AF CRC-TR-229

AFCRL- TR-60-229

AD662019

(PB 150930)

TECHNICAL FINAL REPORT

under Contract AF 61 (514) - 963

submitted by

Institute of Meteorology
University of Stockholm

(December 31, 1959)

The geophysical research reported in this document has been sponsored by the Geophysics Research Directorate of Air Force Cambridge Research Center, AIR RESEARCH AND DEVELOPMENT COMMAND, UNITED STATES AIR FORCE, through the European Office, ARDC.

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MONITORING AGENCY DOCUMENT NO. AFCRL TR 60-229

ASTIA DOCUMENT NO.

TITLE OF REPORT:

" Research on stratospheric meteorology "

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TECHNICAL (FINAL) REPORT

Contract No. AF 61 (514) - 963 , through the European Office, ARDC in Brussels, Air Research and Development Command, USAF.

DATE OF REPORT

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CREDIT LINE

"The research reported in this document has been sponsored by
the European Office, Air Research and Developmend Command, Shell Building, 47, rue Cantersteen, Brussels, Belgium.

Summary description of the whole investigation under contract and synopsis of the results.

The synoptic investigation carried out under this contract during the contract time from October 1956 to December 1959 at the International Institute of Meteorology, Stockholm/Sweden, had at its main aim the detailed synoptic study of the total atmospheric space from the ground up to the upper stratospheric layers which are in reach by our present radiosonde network. The whole study has been concentrated on two special selected periods in mid-winter (1 to 7, and 8 to 13, January 1956) and one period in mid-summer (15 to 20, July 1957, already using IGY-data). Methodically this investigation differs very much from other earlier synoptic-aerological studies because of its worldwide extent and because of the extensive use of the totally available radiosonde material of the Northern Hemisphere. In addition it also differs because the data have not been averaged as in most studies according to latitude or longitude circles but rather according to the main outline of the cores of the atmospheric westwind drift. Thus, the mass field of the atmosphere in three dimensions has been investigated by taking the flow field and its general character as the reference system relative to which an averaging process has been performed.

As far as our knowledge goes, there has never been any synoptic study so far in meteorological synoptic research which was principally based on all available and individually plotted radiosond ascents of the Northern Hemisphere making thus use of all the elements reported as temperature as a function of pressure, pressure as a function of height and the detailed reported wind direction and wind speed particularly of reported maximum winds.

The outcome of all this rather extensive research has either been published in various journals or has been presented in form of research reports to the European Branch of GRD in Brussels (see Appendix I, List of the papers concerning this contract).

In the following we give a summary of the work accomplished under this contract summarizing with respect to the time interval during which the various parts of the work have been performed and stating the papers containing the results:

(A) During the period from October 1956 to March 1957 we have first concentrated on the radiosonde material of a single special day (Jan.1, 1956) and the results are presented in paper no. 1 of Appendix I. The summary of the work is as follows:

Abstract All available soundings for Jan. 1, 1956 have been studied with respect to their tropospheric and stratospheric vertical temperature structure and with emphasis on the height and form of the tropopause. It was found that the latitudinal influence on the total vertical structure of the soundings was not dominating, but rather the location of the soundings with respect to the two main westwind cores (polar front jet and subtropical jet) of the general westerlies seemed to be important. A selection of soundings according to this principle (north of polarfront jet, between the two jets, south of subtropical jet) made it possible to distinguish between different types of soundings. This fundamental difference is discussed and will form the basis for future studies of worldwide circulation changes. Furthermore, the hemispheric distribution of tropopause height and temperature has been mapped and studied. Each of the above mentioned types possesses a typical vertical temperature structure and a characteristic tropopause height. Two rapid changes in tropopause height (tropopause breaklines) strongly interrelated with the location of the main westwind belts separate areas possessing an almost uniform tropopause level. Inside each of these areas a remarkable quasi-barotropic state of the upper atmospheric layers is shown. Similar discussion has been devoted to the nature of the soundings inside or close to the jet cores or close to the breaks in tropopause height. Here the soundings have a more complicated vertical structure and their interpretation becomes difficult.

Principal accomplishments: In this study it was first realized that the tropopause was not uniformly sloping between equator and pole but rather possesses a three-type subdivision into polar tropopause, mid-latitude tropopause and tropical tropopause. Here we stated for the first time that the two jets are continuous around the whole world, and the tropopause level is systematically interrupted in connection with the jet core location (first break at the polarfront jetstream and a second break or overlap of the mid-latitude and the tropical tropopause at the location of the subtropical jetstream. It was already in this paper that we implicitly stated the existence of an additional strong tropospheric frontal system besides the well-known polarfront, and seperating a moderate mid-latitude air mass from the pure tropical air.

As a second outcome during the same period we have extended the work to a seven-days interval (Jan. 1 to 7, 1956) and the results were as follows (paper no. 2 of Appendix I):

Abstract: In continuation of a recent paper (Defant-Taba, see no.1, App. I) a seven-days period (Jan. 1 to 7, 1956) has been investigated on a hemispheric scale. During this period strong worldwide changes in the circulation type occurred. Such a circulation change from a meridional into a zonal type (meridional and zonal with respect to the form of the polarfront jet) has been studied for the first time by means of hemispheric tropopause maps and by using a special technique of dividing the total available material of atmospheric soundings into three different groups as described in the previous paper. The following points summarize the results:

1. The hemispheric tropopause maps are most suitable for obtaining a clear picture of the three-dimensional structure of the atmosphere. The distribution of cold and warm air masses can easily be seen. Breaks in tropopause heights give an immediate impression of the location and form of the two principal westwind maxima of the westerlies (polarfront jet and subtropical jet). The concentrations in the temperature field of the tropopause level are a further guide for locating the main westwind cores. This temperature field makes it possible to picture the meandering structure of the wind belts and shows the inderaction of tropical and mid-latitude circulation. Clear distinction has to be made between subtropical and polar jet.
2. By the use of this method strong northward displacements of the tropical atmosphere have been shown. At the same time a strong meandering behaviour of the subtropical jet appears. These tropical impulses play a dominating role in producing pronounced changes in mid-latitude circulation and for the generation of large blocking anticyclones.
3. During the seven-days period the character of soundings in the three selected groups (north of polar jet, between the two jets and south of the subtropical jet) remained nearly unchanged.
4. Whenever the tropical atmosphere shifted northward stations inside deep troughs having a polar isothermal structure in the stratosphere showed a strong cooling in the higher stratosphere, and at the end of the period even the polar stations showed the same feature. It is thought that this cooling is mainly due to advection from the tropical regions, but outgoing radiation and vertical motion may contribute.
5. A possible cyclic change and three-dimensional interaction of the three different parts of the atmosphere is outlined at the end of the paper.

Principal accomplishments: In this paper we have presented for the first time hemispheric analysis of the tropopause level for a succession of days. The previously mentioned three-fold character of the tropopause level was kept throughout these seven days in mid-winter. We have further shown that there exist three natural atmospheric spaces (the polar space north of the sloping polarfront and north of the location of the polarfront jet; the mid-latitude space south of this sloping polarfront having a further sloping tropospheric frontal system as its southern limit, the subtropical front. It is situated between the two jet cores. And finally the tropical space having the subtropical front as its northernmost limit and extending southward to the equator or more precisely to the equatorial trough zone). There are thus three essentially different air masses present in the atmosphere occupying these three tropospheric spaces in strict contrast to the old idea of only two principal air masses: the polar and the tropical air mass. This outcome is decisive and fundamental for future analysis. It further was shown that the quasi-permanent outline of the subtropical jet in mid-winter near the latitude circle 30°N got strikingly disturbed and the subtropical jet core was shown to deviate strongly towards north from its more permanent winter location. As a consequence a part-time approach of the two principal jet cores simultaneously with an approach or even a join of the two main tropospheric frontal systems took place, a fact which is of great importance for the onset of strong cyclogenetical processes and for the generation of large blocks or blocking anticyclones over the mid-latitude belt. It has also been shown that blocking situations are caused by unstable behaviour of the subtropical jet and not,as was thought earlier,by unstable behaviour of the polarfront jetstream.

(B) During the period from April 1957 to November 1957 some detailed synoptic studies have been carried out to investigate more in detail the wind and temperature field and the essential features of such a blocking situation in existence over Europe during Jan. 1 to 4, 1956. The results were as follows (paper no.3 of App. I):

Abstract: In two previous papers (no. 1 and 2 of App. I) it was shown that the atmospheric soundings for Jan. 1 to 7, 1956 could be divided into five distinct groups. The following points summarize the results of these investigations:

1. The tentative conclusion established in these two articles that hemispheric tropopause maps appear more suitable for depicting

changes in the three-dimensional structure of the atmosphere than constant level charts was strengthened.

2. Evidence that tropical impulses play a dominant role in producing pronounced changes in mid-latitude circulation was established.

3. The conservation of the general character of the soundings in the five selected groups: north of the polarfront jet (tropopause lower than 300 mb), between polarfront and subtropical jet (tropopause between 220 and 270 mb); south of subtropical jet (tropopause higher than 100 mb(100 to 80 mb)); exactly in the polarfront jet(no definite tropopause); exactly in the subtropical jet (double tropopause, the lower one near 200 mb, the higher one near 100 mb) was ascertained.

In this article detailed vertical cross-sections, vertical wind and temperature distributions are presented and discussed for a case of a pronounced invasion of tropical air into Europe (subtropical impulse). It was shown that a sharp distinction can be made between the polarfront jet and another jet phenomenon, usually referred to as the subtropical jet. It is shown that the subtropical jet or a branch of it meanders exceptionally far northward over the Atlantic Ocean and even enters Europe. The meandering behaviour of the subtropical jet seems to be closely related to the formation of a blocking situation. Certain important aspects are mentioned at the end of the paper referring to the total structure of soundings, the form and height of the tropopause, the structure of polarfront jet and subtropical jet and their relation to breaklines or sharp discontinuous slopes in the tropopause level. Reference is also made to the importance of superposition or the approach of the two jets for cyclogenesis.

Principal accomplishments: In this paper it has been demonstrated how consistent vertical cross-sections can be constructed if all the tools developed during the earlier studies are used. It has been demonstrated that a blocking situation consists of an immensely large anticyclonic body of tropical air embedded into either mid-latitude air masses or polar air. Such tropical air masses excluded from their tropical source are surrounded in a circulartory way by the subtropical jet or a branch of it. These blocks have a rather elevated tropopause on top with exceedingly cold tropopause temperature.

(C) During the time from April 1957 to January 1958 we have also extended the work to a further mid-winter period essentially different in character for the previous one. Besides similar working tools we have here first established characteristic-statistical properties of the of the three-dimensional temperature field separately for each of the naturally by the fronts and the jets subdivided atmospheric spaces. Thereby a rather useful material of characteristic space-averaged temperatures became available for all heights in tropo- and stratosphere for mid-winter. Also statistics on tropopause height and temperature have been done. The results were as follows (paper no.4 of App.I):

Abstract:

I. An introductary outline of earlier work on hemispheric changes in the type of the General Circulation by means of tropopause maps is presented to make the reader familiar with the main features of tropopause and its intimate relation to the main wind cores of the westerlies. The prehistory of the period from Jan. 8 to 13, 1956, which we studied in this paper, is described in detail.

II. Tropopause maps for each day (Jan. 8 to 13, 1956) are presented. By means of these maps it was shown how the zonal type of circulation (zonal with respect to a zonal polarfront jet) changes during this period into a meridional (meandering) type. The behaviour of subtropical jet (in a characteristic way different from that of polar-front jet) is simultaneously discussed.

III. A selection of soundings with respect to the breaklines in the tropopause level or according to the location of the belts of maximum wind of the westerlies was made for each day. Different characteristic groups of soundings (different with respect to the total vertical temperature distribution and the form as well as height of the tropopause) are discussed and characteristic mean soundings for each group are computed.

IV. A computation of standard deviation of the individual vertical temperature distributions from the mean one for each group is presented. An important conservatism of the sounding type in each group is shown. Meridional temperature differences between different groups inform the reader about the possible locations of baroclinicity in tropo- and stratosphere.

V. A statistical investigation of tropopause level by use of 2513 atmospheric soundings from Jan. 1 to 13, 1956, is presented. The structure of the tropopause is discussed.

VI. A synoptic vertical cross-section clarifies the relationship between the subtropical break in the tropopause level und the existence of the subtropical jet. Important aspects of the form of the subtropical jet are outlined.

Principal accomplishments: In addition to the already during the previous works accomplished facts we have in this paper also shown by statistical means that the atmospheric soundings with respect to the vertical temperature distribution can be grouped together for naturally subdivided atmospheric spaces. This three-fold character of the atmosphere together with the three-fold outline of the tropopause level has once more been demostrated throughout this special period. The statistically evaluated space-average temperatures at every level of each space have been listed in a table and form by itself a basic fundament and guide in practical three-dimensional analysis. In addition the standard deviations from the space means evaluated for every level in each space demonstrate the variability in temperature separately for each space in each tropospheric and stratospheric level. These figures show convincingly that the polar space is least conservative in winter, while the other spaces keep the space-average temperature rather well. Only in the lower tropospheric layers there is less conservatism. In each space the atmosphere is shown by means of these numbers to be most barotropic in the upper part of the troposphere. A further accomplishment is the tropopause statistic which may be considered a first attempt only. However, it demonstrates one circumstance clearly. That is the existence of a fundamental break in the tropopause over the subtropical latitude belt, where the mid-latitude tropopause and the tropical one overlap. It is shown that the core of the subtropical jet has a strong relation to this break.

(D) During the time interval from January 1958 to June 1958 the author has concentrated on summarizing the general impressions we have gained during the previously mentioned works, while the rest of the group has started the pre-preparation of the data for investigation of a comparative summer period. This summary has the following contents (see paper no. 5 (in German language) and paper no. 6 (English translation) listed in App. I):

Abstract: First a general introduction is presented. In paragraph 2) the three-dimensional atmospheric field of motion is discussed summarizing on the one hand about the main features of the westwind drift (polarfront and subtropical jetstreams) and on the other hand about the main constituents of the tropical circulation regime. In para. 3) it is outlined how the temperature field can be investigated relative to the main outline of the westerlies. It is discussed how the radio-

sonde ascents can be grouped and how characteristic average values for different spaces can be obtained together with the possible standard deviations from these means. The tropopause and its principal character as shown to exist by the previous works is dealt with. In para. 4) the normal state of the general atmospheric circulation in Northern Hemisphere winter is outlined, and in para. 5) the author has attempted to deal with the possible disturbances of this normal state. Here especially subtropical impulses were discussed leading to the generation of a zonal circulation type in the polar jetstream, and finally synoptic events during the breakdown of this type and its transition to a meridional or meandering circulation type is discussed. In para. 6) the author has touched briefly the reasons why we observe two hemispheric jets and why we have two entirely different circulation regimes (the westwind drift and the tropical circulation regime) side by side.

