levels because a more rapid means of overseas supply transportation is used, it is essential to take into account the added transportation cost. This is given by

$$\begin{bmatrix}\text{Added}\\ \text{Transportation}\\ \text{Cost}\end{bmatrix} = M_a(j)(W_{ij})c_{ta} - M_s(j)(k_3W_{ij})c_s \qquad (3)$$

where

W_{ij} = the yearly weight of transported commodity in short tons[1]

c_{ta} = the cost of airlift in dollars per ton-mile,

c_{ts} = the cost of sealift in dollars per ton-mile, and

$$k_3 = \frac{\text{sealift weight}}{\text{airlift weight}}$$

The factor, k_3, represents the fact that packaging for sealift is usually heavier than that used for airlift.

In Equation (3), the first term expresses the yearly cost of airlifting the ith commodity class supplies, and the second term expresses the sealift cost for the same. A given commodity class, say the ith class, should be eligible for airlift from economic considerations if

$$k_1 A_{ij}[\Delta\bar{t}(j)+\Delta(\bar{I}_{os}(i,j)/\lambda_y(i,j))]+M_a(j)W_{ij}[k_2k_3c_{ts}-c_{ta}] \geq 0 \qquad (4)$$

where

k_1 = an inventory expense factor that incorporates the rate of interest on monetary investment, obsolescence, losses, storage and handling cost differences, differences in packaging cost, etc.

A_{ij} = the yearly dollar value of the corresponding demand.

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[1]In this document, tons is used to mean short tons and miles to mean nautical miles.

Rearranging the terms of Equation (4)

$$\frac{A_{ij}}{W_{ij}} \geq - \frac{M_a(j)(k_2k_3c_{ts} - c_{ta})}{k_1\left[\Delta\bar{t}(j) + \Delta(\bar{I}_{os}(i,j)/\lambda_y(i,j))\right]} \qquad (5)$$

Let

$$\alpha_{ij} = \frac{A_{ij}}{W_{ij}} \quad \text{and} \quad \beta_{ij} = - \frac{M_a(j)(k_2k_3c_{ts} - c_{ta})}{k_1\left[\Delta\bar{t}(j)+\Delta(\bar{I}_{os}(i,j)/\lambda_y(i,j))\right]}$$

where α_{ij} and β_{ij} are expressed in units of dollar per pound.

Equation (5) can be used to determine whether it will be more economical to ship a given commodity class to a given overseas supply distribution activity by aerial means or by surface (sea) means. A commodity class for which Equation (5) holds is referred to as being eligible for airlift. This test may be applied to n commodity classes which can be arranged by some predetermined but arbitrary order. Let n_a denote the set of commodity classes which are eligible for airlift, such that $n_a \epsilon n$. Then the total net annual savings, R_{ta}, that can be realized by shipping the eligible n_a classes of line items, can be obtained by the following relationship,

$$R_{ta} = k_1 \sum_{i,j} \sum_{\epsilon n_a} A_{ij} \left[\Delta\bar{t}(j)+\Delta(\bar{I}_{os}(i,j)/\lambda_y(i,j))\right] + \sum_{i,j} \sum_{\epsilon n_a} M_a(j)W_{ij}(k_2k_3c_{ts}-c_{ta}) \qquad (6)$$

In the treatment above it should be noted it was assumed that shipping costs are dependent on weight alone. Nevertheless, the same approach can be generalized to take into account the dependence on weight or volume as applicable. To do that assume that data are available on the volume per short ton for each line item to be considered. Let

$$H_{ij} = \max \left\{ M_a(j)W_{ij}[k_2k_3c_{ts}-c_{ta}], M_a(j)V_{ij}[k_2'k_3'c_{ts}'-c_{ta}'] \right\} \quad (8)$$

where V_{ij} is the yearly demand (packaged volume) in measurement tons of the ith line item at the jth supply distribution activity. The primed symbols represent quantities equivalent to unprimed symbols for a situation approached from a volume point of view. Equation (4) then becomes

$$k_1A_{ij}[\Delta\bar{t}(j) + \Delta\bar{I}_{os}(i,j)/\lambda_y(i,j)] + H_{ij} \geq 0. \quad (9)$$

From here on the analytical methodology can be completed readily in a manner equivalent to that given above.

In the event that the required data are available, there is no question that the approach considering cost to depend on the weight or volume, as applicable, will yield more accurate results than the one based only on weight. The use of very large cargo aircraft may minimize the effect of volume considerations. In case of secondary items, which are relatively small in volume, the effect may be negligible. Since secondary items constituted the only items of interest to this study, as will become evident later in the discussion, the volume consideration was not required.

IV. PARAMETER EVALUATION

In order to apply the inventory analysis model, it was necessary first to evaluate its parameters properly. Estimates of these parameters were based on the best obtainable data. When adequate data were not available, approximations had to be used. For some parameters, only a reasonable range of values could be determined.

A. YEARLY DOLLAR DEMAND AND TONNAGE

The only commodity classes of interest in this study were those consisting of stocked type line items. This is because inventory control policies in effect at the time of the study allowed pipeline inventory reduction savings only in case of stocked type line items. Hence, A_{ij} and W_{ij} should represent the yearly dollar demand and tonnage of the *i*th commodity class consisting of stocked type line items in the *j*th supply distribution activity.

B. THE INVENTORY EXPENSE FACTOR - k_1

The inventory expense factor, k_1, combines several factors affecting the airlift and sealift cost comparisons. The joint effect of these factors can be expressed as a percentage of the average dollar value of corresponding inventories. A review of these factors and their joint effects is given below.

The use of a slower mode of transportation for the movement of resupply, as stipulated by Alternative 1, results in a longer average requisitioning leadtime. This, in turn, results in higher inventories in the supply distribution system as a whole. These higher inventory levels lead to certain costs. It is customary to lump such cost items together and to refer to them as an inventory holding cost. The inventory holding cost (referred to here as the inventory expense factor) includes such cost items as: (a) storage cost, (b) obsolescence and/or excess cost, (c) losses from breakage, spoilage, damage, and a variety of similar factors, and (d) interest cost.

The principal contributions to the holding cost appear to come from the cost of obsolescence or excess and the cost of interest.

For present purposes, it was considered best to treat the k_1 factor as a parameter. Computations have been carried out for $k_1 = 0.20$, $k_1 = 0.25$, and $k_1 = 0.30$.

The above range of values used for the k_1 factor has been arrived at after a careful examination of the available pertinent sources of information. The associated data obtained from each source are given below as a percentage of the average dollar value of corresponding inventories.

Harbridge House Study[1] on the costs of supply operations:

Cost Element	Station Level: Ft. Devens (%)	Station Level: Ft. Meade (%)	Depot Level (%)
Storage & Handling	3.00	4.70	0.98
Interest	4.00	4.00	4.00
Losses	0.00	0.20	0.39
Obsolescence or Depreciation	8.00	6.40	11.46
TOTAL	15.00	15.30	16.83

[1]Harbridge House, Inc., Economic Inventory Policy Report #2, The Costs of Supply Operations, 1959.

- In costing storage and handling operations, the following elements of cost were included appropriately: civilian and military labor, supplies and materials, equipment amortization, repair and utilities and plant depreciation.
- Interest was costed on the basis of DOD Instruction 4140.11 specifications. Accordingly the interest paid by the government on capital investment in inventories was taken at 4 percent annually of average inventory value.
- The cost of losses included those from breakage, pilferage, spoilage and a variety of similar factors.
- The obsolescence cost element was based on the actual costs of generating and disposing of excess against a given inventory level. In arriving at this cost element, excess stocks have been defined to include only those quantities of stock for which definitive disposal action has been taken. The obsolescence cost included labor and material costs incurred in disposing the excess stock, plus the value of excess items sold or salvaged less any credit from sales.

Thomson M. Whitin[1] reports that he had sent hundreds of questionnaires to commercial and industrial companies. He also gives three replies which he picked out as being typical.

Cost Elements	Company A (%)	Company B (%)	Company C (%)
Storage & Handling	4.00	8.01	5.30
Interest	6.00	6.00	6.00
Losses	4.50	0.20	--
Obsolescence	5.00	3.43	4.00
TOTAL	19.50	17.64	15.30

L. P. Alford and J. R. Bangs, Production Handbook, list a comparable table obtained by Parrish for industrial concerns and expressed in terms of percent per annum of the cost of the average inventory on hand.

[1]Thomson M. Whitin, The Theory of Inventory Management, Princeton University Press, 1957.

Cost Element	Percent
Storage & Handling	3.25
Interest	6.00
Losses	0.25
Obsolescence	15.00
TOTAL	24.50

The Inventory Control Division of the Bureau of Supplies and Accounts, Department of the Navy, costed the above cost elements as follows:

Cost Element	Percent of Value of Average On-Hand Inventory
Storage & Handling	1.00
Interest	4.00 - 6.00
Losses	1.00
Obsolescence	10.00 - 13.00
TOTAL	16.00 - 21.00

It should be noted that, if demand and pipeline time are constant, storage cost should not be included in the k_1 factor when it is related to pipeline inventories. The storage space requirements depend on the maximum on-hand inventory, and if the demand and pipeline time are constant, then the maximum on-hand inventory will be equal to the sum: $I_R + I_s + I_o$. If the demand and pipeline time are stochastic, then at times the maximum on-hand inventory could include part of what is normally the pipeline inventory and hence be larger than the sum: $I_R + I_s + I_o$. Good examples for that would be situations where some supplies arrive earlier than expected or the demand is much less than the expected demand. In addition to presenting a situation in which more storage space will be required, the pipeline inventories may also subject the depot labor force to extra paper work in processing and tracking down requisitions, and other such activities. These considerations have led to the belief that the storage cost item should be taken into account in the case presented here.

C. THE SURFACE-TO-AIR DISTANCE NORMALIZING FACTOR - k_2

Generally speaking, supplies transported by surface means travel larger distances than those transported by airlift. It has been decided to let

$$k_2 = \frac{\text{Surface Distance}}{\text{Aerial Distance}}$$

D. THE SEALIFT-TO-AIRLIFT WEIGHT NORMALIZING FACTOR - k_3

Aside from being costlier, packaging for sealift is usually also heavier than that used for airlift. Hence, it was decided to let

$$k_3 = \frac{\text{Sealift Weight}}{\text{Airlift Weight}} .$$

E. PIPELINE TIMES OR REQUISITION LEADTIMES

In this analysis the requisitioning leadtime for airlift and sealift will reflect present practices. Consequently, two sources of available information were used. First were the Maximum Overseas Order and Shipping Time Allowances for Troop Support tables.[1] Pertinent information from these tables is given in Table 2. Issue Priority Groups 1 and 2 in that table refer to requisitions whose priority designators are 1 through

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[1]Military Standard Requisitioning and Issue Procedures (MILSTRIP) DOD Instruction 4140.17, 23 January 1962, and AR 725-50, CID.

