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M.H. BANK

FATIGUE OF AIRCRAFT STRUCTURES

This Publication Modifies and Expands Many of the Concepts
of NAVWEPS 00-25-534 dated 1954, Revised 1960

NAVAL AIR SYSTEMS COMMAND
DEPARTMENT OF THE NAVY

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FATIGUE OF AIRCRAFT STRUCTURES

by

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Battelle Memorial Institute

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PREFACE

Prevention of fatigue failure in structural parts has been an important concern in aircraft engineering for many years. Technological developments continually bring out new materials, new fabrication processes, improved design concepts, and additional information about service requirements. Hence, engineering procedures for prevention of fatigue need continual review. In recognition of this, the Naval Air Systems Command of the Department of the Navy has sponsored, in addition to numerous technical reports, the preparation of several books on fatigue.

In 1941, the Bureau of Aeronautics sponsored the preparation of a book on *Prevention of the Failure of Metals Under Repeated Stress.* This book was prepared by Battelle Memorial Institute with the cooperation of many individuals interested and experienced in the field. It was published by John Wiley & Sons.

In 1955 the Bureau of Aeronautics authorized Battelle to prepare a book, bringing up to date information on the fatigue behavior of materials and including some discussion of design allowance for stress concentrations and complex loading conditions. This book, titled Fatigue of Metals and Structures, was prepared by H. J. Grover, S. A. Gordon, and L. R. Jackson with help from others on the Battelle staff and from numerous individuals in government agencies and aircraft companies. It was published by the Government Printing Office and subsequently published by Thames & Hudson (London, England) and by Longmans, Green (Toronto, Canada).

Quite recently the former Bureau of Naval Weapons sponsored the preparation of a small volume, "Tips on Fatigue" by C. R. Smith, to set forth in simple language some outstanding factors in fatigue. It was planned to help junior designers, shop men, and inspectors to recognize items critical in prevention of fatigue failure.

The present book is another contribution sponsored by the Navy. The objective differs from those of the previously mentioned books. In contrast to the 1941 and 1955 books, the present volume includes information accumulated, particularly in the past decade, on topics of recent interest, such as low-cycle fatigue, acoustic fatigue, and fatigue testing of full-scale structures. Somewhat more emphasis is placed on pertinence to structures in contrast to fatigue test data on materials in small coupons. In comparison with "Tips on Fatigue," the present book is intended to have a wider coverage of engineering details.

Design procedures to prevent fatigue failure are empirical and can, unfortunately, achieve varying results. In most practical situations there are various alternative procedures which can be evaluated only relatively. For conciseness, only some of the procedures in use by aircraft designers at the time of preparing the manuscript have been selected for discussion in this book. Hopefully, many improvements will develop. Accordingly, the data and procedures presented herein should be regarded as starting points for design consideration but not as inviolate rules. It is planned that the book provide a useful background for engineering to prevent fatigue failure. It is not intended that any portion of the book should relieve an engineer from the challenge and responsibility of always using the best information available to provide structural reliability.

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CHAPTER I. INTRODUCTION

BACKGROUND

Fatigue failure—cracking of metal under repeated stressing—was discovered in the railroad industry. This industry presented some of the first situations where extensive repetition of mechanical loading of metal parts caused failures. As sources of vibration and of dynamic loading of materials have increased, fatigue failures have become increasingly important in engineering.

Perhaps nowhere is the prevention of failure by fatigue more

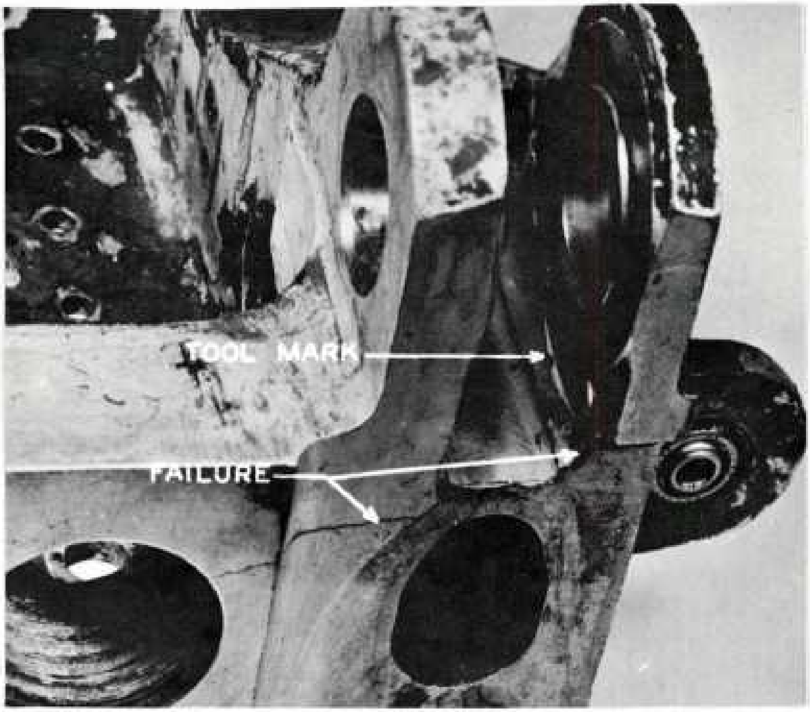


Figure 1.—Fatigue failure of wing-fold fitting.

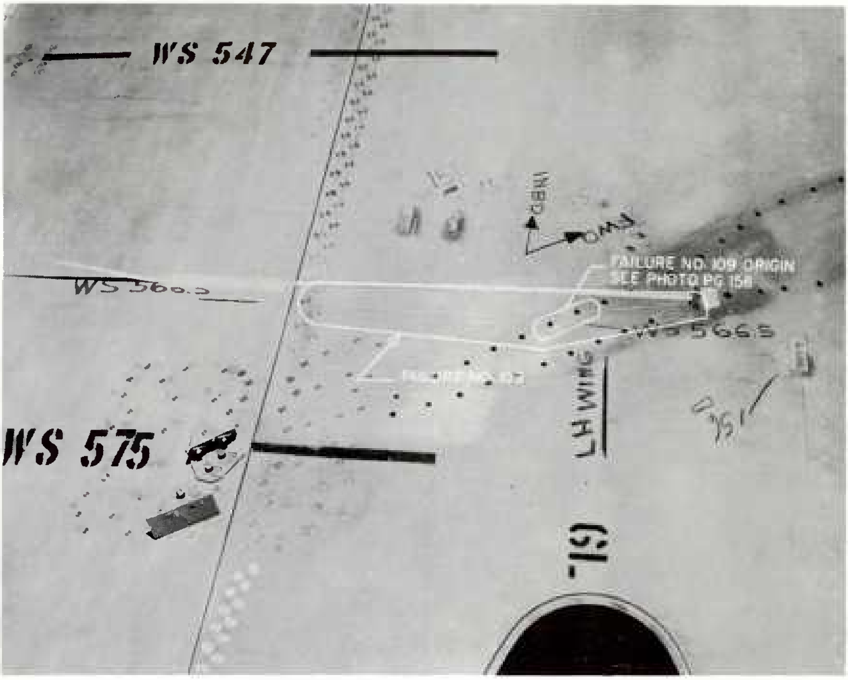


Figure 2.—Fatigue failure in wing panel—crack through holes (see fig. 3).

important than in the aerospace industry. Here, sources of dynamic stressing are plentiful but design must avoid penalties of overweight.

Past records furnish many illustrations of failures ascribed to fatigue. Figures 1 through 14 show examples illustrating a number of points:

1. Fatigue failures have occurred in many parts of aerospace structures.
2. These failures have involved different materials and diverse conditions of loading and of environment.
3. The failures usually start at some local stress raiser such as a bolt hole, a fillet, a flange, a rivet, or a tool mark.
4. Cracks tend not only to start at a stress raiser but also to propagate through others.
5. In some instances, fracture surfaces afford clear indications of fatigue crack progression; however, such evidence is not always clear.

While none of the examples in figures 1 through 14 were the direct cause of a catastrophic failure, catastrophies have occurred as a direct result of fatigue in routine operations of airplanes. Other fatigue problems have necessitated frequent replacements and extensive redesigns. These have been costly in time and in money and, for military operations, have posed very important logistic problems.

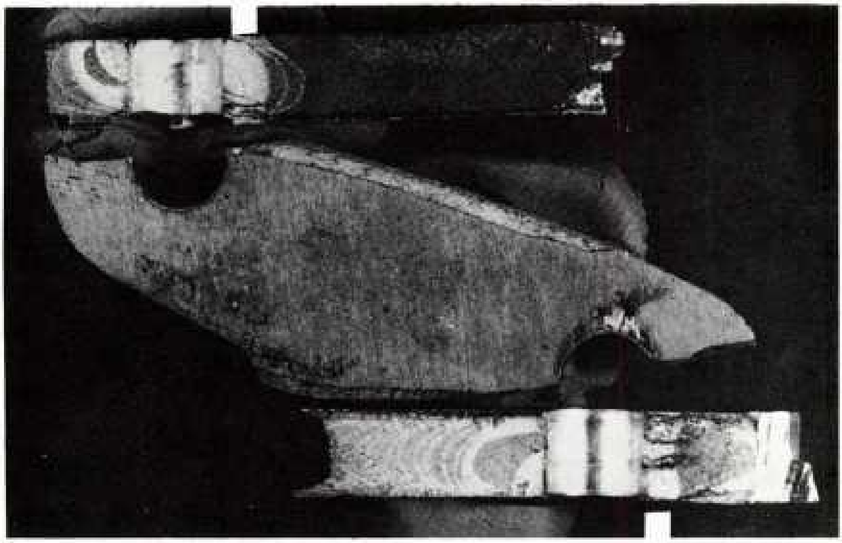


Figure 3.—Fracture surfaces—"beach" marks of section indicated in figure 2.

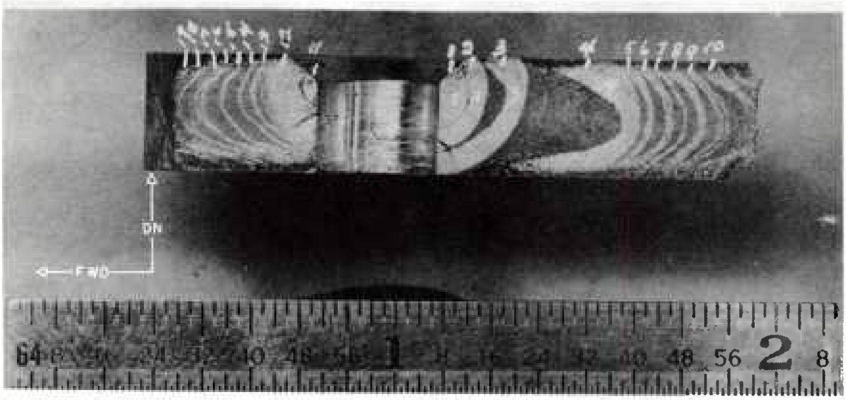
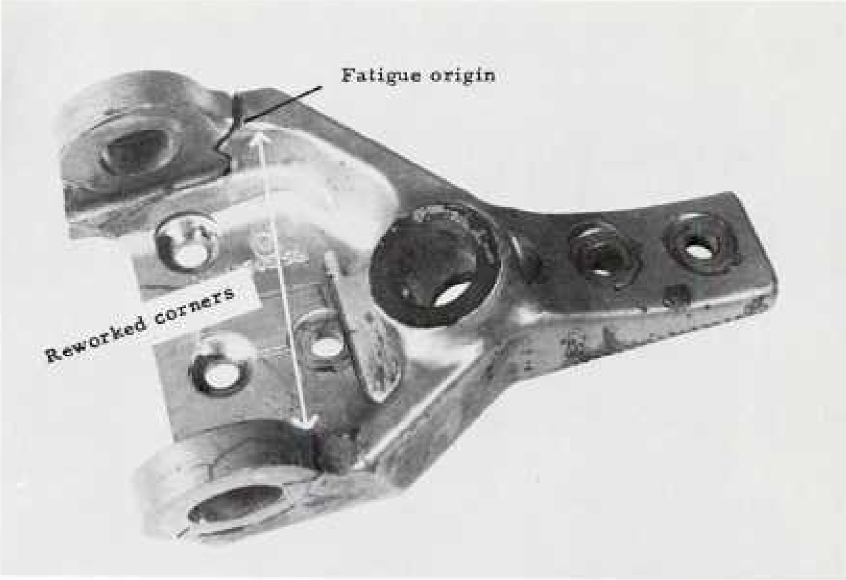
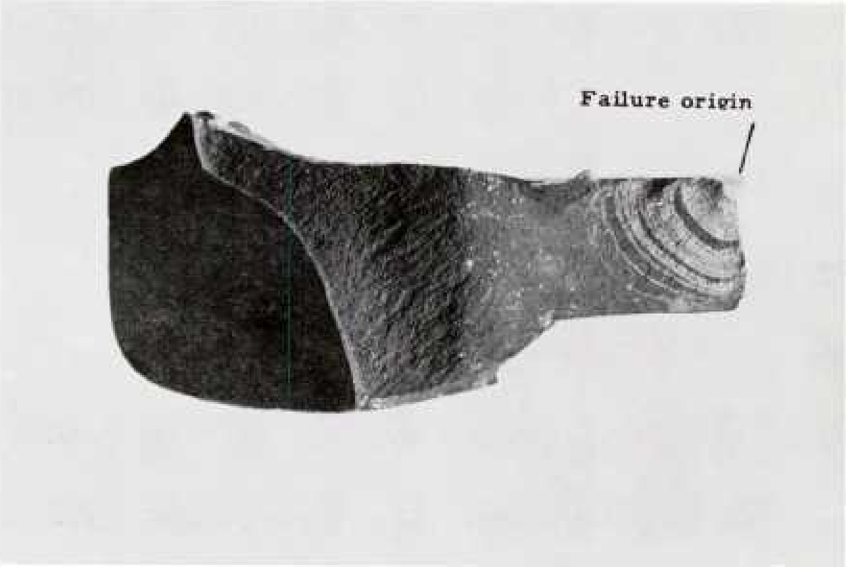


Figure 4.—Fracture surface of fatigue failure of wing panel—clear lines of progression of crack at each side of hole.

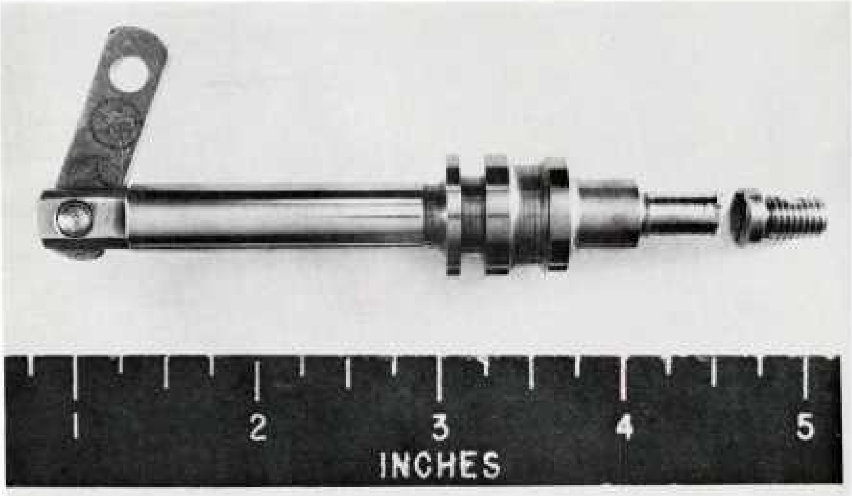


Top: Cracked bracket.

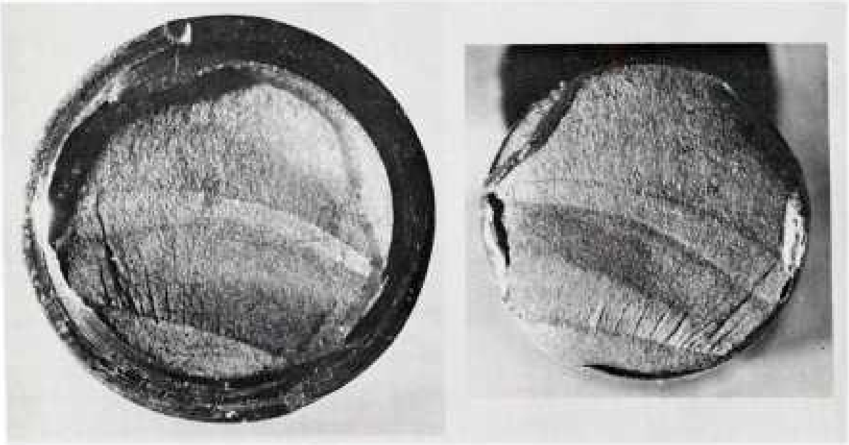


Bottom: Fatigue fracture surface—"beach" marks around origin.

FIGURE 5.—Failed door-cylinder bracket.



Top: Break at right.



Bottom: Fracture face—contour marks indicate progressive nature (however, contrast with "beach" marks in fig. 8).

FIGURE 6.—Fatigue failure in booster-valve modulator piston.

The examples shown have been limited—partly to avoid undue length in this chapter. Additional illustrations are available in the literature (see, for example, references *1* and *2*), in the files of aerospace companies, and in the records of government agencies. It cannot be recommended too strongly that an engineer concerned with the prevention of fatigue study numerous examples of fatigue failures, both from laboratory tests and from service. Whenever possible he

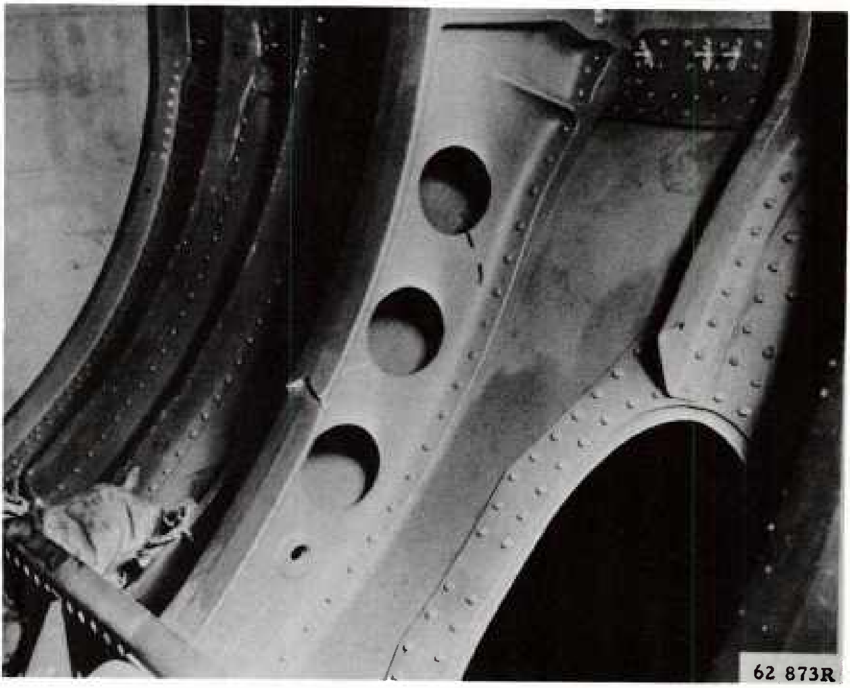


Figure 7.—Fatigue failure in a fuselage ring—crack from large hole to stiff flange.

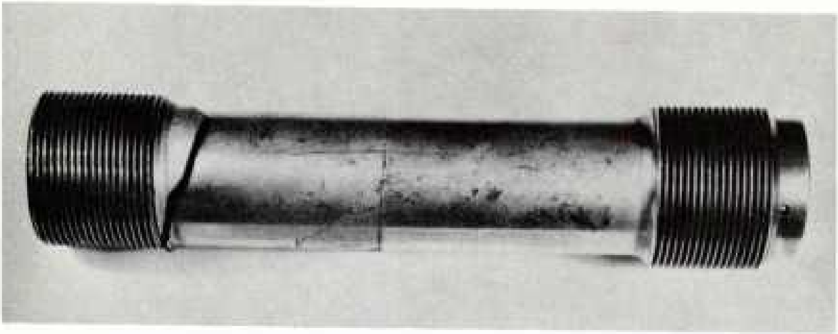
should examine the physical part involved. An appreciation for many factors in fatigue can be developed by extensive study of previous fatigue failures.

SOME TRENDS

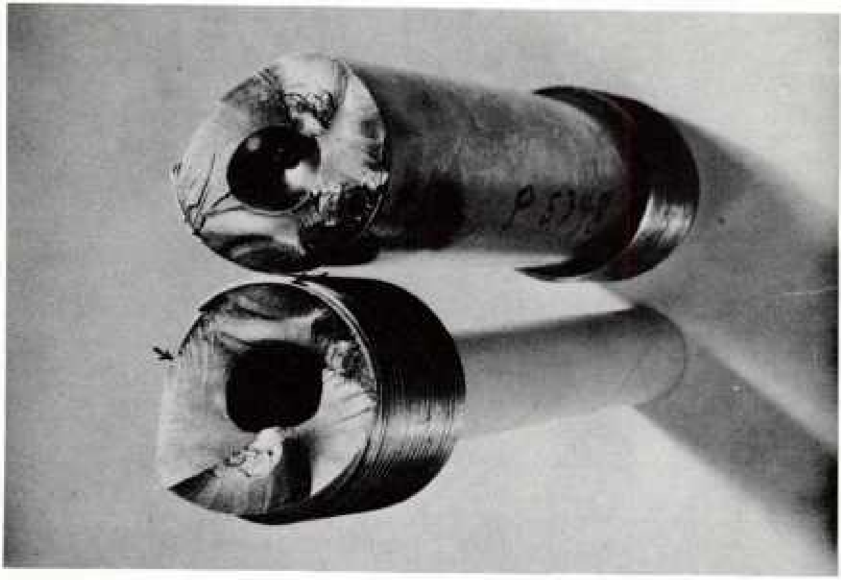
While fatigue has been a serious problem in the past, there is reason to expect that it will present increasingly serious problems in the future. Trends in design and in operations indicate new complexities are certain to arise. Some of these trends are: higher design stresses, requirements for increased performance, and demands for increased operational flexibility. Moreover, special flight vehicles, such as rotary-wing aircraft, VTOL and STOL aircraft, supersonic transports, and missiles and space vehicles, present special problems.

In the past several years, the design-limit load factor has gone down;* this trend has decreased but has not disappeared altogether. In some instances, development of new materials of higher static strength (but not proportionately higher fatigue strength) has produced an increase in design allowable static stresses. These trends, which increase the possibility of fatigue,* were brought about by the continual need for

* THUS MORE OF FLIGHT TIME IS CLOSER TO LIMIT LOAD — IE STRAIGHT & LEVEL, A 2G A/C IS AT 0.5 LIMIT LOAD, WHERE A 6.4G A/C IS AT 0.156 LIMIT... and w/ S-n diagram...!



Top: Origin of crack.

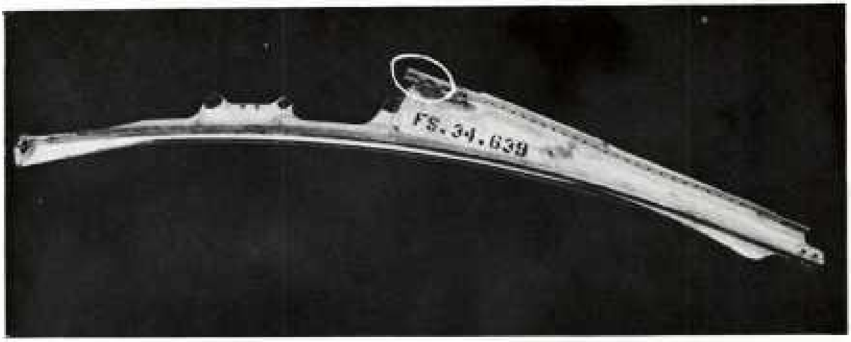


Bottom: Fracture surface.

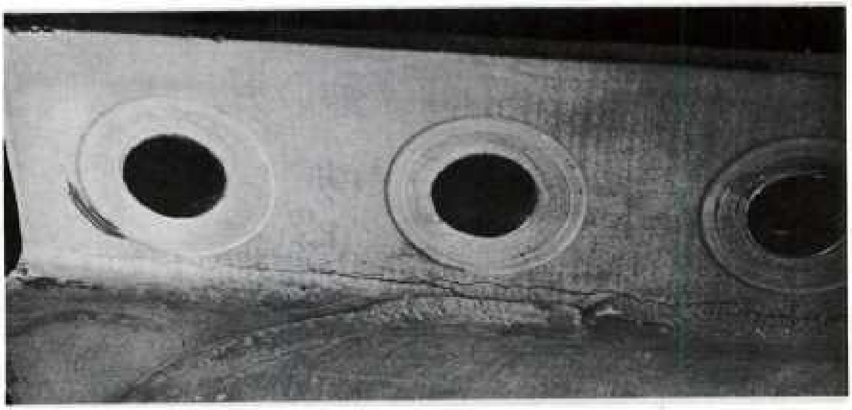
FIGURE 8.—Fatigue in landing-gear bolt.

weight saving and high performance. Although these objectives remain, there is concern for the competing objective of high reliability over long service lifetimes.

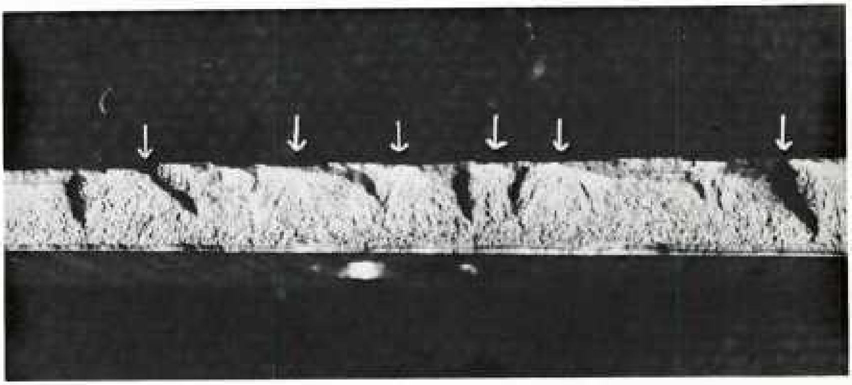
Figure 15 shows the trend of transport aircraft operating speeds over a period of about three decades. This trend contributes relentlessly to more severe gust and maneuver loadings in flight. The corresponding trend in takeoff and landing speeds results in more severe taxiing, ground-maneuvering, and landing dynamic loadings. Catapult takeoffs and arrested landings are particularly severe for carrier-based aircraft.



Top: As received—failure in circled area.



Middle: Crack at flange (2X magnification).



Bottom: Fracture surface (5X magnification).

FIGURE 9.—Fatigue failure in a flap-track flange.

Figure 10.—Fatigue failure of wing main spar—growth through areas of stress concentration (see also fig. 11).

Another design trend is toward flight at higher altitudes. This requires increased pressurization loading on fuselage sections and influences other loadings.

New designs bring new emphasis on dynamic loadings. VTOL and STOL aircraft introduce particular problems. High-speed jet aircraft have made acoustic fatigue an increasing concern. The supersonic transport adds concern for elevated-temperature fatigue and for thermal-cycling fatigue. Missiles and space vehicles have shown some problems in very low-cycle fatigue and in fatigue at cryogenic temperatures. In general, the range of concern (low-cycle, high-frequency, high- and low-temperature) is increasing.

At least as important as some of the design trends are operational trends.

About 20 years ago, a commercial transport airplane might have flown less than 2,000 hours a year; today a commercial airplane may fly 4,000 hours or more a year. Air Force use of one type of aircraft increased 50- to 100-percent over the past three years. Such increasing usage means significant increase in the calendar rate of accumulation of fatigue damage.

Moreover, aircraft assignments vary and often include missions different from those in the planning and developmental stage. Figure 16 shows variations in the frequency of maneuver loadings of identical airplanes flown by two different squadrons. Mission assignments may

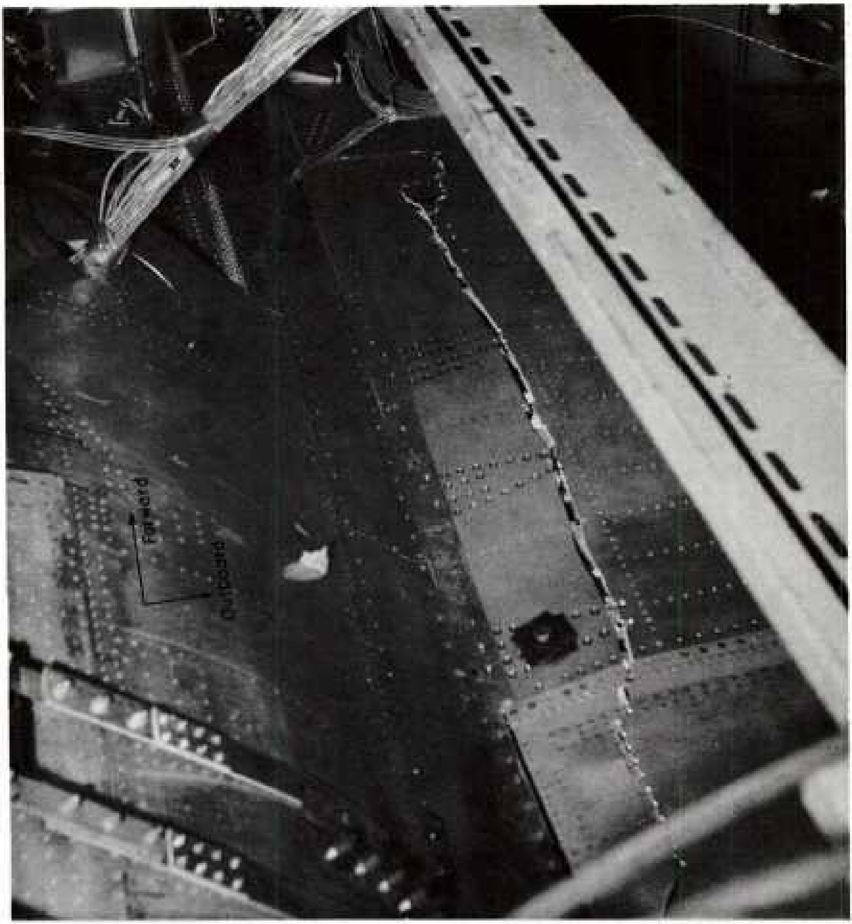
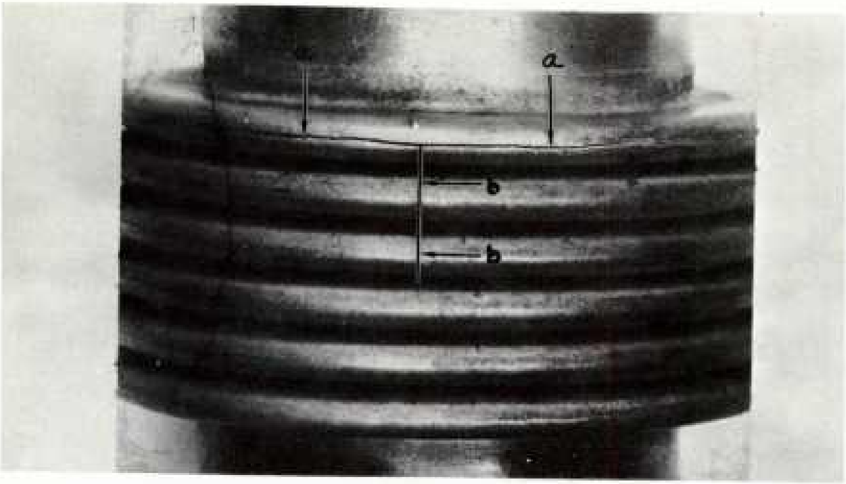


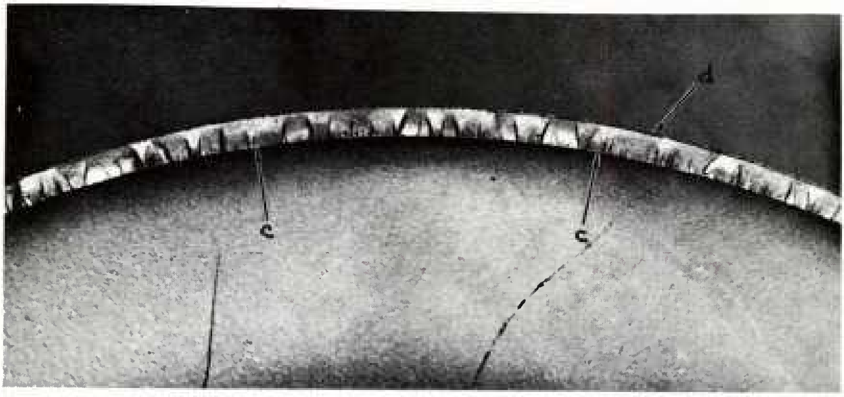
FIGURE 11.—Fatigue failure of lower wing surface after spectrum loading.

include severe environments such as in low-level radar penetration or in weather observation. The increasing trend toward variation of missions requires increasing concern for the accumulation of fatigue damage.

A few years ago, there was a "dividing line" between large aircraft (transports, bombers) and small aircraft (fighters, reconnaissance planes, etc.). The former were given qualitative consideration for possible fatigue from gust loadings; the latter, being designed to withstand occasional high maneuver loads, were thought to be less critical in fatigue. This dividing line is no longer so clear. The diversities of military use (for example, low-level attack) mean that loadings are



Top: Crack caused by surface wrinkles and laps (2½X magnification).



Bottom: Surface of crack along *a–a* (8X magnification).

FIGURE 12.—Fatigue in bellows from a lox duct.

no longer so clearly separable. Hence, there is a trend toward consideration of fatigue in more types of aircraft.

Maintenance problems increase. Larger or more complex aircraft are more difficult to inspect for possible fatigue damage and are more costly to repair or change. Costs are serious even for military budgets. One testing and repair program on a bomber is estimated to have cost over $200 million. The cost of numerous minor alterations and repairs on various aircraft is difficult to determine; one estimate is that the

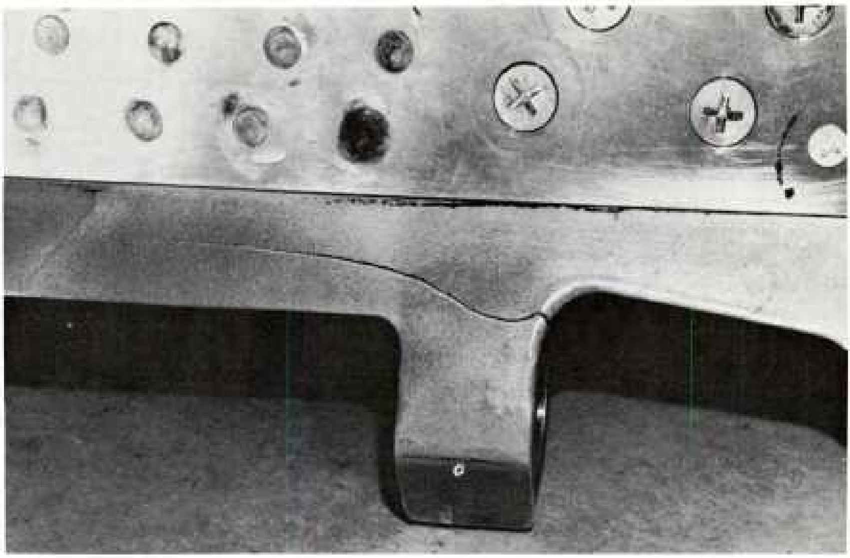


FIGURE 13.—Fatigue crack in lug.

Navy spent over $150 million in one year. Such costs emphasize the importance of design to prevent fatigue failures.

All of these trends require that the aircraft engineer be increasingly cognizant of fatigue problems and of all feasible means of preventing fatigue failure.

DESIGN APPROACHES TO PREVENT FATIGUE

Some materials (for example, low-alloy steels at room temperature in noncorrosive environment) show no tendency to fatigue cracking if peak stresses are less than some value called the fatigue limit.(1) Hence, a steel part can, in principle, be designed for *indefinitely long life*. Some parts, such as engine components or controls, are so designed. Aluminum alloys, widely used for aerospace structures, do not show a clear fatigue limit, but a tendency to fail at quite low stresses with a sufficient (sometimes many millions) number of repetitions. Moreover, weight considerations often prohibit designing for extremely low stress under infrequent high loads. From these considerations arose the concept of *safe-life design,* that is, planning for a finite but usefully

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(1) Fatigue limit is defined in chapter 3. The value varies with type of stressing and with type of stress concentration as well as with material composition and metallurgical structure.

FIGURE 14.—Fatigue crack originating at spline.

long lifetime after which the aircraft is to be retired or critical parts are to be replaced. This safe-life design concept raises two important questions. First, since there are uncertainties in the expected loads and stresses, and since there is known to be scatter in the fatigue behavior of any material, what kind of statistical basis should be used? Second, since consideration of the loading of many parts immediately implies a spectrum of stresses, what design principles should be used for the cumulative-damage loads of varying magnitudes?

The difficulty of these questions led, several years ago, to a different approach called *fail-safe design*. This approach can be illustrated by

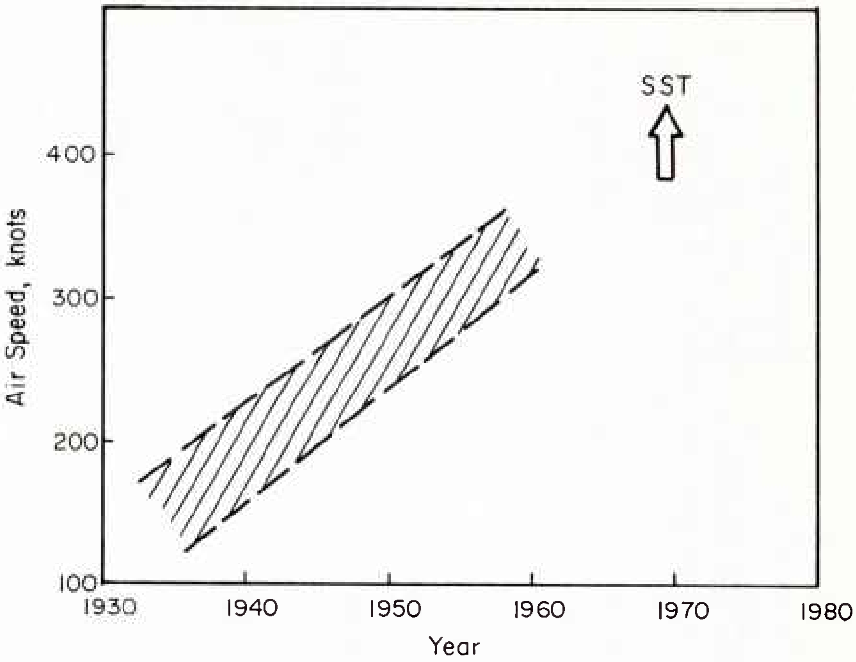

FIGURE 15.—Trend for increased airspeed in transport aircraft.

a structure with planned multiple load paths such that, if one part fails by fatigue, other parts will sustain necessary loads until the vehicle can land and the structure can be repaired. Another illustration is a panel so designed that, if a fatigue crack starts, it will propagate slowly enough that the flight can be completed, the crack discovered, and the panel replaced or repaired before any catastrophe occurs. At first it was hoped that this approach might prevent the statistical uncertainties connected with safe-life design. However, additional thought shows need for careful consideration of fatigue lifetimes and probabilities, even in a fail-safe design. For example, the panel must be designed so that (1) a potentially dangerous small crack is detectable and (2) a crack too small to be detected on one inspection will not propagate to catastrophic proportions before the next inspection.

At present, the indefinitely long-lifetime approach has limited use in aircraft because relatively few materials have dependable fatigue limits, and penalties for overweight prevent designing for extremely low peak stresses. The two approaches of safe-life and of fail-safe are both widely employed. Which approach is emphasized in a specific part depends on many considerations such as material properties, manufacturing and assembly possibilities, inspection accessibility,

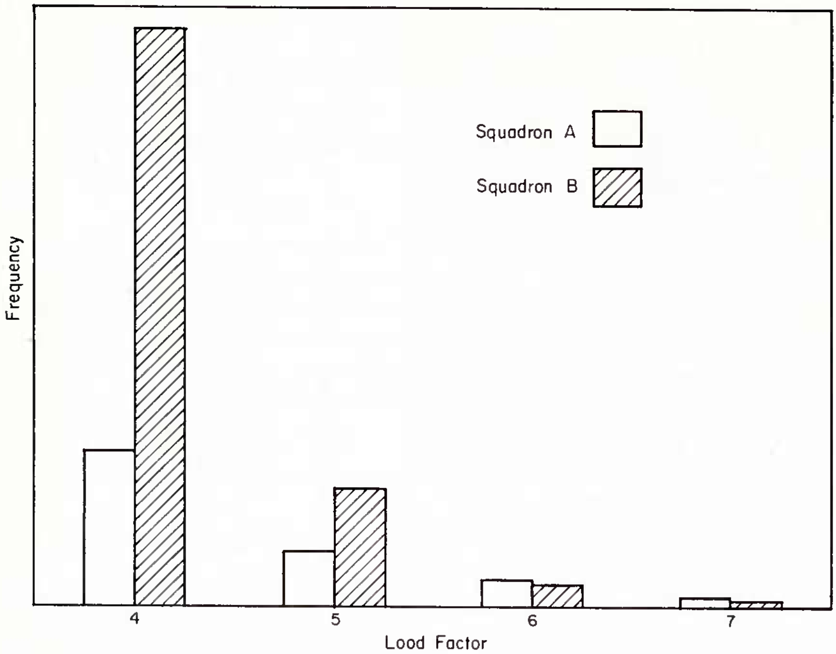

Figure 16.—Comparison of maneuver loads on the same model aircraft in different squadrons.

safety requirements, and weight limitations. In many situations neither method is used alone but rather a combination of the two.

AVAILABLE ENGINEERING INFORMATION

Pursuit of any design approach must be based upon the information currently available. In design to prevent fatigue, as in other engineering design, the available information is largely empirical, and actual procedures are based on approximate rules. The engineer must develop a "feel" for the factors likely to be critical in any specific situation and then utilize, with all the skill and judgment at his command, the scattered information seemingly pertinent to his particular needs.

Information concerning prevention of fatigue comes primarily from three sources: (1) laboratory fatigue tests on material coupons, (2) laboratory fatigue tests on aircraft components, and (3) fatigue tests and service experience on actual aerospace vehicles. It seems convenient to arrange the information in this book in that order.

Accordingly, the following nine chapters are concerned mainly with the information that has been accumulated from laboratory investigations of the behavior of material coupons. Such studies have proved useful in:

1. Comparing different materials, different processing treatments, and different surfacing treatments.
2. Comparing behavior under different types of stressing or under different environmental conditions.
3. Guiding the development of new materials.
4. Investigating basic mechanisms of fatigue.

The results accumulated in laboratory investigations over several decades have provided considerable understanding of fatigue problems.

In general, data from tests of material coupons are not directly applicable for quantitative prediction of the behavior of complex structures. This is discussed more completely in chapter XI, which, with the four chapters following it, presents information accumulated from many laboratory studies of components. This information on components is helpful toward:

1. Revealing unanticipated stress concentrations and other design faults.
2. Indicating various fabrication problems.
3. Comparing designs, comparing materials for a specific design, and comparing fabrication processes.
4. Establishing design criteria.
5. Obtaining correlation between material laboratory test behavior and service experience.

The latter two objectives may not be fully realized in many situations.

Realistic appraisal of the behavior of an aircraft requires additional considerations relating to its expected loading and environment and the complex interactions among its components. The final five chapters cover some of these additional considerations and contain information about some approaches that are currently in use.

Some additional information, not needed for continuity in the text, is included in four appendixes on (1) statistics in regard to fatigue, (2) fatigue behavior under combined stresses, (3) illustrative data on several materials, and (4) processing of alloys in aerospace structures.

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1. Smith, C. R., "Tips on Fatigue," NAVWEPS 00–25–559, U.S. Government Printing Office, 1963.

2. Longsan, J., *A Photographic Study of the Origin and Development of Fatigue Fractures in Aircraft Structures,* Her Majesty's Stationery Office (London), 1962.

CHAPTER II.

THE NATURE OF FATIGUE

Fatigue under repeated loading was established as part of the testing repertory of the engineer about 100 years ago. This was a direct consequence of the experiments of Woehler (*1*) in connection with axles of railway cars. Since then, much effort has been devoted to attempts to understand the process of fatigue well enough to eliminate or to control the problem of failure under repeated loading.

Approaches in seeking such understanding have been extremely varied in nature. They have included development of empirical relations, phenomenological analyses, and meticulous metallurgical studies. Approaches of each type have contributed to engineering by focusing attention on factors critical in design. Current progress promises additional help toward the goal of increased reliability of materials and structures against unanticipated fatigue failure.

The objectives of this chapter are to illustrate the diverse approaches that have been taken, to indicate the current status of basic studies, and to suggest in what manner the developing understanding of the nature of fatigue is relevant to engineering.

EMPIRICAL RELATIONS

Attempts have been made to correlate fatigue strengths of metals with almost every other conceivable property of interest in engineering: with static tensile ultimate strength, with static tensile yield strength, with proportional limit, with internal friction, etc. For some materials there appears to be a limited correlation between fatigue strength and static tensile strength; no such correlation appears generally valid for materials of wide differences in composition and alloy structures. For the most part, fatigue strength and other strength properties do not show a dependable correlation.

Other physical properties (density, electrical conductivity, etc.) have proved completely disappointing as engineering indexes of fatigue strength.

Experimental measurements of fatigue behavior and empirical relations for interpolation and extrapolation of data form the basis of engineering to resist fatigue. Such data and relations are the subject of subsequent chapters. However, it is pertinent to emphasize here an outstanding limitation of this approach. This limitation is the uncertainty about the validity of extrapolation of empirical relations beyond the limited range of available test data. The desire for more certainty about the range of applicability of suggested relations is a major reason for engineering interest in more basic understanding of the process of fatigue.

PHENOMENOLOGICAL APPROACHES

Early Studies

THIS IS STILL HEARD!

A recent paper contains an account of early observations of fatigue failures and of speculations concerning their cause (2). One of the first speculations, that "metals crystallize in service," persisted until the early part of the present century. This idea was partly overridden by recognition of the importance of the *repetition of stress* and was finally eliminated by establishing, through use of the optical microscope and of X-ray, the *inherent crystalline nature* of metals in the solid state.

During the first quarter of the present century, work with the optical microscope brought further insight into the failures of metals by slip along certain crystallographic planes. Many attempts were made to study the behavior of grain boundaries in polycrystalline metals and alloys subjected to cyclic loading. Conjectures that slip converted a material to a different thermodynamic state were advanced as explanations of the irreversibility apparently required for fatigue. The work of such men as Bielby, Ewing, Barstow, Rosenhain, and Haigh was outstanding. Much of the experimental evidence which they obtained has been improved and altered by use of more modern techniques; however, the importance of *slip along crystallographic planes* has remained. In the period 1920 to 1935, Gough and his colleagues carried out extensive experiments of significance in the understanding of the basic nature of fatigue (see reference *3* for an excellent summary). Some of the important conclusions drawn were:

1. In ductile materials fatigue failure is a result of discontinuities which are a consequence of slip; in brittle materials it results from discontinuities inherent in the material.
2. Resolved shear stress along crystallographic planes is important to the fatigue process.
3. Single crystals exhibit the same fatigue behavior as polycrystals; hence, grain boundaries may influence fatigue but are not the sole source.

Some of these conclusions have been slightly modified and considerably extended by later studies, but they remain essentially valid. A major contribution was the identification of the intrinsic importance of the crystalline structure.

In about the same period, Griffith (*4*) suggested a quantitative explanation of some observations concerning the failure of glass under steady load. He suggested that small microscopic cracks were inherent in the material and that failure involved the growth and coalescence of these cracks. While Griffith's original work pertained to brittle materials, the emphasis on inherent defects (not necessarily the crack type suggested for brittle materials) has been used in many subsequent speculations concerning fatigue.

Recognition of the importance of stress repetition suggested an irreversible process: An early stress cycle must produce some irreversible change if a later cycle is to cause failure. In 1881 Bauschinger (*5*) found evidence of *hysteresis loops* in quasistatic stress-strain relations and suggested the pertinence of such behavior to fatigue. This suggestion is still valid, although it has proved less fruitful than originally hoped. It is deficient in at least two respects as an adequate explanation of fatigue. First, experimental evidence shows the material properties revealed by quasistatic tests do not correlate well with fatigue behavior because the latter is extremely sensitive to local imperfections, while in quasistatic tests they are "usually averaged out." Second, mechanical hysteresis itself needs further explanation on a more basic level.

Cumulative Strain Hardening

The work of Gough focused attention upon strain hardening and the exhaustion of ductility. Shortly afterward, Orowan (*6*) developed a theory based upon this idea. Despite some limitations, which will be noted later, this concept retains considerable value and merits description. The idea suggested by Orowan is illustrated by the schematic diagram in figure 17.

Figure 17 (a) shows a small volume of a polycrystalline solid under an applied tensile stress. Grain A is oriented to yield at a relatively low value of stress; it is wholly surrounded by B grains oriented to resist higher stresses. Figure 17 (b) shows the relative stress-strain behavior of grains A and B. Note that the suggestion of different grains is unimportant. Grain A might be any kind of defect in an otherwise elastic matrix. Orowan's term was a "plastic inhomogeneity." Now, consider varying the applied load sinusoidally. The whole element shown in figure 17 (a) will vary through almost a fixed strain cycle since it is (with the exception of the single small grain A) elastic. Suppose the

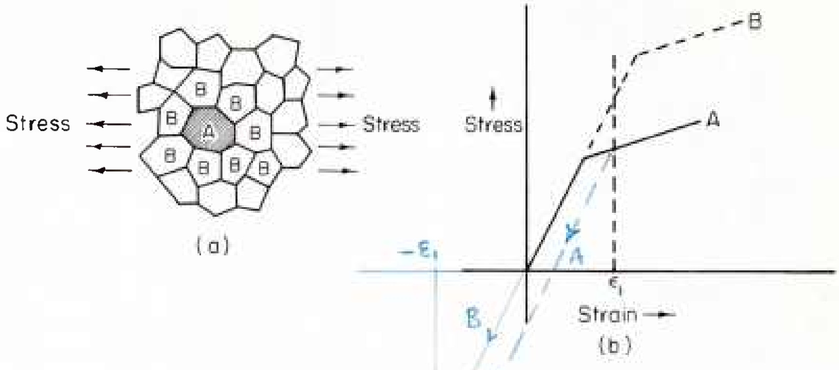

FIGURE 17.—Schematic diagram of Orowan's theory of strain hardening.

cycle of strain (±ε) is such that, if all grains supported their share, B grains would never exceed their elastic limit but grain A would. On the first loading, grain A will deform plastically and work-harden so that, on reversing the load to an equal strain in the opposite direction, the "weak grain" will undergo a higher stress. Repetition of load cycles and cumulative strain-hardening will cause an increase in the local stress on grain A. With successive cycling, the stress will tend to be distributed among the B grains. If this happens before the grain A fractures, the range of strain is "safe" (below the fatigue limit).

This is an inadequate explanation for fatigue for several reasons. First, it is admittedly phenomenological and based upon empirical shapes of stress-strain relations; a more basic mechanism for strain hardening is needed. Second, as Orowan has pointed out, this picture requires additional assumptions to allow for all observations about fatigue; for example, the influence of mean stress is not readily explained. Third, as Gohn (2) emphasizes, some materials sometimes exhibit strain softening in the fatigue region.

Nevertheless, it is quite certain that local "plastic inhomogeneities" and redistribution of stresses are important factors in most, if not all, fatigue situations. In some instances this view may provide an almost complete explanation of behavior in terms quite adaptable to more detailed description (for example, in terms of descriptions in the crystal). In others the Orowan mechanism may be mainly concomitant, influencing the form of results but not accounting for all observed fatigue behavior.

Theories Based on Preexistent Flaws

As already mentioned, Griffith suggested preexisting cracks as a major item in the static fracture of glass. If cracks are formed or propagated by some time-dependent process (such as atmospheric corrosion), then observed static strengths will be time-dependent. According to Griffith's original suggestions, a crack will grow under a nominal tension stress:

$$S = \sqrt{KE/l}$$

where

K = a constant for a specific material in a specific state,
E = Young's modulus, and
l = the half-length of the crack.

Suppose there are many submicroscopic cracks in a piece of metal to be subjected to repeated stressing, and some of these are long enough to grow during the maximum tensile portion of each stress cycle. Each of these cracks will grow increasingly fast as the number of cycles increases, and, at some time, two or more will coalesce. Eventually a visible crack will appear. This explanation has been offered for many of the observations of fatigue behavior. Some aspects are discussed in chapter V.

Of course, flaws other than cracks may preexist and may provide the local stress concentration that is necessary to start a small crack. Subsequent behavior can be cumulatively catastrophic, with susceptibility to damage increasing with the number of repetitions of loading. Formulation of semiempirical relations may then be made on the basis of statistics. Preexistent flaw approaches require more detailed description of the type of flaw than is usually available. However, there is little doubt that such flaws exist and complicate most fatigue experiments even though they may not themselves provide a full explanation of the basic mechanism.

Statistical Approaches

Afanasev (*7*) advanced one of the early "statistical theories" of fatigue failure. Freudenthal (*8*) and many others have made notable contributions.

In general, these "theories" suppose that the disruptive actions are on a microscopic or submicroscopic scale—too small to be observed using simple techniques. Because of the nonhomogeneity of metal on such a scale, and of the imperfection of measurement and control of stress in testing, it is likely that some disruptive and irreversible action will occur in any cycle—even though this may not have happened on a previous, supposedly similar cycle. So, it is suitable to consider the probability of some irreversible action, such as fracture of an atomic bond, which increases with increasing number of cycles.

For example, suppose $\phi(\tau)$ is the distribution function of position of strength of atomic bonds, and $\psi(S)$ is the distribution function of bond forces under the stress, S. Then one can write for the probability of disruption:

$$p(S) = {}_0\int^{s} \psi(S) \left[{}_0\int^{s} \phi(\tau)\, d\tau \right] dS. \qquad (1)$$

The probability of surviving N cycles without fracture can be written for p,

$$p_N(S) = [1 - p(S)]^{mN}, \qquad (2)$$

if the failure of m bonds is required for fracture and if the $\psi(S)$ are considered as independent probabilities. It has been shown (*8*) that this approach may be considered on the basis of the statistics of extreme values to lead to a reasonable logarithmic form for the S–N relation. The statistical approach also suggests, as has been pointed out by Freudenthal, implications concerning "size effect" and "cumulative damage."

Nevertheless, the statistical approach cannot provide a wholly satisfactory theory of fatigue. The ideas noted above, containing the implication of independent probabilities of bond rupture, might approximate the behavior of a brittle material where breakage of one bond may not drastically influence adjacent bonds. Incorporation of the suspected behavior of interaction of local defects in actual metals is beyond the capabilities of current statistical theories. Some of the "conclusions" from the statistical approach are, likewise, less informative than they appear at first. For example, probability considerations of any strength property imply a lower macroscopic strength for larger specimens, but they do not directly imply the magnitude of this "size effect" or the factors which govern the magnitude. Fatigue strength might be low at a thickness of a few hundred atomic distances, with no further decrease of engineering significance anticipated as thickness increases over such practical ranges as ¼-inch to 1 inch. This information requires characterization of the quantitative expressions for such distribution functions as $\phi(\tau)$ and $\psi(S)$; this characterization is not likely to come, in any unique sense, from the statistical approach.

For providing insight into the atomic mechanism of fatigue, the statistical approach may be, as Gough once said, "simply irrelevant." However, as indicated in subsequent chapters and in appendix C, *statistical allowance* for variables in fatigue is most important in the interpretation of experimental data and in the extrapolation from data on similar samples in predicted behavior of *engineering structures*.

Theories of Progressive Deterioration

Many microscopic mechanisms, other than slip, twinning, and strain

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hardening, have been proposed to explain the irreversible action inherent in fatigue.

Freudenthal and Dolan *(9)* have suggested "fragmentation of crystals" with resulting deterioration of structure. Shanley *(10)* considered "progressive unbonding" in reversed slip. Freudenthal *(11)* suggested local "heat flashes" as sources of irreversible damage.

Feltner *(12)* and Blatherwick *(13)* have proposed that in the low and intermediate fatigue range the stress is not a valid criterion to use as a measure of lifetime. Morrow has proposed that in the low-cycle range a more valid criterion for fatigue design is the microplastic strain energy. This shifts the emphasis to measuring and understanding the microplastic strain energy.

Many of the mechanisms mentioned above are either plausible or backed by experimental observations for some materials. None has been experimentally validated to a satisfactory degree to be acknowledged as an approach of major importance.

STUDIES OF METALLURGICAL DETAILS

Attempts to develop empirical relations and phenomenological theories have revealed the importance of localized behavior of the crystalline structure. Recent investigations toward more basic understanding have been concerned mainly with observations in microscopic detail. Grosskreutz *(14)* has reviewed the "micromechanisms of fatigue" and Schijve *(15)* has summarized such work with particular reference to aluminum alloys.

Possibility of Different Mechanisms

Wood and his colleagues *(16)* used X-ray techniques to look for structural changes under cyclic stressing in contrast to those under static stressing. Figure 18 shows their suggestion of differences in a high-stress *(H)* low-cycle region and a low-stress *(F)* high-cycle region. In the former, many features resemble effects under static loading; in the latter, some detailed differences may be noted. Accordingly, Wood suggests that different mechanisms may operate at high stress levels and at low stress levels.

Laird and Smith *(17)* consider the importance of different mechanisms questionable. They suggest that the *H* region represents rather a hastening of the fatigue process and a wider distribution corresponding to a larger volume of plastically deformed material. Current interest in low-cycle fatigue (see ch. VII) may provide further insight into the value of considering two different mechanisms.

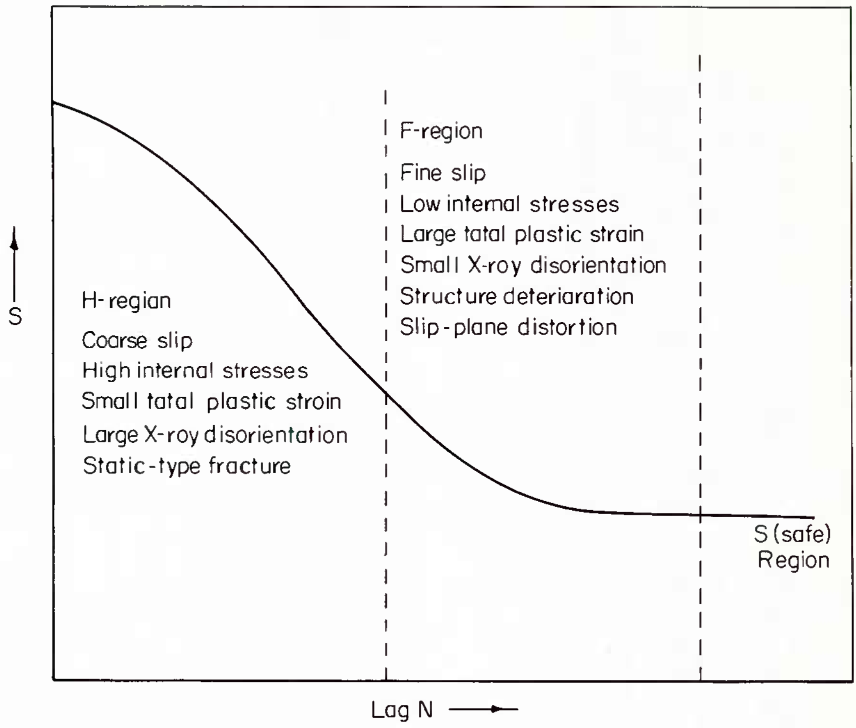


FIGURE 18.—Differences in high-stress and low-stress fatigue (according to W. A. Wood et al.).

The possible importance of considering two distinct mechanisms is not yet entirely clear. Attention is presently directed toward more detailed understanding of behavior in each region.

Stages in the Development of Fatigue

It is now generally agreed that the development of fatigue failure in any metal or alloy subjected to repeated stressing under some fixed amplitude of (macroscopic) stress or strain may be usefully considered in stages (note, for example, reference *18*). These include:

1. Cyclic slip and local strain hardening or strain softening prior to any detectable crack.
2. Development of submicroscopic pores or voids (note reference *19*).
3. Coalescence of these pores to form a microscopic crack or cracks.
4. Growth of the crack or coalescence of cracks to a macroscopic crack.

5. Relatively continuous propagation of the macroscopic crack. This stage may be further subdivided (see ch. V).
6. Final rupture after the macroscopic crack has weakened the piece.

A completely satisfactory theory must account for each stage.

Of the six stages mentioned, the latter two, observable on a macroscopic scale, are being studied in terms of continuum mechanics. The first four are being studied experimentally on a microscopic basis with particular attention to the role of cyclic slip.

The Development of Cyclic Slip

Much careful study has been carried out on pure metals, such as aluminum and copper. These not only are free of some of the microstructural complexities of alloys but are relatively easy to observe for slip bands. One of the major contributions of studies of slip in pure metals has been the distinction of two types:

1. *Fine slip*—movements through relatively few lattice spacings, movements being of the order of 10^{-7} cm.
2. *Coarse slip*—avalanches of fine slip movements, producing bands of the order of 10^{-5} to 10^{-4} cm.

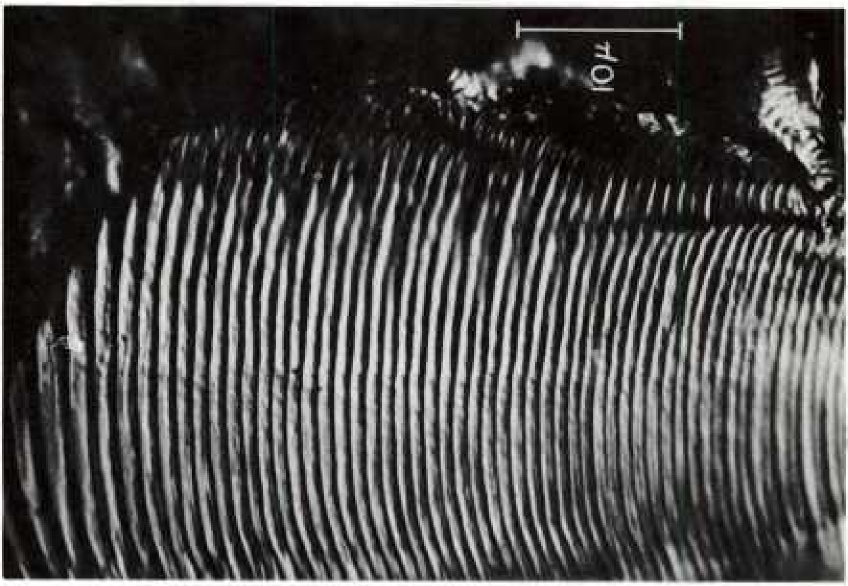
The relative extent of fine slip and of coarse slip appears to be a function of stress amplitude, fine slip being particularly evident at lower stress amplitudes.

The evidence concerning details of slip under cyclic stressing is being obtained by optical and electron microscope studies of surfaces transverse to cracking and of crack surfaces (fractography). Figures 19 and 20 show examples. Figure 19 shows striations in a 7.5 percent Zn–2.5 percent Mg aluminum alloy. The upper photograph shows "ductile striations" frequently observed; the lower photograph shows "brittle striations" observed in a corrosive atmosphere. Figure 20 shows fracture surfaces in a 7178 aluminum alloy. The curvature of the striations in the upper photograph indicates slowing down of the propagation rate at grain boundaries. In the lower photograph alternate regions of "ductile striations" and brittle fracture of second-phase particles can be seen.

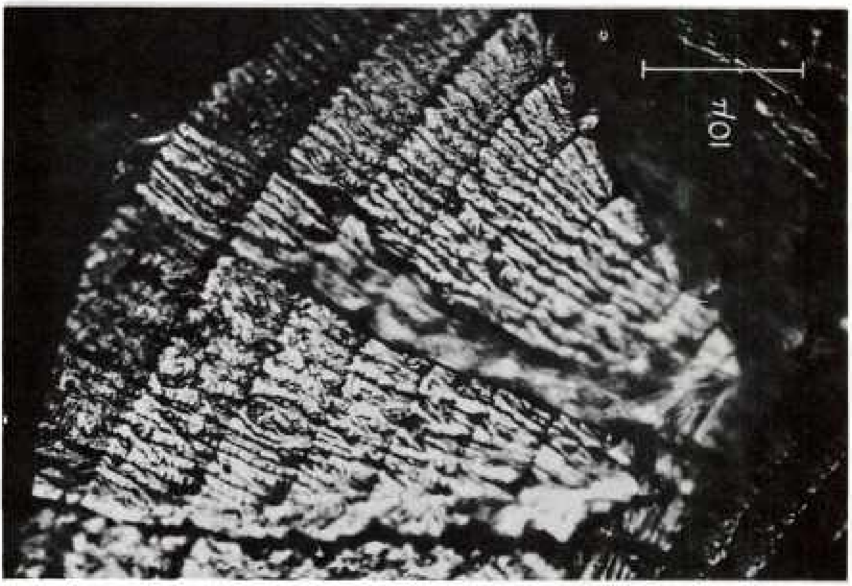
Dislocation Studies

In the past decade there has been considerable progress toward explaining static yielding and slip in terms of line defects or dislocations. It is now realized that dislocation interactions with point defects may be extremely important.

Complex arrays of dislocations produced under cyclic stressing have



Top: "Ductile striations"



Bottom: "Brittle striations" in 3 percent Na-Cl solution.

Photographs through the courtesy of the Royal Aircraft Establishment, Farnborough, England.

FIGURE 19.—Surfaces of fatigue fractures in Al-Zn-Mg alloy.

been observed in electron microscopy of thin films. Figure 21 shows an example. Such arrays and the dislocation dynamics important to the fatigue mechanism are not yet fully understood. Experimental studies have been reported by Grosskreutz (*14*), Forsythe (*20*), Thompson (*21*), Machlin and McEvily (*22*), and others.

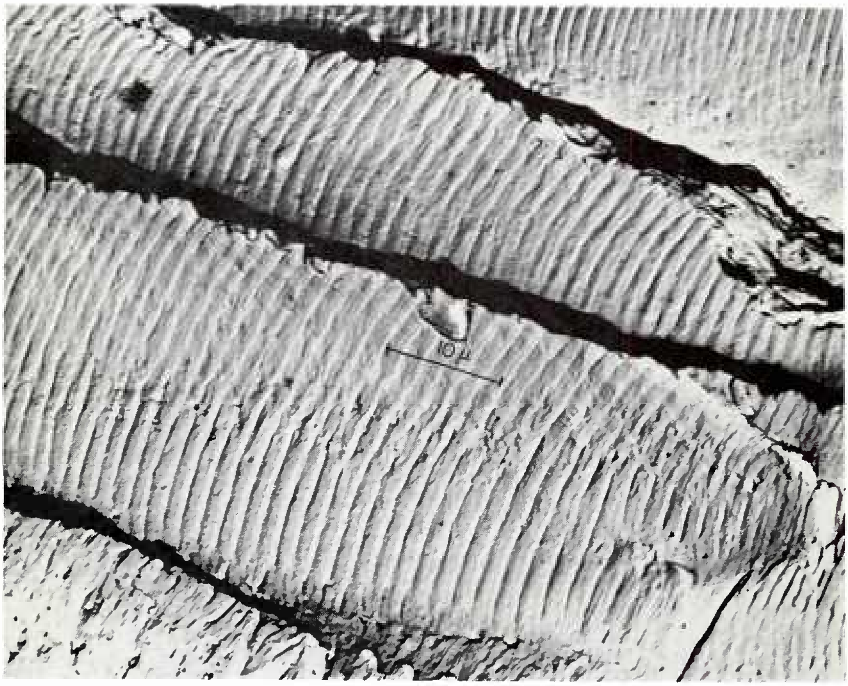
There are a number of speculative models of dislocation movements which may explain various aspects of fatigue (references *21* through *24*).

The upper photograph in figure 21 shows subcells formed in fatigue of pure aluminum and some hints of dislocation tangles at the subcell boundaries. The lower photograph in figure 21 shows, at quite high magnification, a complex array of dislocations.

Other Approaches

Experimental studies at high magnification continue. Some aspects of dislocation behavior in crystals can be studied in nonmetals. See, for examples of studies pertinent to fatigue, references *25* and *26*.

Investigations at cryogenic temperatures presently imply that



a. Striation curvatures showing change in rate at grain boundary.

FIGURE 20.—Fatigue fracture surfaces in 7078 aluminum alloy.

b. Regions of ductile striations and of brittle fracture at second-phase particles.

Photographs through the courtesy of Regis M. N. Pelloux; see Boeing Report DI-82-0169-RI (1963).

FIGURE 20.—Fatigue fracture surfaces in 7078 aluminum alloy—Continued.

mechanisms dependent on thermal diffusion cannot be basic to fatigue (*27*).

The great importance of the surface environment in many instances of fatigue is being recognized (*28, 29*).

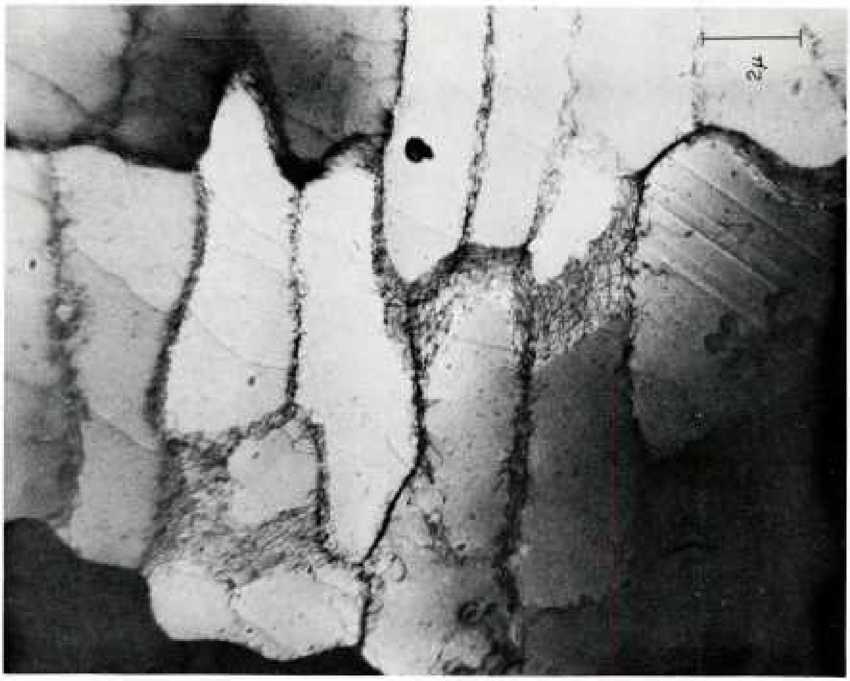
It is to be hoped that these and other avenues of investigation will provide information to eliminate some of the overabundance of models proposed for explaining the early formation of cracks. This represents, at present, a particularly unresolved question in the total picture of the fatigue process.

PRESENT UNDERSTANDING AND PERTINENCE TO ENGINEERING

The foregoing discussions indicate that there is still appreciable confusion regarding the basic nature of fatigue. In a sense, we have recognized the phenomenon for more than a century and still do not understand it. Yet, progress has been made.

It has become clear that fatigue is critically influenced by micromechanisms, not only by the crystalline structure of a metal, but also by defects in this structure. At present it seems likely that a great deal will be understandable in terms of line defects (dislocations). It is equally clear that other details (point defects, gross inclusions, surface interactions) will be important. In fact, of the various types of fracture in engineering materials, fatigue seems to be influenced more significantly by submicroscopic details of metallurgical structures than any other. It should be recognized, however, that even though atomic readjustment in fatigue cracking may become better understood, most fatigue-cracking engineering problems still may not be satisfactorily resolved. It seems evident that basic understanding will involve microscopic and submicroscopic items that can be related to design factors only in some statistical sense.

There are two ways in which our improved understanding about fatigue may be expected to provide engineering help. One is to suggest principles for the improvement of materials and fabrication techniques.



Photographs through the courtesy of J. C. Grosskreutz, Midwest Research Institute, Kansas City, Mo.

FIGURE 21.—Electron microscope transmission photos of fatigued pure aluminum.

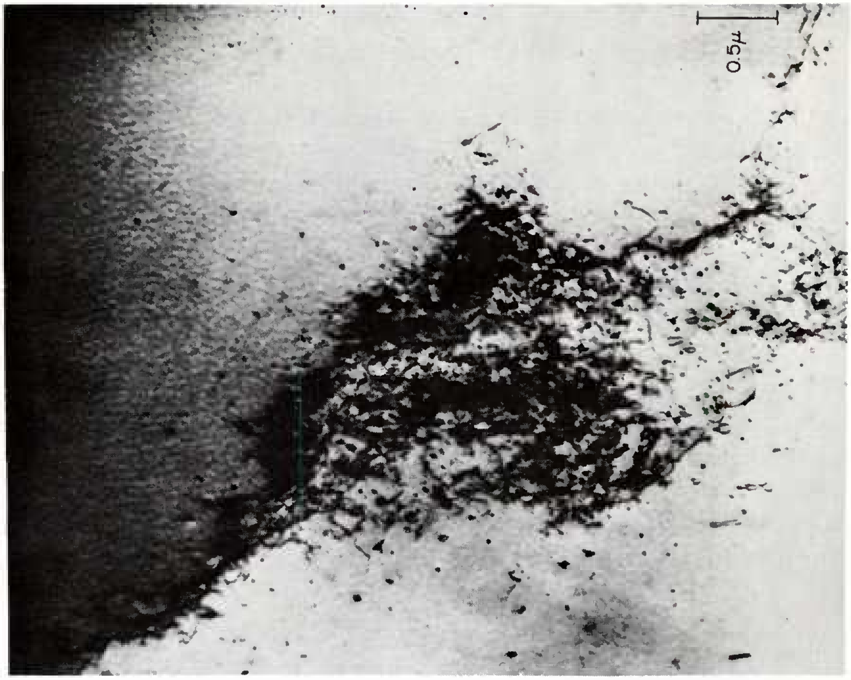


Figure 21.—Electron microscope transmission photos of fatigued pure aluminum—Continued.

These principles may not immediately lead to vastly improved materials; in some instances, materials may already approach economic optima, even though this situation has been reached by trial-and-error methods. Another way in which improved understanding may be helpful is in qualitatively defining the factors most significant in such problems as stress concentration, cumulative fatigue damage, effects of combined stresses, and influences of temperature and environment. Even when our understanding is greatly improved, such effects may have to be determined experimentally, but clearer theoretical mechanisms can be expected to provide guiding rules for optimizing the experiments.

The present status of knowledge indicates quite clearly the complexity of the fatigue processes and is compatible with our observations that fatigue is influenced significantly by many variables difficult to control in practice. This conclusion implies clearly that, to prevent fatigue failure, engineering must be continuously cognizant of the importance of local weaknesses and of the likelihood of "scatter" in engineering values for fatigue strength. It is important to realize that practical problems are not likely to be solved without attention to details—often to a greater extent than may be required to prevent other kinds of fracture. The engineering problem is inherently that of providing maximum reliability against fatigue with some tolerable amount of detailed design, testing, quality control, etc. The important lesson from consideration of present knowledge concerning the phenomenon of fatigue is that this optimization is so difficult that it may be necessary to approach practical problems entirely on a probability basis.

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CHAPTER III.
FATIGUE TESTING NOMENCLATURE AND CONVENTIONS

INTRODUCTION

In the preceding chapter it was indicated that the basic nature of fatigue is becoming better understood in terms of crystalline imperfections. However, even when atomic behavior becomes well understood, the engineer will seldom be able to design in such terms. He will have neither the detailed information nor the computation time to predict the lifetime for a structure from data on a microscopic level. Engineering to prevent fatigue failure is now and will long remain based largely upon experimental test data and upon empirical design relations.

In one sense, this situation is not wholly different from designing against static overloads. The static allowables for materials are determined by measurement rather than by basic theory. Calculations are based upon a combination of theory of elasticity and assumptions (such as isotropy and effective strength under combined stresses) which have been found by experiment to be acceptable approximations and have been justified further by experience.

However, there are some differences in the case of fatigue. Experience has shown that behavior under repeated loading is *more sensitive* than static strength *to local stress* (or strain) *concentrations and to local weaknesses,* so that more careful and detailed allowances for such items must be made. Moreover, in fatigue there are inherently *more stress* (or strain) *variables;* in place of a single quantity such as yield stress, it is necessary to consider (1) stress amplitude, (2) mean stress, and (3) the number of repetitions of stress. Moreover, usually more "scatter" appears in observed values of fatigue strength than in observed values of static strength. For this reason and because of additional uncertainties in expected distribution of loadings, a *probabilistic analysis* of survival has particular significance in design to prevent fatigue failure.

The sensitivity of fatigue behavior to local stresses and to local weak-

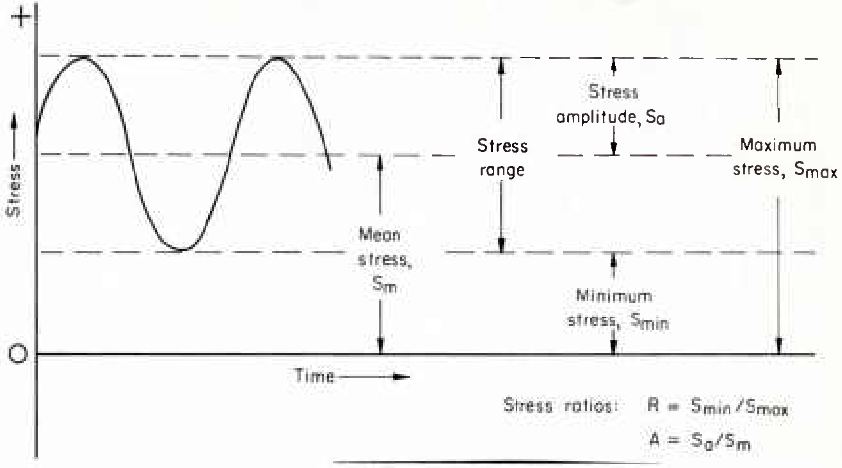

FIGURE 22.—Nomenclature for conventional laboratory fatigue testing.

nesses and the extreme difficulty of analyzing these in a complex structure mean that it is difficult (often impossible) to use laboratory data concerning the fatigue strengths of materials directly for design purposes. Despite this, there are at least two valid reasons for making laboratory fatigue tests of materials. First, such tests, because they isolate variables, are helpful in rating materials. They also indicate the relative importance of various factors in heat treatment, in fabrication, in assembly, and in environment. Therefore, laboratory results have intrinsic (although often only qualitative) value. The second reason, and the reason why such studies are discussed first in this book, is that most of our concepts of design relations have come from laboratory tests and experience. Such ideas as "Goodman diagrams," suggestions for the treatment of "cumulative damage," and so on, have originated from laboratory experiments on simplified specimens. Thus, the nomenclature and definitions of many terms used in engineering are most readily stated and illustrated by reference to results of laboratory tests.

