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SOME ASPECTS OF PREDICTION OF LOAD SPECTRUM FOR AIRPLANES

by

CARL. E. BRÖNN

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SOME ASPECTS OF PREDICTION OF LOAD SPECTRUM FOR AIRPLANES

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Carl E. Brönn

This Report was presented at the Fifth Meeting of the Structures and Materials Panel, held from 29th April to 3rd May, 1957, in Copenhagen, Denmark

SUMMARY

Prediction of load spectra for airplanes hinges on the existence, in a statistical sense, of a regularity in certain features of the overall operation of the airplane. The most important of these features are: A. Objective of operation, e.g. transportation, combat-interception, combat-ground attack, combat-patrol and surface attack, reconnaissance, etc; B. Mission operational pattern, involving mission flight plan (schedule of speeds, altitudes and ranges), mission maneuvering schedule, (schedule of the minimum number and expressed purpose of flight maneuvers required for achievement of the operational objective), maneuvering situations (circumstances pertaining to each particular flight maneuver required for achievement of the operational objective of the mission).

For the purpose of load spectrum forecasting it is necessary to perform a detailed analysis of the features listed under B above for one or a limited set of typical, or average missions. Considerations pertaining to selection of typical missions are reviewed briefly.

Mission patterns are reviewed for typical transport, combat-interception and combat-ground attack operations, and corresponding maneuvering schedules and maneuvering situations are discussed. Procedures for derivation of distributions for maneuvering load factors from some typical maneuvering situations are proposed. Certain important human engineering aspects of the pilot and their influence on the load spectrum are discussed.

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SOMMAIRE

La prédiction du spectre des charges subies par un avion repose sur l'existence, au sens statistique, d'une régularité de certaines caractéristiques dans les conditions d'utilisation de l'avion, dont les plus importantes sont les suivantes: A. But opérationnel, par ex. transport, combat/interception, combat/attaque au sol, combat/patrouille et attaque en surface, reconnaissance, etc; B. Plan opérationnel de mission, comportant programme de vol de mission (vitesses, altitudes et rayons d'actions prévus), programme des manoeuvres de mission (nombre minimum objet exprès des manoeuvres de vol prévues pour réaliser but opération[illegible]); situations de manoeuvres (circonstances relatives à chaque manoeuvre permettant de réaliser le but opérationnel de la mission). La prédiction du spectre des charges demande l'analyse approfondie des caractéristiques énumerées à B ci-dessus pour une seule mission ou pour un groupe restreint de missions types. L'auteur présente de façon sommaire quelques considérations portant sur le choix de missions types et étudie les plans de mission relatifs à des opérations types de transport, de combat/interception et combat/attaque au sol, ainsi que les programmes et les situations de manoeuvre correspondants. Des procédures de dérivation des distributions des facteurs de charge de manoeuvre a partir de l'étude de quelques situations de manoeuvre types sont proposées. La communication se termine en traitant de certains aspects importants de la technique humaine du pilote et de l'influence de ceux-ci sur le spectre des charges.

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NOTATION

A	area
A_M	area of total maneuvering space
$A_{\Delta\bar{n}_{max}}$	area of maneuvering space for which $\Delta n \geq \Delta n_{max}$
$A_{\Delta\bar{n}_{V_{max}}}$	area of maneuvering space for which $\Delta n \geq \Delta n_{V_{max}}$
$A_{\Delta n'}$	area of maneuvering space for which $\Delta n \geq \Delta n'$
d	flight distance
D	distance
$F(\ )$	probability distribution function
$f(\ )$	frequency distribution function
g	acceleration of gravity
i, k	integers
L	average mission length (flight distance from base to target)
L_{max}	mission length with maximum fuel capacity
N	integer
n	load factor
n_{max}	aerodynamically limited maximum load factor, $= V_{max}{}^2/V_{min}{}^2$
$n_{max}(V)$	aerodynamically limited load factor at flight speed V, $= V^2/V_{min}{}^2$
Δn	load factor increment
$\Delta n_{V_{max}}$	load factor increment associated with maneuvers at $V = V_{max}$
$\Delta \bar{n}$	average load factor increment for positioning maneuver
$\Delta \bar{n}_{V_{max}}$	average load factor increment for positioning maneuver at $V = V_{max}$
$\Delta \bar{n}'$	arbitrary load factor increment (average)
$\Delta \bar{n}'_V$	arbitrary average load factor increment associated with flight speed V
$\Delta n_{o_{max}}$	operational maximum load factor increment
$P(\ )$	probability of denoted event

r, φ	polar coordinates
$r_{\Delta \bar{n}}$	position radius vector associated with load factor increment $\Delta \bar{n}$
r_p	maximum distance between initial and terminal points for a positioning maneuver
R	radius of curvature of flight path in turn
R_g	detection range
R_V	maneuvering radius of curvature associated with flight speed V
$R_{V_{max}}$	maneuvering radius of curvature associated with flight speed V_{max}
$R_{F_{max}}, R_{F_{min}}$	maximum and minimum firing range
$R_{\Delta n}$	flight path curvature associated with maneuvering load factor increment Δn
t	time
t_d	flight time over distance d
V	flight speed
V_{max}	maximum flight speed
V_{min}	minimum level flight speed
V_F	velocity of tracking airplane
V_T	velocity of target airplane
$V_{\perp}$	component of relative velocity normal to axis of tracking airplane
x, y	Cartesian coordinates
x_c, y_c	coordinates of flight path center of curvature
X	number of maneuvers required for ground obstacle evasion in low level flight
Y_1, Y_2	number of maneuvers required for evading enemy opposition
y	extension of obstacle normal to initial flight path
γ	heading angle

δ	heading correction
$\left.\begin{matrix}\eta \\ \nu \\ \tau\end{matrix}\right\}$	variables
θ	dive angle

SOME ASPECTS OF PREDICTION OF LOAD SPECTRUM FOR AIRPLANES

Carl E. Brönn*

1. INTRODUCTION

The concept of load spectrum, earlier referred to as *load statistics*, is one of the more recently acquired notions in the technological sciences, and it emerged largely from studies of fatigue strength properties of airplane structures. The word *spectrum* implies the notion of frequency, and a load spectrum can indeed be defined as an inventory of the frequencies with which load peaks of varying magnitudes occur.

It is quite interesting to reflect for a moment over the manner in which the attention has shifted from one part of the spectrum to another.

In the early days, the main concern was proofing of the structure for the occasional and infrequent very high loads, while what fatigue trouble existed usually could be traced to conditions of sustained vibrations within very narrow limits of frequency and stress level. However, as materials with appreciably increased static strength properties were developed and used in conjunction with more refined methods for determination of stress distributions, the effect of the greater number of smaller load fluctuations began to appear in the form of fatigue failures. This necessitated studies of the character of loads capable of producing this type of failure, and present-day evidence is that they are contained within the medium-to-high frequency bands of the spectrum. As the question whether a given structure is critical in fatigue or static loading cannot be answered until both alternatives have been investigated, and because structural weight is at a premium in modern, high-performance airplanes, the importance of reliable advance information on the shape of the load spectrum is readily appreciated.

In parallel with the main objective of designing a structure capable of standing up to a given assemblage of loads, there is a growing tendency towards parametric studies for the purpose of arriving at an optimum design for a set of given operational objectives. Any study of that kind must necessarily include a critical examination of the design limits adhered to in the past in order to establish their validity for the contemplated operational objective. This circumstance enhances the need for evolution of reliable methods for load spectrum prediction.

Finally, at the upper part of the speed ranges contemplated for designs of the immediate future, the thermomechanical strength properties of the structures are rapidly becoming a matter of growing concern. An assessment of these properties can however only be obtained in relation to a known or anticipated load-temperature experience, where, in contrast to the generally accepted notions of pure mechanical fatigue, the time enters as a third parameter. As the general effect of temperature soak is to lower the mechanical strength properties, it becomes all the more necessary to provide realistic estimates of the anticipated load experience at the higher load levels.

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2. GENERAL

Any attempt at forecasting load experience of airplanes depends on the existence, in a statistical sense, of a regularity in certain features of the operation of the airplane. A list of the more important of these features is given below:

A. *Objective of operation*

For example

(a) Transportation (cargo, personnel)

(b) Combat-interception

(c) Combat-ground attack

(d) Combat-patrol, surface attack

(e) Reconnaissance

Etc.

B. *Mission pattern*

characterized by

(a) Flight plan (range, schedule of speeds, altitudes)

(b) Maneuvering schedule (number and expressed purpose of flight maneuvers required for accomplishment of the operational objective)

(c) Maneuvering situations (circumstances pertaining to each flight maneuver required for accomplishment of the operational objective of the mission).

C. *Pilot-airplane combination*

characterized by

(a) Airplane stability and control, in particular dynamic response and control force characteristics

(b) Pilot indoctrination

(c) Pilot's acceleration tolerance

(d) Pilot's motor performance.

The statistical regularity to which reference was made above reflects first the fact that, in the interest of efficiency in performance, a marked specialization of equipment according to objectives of operation does exist. The trend towards specialization is however to some extent offset by a certain number of borderline cases which testify that one design may be successfully employed in the pursuit of quite different operational objectives. In the author's opinion, however, this does not invalidate the main argument. In the first place, such borderline cases cannot be expected to constitute a majority, and secondly, where several operational roles are contemplated in the design stage with no particular preference for any specific one, the solution is either to play it safe and design to that objective which yields the most severe load spectrum, or to adopt an average for all operations as the design target. The first alternative is in general associated with a certain penalty weight-wise, but yields more *stretch potential*, whereas the second furnishes the most efficient solution to the immediate problem.

Next, while no two missions performed toward the identical operational objective can be expected to conform to identical patterns, there certainly exists for one specific type of airplane and a given initial situation, one pattern corresponding to optimum efficiency in the operation.

It appears reasonable to contend therefore, that all possible missions performed towards identical operational objectives will tend to approach the ideal or optimum pattern for each particular situation.

The initial situation pertaining to each individual mission is conveniently described in terms of range (stage length for transport operations) and operating altitude (for interceptor airplanes).

Criteria for evaluating the efficiency of the mission vary in general from one operational objective to another, and must be established separately for each. It can be a question of optimum economy, as is the case for transport operations - adherence to a given route schedule (which itself is fixed by economic considerations) for scheduled transportation, or it can be a matter of timing, which very often is the prime consideration in military operations. Whatever the case may be, as long as the mission concerned is at all pre-planned, it appears reasonably safe to assume that the pattern is laid out with optimum operating conditions for the airplane in mind.

On these premises it seems logical, for the purpose of load spectrum prediction, to base the detailed analysis of the mission pattern on the optimum flight plan for the stage length and eventual operational altitude given as the initial situation.

Finally, as regards the pilot-airplane combination, above, the term *Statistical Regularity* is implied to mean that the results, load factorwise, of experiments in which a great number of pilots were to participate in solving identical maneuvering problems with similar equipment, will tend to cluster around central values.

That this in fact is so, can be surmised from load spectra obtained empirically for a diversity of airplane types and operational objectives[1].

So far, nothing has been said about the relation between the average pilot response, as expressed in actually developed load factor, and the amount of load factor objectively required for solution of the maneuvering problem under consideration. This issue is extremely complicated and belongs to a field which, to the best of the author's knowledge, as yet remains to be explored. Some comments on the aspect will be presented later under the appropriate heading.

At this point it is sufficient to state that a unique relation of the kind mentioned above probably does exist, and that its main function appears to be imposing an upper limit on the load factors developed in maneuvers.

The load experience for any airplane is basically a function of the operational objective which the airplane is employed to achieve and of the mission pattern characterizing the operation, but it is powerfully influenced and modified by the behavior of the pilot in the various maneuvering situations arising as part of the mission. The success of an attempt to predict the maneuvering load spectrum depends on the extent to which it is possible:

(a) To translate the operation of the airplane into an average frequency of maneuvering situations;

(b) To determine for each basically different maneuvering situation the distribution function, hereafter referred to as the 'inherent' distribution function, for the objectively required load factor;

(c) To assess and apply modifying factors accounting for average pilot response in the various situations.

3. MISSION PATTERN ANALYSIS

3.1 Transport Operations

The most important single parameter describing the transport mission is the stage length. This parameter fixes the frequency of the ground-to-air loading cycle and the frequency of the landing load cycle and powerfully influences the flight plan, which in turn determines the anticipated gust load experience. It also exerts a major influence on the maneuvering load experience.

Due to the fact that equipment standardization at present appears to be a major economic factor in the air transportation business, and because most scheduled operators serve networks with a great diversity of stage lengths, it does not at this time appear practical to adopt as a representative stage length that which corresponds to optimum economy of the airplane as a self-contained unit.

It is obvious, therefore, that the *design stage length* for this particular purpose should be determined by a survey of existing and potential route networks.

Next, the flight plan, in terms of loading, optimum air speeds and altitudes, is worked out for this selected stage length. This determines the air distances travelled through the various altitude bands, from which the anticipated gust experience can be worked out[2].

Finally, an inventory and specification of the anticipated maneuvering situations corresponding to this flight plan must be made.

The very simplest case would obtain under VFR with unlimited freedom of track selection. The minimum number of maneuvers per flight would in this case amount to three, mainly one associated with course-setting after take-off, one associated with line-up on the landing runway and finally one performed in the landing flare-out for the purpose of reducing the impact velocity in landing.

