INSTRUCTIONS FOR USING EXPERIMENTAL 0-1 INTEGER LINEAR PROGRAMMING CODE RIP23J

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Numerous requests have been received for copies of the experimental code used to obtain the computational experience reported in Refs. 2 and 3. It should be recognized that this is not a production code. It was developed to test the usefulness of certain innovations applied to a simple Balasian algorithm. The central concern was the rate of increase of solution time as a function of the number of variables, rather than how to achieve the smallest possible execution time for particular problems. For this reason, the simplest possible Balasian algorithm was used as the starting point, and concessions were freely made to programming expediency (e.g., no machine language). It would not be difficult to reduce execution times substantially by reprogramming and introducing some of the more sophisticated tests already available in the literature.

We discuss input in Sec. 1; output in Sec. 2; and give an example in Appendix A, and a program listing in Appendix B. For an outline of the working details of the algorithm, see [1] and [2]. Familiarity with these papers is presumed here.

The program solves integer linear programs of the form

(P) $\qquad$ Minimize cx subject to $b + Ax \geq 0$

$$x_j = 0 \text{ or } 1$$

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where c and x are n-vectors, b is an m-vector, and A is m by n. Any bounded integer linear program can be written in this form, using elementary manipulations if necessary.

1. INPUT

The following parameter and data cards appear for each problem to be run:

(a) Parameter card

(b) S-card(s)

(c) C-card(s)

(d) B-card(s)

(e) A-card(s)

(f) Blank card.

Problems can be stacked by repetition of cards a through f.

Parameter Card

The input parameters are:

M	The number of constraints
N	The number of variables
L	The number of variables in the initial partial solution (L must correspond to the number of entries on the S-card). If L = 0, the initial partial solution is empty. If L < 0, the initial partial solution consists of all variables fixed at the value 0.
SC	Punch 0 if no imbedded linear program is desired (the algorithm then reduces to a simple Balasian algorithm), and 1 if the imbedded linear program is to be used.
KENUM	When intermediate output is used (NØP = 0), the fraction of all 2^n possible solutions that have been implicitly enumerated is printed out every KENUM times that backtracking occurs. KENUM = 20 is reasonable.
ZBAR	If an upper bound $\bar{z}$ on the optimal value of the objective function of (P) is known, put ZBAR = $\bar{z}$ - lcd + 0.0001, where lcd is the least common denominator of the cost coefficients c_j (we assume that $\bar{z}$ is a multiple of lcd). Hence, if all c_j (and $\bar{z}$) are integer, put ZBAR = $\bar{z}$ - .9999. The effect will be that the program looks only for feasible solutions with value < ZBAR. If no upper bound is known, put ZBAR = 0. See Remark 2 below.

ISCMAX	The maximum number of composite constraints that will be carried. ISCMAX = 4 is reasonable.
ISCFR	The frequency with which the imbedded linear program is used. ISCFR = 0 means that it will never be used; ISCFR = j, j a positive integer, means that it will be used every j^{th} time. ISCFR = 1 has proven effective, but frequently a value of 8 or so is even better.
MAXC	If equal to 0, nothing will happen. If equal to 1, all signs on the C- and A-Cards will be reversed automatically when these cards are read in. This is purely a convenience for manuscripting and keypunching for problems with a preponderance of minus signs in C and A.
MAXT	Terminates the calculations after MAXT seconds.
NØP	If equal 1, intermediate output will be suppressed; if equal 0, intermediate output will appear. Normally NØP will be set at 1.
ZKBAR	Put equal to lcd (see ZBAR) minus 1. Thus, if all c_j are integer, put ZKBAR = 0. The effect is that the program looks only for feasible solutions with value at least (ZKBAR + .99999) less than the best feasible solution currently known; this doesn't exclude any optimal solutions. (A solution within Δ of the optimum can be found if desired by increasing the above value of ZKBAR by Δ.)
H1,H2	Arbitrary problem identifiers.

Remark 1: The program is currently dimensioned to use 32,000 words of core in such a way that the following limits must be observed:

$$M + ISCMAX \leq 50$$

$$N \leq 90.$$

Remark 2: If any c_j are negative (after MAXC has changed the input signs, if it has value 1), the program internally makes a trivial change of variables to make such c_j nonnegative: replace x_j by $y_j = (1 - x_j)$ if $c_j < 0$. The problem is solved in terms of the new variables, and the reverse transformation is made at final output in order to recover the solution to the original problem. ZBAR must be set at a value corresponding to the transformed problem when it is desired to use a

known upper bound; hence, when $c_j < 0$ for $j \in J$, put ZBAR = $\bar{z}$ - lcd + .0001 + $\sum_{j \in J} |c_j|$.

The fields and formats of the parameter card are as follows:

Parameter	Column	Format
M	1-3	Integer
N	4-6	"
L	7-9	"
SC	10-12	"
KENUM	13-17	"
ZBAR	18-23	E
ISCMAX	24-26	Integer
ISCFR	27-29	"
MAXC	30-32	"
MAXT	33-37	"
NØP	38-40	"
ZKBAR	41-46	E
H1	47-52	Hollerith
H2	53-58	"

S-Card(s)

The algorithm can start with any initial partial solution (see [1]). When the initial partial solution is desired to be nonempty (L > 0), if x_j is to be fixed at the value one (zero) then "j" ("-j") is entered on the S-card, followed by "B" when an underline is desired. The S-card is divided into 12 fields of 5 columns each: 1-5, 6-10, ..., 66-70. Only the first four columns of each field are to be used except when underlines are desired, in which case "B" must appear in the fifth column of the field.

The special instruction given above in Remark 2 for ZBAR, when a change of variables is made, also applies here. That is, the sign of $\pm$ j or $\pm$ jB must be changed when $c_j < 0$.

C-Card(s)

The values of the c_j must be entered in order (negative values are permissible, as noted above). Each card has six fields of eleven columns read in E-format. The fields are separated by an unread column so that the values of the c_j are in columns 1-11, 13-23,...,61-71.

B-Card(s)

The values of the b_i must be entered in order. The format is exactly the same as for the C-cards.

A-Card(s)

Only nonzero a_{ij} need be entered, and they may be entered in any order. Each value is identified by its row and column. There are four or fewer entries on each of the "A" cards. Each entry has a seventeen column field.

	Columns	Format
Row	1-3	Integer
Column	4-6	Integer
Value	7-17	E

The fields are separated by an unread column so that the matrix subscripts and values of the a_{ij} are in columns 1-17, 19-35, 37-53, 55-71.

2. OUTPUT

The preliminary, intermediate, and final outputs are as follows.

The parameter, "S", "C", and "B" cards are printed in that order (six values to a line for the "C" and "B" cards). Then the complete A matrix is printed (with zeros), row by row. If MAXC = 1, the sign reversals in "C" and "A" will be seen to have occurred. If a change of variables was made internally, the new c, b, and A are printed out (if no change of variables was necessary, the identical c, b, and A are printed out again anyway).

If NØP = 0, intermediate output is produced to reveal the course of the calculations - each feasible solution found, each new composite constraint, data concerning each imbedded linear program, and a summary of progress to date after each KENUM "backtrackings." Since this information is likely to be of little incremental value to the user over the final output information, no detailed explanation is given here.

The final output gives the problem designation; the message "implicit enumeration complete" or "time exceeded" according as termination did or did not occur within MAXT seconds; the total execution time in seconds; the solution (obj. fc. value and a list of which variables equal 1) both before and after the variable change (if no variable change occurred, these solutions are identical); and some statistical information on the course of the algorithm, such as the number of feasible solutions found, the number of times the imbedded linear program was solved, the number of iterations, and the time at which the last feasible solution was found. In the event that no feasible solutions were found, this is indicated by the zeros in the solution after variable change and the statistic "no. feasible solutions 0." In the event that the time limit was exceeded, the final output is preceded by a brief report giving the proportion of all 2^n possible solutions that have been accounted for and the final "state" vector [1], with "B" signifying an underline. All the information needed to restart the calculations is available: make the S-card correspond to the final state vector (set L accordingly), and put ZBAR equal to LEAST Z AFTER VARIABLE CHANGE - 1cd + 0.0001.

Appendix A

EXAMPLE

We shall illustrate the above by solving Petersen's fifth example [4].

For this problem, M = 10 and N = 28. We shall take L = 28, SC = 1, KENUM = 20, ZBAR = 0 (since we will not bother to determine a bound on the objective function), ISCMAX = 4, ISCFR = 1, MAXC = 1 (since we wish to avoid keypunching all the minus signs for c and A), MAXT = 60, NØP = 1, ZKBAR = 4 (since the least common denominator of the c_j is 5), and H1 = PETE 5.

The S-card will contain the numbers 1, 2,..., 28 (we have elected an initial partial solution with all variables fixed at the value 1).

The output is reproduced below.