(E) During the same time interval the author has also carried out a special investigation dealing with hydrodynamic instability brought about by the approach of subtropical and polarfront jetstream. The results were published in an article in the Rossby Memorial Volume (paper no.11 of App. I):

Abstract: Recent investigations of the General Circulation of the atmosphere have shown the importance of strong northward advances of the subtropical atmosphere (subtropical impulses), appearing in connection with sudden northward meandering behaviour of the subtropical jetstream, for the generation of large anticyclonic vortices (blocking anticyclones) over the mid-latitude belt. These processes had strong effect on the form of the polarfront jet, which assumes a more zonal form in northern latitudes (zonal circulation type, high index).

The approach of subtropical and polarfront jet in northern latitudes and the simultaneous approach of tropical and polar atmosphere results in a rather complicated vertical temperature structure of the total atmosphere. Nearly all the meridional temperature contrast of tropospheric layers is concentrated at the strongly inclined polarfront, while in the higher atmosphere a steeply inclined part of the tropopause (opposite inclination relative to that of the polarfront) assumes a frontlike character separating relatively warm stratospheric polar air in the north from extremely cold air occurring in the higher parts of the tropical troposphere in the south.

This special kind of vertical structure of the total atmosphere in combination with double wind maxima (polarfront and subtropical jet)

was observed to explode during a short time of some hours, and at the same time unusual strong cyclogenesis occurred, especially at the northeastern edges of the extensive blocking anticyclones. As a result of the formation of large cyclones drastic changes in the form of the polarfront jet occurred, or, what is the same, the type of circulation changes drastically (from a zonal into a strongly meandering form of polarfront jet, from high to low index circulation).

A combined synoptic-theoretical study of the hydrodynamic equilibrium of these special atmospheric systems by means of a detailed vertical cross section south of Iceland before the onset of strong cyclogenesis on Jan. 8, 1956, 0300Z is the content of this paper. The principal results are the following:

(1) It is demonstrated that the atmospheric system under consideration is hydrodynamical unstable in three special regions: (a) in the middle and upper part of the polarfront and just below the lower boundary of the polarfront, (b) just south of the main core of the polarfront jet and most important (c) in an upper layer between the level of maximum wind and the tropopause.

(2) This is even true if the stratification of the atmosphere is assumed to be statically stable and no anticyclonic curvature of the flow is taken into account. In case of no such restrictions the hydrodynamic equilibrium of the system would be even more unstable.

(3) A computation of the Richardson Number Ri shows that regions with Ri smaller 1 are exactly consistent with the three regions of hydrodynamic instability and agree well with the observed regions of severe turbulence near to the polarfront jet core, especially with respect to the upper instability region just below the tropopause.

(4) Important aspects for drastic changes in the character (zonal or meridional) of the polarfront jet circulation, for strong cyclogenesis respectively are outlined.

<u>Principal accomplishments</u>: It has been demonstrated in this study that the approach of polarfront and subtropical jet and the simultaneous approach or even join of the two main tropospheric and baroclinic frontal systems may easily result in an atmospheric structure which becomes hydrodynamical unstable in the sense put forward on theoretical grounds by van Mieghem. Essentially new is the fact that hydrodynamic instability may occur not only inside the frontal boundaries of the

polarfront and south of the jet core as was so far thought but also in addition in the higher atmosphere between the level of maximum wind and the strongly sloping tropopause. It is particularly of interest to realize from this study that these three places with most likely unstable conditions coincide exactly with those locations where the Richardson Number becomes smaller 1 a criterion for occurrence of turbulence. The painstaking careful prepared synoptic illustrations of various quantities as vertical stability, horizontal and vertical wind shear, Richardson Number and hydrodynamic stability according to van Mieghem's formula deserve attention by the synoptic and theoretical meteorologist.

(F) During the time from June 1958 to January 1959 a comparative summer period has been worked on which we have already mentioned above. Some of the tools have been applied in a more refined form and additional use of the prepared material has been made. The results were as follows (paper no. 8 of App. I):

Abstract and principal content:

The results of this study have been presented in form of six subsequent reports. After a general introduction report 1 deals with the synoptic situation (15 to 20, July, 1957). A worldwide check has been made on the summer tropopause level and for the first time charts of the level of maximum wind have been prepared by searching through every available source about the actual reported maximum wind speed and its direction. We have also succeeded to contruct the topography of the sloping summer polarfront. We have also prepared maps which convincingly demonstrate the location of the summer subtropical jet appearing at a much more northern position near the latitude circle 44°N and its strong interrelation with the subtropical break in the tropopause level which does also exist in summer. In the report 2 of this paper the author has presented mean vertical temperature distributions in each of the special selected atmospheric spaces and a comparison has been made of these average distributions with that obtained for mid-winter periods. In table-form the author has listed these characteristic temperatures for every level of the troposphere and stratosphere. In report 3 of this paper frequency distributions of temperature for various isobaric surfaces in each of the atmospheric spaces selected are presented in order to demonstrate the conservative character of the average temperature in each space. A proof has been given for the existence of three principal air masses as opposed to the system of only two air masses (polar-tropical) also in the summer period. Standard deviations have been worked out for every space and every height for tropo- and stratosphere, and are presented in table-form. In report 4 of this paper the dependence of the summer temperature on latitude in various levels of the naturally subdivided atmospheric spaces informs the reader about the variance of the temperature throughout the total summer atmosphere from the ground up to 30 mb. This dependence of the temperature on latitude is a new item only accomplished for the summer period, and was not done during the previously mentioned winter period. In report 5 the author has presented mean meridional cross-sections of actual and potential temperature from pole to equator for Northern Hemisphere summer based on an evaluation relative to the two principal westwind cores. These sections surpass with their content by far the knowledge which one could abstract from common standard climatological meridional cross-sections based on an averaging process relative to latitude circles. In this subreport the baroclinicity of the summer atmosphere has been determined for the total meridional plane from pole to equator and from the ground up to 30 mb. It was found that the atmosphere is most baroclinic between the frontal boundaries of the polarfront and of the subtropical front and in the close vicinity of these fronts. In addition, it has been shown that the atmosphere is also baroclinic in the stratosphere above the polarfront jet core and above the subtropical jet core. In the remote surroundings of the jet cores the atmosphere is more barotropic especially in the higher troposphere. With regard to the tropical atmosphere it is demonstrated that up to 300 mb the tropical troposphere is of a considerable barotropic character and the vertical stratification is close to vertical instability. However, the higher one goes upwards the more baroclinic the tropical atmosphere becomes reaching a maximum baroclinicity at the tropical tropopause. This is in strict contrast to the other two tropopause levels (the polar- and mid-latitude tropopause) for which the atmosphere shows almost zero baroclinicity. The last report 6 of this paper deals with average values of the vertical gradient of potential temperature in each of the atmospheric spaces. Here the reader has a very valuable guide about the possible average static stability to be found in tropo- and stratosphere of the summer atmosphere. In addition a mean meridional cross-section of the static stability is presented. Such a presentation does not exist in literature in this detailed and complete form. It is also dealt with stability conditions at the level of maximum westerly flow in summer.

Principal accomplishments: The principal accomplishments of this whole paper are the derivation of comparable characteristic temperature values in different air masses and in every level of the tropo- and stratosphere for Northern Hemisphere summer. Furthermore it is definitely checked by an immense amount of data that the main and important features of the General Circulation as polarfront jetstream and polarfront, subtropical jetstream and subtropical front, the threefold subdivision of the tropopause equally exist in summer as in winter. Their hemispheric character can no more be doubted. This report shows also the detailed variance of the atmospheric temperature field in vertical direction in a large number of isobaric surfaces. It thereby gives a clear prove for the main regions of baroclinicity which on the one hand can be found in the troposphere at hemispheric boundary surfaces but also in the stratosphere above where the meridional temperature gradient is not at all uniform but rather two stratospheric zones of strong baroclinic character can be found vertically on top of the polarfront and subtropical jet core.

(G) During the time from January 1959 to May 1959 a special synoptic investigation of the flow and mass field in the closer vicinity of the summer subtropical jetstream has been carried out by the author (paper no. 9 of App. I). The results were as follows:

Abstract: This paper starts in para. 1 with introductory remarks about the fundamental characteristics of the subtropical jetstream. Para. 2 describes the special purpose of the paper, namely to investigate the detailed state of the mass field and field of motion in the close vicinity of the summer subtropical jet. Thereby special emphasis is put on showing the interconnection between subtropical jet, subtropical tropopause break and the subtropical front. In special it is concentrated on the observed long waves realized to exist with the summer subtropical jet core, and it will be aimed to arrive at the principal different conditions of the mass and flow field separately for the front side and the rear side of these waves. Para. 3 describes the methodical approach which consists in studying the temperature field, the height field and the flow field relative to the axis of the subtropical jet and separately for the two waves sides. Conditions in the subtropical jet core are statistically studied. First graphs of wind conditions in the level of maximum wind are prepared and at the same time it is shown that the strongly peaked jet core at each wave side coincides closely with the subtropical beak in the tropopause. Furthermore it is shown that the front side possesses a much higher core velocity than the rear side. Then the whole pressure field, wind field and temperature field has been built up from the ground up to 30 mb, and is presented in a number of refined graphs. By these it is shown that a pronounced frontal surface exists in connection with the subtropical jet in summer, and it is steeply inclined at the front side and less inclined at the rear side, where it mostly does not extend downwards to the surface of the earth. It is then demonstrated that the front side shows strong supergeostrophic winds while at the rear side of the waves conditions are subgeostrophic. This is most so for the jet core and its close vicinity towards both sides, as well as below and above. The remote surroundings towards both sides and the lower troposphere as well as the upper stratosphere possess almost geostrophic conditions. Finally total cross-sections at both wave sides are presented of the wind and temperature field and of the potential temperature field. At the end of the paper the baroclinic character of the summer atmosphere in the vicinity of the subtropical jet is dealt with.

(H) During the same time from January 1959 to June 1959 my colleague, Mr. H. Taba, has completed a work of his own, and the results are presented in paper no. 7 of App.I. They are as follows:

Abstract: All the available wind information over central, west and north Europe, the Atlantic Ocean, the United States and Canada, has been studied for a period of seven days, Jan. 1 to 7, 1956, 0300Z. The axes of the subtropical and polar jet have been chosen as reference coordinates. Mean horizontal and vertical wind profiles have been computed at 100 km intervals from the axis of both jets and are discussed. Based on these profiles, the average wind and the equivalent barotropic level is computed. It is shown that the height of the equivalent barotropic level changes as the distance from the axis of the jet varies. The equivalent barotropic level is shown to have a maximum height of 432 mb at the axis of the subtropical jet and a minimum height of 560 mb at a distance of 1000 km north of the polar jet. Finally, an average equivalent barotropic level is computed and is shown to have a height of 505 mb.

Principal accomplishments: This work has partly synoptic aspects and shows wind profiles in various isobaric surfaces for both the polar and subtropical jet as computed from actual winds and relative to the main jet cores. Mr. Taba has here used the synoptic method developed by the author in the foregoing paper. He obtains average wind conditions in winter of the Northern Hemisphere. The work has also aspects related to numerical forecasting, since the equivalent barotropic level as it was defined by Charney et al. is studied by means of actual data, and it is found, that this level only on the average keeps close to the 500 mb level, but varies considerably with distance from both jet cores in height and varies also from southern to northern latitudes. This outcome is of great interest for the theoretical meteorologist in devising atmospheric models.

(J) During the time from May 1959 to December 1959 the author has made use of the synoptic results of the work mentioned under item (G) to investigate vertical motions and transversal circulations connected with long waves in the summer subtropical jet (see paper no. 10 of App. I). The contents are as follows:

Abstract: In para. 1 an introduction and synoptic estimate of various quantities contained in the equations of motion and in the thermodynamic equations are presented. In para. 2 the author has dealt with vertical motions at both wave sides. First an equation is derived for the computation of the vertical velocity. Then an estimate of the magnitude and a determination of the sign of all the terms involved in the vertical velocity formula is presented. Furthermore, remaining and not considered parts of the vertical velocity formula are discussed and reasons given for their neglection. Finally the result of the computation of the vertical velocity separately for the front and rear side of the long subtropical jet wave is presented in form of graphs. The third paragraph deals with the transversal velocity component and its field distribution. First, a formula for the computation of the transversal component is presented, and a discussion is added of all the terms involved. Then, an example is given for a computation of the transversal component without taking friction into account. After that the influence of turbulent friction on the transversal component is discussed in detail, and finally it is shown how the influence of the denominator ($f - \partial u/\partial y$) affects the outcome of the transversal velocity component. In para. 4 the circulation occurring in the transversal cross-sectional plane through the summer subtropical jet is shown in form of a graph and discussed. This is done separately for the front- and rear side of the subtropical jet wave. In the final paragraph conditions close to the jet core in levels between 400 and 150 mb are studied in detail. The vertical velocity distribution and the vertical divergence distribution exactly along a vertical line vertically through the jet core are computed, and it is furthermore outlined how turbulent friction may take influence on these distributions.

Principal accomplishments: In this work it is shown that a direct transversal circulation must be assumed at the front side of the subtropical jet wave in connection with the supergeostrophic conditions found in the jet core. It is also shown that an indirect circulation occurs at the western or rear side of the wave in connection with the subgeostrophic conditions realized at this side. It is furthermore shown that a strong ageostrophic northward transport must occur in the jet level at the front side, and this becomes even more so when turbulent friction is taken into account. With regard to turbulent friction it could be concluded that it is only of influence in layers between 400 and 150 mb, and only in case of a well-developed jet with strong vertical wind shear. It is pointed out thus, that turbulent friction has little effect at the western and rear side of the wave and strong effect at the front side. It has also been demonstrated that the coefficient of turbulent friction must be assumed to decrease with height and must assume values of 30 g cm^{-1} sec^{-1} or less in the jet level. Otherwise there would occur too large an ageostrophic motion in the level of maximum wind, which is hardly to observe.
Generally speaking, this work presents a most careful determination of vertical velocities in the vicinity of jet flow. The accumulation of data for characteristically similar direction of the jet flow made it possible to arrive at continuous curves of different meteorological elements as the height of isobaric surfaces, the wind component parallel to the jet direction and the temperature in vertical as well as in horizontal direction. Only by means of these curves gradients can be computed with sufficient accuracy to have a reliable estimate of the various terms involved in the equations of motion or involved in formulae of the vertical velocity or the transversal velocity component.