SINUSOIDAL LOADING AND "S-N" CURVES

The most common way of fatigue-testing a material specimen in the laboratory is to subject it to a stress (or strain) varying sinusoidally with time. Figure 22 indicates the quantities ordinarily used to specify the loading.(1) Frequency (except at elevated temperature or in the presence of a corrosive environment) is often chosen for testing convenience. The other characteristics of the sine curve can be specified

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(1) The nomenclature follows that of ASTM (see reference *1*).

by any two of the five stress parameters shown or by any one of these plus one of the stress ratios. Practices common in the aerospace industry are:

1. To run tests at various specified values of R (load ratio) and, for each value of R, to vary S_{max} (maximum stress) for different specimens and to observe and record resulting values of N (number of cycles to failure).
2. To run tests at specified values of S_m (mean stress) and to vary the S_a (stress amplitude) for each value of S_m.
3. To run tests at a constant base load (usually 1g) and to vary S_{max} or S_{min}. For special purposes, other procedures such as "spectrum" or "variable-amplitude" testing are used. Spectrum testing is discussed subsequently.

Figures 23 through 25 show some ways of plotting data from laboratory fatigue tests. At the top of figure 23, values of stress (S_a) are plotted as ordinates and values of number of cycles to failure (N) as abscissas. This plot is called an "S–N" curve. Note the convention of arrows on points at the right; arrows indicate that specimens did not fail in the number of cycles shown. The bottom graph has the same data in terms of S versus log N (a logarithmic scale is nearly always used for lifetimes (in cycles) since it spreads out the shorter lifetimes). At the bottom of figure 23, the experimental curve is dotted to the left of the region of observations toward a value of S_a equal to the ultimate tensile strength (UTS) at one-fourth cycle. This is an artificiality to be used only with caution in recognition that stresses above the yield strength have ambiguous meanings; current work on low-cycle fatigue (see ch. VII) is providing improved methods of representation of data in this region.

A more common practice is to use as abscissa a logarithmic scale extending only a little beyond the range of observed data—as in figure 24. In this figure the bottom curve represents the same data as in figure 23. The top curves illustrate two other points:

1. Many steels show S–N curves that appear horizontal beyond some long lifetime; the stress at which the curve becomes horizontal is called the "fatigue limit" for the material and condition of testing. For a material (such as an aluminum alloy) which may not show a clear fatigue limit, an engineering value sometimes used as an "effective fatigue limit" is the stress at an arbitrarily specified long lifetime (such as 10^8 cycles).
2. The ordinates used for data on notched specimens are usually values of nominal stress at the section through the notch.

Sometimes the ordinate plotted is the ratio of the test stress (amplitude) to the static tensile strength. As indicated in figure 25, this

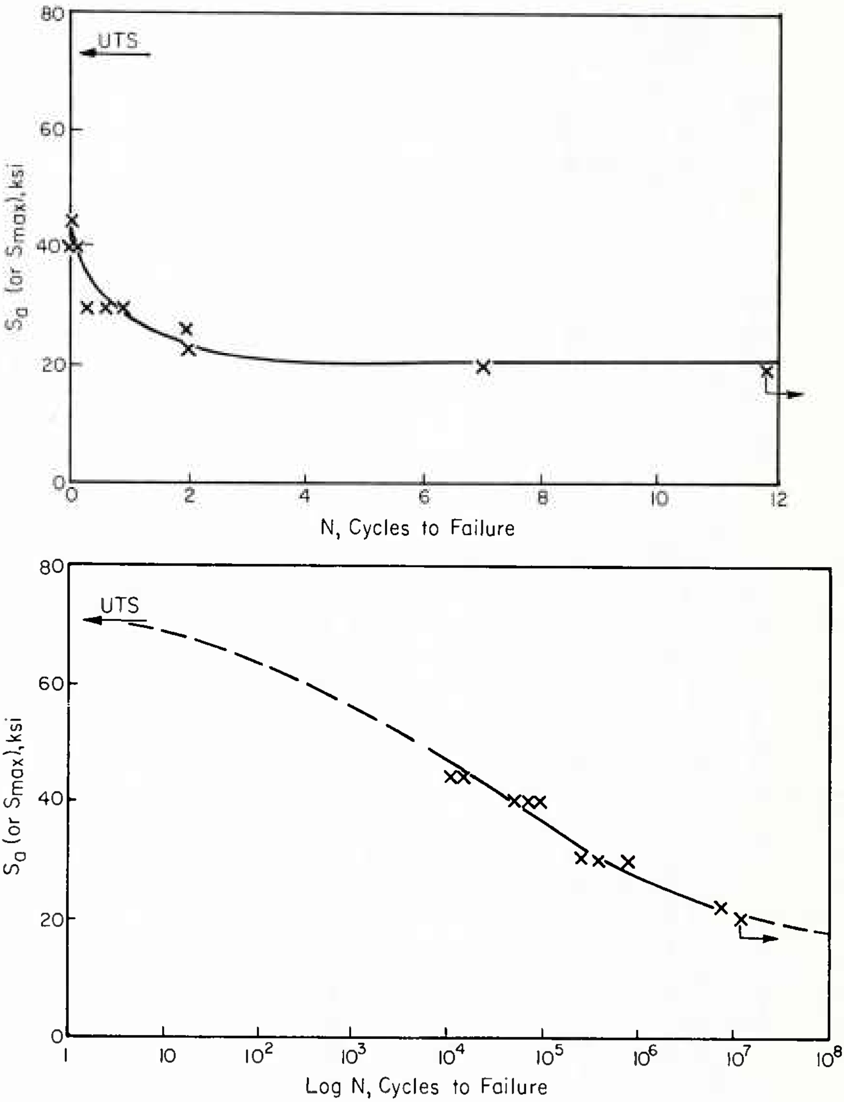

FIGURE 23.—*S*–*N* curves for 7075–T6 aluminum alloy, fully reversed axial loading.

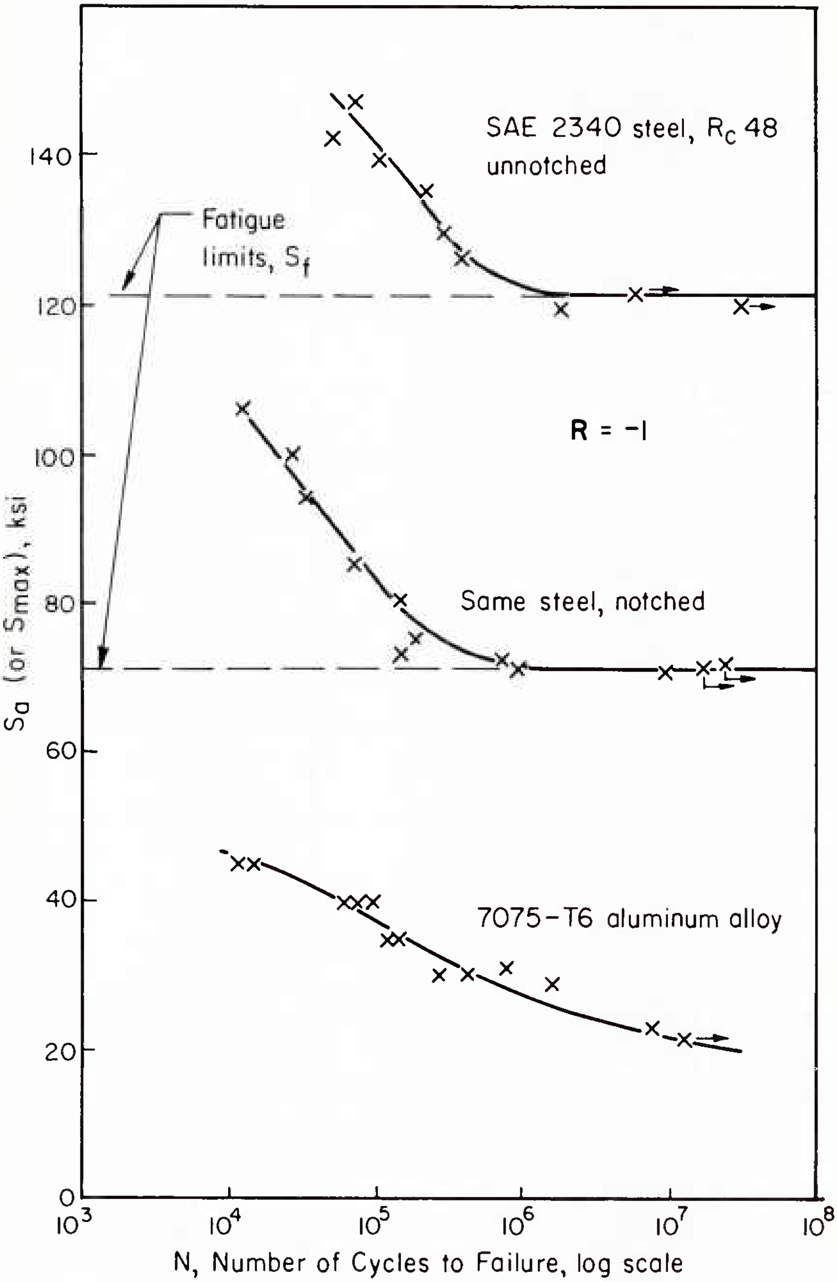

FIGURE 24.—*S–N* curves, fully reversed axial loading.

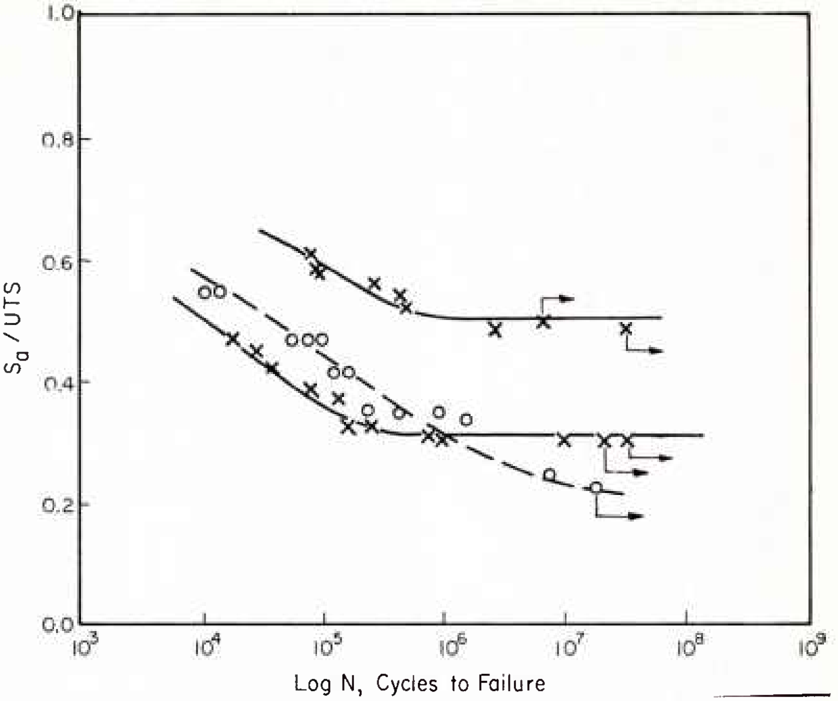

Figure 25.—Another method of plotting "*S–N*" data.

representation affords some comparisons of materials not so apparent otherwise.[2]

"GOODMAN" DIAGRAMS

Figure 26 shows a family of *S–N* curves at different values of the load ratio, *R*. From such extensive data it is possible to construct constant-lifetime diagrams such as that in figure 27, in which the "points" are values at which the curves in figure 26 intersect the dotted line at a particular lifetime value. The solid-line curve through these points provides a design guide for various values of mean stress. Figure 28 shows another representation in terms of S_{max} (upper curve) and of S_{min} (lower curve) against S_m. A plot that shows this relation and contains explicitly many quantities used in design is shown in figure 29.

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[2] Representation as a fraction of static strength (an actual or a design value) is used especially often in representing data from tests of structures.

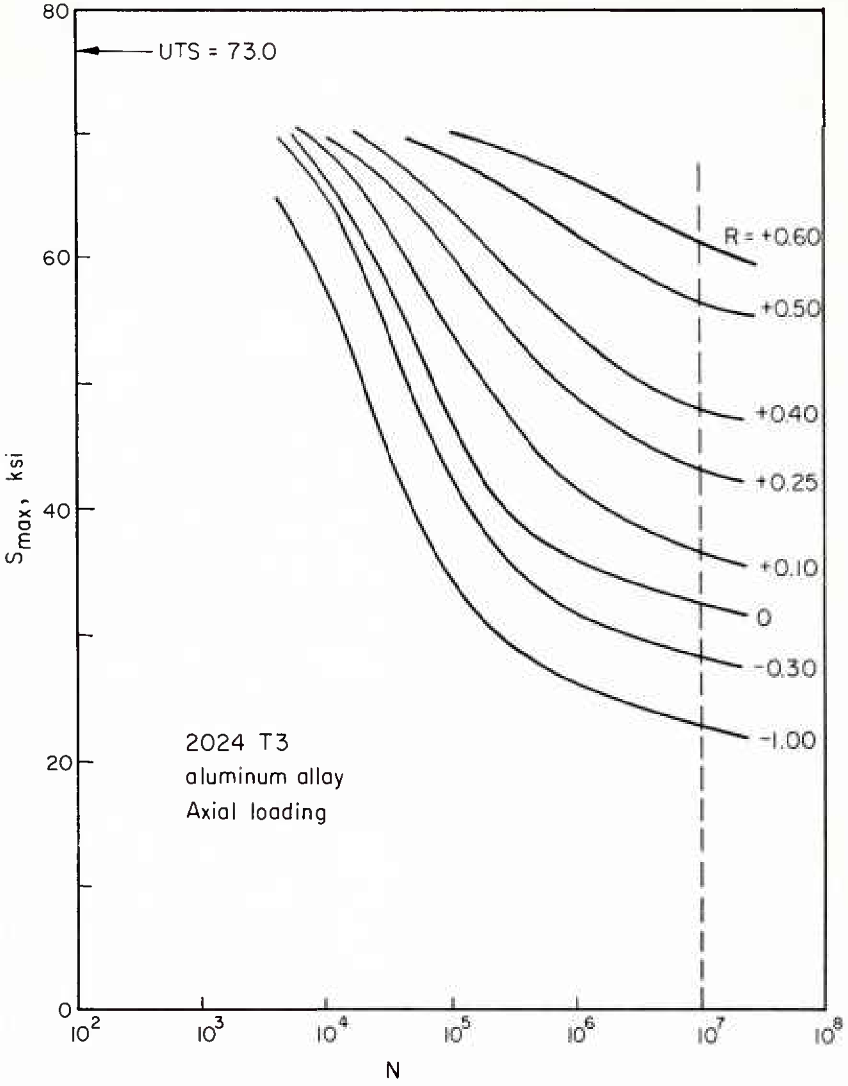

FIGURE 26.—A family of *S–N* curves.

In the United States constant-lifetime diagrams are often called "Goodman" diagrams.(3)

In many instances the designer does not have available such extensive fatigue-test data. In such instances, he may use an approximate curve fitted through such values as he may have (commonly, S_a for

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(3) After J. Goodman who actually suggested a linear relation between S_m and S_a.

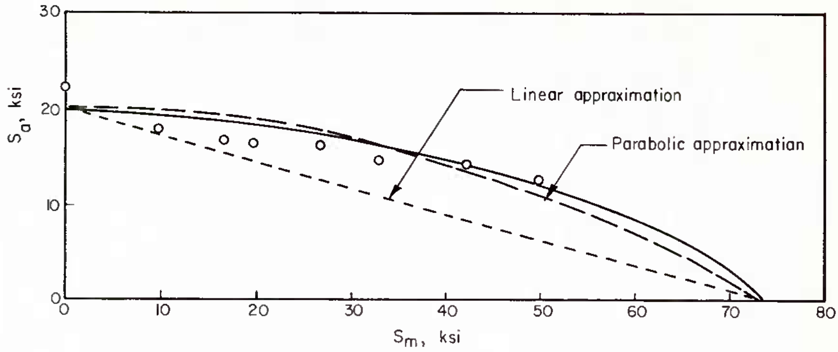

FIGURE 27.—A constant-lifetime (Goodman) diagram.

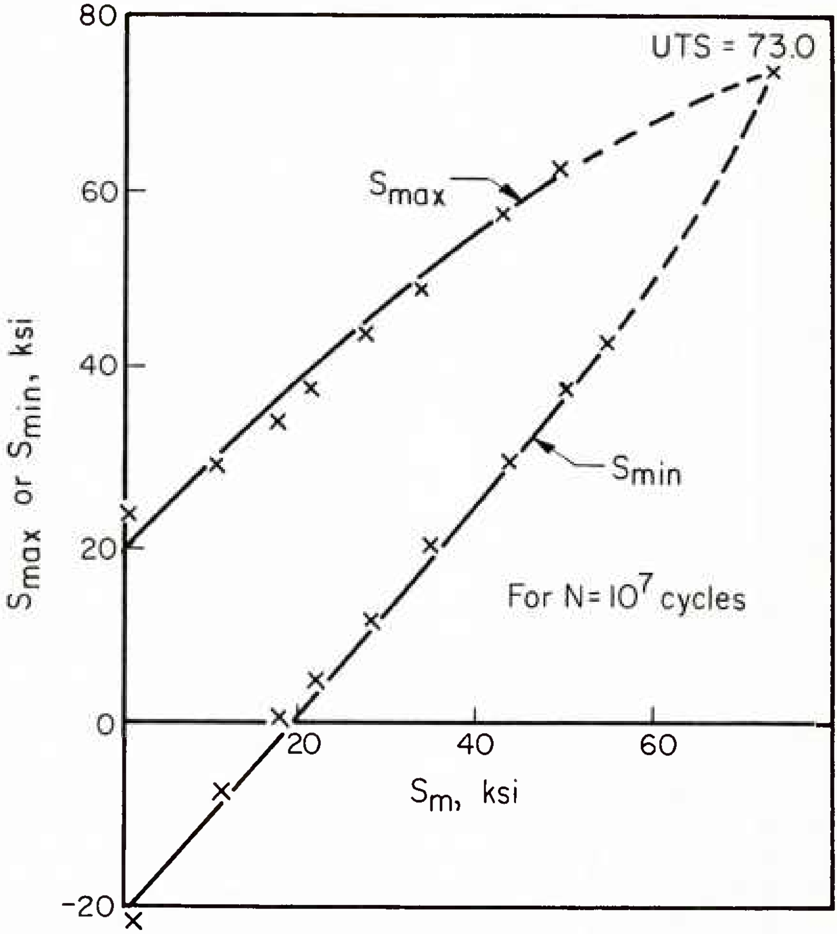

FIGURE 28.—Another constant-lifetime diagram—same data as in figure 27.

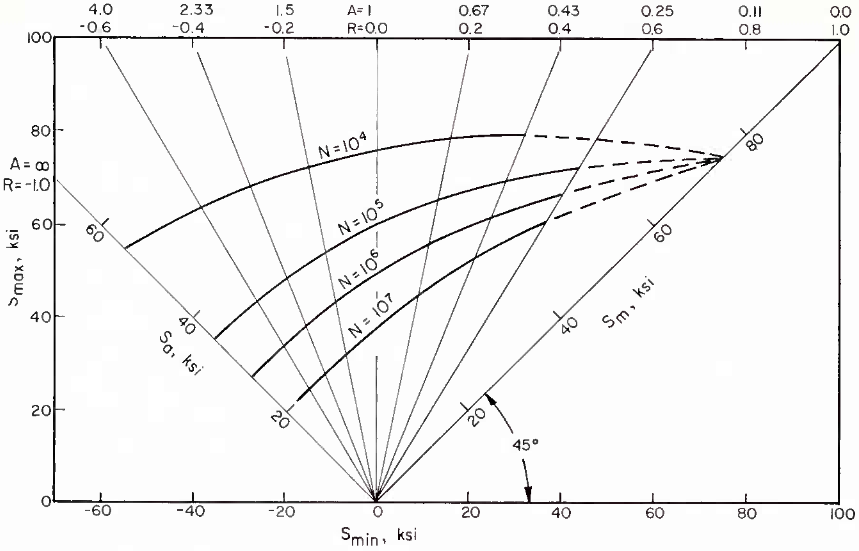

Figure 29.—Constant-lifetime diagram combining features of diagrams in figures 27 and 28.

fully reversed loading, and $S_m = S_u$ for $S_a = 0$). Two simple approximations are:

1. Linear approximation: $S_a = A\,(1 - S_m/S_u)$
2. Parabolic approximation: $S_a = A\,(1 - S_m^2/S_u^2)$

where

$$A = S_a \text{ for } S_m = 0;$$
$$S_u = \text{ultimate tensile strength.}$$

These are indicated as dotted lines in figure 27. For the data represented in figure 27, the parabolic approximation affords a close fit, but no simple relation is known to fit all materials under all loading conditions. Figure 30 shows suggestions (from reference 2) which may be used as rough guides in the absence of many data.

COMBINED STRESSES AND OTHER FACTORS

To this point, emphasis has been on uniaxial stress. In a structural part, this simplicity seldom exists; bending and twisting are likely to be present as well as tension.

Considerable effort has been devoted to devising a method for predicting fatigue strength under any system of stresses when data are available for one type of stress (such as unidirectional). No widely acceptable procedure has been developed. The present status of the

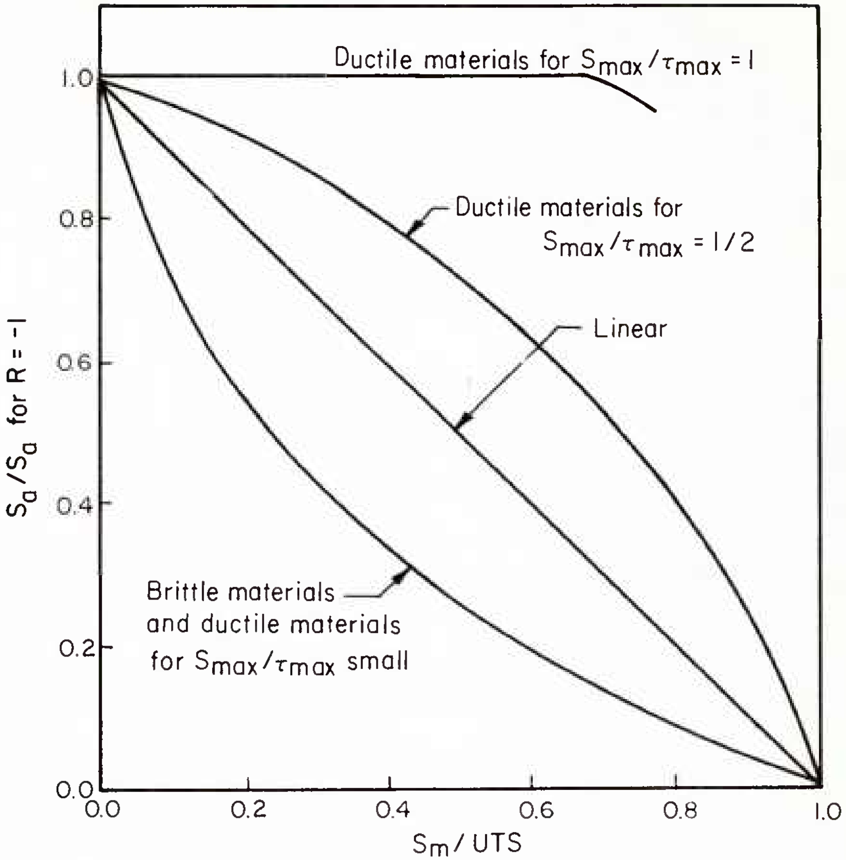

FIGURE 30.—Schemes for approximation of Goodman diagrams.

search is reviewed briefly in appendix A, and some references for further reading are listed there. Although there are no widely accepted conventions in multiaxial-fatigue testing, there is a tendency to follow the "combined-stress strength theories" developed for static design.

The emphasis in the following discussion in this chapter and in subsequent chapters concerning laboratory fatigue tests on materials is in terms of uniaxial stressing. For some situations, results of such tests are directly applicable; for other situations, uniaxial test results give relative values for different materials or processes that can be used as design guides in more complex loading. However, it should be emphasized that fatigue behavior under multiaxial stressing is seldom quantitatively predictable from laboratory results. It is necessary to define the fatigue strength of a material with specification for the type of stressing as well as for the combination of mean stress and alternating stress and the lifetime involved.

Many other factors influence fatigue behavior and necessitate consideration in use of the data in design. Among these are: stress concentrations, crack-propagation rates, cumulative damage behavior, temperature, corrosion, and fretting. These are discussed in subsequent chapters, and definitions and nomenclature are given as appropriate.

SCATTER AND STATISTICS

Up to this point we have considered the idealized situation in which, if all the pertinent factors are delineated, the fatigue strength of a material (or component) can be considered determinable. Actually, if many samples are tested, the results do not fall on a smooth *S–N* curve. Figure 31 illustrates the "scatter" that is more likely to be observed. When it is recalled that fatigue behavior is dependent on minute imperfections always existing in metals and alloys (even the best that can be produced under laboratory conditions), such scatter is not surprising.

Variability in static strength values has long been recognized in design. There are, however, some additional factors of particular concern in fatigue:

1. The scatter for fatigue strengths is larger than for static strengths. For the material used in figure 31, the ultimate tensile strength had a maximum of 100.6 ksi and a minimum of 94.5 ksi—a range of about ±3 percent about the mean. If one takes, as a crude measure of uncertainty of fatigue strength, the vertical distance between dotted lines in figure 31, a range of about ±17 percent is found at 500,000 cycles.
2. In regions of low slope of the *S–N* curve, the scatter implies a very large variation in lifetime. In the results shown in figure 31, the range at 32,000 psi is from 300,000 cycles to 4,000,000 cycles—somewhat more than an order of magnitude.
3. As might be expected and as is illustrated more completely in later chapters, very many factors influence fatigue lifetime significantly, so that the variation found in a laboratory test of a material is extremely difficult to relate to service conditions. The scatter found in fatigue tests of complex structures often seems less than that in the laboratory test specimens of the material involved. This may be rationalized by supposing that every structure contains weak places and these crack according to the lower part of the material scatter

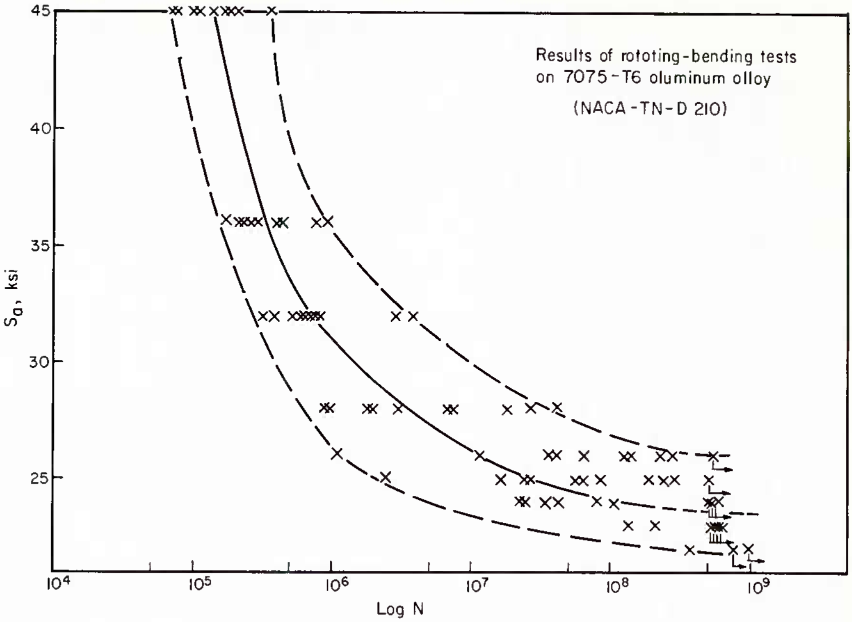

Figure 31.—Scatter in fatigue-test data.

band. Another factor is the likelihood of imperfect surface finish and assembly of components. (See later chapters on components and structures.)

Thus, it is imperative to take cognizance of the statistical nature of fatigue-test results.

Whenever there is scatter in the results of a test to failure, a serious question arises as to what value to use in design. This may be easily illustrated by considering the nine data points at the 36,000 psi level of figure 31. For the nine specimens, there was apparently a 50-percent chance that any one would survive 306,000 cycles (the median value through which the curve is drawn). What is the chance of another specimen (the material to be used in the structure) surviving this many stress repetitions? What is the lifetime which has a higher probability than 50 percent for survival and which might be acceptable in practice? The reasoning employed in statistics (discussed briefly in app. C) permits more precise formulation of such questions and, when enough data are available, useful answers. It is possible to plot, from information such as shown in figure 31, probability-of-survival (*P–S–N*) curves as illustrated in figure 32.

As the very many data needed for describing materials in such detail become available, it can be anticipated that statistical repre-

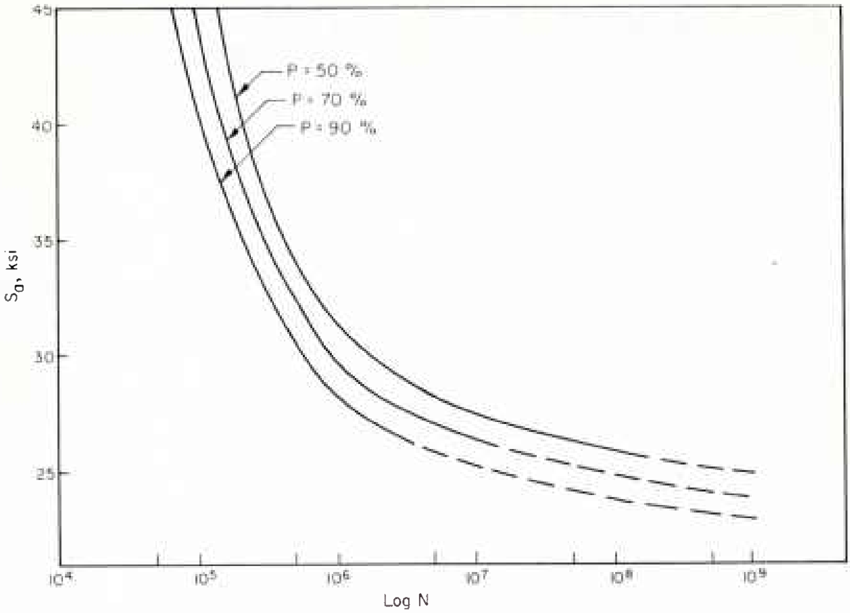

Figure 32.—*P-S-N* curves.

sentations of fatigue properties will become more extensive. A start toward this is provided in reference *3*. However, there are not yet enough data for statistical qualification of the fatigue of many materials subjected to various conditions.

The role of statistical analysis in engineering to prevent fatigue failure is difficult to assess. On one hand, there is the argument that scatter and probability considerations are inherent in the fatigue problem and that no analysis can be dependable without the use of statistical reasoning. On the other hand, economic considerations make it wholly impractical to run a large number of fatigue tests on a complex structure; for this reason mathematical statistics are sometimes avoided in engineering practice. There are at least two potential uses of mathematical statistics for engineering. One is the use of statistical methods in laboratory comparisons of materials or of fabrication practices when it is economically possible to test a reasonably large number of supposedly identical speciments (*4*). The other is the use of mathematical statistics in analytical studies of methodology in design (*5*). In this second use of statistics, it may be unreasonable to suppose that a measure of scatter (for instance, the standard deviation of log *N* divided by the mean value of log *N*) for a material is equivalent to that for a structural component fabricated from this material. Other factors in the fabricated structure may override the variability inherent in the material. Such factors must be considered in any stochastic treatment of the probability of survival of a structure.

For the most part, subsequent discussion in this book will avoid the use of statistical terms. Fatigue-strength values are usually nominal stresses at median lifetimes, although the median values may be crudely estimated. This is largely an admission of the lack of information sufficient to warrant elaborate statistical analysis. However, the existence of variability in material should be emphasized as a precaution to be considered in the application of results from laboratory fatigue tests.

AVAILABLE DATA ON FATIGUE PROPERTIES OF MATERIALS

Most of the information from laboratory fatigue tests of materials has been obtained under expedient limitations such as:

1. With simple types of applied stress.
2. With few specimens from a statistical point of view.
3. In what might be called an "intermediate" range of lifetime.
4. At cyclic ranges of speed from about 1,000 cpm to 3,000 cpm (with rotating-bending tests on small specimens frequently run at higher speed, such as 10,000 cpm). Much lower speeds are used for low-cycle tests (see ch. VII).
5. With rupture as a criterion of failure.
6. With specimens whose size, shape (including notch configuration), and surface finish are not yet entirely standardized.

An engineering objective is to deduce, from an accumulation of such data, practical information such as the relative merits of candidate materials for a structural part. Such deduction nearly always requires interpolation among or extrapolation from available laboratory fatigue test results.

Some of the implications of the first two limitations listed have been mentioned. Data under a few conditions of mean stress or of R-values are extended by Goodman diagrams according to the information available. There are suggestions of limited merit for estimating behavior for one stress system (such as torsion) from data under another (such as uniaxial tension). Cumulative damage, under varying-amplitude stressing, is discussed in chapter VI. Scatter can be handled by techniques of statistics although often only with the help of unproved assumptions.

The other limitations in the list above are sometimes overlooked but deserve attention. Several points can be illustrated by reference to

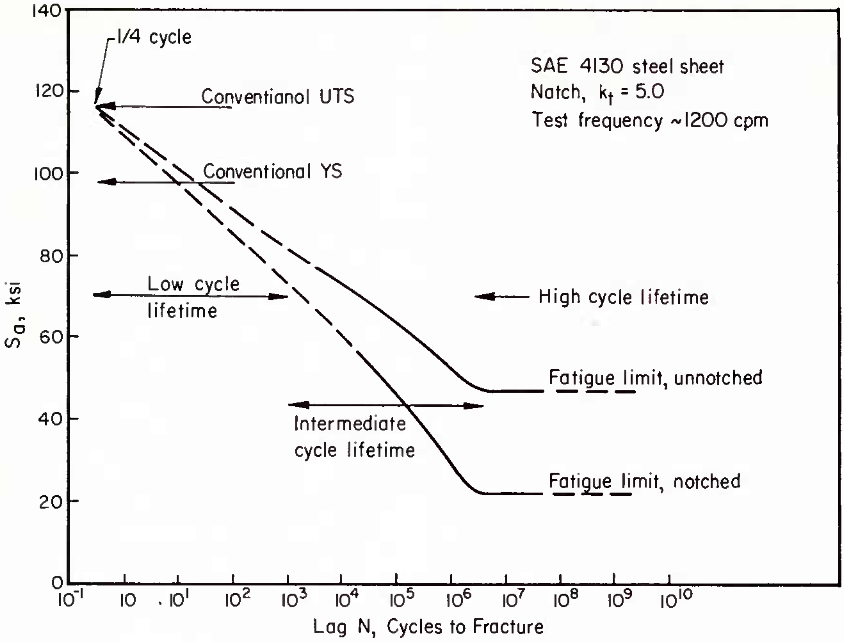

FIGURE 33.—Results of a conventional laboratory fatigue test.

figure 33 in which the solid lines show the results of a reasonable typical laboratory fatigue test on a steel commonly used in aerospace structures. With the testing speed used, it was almost meaningless to observe lifetimes less than about 1,200 cycles and too time-consuming to run to more than about 12,000,000 cycles. Extrapolation to shorter lifetimes is very questionable, partly because the nominal stress values are higher than the proportional limit for the steel. "Low-cycle" fatigue studies are relatively recent, and some of the procedures involved are discussed in chapter VII. Extrapolation to very long lifetimes is of interest when such things as acoustic fatigue or the possible effect of very small gust loadings on an airframe are being considered. There are not many test results available to justify a long extrapolation. Moreover, there are current concerns about the possibility of damage at stresses below the fatigue limit. For example, it is known that a crack started at a relatively high stress may propagate at a stress well below the fatigue limit for an unnotched specimen.

The effect of speed of testing is usually considered negligible (1) for most metals and alloys in the intermediate-lifetime range, (2) at room temperature, and (3) in a noncorrosive environment. It must be emphasized that for other conditions there are *speed effects* (usually fewer cycles being endurable at a specified stress level for a slower rate of loading). The limitations within which speed of cycling is unimportant are not sharply defined.

Unless otherwise stated, the "cycles to failure" usually denote rupture of the laboratory test piece. This is a convenient criterion, but it is influenced by such factors as the size of the specimen, the type of testing machine (constant-load or constant-deflection), and the kind of material (brittle or ductile). There can be distinct differences between failure criteria; for example, in some cases the first visible crack might be considered failure while in other cases failure might be considered rupture. In some instances, use of different criteria of failure may provide quite different comparative ratings of materials.

Some aspects of notch-size effect are noted in the next chapter, and some aspects of surface finish are mentioned in appendix D. For the present, it must be emphasized that there are no widely accepted standards. Consequently, comparison of the results of tests made with different specimen surfacing or of tests with different specimen configurations is speculative.

TESTING COMPONENTS AND STRUCTURES

Fatigue tests of materials follow some conventions that have become fairly well established (plotting of S–N curves and of Goodman diagrams, and statistical analysis when data are sufficient). Extensions to high-stress and low-cycle conditions, to combined-stress loadings, and to wider ranges of frequency and environment are under way, and conventions and nomenclature are being established for these.

Component testing involves additional complications and is accordingly much less standardized. Testing is usually limited by economics of time and cost to few specimens, and there is little chance for studying scatter. Often the design objectives determine the type and range of loading, so that tests may be run at a selected mean load for several values of load amplitude. Detailed stress analysis is frequently extremely difficult; accordingly, in place of S_a or S_a/UTS, a common variable is load as a fraction of design limit load. Results are often plotted in analogy to those indicated for material tests; for example, load versus log N curves in analogy to S-log N curves.

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CHAPTER IV.
STRESS CONCENTRATIONS

INTRODUCTION

Fatigue failures in structural parts always occur at regions of high local stress called "stress concentrations."

There are many potential sources of stress concentration in a structure. Some may be in the material itself; for example, inclusions or regions of local variation in heat treatment or, in a composite material, locations where elements of different properties join. Variations in local stress may also occur at spots where previous stressing (for example, in forging or in extrusion) has produced differences in coldwork and residual stresses. Many stress concentrations are associated with design configurations: fillets, holes, groves, pin-connections, etc. Such types of stress concentration may be considered geometrical stress concentrations. Still more causes of high local stresses may occur in fabrication and assembly: rough surfaces, imperfect alignment of bolts or rivets, as well as accidental scratches, gouges, and dents. An extreme and classic example is the stress concentration produced by an inspector's indentation stamp of approval. Still further complications frequently arise in service, such as fretting, local corrosion pitting, and local damage caused by interference and unanticipated impact loads.

It is clearly unreasonable to make engineering stress analyses so detailed as to consider each cubic millimeter of a large and complex structure. Therefore, the usual procedure is to compute "nominal stresses," considering large volumes as homogeneous (and usually isotropic), and to provide some kind of allowance for stress concentrations by appropriate "stress-concentration factors." This procedure originated and is most easily discussed in regard to geometrical stress concentrations.

ELASTIC STRESS-CONCENTRATION FACTORS

It has been noted that fatigue is dependent upon plastic behavior, even though this may be so localized that most of a fatigue-failed part was stressed elastically and the fracture appears "brittle." Therefore, elastic stress-concentration factors cannot be expected to provide all information pertinent to fatigue behavior. Local plastic regions are very important.

Nevertheless, there are two good reasons for starting our discussion with a review of stress-concentration factors in the elastic region. One is that a great deal of information is available about these, and they are often discussed in the literature. A more important reason is that they are used as guideposts in several ways: (1) as the basis of considerable nomenclature, (2) as indicators of the relative severity of various notches, and (3) as a starting point for some considerations of local plasticity near a notch. The latter use requires precautions, some of which are noted in a later section.

Open Central Circular Hole in a Sheet in Tension

One of the shapes for which extensive analysis has been carried out is that of a central hole in a sheet under tensile loading. For the sheet very thin (so the problem can be considered in two dimensions—that is, a state of "plane stress") and very wide and long compared to the radius, ρ, of the hole, and subject to uniform tensile stress in the y-direction, the stress field is given by

$$S_r = \textit{radial stress} = \frac{S_n}{2}\left[\left(1 - \frac{\rho^2}{r^2}\right) - \left(1 - 4\frac{\rho^2}{r^2} + 3\frac{\rho^4}{r^4}\right)\cos 2\theta\right]. \tag{1}$$

$$S_\theta = \text{tangential stress} = \frac{S_n}{2}\left[\left(1 + \frac{\rho^2}{r^2}\right) + \left(1 + 3\frac{\rho^4}{r^4}\right)\cos 2\theta\right]. \tag{2}$$

$$S_{r\theta} = \text{shear stress} = \frac{-S_n}{2}\left(1 + 2\frac{\rho^2}{r^2} - 3\frac{\rho^4}{r^4}\right)\sin 2\theta, \tag{3}$$

where

S_n = stress remote from hole = $P/2wt$
$2w$ = sheet width
t = sheet thickness.

Here the polar coordinate system has its center at the center of the hole, and $\theta = 0$ in the positive x-direction.

Along the x-axis,

$$S_r = S_x = \text{stress transverse to } P = 3\,\frac{S_n}{2}\left(\frac{\rho^2}{x^2} - \frac{\rho^4}{x^4}\right). \tag{4}$$

$$S_\theta = S_y = \text{stress in P-direction} = \frac{S_n}{2}\left(2 + \frac{\rho^2}{x^2} + 3\frac{\rho^4}{x^4}\right). \tag{5}$$

$$S_{r\theta} = S_{xy} = 0. \tag{6}$$

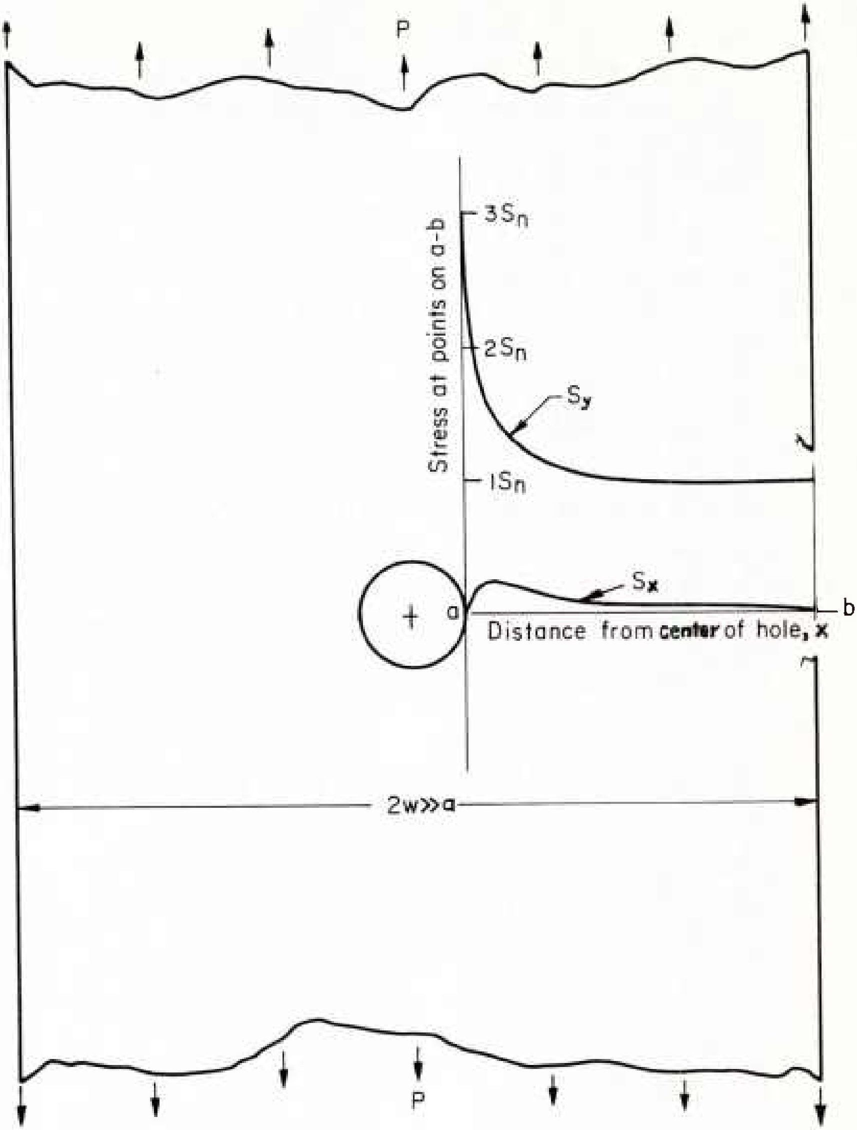


Figure 34.—Stress distribution around a hole in a thin sheet under tension.

Figure 34 shows a plot of equations 4 and 5. The maximum *tensile stress* occurs at the edge of the hole and has a value of

$$S_{max} = S_{y.x=\rho} = 3S_n.$$

The ratio

$$K_t = S_{max}/S_n = 3 \tag{7}$$

is the value usually given for the stress-concentration factor of a circular hole in an infinitely wide sheet.

The maximum compressive stress for tension loading occurs at $\theta = \pi/2$, $r = \rho$, is transverse to the loading direction, and has the value

$$S_x = -S_n. \tag{8}$$

All the above equations are valid if the load, P, is compressive, except for the proper sign reversal. Thus, for *compression loading*, the *maximum* tensile stress is given by equation 8. This means that, if fatigue cracks propagate perpendicular to the direction of maximum tension stress, one might expect:

1. For zero-tension loading, cracking in the x-direction outward from the edges of the hole
2. For zero-compression loading, cracking in the y-direction outward from the edges of the hole
3. For tension-compression loading, a cracking direction dependent on the load ratio and on the isotropy of strength properties of the sheet.

This behavior has been observed.

The equations above are appropriate strictly to a hole in an infinitely wide sheet. Howland (*1*) has calculated the values shown in table 1 for a central circular hole in a thin sheet of finite width under tension loading. Values of K are appreciably higher when the hole diameter becomes greater than about 1/10 the sheet width. As implied by entries in the last column, this increase is due to the net-section nominal stress becoming significantly higher that the gross-section nominal stress.

Other Conditions Involving Circular Holes

Stress distributions have been worked out, largely on the basis of approximations and numerical computations, for a number of situa-

TABLE 1.—*Stresses at edge of circular hole in sheet of finite width*

$2\rho/w$	$K_t = S_{y,\,max}/S_n$	$K_t\ (1 - \rho/w)$ (a)
0	3.00	3.00
.1	3.03	2.73
.2	3.14	2.51
.3	3.36	2.35
.4	3.74	2.24
.5	4.33	2.17
	ρ = hole radius	
	$2w$ = sheet width	
	$S_n = P/2wt$	

(a) This may be considered as a differently defined stress-consideration factor:

$$K' = \frac{S_{max}}{S_{nom}\ (\text{net section})}.$$

tions involving holes. Savin (*2*) has an extensive account of these, and many of the results are conveniently summarized in a series of articles by Griffel (*3*). Two examples will illustrate some important points.

Figure 35 shows some values of peak stress in a wide plate containing two circular holes and loaded in tension. The stress is particularly high at the outermost edges of the holes.

An effective method for reducing the stress concentration around a hole is to weld a ring around the hole. While details of numerical examples given in references *2* and *3* are somewhat lengthy, some of the conclusions can be listed briefly:

1. The influence of a strengthening ring is quite local and practically disappears about five diameters away.
2. With increasing rigidity of the ring (relative to the plate), the stress concentration in the plate decreases, but the stress in the ring increases.
3. It is often possible to choose a ring of such rigidity that the stress-concentration factor in the plate with the strengthened hole is reduced to unity.

These conclusions apply only to a ring integrally fastened in the hole and not to press-fitted rings.

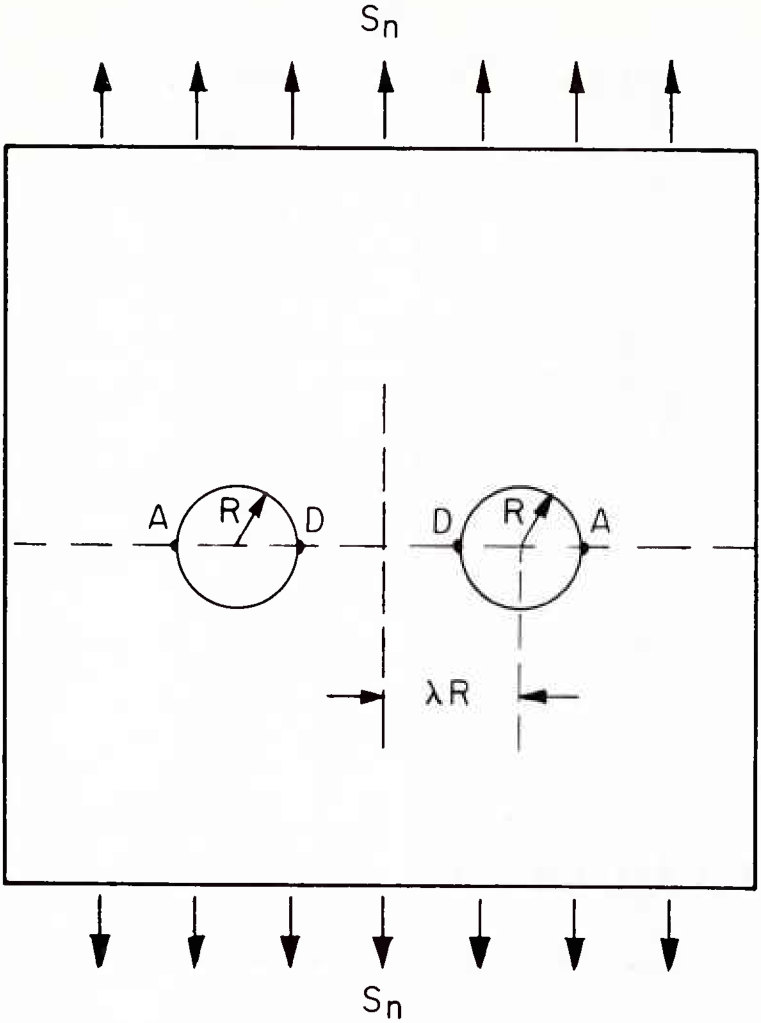
It must be emphasized that when load is transmitted into a sheet through a hole (pin-loading) much higher stress-concentration factors usually apply than for open holes. Theocaris (*4*) gives values shown in figure 36 for a strip loaded by a moderately close-fitting pin in a circular hole. Persson (*5*) has discussed the factors involved in the elastic stress distribution around a close-fitting pin. In addition to the simple parameter ρ/w in figure 36, the following are likely to be important in practical cases: the edge distance, the relative moduli of pin and sheet, and residual stresses in any forced fit.

Square and Rectangular Cutouts

A number of situations involving holes of other than circular shape have been investigated. Details are too extensive for enumeration here, but much information is available in references *2* and *3*. One outstanding generality is that sharp corners are to be avoided to keep stress-concentration factors low.

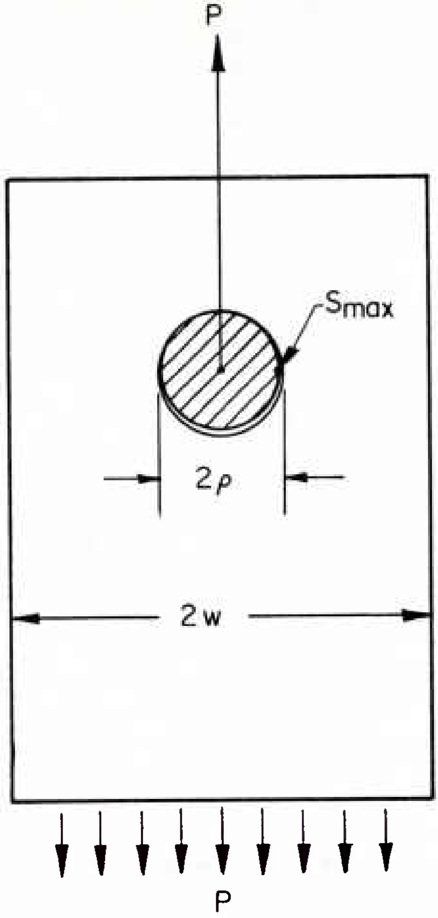
Central Elliptical Hole in a Sheet in Tension

Another geometry for which extensive analysis has been carried out is that of an elliptical hole in a wide, long, thin sheet under tensile loading. Equations for the entire stress field are complex (see reference *6*). However, for an ellipse whose major axis is along the x-axis, in a



λ	S_A/S_n	S_D/S_n
0	3.87	∞
1.5	3.15	3.26
2	3.07	3.02
3	3.02	2.99
5	3.00	3.00
∞	3.00	3.00

FIGURE 35.—Stress concentrations for two adjacent holes.



$S_n \equiv P/2wt$

$\frac{\rho}{w}$	$K_t = S_{max}/S_n$
0.2	6.55
0.3	5.25
0.4	4.96
0.5	5.06

FIGURE 36.—Stress concentration in a pin-loaded hole.

sheet loaded in the *y*-direction, the stresses along the *x*-axis are given by

$$S_x = S_n\{-A + Ax[x^2 - a^2 + a\rho]^{-1/2} - Ba^2x[x^2 - a^2 + a\rho]^{-3/2}\} \tag{9a}$$

$$S_y = S_n\{1 - C + Cx[x^2 - a^2 + a\rho]^{-1/2} + Ba^2x[x^2 - a^2 + a\rho]^{-3/2}\} \tag{9b}$$

$$S_{xy} = 0$$

where $A = (1 - \sqrt{\rho/a})^{-2}$

$B = (1 - \sqrt{\rho/a})^{-1}$

$C = (1 - \sqrt{\rho/a})^{-2}(1 - 2\sqrt{\rho/a})$

a = semimajor axis, b = semiminor axis; $(a > b)$.

ρ = radius at $x = a, y = 0$

Figure 37 illustrates these stress distributions.

The stress S_y has, at $x = a$, a maximum value

$$S_{y\,max} = S_n \ (1+2\sqrt{a/\rho}),$$

so that the stress concentration factor for an elliptical hole, under tension loading perpendicular to the major axis, is often written

$$K_t = S_{y\,max}/S_n = 1+2\sqrt{a/\rho} \tag{10}$$

This shows the high value to be expected for a long, sharp ellipse.

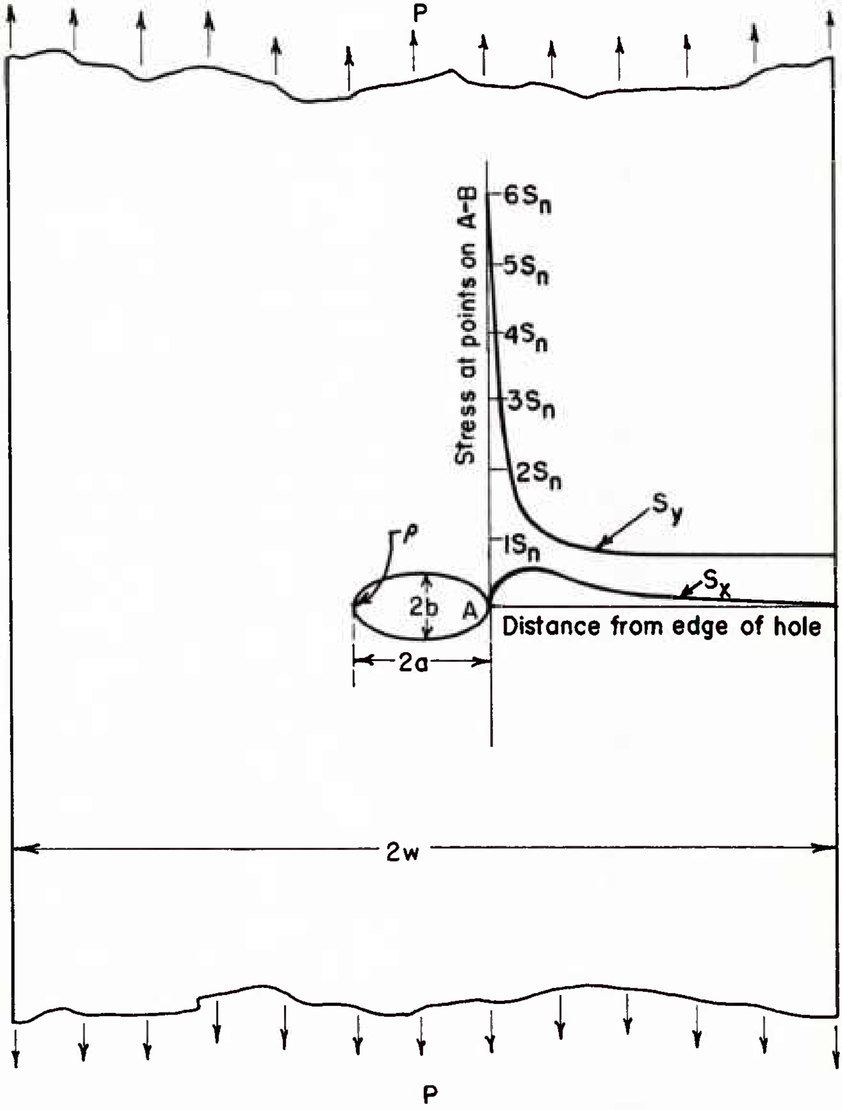


FIGURE 37.—Stress concentration at elliptical hole in thin sheet under tension.

For an ellipse for which $\rho/a \rightarrow 0$, which has been considered an approximate description of a crack, the second term in each of the equations 9a and 9b is dominant for a region near the crack tip and gives values close to the approximations developed by Irwin (see reference *18*).

Stress concentration factors have been developed for elliptical holes under other types of loading (see reference *11*).

Calculation of the stress distribution around an ellipse in a sheet of finite width presents extreme difficulty. An approximation suggested by Dixon (*8*) is

$$K_t = S_{max}/S_n = \left(1+2\sqrt{a/\rho}\right)(1-a^2/w^2)^{1/2}, \qquad (11)$$

where

$$S_n = \text{gross area nominal stress} = P/2wt.$$

Experiments indicated this to be valid for small values of a/w (less than about 0.4).

Stress Concentrations for Other Geometrical Notches

Figure 38 shows the stress distribution near a deep hyperbolic notch in a round bar under tensile loading along its axis. Such a three-dimensional notch introduces another factor—a sometimes significant biaxiality of stress at the notch root and a triaxial stress-state just inside. One(1) stress-concentration factor may be written

$$K_t = \frac{S_{max}}{S_n} = \frac{1+n}{1-0.6n+n^2}\,(n^2-0.2n+2.5), \qquad (12)$$

where

$S_{max} = S_l$ at $r = d/2$,
$S_n = 4P/\pi d^2$,
$n = (1+d/2\rho)^{1/2}$,
$\nu = \text{Poisson's ratio} = 0.3$.

Note that in this case S_n equals the nominal net-area stress at the section through the notch root.

Figures 39 through 42 show values of K_t for four other situations illustrative of the information available. More data of this kind can be found in references *10, 11,* and *12*. However, the four examples indicate several points:

---

(1) Attention is usually focused on the highest tensile stress, and the K_t most often quoted is the ratio of this to a nominal tensile stress. Other factors might be defined; for example, the ratio of the highest shearing stress to some nominal stress. As previously mentioned for the case of a sheet with a circular hole, it may sometimes be important to consider the entire stress pattern to bring out factors critical in fatigue.

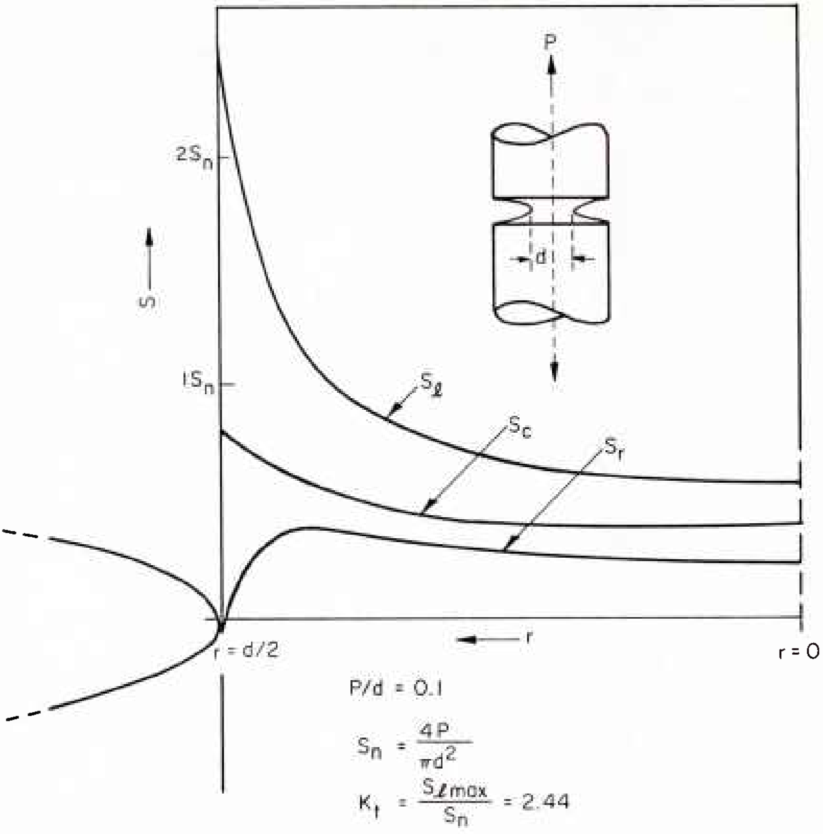


FIGURE 38.—Stress distribution in round bar with deep grove under tension.

1. The value of K_t may vary with the kind of loading. (Compare values in tension, bending, and torsion for a filleted shaft with $r/d = 0.12$ and $D/d = 2$.)
2. In using such graphs, care should be taken to note whether the referenced nominal stress is defined on the basis of gross area or of net area. (For the round hole discussed previously, the nominal stress was in terms of gross area; for the filleted round bar, the nominal stress is in terms of the smallest cross section.)
3. Values are usually very sensitive to the notch root radius. Very high values may be associated with small values of r—sharp notches are dangerous.

As discussed subsequently, direct use of values of K_t in design to prevent fatigue failure is subject to many limitations and some uncertainties. However, values such as illustrated here and more extensively

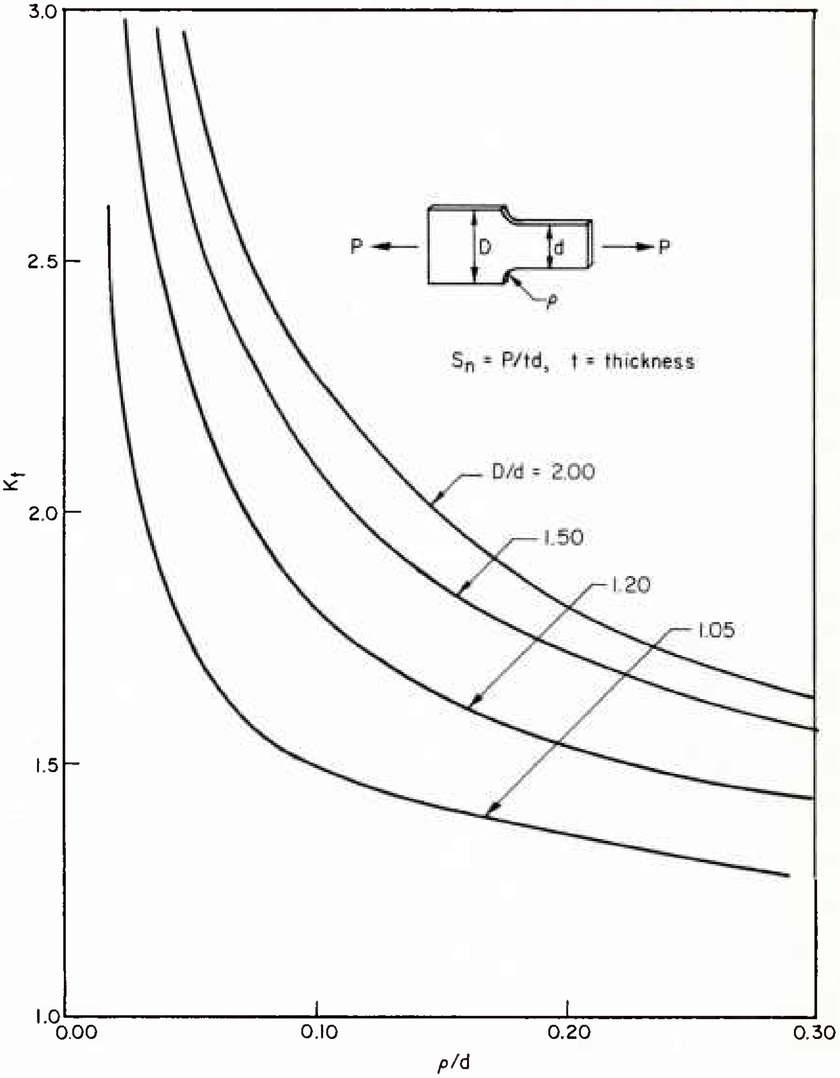


Figure 39.—Stress-concentration factors for flat bar with fillet under tension (based on photoelastic data).

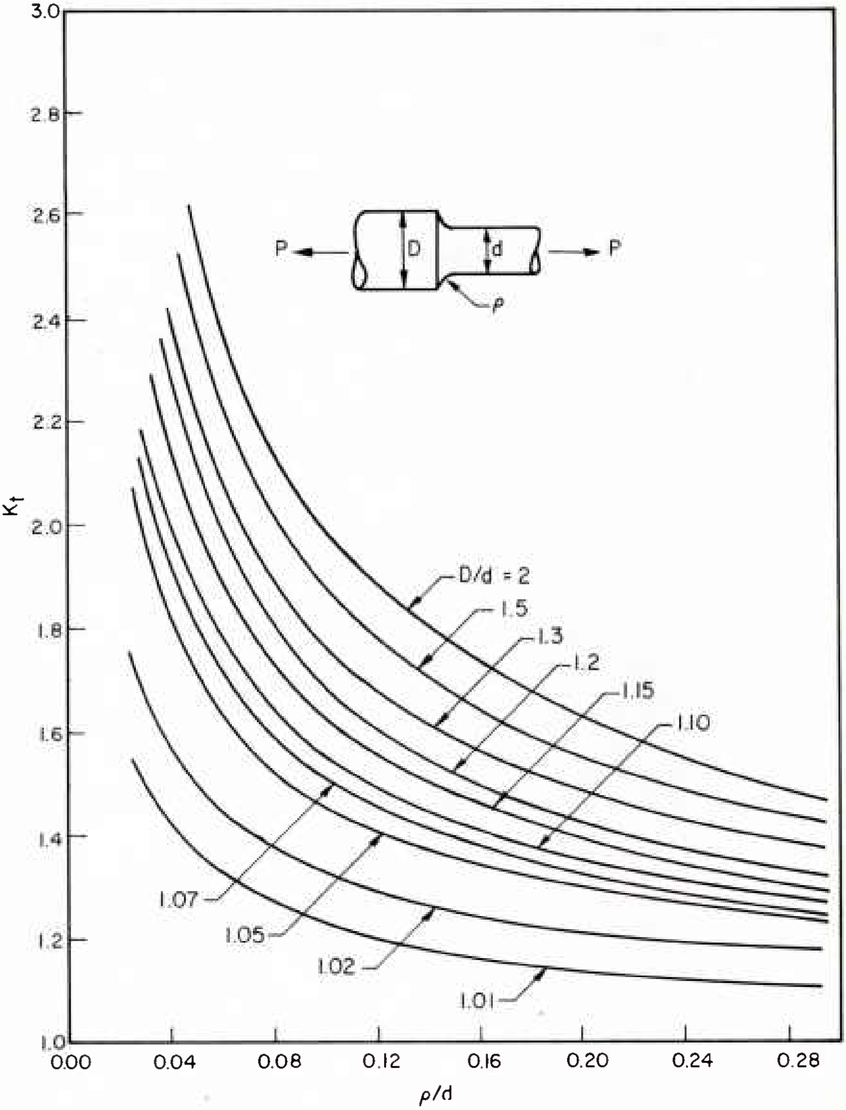


FIGURE 40.—Stress-concentration factors for a round bar with fillet under tension.

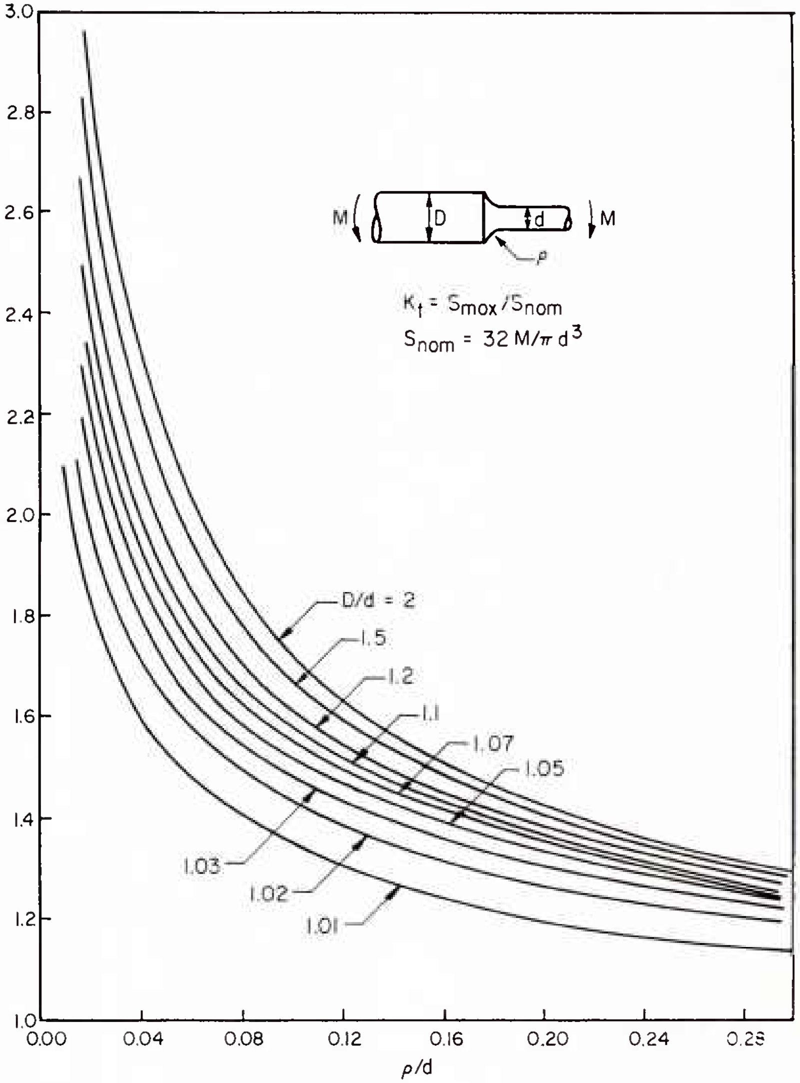


FIGURE 41.—Stress-concentration factors for a round bar with fillet under bending.

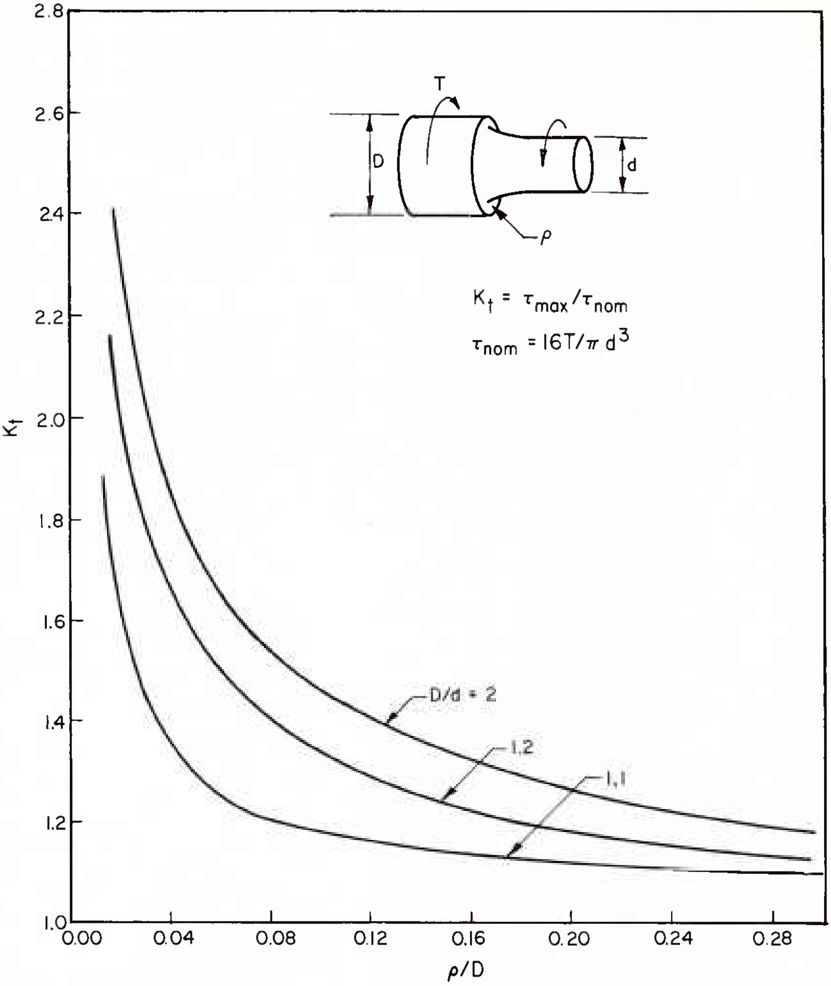


FIGURE 42.—Stress-concentration factors for a round bar with fillet under torsion.

tabulated in the literature are useful in avoiding unduly high local stresses, in comparing design configurations, and in furnishing indexes against which results of tests of components may be compared.

STRESS AND STRAIN CONCENTRATION IN THE PLASTIC REGION

Quick calculations using the elastic-stress-concentration factors quoted in previous illustrations imply that, if a specimen with a sharp notch is moderately loaded, the material near the notch root may be stressed beyond its yield strength. In this event, the stress distribution will be different from that in the elastic case, and neither the local-stress distribution nor the stress-concentration factor can be accurately evaluated on such bases as previously described. Local yielding redistributes stress; so the peak stress is lower in ratio to the nominal stress than it would be if all the material remained elastic.

Rigorous mathematical treatments of stress distribution around notches under conditions in which local regions are plastic are extremely difficult. Figure 43 (adapted from reference 2) shows by shaded areas the plastic regions (sometimes called plastic "enclaves") at the edges of a hole in an aluminium-alloy sheet loaded in tension to a nominal stress of 6/10 of its yield stress. Within this region the stress and strain distributions follow the plastic characteristics of the material, while in the remainder of the sheet, the distributions follow

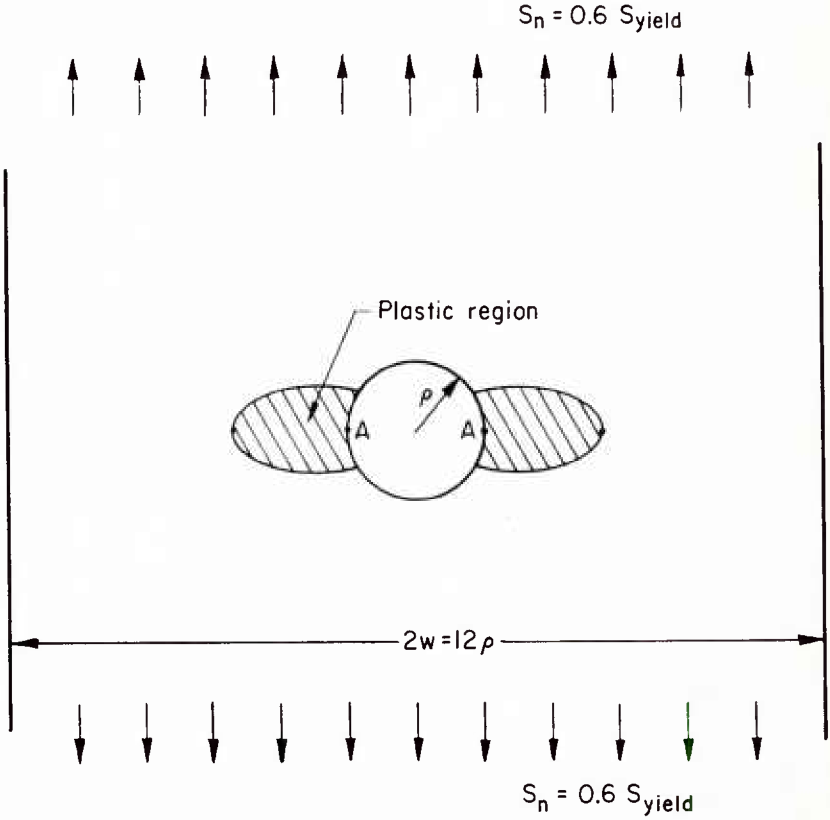


FIGURE 43.—Plastic regions around a stress concentration.

its elastic behavior. The laws of behavior in each situation can be fairly well described mathematically. A major difficulty is that the boundaries of the plastic regions are not easily determinable.