10 28 28 1 20 0. 4 1 1 60 1 0.4000E 01 PETE 5

M= 10 N= 28

1	2	3	4	5	6	7	8	9	10	11	12	13	14
15	16	17	18	19	20	21	22	23	24	25	26	27	28
-1.0000000 E 02	-2.2000000 E 02	-9.0000000 E 01	-4.0000000 E 02	-3.0000000 E 02	-4.0000000 E 02								
-2.0500000 E 02	-1.2000000 E 02	-1.6000000 E 02	-5.8000000 E 02	-4.0000000 E 02	-1.4000000 E 02								
-1.0000000 E 02	-1.3000000 E 03	-6.5000000 E 02	-3.2000000 E 02	-4.8000000 E 02	-8.0000000 E 01								
-6.0000000 E 01	-2.5500000 E 03	-3.1000000 E 03	-1.1000000 E 03	-9.5000000 E 02	-4.5000000 E 02								
-3.0000000 E 02	-2.2000000 E 02	-2.0000000 E 02	-5.2000000 E 02										
9.3000000 E 02	1.2100000 E 03	2.7200000 E 02	4.6200000 E 02	5.3200000 E 02	5.7200000 E 02								
2.4000000 E 02	4.0000000 E 02	4.7000000 E 02	4.9000000 E 02										
-8.0000000 E 00	-2.4000000 E 01	-1.3000000 E 01	-8.0000000 E 01	-7.0000000 E 01	-8.0000000 E 01								
-4.5000000 E 01	-1.5000000 E 01	-2.8000000 E 01	-9.0000000 E 01	-1.3000000 E 02	-3.2000000 E 01								
-2.0000000 E 01	-1.2000000 E 02	-4.0000000 E 01	-3.0000000 E 01	-2.0000000 E 01	-6.0000000 E 00								
-3.0000000 E 00	-1.8000000 E 02	-2.2000000 E 02	-5.0000000 E 01	-3.0000000 E 01	-5.0000000 E 01								
-1.2000000 E 01	-5.0000000 E 00	-8.0000000 E 00	-1.8000000 E 01										
-8.0000000 E 00	-4.4000000 E 01	-1.3000000 E 01	-1.0000000 E 02	-1.0000000 E 02	-9.0000000 E 01								
-7.5000000 E 01	-2.5000000 E 01	-2.8000000 E 01	-1.2000000 E 02	-1.3000000 E 02	-3.2000000 E 01								
-4.0000000 E 01	-1.6000000 E 02	-4.0000000 E 01	-6.0000000 E 01	-5.5000000 E 01	-1.0000000 E 01								
-6.0000000 E 00	-2.4000000 E 02	-2.9000000 E 02	-8.0000000 E 01	-9.0000000 E 01	-7.0000000 E 01								
-2.7000000 E 01	-1.7000000 E 01	-8.0000000 E 00	-2.8000000 E 01										
-3.0000000 E 00	-6.0000000 E 00	-4.0000000 E 00	-2.0000000 E 01	-2.0000000 E 01	-3.0000000 E 01								
-8.0000000 E 00	-3.0000000 E 00	-1.2000000 E 01	-1.4000000 E 01	-4.0000000 E 01	-6.0000000 E 00								
-3.0000000 E 00	-2.0000000 E 01	-5.0000000 E 00	-0.	-5.0000000 E 00	-3.0000000 E 00								
-0.	-2.0000000 E 01	-3.0000000 E 01	-4.0000000 E 01	-1.0000000 E 01	-0.								
-5.0000000 E 00	-0.	-0.	-1.0000000 E 01										
-5.0000000 E 00	-9.0000000 E 00	-6.0000000 E 00	-4.0000000 E 01	-3.0000000 E 01	-4.0000000 E 01								
-1.6000000 E 01	-5.0000000 E 00	-1.8000000 E 01	-2.4000000 E 01	-6.0000000 E 01	-1.6000000 E 01								
-1.1000000 E 01	-3.0000000 E 01	-2.5000000 E 01	-1.0000000 E 01	-1.3000000 E 01	-5.0000000 E 00								
-1.0000000 E 00	-8.0000000 E 01	-6.0000000 E 01	-5.0000000 E 01	-2.0000000 E 01	-3.0000000 E 01								
-1.0000000 E 01	-5.0000000 E 00	-3.0000000 E 00	-2.0000000 E 01										
-5.0000000 E 00	-1.1000000 E 01	-7.0000000 E 00	-5.0000000 E 01	-4.0000000 E 01	-4.0000000 E 01								
-1.9000000 E 01	-7.0000000 E 00	-1.8000000 E 01	-2.9000000 E 01	-7.0000000 E 01	-2.1000000 E 01								
-1.7000000 E 01	-3.0000000 E 01	-2.5000000 E 01	-1.5000000 E 01	-2.5000000 E 01	-5.0000000 E 00								
-1.0000000 E 00	-1.0000000 E 02	-7.0000000 E 01	-5.5000000 E 01	-2.0000000 E 01	-5.0000000 E 01								
-1.5000000 E 01	-1.5000000 E 01	-6.0000000 E 00	-2.0000000 E 01										
-5.0000000 E 00	-1.1000000 E 01	-7.0000000 E 00	-5.5000000 E 01	-4.0000000 E 01	-4.0000000 E 01								
-2.1000000 E 01	-9.0000000 E 00	-1.8000000 E 01	-2.9000000 E 01	-7.0000000 E 01	-2.1000000 E 01								
-1.7000000 E 01	-3.5000000 E 01	-2.5000000 E 01	-2.0000000 E 01	-2.5000000 E 01	-5.0000000 E 00								
-2.0000000 E 00	-1.1000000 E 02	-7.0000000 E 01	-5.5000000 E 01	-2.0000000 E 01	-5.0000000 E 01								
-2.0000000 E 01	-1.5000000 E 01	-6.0000000 E 00	-2.0000000 E 01										
-0.	-0.	-1.0000000 E 00	-1.0000000 E 01	-4.0000000 E 00	-1.0000000 E 01								
-0.	-6.0000000 E 00	-0.	-6.0000000 E 00	-3.2000000 E 01	-3.0000000 E 00								
-0.	-7.0000000 E 01	-1.0000000 E 01	-0.	-0.	-0.								
-0.	-0.	-3.0000000 E 01	-1.0000000 E 01	-0.	-1.0000000 E 01								
-1.0000000 E 01	-5.0000000 E 00	-0.	-1.0000000 E 01										

-3.0000000 E 00 -4.0000000 E 00 -5.0000000 E 00 -2.0000000 E 01 -1.4000000 E 01 -2.0000000 E 01
-6.0000000 E 00 -1.2000000 E 01 -1.0000000 E 01 -1.8000000 E 01 -4.2000000 E 01 -9.0000000 E 00
-1.2000000 E 01 -1.0000000 E 02 -2.0000000 E 01 -5.0000000 E 00 -6.0000000 E 00 -4.0000000 E 00
-1.0000000 E 00 -2.0000000 E 01 -5.0000000 E 01 -3.0000000 E 01 -5.0000000 E 00 -2.0000000 E 01
-2.0000000 E 01 -1.0000000 E 01 -1.0000000 E 01 -2.0000000 E 01

-3.0000000 E 00 -6.0000000 E 00 -9.0000000 E 00 -3.0000000 E 01 -2.9000000 E 01 -2.0000000 E 01
-1.2000000 E 01 -1.2000000 E 01 -1.0000000 E 01 -3.0000000 E 01 -4.2000000 E 01 -1.8000000 E 01
-1.8000000 E 01 -1.1000000 E 02 -2.0000000 E 01 -1.5000000 E 01 -1.8000000 E 01 -7.0000000 E 00
-2.0000000 E 00 -4.0000000 E 01 -6.0000000 E 01 -5.0000000 E 01 -2.5000000 E 01 -2.5000000 E 01
-2.5000000 E 01 -1.5000000 E 01 -1.0000000 E 01 -2.8000000 E 01

-3.0000000 E 00 -8.0000000 E 00 -9.0000000 E 00 -3.5000000 E 01 -2.9000000 E 01 -2.0000000 E 01
-1.6000000 E 01 -1.5000000 E 01 -1.0000000 E 01 -3.0000000 E 01 -4.2000000 E 01 -2.0000000 E 01
-1.8000000 E 01 -1.2000000 E 02 -2.0000000 E 01 -2.0000000 E 01 -2.2000000 E 01 -7.0000000 E 00
-3.0000000 E 00 -5.0000000 E 01 -6.0000000 E 01 -5.5000000 E 01 -2.5000000 E 01 -3.0000000 E 01
-2.5000000 E 01 -1.5000000 E 01 -1.0000000 E 01 -2.8000000 E 01

1.0000000 E 02 2.2000000 E 02 9.0000000 E 01 4.0000000 E 02 3.0000000 E 02 4.0000000 E 02
2.0500000 E 02 1.2000000 E 02 1.6000000 E 02 5.8000000 E 02 4.0000000 E 02 1.4000000 E 02
1.0000000 E 02 1.3000000 E 03 6.5000000 E 02 3.2000000 E 02 4.8000000 E 02 8.0000000 E 01
6.0000000 E 01 2.5500000 E 03 3.1000000 E 03 1.1000000 E 03 9.5000000 E 02 4.5000000 E 02
3.0000000 E 02 2.2000000 E 02 2.0000000 E 02 5.2000000 E 02

