ARL 67-0187
SEPTEMBER 1967

Aerospace Research Laboratories

A CRITICAL REVIEW OF HETEROGENEOUS MIXING PROBLEMS

ANTONIO FERRI
NEW YORK UNIVERSITY
BRONX, NEW YORK

Contract No. AF 33(615)-2215
Project No. 7064

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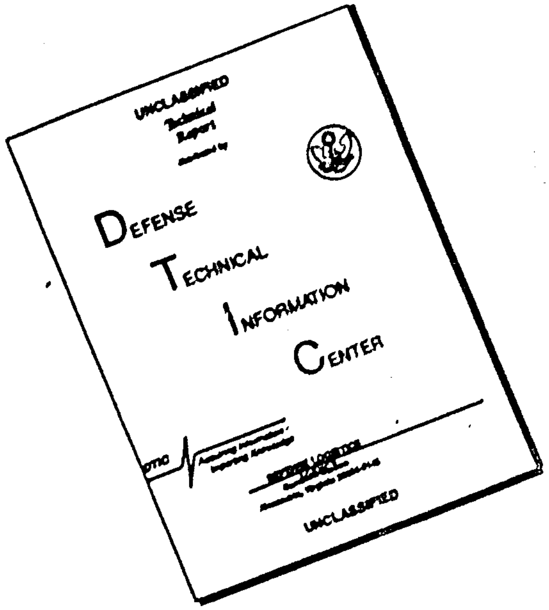

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400 - November 1967 - CO455 - 13-270

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UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

FOREWORD

This interim report was prepared by Dr. Antonio Ferri, Director, N.Y.U. Aerospace Laboratories, and presents research carried out under Contract No. AF33(615)2215, "Boundary Layer Characteristics for Hypersonic Flow in the Presence of Mass Addition", Project No. 7064.

ABSTRACT

A critical review of mixing problems is presented. The limitations of boundary layer types of analyses are discussed. A method of analysis which does not use boundary layer approximation is outlined. Difficulties of correlating experimental data attributable to not-well-defined boundary conditions are described. New phenomena related to mixing with pressure gradient and chemical reaction are presented.

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NOMENCLATURE

D	diffusion coefficient
h	enthalpy
H	total enthalpy
p	pressure
q	velocity
R	gas constant
T	absolute temperature
W	molecular weight
α	mass fraction
μ	viscosity coefficient
ρ	density
θ	streamline direction
$\dot{Y}_i$	mole rate of production of species i
M	Mach number

Subscripts

i	chemical species
n	normal coordinate
s	streamwise coordinate

1. INTRODUCTION

The problem of mixing of two streams is extremely complex. Classical aerodynamics has investigated for many years the problem of mixing of two coaxial gas streams. However, the quantitative understanding of such physical phenomenon is still incomplete even for laminar mixing where the transport properties are known, and the progress made both by theoretical and experimental investigation is still not even satisfactory.

From an analytical point of view, the difficulty arises because of the simplication introduced in the equations. In addition, the lack of knowledge of transport properties increases the difficulties of the problem for turbulent mixing. Classically, mixing problems are analyzed by introducing all simplifications used in boundary layer theory. Unfortunately, the introduction of such simplifications in mixing problems are not necessarily justified and acceptable for the determination of important physical properties of the phenomena. Several questions can be raised with respect to the use of the boundary layer equations in mixing problems. The introduction of the boundary layer equations in the analysis of flow near a wall permits eliminating the second momentum equation. It can be shown that this simplification is not important in the determination of the variation of momentum along streamlines, and therefore of the skin friction which is one of the important quantities to be determined in boundary layer analysis. However, in mixing problems, the most interesting information required from the analysis is the rate of diffusion of a fluid of one stream into the fluid of the other, and such motion is controlled by the second momentum equation. As a consequence, the use of boundary layer equations can be justified only when the characteristics of the two streams (velocity, density, composition-temperature) are not too different. If the two streams have physical properties which are widely different, then the scale for the determination of the order of magnitude, which is the thickness of the layer affected by the flow, becomes extremely large. Typical indication of the inadequacy of the approximation of the boundary layer theory is given by the results of the analysis of a jet discharging in a quiescent atmosphere made with this approximation. In this case, in the external flow the velocity component normal to the axis of the jets are larger than the tangential component in the proximity of the origin of mixing, therefore, the description of the motion of the external flow cannot be represented by the simplified equations. The normal component of the velocity is directly related to the mass of the flow entrained by the jet, and therefore is a quantity that must be determined with satisfactory precision.

When the approximation of the boundary layer theory cannot be justified for the analysis of mixing problems, then, as a consequence, the boundary conditions for a mixing problem cannot be specified in a simple form at the station where mixing begins if the two streams are subsonic. In this case, the mixing process induces velocity components which are significant upstream of the initial mixing region.

For the same reason it is very difficult to perform significant experiments of mixing when the two streams are quite different

In this case, the characteristics of the experimental apparatus upstream of the mixing region have important effects on the mixing phenomena near the origin of mixing. This point is often neglected in presenting experimental results and in comparing experimental results. When the two streams are subsonic even for the case of the mixing of two low velocity streams of the same fluid, the initial conditions cannot be defined by giving the average properties of the two streams as, for example, the average velocities measured in terms of stream tube area and mass flow, but are defined only if the complete velocity distribution and pressure distribution at the station at the beginning of the mixing are given. Such distributions can change significantly from experiment to experiment because they are affected by the experimental apparatus upstream of the mixing region; therefore, presentation of experimental data on the basis of simple parameters as average velocity ratio or average velocity difference of the two jets is not justified because such terms do not define either physical properties or the experiment.

2. ANALYSIS OF COMPLEX VISCOUS FLOW PROBLEMS

When the properties of the two mixing streams are quite different the solution of more complex sets of equations is required where pressure gradients in two perpendicular directions are considered to be of the same order and where viscous effects are considered. Such type of analysis is of great importance in many problems of practical interest besides the chemical problem of coaxial mixing of nonreacting streams. Typical examples of phenomena where such type of analysis is required are: (a) The mixing of two reacting streams where the energy released by the reaction produces important pressure perturbation that cannot be neglected; (b) High speed, low density flow around bodies; in a wide Reynolds number range, for many practical shapes the flow still can be analyzed with the assumption of continuous medium, but gradients of velocity are important in the flow fields and therefore important viscous effects are present in all flow fields; (c) Boundary layer with large rates of ablation where the blowing velocity is large; (d) Boundary layer with chemical reactions where the chemical enthalpy of reaction is of the same order of the total enthalpy of the flow; (e) Hypersonic boundary layers with large pressure gradients along the streamline. For such problems the variation of pressure in the boundary layer thickness is of the same order of the variation between two distant points along a given streamline because M is large. In this case, the static pressure varies strongly in boundary layer in normal direction to the wall and cannot be neglected. The effect of such pressure gradient affects strongly the velocity profile and therefore all the viscous effects. An example of such pressure variation is shown in Fig. 1. The upper part of the figure shows the experimental apparatus, while the lower part shows the pressure variat on and velocity distribution in the boundary layer at a station of the body. The pressure at that station is 27 times the initial static pressure. The flow is initially at $M = 6$, and the body is axially symmetric. The boundary layer thickness is of the order of 0.35". The experiments have been performed at the Laboratory of New York University by Mr. Hoydysh (Doctoral candidate).

The solution of viscous problems where more complex equations are required can be approached along two different lines. The classical line is the perturbation method where higher order terms are obtained as second approximation. The calculation starts with the boundary layer approximation which is assumed as first order solution in the determination of the flow. This method permits us to obtain, in some instances, a second order correction; however, such an approach is useful only if the first order solution gives a sufficiently good approximation of the flow field, then the second approximation can be used to improve the accuracy of the results. For many of the problems described such an approach is not very promising.

A second approach suggested by the author and Dr. Moretti[1] consists in simplifying the equation of motion on the basis of the following physical principle. The viscous effects are produced by gradients. In many physical problems such as those previously described, the variation of physical properties along streamlines is much smaller than in a direction normal to the streamlines, then the equations that govern the motion of the fluid can be written with good approximation in the following form:

In the two-dimensional case, the momentum equations for the above formulation are:

$$\rho q q_s + p_s = A, \tag{1}$$

$$\rho q^2 \theta_s + p_n = 0, \tag{2}$$

where ρ is density, p is pressure, q and θ the modulus and the direction of the velocity relative to engine axis. The quantity A is given by

$$A = (\mu q_n)_n, \tag{3}$$

where μ is the viscosity.

The subsequent solution of the compatibility equations requires that A be constant or of small relative variation during an integration step. Then the initial computation can be made approximately using the initially known value A^o, followed by iteration to obtain a more accurate mean value $\bar{A}$.

The continuity equation reads

$$q\rho_s + \rho q_s + \rho q \theta_n = 0, \tag{4}$$

and the energy equation can be stated (assuming for simplicity that the Lewis and Prandtl numbers are equal to unity)

$$H_s = \frac{1}{\rho q}(\mu H_n)_n, \tag{5}$$

where H is the total enthalpy

$$H = h + \frac{q^2}{2}, \tag{6}$$

h being the static enthalpy.

The diffusion along the streamline has been neglected in comparison with lateral diffusion.

In order to complete the description of the system we express the enthalpy by means of the equation

$$h_s = \sum_{i=1}^{N} h_i \alpha_{is} + T_s \sum_{i=1}^{N} \alpha_i \frac{dh_i}{dt} \tag{7}$$

where α_i is the mass fraction of species i. From the equation of state we obtain further

$$p_s - \frac{p\rho_s}{\rho} - \frac{pT_s}{T} - R\rho T \sum_i \frac{\alpha_{is}}{W_i} = 0, \tag{8}$$

R being the gas constant and W_i the molecular weight of species i.

The species conservation equations must take into account the mixing of species between streamlines and the change in mass fraction of the species due to chemical reaction along the streamline. Thus,

$$\alpha_{is} = \frac{1}{\rho q} \frac{\partial}{\partial n} \rho D \frac{\partial \alpha_i}{\partial n} + \frac{W_i \dot{\gamma}_i}{\rho q} , \tag{9}$$

where, again for simplicity, a single binary diffusion coefficient D has been assumed. For unit Lewis number we can substitute

$$\rho D = \mu \tag{10}$$

The subsequent algebraic manipulations to put the equations into suitable characteristic form are tedious, so details should be sought in Ref. 6. It is shown that the third equation to be used with (1) and (2) can be put in the form

$$\left(q + \frac{F}{q}\right) q_s + \frac{F}{p} p_s + F\theta_n = E, \tag{11}$$

where

$$F = T \sum \alpha_i \frac{dh_i}{dT} ,$$

$$E = \frac{1}{\rho q} (\mu H_n)_n - \sum_i h_i \alpha_i + \frac{R\rho FT}{p} \sum_i \frac{\alpha_i}{W_i} .$$

It is seen that the viscosity does not appear in (1), (2) and (11) except in the right-hand side and so does not directly affect the streamline and characteristic directions and the location of the net intersections.