Table 2. MAXIMUM OVERSEAS ORDER AND SHIPPING TIME ALLOWANCES FOR TROOP SUPPORT REQUISITIONS

Area[a]	A	B	C	A	B	C	A	B	C	A	B	C	Responsible Agency
Issue Priority Group	1 (Airlift)			2 (Airlift)			3 (Sealift)			4 (Sealift)			
Priority Designators	1 Through 3			4 Through 8			9 Through 15			16 Through 20			
Emphasis Required	7-Day Workweek 24-Hour Work-day--Issue Stocks to "0" Balance			7-Day Workweek 24-Hour Work-day--Issue Stocks to "0" Balance			Regular Workweek Regular Shift Workday--Issue to Control Level			Regular Workweek Regular Shift Workday--Issue to Control Level			
Cycle Segments	Consecutive Hrs.			Calendar Days			Calendar Days			Calendar Days			
1. From transmittal of requisitions by oversea command to receipt by initial source of supply.	0	0	0	0	0	0	1	1	1	1	1	1	Oversea command
2. From receipt of requisition by initial source of supply until supplies are released to carrier.	24	24	24	3	3	3	10	10	10	12	12	12	Initial source and shipping activities
Note: This time is divided as follows:													
Stock control activity	(8)	(8)	(8)	(1)	(1)	(1)	(3)	(3)	(3)	(4)	(4)	(4)	Initial source
Shipping activity	(16)	(16)	(16)	(2)	(2)	(2)	(7)	(7)	(7)	(8)	(8)	(8)	Shipping activity
3. From date supplies are turned over to carrier until receipt at oversea terminal.	88	88	88	8[a]	8[a]	8[a]	28	28	38	37[b]	37[b]	52[b]	Transportation agency
Time depot to terminal	(24)	(24)	(24)	(2)	(2)	(2)	(5)	(5)	(5)	(9)	(9)	(9)	
Time in terminal	(12)	(12)	(12)	(2)	(2)	(2)	(5)	(5)	(5)	(10)	(10)	(10)	
Time in transit (air or sea)	(52)	(52)	(52)	(4)	(4)	(4)	(18)	(18)	(28)	(18)	(18)	(33)	
4. From receipt at oversea terminal to recorded receipt by requisitioner.	56	56	56	4	4	4	11	6	6	15	10	10	Oversea command
Time in discharge	(24)	(24)	(24)	(2)	(2)	(2)	(3)	(3)	(3)	(7)	(7)	(7)	
Terminal intratheater movement and recording in requisitioners records	(32)	(32)	(32)	(2)	(2)	(2)	(8)	(3)	(3)	(8)	(3)	(3)	
Total		168 hours			15 days		50	45	55	65	60	75	

Source: DoD Instruction 4140.17, 23 Jan 62, Military Standard Requisitioning and Issue Procedures (MILSTRIP) and AR 725-50, CID.

Note: When not otherwise reflected, transmission times are charged to senaer.

[a]Area A: Europe
Area B: Alaska, Bermuda, Caribbean, Hawaii, and South America
Area C: All other overseas areas

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8 and which qualify under present practices to be transported by aerial means. Priority Designators associated with a given requisition in these cases are based solely on the criticality of the need rather than on an economical criterion. Issue Priority Groups 3 and 4, on the other hand, refer to requisitions whose Priority Designators are 9 through 20 and which qualify only for surface (sea) transportation. Table 2 gives the maximum allowed pipeline time as a function of the Issue Priority Group as well as the area where the requisition originated. Data available on requisition processing performance indicate that the maximum allowed times have been met in about 80 percent of the requisitions. Generally speaking, it has been observed that the trend marking this performance measure shows constant improvement.

The second source of information consisted of the present practices maintained by the involved inventory control activities. After considering both sources of information, 13 weeks and 2 weeks were the determined sealift and airlift pipeline times, respectively. Because there were no better data, these values were used for both the European and Pacific general geographic areas. While the sealift pipeline time was based primarily on current inventory control procedures, the airlift pipeline time was based on the maximum overseas order and shipping time allowances table.

F. AIRLIFT AND SEALIFT DISTANCES

U.S. Military Services' Supply Distribution Elements in Europe and those in the Pacific are located at different points within the theaters. Similarly, the depots in CONUS are sited at different locations. Resupply of various items involves multi-embarkation and multi-debarkation points. In an item-by-item analysis, it would be possible to determine the exact corresponding embarkation and debarkation points;

however, in a gross analysis, it is necessary to determine reasonable estimates for the average distances involved.

The airlift distances from CONUS to Europe and the Pacific were estimated to be 3500 and 6000 miles, respectively. In arriving at these figures, consideration has been given to (1) the location of the overseas facilities drawing the largest resupplies and (2) the fact that most initial embarkation and debarkation points are not located along the coasts.

The sealift distances were obtained from the airlift distances by applying the k_2 factor discussed earlier in this section. Included in the airlift and sealift distances were the inland transportation distances from depot to airport or from depot to seaport.

G. COSTS FOR MOVING CARGO BY AIRLIFT AND SEALIFT

A list of the cost elements that were taken into account in deriving the military airlift and sealift movement costs is given below.

COST ELEMENTS FOR MOVEMENT BY AIRLIFT AND SEALIFT

	Cost Elements
1.	Packaging
2.	CONUS Line Haul
3.	CONUS Port Handling
4.	Intercontinental Transit
5.	Overseas Port Handling
6.	Overseas Line Haul

In obtaining the military airlift cost, it was assumed that the loading space utilization for outbound/inbound cargo aircraft is 80 percent. In the interest of getting meaningful comparison, an attempt was made to obtain these costs estimates on as comparable a basis as possible. The difference between the military and commercial airlift costs are accounted for largely by the inclusion of aircraft depreciation cost and higher allowed daily utilization in the case of commercial case.

H. SAFETY AND ON-SHELF INVENTORY LEVELS

The safety stock level needed in order to insure a prescribed degree of performance will depend largely on the leadtime (pipeline time) and demand distributions and their defining parameters. The safety stock serves as a buffer against the variability inherent in the demand and leadtime distributions. When the leadtime is reduced, reductions will be possible in the average pipeline level (on-order inventory) and in the safety level without degrading the performance. Performance here is taken to mean the same as availability, or probability of not having a stockout. The reduction in pipeline inventory, which, in the present study represents the largest savings, can be obtained readily. This is so since, regardless of the demand and leadtime distributions, it will always be equal to the product of mean demand and the pipeline time differential (assuming of course that demand and leadtime are independent). However, in order to estimate the total savings that could be achieved by reducing the pipeline time, while still maintaining the same level of performance, it is necessary also to evaluate the reduction in mean on-shelf inventory that will result from the required lower safety level. The simulation results given in Section B of Chapter II and the appendix have dealt with these aspects at length. Here it will suffice to include a discussion of the main aspects affecting the final results of interest in this study.

For the purposes of this study, the leadtime distribution was assumed to be normal with coefficient of variation, σ_t/μ_t having values of 0.3 and 0.2. The demand distribution was assumed to be either Poisson or normal with a range of coefficients of variation, σ_x/μ_x, between 0.0316 and 0.316. These assumed distributions and the ranges of values of their parameters were selected in order to get first order approximations of the ultimate results sought. In fact, it is felt that they actually introduced a higher degree of variability than would the real situation. These most likely values yielded estimates of the required safety stocks, as a function of availability, which are on the high side. Tables 3 and 4 present the results of interest as a function of two availabilities: $P_o' = 0.95$, and $P_o' = 0.99$. More detailed results are given in the appendix.

For the sample results included in this report, it was assumed that the prescribed availability is $P_o' = 0.95$. Table 3 gives the safety levels that were required for some sample cases, and the resulting mean on-shelf inventories. It should be noted that the savings to be incurred by reducing the pipeline time will be a function of the mean on-shelf and in the pipeline inventories. The mean on-shelf for $P_o' = 0.95$ varied from 5.20 to 6.30 with the average value being 5.72 weeks of supply for the sealift case. The corresponding values for the airlift case were respectively 2.90, 3.30, and 3.02 weeks of supply. The allowed average reduction in on-shelf inventory that can be achieved by going from sealift to airlift is then 2.69, while its range is 2.00 to 3.00 weeks of supply.

Table 3. SAFETY LEVELS AND ON-SHELF PLUS ON-ORDER LEVELS REQUIRED FOR AVAILABILITY OF $P_o' = 0.95$

(QUANTITIES ARE PRESENTED IN TERMS OF MEAN WEEKLY DEMAND)

				Safety Level $\bar{I}_s/\lambda_w$		Safety Level Differential $\Delta(\bar{I}_s/\lambda_w)$	Mean On-Shelf Inventory $\bar{I}_{os}/\lambda_w$		On-Shelf Inventory Differential $\Delta(\bar{I}_{os}/\lambda_w)$	Mean On-Shelf & On-Order Inventory $(\bar{I}_{os}+\bar{I}_p)/\lambda_w$		On-Shelf & On-Order Inventory Differential $\Delta[(\bar{I}_{os}+\bar{I}_p)/\lambda_w]$
LDD	DD	RP	$\bar{I}_o/\lambda_w$	$\bar{t}_s=13$	$t_a=2$	$\Delta\bar{t}=11$	$\bar{t}_s=13$	$t_a=2$	$\Delta t=11$	$t_a=13$	$t_a=2$	$\Delta t=11$
N	p	1	4.3	3.8	0.7	3.1	6.3	3.2	3.1	19.3	5.2	14.1
N	p	1	4.3	3.5	0.7	2.8	6.0	3.1	2.9	19.0	5.1	13.9
N	p	1	4.3	3.0	0.5	2.5	5.3	3.2	2.1	18.3	5.2	13.1
N	p	1	4.3	2.6	0.6	2.0	4.9	2.9	2.0	17.9	4.9	13.0
N	N	1	4.3	2.6	0.5	2.1	5.2	2.2	3.0	18.2	4.2	14.0
N	N	1	4.3	3.4	0.7	2.7	5.[illegible]	[illegible].1	2.8	18.9	5.1	13.8
N	N	1	4.3	3.6	0.9	2.6	6.2	3.3	2.9	19.2	5.3	13.9
N	N	1	4.3	3.7	0.8	2.9	6.2	3.3	2.9	19.2	5.3	13.9
N	N	1	4.3	2.9	0.6	2.3	5.5	3.0	2.5	18.5	5.0	13.5
Average value				3.23	0.67	2.62	5.72	3.02	2.69	18.72	5.03	13.69
Minimum value				2.60	0.50	2.00	5.20	2.90	2.00	17.90	4.20	13.00
Maximum value				3.80	0.90	3.00	6.30	3.30	3.00	19.30	5.30	14.10

LDD = Leadtime distribution
DD = Demand distribution
RP = Review period
$\bar{I}_o$ = Mean reorder quantity

N = Normal distribution function
p = Poisson distribution function
λ_w = mean weekly demand

12-20-66-7

Table 4. SAFETY LEVELS AND ON-SHELF PLUS ON-ORDER LEVELS REQUIRED FOR AVAILABILITY OF $P_o' = 0.99$

(QUANTITIES ARE PRESENTED IN TERMS OF MEAN WEEKLY DEMAND)