Figure 44 shows the distributions of stress and strain (in the loading direction) along a transverse line through the center of the sheet in the example of figure 43. For comparison, dotted lines indicate the distributions (from equation 5) that would exist if the material had remained elastic. Two effects of the local plasticity are: (1) a higher strain and (2) a lower stress near the edge of the hole than elastic theory would predict.

Relatively few mathematical solutions of elastic-plastic stress and strain distributions around notches have been reported. Dixon *(13)* has described results of a digital computer calculation of the stress field near the tip of a crack in a sheet of aluminum alloy loaded in tension. However, even numerical solutions are difficult to obtain and are scarce for situations of concern in regard to fatigue.

Several years ago, Stowell suggested, for a circular hole in a very wide plate under tension, a simple approximate treatment *(14)*. The stresses are written

$$S_r = \frac{S_n}{2}\left[\left(1-\frac{\rho^2}{r^2}\right) + \frac{E_S}{E}\left(1-4\frac{\rho^2}{r^2}+3\frac{\rho^4}{r^4}\right)\cos\theta\right], \qquad (13)$$

$$S_\theta = \frac{S_n}{2}\left[\left(1+\frac{\rho^2}{r^2}\right) - \frac{E_S}{E}\left(1+3\frac{\rho^4}{r^4}\right)\cos 2\theta\right], \qquad (14)$$

$$S_{r\theta} = -\frac{S_n}{2}\frac{E_S}{E}\left(1+2\frac{\rho^2}{r^2}-3\frac{\rho^4}{r^4}\right)\sin 2\theta, \qquad (15)$$

where

E = elastic modulus
E_S = secant modulus at r,θ.

These reduce to equations 1 to 3 when the material is elastic ($E_S = E$).

$$\begin{aligned} S_{max} &= S_\theta \qquad \text{at } \theta=0,\ r=\rho \\ &= S_n(1+2\ E_S/E). \end{aligned}$$

Thus, one may define

$$K = \frac{S_{max}}{S_n} = 1 + 2\ \frac{E_S}{E} \qquad (16)$$

This reduces to $K_t = 3$ for the elastic case. Figure 45 shows values computed for a specific case and compared with values obtained by measurements with very small strain gages. Hardrath & Ohman *(15)*

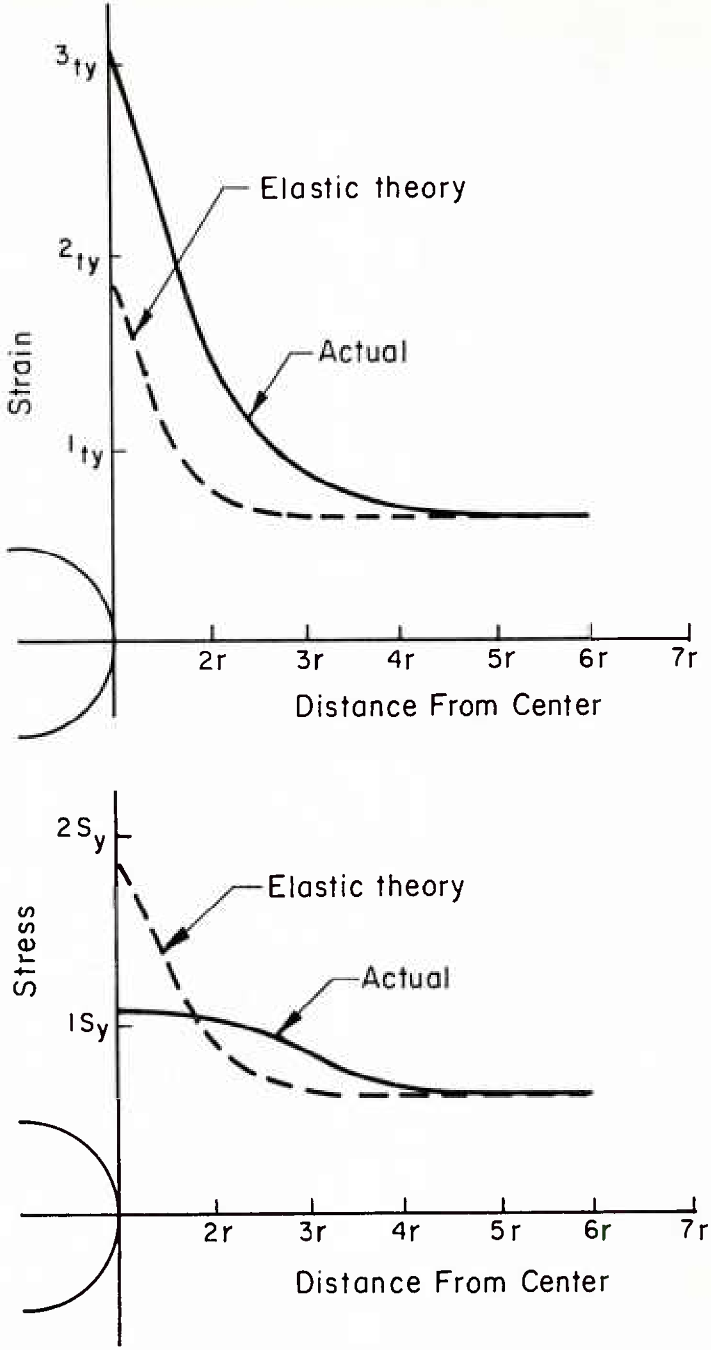


FIGURE 44.—Experimentally determined distributions around a hole with local yielding.

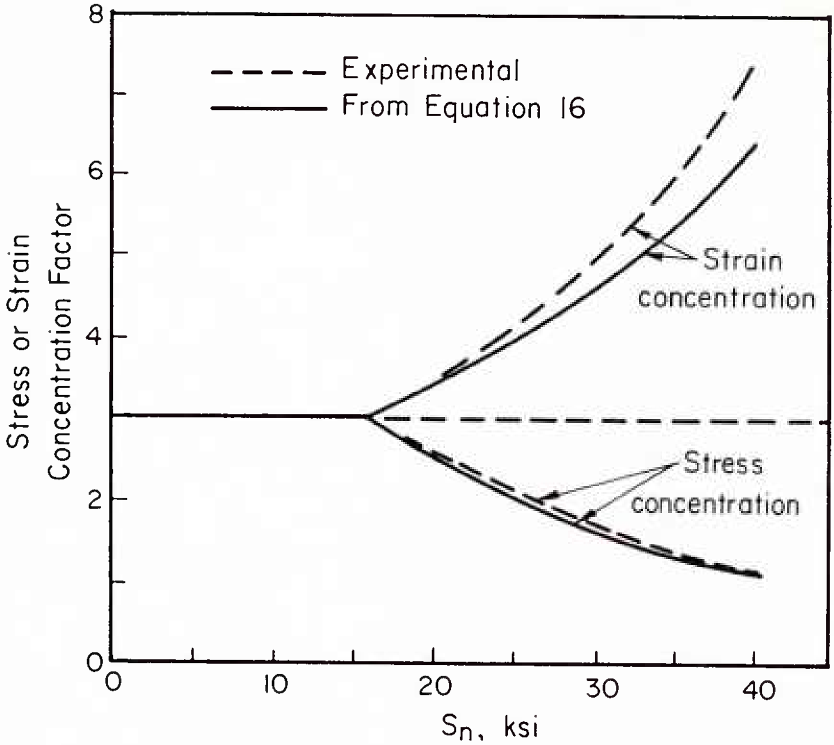


FIGURE 45.—Effect of plasticity on stress concentration.

suggested generalization of Equation 16 to define a "plastic stress-concentration factor":

$$K_p = 1 + (K_t - 1) \quad E_S/E, \tag{17}$$

where K_t is the theoretical elastic stress-concentration factor for the geometrical notch involved.

Dixon and Straunigan (*16*) have reported results of photoelastic measurements at the tips of sharp cracks. They observed that for many materials (mild steel being a notable exception) the elastic-plastic shear strain could be predicted by

$$Y = \left(\frac{E}{E_S}\right)^{1/2} Y_0, \tag{18}$$

where Y_0 is the value from an elastic solution.

Crews & Hardrath (*17*) have applied equation 17 and a similar relation for unloading to estimate the stresses at the root of edge-notched specimens of 2024—T3 aluminum alloy. Values thus computed were in good agreement with experimental data from small strain gages. In addition, it was noted that the local stresses stabilized quickly (in less than 30 cycles under some stress conditions).

A number of approximations have been suggested for estimating the length, S, of the plastic region in front of an advanced crack. Irwin *(18)* suggests

$$S = \frac{a}{2}\left(\frac{S_n}{S_y}\right)^2. \tag{19}$$

Dugdale *(19)* suggests

$$S = \frac{2a\ \sin^2(\pi S_n/4S_y)}{1-2\ \sin^2(\pi S_n/4S_y)}. \tag{20}$$

The first has been observed to fit approximately the behavior of an aluminum alloy, while the second fits some observations on a mild steel. *(13)*

An aspect of plastic deformation near a notch that is important in cyclic loading is the residual stress remaining after unloading from some high value. Figure 46[2] illustrates this for a notched sheet of an aluminum alloy under zero-to-tension loading. The curve marked S_{max} shows the longitudinal stress along a transverse section through the center. Near the notch, S_{max} exceeds the proportional limit, S_p, and there is a region (cross-hatched) of yielding. Upon unloading to zero, the stress at the edge of the notch goes to a compressive value. Hence, not only the stress amplitude but also the mean stress at the notch was affected by the combination of stress concentration (diminished from K_t by plastic deformation) and residual stresses introduced by the local deformation.

Details of elastic-plastic behavior in the regions of stress-concentrations are exceedingly difficult to calculate by rigorous mathematical techniques. The approximate estimations mentioned and the experimental indications of localized plastic zones, residual stresses and strain-hardening to an equilibrium state, clearly indicate qualitative behavior of undoubted significance to notch-fatigue behavior.

FATIGUE STRENGTH REDUCTION BY GEOMETRICAL STRESS CONCENTRATION

There is extensive evidence that geometrical stress-concentrations usually reduce greatly the (nominal stress) fatigue strengths observed in laboratory tests. The reduction appears to depend on several parameters, including the severity of notch (as indicated by an elastic stress-concentration factor, K_t), the notch sharpness (as indicated by

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[2] The values are based upon actual measurements.

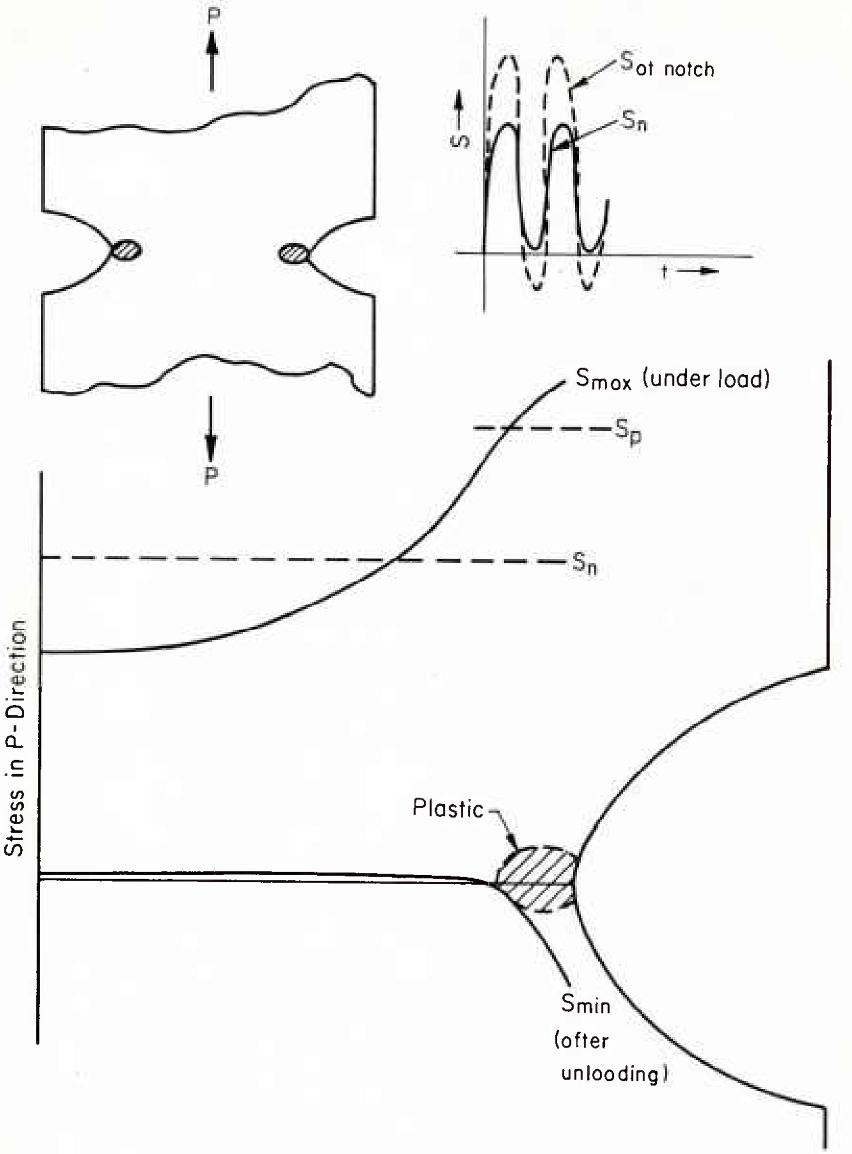

Figure 46.—Residual stress after unloading a notched specimen.

root radius), any residual stresses introduced in preparing the specimen, the loading, and the environment (including temperature).

The following examples of laboratory fatigue test results illustrate the evidence available. The examples are divided into two groups: (1) fully reversed loading and (2) loading not fully reversed. For the first, conventions have developed in terms of a "fatigue-notch factor"; for the second, there are some ambiguities in nomenclature and engineering convention.

Fully Reversed Loading

Figure 47 shows fatigue curves for sheet specimens (without a notch and with a notch) under fully reversed axial loading. It is customary to denote the ratio

$$K_f = \frac{\text{nominal maximum stress, unnotched specimen}}{\text{nominal maximum stress, notched specimen, same lifetime}}$$

as a "fatigue-notch factor". If fatigue failure were wholly determined by the maximum local stress, one might expect that K_f would equal K_t (or possibly K_p at high stresses). Figure 48 shows values of K_p, K_t, and K_f for the specimen configuration of the tests for figure 47. In this figure the agreement between K_f and K_p is within experimental error for the mild ($K_t = 2$) notch, but not for the sharp ($K_t = 5$) notch. Figure 49 shows values of K_f and of K_t for long lifetimes and low stresses for two common materials in sheet form and with edge notches. Here is more evidence of increasing difference between K_f and K_t with increasing notch severity.

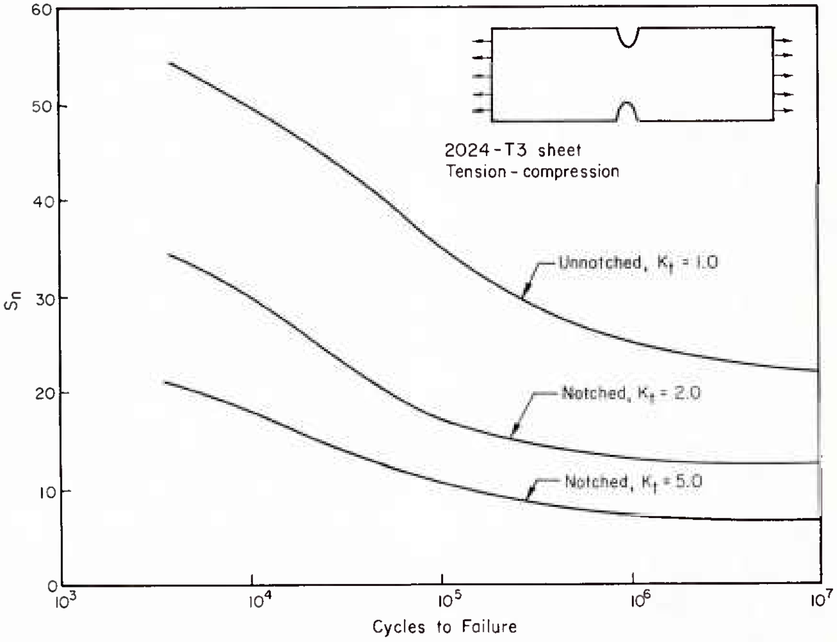

Figure 47.—Effect of notch on observed fatigue strength:

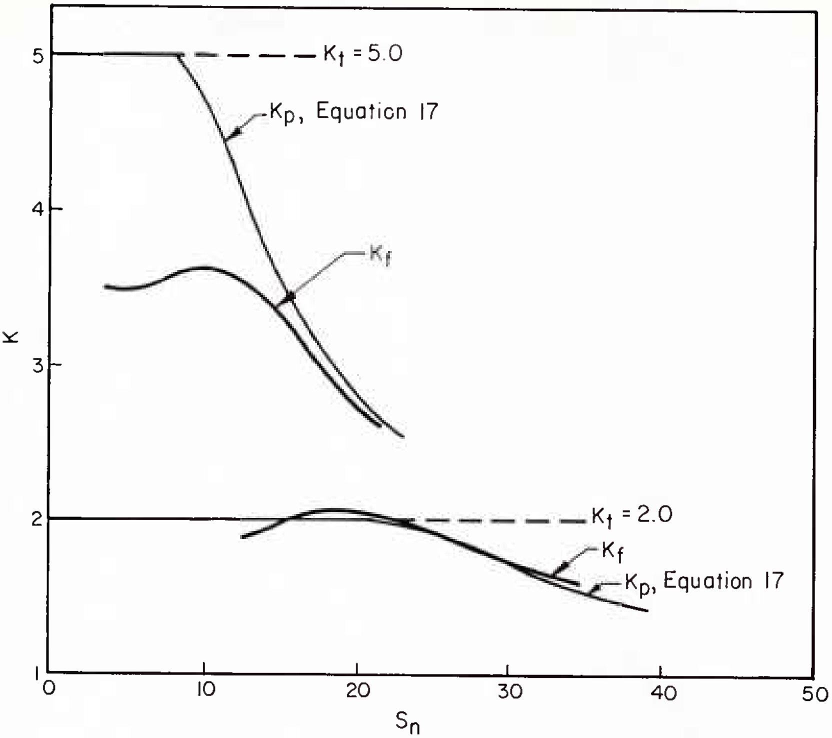

FIGURE 48.—Fatigue-notch factors (from fig. 47).

Neuber (*10*), in a theoretical treatment of stress concentration by notches, emphasized the fact that the sharper the notch (that is, the smaller its radius), the higher the stress gradient near the notch. This has been involved in different explanations of the divergence between K_f and K_t.

Neuber suggested that when there is a very high stress gradient the usual assumptions of isotropy and homogeneity are invalid. He proposes a model in which conventional elasticity relations are used down to regions of a limiting size (of linear dimensions ρ'); a high gradient across this material "building block" is not allowable. Speculations of this kind lead to a suggested reduced "effective stress-concentration factor":

$$K_N = 1 + \frac{K_t - 1}{1 + \sqrt{\rho'/\rho}}, \tag{21}$$

where ρ is the notch radius. Neuber suggested a value of about 0.018 inch for ρ'. Kuhn (*17, 18*) has analyzed fatigue test results for notched

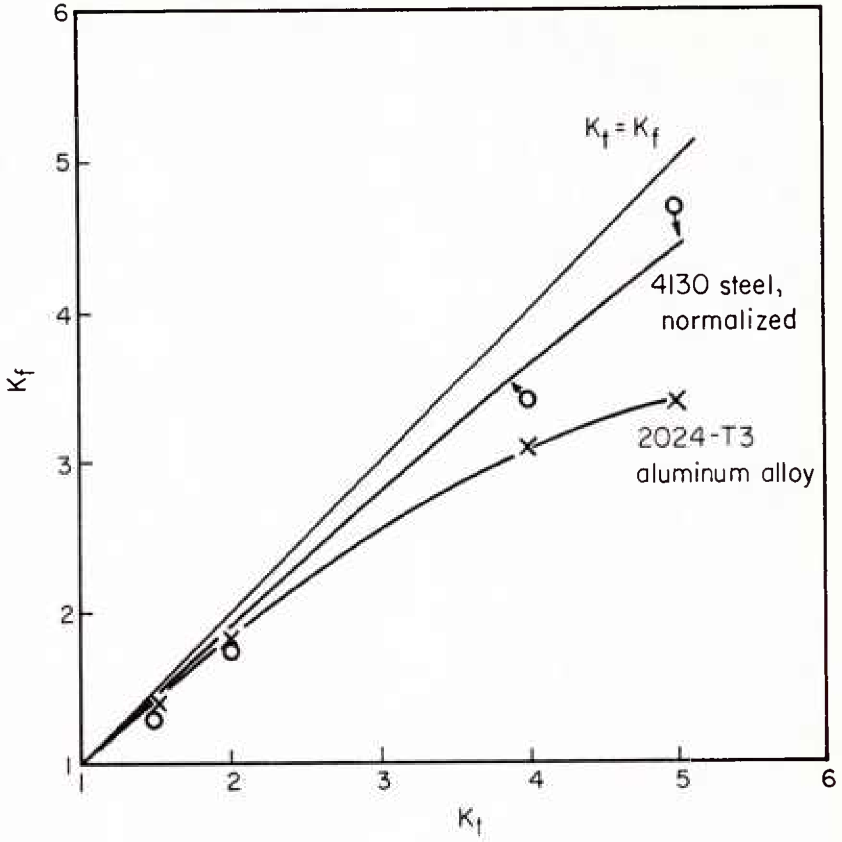

Figure 49.—Notch factors for two materials at 10^7 cycles of tension-compression.

specimens of several steels and aluminum alloys to find values for ρ' that might allow K_N to be used in design for K_f. He concluded that 0.02 inch fits most aluminum alloys (for some of the weakest, a larger value is suggested) and that the values shown in figure 50 are suitable for common low-alloy steels. Figure 51 shows results of fatigue tests on an aluminum alloy in which K_f values actually decreased with decrease in ρ as predicted (although not quantitatively) by equation 21.

Peterson (*8*) suggested an empirical allowance for the observed nonlinear variation of K_f with K_t. This involves the definition of a "fatigue-notch-sensitivity index,"

$$q=\frac{K_f-1}{K_t-1}. \tag{22}$$

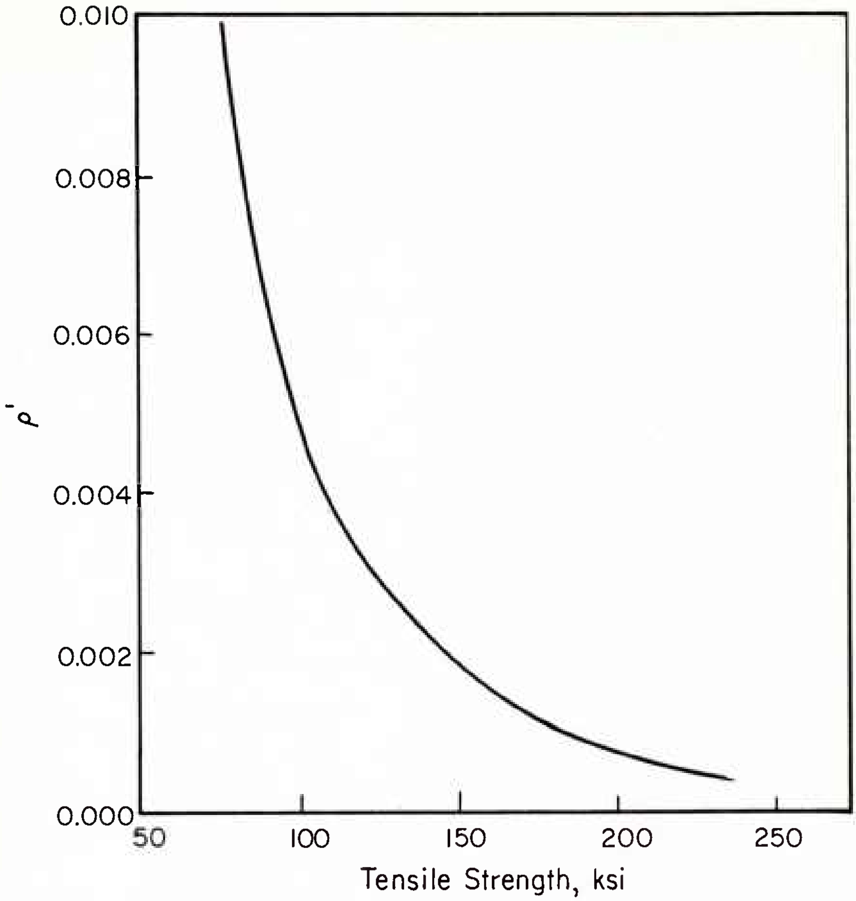

FIGURE 50.—Values of Neuber parameter for ordinary steels.(20)

This index varies from zero for a material insensitive to notches to unity for a material with "full" sensitivity. Figure 52 shows observed values for q for several steels. Equation 21 would indicate

$$q = \frac{1}{1 + \sqrt{\rho'/\rho}}. \tag{23}$$

Values from equation 23, using the indicated values of ρ', are shown as dotted lines in the figure.

Peterson has also suggested an approach based on failure at a distance δ inside the surface; this involves the stress gradient and leads to

$$q = \frac{1}{1 + a/\rho}, \tag{24}$$

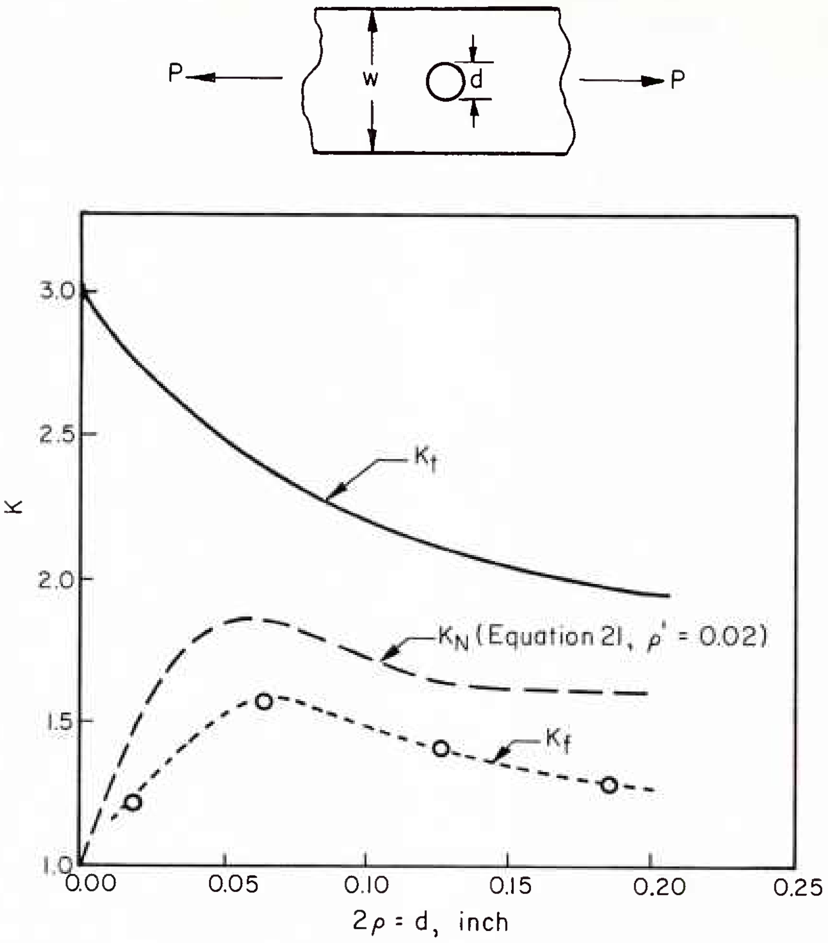

FIGURE 51.—Fatigue-notch factors in a particular test.

where "*a*" is a material constant. Heywood (*19*) suggested an alternative formula with allowance for notch radius in addition to elastic stress-concentration factor.

Thus, results of reversed-loading fatigue tests on notched specimens have been reported in terms of empirical values of K_f, and there have been a number of suggested formulations of K_f in terms of K_t (the elastic stress-concentration factor) and of ρ (the notch radius). Suggestions for incorporating the notch radius usually involve an additional parameter (ρ', δ, or a, for example). This may reflect some plasticity response of the material, but in an indirect manner. It has been speculated that the additional material parameter may relate to micro-

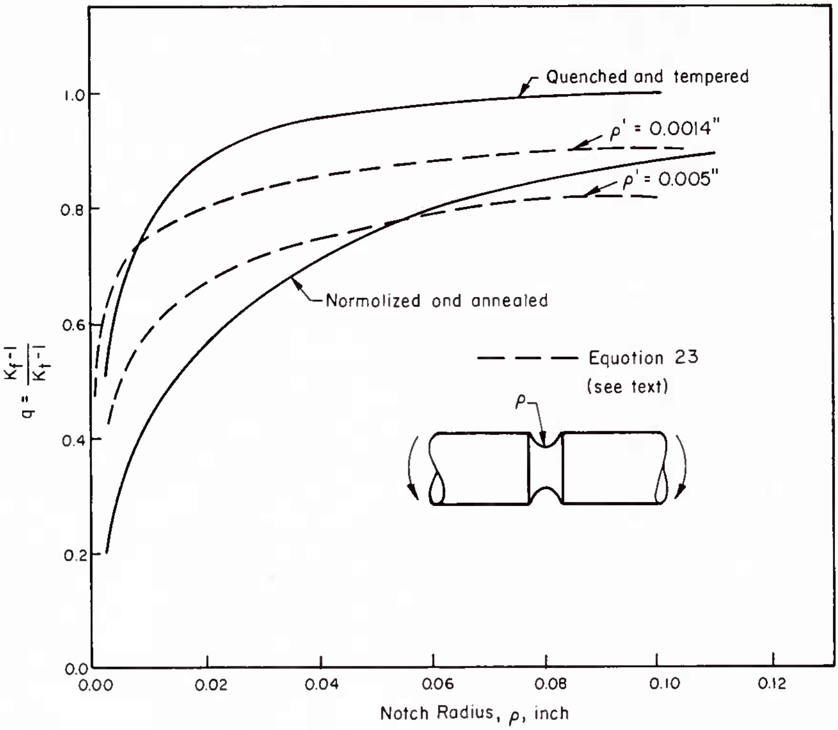

FIGURE 52.—Fatigue-notch sensitivities for ordinary steels in rotating-bending.

scopic inhomogeneity, to effects of stress gradient, and to subsurface nucleation.

Loading Not Fully Reversed

There are probably more situations in which a structural part is subject to a combination of steady load and alternating load than situations of fully reversed loading. There is less certain information about methods for making design allowance for notch effects under such loading than for fully reversed loading. However, figure 53 illustrates a procedure often used. The upper curves are for unnotched specimens and the lower curves for notched specimens at two lifetimes indicated. The "points" are derived from the upper curves by dividing ordinates by K_N(3). This means applying a stress-concentration factor only to the stress amplitude. This procedure is questionable in meaning, since

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(3) Another procedure, usually more conservative, is to use K_t. When data are available, it is possible to use values of K_f for zero mean nominal stress.

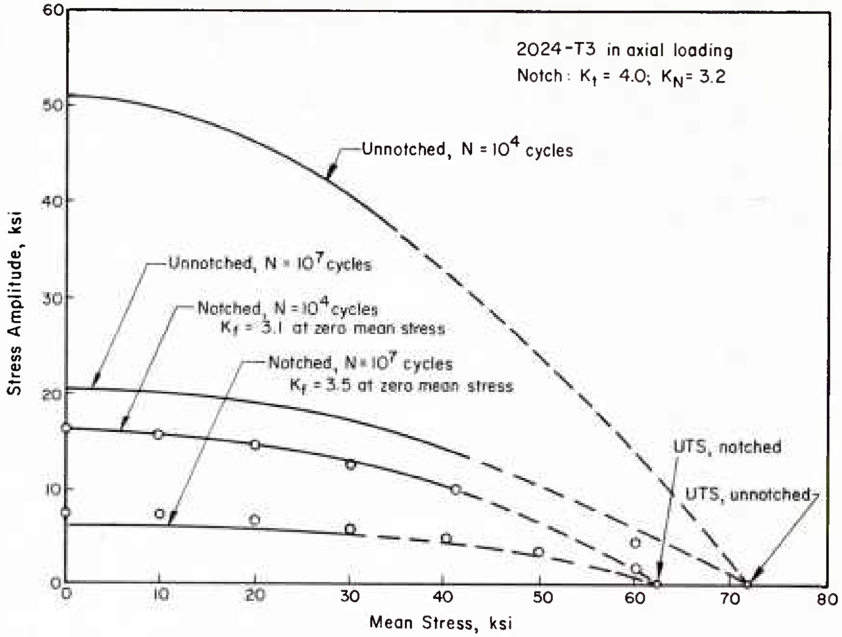

FIGURE 53.—A constant-lifetime diagram for notched specimens.

there is evidence that a notch affects both mean stress and stress amplitude. However, there are partly-compensating effects: (1) at higher mean stress the maximum stress is high enough that local plasticity makes the actual stress-concentration factor less than K_t, and (2) at low mean stress the observed fatigue strength is not as sensitive to mean stress as to stress amplitude. In practice, it appears that application of an effective fatigue-strength-reduction factor to just the stress amplitude often provides a reasonable prediction of the nominal stress amplitude that a notched specimen will withstand.

There are several possibilities for defining a fatigue-strength-reduction factor in situations of combined steady load and varying load. For example,

$$K_f = \frac{\text{stress amplitude, unnotched specimen}}{\text{stress amplitude, notched specimen, same mean stress}}$$

or

$$K_f' = \frac{\text{maximum stress, unnotched specimen}}{\text{maximum stress, notched specimen, same } R}$$

are two commonly used definitions. It should be emphasized that the values of K_f and of K_f' may be significantly different. If K_t is constant for a notched specimen, then

1. if the Goodman diagram is a straight line,

$$K_f' = K_f;$$

2. if the Goodman diagram is parabolic (Gerber),

$$K_f' = K \qquad \text{at } R = -1$$

but decreases with decreasing R toward

$$K_f' = 1 \qquad \text{at } R = +1.$$

For many materials K_f' is significantly less than K_f within the range $0 < R < 1$. Of course, either definition (or any of several alternatives) is valid and may be useful if enough data are available to make only small interpolations or extrapolations necessary.

In summary to this point, it appears that laboratory tests of simply notched specimens show that:

1. A notch usually lowers the cyclic load a specimen can withstand far more than is attributable to its reduction of the cross section. The fatigue-notch factor may, especially under reversed stressing, be as high as the theoretical stress-concentration factor for the notch.
2. For high stresses that produce local yielding, the theoretical stress-concentration factor is less than that for elastic stress distribution; the fatigue-notch factor is also often lower for high loads.
3. For very sharp notches, there appears to be an influence of notch radius that differs for different materials.
4. For loading under combined steady and alternating stresses, analysis is uncertain, but application of a fatigue-notch factor to the nominal stress amplitude alone is a common expedient.

Stress Concentrations in Structures

In an actual structure, there may be many additional factors that influence the local stress in relation to the applied load, such as residual stresses from fabrication and assembly, fretting, welding, and redistribution of stress under service conditions. Usually some of these factors are unknown. Consequently, values of K_f from laboratory tests on simply notched specimens can seldom be used to predict with accuracy the lifetime of a structure.

Values of K_f from fatigue tests of simply notched specimens are used in engineering for such purposes as:

1. Making preliminary design estimates when no other information is available.
2. Delineating factors critical in a structure.
3. Assessing the relative fatigue-notch sensitivity of materials.
4. Preparing critical tests of a structure and making indexes of attainable performance.

For any of these purposes, it is important to consider the several factors shown by laboratory fatigue tests to be significant in the fatigue behavior of a material in the region of a geometric stress concentration. The importance of local stresses, including residual stresses in fatigue of components, is illustrated by such studies as those of Smith (*23*) on interference fasteners.

The present state of knowledge concerning stress concentrations in fatigue may be summarized as follows. A considerable amount of information exists concerning the effects of several types of notches upon the fatigue behavior of various materials. Some factors are qualitatively well established. There is no excuse for neglecting fatigue-notch sensitivity in design of structures. However, a great deal of judgment and, usually, verification by component testing is necessary for effective use of empirical parameters such as fatigue-notch factors to predict the fatigue lifetime for an aerospace structure.

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CHAPTER V. CRACK PROPAGATION AND RESIDUAL STRENGTH

BACKGROUND

Ten or more years ago, there was relatively little interest in propagation of fatigue cracks. It was considered that the "propagation stage" was a small fraction of the fatigue lifetime, and main attention was devoted to the "initiation stage." In conformity with the safe-life design philosophy, the emphasis was on preventing the start of a fatigue crack. Several factors have changed this emphasis.

One factor is that, as tools for detecting cracks improve, there is increasing recognition that a large fraction of total lifetime may be in the propagation stage. In some instances, very small cracks may exist at the start of service loading; in other cases, cracks may develop in 1 to 10 percent of the total recorded lifetime to fracture. On the other hand, there have been a number of observations (see, for example, reference *1*) of cracks which do not propagate to fracture. Thus, it becomes of practical significance to attempt to formulate the conditions that determine whether or not a crack will be catastrophic.

Concurrent with interest along these lines, there has been increasing emphasis on crack growth criteria under static loading. A whole field of research related to brittle fracture of metals has arisen. Studies of brittle fracture show that the fractional reduction in static strength by a sharp notch, such as a fatigue crack, is a property that varies greatly from one material to another. Such conditions have developed concurrently with growing emphasis on fail-safe design. In this design approach, it is imperative to consider whether a crack, if one exists, can grow, between inspection times, to a catastrophic size. It is therefore important to consider the relative evaluation of materials on the basis of their resistance to catastrophic crack propagation.

Finally, there is interest in attempting to understand the mechanism of crack propagation (under static or impact or fatigue loading) in terms of more fundamental factors such as the behavior of dislocation arrays. This interest will not be emphasized in the present chapter because basic understanding seems insufficient to afford much help in current design problems. Rather, the subsequent discussion contains some account of quantitative relations that have been suggested in terms of engineering parameters and very brief references to more basic approaches.

CRACK PROPAGATION UNDER "STATIC" LOADING

It is convenient to start with some account of studies of crack propagation under steady (or monotonically increasing) load. These studies not only furnish some background for considering behavior under fatigue loading, but also provide an approach to consideration of "residual static strength" that is important in fail-safe design.

The Griffith Crack (in Brittle Materials)

Many years ago, Griffith (2) discussed the (static) strength of inherently brittle materials (such as glass) on the basis that they might contain, from the time of fabrication, small cracks. His argument may be understood by considering the stored elastic-strain energy in a plate subject to uniform tension. If we consider a wide plate containing an elliptical crack and under tension(1), as indicated in figure 54, the decrease in total elastic-strain energy per unit thickness is approximately

$$\begin{aligned} U_e &= \text{strain energy of plate without elliptical hole}\\ &\quad\ \text{minus strain energy of plate with elliptical hole} \qquad (1)\\ &= \frac{\pi \ell^2 S_n^2}{E}, \end{aligned}$$

where ℓ = half-length of ellipse(2)

S_n = nominal stress far from hole = P/wt

E = Young's modulus for the material.

The surface energy of the two faces of the crack is

$$U_s = 2\,(2\ell\ T) = 4\ell\ T,$$

where T = surface tension for unit thickness.

The total change in potential energy for the presence of the crack is

$$U = U_e - U_s = \frac{\pi \ell^2\ S_n^2}{\mathrm{E}} - 4\ell\ T.$$

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(1) Griffith actually used, as one of his approximations, a stress-concentration factor more appropriate to biaxial tension. This is $K_t = 2\sqrt{\ell/\rho}$.

(2) In this chapter, the symbol ℓ is used for the half-length of a central crack and also for the semimajor axis of the ellipse representing the crack for analytical purposes. In chapter IV, *a* was used for the semimajor axis.

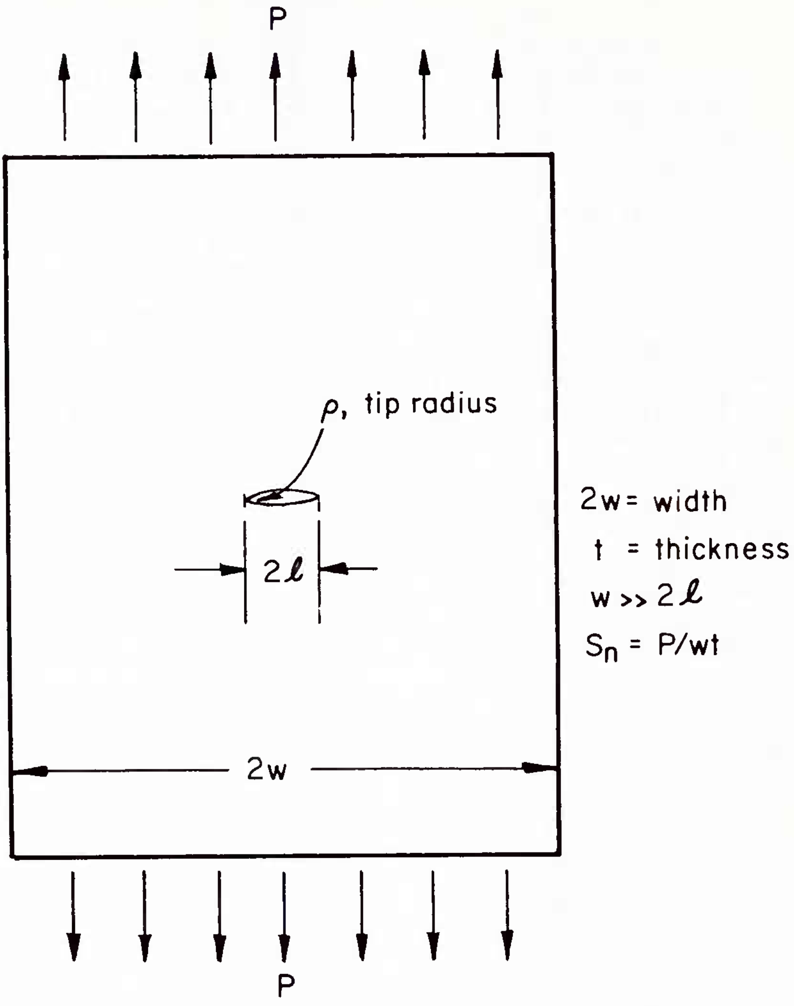

FIGURE 54.—Notation for an elliptical "crack."

A condition for the crack to grow is that

$$\frac{\partial U}{\partial \ell} \geqq 0,$$

or that

$$S_n \geqq \sqrt{\frac{2ET}{\pi \ell}}. \tag{2}$$

Thus, a small crack needs a relatively large stress to satisfy energy requirements for growth.

A similar conclusion may be reached by another argument. The maximum stress at the end of an elliptical crack in a sheet under tension is approximately

$$S_{max} = S_n\,(1 + 2\sqrt{\ell/\rho}), \qquad (3)$$

where ρ = radius of curvature of crack

Now, if $\sqrt{\ell/\rho} >> 1$, and
S_n is constant, and
S_{max} is fixed (a maximum stress criterion of failure), then

$$S_n \doteq \frac{S_{max}\sqrt{\rho}}{2}\sqrt{1/\ell} = \frac{\text{constant}}{\sqrt{\ell}}. \qquad (4)$$

So, again it appears that for a specified applied stress a large crack will grow, but one too small will not.

The Griffith condition (equation 2) does not give an idea of propagation velocities even in an isotropic brittle material. It does suggest that S_n must be greater than the stability value (given by the equal sign in equation 2), and one may consider that the excess elastic strain energy is used to provide kinetic energy for the crack propagation. However, other factors may also be important in cleavage fracture of brittle materials, and a full discussion is outside the present interest.

Cracks in Ductile Metals

In a ductile metal, there will be at least one other significant energy besides the surface energy that must be supplied by the decrease in elastic strain energy as a crack grows. That is the plastic energy in the region immediately around the crack. Orowan (*3*) noted that this may be much greater than the surface tension energy. Now, it is commonly observed that the plastic lip along a crack is reasonably uniform, so one may assume an energy increase, U_p, proportional to ℓ. Then

$$U = \frac{\pi \ell^2 S_n^2}{E} - 4\ell\,T - 4\ell p,$$

and equation 2 becomes

$$S_n \geqq \sqrt{\frac{2E\,(T + p)}{\pi \ell}}, \qquad (5)$$

which provides the same functional relation between S_n and ℓ as equation 2, with allowance for the energy expended in plastic deformations.

Friedel (*4*) has given an extensive discussion of the theoretical influence of plastic deformation on the propagation of a crack under unidirectional loading. He concludes that the most important consideration is whether or not plastic relaxation of the high stress at the crack tip occurs. Details of this are complex and not yet fully established.

Irwin (*5*) extended the Griffith-Orowan considerations. Using stress functions of a type suggested by Westergaard, he noted that important features of the elastic stress field around a sharp notch can be characterized by a parameter K dependent on specimen geometry and loading. For example, the stresses near an end of a central slit of length $2l$ in a wide sheet under tensile load perpendicular to the slit are(3) (for plane stress)

$$S_x = \frac{K}{\sqrt{2\pi r}} \frac{\cos\theta}{2}\left(1 - \frac{\sin\theta}{2}\frac{\sin 3\theta}{2}\right) - \sigma_{0x}.$$

$$S_y = \frac{K}{\sqrt{2\pi r}} \frac{\cos\theta}{2}\left(1 + \frac{\sin\theta}{2}\frac{\sin 3\theta}{2}\right)$$

$$S_{xy} = \frac{K}{\sqrt{2\pi r}} \frac{\cos\theta}{2}\frac{\sin\theta}{2}\frac{\cos 3\theta}{2}. \qquad (6)$$

Here r, θ are polar coordinates about a point at the tip of the crack, and $\theta = 0$ is the direction (x) of the crack. The point is that (neglecting the value σ_{0x}) the stress at every position reflects the geometry of the slit specimen and the external loading through the single parameter K.

Irwin suggests that a critical value of K—the K_c at which brittle (fast) cracking starts—may be a useful material parameter. Integration of the energy around a crack can provide a relation between K and the specimen and crack geometry, and the loading. For the central slit

$$K = S_n\sqrt{\pi l}. \qquad (7)$$

Thus, if one measures, for a specific nominal stress, the critical length $_c$, he can evaluate K_c as a material characteristic. This illustration has been somewhat oversimplified to emphasize the main principle of a critical value of K being a possible material characteristic. A recent review (*6*) pictures some of the additional complications and provides number of additional references.

It has been noted that the Griffith-Orowan-Irwin approach can be related to the Neuber elementary particle concept (see, for example, reference 7).

The idea of susceptibility of a material to brittle fracture has one very important relevance to design to prevent fatigue failure: the possibility of estimating the residual static strength of a panel after a fatigue crack has grown to some length. Table 2 (adapted from reference *8*) shows some illustrative data on the rupture strengths of cracked flat

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(3) The approximations in equations (6) are currently the subject of considerable discussion.

panels under tension. The gross-area nominal stress for rupture apparently is not constant but decreases with increasing crack length. The net-area nominal stress also decreases with increasing crack length, but significantly less. The Irwin K_c value is still more nearly constant in this example.

TABLE 2.—*Residual strength of cracked sheet*
Material: 2024–T3 Alclad
Panel Dimensions: 20 inches wide, 40 inches long, 0.040 inch thick

, inches	S_n (gross), ksi	S_n (net), ksi	K_c (a), ksi × in.$^{1/2}$
1.48	40.2	47.2	86.7
1.46	40.3	47.2	86.4
1.65	38.4	45.9	88.8
1.62	38.4	45.8	87.5
2.64	30.0	40.7	89.0
2.41	29.4	38.8	83.0
3.77	22.9	36.7	86.0
4.42	24.0	42.8	90.0
4.68	19.1	35.8	85.0
4.36	19.7	35.0	83.0
5.67	14.3	33.0	79.0
6.46	14.1	37.6	89.5

(a) $K_c = S_{n(gross)} \sqrt{\pi \ell} \times C'$.

Other data on residual static strength of fatigue-cracked specimens have been developed at NASA (see, for example, reference *9*) and interpreted in a somewhat different way—in terms of an adjusted Neuber stress-concentration factor. Somewhat similar conclusions may be drawn: (1) The strength decreases more rapidly than can be accounted for by the increase in net nominal stress, (2) allowance for increasing stress concentration with increasing crack length helps systematize the observations, and (3) different materials (for example, 7075 and 2024 aluminum alloys) show different sensitivities to the stress concentration of the crack.

The ideas of a critical-stress intensity factor (K_c) or of a notch-sensitivity correction or of an energy-release rate (G_c of Griffith) have been helpful. However, further study has shown that no unique value of such a quantity as K_c characterizes completely the brittle fracture nature of a material. Corrections for panel width and thickness help, but different modes of cracking (shear and tensile, for example) require different K_c values; each K_c varies with temperature; fast and slow cracks behave differently; branching cracks require additional considerations.

Consequently, engineering design in regard to residual static strength remains largely empirical. A good example of measurements on panels (flat and curved, plain and stiffened) is given in reference *8* together with empirical formulae for interpolation. Extrapolation of any such formulae must always be considered questionable. The theories based on elastic-stress concentration have provided the significant precaution that the brittle response of a material to a crack is not predictable from conventional stress-strain data. Cracking tendencies must be evaluated experimentally, and, at least until more theoretical understanding is available, under conditions (loading and geometry) suitable to the design purpose.

CRACK PROPAGATION UNDER REPEATED LOADING

The problem of developing any sort of theory of crack propagation under repeated loading is extremely difficult since there is scarcely any well-established mechanism for fatigue. Such development is currently in a transitional stage. Accordingly, the following brief account of theoretical considerations is illustrative rather than complete and planned to provide a framework for the empirical formulations which are subsequently discussed.

Theoretical Considerations

Head (*10*) developed a theory of crack propagation under cyclic loading on the basis of Orowan's theory of fatigue failure (see ch. II). He focused attention on a small volume of material (of length a in the crack direction) just ahead of the advancing crack tip. As the load increases the stress rises to the yield stress, and this volume becomes plastic. With successive cycles, the material strain-hardens and loses its ductility until, after sufficient stress cycles, it fractures, and the crack becomes longer. On the basis of this physical picture, highly approximate computations give

$$dl/dN = Cg(S)\,a^{-1/2}\,l^{3/2}, \qquad (8)$$

where N = number of cycles
$g(S)$ = function of stress amplitude
a = length of the plastic element assumed in the model.

Hult (*11*) considered the radial growth of a crack in an idealized elastic-plastic material subjected to reversed torsion. He used the criterion that material ahead of the crack would fracture when the accumulated total plastic strain reached a critical value. This resulted in where γ_n = the nominal amplitude of torsional stress.

$$dr/dN = K\gamma_n^2 r, \tag{9}$$

Others (see, for example, references *12* through *14*) have derived relations of the form

$$dl/dN = CS_a^x l^y, \tag{10}$$

where C is a "material constant" and various values have been given to the exponents x and y. In nearly every case, some aspect of stress concentration at the tip of the advancing crack has been concerned. Other criteria have included: cumulative plastic strain, a fixed value of shear-strain energy, a peak value of local strain, and dimensional analysis. Values suggested for x have included 2, 3, 4, and $3+m$ ($m=$ strain hardening coefficient); values for y have included 0, 1, and $x/2$.

Meanwhile (see references *14* and *15*) examination of details with high optical (and electron optical) magnification has afforded some suggestions. It appears possible that more than one mechanism may be involved—for example, a slip process strongly influenced by shear stresses in an early stage and a different fracture process much influenced by local tensile stress in a later stage. The relative length of each stage may be related to the strain-hardening characteristics of the metal and to the stress level at which the fatigue crack is being propagated. Limited studies, in detail, of polycrystalline metals show apparent variations as a crack travels through differently oriented grains (*16*) and around inclusions.

Consequently, a detailed theory of crack propagation under repeated stressing is not available. Engineering must depend mainly upon empirical relations which may be somewhat guided by the ideas of currently imperfect theory.

Empirical Relations for Fatigue Crack Growth

Frost and his colleagues (*17, 18*) have reported a number of measurements of fatigue crack growth in panels under constant-amplitude axial loading. These have been analyzed in terms of the relation

$$dl/dN = (P + QS_m)\, S_a^3 l, \tag{11}$$

where S_m and S_a refer to the nominal gross area stress and where the constants P and Q are characteristics of each sheet material. For some materials, a small value was reported for Q; for these, the rate of propagation seemed relatively insensitive to mean stress. Figure 55 is a plot adapted from some of this work; note that:

1. Over a good portion of the test, the data lie close to a straight

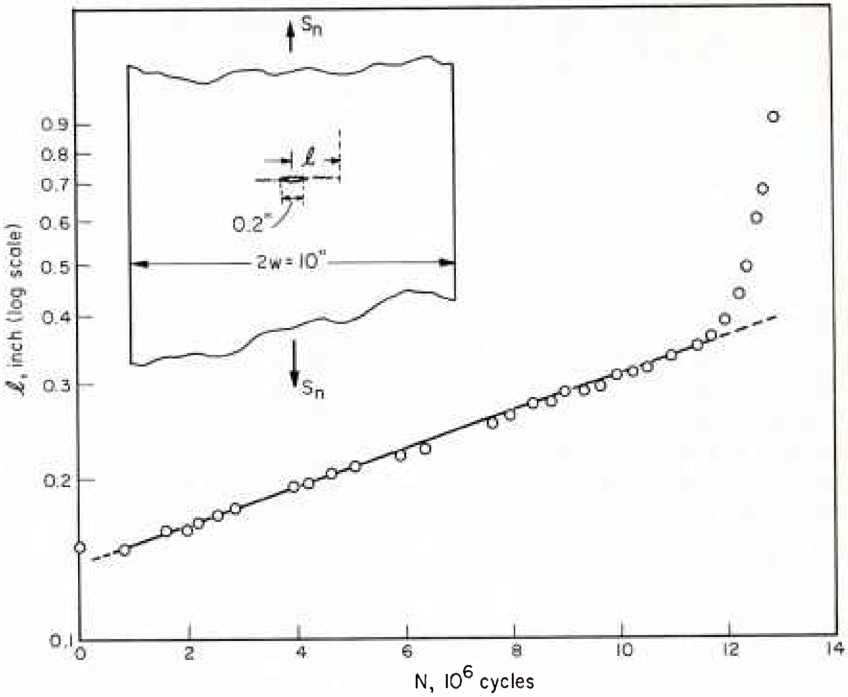

FIGURE 55.—Results of axial-load (S_n = 56.0 ± 2.24 ksi) tests on cold-rolled mild steel (see ref. 18).

line on the plot of log l versus N. This is compatible with equation 11.

2. At an early stage, points deviate as if the early propagation were different. This is discussed later.

3. Values fit only to a total crack length about one-tenth of the panel width. Frost suggests that equation 11 should not be considered valid beyond this distance (the finite width of the panel then becomes significant).

Weibull (*19*) describes tests in which the nominal net area stress amplitude was held constant and suggests

$$dl/dN = K\,S^{x}_{a,\,net}. \tag{12}$$

He also notes cases in which an early stage appears to follow a different growth rate relation than fits later behavior.

Hardrath (*20*) and others at NASA have interpreted data in terms of an effective-stress-concentration factor at the crack tip. Thus an empirical expression of the type

$$\log\, dl/dN = C_1K_NS_{net} - C_2 - \frac{C_3}{K_NS_{net} - C_4}, \tag{13}$$

where K_N is an effective-stress-concentration factor and the C's are experimentally determined constants is found to fit data.

A number of investigators have suggested fitting data (under fixed-amplitude loading) to an expression of the type

$$d\ell/dN = CS_a^x \ell^y, \tag{14}$$

and several have suggested (on consideration of the energy near the crack tip)

$$d\ell/dN = f(S_a \ell^{1/2}). \tag{15}$$

Figure 56 shows results of zero-to-tension tests on sheets of 6061 aluminum alloy with three different loading conditions: (1) constant amplitude of nominal gross-area stress, (2) constant amplitude of nominal net-area stress, and (3) constant amplitude of a quantity intended to characterize the local stress near the crack tip. It may be noted that:

1. a good portion of the data for the first condition may be fitted to an expression like equation 11;
2. a good portion of the data for the second condition may be fitted to equation 12;
3. a good portion of the data for the third condition fits a straight line on the plot as drawn.

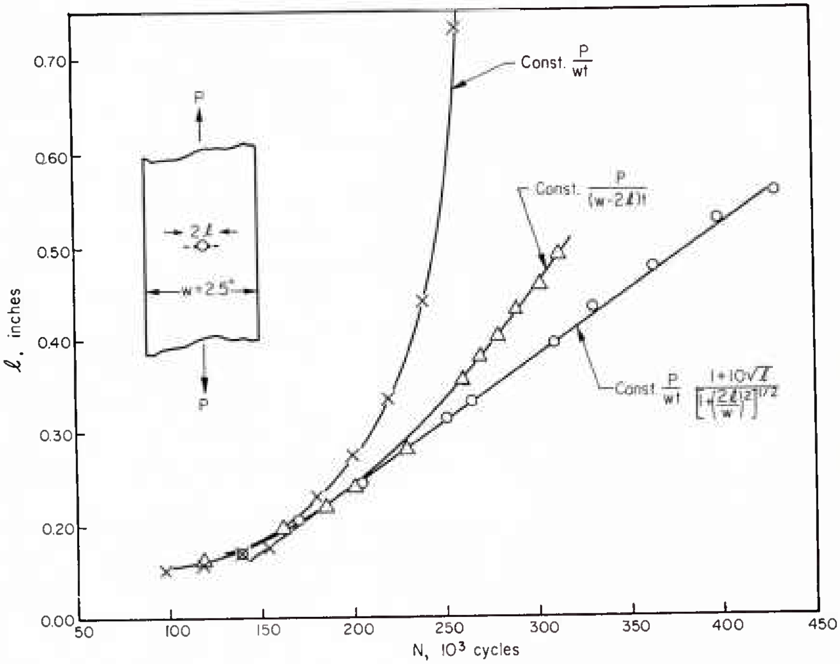

FIGURE 56.—Results of zero-to-tension tests on an aluminum alloy sheet.

The moral is that such data are inadequate *(21)* to "prove" any theory. It may be further noted that data to about $N = 130$ kilocycles do not fit with latter trends—an implication of a possible first stage that may differ in rate from a later stage.

In the whole picture of empirical formulations, there is much analogous to the situation concerning brittle fracture and residual static strength. Various means have been tried to allow for stress intensity and material response to this at the tip of the advancing crack. These have had limited success, and no universally applicable relation among such quantities as S_a, S_m, l, K_T has been developed. Nevertheless, the experiments have shown clearly that fatigue-crack propagation rates can vary greatly with different loading conditions on a material or, for the same loading condition, with different materials. Thus fatigue-crack propagation sensitivities must be considered in design; at present, the only dependable way is through experiment.

SOME ADDITIONAL CONSIDERATIONS

Since aircraft structures are subject to variable amplitude loading, the question of crack propagation under such loading has been considered. Figure 57 shows some results reported by Schijve *(22)* which show that a crack grown to a certain length at a high stress level grew slowly after the stress amplitude was lowered but, after a delay period, assumed a growth rate characteristic of its length at the lower stress amplitude. When the stress levels were reversed (the lower level first), there was hardly observable delay in assuming a rate characteristic of the second stress amplitude. A similar trend has been observed by others. This effect is believed to be due mainly to the cold worked plastic region and the residual stress remaining after unloading from the cycle preceeding the change in stress amplitude.

Valluri, in a number of publications (see, for example reference *23*), has suggested a "unified engineering theory of fatigue" based upon considerations of crack propagation rates. While most of his detailed relations are highly questionable, the broad framework is important.

Consider the sheet specimen in figure 58 under constant-amplitude loading (say, zero to P). The total process to rupture may be usefully considered in stages:

1. For a period of N_1 cycles, there is accumulating damage that results in the *initiation* of a small crack. This will be greatly influenced by the stress concentration, residual stress, and material condition near the notch.
2. In subsequent N_2 cycles, there is a *first stage* of growth. During part of this, there may still be an influence of the stress field around the notch.

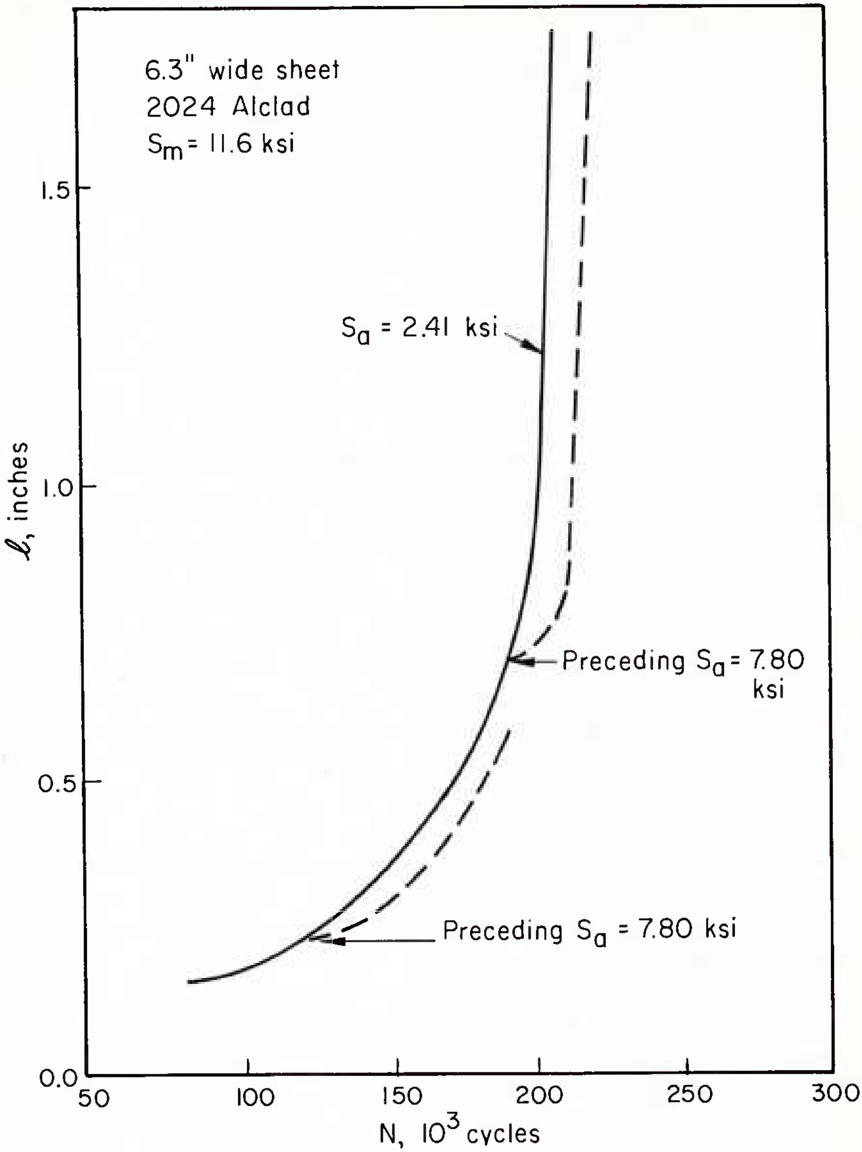

FIGURE 57.—Effect of changing stress amplitude on crack propagation.

3. For N_3 further cycles, there may be a *second stage* of growth which is relatively stable and may follow, at least to a reasonable approximation, some simple relation such as proposed in the empirical rules mentioned. In this stage, the crack is its own stress raiser

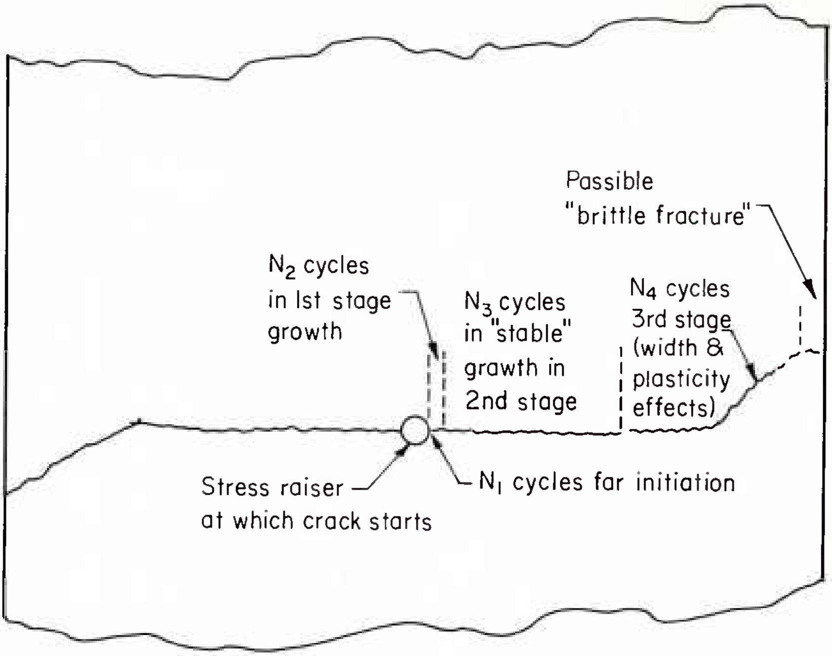

FIGURE 58.—Some possible stages in fatigue
N = total cycles to rupture
$= N_1 + N_2 + N_3 + N_4$.

and may be expected to propagate at the same rate as in an unnotched sheet.

4. At a still later stage, the growth may be strongly influenced by plasticity all across the remaining net section—for N_4 cycles.

5. Finally, after the crack has reached a critical length, l_c, the residual strength may be low enough that rupture occurs on the next load cycle.

Valluri has considered only stages (3) and (5), and these in a doubtful manner. But the important idea is that this total consideration of several successive steps may ultimately be helpful. In some situations it may prove possible to neglect some steps and thus simplify the analysis. In any event, knowledge of the several possible different stages helps avoid unrealistic extrapolations from observations under one set of conditions to prediction of behavior under other conditions, for which the relative importance of different stages might be expected to be different. Manson *(24)* has reemphasized the possible engineering usefulness of separating analysis into at least the two stages of crack initiation and crack propagation.

In some respects, analysis of crack propagation in structures adds very many complications. There may be large changes in the tip stresses; there may be additional stress raisers that sometimes speed up the crack or that are effective crack-stoppers. Yet some experiments (see references *8 to 20*) imply that: (1) analysis analogous to the empirical suggestions for simple sheets may be useful in correlating data on composite structure and (2) practical advantages may accrue from intentional use of crack-arresting methods.

Finally, it should be emphasized that some design trends require particular consideration of crack sensitivity under high loads and of the possibility of rapid crack growths under repeated low loads. The increasing use of high (static) strength materials required increased precaution against possibilities of rapid fatigue crack propagation and low residual static strength. The use of thick-skin, integrally-milled, monolithic construction and of heavy forgings tends to provide few crack-stoppers. For some situations even a very small crack may be catastrophic and careful assessment of crack sensitivity is imperative.

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CHAPTER VI. CUMULATIVE DAMAGE

BASIC CONSIDERATIONS

Many laboratory tests to evaluate materials and effects of various design parameters are carried out under constant stress (or strain) amplitude loading. That is, a specimen is run at some condition of $S_m \pm S_{a1}$ until it fails in N_1 cycles; a second specimen is run at $S_m \pm S_{a2}$ to failure at N_2 cycles; other specimens at other values of S_a. The results provide an S–N curve. Such a curve (with proper allowance for statistical variation) can be used to predict the lifetime of a part subject to loading at $S_m \pm$ any S_a within the range of the experiment.

Of course, most structural parts may be expected to undergo in service more complex loading. Even if the mean stress, S_m, remains constant or if its variation can be neglected, a part is often required to withstand a whole spectrum of values of the stress amplitude, S_a. Perhaps even more important, the loading for which the part must be designed can often only be described statistically; no two parts can be be expected to undergo exactly the same sequence of values of S_a.

The basic problem of how to allow for this complexity may be approached in either of two ways. One can seek a "theory of cumulative damage" which will permit estimation (from S–N data obtained under simple laboratory tests) of the behavior of the structural part under any specified set of more complex variable-amplitude loadings. Alternatively, one may seek some spectral loading representative of service conditions and may make all laboratory evaluations under this load spectrum. In view of the fact that the anticipated load history is often inherently statistical in nature, this alternative still requires consideration of the possible effects of different sequences of individual loads. Consequently, a large number of investigations of cumulative fatigue damage have been carried out.

Factors involved in cumulative fatigue damage include the following:

1. *Material Response.*—Laboratory tests indicate changes in metallurgical structure may occur under repeated loading. "Over aging," sometimes accelerated by cyclic stressing, is an example. Thus, even an unnotched specimen, prior to detectable cracking, may have a complex response to a spectrum of stress amplitudes.

2. *Crack propagation.*—Stresses incurred after cracking may have a different effect than the same stresses before cracking.
3. *Environmental effects.*—Corrosion and creep at elevated temperature may change the fatigue response of a specimen by amounts depending upon the time of exposure; stresses incurred after long exposure may contribute differently to damage than the same stresses at an earlier time.
4. *Stress redistribution.*—Especially in a notched specimen or in a structure with one or more stress-raisers, the same loads or nominal stresses may produce different local strains after redistribution under some high load than before this high load. This point is emphasized in a subsequent section on structures.

If such factors as these are kept in mind, the many approaches to analysis of cumulative fatigue damage may be viewed with reasonable perspective.

SUGGESTED APPROACHES FOR DAMAGE PREDICTION

Defining "damage" in a manner suitable for engineering design is extremely difficult. Many attempts have been made to use the cycle-ratio measure. These attempts started with Palmgren and Miner, whose approach (still widely used) is outlined first in the following summary. A different basic concept is the idea of a "damage line" and, subsequently, a different *S–N* curve. Approaches such as those of Henry or of Manson may be considered with respect to this data. Still other methods are hard to classify and illustrate the difficulty of a fruitful measure of damage or of rate of damage accumulation.

No method (except the simple approach of Miner) is widely accepted in current engineering. However, acquaintance with several suggested approaches is desirable, both for general background and because each may throw some light upon the factors involved in design to resist cumulative-fatigue damage. The following discussion contains brief notes on many approaches that have been advanced, some references for further reading, and notes on apparent relative merits of different suggested viewpoints.

The Palmgren-Miner Approach

One of the earliest approaches was suggested by Palmgren (*1*) and independently discovered and extended by Miner (*2*). We henceforth call it the Miner approach.

Suppose that constant-amplitude tests have shown a lifetime N_i cycles for a stress-amplitude S_i. If a specimen is run n_i ($< N_i$) cycles, the cycle-ratio is defined

$$\mathrm{CR}_{S_i} = n_i/N_i. \tag{1}$$

the Miner approach is that failure occurs when

$$\sum_i (\mathrm{CR})_{S_i} = \sum_i n_i/N_i = 1 \tag{2}$$

For obvious reasons, this is sometimes known as the "linear cumulative cycle ratio" theory.

There are some outstanding aspects of this theory:

1. It only requires knowledge of the S–N curve (or curves if the loading involves a considerable area of the Goodman diagram).
2. It is simple analytically.
3. It implies no effect of order-of-occurrence of high loads and low loads.
4. It implies no effects at values of S_i below the fatigue limit (where $N_i \rightarrow \infty$).

The latter two items represent particular limitations in applicablility.

Figure 59 shows some experimental results of two-level fatigue tests on a simple material coupon. The solid line shows the prediction of equation 2. The dotted lines through experimental "points" do not conform to the Miner prediction but show a strong indication of an effect of the order-of-occurrence of loads.

Table 3 shows arbitrarily selected data that illustrate observed discrepancies from unity of the cumulative cycle ratio, $\Sigma\, n_i/N_i$. The total impression is that this value is often greater than unity and that the approach therefore is conservative—it may be highly overconservative. Sometimes the summation is less than unity, and the approach is nonconservative. The conditions for nonconservatism are not sharply defined.

The "Damage-Line" Approach

One of the early attempts to study damage was that by French (*6*). Figure 60 illustrates French's suggestion of a "damage line" for a material having a fatigue limit. The solid line is the S–N curve obtained by constant-amplitude tests of "virgin specimens." Several specimens were then prestressed to various fractions of their normally expected lifetimes at selected S_i above the fatigue limit. Each was run at the fatigue limit and either failed (indicating damage) or did not fail. The dotted line drawn between points for the two different classes of specimens (as indicated on the figures) was called a "damage line." Presumably, any specimen cycled below the damage line could be considered undamaged.

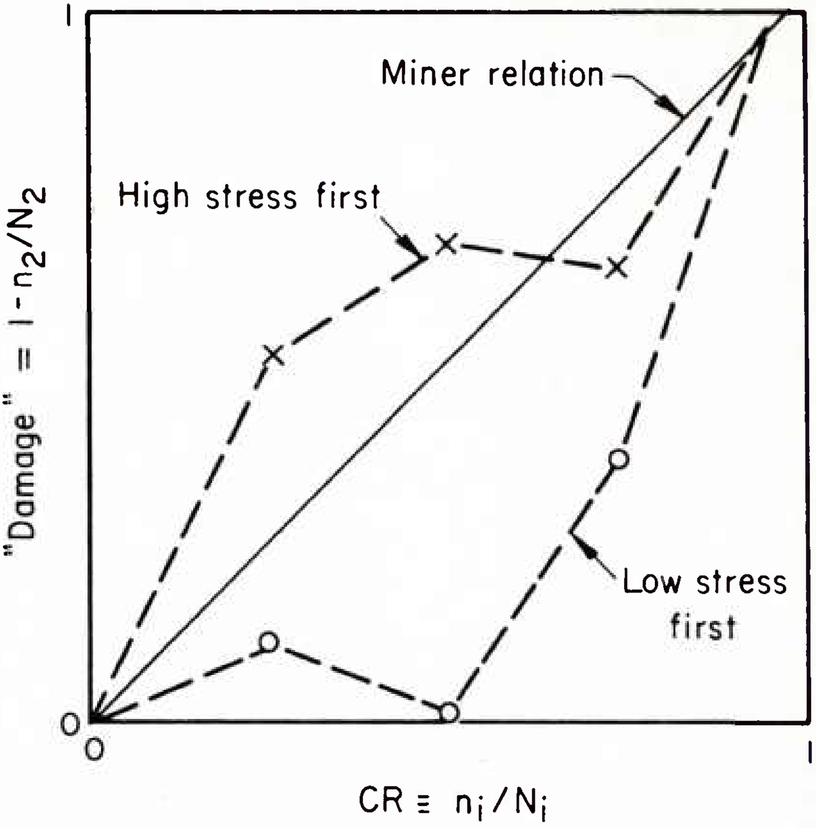

Figure 59.—Effect of order of stress levels in two-step test.

Table 3.—*Illustrative Cumulative Cycle Ratios*

Material and type of specimen	Loading	$\sum_i n_i/N_i$	Reference
24S–T, unnotched	2-step, high first	0.8 to 3.8	*3*
	2-step, low first	.6 to 1.1	
24S–T, with edge notch	2-step, high first	.6 to $>$ 15	
	2-step, low first	.8 to 1.5	
24–S–T, riveted joint	2-step, high first	1.0 to $>$ 17	
	2-step, low first	.6 to 1.2	
2024–T3 Alclad, riveted joint	5 to 10 steps	1.1 to 2.9	*4*
7075–T6 Alclad, riveted joint	5 to 10 steps	.7 to 3.7	
7075–T6, box beam	4-step spectrum	6.2	*5*

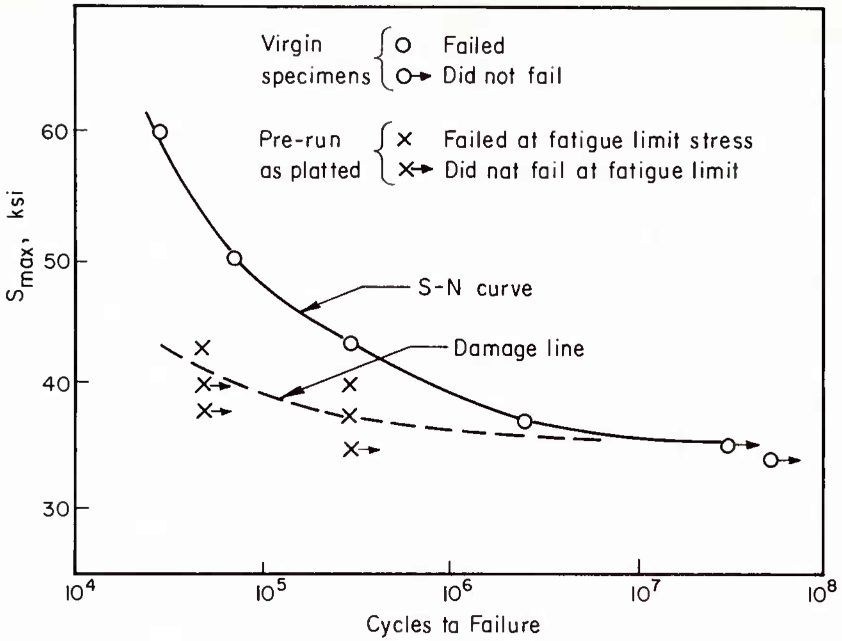

FIGURE 60.—French's damage line.

The French concept of a fatigue-limit damage-line is applicable only in limited situations, mostly of no interest in aircraft. However, the concept of a damage-line of some sort has interest as an alternative to a cycle-ratio concept and has been the basis of more than one suggested approach to making allowance for cumulative damage.

Other Suggested Methods

One report (7) lists 20 proposed methods for the prediction of fatigue lifetime under variable-amplitude loading. This does not include all variants nor some suggestions made since the writing of the report. Table 4 shows a number of the proposed approaches—mainly in chronological order. Details may be found in the references cited in the table.