(K) During the time from June 1959 to December 1959 Mr. Taba has carried out research on divergence field and vertical motion by means of continuity and vorticity equation.for a special case over the Atlantic and Europe. The results are presented in paper no. 12 of App. I.

Abstract: In part I, the divergence field over part of the Atlantic and Europe on Jan. 2, 1959, 0300 GMT, is computed from continuity equation as derived for an isobaric level. The computation is performed for various isobaric levels at 100 mb intervals from 1000 to 100 mb. The results show that the pattern of divergence agrees well with the surface synoptic situation. Integrating the divergence obtained at each level with respect to pressure, the vertical velocity field is obtained and discussed. In part II the divergence field of the simplified vorticity equation is computed. It is observed that the divergence field of the vorticity equation has an opposite sign as compared with the continuity equation in the frontal region. An attempt has been made to demonstrate that this difference is due to the large interval of time which was used for computation of the term $\partial\zeta/\partial t$. In section III it is assumed that the variation of wind with height can be expressed by the relation $\mathbb{V} = A(p)\cdot\overline{\mathbb{V}}$. A formula is derived for divergence computation. It is shown that the divergence field computed from this formula is approximately one order of magnitude smaller than the fields obtained in part I and II.

APPENDIX I

List of published papers and reports

Contract AF 61 (514) - 963, sponsored by Air Research and Development Command USAF, through the European Office, ARDC in Brussels.

1) Defant, Fr. and Taba, H. (1957):
The threefold structure of the atmosphere and the characteristics of the tropopause.
Tellus, 9, no.3, pp. 259 - 274.

2) Defant, Fr. and Taba, H. (1958):
The strong index change period from January, 1 to 7, 1956.
Tellus, 10, no.2, pp. 225 - 242.

3) Defant, Fr. and Taba, H. (1958):
The details of wind and temperature field and the generation of the blocking situation over Europe (Jan.1 to 4, 1956).
Beiträge zur Physik der Atmosphäre, 31, Heft 1/2, pp. 69 - 88.

4) Defant, Fr. and Taba, H. (1958):
The breakdown of zonal circulation during the period Jan. 8 to 13, 1956, the characteristics of temperature field and tropopause and its relation to the atmospheric field of motion.
Tellus, 10, no.4, pp. 430 - 450.

5) Defant, Fr. (1958):
Die Allgemeine Atmosphärische Zirkulation in neuerer Betrachtungsweise.
Geophysica, 6, no. 3/4, pp. 189 - 217, (Palmén 60.Birthday Festival Volume).

6) Defant, Fr. (1958):
The General Circulation of the atmosphere in the light of more recent investigations.
Project Report under contract AF 61 (514) - 963, to the European Office, ARDC in Brussels, Technical Note No. 3, pp. 1 - 38.

7) Taba, H. (1959):
The horizontal and vertical wind profiles of the subtropical and polar jet for Jan. 1 - 7, 1956, and the variation of the equivalent barotropic level.
Project Report under contract AF 61 (514) - 963, to the European Office, ARDC in Brussels, Technical Note No. 4, pp. 1 - 14.
To be published in Tellus, 11, no. 4.

8) Defant, Fr. (1959):
Investigations on the detailed structure of the atmosphere in Northern Hemisphere mid-summer (15 - 20 July, 1957, IGY-Data) consisting of 6 sub-reports.
Project Report under contract AF 61 (514) - 963, to the European Office, ARDC in Brussels, Technical Note No. 5, pp. 1 - 92.

9) Defant, Fr. (1959):

A synoptic investigation of the flow and mass field in the close vicinity of the summer subtropical jet stream relative to the jet axis.

Project Report under contract AF 61 (514) - 963, to the European Office, ARDC in Brussels, Technical Note No. 6, pp. 1 - 49.

10) Defant, Fr. (1959):

On vertical motions and transversal circulations at the eastern and western side of long wave disturbances in the summer subtropical jetstream.

Project Report under contract AF 61 (514) - 963, to the European Office, ARDC in Brussels, Technical Note No. 7, pp. 1 - 53.

11) Defant, Fr. (1959):

On hydrodynamic instability caused by an approach of subtropical and polarfront jetstream in Northern Latitudes before the onset of strong cyclogenesis.

The Rossby Memorial Volume, Rockefeller Institute and Oxford University Press, New York 1959, pp. 305 - 325.

12) Taba, H. (1959):

A comparative study of divergence and vertical motion by means of the continuity and vorticity equation.

Project Report under contract AF 61 (514) - 963, to the European Office, ARDC in Brussels, Tedhnical Final Note, pp. 1 - 15.

A COMPARATIVE STUDY OF THE FIELDS OF DIVERGENCE AND VERTICAL MOTION BY MEANS OF THE CONTINUITY AND VORTICITY EQUATIONS

by

H e s s a m   T a b a

This report is part of investigation on the General Circulation of the Atmosphere, with special emphasis on upper atmospheric layers, sponsored by Air Research and Development Command USAF, under Contract AF 61 (514) - 963, through the European Office, ARDC in Brussels.

A comparative study of the fields of divergence and vertical motion by means of the continuity and vorticity equations.

By

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Abstract.

In part I, the divergence field over part of the Atlantic and Europe on Jan. 2, 1956, 0300 GMT, is computed from continuity equation as derived for an isobaric level. The computation is performed for various isobaric levels at 100 mb intervals from 1000 to 100 mb. The results show that the pattern of divergence agrees well with the surface synoptic situation. Integrating the divergence obtained at each level with respect to pressure, the vertical velocity field is obtained and discussed.

In part II the divergence field of the simplified vorticity equation is computed. It is observed that the divergence field of the vorticity equation has an opposite sign as compared with the continuity equation in the frontal region. An attempt has been made to demonstrate that this difference is due to the large interval of time which was used for computation of the term $\frac{\partial \zeta}{\partial t}$.

In section III it is assumed that the variation of wind with height can be expressed by the relation $V = A(p) \cdot \bar{V}$. A formula is derived for divergence computation. It is shown that the divergence field computed from this formula is approximately one order of magnitude smaller than the fields obtained in part I and II.

Introduction.

It has been generally believed that as the divergence is usually one order of magnitude smaller than the vorticity, its computed value is quite unreliable. Furthermore it has been often mentioned that the existing errors of the wind field would make any computation of divergence if not impossible, very difficult, and that the superimposed small-scale perturbations have bigger divergences than the large-scale motions. Recent studies such as by Landers (1) , Murokami (2) , Rex (3) have established that one can obtain quite a representative three-dimensional picture of the divergence field by the use of observed wind. This is because the analysis of the observed wind gives a more representative picture of the existing flow and one can expect that the vertical motion of this flow should correlate better with the existing synoptic situation. As far as the observational errors are concerned, one can expect continuous improvement in the construction of the wind measuring instruments. Regarding the elimination of the effects of smaller scale motions such as gravity waves, it seems that the density of the present observation stations is good enough (except over the ocean) for this purpose, provided that the scale of disturbances is not smaller than the distance between stations.

Analysis of the data and divergence computation.

In this study the observed wind data have been used as a basis for computations. The available wind reports for January 2nd 1956 0300 GMT were plotted at levels of 1000, 900, 800, 700, 600, 500, 400, 300, 200 and 100 mb. Fig. 1 shows the stations reprting upper winds and the working region. A flow pattern, isotach analysis was performed on each of the above surfaces. The grid points where

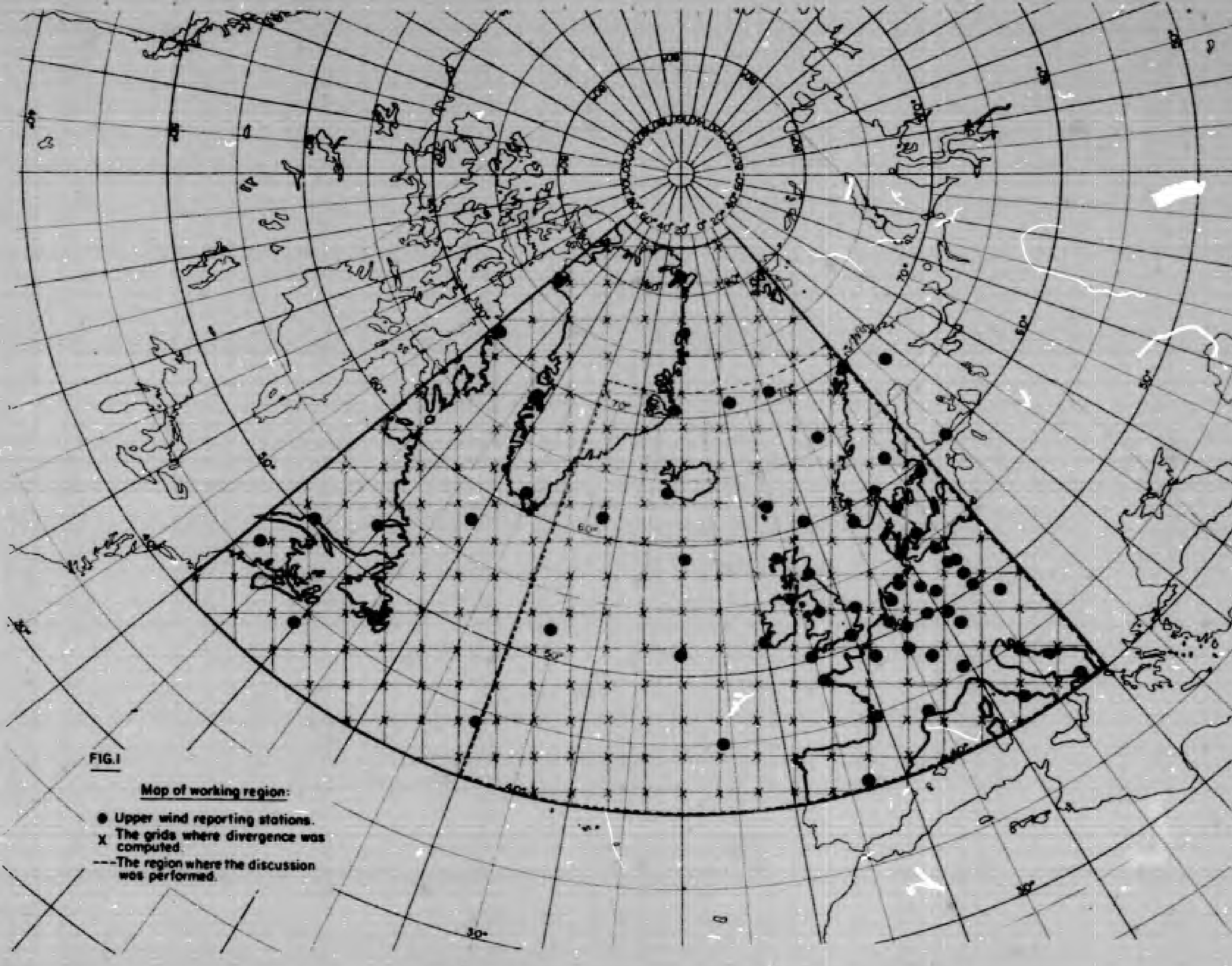


FIG.I

Map of working region:

divergence computations were made are indicated by crosses in Fig. 1. The distance between the grid points is approximately 300 km.

For each grid point from the wind vector, the components u and v were read, and finally the value of divergence from the following two formulae

1) $$\nabla \cdot \mathbb{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

2) $$\nabla \cdot \mathbb{V} = - \frac{\partial \zeta / \partial t + \mathbb{V} \cdot \nabla \zeta + \mathbb{V} \cdot \nabla f}{\zeta + f}$$

was computed. The computations were performed by the Swedish computer BESK. As the number of reporting stations were not enough over the western part of the working region, and the analysis was not reliable, it was decided to exclude this area while discussing and verifying the results. The synoptic situation is as follows: The subtropical jet has advanced over the Atlantic resulting in an extension of warm air in the upper levels in the mid-troposphere over the ocean. At the surface a frontal zone is running from a cyclone centered over Iceland southward to Spain and from there eastward to another cyclone centered over Germany. Fig. 2 shows the cyclone and the frontal zone together with the precipitaion areas and regions of isallobaric higs and lows.