The quantities related to transport properties are represented by the terms in the right-hand side of Eqs. (1), (2) and (11). Such quantities are functions of the coordinates, and of the gradients of local flow properties and therefore can be determined only by the solution of the differential equations. However, in practical numerical solutions, the values of such terms can be estimated with sufficient precision from the values of the same quantities at neighboring points. On the basis of such an estimate a numerical value can be determined for the local value of such quantities. Then the left-hand side of the equations can be solved by using known methods of analysis for inviscid flows. When the flow is determined in first approximation, a second approximation is feasible where these terms in the right-hand side of Eqs. (1), (2) and (11) are calculated on the basis of the local flow properties determined from the first approximation. Such an approach is very useful when the "inviscid" part of the flow contributes appreciably to the local variation of flow properties, while the effect of transport phenomena is smaller because it is more gradual. In this case boundary layer type of equations cannot give a satisfactory first order approximation. This approach has been used for the case of supersonic flows[1,2]. In this case, the inviscid part is solved with the method of characteristics by means of a difference method, by determining as a first step the effect of terms containing the transport properties at the two initial points, 1 and 2 of Fig. 2, where the properties are known. Such terms are reevaluated in second approximation at point 3, and average values between 1, 3 and 2 and 3 are used. Dr. Moretti has applied such an approach to problems of supersonic mixing with chemical reaction. Results of calculations for such problems are shown in Figs. 3 and 4. The problem analyzed is mixing with combustion of two coaxial axially symmetric streams of hydrogen and air. The two jets are initially parallel and at the same static pressure; however, the mixing and combustion produce waves that propagate in the flow field. In Fig. 3 the isobars are shown. The combustion produces pressure waves that propagate in the flow outside of the mixing region. In Fig. 4, the isotherms for the same case are shown. The pressure field outside of the mixing region is obtained as a result of the analysis. If the same problem is analyzed by means of classical mixing type of analyses based on boundary layer equations, then the pressure field outside of the mixing region must be assumed as given and cannot be determined analytically.

Another application of the approach described is related to the determination of the low density supersonic flow around bodies as shown in Fig. 5. The Reynolds number is small, therefore the displacement of the boundary layer affects the pressure downstream of the nose. In addition, because of the curvature of the shock, the velocity gradients in the flow field are finite and the shock thickness cannot be neglected, therefore the viscosity is important in all flow fields. Such types of flow are usually analyzed by neglecting the viscosity between the shock and the body surface, and by determining the viscous effects at the wall with boundary layer analysis in the presence of external vorticity. Such an approach requires some discontinuity between the two flow regions independently of how the analysis is performed. The discontinuity in assumptions of the two flow fields is not justified due to the presence of curvature of the shock. The transport properties can be considered in all flow fields if the flow is analyzed with the approach described before. In a small layer near the wall, where the curvature of the streamline is small, boundary layer types of analyses are used. The flow at the outer edge is supersonic. Such a layer is much smaller than the boundary layer, therefore, boundary layer type of equations are sufficiently accurate. The matching with the external flow takes into account quantities and derivatives both for velocity and enthalpy. This can be done because transport properties also are considered in the equations that define the external flow. The external flow is analyzed by means of the method of characteristics in presence of transport effects as described previously. The conditions at the shock take into account the effect of shock thickness as discussed in the doctoral thesis of R. R. Chow[3], (this work has been performed by Dr. E. Krause at NYU as his doctoral thesis). The basic equations for the three regions are indicated in Figs. 5-7. Such type of analysis can be applied in the same way to the analysis of boundary layer with large pressure gradients, when the transport properties are known.

The flow properties near the wall are determined with boundary layer equations as shown in Fig. 6, while the outer flow which is supersonic is analyzed with the equations of Fig. 7.

3. TURBULENT MIXING

In addition to the difficulties of an analytic nature, additional difficulties are present in the case of turbulent mixing, due to lack of knowledge of the physical laws that regulate the transport properties in turbulent flows. The transport properties for turbulent flow are affected by many different characteristics of the flow; at present, even the most important parameters which affect such transport properties are not completely determined. Because of the complexity of the phenomenon, it is traditional to attempt to relate these transport properties to a few of the global flow properties and to determine their values experimentally as a function of such parameters. However, the approximate dependence of such properties on gross values of the flow parameter is not well understood. The reasons are several. Such dependence is obtained on the basis of experiments. The experimental work available is limited, and many of the experiments are not satisfactorily defined for such a case. In many of the experiments even the statement that the mixing is turbulent must be carefully scrutinized. In boundary layer investigations the turbulent boundary layer has a region of laminar motion near the wall while at some distance from the wall random turbulent fluctuations exist and farther on they disappear. The persistence of turbulent motion is related to a Reynolds number for which a characteristic velocity, density, and length that defines the experiment must be defined.

In a mixing of two coaxial jets, two characteristic velocities exist, two different densities must be considered, and a characteristic length is difficult to define. For some flow conditions the random motion can persist in one of the two streams while the motion in the other stream can be essentially of laminar type. It is therefore probable that the transition for a purely laminar to a completely turbulent mixing is very gradual, and the transitional conditions could cover a wide range of combination of Reynolds numbers of the two streams.

In this case the determination of presence of turbulence, for example, by means of a hot wire anemometer, in the high velocity stream, does not necessarily define a purely turbulent mixing, but could indicate a transitional region. For example, it is probable that the type of phenomena obtained when both streams have large velocity and density, and therefore high Reynolds numbers, are different from the profiles when the streams have a low density and the same velocity as in the first case, even if one stream is already turbulent while the other is initially laminar. The problem is more difficult when one of the streams has very low velocity and therefore very low Reynolds number.

A second problem that makes it difficult to determine the dependence of the transport properties on gross flow properties is related to the difficulty of accurately defining the initial conditions and boundary conditions. Usually the properties of the two streams are defined in terms of average properties at the initial sections. Such properties are: chemical species of the flows, enthalpies, and velocities. Often, average values of such quantities are assumed to be representative, which corresponds to step function distributions for the three quantities. In addition, the static pressure external to the mixing is assumed to be practically constant. However, the actual conditions are much more complex, especially for subsonic mixing. The mixing, especially when the velocites of the two jets are very different, tends to produce large longitudinal and radial pressure gradients and therefore large velocity gradients are present in the region near the origin of mixing. Such effects travel upstream and induce a flow field far from uniform, therefore the flow and properties at the beginning of mixing are far from the average values assumed, therefore the gradients, which are related to the transport phenomena, in this region cannot be defined only by simple average parameters. In addition, the presence of boundary layers modify substantially the average distribution both of velocity and enthalpy. The heat transfer across the wall that divides the coaxial jets can have serious effects on the flow in the near region of mixing in experiments where the enthalpies of the two streams are different. These effects are reduced when the external stream is supersonic and the axial stream has a cylindrical entrance region. In this case the flow at the beginning of the mixing can be defined fairly accurately, independently of the mixing effects. In subsonic flow, an additional problem must be considered which is related to the interference of the boundary of the external jet as indicated by Dr. Abramovich in his paper[4].

In subsonic flow the turbulence generated at the boundary of the external jet can propagate and affect the mixing region. Again, such an effect can be eliminated in supersonic flow if the mixing takes place upstream of the Mach cone from the boundary. (This was the case in the experiments performed under my supervision, contrary to what has been stated in Ref. 4.) The presence of induced flow in subsonic streams is strongly dependent on the boundary conditions, and therefore, for subsonic flow it is more difficult to obtain information on the basis of simple parameters. For example, when the velocity of the external stream is much smaller than the velocity of the other jet, it seems arbitrary to try to define the properties of mixing on the basis of quantities related to two average velocities, while the velocities in the immediate region outside of the mixing are quite different than such values.

The quantity that is less affected by the flow field distribution and by the boundary conditions is the concentration of species which is given at the beginning of mixing always as a step function, therefore in heterogeneous mixing this quantity is more uniformly representative of the initial conditions.

4. PRESSURE GRADIENT EFFECTS

From an experimental point of view, axially symmetric jets are usually preferred to "two-dimensional" type of jets because of the difficulty of producing really two-dimensional jets with finite side boundaries. However in axially symmetric supersonic jets problems of other nature are present that often are neglected. Adverse pressure gradients in a supersonic stream are transmitted by waves. In axially symmetric flow, waves become stronger at the axis of the jet. Such focusing of waves produces much larger pressure gradients at the axis than at some distance of the axis; such radial pressure variation strongly affects the velocity gradients because it changes the radial velocity profile, and therefore affects the mixing. Small gradients usually are always present in wind tunnels, or in wakes, even at some distance of the body especially if the body travels in supersonic stream. Such small gradients can strongly affect the mixing process and cannot be neglected as has been usually done. In order to indicate the importance of such affects an experiment has been performed, at my request, by Dr. Zakkay at New York University Laboratory. A wake has been produced experimentally at $M = 3.89$, at high Reynolds numbers. The Mach number along the axis in a region of the wake has been determined for the case of external pressure practically uniform at the wall of the tunnel. In a second test, a strip of the thickness of the order of 1/20 mm (scotch tape) has been glued at the wall of the tunnel. The measurements have been repeated. The difference in results between the two experiments is shown in Fig. 8. The pressure disturbance produced by the tape at some distance from the axis is small and of the order of precision of experiments. However, the effect at the axis is significant. The effects of pressure gradients can be very large in mixing phenomena, and can modify substantially the flow field properties of the flow.

5. MIXING WITH ADVERSE PRESSURE GRADIENTS

The presence of longitudinal pressure gradients is of extreme interest because it can produce flow fields which are quite different from those corresponding to zero pressure gradients. Consider the mixing of two heterogeneous coaxial streams. Assume that an adverse pressure gradient is imposed in the outside flow and is extended to a length such that the total pressure rise can produce reverse flow in the stream at lower Mach number if mixing is neglected. The mixing can accelerate the lower Mach number stream, and therefore, the ability of such a stream to traverse the pressure gradient depends on the amount of shear force produced by the mixing process. In Fig. 9 the result of such analysis is shown for two assumed values of eddy viscosity, one having twice the value of the other.

For the larger value of the eddy viscosity the flow goes through the pressure gradient while for the smaller value reverse flow occurs.

Figure 10 gives the Mach number profiles at several stations for the two cases.

Such an investigation can be used for the experimental determination of average values of turbulent eddy viscosity.

The presence of adverse pressure gradients in the presence of mixing is the determining factor for the definition of the flow field in the region of the base flow behind a body. The flow at the base of a body immersed in an external stream is the limiting case of coaxial mixing, and corresponds to the case in which the momentum of the central jet is zero. If the impulse of the central jet increases from zero to a finite value the flow field changes gradually from the flow corresponding to a base flow in the mixing region to the flow corresponding to the mixing of two parallel streams, both having large velocity.

The flow corresponding to this transition is very complex. An analysis of such flow indicates the existence of flow fields that are not well known at present. A qualitative description obtained from an experimental investigation performed at NYU by Dr. Zakkay is presented in Fig. 12 and following (Ref. 5). The experimental apparatus is shown in Fig. 11. The external flow is supersonic at $M = 3.89$. The central body is a streamline of the nozzle flow for uniform flow at the beginning of the test section. The external gas is air, while different constituents have been used in the central flow. In Fig. 12, four steps of the transition from base flow to a mixing type of flow are represented. In the upper figure, no flow is injected at the bases. The flow recirculates in the cavity. In Fig. 12b a small amount of flow is injected uniformly at the base through a porous surface placed at the end of the jet. Because of the radial and longitudinal pressure gradients a vortical region which has a ring shape is produced near the axis and the rear stagnation point moves from the center of the base somewhat downstream of the base. If the mass flow injected increases, the recirculating region gradually moves downstream until the stagnation pressure of the injected gas is sufficiently high to overcome the adverse pressure gradient produced by the external flow. Then the recirculating region moves away and disappears. The pressure at the base of the central body increases with the mass flow injected until a point where the recirculating region moves downstream and disappears and then remains roughly constant. In Fig. 13 the pressure is given as a function of the non-dimensionalized coefficient for different gases.