Comments on Various Approaches

Nearly all of the quantitative procedures suggested for prediction of fatigue lifetime under variable amplitude loading are essentially empirical relations. Many involve weighting factors on the cycle-ratios of the Palmgren-Miner approach. Some authors have claimed superiority of

TABLE 4.—*Some approaches to cumulative damage*

Investigator	General approaches	Reference
Palmgren-Miner	Linear summation of cycle ratio	*1, 2*
French	Fatigue-limit damage-line	*6*
Kommers	Experimental study of steels—idea of "damage" curves.	*8*
Richart and Newmark	Development of damage curves in terms of $(n_i/N_i)^x$.	*9*
Marco and Starkey	do	*10*
Shanley	Importance of crack propagation; reduction to lower *S–N* curve after each set of n_i.	*11*
Henry	Specific relation for reduction of *S–N* curve in terms of reduced fatigue limit for steels.	*12*
Smith	Emphasis on residual stress after a high-stress-level loading; primarily applicable to structural parts.	*13*
Corten and Dolan	Damage related to n/N and to stress level; emphasis on highest stress level.	*14*
Grover	Separation of stages of crack initiation and of crack propagation.	*15*
Freudenthal and Heller	Weighting factor on (n/N) to allow for previous stress history.	*16*
Manson	Reduced *S–N* curves with particular form of reduction.	*17*
Valluri	Crack propagation and specific assumption for fracture.	*18*
Swanson	Considers different material behavior in different regions of *S–N* curves.	*19*
Halford and Morrow	Cumulative plastic strain for low-cycle fatigue.	*20*

their procedures over that of Miner; apparent superiority is often achieved by adjusting empirical constants to fit particular data.

Comparison of various approaches on the basis of compatibility with experimental data (see, for examples, references *7, 22,* and *23*) suggests that:

1. The Miner approach is often overconservative; it may, in an extreme case, predict a lifetime as short as 6 percent of that observed. On the other hand, a few instances have been reported in which this approach was unconservative.
2. Other approaches are not clearly better⁽¹⁾ except those which require additional data for correction factors.

Consequently, the Miner approach is widely used in aircraft engineering with necessary allowance for its limitations.

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(1) Note, however, the following section on "Cumulative Damage in Structures."

There are some suggested approaches which provide optimism for future development of improved procedures.

An approach advocated by Smith (reference *13*) for structural parts, emphasizes the nonlinearity between the local stress (at a stress-raiser) and the nominal stress usually considered in engineering. This approach is described in the next section since, in Smith's formulation, it concerns only the relation between a part with a stress-raiser and the basic material. For the behavior of the basic material, Smith uses a Miner approach (although he notes that his relation of the structural behavior to the material behavior could be adapted to other material behavior approaches). The mechanism on which he focusses attention is the local stress (particularly the residual stress after unloading) produced at a point of stress-concentration.

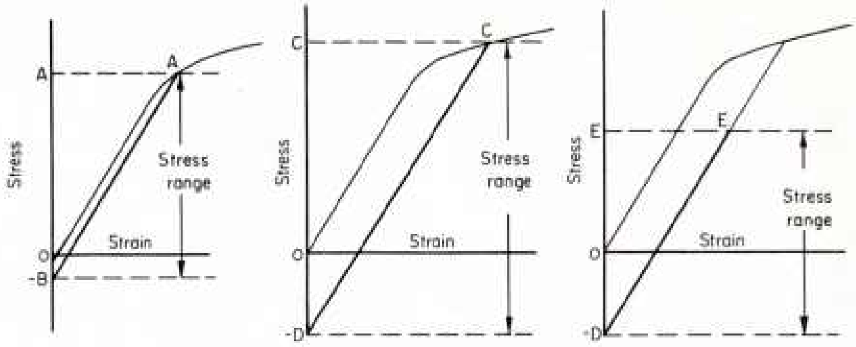
Grover (reference *15*) has suggested that cumulative damage be considered in two steps: (1) that up to crack initiation, and (2) that in the stage of crack propagation. His speculation that, for each stage, a Miner relation might hold is now known to be invalid. As noted in chapter V, there is evidence of delay periods in crack propagation after changing from a high stress-amplitude to a lower stress-amplitude. This hesitation may be largely due to the residual stress at the crack tip and compatible with Smith's emphasis upon the difference between a notched specimen (which exists after a crack is developed) and an unnotched one.

Morrow (reference *20*) is exploring the idea of accumulation of local plastic strain as a criterion for fatigue damage.

These three suggestions have one concern in common: emphasis upon the local stress (and strain) conditions with allowance that this is not likely to be linearly related to the nominal stresses used, not only in engineering but also in most laboratory experiments to study cumulative damage relations. Consequently, there are limited data suitable to assess the compatibility of approaches in terms of local stresses and strains with experience. The few data available suggest that some of the apparent discrepancies between the Miner approach and experience may be eliminated or greatly reduced when account is taken of such local conditions. However, it remains to develop a procedure for accounting for nonlinearities between local stresses (or strains) and applied loads within engineering limitations.

CUMULATIVE DAMAGE IN STRUCTURES

When the fatigue behavior of a structure is analyzed, the input information is usually in terms of loads (or of nominal stresses) rather than in terms of localized stresses. Most aircraft structures have one or



Low Load Amplitude
(a)

High Load Amplitude
(b)

Low Load Amplitude
(c)

Figure 61.—Schematic representation of local stress at point of stress concentration under various loadings—see text.

more stress-raisers, and it is important to realize that there is not likely to be a fixed one-to-one correspondence between the stress at one of these and the applied load.

Figure 61 is a schematic illustration of one important aspect of the difference between local stress and nominal stress which is proportional to load. The figure might apply to a structure of an aluminum alloy having the stress-strain curve shown. If there is a point of stress concentration then as the load is increased from zero, even to a relatively low value, the local stress might rise to the level A in figure 61 (a)—beyond the elastic limit of the alloy. Upon unloading, this local region which yielded plastically is "squeezed" by its elastic surroundings to a compressive stress, $-B$. Subsequent cycling, from zero to the same nominal stress, would (to a first approximation) cause local stress variation between $-B$ and $+A$. As suggested in figure 61 (b), a similar result might be anticipated for a higher load amplitude, except that there would be greater plastic strain and greater residual compressive stress, $-D$.(2)

Now if, after one or more cycles of high load, the structure is subjected to a low load amplitude, the local stress may vary from $-D$ to $+E$ as in figure 61 (a). The residual compressive stress remaining after the high load causes the mean value of local stress to be less than for a virgin specimen under the same load amplitude. This favorable "strengthening" by the previous high load cycle will cause $\Sigma n_i/N_i$ (where N is based upon constant-amplitude-tests) to be greater than

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(2) For very high loads, or for cycles from compression to tension, there may be also compressive yielding. In the present example, this possibility is omitted.

unity. This is the main principle of Smith's approach (*13*).

Such redistribution of local stress has been shown to occur in notched specimens (see ch. IV) and there is somewhat less direct evidence that it also occurs in structures. Allowance for this effect, even by relatively crude methods, has been shown helpful in analyzing the behavior of structures under variable amplitude loading. Presently, such methods are in a stage of development,⁽³⁾ and it may be anticipated that improved methods will help in analyzing cumulative damage in structures.

There are, of course, still other factors in the behavior of structures under variable-amplitude loading. One is the presence of stress raisers having unknown stress concentration factors. Another is the likelihood of unknown residual stresses from fabrication and from assembly. Still another is the possibility of fretting and corrosion. In view of such uncertainties, it is increasingly common practice to subject structures to "simulated-service" testing under varying load amplitudes. Such tests are discussed later in chapter XIX. Here it is relevant to note that some cumulative damage approach is still necessary for (1) preliminary design, (2) help in planning the simulated service test, (3) aid in interpreting results of the test and (4) guidance in extrapolating results of one or a very few tests to other possible service conditions.

For structures, the Miner approach remains the most widely used cumulative damage analysis. It appears that this approach may be more relevant to structures as better methods are devised to account for redistribution of local stresses as load-amplitudes are changed. Other factors (such as fabrication stresses and fretting) can be important and, presently, require experimental evaluation under an appropriate load spectrum.

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CHAPTER VII. LOW-CYCLE FATIGUE

Not many years ago, interest in fatigue in aircraft was directed mainly toward lifetimes involving many repetitions of loading. Items such as propellers and engine parts were designed for very many cycles, often more than 10 million, while wing-structure design for many loadings, perhaps 1 million, by small gusts was considered critical.

Recently, the importance of design to withstand relatively few (say, 5 to 5,000) cycles of high loading has been recognized. Experience with reactor pressure vessels and other components subjected to periodic thermal cycling stimulated some of the early studies of low-cycle fatigue. Other circumstances involving either thermal cycling or mechanical cycling are many. In missiles, attention has been directed toward low-cycle fatigue in liquid-propellant tanks and in solid-propellant rocket motor cases. Proof tests of such pressure vessels are made at various stages in manufacture; failures may occur after a few (perhaps 3 to 10) such loadings. In aircraft, pressurization of the fuselage causes a once-per-flight loading cycle. In a reasonable service lifetime this represents relatively few cycles (perhaps 5,000 to 10,000) in comparison with mild gust cycles (more than 1,000,000). Ground-air-ground cycles, some landing-impact loads, arresting loads on carrier-based vehicles, and some maneuver loadings are other potential sources of infrequently repeated high stress.

In chapter IV it was emphasized that high local stresses and strains in the plastic region can be produced at stress concentrations by even low nominal stresses. Consequently, it is quite possible to produce fatigue in a structure with a few cycles of loads that are not very high on the basis of nominal stress "static design." This danger tends to increase with increasing use of high-yield-strength materials which often have high fatigue-notch sensitivity.

Thus, as design grows more sophisticated, the concern for reliability under a few high loadings increases. This chapter outlines some of the particular design considerations required with respect to such low-cycle fatigue.

SOME BASIC CONSIDERATIONS

The term "low-cycle fatigue," while ambiguous, has been widely used to describe situations of high local stresses which result in failure after less than about 10,000 repetitions. The term may be employed from various points of view:

1. From the point of view of basic mechanisms, there appear to be some different characteristics in fatigue at high stresses and at low stresses.
2. From a testing point of view, there are different factors that require consideration when the nominal stress is above the engineering yield stress. Figure 62 illustrates that this may occur in the neighborhood of 10,000 cycles or less.
3. From the design viewpoint, the intended meaning of the term low-cycle may vary with the items being considered and with its end use; perhaps from 2 to 5 cycles when the effect of several proof tests is a consideration, from 5 to 500 cycles when occasional high loads must be considered, and from 500 to 10,000 cycles in the case of once-per-flight loadings.

For subsequent discussion in this chapter, a precise definition will be unnecessary. However, it is helpful to keep in mind the second of the viewpoints noted: the distinction between a stress level where most of the material is elastic and a higher level (for low-cycle fatigue) where the strain has a significant plastic component.

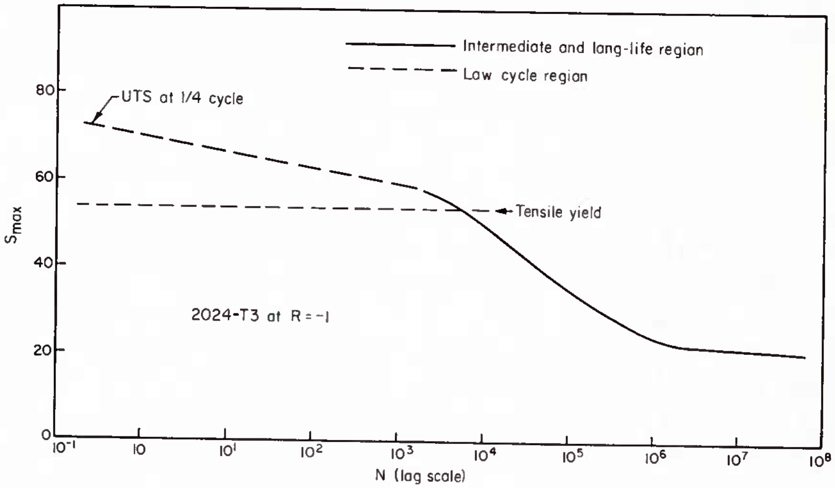

FIGURE 62.—One characterization of "low-cycle" fatigue.

Figure 63 shows a hysteresis loop that might be traced by a single cycle of reversed loading at a high stress level capable of producing low-cycle fatigue. Identified on the figure are the total strain amplitude, $\Delta\varepsilon_t$, and the elastic strain amplitude,

$$\Delta\varepsilon_e = \frac{\Delta S}{E}.$$

The difference between these is usually considered to be a measure of the plastic strain amplitude,

$$\Delta\varepsilon_p = \Delta\varepsilon_t - \Delta S/E. \tag{1}$$

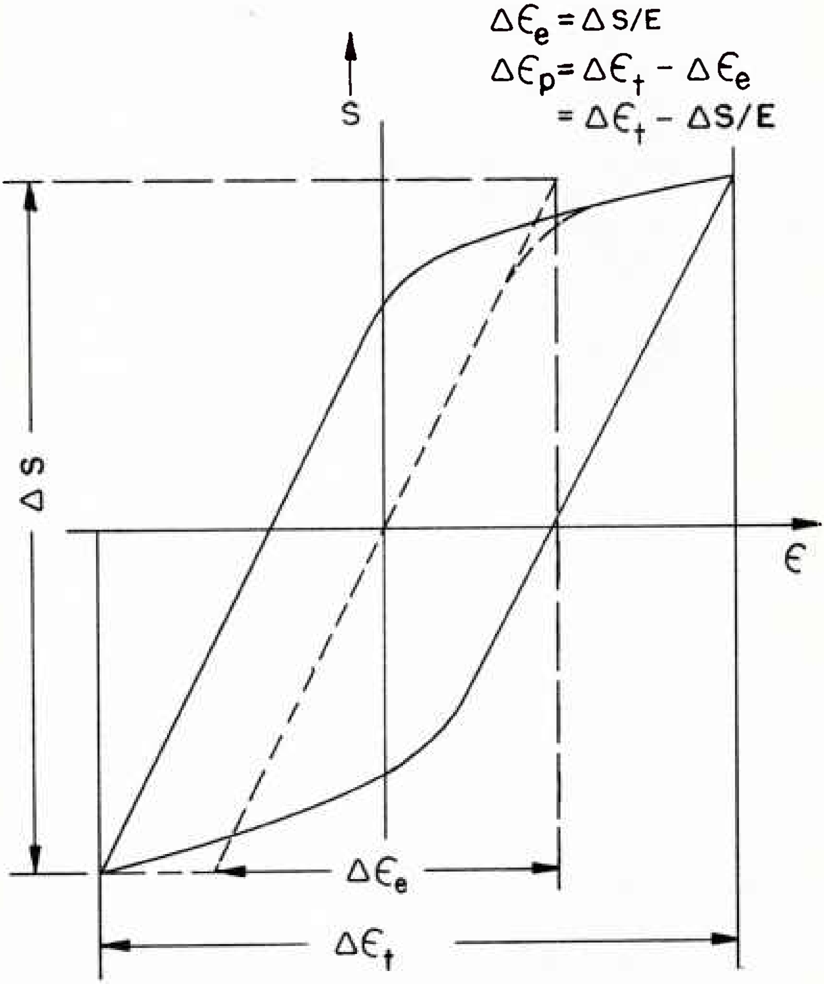

FIGURE 63.—A single cycle of loading and nomenclature often used in low-cycle fatigue.

Fatgue tests may be run at constant load amplitude. In this event, the hysteresis loop varies during subsequent cycles so that the strain amplitudes $\Delta\varepsilon_t$ and $\Delta\varepsilon_p$ change as the test proceeds. However, the tests may be run at a constant strain amplitude, in which case the stress amplitude will vary as cycling continues. These two kinds of testing, subsequently called (1) *strain cycling* and (2) *load cycling,* should be distinguished in design to prevent low-cycle fatigue.

REPRESENTATION OF STRAIN-CYCLING DATA

Manson (*1*) suggested that in the low-cycle range many data could fitted by the relation

$$\Delta\varepsilon_p = MN^z, \tag{2}$$

where M and z are material constants. Coffin (*2,3,4*) suggested a similar relation with z = a universal constant = $-1/2$ and with M related to the fracture ductility in a tensile test. If the tensile test is considered to represent $N = 1/4$ (just the increasing quarter-cycle of a sine curve), Equation 2 can be written

$$\Delta\varepsilon_p = \varepsilon_{f/2} N^{-1/2}, \tag{3}$$

where ε_f is the "fracture ductility" (the plastic strain at tensile fracture). Then a graph of log $(\Delta\varepsilon_p)$ against log N would appear as a

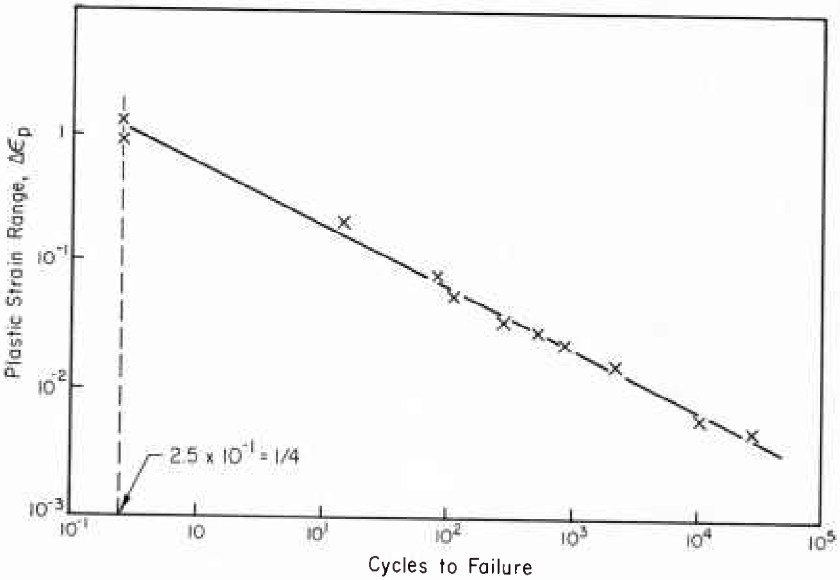

FIGURE 64.—Low-cycle fatigue test results for a stainless steel.

straight line of slope $-1/2$. Some results of Coffin's work that conform to this are shown in figure 64. A number of experiments interpreted according to this relation have been described (*2–6*).

Several investigators have questioned: (1) the universality of $z = -1/2$, (2) the universality of M, and in particular, the extrapolation to $\Delta\varepsilon_p = \varepsilon_f$ at 1/4 cycle. At present equation 3 can be considered only as an approximation which is useful when data are lacking for a more exact characterization.

Weiss and his coworkers (*7*), in analysis of their data and of data reported by Pian and D'Amato (*8*), suggested for nonzero mean strain that

$$\Delta\varepsilon_t = (\varepsilon_f' - \varepsilon_o)\, N^{-1/2}, \qquad (4)$$

where ε_f' may be obtained by curve-fitting and is close to, but not necessarily identical with, the static fracture ductility, and ε_o is the mean strain during the cycle. Note that, in contrast with Coffin's suggestion, the total strain range rather than the plastic strain range is suggested. This would make an appreciable difference at low strain amplitudes where the elastic strain amplitude, $\Delta\varepsilon_e$, is a significant part of $\Delta\varepsilon_t$.

Figures 65 and 66 show data for two materials plotted in accordance with equation 4. Note that the results fit discrete curves for various values of the strain ratio, R, and that this separation is analogous to the

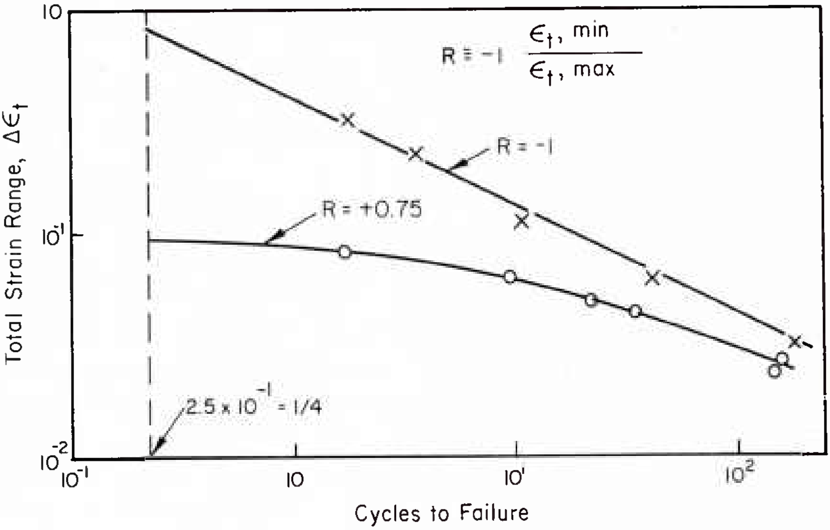

FIGURE 65.—Strain-cycling fatigue-test results for 2024–T4 aluminum alloy (lines drawn from equation 4 with ε_f' adjusted to fit for $R = -1$).

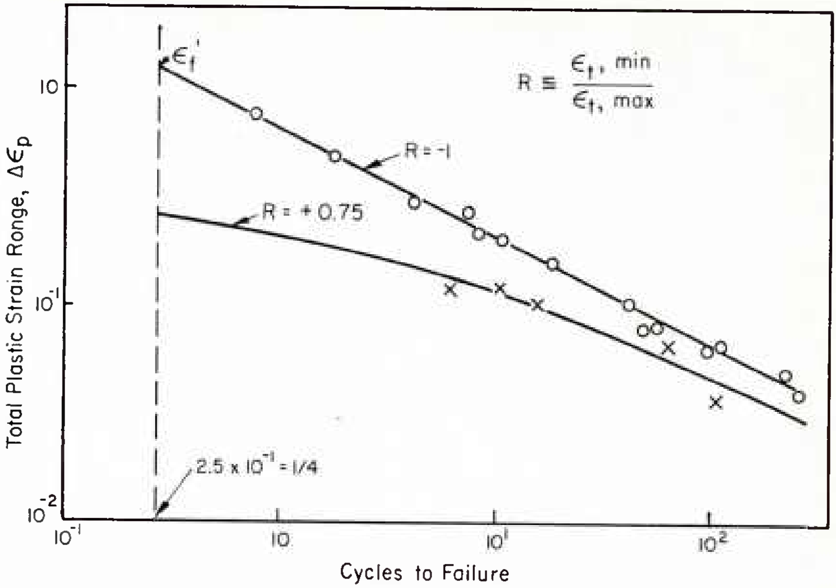

Figure 66.—Strain-cycling fatigue-test results for A-302 steel (lines drawn from equation 4 with ε_f' adjusted to fit for $R = -1$).

separation of R-curves and S–N diagrams for long lifetimes. However, for the latter the R-curves approach each other at lower values of N, while in figures 65 and 66 the different R-curves come together at larger values of N.

REPRESENTATION OF LOAD-CYCLING DATA

Two NASA studies (*9, 10*) illustrate the behavior of materials subjected to constant-load cycling. These studies were conducted with axial loading and with particular care to keep the stress amplitude constant during each test. Illustrative results are shown in figures 67 and 68. The former shows observations on unnotched sheet specimens of two aluminum alloys; the latter shows data for unnotched and for notched sheet specimens of a stainless steel. In figure 68 the early drop in fatigue strength that the sharply notched specimen shows (below 100 cycles) deserves some emphasis. This indicates one important aspect of low-cycle fatigue for many aircraft structures which contain significant stress raisers.

Benham and Ford (*11*) have examined the relationship of data obtained in load cycling to data obtained in strain cycling. Figure 69

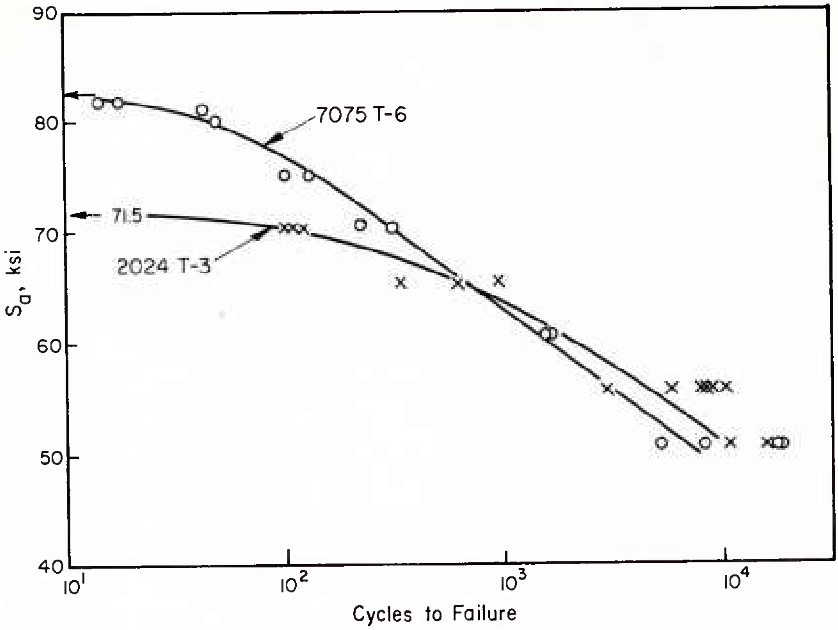

FIGURE 67.—Load-cycling fatigue-test results for two materials (fully reversed loading).

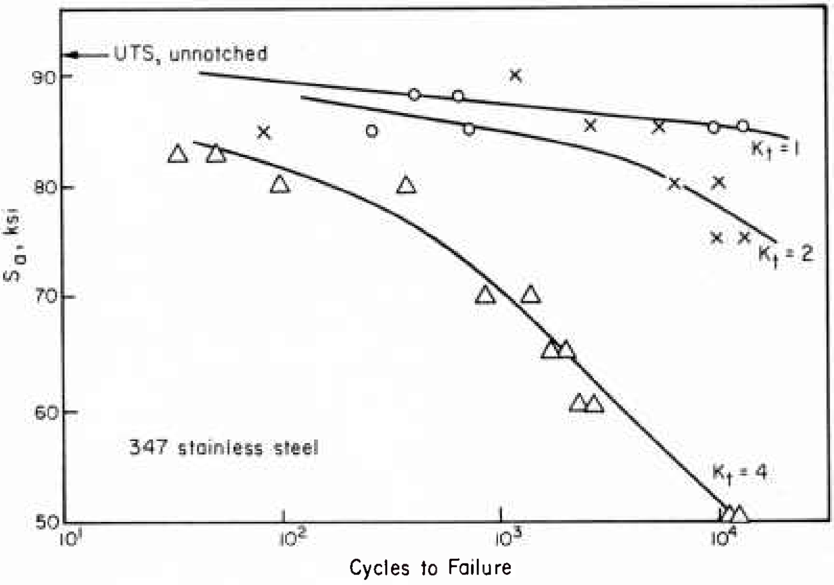

FIGURE 68.—Load-cycling fatigue-test results for notched and unnotched specimens (fully reversed loading).

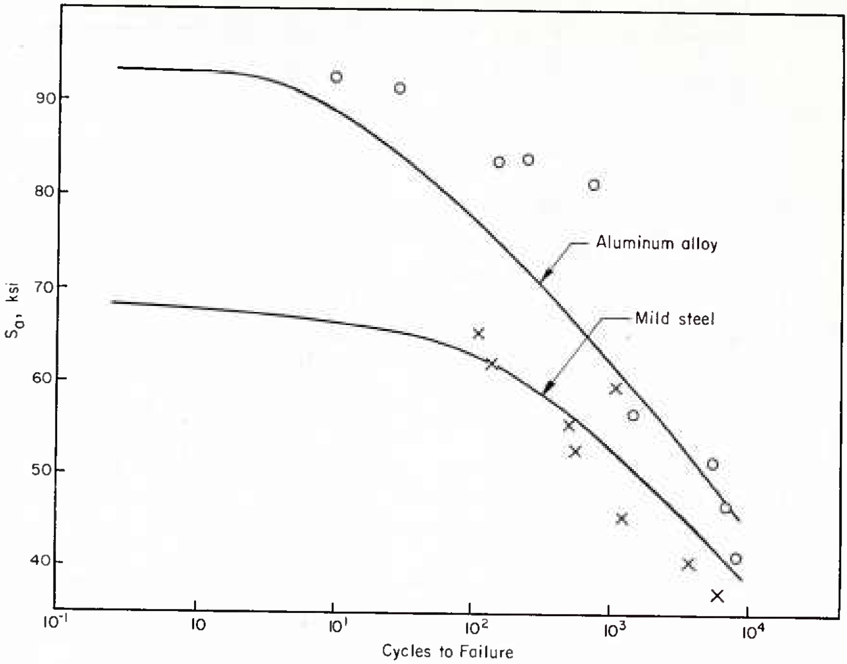

FIGURE 69.—Load-cycling curves and data from strain-cycling tests.

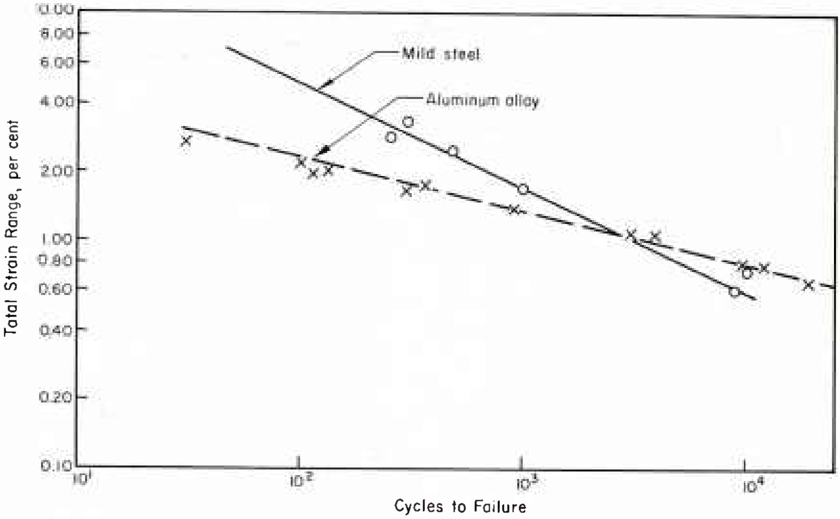

FIGURE 70.—Strain-cycling curves and data from load-cycling tests.

shows illustrative results. For two materials (an aluminum alloy and a mild steel) there are lines representing results of load-cycling experiments with "points" computed from strain-cycling experiments. In computing these points for a specified range of load, it was found necessary to take a strain range corresponding to a sort of equilibrium value after a number of cycles and *not* the strain range from a static stress-strain curve. It should be noted that the "stress" in figure 69 is the conventional engineering value based upon original cross section and thus is "fictitious" at the very low-cycle—high-stress portion. Figure 70, from the same source, shows strain-cycling curves with "points" computed from load-cycling tests. Comparison of figures 69 and 70 reveals a *practical precaution*: one metal may appear superior to another in a load-cycling test but inferior in a strain-cycling test unless hysteresis data are available to provide a proper stress-strain relationship for relating the two different loading responses.

DESIGN TO PREVENT LOW-CYCLE FATIGUE

A task group of the ASME boiler and pressure vessel committee considered the possibility of a practical procedure for design in the very-low-cycle region. Such a procedure is discussed by Tavernelli and Coffin (*6*) essentially as follows.

If one uses elastic stress analysis for a structure in which there is considerable plastic deformation, the computed strains, but not the stresses, may approximate those observed.(1) Manson (*12*) has shown this to be a good approximation for various cases of external loading and of thermal loading. The strains so estimated can be used in a Coffin-type equation:

$$\Delta\varepsilon_t = \Delta\varepsilon_p + \Delta\varepsilon_e = CN^{-1/2} + \Delta S/E. \qquad (5)$$

It is next assumed that:

$$C = \varepsilon_f = \tfrac{1}{2} n\ell \frac{100}{100 - RA},$$

RA = percent reduction in area in static test,

$\Delta S = 2S_e$, and

S_e = fatigue limit ($R = -1$ at 10^7 cycles).

Figures 71 and 72 show observed values in comparison with values computed from equation 5. In many cases the computed results are conservative. Manson (*12*) has discussed the shortcomings of the approxi-

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(1) See the precaution indicated in the work by Benham and Ford in the preceding section.

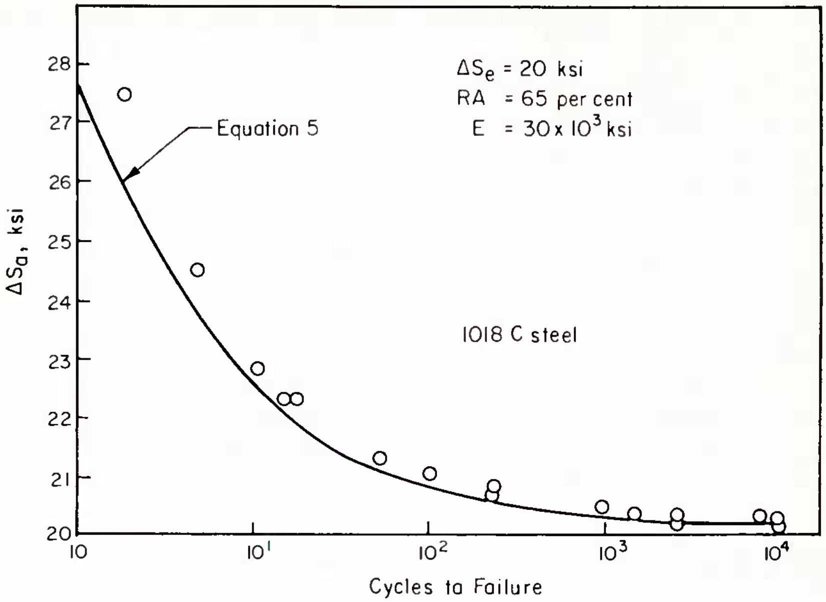

Figure 71.—Approximate calculation of low-cycle fatigue for a steel.

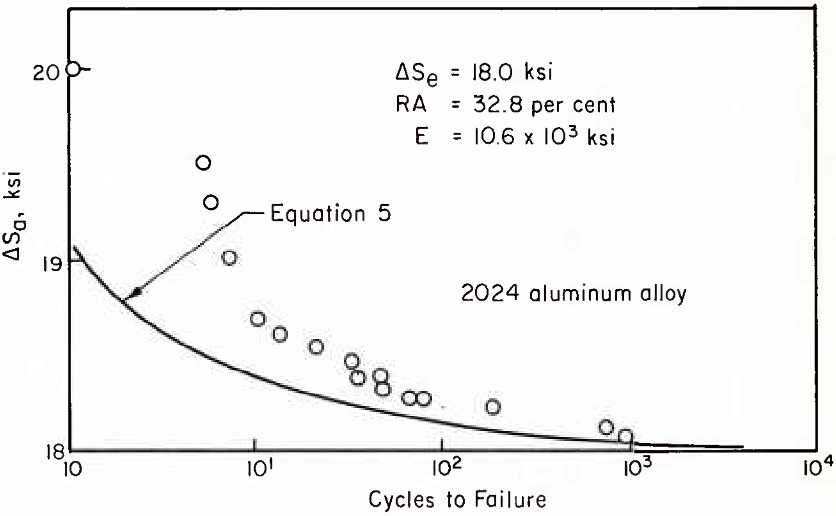

Figure 72.—Approximate calculation of low-cycle fatigue for an alluminum alloy.

mate procedure suggested above and has suggested a more elaborate empirical approach.

Many situations of concern in structural design have been less extensively explored in the low-cycle range than in the long-lifetime range. These include behavior under combined stress and cumulative damage under varying stress (or strain) loading. Studies in these areas are in progress, but obtaining results of general applicability in design may require much time and effort.

In summary, in the past decade there has developed increased recognition of the engineering importance of low-cycle fatigue, and consequently some approaches have been suggested for analysis of behavior in this region. For unnotched material specimens consideration of strains rather than of nominal stresses has been helpful in correlating experimental results. Approximations such as those in equation 5 provide procedures that appear useful in some instances. Such approximations must be used with care and should be expected to afford only rough estimates and a framework against which test observations can be viewed.

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CHAPTER VIII. EFFECTS OF TEMPERATURE ON FATIGUE

TRENDS AND SOME FACTORS INVOLVED

Recent developments—such as supersonic aircraft and reentry space vehicles—focus engineering concern on the behavior of structural materials at elevated temperature. On the other hand, containers for such fuels as liquid oxygen and liquid hydrogen and valving equipment to control their flow bring attention to materials behavior at very low temperatures.

Figure 73 shows approximate values for some mechanical properties of an aluminum alloy of particular current interest over a range of temperatures. The fatigue strength indicated appears to follow the course of the curve for ultimate tensile strength. If one were to predict the fatigue curve from the tensile-ultimate-strength curve, assuming that the ratio is everywhere the same as at room temperature, he would have a generally conservative prediction. This procedure is, however, less dependable than it might appear because it does not take realistic account of several complicating factors.

At high temperatures the ultimate tensile strength of an alloy may be significantly dependent on the strain rate used in its determination and even more dependent on the exposure time at temperature prior to and during the measurement. Moreover, the strain in a metal under steady stress may increase very much with time; this is the phenomenon called *creep*. If the strain is held fixed, the stress may decrease with time; this is called *stress relaxation*. Under a combination of steady and alternating stress, we expect a complex behavior pattern involving creep, stress relaxation, fatigue, and possible interactions of these. Practical experience shows this complex behavior does, in fact, occur. Consequently, design to prevent fatigue (and creep or other causes of failure to perform a necessary function) is complicated.

At low temperatures some alloys are known to be subject to "brittle fracture." This tendency suggests that notch-sensitivity and fatigue-crack propagation rates may warrant careful consideration under conditions of low temperature.

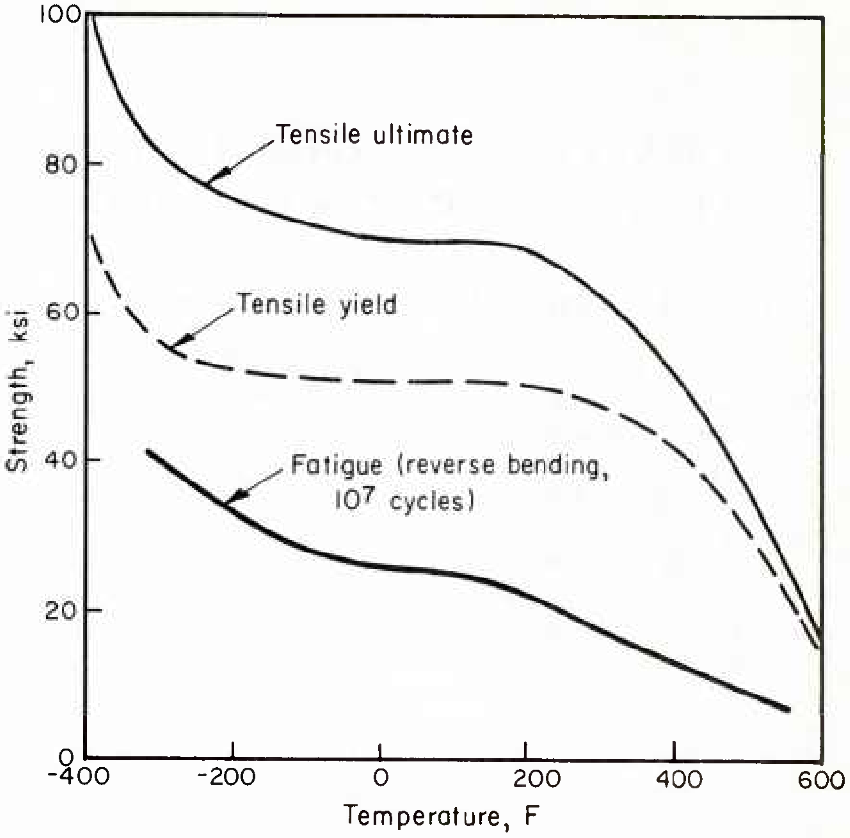

FIGURE 73.—Trends with temperature for an aluminum alloy.

There is a third area to be considered in regard to the effect of temperature on fatigue—that of *thermal stresses* and "thermal fatigue." The following sections contain some discussion of these topics and references for additional reading about them. Most emphasis is given to high-temperature behavior, primarily because most information is available in this area.

CREEP, CREEP RUPTURE, AND STRESS RELAXATION

Figure 74 is a schematic illustration of the behavior of an unnotched specimen subjected to a constant stress, the resulting strain being

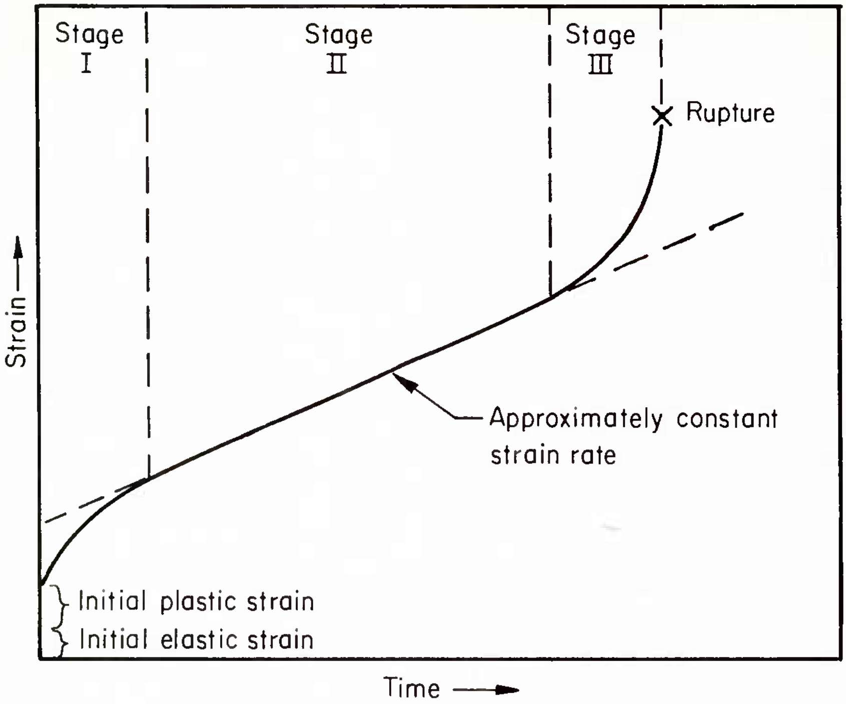

FIGURE 74.—Schematic creep curve.

observed over a period of time.(1) After an almost instantaneous strain, there is a stage (primary creep) during which the time rate of increase in strain changes. This rate settles down in a secondary stage to a constant (minimum) value, which is dependent on the stress. A widely used empirical generalization is

$$\left[\frac{d\varepsilon}{dt}\right]=B_{\sigma}^{n}, \tag{1}$$

where ε represents the true strain (log l/l_0) and σ represents the true stress (force divided by actual area rather than initial area of cross section). Finally, in third-stage creep, there is a rapid increase that ends in catastrophic rupture. If a number of specimens are tested, each at a different stress level, the times to rupture will increase with decreasing

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(1) Often the test condition is constant load, rather than constant stress, and the measurement is relative extension of the specimen length rather than strain.

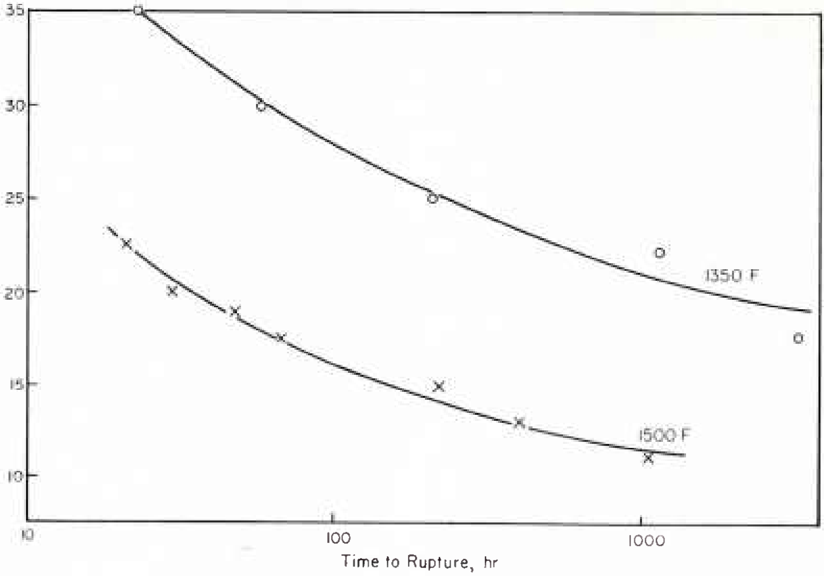

Figure 75.—A stress-rupture curve (N–155 alloy).

stress; a plot of stress against t_r, time to rupture, is called a *stress-rupture curve.*(2) Figure 75 is an example.

A stress-rupture plot may be used to design against rupture. However, prior to rupture a bit of material may elongate or deform by creep. In some circumstances, creep beyond a specifiable amount is not tolerable: excessive deformation may not be permissible or deformation in some region may result in unduly shifting load to an adjacent member. When a design limitation on creep deformation is important, curves such as those in figure 76 may be constructed.

Creep also affects the apparent "static notch sensitivity" of metals and alloys at high temperatures. Under some conditions a notch may produce an apparent weakening while under other conditions a notch may produce an apparent strengthening of a material. This is illustrated in figure 77 where the specimens with notches of small radius seem weaker (in terms of *nominal* stress) than unnotched specimens, while the specimens with larger radius notches seem stronger than the unnotched. This behavior has been explained in terms of the triaxiality of stress near the notch root and of a shear-stress criterion for creep. If there is enough creep ductility in the material, local creep will lower

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(2) Various parametric relations have been proposed to improve extrapolation of creep-rupture data. A widely used (the Larson-Miller) method involves plotting stress against

$$(T + 460)\ (20 + \log t_r)\,,$$

where T is temperature in degrees Fahrenheit, and t_r is time to rupture in hours.

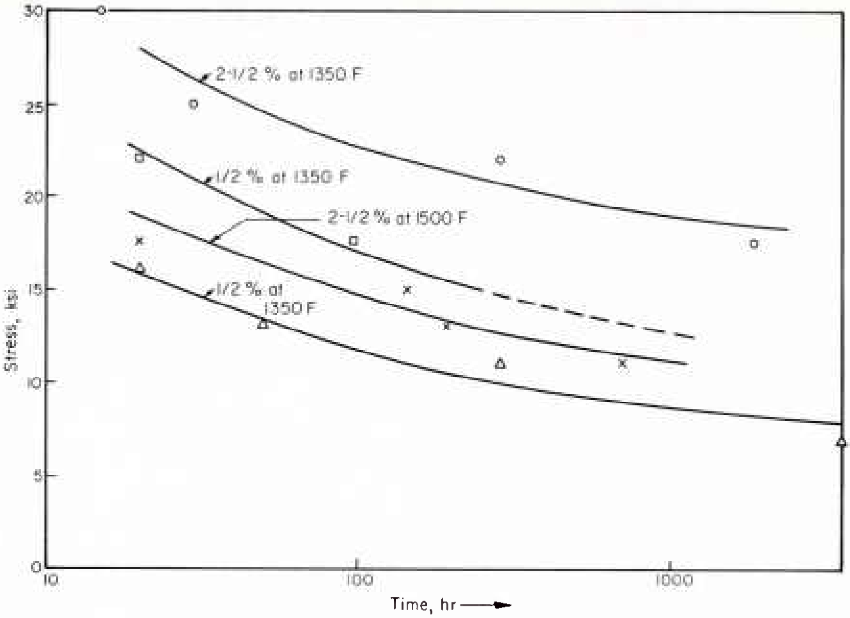

Figure 76.—Times to specified per cent deformation (N-155 alloy).

the stress concentration and reduce the shear stress available to produce creep rupture. If, however, the material has low creep ductility or the notch is very sharp, failure will occur before the stress system is sufficiently altered.

Further details on the creep behavior of engineering materials may be found in the literature (*1*). The points discussed above afford some background for consideration of creep and fatigue. First, we consider the two processes independent of each other; next, we consider interactions.

APPROXIMATION: CREEP AND FATIGUE INDEPENDENT

A simple design procedure is to assume that fatigue and creep (or creep rupture) are independent of each other but that a design objective is to prevent either. To illustrate this, let us make three assumptions *whose validity we may question later:*

1. Under fully reversed stress, fatigue is overriding and creep is negligible; under steady stress, creep rupture (or creep deformation) is limiting.
2. Fatigue is cycle-dependent but insensitive to speed of cycling.
3. The two processes may be considered by separation of the stress components (alternating and mean).

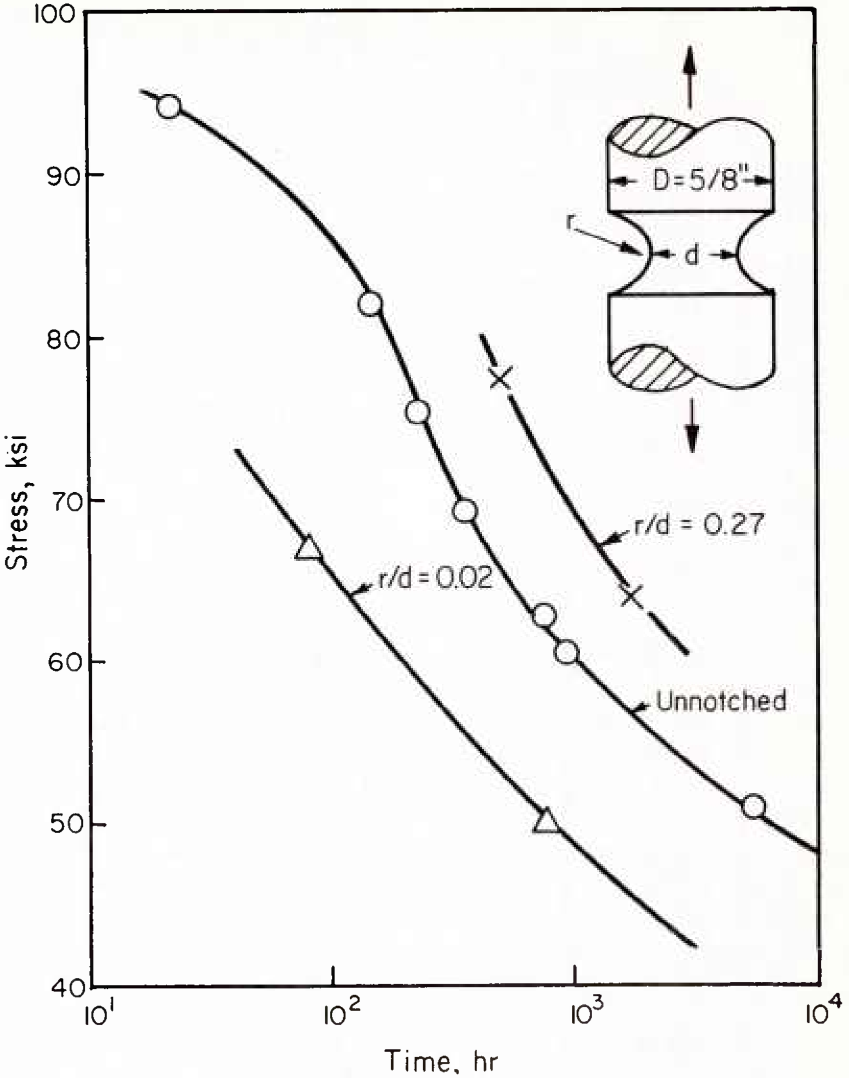

FIGURE 77.—Stress-rupture data illustrating notch effects (refractalloy 26, 1,200° F).

Now let us examine the behavior of unnotched specimens of a high-temperature alloy under two conditions: (1) where creep rupture is a design criterion, and (2) where creep deformation is a design criterion.

Figure 78 shows a plot of S_a versus S_m for three selected times to failure. The solid lines fit the relation

$$\left[\frac{S_a}{S_f}\right]^2 + \left[\frac{S_m}{S_R}\right]^2 = 1, \tag{2}$$

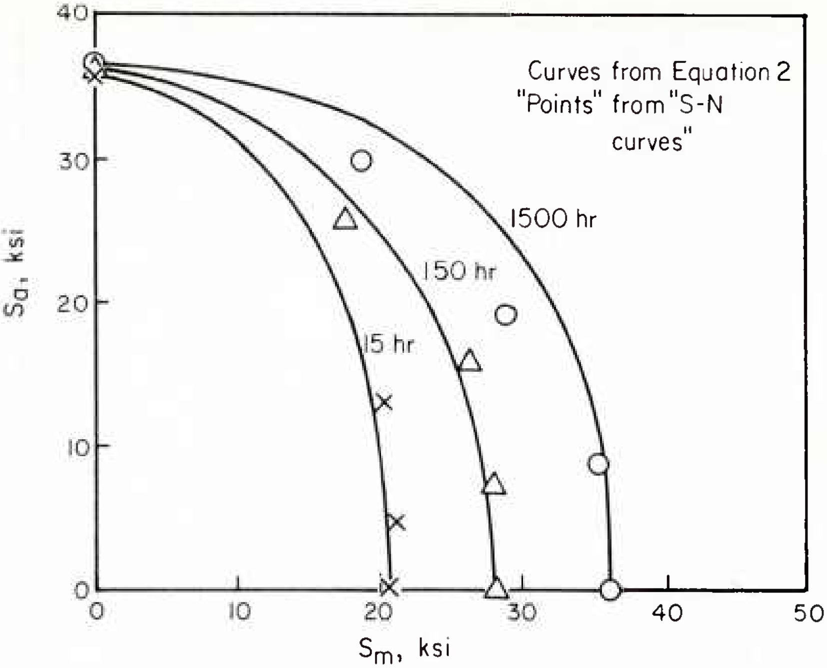

FIGURE 78.—Illustrative data for N–155 alloy at 1,350° F, tested at 3,600 cpm stress combinations for rupture.

in which S_f is the fatigue strength under full reversed load for N appropriate to the times listed,⁽³⁾ and S_R is the creep-rupture strength. The "points" are values interpolated from "S-N curves."

Figure 79 is a similar diagram drawn not for fatigue fracture but for creep to 2-percent deformation. The solid lines fit

$$\left[\frac{S_a}{S_f}\right]^2 + \left[\frac{S_m}{S_c}\right]^2 = 1, \tag{3}$$

where S_c is the creep strength for deformation to 2 per cent in the times listed.

Clearly such diagrams can be constructed for a variety of conditions: creep or creep-rupture limitations, notched or unnotched specimens, various temperatures, etc. If we make full use of assumption (2), the data to obtain S_f can be taken at any convenient speed of cycling. Moreover, a diagram may be drawn by use of relations 2 and 3 with quite limited data or, if there are data available from tests at various stress combinations, the diagram can be constructed as a purely

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(3) E.g., $N_{150\ hours} = 9{,}000 \text{ min} \times 3{,}600 \text{ cpm} = 32.4 \times 10^6$ cycles.

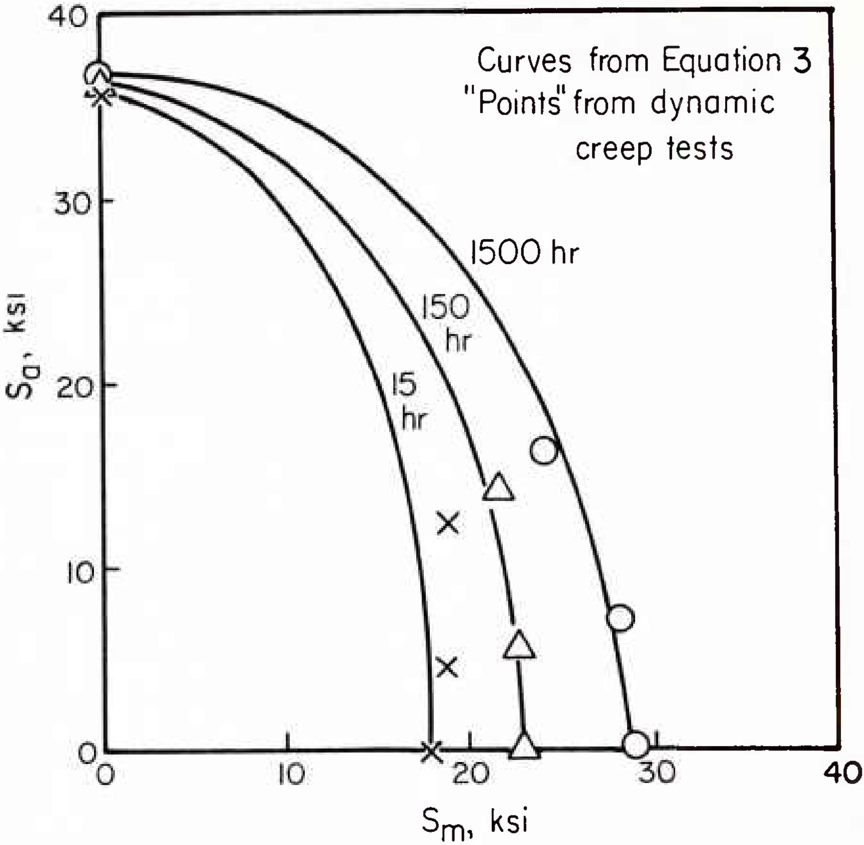


FIGURE 79.—Illustrative data for N–155 alloy at 1,350° F, tested at 3,600 cpm stress combinations for 2 percent creep.

empirical plot (which may not turn out to be a family of parabolas). It should be emphasized that even empirical plots cannot be extrapolated dependably beyond the ranges of frequency, temperature, stresses, and time for which data are available.

Other methods based upon empirically determined "end-points" (S_f and S_R) have been suggested (see, for example, reference *2*). These, like the parabolic fitting, have little foundation and have not been shown to fit all data well. A parametric method for correlating *S–N* curves has also been suggested (*3*).

The assumption that creep and fatigue are independent is a simplification that can be misleading in design.

Both present knowledge of the basic mechanisms probably involved in the two deformation processes and experimental evidence imply the likelihood of either process influencing the other. Some of the interaction possibilities have been discussed by Freudenthal (*4*) and by Lazan (*5*).

A few examples of lack of correlation between prediction of simple (no interaction) concepts and experimental results are given here to illustrate the importance of design precaution with respect to unwarranted extrapolation.

If it is assumed that the creep rate varies with stress according to equation 1 and the loading is sinusoidal according to

$$S = S_m + S_a \sin \omega t,$$

one may compute, by numerical integration and by use of values of B and n from static loading, the creep to be expected under cyclic loading (*6*). In some situations computed values agree with values observed under cyclic loading. In other cases, as illustrated in figure 80,

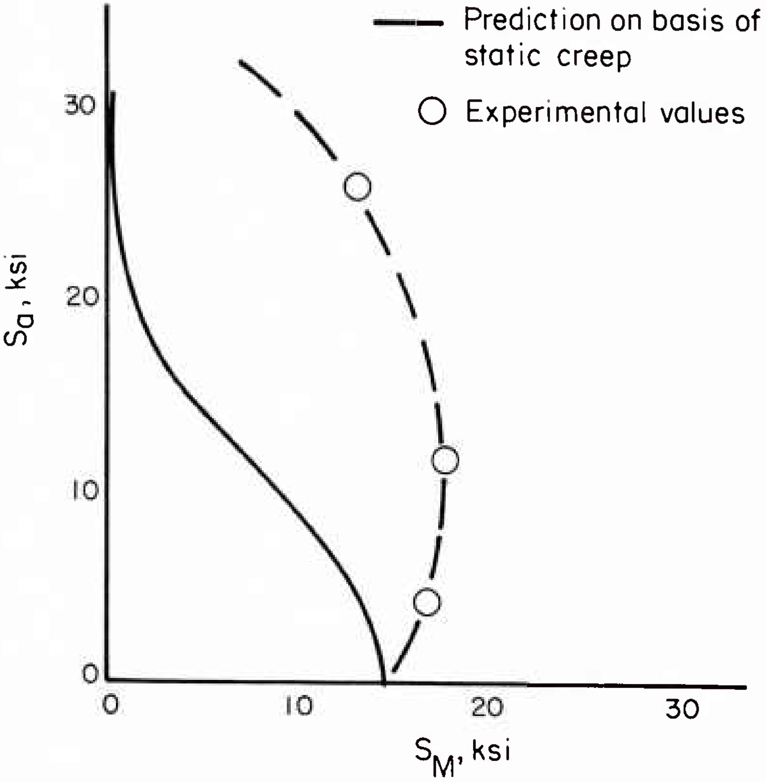


FIGURE 80.—Example of poor correlation between computed and observed dynamic creep in a "superalloy."

there may be large differences between such a prediction and experimental results. Lazan (*5*) has considered several oversimplifications in this computation, such as neglect of primary creep. Finding most of these oversimplifications unlikely to explain the discrepancies, he suggested that "synergistic" interaction mechanisms are important.

There have been a number of reported experiments in which values predicted by neglect of interaction were conservative; in other situations there may be close agreement or the computation may be nonconservative.

Strikingly large creep may occur under cyclic stressing even with zero mean stress. Figure 81 (*7*) illustrates this "cyclic-strain-induced creep." Dynamic creep may be influenced by various changes in load and temperature (*8*).

The conditions of interaction important to engineering design are neither well known from theory nor fully established experimentally. In a recent investigation (*9*) special attention was given to the effect of frequency on creep of a super alloy; results indicated a relatively narrow range of frequency in which the effect was large and this range varied with stress and temperature conditions.

Cumulative damage under varied stress amplitudes has been studied to a limited extent. Some results under conditions in which creep

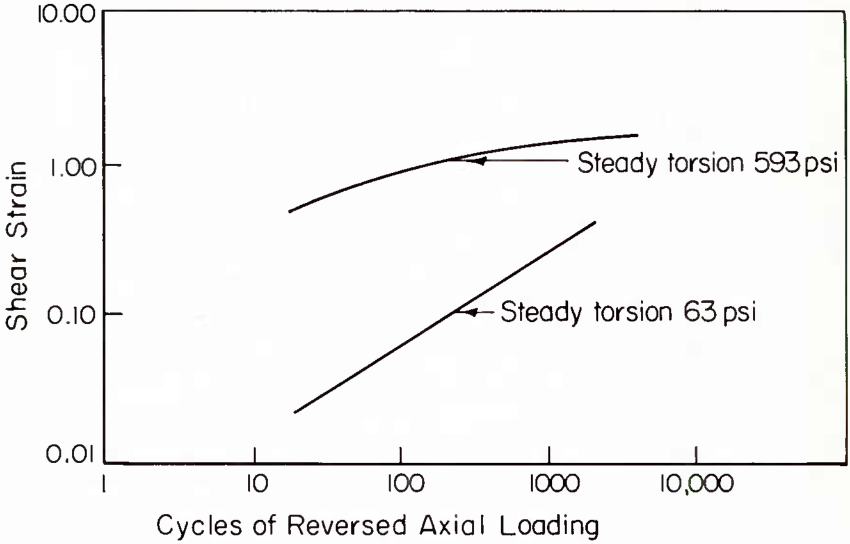


Figure 81.—Enhancement of torsional creep by reversed axial loading (material is 2S aluminum—see references 7 and *8*)..

might not be important are given by Rey (*10*). The interaction between creep and fatigue leads to complications in definition of a damage parameter. For some discussion of this, see previously cited references *4* and *5*.

In summary, creep may cause a material to deteriorate or strengthen with respect to fatigue, and fatigue may accelerate creep. Either process may produce changes in metallurgical structure which would be expected to influence the material reaction to the other process. For such reasons, values indicated in examples of various points in this chapter or values in the literature must be considered illustrative of possible material behavior but not directly usable in precise design. Moreover, design diagrams such as figures 78 and 79 must be used only with the precaution that extrapolation to different conditions of frequency or of time may be invalid. The main contribution of existing background is guidance in experiments for obtaining design values.

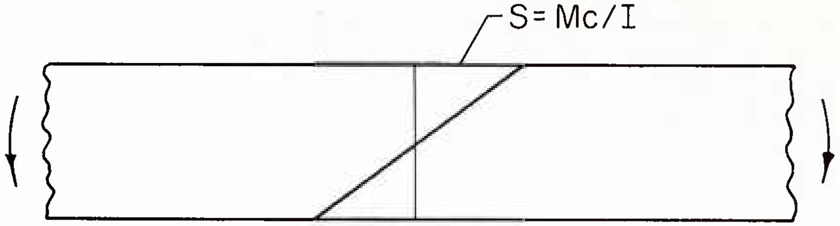
ADDITIONAL FACTORS IN HIGH-TEMPERATURE FATIGUE OF STRUCTURES

Whenever creep is involved in a structural part, the theory of linear elastic behavior is inadequate to compute stress and strain distribution. Even analysis of a cantilever beam under a constant end-load becomes complex; the stress distribution may vary with time from that of figure 82a through that of 82b toward that of 82c. Time-dependent stress analyses for a number of cases have been programmed for digital computers (see, for example, reference *11*). The point is that incorporation of empirical fatigue-creep data such as previously illustrated can involve extremely complex analysis in a relatively simple structure. In such analysis the time-dependent strain distribution may be extremely important.

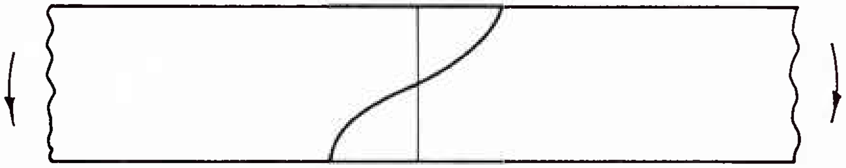
In most actual situations the stress field is biaxial or triaxial at critical locations. Relatively little is known about combined-stress fatigue at temperatures high enough for creep to be significant (see references *4*, *5*, and *12*).

FATIGUE BEHAVIOR AT LOW TEMPERATURES

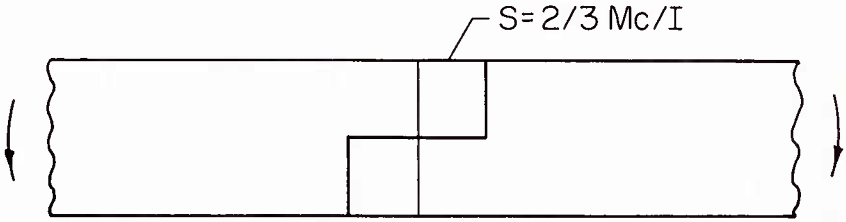
For almost all materials the fatigue strengths at low temperatures are higher than that at room temperature. In general, the ratio of fatigue limit (under fully reversed stressing) to the tensile ultimate will also be as high as or higher than at room temperature.



a. Elastic Stress Distribution



b. Elastic-Plastic Stress Distribution



c. Plastic Stress Distribution

Figure 82.—Possible stress distributions in bending—creep rate varies with stress.

This observation has significance in regard to possible basic mechanisms (*13*) since it means that there are fatigue mechanisms still operative at temperatures at which certain processes (such as diffusion) would be expected to be negligible. Its engineering significance is that, when alternating stresses are concerned, there must be consideration of prevention of fatigue even at cryogenic temperatures.

There are, of course, relatively few test data at very low temperatures. References *14, 15,* and *16* (together with reference *13*) illustrate the data available. These references indicate:

1. There are few data obtained under combined steady and alternating stress.
2. There are few low-cycle data (although these have significance for some cryogenic pressure vessels).
3. There are virtually no data indicative of statistical variation.
4. There are very few data obtained under combined stress conditions.

While there are some data on fatigue-notch sensitivity at low temperatures, these are rather inadequate to provide confidence in design.

The present knowledge about fatigue behavior at low temperatures can be summarized as follows. Enough surveys have been made to indicate the general presence of fatigue phenomena at temperatures down as low as −423° F. Studies have shown no particularly alarming trends. However, in anticipation of more concern with this environment in the near future, much more study will be needed.

THERMAL FATIGUE

The term "thermal fatigue" was originally used to denote failure that can occur in a noncubic lattice material when the temperature is fluctuated slowly without external constraints (*17*). The application of the term has been extended to include failures caused by thermal stresses from temperature gradients and external constraints; such failures can occur in the cubic metals more commonly used in aircraft structures. In the present discussion, the term thermal fatigue will be used with the latter connotation.

One of the first to emphasize the study of thermal fatigue was Coffin (*18*). He used a tubular specimen between two end plates connected by rigid columns. Heating and cooling the specimen produced strain of the order of

$$\pm\Delta\varepsilon = \alpha\Delta T,$$

where α = thermal expansion and ΔT is the range of the temperature cycle. It may be noted that in these experiments the stress was compressive at the high temperature and tensile at the low temperature. This arrangement provided some of the first low-cycle, elevated-temperature test data. More recently, tests have been carried out in equipment capable of independent variation of temperature and loading (*19*). To a first approximation, it appears that the resistance to alternating strain when the temperature is fluctuated in phase with the applied strain cycle is about the same as when the temperature is held constant near the maximum temperature all the time. Consequently, some information helpful in regard to thermal fatigue can be obtained from mechanical low-cycle fatigue tests. (See chapter VII.)

Some general guidance in consideration of thermal fatigue in structures can be obtained by considering individual factors. The thermal expansion coefficient, α, and the constraints will directly influence the range of strain for a specific temperature fluctuation. The temperature gradients and the local temperature fluctuations will be influenced by the thermal conductivity, k, of the material. It is generally desirable to have a low value of α and a high value of k to minimize thermal stresses.

Manson has dicussed allowance for thermal stress in design in a series of papers (*20*).

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CHAPTER IX. CORROSION AND FRETTING

CORROSION AND FATIGUE

A most important limitation in the selection of materials for aircraft structures is imposed by the necessity for the materials to resist detrimental chemical reaction with the environment in which they must operate. For present purposes, we may consider the term corrosion to mean all such reactions.

Essentially all aircraft structural parts are exposed for a considerable time to "normal atmospheric air." This is corrosive to the high-strength materials commonly required for such parts. However, there are many conditions possibly more serious with respect to corrosion. For example:

1. Not only carrier-based but land-based aircraft may frequently be subjected to a *marine atmosphere.*
2. Landing and takeoff conditions may include exposure to runway *de-icing salts.*
3. Flights involve exposure to low temperatures with *condensation* upon landing and return to warmer ambients.
4. *Engine exhaust gas* impingement subjects parts to a highly corrosive environment.
5. In extreme cases, *bacteria* in fuél tanks and from sanitary and cooking facilities can destroy structural parts.

Hence, the possibilty of corrosion must be considered in the selection and evaluation of aircraft structural materials.

Corrosion may be considered as essentially an electrochemical process. In the analysis of many corrosion problems, one important factor is the amount of electrical potential between dissimilar metals immersed in an electrolyte. Table 5 shows the electromotive series of a number of the elements with respect to hydrogen. When two metals differing widely in position in this series (as aluminum and iron) are joined in the presence of an electrolyte, a cell is formed, and there is a tendency for the less noble metal to corrode. For example, if a steel part is in contact with aluminum in an electrolyte (which might be salt spray), the aluminum will corrode although the steel will remain relatively unaffected. Thus, the use of dissimilar adjacent materials in aircraft structures is to be avoided.

The electrochemical process set up by two dissimilar metals in contact with each other and immersed in an electrolyte is called *galvanic corrosion.* While the electromotive series in table 5 indicates roughly the trends for such a process, the performance of two metals or alloys may differ in different electrolytes. Table 6 shows a galvanic series of metals and alloys in sea water under particular conditions. Note that the metals are arranged in generally the same order as in the electromotive series shown in table 5, but there are differences in chemical constituents.

TABLE 5.—*Electromotive series*

Element	Standard electrode potential, $E_m{}^0$, volts, at 25° C
Potassium	−2.922
Sodium	−2.712
Magnesium	−2.34
Aluminum	−1.67
Zinc	− .762
Chromium	− .71
Iron	− .440
Cadmium	− .402
Nickel	− .250
Tin	− .136
Lead	− .126
Hydrogen	.000
Copper	.345
Silver	.800
Palladium	.83
Platinum	1.2
Gold (valence 3)	1.42
Gold (valence 1)	1.68

NOTE: This series holds for metals in particular concentrations of their own salts. In other electrolytes, involving other reactions, performance may be different. Hence, the electromotive series can provide only a rough guide concerning the possibilities of corrosion. Compare the galvanic series in table 6.

It is important to recognize that corrosion reactions, even though they may be entirely electrochemical, are influenced by many factors in addition to the potentials indicated in the electromotive series. These factors include: inhomogeneities in the metal surfaces and in the electrolyte, solubility of corrosion products, polarization by reaction products, effects of stress, and development of stress concentrations.

TABLE 6.—*A galvanic series in sea water* (a)

Metal or alloy	Potential, (b) volts
Zinc	1.03
Alclad 3S	.94
61 S–T aluminum	.76
Carbon steel	.61
Copper	.36
Nickel	.20
Titanium	.15
Silver	.13

(a) At 77° F., flowing at 13 fps., after 14 days (see reference 1).
(b) Relative to saturated Calomel Half-Cell.

As might be expected, the complexity of electrochemical corrosion affords varied types of observed behavior. Classification is sometimes confusing, but some of the widely used terms pertinent to the relation of corrosion and fatigue should be kept in mind. Some terms which are frequently used are: uniform corrosion, pitting corrosion, intergranular corrosion, stress corrosion, and corrosion fatigue.

Uniform corrosion results in an even thinning of the metal. The attack may be either rapid or slow. Uniform corrosion is typical of the type of corrosion occurring in many acids and pickling solutions. This quite common form of corrosion is easily evaluated by weight-loss determinations or by micrometer measurements of thickness. Its main effect on fatigue strength is the increase in stress resulting from a decrease in section of a part.

Pitting corrosion occurs when some areas corrode faster than others. While a pit may start as a result of some surface imperfection, most severe pitting is normally associated with environmental reactions. The major causes are metal-ion concentration cells and differential aeration cells. In the first case, a potential is developed between areas on a metal surface in contact with solutions of different metal-ion concentration. The area in contact with the dilute solution becomes the anode and the metal at this point corrodes. In differential-aeration cells the areas exposed to a high concentration of oxygen tend to become cathodic. Conversely, those areas in contact with low concentrations of oxygen become anodic. Pitting is a very common form of corrosion and, as noted later, plays an important role in fatigue life.

In addition to the environment, the internal structure of the metal or alloy may influence corrosion. For example, the presence of inhomogeneities such as inclusions, grain boundary constituents, bonding, and different alloy phases may result in selective attack. Probably the most familiar form of selective attack is *intergranular corrosion.* This is essentially selective attack of the grain boundaries or adjacent metal. The mechanism involves a difference in potential between the grain and the grain boundary. Aluminum alloys of the 2024 type, containing copper-rich grain-boundary phases, and certain AISI-300 series stainless steels, improperly heat-treated and containing intergranular carbides, are susceptible to intergranular corrosion in some environments.

A combination of steady tensile stress (applied or residual or both) and a corrosive environment may lead to *stress-corrosion* cracking. High-strength alloys are particularly susceptible. Figure 83 shows a crack in a longeron splice fitting of 7079—T6 aluminum alloy which was analyzed as caused by stress corrosion and aggravated by fatigue. Steels heat treated to tensile yields above 200 ksi may be susceptible to stress corrosion. The delayed failure of high-strength steels under steady tension in an aqueous environment has been described in terms of hydrogen, produced by a corrosion reaction, acting as an embrittling agent. It is good practice to require that parts of high-strength materials and the

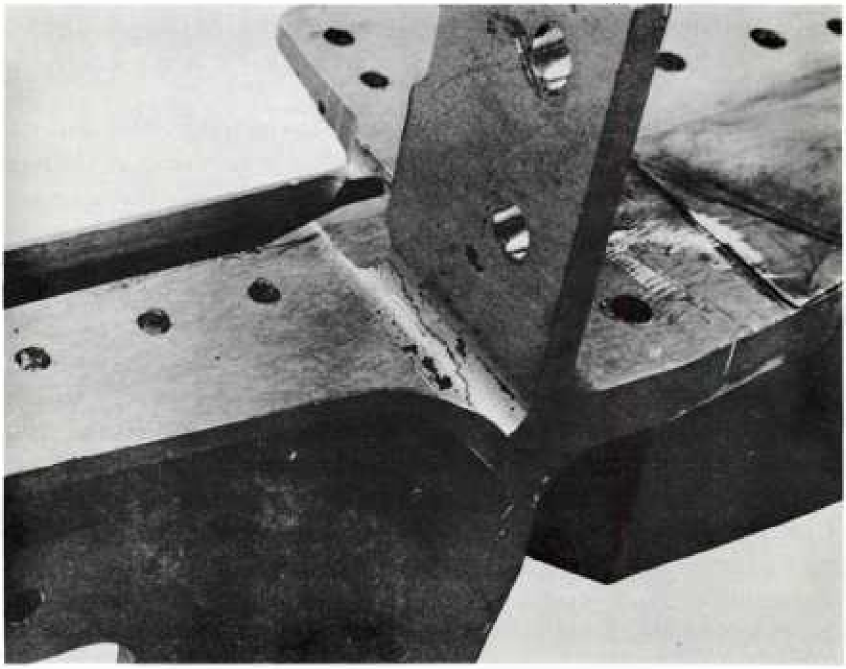


Figure 83.—Stress-corrosion fatigue crack in longeron fitting.

techniques by which they are assembled or installed be such that sustained surface tensile stresses and stress concentrations are minimized. Corrective practices such as stress-relieving heat treatments, optimum grain flow orientation, and shot peening or other surface working may be necessary to minimize the hazards of stress corrosion.

The combined action of cyclic stress and corrosion produces cracking failures termed *corrosion fatigue*. Some of the ways in which corrosion and fatigue may be interrelated can be visualized. For example, corrosion pits or cracks provide stress raisers which can make a corroded part more susceptible to fatigue. On the other hand, a fatigue crack may provide a local, highly stressed area which is unusually susceptible to corrosion. However, the most serious concern is the continuous interaction between the processes of fatigue damage and corrosion.

CORROSION FATIGUE OF SOME AIRCRAFT MATERIALS

There are at least two ways in which fatigue may be concerned with corrosion: (1) corrosion while under no load may cause pits, intergranular cracks, or other "notches" which lower resistance to subsequent fatigue under repeated loading, and (2) simultaneous corrosion and fatigue may be extremely detrimental. The latter process, which is called here *corrosion fatigue*, is the subject of most of the subsequent discussion.