In the general case, however, the mission would have to conform to airport traffic patterns in take-off as well as in landing, both under VFR and IFR conditions, while the en-route track would be in compliance with an airway system.

Under VFR conditions, entry into an airway lane would require a minimum average of 2½ turns per take-off, assuming full freedom in take-off procedure.

Under IFR conditions, an instrument departure procedure might have to be followed. This involves a sequence of turns, the number of which varies with the airway system converging on the airport, topography of surrounding terrain and traffic intensity of the time of take-off. Irrespective of traffic intensity, whenever an instrument departure procedure is required, the minimum number of turns would be 2½. Depending on traffic intensity, a certain number of holding circuits, each consisting of two legs on opposite courses connected by 180° turns, would be required. In addition, two turns for entry into and exit from the holding pattern are likely to be required.

The number of en-route maneuvers depends on the average number of course-shifts occurring along a stage length segment of the airway system. This number can be determined with fair accuracy hy a simple sampling method.

Considering finally the landing segment of the mission, again under VFR, a minimum average of 2½ turns is required for exit from airway lane and line-up with runway. IFR conditions will require adherence to the established IFR procedure, which on the average can be estimated at an additional 4 turns, exclusive of turns required for establishing, maintaining and leaving a holding pattern. The average number of holding circuits can be obtained by reference to operational statistics. There are several indications that present air traffic procedures may have to be modified to accomodate the type of transports under consideration for the future.

Speculation on that score is however somewhat outside the scope of this paper, the main intention of which is to indicate approaches rather than to give ready-cut solutions. All figures given are therefore strictly illustrative only. In order to arrive at the average number of maneuvering situations per flight, an allowance for the number of holding circuits per IFR-flight and an averaging between the number of VFR-and IFR-flights must be made. Assuming an average of 1 holding circuit per IFR-flight and a 50-50 distribution of VFR-IFR, we obtain the results as shown in Table I.

3.2 Combat Operations - Interception

3.2.1 To provide the proper background, a few observations and assumptions of a more general nature will be given. The airplane taken as example is assumed to be designed as an integral part of a comprehensive air defense system. The features of that system which are important in these considerations are (see Figure 2):

(a) Interceptor operations are initiated whenever unidentified airplanes trangress certain air defense zone perimeters;

(b) Outside a certain zone surrounding the intruding airplane, vectoring of the interceptor towards the target is accomplished by ground control, which instructs the pilot to perform such maneuvers as are necessary to move the interceptor within spotting range of the target, on a bearing within such limits that it is practical to proceed with attack maneuvers;

(c) The interceptor having arrived within the spotting range, the subsequent maneuvers are controlled or monitored by the pilot. The purpose of these maneuvers is first, to place the interceptor within firing distance of the target on a heading corresponding to accurate aim of the armament and second, to avoid collision with debris resulting from the attack, or to avoid enemy fire;

(d) It is assumed that one offensive pass at the target exhausts the armament and fuel supply to the extent that return to base for re-arming and refuelling is necessary. The main concern in the remainder of the mission is therefore to get the interceptor quickly back to base in anticipation of further missions. Maneuvers pertaining to this phase are assumed conforming to ground control instructions, or to established navigational procedure.

Table II gives a summary of the maneuvering situations which in the author's opinion merit consideration for the purpose of load spectrum prediction for interceptor airplanes.

3.3 Combat-ground Attack

The objective of a ground attack mission is to deliver from the air a certain type of cargo at a pin-point location on the ground. A characteristic feature of the operation is that the mission in general will have to be carried out in the face of enemy opposition. This feature has a strong influence on the general character of the flight plan and on the maneuvering experience in the mission. Another pertinent feature is the type of cargo delivered, whether it is capable of being directed to the target in free flight by remote control from the carrier airplane, or whether it must be released on a further uncontrollable flight path designed to terminate in the target.

Length of mission: Distance from take off to target can vary between the maximum obtainable radius of action with maximum fuel capacity and a minimum, which for all practical purposes can be considered equal to zero. The largest distances are generally only obtainable at the expense of a reduced military load, so beyond a certain point the efficiency of the operation in terms of load carrying capability drops off.

As in general there are no means for predicting any predominant location of target opportunities relative to arbitrarily selected base locations, the best assumption is that target opportunities are evenly distributed around the base. This implies a constant density of target distribution, and that the average mission length L (distance from base to target) is

$$\frac{1}{\sqrt{2}} (L_{max})$$

where L_{max} = mission length with maximum fuel capacity.

Flight plan (see Figure 4):

For reasons of optimum performance (maximum military load for a given mission) a flight plan which allows the best fuel economy compatible with other operational requirements can usually be assumed. This yields conditions for determining operational cruise altitudes and rates of climb to altitude.

Operational requirements of major importance are:

(a) Avoiding of enemy interference both from the air and the ground;

(b) As easy and certain identification of designated targets as possible;

(c) Greatest possible accuracy in delivery of cargo.

The first-mentioned requirement implies a flight plan minimizing the probability of being detected on the part of the flight leading to the target area. This is achieved in general by a low level approach.

The second requirement implies the necessity for performing a climb to an altitude which permits easy and rapid scanning of the target area in order to locate specific target pin points.

Finally, the third requirement implies, in the case of non-controllable missiles a dive toward the target to establish the correct flight path for the cargo. In the first part of this dive, a certain amount of 'jinking' may be necessary to distract enemy opposition.

After completion of the attack, the main objective is to get out as fast as possible and return to base. This implies a fast climb to the optimum altitude for the new weight condition, followed by cruise, let down, and normal traffic procedure preparatory to landing.

The question of the flight distances involved in the various flight plan segments is very important, but only some very general comments can be offered.

The length of the low-level part of the flight depends obviously on the anticipated means for detection which are at the opponent's disposal, and on the topography of the terrain covered by the flight. The characteristic dimensions of the scanning segment are largely determined by the anticipated types and intensity of opposition, in conjunction with known data for aircrew proficiency in this form of operational activity.

Maneuvering Situations:

The number of maneuvering situations required for achievement of the operational objective can be divided in two classes. The first of these contains all maneuvering situations which are caused by adherence to the *general* features of the flight plan in Figure 4. The second class contains all maneuvering situations arising from special characteristics of certain parts of it, in particular the low-level and attack segments.

Maneuvering situations belonging to the first class are easily enumerated. They have been entered in Table III, which is self-explanatory. The second class merits some comments.

A part of the maneuvering situations within this class arises from the requirement that the airplane shall keep within a certain maximum distance from the ground level. This suggests that the load factor experience related to this flight segment is a function of flight speed, required proximity to the ground, and the ground profile of the flight track. A general representation of ground profiles would seem possible by means of power spectral density methods[3]. The pilot-airplane combination could then be conceived as a filter operating on that power-spectral density distribution such as to suppress the higher frequency components, thus producing a power spectral density distribution for the flight path, from which a maneuvering load factor distribution could conceivably be derived.

The remaining part of maneuvering situations belonging to the second class arises from the necessity for distracting and evading enemy opposition during the period required for reconnoitering the target area and for the attack dive. Again, due to the randomness of the factors involved in the problem, it appears that an approach based on study of the power spectral density distribution characterizing the flight path is likely to yield useful results. To the author's mind, the flight path power spectral density distribution ought to reflect an optimum game-theoretical solution of the tactical problem involved.

A summary of the maneuvering situations which should be considered in a ground attack mission is given in Table III.

4. PROBABILITY DISTRIBUTION OF LOAD FACTORS FOR SOME MANEUVERING PROBLEMS

4.1 The fact that different models of airplanes engaged in similar operational activities tend to experience essentially similar load spectra leads one to suspect that the basic background for the load spectra is similar for all models, and is primarily a function of the maneuvering problems peculiar to the type of operation concerned. In the following an attempt is made to classify some typical maneuvering problems and also to determine the 'background distribution' inherent in each class.

A proposed list of basically different classes is given in Table IV and is by and large self-explanatory. One item which merits particular comment, however, is the distinction between the classes of 'corrective' and 'navigational' problems. The reason for this is that the time available for solving 'corrective' problems may vary within wide limits and thus occasion a great variation in required rates of turn.

4.2 Positioning Problems

We establish a coordinate system with origin in the initial location of the airplane and positive x-axis coinciding with the velocity vector (see Figure 5).

The maneuvering problem involved consists in determining the optimum flight path for transfer of the airplane from the origin to the point (r,φ), such that the heading at that point is γ.

As there is an infinite number of flight paths capable of achieving the objective as defined by the initial and final conditions only, some qualifying criteria for evaluating the goodness of any particular solution must be established. The following qualifying criteria are proposed:

(a) The number of distinct maneuvers involved should be a minimum;

(b) The load factor required to perform the transfer should be a minimum;

(c) The objective should be accomplished within a given time interval.

Disregarding at first the last criterion, it is obvious that the first two criteria are satisfied by a solution corresponding to motion along a flight path which consists of circular arcs with equal radii of curvature. The magnitude of the maneuvering load factor is related to the flight speed V and curvature of the flight path R by

$$\Delta n = \frac{V^2}{g} \cdot \frac{1}{R} \tag{1}$$

A relationship between the radius of curvature and the geometrical parameters characterizing the situation is obtained from the relations

$$x_c^2 + (y_c - R)^2 = 4R^2$$

$$x_c = r\cos\varphi - R\sin\gamma$$

$$y_c = r\sin\varphi - R\cos\gamma$$

$$\gamma = \varphi + \delta \qquad [-\delta_{max} \leqslant \delta \leqslant +\delta_{max}]$$

yielding

$$\frac{r}{R} = [\sin\varphi - \sin\delta] \pm [(\sin\varphi - \sin\delta)^2 + 2(1 - \cos(\varphi + \delta)]^{\frac{1}{2}} \tag{2}$$

To proceed further, it is necessary to make certain assumptions with regard to the range of variation and the distribution of δ. As these assumptions are closely tied in with the operation of the defense system of which the airplane considered is a part, the author believes that in particular cases no great difficulty should be experienced in substantiating these assumptions.

For the present purpose, it is assumed that

(a) δ varies within the range $-\frac{\pi}{2} < \delta < +\frac{\pi}{2}$

and (b) δ is uniformly distributed within the range.

It is then possible to calculate a mean value

$$\left(\frac{r}{R}\right)_\delta = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{r}{R}(\varphi,\delta)\,d\delta = \frac{\bar{r}}{R}(\varphi) \tag{3}$$

With
$$\overline{\Delta n} = \frac{V^2}{g}\frac{1}{R} = \frac{1}{r}\frac{V^2}{g}\frac{\bar{r}}{R}(\varphi) \tag{4}$$

where $\overline{\Delta n}$ = mean maneuvering load factor increment required for a positioning maneuvers between (o,o) and (r,φ)

then
$$r_{\overline{\Delta n}} = \frac{1}{\Delta n}\frac{V^2}{g}\cdot\frac{\bar{r}}{R}(\varphi) \tag{5}$$

from which contours of constant maneuvering load factor increments at constant speed V can be plotted. It is thus possible to map the space surrounding the airplane in terms of average incremental load factors required for transfer of the airplane at constant speed from the origin to any arbitrary position (r,φ).

The scale factor for the mapping depends on the speed. Considering that the maximum obtainable load factor (disregarding structural and physiological limitations) is determined by the ratio:

$$n_{max} = \left(\frac{V_{max}}{V_{L_{min}}}\right)^2 \tag{6}$$

and that the maximum obtainable load factor at any speed V can be written in terms of n_{max} and V as

$$n_{max_{(V)}} = n_{max}\cdot\left(\frac{V}{V_{max}}\right)^2 \tag{7}$$

it appears logical to base the mapping on Equation (5) with $V = V_{max}$.

Considering next that the extension of the maneuvering space is far greater laterally than vertically, one can assume all maneuvers as being carried out in the horizontal plane, such that

$$n = (1 + \Delta n^2)^{\frac{1}{2}} \tag{8}$$

From Equations (5), (6) and (8):

$$r_{\overline{\Delta n}_{max}} = \frac{1}{\left[\left(\frac{V_{max}}{V_{L_{min}}}\right)^4 - 1\right]^{\frac{1}{2}}}\cdot\frac{V^2_{max}}{g}\cdot\frac{\bar{r}}{R}(\varphi) \tag{9}$$

which defines an inner boundary of the maneuvering space which need be considered.

The outer boundary defines the maximum range within which a positioning maneuver would be called upon and its magnitude can be determined from a game-theoretical study of the tactical problem involved. It is certainly less than the radius of the defense zone perimeter and is denoted r_p. The total maneuvering space is then

$$A_M = \frac{\pi}{2}r_p^2 - \int_{-\pi/2}^{+\pi/2}\frac{1}{2}r^2_{\overline{\Delta n}_{max}}\cdot d\varphi \cong \frac{\pi}{2}\left[r_p^2 - .584r^2_{\overline{\Delta n}_{max,\varphi=\pi/2}}\right]$$

In the absence of further information, target points (r,φ) for positioning maneuvers are best assumed uniformly distributed over the maneuvering space.