The effect of pressure gradients becomes more critical when chemical reactions take place, because in this case the release of energy due to chemical reaction increases the speed of sound, therefore decreases the Mach number making easier the production of reverse flow. It is interesting to observe that while in the case of mixing without reaction the initial point for reverse flow is the center of the jet, as shown in Fig. 14., in the case of chemical reaction the initial region of reverse flow can be in the combustion region that is at some distance from the axis. Therefore, the effect of pressure gradient for mixing with chemical reaction can be different than for frozen flows. Consider, for example, the mixing problem shown in Fig. 14. A jet of hydrogen at M = 1.8 is injected into a supersonic stream of air. The pressure and temperature of the air are sufficiently high so that chemical reaction takes place. The speed at the center of the hydrogen jet decreases because of the pressure gradient, however, the pressure gradient is small and therefore the stream remains supersonic. In the region of combustion the temperature increases and the Mach number decreases, here the flow becomes locally subsonic and therefore more sensitive to pressure gradients. The pressure gradient continues, here the flow reaches locally zero velocity and reverse flow is produced at some distance from the axis. Fig. 15 gives some velocity profiles before reverse flow occurs. When reverse flow is generated, two opposite vortical rings are produced (shown schematically in Fig. 16), which are very unstable. Experimental evidence of such flow is shown in Fig. 18. The flame spreads very rapidly radially as soon as reverse flow is produced by adverse pressure gradients.

Such type of phenomena is of great interest in many practical problems, for example, in boundary layer in presence of ablation when the ablative material reacts with the air. The flow for such phenomena is difficult to describe and analyze because the adverse pressure gradients can change strongly even the qualitative characteristics of the flows.

6. CONCLUSIONS

From these considerations it appears evident that the problem of mixing of heterogeneous gases is far from being understood quantitatively. The analytical approaches available are, in many cases, not satisfactory; the experimental data are insufficient, and often not accurately presented. Many new phenomena, some characteristic of the heterogeneity of the two streams are present because of the production of adverse pressure gradients, which even if small, often can change strongly the flow. Therefore, it can be concluded that additional work in this relatively new field is required.

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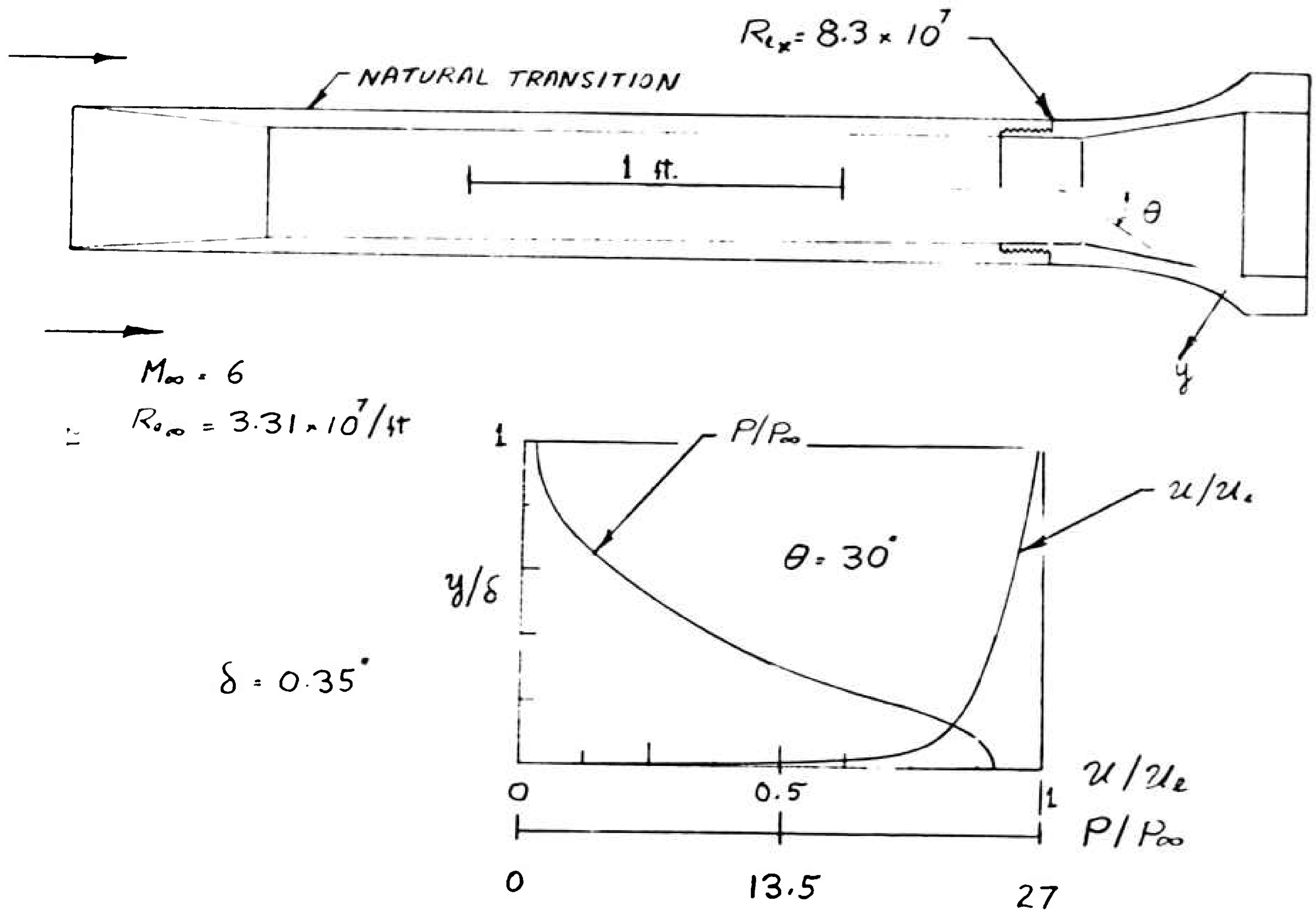