-4.9700000 E 02 -7.7600000 E 02 -4.5000000 E 01 -1.8000000 E 02 -2.5400000 E 02 -2.4900000 E 02
1.3000000 E 01 -9.6000000 E 01 -2.1900000 E 02 -2.5500000 E 02

8.0000000 E 00 2.4000000 E 01 1.3000000 E 01 8.0000000 E 01 7.0000000 E 01 8.0000000 E 01
4.5000000 E 01 1.5000000 E 01 2.8000000 E 01 9.0000000 E 01 1.3000000 E 02 3.2000000 E 01
2.0000000 E 01 1.2000000 E 02 4.0000000 E 01 3.0000000 E 01 2.0000000 E 01 6.0000000 E 00
3.0000000 E 00 1.8000000 E 02 2.2000000 E 02 5.0000000 E 01 3.0000000 E 01 5.0000000 E 01
1.2000000 E 01 5.0000000 E 00 8.0000000 E 00 1.8000000 E 01

8.0000000 E 00 4.4000000 E 01 1.3000000 E 01 1.0000000 E 02 1.0000000 E 02 9.0000000 E 01
7.5000000 E 01 2.5000000 E 01 2.8000000 E 01 1.2000000 E 02 1.3000000 E 02 3.2000000 E 01
4.0000000 E 01 1.6000000 E 02 4.0000000 E 01 6.0000000 E 01 5.5000000 E 01 1.0000000 E 01
6.0000000 E 00 2.4000000 E 02 2.9000000 E 02 8.0000000 E 01 9.0000000 E 01 7.0000000 E 01
2.7000000 E 01 1.7000000 E 01 8.0000000 E 00 2.8000000 E 01

3.0000000 E 00 6.0000000 E 00 4.0000000 E 00 2.0000000 E 01 2.0000000 E 01 3.0000000 E 01
8.0000000 E 00 3.0000000 E 00 1.2000000 E 01 1.4000000 E 01 4.0000000 E 01 6.0000000 E 00
3.0000000 E 00 2.0000000 E 01 5.0000000 E 00 0. 5.0000000 E 00 3.0000000 E 00
0. 2.0000000 E 01 3.0000000 E 01 4.0000000 E 01 1.0000000 E 01 0.
5.0000000 E 00 0. 0. 1.0000000 E 01

5.0000000 E 00 9.0000000 E 00 6.0000000 E 00 4.0000000 E 01 3.0000000 E 01 4.0000000 E 01
1.6000000 E 01 5.0000000 E 00 1.8000000 E 01 2.4000000 E 01 6.0000000 E 01 1.6000000 E 01
1.1000000 E 01 3.0000000 E 01 2.5000000 E 01 1.0000000 E 01 1.3000000 E 01 5.0000000 E 00
1.0000000 E 00 8.0000000 E 01 6.0000000 E 01 5.0000000 E 01 2.0000000 E 01 3.0000000 E 01
1.0000000 E 01 5.0000000 E 00 3.0000000 E 00 2.0000000 E 01

5.0000000 E 00 1.1000000 E 01 7.0000000 E 00 5.0000000 E 01 4.0000000 E 01 4.0000000 E 01
1.9000000 E 01 7.0000000 E 00 1.8000000 E 01 2.9000000 E 01 7.0000000 E 01 2.1000000 E 01
1.7000000 E 01 3.0000000 E 01 2.5000000 E 01 1.5000000 E 01 2.5000000 E 01 5.0000000 E 00
1.0000000 E 00 1.0000000 E 02 7.0000000 E 01 5.5000000 E 01 2.0000000 E 01 5.0000000 E 01
1.5000000 E 01 1.5000000 E 01 6.0000000 E 00 2.0000000 E 01

5.0000000 E 00 1.1000000 E 01 7.0000000 E 00 5.5000000 E 01 4.0000000 E 01 4.0000000 E 01
2.1000000 E 01 9.0000000 E 00 1.8000000 E 01 2.9000000 E 01 7.0000000 E 01 2.1000000 E 01
1.7000000 E 01 3.5000000 E 01 2.5000000 E 01 2.0000000 E 01 2.5000000 E 01 5.0000000 E 00
7.0000000 E 00 1.1000000 E 02 7.0000000 E 01 5.5000000 E 01 2.0000000 E 01 5.0000000 E 01

```
2.0000000 E 01   1.5000000 E 01   6.0000000 E 00   2.0000000 E 01

0.               0.               1.0000000 E 00   1.0000000 E 01   4.0000000 E 00   1.0000000 E 01
0.               6.0000000 E 00   0.               6.0000000 E 00   3.2000000 E 01   3.0000000 E 00
0.               7.0000000 E 01   1.0000000 E 01   0.               0.               0.
0.               0.               3.0000000 E 01   1.0000000 E 01   0.               1.0000000 E 01
1.0000000 E 01   5.0000000 E 00   0.               1.0000000 E 01

3.0000000 E 00   4.0000000 E 00   5.0000000 E 00   2.0000000 E 01   1.4000000 E 01   2.0000000 E 01
6.0000000 E 00   1.2000000 E 01   1.0000000 E 01   1.8000000 E 01   4.2000000 E 01   9.0000000 E 00
1.2000000 E 01   1.0000000 E 02   2.0000000 E 01   5.0000000 E 00   6.0000000 E 00   4.0000000 E 00
1.0000000 E 00   2.0000000 E 01   5.0000000 E 01   3.0000000 E 01   5.0000000 E 00   2.0000000 E 01
2.0000000 E 01   1.0000000 E 01   1.0000000 E 01   2.0000000 E 01

3.0000000 E 00   6.0000000 E 00   9.0000000 E 00   3.0000000 E 01   2.9000000 E 01   2.0000000 E 01
1.2000000 E 01   1.2000000 E 01   1.0000000 E 01   3.0000000 E 01   4.2000000 E 01   1.8000000 E 01
1.8000000 E 01   1.1000000 E 02   2.0000000 E 01   1.5000000 E 01   1.8000000 E 01   7.0000000 E 00
2.0000000 E 00   4.0000000 E 01   6.0000000 E 01   5.0000000 E 01   2.5000000 E 01   2.5000000 E 01
2.5000000 E 01   1.5000000 E 01   1.0000000 E 01   2.8000000 E 01

3.0000000 E 00   8.0000000 E 00   9.0000000 E 00   3.5000000 E 01   2.9000000 E 01   2.0000000 E 01
1.6000000 E 01   1.5000000 E 01   1.0000000 E 01   3.0000000 E 01   4.2000000 E 01   2.0000000 E 01
1.8000000 E 01   1.2000000 E 02   2.0000000 E 01   2.0000000 E 01   2.2000000 E 01   7.0000000 E 00
3.0000000 E 00   5.0000000 E 01   6.0000000 E 01   5.5000000 E 01   2.5000000 E 01   3.0000000 E 01
2.5000000 E 01   1.5000000 E 01   1.0000000 E 01   2.8000000 E 01
```

```
     PETE  5

 IPLICIT ENUMERATION COMPLETE      TOTAL TIME=   4.371

LEAST Z AFTER VARIABLE CHANGE = 3.0950000 E 03
     0     0     0     4     5     6     7     8     0    10    11    12    13     0     0
     0     0     0     0     0     0     0     0    24     0     0     0     0

LEAST Z BEFORE VARIABLE CHANGE =-1.2400000 E 04
     1     2     3     0     0     0     0     0     9     0     0     0     0    14    15
    16    17    18    19    20    21    22    23     0    25    26    27    28

NO. FEASIBLE SOLUTIONS   24
ZS GE ZBAR    5 TIMES
CONSTRAINT INFEASIBLE   15 TIMES
AUGMENTATION IMPOSSIBLE    2 TIMES
AUGMENTATION POSSIBLE   19 TIMES
INTEGER DUALS    0 TIMES
LP FATHOMED    2 TIMES
LP CALLED   21 TIMES
NO. ITERATIONS   95
LAST FEASIBLE SOLUTION AT   4.132  SECONDS
```