Part I

Divergence computation from equation (1)

The divergence is obtained from the equation

$$\mathrm{Div}\, \mathbb{V} = \frac{u_2 - u_4 + v_3 - v_1}{2d}\, m$$

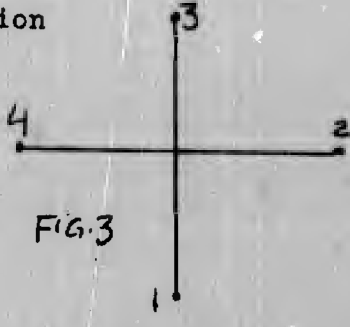


FIG.3

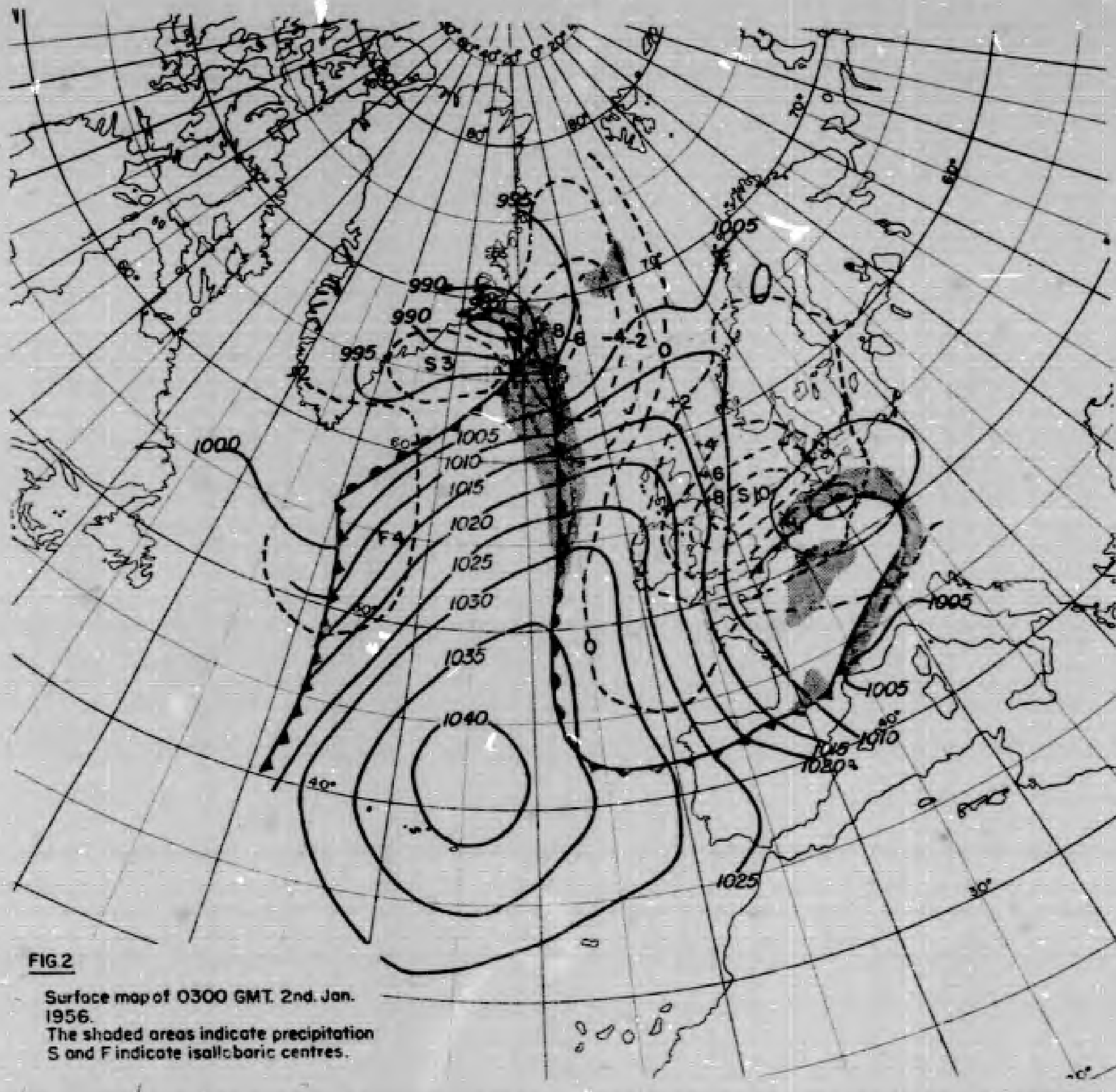


FIG.2

Surface map of 0300 GMT. 2nd. Jan. 1956.
The shaded areas indicate precipitation
S and F indicate isallobaric centres.

where u and v are the wind components at each grid point, d is the grid size, and m the map factor (Fig. 3). Although the wind-field was not smoothed, the divergence-field was not complex. However, as it was intended to compare this field with the one obtained from equation (2), in order to eliminate small irregularities and facilitate the comparison, it is decided to apply smoothing in both cases. A similar method was used by Landers . It was noticed that the smoothing did not change much the position of the centres although the magnitude was somewhat reduced. The position of the line of zero divergence was almost unchanged.

Fig. 4 shows the distribution of $\frac{\partial \omega}{\partial p} = -\nabla \cdot V$ at 700, 600, 500, 400 and 300 mb. The areas with positive sign are where we have convergence, and the negative signs indicate divergence. A comparison of the convergent and divergent areas with the fronts indicated on Fig. 2 shows that this distribution is what one should expect. A study of Fig. 4 reveals that there are three essential features to be noticed. First the divergence centre located over Central Europe. Second, the convergence area over west of England and south-east of Iceland. Finally the divergence pattern over south-east of Greenland extending southward. As can be seen from the figure, these patterns change their shape, intensity, and, in parts of the map, also their sign with height. The convergence area over England has its maximum value 4.7×10^{-5} sec^{-1} at 400 mb, and the divergence area over north-west Europe is most intense at 300 mb, and has a value of 3.8×10^{-5} sec^{-1}. At this point maybe one should mention something about the order of magnitude of divergence. Evidently the scale of motion is a decisive factor in divergence computation. Therefore we can only talk about the motions the scale of which is resolvable by the stations network.

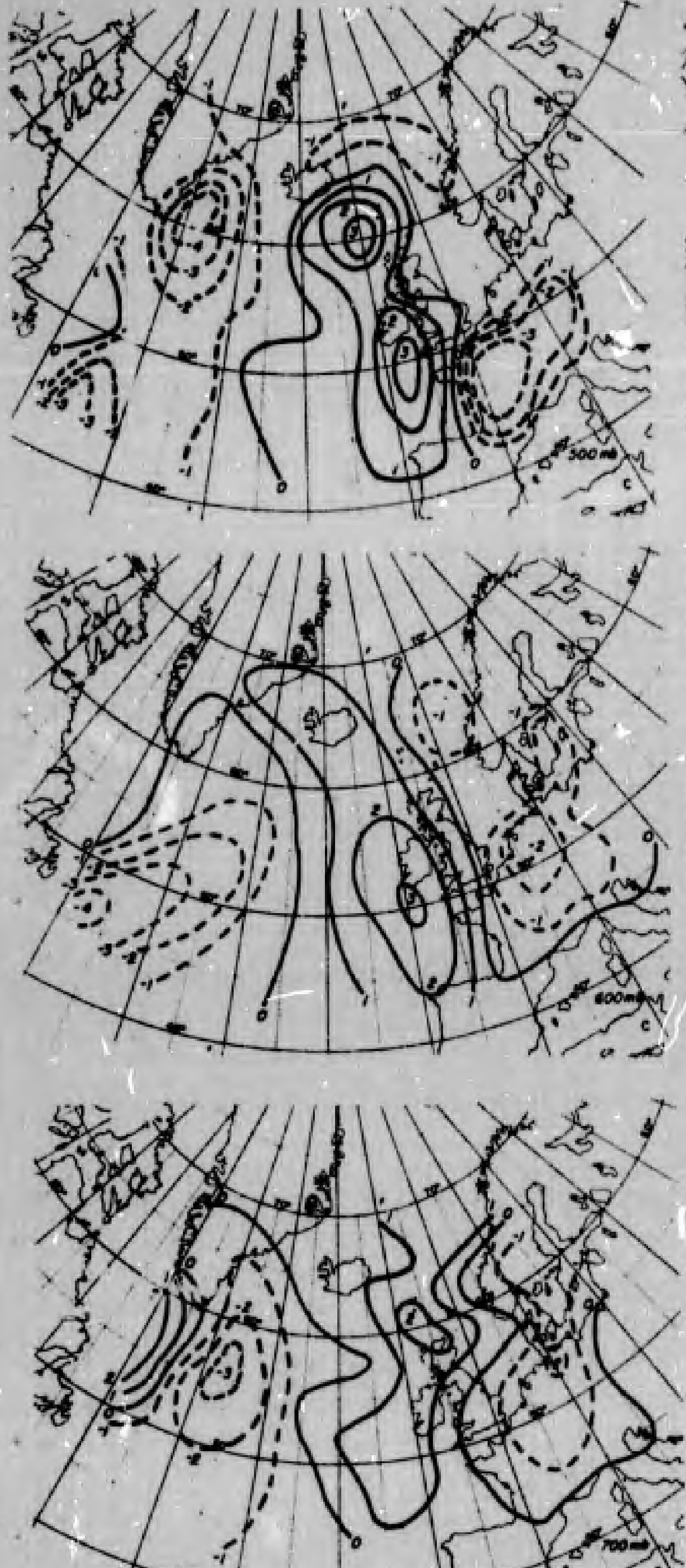


Fig. 4

The distribution of $-\nabla\cdot V$ 700,500,400 and 300 mb at 0300Z 2nd Jan. 1956. The units are expressed 10^{-5} sec^{-1}. Dashed lines show divergence and full lines convergence.

The value of $|\overline{\nabla\cdot V}|$ is computed for every level and shown in the table below and FIG 5

The value of $|\overline{\nabla\cdot V}|$ for different levels

mb	1000	900	800	700	600	500	400	300	200	100
$10^{-5}sec^{-1}$	0.5	0.7	0.8	1.0	1.2	1.2	1.7	1.7	1.4	0.2

As one can see this has a minimum value of 0.5×10^{-5} sec^{-1} at 1000 mb and a maximum value of 1.7×10^{-5} sec^{-1} at 400 mb. Compared with the following values which are found in the literature, it seems that 10^{-5} sec^{-1} is the correct magnitude using the observed winds.

(4) Fleagle 1×10^{-5} sec^{-1}

Hess 2×10^{-5} sec^{-1}

Landers 1.5×10^{-5} sec^{-1}

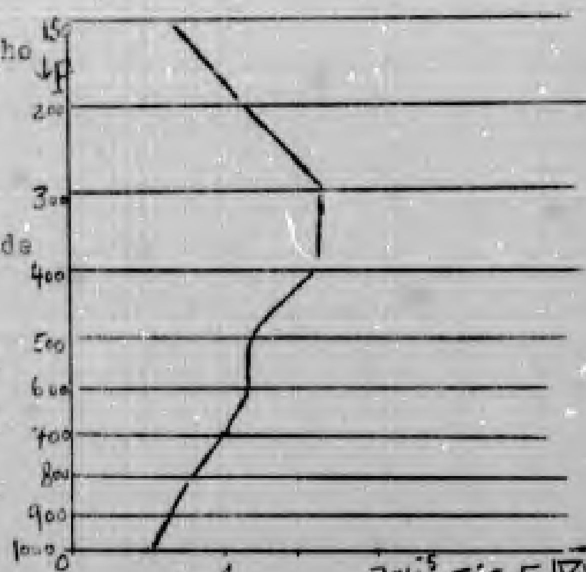


FIG 5 $|\overline{\nabla\cdot V}|$

<u>Computation of vertical velocity and its correlation with the surface analysis and precipitation.</u>

The vertical velocity at any isobaric level can be obtained from the following equation:

$$\omega_p = \omega_{p_1} + \int_p^{p_1} \nabla\cdot V \, dp$$

with the boundary conditions $\omega = 0$ at $p = p_0$ and $p = 0$ where p_1 is the pressure at the lower level and p the pressure at any higher level. For the integral to the right $\overline{\nabla \cdot V}(P_1 - P)$ was substituted where the bar indicates the average value from the level P_1 to P.

The vertical velocities at different levels are shown in Fig. 6. There are three main centres at all levels. First the area of positive ω over north-west Europe and east of England. This is a region of subsidence. A comparison of this pattern with the surface analysis shows that this subsiding region is very well indicated by the ridge of high pressure and the surface tendency (pressure rise). Secondly the area of negative ω or the region of upward motion. Again a look at the surface map indicates that the air rises in the warm sector and specially along the warm front. Notice especially the areas of precipitation and the alongated shape of the negative ω. Thirdly the area of positive ω over the southwestern region which coincides with the high pressure system. Finally one should notice the subsidence over south of Greenland which is indicated by the surface pressure rise. It seems that the pattern of vertical velocity correlates very well in shape and location with the synoptic situation. This is about as good a result which one could expect.

Regarding the correctness of the values of the ω given here, one can only mention what has been said by the other authors as well that an observational wind error of 1 m/sec at each grid point causes an error of 0.8 x 10^{-5} sec^{-1} in the value of divergence. Besides as it is well known one can not avoid committing certain mistakes due to vertical integration.

To study the magnitude of the ω obtained, we can as an example look at 700, 600 and 500 mb. At 700 mb we have a value of -3.6 g cm^{-1} sec^{-3}. This is approximately 36 mb per three hours.

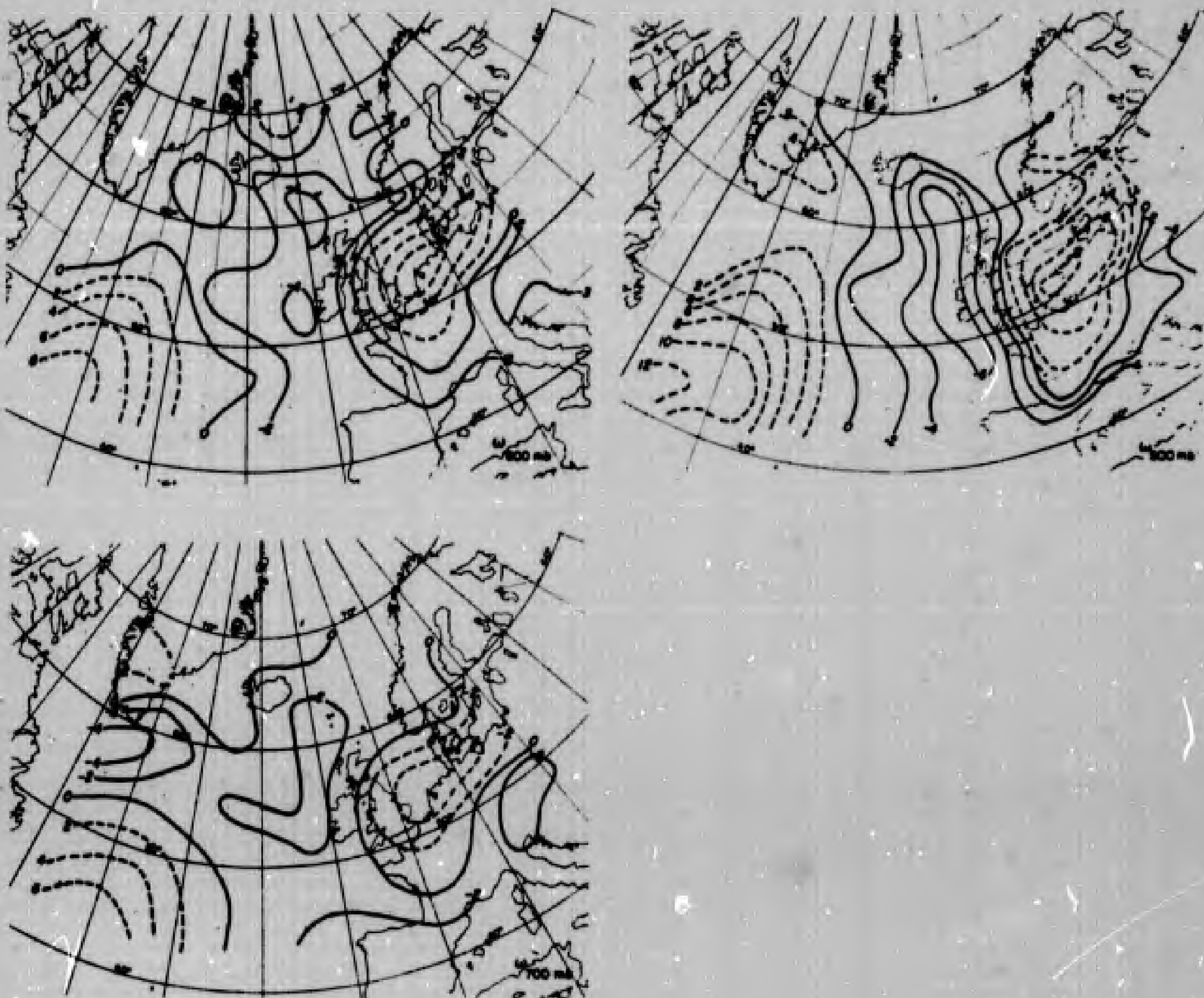


Fig. 6

The distribution of ω at 700, 600 and 500 mb. The isolines are expressed in g cm^{-1} $sec.^{-3}$

Full lines indicate upward and dashed lines downward motion.