From this, a distribution function for load factor increments in maneuvers at maximum speed is determined by observing that the probability of occurrence of a load factor increment, $\geqslant \overline{\Delta n}_{V_{max}}$ is equal to the probability of a maneuvering target (r,φ) occurring within the area requiring a load factor increment $\geqslant \overline{\Delta n}_{V_{max}}$ to be obtained. The probability of that event is identical with the ratio of that area to the total area within which a maneuvering target can at all occur. We have thus

$$F\left(\overline{\Delta n}_{V_{max}}\right) = 1 - P\left(\Delta n \geqslant \overline{\Delta n}_{V_{max}}\right)$$

$$F\left(\overline{\Delta n}_{V_{max}}\right) = 1 - \frac{A_{\overline{\Delta n}_{V_{max}}} - A_{\overline{\Delta n}_{max}}}{A_M} \tag{11}$$

If the maneuver is carried out at a speed $V < V_{max}$, the load factor increments associated with the areas $A_{\overline{\Delta n}_{V_{max}}}$ are decreased, as

$$\overline{\Delta n} = \left(\frac{V}{V_{max}}\right)^2 \overline{\Delta n}_{V_{max}} \tag{12}$$

The distribution function for $\overline{\Delta n}$ can be obtained as follows (see Figure 7): Solving for V/V_{max} in Equation (12), we obtain

$$\frac{V}{V_{max}} = \left(\frac{\Delta n}{\Delta n_{V_{max}}}\right)^{\frac{1}{2}} \tag{13}$$

Equation (13) is plotted on a graph as $V/V_{max} = f(\overline{\Delta n}_{V_{max}})$ with incremental values of Δn as parameter, each (V/V_{max})-line terminates at a value $(V/V_{max})_{min}$ determined by the requirement

$$\left(\frac{V}{V_{max}}\right)_{min} \geqslant \frac{V_{L_{min}}}{V_{max}} (1 + \Delta n^2)^{\frac{1}{4}} \tag{14}$$

The diagram is completed by plotting the distribution functions $F(V/V_{max})$ and $F(\overline{\Delta n}_{V_{max}})$ against the pertinent variables.

With reference to Figure 7, all values of the variables V/V_{max} and $\overline{\Delta n}_{V_{max}}$ which can combine to yield values for $\overline{\Delta n} \leqslant$ say $\overline{\Delta n}^1$ are located between the curves $V/V_{max} = f_1(\overline{\Delta n}_{V_{max}})$ and $(V/V_{max})_{min} = f_2(\overline{\Delta n}_{V_{max}})$.

A certain number of these combinations are contained within the element of area whose upper and lower boundries are $\overline{\Delta n}^1 = \text{const}$ and $(V/V_{max})_{min}$ respectively and which is bounded laterally by $\overline{\Delta n}_{V_{max}}$, $(\overline{\Delta n}_{V_{max}} + d(\overline{\Delta n}_{V_{max}}))$.

The probability of occurrence of a particular value, say

$$\overline{\Delta n}_{V_{max}} \leqslant \eta \leqslant \left(\overline{\Delta n}_{V_{max}} + d\left(\overline{\Delta n}_{V_{max}}\right)\right) \qquad \text{is}$$

$$dP(\eta) = F\left(\overline{\Delta n}_{V_{max}} + d\left(\overline{\Delta n}_{V_{max}}\right)\right) - F\left(\overline{\Delta n}_{V_{max}}\right) = f\left(\overline{\Delta n}_{V_{max}}\right) d\left(\overline{\Delta n}_{V_{max}}\right) \qquad (15)$$

The probability of occurrence of a value ν such that

$$\left(\frac{V}{V_{max}}\right)_{\overline{\Delta n}_{V_{max}}} \ngtr \nu > \left(\frac{V}{V_{max}}\right)_{min, \overline{\Delta n}_{V_{max}}} \qquad \text{is}$$

$$P(\nu) = F\left[\left(\frac{V}{V_{max}}\right), \overline{\Delta n}_{V_{max}}\right] - F\left[\left(\frac{V}{V_{max}}\right) = \left(\frac{V}{V_{max}}\right)_{min, \overline{\Delta n}_{V_{max}}}\right] \qquad (16)$$

The probability of coincidence of these two independent events is then, according to the theory of probability, equal to the product of the probabilities of each event occuring separately[4], i.e;

$$dP(\eta, \nu) = dP(\eta) \cdot P(\nu) \qquad (17)$$

$$dP(\overline{\Delta n} \leqslant \overline{\Delta n}') = \left[F\left(\frac{V}{V_{max}}\right)_{\overline{\Delta n}_{V_{max}}} - F\left(\frac{V}{V_{max}}\right)_{min, \overline{\Delta n}_{V_{max}}}\right] \cdot f\left(\overline{\Delta n}_{V_{max}}\right) d\left(\overline{\Delta n}_{V_{max}}\right) \qquad (18)$$

and

$$P(\overline{\Delta n} \leqslant \overline{\Delta n}') = F(\overline{\Delta n}')$$

$$= \int_0^{\overline{\Delta n}'_{V_{max}}} \left[F\left(\frac{V}{V_{max}}\right)_{\overline{\Delta n}_{V_{max}}} - F\left(\frac{V}{V_{max}}\right)_{min, \overline{\Delta n}_{V_{max}}}\right] \cdot f\left(\overline{\Delta n}_{V_{max}}\right) d\left(\overline{\Delta n}_{V_{max}}\right) \qquad (19)$$

$\overline{\Delta n}'_{V_{max}}$ denoting the abscissa at intersection of $\overline{\Delta n} = \overline{\Delta n}'$ with $(V/V_{max})_{min}$

In order to carry out the calculation, it is necessary to have some idea of the distribution function for V/V_{max}.

This appears possible by referring to the timing criterion mentioned earlier.

Obviously if the criterion of minimum load factor is adhered to, the only way of achieving a specified timing is by speed control. The speed can vary between the limits

$$V_{L_{min}} \leqslant V \leqslant V_{max}$$

and the time available for any particular maneuver can accordingly vary between the limits

$$\frac{d}{V_{L_{min}}} \geqslant t_d = \frac{d}{V} \geqslant \frac{d}{V_{max}} \tag{20}$$

It is very likely that some prevailing distribution of speed may be shown to exist. However, with no prior information in this regard, the best assumption is that the actual maneuvering times are uniformly distributed over the available interval, such that

$$\frac{1}{V_{L_{min}}} \geqslant \frac{1}{V} \geqslant \frac{1}{V_{max}}$$

and

$$F\left(\frac{1}{V}\right) = \frac{\frac{1}{V} - \frac{1}{V_{max}}}{\frac{1}{V_{L_{min}}} - \frac{1}{V_{max}}} = \frac{V_{L_{min}}}{V_{max} - V_{L_{min}}}\left(\frac{V_{max}}{V} - 1\right) \tag{21}$$

from which a distribution function $F(V/V_{max})$ easily is obtained by observing that

$$F\left(\frac{V}{V_{max}}\right) = 1 - F\left(\frac{V_{max}}{V}\right) = 1 - F\left(\frac{1}{V}\right)$$

4.3 Corrective Maneuvers

The maneuvering problems specifically assigned to this class are all characterized by having a definite limit imposed on the time during which the maneuver must be performed, essentially regardless of the amount of correction involved.

The basic or inherent distribution function for the maneuvering load factor increments can be estimated from a few basic data which characterize the type of airplane and the operation involved.

Assume that the available time for performing the maneuver is distributed in some manner within the interval

$$0 \leqslant t \leqslant t_{max}$$

and that the magnitude of the required heading correction varies within the interval

$$0 \leqslant \delta \leqslant \delta_{max}$$

The required rate of change of heading in any particular maneuver is

$$\frac{\delta}{t} \approx \frac{d\delta}{dt} = \frac{V}{R} \tag{22}$$

Now as $\Delta n = \frac{V^2}{gR} \leqslant \left[\left(n^2_{max}\left(\frac{V}{V_{max}}\right)^4 - 1\right)^{\frac{1}{2}}\right]$

$$\Delta n = \frac{V}{g}\frac{d\delta}{dt} \approx \frac{V}{g}\frac{\delta}{t} \tag{23}$$

we have for constant values of Δn

$$\delta = \frac{g}{V}\cdot\Delta n\cdot t \tag{24}$$

i.e. constant values of Δn correspond (for V = const) to a linear relation between δ and t.

Provided that distribution functions for δ and t are available, distribution functions for Δn at any V = const can be derived from Equation (24). The procedure is as follows (See Figure 9):

The first step is to determine the distribution function for Δn at constant V.

To do this, it is convenient to calculate the probability of obtaining load factor increments which are *larger* than a given value, say $\Delta n'_V$

The condition $\Delta n \geqslant \Delta n'_V$ is obviously realized for all combinations of δ and t contained within the element of area bounded by

$$\delta = \delta_{max}, \quad \delta = \frac{g}{V}\Delta n'_V\cdot t$$

and t, $(t + \Delta t)$

The probability of obtaining a value $t \leqslant \tau \leqslant t + \Delta t$ is

$$\Delta P(\tau) = F(t + \Delta t) - F(t)$$

The probability of obtaining a value $\delta_{max} \geqslant \delta \geqslant \delta_t$ is

$$P(\delta) = F(\delta_{max}) - F(\delta_t) = 1 - F(\delta_t)$$

The probability of coincidence of these independent events is, according to the theory of probability[4],

$$\Delta P(\tau,\delta) = \Delta P(\tau)\cdot P(\delta) \underset{\Delta t \to 0}{\to} (1 - F(\delta_t))\cdot f(t)dt \tag{25}$$

By integrating Equation (25) from t = 0 to $t = t_{max}\left(\text{or } t = \frac{V}{g}\cdot\frac{\delta_{max}}{\Delta n'_V}\right.$, whichever is the smaller$\left.\right)$ we obtain the probability of occurrence of all possible combination of δ and t which make $\Delta n > \Delta n'_V$, *including all combinations of δ and t for which*

$$\Delta n > \left[n^2_{max} \cdot \left(\frac{V}{V_{max}}\right)^4 - 1 \right]^{\frac{1}{2}} = \Delta n_{max_{(V)}}$$

However, as the latter set of combinations represents physically impossible combinations, they must be discounted, which is achieved by subtracting the value

$$\int_0^{\frac{V}{g}\frac{\delta_{max}}{\Delta n_{max_V}}} (1 - F(\delta_t))\, f(t)dt$$

from each evaluation of the integral of Equation (25). We have thus:

$$P(\Delta n \geq \Delta n'_V) = 1 - F(\Delta n'_V)$$

$$\left[\int_{t=o}^{t = \begin{cases} t_{max} \\ \frac{V}{g}\frac{\delta_{max}}{\Delta n'_V} \end{cases}} (1 - F(\delta_t))\, f(t)dt - \int_{t=o}^{t = \frac{V}{g}\frac{\delta_{max}}{\Delta n_{max_V}}} (1 - F(\delta_t)) \cdot f(t)dt \right] \qquad (26)$$

The next and final step in determining the 'inherent' distribution function for corrective maneuvers consists in allowing for speed variations. This is achieved by noting that for constant relationship δ/t we have

$$\frac{\delta}{t} = \frac{V}{R_V} = \frac{V_{max}}{R_{V_{max}}}$$

further, with

$$\Delta n = \frac{V^2}{gR_V}$$

$$\Delta n_{V_{max}} = \frac{V^2_{max}}{gR_{V_{max}}}$$

$$\frac{\Delta n}{\Delta n_{V_{max}}} = \frac{V^2}{V^2_{max}} \cdot \frac{R_{V_{max}}}{R_V} = \frac{V^2}{V^2_{max}} \cdot \frac{V_{max}}{V} = \frac{V}{V_{max}} \qquad (27)$$

i.e. for unchanged relations δ/t, the load factor increment required for maneuver at speed V is proportional to the load factor at max speed maneuver by the speed ratio.

Equation (27) is represented by plotting V/V_{max} as a function of $\Delta n_{V_{max}}$ with incremental values of Δn as parameter. (See Figure 9). The lines of Δn = const terminate at values of $(V/V_{max}) = (V/V_{max})_{min}$ determined by the condition

$$\left(\frac{V}{V_{L_{min}}}\right)^2 \geqslant (1 + \Delta n^2_V)^{\frac{1}{2}} \tag{14}$$

$$\left(\frac{V}{V_{max}}\right)_{min} \geqslant \frac{V_{L_{min}}}{V_{max}} (1 + \Delta n^2_V)^{\frac{1}{4}} \tag{28}$$

Completing the diagram by also representing the distribution function $F(V/V_{max})$, $F(\Delta n_{V_{max}})$, the 'inherent' distribution of maneuvering load factor increments is determined in the same manner as outlined earlier:

$$\Delta n_{V_{max}} = \Delta n\left(\frac{1 + \Delta n^2}{1 + \Delta n_{max}}\right)^{\frac{1}{4}}$$

$$F(\Delta n) = \int_{\Delta n_{V_{max}} = 0} \left[F\left(\left(\frac{V}{V_{max}}\right) \Delta n_{V_{max}}\right) - F\left(\left(\frac{V}{V_{max}}\right) min, \Delta n_{V_{max}}\right)\right] \cdot f\left(\Delta n_{V_{max}}\right) d\left(\Delta n_{V_{max}}\right) \tag{29}$$

Some comments on the basic variables characterizing the situation may be in order.