Appendix B

LISTING OF RIP23J

```
$IBFTC RIP23J                                                          00000010
      DIMENSION A(50,90),JF(50,90)                                     00000020
      DIMENSION B(100),C(100),BS(100),S(100),SB(100),NS(100),NF(100)   00000030
      DIMENSION ITEMP(4),JTEMP(4),ATEMP(4),SMAX(100),SMAXB(100),T(100) 00000040
      DIMENSION CS(100),H(100)                                         00000050
      DIMENSION XL(90),D(90),E(90,90)                                  00000060
      DIMENSION JH(100),XX(100),Y(100),PE(100),KO(6)                   00000070
      INTEGER S,SMAX,SC,T                                              00000080
      COMMON /BLS/MS(90),ZBAR                                          00000090
      DATA BCIB/6HB      /                                             00000100
      DATA BLANK/6H      /                                             00000110
  100 DO 110 I=1,90                                                    00000120
      H(I)=0.0                                                         00000130
      B(I)=0.0                                                         00000140
      C(I)=0.0                                                         00000150
      BS(I)=0.0                                                        00000160
      S(I)=0                                                           00000170
      SB(I)=BLANK                                                      00000180
      NS(I)=0                                                          00000190
      NF(I)=0                                                          00000200
      SMAX(I)=0                                                        00000210
      SMAXB(I)=BLANK                                                   00000220
      T(I)=0                                                           00000230
      DO 110 J=1,50                                                    00000240
      A(J,I)=0.0                                                       00000250
      JF(J,I)=0.0                                                      00000260
  110 CONTINUE                                                         00000270
      II=0                                                             00000280
      NCON=0                                                           00000290
      NRED=0                                                           00000300
      NAUG=0                                                           00000310
      NOPT=0                                                           00000320
      NID=0                                                            00000330
      NAP=0                                                            00000340
      NLPF=0                                                           00000350
      NSIMP=0                                                          00000360
      NFATH=0                                                          00000370
      NENUM=0                                                          00000380
      NTCE=0                                                           00000390
      ITB=0                                                            00000400
      IPOST=1                                                          00000410
      IINS=5                                                           00000420
C                                                                      00000430
C  READ A NEW SET OF DATA                                              00000440
C  PARAMETER CARD FIRST                                                00000450
C  'S' CARD THIRD                                                      00000460
C  'C','B','A' MATRICES FOLLOW 'S'                                     00000470
C                                                                      00000480
C  MINIMIZE SUM C(J)*X(J)                                              00000490
C  CONSTRAINTS ARE B(I)+SUM A(I,J)*X(J) GE ZERO                        00000500
      READ 9000,M,N,L,SC,KENUM,ZBAR,ISCMAX,ISCFR,MAXC,MAXT,            00000510
     *          NOP,ZKBAR,H1,H2                                        00000520
 9000 FORMAT (4I3,I5,E6.0,3I3,I5,I3,E6.0,2A6)                          00000530
      PRINT 9993                                                       00000540
      PRINT 9001,M,N,L,SC,KENUM,ZBAR,ISCMAX,ISCFR,MAXC,MAXT,           00000550
     *          NOP,ZKBAR,H1,H2                                        00000560
 9001 FORMAT (4I3,I5,1X,E11.4,3I3,I5,I3,E11.4,1X,2A6)                  00000570
      IF (IINS.EQ.0) IINS=9999                                         00000580
      IF (MAXT.EQ.0) MAXT=999999                                       00000590
      MAXT=1000*MAXT                                                   00000600
```

```
      MO=M                                                              00000610
      M1=MO+1                                                           0000062C
      JSCFR=ISCFR                                                       00000630
      ZKBAR=ZKBAR+.99999                                                00000640
      PRINT 9010,M,N                                                    00000650
 9010 FORMAT (3HOM=,I3,2X,2HN=,I3)                                      00000660
      PRINT 9992                                                        00000670
 9991 FORMAT (1H )                                                      00000680
 9992 FORMAT (1H0)                                                      00000690
 9993 FORMAT (1H1)                                                      00000700
      L1=L                                                              00000710
      IF (L.LE.0) L1=0                                                  00000720
      READ 9100,((S(K),SB(K)),K=1,L1)                                   00000730
 9100 FORMAT (14(I4,A1))                                                00000740
      IF (L.GE.0) GO TO 130                                             00000750
      L=N                                                               00000760
      DO 120 K=1,N                                                      00000770
  120 S(K)=-K                                                           00000780
  130 CONTINUE                                                          00000790
      READ 9200,(C(J),J=1,N)                                            00000800
 9200 FORMAT (6(E11.0,1X))                                              00000810
C*********************************                                    00000820
      IF (MAXC.EQ.0) GO TO 141                                          00000830
      DO 140 J=1,N                                                      00000840
  140 C(J)=-C(J)                                                        00000850
  141 CONTINUE                                                          00000860
      READ 9200,(B(I),I=1,M)                                            00000870
  200 READ 9400,((ITEMP(K),JTEMP(K),ATEMP(K)),K=1,4)                    00000880
 9400 FORMAT (4(2I3,E11.0,1X))                                          00000890
      END=0.0                                                           00000900
      DO 250 K=1,4                                                      00000910
      KI=ITEMP(K)                                                       00000920
      KJ=JTEMP(K)                                                       00000930
      IF (KI.EQ.0) GO TO 250                                            00000940
      IF (KJ.EQ.0) GO TO 250                                            00000950
      KJF=NF(KI)+1                                                      00000960
      NF(KI)=KJF                                                        00000970
      JF(KI,KJF)=KJ                                                     00000980
C*********************************                                    00000990
      IF (MAXC.NE.0) ATEMP(K)=-ATEMP(K)                                 00001000
      A(KI,KJ)=ATEMP(K)                                                 00001010
      END=1.0                                                           00001020
  250 CONTINUE                                                          00001030
      IF (END.NE.0.0) GO TO 200                                         00001040
      PRINT 9992                                                        00001050
      PRINT 9500,((S(K),SB(K)),K=1,L)                                   00001060
 9500 FORMAT (14(3X,I4,A1))                                             00001070
      PRINT 9992                                                        00001080
      PRINT 9600,(C(J),J=1,N)                                           00001090
      PRINT 9992                                                        00001100
      PRINT 9600,(B(I),I=1,M)                                           00001110
      PRINT 9992                                                        00001120
      DO 251 I=1,M                                                      00001130
      PRINT 9600,(A(I,J),J=1,N)                                         00001140
      PRINT 9991                                                        00001150
  251 CONTINUE                                                          00001160
      PRINT 9992                                                        00001170
C                                                                       00001180
C  ALL DATA READ FOR THIS RUN                                           00001190
C                                                                       00001200
```

```
      DO 255 J=1,N                                                      00001210
      CS(J)=C(J)                                                        00001220
      IF (C(J).GE.0.0) GO TO 255                                        00001230
      C(J)=-C(J)                                                        00001240
      DO 253 I=1,M                                                      00001250
      B(I)=B(I)+A(I,J)                                                  00001260
  253 A(I,J)=-A(I,J)                                                    00001270
  255 CONTINUE                                                          00001280
      PRINT 9600,(C(J),J=1,N)                                           00001290
      PRINT 9992                                                        00001300
      PRINT 9600,(B(I),I=1,M)                                           00001310
      PRINT 9992                                                        00001320
      DO 260 I=1,M                                                      00001330
      PRINT 9600,(A(I,J),J=1,N)                                         00001340
      PRINT 9991                                                        00001350
  260 CONTINUE                                                          00001360
 9600 FORMAT (6(2X,1PE15.8))                                            00001370
      IF (ZBAR.GT.0.0) GO TO 300                                        00001380
      ZBAR=0.0                                                          00001390
      DO 275 J=1,N                                                      00001400
  275 ZBAR=ZBAR+C(J)                                                    00001410
  300 ZS=0.0                                                            00001420
      DO 325 I=1,M                                                      00001430
  325 BS(I)=B(I)                                                        00001440
      DO 330 J=1,N                                                      00001450
  330 NS(J)=J                                                           00001460
      IF (L.EQ.0) GO TO 400                                             00001470
      DO 375 K=1,L                                                      00001480
      J1=S(K)                                                           00001490
      K1=IABS(J1)                                                       00001500
      NS(K1)=0                                                          00001510
      IF (J1.LE.0) GO TO 375                                            00001520
      ZS=ZS+C(J1)                                                       00001530
      DO 350 I=1,M                                                      00001540
  350 BS(I)=BS(I)+A(I,J1)                                               00001550
  375 CONTINUE                                                          00001560
  400 CONTINUE                                                          00001570
      IF (M0+ISCMAX.GT.50) ISCMAX=50-M0                                 00001580
      I1=M0+ISCMAX                                                      00001590
      DO 425 I=M1,I1                                                    00001600
      NF(I)=N                                                           00001610
      DO 425 J=1,N                                                      00001620
      JF(I,J)=J                                                         00001630
  425 CONTINUE                                                          00001640
      CALL DATIME (0,IT0)                                               00001650
      IT1=IT0                                                           00001660
      GO TO 1910                                                        00001670
C                                                                       00001680
C INITIALIZATION COMPLETE                                               00001690
C                                                                       00001700
 1000 CONTINUE                                                          00001710
      IF (SC.EQ.0) GO TO 2400                                           00001720
C SURROGATE CONSTRAINTS GO HERE                                         00001730
      JSCFR=JSCFR+1                                                     00001740
      IF (ISCFR.GT.JSCFR) GO TO 2400                                    00001750
      ML=N-L                                                            00001760
      IF (ML.LE.1) GO TO 2400                                           00001770
      JSCFR=0                                                           00001780
 1050 DO 1060 J=1,N                                                     00001790
 1060   MS(J)=0                                                         00001800
```

```
      NSIMP=NSIMP+1                                                          00001810
      IF(L.EQ.0) GO TO 1076                                                  00001820
      DO 1075 I=1,L                                                          00001830
        J=IABS(S(I))                                                         00001840
1075    MS(J) =-S(I)                                                         00001850
      IF (NOP.NE.0) GO TO 107(                                               00001860
      PRINT 3600,((S(K),SB(K)                                                00001870
1076  CALL SIMPLE (II, N,MO,A,C,        D,XL,D,JH,XX,Y,OBJ,E,NOP)            00001880
      IF (NOP.NE.0) GO TO 1077                                               00001890
      PRINT 9600,OBJ,ZBAR                                                    00001900
1077  CONTINUE                                                               00001910
      II=II+IPOST                                                            00001920
      IF (KO(1).EQ.2) GO TO 3400                                             00001930
      IF (KO(1).EQ.4) GO TO 100                                              00001940
      IF (KO(1).EQ.6) GO TO 1500                                             00001950
      VLPS=-OBJ                                                              00001960
      IF (VLPS.LE. (-ZBAR))GO TO 1499                                        00001970
      DO 1350 I=1,N                                                          00001980
      IF (D(I).NE.AINT(D(I)).AND.NS(I).NE.0)  GO TO 1500                     00001990
1350  CONTINUE                                                               00002000
      DO 1450 J=1,N                                                          00002010
      IF (NS(J).EQ.0) GO TO 1450                                             00002020
      I=J                                                                    00002030
      L=L+1                                                                  00002040
      NS(J)=0                                                                00002050
      SB(L)=BCIB                                                             00002060
      IF (D(I).NE.0.0) GO TO 1400                                            00002070
      S(L)=-J                                                                00002080
      GO TO 1450                                                             00002090
1400  S(L)=J                                                                 00002100
      ZS=ZS+C(J)                                                             00002110
      DO 1425 I1=1,M                                                         00002120
1425  BS(I1)=BS(I1)+A(I1,J)                                                  00002130
1450  CONTINUE                                                               00002140
      NID=NID+1                                                              00002150
      GO TO 2320                                                             00002160
1499  KO(1)=6                                                                00002170
1500  IF (ISCMAX.LE.0) GO TO 1599                                            00002180
      BMP1=ZBAR                                                              00002190
      DO 1505 I=1,MO                                                         00002200
1505  BMP1=BMP1+XL(I)*B(I)                                                   00002210
      IF (ABS(BMP1-B(M)).LE.0.0005) GO TO 1599                               00002220
      IF (M-MO.LT.ISCMAX) GO TO 1520                                         00002230
      DO 1510 I=M1,M                                                         00002240
      B(I)=B(I+1)                                                            00002250
      BS(I)=BS(I+1)                                                          00002260
      DO 1510 J=1,N                                                          00002270
1510  A(I,J)=A(I+1,J)                                                        00002280
      M=M-1                                                                  00002290
1520  B(M+1)=BMP1                                                            00002300
      DO 1550 J=1,N                                                          00002310
      ZJH =XX(J)                                                             00002320
      IF (JH(J).GE.(-N)) ZJH=-ZJH                                            00002330
      IF (JH(J).GT.0) ZJH=0.                                                 00002340
1550  A(M+1,J)=ZJH                                                           00002350
      M=M+1                                                                  00002360
      BS(M)=B(M)                                                             00002370
      DO 1575 K=1,L                                                          00002380
      K1=S(K)                                                                00002390
      IF (K1.LE.0) GO TO 1575                                                00002400
```