				Safety Level $\bar{I}_s/\lambda_w$		Safety Level Differential $\Delta(\bar{I}_s/\lambda_w)$	Mean On-Shelf Inventory $\bar{I}_{os}/\lambda_w$		On-Shelf Inventory Differential $\Delta(\bar{I}_{os}/\lambda_w)$	Mean On-Shelf & On-Order Inventory $(\bar{I}_{os}+\bar{I}_p)/\lambda_w$		On-Shelf & On Order Inventory Differential $\Delta[(\bar{I}_{os}+\bar{I}_p)/\lambda_w]$
LDD	DD	RP	$\bar{I}_o/\lambda_w$	$\bar{t}_s=13$	$t_a=2$	$\Delta\bar{t}=11$	$\bar{t}_s=13$	$\bar{t}_a=2$	$\Delta t=11$	$t_a=13$	$t_a=2$	$\Delta\bar{t}=11$
N	p	1	4.3	6.0	1.8	4.2	8.4	4.1	4.3	21.4	6.[illegible]	15.3
N	p	1	4.3	6.0	1.5	4.5	8.3	3.8	4.5	21.3	5.8	15.5
N	p	1	4.3	5.0	1.3	3.7	7.4	3.7	3.7	20.4	5.7	14.7
N	p	1	4.3	4.3	1.3	3.0	6.6	3.3	3.3	19.6	5.3	14.3
N	N	1	4.3	4.5	1.3	3.2	6.9	3.7	3.2	19.9	5.7	14.2
N	N	1	4.3	5.5	1.7	3.8	8.0	4.1	3.9	21.0	6.1	14.9
N	N	1	4.3	6.2	1.8	4.4	8.7	4.2	4.5	21.7	6.2	15.5
N	N	1	4.3	5.4	1.4	4.0	7.9	3.8	4.1	20.9	5.8	15.1
N	N	1	4.3	5.0	1.5	3.5	7.6	3.9	3.7	20.6	5.9	14.7
Average value				5.33	1.51	3.81	7.76	3.84	3.91	20.76	5.84	14.91
Minimum value				4.30	1.30	3.00	6.60	3.30	3.20	19.60	5.30	14.20
Maximum value				6.20	1.80	4.50	8.70	4.20	4.50	21.70	6.20	15.50

LDD = Leadtime Distribution Function
DD = Demand Distribution Function
RP = Review period in weeks
$\bar{I}_o$ = Mean Reorder Quantity

N = Normal Distribution function
p = Poisson distribution function
λ_w = Mean weekly demand

12-20-66-8

V. ANALYSIS AND RESULTS

The method of analysis that could be applied and the type of results obtained depended heavily on data availability and the problems involved in compiling it. Current inventory control procedures followed by the U.S. Military Services supply systems consider only the stocked type line items to be a function of pipeline times. In other words only items with a recurrent demand are subject to order and shipping time allowances. Hence the analysis presented in this paper has been concerned only with the stocked type line items. The line items that fall in this general category are repair parts, and repairable and consumable line items. The stocked type line items constitute more than 90 percent of the line items in the military supply systems and number over 2-1/2 million.

Within the limitations of time and effort imposed on this study, it was not practical to approach the problem by an item-by-item analysis. In addition to that, it was quite doubtful whether such detailed analysis could have actually yielded more accurate projections for economic air cargo requirements in the mid 1970s. For the purpose of this study, it was necessary to develop a methodology that could be used readily in conjunction with the available data to obtain first order approximations of the results sought. Hence the approach in which commodities are grouped into classes has been utilized for this purpose.

The methodology given in Chapters II and III is presented in terms of line items. If each commodity class is regarded as being composed of homogeneous line items, the same approach can be applied to an analysis based on commodity classes. This approach yields results whose accuracy depends largely on the degree of homogeneity of the line items within each commodity class. The degree of homogeneity is dependent on such characteristics as unit weight, unit cost, and related inventory control procedures. In application of the commodity class approach, an attempt has been made to group as many homogeneous commodities as possible with as great homogeneity as the available raw military data permitted. Consequently, about 800 commodity classes have been used for this analysis. The modified data have been compiled and tabulated with respect to each Military Service by geographical area as shown in Table 5.

Table 5. MODIFIED DATA LISTING FOR A GIVEN MILITARY SERVICE IN A GIVEN GEOGRAPHICAL AREA

Commodity Class	Annual: Dollar Demand (A_{ij}) ($)	Annual: Weight (W_{ij}) (tons)	Rating α_{ij} ($/pound)
1	A_{11}	W_{11}	α_{11}
2	A_{21}	W_{21}	α_{21}
3	A_{31}	W_{31}	α_{31}
4	A_{41}	W_{41}	α_{41}
.	.	.	.
.	.	.	.
.	.	.	.

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Commodity Class	Annual		Rating α_{ij} ($/pound)
	Dollar Demand (A_{ij}) ($)	Weight (W_{ij}) (tons)	
1	A_{11}	W_{11}	α_{11}
2	A_{21}	W_{21}	α_{21}
3	A_{31}	W_{31}	α_{31}
4	A_{41}	W_{41}	α_{41}
.	.	.	.
.	.	.	.
.	.	.	.

The method given above has been used to determine the potential "net yearly savings" that the Department of Defense could realize under various airlift and sealift cost assumptions by airlifting all the commodity classes that qualify for airlift. Results in terms of the difference between airlift and sealift costs are presented for three different values of the inventory expense factor k_1 in Figure 7.

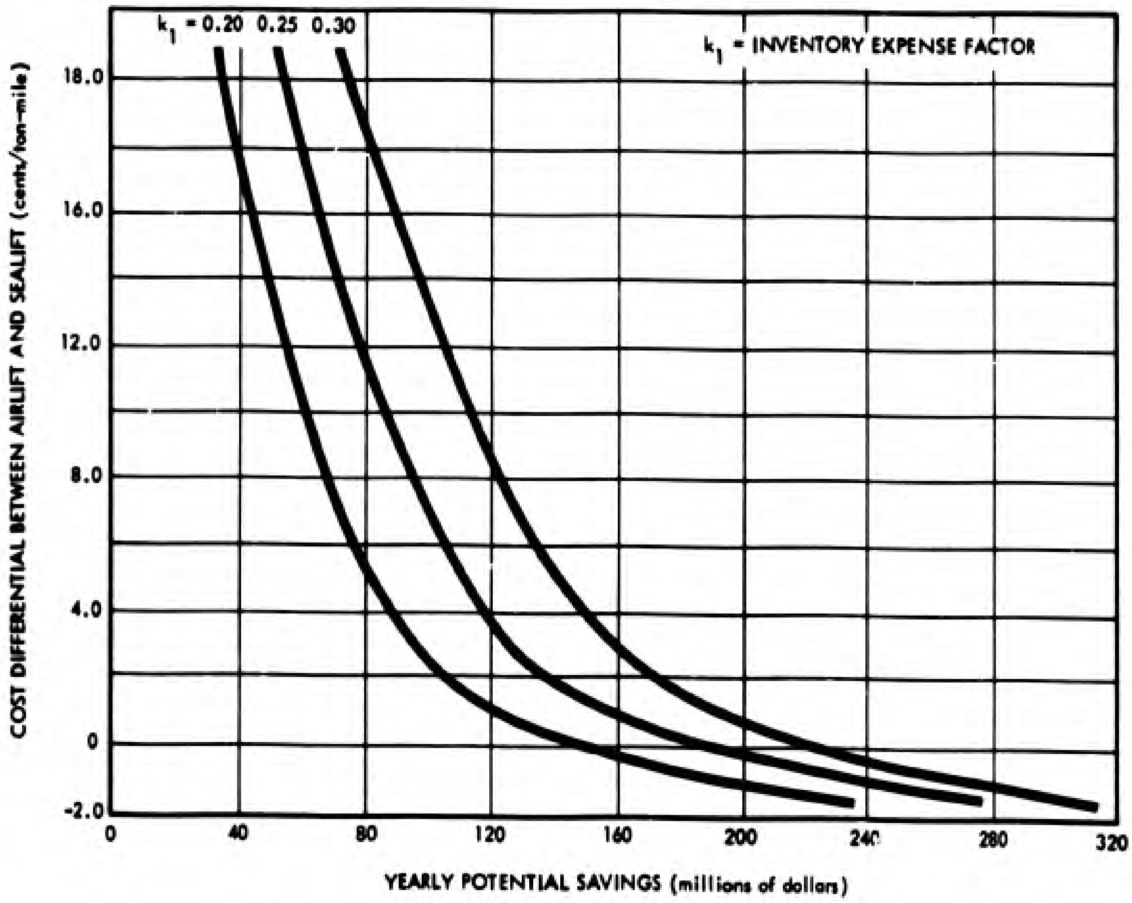

FIGURE 7. DOD Total Yearly Potential Savings

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Appendix

GENERALIZATION OF THE SIMULATION RESULTS

GENERALIZATION OF THE SIMULATION RESULTS

From studying the various simulated cases and noting the differences in input conditions and the resulting differences in output, we can made some general conclusions as follows.

The Effect of Service Level. We see that in all tables, as we would expect, the higher the average availability, $\overline{P'_o}$, the higher the safety level I_s required to achieve this service level, and, thus, the higher the corresponding average on-shelf inventory level $\overline{OS}$. However, in addition, considering the reductions in I_s and OS achievable when mean leadtime μ_t is reduced, we see that the highest reductions occur for the highest $\overline{P'_o}$ values, indicating that the greatest savings are achieved when operating at high service levels. Since in reality most systems operate at high average availability levels (usually $P'_o \geq 0.90$), reducing μ_t can effect some significant savings in OS.

The Effect of Mean Demand μ_x. When measuring OS, I_s, and P'_o in terms of μ_x, that is in weeks of supply, the absolute value of μ_x appears to have no effect. Comparing Tables A-9 and A-10 which differ only in μ_x (in the first case μ_x=10, in the second μ_x=1000) we see the results are almost identical. What little differences do exist in these two tables can probably be attributed to errors in reading graphs, rounding off and simulation variation. Intuitively, this makes sense since we can look at μ_x as a scaling factor, that is, instead of measuring in units we measure in hundreds of units or instead of measuring in pounds we measure in hundreds of pounds, etc.

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In addition to the two cases mentioned above, several additional points were run for Case A9 with μ_x-1000 and the results again were similar.

In order to study the effect of such factors as μ_x, σ_x, and σ_t, it is necessary to compare cases where only the factor under consideration is varied (in the case of μ_x it was necessary to compare Tables A-9 and A-10). In order to observe all factors, comparisons of Tables A-5 through A-19 on a two-by-two basis are necessary. To facilitate these comparisons, Table A-1 is included to show the differences in $\overline{OS}$ at $\overline{P_o'}$=0.95 for all combinations of pairs of cases represented by Tables A-5 through A-19. This is equivalent to comparing, on a pairwise basis corresponding $\overline{OS}/\mu_x$ vs. $\overline{P_o'}$ curves at the $\overline{P_o'}$=0.95 point.[1] Table A-1 is ranked in the order of the greatest difference down to the least difference. Table A-1 readily shows that μ_x has little effect since the combination of Cases A5 and A6 appears last in the table.

<u>The Effect of Leadtime μ_t.</u> The mean leadtime, μ_t, has a significant effect on system performance and inventory levels, and in fact, its effect was the main goal of the study. Both the curves and the tables show how reducing μ_t increases systems performance for a given inventory level, or decreases inventory level for a given system performance. On the I_s and $\overline{OS}$ vs. P_o' graphs, a curve lower and to the right gives a better system performance as described above. The reduction tables also clearly show this. For example, consider Table A-5, the row associated with an $\overline{P_o'}$=0.95. We see reducing μ_t from 13 to 6 allows a reduction of I_s from 3.8 to 2.0 weeks of supply. We can also see the effect of μ_t in Table A-1. The differences

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[1]Differences in $\overline{OS}$ and $\overline{I}_s$ values between different cases will be referred to as $\Delta(\overline{OS})$ or $\Delta(\overline{I}_s)$ and are not to be confused with reduction in OS and I_s within a case due to a decrease in μ_t.