Using the relation $W = -\frac{\omega}{g\rho}$ we see that this corresponds to approximately 4 cm per second downward motion.

Fig. 7 shows the distribution of W in cm/sec for levels 700 and 500 mb.

Part II

Computation of divergence from vorticity equation.

The simplified vorticity equation

$$\frac{\partial \xi}{\partial t} + \mathbb{V} \cdot \nabla \xi + \mathbb{V} \cdot \nabla f = -(f+\xi)\nabla \cdot \mathbb{V}$$

is used for computing the divergence. The same data as in part I, that means the observed wind has been used in order to make it easier to compare the two patterns. To calculate the term $\frac{\partial \xi}{\partial t}$ two more series of maps, 12 hours before and 12 hours after, were prepared and analyzed. Therefore

$$\frac{\partial \xi}{\partial t} = \frac{1}{2\Delta t}\left[(\xi)_{t+12} - (\xi)_{t-12}\right]$$

where Δt = 12 hours and ξ_t denotes the value of relative vorticity at time t. The maps of $-\nabla \cdot \mathbb{V}$ were analyzed and are presented in Fig. 8. It was very surprising to see that the divergence pattern obtained from the continuity equation disagreed completely in sign, with the one obtained from the vorticity equation, ahead of the surface cyclone south-east of Iceland and over England. As an example compare the 500 mb patterns obtained from continuity equation and vorticity equation. Notice the negative area west of England in Fig. 8 and positive area of Fig. 4. This disagreement of sign was at first very surprising. It was thought that there

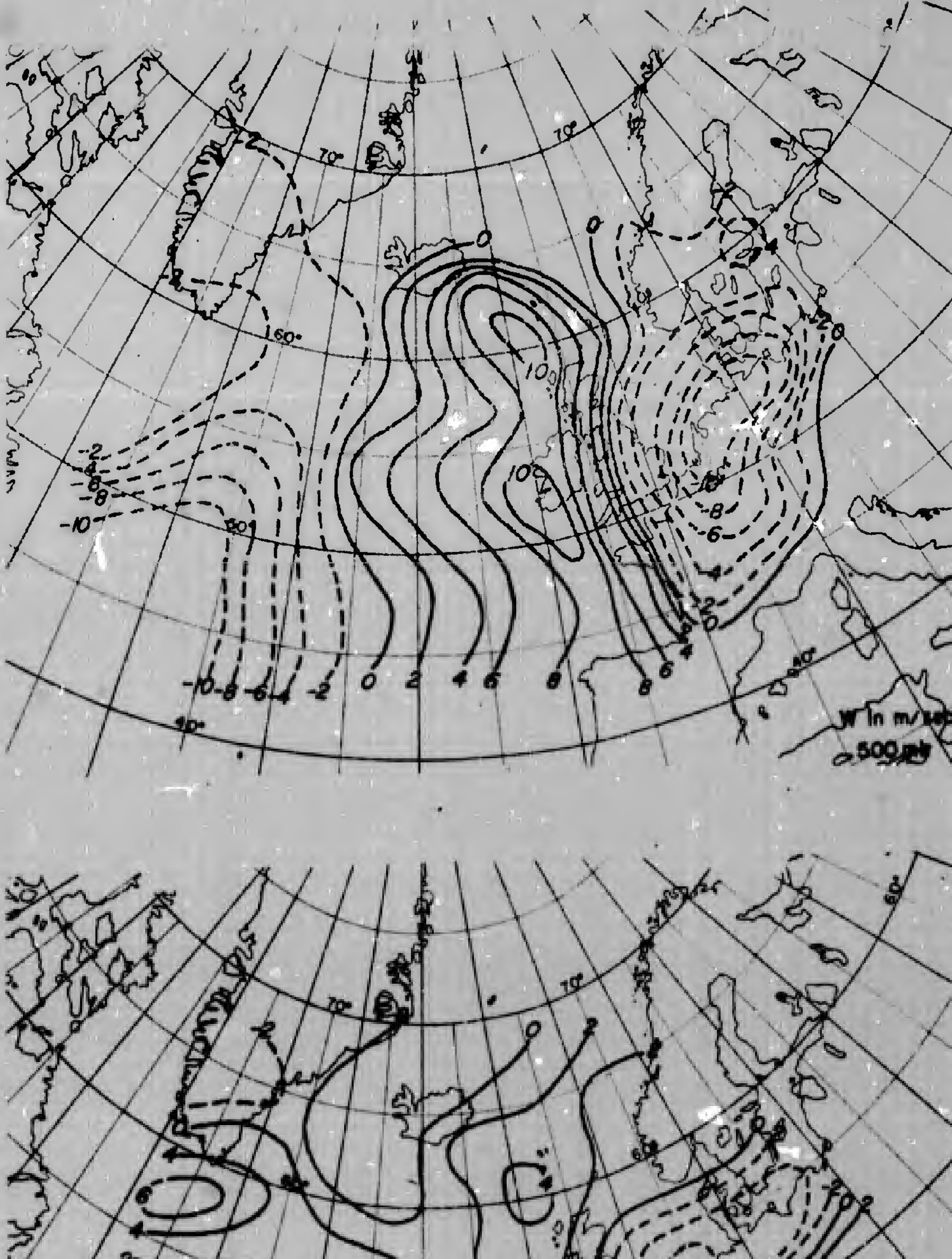
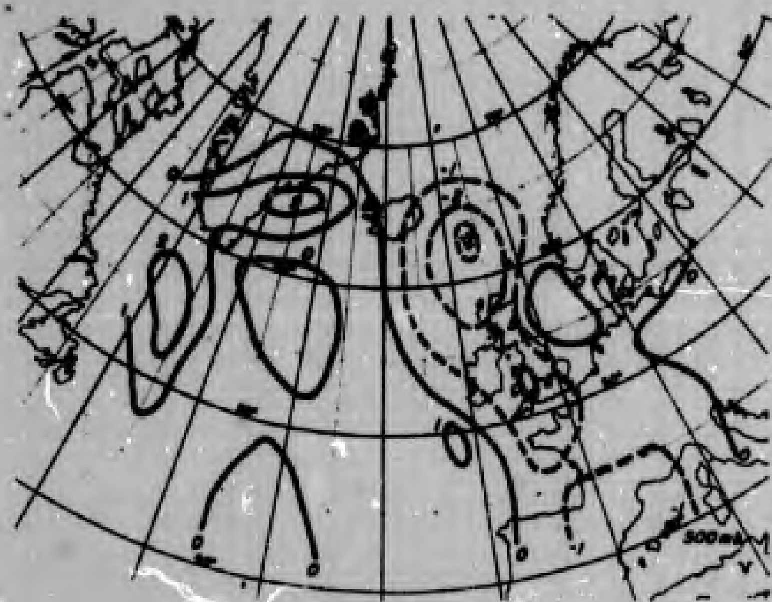
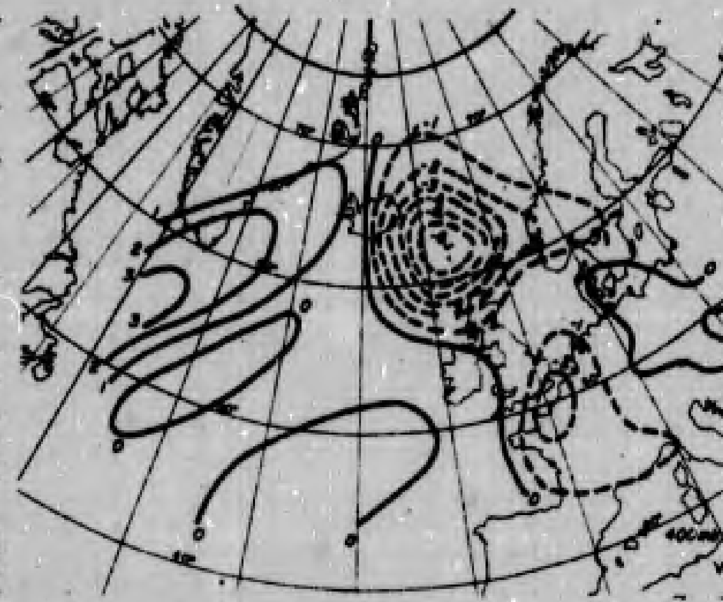


FIG.7

Distribution of vertical velocity on 2nd. Jan.1956 0300 Z at 700 and 500 mb. Vertical velocity is expressed in cm sec^{-1}.

Full lines indicate upward and dashed lines indicate downward motion.





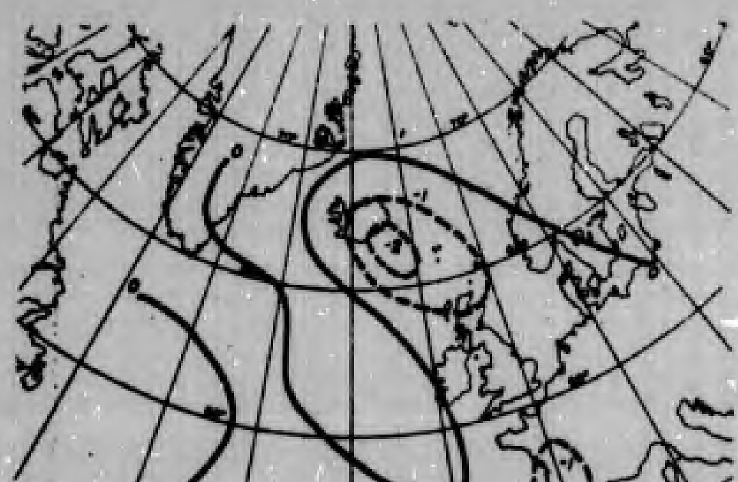


Fig. 8

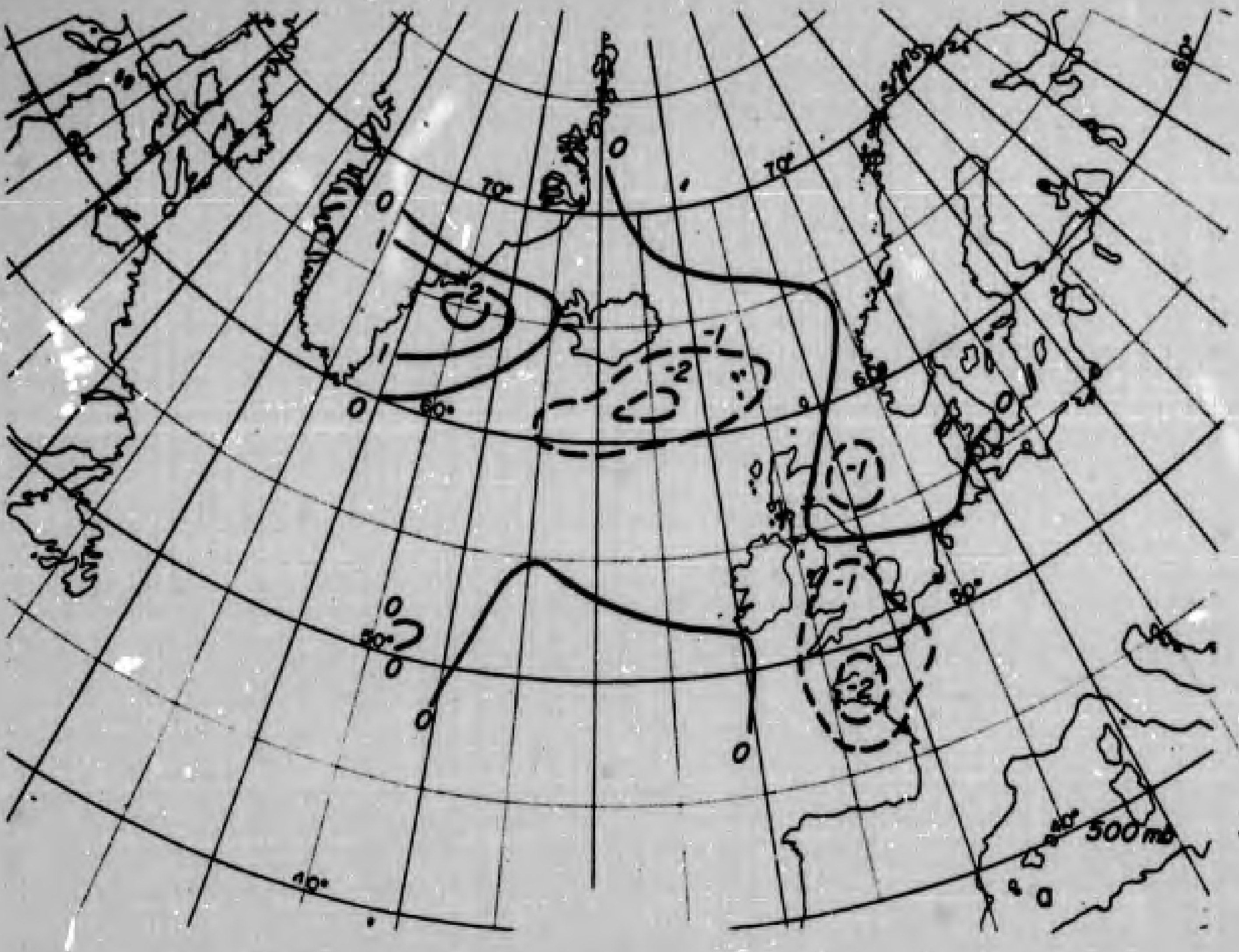
The distribution of $-\nabla \cdot V$ computed from simplified vorticity equation, for the levels of 700,600,500,400 and 300 mb.
The units are 10^{-5} sec.$^{-1}$. Full lines indicate convergence and dashed lines divergence.

must be a sign error in the computations. This possibility was eliminated by going through all the calculations. Besides if there was a sign error, then the two patterns should disagree everywhere on the map, while they differed mainly ahead of surface cyclone. The question was then, what was the cause of this difference. This necessitated a systematic investigation and it was decided to compute each term in the vorticity equation by itself and compare it with the other terms.