Even for metals which appear to have fatigue limits when tested in air, usually there is *no such limit* found with corrosion fatigue. Uniform corrosion by thinning the section will eventually produce increased stress amplitude for a fixed load amplitude; corrosion pitting will produce stress concentrations and will still further increase local stress amplitudes. The decrease in fatigue strength due to corrosion will be greater at large numbers of cycles; the corrosion process has more time to work. Figure 84 illustrates this.

For a similar reason, we expect a speed effect in corrosion fatigue. Corrosion is a rate process that is time-dependent; fatigue, on the other hand, seems dependent mainly on the number of stress repetitions rather than on time. Figure 85 illustrates this "speed effect." In making a corrosion-fatigue test and in interpreting test results, the speed of testing is most important. This presents a real problem in interpretation of results of accelerated testing. It is unrealistic to accelerate the speed or load repetition unless there is some knowledge (usually missing) of how to correspondingly accelerate the corrosive effects of the environment.

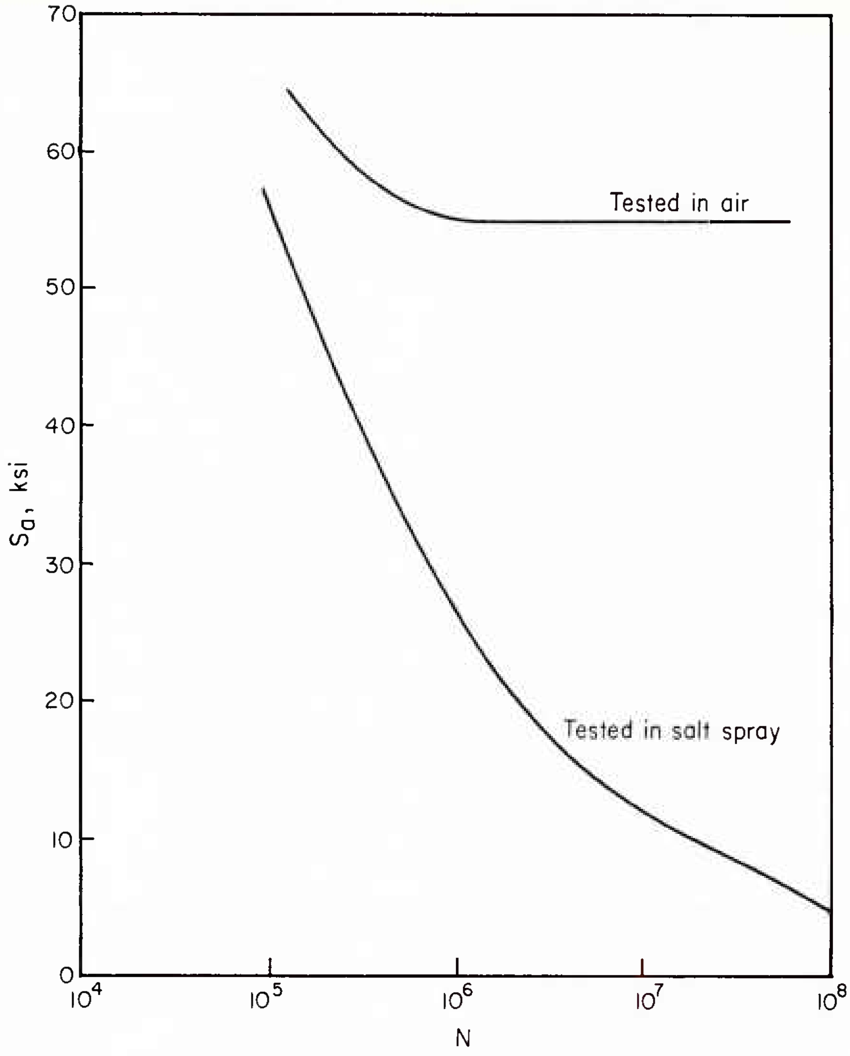


FIGURE 84.—Corrosion fatigue of 1050 steel (fully reversed axial loading at 200 cpm —spray of 3 percent salt solution at room temperature).

Thus, the many factors involved in corrosion fatigue include all of those that influence corrosion, plus all of those that influence fatigue, plus some interactions as yet only partly understood.

Some Illustrative Corrosion-Fatigue Data

Gilbert (*2*) has summarized corrosion-fatigue work to about 1956 and includes a good bibliography to that date. Harris (*3*) and Forrest (*4*) report some later work and references.

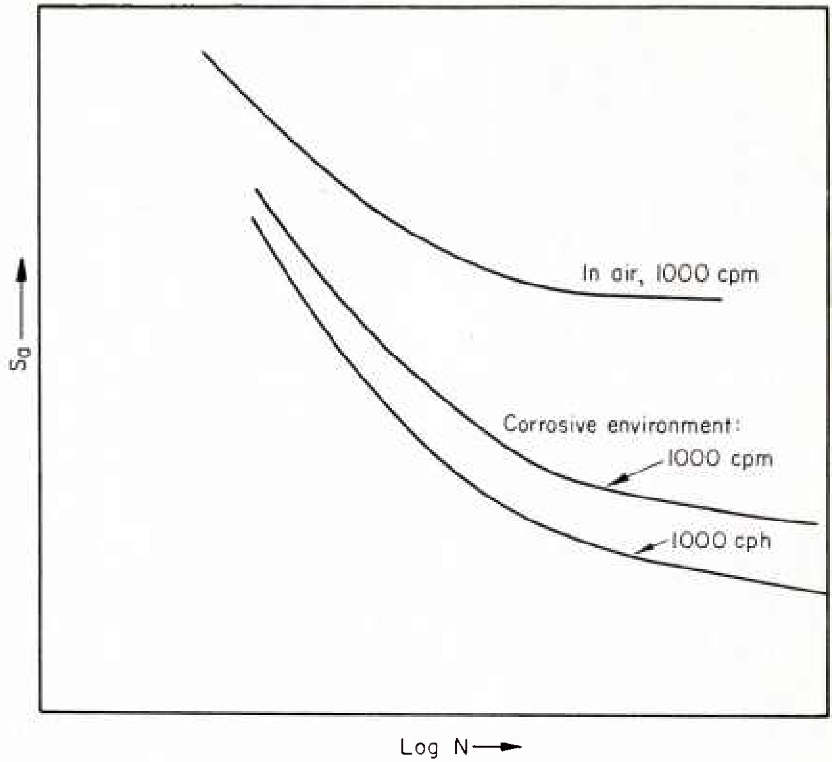


FIGURE 85.—Speed effect in corrosion fatigue.

Several test results are listed in table 7. These indicate a number of features:

1. The loss of fatigue strength in the presence of a corrosive environment can be very great.
2. Many factors influence the magnitude of the loss: the type of corrodent, the type of stressing, the type of heat treatment.
3. Generally, materials more susceptible to stress-free corrosion are also more susceptible to corrosion fatigue.

Perhaps the most important observation from the few examples in the table is that so many and such varied factors affect corrosion fatigue that is dangerous to attempt quantitative generalities.

Many structural parts in aircraft and in missiles may undergo stress-free corrosion and subsequent repeated stressing. Corrosion pits can serve as stress concentrations which result in fatigue cracking. Table 8 shows some results of rotating-bending fatigue tests on 2024—T4 alum-

TABLE 7.—*Some results of corrosion-fatigue tests* (a)

Material	Condition	Ultimate tensile strength, ksi	Fatigue test conditions (b)		Cycles	Fatigue strength, ksi		Reference
			Speed, cpm	Corrodent		Air	Corrodent	
0.2 steel	Annealed	71.2	1,300	Sea water	10^8	33.4	4.3	5
0.5 steel	do	141.0	2,200	3% NaCl spray	5×10^7	56.0	6.3	4
SAE 1035	...	87.3	1,750	6.8% NaCl	10^7	40.5	24.6	6
Do	...	87.3	1,750	6.8% NaCl + H_2S	10^7	40.5	10.5	6
SAE 3140	Hot-rolled	128.0	1,400	Tap water	10^7	64.0	34.0	7
SAE 3140 (notched)	do	128.0	1,400	do	10^7	31.2	15.9	7
SAE 3140	Q & T	166.0	1,400	do	10^7	90.0	13.0	7
18–8 stainless	...	148.5	2,200	3% NaCl spray	5×10^7	53.2	35.4	4
Spring steel	Annealed	98.0	2,200	Tap water	2×10^7	42.1	21.9	2
Do	Q & T	146.0	2,200	do	2×10^7	69.0	15.9	2
Do	Q & T	179.0	2,200	do	2×10^7	77.7	15.9	2
2024 aluminum	T–4	67.4	...	3% NaCl	...	20.7	4.0	...

(a) These results have been collected from diverse sources involving varied test conditions. They illustrate possible effects of corrosion fatigue but should not be used as design values.

(b) All specimens in rotating-bending.

TABLE 8.—*Fatigue test results on corrosion-pitted 2024–T4 aluminum alloy*

Exposure time	Average cycles to failure	Average surface roughness, rms
0	5.60 × 10^6	8.6
4 hours	1.70	12.7
1 day	.75	23.2
4 days	.49	56.9
32 days	.43	138.2

NOTES: 1. Derived from reference *8*. 2. Tested in rotating-bending at 3,600 cpm. 3. Exposed to spray of 20% NaCl in de-ionized water at 95° + 2° F.

inum alloy specimens after exposure to a salt spray which produced pitting. In these tests there was good correlation between observed lifetimes and estimates based upon stress concentrations evaluated from measurements of pit depths (and indicated by surface roughness). Such a direct correlation may not be found in all situations.

OIL
REPELLANT?

There are indications that even a "normal" atmosphere influences fatigue strength. Years ago Gough showed that fatigue lifetimes of many common metals and alloys may be significantly longer (2 to 10 times) in vacuum than in air. This has been reemphasized recently (references *9* to *11*). Some coatings of oleophobic films have been found to increase the fatigue lifetimes of aluminum in ordinary moist air. Such coatings have not yet found significant application in aircraft.

Mitigation of Corrosion Fatigue

There are two general principles for minimizing the hazard of corrosion fatigue. The first is to protect against corrosion in any anticipated environment. All the techniques of the corrosion expert can be utilized, such as proper choice of materials, coatings and surface protection, and separation of electrochemically dissimilar metals. The second principle is to keep tension stresses, including residual stress, low.

In particular instances, certain techniques, including the use of anodic protective coatings and surface-compressive stresses, may be helpful. However, these require extremely careful control for dependability and permanence.

In testing to evaluate the merit of any specific procedure, particular attention is required in regard to the relative time scales of the test and of the anticipated service life.

FRETTING AND FRETTING CORROSION

Fretting, which may be considered as localized wear, occurs at areas of contact of two metals which are in relative motion. It involves such details as:

1. Highly localized frictional forces and stresses at asperities.
2. Consequent possibilities of local transient cold-welding.
3. Possible highly localized temperatures.
4. Accumulation of wear debris.

The local clean surfaces and the finely divided particles in the debris are particularly susceptible to oxidation (or other chemical reaction). Hence, there is often *fretting corrosion*; this adds complication.

In a number of instances the fretting debris has been shown to include oxides of the parent metal. Thus, fretted steel often shows the "rust color" characteristic of Fe_2O_3; fretted aluminum may appear black in reflected light. There is ample evidence that, depending upon the environment, chemical reaction may occur. However, the nature of the corrosion products will vary with circumstances. Thus, it is not easy to characterize a "typical" appearance of fretting corrosion, but, in a specific situation, much can be inferred from the wear appearance.

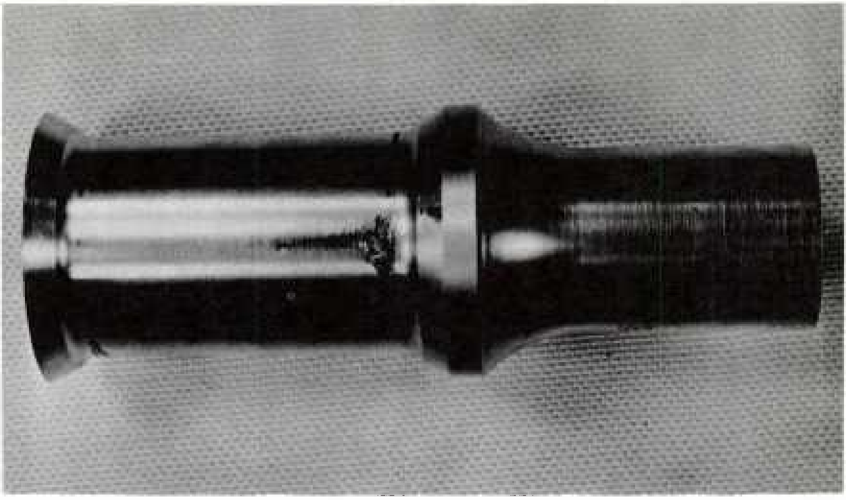
The oxides or other corrosion products may be very abrasive. Under high repeated stresses, the pits and the abrasive action may serve to initiate a fatigue crack. In some instances the debris has been considered to exert a wedging action in a crack and thus to accelerate fatigue crack propagation.

Figure 86 shows (a) fretting on an engine-mount pin during a cyclic-loading test and (b) the result of turning the pin to a new position wherein the fretted surface was on the tension side. Incidentally, this pin was of a high-strength (280–300 ksi) steel and correspondingly notch sensitive.

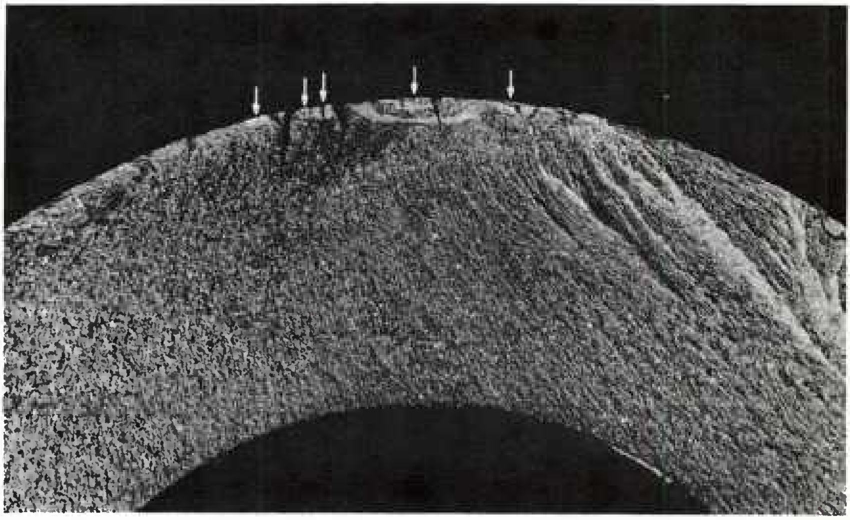
Commonly, fretting is evident in fatigue failures of pin connections and suspect of initiating or accelerating the failure. Figure 87 shows fretting in a laboratory fatigue test of a lug connection. In this case, indirect evidence indicated the significant contribution of fretting to the lowering of fatigue lifetime.

Fretting and, depending upon the environment, fretting corrosion are likely to occur in splines, keyways, and press-fitted members where relative motion is not intended. It occurs commonly in bolted joints and in riveted joints. It is a contingency to be considered in every assembled structure.

Accounts of the current state of knowledge about fretting and fretting corrosion are given in references *12, 13,* and *14*. These accounts and continuing research indicate the importance of these factors in limiting



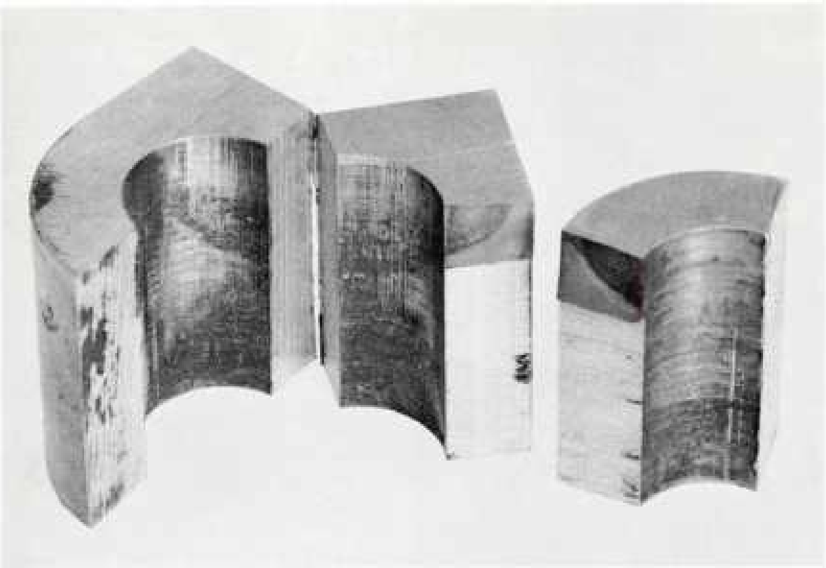
Top: Fretting on compression surface.



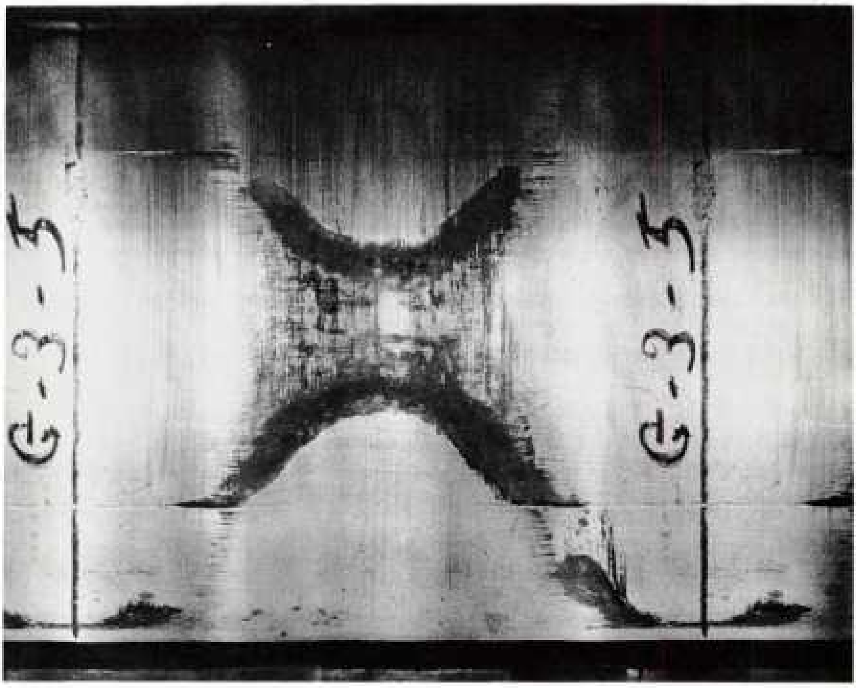
Bottom: Fatigue origins after turning pin so that fretting region was on tension surface.

FIGURE 86.—Fretting on engine-mount pin.

fatigue lifetimes. They also indicate qualitative procedures for minimizing the factors. These include design for low concentration of surface strain and use of coatings such as molybdenum disulphide and tungsten carbide to provide suitable surfaces. Quantitative prediction of the amount or effect of fretting and of fretting corrosion is virtually



Top: Fatigue crack starting at edge of lug.



Bottom: Fretting pattern on interior of lug extending to the **origin** of **fatigue crack**.

FIGURE 87.—Fretting in laboratory tests of a **lug connection**.

impossible except on the basis of tests; this is, in fact, one important reason for the desirability of component tests. In tests planned to include an evaluation of fretting, conditions must be carefully designed to include pertinent factors of service loading and environment. The latter may be especially critical if fretting corrosion is an important contingency; in this case, as in any situation involving corrosion, testing time and speed can be important.

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CHAPTER X. ACOUSTICAL FATIGUE

NATURE AND GROWING IMPORTANCE

In the past few years a new type of environment, high-intensity "noise," has become a potential cause of the destruction of structural parts. A variety of aerodynamic and thermodynamic conditions may generate noise. However, a principal cause in high-speed aircraft and in missiles is aerodynamic turbulence. Sources of such turbulence may be classified into two broad categories: (1) propulsion systems and (2) high speed.

In the first category are jet exhaust noise, noise from turbomachinery, combustion noise, and jet-nozzle-instability noise. Noise in this category contributes by direct acoustical vibration. In the second category are such noises as turbulent-boundary-layer noise, wake noise, and noise due to oscillating shocks. Contributions from this category are strongly dependent on the aircraft configuration and speed characteristics.

These sources may generate high levels of sound pressure. The overall sound power from a jet may reach 180 db(1) in the afterburner condition. This affords high sound-pressure levels at some locations near the nozzle. As higher exhaust velocities are developed, the total acoustic power increases very rapidly.

The propulsion system noise intensity is generally greatest under ground run-up conditions and decreases with increasing speed in flight. On the other hand, the noise spectrum generated by boundary-layer turbulence around the airframe increases with increasing speed. Over much of the surface of an aircraft in flight, the primary contributor to the noise environment is the turbulent boundary layer.

The noise associated with the turbulent flow in the wake of a flight vehicle is not well understood quantitatively, but it increases with increasing Mach number. Estimations indicate that it may not be important in powered flight but may be very important in a reentry vehicle. Other sources of noise are even more difficult to describe in general terms. However, it seems clear that the importance of high-intensity noise environments will increase in the future.

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(1) Reference 10^{-13} watts.

The ultimate objective of engineering is to establish design rules to allow structural components to withstand the environments in which they must function. For prevention of fatigue failure in the environment of high-intensity noise, the objective may be considered in terms of three subareas:

1. Description of the acoustic field.
2. Determination of the resultant stresses in the structure.
3. Evaluation of the fatigue life of the material.

Each area presents difficulties, and it is particularly challenging to attempt to understand the necessary interrelations among concepts in acoustics, in dynamic stress analysis, and in fatigue of materials. Progress toward complete theoretical analysis is understandably slow.

In view of this complexity and in view of the extensive current efforts from different viewpoints, this chapter cannot contain either a complete summary or design rules of extensive applicability and permanent validity. The subsequent discussion is intended, rather, to indicate some of the factors involved, to provide some references for further study, and to suggest the growing importance of acoustic fatigue in flight vehicles.

DESCRIPTION OF AN ACOUSTIC FIELD

An acoustic field comprises variations of pressure in the environment which produce varying stresses in a structure within the field. These pressure variations are complex, and their description in terms suitable for estimation of stresses is difficult.

Figure 88 shows a *spatial distribution of sound level* from a turbojet engine. This distribution varies with operating conditions. Intensities are usually highest at warm-up, fall away during takeoff, and are much lower in flight. At any location in space there is a distribution of frequencies; figure 89 illustrates this. This distribution is different at different locations; energy is differently distributed in frequency in the hydrodynamic (near) field and in the acoustically radiated (far) field. In many locations of interest the "mixed fields" can be characterized only by measurement and empirical formulas. However, analysis of pressure intensities and frequencies is not enough. The pressure fluctuations in adjacent regions may also differ *in phase*. The loading on the structure depends on the *correlation* of pressure variations in adjacent regions.

The complexities of these distributions and the problems involved in determining them have led to descriptions in statistical terms: average energy densities or root-mean-square values of pressure fluctua-

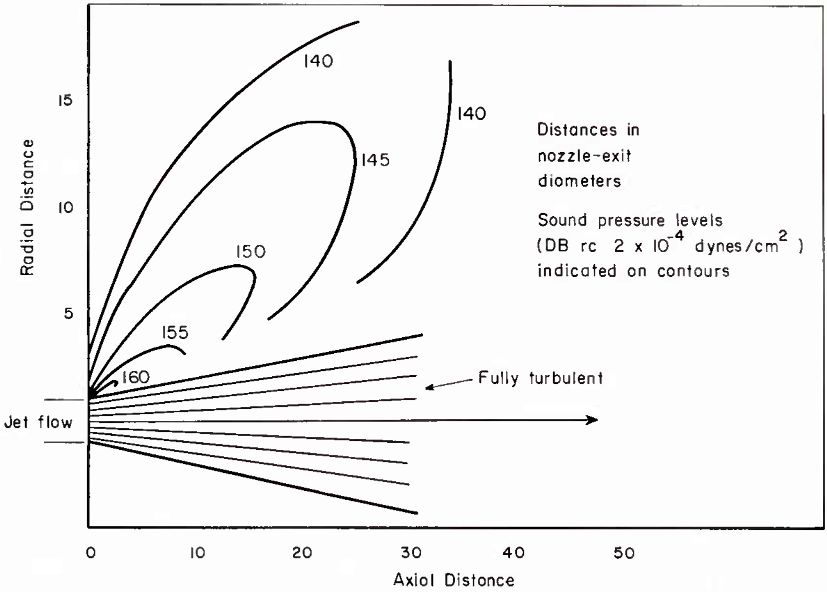

FIGURE 88.—Example of space distribution of sound-power level in sound field of a turbojet engine.

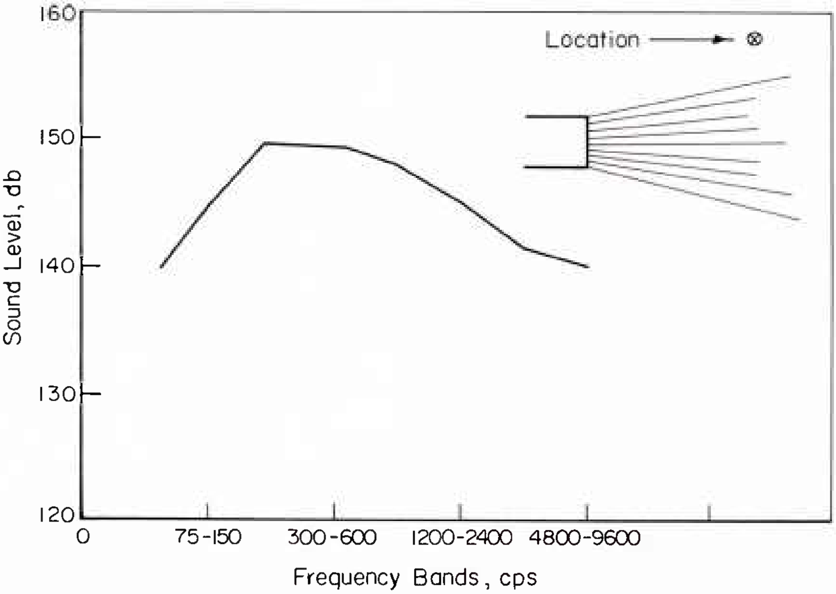

FIGURE 89.—Example of octave-band frequency distribution of acoustic energy at a point in the noise field of a jet.

tions over frequency ranges (bands) and average correlation coefficients. The literature (references *1* and *2,* for example) contains an accumulation of data and analyses of sound environments in such terms, and more information is continually being contributed.

DETERMINATION OF STRESSES IN STRUCTURAL PARTS

In classical mechanics of vibration, the responses of a structure to random loading have been considered often by detailed computation of the individual resonance modes in which the structure could vibrate. In principle, if there is knowledge of the driving force at each resonant frequency (and, of course, of the inertia and damping available) the deflections and stresses can be computed. For even a simple cantilever beam, there may be an immense number of modes that could be considered. There may, of course, be interactions that influence the stress picture. For a complex structure (for example, panels with stiffeners) in a complex field of driving forces (such as the near sound field of a jet engine), the difficulty of a calculation of this kind becomes prohibitive—even with modern computers. There are, in practice, other complications: the interaction of the vibrating panel with the sound field, uncertainties of correlation between modes of adjacent panels, complexities of joints, etc.

In view of this, there is an increasing tendency to seek to relate the properties of the sound field and the resulting structural stresses in a statistical manner. Values of root-mean-square stress are identified with values of correlated energy density in frequency bands and a (Rayleigh) probability density function is used to estimate the excursions of maximum stress. (References *3, 4,* and *5* indicate some approaches.) Experimental studies by Clarkson and Ford *(6, 7, 8)* reveal some of the complications in theoretical approaches. There is, at present, no indication of finality in the search for a wholly acceptable theory and no satisfactory theory of the material stresses in a practical structure subjected to a noise field.

All approaches require empirical determination of some factors (the damping of the structure, the effects of stiffeners and joints, etc.). Accordingly, experimental determinations of some aspects of structural response are necessary. One approach is to test the structure under some simulation of service environments. This focuses attention upon theory in regard to interpretation from a test under some selected condition to the random expected service environment.

EVALUATION OF FATIGUE LIFE

If the stress history at all critical points were known with reasonable certainty, the chief remaining question in fatigue evaluation would be the proper relation for cumulative damage. At present the only relation used in acoustic fatigue studies is that of Miner-Palmgren. (See ch. VI). Despite questions about the validity of this relation, it apears to be as good as warranted in view of the many uncertainties in analysis of the sound field and of the structural response. An approximate procedure often used in preliminary design estimates is to estimate a "random *S-N* curve" from an available discrete frequency *S-N* curve by assuming a Rayleigh distribution of stress peaks and the Miner damage rule. The stress in the "random curve" is the rms stress.

Actual testing under a random-load excitation begs the question of the proper cumulative-damage rule. On the other hand, it raises sharply the question of how to characterize a proper sort of random loading (as emphasized by Weibull in reference *2*).

Because of the uncertainties in material fatigue response and in structural reaction to a noise field, prediction of acoustic fatigue lifetime currently requires considerable testing.

ACOUSTIC FATIGUE TESTING

Forney (*1, 2*) has given reviews of acoustical fatigue testing procedures. At present facilities and practices are being developed at such a rate that a detailed account can be of only transient value. However, some brief notes concerning acoustic test facilities and testing problems will serve to indicate the scope of acoustic fatigue concerns.

Various sound sources are used to excite structural response. *Discrete-frequency sirens* are widely used in component tests because they are relatively economical and simple to operate and, at the same time, capable of high acoustic output. They can operate at 155 to 175 db over a frequency range from 50 to 10,000 cps, but most are designed for a range from 100 to 1,000 cps. In general, sirens are less accurate the more complex the panel, since they do not excite the multimode response that may be important in a random noise field.

Jet engines have been used widely since they frequently are set up for operation for other reasons. They can produce sound pressure levels as high as 155 db in octave bands over the frequency range from 50 to 2,000 cps. The random and broad-band output is desirable and realistic for exciting multimode response. The operation is relatively expensive, and this source is less flexible than some others.

Air jets have many potentialities *if* a sufficiently large air supply is available. *Electrodynamic speakers* may be convenient to establish patterns of vibration and locations of high stress. However, most speakers cannot produce the high sound-pressure levels available from sirens or jet engines. *Modulated-air-flow speakers, arrays of sirens* and *broadband generators* have been used to a limited extent.

When a discrete frequency source is used, care must be taken in interpretation of results with respect to estimation of the damaging effect of random loading. Figure 90 shows results of tests using a siren and an air jet to test a particular type of panel. The lower curve for the air jet indicates the contribution of more than one mode of response.

Orientation of a test specimen in an acoustic field and mounting of the specimen to insure suitable boundary conditions are important concerns in testing procedure. Measurements of the acoustic field (including correlation values) and of the structural response (in detail) are difficult to obtain and to interpret.

The art of acoustic fatigue testing is in a stage of extensive development of sound sources, acoustic measurement techniques, specimen design, procedural details, and interpretation.

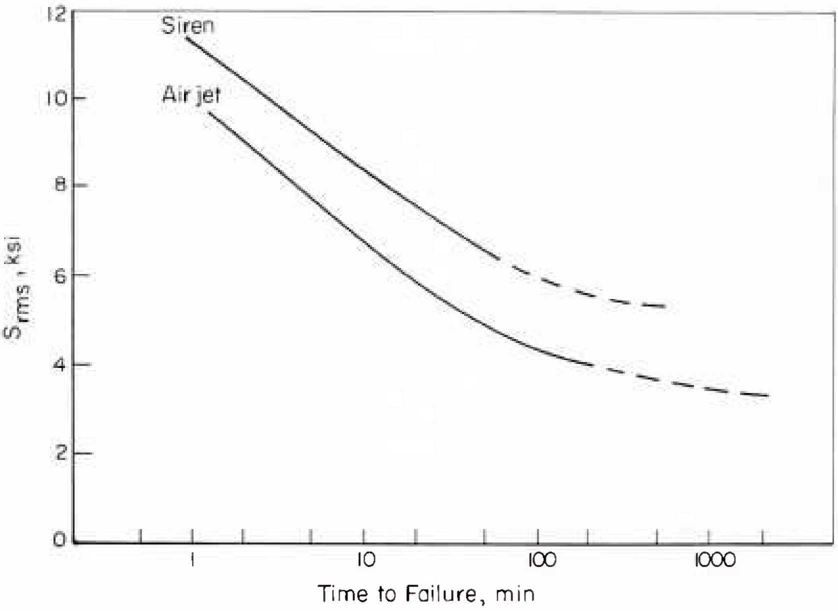

FIGURE 90.—Fatigue lifetimes of 0.032-inch panels tested by two methods.

DESIGN CONSIDERATIONS INDICATED BY COMPONENT TESTS

A high percentage of the acoustic fatigue problems encountered to date has been in structural parts built up from light-gage material; for example, in skin surface panels. It is difficult to add sufficient stiffness and damping to some conventional designs without a weight penalty. Accordingly, research studies have emphasized the acoustic fatigue behavior of skin and skin-panel configurations.

A number of factors were studied in a NASA investigation (*9*) under both discrete loading and random loading of 11-inch by 13-inch aluminum alloy panels. Results indicated:

1. For unstiffened panels attached at discrete points to relatively rigid frames, fatigue lifetime increased with increasing skin thickness, increasing curvature, and increasing differential pressure.
2. For skins with bent flange ribs, decreasing the space between stiffeners and increasing rib stiffness were effective in increasing the ability of the panel to withstand sound-spectrum pressure levels.
3. The design of skin-rib joining was important.

Other reports (*10, 11*) have concerned various features of panels: skin-rib joining, rib configuration, honeycomb structures, etc. It is extremely difficult to generalize presently available conclusions. The items mentioned above should be considered illustrative of some of the current studies rather than conclusive for design purposes.

One widely discussed topic in design is that of damping (see, for example, reference *2*). There are two basic kinds of damping: (1) acoustic damping and (2) structural damping. The first depends upon the integrated model response of an area to the exciting sound field. This is very difficult to analyze in a way helpful to design, but some useful principles may be ultimately forthcoming. Structural damping is also complex. Some sources are: hysteretic damping of the material itself, slip at interfaces (for example, riveted joints), and surface coatings for intentional viscoelastic damping. Some of the theoretical principles are discussed in reference *12,* which also contains some experimental results indicating the range of results obtainable with different damping materials in sandwich beams. There is one important precaution: adding damping material may be very effective for one mode of response and quite ineffective for others—added viscoelastic material may be wasteful (if not harmful) without sound understanding of the critical model responses.

Design procedures for proper stiffness and damping within such other limitations as weight and strength have not been worked out for many practical configurations in all types of acoustic fields. It may be anticipated that design to resist acoustic fatigue will, for considerable time, be based upon experience (partly qualitative) and subject to extensive testing verification for circumstances under which a part is expected to have much exposure to a field of high intensity (for example, sound-level pressures in excess of 140 db overall rms).

RECAPITULATION

As the preceding account suggests, acoustic fatigue is a field of increasing concern, and one of very great complexity. At present a subarea of particular significance is that of structural response to a sound field which can be estimated only in statistical terms. Accordingly, the uncertainties in the fatigue of materials (even in cumulative damage behavior) are of relatively little importance. While there are increasing data on some details of panel design, joint design, and damping, there are few quantitative rules. Experimental testing seems necessary, and even this approach requires unusual care since a simulated service environment is difficult to specify and to obtain.

From the general viewpoint of structural fatigue, acoustic fatigue is a highly specialized subject. It is of relatively recent practical concern. However, as sources of excitation increase, the importance of acoustic fatigue may be expected to grow.

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CHAPTER XI. FACTORS IN THE FATIGUE BEHAVIOR OF COMPONENTS

This chapter and the following four chapters are concerned with the fatigue behavior of "components." The word "component" here refers to a part of a structure which, for engineering reasons, is conveniently regarded as a unit. The engineering reasons may vary, and, consequently, the connotations of the word component are somewhat broad. For example, certain small items—such as bolts and screws—are sometimes called components, but they are so widely used that it is desirable to assess the influence of pertinent factors, such as thread design and fabrication process, in as general a manner as possible. On the other hand, with such a part as a wing-flap support fitting for which the loads and environment can be reasonably well defined in advance of the construction of an entire aircraft, it is sometimes helpful to evaluate design and/or fabrication factors in very specific terms. Between such extremes, there are many units which in these chapters are also called components.

The specific purpose of the present chapter is to consider some of the objectives of fatigue testing components. This includes consideration of some precautions in testing and in the engineering use of test results.

FATIGUE TESTS OF MATERIALS

A good starting point in the discussion of the fatigue testing of components is to review the advantages and the limitations of laboratory fatigue tests on material coupons.

Such tests are comparatively economical and, partly for this reason, extensively performed to provide approximate fatigue characterizations of materials. The results are sometimes considered as indicating an upper limit to the potentialities of a material and as providing a "first screening" in the selection of materials. If the coupon test results show low fatigue strength, the usefulness of the material is questionable. If however, tests with small coupons yield high fatigue-strength values, further study is warranted to determine whether high fatigue strength can also be attained in parts constructed from materials of other heats. Components that are shop fabricated and welded or joined or assembled with available techniques must finally be tested to provide sufficiently reliable data.

Material fatigue tests are usually run on carefully prepared specimens under controlled conditions of specimen design, fabrication, and surface finish, and under precise loading and load control. Various conditions can be intentionally and systematically varied so that effects of such factors as heat treatment, surface finish, notch severity and nature, and level of loading can be isolated and independently studied. This advantage is important for the development of improved techniques for using a material.

However, the simplified conditions that permit isolation of some variables make necessary important precautions against the direct use of fatigue-strength values from material coupon tests in design—except in the extremely rare situation in which a component is to be *made from the same heat of material, fabricated in the same way, to a similar size and shape,* and *loaded in service* in an *essentially identical manner.*

Design applicability of data from fatigue tests on small coupons is limited because of the uncertainty that details in a component correspond to those in the coupon. These uncertainties are illustrated in the following discussions of material factors, fabrication factors, stress and environmental factors, and failure criteria.

Material Factors

It has been noted in previous chapters that fatigue behavior is sensitive to details (not always easily detectable) in metallurgical materials. Conventional specifications do not insure that material purchased for a structure is identical in fatigue resistance to some other lot for which fatigue data may be available. Hence, data in the literature cannot be considered completely representative.

Some of the factors that have been shown to affect fatigue strength are metallurgical structure, residual stresses, and surface conditions. In a component these may be significantly different, singly or in combination, from their values in coupons which have been tested. When the nature and extent of differences are not known, the fatigue strength of the component cannot be accurately predicted from reported data from coupon tests.

Metallurgical details pertinent in the establishment of fatigue expectations for components on the basis of data obtained with coupons are complex. They differ from one alloy system to another (see app. D for examples). Metallurgical phases—grain size and orientation, inclusions and inhomogeneities—may vary from one place to another in a large casting, forging, or extrusion. In some instances a thorough metallurgical investigation of a component might provide convincing evidence. In many other instances such an investigation might be too expensive, difficult, or inconclusive. Then a test of the actual component would be more appropriate.

Residual stresses may be introduced in a component by heat treatment or by the fabrication process. These may give rise to other differences between the component and a coupon, particularly since coupons can be relatively free from residual stresses.

Surface conditions are usually carefully controlled on material coupons. Surface finishes on complex parts often cannot be produced in the same way and with identical smoothness, working of surface layers, and residual stresses. The exact identification of the surface conditions of a component and of a material test coupon can be difficult or impossible.

There have also been observations of an apparent size effect in fatigue. Large monolithic structures show significantly lower fatigue strength (especially in bending) than do small test coupons. Some of this may be explainable statistically. Some results may reflect the influence of differences in surface finish or in residual stress. In any event, this means an added precaution in the use of results of tests on small specimens to predict behavior of large components.

Fabrication Factors

Shop fabrication of actual parts usually involves factors not duplicated in the preparation of material test coupons. Some of these may be considered items of quality control: care in machining, in surface finish, etc. In principle, variations can be eliminated by sufficient care and elaborate inspection. In practice, such care may be impractical.

However, there are also factors that may be technically unreasonable or impossible to duplicate. Production techniques of forging, extrusion, punching, and the like may influence local strength, residual stresses, and surface finish in ways not predictable in detail and not quantitatively assessible in regard to fatigue strength.

The specific practices in a particular production shop may be very significant in the fatigue strength of a specific component constructed of a specific material. Such factors as details of heat treatment, grinding after heat treatment, rolling, shot peening, and surfacing (as by carburizing or nitriding for steels) may have a great influence. Some examples are given in appendix D. However, information on such factors cannot always be quantitatively related to a specific situation. Hence, background data on materials often cannot allow for the combined effects of various fabrication processes.

Among some very important fabrication or assembly factors are those in connection with joints. Many aircraft components include welded, bolted, riveted, or adhesive joints. These bring into the fatigue picture items not included in laboratory tests of material coupons.

Stress and Environmental Factors

Despite the extensive development of methods of stress analysis, many actual parts can be analyzed only to an approximation not sufficiently accurate for prediction of fatigue behavior. Multiple stress-paths exist in many actual components, in contrast to the simple stress distribution planned in the design of a material test coupon. A simple example is a riveted lap-joint in tension. An engineering approximation that each rivet carries the same amount of load is known to be inaccurate. Stresscoat patterns not only show the theoretically (but not quantitatively) predicted higher stresses around certain edge rivets but also indicate that these are places where fatigue cracks start. Even with methods of elastic-plastic analysis which are available for stress analysis in such detail, the results of the analysis may be unrealistic unless all details of fabrication are taken into account. If this is possible the analysis may still be unrealistic in that it costs more than the fatigue test of a joint would cost.

Environmental factors include temperature, corrosive surroundings, fretting, and radiation. The effects of these upon fatigue have been somewhat delineated by fatigue tests upon material coupons. In chapter IX some of the possible interactions between cyclic stressing and corrosion and some of the complications in fretting corrosion were pointed out. It is usually impossible to assess such complications quantitatively in a component of any complexity. Therefore, effects of environment on components can be predicted, at best, only qualitatively from fatigue tests of material coupons.

Failure Criteria

At least until recent years, fatigue tests of material coupons were carried out to fracture. Lately, as mentioned in chapter V, studies of crack propagation have been increasing. Less than complete fracture of a component may be critical in terms of component functioning; excessive deformation or cracking may cause overload on another component.

There probably is no simple and universal criterion of fatigue failure, but the important point in this discussion is that it is not always possible to extrapolate from laboratory tests of material coupons to critical failure of a component. In the component the phases of crack initiation and propagation may differ from those in the coupon.

Cumulative damage under loads of variable amplitude can hardly be considered well understood for simple specimens. In complex structures, moreover, there is evidence that stress distribution may change with load level to a degree not reached in a simple coupon.

Table 9 recapitulates the items mentioned in illustration of factors not readily translatable to structures from tests of material coupons. These furnish strong justification for fatigue tests of actual components.

Table 9.—*Some differences between components and coupons*

Material factors	Metallurgical structure. Residual stresses. Surface condition. Size effect.
Fabrication factors	Quality control. Production processes. Assembly processes.
Stress and environmental factors	Multiple stress paths. Complex stress concentrations. Temperature distribution. Corrosion details. Fretting. Radiation.
Failure criteria	Crack initiation. Crack propagation. Cumulative damage. Changing stress distribution.

FATIGUE TESTS OF COMPONENTS

Some Objectives

The limitations mentioned in regard to design applicability of data from fatigue tests of simple material coupons suggest some objectives for fatigue tests of components. These include:

1. Revealing material and fabrication problems. Figure 91 shows a failure in a 7075–T6 forging in which the grain direction, although specified as longitudinal, was more nearly transverse. A substantial life increase was obtained by manufacturing the part with the grain running longitudinally. Other examples of component tests revealing material and fabrication problems are scattered throughout this book and can be found in published literature and in unpublished records.
2. Revealing unantipicated stress concentrations and related prob-

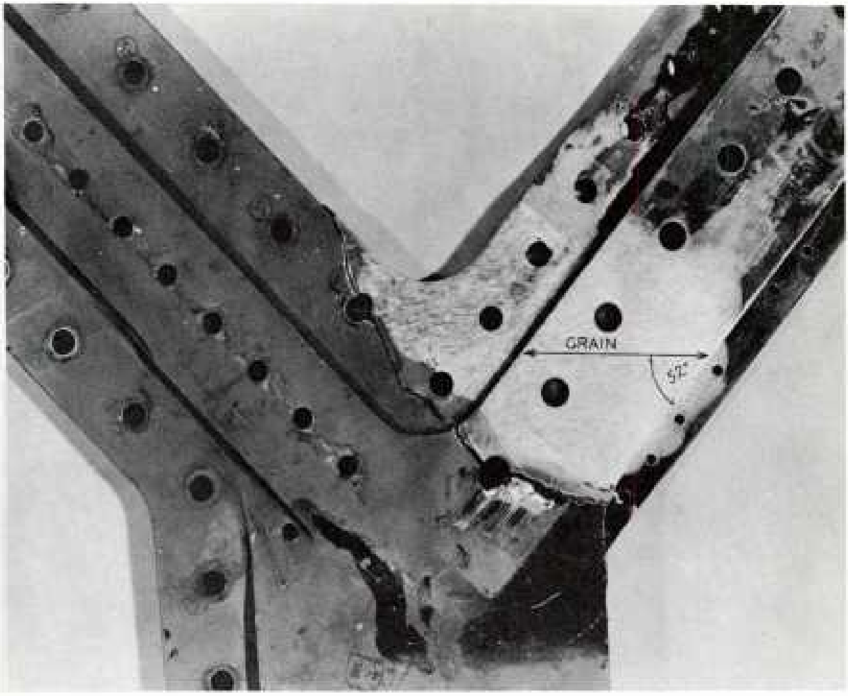


Figure 91.—A failure in a 7075–T6 forging.

lems. Figure 92 shows examples of design and assembly oversights that are only too easy to make.

3. Comparing designs, comparing the behavior of different materials in a specific design, and comparing the effects of different fabrication processes. These are common objectives, and much of the information in the following four chapters illustrates studies made with these objectives.
4. Establishing design criteria.
5. Seeking correlation between component test behavior and service experience.

The latter two objectives may not be met fully in many situations. Some work toward these two objectives and some of their complexities (particularly the many parameters involved) are discussed in the following chapters.

Precautions in Fatigue Tests of Components

Fatigue tests of components require care in planning, in performance, and in interpretation of results. To a considerable extent, specific precautions are related to the specific objective of a test.

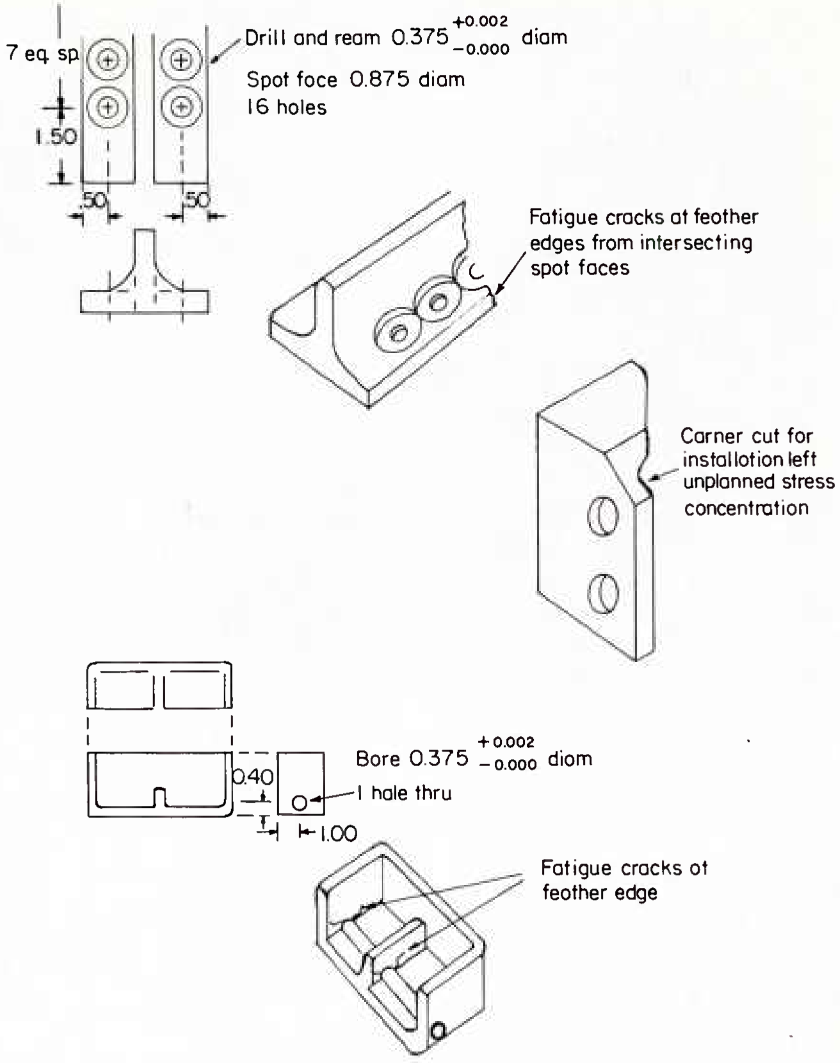


Figure 92.—Unexpected stress concentrations (of course, they might have been foreseen—component fatigue tests can bring out such errors).

Tests to reveal material and fabrication problems require that material and fabrication factors be representative of those planned for manufactured components. Use of a special heat or a special fabrication process for fatigue test specimens—different from that intended for actual components—is unrealistic. Neglecting to use truly representative tests has, in the past, brought about costly retrofits and design and material changes.

Comparison tests require attention to control of all parameters except that intentionally varied *and* care that all conditions be as representative as possible of those expected in service. For example, comparison of two materials in a particular component is unrealistic unless the mounting, the loading, and the environment are appropriate to the design conditions. Comparison in a high-load, low-cycle test may not give results suitable for low-load, long-lifetime service.

The latter two objectives, establishing design criteria and correlating test and service behavior, bring out an important *limitation* of *component fatigue testing*. Just as the test is particularly suitable to indicate the behavior of a specific material with a specific treatment, which has been fabricated into a specific design and which must operate under a specific condition of loading and environment, so the results are unsuitable for extrapolation to other circumstances. The inclusion of design and fabrication factors which are hard to identify makes the test data valuable for a specific situation and hard to extend to other circumstances.

Data on certain classes of components are presented in the next four chapters. These must be viewed with the foregoing precaution in mind. For example, data from axial-loading fatigue tests on riveted lap-joints may not be directly applicable to the behavior of a box-beam (see ch. XV). The lap-joint data may, however, provide some useful comparisons of materials or of riveting techniques which may be qualitatively extrapolatable to many structural design requirements. All data from laboratory tests should be viewed as illustrative of trends and as applicable to prediction of service behavior only with consideration for every conceivable difference of significance between the test piece and testing conditions and the actual component and service conditions.

Thus, it is important to use such data as those in the following four chapters (and in the references provided) with care in every practical application.

CHAPTER XII. FATIGUE BEHAVIOR OF FASTENERS AND OF MECHANICALLY FASTENED JOINTS

While welding and adhesive bonding are used extensively in aircraft and in aerospace structures, mechanical fasteners remain most important. In some instances their inherent strength makes them desirable; in others, their capabilities for replacement and alterations. Consequently, it is important to understand some of the factors in the design, manufacture, and use of fasteners and mechanically fastened joints that influence the fatigue resistance of structures containing them.

Since mechanical fasteners are among the oldest means of joining, they have been studied extensively. Many types of screw fasteners, rivets, and special mechanical fasteners have been developed and are available. Detailed consideration of these and of their use in joints would require several volumes; in this chapter the primary objective is to provide an introduction to the technology of mechanical fasteners and some principles that particularly influence their behavior in fatigue.

To provide some plan in the discussion, we will start with bolts and studs, giving some emphasis to their use in tension. Brief consideration of shear bolts will be followed by mention of some of the many special fasteners. The discussion of pins and lugs will bring out further factors concerning shear fasteners. Then, in consideration of multibolt joints, distribution of stress among fasteners will be introduced. Rivets, which are only used in shear, will be discussed in the final section on riveted joints. Each topic essentially will introduce new factors in regard to stress concentrations in fatigue.

BOLTS AND STUDS

Screw fasteners are the most important class of mechanical fasteners. Since they are also the oldest, literature concerning their fatigue behavior is voluminous. There are several good summary accounts (references *1* through *5*). Special details, particularly in connection with recent developments, are in current reports, including those of some of the progressive manufacturers.

Bolts may be considered in two classes: tension bolts and shear bolts. While there are differences in design, the same factors are generally important in regard to fatigue. It will be convenient to focus attention first upon the bolts and studs to resist tensile fatigue, and subsequently to mention shear bolts. The factors influencing the behavior of either type can be grouped under three headings: geometrical factors, fabrication details, and assembly practices.

Some Geometrical Factors

Figure 93 shows schematically three regions which are likely to be critical for fatigue in a bolt subjected to tension-tension loading. These are:

1. At the first engaged thread where the transfer of load from the nut is particularly high.
2. At the thread run-out on the shank where there may be a high geometrical stress concentration.
3. In the head region. The figure implies just the juncture of head and shank; as noted subsequently, there are other regions of potential concern in the head.

Of course, other areas may be subject to fatigue if there are metallurgical or surface flaws or localized loads.

Locations 1 and 2 are concerned with the screw thread, and particular attention has been devoted to factors in design and factors in forming and surfacing threads. Considerations in design of a screw thread include the following:

1. Root radius—a generous radius is desirable, particularly for notch-sensitive materials.
2. Flank angle—there is some indication that variations from 45 to 65 degrees produce little effect in fatigue strength. But increasing the angle to 90 degrees may be helpful.
3. Pitch—this seems relatively noncritical.

Examples of the effect of such parameters on fatigue strength may be found in references *1* through *6*. Present aircraft practices based upon considerable testing and experience are indicated in MIL–S–7742A, MIL–B–7838A, and MIL–S–8879. A point worth emphasis is that it is the *actual* geometrical details of the finished bolt or stud rather than the theoretical design values that count; good inspection and quality control are necessary.

Possibility of failure at location 1 (fig. 93) is dependent on both screw and nut. The engaged threads do not share the load equally, and

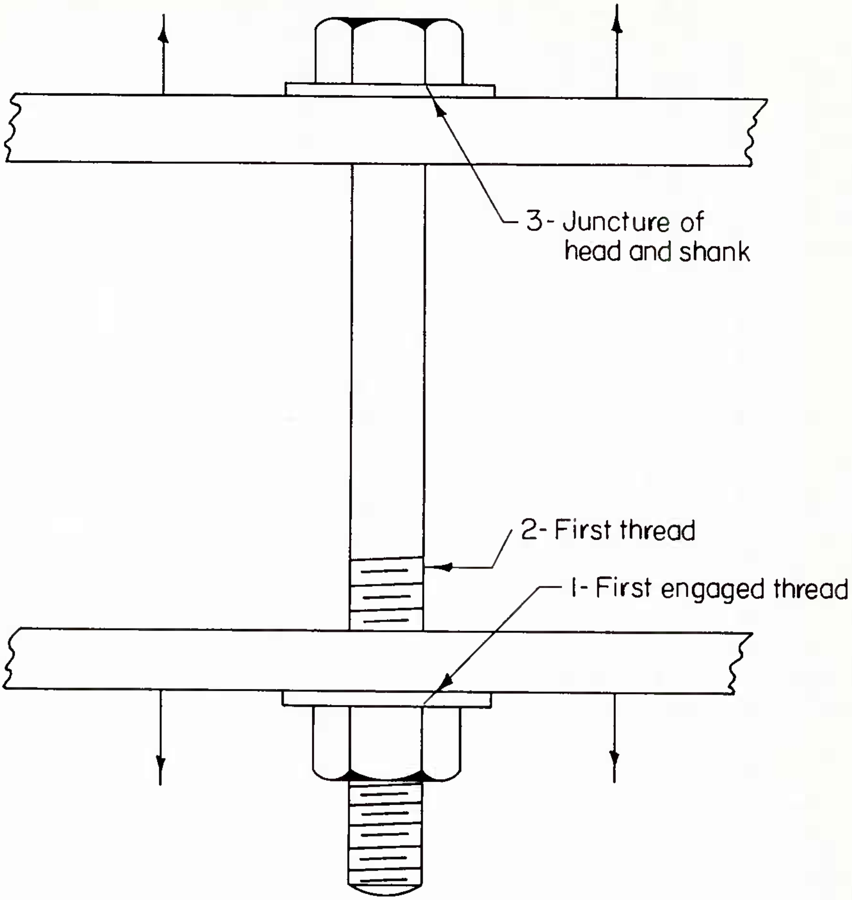


FIGURE 93.—Three locations for fatigue cracks in bolts in tension-tension.

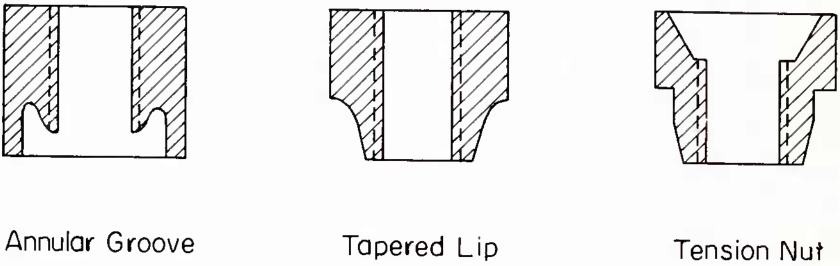


FIGURE 94.—Some nut designs for improving load distribution.

most failures occur in the first engaged thread. Figure 94 shows some nut designs for obtaining better load distributions among threads and, hence, better fatigue strength of the bolt-nut combination. Other possibilities for decreasing the stress concentration in the region of the first engaged thread are: (1) use of a nut of different elastic modulus, (2) use of a tapered thread, (3) use of a differential-pitch thread on nut and bolt. An analogous situation occurs in studs. The important point is that any means of decreasing the stress concentration at the first engaged thread will usually improve the fatigue strength significantly.

The possibility of fatigue failure at the first thread (location 2 in fig. 93) is due to the fact that this thread does not have the benefit of "interfering stress concentrations" on both sides. Particularly when bending stresses are anticipated, this situation can be helped by putting a smooth groove in the shank just beyond the first thread and slightly deeper than the thread roots. Drilling the end part-way up has also been suggested. Most aircraft bolts rely on careful thread run-out to avoid the likelihood of failure at this location.

When there are no unengaged threads, so that Locations 1 and 2 coincide, there is a particularly dangerous situation for fatigue.

Failure at location 3 is often related to fabrication, but some aspects of geometry are important. These include:

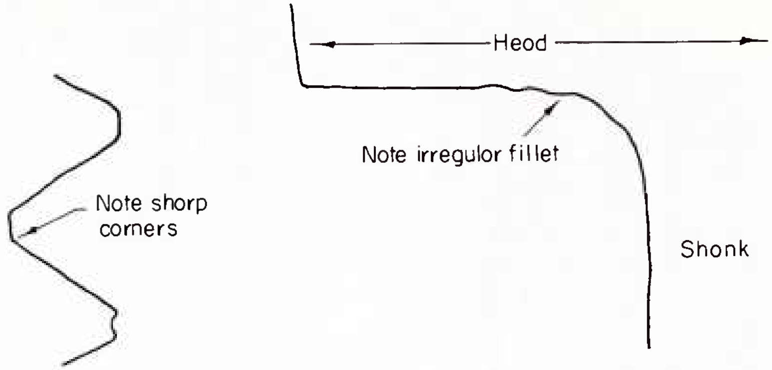
1. The shape of the fillet where the shank joins the head (which should have as generous a radius as possible).
2. The flatness of the mating surface of the head and the accuracy with which this surface is perpendicular to the axis of the shank.
3. Possible sharp corners or stress-raising tool marks. For internal-torquing bolts, these may be serious on the inside as well as on the outside of the head.

In the head region, care in these geometrical details is particularly important to fatigue reliability.

Present design technology is advanced to the degree where much care is taken in specifying geometrical details in thread contour, thread run-out, and shank-to-head fillet radius. Nevertheless, failure to keep very close control can give rise to such improper details as shown in figure 95. This figure shows contours from comparator photographs of actual bolts which showed low fatigue lifetimes. Extreme care in details of geometry is needed to produce bolts or studs having optimum resistance to fatigue.

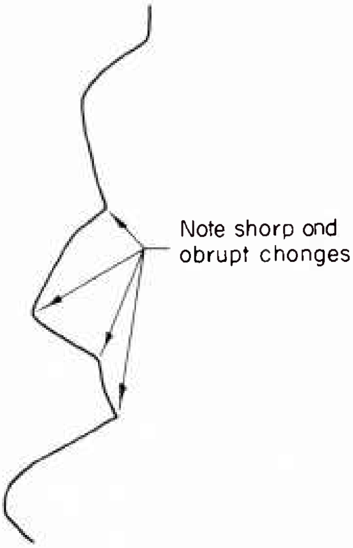
Some Fabrication Details

In addition to geometrical details which are a concern in both design and fabrication, there are numerous metallurgical details important in the production of fatigue-resistant bolts.



Poor Threads

Poor Head Fillet

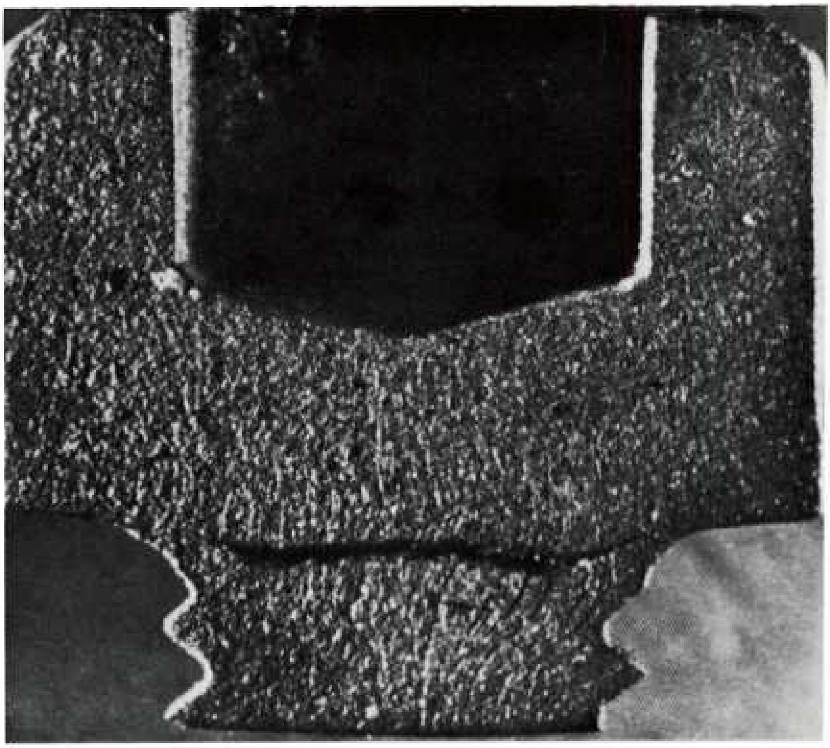


Poor Thread Runout

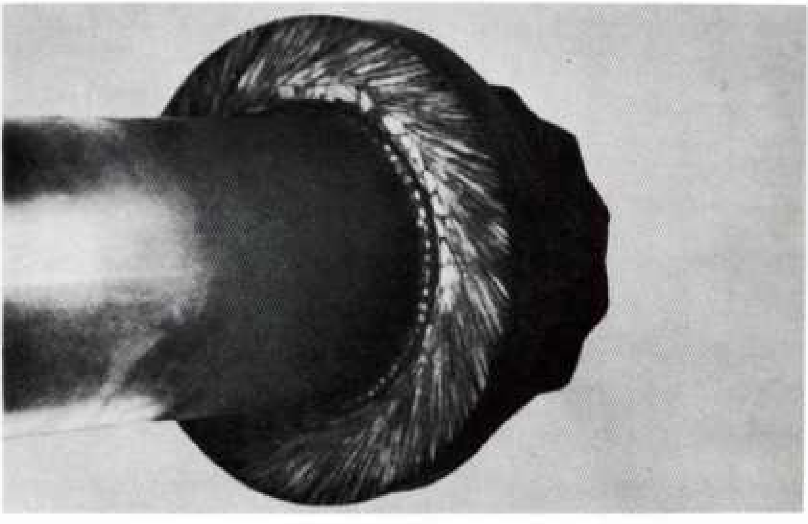
FIGURE 95.—Tracings of comparator photos of bolts showing poor geometrical details —these bolts have low fatigue lifetimes.

Aircraft bolts are available from many materials: steel bolts in strengths from 160 ksi to 300 ksi, titanium fasteners from 150 ksi to 200 ksi, stainless-steel bolts to about 200 ksi at 1200° F. The choices, from the viewpoint of end use, involve, in addition to economy, such concerns as: strength over the temperature range of intended service, effective modulus and thermal-expansion coefficient (with respect to joint material), and corrosion resistance.

The manufacture of a bolt or stud includes a number of opera-



Top: Defects in forging the head region.



Bottom: Grinding burns brought out by etching.
FIGURE 96.—Some faults in the manufacture of bolts.

tions: head section forming, thread forming and finishing, and (frequently) plating for corrosion resistance. Heat treatment at more than one stage may be involved. Each operation must be tailored to the response of (the particular heat of) the selected material. Numerous fatigue failures have occurred on account of inadequate attention to these details.

A common fabrication procedure is to form the bolt head by upsetting of bar stock. This leaves flow lines which may be unfavorably oriented. A finishing operation may involve grinding. Grinding burns in the head-shank fillet have caused fatigue failure there. A final rolling is usually desirable. Figure 96 shows (a) an extreme case of poor forging and (b) grinding burns in the head region.

Procedures of forming and processing screw threads can be extremely important in the resulting fatigue strength of a fastener. Table 10 illustrates some of the variations in fatigue strength in laboratory tests of threaded specimens of a low-strength steel. The tests were in reversed bending, which is not typical of the loading incurred by bolts in most aircraft applications. On the other hand, the higher strength steels now widely used are even more susceptible to some defects in fabrication than the steel in these specimens. The importance of cold rolling after heat treatment and of care in plating and in avoiding decarburization cannot be overlooked.

TABLE 10.—*Results of some laboratory tests on screw threads with different processing*

Item(a)	Processes(b)	Reversed-bending fatigue strength at 10^7 cycles,(c) ksi
1	Heat-treated, ground, machine cut....................	52.6
2	Heat-treated, ground, cold-rolled......................	58.0
3	Heat-treated, ground, cold-rolled, Cd plate	46.1
4	Cold-rolled, heat-treated, Cd plate....................	39.5
5	Cold-rolled, heat-treated, Cd plate, 1-mil decarb........	26.3
6	Cold-rolled, heat-treated, Cd plate, 2-mil decarb........	25.1

(a) UTS of heat-treated steel about 135 ksi.
(b) Process in order listed.
(c) Loading (as well as steel) not typical of aircraft bolts.

Some of the fabrication defects that have contributed to low fatigue strength in threaded parts are: seams or laps in the root area, decarburization or excessive carburization along the thread profile, and hydrogen embrittlement from improper plating procedures.

Some Considerations in Assembly

Even a good fastener may provide low fatigue strength with improper assembly, such as: a poor choice of nut, an improperly drilled hole, poor spot-facing, or inadequate torquing.

An exploratory investigation of several bolt-nut combinations (7) showed a variation in weighted minimum fatigue lifetime of more than ten to one. Factors of apparent significance included: nut strength (in relation to bolt strength), nut height, lubricants, hardness of washer, angularity of nut-bearing surface, preload and tightening torque, and the number of unengaged threads above the nut surface.

A great deal has been written about the importance of preload of a bolt in resisting fatigue. Some of the factors involved can be pictured by imagining a spring inserted between the plates in figure 93, supposing the bolt tightened to some preload against this spring and then an alternating load being applied to the plates. The result is that the mean stress on the thread will be higher, but the stress amplitude smaller than if the bolt had not been so preloaded. Since fatigue of a material is commonly more sensitive to stress-amplitude than to mean stress, the final effect will usually be to increase the fatigue life of the assembly (assuming the spring does not fail).

Tightening by torque wrenches is one method of preloading bolts. This procedure is undependable since the torque-tension relation is influenced by many variables such as bolt and nut thread geometry and surface; lubricants; mating of nut, washer, and structural surface; and possible subsequent loosening under service loads. Self-aligning nuts and preloading washers afford another means of preloading. As higher strength bolts are developed and as attempts are made to use these to their greatest capacity, bolt tightening may require much care to develop optimum assembly conditions.

SHEAR BOLTS

Many factors in design, in fabrication, and in assembly have been noted to be important to the fatigue resistance of tension bolts.

Quite similar items are pertinent to the fatigue strength of shear bolts. These differ from tension bolts primarily in head and nut dimensions. In tension bolts, the head is designed to be strong enough to develop the full tensile strength of the shank or threaded portion; for bolts in shear, such head dimensions may not be necessary. However, the areas critical in the fatigue of bolts in shear include the same ones (fig. 93) critical in tension, and the factors affecting the fatigue sensitivity are similar. In fact, a shear fastener must have some resistance to tensile fatigue (to withstand prying action, for instance) and may have a tension-fatigue specification even though it is chosen for shear-fatigue resistance.

SPECIAL FASTENERS

There is a confusing variety of fasteners that are neither conventional bolts nor rivets (riveted joints are discussed subsequently). These include certain proprietary nut-bolt combinations (Taper-Shank fasteners or Hi-Loc fasteners, for examples), swage-lock combinations (Huckbolts or Hi-Shear rivets), blind rivets and blind bolts, etc. The variety testifies to the ingenuity of fastener manufacturers.

A number of the special fasteners have, as one means of increasing resistance to fatigue, provision for obtaining a high and controllable clamping force. For example, Hi-Loc fasteners provide this by a "torque-off" collar; Huckbolts by carefully planned swaging of a special collar. Such provision often depends upon meticulous care in manufacture of the fastener and upon care in its use. Provision for care in use is usually made by the manufacturer in the design of a special driving tool and/or instructions for driving. Quality control, both in manufacture and in driving, is particularly important to the fatigue strength of a joint made with most types of special fasteners.

Many of the special fasteners are covered either by a broad military specification or by a somewhat more specific NAS document. These specifications indicate concern for fatigue; they include such items as the following:

1. Metallurgical requirements: avoiding inclusions, laps, or seams; checks against corrosion or stress-corrosion; heat treatment, etc.
2. Processing requirements: fillet blending and rolling, geometrical conformance, flow lines, plating, etc.
3. Performance testing: static and fatigue (both tension-tension and shear), clamping force and ability to resist relaxation of this; hole-filling ability, etc.

Not every fastener specification contains all the items listed above, but most contain many of them.

The choice of a special fastener for a specific situation and the qualifications for its use (which may require joint or component testing) are exacting tasks. Many companies have a special internal group to evaluate characteristics and uses of special fasteners. Evaluation should usually include consideration of fatigue; in a specific situation this will require detailed consideration of the joint involved. The principles of balance between shear and tensile fatigue strength, proper clamping force, and suitable deformation characteristics for good fatigue resistance are helpful but cannot substitute for pertinent experience and careful judgment.

LUGS AND PINS

Figure 97 shows some values of stress-concentration factor for a pin-loaded sheet or plate (see reference *6* for more details). It is immediately apparent that high values of stress concentration can exist under

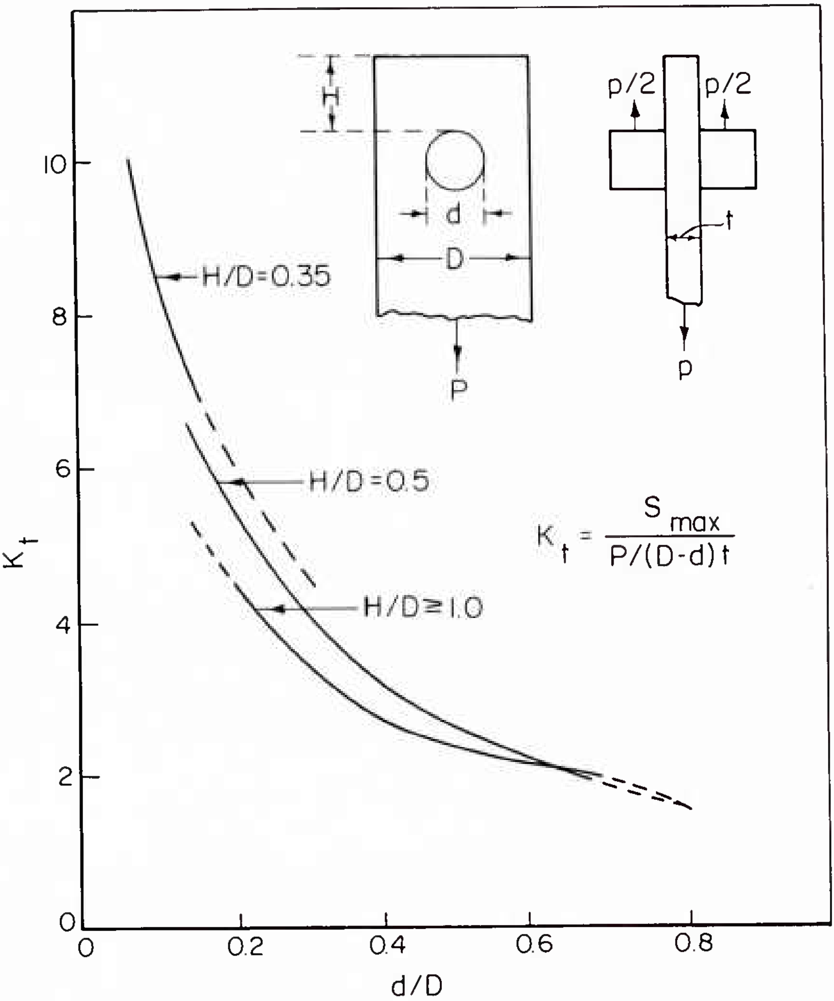


Figure 97.—Stress-concentration factors for pin loading (factors for small clearance between hole and pin).

such loading. Since fatigue behavior is very sensitive to stress concentration, a pin-lug combination would be expected to be critical in fatigue.

There are a number of additional factors in a practical consideration of a loaded lug such as shown in figure 98. These include:

1. Effects of *pin fit.*—Clearances often increase stress concentration and may increase fretting and thus tend to decrease fatigue strength. Interference fits can introduce favorable residual stresses and increase effective stiffness of the joint and tend to increase fatigue strength.
2. Effects of *lug thickness.*—For large values of t_1/D, pin bending causes a significant increase in stresses near the faces.
3. Effects of *material.*—Often the pin material, M_3, is different from the lug materials (M_1 and M_2). Differences in elastic moduli influence the stress distribution; material qualities, including the surface finish, influence fretting.
4. *Clamping effects.*—As will be discussed later, for bolted joints, pretensioning the bolt, with resulting friction distributing the load,

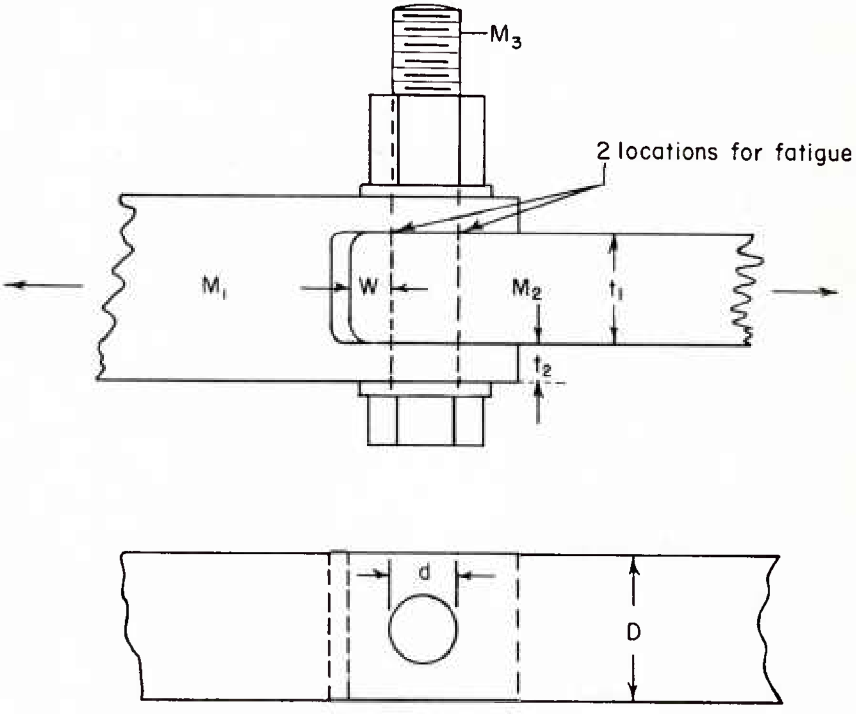


Figure 98.—Sketch of lug showing some parameters.(8)

may afford significant improvement in fatigue strength. Extreme clamping can be detrimental.

5. Fatigue characteristics can be improved by *other methods* including: use of tapered pins, use of bushings, use of jointing compounds (such as molybdenum disulfide or lithium stearate).

The many design variables make it unrealistic to suggest quantitative measures of the effect on fatigue strengths of actual lug joints of the factors shown in figure 98 and of the additional ones mentioned above. Heywood (*6*) gives formulas which may be used to provide design approximations in the absence of directly pertinent experience. However, these formulas do not take complete account of fretting, pin-bending, and other details which may be very critical.

Several investigations of pin-jointed lug connections have been reported. One fairly extensive study of a joint like that in figure 98 involved variation of a number of design parameters and testing under steady tension plus alternating bending (*8*). Figure 99 shows a failure in such a joint and indicates fretting at the inside of the hole (where failure originated in this specimen) as well as on the pin. A result of this investigation was that each of the variables mentioned appeared to have a significant influence on fatigue strength, and there appeared to be some optimum value for each of the parameters investigated.