Table A-1. DIFFERENCES AMONG CASES A1 TO A15 IN $\overline{OS}$ FOR $\overline{P}'_0 = 0.95$

Rank	Cases[a] Compared	Diff. in OS $\mu_t=13$	$\mu_t=2$	Rank	Cases[a] Compared	Diff. in OS $\mu_t=13$	$\mu_t=2$	Rank	Cases[a] Compared	Diff. in OS $\mu_t=13$	$\mu_t=2$	Rank	Cases[a] Compared	Diff. in OS $\mu_t=13$	$\mu_t=2$
1	A11, A15	7.1	1.9	28	A10, A13	3.0	0.4	55	A4, A10	1.3	0.0	82	A1, A8	0.4	0.1
2	A11, A14	7.1	1.8	29	A11, A12	2.9	0.4	56	A9, A12	1.2	0.3	83	A7, A8	0.4	0.1
3	A10, A15	7.0	1.9	30	A10, A12	2.8	0.4	57	A7, A13	1.1	0.3	84	A5, A13	0.4	0.0
4	A10, A14	7.0	1.8	31	A1, A11	2.8	0.3	58	A1, A9	1.1	0.2	85	A6, A13	0.4	0.0
5	A4, A15	5.7	1.9	32	A1, A10	2.7	0.3	59	A2, A4	1.1	0.2	86	A6, A8	0.3	0.2
6	A4, A14	5.7	1.8	33	A5, A11	2.7	0.4	60	A3, A12	1.1	0.1	87	A5, A8	0.3	0.2
7	A9, A15	5.4	1.8	34	A6, A11	2.7	0.4	61	A5, A9	1.0	0.3	88	A1, A2	0.3	0.1
8	A9, A14	5.4	1.7	35	A6, A10	2.6	0.4	62	A6, A9	1.0	0.3	89	A1, A13	0.3	0.1
9	A3, A15	5.3	1.6	36	A5, A10	2.6	0.4	63	A4, A8	1.0	0.2	90	A4, A9	0.3	0.1
10	A3, A14	5.3	1.5	37	A2, A11	2.5	0.2	64	A1, A3	1.0	0.0	91	A7, A9	0.3	0.0
11	A7, A15	5.1	1.8	38	A2, A10	2.4	0.2	65	A7, A12	0.9	0.3	92	A2, A5	0.2	0.2
12	A7, A14	5.1	1.7	39	A8, A11	2.4	0.2	66	A3, A6	0.9	0.1	93	A2, A6	0.2	0.2
13	A2, A15	4.6	1.7	40	A8, A10	2.3	0.2	67	A3, A5	0.9	0.1	94	A3, A7	0.2	0.2
14	A2, A14	4.6	1.6	41	A7, A11	2.0	0.1	68	A1, A7	0.8	0.2	95	A5, A12	0.2	0.0
15	A6, A15	4.4	1.5	42	A7, A10	1.9	0.1	69	A2, A9	0.8	0.1	96	A6, A12	0.2	0.0
16	A6, A14	4.4	1.4	43	A3, A11	1.8	0.3	70	A5, A7	0.7	0.3	97	A12, A13	0.2	0.0
17	A5, A15	4.4	1.5	44	A4, A13	1.7	0.4	71	A6, A7	0.7	0.3	98	A3, A9	0.1	0.2
18	A5, A14	4.4	1.4	45	A3, A10	1.7	0.3	72	A8, A13	0.7	0.2	99	A1, A5	0.1	0.1
19	A1, A15	4.3	1.6	46	A9, A11	1.7	0.1	73	A2, A3	0.7	0.1	100	A1, A6	0.1	0.1
20	A1, A14	4.3	1.5	47	A9, A10	1.6	0.1	74	A8, A9	0.7	0.1	101	A1, A12	0.1	0.1
21	A12, A15	4.2	1.5	48	A4, A12	1.5	0.4	75	A2, A13	0.6	0.2	102	A2, A8	0.1	0.1
22	A12, A14	4.2	1.4	49	A2, A4	1.4	0.3	76	A3, A8	0.6	0.1	103	A10, A11	0.1	0.0
23	A8, A15	4.1	1.7	50	A9, A13	1.4	0.3	77	A4, A7	0.6	0.1	104	A14, A15	0.0	0.1
24	A8, A14	4.1	1.6	51	A4, A11	1.4	0.0	78	A2, A7	0.5	0.1	105	A5, A6	0.0	0.0
25	A13, A15	4.0	1.4	52	A4, A5	1.3	0.4	79	A8, A12	0.5	0.0				
26	A13, A14	4.0	1.4	53	A4, A6	1.3	0.4	80	A3, A4	0.4	0.3				
27	A11, A13	3.1	0.4	54	A3, A13	1.3	0.1	81	A2, A12	0.4	0.2				

[a]Cases A1 to A15 are defined in Tables A-5 to A-19.

11-2-66-16

when μ_t=2 are much smaller than when μ_t=13.

Reducing μ_t reduces the system variability. Variation in the system comes from two sources, demand and lead time. Reducing mean leadtime reduces the variation of demand over the leadtime. This is easiest to see in the constant leadtime case. If the leadtime is constant at μ_t, then the standard deviation of demand over the leadtime is $\sqrt{\mu_t}\sigma_x$, where σ_x is the standard deviation of demand over one period and μ_t is expressed in number of periods. Reducing μ_t to a lower value, μ_t', gives a standard deviation of demand over the leadtime of $\sqrt{\mu_t'}\sigma_x$. The reduction in standard deviation is σ_x $(\sqrt{\mu_t}-\sqrt{\mu_t'})$. Since one purpose of safety stock is to guard against this variation, reducing μ_t allows us to reduce safety levels and still provide previous system performance.[1] We will show below how the standard deviation of demand over the leadtime can be calculated for stochastic leadtimes.

The Effect of σ_x. Keeping all things constant but increasing σ_x increases the system variation and hence yields poorer performance. Since in considering σ_x, all other factors will be held constant, we can look at the coefficient of variation of demand, v_x, which is equal to σ_x/μ_x. The pairs of cases where only v_x differs are given by tables (A-5, A-6) (A-7, A-8), and (A-9, A-10). For the first two pairs, v_x is changed from 0.316 (A-5 and A-6) to 0.0316 (A-6 and A-8) and for the third pair, v_x is changed from 0.3 (A-9) to 0.2 (A-12). Observing Table A-1, we see that for the greatest change in v_x (Cases A1 vs. A2 and A3 vs. A4) they appear rather low in the table listing ranking 88 and 80, respectively, out of 105. For Cases A5 and A8, where v_x changes from 0.3 to 0.2, the ranking is 86. It appears strange that A5 vs. A8 falls between the

[1]The other purpose of safety stock, as mentioned previously, is to provide for average usage and demand fluctuation for one period in addition to the leadtime.

other two in ranking; however, the distribution shapes may have an effect here. For A5 vs. A8 both demand distributions are normal with mean 10.0. In the others, A1 and A3 are Poisson with mean 10, while A2 and A4 are Poisson with mean 1000. Although we have stated that μ_x has no effect, a Poisson with μ_x=1000 is almost identical to a normal distribution while a Poisson with μ_x=10 is still considerably skewed. Furthermore, there is not a great deal of difference between a rank of 80 and 88 in Table A-1; this difference could be attributed to rounding off or simulation error.

We can conclude that while v_x does have an effect, it appears less than some of the other factors.

<u>The Effect of σ_t or v_t.</u> The leadtime variation appears to have a strong influence on system performance. The cases in which we can get an estimate of the magnitude of this effect are listed in Table A-2 below, with their ranking in Table A-1 and their change in the coefficient of variation of leadtime v_t. We see the greater the $\Delta(v_t)$, the higher the rank. Even for the smallest $\Delta(v_t)$'s, ($\Delta(v_t) = 0.1$), the ranks were between 59 and 74. These were above those for the largest $\Delta(v_x)$ described above which equalled 0.2844. Hence, the performance curves appear to be more sensitive to v_t than v_x.

Thus far we have been looking at μ_x, μ_t, v_x and v_t separately. It is intuitively appealing that the most important factor on system performance would be the total "system variation", that is, the variation of demand over a leadtime. This variation depends upon all the above factors. Since we observed that μ_x is only a scaling factor, we will be interested in computing σ/μ_x, where σ is the standard deviation of demand over the leadtime. The following equation gives σ/μ_x.

$$\sigma/\mu_x = \sqrt{\mu_t v_x^2 + \mu_t^2 v_t^2} \,. \qquad (7)$$

From this equation we see that μ_t and v_t generally play a more important role than v_x in determining σ/μ_x.

Table A-2. $\Delta(\overline{OS})$ RANKING VS. $\Delta(v_t)$

Case Comparisons	$\Delta(v_t)$	Rank in Table A-1
A1, A3	0.1	64
A2, A4	0.1	59
A8, A9	0.1	74
A5, A7	0.1	70
A1, A10	0.3	32
A2, A11	0.3	37
A3, A10	0.2	45
A3, A11	0.2	43
A1, A14	0.7	20
A2, A15	0.7	13

Now let us look at Table A-1. If our conjecture is correct, the highest ranking comparisons in Table A-1 should correspond to cases which have the greater change in σ/μ_x while the lowest ranking comparison should have the least. Table A-3 is a sampling from Table A-1 showing rank and change in σ/μ_x. We see that in general the greatest $\Delta(\overline{OS})$ are associated with the greatest $\Delta(\sigma/\mu_x)$ and conversely the smallest $\Delta(\overline{OS})$ are associated with the smallest $\Delta(\sigma/\mu_x)$.

Let us now choose from Tables A-5 to A-19, the best performance situation, medium performance situations, and worst performance situation. We will define best performance as the situation which, for an $\overline{P'_o}=0.95$ has the lowest $\overline{OS}$ value. We

Table A-3. EFFECT OF σ/μ_x

Rank	Cases Compared	$\Delta(\sigma/\mu_x)$ (μ_t = 13)	$\Delta(\sigma/\mu_x)$(μ_t = 2)
1	A11, A15	11.918	1.586
6	A4, A14	11.180	1.647
13	A2, A15	9.115	1.398
35	A6, A10	2.910	0.288
46	A9, A11	2.186	0.066
60	A3, A12	1.230	0.150
77	A4, A7	2.016	0.181
86	A5, A8	0.085	0.062
97	A12, A13	0.020	0.015
105	A5, A6	0	0

Table A-4. SYSTEM PERFORMANCE VS. σ/μ_x, $\overline{P_o^T}$=0.95

Table	$\overline{\overline{OS}}/\mu_x$	μ_t	σ/μ_x
A11	2.9	2	0.414
A12	3.3	2	0.750
A10	3.6	13	1.140
A2	4.3	6	1.803
A14	4.7	2	2.050
A9	5.2	13	2.600
A7	5.5	13	2.820
A8	5.9	13	3.965
A5	6.2	13	4.050
A15	10.6	13	13.020

will take a sample out of these tables and see how they compare with their associated σ/μ_x values. The results are presented in Table A-4 above. Thus, we see that there is a strong correspondence between σ/μ_x and performance as we would expect.

This rather strong relation between σ/μ_x and $\overline{OS}$ can be used to give us a first approximation predictor of what effect a change in system parameters (in our study we are mainly interested in a change in μ_t) would have on the on-shelf inventory level. In addition, there is an equally strong relation between σ/μ_x and I_s. Table A-4 was extended for all situations represented in Tables A-5 to A-19. A similar table was prepared for an $P_o'=0.90$. In addition, these tables included I_s/μ_x values as well. The results were plotted in Figure A-1. The point scatter is relatively small and the curves were eyed in. Unfortunately, no values of σ/μ_x were available in the 4 to 13 range. The point scatter is attributed to the effects of factors not included in σ/μ_x, such as distribution shape. However, the small amount of point scatter would seem to indicate that these other factors are secondary effects. No statistical analysis was performed at this time to determine the extent of the point variation. However, from the graph this does not appear too great.