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      BS(M)=BS(M)+A(M,K1)                                              00002410
 1575 CONTINUE                                                         00002420
      IF (NOP.NE.0) GO TO 1599                                         00002430
      PRINT 1598,M                                                     00002440
      PRINT 9600,(A(M,J),J=1,N),B(M),BS(M)                             00002450
 1598 FORMAT (22HOSURROGATE CONSTRAINTS,2X,I4)                         00002460
 1599 IF (KO(1).EQ.6) GO TO 3400                                       00002470
 1900 GO TO 2400                                                       00002480
 1910 IJK=0                                                            00002490
 1920 CONTINUE                                                         00002500
      IF (ZS.GE.ZBAR) GO TO 3100                                       00002510
      DO 1950 I1=1,MO                                                  00002520
 1950 IF (BS(I1).LT.0.0) GO TO 1980                                    00002530
      GO TO 2320                                                       00002540
 1980 CONTINUE                                                         00002550
      DO 2000 J=1,N                                                    00002560
      IF (NS(J).EQ.0) GO TO 2000                                       00002570
      IF (ZS+C(J).LT.ZBAR) GO TO 2000                                  00002580
      NS(J)=0                                                          00002590
      L=L+1                                                            00002600
      SB(L)=BCIB                                                       00002610
      S(L)=-J                                                          00002620
 2000 CONTINUE                                                         00002630
      KINS=0                                                           00002640
      IF (IJK.EQ.1) GO TO 2220                                         00002650
      IF (IJK.EQ.2) GO TO 1000                                         00002660
      IJK=1                                                            00002670
      IF (M.LT.M1) GO TO 2025                                          00002680
      MSC=0                                                            00002690
      I1=M1                                                            00002700
      I2=M                                                             00002710
      GO TO 2050                                                       00002720
 2025 MSC=1                                                            00002730
      I1=1                                                             00002740
      I2=MO                                                            00002750
 2050 DO 2220 I=I1,I2                                                  00002760
      Q=BS(I)                                                          00002770
      DO 2100 J=1,N                                                    00002780
      IF (NS(J).EQ.0) GO TO 2100                                       00002790
      IF (A(I,J).GT.0.0) Q=Q+A(I,J)                                    00002800
 2100 CONTINUE                                                         00002810
 2110 IF (Q.LT.0.0) GO TO 3000                                         00002820
      K=NF(I)                                                          00002830
      DO 2200 K1=1,K                                                   00002840
      J1=JF(I,K1)                                                      00002850
      IF (NS(J1).EQ.0) GO TO 2200                                      00002860
 2120 IF (Q.GE.ABS(A(I,J1))) GO TO 2200                                00002870
      NS(J1)=0                                                         00002880
      L=L+1                                                            00002890
      SB(L)=BCIB                                                       00002900
      IF (A(I,J1).GT.0.0) GO TO 2150                                   00002910
      S(L)=-J1                                                         00002920
      GO TO 2200                                                       00002930
 2150 S(L)=J1                                                          00002940
      ZS=ZS+C(J1)                                                      00002950
      DO 2175 I1=1,M                                                   00002960
 2175 BS(I1)=BS(I1)+A(I1,J1)                                           00002970
      KINS=KINS+1                                                      00002980
 2200 CONTINUE                                                         00002990
      IF (KINS.GE.IINS) GO TO 1920                                     00003000
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 2220 CONTINUE                                                          00003010
      IF (MSC.EQ.0) GO TO 2025                                          00003020
      IF (KINS.EQ.0) GO TO 1000                                         00003030
      IJK=2                                                             00003040
      GO TO 1920                                                        00003050
C  4A                                                                   00003060
 2320 CONTINUE                                                          00003070
      IF (M.EQ.MO) GO TO 2340                                           00003080
      DO 2325 I=M1,M                                                    00003090
      B(I)=B(I)+ZS-ZKBAR-ZBAR                                           00003100
 2325 BS(I)=BS(I)+ZS-ZKBAR-ZBAR                                         00003110
 2340 ZBAR=ZS-ZKBAR                                                     00003120
      DO 2350 J=1,N                                                     00003130
 2350 SMAX(J)=S(J)                                                      00003140
      GO TO 3300                                                        00003150
 2400 K1=0                                                              00003160
      DO 2500 J=1,N                                                     00003170
      IF (NS(J).EQ.0) GO TO 2500                                        00003180
      IF (ITB.EQ.0) GO TO 2430                                          00003190
      IF (ZS+C(J).GE.ZBAR) GO TO 2500                                   00003200
      DO 2450 I=1,M                                                     00003210
      IF (A(I,J).LE.0.0) GO TO 2450                                     00003220
      IF (BS(I).GE.0.0) GO TO 2450                                      00003230
 2430 CONTINUE                                                          00003240
      K1=K1+1                                                           00003250
      T(K1)=J                                                           00003260
      GO TO 2500                                                        00003270
 2450 CONTINUE                                                          00003280
 2500 CONTINUE                                                          00003290
      IF (K1.EQ.0) GO TO 3200                                           00003300
      NAP=NAP+1                                                         00003310
      P=-1.0E10                                                         00003320
      DO 2575 K=1,K1                                                    00003330
      J=T(K)                                                            00003340
      P1=0.0                                                            00003350
      DO 2550 I=1,M                                                     00003360
      P2=BS(I)+A(I,J)                                                   00003370
      IF (P2.GE.0.0) GO TO 2550                                         00003380
      P1=P1+P2                                                          00003390
 2550 CONTINUE                                                          00003400
      IF (P1.LE.P) GO TO 2575                                           00003410
      P=P1                                                              00003420
      J1=J                                                              00003430
 2575 CONTINUE                                                          00003440
      NS(J1)=0                                                          00003450
      L=L+1                                                             00003460
      S(L)=J1                                                           00003470
      ZS=ZS+C(J1)                                                       00003480
      DO 2600 I=1,M                                                     00003490
 2600 BS(I)=BS(I)+A(I,J1)                                               00003500
      H(L)=H(L)+1.0                                                     00003510
      GO TO 1910                                                        00003520
 3000 NCON=NCON+1                                                       00003530
C     PRINT 3010,I                                                      00003540
 3010 FORMAT (1HO,I3,26H(TH) CONSTRAINT INFEASIBLE)                     00003550
      GO TO 3500                                                        00003560
 3100 NRED=NRED+1                                                       00003570
C     PRINT 3110                                                        00003580
 3110 FORMAT (33HOZ CANNOT BE REDUCED (ZS GE ZBAR))                     00003590
      GO TO 3500                                                        00003600
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 3200 NAUG=NAUG+1                                                       00003610
C     PRINT 3210                                                       00003620
 3210 FORMAT (25HONO AUGMENTATION POSSIBLE)                             00003630
      GO TO 3500                                                       00003640
 3300 NOPT=NOPT+1                                                       00003650
      CALL DATIME (0,IT3)                                              00003660
      IF (NOP.NE.0) GO TO 3500                                         00003670
      PRINT 3310,ZS                                                    00003680
      PRINT 3600,((S(K),SB(K)),K=1,L)                                  00003690
 3310 FORMAT (23H0 BETTER SOLUTION FOUND,5X,2HZ=,1PE15.8)               00003700
      GO TO 3500                                                       00003710
 3400 NLPF=NLPF+1                                                       00003720
      GO TO 3500                                                       00003730
C 4B                                                                   00003740
 3500 CONTINUE                                                         00003750
      NENUM=NENUM+1                                                    00003760
      IF (NENUM.LT.KENUM) GO TO 3530                                   00003770
      NENUM=0                                                          00003780
 3505 CONTINUE                                                         00003790
      ENUM=0.0                                                         00003800
      DO 3510 K=1,N                                                    00003810
 3510 IF (SB(K).EQ.BCIB) ENUM=ENUM+.5**K                               00003820
      CALL DATIME (0,IT2)                                              00003830
      ELT1=IT2-IT0                                                     00003840
      ELT2=IT2-IT1                                                     00003850
      IT1=IT2                                                          00003860
      ELT1=ELT1/1000.0                                                 00003870
      ELT2=ELT2/1000.0                                                 00003880
      IF (IT2-IT0.LT.MAXT) GO TO 3515                                  00003890
      MAXT=-1                                                          00003900
      GO TO 3517                                                       00003910
 351. CONTINUE                                                         00003920
      IF (NOP.NE.0) GO TO 3700                                         00003930
 3517 CONTINUE                                                         00003940
      PRINT 3520,ENUM,ELT1,ELT2,L                                      00003950
 3520 FORMAT (1H0,F10.5,38H OF THE SOLUTIONS HAVE BEEN ENUMERATED,5X,   00003960
     *        15HTIME IN SECONDS,2X,5HTOTAL,F8.3,2X,7HELAPSED,F8.3,    00003970
     *        5X,2HL=,I3)                                              00003980
 3530 CONTINUE                                                         00003990
      IF (MAXT.LT.0) PRINT 3600,((S(K),SB(K)),K=1,L)                   00004000
 3600 FORMAT (15(2X,I4,A1))                                            00004010
      IF (MAXT.LT.0) GO TO 3738                                        00004020
C  4B                                                                  00004030
 3700 NPATH=NPATH+1                                                    00004040
 3710 IF (SB(L).EQ.BLANK) GO TO 3900                                   00004050
      J=IABS(S(L))                                                     00004060
      NS(J)=J                                                          00004070
      IF (S(L).LT.0) GO TO 3735                                        00004080
      ZS=ZS-C(J)                                                       00004090
      DO 3725 I=1,M                                                    00004100
 3725 BS(I)=BS(I)-A(I,J)                                               00004110
 3735 SB(L)=BLANK                                                      00004120
      S(L)=0                                                           00004130
      L=L-1                                                            00004140
      IF (L.GT.0) GO TO 3710                                           00004150
C  FINISHED                                                            00004160
 3738 CONTINUE                                                         00004170
      PRINT 3739,H1,H2                                                 00004180
 3739 FORMAT (1H1,5X,2A6)                                              00004190
      DO 3740 J=1,N                                                    00004200
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3740 S(J)=0                                                                    00004210
     DO 3742 J=1,N                                                             00004220
     K=IABS(SMAX(J))                                                           00004230
     IF (K.EQ.0) GO TO 3744                                                    00004240