With reference to Figure 2 it is evident that the time available for achieving the correction is limited in the extreme by the difference between spotting and minimum firing distance and the minimum closing speed. However, as chasing attacks are likely to involve near maximum speeds of both target and interceptor airplanes, it seems probable that consideration of the distances mentioned and the speed difference at maximum speeds gives the best estimate.

4.4 Tracking Maneuvers

The objective of tracking maneuvers is to keep the maneuvering airplane headed on a moving object until the distance between target and airplane has been reduced below a certain maximum value.

In the general case, the target object must be assumed moving along a curved flight path with a randomly varying radius of curvature. The ratios between the radius of curvature of the target flight path and other characteristic dimensions involved in the problem are therefore very important parameters.

However, as it is believed that in many important cases the ratios mentioned are sufficiently large to permit use of results from investigation of a simplified problem, the attention is here focused on the particular case with the target moving on a straight line track.

The geometry of the situation is shown on Figure 10. In order to maintain the desired heading on the target, the airplane must at any instant turn the rate

$$\frac{d\gamma}{dt} = \frac{V_\perp}{r} = \frac{V_T \sin\varphi}{r} \tag{30}$$

The corresponding increment in maneuvering load factor is

$$\Delta n = \frac{V_F}{g} \cdot \frac{d\gamma}{dt} = \frac{1}{g} \frac{V_F V_T}{r} \sin\varphi \tag{31}$$

$$n = (1 + \Delta n^2)^{\frac{1}{2}}$$

Conversely, the location of points in the space surrounding the target, in which a load factor increment Δn is required for maintaining of heading, is given by

$$r_{\Delta n} = \frac{1}{g} \cdot \frac{V_F V_T}{\Delta n} \sin\varphi \tag{32}$$

which corresponds to circles with radius

$$R_{\Delta n} = \frac{V_F V_T}{2g\Delta n} \tag{33}$$

with the target velocity vector V_T as common tangent. The target carries along with it a 'load factor-calibrated' space, and the load factor experience of any airplane approaching the target in a tracking maneuver depends on the approach angle φ and the depth of penetration into that space.

Characteristic dimensions:

Disregarding structural and physiological limitations in the airplane and its crew, the theoretical load factor which can be developed by the airplane depends on the available speed range and is

$$n_{max} = (1 + \Delta n^2)^{\frac{1}{2}}_{max} = \left\{\frac{V_{max}}{V_{L_{min}}}\right\}^2$$

$$\Delta n_{max} = \left[\left(\frac{V_{max}}{V_{L_{min}}}\right)^4 - 1\right]^{\frac{1}{2}}$$

This corresponds to a circle

$$R_{\Delta n_{max}} = \frac{V_{max} \cdot V_T}{2g\left[\left\{\frac{V_{max}}{V_{L_{min}}}\right\}^4 - 1\right]^{\frac{1}{2}}} \tag{34}$$

defining the outer boundary of a theoretically inaccessible region in the vicinity of the target.

Other important dimensions are the maximum and minimum firing distances, denoted by $R_{F_{max}}$ and $R_{F_{min}}$. Circles with these radii and center in the target define an area surrounding the target containing the termination points of all tracking maneuvers. The inaccessible region mentioned earlier may or may not extend into this region, depending on available speed margins and speed ranges at the altitude considered.

In the case where it does not, a characteristic operational maximum load factor increment is defined by

$$\Delta n_{o_{max}} = \frac{V_F \cdot V_T}{g\, R_{F_{min}}} \tag{35}$$

Termination points:

All tracking maneuvers must terminate at points (r, φ) located inside the maneuvering area which is bounded by $R_{F_{min}} \leqslant r \leqslant R_{F_{max}}$.

As the load factor required for tracking increases with decreasing r, the load factor experience will obviously be a function of the distribution of the termination distances. It is possible that a distribution function for r can be deduced by a closer analysis of the tactical problem involved, taking various types of armament etc., into account.

In the aesence of any information on that score however, the best estimate is obtained by assuming the termination distances as being evenly distributed over the interval $R_{F_{min}} \leqslant r \leqslant R_{F_{max}}$.

The same reasoning applies to the distribution of termination approach angles, φ, with the result that a best estimate is obtained by assuming the termination points to be uniformly distributed over the maneuvering space.

Load factor distribution function:

For the case of uniformly distributed termination points, the probability of obtaining or exceeding a given load factor incrememt $\Delta n'$ is equal to the ratio between the area of the maneuvering space in which $\Delta n'$ or higher load factor increment is required, and the area of the total maneuvering space:

$$P(\Delta n \geqslant \Delta n') = 1 - F(\Delta n') \quad \frac{A_{\Delta n'}}{\frac{\pi}{2}\left(R^2_{F_{max}} - R^2_{F_{min}}\right)} \tag{36}$$

(For reasons of symmetry only the half-plane $-\frac{\pi}{2} \leqslant \varphi \leqslant +\frac{\pi}{2}$ need be considered).

Dropping the prime,

$$F(\Delta n) = 1 - \frac{A_{\Delta n}}{\frac{\pi}{2}\left(R^2_{F_{max}} - R^2_{F_{min}}\right)} \tag{37}$$

If so desired, corrections to the 'raw' distribution function $F(\Delta n)$ (based on $V_{F_{max}}$ and $V_{T_{max}}$ estimates) can be made by methods essentially similar to those outlined previously. However, in the absence of specific requirements for timing of the termination of tracking maneuvers, no particular benefit from speed reduction appears to exist for either of the parts involved. There are, on the other hand, considerable advantages associated with operation at high speeds.

It is suggested, therefore, that the distribution function $F(\Delta n)$, as derived from consideration of maximum speed conditions, can be accepted as a useful approximation to the 'inherent' distribution function.

4.4 Evasive Maneuvers

Ther term evasive maneuver is here implied to mean a maneuver designed to prevent collision with objects in the air or with the ground. As the situation is somewhat different in the two cases, they will be reviewed separately.

4.4.1 Evasion of Mid-air Collisions

The need for an evasive maneuver is established whenever the pilot recognizes the danger of collision if continuing on the original course.

The pertinent quantities are:

(a) The range at which the danger is realized, D

(b) The transverse extension of the obstacle, y

(c) The speed of the airplane, V.

The geometrical features of the situation are shown in Figure 11, from which the following observations can be made. The smallest load factor required for clearing the obstacle is given by:

$$\Delta n = \frac{V^2}{gR} \; ; \qquad 2\,R\,\sin^2\varphi = y$$

$$\sin\varphi \approx \frac{y}{D}$$

$$\Delta n \cong 2\,\frac{y}{D}\cdot\frac{V^2}{gD} \tag{38}$$

A distribution function for the minimum required load factor increment can be derived from Equation (38) if distribution functions for the quantities D and y are available.

The pertinent question is then what distribution functions appear most reasonable for these quantities. Observing that the actions of the pilot are governed by what he thinks he sees rather than by what exists objectively, it appears reasonable to accept as independent variables the quantities y/D and D rather than y and D.

The variable y/D measures the field of view occupied by the obstacle and is immediately observable by the pilot. Theoretically, this variable possesses no upper limit, but it is believed that, for practical purposes, an upper limit can be estimated from circumstances pertaining to the type of operation involved. Inside this limit, a uniform distribution may be assumed, in the absence of better data. Limitation on the maximum range D which need to be considered should also be obtainable from consideration of operational circumstances., e.g. in the case of interception missions, it is believed that the maximum firing range should provide a reasonable estimate, because this is the maximum distance at which any activity is undertaken which ultimately might produce any opportunity for mid-air collision. Again, in the absence of better information, the assumption of a uniform distribution

$$0 \leqslant D \leqslant R_{F_{max}}$$

appears to yield a best estimate. Determination of the distribution function for the maneuvering load factor increment follows closely the routine outlined in Figure 12.

4.4.2 Dive Pull-out

The following analysis pertains to maneuvers which are carried out for the specific purpose of avoding collision with the ground.

The geometry of the situation is shown in Figure 13. The airplane is initially in a straight-line flight path inclined at an angle θ to the ground plane. At a distance D from the imminent point of impact a pull-out maneuver is initiated in order to avoid collision. The minimum load factor increment at which this can be achieved is determined by

$$\Delta n = \frac{V^2}{gR}$$

where

$$R tg \frac{\theta}{2} = D$$

and

$$R = D \cot g \frac{\theta}{2}$$

and consequently

$$\Delta n = \frac{V^2}{gD} tg \frac{\theta}{2} \tag{39}$$

$$n = 1 + \Delta n$$

(Plane of motion essentially vertical)

Distribution functions for Δn can be calculated following the procedure which has been outlined previously, provided that data for the range and variation of the basic parameters θ and D are obtainable. The dive angle θ is obviously limited to within the range

$$0 \leqslant \theta \leqslant \frac{\pi}{2}$$

The distribution of θ within this range is most likely a function of the type of operation and the tactics involved. By assuming a uniform distribution, no special tactic is favored at the expense of others equally possible.

The range of variation for D is also a function of operational and tactical considerations, and the maximum value D_{max} must be determined with reference to possible types and uses of armament. Having established that value however, a uniform distribution of D within the interval $0 \leq D \leq D_{max}$ would appear acceptable.

In maneuvers of this kind, one would *a priori* expect a strong correlation between dive angle θ and speed V. This correlation would however tend to be alleviated by the prevailing use of dive brakes for speed control in dives, and an average dive speed could, with probably no great loss in accuracy, be adopted for calculation purposes.

5. RESULTANT PROBABILITY DISTRIBUTION FOR 'INHERENT' MANEUVERING LOAD FACTORS

Having obtained distribution functions for all different classes of maneuvering problems occurring in the average mission, the resultant distribution function for all maneuvers is determined as the weighted mean of all distribution functions.

Suppose that the mission analysis results in, say, N different classes of maneuvering situations. Suppose further that each class contains i_k (k = 1, 2, ...N) maneuvering problems characterized by the inherent distribution functions

$$F_k(n), \qquad (k = 1, 2, ...N)$$

The resultant inherent probability distribution for the maneuvering load factor experience will then be

$$F_R(n) = \frac{\sum_{k=1}^{k=N} i_k \cdot F_k(n)}{\sum_{k=1}^{k=N} i_k} \tag{40}$$

6. PILOT-AIRPLANE INFLUENCE ON THE PROBABILITY DISTRIBUTION OF MANEUVERING LOAD FACTORS

The resultant inherent probability distribution of maneuvering load factors can be expected to represent quite accurately the requirements for 'functional' load-carrying ability in the airplane, provided that the analysis of operation, mission and maneuvering problems is carried out with due regard to pertinent facts and circumstances.

The probability distribution of *actually* experienced loads is, however, bound to differ somewhat from the probability distribution of functionally required loads, for a number of reasons. The most important of these are:

(a) The inherent probability distribution is obtained by considering steady or quasi-steady states, whereas actual peak loads very often occur as transient responses. The dynamic response characteristics of the airplane can therefore be expected to exert a powerful influence on the probability distribution of actual loads;

(b) Even if transient dynamic effects could be eliminated, the pilot would nevertheless exert his own judgment and capability when applying load factor in response to a given situation. The extent to which the applied load factor agrees with the required, is characterized by what could be called the pilot's 'response-fidelity function'. Since design criteria obviously should be based on an anticipated probability distribution of *actual* loads, the problem of determining the response-fidelity function for human pilots merits considerable interest.

The response-fidelity function is represented in Figure 14, in which the coordinate axes of the upper half are calibrated in terms of maneuvering load factor. Let the abscissa represent functionally required load factor, i.e. maneuvering load factors which are necessary for the correct solution of *any* maneuvering problem pertinent to the operation, and let the ordinate represent the load factor actually developed by the pilot in response to that problem. A 100% degree of response-fidelity is thus represented by a straight line through the origin at 45° inclination. It is probably correct to assume that the average degree of response-fidelity remains near the 100% value within some region extending from the origin. However, as the functionally required load factor increases beyond that region, a consistent tendency to develop lower-than-required factor can be expected to emerge, the tendency increasing as required load factor continues to increase.

The author would like to point out that he is not at present aware of any data from which response-fidelity functions could be constructed, but that he is forced to infer the existence of this class of functions from the general tendencies of available probability distributions for maneuvering load factors.

The advantage of data representation in this form for practical design work is apparent with reference to the lower half of Figure 14. Here the lower ordinate axis is calibrated in terms of probability of obtaining or exceeding given load factor values, $(1 - F(n))$. Having plotted $(1 - F_R(n))$ in this quadrant, and observing that this curve indicates the relative frequency of occurrence of occasions in which load factor equal to or exceeding n is required, it is easy to correlate this with the corresponding actual load factor experience, by a transfer of constant probability values over the difference between required and actually developed load factor.

Some factors which are believed to have a strong influence on the shape of the response-fidelity function are:

(a) *Acceleration tolerance*

This is a very influential factor, but it is rather difficult to handle due to the fact that it is characterized by the tolerable acceleration-impulse (n.g.t) and therefore indeterminate as far as acceleration is concerned, unless reference can be made to a specified time of exposure.

(b) *Physical strength*

This is a factor which assumes importance when achievement of load factor is made contingent on a certain physical effort by the pilot. Here again we are confronting a certain indeterminacy caused by the dynamic response characteristics of the airplane. Thus, while a one-to-one correspondence between steady state acceleration and control force and deflection exists, no such correspondence exists in general for transient states.