In the first place let us look at our simplified vorticity equation. Neglecting the divergence term, the equation simply states that the vorticity of individual parcels of air is conserved along the trajectory. This means that

$$\frac{\partial \xi}{\partial t} + \left[\mathbb{V} \cdot \nabla \xi + \mathbb{V} \cdot \nabla f \right] = 0$$

whic again means that the time change of vorticity and the advective part must have opposite sign (they must balance). The term $\frac{\mathbb{V} \cdot \nabla \xi + \mathbb{V} \cdot \nabla f}{\xi + f}$ was computed for 500 mb and is presented in <u>Fig. 9a</u>. As one can see this term has a negative sign with a maximum absolute value of 2.3 $10^{-5}\mathrm{sec}^{-1}$ immediately south-east of Iceland. This suggests that as a first guess the term $\frac{\partial \xi / \partial t}{\xi + f}$ should have a positive sign over this region. Next the term $\frac{\partial \xi / \partial t}{f + \xi}$ was computed over the area of disagreement and analyzed, and is shown in <u>Fig. 9b</u>. As can be seen from the figure, this term also has negative sign over the mentioned region with an absolute maximum value of 2 10^{-5} sec^{-1} displaced somewhat towards south-east. Now one can ask the question if this term is really the error introducing term. Or in other words, can one really use a 24 hours interval in order to calculate the time rate of vorticity variation. To answer this question we proceeded



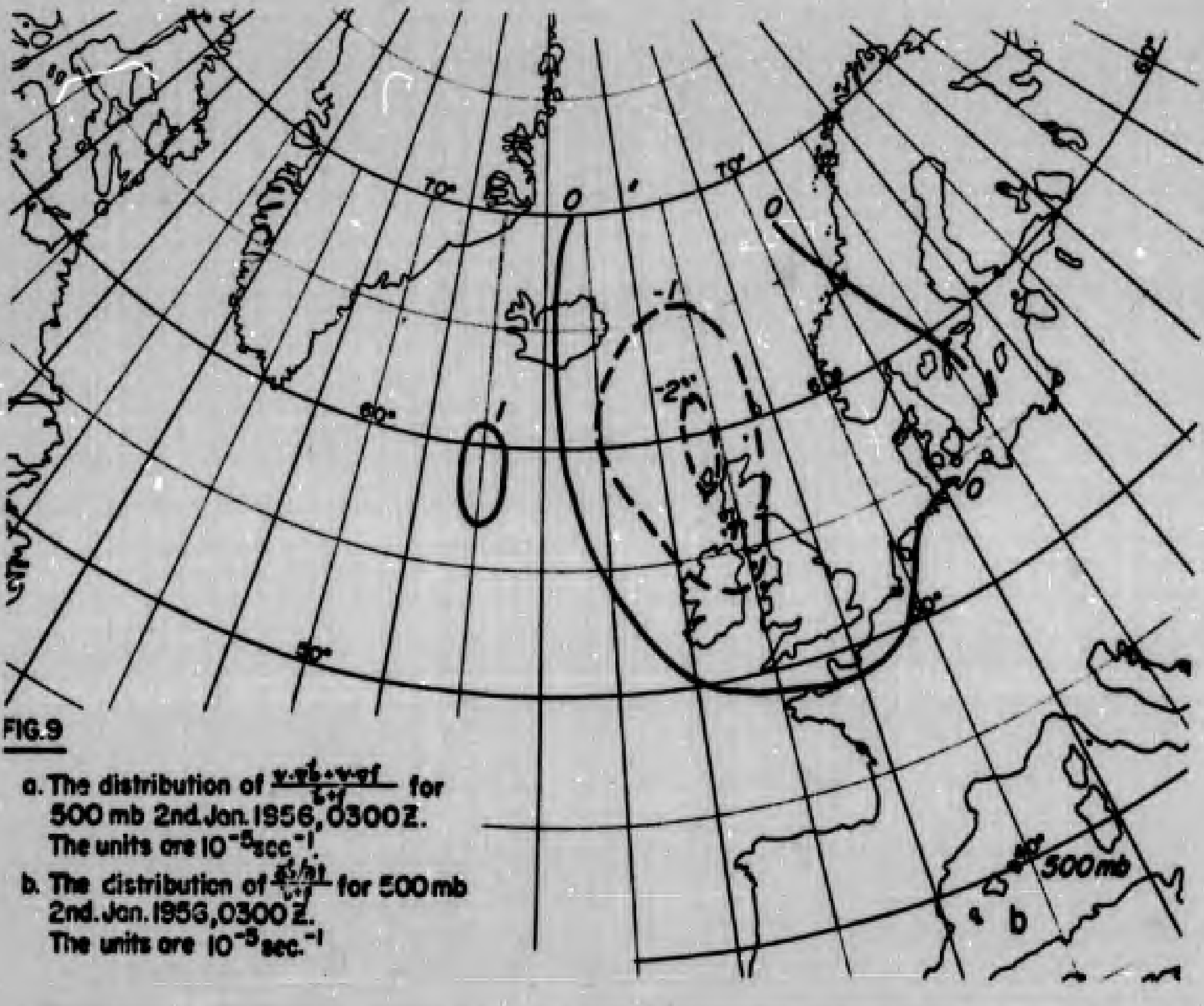


FIG.9

a. The distribution of $\frac{V\cdot\nabla\zeta + V\cdot\nabla f}{\zeta+f}$ for 500 mb 2nd.Jan.1956, 0300 Z. The units are 10^{-5} sec^{-1}.

b. The distribution of $\frac{\partial\zeta/\partial t}{\zeta+f}$ for 500 mb 2nd.Jan.1956, 0300 Z. The units are 10^{-5} sec.$^{-1}$

as follows. Since we are more or less satisfied with the result obtained from continuity equation and furthermore the sign of the vertical motion agrees rather well with the existing synoptic situation, let us assume that the value of divergence obtained from equation (1) is correct. Now we substitute this in equation (2) and solve for the time variation of vorticity. Fig. 10 shows the pattern suggested by the distribution of divergence obtained from continuity equation. The figure shows that the sign of this term is positive.

At this stage it might be said that as the vorticity equation is used in a simplified form without taking the vertical advection of vorticity and the twisting term into consideration the absence of these terms might be completely or partly the cause of the sign error. To investigate this, the above mentioned terms were computed separately. Here the values of ω and $\frac{\partial \omega}{\partial p}$ are taken from our computations by means of continuity equation. Figs. 11a and 11b show the distribution of these two terms. The important thing to notice is that these two terms have almost the same magnitude but opposite sign. This shows that they cancel each other more or less which demonstrates very well the correctness of the assumption that their integrated value approaches zero over sufficiently large area. Substituting the values of these terms with the ones computed before into the complete vorticity equation (complete except frictional term), we obtain the value of $\frac{\partial \zeta}{\partial t} / \zeta + f$ with the correct sign. This is shown in Fig. 12. This figure is not very different from Fig. 10 because the sum of the twisting and vertical advection of vorticity terms was very small. It is seen from Fig. 12 that the computed value of the time change of vorticity has a positive sign almost exactly over the region where

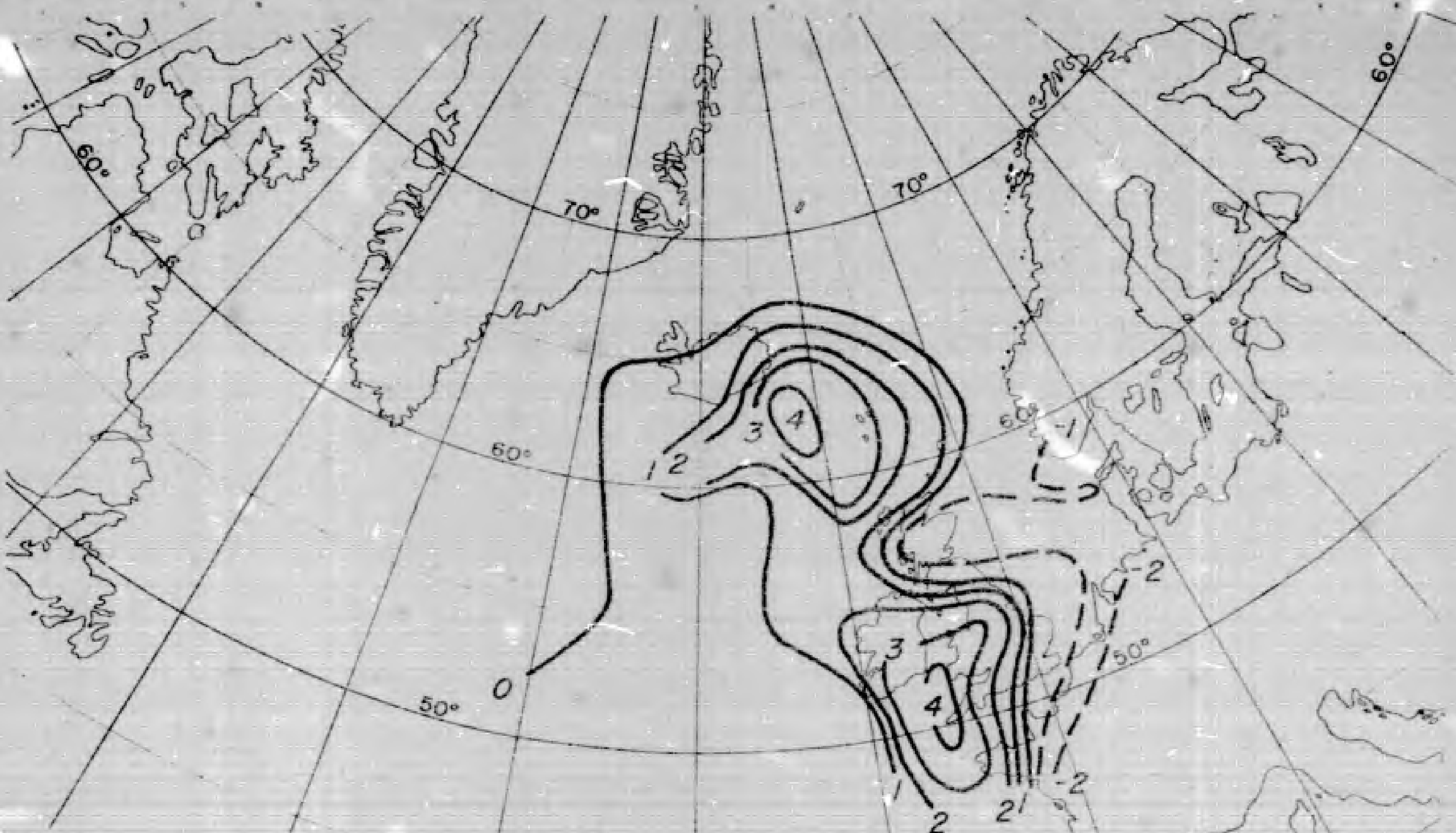


FIG. 10

The distribution of $\frac{\partial\zeta/\partial t}{\zeta+f}$ computed from simplified vorticity equation.
Here we have substituted for $\nabla \cdot \mathbb{V}$ from continuity equation.

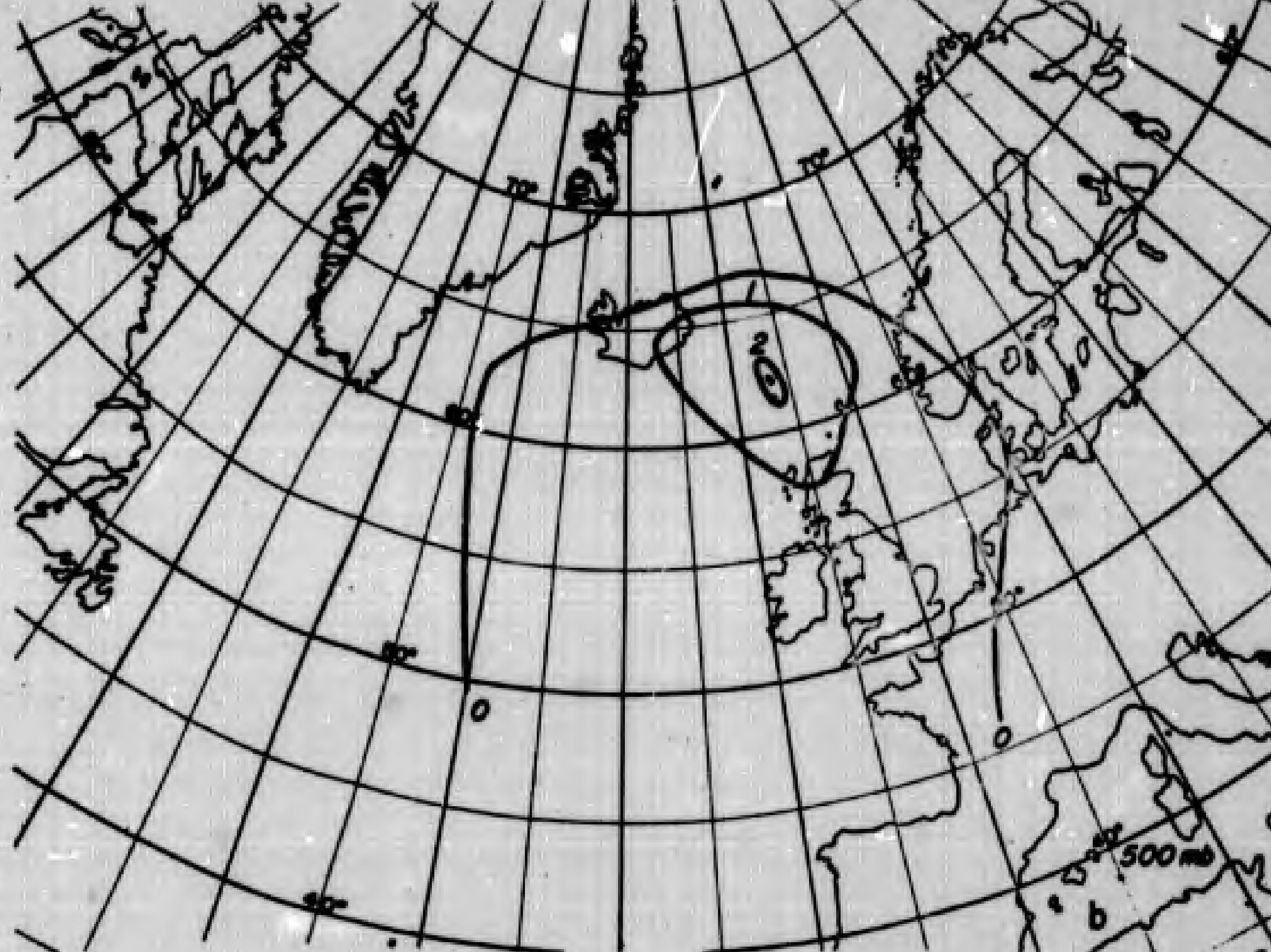


FIG. 11

a. The distribution of [illegible] for 500 mb Jan. 2nd. 1956, 0300 Z. The units are 10^{-5} sec^{-1}

b. The distribution of [illegible] for 500 mb Jan. 2nd. 1956, 0300 Z. The units are 10^{-5} sec.$^{-1}$