FIGURE 1: EXPERIMENTAL MEASUREMENTS OF PRESSURE GRADIENTS ACROSS THE BOUNDARY LAYER

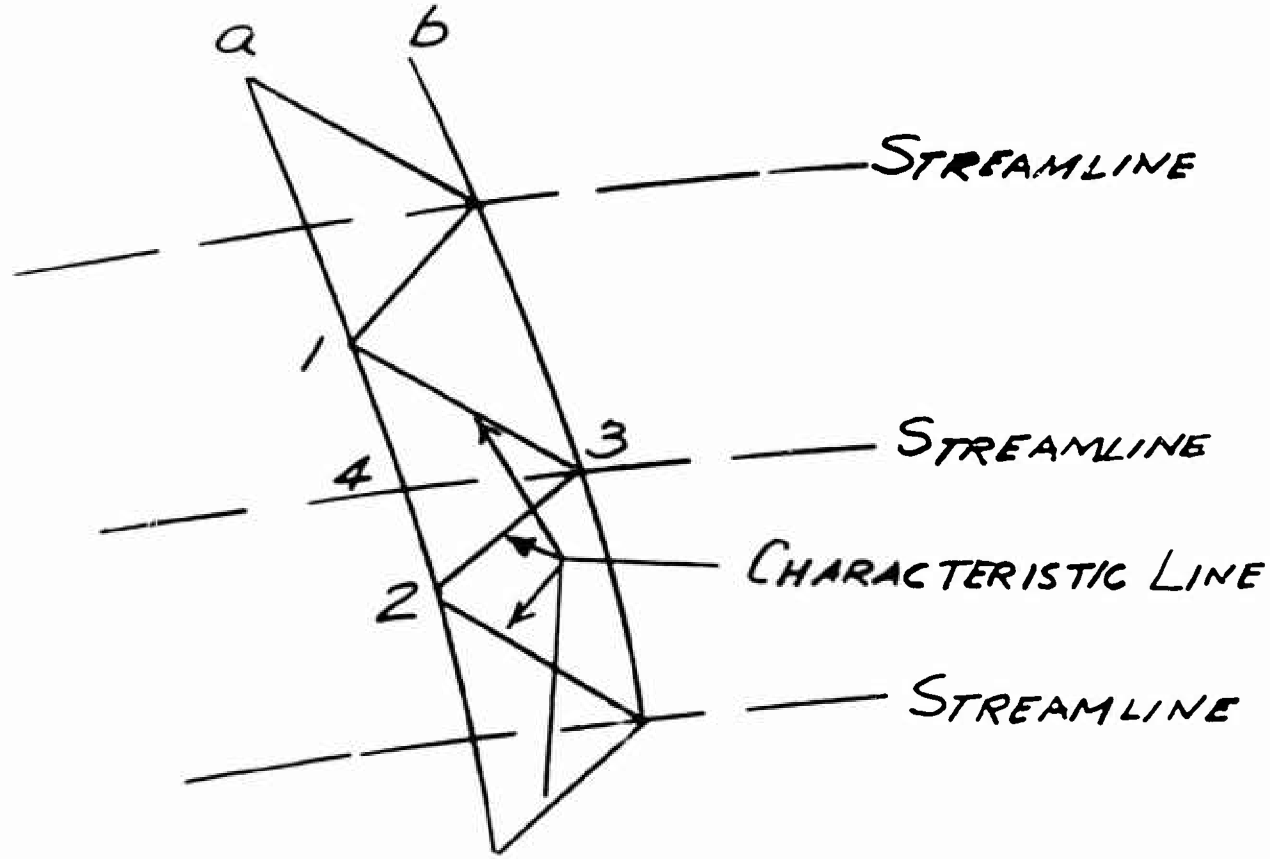


FIGURE 2: CALCULATION NET FOR ANALYSIS OF VISCOUS FLOW WITH PRESSURE GRADIENTS

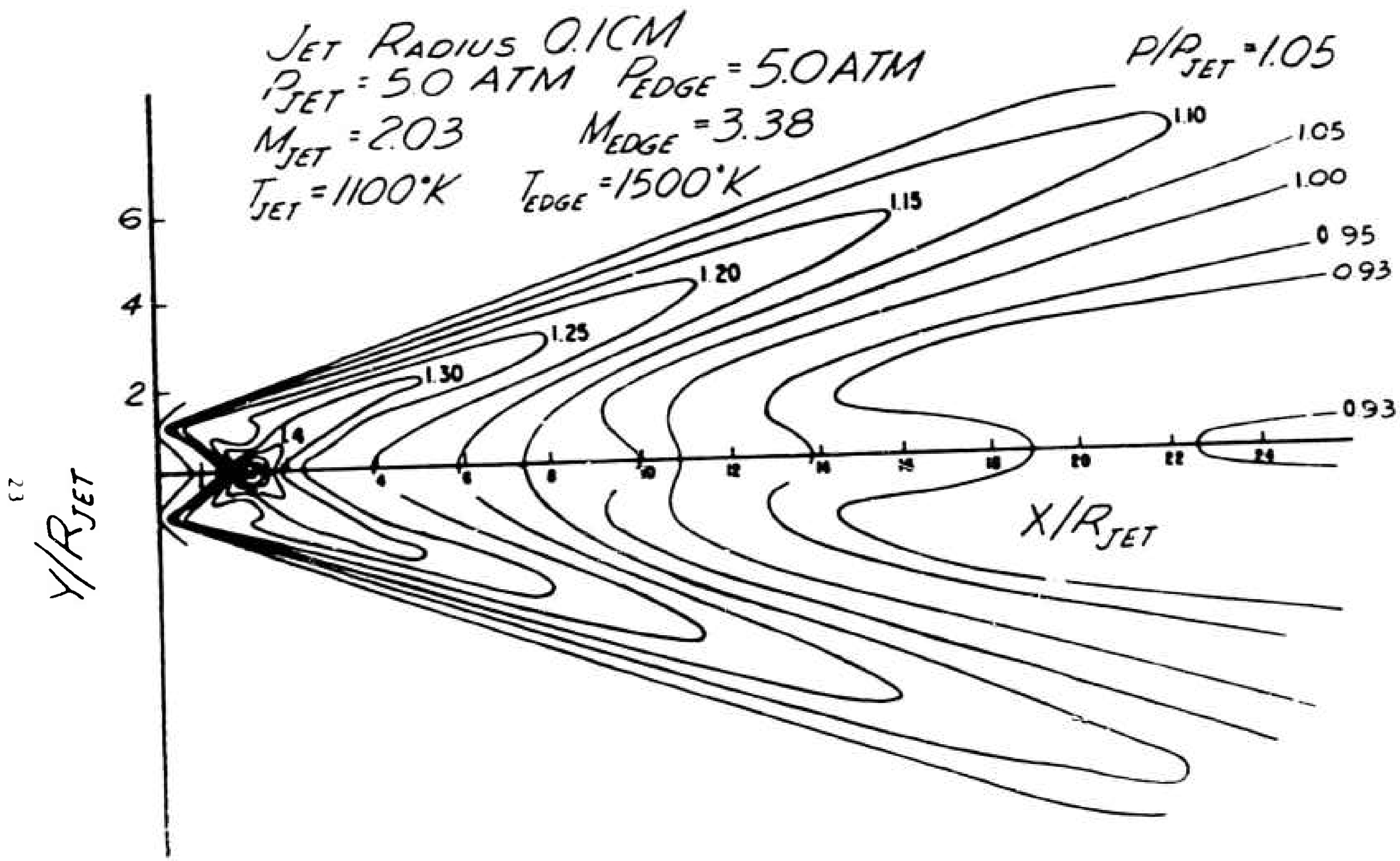


FIGURE 3: PRESSURE FIELD FOR A FREE JET OF HYDROGEN IN AIR COMPUTED BY A METHOD OF CHARACTERISTICS WITH VISCOSITY AND FINITE RATE CHEMISTRY

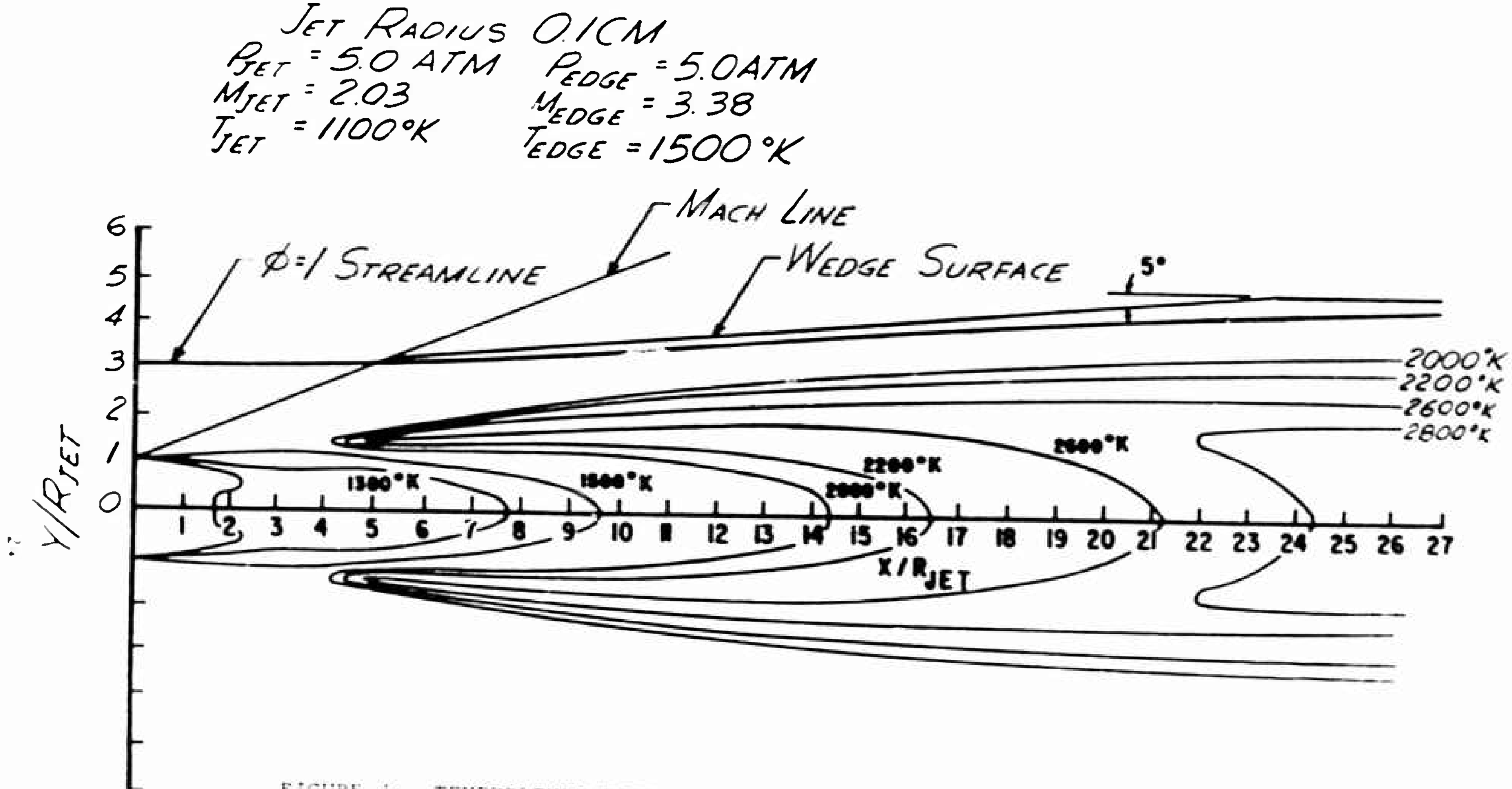
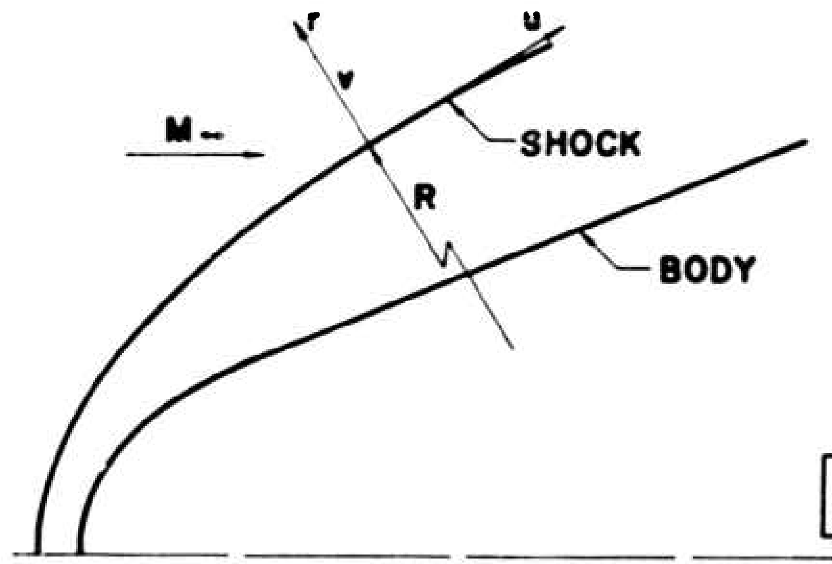


FIGURE 4: TEMPERATURE FIELD FOR A FREE JET OF HYDROGEN IN AIR COMPUTED BY A METHOD OF CHARACTERISTICS WITH VISCOSITY AND FINITE RATE CHEMISTRY

SHOCK EQUATIONS



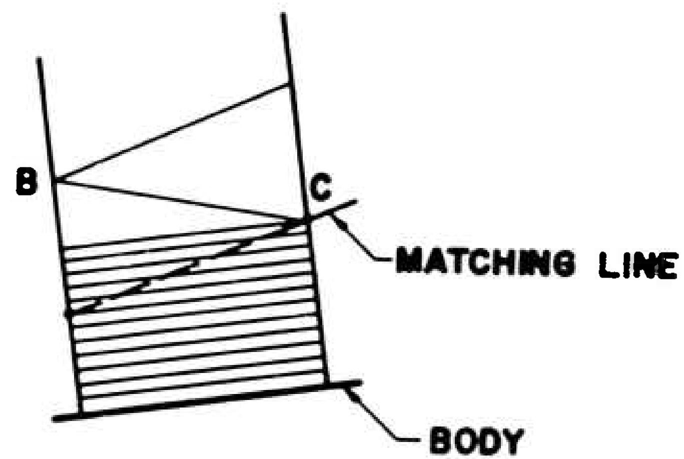
$$\left[\rho v\right]_R = \rho_\infty v_\infty$$

$$\left[p - \rho_\infty v_\infty v - \left(\mu' + \frac{4}{3}\mu\right)\frac{\partial v}{\partial r}\right]_R = p_\infty - \rho_\infty v_\infty^2$$

$$\left[\rho_\infty v_\infty u - \mu \frac{\partial u}{\partial r}\right]_R = \rho_\infty v_\infty u_\infty$$

$$\left[\rho_\infty v_\infty h_s - k\frac{\partial T}{\partial r} - \left(\mu' + \frac{4}{3}\mu\right) v \frac{\partial v}{\partial r} - \mu u \frac{\partial u}{\partial r}\right]_R = \rho_\infty v_\infty h_{s_\infty}$$

FIGURE 5: SHOCK EQUATIONS



MATCHING

DETERMINE LOCATION OF THE POINT C BY TESTING

TEST:

$$\left| \frac{\partial q}{\partial n_{CH}} - \frac{\partial q}{\partial n_{BL}} \right| < \varepsilon$$

$$\left| \frac{\partial T}{\partial n_{CH}} - \frac{\partial T}{\partial n_{BL}} \right| < \varepsilon$$