BOLTED JOINTS

Multibolt joints involve the factors mentioned for pin-lug connections plus factors involved with the distribution of load among the fasteners. Figure 100 illustrates this for idealized situations in "double-shear" joints. Fatigue failure of such joints may occur in several ways:

1. By cracking of sheet at one of the outer (most highly loaded) holes.
2. By cracking of the sheet initiated by fretting near (for example, at the edge of a washer) one of the outer holes.
3. By bearing in the sheet (under a high mean load); this is rare.
4. By shear of a bolt; this is rare.

An important principle in design is to plan the most uniform distribution of stress feasible under other restrictions. Another principle, particularly important in regard to fatigue at long lifetimes under relatively low stresses, is to minimize fretting.

The variables available to the designer include: (1) choice of materials; (2) geometry of joint; (3) geometry of fastener array or bolt pattern; and (4) precautions as to bolt fit and bolt tightness.

The choice of material is usually dictated by other requirements. Very often, steel bolts are used to join aluminum alloy parts. The

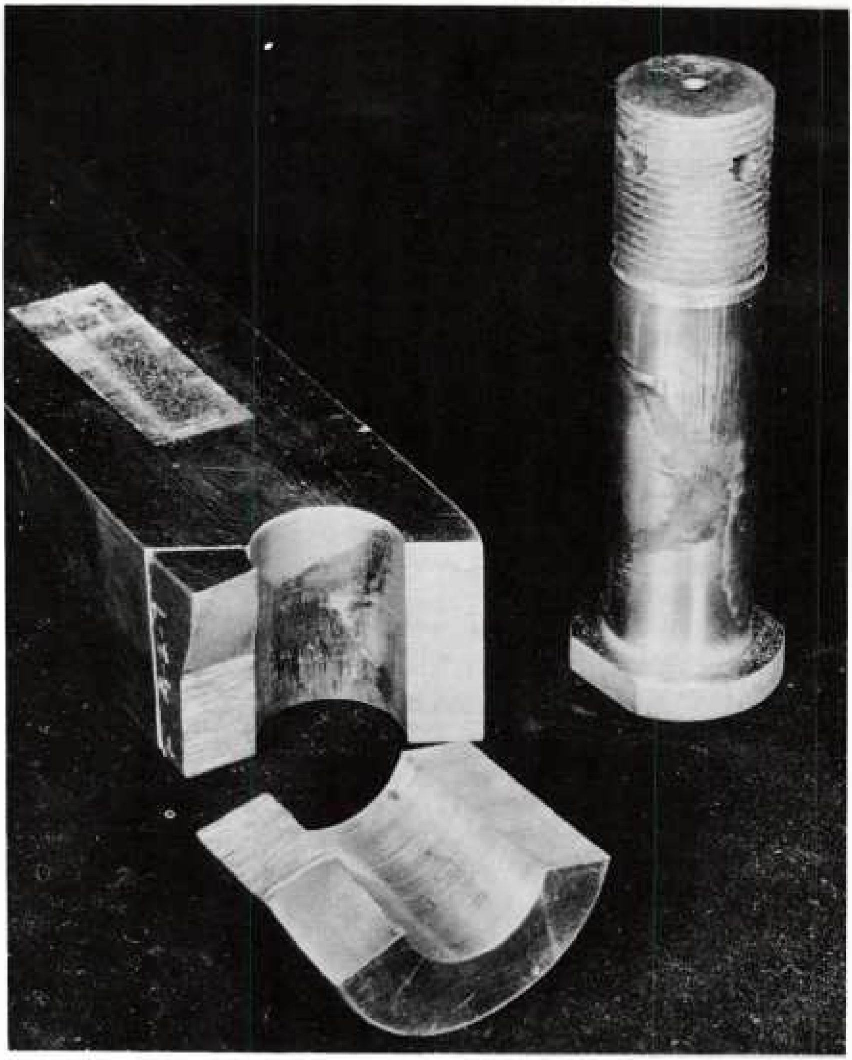


Figure 99.—Failure in a pin joint.[8]

difference in elastic modulus between the steel and the aluminum influences stress distribution. The steel bolts deform relatively little (especially for bolt diameters large compared with sheet or plate thickness), and this influences load distribution and stress concentrations. Under certain circumstances, steel bolts "Brinell" into holes in the softer aluminum alloys. There is nearly always local fretting under washers and around bolt holes—affected by the materials. Detailed analysis of material properties in a multibolt joint is very complex, and few kinds of joints have been analyzed from first principles.

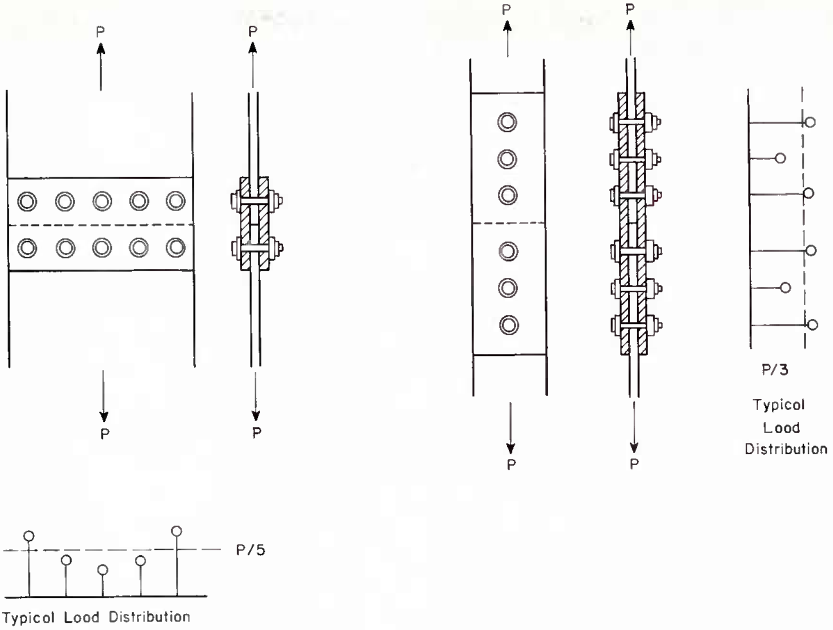


FIGURE 100.—Illustration of unequal load distributions among bolts in a multibolt joint.

Note: These are *not* good joint designs for fatigue resistance.

Much attention has been given to joint geometry. Figure 101 shows a few types of joints studied in one extensive investigation (*9*). Table 11 gives some results observed for 7075–T6 aluminum alloy joints of these types with steel bolts. Note that: (1) joints of high static strength were not necessarily best in fatigue (compare B with C); (2) symmetrical joints were better in fatigue than unsymmetrical joints (compare A with D, B with C). Although it does not show in this table,

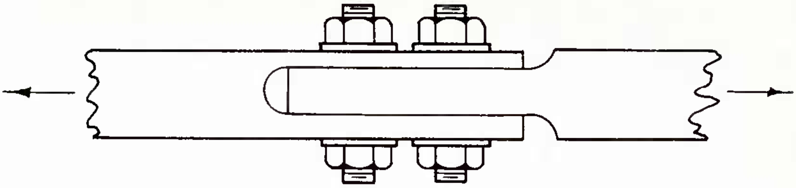
TABLE 11.—*Fatigue behavior of particular joints* (a)

Joint type (b)	Static strength, pounds	Lifetime under 16,000 ±5,330 pounds, cycles
A Double shear	115,250	3,427,000
B Single scarf	107,250	210,800
C Double scarf	76,600	> 26,000,000
D Single shear	75,900	42,000

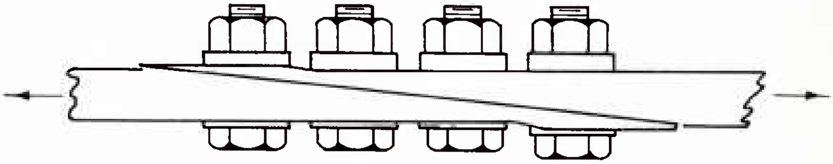
(a) 7075–T6 aluminum alloy.

(b) See figure 101 for geometries.

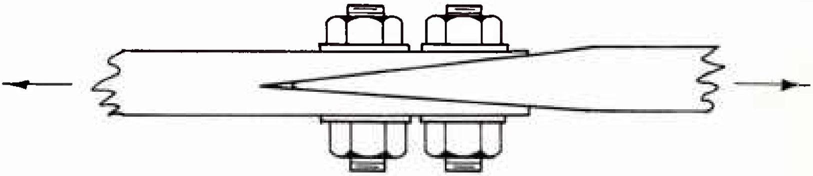
NOTES: 1. See reference 8 for further details and results for other types of joints. 2. These values should not be used for design.



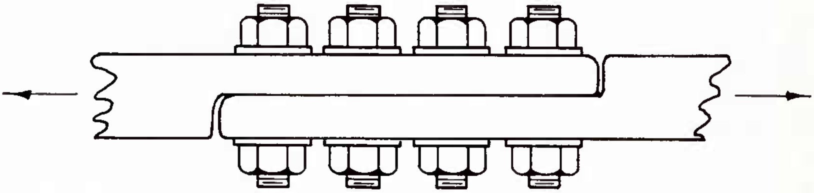
a. Double Shear



b. Single Shear



c. Double Scarf



d. Single Shear

FIGURE 101.—Different joint geometries.

variations in joint design were more significant than use of different aluminum alloys (2014–T6 and 2024–T6).

Figure 102 shows some results reported from laboratory axial-loading fatigue tests of lap joints (*10*). The sheet material was 24ST Alclad; bolts were steel. The figure shows increasing fatigue strength with

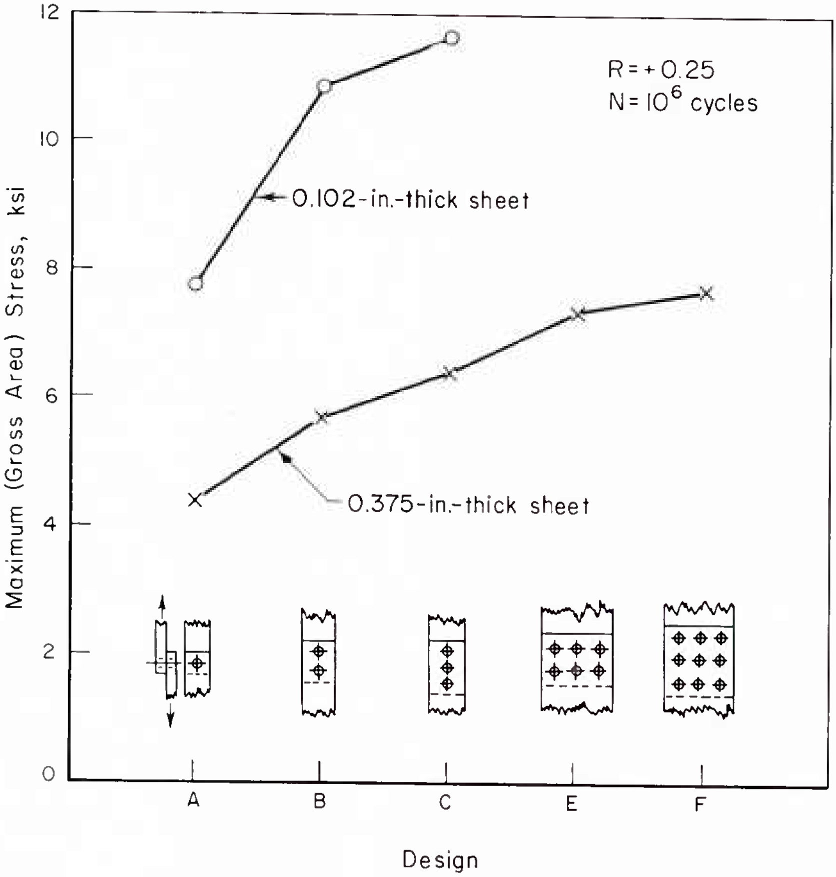


Figure 102.—Fatigue strength of some lap joints.

increasing number of bolts. However, although this is not apparent in the figure, the increase in long-lifetime fatigue strength was not directly proportional to the increase in static strength. Another observation in the same investigation, but not evident in figure 102, was that variation in initial bolt fit from a small clearance to an interference fit appeared to have little influence on the fatigue strength of the joint. This was attributed to the fact that, after very few cycles, the hard steel bolts had Brinelled their way into the aluminum sheet to some fit characteristic of the loading. While observations from studies of this type can bring out trends worth consideration, these cannot be applied directly to most aircraft joints. These joints are usually more complex in design (for example, involving a thin skin and a thicker spar cap) and in loading (for example, involving bending combined with tension) than specimens in most laboratory tests. However, the laboratory studies do indicate the importance of bolt pattern.

Prestressing has been shown to be beneficial in specific joints; overloading may introduce favorable residual stresses or may help distribute loads in a multibolt joint. The amount of increase in fatigue strength of bolted joints by prestressing to a high load has not been clarified to an extent to provide much design help.

An effective method of increasing the fatigue strength of a bolted joint is by tight clamping so that much of the load is transferred by friction at the contacting surfaces. Heywood (*6*) shows more than five times the stress amplitude for a fixed mean stress and lifetime (2×10^6 cycles) for a tightly clamped joint as for one "typically" assembled. There are, however, some precautions to be followed in seeking high fatigue strengths by clamping. One involves the problem of insuring tightness and tightness retention. Another is the limitation that friction-holding is more profitable for thin plates than for thick ones (heavy lugs, for example) in which pin bending may be critically important. Still another concern is that clamping, unless done with great care, may increase the hazard of fretting.

Interference fits are sometimes advantageous, particularly in a single hole, to insure alignment or to strengthen a critical area. Tapered bolts are of interest in this regard. If increased fatigue strength is sought by such means, it is necessary to use particular care in fabrication and assembly and to check results by testing.

Some limited investigations of inserts and of jointing compounds for bolted joints have been reported (*6*).

In summary, many factors influence the fatigue behavior of a multibolt joint: Bolt and plate material and geometry, bolt pattern, bolt tightness and fit, possible inserts and/or jointing compounds, etc. It is not possible to analyze all these quantitatively for complete optimization, particularly since details of machining and assembling joints are important and can be critical. Recognition of the factors mentioned here and suggestions in the references cited provides guides toward avoiding bad practices; to obtain the best joint for a specific purpose, a testing program developed in consideration of these guides may be required.

RIVETED JOINTS

Riveted joints are perhaps the most widely used in aircraft structural parts. They have, in common with bolted joints, problems of load distribution. The details, particularly important in fatigue, differ largely on account of inherent differences in the nature of rivets and bolts. Several points of difference warrant consideration:

1. Rivets commonly used in aircraft are of an aluminum alloy with an elastic modulus similar to that of the sheets or plates being jointed. This permits a different deformation pattern than aluminum alloy plates joined by steel bolts.
2. For many reasons (economy, relatively low shear strength compared with that of steel bolts, etc.), a relatively large number of small-diameter rivets may be used in a joint.
3. The relatively low yield point (compared with that of steel bolts) permits less clamping force per rivet than per bolt. For this same reason a properly driven rivet fills the hole to a degree obtained only with much care with a bolt.
4. The details of geometry of a rivet vary.
5. Riveting is usually a semiautomatic process—bolts must be torqued individually.

Such considerations affect local stress concentrations and load distributions among fasteners and, hence, fatigue behavior.

A great deal of the available information on the fatigue behavior of riveted joints has been obtained in *tension-tension* fatigue tests of *lap* joints. Figure 103 shows somewhat typical results. Above and to the

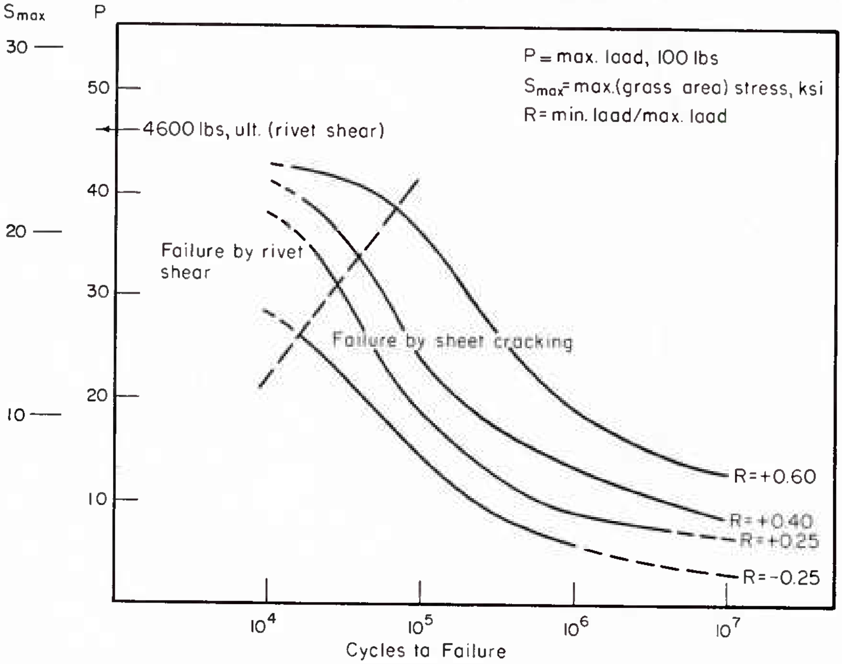


FIGURE 103.—Fatigue strength of riveted lap joints.

left of the dotted line, failure was (like static failure) by rivet shear; below and to the right it was cracking of the sheet. This indicates that long-lifetime fatigue cracking is different from low-cycle and static failure; hence, it is not surprising to find that some design parameters affect the (long-lifetime) fatigue strength differently than they affect the static strength of riveted joints.

When a multirow pattern is used in a lap joint, edge rivets get the highest loads; this is strikingly shown in the Stresscoat pattern in figure 104. This high stress concentration means that, as the number of rivets in a joint is increased, the long-lifetime fatigue strength would not be expected to increase proportionately. Some experimental results that bear out this expectation are shown in table 12. The values listed in this table should not be considered useable in design, but they are values which demonstrate the importance of stress concentration in low-stress, high-cycle fatigue.

Heywood (*6*) suggests, for lap joints under tension-tension loading, the following patterns:

For one row: $p = 3d, \quad d = 3t$

For two rows: $p = 4d, \quad d = 4\frac{1}{2}t, s = 3\frac{1}{2}t$

where

p = distance between rivets in a row

d = rivet diameter

t = sheet thickness

s = distance between rows.

These values can be considered only as guides illustrating trends which can be influenced by other factors (sheet and rivet materials, type of rivet, clamping effects, etc.). Further examples of laboratory tests on

TABLE 12.—*Effect of number of rows of rivets on the fatigue strengths of lap-joint specimen*

Number of rows (a)	Lap width, inches	Static strength,(b) ksi	Fatigue strength at indicated cycles,(c) ksi	
			10^4 cycles	10^7 cycles
..........plain sheet..........		69	68	25
1	1.0	26	21	4
2	1.5	47	31	8
3	2.0	55	33	10

(a) All specimens of 24ST Alclad, 0.040-inch thick, 4.5 inches wide.

(b) Strength values in terms of nominal gross-area stress.

(c) Values of S_{max} for $R = +0.25$.

NOTE: These values are illustrative of effects of rivet pattern but should not be used in design.

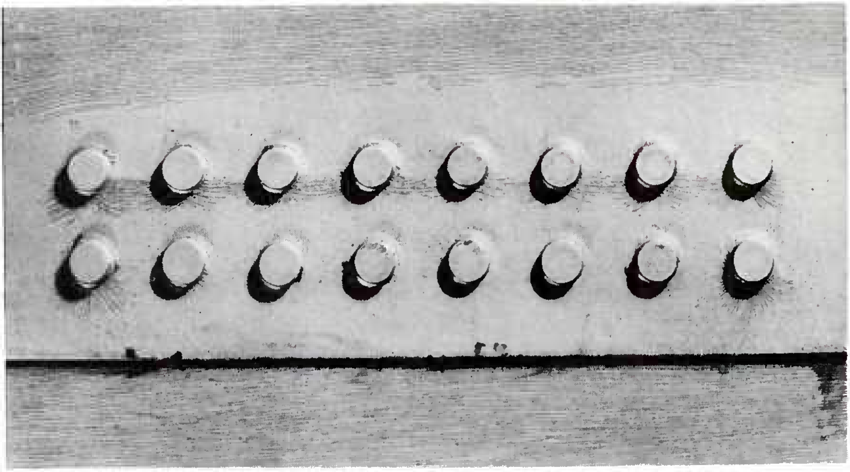


FIGURE 104.—Stresscoat pattern in riveted lap joint under tension (note higher stresses near edge rivets).

effects of patterns are in reference *11*. This reference also contains some of the many other conceivable means of minimizing stress concentration: splice plates, "idle" rivets, and preloading. Most methods have limitations and can be safely used in actual design only on the basis of tests.

The discussion to this point has omitted account of the effect of *type* of rivet. Examples of effects of pattern were self-consistent in using the same type of rivet for comparative tests of patterns. There are many types of rivet for use in various situations. Figure 105 illustrates some different types. There are also varied ways of drilling and reaming holes, varied methods of counterboring and dimpling, and variations in details of driving rivets. Studies of the effects of such factors have led to confusing results. This is not surprising, since the number of combinations and permutations of the variables is very large, and control of some is extremely difficult. The important point is that details such as poor patterns, dimpling, countersinking, or riveting, that increase stress concentration, degrade fatigue strength. On the other hand, the

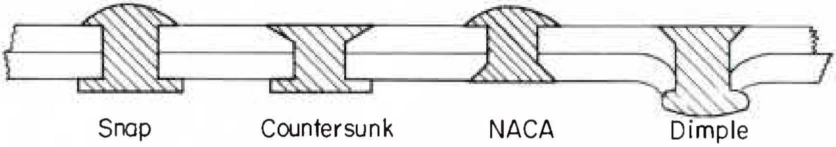


FIGURE 105.—Four types of rivets.

fatigue integrity is enhanced by a well designed pattern of rivets, by good clamping forces, by good fits, and by good riveting.

Relatively few data exist on other than lap joints. However, use of a stiffening plate on a lap joint has been shown to increase the tension-tension fatigue strength at 10^6 cycles as much as 90 percent, and double-strap joints are much stronger than lap joints.

The limited evidence available indicates that the same principles of design hold for other than lap joints.

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CHAPTER XIII. THE FATIGUE BEHAVIOR OF WELDED JOINTS AND OF ADHESIVE-BONDED JOINTS

The fatigue properties of welded joints have been of interest in many areas of engineering, and the literature concerning these is voluminous. References *1* through *4* are summary accounts.

The art of welding is very highly developed. There are many methods: arc-welding, arc-welding in controlled atmospheres, flash-welding, pressure-welding, "electroslag" welding, etc. Each method has many possible variations in procedure: simple arc welding may vary in electrode material, rate of feed, number of passes; spot welding may vary in electrode pressure and in current and voltage cycles. For each material, different methods and different procedures may produce different details in a welded joint. Since fatigue is affected by details, these differences can be significant in their effect upon fatigue strength.

Brazing and adhesive bonding are of increasing interest, not in direct competition to welding, but in specific applications such as the construction and joining of honeycomb panels or composite materials. These methods of joining have been studied with respect to fatigue somewhat less extensively; however, some information is available concerning factors involved in the fatigue of brazed joints, of adhesive-bonded joints, and of composite materials.

This chapter illustrates some of the main principles involved in the fatigue behavior of welded, brazed, and adhesive-bonded joints. There are examples of observed fatigue strengths (particularly for the welded joints) ; however, the aircraft engineer should, for any specific application, consider actual metallurgical and mechanical factors to be anticipated, rather than count on obtaining fatigue strengths reported by others (on different heats of metal, with many possible differences in fabrication) .

BUTT ARC WELDS

Perhaps the simplest type of joint in which many of the factors pertinent to fatigue can be encountered is the transverse butt-welded plate (shown schematically in fig. 106) . Note:

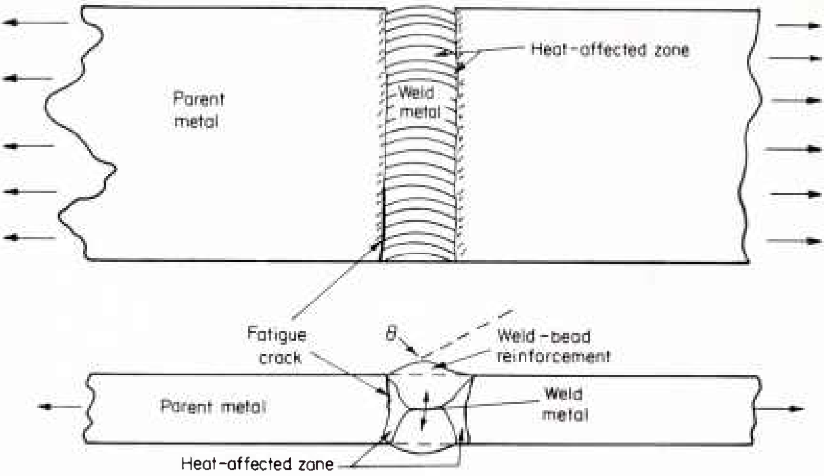


Figure 106.—A transverse butt weld.

1. The weld bead reinforcement is a change in section and, therefore, a source of stress concentration. In good welds, fatigue cracks usually start at the region of peak stress caused by this geometry.
2. There are (particularly if the joint is not heat-treated after welding) three regions of potentially different metallurgical structure: the cast weld metal, an intermediate zone that is "heat-affected" by the welding, and the unaffected parent metal.

For a good quality weld in a mild steel, the weld metal and the heat-affected zone may be as strong as the parent metal. In this case, the stress-concentration effect of the weld bead is the main cause of lowering the fatigue strength of the joint. Newman and Gurney (*5*) have shown that increasing the angle θ can increase the tension-tension fatigue strength by more than 50 percent. Grinding off the weld bead may also significantly increase the fatigue strength of a transverse butt weld (*4*). Grinding has practical limitations. It may expose subsurface defects, and it may produce undesirable surface roughness and undesirable residual stresses.

A similar stress-concentration effect has been shown for longitudinal welds (such as indicated in fig. 107). For such a weld a point of stress concentration is at the end or "toe"; another stress raiser may be a roughness in the bead.

For joints in steels of high hardness or in high-strength aluminum alloys, the weld metal and/or the heat-affected zone may be weaker than the parent metal. In such a situation a butt-welded joint, even with no defects and with the weld bead removed, may be weaker, both in static loading and in fatigue, than an unwelded piece.

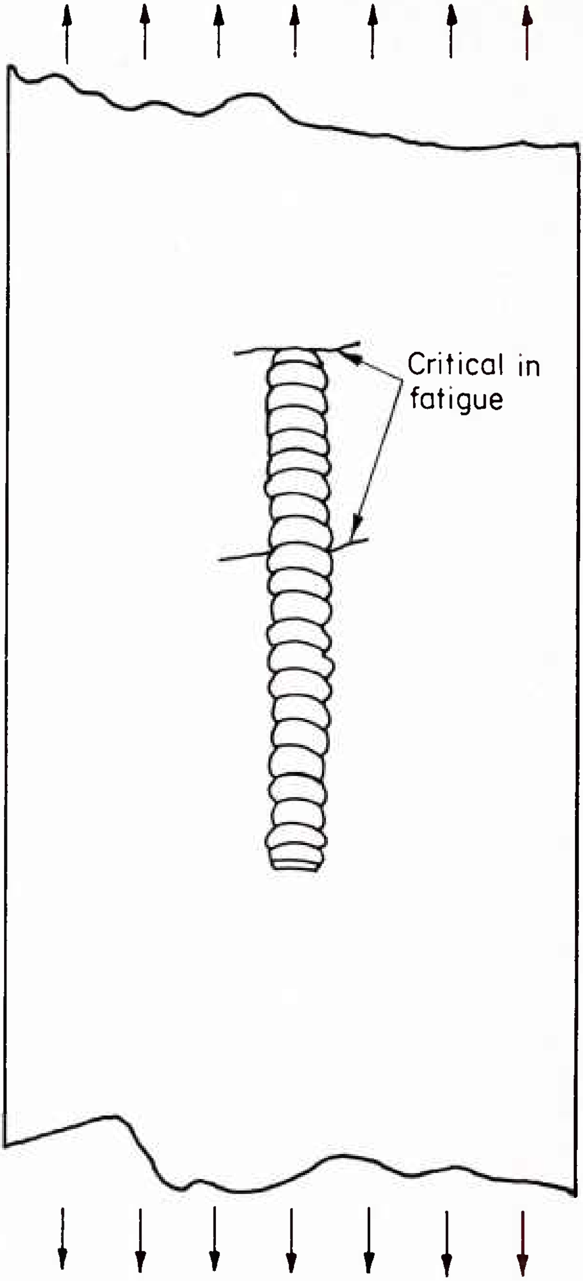


Figure 107.—A longitudinal weld.

TABLE 13.—*Fatigue strengths for some transverse butt-welded joints*

Item No.	Material			Joint		Fatigue strength		
	Type	UTS, ksi	Thickness, inches	Welding	UTS, ksi	Type test	S_{max},[a] ksi	Reference
				Steel				
1	BS15, 0.18C	63	½	Manual arc		$Ax, R = 0$	26	5
2	BS15, 0.18C	63	½	Manual bead off		$Ax, R = 0$	36	5
3	Structural C	61		do		$Ax, R = 0$	21	6
4	A242	76		do		$Ax, R = 0$	26	6
5	0.3% Si	81		do		$Ax, R = 0$	24	6
6	T-1	105		do		$Ax, R = 0$	19-36	6
7	Ni-Cr-Mo	105	½	Electrode E12015		$Ax, R = 0$	21	7
8	Ni-Cr-Mo	105	½	do		$B, R = -1$	14	7
9	Ni-Cr-Mo	105	½	Electrode bead off		$B, R = -1$	28	7
				Aluminum				
10	NS 4-1/2H	36	¼	Inert gas	29	$Ax, R = -1$	6.2	8
11	NP 5/6M	44	¼	do	42	$Ax, R = -1$	7.6	8
12	NP 30WP	48	¼	do	26	$Ax, R = -1$	6.7	8
13	14S-T6 A1	67	⅜	Flux-coated electrode	34	$Ax, R = 0$	5-8	9
14	61S-T6	45	⅜	do		$Ax, R = 0$	5-8	9
15	61S-T6	45	⅜	Inert gas		$Ax, R = 0$	6-10	9

(a) 2 x 10^7 cycles for steels ; 10^7 cycles for aluminum alloys.

Some values reported from laboratory tests are listed in table 13. These illustrate two points mentioned:

1. Removing the weld reinforcement generally improved fatigue strength (compare items 1 and 2, 8 and 9).
2. Higher strength parent metals do not necessarily provide higher strength welded joints.

It should be noted that laboratory tests such as these are usually made on specimens prepared and tested under ideal conditions; in actual practice there are likely to be welding defects. These may include:

1. Surface defects: cracks, undercutting, overlap, underbead defects, lack of complete penetration, arc strikes, or burning, etc.
2. Internal defects: cracks, lack of complete penetration, lack of fusion, voids or pores, oxide inclusions, etc.

An example of the effects on fatigue of intentional weld defects is shown in figure 108; the detrimental effect of small amounts of defect

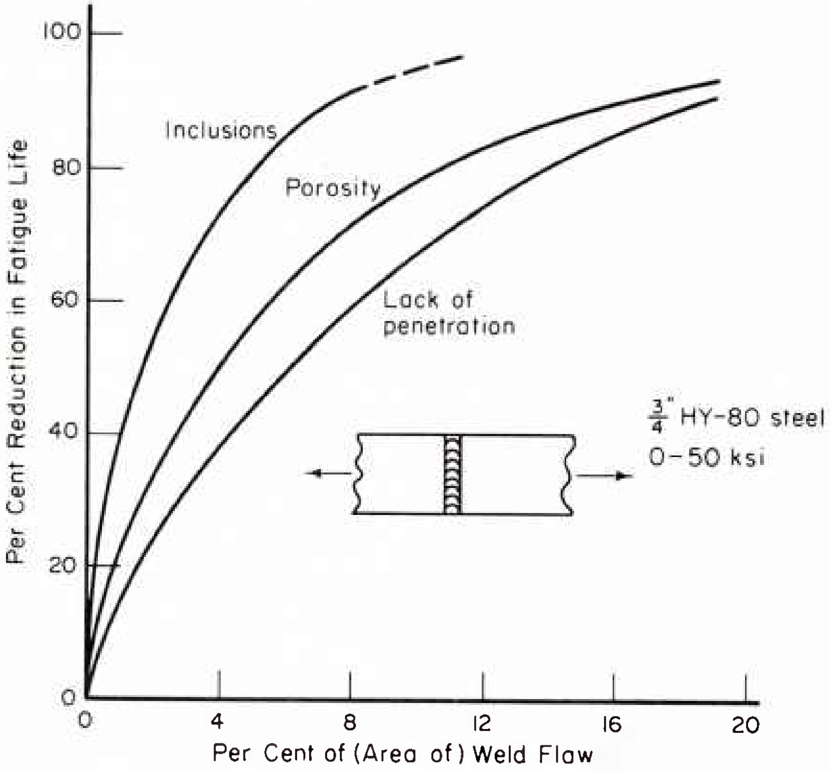


FIGURE 108.—Effects of defects. (Note: These are only illustrative.)

is striking. It would be expected that some materials, especially sensitive to small cracks, would be more affected by a particular size of defect than other materials; information as to defects "tolerable" in different materials is not adequate for exact design. Moreover, small defects are difficult or impossible to detect with certainty (and with reasonable economy).

Introduction of favorable (compressive) residual stresses has been shown to increase the fatigue strength of a weld. Such stresses may be introduced thermally by spot-heating or mechanically. These procedures are not well enough established to be wholly dependable.

FILLET ARC WELDS

In a number of situations, "fillet welds" are useful. Some of the varied geometries that have been used in fatigue investigations are shown in figure 109. DeLeiris and Dutilleul (*10*) found that mild steel specimens of types A and B had equal fatigue strengths; the prevailing effect was the geometrical stress concentration at the fillet. The *Welding Handbook* (*11*) shows rather similar fatigue strengths for specimens of types C, D, and E. Munse and Stallmeyer (*12*) have compared beams with cover plates of different design; they found significant design effects (for example, type G appeared stronger than either type F or type H) and butt-welded joints with a tapered transition from the larger to the smaller plate were stronger than any of the fillet-welded joints.

Because of the high stress concentration inherent in the fillet weld, considerable benefit can be obtained from stress-relieving treatments and from the introduction of favorable residual stresses (*13*).

FLASH WELDS

Flash welds involve, in addition to the joining of material by local melting of the metal, an upset of the metal in the softened condition. A common result is the "outbent-fiber" characteristic shown in the macrograph in figure 110. Intuition suggests that such a metallographic structure would be dangerous in fatigue. Results of several investigations bear this out to the following extent:

1. The upset region is a stress concentration. If the weld is very good, the fatigue strength of the joint is improved by carefully machining this off.
2. The nature of the outbending fibers makes possible a line of incomplete fusion and lines of segregated constituents in directions

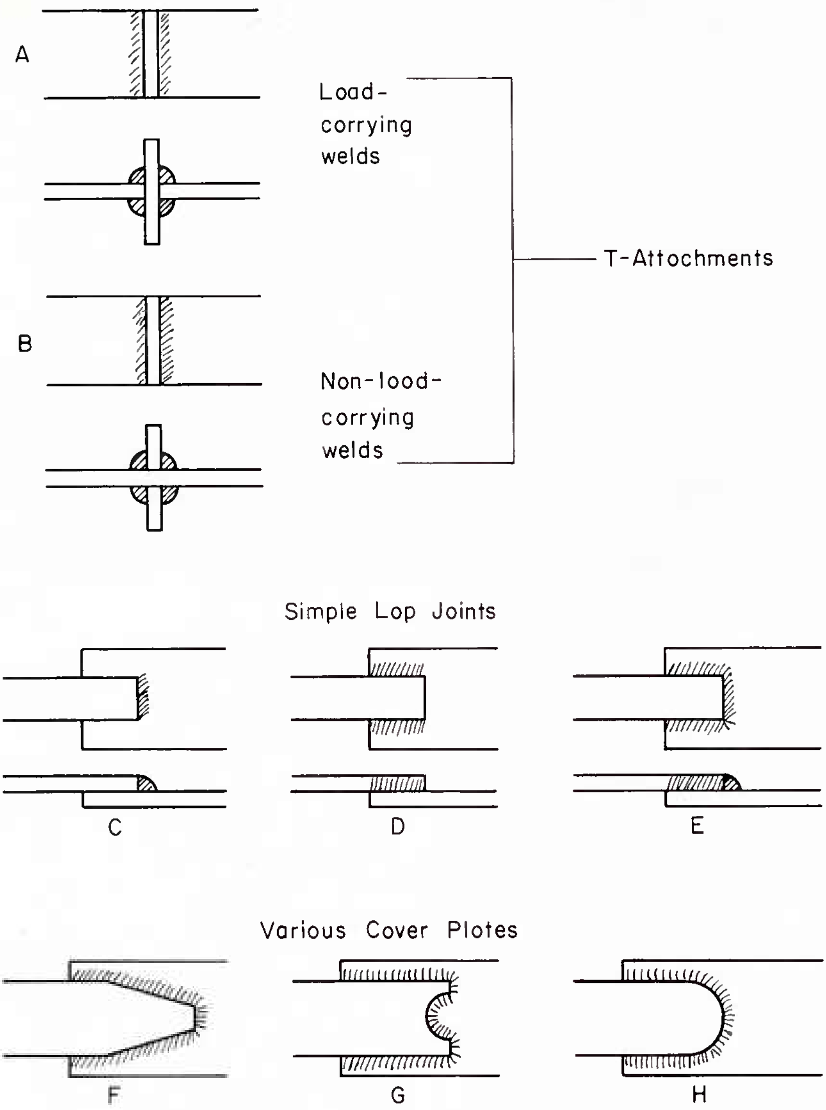
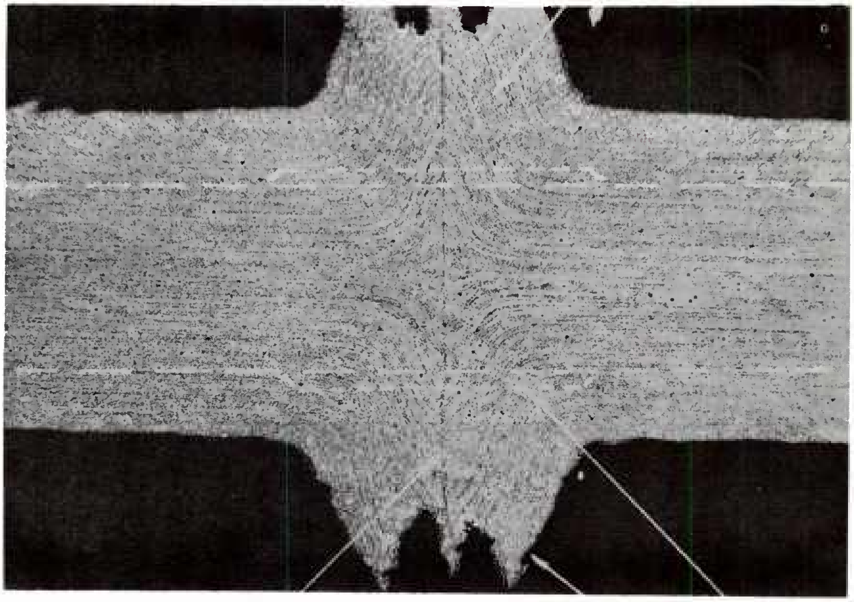


FIGURE 109.—Several types of fillet welds.

unfavorable to stress flow under common loadings. These can lead to low fatigue strength.

3. With very good welding and careful removal of the outbent projection, a flash-welded joint may be quite strong in fatigue. This is subject to the limitation of the relative strength of the weld metal and the parent metal for metals heat treated to high strength.



Arrows show:
Top: Outbent fibers.
Bottom: Weld line, flash reinforcement, and contour for test specimen.

FIGURE 110.—Cross section of a flash-welded joint.

The fatigue properties of flash-welded joints thus involve the same basic factors of stress concentration and metallurgical structure that are of concern in arc welds (*14*). Grain growth and defects are factors. The main different feature is the metallurgical structural flow lines involved in the procedure.

RESISTANCE WELDS

Spot welding offers advantages of high mechanization in joining reasonably thin materials to each other or (with some limitations) a thin panel to a much thicker stiffener. Usually (that is, not considering seam welds), it involves some of the factors mentioned previously for riveted and bolted joints: the stress concentration inherent in joining sheets at discrete points. In addition, it involves the factors inherent in welding—variability of metallurgical structure from the fusion zone through heat-affected metal to parent metal.

As for riveted joints, many of the fatigue investigations on spot-welded joints have been made under tension-tension loading of lap joint specimens. Figure 111 shows (a) a static failure of such a specimen, (b) a fatigue failure in this type of specimen, and (c) a cross-section view of an individual spot weld. Note that:

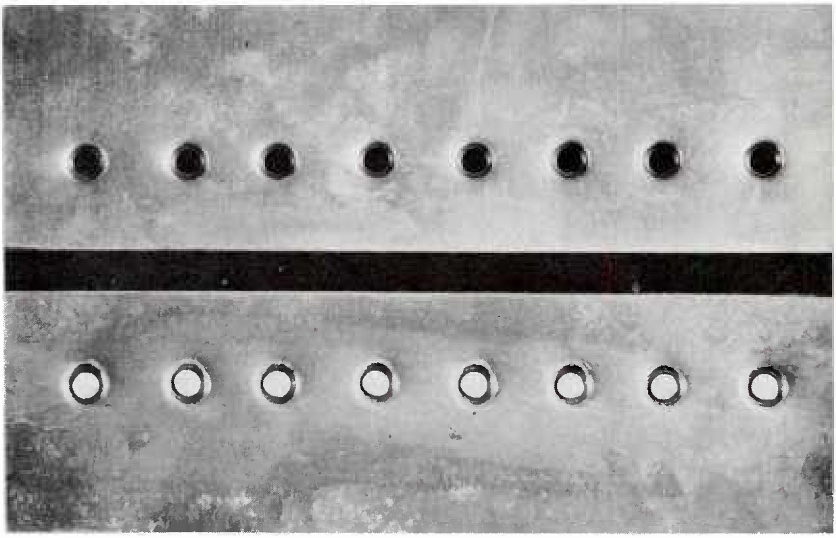
1. As for riveted specimens, the (long-lifetime, low-stress) fatigue failure is by cracking through regions of stress concentration. Static failures (and low-cycle fatigue failures) are often (as in this illustration) by a different mode.
2. The origin of cracking is often (as in fig. 111 (c)) near the notch of the weld and the faying surfaces of the sheets. This is a spot of high stress, because of the geometry of the weld, and also is a spot of potential material weakness.

While the figure shows a particular lap joint, these characteristics are typical of spot-welded joints.

Table 14 shows some fatigue-strength values reported for spot-welded lap joints. These may be compared in certain respects with values in table 12 for riveted lap joints:

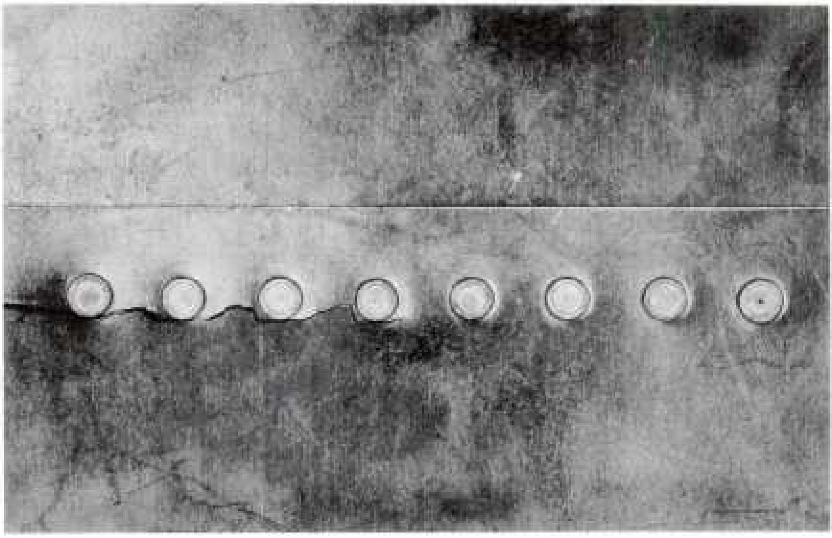
1. The fatigue strength at long lifetime is a small fraction of the static strength.
2. The increase in number of rows of spots increases the fatigue strength less than it does the static strength.

The results in table 14 and in table 12 should not be taken as necessarily representative of the relative merits of riveting and spot welding; in neither case is it clear that fully optimum conditions prevailed.

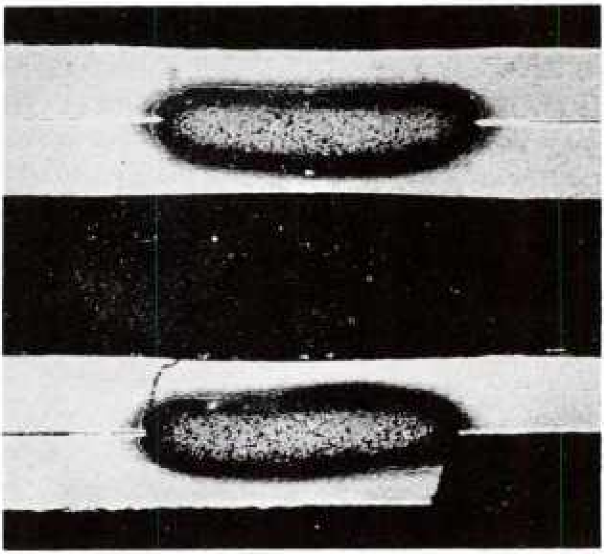


a. **Shear failure in static tests and in low-cycle fatigue.**

FIGURE 111.—Fatigue in spot-welded lap joints.



b. Sheet cracking in long-lifetime fatigue.



c. Spot welds showing fatigue crack starting at region of peak stress.

FIGURE 111.—Fatigue in spot-welded lap joints—Continued.

TABLE 14.—*Effect of number of rows of spot welds on fatigue strength of lap joints* [a]

Number of rows	Lap, inches	Static strength, [b] ksi	Fatigue strength: S_{max}, ksi at $R = +0.25$	
			10^4 cycles	10^7 cycles
..........plain sheet..........		67.4	65.0	24.0
1	1	19.0	10.0	3.6
2	1.5	34.6	15.5	5.5
3	2	44.3	27.5	7.0

[a] Spot welds ¾-inch apart in each row. Joint 5.0-inch wide in 0.040-inch thick 25ST Alclad.

[b] Strength values ae in terms of nominal stress in the sheet away from the joint.

NOTE: These values should not be used in design (see text).

Artificially induced residual stresses can improve the fatigue strength of spot-welded joints (*15*). As is true for other welded joints, procedures for such improvement have not yet been developed to a stage where they can be used as procedures in quantity production.

One of the most serious limitations in spot-welded joints for reliability under fatigue loading is that small defects in the periphery can be very serious but very difficult to avoid or to detect. Defects in the weld nugget, where stress is usually lower, are often less important. Consequently, considerable effort has been expended toward finding nondestructive tests that might correlate with fatigue behavior and/or show defects known to be significant in fatigue. Several methods are reviewed in recent literature. The present status is as follows:

1. There are some very helpful nondestructive tests.
2. If the available tests were used with sufficient thoroughness to provide assurance as to weld quality in relation to fatigue resistance, the effort would probably prove unreasonable in cost and time.

Much of the background information available, and the discussion to this point, have been based upon lap joints. Some of the observations from tests of lap joints cannot be extended directly to other configurations. For example, Kalbfleisch and Weismantel (*16*) describe some experiments on spot-welded corrugated sandwich panels. In these the test results are in terms of pounds per spot (not a good measure for the varied patterns in lap-joint data such as listed in table 14). The loading of welds in the sandwich panel is apparently well distributed by the geometry. Studies of numerous other types of spot-welded joints (butt joints with cover plates, attachments to stiffeners, etc.) have been reported. These all show:

1. Stress distribution among spots is significant. This is often influenced by the relative stiffness of the spot-weld region and the surrounding material (and may not be identical with that of a riveted joint having a similar pattern).
2. Stress concentration at the individual spot is critical. This may vary, because of local yielding, with stress level; it is greatly influenced by possible defects.
3. The local strength at the region where cracking starts is dependent on welding procedure. Surface preparation, details of welding cycle, and post-weld treatments influence this.

Thus, spot-welded joints can vary in resistance to fatigue from being quite good to being very poor—depending upon details which are not easy to categorize or to control.

Tight seams may be produced:

1. by step-by-step welding with spots closely adjacent or with spots overlapping,
2. by roller welding with a wheel moving continuously but producing spots which overlap, or
3. by continuous current producing an essentially continuous seam.

The latter procedure is seldom used because it is difficult to obtain good penetration.

Some differing features in seam welds and in spot welds are evident when we consider the overlapping of two spot welds in a step-by-step process. Heat and pressure effects in forming a single spot will influence the previously formed spot. This provides an opportunity for contraction cracking that may be quite different from the situation where spots are far enough apart to be metallurgically independent of each other. The result is that the fatigue behavior of seam welds is not readily predictable from fatigue tests data on spot welds in the same material.

Some of the factors involved in seam welding are indicated in a report on a study of a particular group of materials in reference *17*.

WELDS IN STRUCTURES

Structural uses of welding frequently involve more complex situations than the simple butt-welded or fillet-welded specimens for which laboratory fatigue test data exist. Spot-welded constructions of interest in aircraft are almost never simple lap joints. The applicability of such observations as those in the preceding pages brings up some questions difficult to answer.

A number of investigations on the fatigue behavior of welded structures such as beams and girders have been reported; examples are in references *12* and *18*. Table 15 shows some illustrative results on arc-welded mild steel. The importance of detail design is clear; incidentally, this is one reason why it is difficult to extend results obtained on one structural specimen to a different design. Also indicated in the table is the fact that, even with good welding such as is obtainable in the laboratory, fatigue strengths lower than might be estimated from the section geometry are usually obtained. Not shown are the further possible decreases in fatigue strength from defects such as might occur in production welding.

TABLE 15.—*Results of fatigue tests on welded mild steel*
[Zero-to-tension at 2x10^6 cycles]

Type	S_{max}, ksi
Rolled I-beam	31.2
Welded beam	26.0
Welded beam with stiffeners	23.0
Welded beam with butt-welded transitions	19.0
Welded beam with splice	19.0
Welded beam with partial-length cover plate	12.5

There have been several reports of bending-fatigue investigations on welded tubes; results are generally in accord with those that might be expected from plate test specimens with allowance for the greater difficulties in welding and testing tubular structures with precision. Some investigations on welds in pressure vessels have also been reported; more work along this line is in progress.

Koziarski (*19*) notes some of the complications in the analysis of fatigue resistance of welded structures in aircraft. He emphasizes that laboratory test results on sample specimens may give only qualitative information in view of the enormous number of variables in a welded joint with the configuration and loading conditions of frequent practical interest.

ADHESIVE-BONDED JOINTS

There is continuous development of synthetic adhesives with improved strength properties and development of increasing reliability in bonding metals with these. Adhesive-bonded joints have attractive possibilities: (1) adaptability to quantity production and (2) low stress concentrations and hence potentially good behavior in fatigue.

Most studies of the fatigue behavior of such joints have been on lap-joint specimens such as treated in figure 112. For such a specimen the average tensile stress in the metal is

$$S_{me} = P/wt,$$

while the average(1) shear stress in the adhesive is

$$\tau_{ad} = P/wl$$

Experience shows that the joint fails either by shear of the adhesive (almost always the mode of static failure and of low-cycle tension-fatigue failure) or by tensile cracking of the sheet (common in long-life fatigue). The ratio $S_{me}/\tau_{ad} = l/t$ would be expected to be an important factor. Heywood (*20*) shows collected data in terms of a "joint factor," $\sqrt{t}/l$ (see fig. 112). Wang (*21*) also shows results that indicate an effect of t/l (see fig. 113). In view of the many factors (elastic modulus of sheet, elastic modulus of adhesive, thickness of adhesive, and details of surfacing and bonding), there is not enough information to establish many trends even qualitatively.

A problem in practical use of adhesive-bonded joints is the difficulty of insuring uniform quality. Small variations in procedure can lead to such defects as voids, unbonded areas, etc. These cannot be detected readily by (economical) nondestructive methods. Hence, scatter may be large and reliability low.

The present status of adhesive-bonded joints may be summarized as follows:

1. At best, joints in thin sheets may be quite good in fatigue—comparable in some instances to riveted or to spot-welded joints.
2. Joints may be relatively weak, and the conditions for avoiding this possibility are not yet well established. Moreover, the conditions may involve extremely careful control and inspection.

Hence, use of adhesive bonding in fatigue-sensitive areas must be viewed with caution.

BRAZED JOINTS

Brazing may be considered as having some of the aspects of adhesive bonding. The brazing material usually has strength and elastic modulus significantly different from values for the pieces of material being joined. In this sense the analysis of a brazed joint has similarities to that of an adhesive-bonded joint. There are, however, some factors that differ (at least in degree):

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(1) The stress distribution in the region of the lap is complex. Wang (*21*) gives an interesting analysis with reference to modes of failure.

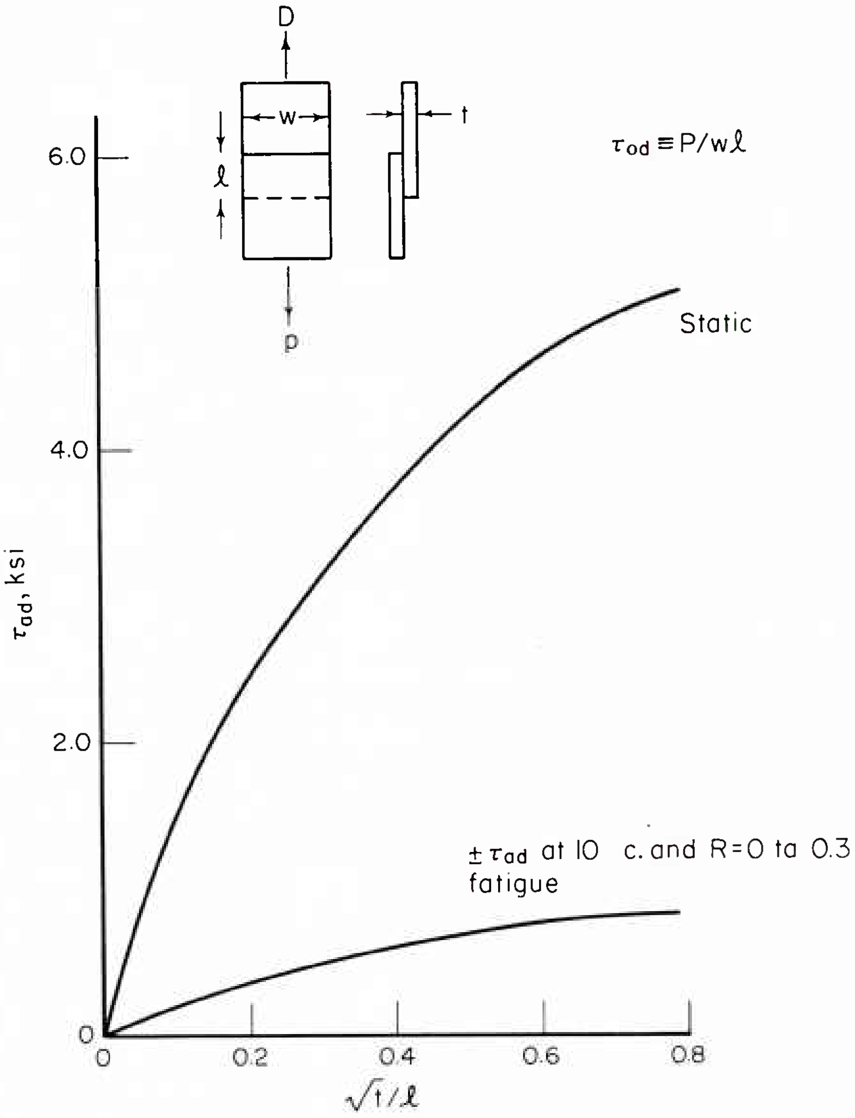

FIGURE 112.—Some variations in strength of adhesive-bonded lap joints with thickness and lap (joints of aluminum alloy sheets bonded with redux and with uraldite—adapted from reference 20).

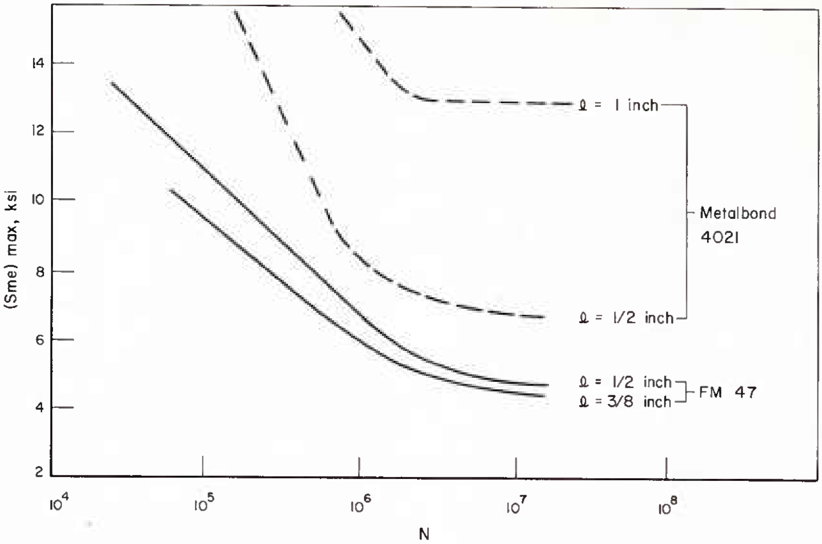

Figure 113.—Fatigue test results on lap joints in 2024–T3 aluminum alloy with adhesive bonding (tests at $R = +0.1$).

1. Brazing is usually done at a higher temperature than adhesive bonding. The difference in coefficient of thermal expansion between brazing alloy and parent metal may influence the condition of the final joint, particularly with respect to residual stresses.
2. There may be a metallurgical interaction at the surfaces between braze and metal. The mechanical properties of a region of such interaction may be difficult to define.
3. Possible corrosion interactions with in-service environment will vary.

For such reasons a brazed joint has some characteristics of an adhesive-bonded joint, some resemblance to a welded joint, and some complex characteristics of its own. Specific details are important in the brazing of each material: metal being joined, technique of joining, and geometry of the joint. Hence, generalizations are of questionable value.

Some high-strength brazed joints have been developed for use at high temperatures (see, for example, reference *23*). Figure 114 shows results that can be obtained at room temperature in a simple joint with careful brazing. In actual practice there are many difficulties in control to avoid porosity, spotty adherence, unfavorable residual stresses, etc. The only reasonable check is actual fatigue testing of shop-produced brazed joints.

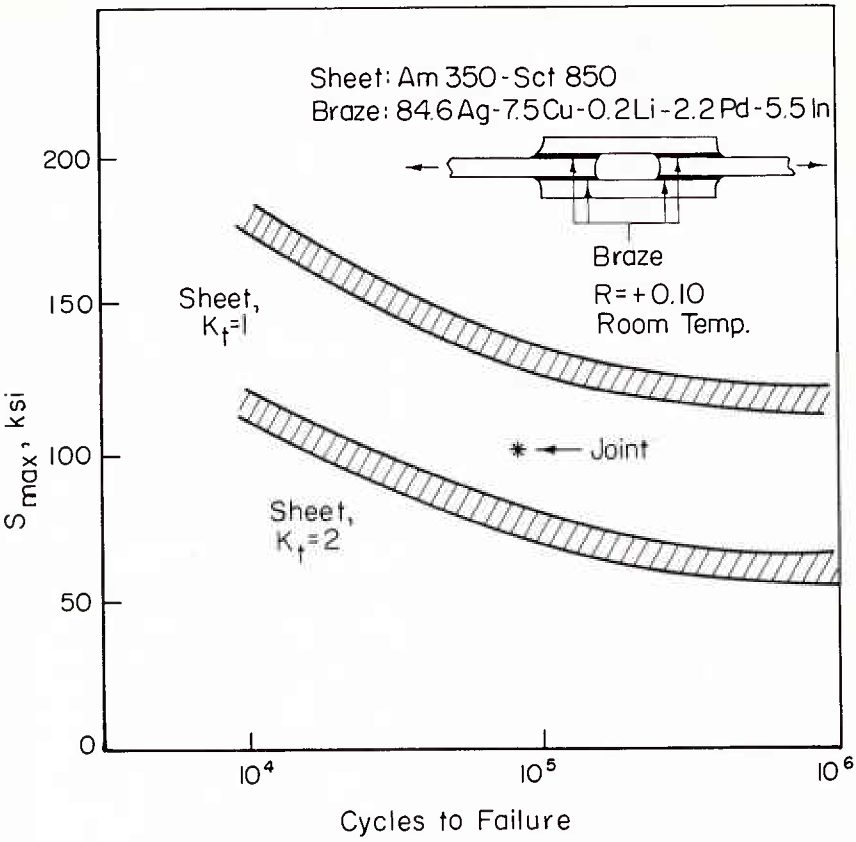

Figure 114.—Example of axial-load fatigue strength of a brazed joint.

An important use of brazing is the production of such items as honeycomb sandwich panels; these are mentioned briefly in the following section.

COMPOSITE MATERIALS AND PANELS

Particularly in some aerospace applications, there is increasing use of laminated and other composite materials. These are sometimes subject to vibrations which make long-life fatigue one possible mode of failure. In such uses as pressure vessels, proof-testing includes concern for very low-cycle fatigue.

Table 16 gives results of reversed axial-loading tests on several plastic laminates. The influence of fiber direction illustrates one of the additional complicating factors that may be significant in the fatigue prop-

TABLE 16.—*Results of reversed axial-loading tests on several plastic laminates*

[Axial loading, R = −1,900 cpm]

Specimen		Test conditions		Static strength, ksi		S_a at indicated cycles, ksi		
Reinforcement	Lay-up [a]	Angle, [b] degrees	Environment [c]	Tension	Compression	10^3	10^5	10^7
			Epoxy laminates					
181-Volan A	*P*	0	*D*	48.2	50.6	29.0	20.6	15.5
Do	*P*	0	*W*	43.4	51.9	25.4	15.5	12.3
Unwoven fiber	*P*	0	*D*	63.9	75.7	33.3	27.5	21.6
Do	*C*	0	*W*	59.4	66.1	29.6	20.8	11.8
Do	*C*	45	*W*	26.6	21.9	7.0	4.0	3.0
Do	*C*	90	*W*	64.6	61.3	....	19.4	13.7
Do	*I*	0	*W*	51.1	55.8	26.3	16.4	9.6
Do	*I*	30	*W*	39.2	45.8	17.5	9.4	6.0
Do	*I*	120	*W*	50.7	58.0	....	17.3	10.0
Do	*P*	0	*W*	....	78.2	36.5	24.5	12.5
Do	*P*	90	*W*	1.2	11.6	0.9	0.7	0.3
			Phenolic laminates					
Asbestos Material	*P*		*D*	48.6	27.7	23.2	20.4	17.6
Do	*P*		*W*	42.8	22.4	15.3	14.0	13.0
Do	*C*		*D*	41.5	22.8	19.5	15.0	15.0
Do	*C*		*W*	38.4	17.8	13.8	11.7	11.1

(a) Lay-up of fibers: *P*—parallel, *C*—cross, *I*—isotropic.

(b) Angle to surface fibers.

(c) *D*—dry—conditioned at 73° F. and 50 percent relative humidity.
W—wet—conditioned at 100° F. and 100 percent relative humidity.

erties of such materials. Many details (type of plastic, type of reinforcement, orientation of reinforcement, conditioning treatment, etc.) must be considered important to the fatigue strength of laminates (see, for example, reference *23*).

Honeycomb panels have been used in aircraft for their resistance to compressive buckling loads, to shear loads, and to acoustic loads. For the latter, their stiffness/weight ratio and their damping characteristics are valuable. The fatigue resistance of honeycomb panels has been reported only in isolated instances, and presently available information is scarcely generalizable.

At this time there are varied test specimens and test procedures, so that there is not yet available a large body of systematic information from which general design principles can be drawn. References *24* and *25* are illustrative of some of the approaches. The sandwich panel has many possible weak spots: in the core, in the cover plates, and in joints and attachments. It currently appears that a combination of elaborate stress analysis (by digital computers) and design and testing of representative, relatively simple coupons (embodying the critical stress raisers) may ultimately provide a sound engineering background for development of such components with acceptable fatigue characteristics.

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CHAPTER XIV. BEARINGS, GEARS, AND MONOLITHIC COMPONENTS

To this point in our discussion, we have reviewed available background on the fatigue behavior of joints of various kinds. While these are extremely important components in aircraft structures, there are other units of importance, such as controls, on which there is also some fatigue background. These include bearings (both rolling-element types and plain journal bearings), gears, shafts, and springs. Factors important in the fatigue behavior of such components are reviewed in this chapter.

JOURNAL BEARINGS

"Plain" journal bearings, which have been used for centuries, seem to have extremely complex mechanisms of failure. The principal fluctuating stress is apparently compressive (and hence unlikely to cause fatigue), but there are numerous secondary stresses. These include longitudinal and circumferential stresses from imperfect alignment and/or bending of the bearing liner, fluctuating thermal stresses, stresses from lubricant action, stresses from local irregularities in material or surface finish, and stresses because of local anisotropy of material.

Fatigue failure in journal bearings usually involves multiple cracking of the liner material and often spalling and subsequent disintegration. To evaluate separately such characteristics as fatigue resistance and wear resistance is difficult. Moreover, some essential requirements of a good liner material—such as wear resistance, resistance to seizure, low coefficient of friction, and corrosion resistance—are sometimes incompatible with high resistance to fatigue under the rolling load stresses in a bearing. Table 17 (adapted from reference *1*) illustrates this by showing results of fatigue tests of bearings to a selected lifetime and criterion of liner-cracking, with estimates of other properties. Reference *2* contains results of further investigations of journal bearings.

TABLE 17.—*Some characteristics of liquid-lubricated sleeve bearings*

Lining material	Fatigue strength, ksi	Shaft wear	Seizure Rating	Corrosion susceptibility
Tin-base babbett	1.9	Low	Low	Low.
Copper-lead	3.4	High	High	High.
Aluminum-tin	4.6	Medium ...	Medium ...	Low.
Lead-bronze	5.5	High	High	High.

NOTES: 1. Thickness: 12 mils in all cases. 2. See reference 1 for details. 3. These values are not suitable for design; many other factors are important, and the particular testing conditions were intentionally severe.

In addition to the selection of liner materials, the use of lubricants is a major factor in journal bearings. Liquid lubricants (including "heavy greases") take away heat and modify the stresses by providing an elastohydrodynamic film. For elevated temperatures, baked-on solid lubricants have been studied. These do not provide the heat sink or the hydrodynamic film afforded by the liquid, but they have shown potential for improving short-lifetime fatigue resistance of the bearing surface.

Even the bearing expert cannot assess all the factors in the fatigue performance of a specific design of sleeve bearing except on the basis of experience. Consequently, actual tests are a requirement if the fatigue performance is an important question. Tests are usually statistical in nature,⁽¹⁾ but the pertinent variables are many; materials of shaft, casing, and liner, dimensions and tolerances, lubricants, speed and other external forces, and environments.

BALL AND ROLLER BEARINGS

One of the important factors in a ball or roller bearing is the subsurface shear stress (Hertzian stress) indicated schematically in figure 115. Determining the depth at which the maximum range of shear occurs requires complex calculation. However, the factors involved and the order of magnitude can be estimated from the expressions for the elastic stress beneath a ball of radius R pushed with a force P on a plane $(R_1 = R, R_2 = \infty)$. If both ball and plane are steel,⁽²⁾ the maximum elastic shearing stress is

$$\tau_{max.} = 1.15 \times 10^4 \sqrt{P/K^2} \qquad (1)$$

at a depth

$$y = 1.26 \times 10^{-2} PR, \qquad (2)$$

⁽¹⁾ In this respect, testing of journal bearings is similar to the testing of roller bearings mentioned in the next section.

⁽²⁾ The numerical constants include values for the elastic moduli of steel, and the "constants" have dimensions.

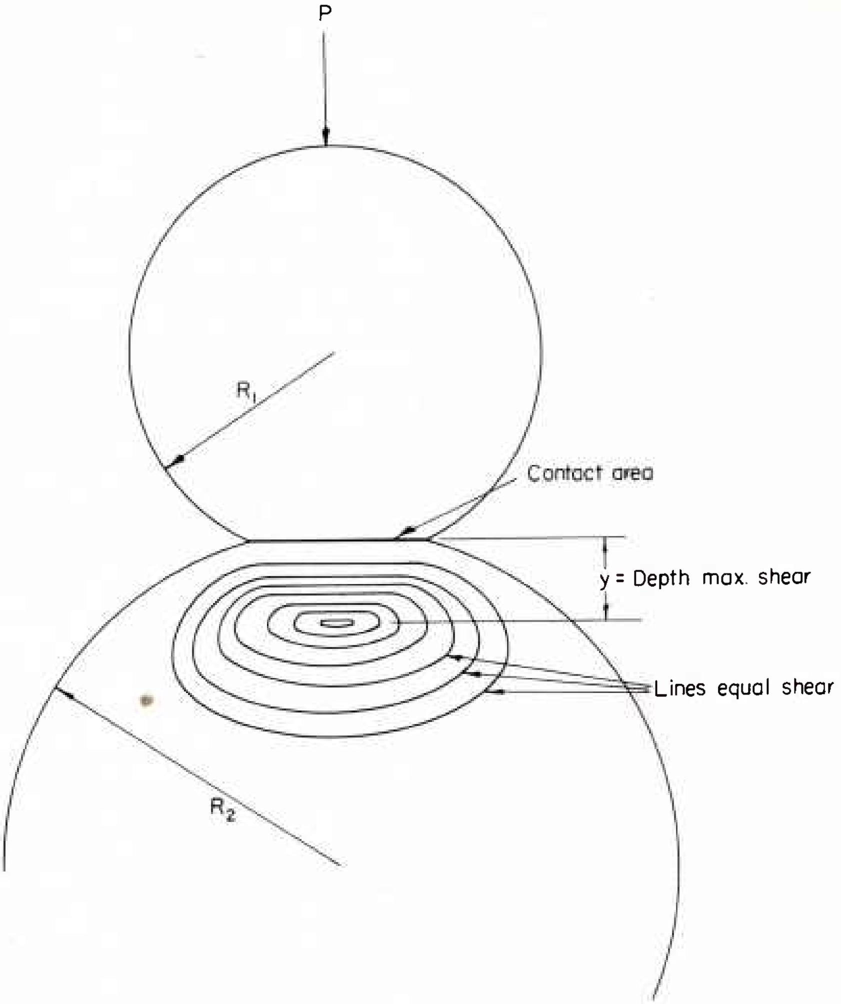

FIGURE 115.—Subsurface shear in cylinders pressed together.

where τ is in psi, P in pounds, R and y in inches. For example, a 1/8-inch ball with a force $P=50$ pounds would have a shearing stress of about 190,000 psi at about 0.0013 inch beneath the surface. The ball rolling repeatedly over the same spot in the surface might produce a subsurface crack at this depth. Subsequently, the crack might progress toward the surface and a small piece (1/1,000-inch thick) might "spall" out.

Evaluation of the stress distribution, even for a simple geometry, is complex when there is included any account of surface friction, plasticity, or similar real complications. For some theoretical discussions see references *3, 4,* and *5.* Details of the state of stress relevant to initiation of fatigue can only be estimated. However, it is clear that subsurface shear stress is important. Moreover, many of the relevant parameters are illustrated by relatively simple elastic formulas such as equations 1 and 2.

Thus, fatigue under rolling load includes the complication of a small volume of subsurface material under alternating shearing stresses and, at the same time, under a high triaxial compressive stress. Evaluation of the local stress state is very approximate since there are usually involved such factors as: plastic deformation, tangential surface stresses, effects of lubricant film, corrosion surface residual stresses, and metal inhomogeneities. Stress analysis can provide merely a general guide. The fatigue behavior of a roller bearing is therefore determined by test.

Life tests of bearings show, as might be expected, considerable scatter. Accordingly, a common practice is to run a number of tests(3) and interpret the results according to a statistical "Weibull relation." This relation can be written

$S =$ probability of survival of a bearing to a lifetime

$= e^{-\ell m/a}$

where

$a =$ constant characteristic of the fatigue life of the sample

$m =$ constant characteristic of the dispersion of life in the sample.

Figure 116 shows a Weibull plot from such bearing tests and indicates a B-10 lifetime (expected lifetime for 10 percent failures) for the test conditions involved.

While statistical analysis of many bearing test results has proved very useful, this procedure has drawbacks not the least of which is cost in time and effort. Consequently, many attempts have been made to develop simplified test rigs; studies with special setups are providing considerable background concerning the complexities of contact fatigue. (See, for example, reference *6.*) This background provides help in improvement of roller bearings; however, no simple test yet appears wholly dependable for prediction of actual bearing life under service conditions.

One current area of research is toward development of ball and/or roller bearings for use at elevated temperatures. This involves con-

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(3) For practical reasons, bearing tests are often accelerated by use of high loads and/or speeds compared to expected service. Corrections can be made to allow for this.

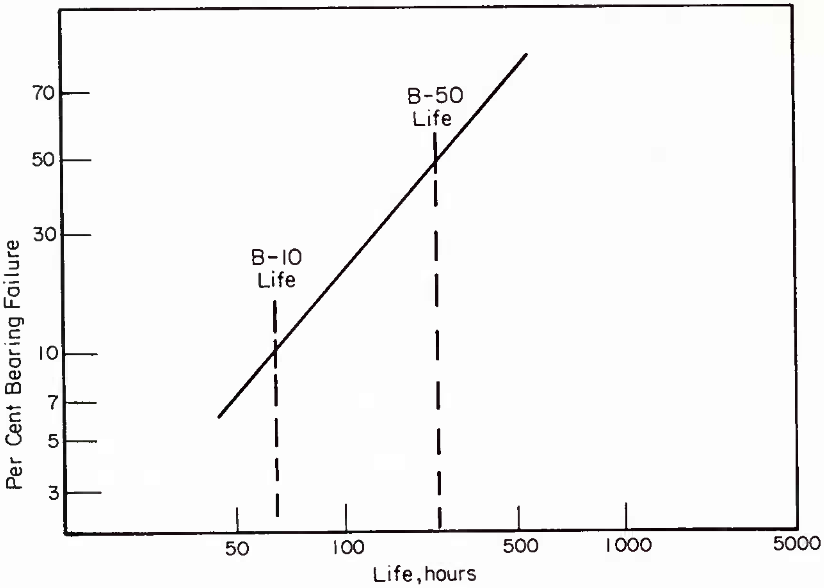


FIGURE 116.—Weibull plot of results of ball-bearing fatigue tests.

sideration of bearing materials and lubricant requirements (see, for example, references *6* and *7*).

GEARS

Figure 117 shows schematically two types of failure that have been observed in spur gears. Two critical locations are: (a) the tooth root, where there may be high stress concentration and fatigue cracking, and (b) the contact area of the tooth, where failure under repeated stress may involve pitting, wearing, spalling, etc.

Failure at position (a) is illustrated in figure 118. This kind of failure can be considered as bending fatigue of a cantilever (the single tooth) under repeated loading at the contact area away from a relatively fixed point (the tooth end). Factors that influence this are (1) design to avoid overstressing, (2) details of radius of curvature and of surface smoothness affecting stress concentration, and (3) analysis of hardening that may introduce favorable or unfavorable metallurgical structure and residual stresses at the tooth root.

Failure at position (b) is usually by some sort of pitting from the relative rolling and sliding contact of the mating teeth. The details of surface pitting are still the source of much discussion (*8,9*). Cer-

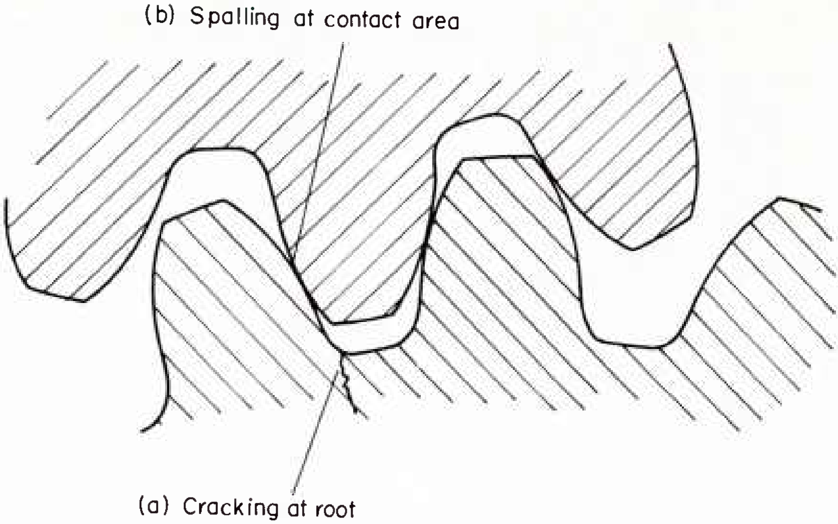


Figure 117.—Two locations for failure in spur gear.

tainly, many factors are involved: surface compression and frictional stresses, subsurface Hertzian stresses, residual stresses from surface hardening, local effects of lubricants, local thermal stresses, etc. Reasonably detailed analysis of these many factors and the mechanism (or, more likely, mechanisms) of gear-tooth surface failures may require many more years of study. Meanwhile, various practical means of influencing design life are known. First, suitable design minimizes Hertzian stresses, sliding velocity, and effects of slight misalignment. Some items of profile modifications and crowning and taper are mentioned in the references cited. Then, metallurgical factors are important and include choice of hardness to permit good wear resistance but some ductility to minimize contact stresses, particularly clean (inclusion-free) metal, and optimum surface hardening (with proper allowance for depth or hardened layer and for residual stress patterns). Finally, fabrication precautions are significant and include providing for close tolerances and surface finish (including run-in).