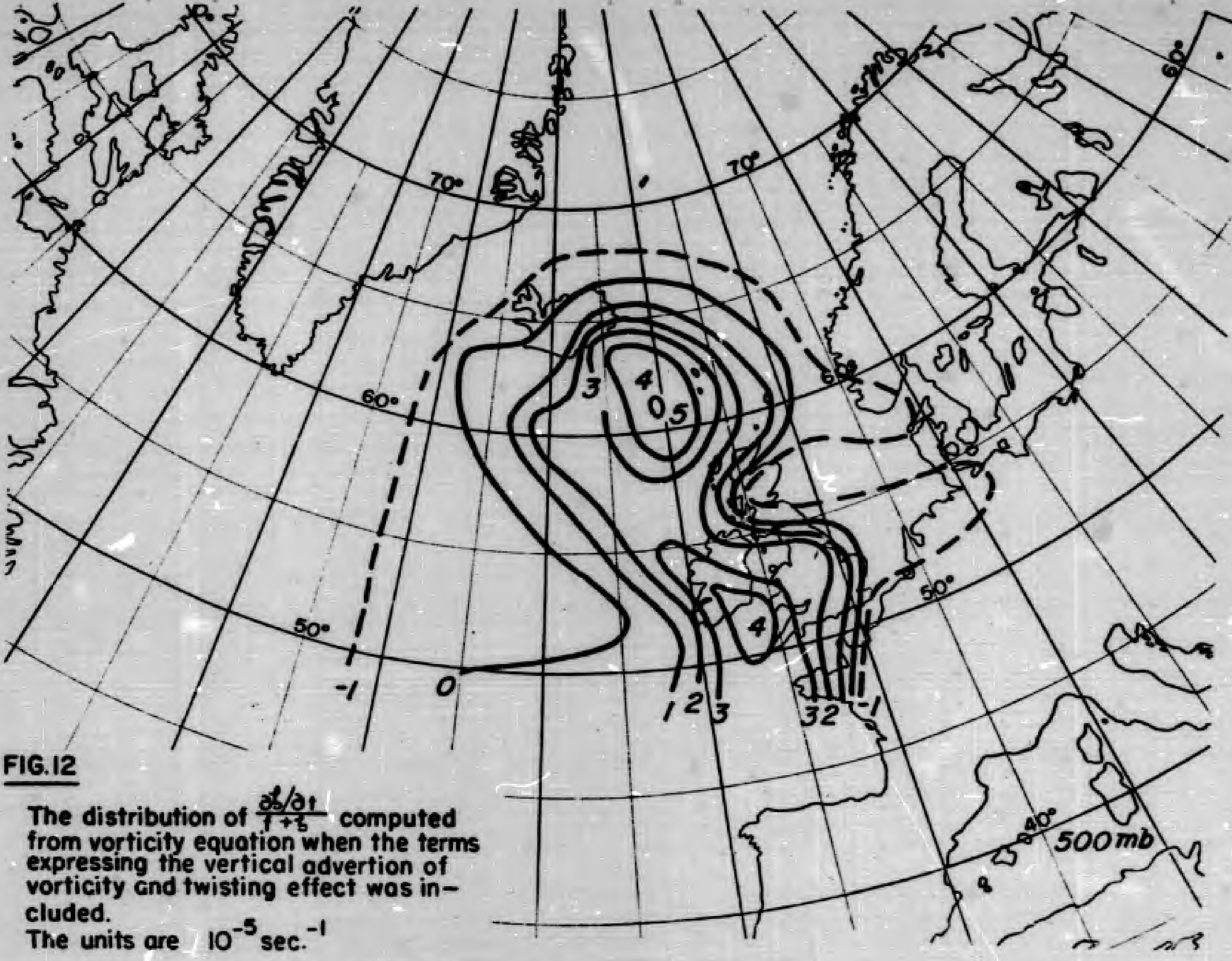


FIG. 12

The distribution of $\frac{\partial \zeta / \partial t}{f+\zeta}$ computed from vorticity equation when the terms expressing the vertical advertion of vorticity and twisting effect was included.
The units are 10^{-5} sec.$^{-1}$

a centered difference method of computation gave negative value. There are two different sources which might contribute to this sign difference:

1) friction has been neglected
2) the mathematical error which can be divided in two parts:
   a) the errors due to the existence of second derivatives
   b) the errors due to large time interval.

Now there is the question which of the factors 1), 2a) or 2b) can cause more error. As far as friction is concerned, although one cannot neglect its effect alltogether, nevertheless it is rather hard to believe that neglecting this term could be so serious. Regarding the errors due to second derivatives, we know that these errors are quite random and non-systematic. Although some meteorologists are of the opinion that they could cause serious trouble, nevertheless, if we are to believe this, we are also forced to believe that numerical forecasting as a whole would be in a serious predicament. However, we know that this equation is always used with more or less successful results. The last error source is just what was mentioned at the beginning, namely, that 24 hours is a rather long time interval and the centered time variation over this period may not be a representation of what happens in the short time interval around the time t at which the instantaneous values of other terms in vorticity equation are computed. It is interesting to look at the vorticity distribution over the area of disagreement. Fig. 13 shows the vorticity pattern of Jan. 2nd, 1956, 0300 GMT together with the patterns of 12 hours before and 12 hours after. What we did for the computation of $\frac{\partial \zeta}{\partial t}$ was to approximate this by $\frac{\zeta_{t+12} - \zeta_{t-12}}{2\Delta t}$. Now ζ_{t+12} is negative and bigger (over the region of disagreement) than ζ_{t-12} which

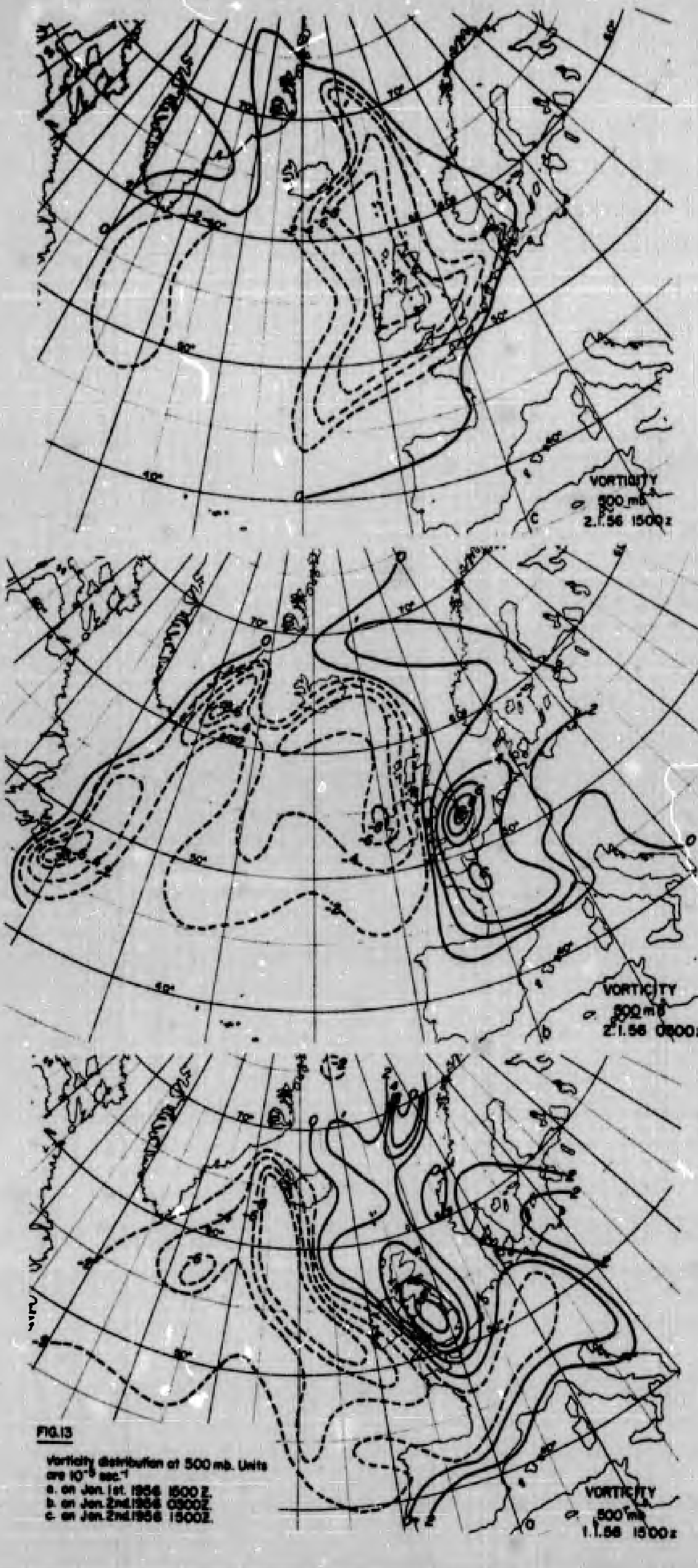


FIG.13

Vorticity distribution at 500 mb. Units are 10^{-5} sec.$^{-1}$
a. on Jan. 1st. 1956 1500 Z.
b. on Jan. 2nd. 1956 0300Z.
c. on Jan. 2nd. 1956 1500Z.

is both positive and negative over the region of discussion. What has happened in reality is that in the short time interval around time t an abrupt change of vorticity sign has taken place which could not be detected by taking long time interval.

Another point of interest is that the magnitude of relative vorticity is as high as 0.8×10^{-4} sec^{-1}. This shows that ζ is not at all negligible compared to f as it is assumed in most of the models constructed for the numerical forecasting.

Conclusion.

1) The magnitude of vertical velocities computed from continuity equation may not be quite correct, because there exist errors in wind observation, truncation errors in which the errors due to vertical integration are included. Experience shows that this method of divergence and vertical velocity computation is so far the most correct and least complicated one. It is also true that in deriving continuity equation for a constant pressure surface no assumption has been made (except the hydrostatic balance) and that the wind values used are all instantaneous values. However, one must not forget that the correctness of the result depends of the density of the reporting stations.

2) Due to the fact that the vorticity equation contains a term which is time dependent and cannot be evaluated from instantaneous values, certain care should be taken in the use of this equation. To study individual disturbances of the scale of a cyclone, a 24 hours time interval is rather a long time for computing the time variation of the vorticity.

Part III

This is an attempt to compute the divergence field from vorticity equation without including the term $\frac{\partial \xi}{\partial t}$. One way to do this is simply to neglect this term alltogether. However, since we are aware of the importance of the time rate of vorticity changes, this does not seem to be permissible. By introducing the same assumption about the vertical variation of the wind as was done by Charney, (5), a somewhat simple formula can be obtained; we proceed as follows. The first two equations of motion in a simplified form are:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \omega\frac{\partial u}{\partial p} - fv = -g\frac{\partial z}{\partial x}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \omega\frac{\partial v}{\partial p} + fu = -g\frac{\partial z}{\partial y}$$

Next multiply the continuity equation for an isobaric surface by u and v respectively and add to the two equations of motion. Here we assume that the wind variation with height is given by the relation

(1) $$\mathbb{V} = A(p)\cdot\bar{\mathbb{V}}$$

Introducing this into the equations we get

$$A\frac{\partial \bar{u}}{\partial t} + A^2\frac{\partial \bar{u}\bar{u}}{\partial x} + A^2\frac{\partial \bar{u}\bar{v}}{\partial y} + \frac{\partial(\bar{u}\omega A)}{\partial p} - Af\bar{v} = -g\frac{\partial z}{\partial x}$$

$$A\frac{\partial \bar{v}}{\partial t} + A^2\frac{\partial \bar{u}\bar{v}}{\partial x} + A^2\frac{\partial \bar{u}\bar{v}}{\partial y} + \frac{\partial(\bar{v}\omega A)}{\partial p} + Af\bar{u} = -g\frac{\partial z}{\partial y}$$

Let us integrate these equations with respect to pressure and assume that $\omega = 0$ at $p = 0$ and $p = p_0$ where p_0 is pressure at the surface. Remembering that $\overline{A} = 1$ the integrated equations become:

$$\frac{\partial \overline{u}}{\partial t} + \overline{A^2}\,\overline{u}\,\frac{\partial \overline{u}}{\partial x} + \overline{A^2}\,\overline{v}\,\frac{\partial \overline{u}}{\partial y} + \overline{A^2}\,\overline{u}\left(\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y}\right) - f\overline{v} = -g\int_{p_0}^{p_0} \frac{\partial z}{\partial x}\,dp$$

$$\frac{\partial \overline{v}}{\partial t} + \overline{A^2}\,\overline{u}\,\frac{\partial \overline{v}}{\partial x} + \overline{A^2}\,\overline{v}\,\frac{\partial \overline{v}}{\partial y} + \overline{A^2}\,\overline{v}\left(\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y}\right) + f\overline{u} = -g\int_{p_0}^{p_0} \frac{\partial z}{\partial y}\,dp$$

The terms inside paranthesis are zero due to continuity equation and boundary conditions. By cross differentiation one can derive the vorticity equation for the mean atmosphere

$$\frac{\partial \overline{\xi}}{\partial t} + \overline{A^2}\,\overline{\mathbb{V}}\cdot\nabla\overline{\xi} + \overline{\mathbb{V}}\cdot\nabla f = 0$$

This is evidently the same equation derived by Charney by using the vorticity equation, where the vertical advection of vorticity and twisting term are neglected and it is assumed that $\xi \ll f$. As it is shown here, these assumptions are not necessary. Next let us take the vorticity equation in the simplified form

$$(2)\qquad \frac{\partial \xi}{\partial t} + \mathbb{V}\cdot\nabla\xi + \mathbb{V}\cdot\nabla f = -(\xi + f)\,\nabla\cdot\mathbb{V}$$

Introduce the relation (1)

$$(3)\qquad A\,\frac{\partial \overline{\xi}}{\partial t} + A^2\,\overline{\mathbb{V}}\cdot\nabla\overline{\xi} + A\,\overline{\mathbb{V}}\cdot\nabla f = -(f + A\overline{\xi})\,\nabla\cdot\mathbb{V}$$

Multiplying the equation (2) by A and subtract from (3), a relation for divergence can be obtained

$$(4)\qquad \nabla\cdot\mathbb{V} = -\left(A^2 - A\overline{A^2}\right)\frac{\overline{\mathbb{V}}\cdot\nabla\overline{\xi}}{f + A\overline{\xi}}$$

This equation has been used before, however, without the term ξ in the denominator. From our discussion of part II, one can see that it is important to have this term in the equation.