CALCULATION OF "EDGE CONDITIONS" AT POINT C ASSUME θ_{c_1}

COMPUTE q_c, p_c, T_c FROM B-CHARACTERISTIC

RECOMPUTE θ_{c_2} FROM BOUNDARY LAYER

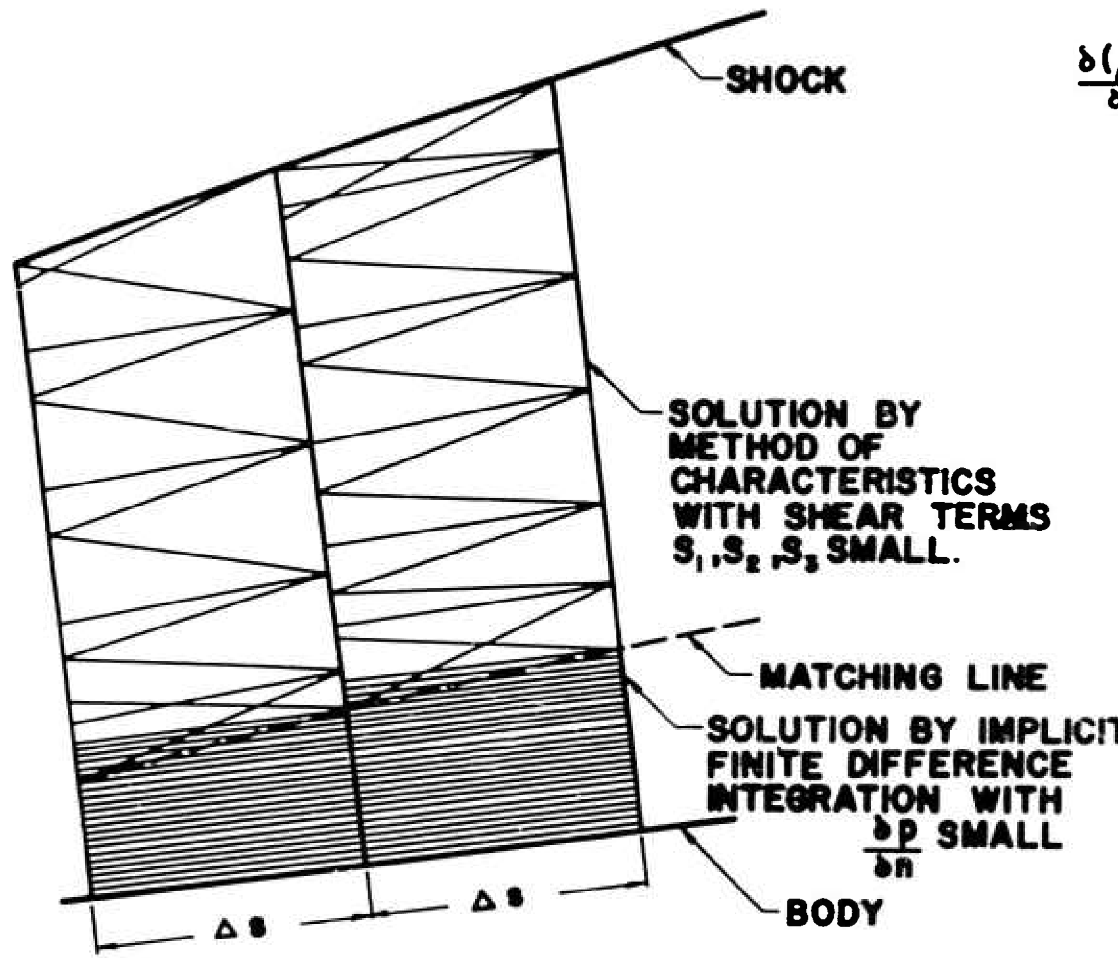
TEST:

$$\left| \theta_{c_1} - \theta_{c_2} \right| < \varepsilon$$

FIGURE 6: MATCHING ANALYSIS BETWEEN BOUNDARY LAYER AND FLOW WITH PRESSURE GRADIENT

FIGURE 7:

INTEGRATION OF FLOW EQUATIONS



$$\frac{\partial(\rho q)}{\partial s} + \rho q \frac{\partial \theta}{\partial n} + j \frac{\rho q}{r} \sin \theta = 0$$

$$\rho q \frac{\partial q}{\partial s} + \frac{\partial p}{\partial s} = S_1$$

$$\rho q^2 \frac{\partial \theta}{\partial s} + \frac{\partial p}{\partial n} = S_2$$

$$c_p \rho q \frac{\partial T}{\partial s} - q \frac{\partial p}{\partial s} = S_3$$

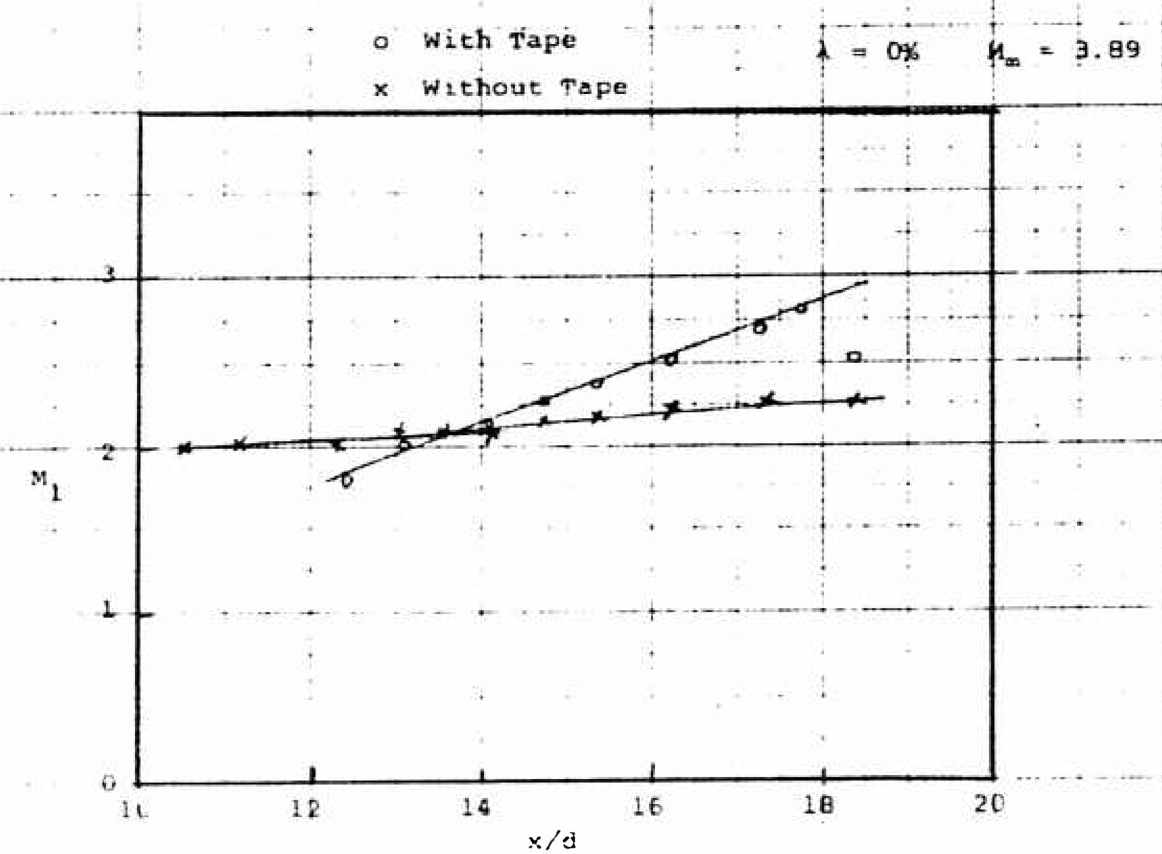
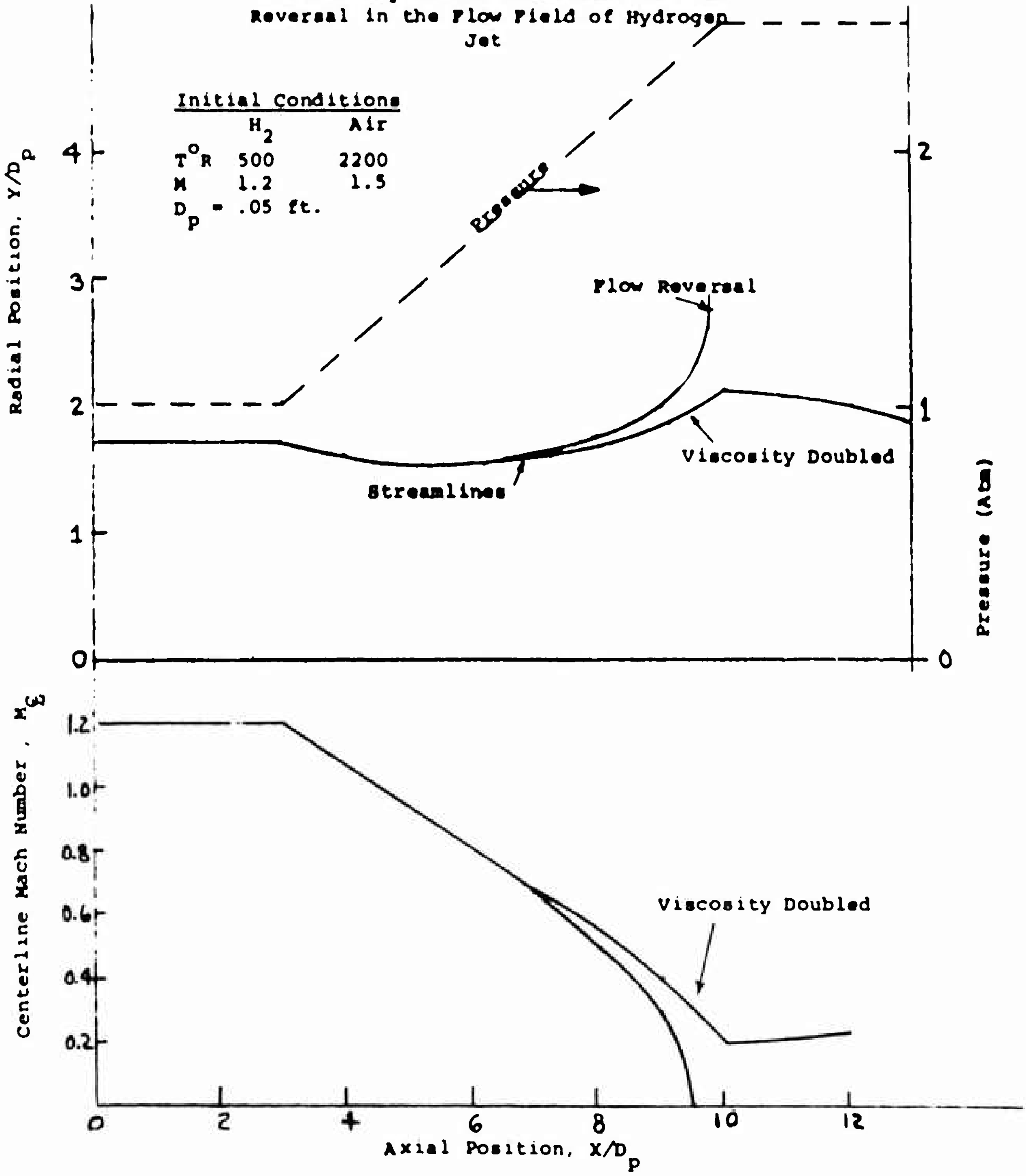


FIGURE 8: MACH NUMBER DISTRIBUTION AT THE AXIS OF WAKE WITH OR WITHOUT SMALL DISTURBANCES

FIGURE 9:

External Flow Streamline and the Centerline Mach Number Distribution Showing the Effect of Viscosity on the Occurrence of Flow Reversal in the Flow Field of Hydrogen Jet