3742 S(K)=1                                                                    00004250
3744 DO 3746 K=1,N                                                             00004260
     IF (S(K).NE.0) GO TO 3746                                                 00004270
     SMAX(J)=-K                                                                00004280
     J=J+1                                                                     00004290
3746 CONTINUE                                                                  00004300
     CALL DATIME (0,IT2)                                                       00004310
     ELT1=IT2-IT0                                                              00004320
     ELT1=ELT1/1000.0                                                          00004330
     IF (MAXT.LT.0) GO TO 3752                                                 00004340
     PRINT 3750,ELT1                                                           00004350
3750 FORMAT (30HOIMPLICIT ENUMERATION COMPLETE,5X,11HTOTAL TIME=,F8.3)         00004360
     GO TO 3758                                                                00004370
3752 PRINT 3755,ELT1                                                           00004380
3755 FORMAT (14HOTIME EXCEEDED,5X,11HTOTAL TIME=,F8.3)                         00004390
3758 CONTINUE                                                                  00004400
     ZBAR=ZBAR+ZKBAR                                                           00004410
     PRINT 3760,ZBAR                                                           00004420
3760 FORMAT (32HOLEAST Z AFTER VARIABLE CHANGE =,1PE15.8)                      00004430
     I=0                                                                       00004440
3800 DO 3810 K=1,N                                                             00004450
3810 T(K)=0                                                                    00004460
     DO 3820 K=1,N                                                             00004470
     K1=IABS(SMAX(K))                                                          00004480
3820 IF (SMAX(K).GT.0) T(K1)=K1                                                00004490
     PRINT 3830,(T(K),K=1,N)                                                   00004500
3830 FORMAT (15(4X,I3))                                                        00004510
     IF (I.NE.0) GO TO 3845                                                    00004520
     ZBAR=0.0                                                                  00004530
     DO 3835 J=1,N                                                             00004540
     K=IABS(SMAX(J))                                                           00004550
     IF (CS(K).LT.0.0) SMAX(J)=-SMAX(J)                                        00004560
     IF (SMAX(J).GT.0) ZBAR=ZBAR+CS(K)                                         00004570
3835 CONTINUE                                                                  00004580
     PRINT 3840,ZBAR                                                           00004590
3840 FORMAT (33HOLEAST Z BEFORE VARIABLE CHANGE =,1PE15.8)                     00004600
     I=1                                                                       00004610
     GO TO 3800                                                                00004620
3845 CONTINUE                                                                  00004630
     ELT3=IT3-IT0                                                              00004640
     ELT3=ELT3/1000.0                                                          00004650
     NITER=NFATH+NFATH-1                                                       00004660
     PRINT 3850,NOPT,NRED,NCON,NAUG,NAP,NID,NLPF,NSIMP,NITER,ELT3              00004670
3850 FORMAT (23HONO. FEASIBLE SOLUTIONS,I5/                                    00004680
    *        11H ZS GE ZBAR,I5,6H TIMES/                                       00004690
    *        22H CONSTRAINT INFEASIBLE,I5,6H TIMES/                            00004700
    *        24H AUGMENTATION IMPOSSIBLE,I5,6H TIMES/                          00004710
    *        22H AUGMENTATION POSSIBLE,I5,6H TIMES/                            00004720
    *        14H INTEGER DUALS,I5,6H TIMES/                                    00004730
    *        12H LP FATHOMED,I5,6H TIMES/                                      00004740
    *        10H LP CALLED,I5,6H TIMES/                                        00004750
    *        15H NO. ITERATIONS,I5/                                            00004760
    *        26H LAST FEASIBLE SOLUTION AT,F8.3,9H  SECONDS)                   00004770
     GO TO 100                                                                 00004780
3900 SB(L)=BCIB                                                                00004790
     S(L)=-S(L)                                                                00004800
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      J=IABS(S(L))                                                      00004810
      IF (S(L).GT.0) GO TO 3950                                         00004820
      ZS=ZS-C(J)                                                        00004830
      DO 3925 I=1,M                                                     00004840
 3925 BS(I)=BS(I)-A(I,J)                                                00004850
      GO TO 1910                                                        00004860
 3950 ZS=ZS+C(J)                                                        00004870
      DO 3975 I=1,M                                                     00004880
 3975 BS(I)=BS(I)+A(I,J)                                                00004890
      GO TO 1910                                                        00004900
      END                                                               00004910
$IBFTC SIMPLE                                                           00004920
C  AUTOMATIC SIMPLEX       REDUNDANT EQUATIONS CAUSE INFEASIBILITY      00004930
      SUBROUTINE SIMPLE(INFLAG,MX,NN,A,B,C,KO,KB,P,JH,X,Y,OBJ,E,NOP)    00004940
      REAL B(1),C(1),P(1),X(1),Y(1) ,OBJ                                00004950
      REAL E(90,90)                                                     00004960
      INTEGER INFLAG,MX,NN,KO(6),KB(1),JH(1)                            00004970
      EQUIVALENCE (XX,LL)                                               00004980
C THE FOLLOWING DIMENSION SHOULD BE THE SAME HERE AS IT IS IN CALLER.   00004990
      REAL A(50,90)                                                     00005000
      REAL AA,AIJT,BB,COST,DT,RCOST,TEXP,TPIV,TY,XOLD,XX,XY,YI,YMAX,EM  00005010
      INTEGER I,IA,INVC,IR,ITER,J,JT,K,KBJ,LL,M,N,JT2                   00005020
      INTEGER NCUT,     NUMVR,NVER,NUMPV                                00005030
      LOGICAL TRIG,VER                                                  00005040
      LOGICAL FINV,FFRZ,SCH                                             00005050
      COMMON /BLS/MS(90),ZBAR                                           00005060
      DIMENSION NF(90)                                                  00005070
C          SET INITIAL VALUES, SET CONSTANT VALUES                      00005080
      FINV = .FALSE.                                                    00005090
      TRIG = .FALSE.                                                    00005100
      ITER = 0                                                          00005110
      LPSEQ = LPSEQ+1                                                   00005120
      NUMVR = 0                                                         00005130
      NUMPV = 0                                                         00005140
      M = MX                                                            00005150
      N = NN                                                            00005160
      TEXP  =  .5**16                                                   00005170
      NVER  =  M/2  +  5                                                00005180
      NCUT  =  4*M + 10                                                 00005190
      IF (INFLAG.EQ.0) GO TO 1410                                       00005200
C        IMPOSE CORRECT TEMPERATURE ON ROWS                             00005210
      FFRZ =.TRUE.                                                      00005220
      DO 1960 L=1,M                                                     00005230
        IF (MS(L).EQ.NF(L)) GO TO 1955                                  00005240
        IF(MS(L)*NF(L).GT.0.OR.(MS(L).EQ.0.AND.X(L).GE.0.)) GO TO 1950  00005250
        I=L                                                             00005260
        IF (NF(L).NE.0) GO TO 1925                                      00005270
 1920   IF (JH(I).GT.0) GO TO 1930                                      00005280
C         IF JH DISAGREES WITH MS DO SPECIAL PIVOT                      00005290
        IF (MS(L).GT.0.AND.JH(L).GE.(-M)) GO TO 1950                    00005300
        IF (MS(L).LT.0.AND.JH(L).LT.(-M)) GO TO 1950                    00005310
C  SPECIAL PIVOT, SWITCH SINGLETONS                                     00005320
 1925   DO 1926  J=1,M                                                  00005330
          P(J) = P(J) + E(I,J)                                          00005340
          E(I,J) = -E(I,J)                                              00005350
 1926   CONTINUE                                                        00005360
        OBJ = OBJ + X(I)                                                00005370
        X(I) = - X(I)                                                   00005380
        JHL = JH(L)                                                     00005390
        IF (JHL.GE.(-M)) JH(L) = -L-M                                   00005400
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      IF (JHL.LT.(-M)) JH(L) = -L                                          00005410
      GO TO 1950                                                           00005420
C     DO FULL PIVOT ON SINGLETON                                           00005430
 1930 JT = -I                                                              00005440
      COST = P(I)                                                          00005450
      IF (MS(I).GT.0) GO TO 1931                                           00005460
      JT = JT-M                                                            00005470
      COST = 1.-COST                                                       00005480
 1931 EN = 1.                                                              00005490
      GO TO 630                                                            00005500
C       GET COLUMN(JT)                                                     00005510
 1932 SCH = .FALSE.                                                        00005520
      IF (COST.GT.0.) GO TO 1938                                           00005530
 1935 GO TO 1000                                                           00005540
C       SELECT ROW(IR)                                                     00005550
 1936 IF (IR.NE.0.OR.SCH) GO TO 1940                                       00005560
      SCH =.TRUE.                                                          00005570
 1938 EN =-EN                                                              00005580
      DO 1937 J=1,M                                                        00005590
        Y(J) = -Y(J)                                                       00005600
 1937 CONTINUE                                                             00005610
      GO TO 1935                                                           00005620
 1940 IF((SCH.AND.ABS(COST).GT.TPIV).OR.IR.EQ.0)  GO TO 1980               00005630
 1941 IF (EN.GT.0.)  GO TO 1945                                            00005640
      DO 1942 J =1,M                                                       00005650
        Y(J) = -Y(J)                                                       00005660
 1942 CONTINUE                                                             00005670
 1945 GO TO 901                                                            00005680
C       PIVOT(IR,JT)                                                       00005690
 1950 NF(L) = MS(L)                                                        00005700
 1955 IF(JH(L).LT.0) GO TO 1960                                            00005710
      IA=JH(L)                                                             00005720
      KB(IA)=L                                                             00005730
 1960 CONTINUE                                                             00005740
      FFRZ = .FALSE.                                                       00005750
      GO TO 910                                                            00005760
C*          START WITH SINGLETON BASIS                                     00005770
 1410 DO 1402  J=1,N                                                       00005780
        KB(J) = 0                                                          00005790
 1402 CONTINUE                                                             00005800
      FFRZ =.FALSE.                                                        00005810
 1400 DO 1401  I = 1,M                                                     00005820
        JH(I) =-I                                                          00005830
        NF(I) = MS(I)                                                      00005840
        IF (NF(I).LT.0.OR.(NF(I).EQ.0.AND.B(I).LT.0.)) JH(I)=-I-M          00005850
 1401 CONTINUE                                                             00005860
C*          CREATE INVERSE FROM 'KB' AND 'JH'      (STEP 7)                00005870
 1320 VER = .TRUE.                                                         00005880
      INVC = 0                                                             00005890
      NUMVR   =  NUMVR  +1                                                 00005900
      TRIG = .FALSE.                                                       00005910
      OBJ = 0.                                                             00005920
      DO 1113  I = 1,M                                                     00005930
        DO 1151  J=1,M                                                     00005940
          E(J,I) = 0.                                                      00005950
 1151   CONTINUE                                                           00005960
        IF (JH(I).LT.(-M)) GO TO 1111                                      00005970
        IF (JH(I).GT.0) JH(I) = 0                                          00005980
        E(I,I) = 1.                                                        00005990
        P(I) = 0.                                                          00006000
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        X(I) =  B(I)                                                          00006010