These types of curves provide us with a quick empirical means of evaluating a change in μ_t, σ_x or σ_t, providing we have an estimate of these values. For example, suppose we are operating with a μ_t of 13 and the μ_x, σ_x, and σ_t are such that $\sigma/\mu_x=4$. Thus according to Figure A-1, for a P_o' of 0.95, I_s/μ_x should be approximately 3.4 and OS/μ_x should be approximately 6.3. Now if we reduce μ_t to 2, say, this reduces σ/μ_x to .75. Thus, to still achieve an $\overline{P_o'}$ of 0.95, I_s/μ_x must be 0.9 and the resulting OS/μ_x would be 3.3. We could expect then a savings in OS of about 3 weeks of supply.

This type of empirical approach is very appealing in that a great deal of simulation time could be saved in actual applications. Once such curves are generated, they could be used in a variety of situations to provide quick estimations. For example, Figure A-1 would apply to any situation where the review period is one and S-s is $4.3\mu_x$.

Let us now consider the role σ/μ_x might play in influencing the amount of reduction achievable in OS when μ_t is decreased. The case with the largest σ/μ_x value studied is represented by Table A-18 while the case with the least value is represented

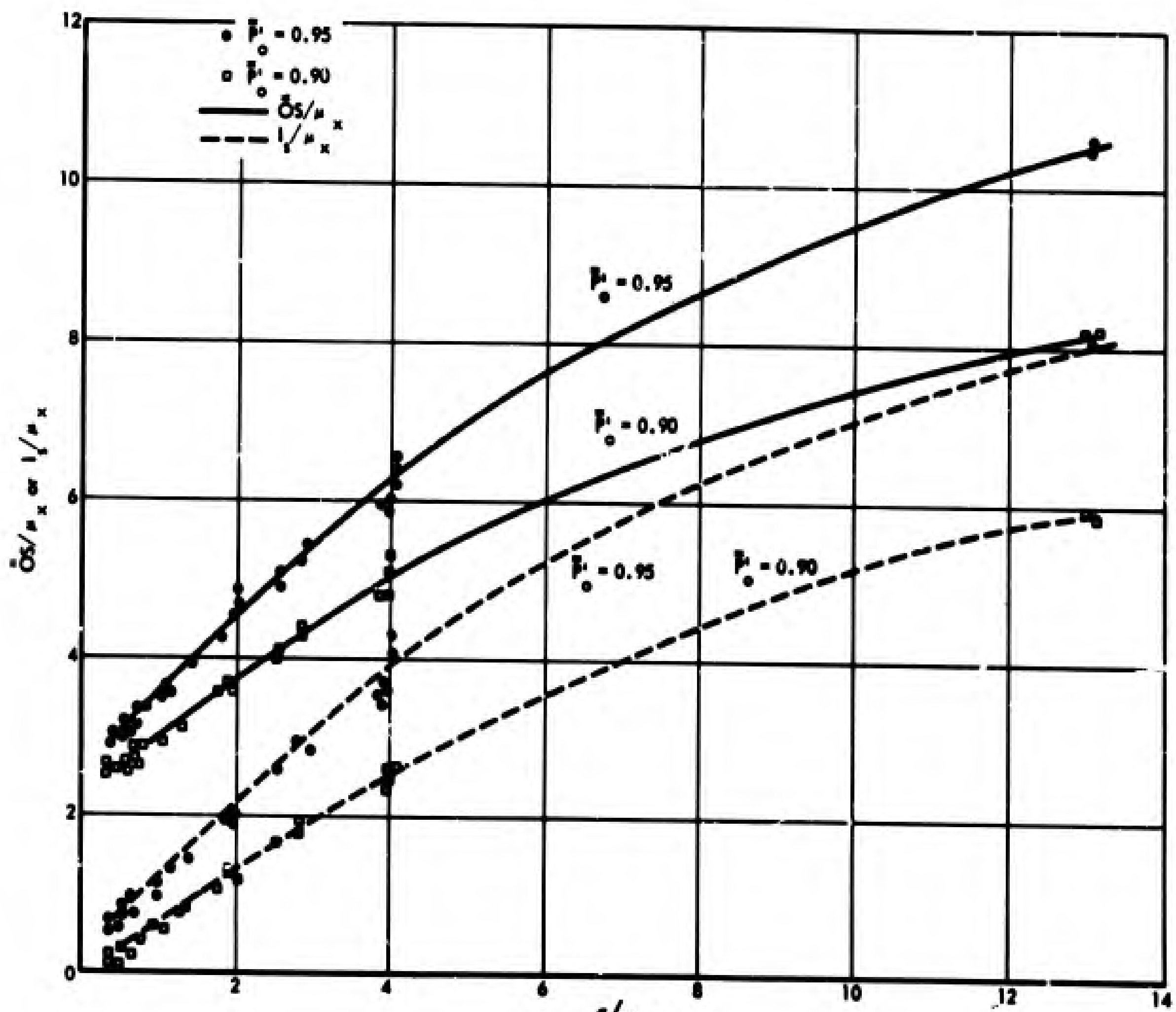

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FIGURE A-1. $\bar{\bar{O}}S/\mu_x$ and I_s/μ_x Versus σ/μ_x

by Table A-15. Comparing results for $\overline{P_o'}$=0.95, we see the allowable reductions in I_s/μ_x are 6.1 and 0.7 respectively while the resulting reductions in OS/μ_x are 5.9 and 0.6, respectively. As expected, it appears rather convincingly that larger reductions in I_s/μ_x and $\overline{OS}/\mu_x$ could be achieved in cases with larger σ/μ_x values, i.e., larger system variability. For Case A14, cutting μ_t from 13 to 2 reduced σ/μ_x from 13.18 to 2.05, while for Case A11, σ/μ_x is reduced from 1.08 to 0.41. Thus we see in Case A14 there was greater reduction in σ/μ_x and correspondingly greater reductions in I_s and OS. We can conclude then, that we achieve greater savings in OS for a reduction in μ_t when we are dealing with cases of larger "system variability" for which σ/μ_x appears to be an adequate measure. This effect can also be shown from graphs such as Figure A-1.

<u>The Effect of I_o.</u> Up to now, all runs that were considered had an I_o/μ_x=4.3, that is, I_o was approximately one month of supply. To study the effect of I_o, the conditions of Case A9 were held constant except for I_o which was set at μ_x, $2\mu_x$, $6\mu_x$ and $8\mu_x$. These results are presented in Tables A-23 to A-26, respectively. These, together with Table A-13, will give us an indication as to the effect of I_o when all other conditions are held constant. The I_s/μ_x and $\overline{OS}/\mu_x$ values for $\overline{P_o'}$=0.95 from Tables A-23 to A-26 were plotted against I_o in Figure A-2 to show the effect of increasing I_o. We see that as I_o increases, I_s/μ_x decreases, as we would expect; that is, we require less safety stock since the average order size is larger causing fewer orders to be placed and putting us in a vulnerable position less often. Nevertheless, we see that $\overline{OS}$ increases. The reduction allowed in I_s due to larger order sizes which would tend to decrease OS is more than offset by the larger order size itself which tends to increase OS. However, the net increase in OS as I_o increases is far less than a one-to-one relationship, although it does appear linear.

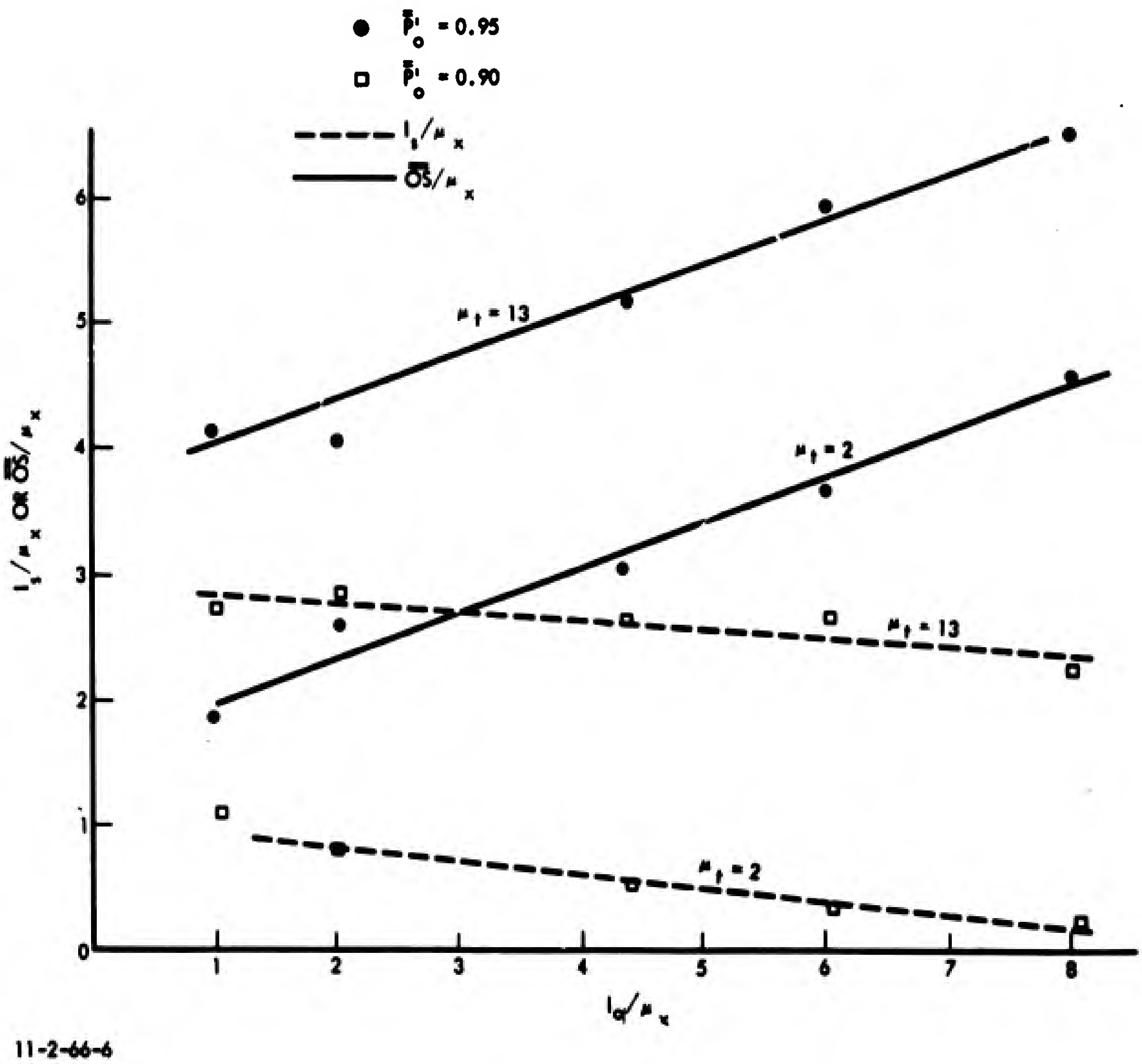

FIGURE A-2. Effect of I_o

In order to assess the importance of the I_o effect, let us compare $\Delta(\overline{OS}/\mu_x)$ values for $\bar{P}_o'=0.95$ with Table A-1. We see that for $\mu_t=13$, the $\Delta(\overline{OS}/\mu_x)$ between the $I_o=\mu_x$ and $I_o=8\mu_x$ cases from Tables A-23 and A-26 is equal to 2.4 and would receive a ranking of around 40, indicating only a moderate effect. However, up to now, we see from Table A-1 that all effects have a strong interaction with μ_t and the Δ's for $\mu_t=2$ were considerably smaller than 2.6 and those for $\mu_t=13$. This is not true here. The $\Delta(\overline{OS}/\mu_x)$ for $\mu_t=2$ is of the same order as that for $\mu_t=13$. Thus, the effect of I_o seems to be independent of μ_t. Intuitively, this makes sense since increasing I_o causes fewer orders to be placed which results in being in a vulnerable position less often and increases the system availability. This is paid for by an increase in OS. Changing the leadtime has no effect on the number of orders placed; only the average demand affects this. Since all values are in terms of weeks supply, the effect of μ_x is normalized.