(c) *Pilot indoctrination*

This is a factor which appears to alleviate to some extent the indeterminacy introduced by the dynamic response charactetistics of the airplane.

(d) *Psychological factors*

In this class are grouped all those influences which induce the pilot to go ahead and achieve the objective in spite of physical discomfort and danger. They do not, at the present time, appear adaptable to engineering calculations.

It appears to the author that two approaches are open for attempts at determination of the response-fidelity function. One is to determine inherent probability distributions of maneuvering load factors for operations in which extensive empirical data on probability distribution have already been collected, and apply the reverse of the procedure outlined above. The second is to conduct experiments in order to record the response-fidelity directly under representative conditions. Due to the existing variance between human individuals, a great number of tests with an appreciable collection of individuals would have to be made.

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TABLE I

Transport Operation: Mission Analysis
Maneuvering Situation Characteristics

Flight Segment	*Speed*	*Altitude*	*Number of Maneuvers*	*Purpose and Extent of Maneuver*		*Characteristics Mean Values*
T.O. and Initial Climb		0-5000	1¼	Instr. dep. proc. turn	$0\text{-}360^{\circ}$	Standard rate of turn: 3°/sec
Climb		5000-Cruise alt.	2½	Turn into airway	$0 - \pm 180^{\circ}$	"
Cruise	Operational cruise speed	Cruise alt.	4	Airway course shift	$0 - \pm 90^{\circ}$	"
Cruise	Operational cruise speed	Cruise alt.	½	Identification turn	$\pm 90^{\circ}$	"
Descent	Best fuel economy speed	Stacking alt.	½	Holding cir. entry turn	$0 - 180^{\circ}$	"
Descent	Best fuel economy speed	Stacking alt.	1.0	Holding circuit turn	180°	"
Descent	Best fuel economy speed	5000-1000	½	Holding circuit turn	$0 - 180^{\circ}$	"
Approach	Appr. speed	1000-0	3¼	Approach traffic pat. turns	0 - 360	"
Landing	Landing speed		1	Landing flare-out		Glide path rate of descent duration of flare

TABLE II

Mission Analysis: Combat-interception

Flight Segment	*Maneuvering Situations*					
	Number	*Type*	*Purpose*	*Speed*	*Altitude*	*Remarks*
Take-off climb	1	Turn	Navigational, to establish initial course	T.O. - climb	~ zero	1) Range of change in heading: 0 - 180° 2) Rates of turn distributed around a central, standard value
Cruise	2	Turn	Positioning for interception	Min. level flight - max. speed	Average operational	3) Turning radius depending on distance and bearing to desired position 4) Timing assumed achieved by speed control
Interception	1	Turn	Correction of initial aiming error	As above	As above	5) Reqd. load factor depending on magnitude of correction and on available time
	1	Turn	Tracking	Max. combat	As above	6) Reqd. load factor depending on: speed ratio and location relative to target
	1	Turn pull-up	Evasive, break-away	As above	As above	7) Reqd. load factor depending on proximity to and size of obstacle
Cruise	1	Turn	Navigational, heading for base	Normal cruise	Optimum cruise	As 1) and 2) above
Cruise	1	Turn	Navigational, identification	As above	As above	As above
Let-down and Approach	3	Turn	Navigational, conforming to standard traffic pattern	Approach speed	Optimum cruise - zero	As above
Landing	1	Pull-up	Checking of vertical descent	Landing speed	Zero	8) Reqd. load factor depending on speed, glide angle, timing of maneuvre

TABLE III

Mission Analysis: Combat-ground attack

Flight Segment	Maneuvering Situations					
	Number	Type	Purpose	Speed	Altitude	Remarks
Take-off climb	1	Turn	Navigational, to establish initial course	T.O. - climb	~ zero	1) Range of change in heading: 0 - 180° 2) Rates of turn distributed around central standard value
Cruise at altitude	1	Turn	Navigational, dive entry	Optimum combination for miss. N and segm. T lengths considered		3) Turning angle range 0 - ± 90° 4) Rate of turn ~ standard
Descent	1	Pull-up	Dive recovery	Dive speed	Zero	5) Reqd. load factor depending on dive angle, ground proximity in pull-out
Low-level cruise	X	Turn, pull-up, push-over	Navigational, clearing of obstacles in flight path	Optimum for flight and segm. T lengths		6) Man. load experience depending on topography of flight track and reqd. ground clearance
Low-level cruise	1	Pull-up	Entry of climb to reconnoitering altitude	Max. level flight	Zero	7) Reqd. load factor depending on situation geometry
Climb	1	Push-over	Resuming level flight	Max. level less climb speed loss	From tactical considerations	8) Load factor possibly a function of climb angle
Target area scanning	Y_1	Turn, pull-up, push-over	Evading enemy opposition	Max. level flight	As above	9) Man. load experience depending on enemy opposition
	1	As above	Entry into attack dive	As above	As above	10) Reqd. load factor depending on situation geometry
Attack dive	Y_2	As above	Evading enemy opposition	Combat dive	Scanning altitude to zero	As 9) above
	1	Pull-up	Recovery from dive	As above	~ zero	As 5) above
Climb	1	Turn	Navigational	Climb speed	Zero to optimum cruise	As 2) above
Cruise	1	Turn	Navigational identification	Cruise speed	Optimum cruise	As 2) above
Let-down and approach	3	Turn	Navigational, as reqd. by standard traffic pattern	Approach speed	Optimum cruise to zero	As 2) above
Landing	1	Pull-up	Checking of vertical descent	Landing speed	Zero	11) Reqd. load factor depending on situation geometry

TABLE IV

Classes of Maneuvering Problems

Class Description	*Type of Maneuver*	*Operational Purpose*	*Remarks*
Positioning	Turn	Transfer of airplane between states of initial and desired positions and headings	Timing usually important
Corrective	Turn, Pull-up, Push-over	Reduction to zero of difference between prevailing and desired direction of flight path	Timing important, correction must be accomplished within limits of available time
Tracking	Turn, Pull-up	To maintain a flight path which at any instant keeps the airplane headed on a moving object	Timing irrelevant
Evasive	Turn, Pull-up	To avoid collision with obstacle, mid-air or ground	Distance, angle of approach to, and size of object important
Navigational	Turn	To accomplish heading changes as required for navigational reasons	Usually performed at standard rates of turn