Equation 4 has been used for divergence computation. The values of A and A^2 and A^3 are obtained from the mean distribution of A(P) and $A^2(P)$ computed by Taba (6) .

Fig. 14 shows the divergence distribution computed from relation (4). Due to model assumption the divergence changes sign at 500 and 130 mb level. At these levels ω has extreme values.

There are some points of interest:

1) The divergence distribution is on the whole smaller (10^{-6}) in this case than what was obtained from continuity equation and vorticity equation.

2) The agreement with the surface cyclone and frontal zone is better in this case than the case of vorticity equation. However, it is not as good as the pattern obtained from the continuity equation.

Next equation (4) was integrated upwards to obtain vertical velocity distribution at different levels. Fig. 15 shows the distribution of ω at 700 and 500 mb. The full lines indicate upward and the dashed lines downward motion.

Finally from the formula $W = -\dfrac{\omega}{g\rho}$ the value of W in cm/sec is computed and presented in Fig. 16.

Discussion of the result.

1) As we know the assumption that the wind variation with height can be expressed through the equation (1) is far from being realistic. This is specially true in the frontal zone as the vertical wind variation is most pronounced in these regions. Therefore one cannot expect that the equation (4) should give a

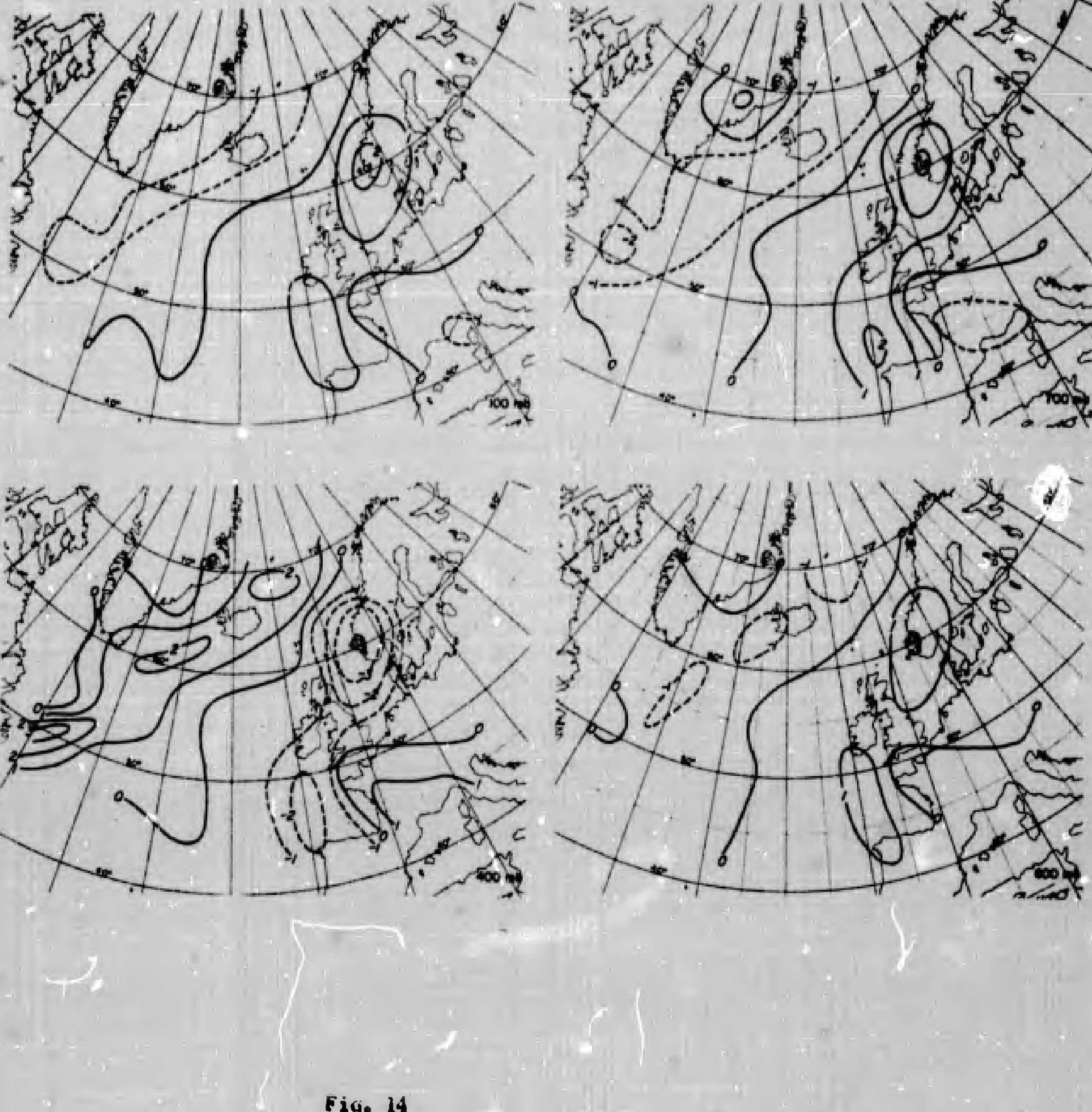


Fig. 14

The distribution of $-\nabla\cdot V$ computed from the formula $-\nabla\cdot V = \left(\bar{A}^{2} - A\,\overline{A^{2}}\right)\dfrac{\overline{V\cdot\nabla\xi}}{f + A\bar{\xi}}$
The units are 10 sec.

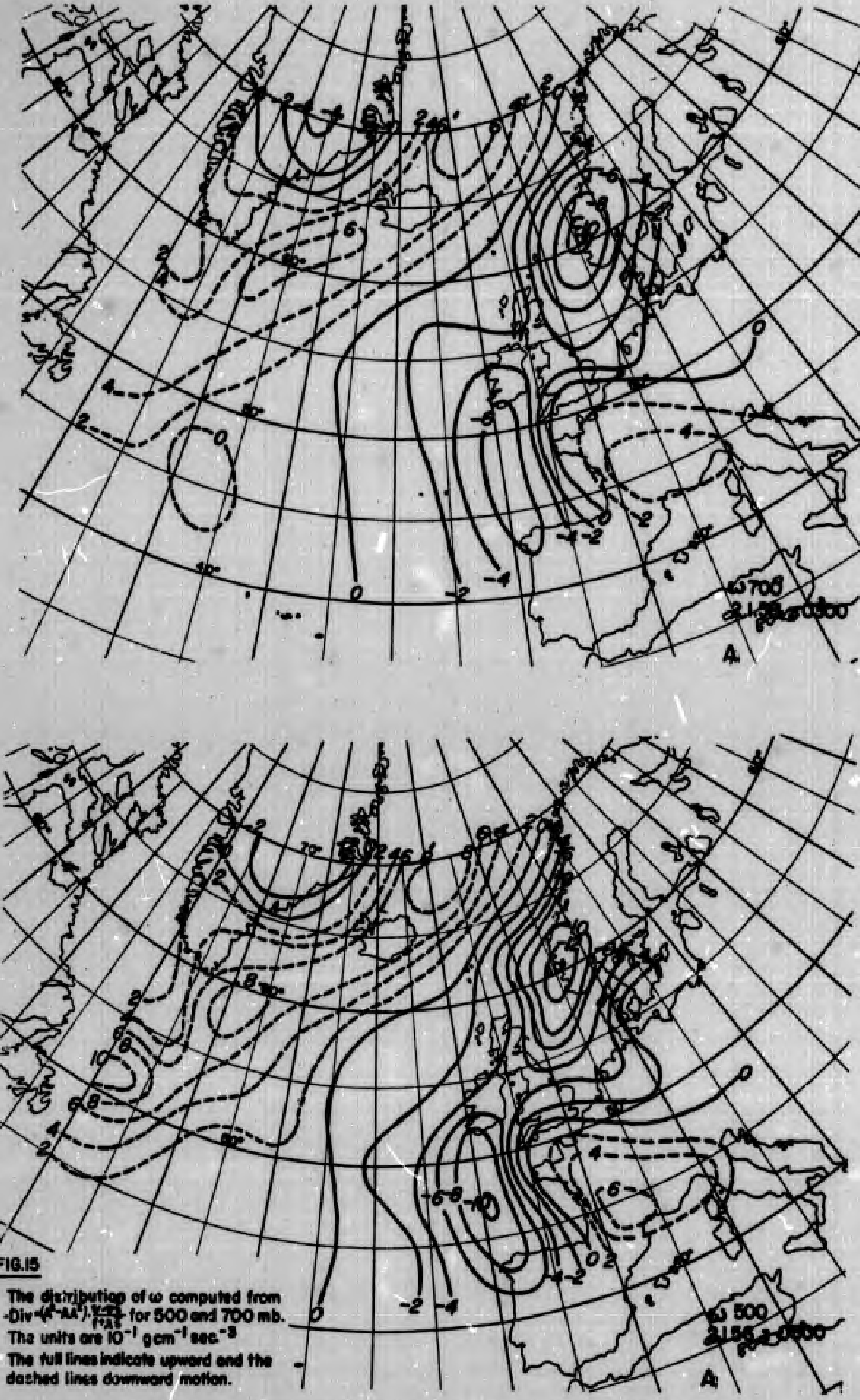


FIG.15

The distribution of ω computed from -Div [illegible] for 500 and 700 mb. The units are 10^{-1} g cm^{-1} sec.$^{-3}$ The full lines indicate upward and the dashed lines downward motion.

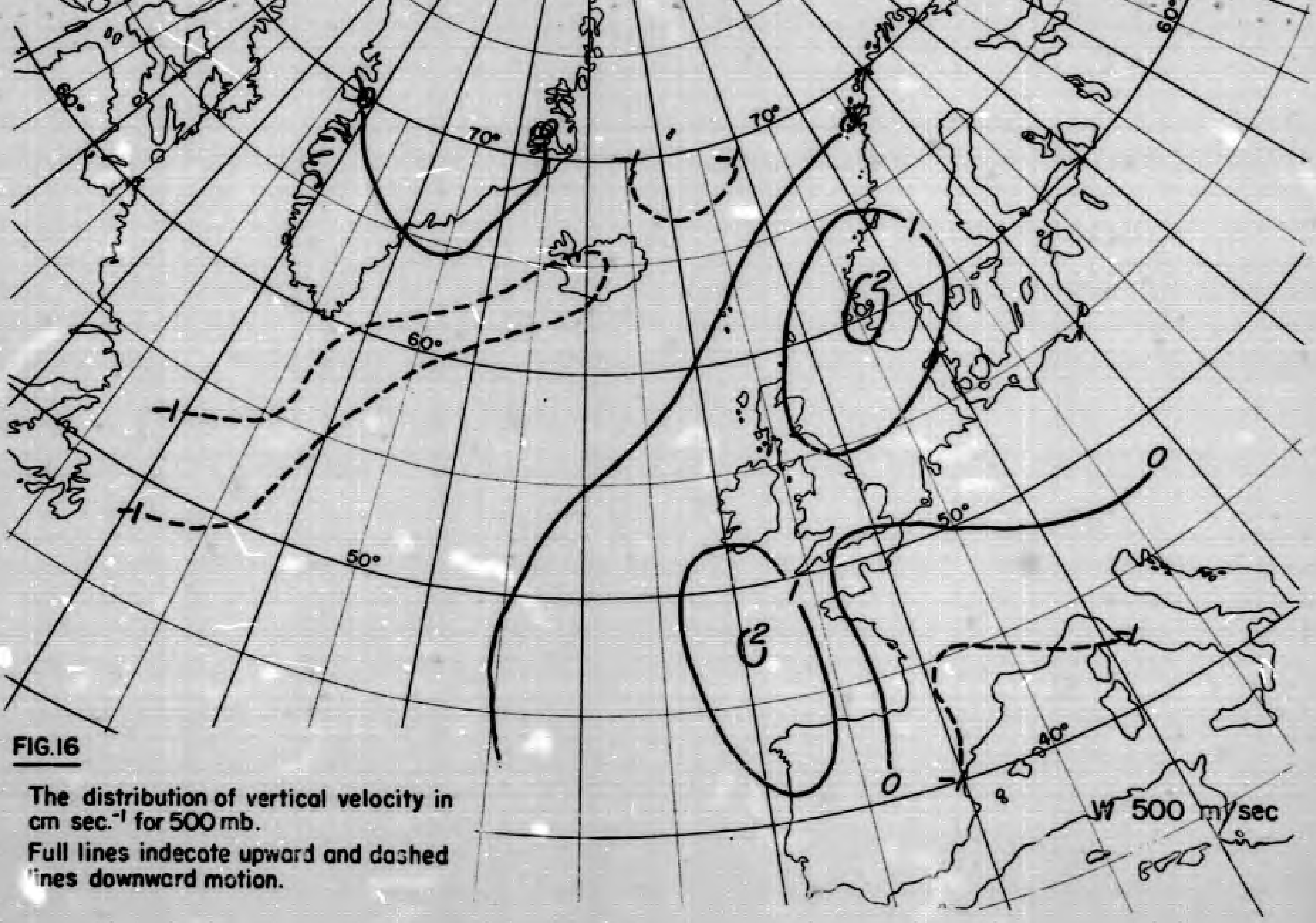


FIG.16

The distribution of vertical velocity in cm sec.$^{-1}$ for 500 mb.
Full lines indecate upward and dashed lines downward motion.

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