Thus the production of a gear to resist fatigue involves not only good design but very careful heat treatment, with particular care for such items as proper carburization and surface hardening. Extreme care in surface finish is also required, for example, grinding to a low rms without leaving surface burns or residual surface tension stresses or locally removing a thinly hardened surface.

A discussion of some of the complex factors involved in estimating the fatigue life of gears is in a paper by Coleman (*10*). The final

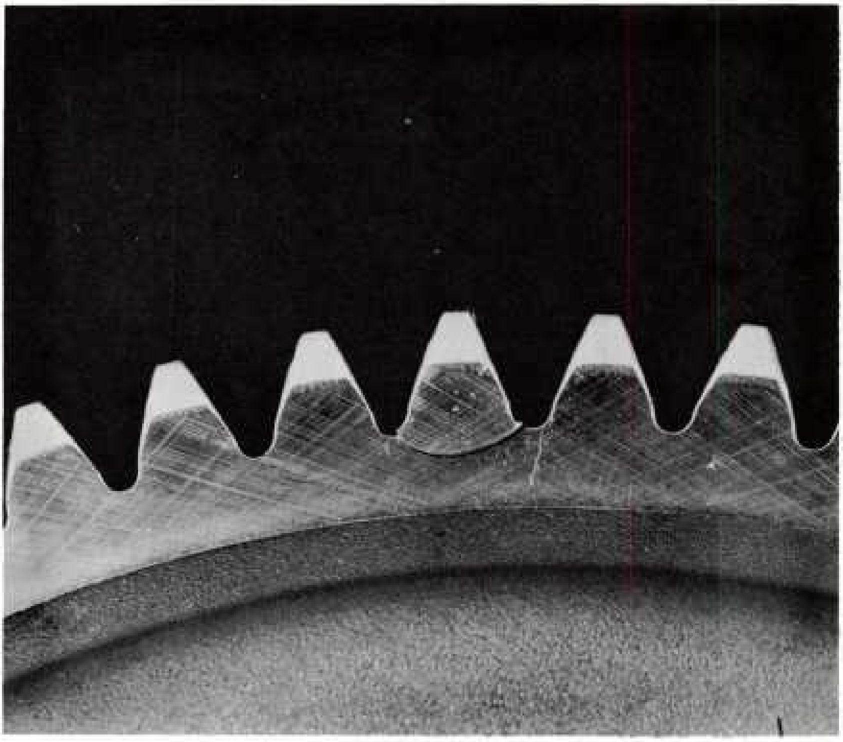


FIGURE 118.—Fatigue failure at root of a high-speed gear (note origin at stress concentrations near root of tooth).

criterion of the serviceability of a gear set is service experience. Short of this, simulated-service testing is almost mandatory.

AXLES AND CRANKSHAFTS

A great deal of study of the fatigue behavior of axles and of crankshafts has been carried out by and for the automotive industry. While much of this is of limited interest in aircraft, some important ideas have emerged concerning such general-interest factors as: size effects, effects of press fits, and effects of surface treatments such as rolling or shot peening.

Several investigations (*11*) have shown that large-diameter shafts may have significantly lower fatigue strength than small-diameter ones of the same material. Part of this appears to be attributable to metallurgical and fabrication effects since it is not predicted by extrapolation from small laboratory tests (*12*). Also important is the "notch-size effect" (see ch. IV); many experiments on filleted large shafts indicate fatigue-notch factors nearly as large as theoretical stress-concentration factors.

Other stress concentrators particularly pertinent to shafts are: transverse holes such as oil holes, splines and keyways, and press-fitted members such as wheels or collars. For all of these the usual principles of design to minimize stress concentration are important. For press-fitted members, some special designs that have been suggested for minimizing fatigue weakness are shown in figure 119. Some of these are discussed in reference *13*.

Introduction of surface compressive stresses by shot peening and by rolling (more commonly used in shafts) has been shown to be beneficial.

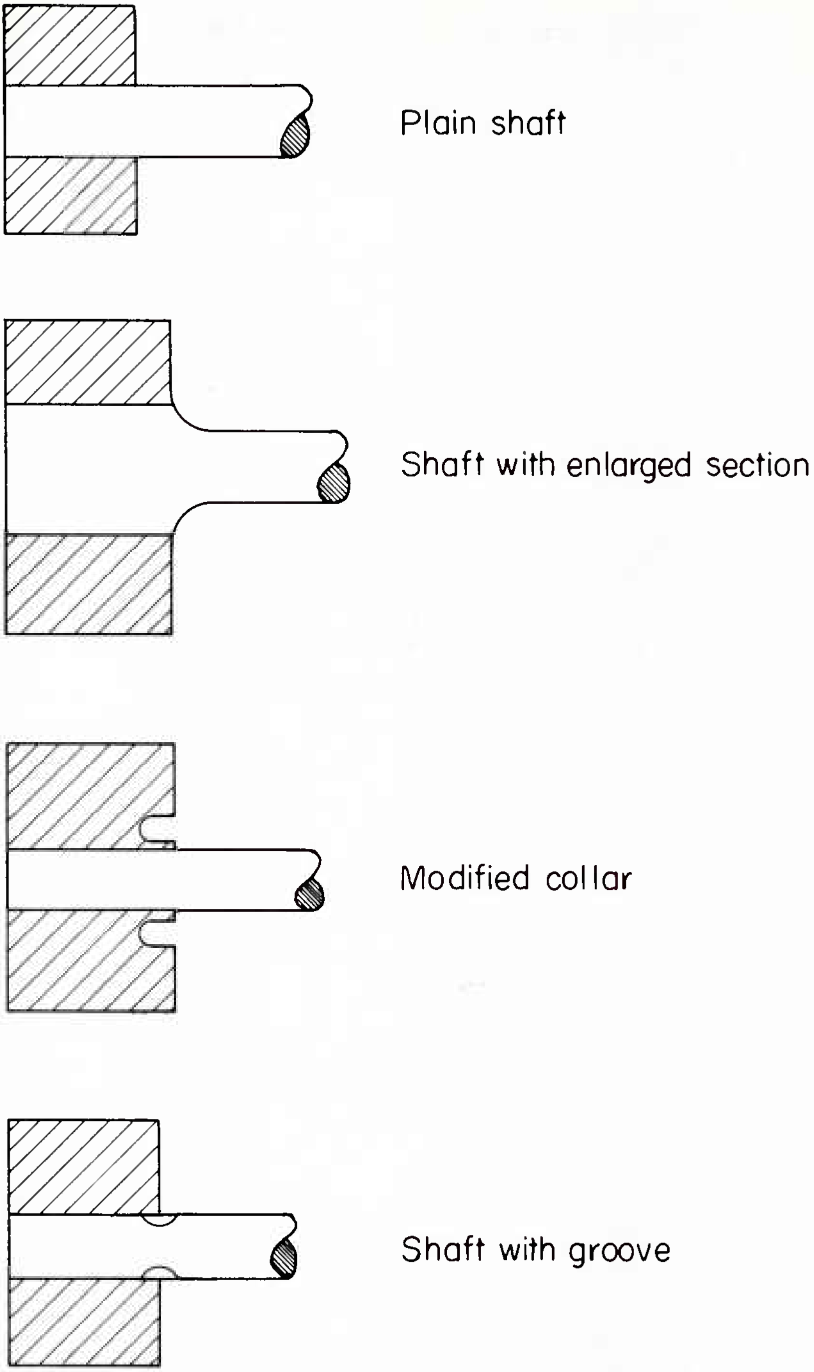
There have also been a number of studies of the fatigue behavior of crankshafts. These present problems with respect to fabrication, which usually includes forging to some approximation of final shape and then considerable machining. A recent investigation (*14*) with a large rotating-bending fatigue machine included examination of some of the factors involved in a typical crankshaft. Control of forging flow lines, of final dimensional tolerances (including generous fillets), and of surface treatment can reduce fatigue-notch factors from more than eight to a little less than the theoretical-stress-concentration factor (in the range of 3.5 to 4 for a "typical" design).

SPRINGS

Springs are commonly expected to withstand fluctuating stresses, and there have been fatigue studies of helical springs, of leaf springs, and of torsion bars. Examples of service failures are given in reference *15*.

In helical springs as well as in torsion bars, the important stress is mainly torsional. Watkinson (*15*) gives torsional fatigue data for a number of spring steels. He also indicates the detrimental effect of decarburization and the helpfulness of shot peening the surface. Coates and Pope (*16*) give results of tests on steel compression-type coil springs: (1) a mean-stress effect quite like that to be expected from tests on small polished bars, (2) an increase from prestressing (scragging), and (3) a still larger increase from shot peening. The latter operations usually introduce favorable residual stresses.

Leaf springs behave, in fatigue, as would be expected from cantilever tests of small specimens. However, especially in multileaf springs, such as widely used in automobile suspensions, there are additional factors: (1) surface finish as determined by production processes, (2) assembly, again influenced by production practice, and (3) effects of surface



Plain shaft

Shaft with enlarged section

Modified collar

Shaft with groove

FIGURE 119.—Some designs to alleviate stress concentration at press-fitted member.

friction. The surface rubbing of one leaf over another may decrease the effectiveness of shot peening.

FACTORS IN THE FATIGUE BEHAVIOR OF MONOLITHIC PARTS

The foregoing examples have been essentially monolithic components. It might be expected that the fatigue strength of any one could be estimated from data on the material and detailed stress analysis. Although this expectation is probably sound in principle, the examples indicate several factors that make this unrealistic.

First, each item has complex local stress details. Bearings and gears involve contact loading with high surface-compressive stress and apparently significant subsurface shear. In the presence of some amount of tangential stress, the details of local stress are extremely hard to evaluate. Shafts may involve size effects, press fits, and fretting, and consequently detailed stresses are also difficult to assess. Therefore, a background of fatigue-test information on each type of component is extremely important in identifying critical locations in the part and critical loading conditions for fatigue.

Second, many items have specific fabrication factors that influence fatigue behavior. Journal bearings are sensitive to the material and procedure used in the insertion of liners and to details of lubrication. Roller bearings and gears are significantly influenced by surface hardness, smoothness, and residual stress. Gears are critical in profile, surface hardness, and smoothness. Shafts are also sensitive to surface metallurgy and to mechanical perfection. Again, fatigue-test information in the literature furnishes background on the fabrication (forging, machining, heat treating, etc.) conditions that, within normal production, appear most influential in regard to fatigue behavior.

Finally, loading and environment are important. A background of laboratory tests can give information concerning the range of variation of lifetime with possible variations of loading and environment as well as with such factors as geometrical tolerances, alignment, and the like.

Thus, the information in the literature provides important indications of which locations, fabrication details, and operational factors are important. However, except when there are extensive data (as in the statistical test values for some bearings), the information may not be adequate for wholly dependable lifetime or reliability prediction. It is desirable, particularly when first estimates show a marginal expectation, to test a specific component. Here the background information can be used to help plan a critical test and to indicate the variables most likely to be important.

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CHAPTER XV. THE FATIGUE BEHAVIOR OF BUILT-UP STRUCTURES

For many of the components discussed in the preceding three chapters, analysis has been adequate to indicate the most severe stress raisers and the probable location of fatigue cracking. For bolts and studs, three critical positions were noted; for a simple joint, the location critical in fatigue is often self-evident; for a spur gear, the tooth-root and the tooth-contact areas appear to be important spots.

In aircraft, entire structural sections—such as wing, tail, or fuselage—are composites with many fasteners, stiffeners, cutouts, and other stress raisers. There are frequently multiple load paths. In addition to variables in the design, material, and fabrication of subcomponents, there are possible variables in the assembly of such large sections.

Several research investigations of large composite structures have been conducted. Some of these have had the broad objective of providing general background concerning the fatigue behavior of built-up structures. Examples of such investigations are given in this chapter. The results of programs of this type provide some indications of the pertinence and of the limitations in applicability of information concerning the fatigue behavior of such subcomponents as bolted, riveted, or welded joints to the behavior of a built-up assembly.

References *1*, *2*, and *3* contain descriptions of many investigations of built-up structures. The following brief accounts of selected examples illustrate the type of studies performed and the nature of information derived from such programs.

BEAM SPECIMENS

One of the earliest investigations of built-up aircraft structures was a study of new and of "used" beams of 24S–T aluminum alloy cut from wings of biplanes (*4*). The used beams did not show significantly lower fatigue strength than the unused ones. In the light of later knowledge, the probable service damage may have been insignificant. Figure 120 shows one result: values of K_f for beam specimens(1) showed different

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(1) All tests were at $R = -1$; hence, K_f is defined as the ratio of the stress amplitude for an unnotched sheet to the nominal stress amplitude, at the location of cracking, for beam or coupon at the same lifetime.

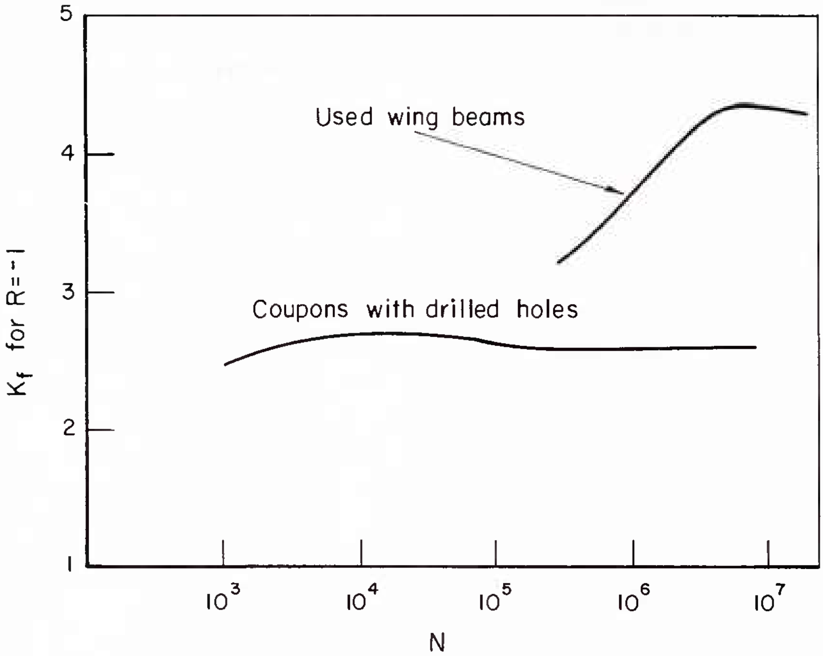

Figure 120.—Fatigue-notch factors for wing beams.[4]

variations with lifetime than did values for simply notched sheet coupons. The beams had, for long-lifetime failure, relatively high fatigue-notch factors.

A later study at the same laboratory involved a beam specimen of two U-shaped sections bolted to a steel splice bar to form a box having riveted cover plates (*5*). These specimens were also tested in reversed bending. Figure 121 shows *S–N* curves for (1) unnotched sheet coupons, (2) sheet coupons notched with "idle" rivets, and (3) the box-beam specimens. For the idle-rivet coupons, K_f is about 1.2 from 10^4 to 10^7 cycles; for the box-beam, K_f increases from about 2 at 10^4 cycles to about 6 at 5×10^5 cycles. As for the previous illustration, one conclusion is the inadequacy of applying, to a complex structure, a single value of fatigue strength reduction factor obtained from coupons with a stress-raiser which does not duplicate all significant features of stress and strain concentrations in the structure. In these experiments it was noted also that the mode of failure in fatigue of the beams was different from that of static failure so that ratios of fatigue strength to static strength of a structure cannot be expected to have clear-cut physical significance. Such conclusions concerning the complexities of applying

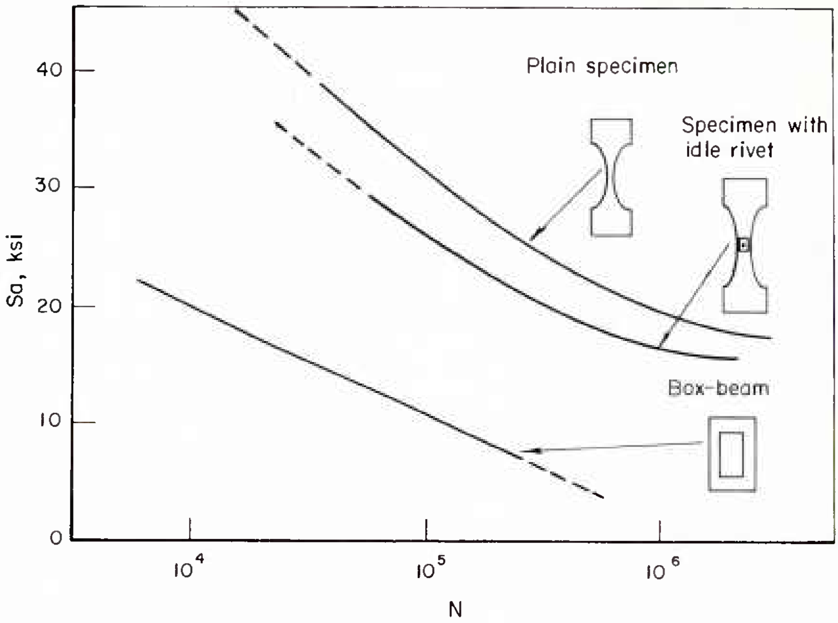

Figure 121.—Results of reversed-bending fatigue tests of box-beams.

K_f factors and the questionable significance of ratios of fatigue strength to static strength of structures are widely recognized today.

Some years later a study of two types of beam specimens was reported (*6*). These were a riveted box-beam and a riveted I-beam tested under repeated, but not fully reversed, bending. A moderately detailed stress analysis was made. Figure 122 shows some of the reported results. Note that, as in the investigation first cited, the beams show values of K_f increasing with increasing lifetime in contrast to the behavior of simply notched coupons. The values of K_f for the beams, computed on the basis of nominal longitudinal stresses, appeared rather high at long lifetimes. Strain gages on the beams in the neighborhood of fatigue cracking showed "secondary" transverse stresses of appreciable magnitude. Figure 123 shows results of bending-fatigue tests on the box-beams and of tension-fatigue tests on coupons designed to simulate the stress concentration at the location of fatigue cracking. For all three the longitudinal stresses were similar; however, coupon B had transverse stresses like those in the beam while coupon A did not. A similar indication of the importance of transverse stresses was found for the I-beam.

A more complex box-beam specimen with features of interest in connection with future supersonic flight vehicles is described in refer-

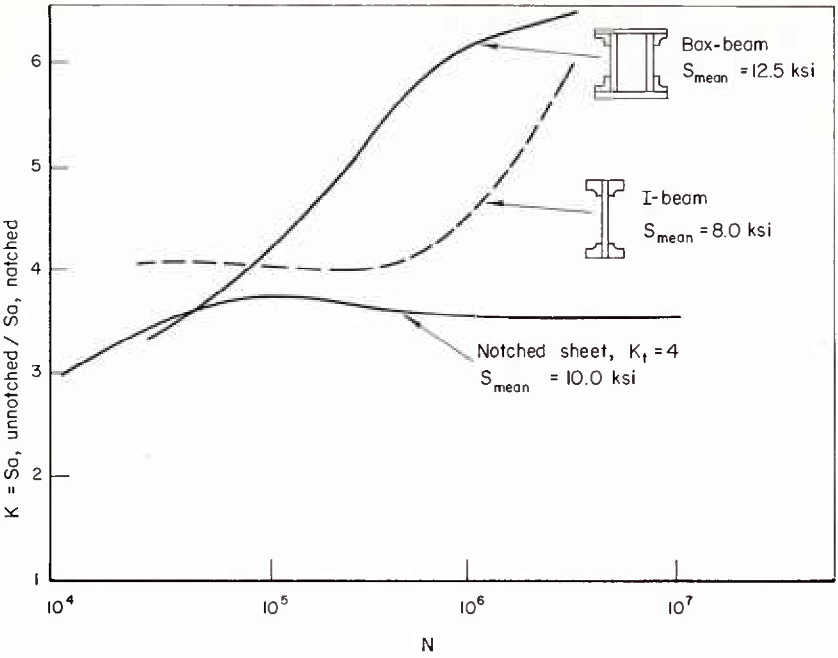

FIGURE 122.—Reported results of stress analysis of a riveted box-beam and a riveted I-beam.

ence 7. This consisted of three corrugated spar webs with compression flanges and with four loading bulkheads and two corrugated covers. The material was PH15–7 Mo corrosion-resistant steel. Each specimen was tested so that the center section was in pure bending, and the cover panel (critical in fatigue) was in tension. The test involved a loading of some complexity: a spectrum of load amplitudes with heating and cooling.

Results were compared with results of axial-loading tests of a sandwich-joint specimen simulating aspects of the cover plate. Somewhat limited data showed close agreement for the beam and the sandwich specimen, and relatively close agreement with a simply notched coupon tested at a higher mean stress (chosen on the basis of a particular analysis). This investigation suggests that (1) fatigue behavior of structures can be related to fatigue behavior of simpler notched specimens if critical features of stress and strain details are suitably correlated, and (2) such correlation may require considerable sophistication in analysis and in testing.

The continuing studies of fatigue of composite specimens of the box-beam type are further illustrated by experiments recently com-

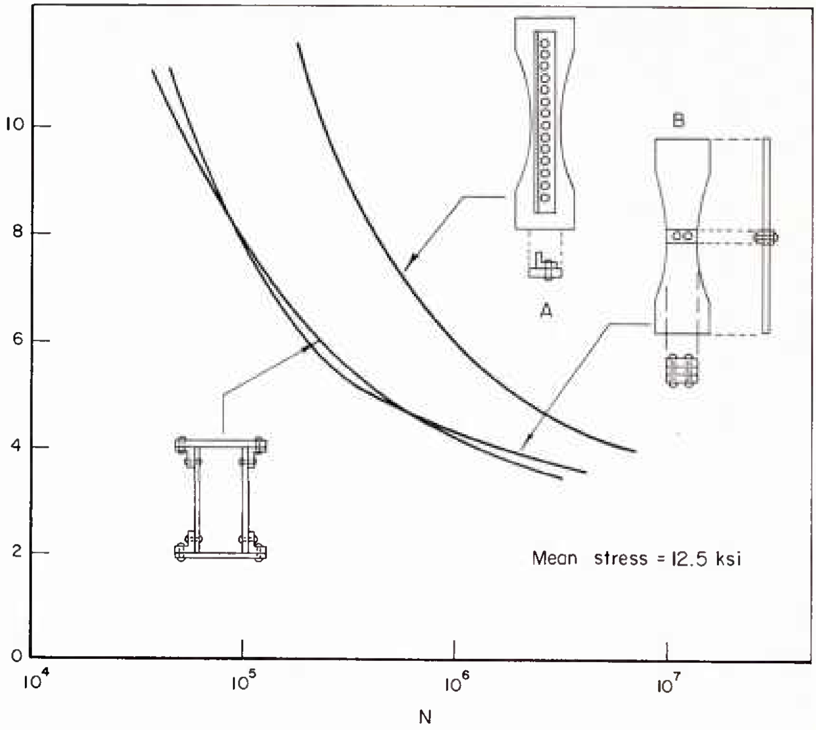

FIGURE 123.—Coupon elements simulating fatigue behavior of a box-beam.

pleted at the National Bureau of Standards (*8*) and currently being extended at the Naval Air Engineering Center. Figure 124 shows the results of constant-amplitude tests on the simple box-beam used: the lower curve is approximately that for the beam specimen, while the upper curve is that (obtained by extrapolation from data in the literature) for unnotched specimens of the 7075–T6 material at the same constant minimum stress. If the ordinates of the two curves are divided, a fatigue-notch factor(2) is obtained that increases from about 1.7 at 10^4 cycles to about 2.7 at 5×10^9 cycles. This is similar to trends noted previously for beam specimens. The main purpose of this investigation was examination of behavior of the beams under variable-amplitude loading. Some conclusions were: none of the current theories of cumulative damage (see ch. VI) provided very close pre-

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(2) This is a somewhat unconventional fatigue-notch factor:

$$K_f = \frac{S_{max.} \text{ for unnotched sheet}}{S_{max.} \text{ for beam at same } S_{min.} \text{ and } N}.$$

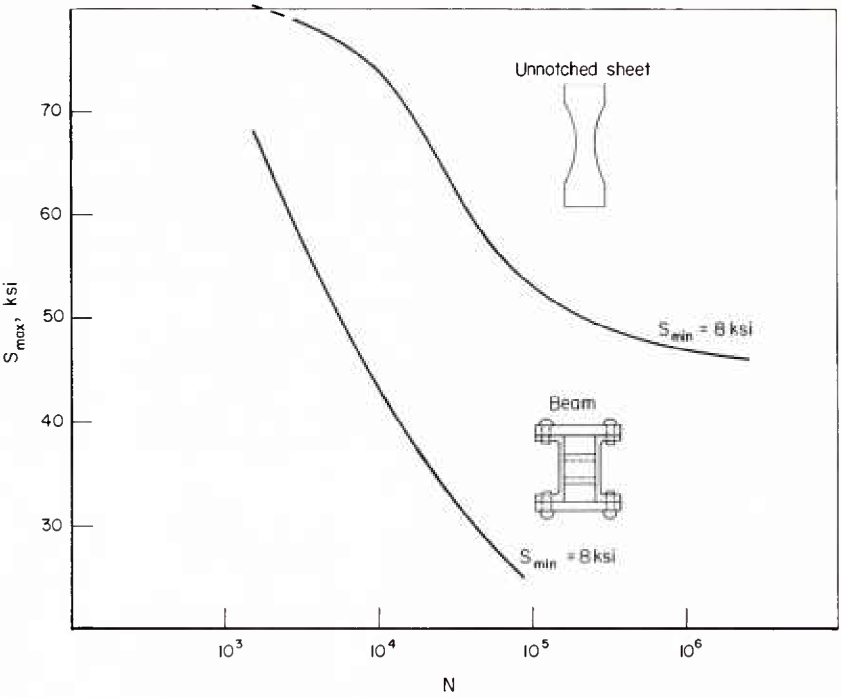

Figure 124.—Results of other box-beam tests.(8)

dictions of the experimental lifetimes; static prestressing at 100 percent limit load improved lifetimes at and below 60 percent limit load if prestressing was in the same direction; prestressing in the opposite direction reduced fatigue lifetime; periodic understressing at 25 percent limit load produced no significant changes in fatigue life under loads in the same direction. There were indications that the spectrum could be truncated for positive loads below some level (of the order of 22 percent of ultimate strength) ; however, relatively small negative loads were damaging.

At present, studies of this type are being extended at the Aeronautical Structures Laboratory of the Naval Air Engineering Center. Some interesting apparent effects of block size in spectrum tests have been reported.(3) These observations could influence selection of spectrum details for full-scale tests of aircraft.

Another factor which has been and is being further explored with box-beam specimens is resistance to crack propagation. There have

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(3) Oral discussion by M. S. Rosenfeld at February 1964 meeting of ASTM Committee E–9.

been measurements of the rate of crack propagation under repeated bending of simple boxes with stiffened cover plates (*9*). For two cover-plate materials, 7075–T6 and 2024–T3, cracks propagated faster in 7075–T6. For three types of stiffener, integral, riveted, and riveted and bonded, the rate of propagation was largest for the integral stiffeners. Static tests to measure residual strengths were also conducted (*10*). Beams with stringers bonded had highest residual static strengths for both materials, while 7075–T6 beams with integral stiffeners had lowest. Predictions based upon the Hardrath Formula (see ch. V) agreed fairly well with experimental results.

Critchlow (*11*) describes a study of a number of types of specimens (panels, box beams) with regard to residual strength after cracking. Analysis is, for each type of component, in terms of parameters expected to determine local stresses. For the more complex components, this involved numerical computation.

WING AND TAIL SURFACES

Considerable effort has been devoted to fatigue tests of full-scale aircraft wing and tail surfaces. Many tests have had such limited objectives as comparison of alternative designs, study of the cause of some observed failure, or "proving out" of a specific design. However, some investigations have been directed toward better understanding of the fatigue behavior of built-up structures. The following discussion is limited to brief accounts of studies of the latter type.

Figure 125 shows *S–N* plots derived from a number of tests of full-scale structures. The data in this figure have been adjusted to a common testing condition ($R=0$) so that conclusions based upon comparison of the curves must be considered speculative. It may also be speculatively concluded that:

1. The structures exhibit *S–N* curves of the same general shape as those developed for coupons of the aluminum alloy (2024) of which the structures were made.
2. The *S–N* curves for the structures drop down to quite low values at long lifetimes.
3. The scatter band for any one structure is not excessively wide. This relatively little scatter is frequently observed for structures made with good or superior quality control.
4. There are apparent differences among the structures.

Thus, an overall conclusion might be that actual built-up structures exhibit fatigue behavior with similarities to the fatigue behavior of a notched coupon of the material involved, but there are enough un-

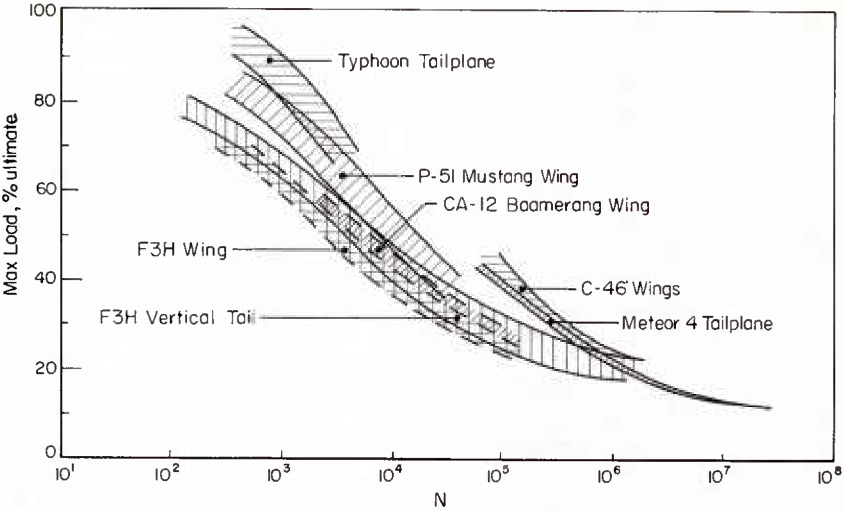

Figure 125.—Results of several investigations on aircraft structures.
Note: Loads *adjusted to* $R = 0$ by data for sheet material.

certainties to make design upon the basis of laboratory data on notched coupons extremely difficult.

More than 15 years ago the Aeronautical Structures Laboratory, Naval Air Engineering Center, started fatigue tests of full-scale aircraft wings. Some of the early work is described in reference *12*. This work showed failures originating, as in box-beam specimens, at points of stress concentration such as rivets and bolts; high values (4 to 6) of K_f (defined in terms of ratios of maximum stress at a specified mean stress) were also noted.

In 1956, 26 new production jet-fighter aircraft, with no service time, were made available to the same laboratory, and an extensive program of studying the fatigue behavior of composite structures was started. The aircraft were disassembled into major subassemblies: wing, horizontal tail, vertical tail, aft fuselage, nose landing gear, and main landing gear. Tests on some of these are still under way, but tests on wing and on tail surfaces have been completed. Table 18 outlines tests that have been completed and indicates the extensive scope of study. Reference *13* contains a summary account, and reference *14* has additional details.

Figure 126 shows a test setup for a typical wing fatigue test, and figure 127 shows a horizontal tail test setup. The latter gives a clear view of the "whiffle-tree" connections to loading pads for providing a distribution of applied load simulating service air loads. This arrangement is commonly used in tests of such subassemblies as wings and tails.

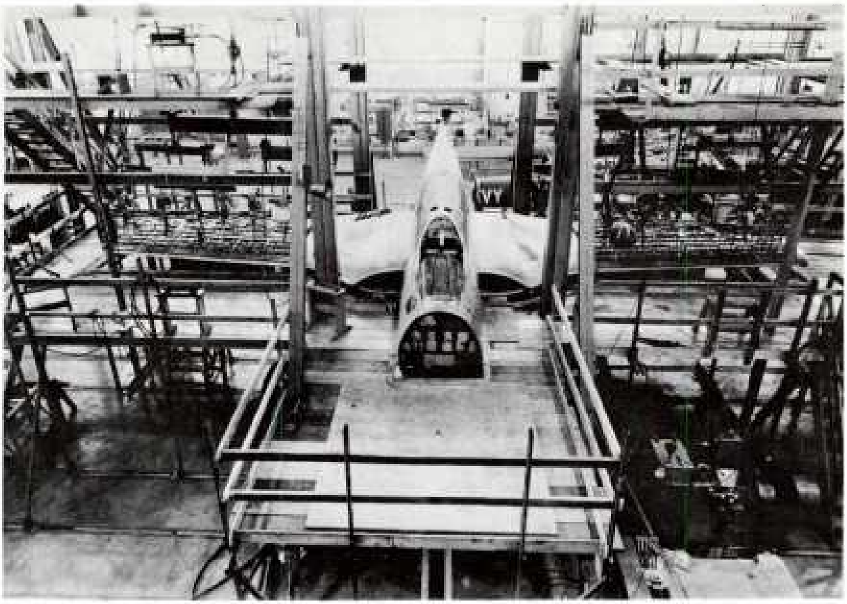

Figure 126.—Setup for a typical wing fatigue test.

Figure 128 shows the results of fatigue tests on the wings and on the horizontal tails. The curves appear to be in reasonable agreement with results of tests on other subassemblies of similar kind. The scatter is small—particularly in view of the observation that failures in the wings

Table 18.—*Outline of portion of an extensive investigation of aircraft subassemblies*

Part	Number of specification	Type of fatigue test
Wing	12	Constant amplitude for each specimen. Maximum varied; minimum constant for series.
	10	Spectral loading, 4 types.
Horizontal tail	24	Constant amplitude for each specimen. Maximum varied; minimum constant for series.
	12	Spectral loading, 4 types.
	6	Spectral loading with prestress.
Vertical tail	8	Constant amplitude, $R = 0$.
	8	Constant amplitude, $R = -\frac{1}{2}$.
		Constant amplitude, $R = -1$.

Notes: 1. All from unused jet-fighter aircraft. 2. See reference 13 for additional information. 3. Other structural subassemblies to be tested in total program.

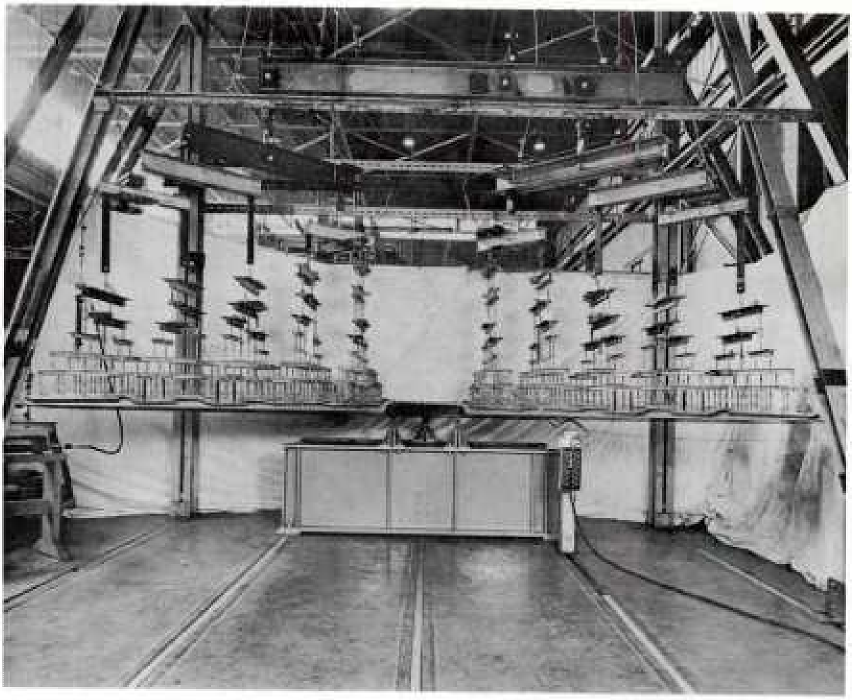

FIGURE 127.—Horizontal tail test setup.

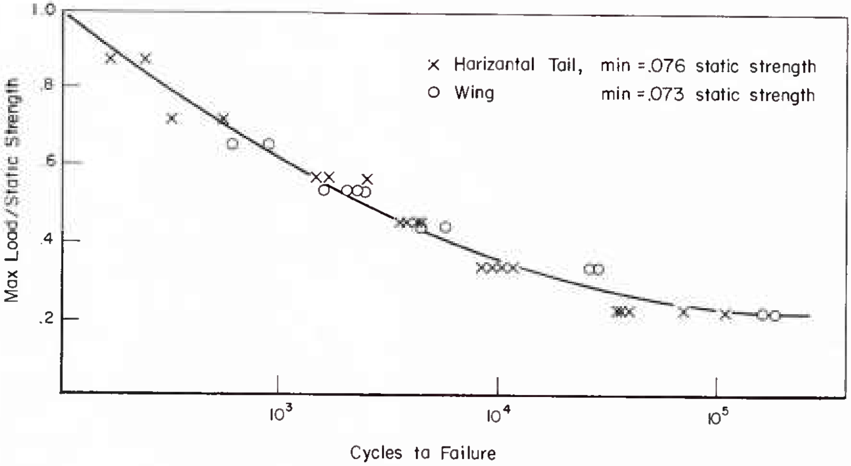

FIGURE 128.—Results of fatigue tests of some aircraft structures.

occurred at different locations.[4] Values for K_f were not quoted; meaningful values are difficult to obtain. However, strain gage readings in the region of failure of the tail section showed (1) appreciable difference in strain-load ratio for different specimens, (2) some variation of local strain with number of cycles, and (3) a nonlinear strain-load relation at high loads. These observations suggest the uncertainty of attempts to define meaningfully an effective K_f for such a structure. The observations also indicate difficulties in the often-suggested use of a strain-sensitive fatigue-warning indicator.

Another item of interest in the Aeronautical Structures Laboratory investigation was spectral loading. Variations of load in a spectrum that seemed relevant to this fighter aircraft were used for the wings and for the horizontal tail. Details would be inappropriate in this chapter (see reference *13*), but several observations were most interesting:

1. For the particular spectrum and block size (corresponding to 20 flight hours) no significant difference was observed between a fixed sequence and a randomized order of loading.
2. Prestress at 73.8 percent static strength increased the lifetime of horizontal tails by 100 percent; however, there are limitations on this method of improving fatigue strength of a full-scale structure so that generally it may be "technically and economically impractical" (*14*).
3. The Palmgren-Miner summation of cycle ratio gave $\Sigma n/N$ values of 2.4 to 8.2. Most other predictions of cumulative-damage lifetime were impractical; a modification of the Corten-Dolan method fitted observations significantly better than the Palmgren-Miner procedure.

The applicability of these observations to other subassemblies or to other loading conditions is admittedly uncertain.

Another extensive program of tests has been carried out at the Langley Laboratories of the NASA on full-scale wings of C–46 airplanes (*15–17*). Figure 129 shows the Langley test setup. The relatively simple loading was possible since it provided a close approximation to service stresses in the 5-foot-long section just outboard of the nacelle, which was the area of interest in this study. The fatigue tests were run at several values of load amplitude ($\Delta n = \pm 0.35$, ± 0.425 ± 0.625, and ± 1.00 g) at a fixed mean of 1.00 g. Table 19 lists some values of K_f observed for the several modes of failure noted. The definition of K_f in the table should be noted closely. Values are not

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[4] 7 at main spar carry-through fitting at station 33.25; 2 at main spar carry-through fitting at station 25.5; 1 at main spar carry-through fitting at station 3.0; 2 at main spar lower flange at station 117.0. "For all practical purposes, the failures (of the horizontal tail section) were identical."

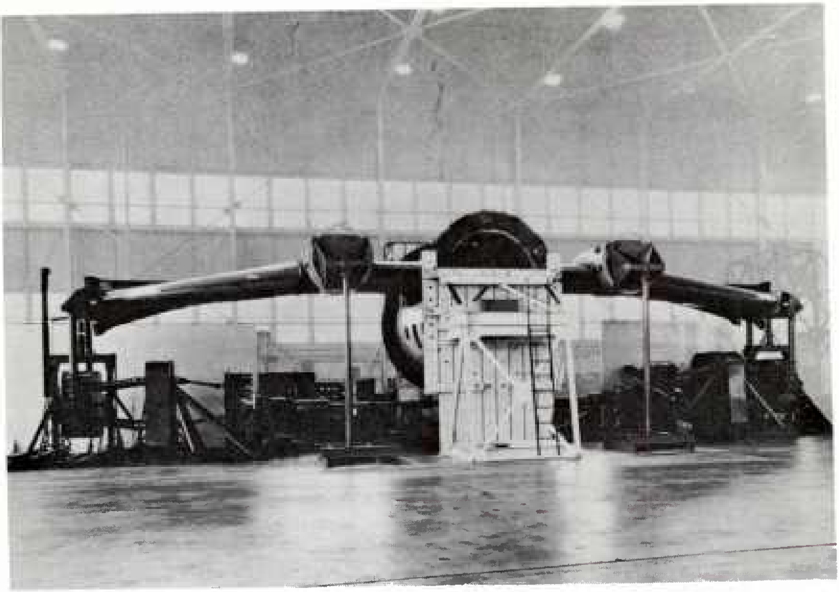

FIGURE 129.—C-46 variable-amplitude fatigue machine tests at NASA, Langley.

readily comparable with others listed in this chapter. However, three points may be observed:

1. K_f had different values for the different modes of failure.
2. Values increased with decreasing load amplitude (and increasing lifetime).
3. Although not shown in table 19, there was considerable scatter for a specific type of failure and load amplitude.

Despite these notes the scatter in lifetimes to first cracking was not great (about 4 to 1). In many cases, attempts were made to measure crack propagation rates. It appeared that crack growth was small until there was a 5 to 9 percent decrease in tension material at any cross section. Cracks reduced the residual static strength more than in proportion to the decrease in tension area. In later tests at the same laboratory, on wings of T–29A airplanes, additional studies of fatigue crack initiation and propagation were made in failures in 7075–T6 skin and extrusions. Observations indicated some sensitivity of this material to fatigue-crack propagation.

A third program on fatigue behavior of wings was carried out at the Aeronautical Research Laboratories at Melbourne, Australia. A summary account is in reference 2. This included tests on some 89 pairs of wings from P—51D airplanes (with varied amounts of service life prior to testing). Some general observations were:

TABLE 19.—*Values of* K_f *for fatigue failures in C-46 wings*

Type of failure	Value of K_f (a) at indicated load amplitude (b)			
	$\Delta n = \pm 1.00$ g	$\Delta n = \pm 0.625$ g	$\Delta n = \pm 0.425$ g	$\Delta n = \pm 0.350$ g
1. Cracks at corners of cutouts	4.0	5.0	5.5	....
2. Riveted tension joints	....	2.6	....	....
3. Riveted shear joints	....	3.5	3.4	....
4. Cutout reinforcements ..	4.3	4.6	5.6	6.2
5. Miscellaneous discontinuities ..	3.4	4.5	3.8	4.0

(a) $K_f = \frac{\text{Maximum stress in unnotched material at same mean stress and number of cycles}}{\text{Maximum stress near failure (from stain-gage remote from local stress concentration)}}$.

(b) Mean load factor, 1.00 g.

1. Failures occurred at varied locations; all were at some point of stress concentration.

2. Effective strength-reduction factors varied with the location of failure (or with the type of stress raiser) and with the loading conditions (for example, the load ratio). Different load spectra produced differences in the prominence of different failure modes.

3. A log-normal distribution provided an approximate fit to the scatter of the results. The standard deviation (for a group tested at a particular load range) varied from 0.02 to 0.31, the mean value being 0.12.

CONCLUSIONS FROM PROGRAMS ON BUILT-UP STRUCTURES

The examples described in the preceding pages and further examples in the references cited indicate the diversity of programs of fatigue tests on built-up structures. Accordingly, to draw detailed conclusions directly applicable to structures other than those tested, or to conditions under which they were studied, is difficult.(5)

Nevertheless, the following points seem rather well established for the built-up aluminum-alloy skin-stiffener structures that have been investigated rather extensively:

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(5) The engineering economics of extensive fatigue testing of built-up structures are such that either the structures are simplified (as some of the box beams) or they are nearly obsolete (as some of the wing specimens) by the time of completion of a program.

1. More than one mode of failure may occur. This implies that use of an effective fatigue-notch factor obtained by a single test related to curves for simply notched coupons may not be valid for the structure.
2. When analysis of results of full-scale testing has delineated the possible types of failure, study of a simulation element may be helpful.
3. At least in some instances, a spectral loading test (with a spectrum appropriate to expected service loadings) is more likely to show up all important types of failure than a constant-amplitude test.
4. Scatter in lifetimes of built-up structures is difficult to relate to scatter in tests of small coupons. In some instances, built-up structures show apparently small scatter; this may be because any very complex structure is likely to have a severe stress concentration and fail at the low edge of a scatter band for coupons. At least at present, safe design requires some allowance for a scatter factor.

Fatigue studies of subassemblies are continuing, and the analysis of the the many data already obtained is continuing. It may be hoped that more definite design principles will be formulated in the next decade.

Meanwhile, caution should be taken in extrapolating "apparent" results. Use of Goodman-type diagrams, "effective" fatigue-notch factors, statistical variances extrapolated from data on small coupons, or cumulative-damage "rules" based on tests of simple specimens, can lead to erroneous conclusions. The important and difficult problem is the identification of local stress conditions with the load factors or nominal stress values in the composite structure. When no important item in such an identification is missing (a secondary stress, a residual stress, a redistribution of stress under loading), the fatigue behavior of the composite seems understandable in terms of the background information on materials and on simple laboratory specimens.

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CHAPTER XVI. THE AIRFRAME AND ITS ENVIRONMENT

Prevention of fatigue failure in an aircraft during its useful lifetime involves a composite of the problems discussed in previous chapters, *plus* consideration of likely loads and other environmental factors, and of the response of all elements of the structure to these. This and the following four chapters are concerned with some current engineering approaches to this challenging requirement. To keep the discussion within bounds, it will be focused upon the airframe structure whose integrity is critical for safety.

The engineering approaches to reliability of the airframe structure, in addition to being of obvious importance, include the principles involved in consideration of fatigue for most other parts. There are some concepts, described in this chapter, that underlie analysis and testing to provide reliability of airframes against fatigue failure. Details of engineering applications, some of which are illustrated in succeeding chapters, vary with individual circumstances.

RELIABILITY CONSIDERATIONS

The loads and other environmental factors to which an aircraft may be subject vary from one flight to another; the loads encountered by two aircraft of the same type cannot be expected to be identical over their respective service lifetimes. However, assuming similar missions, there should exist a probability-of-occurrence distribution which would be typical for the type of aircraft and service experience expected. Moreover, the strengths of airframe elements (such as wings) may be expected to be non-identical and to fall within some probability distribution, influenced by details of material, fabrication, and assembly variables. Hence, there is an increasing tendency to recognize that the reliability of an airframe in its environment can be considered in terms of probability distributions.

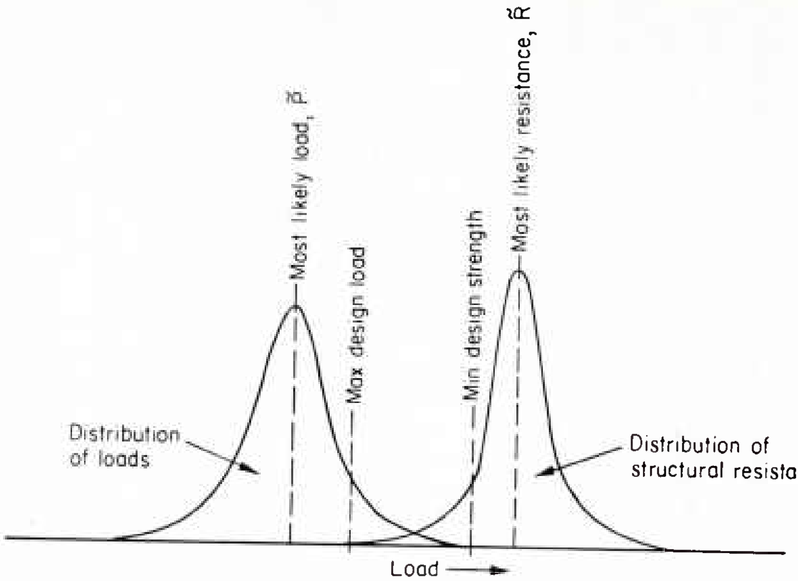

Figure 130.—Schematic diagram of factors involved in reliability against "static overload failure."

Figure 130 shows schematically some aspects of resistance to "static" overloading. The left-hand portion indicates a possible distribution, during the service lifetime, of loads; there is some most likely load *and* some probability of higher as well as lower loads. At the right is a distribution curve for structural resistance or strength; there is some most probable strength *and* a chance of some significantly lower as well as higher strength. The shaded region of overlap of the two distribution curves implies some probability of failure. Several observations may be drawn from consideration of this schematic representation:

1. As long as either distribution function spreads out into the other, there is a finite probability of failure. This may be very low; an objective is to make this probability small compared to other hazards such as pilot error.
2. Just allowing a large difference, $R-P$, in the most likely values of load and of structural resistance does not insure reliability. Allowance must be made for widths of the distribution curves.
3. The critical region in the sketch is the overlapping area⁽¹⁾ of the

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(1) The probability of failure increases (and the reliability decreases) with increase in this shaded area, although the relation is not generally linear.

tails of the distributions. These tails are extremely difficult to establish with precision.(2)

These observations indicate the impracticality of rigorously following statistical probability theory in design for prevention of "static" overload failure. Engineering practice involves (1) assigning upper bounds (such as ultimate load factors) to the load distribution and (2) using conservative values (less than the most probable value) for strength allowables. Such procedures, with continuing review of assigned values with respect to experience, have been very successful.

Two main points in regard to reliability against static overloads are applicable to fatigue. There is need for probability considerations, and these are not now tractable by mathematical statistics without physical considerations to guide extrapolations from available information concerning the distributions.

However, there is a major additional consideration in regard to reliability against fatigue failure. The degradation(3) with increasing time of structural resistance cannot be neglected, even as a reasonable first approximation. Usually, repetition of loads decreases the ability of a material to withstand subsequent loads. Moreover, as discussed in chapter VI, this cumulative damage is dependent upon the spectrum of all applied loads.

Figure 131 shows a schematic diagram illustrating this degradation of structural resistance with increasing time. For simplicity, the distribution of loads has been assumed unchanged with time. However, the distribution of structural resistance is drawn with a lower mean value, a wider dispersion and a skewed shape at the later time. As a result, the region of overlap has increased and the reliability (the probability of nonfailure) has decreased.

. For some purposes (note subsequent comments on fail-safe design) the probability of failure per flight or per interval between inspections is of particular interest. This will increase with increasing hours of service. It is then theoretically possible to select a reasonably low cutoff value, upon reaching which the critical airframe parts are to be replaced.

There have been suggestions (see, for examples, references *1* through *4*) for applying mathematical statistics to the evaluation of reliability

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(2) It is impossible to establish strength distributions by a sufficient number of tests to determine the statistical probabilities in detail. This is in contrast to possibilities for some characteristics of some small, mass-produced components of electronic systems. Consequently, analysis of mechanical reliability of complex structures requires combined probabilistic-physical considerations.

(3) In this discussion, such influences as corrosion, wear, fretting and creep are omitted for simplification and to emphasize the role of cumulative fatigue damage.

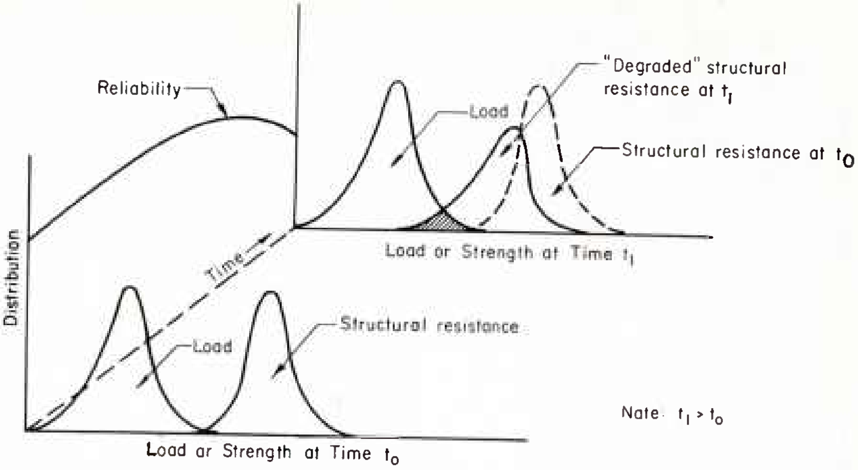

Figure 131.—Schematic diagram of factors involved in reliability against time-dependent (fatigue) failure.

against fatigue failure. These provide increased insight into the factors important in the pursuit of more sharply defined reliability (reference *5*). However, a rigorous probabilistic theory, acceptable to aircraft engineering, has not yet been established (reference *6*).

At present, the general engineering procedure is to allow for the uncertainties in estimation of reliability by procedures involving judgment tempered by experience, in a manner analogous to static strength considerations.

ENGINEERING VIEWPOINTS

Analysis of the dependability of an airframe against fatigue can be discussed in terms of a specific major component (such as wing or fuselage) under specific loadings (such as gusts and maneuvers or pressurization cycles). Let us consider a wing whose fatigue response is critical under gust loadings.

Suppose this structure is subject to a steady load of 1-g with superimposed alternating loads of various amplitudes. Let the most probable number of repetitions of each amplitude in each 100 hours of flight be distributed(4) as shown in the lower (dotted) curve of figure 132. Let the *S–N* curve for the wing structure under a mean load of 1-g be given by the upper (solid line) curve of the same figure. This curve,

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(4) Some questions about such characterization of loads are discussed in the next chapter.

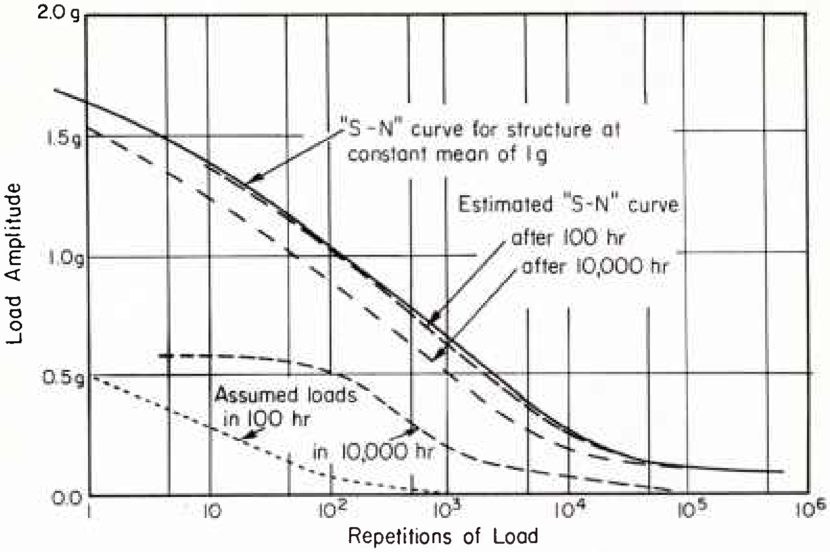

FIGURE 132.—Schematic representation of cumulative fatigue damage (see text).

usually obtained by calculations based on tests of material coupons and of small elements, may be considered as a curve for most probable lifetimes. Now, one may use a cumulative damage rule⁽⁵⁾ to estimate the damage and the remaining fatigue structural resistance after 100 hours and, assuming the load distribution not changed, after 10,000 hours. At some subsequent number of hours the two curves will intersect and the wing presumably will fail.

This sort of calculation provides, within the uncertainties of the assumptions, an estimate of the probable service lifetime. It also suggests (by consideration of the load level of the intersection) an idea as to whether the wing is critical for the low load levels or for higher (but less frequently occurring) loads. But it does not provide an idea of reliability in the sense of the possibility of a wing on a particular aircraft having lower fatigue strength than the "most probable" value and encountering loads of more severity than average, thus failing sooner than others.

There must be added to such an analysis some allowance for the several sources of uncertainty in regard to reliability. There have been two rather different approaches to this requirement.

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(5) For this figure, the Miner rule with reference to the original *S–N* curve was used. Uncertainties in this procedure are noted in ch. VI and an alternative method is described in ch. XVIII.

Safe-Life Evaluations and Scatter Factors

One procedure is to evaluate the expected lifetime of a component (as a wing) by calculations similar in principle to that just illustrated or by a full-scale test under an appropriate load spectrum and to require a margin of safety between the calculated and/or test lifetime and the planned service lifetime. This margin may be represented by a "scatter factor" requirement that the estimated lifetime must be some multiple (the scatter factor) times the designed service lifetime. A minimum value of scatter factor may be dictated by the procuring agency.(6)

Illustrations of somewhat more refined ways of characterizing the load occurrences, of making fatigue damage calculations, and of estimating lifetime by full-scale tests are suggested in the following chapters. These do not override the dependence of reliability in the safe-life approach upon (1) conservatively severe assumptions of load distribution, (2) conservative fatigue allowables, (3) product control to minimize variations among supposedly identical structures, and (4) use of a scatter factor margin. For the near future, it is likely that such concerns of engineering judgment will remain imperative.

The Fail-Safe Approach

An approach, until recently considered an alternative, is to admit the possibility of fatigue cracking, but to prevent such occurrences from being catastrophic by suitably rigorous inspection together with design such that small cracks permit safe flying between inspection periods.

Figure 133 shows schematically some of the factors involved in fail-safe design. There are several items that might be considered in terms of probability distribution:

1. *The detectable length.*—This must be expected to vary with accessibility for inspection, with inspection techniques and equipment available, and with frequency of inspection.
2. *Rate of crack growth.*—This varies with material, with design (which may include "crack stoppers"), and with fabrication as well as with variation in loads encountered.
3. *Critical crack length.*—Both the residual strength of the cracked structure and the loads likely to be met after cracking are variable,

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(6) For example, the following quotation is from MIL–A–8866 (ASG): "These tests and analyses shall include consideration of the probable scatter of fatigue lives of identically manufactured structures. For Navy procured airplanes, the scatter factor used in lieu of specifically applicable data shall be at least 2.0."

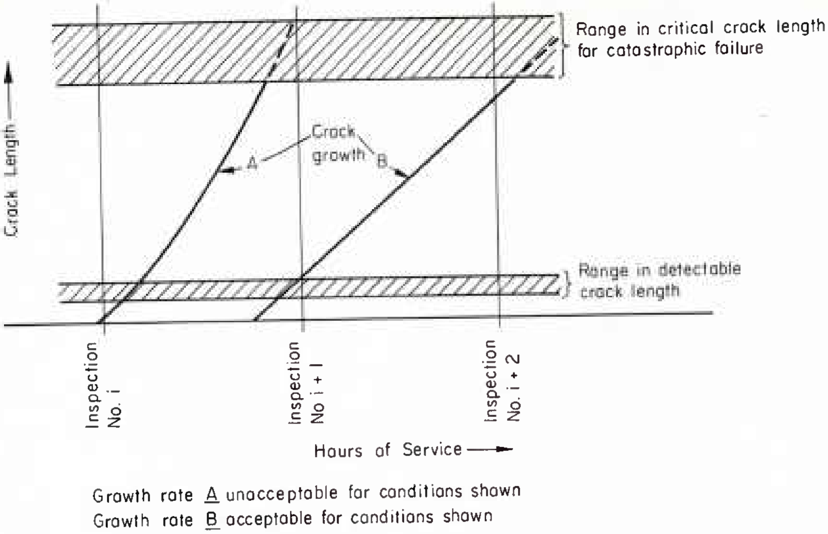

FIGURE 133.—Schematic representation of some factors in fail-safe design.

so that the values of critical crack length must be considered to have scatter.

While approaches taking cognizance of these probability aspects are being developed, current practices are empirical. Emphasis is directed toward demonstrating that a structure with a large defect (a long artificial crack, or the complete severance of one load path in a component of intentional redundancy) can withstand the highest probable load (or the most severe likely load spectrum) until subsequent inspection and repair.

It should be noted that these two approaches toward reliability against fatigue failure are not exact alternatives. The uncertainty in safe-life estimations, with all reasonable provisions of testing and use of scatter factors, is quite enough to warrant, for any structure designed on a safe-life basis, addition of any feasible provisions for fail-safe characteristics. On the other hand, fail-safe designs require sufficient safe-life characteristics to make inspection intervals and replacements dictated by inspection economically and logistically acceptable. Moreover, fail-safe design requires inspection accessibility and full reliance upon inspection and maintenance. Accordingly, current engineering practice involves a mixture of the two approaches with one or the other emphasized according to the type of structure, the weight requirements, the testing and maintenance plans, the procurement specifications, and the overall economics and logistics involved.

ENGINEERING APPROACHES

It has been indicated that rigorous analysis of reliability against fatigue by methods of probability statistics are not yet developed to a degree of general applicability. One limitation not likely to be overcome quickly is lack of sufficient data for direct application of classical statistics. Current practice, including factors of judgment and attempts to utilize all background available for any specific situation, may be improved by use of decision statistics.

In practice, there are many factors which influence detailed procedures in striving for fatigue reliability of an airframe structure. One example has been mentioned: decision as to the emphasis to be put on the fail-safe viewpoint versus the safe-life viewpoint is influenced by such items as inspection accessibility and dependability, weight considerations of alternative configurations, and testing requirements for reliability. The procedure adopted for fatigue evaluation of a structural component is influenced by the nature and extent of available pertinent information. For example, in the stage of preliminary design, fatigue data are not usually available for all components and evaluation has to be based upon extrapolation from data on material coupons or upon related elements. Even after completion of a prototype, there are uncertainties in details of loads (to be resolved by flight tests) and of structural resistance (to be clarified by a full-scale fatigue test).

Whatever the viewpoint and the details of procedure chosen for fatigue reliability, these may be considered in terms of (1) analysis of loads and (2) analysis of resistance to fatigue failure under these loads. Discussion of obtaining and representing information about loads is in the next chapter. Evaluation of structural response to loads may be either by analytical computation or by full-scale testing, and these two topics are discussed in chapters XVIII and XIX, respectively. Modern engineering uses both to get a final evaluation.

Finally, protection against fatigue cannot end with the delivery of vehicles whose reliability has been evaluated by analysis and by one full-scale test. Subsequent responsibility includes further recording of service loads and environments, monitoring and reevaluation in circumstances of mission alterations, inspection (the main safeguard in fail-safe designs), and maintenance and alterations throughout service life. These concerns are emphasized in chapter XX.

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CHAPTER XVII.
LOADS AND ENVIRONMENT

INTRODUCTION

The loads on an airframe structure vary both with the type of aircraft and with the nature of its service usage. The objective of this chapter is to illustrate some considerations in the characterization of these loads in a manner suitable for analysis of expected fatigue lifetime.

Information concerning loads comes from diverse sources and may vary in nature and extent at different stages in development and production. In early stages of design and development, data from past experience on vehicles of similar type, together with specifications of expected flight characteristics and of planned missions, provide the main source of estimation. Flight tests of a prototype later afford confirmation or alteration of the estimated values. Still later, service records or changes in mission assignment may require reappraisal of the load spectra.

Loads from some sources—such as atmospheric turbulence or taxiing over varied terrain—must be estimated on a basis of statistical probability. While the estimated values can be adjusted for anticipated severe service, extreme values that might be encountered by a particular flight vehicle at a specified lifetime are scarcely predictable. In subsequent discussion, we consider the most probable load spectra.

Enumeration of expected loads for fatigue analysis must be not only complete, but also in terms relatable to available fatigue information. Some aspects of this requirement are indicated subsequently by a particular example—loads on a wing spar of a transport airplane. This example is chosen because: (1) much attention has been devoted to analysis of fatigue behavior in wings and many illustrative data are available, and (2) the example includes many items particularly important in description of loads for fatigue analysis. It should not be inferred from this example that other airframe structures may not be critical under service loadings.

Methods of handling environmental factors such as temperature and corrosive surroundings in fatigue analysis have not been developed in widely accepted quantitative formulations. Accordingly, these are discussed briefly in the final section of this chapter, largely to emphasize that the aircraft engineer should be alert to their current and growing importance.

VARIETY OF LOADS

In view of the cumulative-damage effect in fatigue, enumeration of all significantly high loads and of the probable number of repetitions of each is necessary for fatigue analysis.

Table 20 (adapted from reference *1*) lists a number of sources of repeated loads on a fixed-wing aircraft. Some, such as the 1-g lift involved in the ground-air-ground cycle (G–A–G) can be considered to occur once per flight; others, such as vertical accelerations from small gusts may occur very many times per flight. These must all be enumerated in some manner. Note that other sources may be important for some aircraft. For carrier-based aircraft, particular consideration of all launching- and arresting-load effects is necessary. For helicopters, several ground conditions, hovering conditions, conditions of powered level flight and autorotation conditions are to be considered (see, for example, reference *2*). In some structural parts of aircraft, thermal stresses require considerations.

For any type of aircraft, loads vary with mission assignment. Atmospheric turbulence differs in different locales and, over the same route, at different seasons. Taxiing loads vary with terrain. Maneuver loads

TABLE 20.—*Some repeated loads on aircraft structures*

One (a) per flight	Many per flight
1-g Wing lift.	Wing loads from gusts.
Tail balance loads.	Wing loads from maneuvers.
Flap loads.	Fin loads from gusts and maneuvers.
Cabin pressure loads.	Fuselage loads from gusts and maneuvers.
Landing gear impact.	Landing-gear taxiing loads.
Etc.	Propeller, slip-stream or jet-stream loads.
	Engine vibrations.
	Etc.

(a) In some instances this denotes one group of loads per flight.

vary with pilot as well as with mission schedules. Obtaining statistically significant data on loads actually encountered required much effort over a considerable period of time. A review of nearly 20 years of VG and VGH data collection by NASA (reference *3*) summarizes much information on transport airplanes of several types (2- and 4-engine piston, 2- and 4-engine turboprop, and 4-engine turbojet) in various airline service (feeder line, short haul, and long haul). Some of the trends noted are:

1. The frequency of gust accelerations per mile of flight decreased from feeder-line to short-haul to long-haul airplanes. This was attributed mainly to the beneficial effect of higher wing loading and less severe gust environment associated with the higher cruise altitudes for the long-haul airplanes.
2. The frequency of accelerations caused by operational maneuvers during passenger-carrying flights was relatively constant for the different types of transport and of airline service.
3. The landing-impact accelerations were generally higher for the turbine-powered that for the piston transports.
4. The accelerations due to landing impact and to check-flight maneuvers appeared to be significantly influenced by the airline.
5. The in-flight acceleration histories were "unexpectedly consistent when viewed on a flight rather than on a flight-mile basis." This was attributed to the fact that the majority of the repeated loads occurred in the climb and descent phases of flight rather than during cruise.

Some other programs of recording loads and evaluating the records to extend generalizable information about these are mentioned in chapter XX.

Extensive programs, such as mentioned above, have provided much information concerning aerodynamic effects on the c.g. acceleration. This information, in conjunction with analysis and with experience, allows assessment of gust and maneuver in-flight loads upon many parts of the airframe of a fixed-wing aircraft. Some loads are interrelated with the response of the structure to such a degree that they require experimental evaluation for each type of vehicle. Examples include landing gear loads, loads on rotor-blades of helicopters, and engine vibration loads. These may have to be measured under conditions (not always easy to forsee) that include all loads contributing to significant fatigue damage, *and* their probable frequency of occurrence must be estimated. This latter requirement is more important for fatigue than for static strength analysis.

REPRESENTATION OF LOADS FOR FATIGUE ANALYSIS

The enumeration of loads in a manner suitable for either analytical analysis of expected fatigue behavior or for full scale testing contains a number of interesting problems. Many of these can be illustrated by a specific example: the loads on a wing-spar of a transport airplane.

Qualitative Enumeration of Loads

Figure 134 shows loads, in terms of nominal stress on a wing spar, that might be incurred in one flight. The stress varies in compression during taxiing and take-off, rises to a tension stress as the vehicle is airborne, varies about 1-*g* level (slowly diminishing as fuel weight decreases) due to gusts and maneuvers, has an impact stress fluctuation on landing and more ground induced stress fluctuations about a mean value in compression (changing as the vehicle is refueled).

For the next flight of the same airplane, the values would certainly be different in detail. Differences might be extensive due to such items as: a different weight of fuel and cargo, a different flight pattern and correspondingly different maneuvers, more (or less) turbulent air and a rougher (or smoother) runway. Thus, even for a specific structural

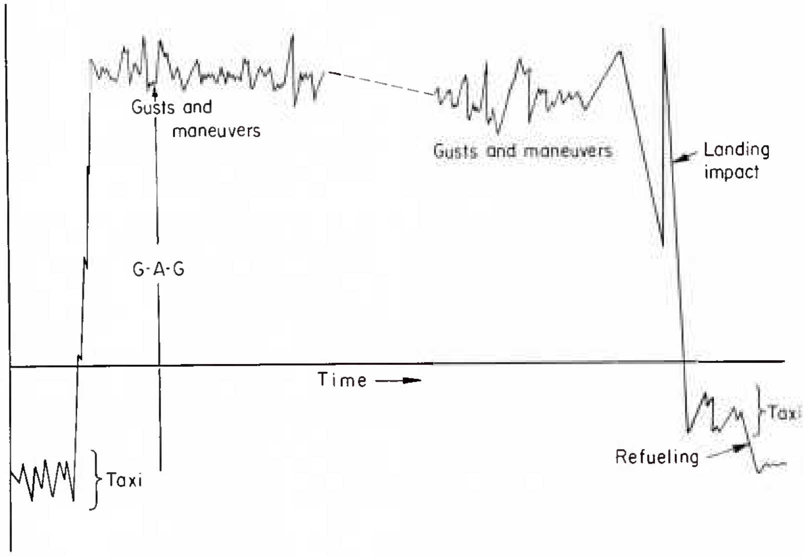


FIGURE 134.—Qualitative representation of loads in an airframe structure in one flight.

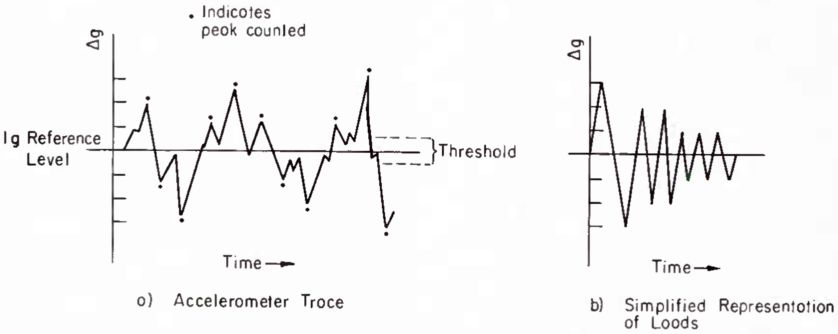
part of a specific vehicle, some flight loads can be specified only on a probability basis from data of past experience.

To indicate several details in representation of loads, it will be convenient to consider separately some of the loadings: gusts and maneuvers, *G–A–G*, landing impacts, and taxiing loads.

Gusts and Maneuvers

Much of the existing data on gusts and maneuvers come from V–G counts and V–G–H records (see ch. XX). It is common practice to organize these in terms of numbers of peaks about a reference (1-g) level. This raises questions as to the definition of a peak. One may count the peaks after each crossing of the 1-g (zero Δg) level, or the peaks after some selected threshold change, or according to some other definition (see, for some discussion, reference *4*). Another decision is now required. Negative peaks do not always follow positive peaks although they do seem to occur about equally often. Can the positive and the negative peaks be grouped together to count as cycles of load about a mean value? Figure 135 illustrates the counting by a threshold definition and by considering that positive peaks and negative peaks can be grouped so that the loading in figure 135b is representative of that in figure 135a. Note that a third assumption is involved here: That the order of high load cycles and of low load cycles can be neglected so that these may be rearranged for convenience. This point will be discussed further in the next chapter.

Figure 136 shows information (adapted from reference *3*) that we will assume appropriate to the wing-spar example. The ordinate is in terms of cumulative frequency of occurrence—the number, per flight-mile, of acceleration increments equal to or exceeding the Δg incre-



a) Accelerometer Trace

b) Simplified Representation of Loads

FIGURE 135.—One method of peak count for gusts and maneuvers (see text).

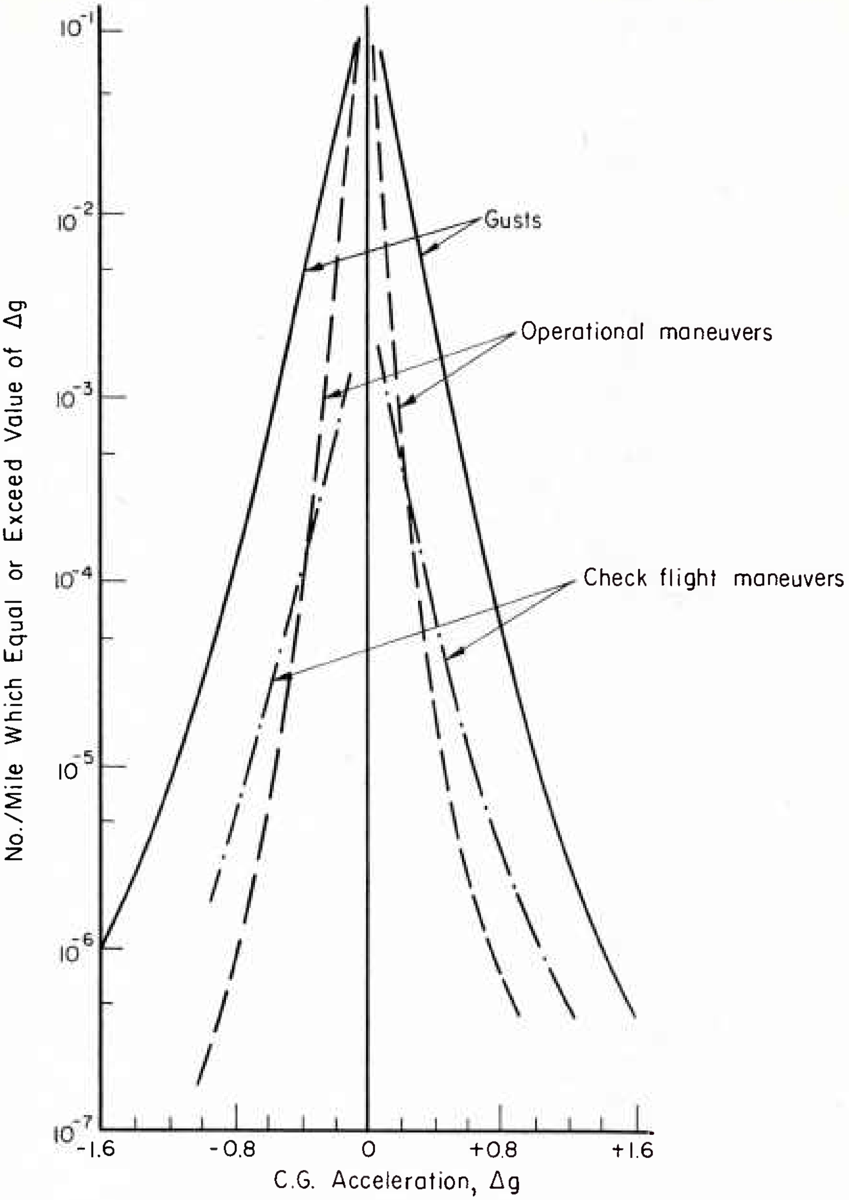


Figure 136.—Assumed occurrences of gust and maneuver loads for illustrative example.

ment values used as abscissae.(1) This representation of data from very many observations is convenient for statistical representation but less convenient for relation to laboratory fatigue test data or for setting up plans for full scale fatigue testing. For these purposes, it is common practice to select a finite number of increments of Δg and to assign frequencies of occurrence to each to match the continuous curve (see histogram in appendix B). Table 21 shows values conforming to figure 136 and additional assumptions that: the 1-g level corresponds to a constant mean stress of 14.0 ksi (neglecting, for simplicity, effects such as decreasing fuel weight); the airplane has a constant air-speed of 280 mi hr. (and, on the average, this corresponds to ground speed); and the average flight time is 2 hours.

The various ways of listing the number of occurrences (per mile, per hour, per flight, etc.) have particular implications when loads from other sources are combined to provide a complete spectrum of loads. In setting up a spectrum for full-scale testing, there are questions about the fractional cycles of loading for short periods (in this example, less than 1,000 hours).

TABLE 21.—*Assumed gust and maneuver loads for illustrative computation*

Δ g	S_a,(a) ksi	Number of loads per:			
		Mile (b)	Hour (c)	Flight (d)	1,000 hours
0.2	2.8	8.8 × 10	14.0	28.0	14,000
.4	5.6	1.1 × 10	3.1	6.2	3,100
.6	8.4	1.3 × 10	.36	.72	360
.8	11.2	2.4 × 10	.067	.134	67
1.0	14.0	5.8 × 10	.016	.032	16
1.2	16.8	1.0 × 10	.003	.006	3
1.4	19.6	4.1 × 10	.001	.002	1

(a) Assuming stress increments proportional to Δ g and that the 1-g stress is 14.0 ksi.
(b) From assuming the numbers for gusts and maneuvers (fig. 134) and developing a histogram to approximate the resulting cumulative frequency curve.
(c) Assuming a constant air-speed of 280 mi/hr.
(d) Assuming an average flight time of 2 hours.

G–A–G Loading

Once per flight, the load on the spar goes from a negative value due to the weight of the wings on the ground to a positive value when the wings support the airplane. If we assume that the nominal stress

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(1) Note the symmetry about Δg = 0 — the indication that positive peaks occur about equally often.

is known (—5.4 ksi) on the ground and (+14.0 ksi) in level flight at constant velocity, we may represent this as one cycle per flight of $S_a = 9.7$ ksi and $S_m = 4.3$ ksi. Other possibilities exist; for example, the *G–A–G* may be considered from the lowest likely negative stress to the highest likely positive stress. The best representation is not certain, and to continue discussion we will use the simple mean ground stress to mean flight stress.

Note that we now have, to combine with the loadings from gusts and maneuvers, one loading that adds two complicating features. First, there is a fixed sort of occurrence—once per flight. Second, the G–A–G is considered a cyclic loading about a mean stress sufficiently different from the 1-g level that the effect of mean stress upon fatigue lifetime must be considered.