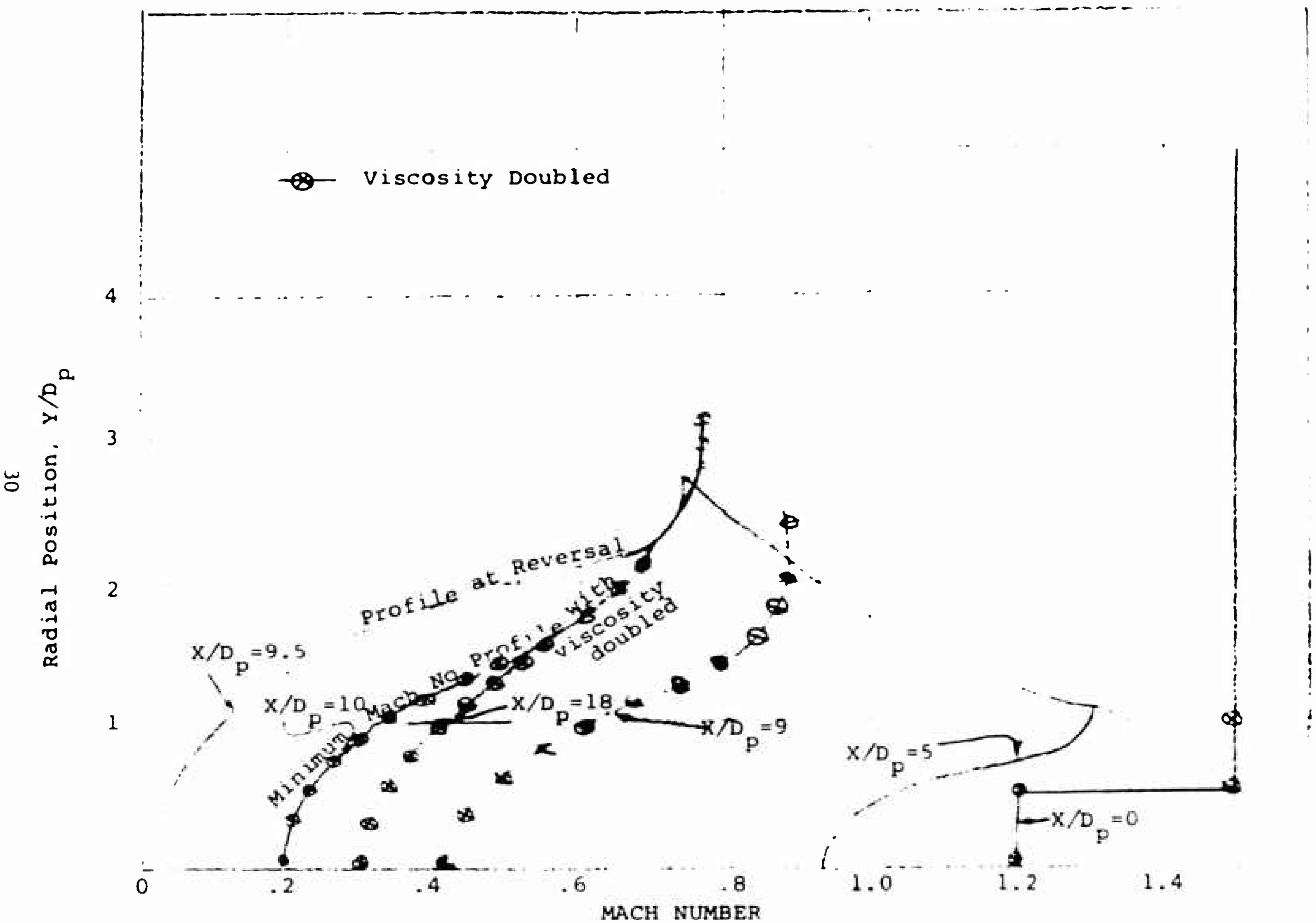


FIGURE 10: MACH NUMBER PROFILES AT SEVERAL AXIAL STATIONS SHOWING THE EFFECT OF VISCOSITY ON FLOW REVERSAL

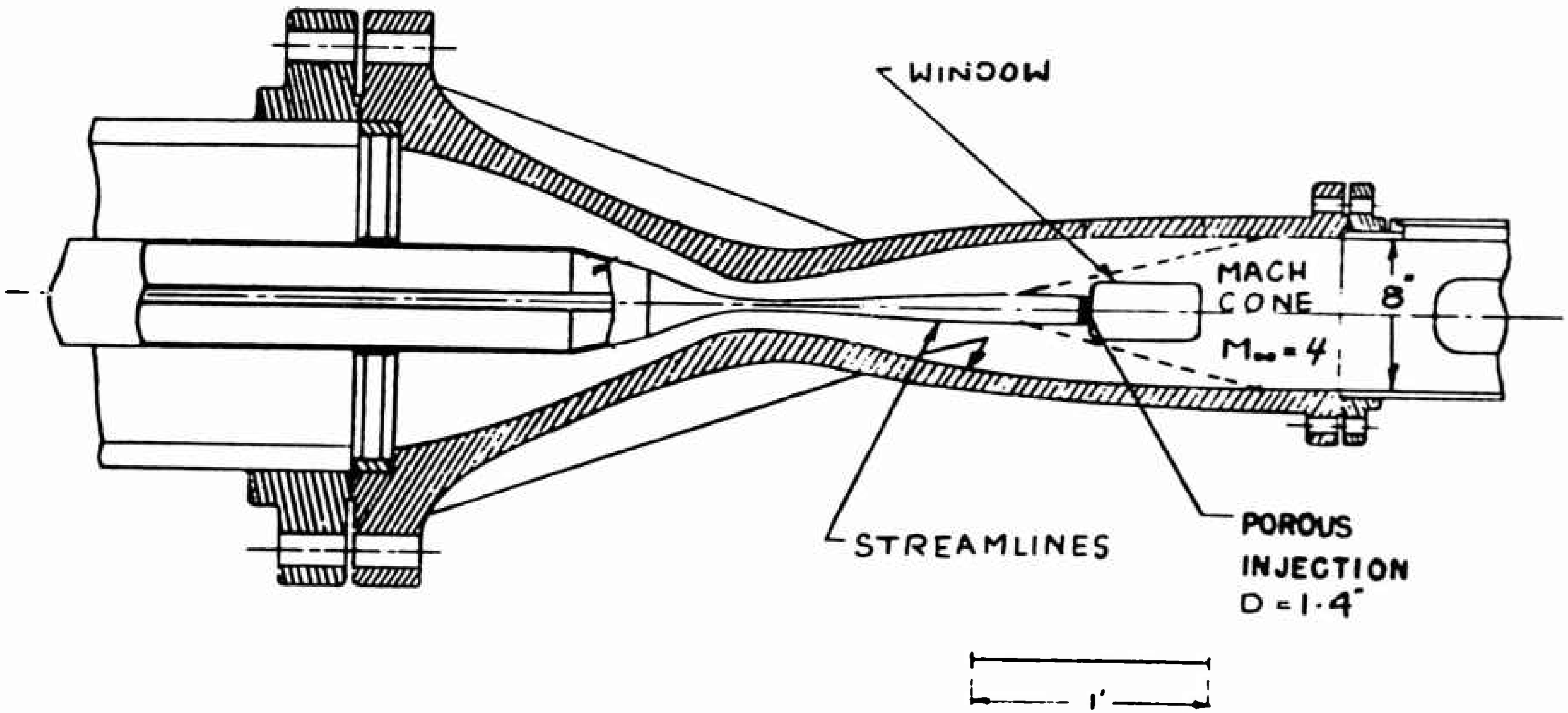


FIGURE 11. Scheme for base injection and coaxial mixing.

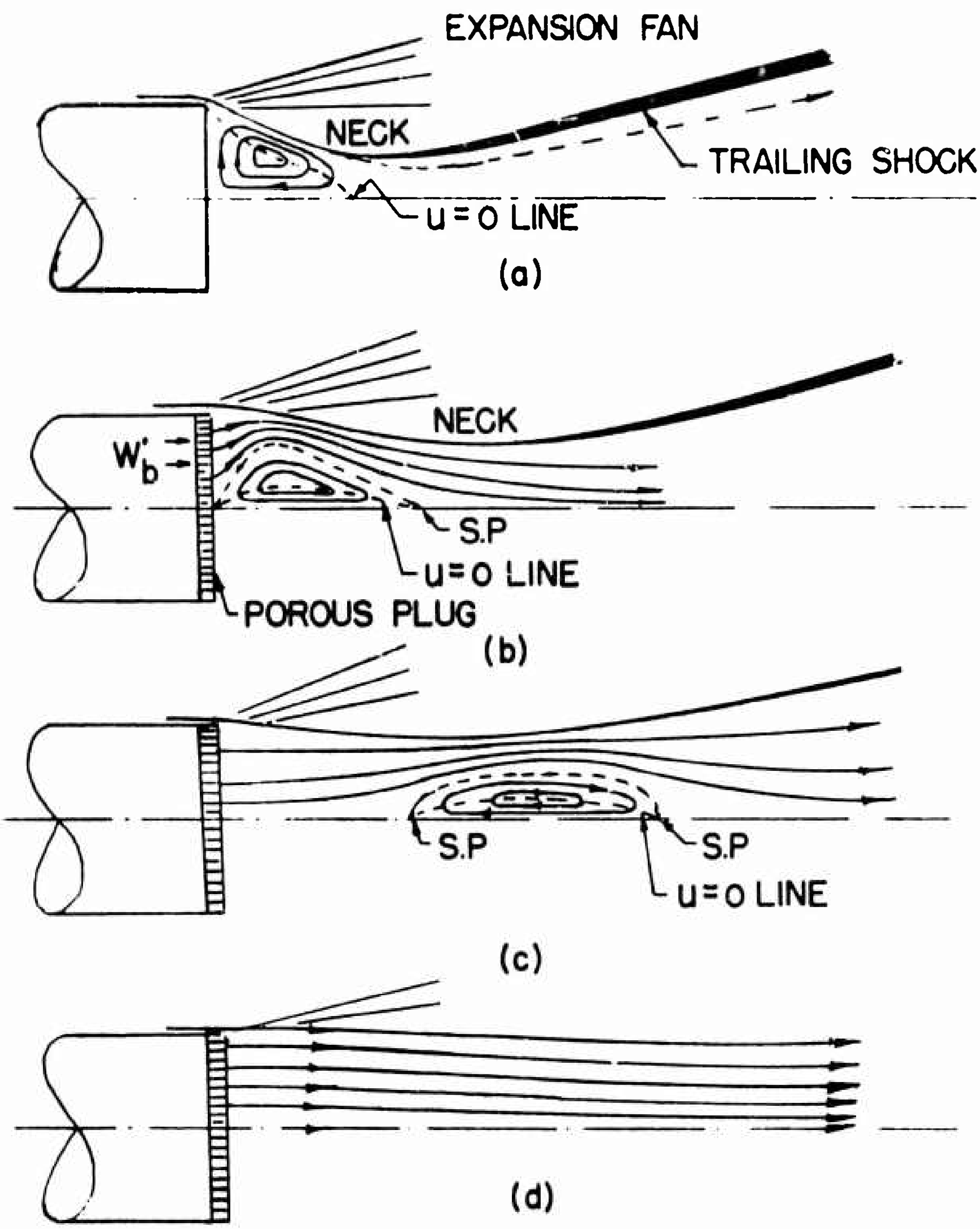
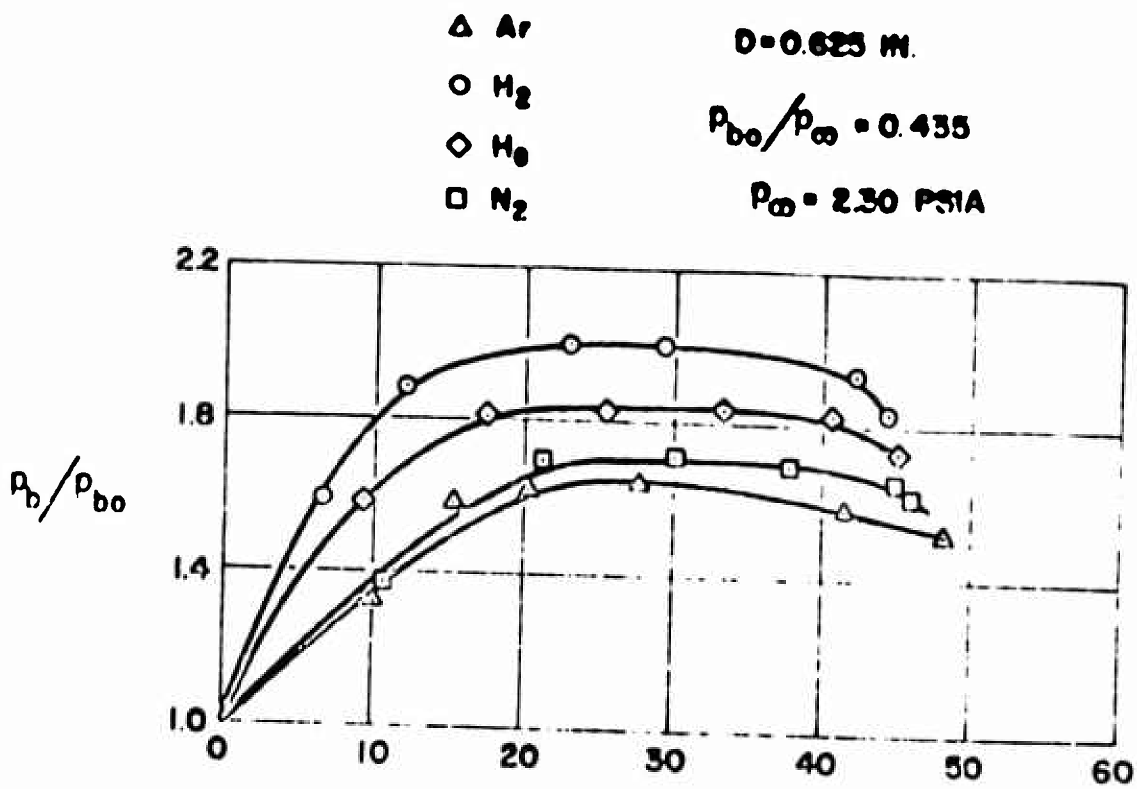