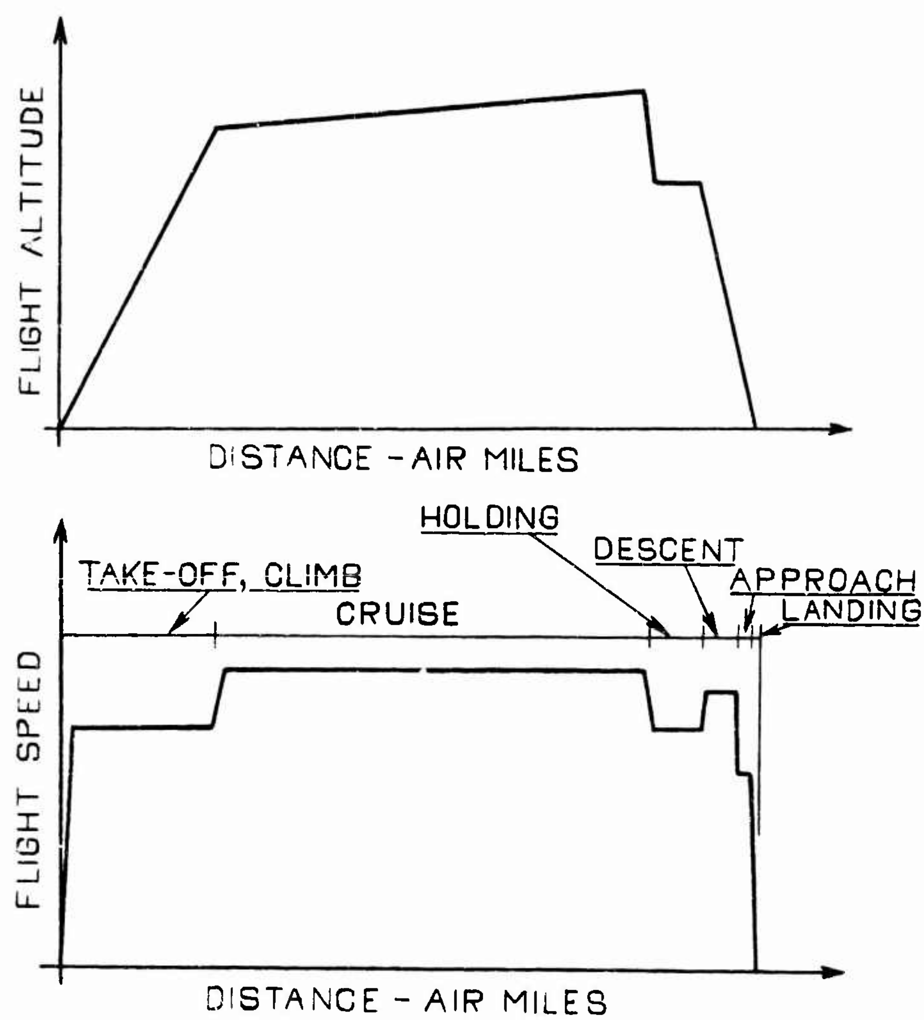


Fig. 1 Average mission profile for transport operation

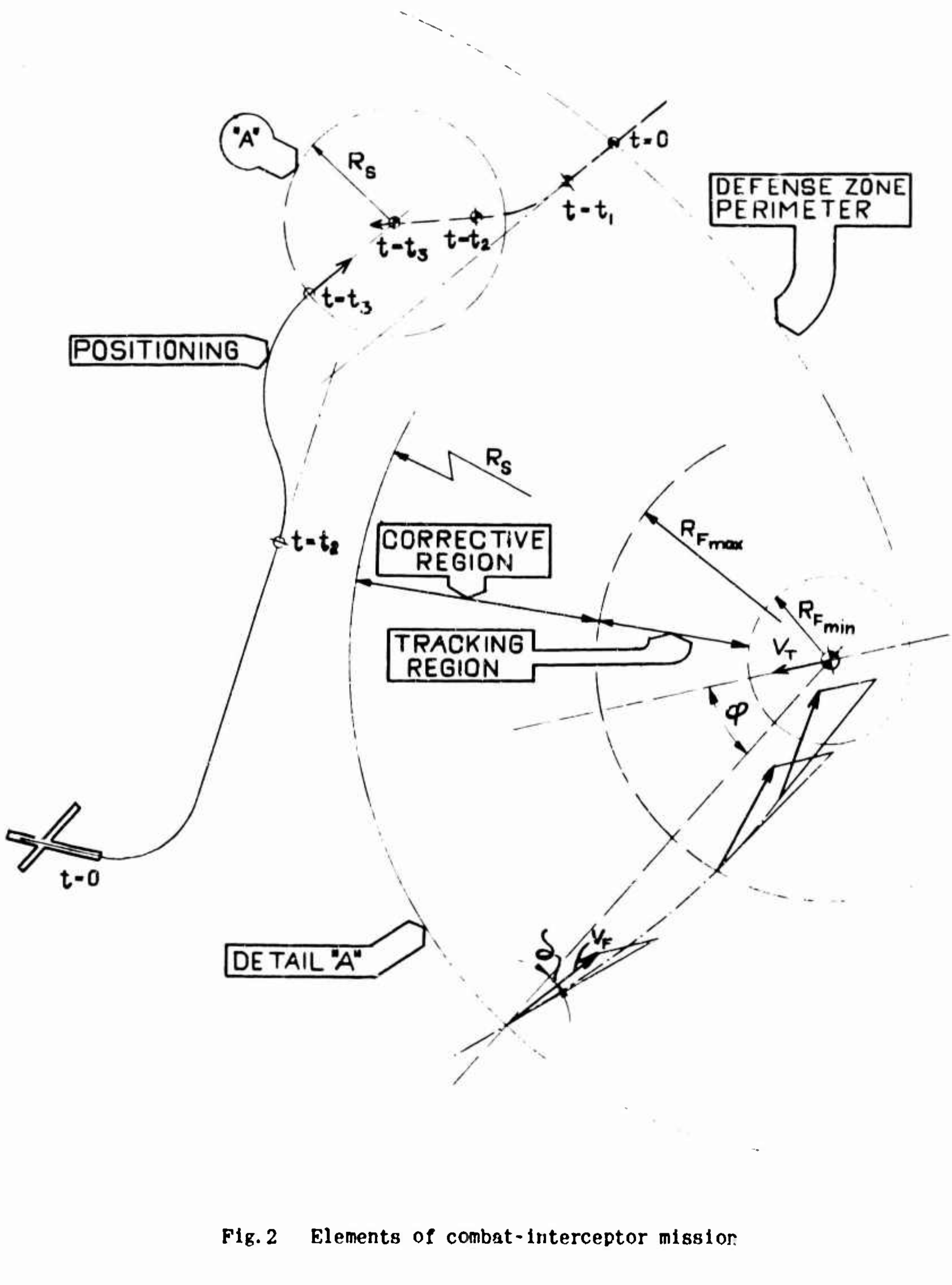


Fig. 2 Elements of combat-interceptor mission

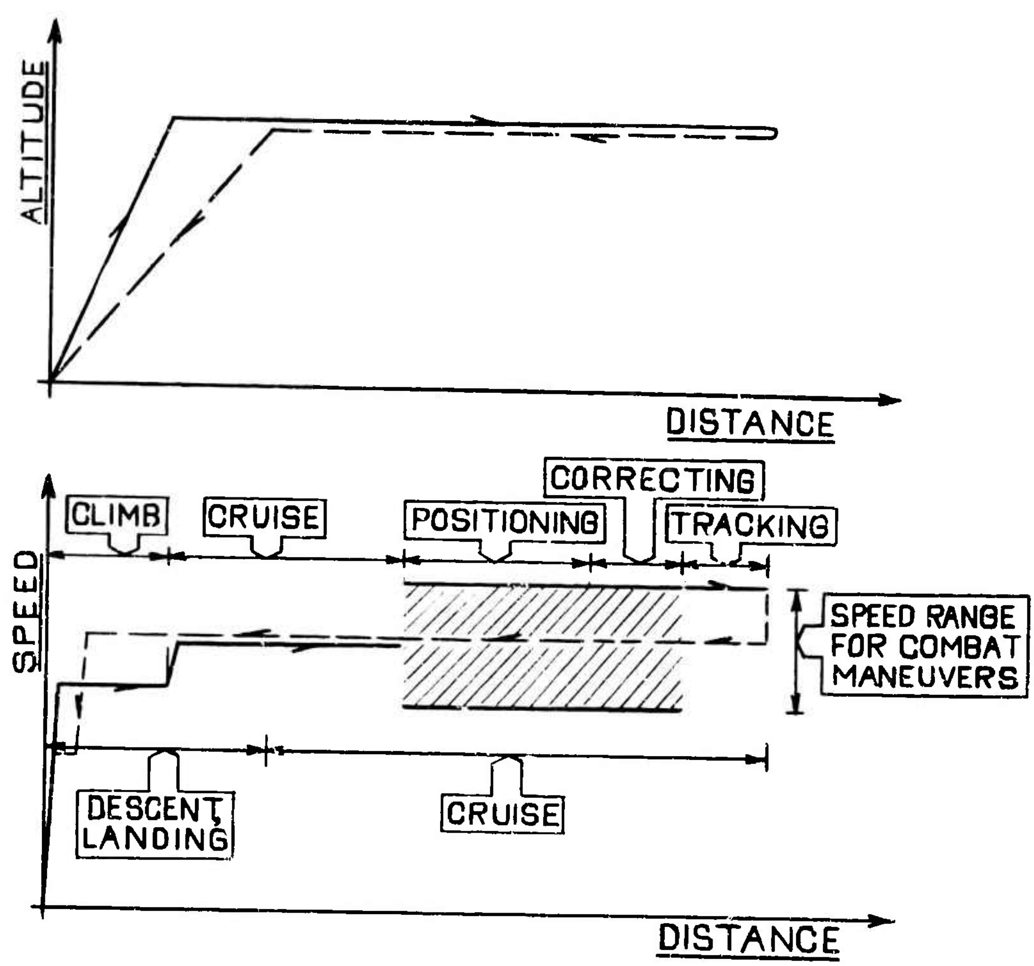


Fig.3 Interceptor mission profile

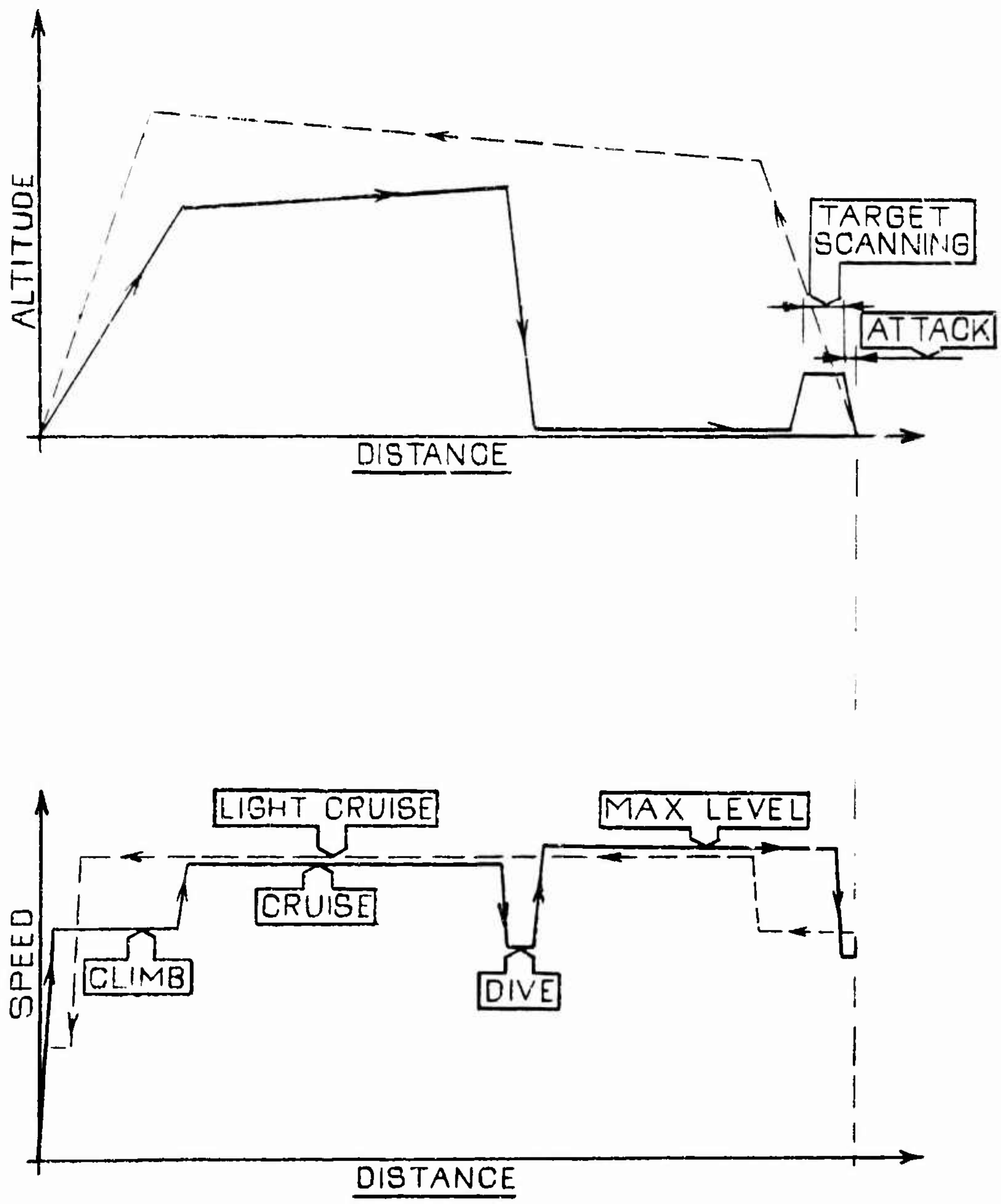
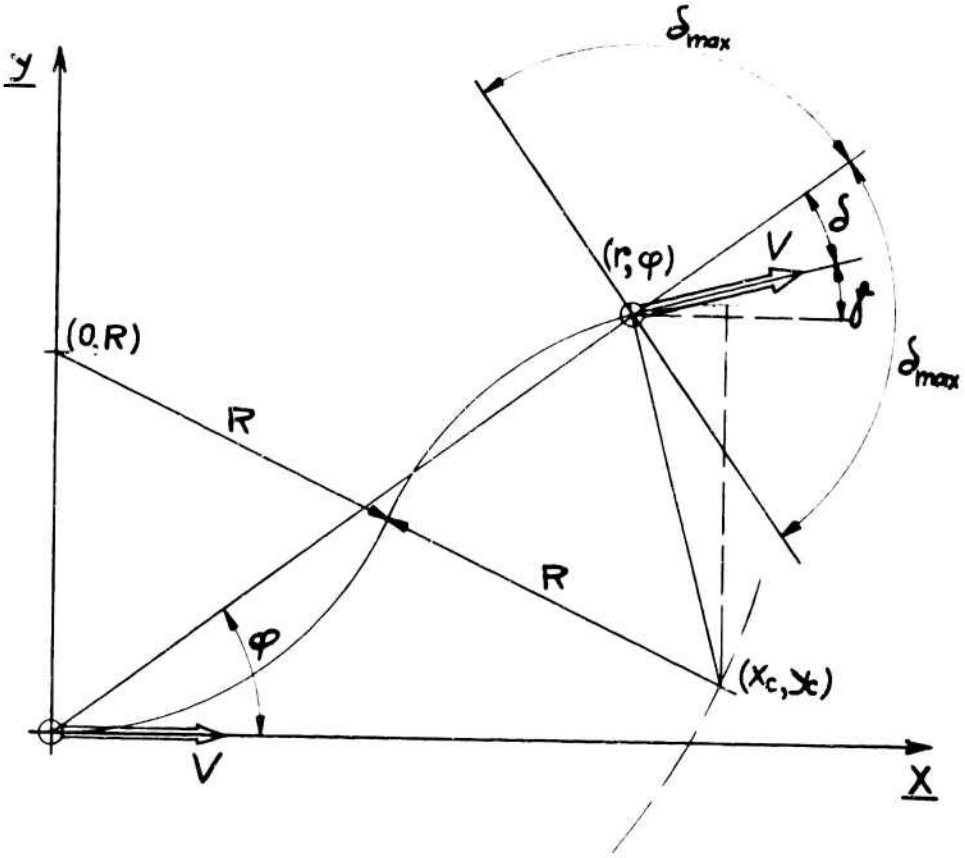


Fig.4 Ground-attack mission profile



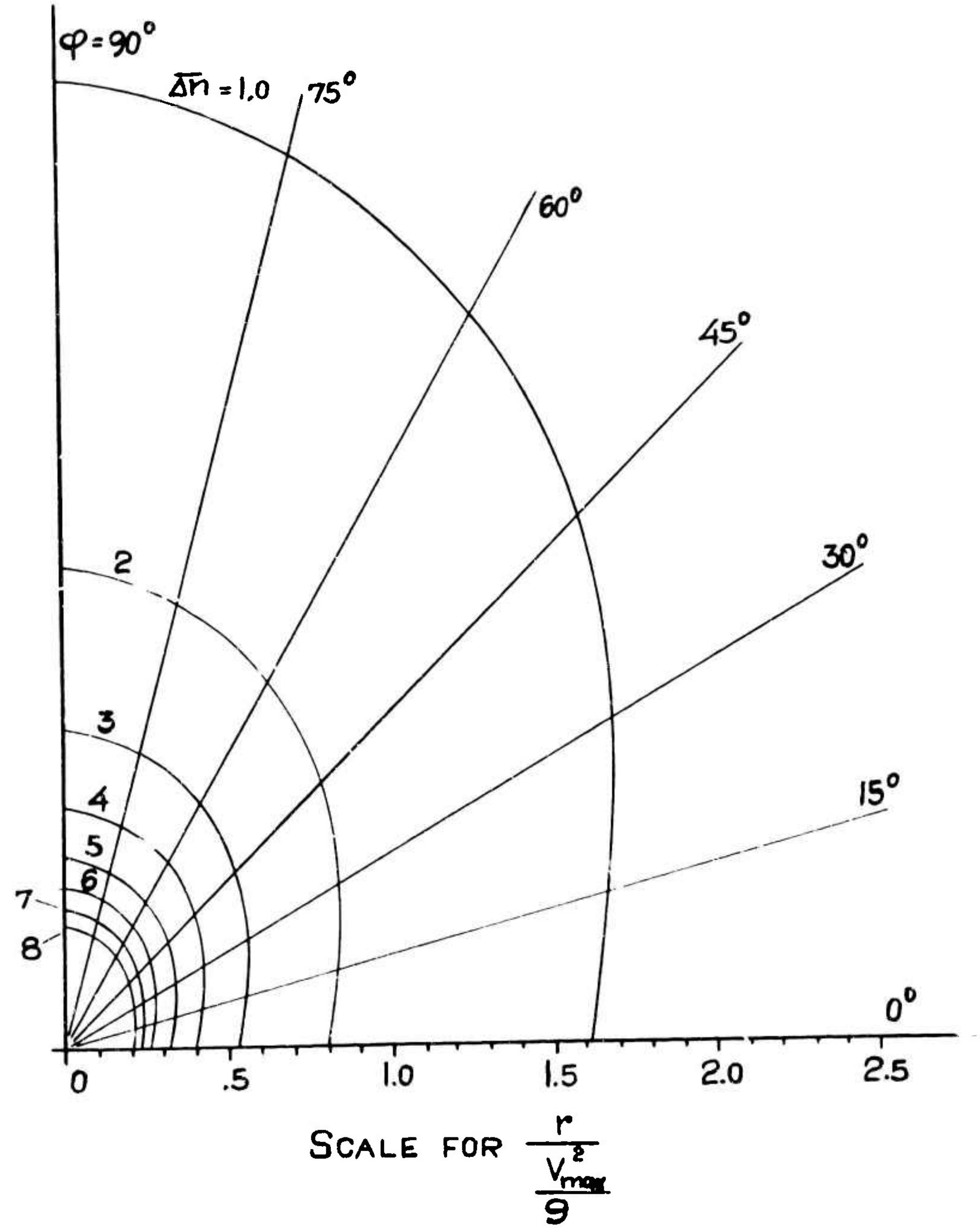
$$\frac{r}{R} = \left[\sin\varphi - \sin\delta \pm \left((\sin\varphi - \sin\delta)^2 + 2(1-\cos(\varphi+\delta)\right)^{\frac{1}{2}}\right] = \frac{r}{R}(\varphi,\delta)$$

$$\frac{\bar{r}}{R}(\varphi) = \frac{1}{2\delta_{max}} \int_{-\delta_{max}}^{+\delta_{max}} \frac{r}{R}(\varphi,\delta)\, d\delta$$

$$\overline{\Delta n} = \frac{V^2}{g} \cdot \frac{1}{\bar{R}} = \frac{1}{r} \cdot \frac{V^2}{g} \cdot \frac{\bar{r}}{R}(\varphi)$$

$$\underline{r_{\overline{\Delta n}}} = \underline{\frac{1}{\Delta n} \cdot \frac{V^2}{g} \cdot \frac{\bar{r}}{R}(\varphi)}$$