        GO TO 1113                                                            00006020
 1111   E(I,I) = -1.                                                          00006030
        P(I)   = +1.                                                          00006040
        OBJ = OBJ  + B(I)                                                     00006050
        X(I)   = -B(I)                                                        00006060
 1113 CONTINUE                                                                00006070
      DO 1102  JT = 1,N                                                       00006080
        IF (KB(JT).EQ.0)  GO TO 1102                                          00006090
        GO TO 600                                                             00006100
C       GET COLUMN(JT)                                                        00006110
 1114   TY = TPIV                                                            00006120
        IR = 0                                                                00006130
        COST = C(JT)                                                          00006140
        DO 1104 I = 1,M                                                       00006150
        COST = COST + A(JT,I)*P(I)                                            00006160
          IF(JH(I).NE.0.OR.X(I).NE.0..OR.ABS(Y(I)).LE.TY) GO TO 1104          00006170
          TY = ABS(Y(I))                                                      00006180
          IR = I                                                              00006190
 1104   CONTINUE                                                             00006200
        IF (IR.NE.0) GO TO 1119                                               00006210
        TY = 0.                                                               00006220
        DO 1105 I = 1,M                                                       00006230
          IF(JH(I).NE.0.OR.X(I).EQ.0..OR.ABS(Y(I)).LE.TPIV) GO TO 1105        00006240
          IF (ABS(Y(I)).LE.TY*ABS(X(I)))  GO TO 1105                          00006250
          TY = ABS(Y(I)/X(I))                                                 00006260
          IR = I                                                              00006270
 1105   CONTINUE                                                             00006280
 1119   IF (IR.NE.0) GO TO 900                                               00006290
C       PIVOT(IR,JT)                                                          00006300
        FINV = .TRUE.                                                         00006310
      IF (NOP.EQ.0) PRINT 1199,LPSEQ                                          00006320
 1199   FORMAT(15HOINVERT FAIL LP,I4)                                        00006330
        GO TO 1410                                                            00006340
 1102 CONTINUE                                                                00006350
C* PERFORM A SIMPLEX ITERATION                                                00006360
 1200 VER = .FALSE.                                                           00006370
  500 DO 503 I = 1,M                                                          00006380
        IF (NF(I).EQ.0.AND.X(I).LT.0.) X(I)=0.                                00006390
  503 CONTINUE                                                                00006400
C*          FIND MINIMUM REDUCED COST                (STEP 3)                 00006410
  599 JT = 0                                                                  00006420
      BB  =  0.0                                                              00006430
      DO 701  J =1,N                                                          00006440
        IF (KB(J).NE.0)   GO TO 701                                           00006450
        DT = C(J)                                                             00006460
        DO 303 I = 1,M                                                        00006470
          DT = DT + A(J,I)*P(I)                                               00006480
  303   CONTINUE                                                            00006490
        IF (DT.GE.BB)  GO TO 701                                              00006500
        BB = DT                                                               00006510
        JT = J                                                                00006520
  701 CONTINUE                                                                00006530
      DO 702 I=1,M                                                            00006540
        IF (JH(I).LT.0) GO TO 702                                             00006550
        IF (P(I).LT.BB) GO TO 703                                             00006560
        IF ((1.-P(I)).GE.BB) GO TO 702                                        00006570
        BB = 1.-P(I)                                                          00006580
        JT = -I-M                                                             00006590
        GO TO 702                                                             00006600
```

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  703   BB=P(I)                                                          00006610
        JT = -I                                                          00006620
  702 CONTINUE                                                           00006630
      COST = BB                                                          00006640
      IF (JT.EQ.0)  GO TO 203                                            00006650
      IF (ITER.GE.NCUT)  GO TO 160                                       00006660
      ITER = ITER +1                                                     00006670
C*          MULTIPLY INVERSE TIMES A(.,JT)          (STEP 4)           00006680
      IF (JT.LT.0) GO TO 630                                             00006690
C     BEGIN SUBROUTINE GET COLUMN(JT)                                    00006700
  600 DO 610  I= 1,M                                                     00006710
        Y(I) = 0.0                                                       00006720
  610 CONTINUE                                                           00006730
      DO 605    I= 1,M                                                 00006740
        AIJT = A(JT,I)                                                   00006750
        IF (AIJT.EQ.0.) GO TO 605                                        00006760
        DO  606  J = 1,M                                                 00006770
          Y(J) = Y(J) + AIJT*E(J,I)                                      00006780
  606   CONTINUE                                                         00006790
  605 CONTINUE                                                           00006800
      GO TO 640                                                          00006810
  630 JT2 = -JT                                                          00006820
      EM = 1.                                                            00006830
      IF (JT2.LE.M) GO TO 631                                            00006840
      JT2 = JT2 - M                                                      00006850
      EM = -1.                                                           00006860
  631 DO 632  I=1,M                                                      00006870
        Y(I) = EM*E(I,JT2)                                               00006880
  632 CONTINUE                                                           00006890
  640 YMAX = 0.                                                          00006900
      DO 620  I  =  1,M                                                  00006910
        YMAX = AMAX1( ABS(Y(I)),YMAX )                                   00006920
  620 CONTINUE                                                           00006930
      TPIV  =  YMAX * TEXP                                               00006940
C     END OF GET COLUMN                                                  00006950
      IF (FFRZ) GO TO 1932                                               00006960
      IF (VER)  GO TO 1114                                               00006970
      RCOST = YMAX/BB                                                    00006980
      IF (TRIG.AND.BB.GE.(-TPIV)) GO TO 203                              00006990
      TRIG=BB.GE.(-TPIV)                                                 00007000
C*          SELECT PIVOT ROW                        (STEP 5)           00007010
 1000 AA = TPIV                                                          00007020
      IR = 0                                                             00007030
 1002 DO 1003 I = 1,M                                                    00007040
        IF (X(I).NE.0..OR.Y(I).LE.AA.OR.NF(I).NE.0) GO TO 1003           00007050
        AA = Y(I)                                                        00007060
        IR = I                                                           00007070
 1003 CONTINUE                                                           00007080
      IF (IR.NE.0) GO TO 1020                                            00007090
      AA = 0.                                                            00007100
      DO 1010  I  = 1,M                                                  00007110
        IF (NF(I).NE.0.OR.Y(I).LE.TPIV.OR.Y(I).LE.AA*X(I)) GO TO 1010    00007120
        AA = Y(I)/X(I)                                                   00007130
        IR = I                                                           00007140
 1010 CONTINUE                                                           00007150
 1020 IF (FFRZ) GO TO 1936                                               00007160
      IF  (IR.EQ.0)  GO TO 207                                           00007170
C*          PIVOT ON (IR,JT)                        (STEP 6)           00007180
  901 IA = JH(IR)                                                        00007190
      IF (IA.GT.0)  KB(IA) = 0                                           00007200
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```
C     BEGIN SUBROUTINE PIVOT(IR,JT)                                    00007210
  900 NUMPV  =  NUMPV  +  1                                            00007220
      JH(IR) = JT                                                      00007230
      IF (JT.GT.0)  KB(JT) = IR                                        00007240
      YI = -Y(IR)                                                      00007250
      Y(IR) = -1.0                                                     00007260
      DO  904  J = 1,M                                                 00007270
        XY = E(IR,J)/YI                                                00007280
        IF (XY.EQ.0.) GO TO 904                                        00007290
        P (J) = P (J) + COST * XY                                      00007300
        E(IR,J) = 0.                                                   00007310
        DO 906  I = 1,M                                                00007320
          E(I,J) =  E(I,J) + XY * Y(I)                                 00007330
  906   CONTINUE                                                       00007340
  904 CONTINUE                                                         00007350
      XY  =  X(IR) / YI                                                00007360
      DO 908  I  = 1, M                                                00007370
        XOLD = X(I)                                                    00007380
        X(I) = XOLD + XY * Y(I)                                        00007390
  908 CONTINUE                                                         00007400
      Y(IR) = -YI                                                      00007410
      X(IR) = -XY                                                      00007420
C     END OF PIVOT                                                     00007430
      OBJ =  OBJ + XY*COST                                             00007440
      IF (VER) GO TO 1102                                              00007450
C       EXCHANGE ROWS IF SLACK PIVOTED IN WRONG ROW                    00007460
      IF  (JT.GT.0.OR.JT2.EQ.IR) GO TO 907                             00007470
      XY = X(IR)                                                       00007480
      X(IR) = X(JT2)                                                   00007490
      X(JT2) = XY                                                      00007500
      DO 909 I =1,M                                                    00007510
        XY =  E(IR,I)                                                  00007520
        E(IR,I) =  E(JT2,I)                                            00007530
        E(JT2,I) = XY                                                  00007540
  909 CONTINUE                                                         00007550
      IA = JH(JT2)                                                     00007560
      JH(JT2) = JT                                                     00007570
      JH(IR) = IA                                                      00007580
      KB(IA) = IR                                                      00007590
  907 INVC   = INVC   +1                                               00007600
C   TO STEP 1 IF NOT INVERTING, TO STEP 7 IF INVERTING                 00007610
      IF (FFRZ) GO TO 1950                                             00007620
      IF (OBJ.GE.ZBAR) GO TO 180                                       00007630
      IF (FINV) GO TO 1200                                             00007640
  910 IF (INVC.GE.NVER) GO TO 1320                                     00007650
      GO  TO  1200                                                     00007660
C*  END OF ALGORITHM, SET EXIT VALUES                  ***             00007670
  207 IF                (RCOST.LE.(-1000.)) GO TO 203                  00007680
C         INFINITE SOLUTION                                            00007690
      K  =  2                                                          00007700
      GO TO 250                                                        00007710
  180 K=6                                                              00007720
      GO TO 250                                                        00007730
C         PROBLEM IS CYCLING PERHAPS                                   00007740
  160 K  =  4                                                          00007750
      PRINT 161,LPSEQ                                                  00007760
  161 FORMAT(31HOITERATION LIMIT EXCEEDED ON LP,I4)                    00007770
      GO TO 250                                                        00007780
C         FEASIBLE OR INFEASIBLE SOLUTION                              00007790
  203 K  =  0                                                          00007800
```