FIGURE 12 Schematic of the transitional phenomena between the base flow with zero injection to that with large injection.



$$K = \left(\frac{1+\gamma}{2}\right)^{\frac{\gamma+1}{2(\gamma+1)}} \sqrt{\frac{R}{\gamma}} \; \frac{\rho_1 u_1}{\rho_e u_e}$$

Figure 13: Normalized Base Pressure Distribution

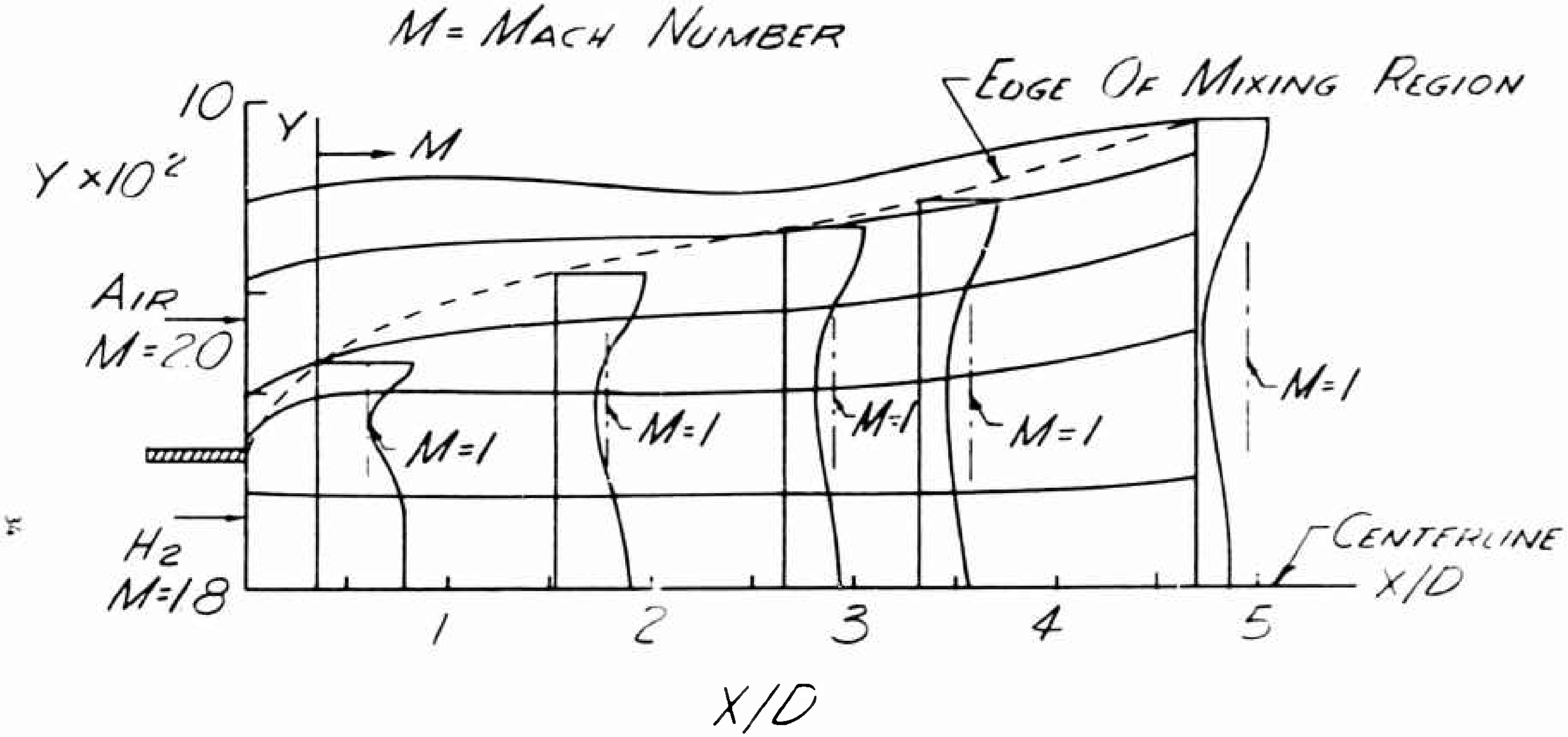


FIGURE 14: FLOW FIELD CALCULATION SHOWING DEVELOPMENT OF RECIRCULATION REGION IN AXISYMMETRIC DIFFUSION-CONTROLLED COMBUSTION WITH ADVERSE PRESSURE GRADIENTS