Let us now consider the effect of I_o on the allowable reductions in I_s and OS when μ_t is reduced. From Tables A-23 to A-26, inclusive, and A-13, as I_o increases we see that for $\bar{P}_o'=0.95$, the reductions achievable in I_s/μ_x are 1.5, 1.9, 2.1, 2.3, 2.0, respectively, and those of $\overline{OS}/\mu_x$ are 2.2, 1.9, 2.2, 2.2, 2.0, respectively. Thus, as I_o increases, the reductions achievable in I_s appear to increase. However, because of the large order sizes, the reductions achievable in actual on-shelf inventory appear to remain relatively constant at approximately 2 weeks of supply, indicating a lack of sensitivity of OS to the size of I_o. It should be noted that this is quite evident from Figure A-2.

<u>The Effect of Review Period</u>. Again using Case A9, several different review periods were studied. These results are presented in Tables A-20, A-21, and A-22 for review periods of 2, 4, and 6 weeks, respectively. Comparing these and Table A-13

we see as the review period is increased from one to two and then to four, we get poorer system performance, that is, for given $\bar{P}_o'$'s, $\overline{OS}$ and I_s increase. However, for a review period of six, we see that performance is better than that for four and almost as good as that for two. This seems surprising at first since intuitively we feel the more often we review inventory, the better should be our performance. However, we must keep in mind that for these cases, since I_o was equal to 4.3 weeks supply, on the average we should place an order roughly every 4.3 weeks. For a review period of one week, we then, on the average, would place an order on every fifth week. For a review period of two weeks, we would place an order, on the average, every sixth week. Thus instead of catching the placing of an order on the fifth, we must wait until the sixth week. Since this is further from the average of 4.3, we expect poorer performance. If the review period were four weeks, then we would have to wait until the eighth week to place an order causing even poorer performance. However, when we increase the review period to six weeks, we can place an order every sixth week, on the average, and hence do better than that for a review period of four. The reason that a review period of six is poorer than that of two appears due to two reasons. First, the more frequent review period allows ordering at intervals closer to 4.3, i.e., at the 6th, 10th, 14th, etc., week. Using a review period of six we can order only at the 6th, 12th, 18th, etc., week. Second, the variation in the demand causes deviations from average usage. The more frequent reviewing helps catch these deviations.

SUMMARY AND CONCLUSIONS

A simulation model describing an s,S periodic inventory control procedure was utilized on a variety of cases to study the effect of reducing mean leadtime on safety stock levels required, in order to maintain given degrees of service, and to study the sensitivity of the system to certain parameters. The following conclusions from this study were drawn:

1. For the operating practice of the military resupply systems studied, reducing the mean leadtime from 13 weeks (sealift) to 2 weeks (airlift) would allow a reduction of approximately 3 weeks of supply in safety stock levels with a corresponding reduction in average on-shelf inventory of approximately 3 weeks supply.

2. The higher the availability specification for system performance, the larger the allowable reduction in safety stock, when mean leadtime is reduced, which in turn produces higher potential savings resulting from lower mean on-shelf inventory.

3. Changes in system performance seem to be most sensitive to mean leadtime and variation in leadtime. While system performance is somewhat sensitive to variation in demand, it does not appear as significant a factor as the leadtime parameters.

4. Changes in system performance appear to be less sensitive to the shape of the distributions, however only a limited number of cases were studied on which to base this conclusion.

5. The mean demand has little effect other than to act as a scaling factor. If all quantities are measured in terms of weeks of supply (assuming a week is the basic time period considered), the absolute value of the mean is not significant. However, if the absolute value of demand is very low, say less than 20 units per year, the difficulties that may result from the effect of rounding off will have to be considered carefully.

6. The standard deviation of demand over leadtime (as given by Equation (7)) appears to allow us to measure the effects of changing the mean leadtime and/or leadtime and demand variations. As these appear to be the most significant factors (for a fixed I_o and review period) this can be a powerful tool for application purposes. In addition to measuring effects

on on-shelf inventory, this tool can indicate what the reduced safety levels should be when leadtime is reduced in order to achieve specified availability levels.

7. Keeping all other conditions fixed, increasing I_o increases the availability level but also increases average on-shelf inventory. If availability level is fixed, increasing I_o allows for smaller I_s. However, even though I_s is reduced, because of larger order sizes, OS increases. This amount of increase appears to be relatively insensitive to the mean value of the leadtime. It should be kept in mind that the I_o value, representing closely the economic order quantity, is dependent also on other factors than availability, such as ordering cost, etc. We are concerned here only with what effect I_o has on performance. The reductions achievable in I_s and OS for fixed performance level when leadtime is reduced do not change appreciably as I_o is increased, even though the actual values of I_s and OS do. The achievable reductions appear relatively insensitive to variations in I_o.

8. Changing the time between inventory reviews has an effect on system performance. A review period of one seems to be best (excluding the possibility of more frequent reviews than once per period). There appear to be two factors which have influence here: the frequency of review and when the review occurs. It is better to have a review occur just after the average time between placing orders (this is roughly equivalent to the number of weeks of supply of I_o) in order to "catch" the inventory position soon after it falls below s. However, if there is a great deal of variation in demand, frequency of review becomes more important in order to catch the deviations from average usage. A review period of one is the best with regard to both these considerations. The costs associated with making a review will, of course, be the determining factor in how frequent reviews are made.

The reductions achievable in I_s and OS for fixed performance level when leadtime is reduced, as is the case with I_o, do not change appreciably as r is varied and hence also appear to be rather insensitive to r, even though the I_s and OS values are dependent on r.

Explanation of symbols used in Tables A-5 to A-26:

Vector Defining Each Case = $(F_x, \mu_x, v_x; F_t, \mu_t, v_t; r, q)$

F_x = demand distribution

μ_x = mean weekly demand

v_x = coefficient of variation of demand

F_t = leadtime distribution function

μ_t = mean leadtime

v_t = coefficient of variation of leadtime

r = review period

q = reorder quantity

Symbols defining distribution functions:
N = normal
P = Poisson
C = constant
U = uniform

$\bar{P}'_o$ = average availability

I_s = safety level

$\overline{OS}$ = average on-shelf inventory

Amt. Red. = Amount reduction in leadtime

Amt. Red. 13-6 = reduction of leadtime from 13 weeks to 6 weeks.

Reorder Quantity is given in terms of weeks of supply.

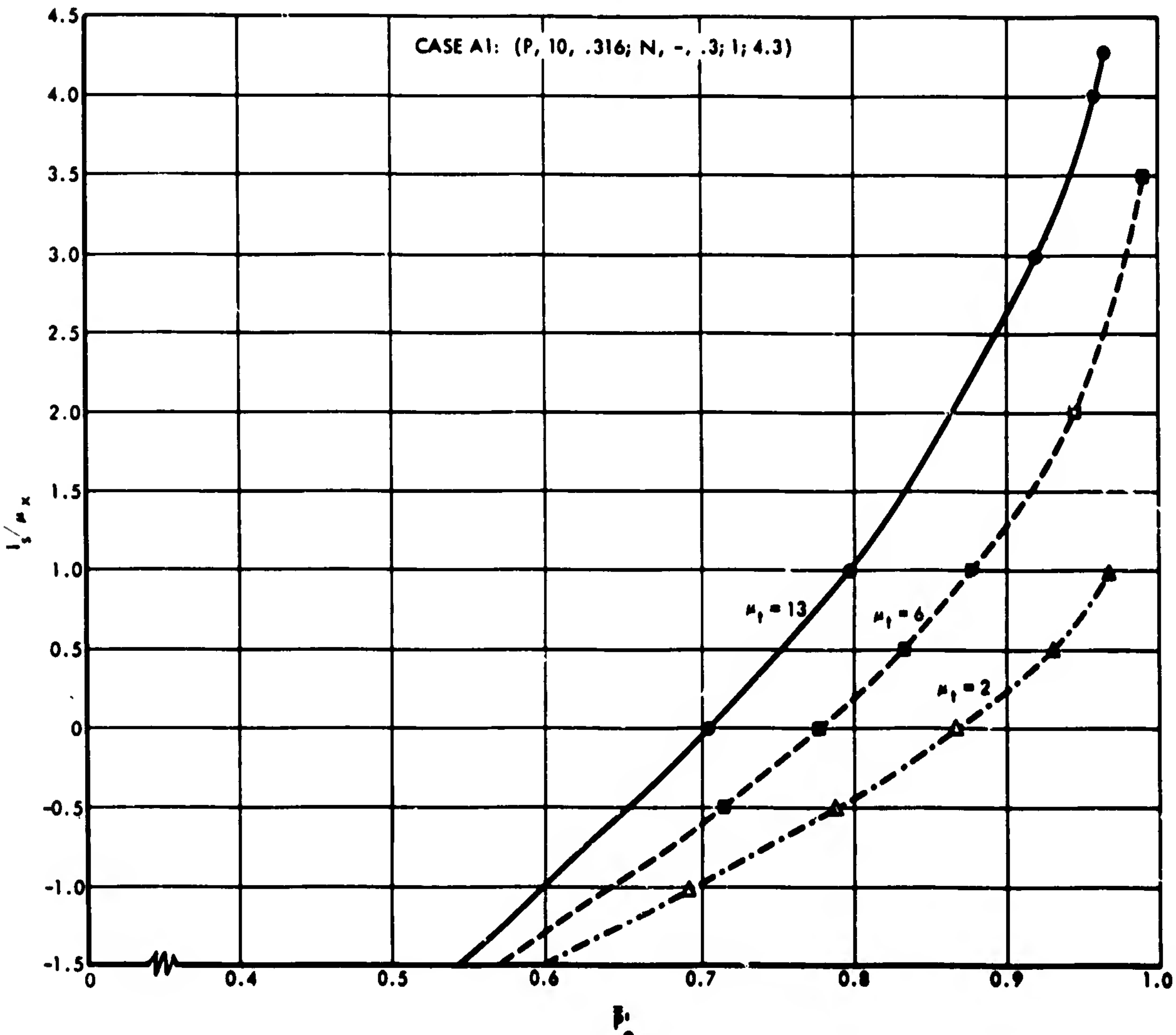

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FIGURE A-3. I_s/μ_x Versus $\bar{\bar{A}}$