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  250 DO 1399  J = 1,N                                                          00007810
        XX  =  0.0                                                              00007820
        KBJ  =  KB(J)                                                           00007830
        IF (KBJ.NE.0)  XX = X(KBJ)                                              00007840
        KB(J) = LL                                                              00007850
 1399 CONTINUE                                                                  00007860
      KO(1) = K                                                                 00007870
      KO(2) = ITER                                                              00007880
      KO(3) = INVC                                                              00007890
      KO(4) = NUMVR                                                             00007900
      KO(5) = NUMPV                                                             00007910
      KO(6) = JT                                                                00007920
      IF (NOP.NE.0) RETURN                                                      00007930
      PRINT 162,LPSEQ,(KO(I),I=1,6)                                             00007940
  162 FORMAT(3H LP,I5,6H  KO  ,6I6)                                             00007950
C     PRINT 1982                                                                00007960
 1982 FORMAT(21HOI  JH NF MS ,P,Y,X,B/1X)                                       00007970
C     DO 1983 I=1,M                                                             00007980
C       PRINT 1984,I,JH(I),NF(I),MS(I),P(I),Y(I),X(I),B(I)                      00007990
 1983 CONTINUE                                                                  00008000
 1984 FORMAT(1X,4I3,4F12.6)                                                     00008010
      RETURN                                                                    00008020
 1980 IF (NOP.EQ.0) PRINT 1981,LPSEQ,L,IR,SCH,COST                              00008030
 1981 FORMAT( 3HOLP,I4,12H FAIL, SLACK,I3,4H IR=I3,5H SCH=L1,3H C=F19.6)        00008040
      IF (IR.NE.0) GO TO 1941                                                   00008050
      GO TO 1410                                                                00008060
       END                                                                      00008070
```

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