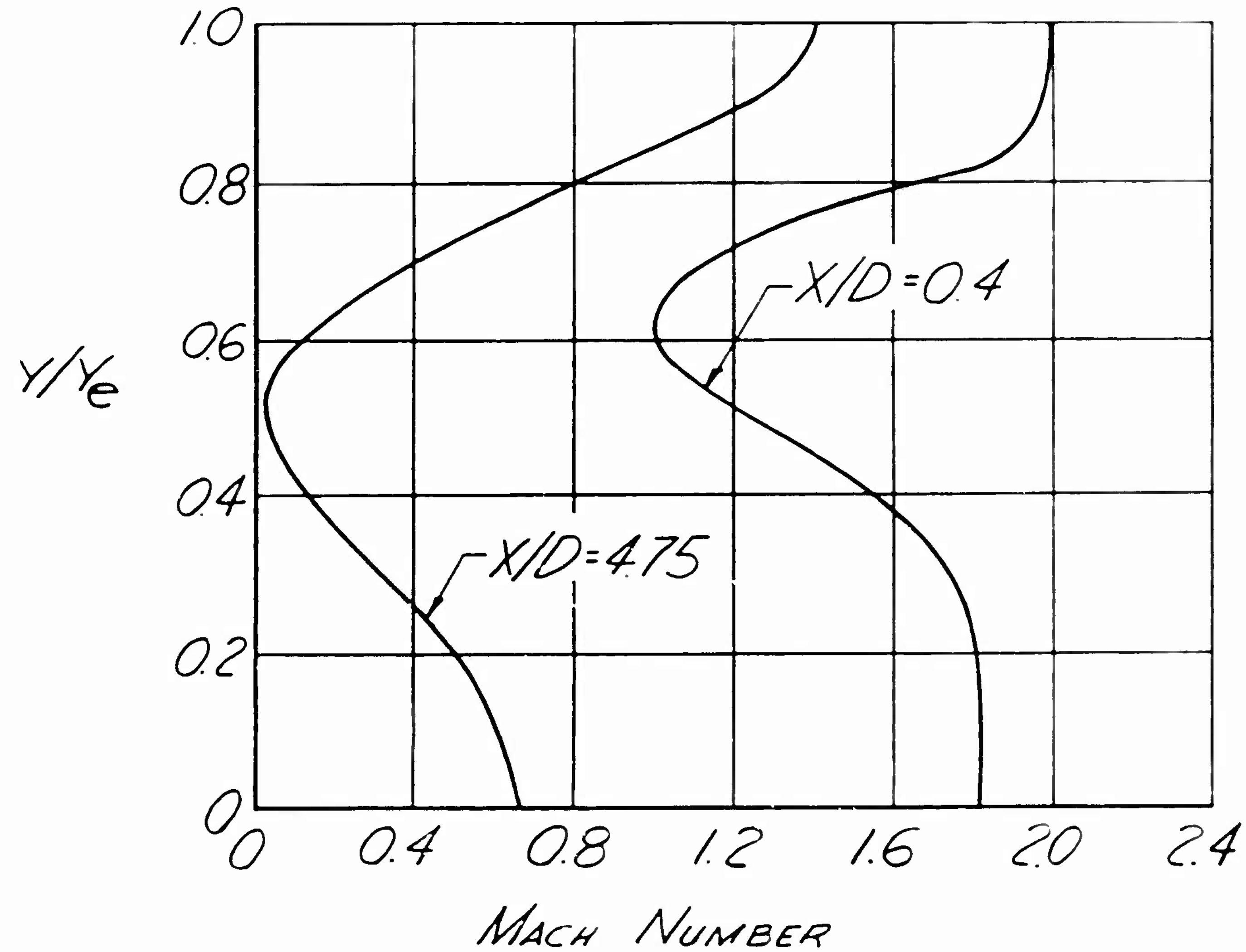


FIGURE 15: MACH NUMBER PROFILES NEAR THE INJECTION POINT AND NEAR THE STAGNATION POINT

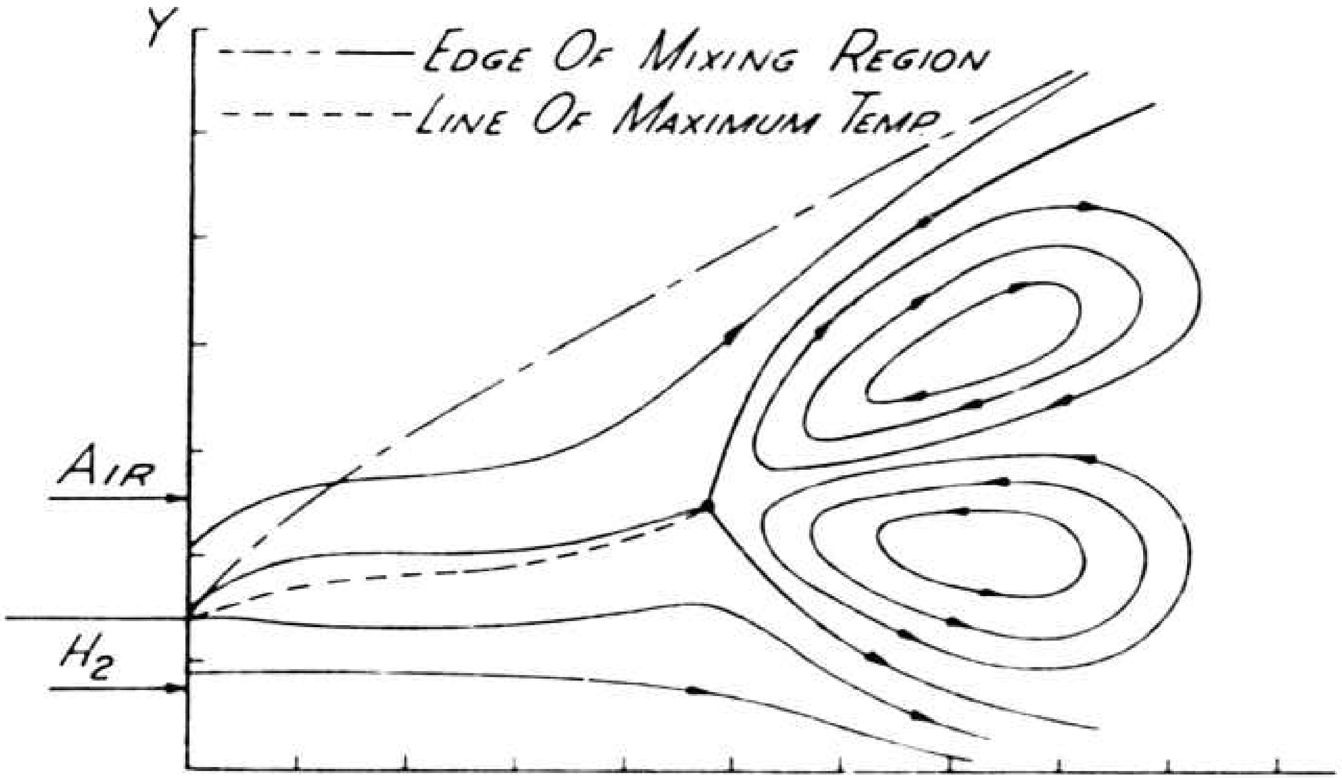


Fig. 16

Schematic Of Off-Axis Stagnation Point And Recirculation Region In An Axisymmetric Diffusion Controlled Combustion Process With Adverse Pressure Gradient

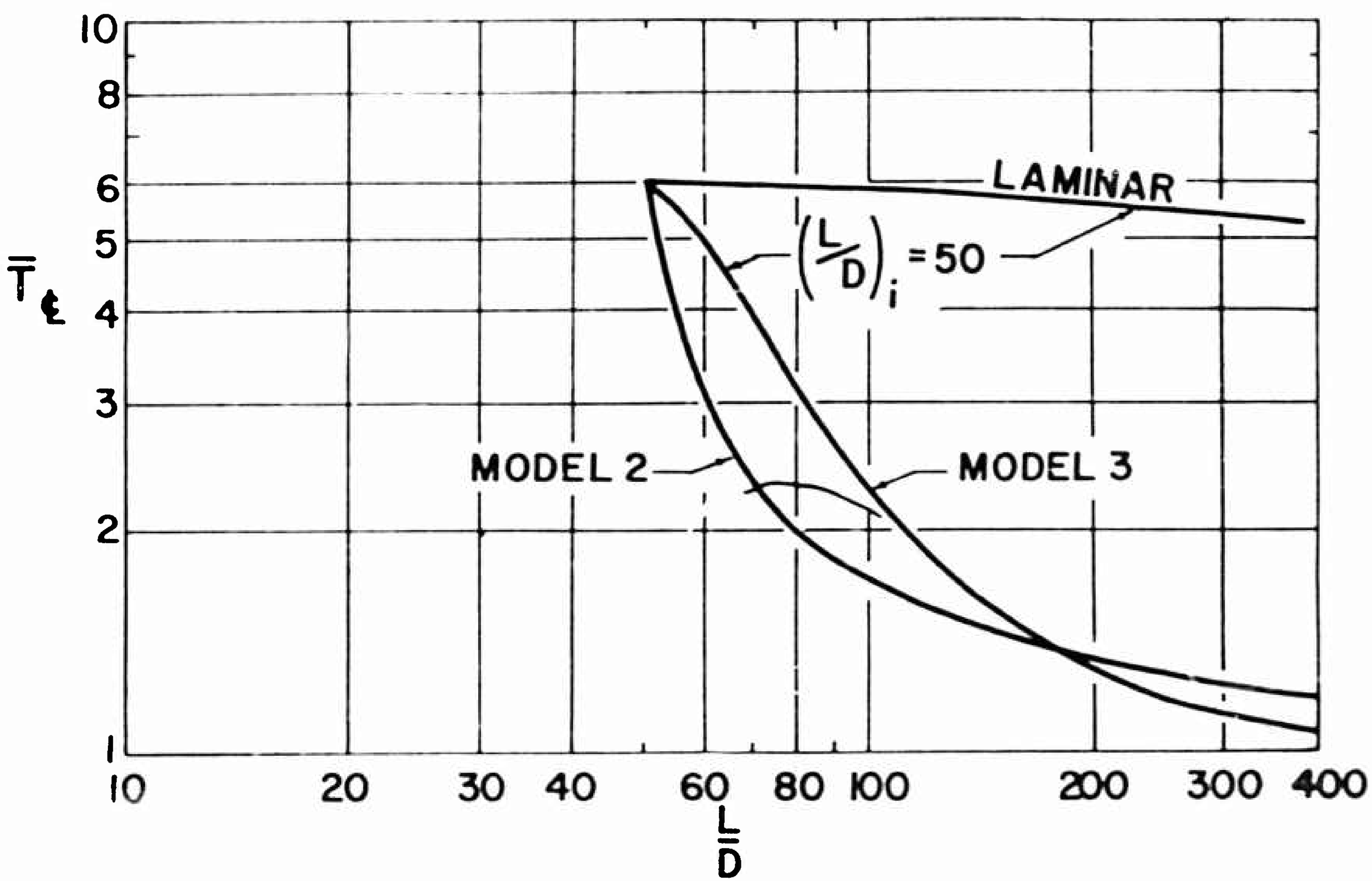


FIGURE 17. Theoretical centerline static temperature distribution

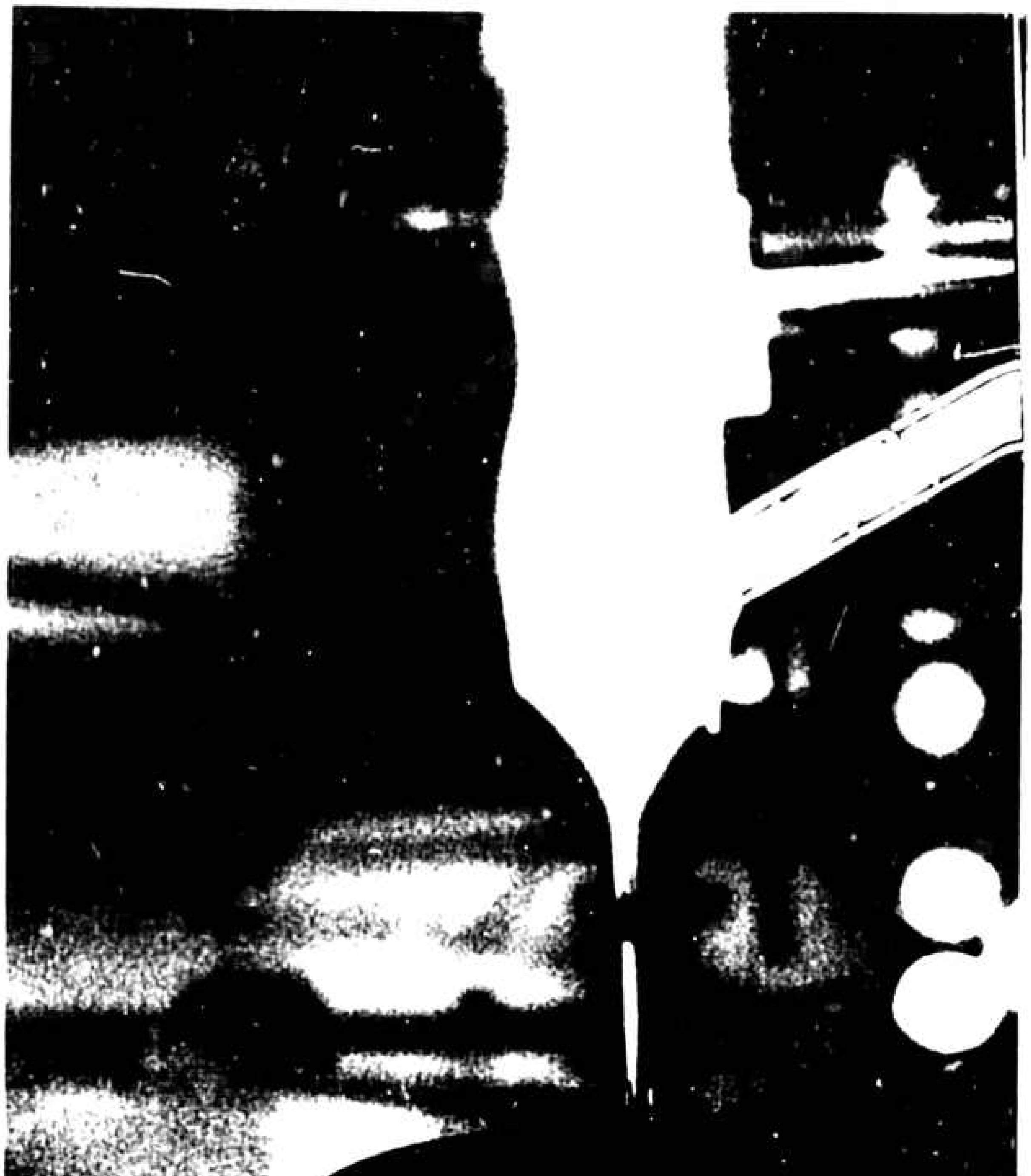


FIGURE 19: SHADOWGRAPH OF FLAME WITH REVERSE FLOW

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3. REPORT TITLE A CRITICAL REVIEW OF HETEROGENEOUS MIXING PROBLEMS	
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Scientific. Interim.	
5. AUTHOR(S) (Last name, first name, initial) ANTONIO FERRI	
6. REPORT DATE September 1967	7a. TOTAL NO. OF PAGES: 45 / 7b. NO. OF REFS: 6
8a. CONTRACT OR GRANT NO. AF33(615)2215 b. PROJECT NO. 7064 c. DoD Element 61445014 d. DoD subelement 681307	9a. ORIGINATOR'S REPORT NUMBER(S) NYU-AA-67-102 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) ARL 67-0187
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13. ABSTRACT

A critical review of mixing problems is presented. The limitations of boundary layer types of analyses are discussed. A method of analysis which does not use boundary layer approximation is outlined. Difficulties of correlating experimental data attributable to not-well defined boundary conditions are described. New phenomena related to mixing with pressure gradient and chemical reaction are presented.

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