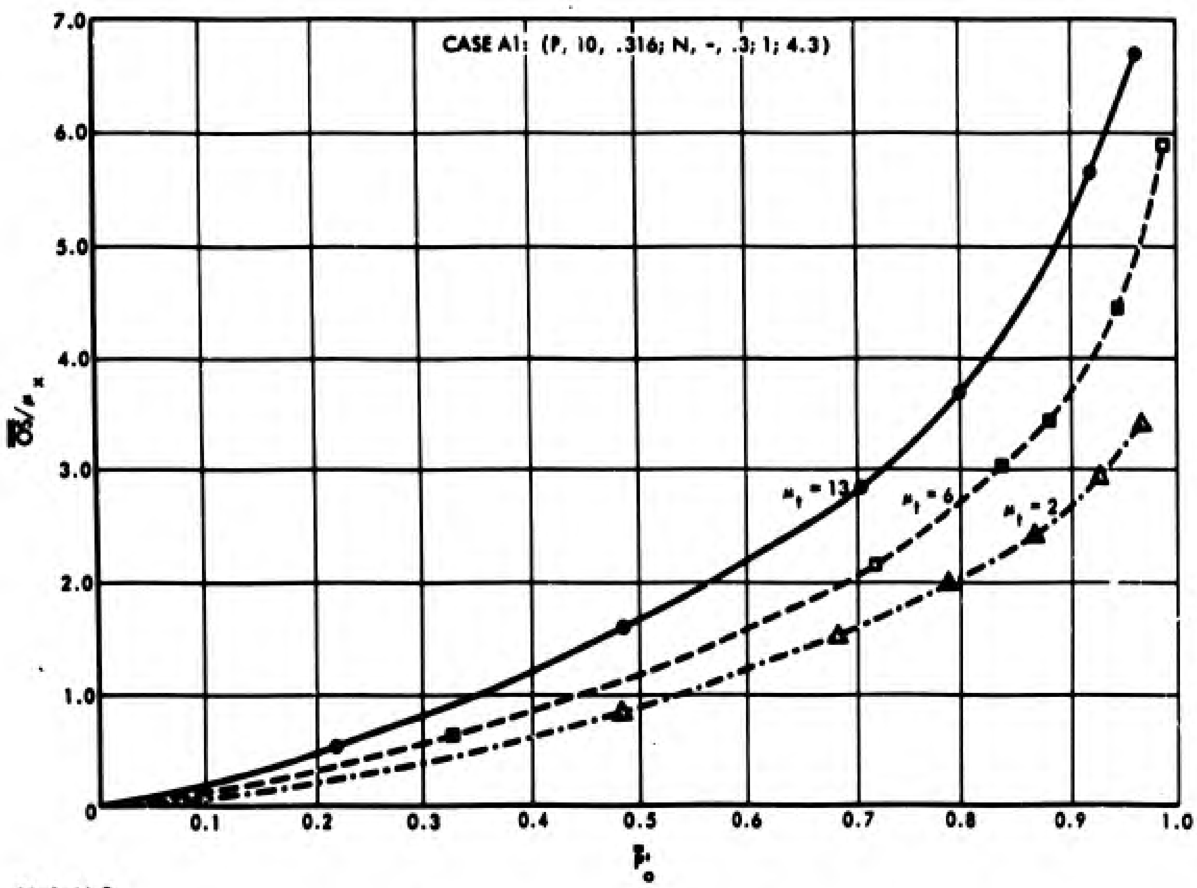

FIGURE A-4. $\overline{\overline{OS}}/\mu_x$ Versus $\bar{\bar{A}}$

Table A-5. CASE A1 ($P, 10, 316; N, \mu_t, .3; 1; 4.3$)

$\bar{P}'_o$	I_s/μ_x						$\overline{OS}/\mu_x$					
	Mean Ldtm.			Amt. Red.			Mean Ldtm.			Amt. Red.		
	13	6	2	13-6	6-2	13-2	13	6	2	13-6	6-2	13-2
0.990	6.0	3.8	1.8	2.2	2.0	4.2	8.4	6.3	4.1	2.1	2.2	4.3
0.965*	4.3	2.4	0.9	1.9	1.5	3.4	6.7	5.0	3.4	1.7	1.6	3.3
0.950	3.8	2.0	0.7	1.8	1.3	3.0	6.3	4.6	3.2	1.7	1.4	3.1
0.900	2.6	1.3	0.3	1.3	1.0	2.3	5.3	3.7	2.7	1.6	1.0	2.6
0.850	1.8	0.8	-0.1	1.0	0.9	1.9	4.4	3.2	2.4	1.2	0.8	2.0
0.800	1.0	0.2	-0.4	0.8	0.6	1.4	3.7	2.7	2.1	1.0	0.6	1.6
0.700	-0.1	-0.6	-0.9	0.5	0.3	0.8	2.8	2.1	1.7	0.7	0.4	1.1

Table A-6. CASE A2 ($P, 1000, .0316; N, \mu_t, .3; 1; 4.3$)

$\bar{P}'_o$	I_s/μ_x						$\overline{OS}/\mu_x$					
	Mean Ldtm.			Amt. Red.			Mean Ldtm.			Amt. Red.		
	13	6	2	13-6	6-2	13-2	13	6	2	13-6	6-2	13-2
0.990	6.0	3.5	1.5	2.5	2.0	4.5	8.3	5.8	3.8	2.5	2.0	4.5
0.967*	4.3	2.4	0.9	1.9	1.5	3.4	6.7	4.7	3.3	2.0	1.4	3.4
0.950	3.5	2.0	0.7	1.5	1.3	2.8	6.0	4.3	3.1	1.6	1.2	2.8
0.900	2.3	1.1	0.3	1.2	0.8	2.0	4.8	3.6	2.7	1.2	0.9	2.1
0.850	1.6	0.6	0.0	1.0	0.6	1.6	4.1	3.0	2.4	1.1	0.6	1.7
0.800	1.0	0.2	-0.3	0.8	0.5	1.3	3.6	2.6	2.1	1.0	0.5	1.5
0.700	0.0	-0.5	-0.8	0.5	0.3	0.8	2.8	2.0	1.6	0.8	0.4	1.2

Table A-7. CASE A3 (P,10,.316;N,μ_t,.2;1;4.3)

$\bar{P}'_o$	I_s/μ_x						$\overline{OS}/\mu_x$					
	Mean Ldtm.			Amt. Red.			Mean Ldtm.			Amt. Red		
	13	6	2	13-6	6-2	13-2	13	6	2	13-6	6-2	13-2
0.990	5.0	2.8	1.3	2.2	1.5	3.7	7.4	5.2	3.7	2.2	1.5	3.7
0.984*	4.3	2.4	1.0	1.9	1.4	3.3	6.7	4.7	3.5	2.0	1.2	3.2
0.950	3.0	1.4	0.5	1.6	0.9	2.5	5.3	3.9	3.2	1.4	0.7	2.1
0.900	1.9	0.8	0.2	1.1	0.6	1.7	4.4	3.2	2.6	1.2	0.6	1.8
0.850	1.2	0.2	-0.1	1.0	0.3	1.3	3.7	2.8	2.3	0.9	0.5	1.4
0.800	0.5	-0.1	-0.4	0.6	0.3	0.9	3.2	2.5	2.0	0.7	0.5	1.2
0.700	-0.4	-0.8	-0.9	0.4	0.1	0.5	2.5	1.9	1.6	0.6	0.3	0.9

Table A-8. CASE A4 (P,1000,.0316;N,μ_t,.2;1;4.3)

$\bar{P}'_o$	I_s/μ_x						$\overline{OS}/\mu_x$					
	Mean Ldtm.			Amt. Red.			Mean Ldtm.			Amt. Red.		
	13	6	2	13-6	6-2	13-2	13	6	2	13-6	6-2	13-2
0.990*	4.3	2.5	1.3	1.8	1.2	3.0	6.6	4.6	3.3	2.0	1.3	3.3
0.950	2.6	1.3	0.6	1.3	0.7	2.0	4.9	3.6	2.9	1.3	0.7	2.0
0.900	1.6	0.7	0.3	0.9	0.4	1.3	4.1	3.0	2.6	1.1	0.4	1.5
0.850	1.0	0.3	-0.1	0.7	0.4	1.1	3.5	2.6	2.3	0.9	0.3	1.2
0.800	0.4	-0.1	-0.3	0.5	0.4	0.7	3.0	2.3	2.0	0.7	0.3	1.0
0.700	-0.4	-0.8	-0.8	0.4	0.0	0.4	2.3	1.8	1.6	0.5	0.2	0.7

Table A-9. CASE A5
(N,10,.3;N,μ_t,.3;1;4.3)

F'_o	I_s/ν_x Mean Ltdm.		I_s/ν_x Amt. Red.	$\overline{OS}/\nu_x$ Mean Ldtm.		$\overline{OS}/\nu_x$ Amt. Red.
	13	2	13-2	13	2	13-2
0.990	6.2	1.8	4.4	8.7	4.2	4.5
0.964*	4.3	1.1	3.2	6.8	3.5	3.3
0.950	3.6	0.9	2.7	6.2	3.3	2.9
0.900	2.6	0.3	2.3	5.2	2.8	2.4
0.850	1.8	-0.1	1.9	4.4	2.4	2.0
0.800	1.1	-0.4	1.5	3.8	2.1	1.7
0.700	0.0	-0.9	0.9	2.9	1.7	1.2

Table A-10. CASE A6
(N,1000,13;N,μ_t,.3;1;4.3)

F'_o	I_s/ν_x Mean Ldtm.		I_s/ν_x Amt. Red.	$\overline{OS}/\nu_x$ Mean Ldtm.		$\overline{OS}/\nu_x$ Amt. Red.
	13	2	13-2	13	2	13-2
0.990	5.5	1.4	4.1	7.9	3.8	4.1
0.963*	4.3	1.0	3.3	6.7	3.4	3.3
0.950	3.7	0.8	2.9	6.2	3.3	2.9
0.900	2.5	0.3	2.2	5.2	2.7	2.5
0.8[illegible]	1.7	-0.0	1.7	4.4	2.4	2.0
0.800	1.0	-0.4	1.4	3.8	2.1	1.7
0.700	-0.1	-0.9	0.8	2.9	1.7	1.2

Table A-11. CASE A7
(N,10,.3;N,μ_t,.2;1;4.3)

F'_o	I_s/ν_x Mean Ldtm.		I_s/ν_x Amt. Red.	$\overline{OS}/\nu_x$ Mean Ldtm.		$\overline{OS}/\nu_x$ Amt. Red.
	13	2	13-2	13	2	13-2
0.990	5.0	1.5	3.5	7.6	3.9	3.7
0.985*	4.3	1.2	3.1	6.8	3.5	3.3
0.950	2.9	0.6	2.3	5.5	3.0	2.5
0.900	1.8	0.2	1.6	4.5	2.6	1.9
0.850	1.0	-0.1	1.1	3.8	2.3	1.5
0.800	0.4	-0.4	0.8	3.2	2.0	1.2
0.700	-0.5	-1.0	0.5	2.5	1.6	0.9

Table A-12. CASE A8
(N,10,.2;N,μ_t,.3;1;4.3)

F'_o	I_s/ν_x Mean Ldtm.		I_s/ν_x Amt. Red.	$\overline{OS}/\nu_x$ Mean Ldtm.		$\overline{OS}/\nu_x$ Amt. Red.
	13	2	13-2	13	2	13-2
0.990	3.6	2.1	1.5	6.1	4.5	1.6
0.972*	4.3	1.0	3.3	6.7	3.4	3.3
0.950	3.4	0.7	2.7	5.9	3.1	2.8
0.900	2.3	0.2	2.1	4.8	2.6	2.2
0.850	1.5	-0.2	1.7	4.1	2.3	1.8
0.800	0.9	-0.4	1.3	3.6	2.0	1.6
0.700	0.0	-0.9	0.9	2.8	1.6	1.2

Table A-13. CASE A9
(N,10,.2;N,μ_t,.2;1;4.3)