Fig.5 Situation geometry for minimum load factor positioning maneuver



$$n = (1 + \overline{\Delta n}^2)^{1/2}$$

Fig.6 Space distribution of mean load factor increment in constant speed positioning maneuver, $V = V_{max}$

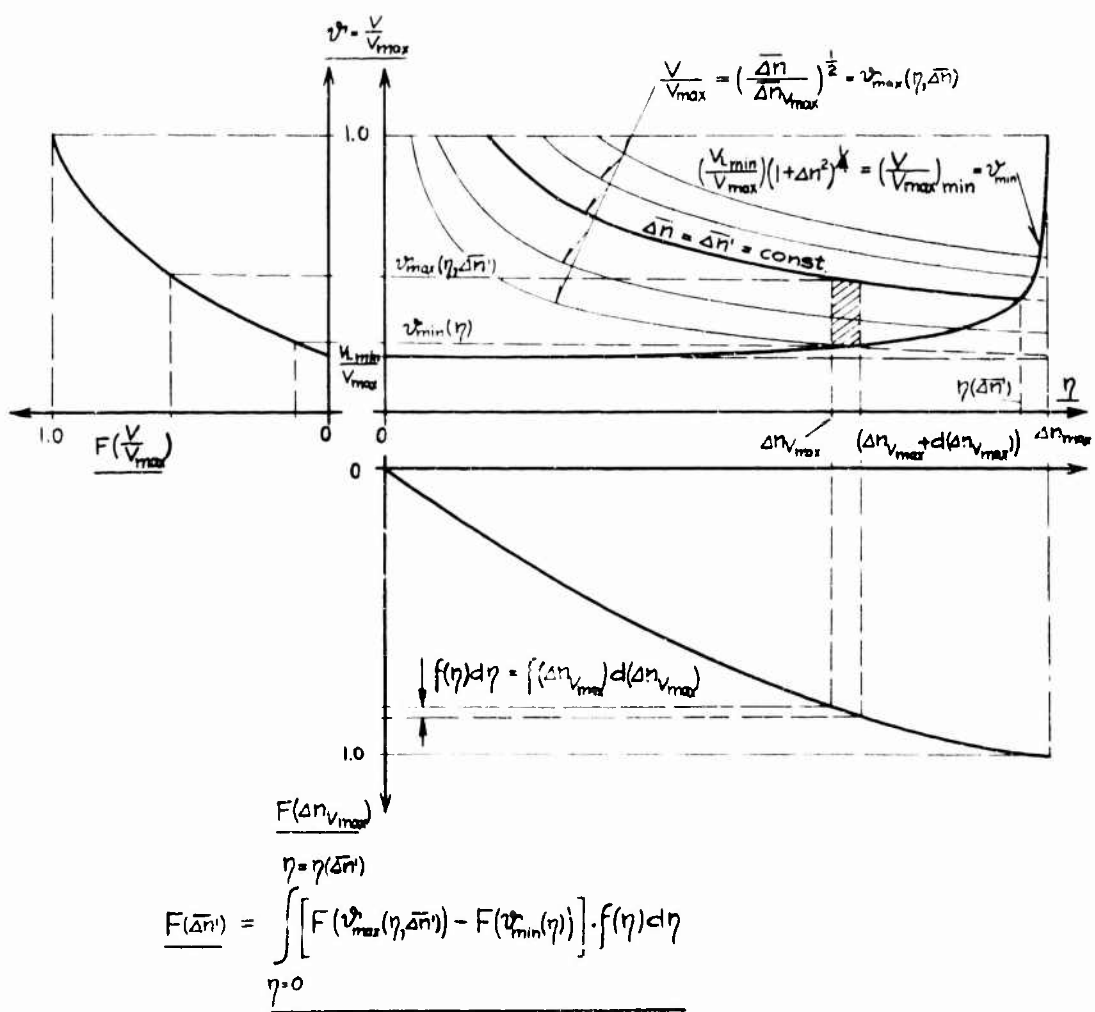
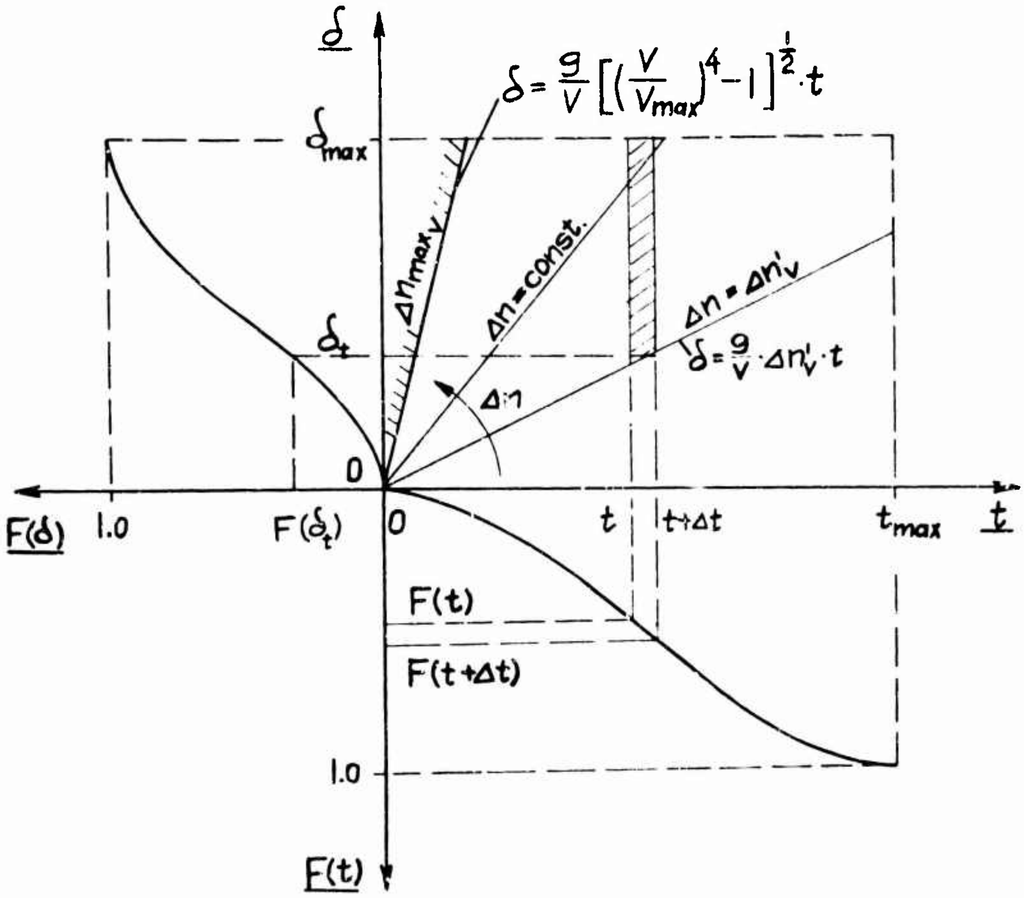


Fig.7 Determination of inherent distribution of maneuver load factors in positioning maneuvers



$$P(\Delta n \geq \Delta n'_V) = \underline{1 - F(\Delta n'_V)}$$

$$= \left[\int_{t=0}^{t=\left\{\begin{array}{l} t_{max} \\ \frac{V}{g}\frac{\delta_{max}}{\Delta n'_V} \end{array}\right.} \left[1 - F(\delta_t)\right] f(t)\,dt - \int_{t=0}^{t=\frac{V}{g}\left(\frac{\delta_{max}}{\Delta n_{max_V}}\right)} \left[1 - F(\delta_t)_{\Delta n_{max_V}}\right] f(t)\,dt \right]$$

Fig. 8 Determination of load factor distribution for constant speed corrective maneuver

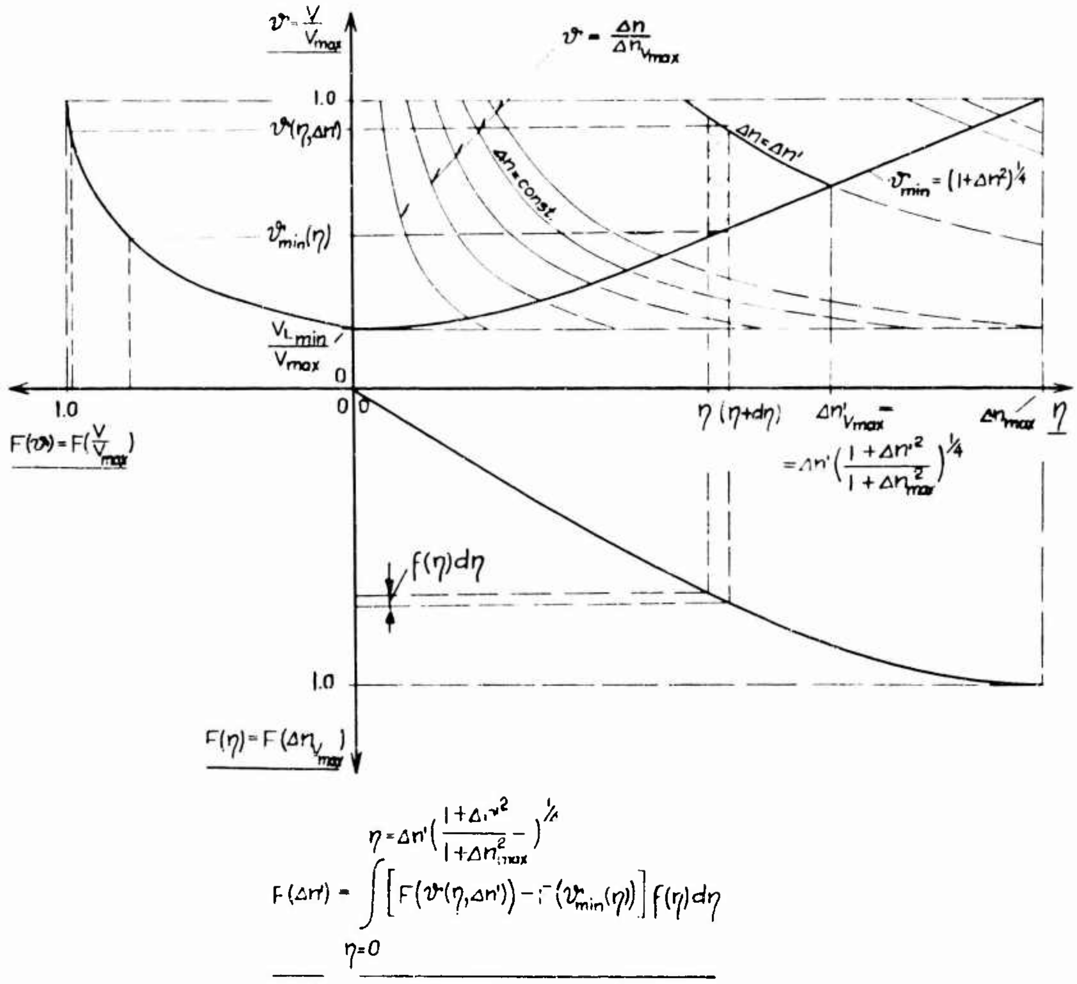


Fig.9 Determination of inherent distribution of maneuver load factors in corrective maneuvers

$$F(\Delta n) = 1 - \frac{A_{\Delta n}}{\frac{\pi}{2}\left(R_{F_{max}}^2 - R_{F_{min}}^2\right)}$$

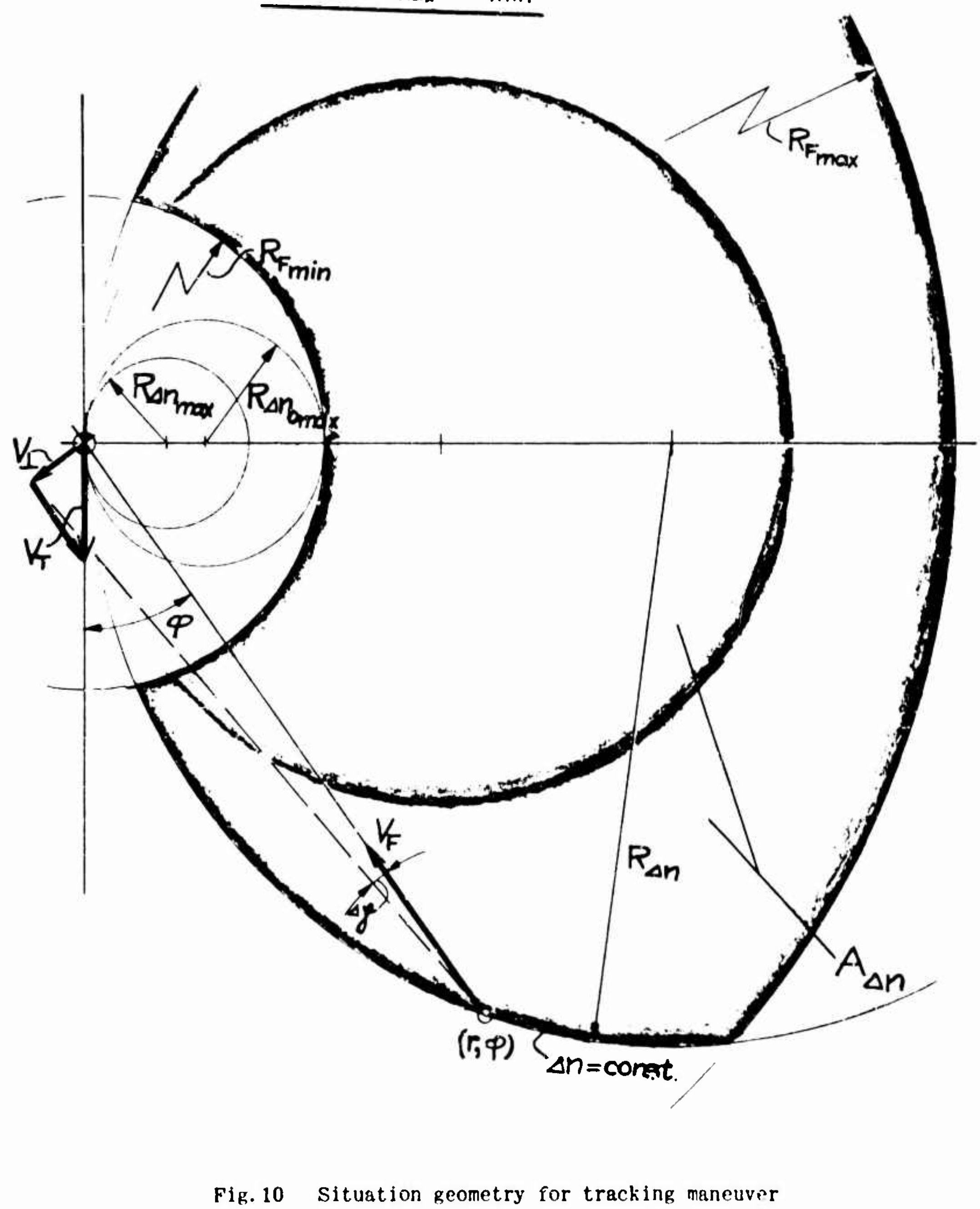


Fig. 10 Situation geometry for tracking maneuver

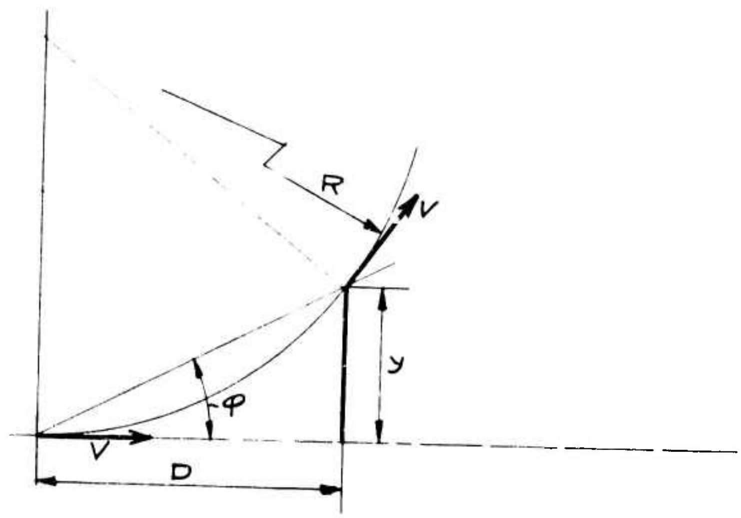


Fig. 11 Situation geometry for collision avoidance

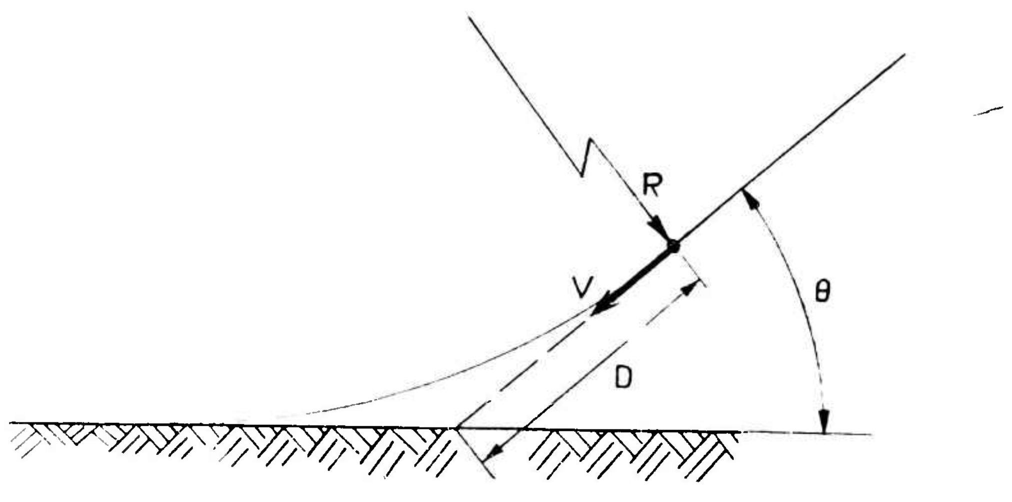


Fig. 13 Situation geometry for dive pull-out

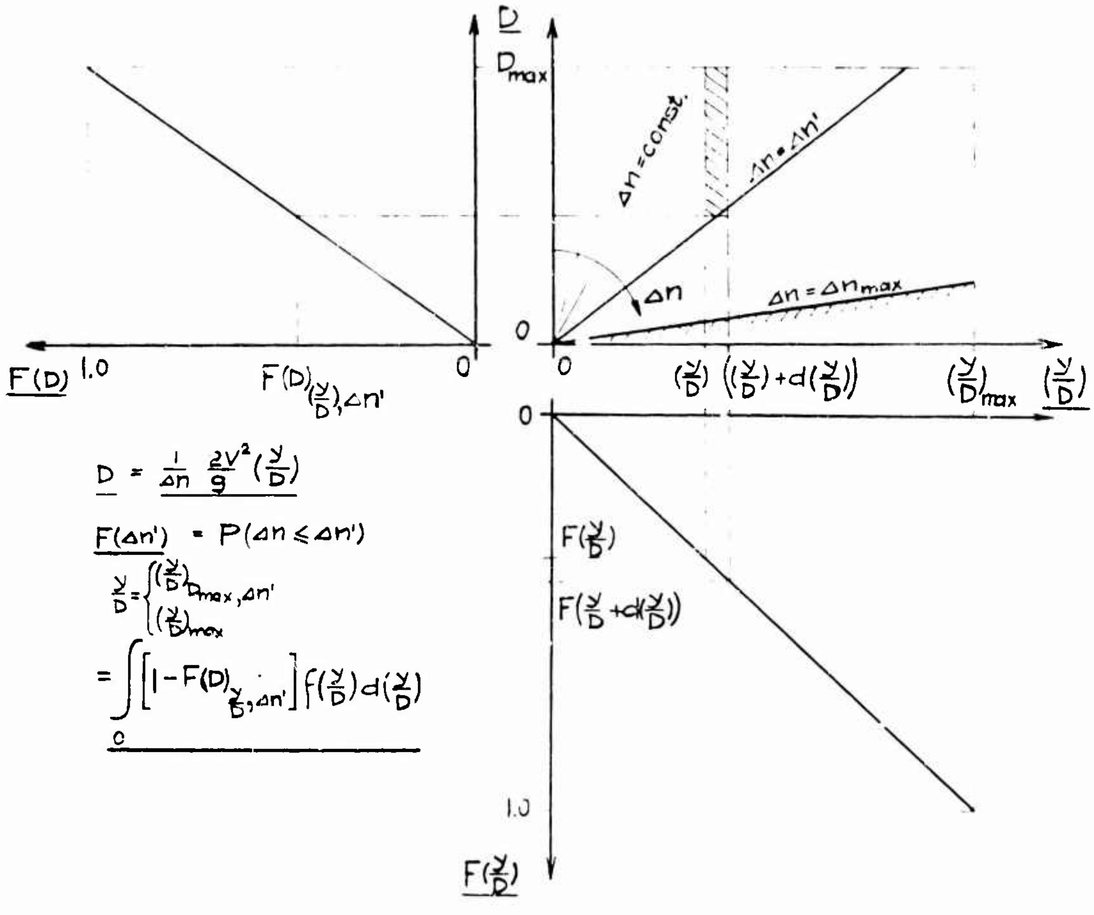


Fig. 12 Determination of distribution function for load factor increments in collision avoidance maneuvers

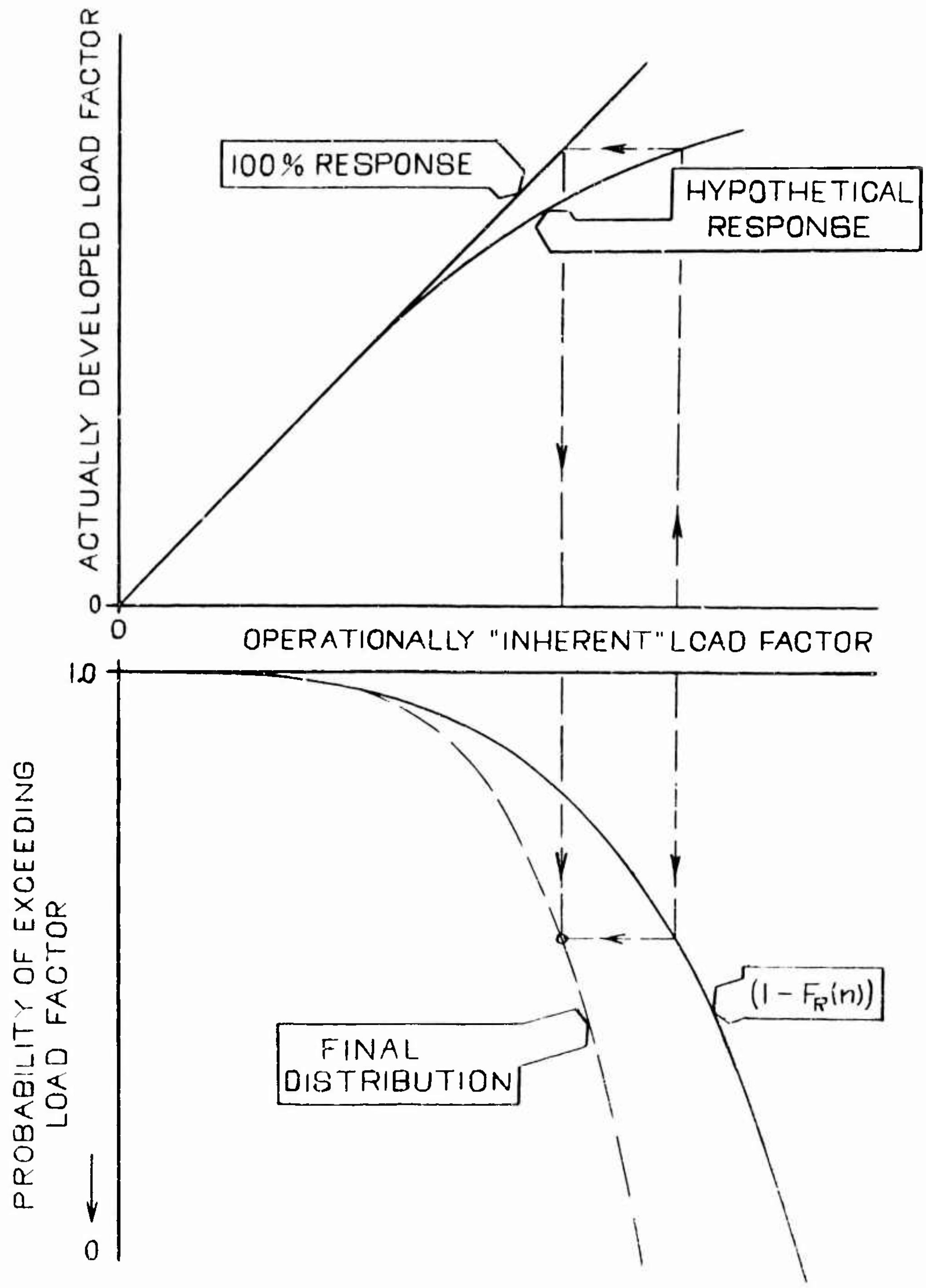


Fig. 14 Effect of pilot's 'response fidelity' on the resultant load spectrum