$\bar{F}'_o$	I_s/μ_x			$\overline{OS}/\mu_x$		
	Mean Ldtm.		Amt. Red.	Mean Ldtm.		Amt. Red.
	13	2	13-2	13	2	13-2
0.990	4.5	1.3	3.2	6.9	3.7	3.2
0.988*	4.3	1.3	3.0	6.8	3.6	3.2
0.950	2.6	0.5	2.1	5.2	3.0	2.2
0.900	1.6	0.2	1.4	4.2	2.6	1.6
0.850	1.0	-0.1	1.1	3.6	2.2	1.4
0.800	0.5	-0.4	0.9	3.1	2.0	1.1
0.700	-0.4	-1.0	0.6	2.4	1.6	0.8

Table A-14. CASE A10
(P,10,.316;C,μ_t,0;1;4.3)

$\bar{F}'_o$	I_s/μ_x			$\overline{OS}/\mu_x$		
	Mean Ldtm.		Amt. Red.	Mean Ldtm.		Amt. Red.
	13	2	13-2	13	2	13-2
~1.000*	4.3	2.5	1.8	6.7	4.9	1.8
0.990	3.6	2.1	1.5	6.1	4.5	1.6
0.950	0.9	0.5	0.4	3.6	2.9	0.7
0.900	0.5	0.1	0.4	3.0	2.5	0.5
0.850	0.2	-0.2	0.4	2.6	2.2	0.4
0.800	-0.1	-0.5	0.4	2.3	2.0	0.3
0.700	-0.8	-0.9	0.1	1.8	1.6	0.2

Table A-15. CASE A11
(N,1000,.3;C,μ_t,0;1;4.3)

$\bar{F}'_o$	I_s/μ_x			$\overline{OS}/\mu_x$		
	Mean Ldtm.		Amt. Red.	Mean Ldtm.		Amt. Red.
	13	2	13-2	13	2	13-2
~1.000*	4.3	2.5	1.8	6.7	4.9	1.8
0.990	2.7	1.3	1.4	5.2	3.7	1.5
0.950	1.2	0.5	0.7	3.5	2.9	0.6
0.900	0.6	0.2	0.4	3.0	2.5	0.5
0.850	0.1	-0.1	0.2	2.5	2.2	0.3
0.800	-0.3	-0.4	0.1	2.2	2.0	0.2
0.700	-0.9	-0.9	0.0	1.8	1.6	0.2

Table A-16. CASE A12
(P,10,.316;U,μ_t,.3;1;4.3)

$\bar{F}'_o$	I_s/μ_x			$\overline{OS}/\mu_x$		
	Mean Ldtm.		Amt. Red.	Mean Ldtm.		Amt. Red.
	13	2	13-2	13	2	13-2
0.990	6.2	1.9	4.3	8.7	4.1	4.6
0.955*	4.3	0.9	3.4	6.7	3.4	3.3
0.950	4.1	0.8	3.3	6.4	3.3	3.1
0.900	2.9	0.4	2.5	5.3	2.8	2.5
0.850	2.0	0.0	2.0	4.5	2.4	2.1
0.800	1.1	-0.3	1.4	3.9	2.1	1.8
0.700	-0.7	-0.6	-0.1	3.0	1.6	1.4

Table A-17. CASE A13
(N,1000,.3;U,μ_t,.3;1;4.3)

$\bar{F}_o$	I_s/μ_x			$\overline{OS}/\mu_x$		
	Mean Ldtm.		Amt. Red.	Mean Ldtm.		Amt. Red.
	13	2	13-2	13	2	13-2
0.990	6.2	1.8	4.4	8.5	4.2	4.3
0.955*	4.3	1.0	3.3	6.7	3.4	3.3
0.950	4.3	0.9	3.3	6.6	3.3	3.3
0.900	2.9	0.4	2.5	5.3	2.8	2.5
0.850	2.0	0.0	2.0	4.5	2.5	2.0
0.800	1.3	-0.3	1.6	3.9	2.1	1.8
0.700	0.1	-0.9	1.0	3.0	1.7	1.3

Table A-18. CASE A14
(P,10,.316;E,μ_t,1.0;1;4.3)

$\bar{F}_o$	I_s/μ_x			$\overline{OS}/\mu_x$		
	Mean Ldtm.		Amt. Red.	Mean Ldtm.		Amt. Red.
	13	2	13-2	13	2	13-2
0.990	12.5	4.5	8.0	15.0	6.8	8.2
0.950	8.1	2.0	6.1	10.6	4.7	5.9
0.900	5.8	1.2	4.6	8.2	3.6	4.6
0.855*	4.3	0.5	3.8	7.0	3.0	4.0
0.850	4.3	0.5	3.8	6.9	2.9	4.0
0.800	3.0	-0.1	3.1	5.9	2.4	3.5
0.700	1.0	-0.7	1.7	4.4	1.8	2.6

Table A-19. CASE A15
(P,1000,.0316;E,μ_t,1.0;1;4.3)

$\bar{F}_o$	I_s/μ_x			$\overline{OS}/\mu_x$		
	Mean Ldtm.		Amt. Red.	Mean Ldtm.		Amt. Red.
	13	2	13-2	13	2	13-2
0.990	12.5	4.5	8.0	14.8	6.6	8.2
0.950	8.3	2.0	6.3	10.6	4.8	5.8
0.900	5.9	1.3	4.6	8.2	3.6	4.6
0.851*	4.3	0.7	3.6	6.9	2.9	4.0
0.850	4.3	0.7	3.6	6.8	2.9	3.9
0.800	3.2	0.1	3.1	5.9	2.4	3.5
0.700	1.2	-0.6	1.8	4.4	1.8	2.6

Table A-20. CASE A16
(N,10,.2;N,μ_t,12;2;4.3)

$\bar{F}_o$	I_s/μ_x			$\overline{OS}/\mu_x$		
	Mean Ldtm.		Amt. Red.	Mean Ldtm.		Amt. Red.
	13	2	13-2	13	2	13-2
0.990	5.5	2.0	3.5	7.6	4.0	3.6
0.975*	4.3	1.8	2.5	6.4	3.8	2.6
0.950	3.5	1.5	2.0	5.4	3.4	2.0
0.900	2.3	0.9	1.4	4.4	2.9	1.5
0.850	1.5	0.5	1.0	3.8	2.5	1.3
0.800	1.0	0.2	0.8	3.2	2.2	1.0
0.700	0.0	-0.4	0.5	2.5	1.7	0.8

Table A-21. CASE A17
$(N,10,.2;N,\mu_t,.2;4;4.3)$

$\bar{F}_o$	I_s/μ_x			$\overline{OS}/\mu_x$		
	Mean Ldtm.		Amt. Red.	Mean Ldtm.		Amt. Red.
	13	2	13-2	13	2	13-2
0.990	7.3	4.2	3.1	8.6	5.4	3.2
0.950	5.0	3.3	1.7	6.3	4.4	1.9
0.926*	4.3	2.9	1.4	5.6	4.1	1.5
0.900	3.9	2.7	1.2	5.0	3.8	1.2
0.850	3.1	2.1	1.0	4.3	3.3	1.0
0.800	2.5	1.6	0.9	3.7	2.9	0.8
0.700	1.2	0.7	0.5	2.9	2.2	0.7

Table A-22. CASE A18
$(N,10,.2;N,\mu_t,.2;6;4.3)$

$\bar{F}_o$	I_s/μ_x			$\overline{OS}/\mu_x$		
	Mean Ldtm.		Amt. Red.	Mean Ldtm.		Amt. Red.
	13	2	13-2	13	2	13-2
0.990	6.0	2.5	3.5	7.8	4.3	3.5
0.963*	4.3	2.1	2.2	6.1	3.7	2.4
0.950	3.9	1.9	2.1	5.9	3.5	2.4
0.900	2.7	1.2	1.5	4.8	3.0	1.8
0.850	2.0	0.9	1.1	3.9	2.6	1.3
0.800	1.3	0.5	0.8	3.4	2.3	0.9
0.700	0.4	-0.1	0.5	2.5	1.9	0.6

Table A-23. CASE A19
$(N,10,.2;N,\mu_t,.2;1;1.0)$

$\bar{F}_o$	I_s/μ_x			$\overline{OS}/\mu_x$		
	Mean Ldtm.		Amt. Red.	Mean Ldtm.		Amt. Red.
	13	2	13-2	13	2	13-2
0.997*	4.3	~2.0	2.3	5.1	2.7	2.4
0.990	4.0	1.9	2.1	5.0	2.6	2.4
0.950	2.6	1.1	1.5	4.1	1.9	2.2
0.900	2.0	0.7	1.3	2.9	1.5	1.4
0.850	1.5	0.5	1.0	2.4	1.3	1.1
0.800	1.1	0.3	0.8	2.0	1.2	0.8
0.700	0.6	0.1	0.5	1.5	1.0	0.5

Table A-24. CASE A20
$(N,10,.2;N,\mu_t,.2;1;2.0)$

$\bar{F}_o$	I_s/μ_x			$\overline{OS}/\mu_x$		
	Mean Ldtm.		Amt. Red.	Mean Ldtm.		Amt. Red.
	13	2	13-2	13	2	13-2
0.992*	4.3	1.8	2.5	5.6	3.0	2.6
0.990	4.2	1.7	3.5	5.5	2.9	2.6
0.950	2.7	0.8	1.9	4.0	2.1	1.9
0.900	2.1	0.5	1.6	3.3	1.7	1.6
0.850	1.5	0.3	1.2	2.8	1.5	1.3
0.800	0.9	0.1	0.8	2.4	1.3	1.1
0.700	0.2	-0.2	0.4	1.9	1.0	0.9

Table A-25. CASE A21 $(N,10,.2;N,\mu_t,.2;1;6.0)$

$\bar{F}'_o$	I_s/μ_x			$\overline{OS}/\mu_x$		
	Mean Ldtm.		Amt. Red.	Mean Ldtm.		Amt. Red.
	13	2	13-2	13	2	13-2
0.990	4.5	1.3	3.2	7.7	4.6	3.1
0.987*	4.3	1.0	3.3	7.6	4.3	3.3
0.950	2.6	0.3	2.3	5.8	3.6	2.2
0.900	1.4	-0.1	+1.5	4.8	3.1	1.7
0.850	0.5	-0.5	1.0	4.1	2.8	1.3
0.800	0.0	-0.8	+0.8	3.6	2.5	1.1
0.700	-1.0	-1.3	+0.3	2.7	2.0	0.7

Table A-26. CASE A22 $(N,10,.2;N,\mu_t,.2;1;8.0)$

$\bar{F}'_o$	I_s/μ_x			$\overline{OS}/\mu_x$		
	Mean Ldtm.		Amt. Red.	Mean Ldtm.		Amt. Red.
	13	2	13-2	13	2	13-2
0.990*	4.3	1.0	3.3	8.6	5.2	3.4
0.950	2.2	0.2	2.0	6.5	4.5	2.0
0.900	0.9	-0.3	+1.2	5.4	3.9	1.5
0.850	0.0	-0.8	+0.8	4.6	3.5	1.1
0.800	-0.9	-1.2	0.3	4.0	3.2	0.8
0.700	-1.4	-	-	3.1	2.6	0.5

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This study demonstrates how, in a supply system, the inventory stock levels needed for a specified level of customer service vary as a function of the requisition lead time and pattern. The study presents a practical method to determine the savings in inventory holding cost made possible by reducing requisition lead time for problems involving several million different line items. The approach developed ~~has been~~ applied successfully to the peacetime military overseas resupply problem involving over 200 supply distribution points with more than two million different line items.

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