Landing Impacts

Repeated landing impact loads may, in some instances, contribute significant fatigue damage. Characterization of these may be difficult because: (1) there are relatively few data for statistical representation like that for gust loadings; (2) there are significant variations with terrain, type of airplane and pilot operation; and (3) the translation of values such as c.g. acceleration increments to nominal stresses may be strongly dependent on structural configuration.

Some illustrative values of c.g. acceleration increments are given for transport aircraft in reference *3*. To provide values to make our example more nearly complete, these have been used with the over-simplified assumption that they are proportional to variations of nominal stress about a 1-g value of 14.0 ksi. Resulting stress values are shown in table 22.

TABLE 22.—*Assumed loads from landing impact and from taxiing for illustrative example*

Load	Δn_z,g's	S_m, ksi (a)	S_a, ksi (b)	No./Flight
Landing Impact	.2	14.0	2.8	0.600
	.4	14.0	5.6	.120
	.6	14.0	8.4	.018
	.8	14.0	11.2	.004
Taxi	.2	−5.4	2.8	8.000
	.4	−5.4	5.6	.800
	.6	−5.4	8.4	.080
	.8	−5.4	11.2	.008

(a) These values based on oversimplified assumptions.

(b) Assumption of linear relation between S_a and Δn_z made primarily for simplicity.

Taxiing Loads

Extensive data samplings for accelerations during ground operations are not available and stresses on a component of the wing structure are not easily related to such accelerations except on the basis of careful stress analysis for a particular airplane configuration. Reference *3* indicates appreciable c.g. acceleration increments for landing roll-out and for takeoff rolls. Values from the illustrative data in that reference with the additional assumption that these correspond to nominal stress amplitudes about a mean stress of –5.4 ksi are included in table 22.

Total Load Spectrum

In the example, there have been several simplifications and some omissions; these include:

1. Stated simplifications in regard to landing loads and ground operation loads.
2. Omission of oscillatory in-flight loads (see reference *3*), of high-frequency, low-amplitude components of the landing impact loads, and of "secondary" loads from sources that are not reflected in c.g. vertical accelerations.
3. Omission of possible nonlinear responses and of possible structural resonant modes.

With these limitations in mind, the information in tables 21 and 22 provides data for a total spectrum of anticipated loads.

There are, however, some decisions yet to be made in using the data for either analysis of expected fatigue lifetime or for planning a full-scale fatigue test. For either use, some consideration of the statistical implications of any spectrum designed from these data should be kept in mind. Some of the data are based upon statistical analysis of the probable frequency of occurrence of loads—from gusts, from operational maneuvers, from check-flight maneuvers, from landing impacts, and from ground operations. The relevance of these probabilities of occurrence to a particular airplane may be uncertain and requires some allowance such as use of a conservatively severe spectrum and a scatter factor. Using the selected spectrum for calculation of probable fatigue lifetime requires decisions about the treatment of the order of occurrence of some of the infrequent high loads; this point is discussed further in the next chapter. Using the spectrum for full-scale testing also requires consideration of the order of application of testing loads; these loads must be arranged with allowance for such limitations as time, cost and available test facilities.

Digital computers offer much help in setting up representations of load spectra; an example is described in reference *6*. This possibility does not overcome such questions as the most suitable representation of the G–A–G, or the best manner of allowing for order of occurrence, or decisions about the pertinence of probable loads to a specific situation.

Government agencies, both civil and military, recognize the problems involved in representation of a load spectrum for fatigue analysis. In some instances (see tables in reference 5), specific spectra may be listed for specific types of aircraft with values adjusted as information accumulates from flight and service records. In general, the total load spectrum for a specific airframe structure of a specific type of aircraft with a specific intended service is a matter for the best judgment and mutual agreement of a number of parties concerned (manufacturer, purchaser, licensing agent, and operator).

ENVIRONMENT

Neither the loads (nominal stresses) nor their effects in producing fatigue damage are independent of some environmental factors; particularly important are temperature and corrosion.

Figure 137 shows a load schedule analogous to that in figure 134 but with indications of temperature regimes such as might be involved at the leading edge of a supersonic transport. We must consider the accumulation of damage due to:

1. Fatigue at low temperatures in ground operations and in some portions of climb and descent.
2. Fatigue from air loads at elevated temperature.

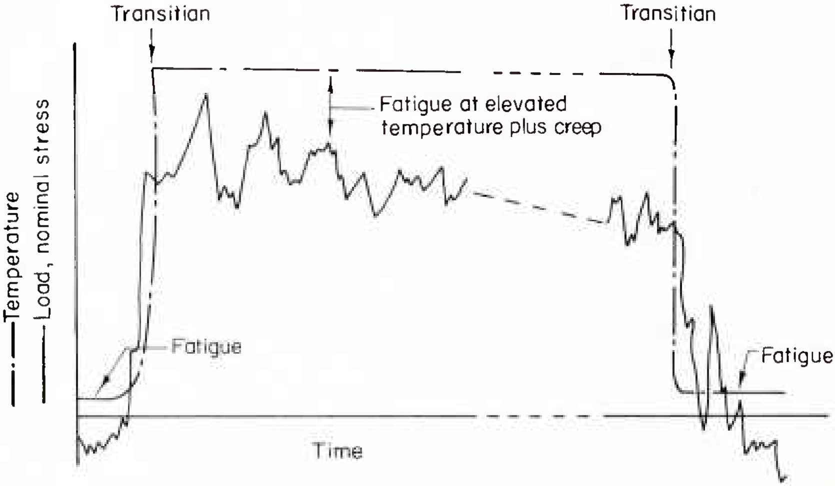

FIGURE 137.—Combination of load and temperature effects.

3. Possible creep at high loads at elevated temperature.
4. Transition loads which include changing temperature and thermal stresses.

A linear interaction formula has been proposed for combining such different contributions to damage (reference 7), and several elaborate computer programs have been developed to handle the involved computations. However, there is not yet a wholly satisfactory theory for allowance for such factors as creep-fatigue interaction. Present practice involves much testing, both of components and of full-scale structures, under patterns of temperature and load designed to simulate service conditions.

Corrosive environment may be a significant factor during service lifetime—especially for aircraft on a carrier or used extensively near a seacoast. Here, too, there is no satisfactory theory for allowing for interaction of corrosion damage and fatigue damage. Present practice includes choice of materials with respect to surroundings, surface protection of materials, and careful inspection.

Both creep and corrosion are time-dependent factors, whereas fatigue is usually considered cycle-dependent and (nearly) independent of frequency. This means that, in component testing and in full-scale testing, there may be an important distinction between (accelerated) test time and real (service) time.

Other factors that may be important relative to fatigue behavior of special parts or components include wear, fretting, and, for space vehicle, such items as ablation, radiation, and meteorite bombardment. The interrelation of most of these with fatigue is not yet well understood.

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CHAPTER XVIII. STRUCTURAL RESPONSE: DESIGN AND ANALYSIS

Detailed procedures in design and analysis of an airframe structural component to withstand assumed loading conditions vary greatly with the background information and experience available. For a component only slightly different from one with a long record of successful performance, the emphasis may be upon analysis of the effects of the differences. A novel design may require detailed study of every conceivable weakness. Since it is not possible to describe every contingency, the following discussion illustrates some important points by (1) enumeration of some situations which have caused fatigue problems in the past, and (2) description of some approaches which have been used to circumvent such problems.

In practice, design and analysis must go hand-in-hand. However, for convenience of discussion, we first enumerate some concerns in design with the assumption that the background of previous chapters makes these examples understandable. Subsequently, some specific approaches to analytical estimation of the fatigue lifetime of a structure are described.

CONCERNS IN DESIGN TO PREVENT FATIGUE FAILURE

The keynote to design to prevent fatigue failure is attention to local stresses and strains. While this has been recognized for a long time, various factors have been overlooked often in the past. Several such items are noted in the following brief discussion, under the convenient subheadings of materials, geometry, and fabrication and assembly.

Materials

Every airframe manufacturer has a file of data on the fatigue properties of materials. These may be tabulated in various ways or may be in the form of *S–N* curves (such as fig. 143) or constant-lifetime diagrams (such as those in app. C). Nevertheless the designer usually faces two difficulties:

1. finding data directly pertinent to his needs in a specific problem;
2. assessing the reliability of the most pertinent data he can find.

To surmount the first need, it is customary to use interpolation and extrapolation of available information and, frequently, additional material testing. The second difficulty may be overlooked, although it is important.

Pertinence of material fatigue test data implies several considerations in addition to the obvious one of the type and level of stresses. Material coupons should be from a lot representative of production conditions, in a suitable heat-treatment or state of cold-work, and with a surface condition appropriate to the component planned. Such precautions have been overlooked in the past. In one instance, a (then) new alloy was tested in fatigue in comparison with material for which there was a background of satisfactory service experience. The plan was to use the new material in a particular heat treatment and, in some instances, with welded joints. Fatigue tests on conventional unnotched specimens, without particular attention to heat treatment to simulate that likely on production parts, indicated satisfactory fatigue performance. Actual parts gave much trouble in full-scale fatigue tests and required costly changes. A very significant part of this trouble could have been forestalled by initial attention to the pertinence of the material fatigue tests.

Particularly in fail-safe aspects of design, fracture toughness and crack propagation rate are useful characteristics of a material. Recognition of these properties and attempts to discover their values are increasingly important in material selection.

Most material fatigue test data involve too few specimens to provide a statistical measure of scatter in reported lifetimes. Reference *1* gives recommendations for statistical planning of fatigue tests; some of the recommendations have not been followed in previous work, and many are difficult to utilize in the usual pressure of time and funds available. Reference *2* gives some collected information on standard deviations reported for various materials. The design engineer should, despite the difficulties, attempt to assess the reliability as well as the pertinence of fatigue data he may use to characterize materials.

Geometry

Geometrical stress concentrations have received much attention. Some of the background for estimating geometrical stress-concentration factors has been discussed in a previous chapter.

However, there have been instances of overlooking some of the many details which can produce high stress concentrations. Figure 92 shows some stress raisers overlooked in design and inspection. The photographs and references in chapter I provide other examples. Sometimes so much attention is focused upon one stress detail that others are neglected. Figure 138 shows a failure in a fitting designed as a shear-carrying joint with neglect of the need for distributing bending loads. A simple change that provided allowance for the "secondary" bending loads eliminated further serious fatigue problems with this fitting.

A particular challenge in design is that of providing necessary joints. Some of the background information concerning fatigue properties of joints has been described in chapters XII through XIV. General design procedures worthy of continual emphasis include:

1. Use simple rivet and bolt patterns since elaborate ones good for static strength may not be equally good for fatigue resistance.
2. Distinguish between tension fastenings and shear fastenings but do not overlook the fact that a joint designed for tension may need to withstand some shear, and one designed for shear may need some resistance to cyclic tension.
3. Consider the importance of proper tolerance between bolt and hole.

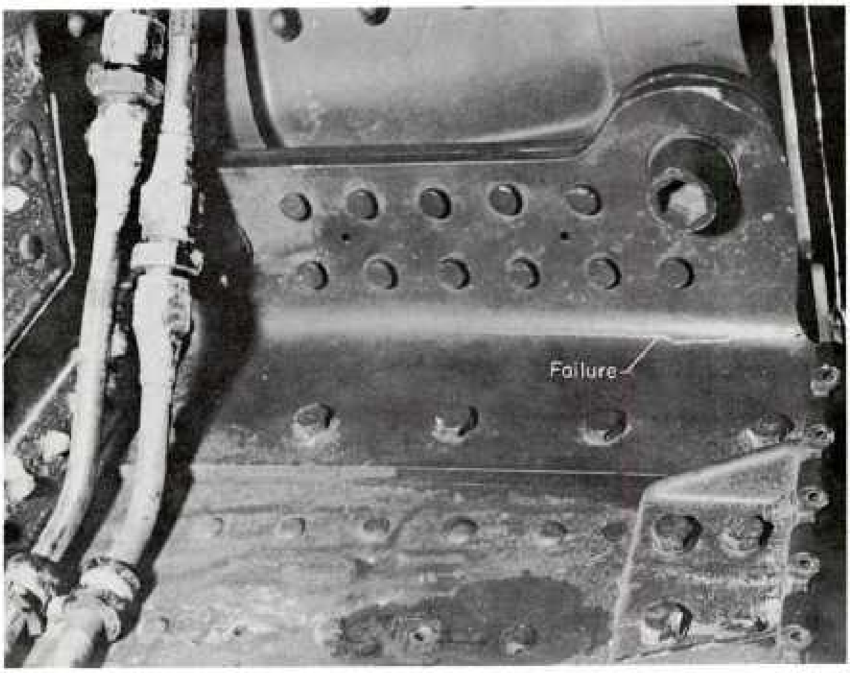


Figure 138.—Failure in shear joint from bending loads.

4. Specify suitable clamping forces.
5. Avoid rough weld beads in critical spots.

Every one of these has been neglected in one program or another with resultant fatigue problems.

Especially when the geometry of a part is such as to have spots of high stress concentration, additional factors may combine to reduce the fatigue strength below tolerable limits. Some factors that have given trouble in the past are:

1. Metallurgical defects: inclusions, large grain size, flow lines poorly oriented with respect to local stress.
2. Surface finish, including residual stress.
3. Secondary stress patterns.
4. Details of joint design.

Many such factors have been detected in component testing before they contributed to catastrophic failure, *but* the required changes have been costly.

Fabrication and Assembly

Design to prevent fatigue failure includes consideration of factors in fabrication and assembly. Numerous examples of fatigue failures attributable wholly or in part to improper fabrication and assembly have been reported. Figure 96 shows poorly fabricated bolts; figure 1 (in ch. I) shows a wingfold fitting in which the mechanic used an oversize milling cutter and "gouged" the fillets. References *3* and *4* contain other illustrations. Figure 139 shows a Y-splice fitting for which the drawing specified a reasonable radius, but the skin was fabricated with a sharp radius to provide a "good" close fit around the Y.

A particularly insidious possibility is a combination of improper design specifications, imperfect fabrication and assembly, and inspection not fully cognizant of fatigue hazards. Figures 140 and 141 show fatigue cracks in a main wingfold fitting. Analysis of the failure in figure 140 indicated that: (1) detail design specified undersized fillets, poorly faired compound fillets, sharp corners, too rough a surface finish, and a cyanide-bath cadmium plating that resulted in hydrogen embrittlement; (2) production allowed decarburization in heat treatment, improper grain orientation and a surface even rougher than specified; (3) inspection let the unfortunate combination get by. The analysis was supported by finding that parts reworked to remove these undesirable features had consistently significantly higher fatigue lifetimes. Figure 141 shows an alternate mode of failure of the same fitting.

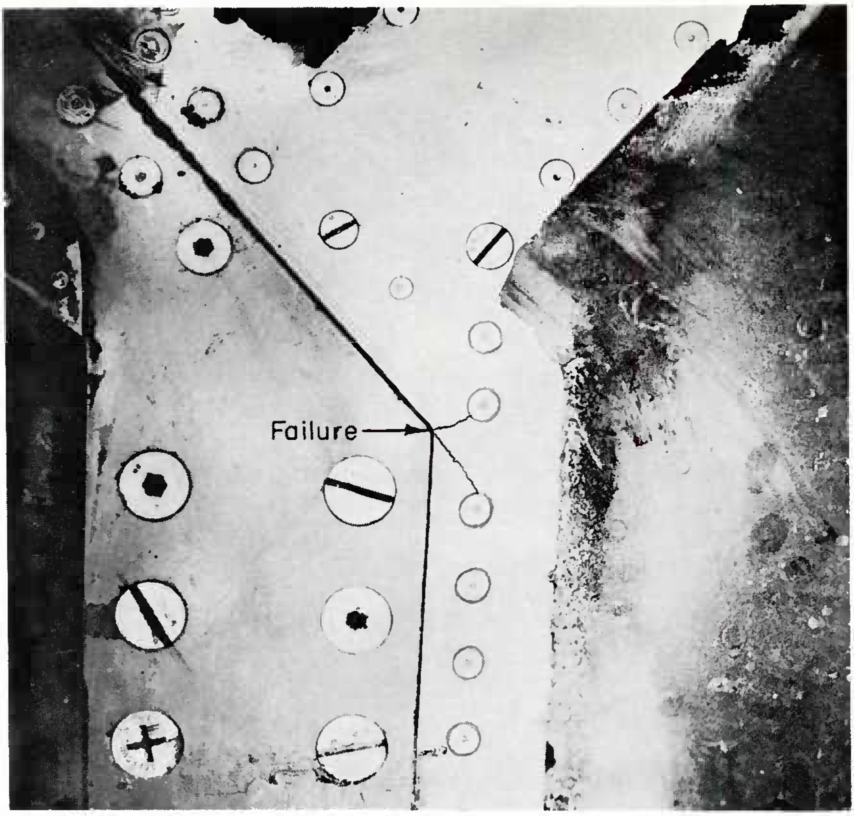


FIGURE 139.—Fatigue failure in Y-splice fitting.

ANALYTICAL EVALUATION OF FATIGUE LIFETIME

Procedures in analytical estimation of the fatigue life expectancy of a structural component vary with the character of the expected loading, with the available fatigue data, and with other background experience that may be available. To indicate some approaches that may be used, it is convenient to consider specific examples.

Constant Amplitude Loading

One important concern in any structural component is the effect of the most critical stress raiser in the component. In practice, it is often necessary to account for such factors as residual fabrication stresses and fretting by test of the component. In principle, some of the relations described in chapter IV can be used to estimate an *S–N* curve for a specimen with a geometrical notch from dimensions of the notch and

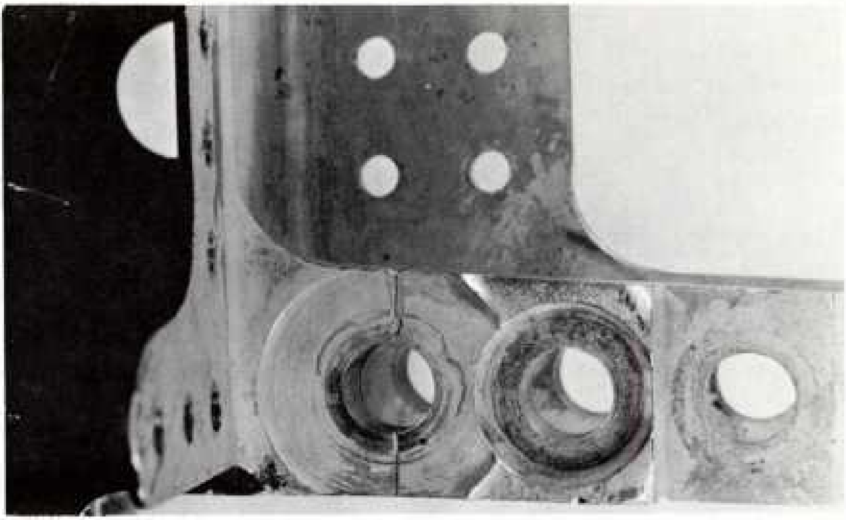
Figure 140.—Fatigue failure in wing-fold fitting—gouged fillet.

data on the material. Moreover, some of the procedures involved in such an estimation can be used for extrapolation and interpolation of data from tests of components that may involve complex stress raisers.

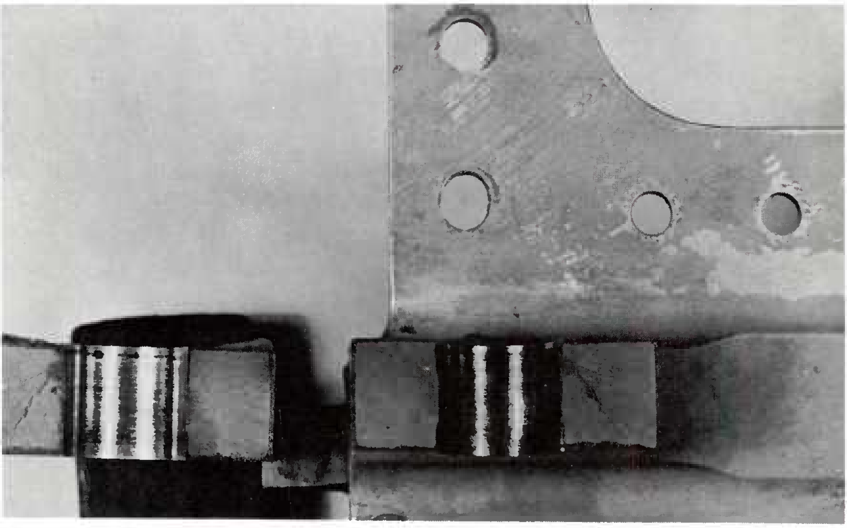
Consider a notched specimen under axial loading. Suppose the stress-strain and the fatigue behavior of unnotched specimens of the material are known. Suppose the notched specimen is cycled through a nominal stress $S_{n,min}$ to $S_{n,max}$. If the peak local stress does not significantly exceed the elastic limit of the material, values of the local stress will vary

$$\text{from} \quad S_{min} = K_{el}\, S_{n,min}$$
$$\text{to} \quad S_{max} = K_{el}\, S_{n,max}$$

where K_{el} denotes the elastic stress-concentration factor. A value for K_{el} may be established by fatigue tests (at low stress levels and long lifetimes) of unnotched and of suitably notched specimens. If such



Top: Crack through hole and symptoms of fretting.



Bottom: Origins of crack.

FIGURE 141.—Fatigue failure in fitting-to-spar attachment combination.

data are not available, K_{el} may be estimated by Neuber's relation

$$K_{el}=1+\frac{K_t-1}{1+\sqrt{A/\rho}} \qquad (1)$$

where K_t, the theoretical stress concentration factor, is known and the correction for notch-size effect, A/ρ, is made from available information about the material constant, A, and the notch radius, ρ.

For a higher value of $S_{n,max}$, the peak stress will exceed the yield strength of the material. The value may be estimated, as suggested in chapter IV, by

$$S_{max} = 1 + (K_{el} - 1)\frac{E_s}{E} S_{n,\,max}\,. \tag{2}$$

For a number of practical situations, the value of $S_{max.}$ may be approximated by simply assigning, as an upper limit, the tensile yield strength, F_{ty}.

Now suppose that the nominal stress is lowered and that the region of stress concentration unloads elastically.⁽¹⁾ Then the minimum local stress may be evaluated by

$$S_{min} = S_{max} - K_{el}\,(S_{n,max} - S_{n,min})\,. \tag{3}$$

However, if the value so estimated is lower than the compressive yield strength, another estimation is in order. This estimation may be made by allowance for yielding in the compressive region by a relation analagous to equation 2, or by the approximation that the local minimum stress cannot be less than the compressive yield strength, F_{cy}.⁽²⁾

A numerical example will illustrate some details. Assume a specimen containing a notch of stress-concentration factor $K_t = 4.00$, and of radius $\rho = 0.057$ in. Let the material have the stress-strain curve shown in figure 142, the fatigue behavior shown in figure 143, and the Neuber constant $\rho' = 0.0144$ in.⁽³⁾ Let us estimate the fatigue lifetimes of such specimens under constant-amplitude loading at a nominal mean stress of 10 ksi. Table 23 shows the details of the computation. Figure 144 shows the S–N curve (the solid line) corresponding to the values in table 23 and (the dotted line) the approximation obtained by limiting the maximum and the minimum values of local stress by F_{ty} and F_{cy} respectively. It is clear that this approximation becomes questionable for high stresses and short lifetimes (as might be expected).

It is instructive to compute the behavior that might be expected for similar specimens, each of which was preloaded to a nominal stress of 30 ksi before testing. The preload would be expected to give a local maximum stress of 73.5 ksi and, after removal of the preload, the local residual stress would be

$$\begin{aligned} S_R &= S_{max} - K_{el}\,S_{n,max} \\ &= 73.5 - 3 \times 30 = -16.5\text{ ksi.} \end{aligned}$$

---

(1) This is, as Smith (reference *5*) notes, an approximation; data reported by Hardrath and Crewes (reference *6*) imply it may often be a close approximation for some instances.

(2) Note that the Bauschinger effect may alter F_{cy}.

(3) These properties are appropriate to aluminum alloy 7075–T6, but should not be considered well established for general design use for this material.

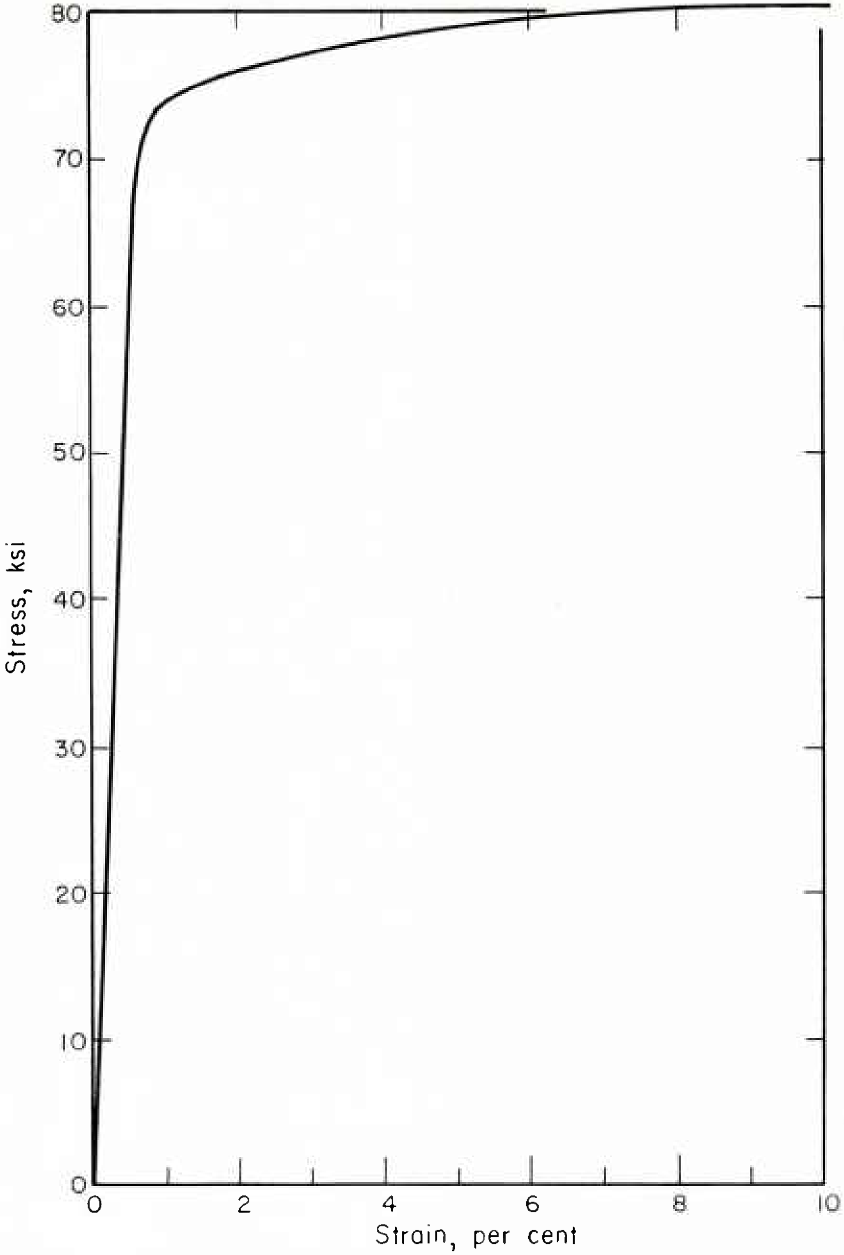


FIGURE 142.—Stress-strain curve assumed for illustrative numerical computation.

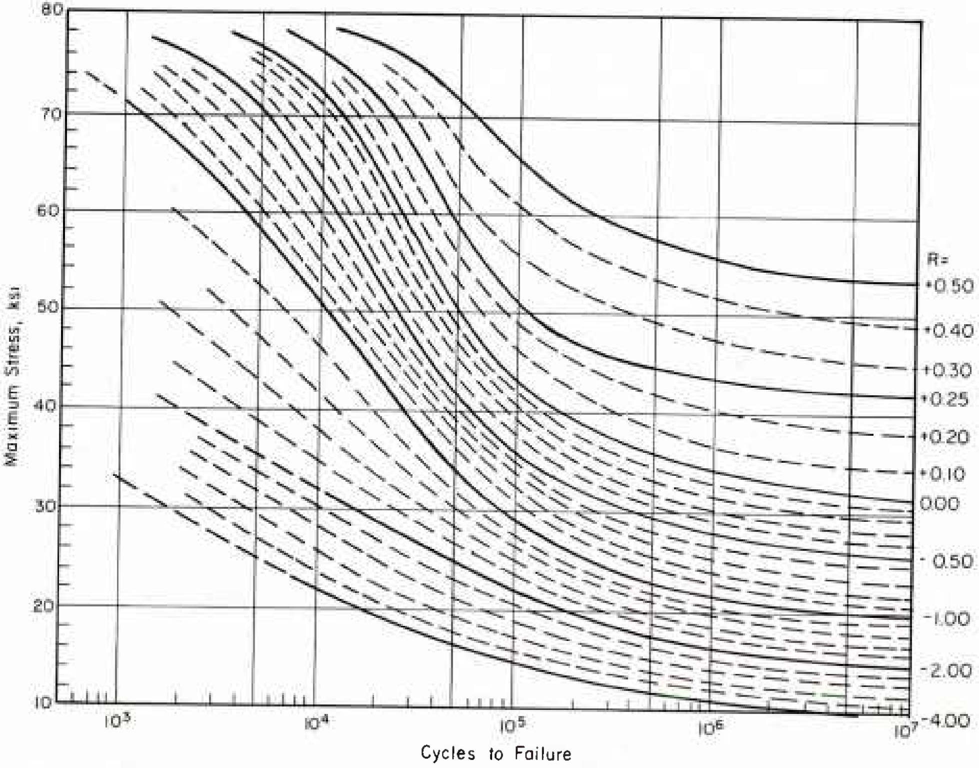


Figure 143.—*S–N* curves assumed for unnotched specimens.

Note: These are taken from Reference 5. They are used here for illustrative calculations, but are not necessarily suggested as representative of the material involved.

TABLE 23.—*Calculated values for constant-amplitude fatigue lifetimes of a particular specimen*

Assume: 1. $\left.\begin{array}{l} K_t = 4.00 \\ \rho = 0.057 \text{ in.} \\ A = 0.0144 \text{ in.} \end{array}\right\} K_{el} = K_N = 1 + \dfrac{4-1}{1+\sqrt{\dfrac{0.0144}{0.057}}} = 3.00$

2. $S_{n,\,mean} = 10.0$ ksi

3. Material data in figs. 142 and 143

$S_{n,max}$, ksi	S_{max},[a] ksi	S_{min},[b] ksi	R (S_{min}/S_{max})	N [c]
15	45.0	15.0	+0.33	$>10^7$
16	48.0	12.0	+ .25	5×10^5
18	51.0	3.0	+ .06	6×10^4
20	60.0	0	0	2.5×10^4
22	66.0	−6.0	− .09	1.3×10^4
25	72.2	−17.8	− .25	7×10^3
30	73.5	−46.5	− .63	2.5×10^3
35	75.0	−72.2[d]	− .96	6×10^2

[a] From equation 2 and fig. 142.

[b] From equation 3.

[c] From fig. 143.

[d] From equation 3, applied to compressive stress with assumption of symmetry in stress-strain curve.

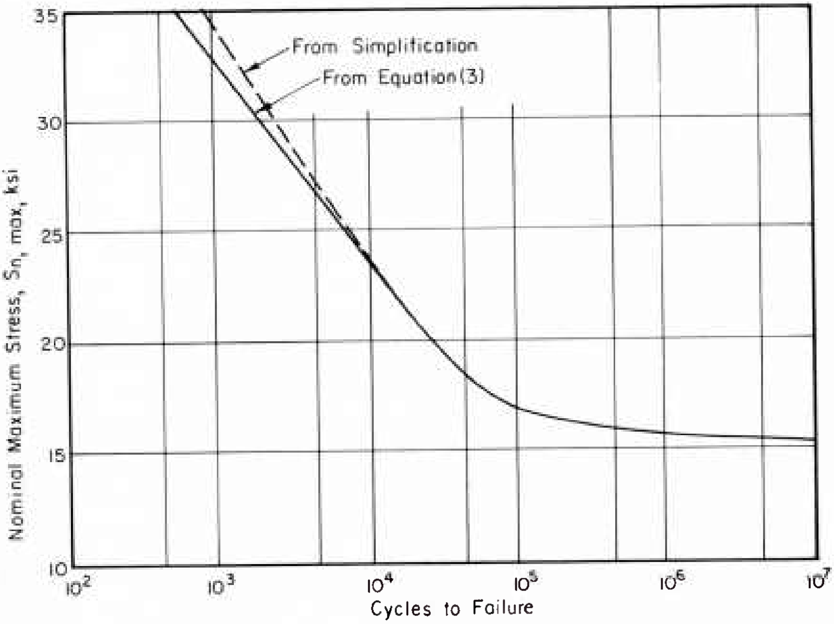


FIGURE 144.—Constant-amplitude *S–N* curve for notched specimens calculated by procedure described in text.

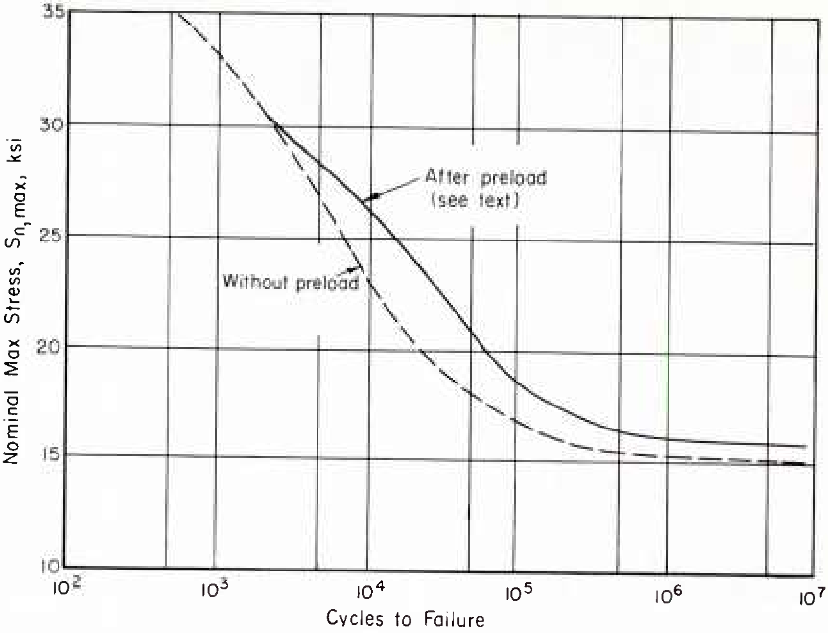


FIGURE 145.—Calculated constant-amplitude *S–N* curve for prestressed specimens.

We assume that all specimens start with this local residual stress and compute the values shown in table 24. For nominal maximum stresses up to the preload value, the local mean stress is lower and the lifetime longer than for virgin specimens. The shape of the *S–N* curve for the preloaded specimens is shown in figure 145. This effect of preload has been observed frequently.

The procedure just described affords a method of estimating the constant-amplitude-loading fatigue behavior of notched specimens from data on unnotched specimens of the material involved. It includes the possibility of accounting for effects of preloading and of residual stress (if known) and, as subsequently shown, is adaptable to variable-amplitude loading conditions. It is currently used by at least one aircraft company.(4) There are uncertainties in details: the use of K_N for notch-size correction, the use of E_s/E for plasticity correction, and the linear-elastic unloading assumption. However, the general approach seems compatible with increasing evidence that fatigue behavior of structures can be usefully interpreted in terms of local strains at points of concentration.

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(4) Grumman Aircraft, whose cooperation in supplying a very clear account of their fatigue evaluation procedures is gratefully acknowledged.

TABLE 24.—*Calculated values for constant-amplitude loading of prestressed specimen*

Assume: 1. $\left.\begin{aligned} K_t &= 4.00 \\ \rho &= 0.057 \text{ in.} \\ \rho' &= 0.0144 \text{ in.} \end{aligned}\right\} K_{el} = 1 + \dfrac{3}{1+\sqrt{\dfrac{144}{570}}} = 3.00$

2. $S_{n,mean} = 10.0$ ksi

3. Material data in figs. 142 and 143

$S_{n,max.}$	$S_{max.}$	$S_{min.}$	R	N
	With preload of 30 ksi: prestress = −16.5 ksi			
15	28.5	−1.5	−0.05	$>10^7$
20	43.5	−16.5	− .38	7×10^4
25	58.5	−31.5	− .54	2×10^4
30	73.5	−46.5	− .63	2.5×10^3
35	75.0	−72.2	− .96	7×10^2
18	37.5	−10.5	− .28	1.5×10^5

Variable Amplitude Loading

The procedure just described can be used for variable amplitude loading by addition of an assumption about cumulative damage. A simple assumption is that the Miner rule holds for the local behavior of the material.

The use of this method of fatigue prediction can be depicted conveniently by a numerical example. Let us consider the wing spar mentioned in the preceding chapter. For convenience, we assume it contains a stress concentration having $K_{el}=3$, like the specimen used as an example for constant amplitude loading, and that it is made of the same material, so that the data in figures 142 and 143 apply. Table 25 indicates assumed loadings based upon tables 21 and 22 of the previous chapter. Note that the infrequent high loadings have been omitted for the first nine flights and numbers of cycles have been listed to the nearest integral values. For these flights, estimated local stresses are within the elastic limit and $\Sigma n/N = 7.7 \times 10^{-5}$ per flight. Now suppose the 10th flight includes one higher gust loading that causes the local stress to exceed the elastic limit and produces a subsequent residual compressive stress. For the values assumed in table 25, this flight would produce $\Sigma n/N = 22.0 \times 10^{-5}$, and the first 10 flights would give $\Sigma n/N = 9.1 \times 10^{-4}$. Now, if the next 10 flights follow the loading pattern of the first 10, these will contribute $\Sigma n/N = 6.8 \times 10^{-4}$.⁽⁵⁾ If all subsequent flights follow a similar loading pattern, the number, x, of flights for failure should be compatible with

$$9.1\times10^{-4} + (x-10) \times \frac{6.8\times10^{-4}}{10} = 1.$$

⁽⁵⁾ This is less than $\Sigma n/N$ for the first 10 flights on account of the residual compressive stress subsequent to the 10th flight.

TABLE 25.—*Calculations of fatigue life expectancy of a particular structural part under a particular variable amplitude loading*

Part: Wing spar with high stress concentration, $K_{el} = 3.00$, of material fitting data in figs. 142 and 143

Source	$S_{n,m}$, ksi	$S_{n,a}$, ksi	n per flight	S_{max}, ksi	S_{min}, ksi	R	N	n/N
				For first 9 flights				
Taxi	−5.4	2.8	8	−7.8	−6.0	+0.8	$>10^7$	......
	−5.4	5.6	1	+0.6	−33.0	−550	$>10^7$	......
G & M ...	+14.0	2.8	28	+50.4	+33.6	+ .67	$>10^7$	......
		5.6	1	+58.8	+25.2	+ .43	1×10^5	1.0×10^{-5}
		8.4	1	+67.2	+16.8	+ .25	2×10^4	5.0×10^{-5}
G–A–G ...	+4.3	9.7	1	+42.0	−16.2	− .39	6×10^4	1.7×10^{-5}
L. Imp. ...	+14.0	2.8	1	+50.4	+33.6	+ .67	$>10^7$	......
							$\Sigma n/N = 7.7 \times 10^{-5}$	
				For 10th flight				
Taxi	−5.4	2.8	8	−7.8	−6.0	+0.77	$>10^7$	......
		5.6	1	+0.6	−33.0	−550	$>10^7$	......
		8.4	1	+9.0	−41.4	−4.6	$>10^7$	......
G & M ...	+14.0	2.8	28	+50.4	+33.6	+ .67	$>10^7$	......
		5.6	1	+58.8	+25.2	+ .43	1×10^5	1.0×10^{-5}
		8.4	1	+67.2	+16.8	+ .25	2×10^4	5.0×10^{-5}
		11.2	1	+72.2 [(a)]	+5.9	+ .08	7×10^3	14.3×10^{-5}
G–A–G ...	+4.3	9.7	1	+38.9	−19.2	− .49	7×10^4	1.4×10^{-5}
L. Imp. ..	+14.0	2.8	1	+47.3	+30.5	+ .65	$>10^7$	......
		5.6	1	+55.7	+22.1	+ .40	3×10^5	$.3 \times 10^{-5}$
							$\Sigma n/N = 22 \times 10^{-5}$	

For 10 flights, $\Sigma \frac{n}{N} = 22 \times 10^{-5} + 9 \times 7.7 \times 10^{-5} = 9.1 \times 10^{-4}$

[(a)] After $S_{max} = 72.2$, assumed local residual stress of −3.1 ksi.

Then, $x = 14{,}700$ flights (corresponding to a service life of 29,400 hours).

Reference to tables 21 and 22 indicates that some flights might be expected to involve still higher loads with consequent higher damage and higher residual stress. An estimate, with allowance for such effects, can be calculated by following the same principles with assumptions as to the order of occurrence of all loads. Use of computers permits such computations to be made easily and the effects of various assumed loading patterns to be thus examined. A conservative estimate may be made by assuming that all loads which would produce favorable residual stresses occur toward the end of the service lifetime.

This kind of calculation can also provide help in planning full scale fatigue tests of structures. In such tests, as noted in the next chapter, it is usually expedient to apply loads in patterns which may be influenced by testing facilities and scheduling requirements. Consequences of varying patterns can be estimated by calculations like the example described and plans for a reasonably critical full scale test may be based upon such estimations. Moreover, it is often desirable to extrapolate results of full scale tests to indicate different results that might be expected for different loadings (such as expected for other missions); the only expedient means of extrapolation may be calculation.

OTHER ESTIMATIONS OF FATIGUE LIFETIME

The detailed approach by reference to data on unnotched specimens with regard for the stress-strain curve is not always used. A simpler and often conservative approach is to use a Miner criterion,

$$\Sigma\, n/N = 1$$

with reference to nominal-stress-lifetime curves for the structural element.

Figure 146 shows *S–N* curves that might be appropriate (6) for the wing spar used in previous examples. Table 26 shows the cumulative damage calculation for the loading in table 23 from these and the assumption that $\Sigma\, n/N = 1$ for lifetimes in terms of nominal stresses. Note that, in this example, the estimated lifetime is a little less than obtained by the more detailed analysis. This somewhat simpler analysis does not contain any allowance for effects of ordering of loads and is thus less informative and, in some instances, less dependable than the more detailed analysis of local stresses and strains. It has two advantages: (1) simplicity, which may be desirable for preliminary estimates, and (2) feasibility, when complete data on the part but incomplete data on the material are available.

A great deal of effort has been devoted to analysis of lifetime expectancy under variable amplitude loading that can be considered to produce uniaxial nominal stresses in absence of significant temperature or other environmental effects. Figure 147, adapted from reference 7, shows a schematic diagram of conditions that would require

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(6) These were obtained by calculation, in the manner just described, from the assumed S–N curves for the material. To this extent they are compatible with information used for the other illustrative calculations.

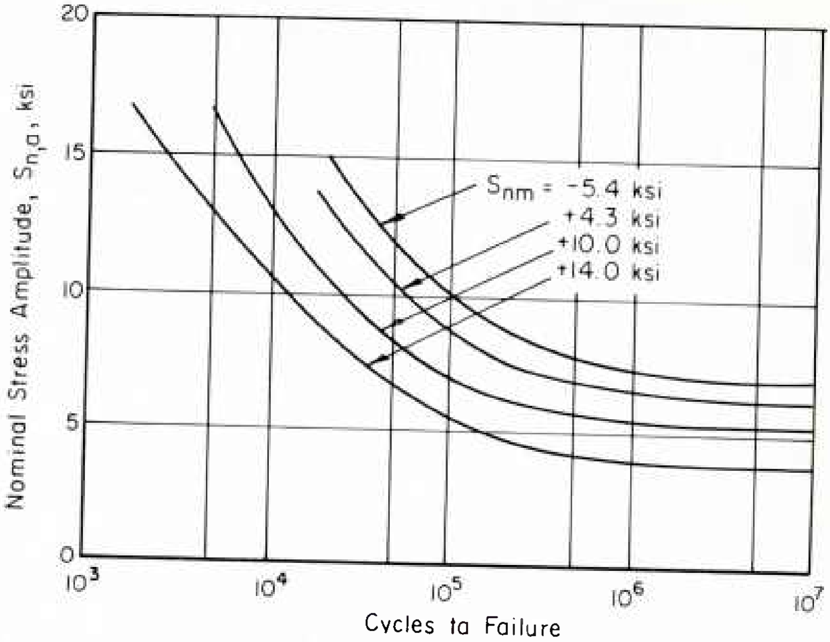

FIGURE 146.—*S–N* curves assumed for wing spar illustrative computation.

TABLE 26.—*Another calculation of fatigue life expectancy of a particular structural part under a particular variable amplitude loading*

Part: Wing spar with *S–N* curves as in fig. 146
Loading: 10 flights as in table 25

Loads			N	n/N
$S_{n,m}$, *ksi*	$S_{n,a}$, *ksi*	n		
−5.4	2.8	80	$>10^7$	....
	5.6	10	$>10^7$	....
	8.4	1	3×10^5	0.3×10^{-5}
+4.3	9.7	10	6×10^4	16.7×10^{-5}
+14.0	2.8	290	$>10^7$	....
	5.6	11	9×10^4	12.2×10^{-5}
	8.4	10	2×10^4	50.0×10^{-5}
	11.2	1	8×10^3	12.5×10^{-5}
				$\Sigma n/N = 91.7 \times 10^{-5}$
				$= 9.2 \times 10^{-4}$

$$\text{Lifetime} = \frac{10}{91.7 \times 10^{-5}} = 10{,}900 \text{ flights.}$$

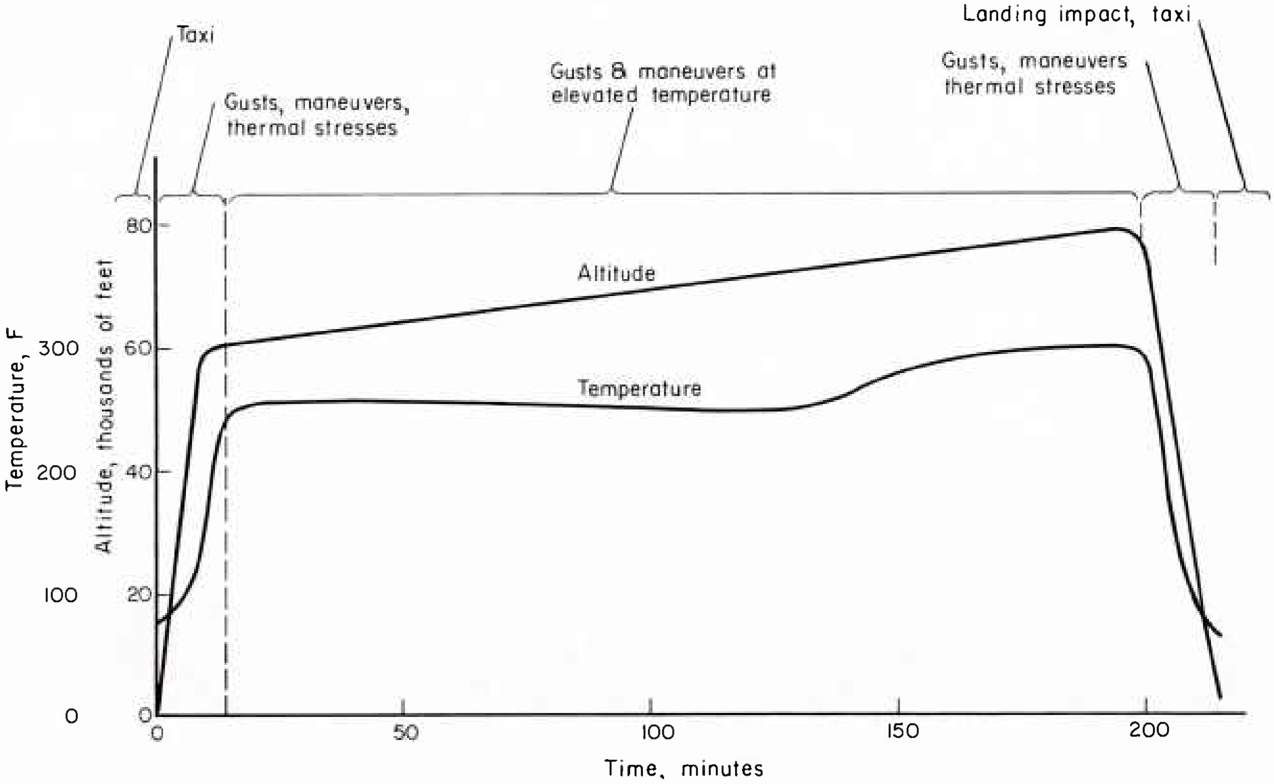

FIGURE 147.—Diagram of temperature condition that might occur on a lower wing panel of a supersonic transport.

consideration in estimating the lifetime of a wing panel in a supersonic transport. The procedure involving detailed analysis of local strains has not yet been developed sufficiently to apply to such a situation. Analysis in terms of a Miner rule for nominal stresses requires: (1) estimation of effects of thermal stresses, and (2) some allowance for creep or other degradation at the elevated temperature. At present, methods of handling such factors are speculative (see ch. VIII). Degradation of resistance to fatigue by corrosive environment is another practical concern for which analytical procedures are inadequate. The current procedure for such complications is to make any feasible estimate as to whether the effects of such factors as temperature and corrosion are likely to be significant; unless there is conviction that these effects are negligible, testing is required.

Some parts of aircraft require fatigue analysis under combined-stress loading. For example, a rotor shaft in a helicopter may be subject to both bending and torsion of varied levels. In such situations, approaches in terms of local strains require further development to be applicable. Cumulative damage rules in terms of nominal stresses are sometimes used with assumptions concerning strengths under combined stresses (see app. A).

RECAPITULATION

Review of the preceding notes on consideration in design suggests the importance of *all* factors that may influence local stresses and strains in the material of which a component is made. It is, of course, imperative that such factors be taken into account in any analytical evaluation of the structural response to repeated loading. In most instances, component or full-scale testing is the only dependable way of including some assessment of all the factors involved.

Analysis also should involve consideration of local strains. In some instances, this consideration may be made on the basis of the stress-strain characteristics of the material and the results of extensive fatigue tests on unnotched specimens of the material. In other situations, such detailed analysis may not be feasible and conclusions from the analysis may be consequently less informative. To whatever extent feasible, analytical evaluations as well as design should take account of local strains at regions of stress concentration.

In both design and analysis, an attempt should be made to consider all factors and to estimate probable errors in every step of computation. This affords a chance of estimating the probable error in a calculated value of fatigue lifetime. Such estimation is sometimes a sobering experience. However, it helps clarify the items contributing most to uncertainty of the final result and spotlights them for further study.

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CHAPTER XIX. STRUCTURAL RESPONSE: FULL-SCALE TESTING

A large part of the task of preventing fatigue in aircraft can be accomplished in design, in testing of small components, and in care during production and fabrication. However, there are at least two important additional considerations: (1) variables in the final assembly process and (2) complex interactions of assembled components under loading. Experience has shown the great value of full-scale fatigue testing on an essentially complete vehicle toward taking these into account.

Full-scale fatigue testing has contributed toward economy as well as toward safety and reliability. Every full-scale fatigue test has resulted in a number of minor fixes and changes which might otherwise have been made at greater cost during operational lifetime.

The increasing emphasis on full-scale fatigue testing in the United States is indicated by the extension in requirements for such testing in various official documents (for example, the MIL–A–8860 series). International attention to full-scale fatigue testing is illustrated in corresponding documents in various countries. A recent symposium (*1*) was devoted to various aspects of the "planning and interpretation of full-scale aircraft fatigue tests."

SOME LIMITATIONS AND OBJECTIVES

Full-scale fatigue testing to validate the capability of an aircraft to undergo anticipated service without major fatigue failures is somewhat different in approach from programs such as mentioned in chapter XV, aimed at developing long-range information. Validation fatigue tests have important limitations and specific objectives.

One limitation is that, for reasons of time as well as of cost, the complete assembly can have only a *single test*. The point is sometimes raised that this test includes, in some respects, multiple specimens (two wings, innumerable riveted joints, etc.). While this is a valuable viewpoint, it is also worth emphasis that there can be only a single loading schedule and, in respect to many interactions, only a single specimen.

For many reasons, the full-scale fatigue test is usually an *accelerated* test. The assembled vehicle is not available until production is well along, and a target date for acceptance is close. If acceptance is contingent upon results of a full-scale fatigue test, there are obvious pressures to speed this up. Sometimes it is necessary to complete a static test on the aircraft and to add resulting changes before the fatigue test; this necessity further aggravates the need for acceleration. Moreover, the fatigue test can provide technical information in advance of possible service difficulties only if the test progresses with sufficient lead-time with respect to service. Accelerated testing usually involves omission of some factors estimated (or hoped) to be relatively unimportant. Thus, many very low loads may be omitted from the test spectrum, although they are expected in service. If the test time is significantly shortened, atmospheric corrosion will not exactly duplicate that of service. In elevated temperature environments, acceleration of test-time may alter the creep behavior from that of real-time exposure. The compromises inherent in acceleration of the full-scale fatigue test comprises another limitation.

A third limitation is the *uncertainty* of some factors in the test. Missions must be based upon agreement; planned missions may change after the test is under way or even after its completion. Loads, even for the assigned missions, may not be known in detail, or it may not be feasible to duplicate their distribution completely. Sometimes stress details are discovered during the test; changes are then incorporated, and the test is continued with an altered specimen. Such uncertainties are, to a large extent, unavoidable, and their existence must be accepted as an engineering reality.

Within such limitations, a full-scale test can have, for a specific vehicle under specific loadings, such objectives as:

1. identifying areas critical in fatigue;
2. indicating modes of potential failure under fatigue loadings;
3. estimating service lifetime before fatigue failure;
4. estimating fail-safe characteristics.

To obtain reasonable fulfillment of these objectives, within the limitations of full-scale, testing requires much care in approach, in planning and conducting the test, and in interpretation of results.

PLANNING A FULL-SCALE FATIGUE TEST

Considerations in planning a full-scale fatigue test include:

1. defining a test-load spectrum;
2. arranging for an appropriate loading system;
3. scheduling the program of observation and recording.

The Load Spectrum

Some of the factors involved in estimating a spectrum of expected service loadings have been noted previously (ch. XVI). There are many sources of information concerning gust loadings, several sources of information about maneuver loadings, and some available background concerning landing loads and taxiing loads. The usual practice for a new vehicle is to define expected missions, collect available data concerning expected loads during these missions, and painstakingly build up an expected load spectrum. An illustration of this procedure is given in reference *2*.

In general, it is possible to define some reasonable average flight-load spectrum. Some items will be uncertain and will have varied interpretations. For a new aircraft with radically different structural and/or aerodynamic characteristics, the assignment of an appropriate load spectrum may be particularly difficult. Consequently, the overall spectrum for any particular vehicle is partly a matter of judgment. The combined efforts of the manufacturer, the buyer, and the certifying agency are needed for a decision concerning a mutually acceptable expected pattern of service loadings. Reduction to a *test-load spectrum* requires some additional decisions.

Since there is to be only one full-scale fatigue test, the test-load spectrum must be considered *representative* in some manner: typical for an "average" airplane in the fleet, or intentionally severe (loadwise or fatigue-damage-wise).

Since the single test must be completed in a time significantly shorter than the anticipated service lifetime, various possibilities of *acceleration* of *loading* must be considered. Some obvious means are:

1. Run the test continuously (except for inspections, minor fixes, etc.). This means *omitting rest periods.*
2. *Accelerate* the *rate* of *loading,* but only to an extent that does not significantly influence dynamic stress distributions or material behavior.
3. Omit some very small load amplitudes which are estimated to contribute negligibly to fatigue damage.(1)
4. *Omit* some *types* of loading which are estimated to be unimportant (usually on the basis of estimated small stress contributions).

The simplifications adopted to reduce testing time and effort to tolerable limits are critically important. However, the choice rests

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(1) An alternative that has been used is to consider a trade-off on an "equivalent damage" basis and use a smaller number of higher amplitude loads rather than to neglect completely the many very small-amplitude loads. The crucial question in this procedure is the basis of estimating "equivalent damage."

often upon engineering judgment and cannot be dictated by fixed rules that hold for all situations.

Another consideration in the test-load spectrum is the *ordering of loads.* This can be illustrated by an example of gust and maneuver loading upon a wing. Figure 148 shows:

(a) A trace that might be expected from a strain-gage recorder during part of a flight and three breakdowns that might be considered for test loading.

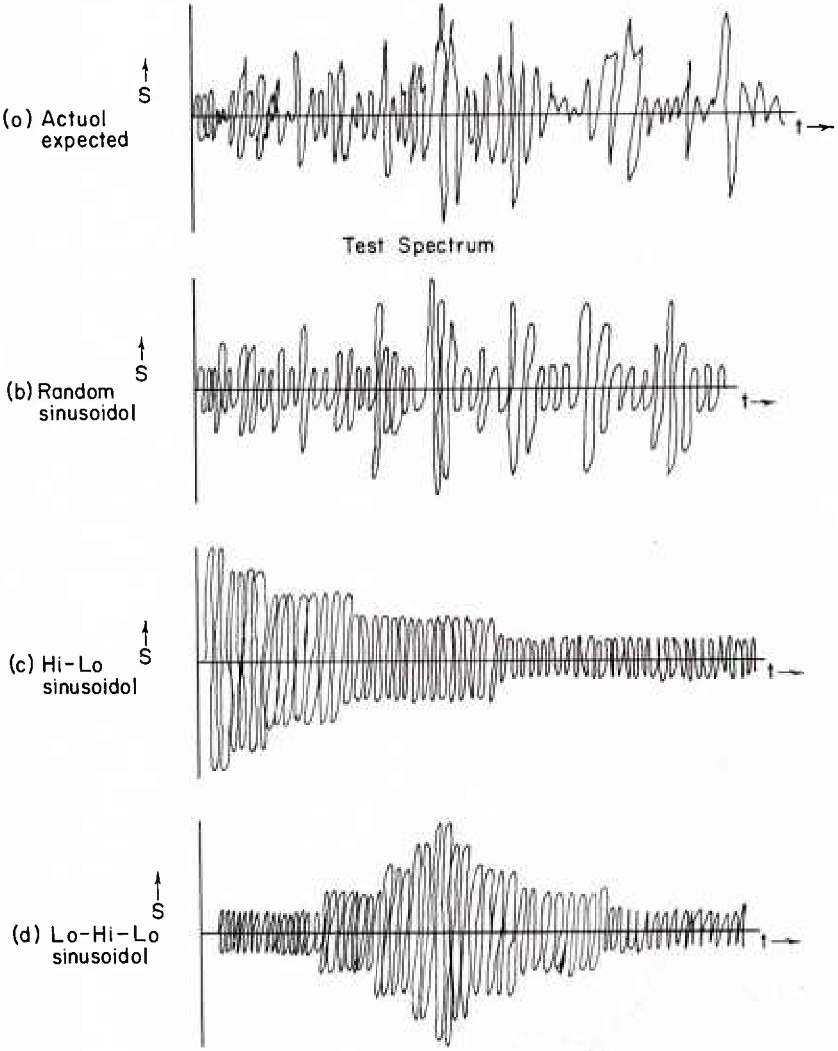

FIGURE 148.—Schematic test spectra for some wing loadings.

(b) A breakdown into sinusoidal components with some specified definition of peaks (see ch. XVI) and further simplification by omission of some very small excursions and by grouping others into four levels (more than this are recommended, but would complicate the illustrative sketch). The ordering titled "random" is that of the trace in item 1.

(c) The same loadings arranged in descending order of load amplitude (called "Hi-Lo").

(d) The same loadings in a different ordering, "Lo-Hi-Lo."

Clearly, other orderings (for example, a Lo-Hi sequence) are possible.

The simply ordered arrangements (of example, (c) and (d) in fig. 148) are usually very much easier to apply than "random" arrangements (such as (b)). Consequently, one practice is to arrange a test schedule in terms of blocks of ordered loadings. The pattern of expected service loadings might be put into the form of average numbers of cycles anticipated in, say, 1,000 hours of service—as shown in the example in table 26. Test loads could be run in a sequence ((b), (c) or (d) of fig. 148), in blocks corresponding to 100 hours or to 20 hours or to 2 hours of service. Note that, for any of the block sizes shown in the table, fractional values occur; for example, in the 20-hour breakdown there are 0.04 cycles of gust and maneuver load amplitudes of 0.87 *g*; this cycle might be used once every 25 blocks. The problem of where in the testing sequence of block loading to insert the infrequently occurring loads is a difficult one, concerning which there are justifiable differences of opinion.

There is a trend toward as complete loading as possible; this would include not just blocks of wing loadings, but tail loadings (both hori-

TABLE 26.—*Load cycles in blocks*

Source	LOADINGS, Δg		NUMBER INDICATED HOURS PER BLOCK			
	Mean	Ampl.	1,000	100	20	2
Gusts and maneuvers	1	0.204	12,000	1,200	240	24
		.296	1,360	136	27.2	2.72
		.396	275	27.5	5.5	.55
		.494	78	7.8	1.56	.16
		.770	6	.6	.16	.02
		.870	2	.2	.04	.01
		.965	1	.1	.02	....
G–A–G	0.642	.570	500	50	10	1
Taxi	— .500	.072	1,500	150	30	3
	— .535	.107	90	9	2.8	.28
	— .571	.143	4	.4	.08	.02

zontal and vertical), landing loadings (including catapult loadings), fuselage loadings, and so on. This includes the effect of interactions, especially if blocks of the various loadings are applied in suitable order. It raises questions concerning practical ordering.

For some loadings of the complete vehicle, an approach to "flight-by-flight" testing is sometimes used (see reference *3*). Computer programming makes feasible somewhat suitable randomization in the loading schedule, so that, for example, *G–A–G* loadings can be inserted in an order relative to gust and maneuver loadings that provides simulation to the nonperiodic ordering of service. However, there remains considerable room for judgment as to how complex a loading is justified in view of other requirements (time and cost), and of uncertainties (such as probable missions and loadings), which may be more significant than some refinements in the test loading schedule.

Thus, the selection of a fatigue-test-load spectrum presents problems varying in detail with the type of airplane and with the assigned missions and maintenance. The problems involve assumptions concerning service loads, compromise in simplification, especially with respect to acceleration of testing time compared with real time, and decisions regarding the order of occurrence of various load amplitudes.

Fatigue-Test Loading Systems

The planning of a full-scale test must next include attention as to how the equivalent loads are to be applied to the structure. The equivalent of aerodynamic loads, of landing loads, and of inertial loads must be represented in suitable phase and with proper distribution over the structure. Physically available means of loading are by resonance, by hydraulic loading, by combinations, and by special methods (such as producing noise fields for acoustic fatigue).

Much testing of large vehicles has been done with hydraulic jacks with special arrangements for distribution corresponding to such service conditions as aerodynamic loads. A previous figure (fig. 127) indicated a "whiffle-tree" arrangement for load distribution; figure 149 shows more clearly a similar arrangement with tension pads bonded to the skin of the surface over which the load is distributed. Figure 150 shows another test setup, with both upper and lower loading systems. Figure 151 provides an idea of the complexity of the test setup for a complete aircraft; in the lower left-hand corner can be seen some of the monitoring equipment for controlling the load applications.

For some aircraft structures, reasonable representation of loads may be exceedingly difficult. An example is the ground loading of helicopter

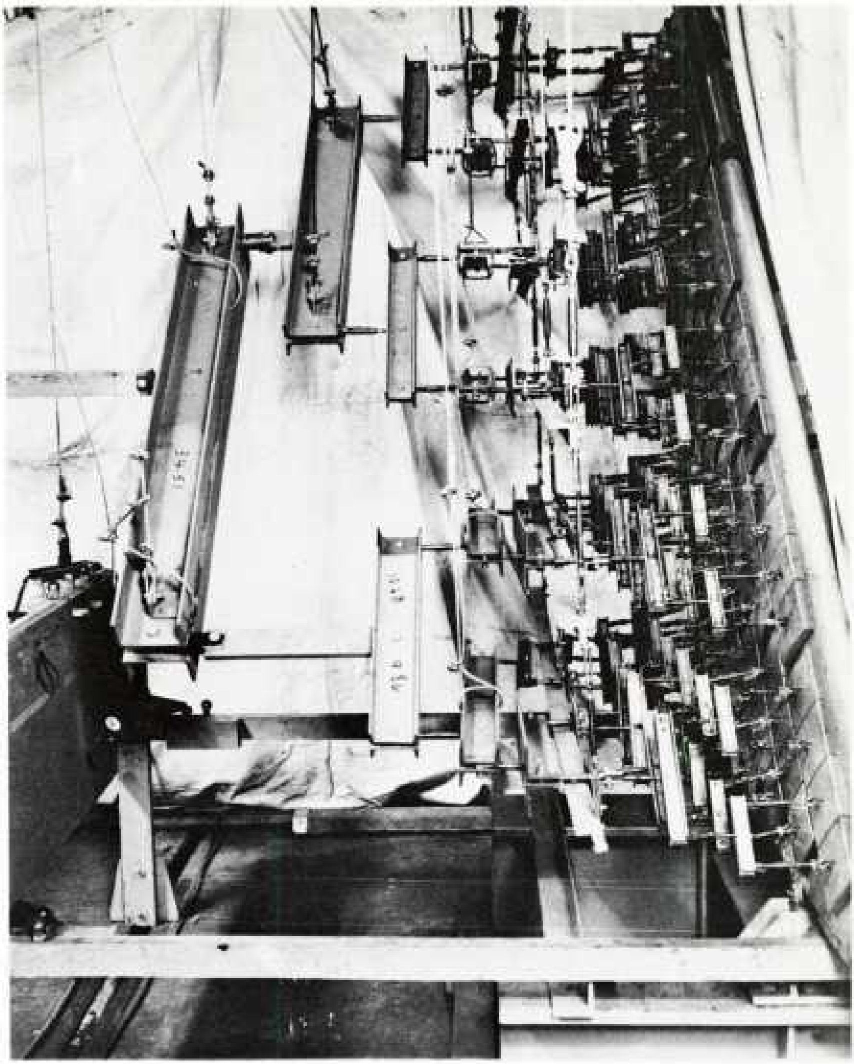

FIGURE 149.—A whiffle-tree load-distribution setup.

rotor systems. To date, to obtain full representation of air load for various conditions of speed and pitch of the rotor blades has been nearly impossible. Figure 152 shows a method under development; the panels under the rotor can be so positioned as to produce appropriate load distributions on the blades (the stress distributions having been determined in a flight test).

Not only mechanical loads but environment should be planned for a simulated-service test. Elevated-temperature fatigue tests of full-scale

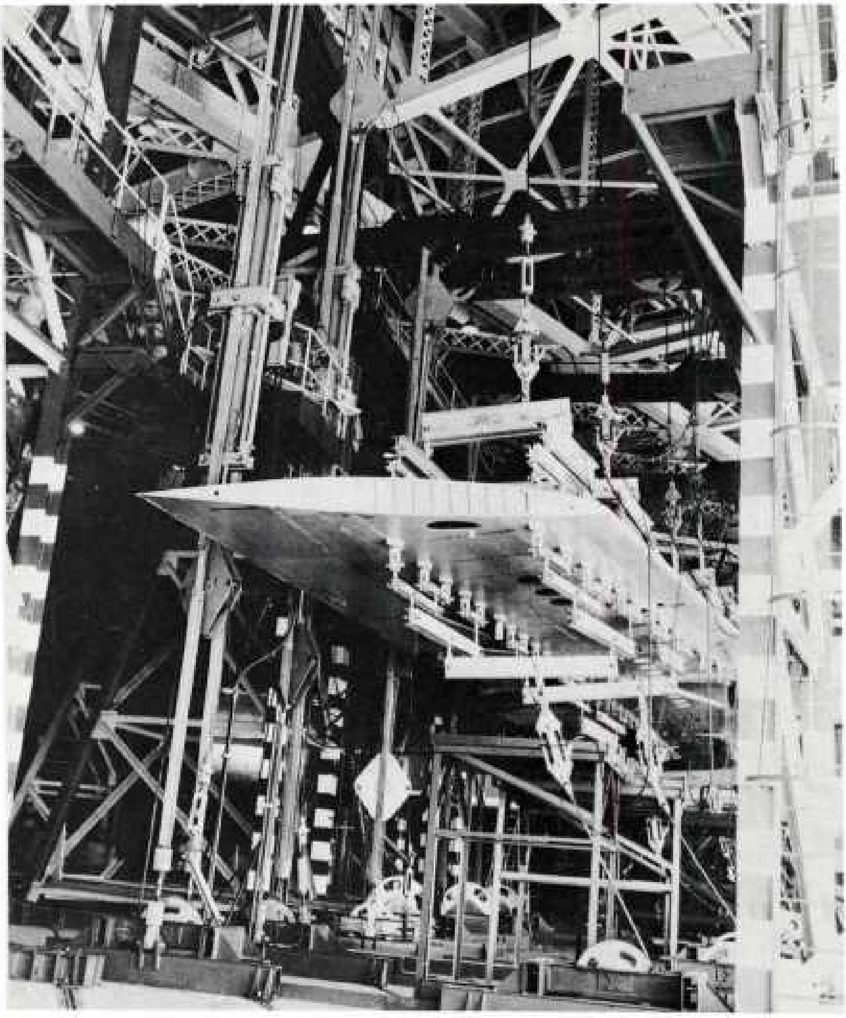

FIGURE 150.—Test setup—right-hand wing, upper and lower loading systems.

aircraft structures are being developed. Figures 153, 154, and 155 show some of the components for power supply, heating, and temperature control for such tests at one laboratory (of the Naval Air Engineering Center). At high temperatures there are problems such as attaching loading pads without upsetting thermal behavior, recording strains and local deflections, and inspecting for small cracks. In elevated temperature testing, the relation between real-time and (accelerated) test-time becomes very important; this problem is currently under study. Some of these problems have not yet been satisfactorily solved.

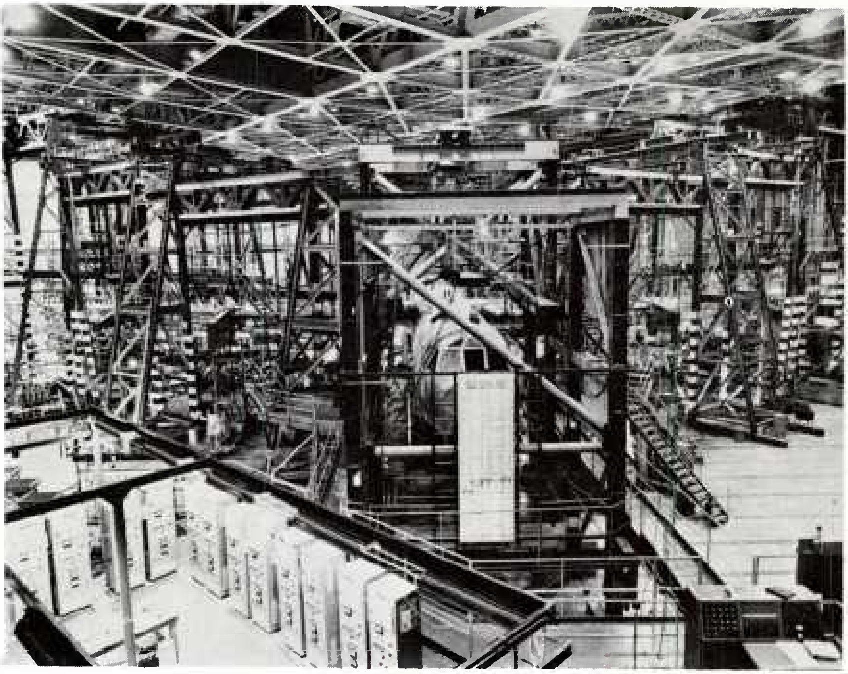

FIGURE 151.—Fatigue test of complete aircraft.

The illustrations suggest some of the procedures that have been used to apply suitably distributed loads to some structures. Other loading may be required, for example: pressure cycling of fuselages, landing-gear impacts, catapult and arresting loads. The details usually require some simplification to be feasible, and much judgment is needed to attain reasonable loading without defeating the objectives of the testing.

The Test Schedule

Details of monitoring and recording loads and of observing and recording behavior vary with the nature of the test airplane, the testing requirements, and the test facilities. However, there are some factors common to most full-scale tests:

1. Load applications must be *carefully monitored* in sequence, in amplitude, and in phase. Both monitoring and recording are often carried out with electronic control consoles.
2. Observations of behavior should be extensive. These may include: continual recording of deflections and strains at selected points, frequent visual inspection, and detailed inspection with such aids as dye penetrants and eddy-current devices at longer intervals.

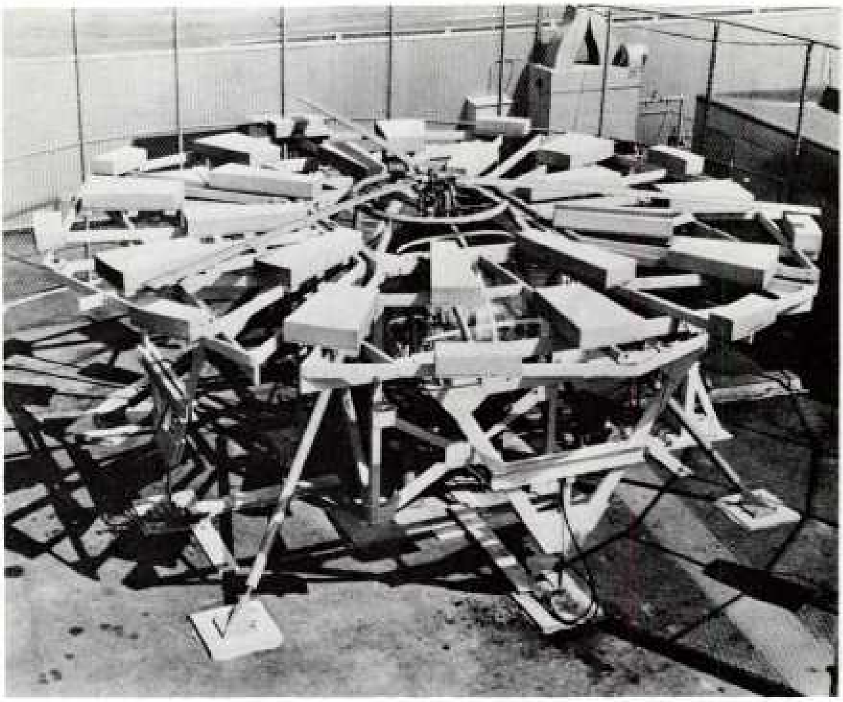

Figure 152.—Testing a helicopter rotor blade.

3. All observations should be *recorded*, but particularly so when small cracks are detected and their progress under further loading becomes a concern.

4. The full-scale test furnishes an opportunity for experimental *stress-analysis* and for the *calibration* of fatigue monitors that might be used in service.

5. Each *failure* (even though minor) requires decision between *continuation* to observe progress of damage, and repair of damage or change in design and continuation, with consideration for effect on surrounding structure.

6. The *duration* of a test is an important concern. To get, with practical equipment, a reasonable total number of cycles on a fighter plane may take 6–12 months; testing a transport may require 12–24 months (or longer).

Each full-scale fatigue test of a complete airplane is an extensive research investigation and should be approached with plans for some flexibility of procedure.

An example of a need for flexibility is the situation where early

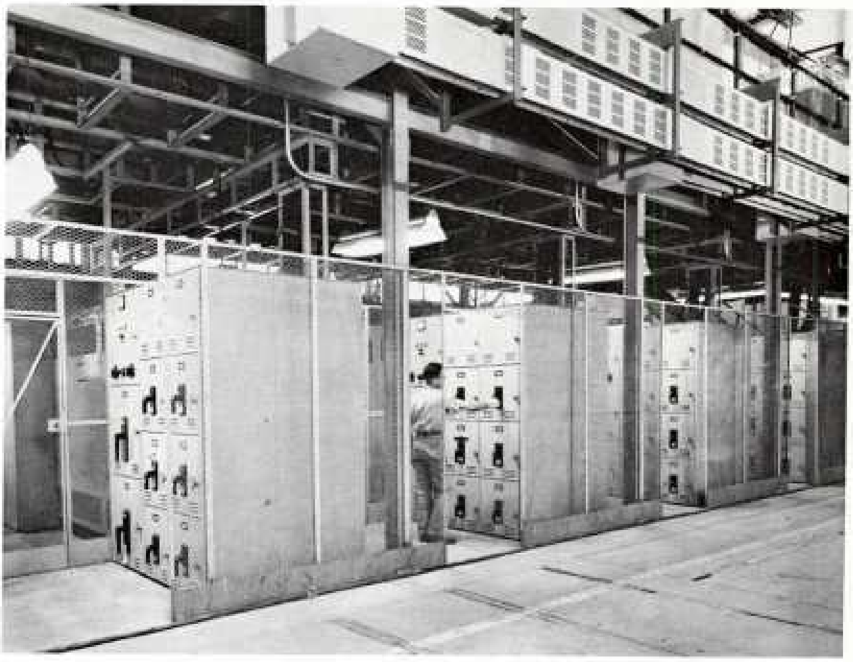

FIGURE 153.—Equipment for elevated-temperature test—power-center components (see also figs. 154 and 155).

flight tests and/or reconsideration of mission requirements make the loading spectra under which the test was started (and perhaps continued for a few months) unrealistic. In such an event, there are conflicting arguments for continuing (for the sake of consistency) the original loading and for changing to the loading schedule now apparently more representative of intended service. There can be no simple answer for all such contingencies. But the fatigue investigator should be aware that problems of this kind are apt to arise.

Test Procedure and Interpretation of Results

Inspection schedules and procedures should be planned carefully at the start, but they often require revision as the fatigue test progresses. Schedules may be planned with regard for:

1. A lead time over planned service inspections.
2. Conservative estimates of possible cumulative damage at critical locations.
3. A trade-off between testing time and time required for detailed inspection.

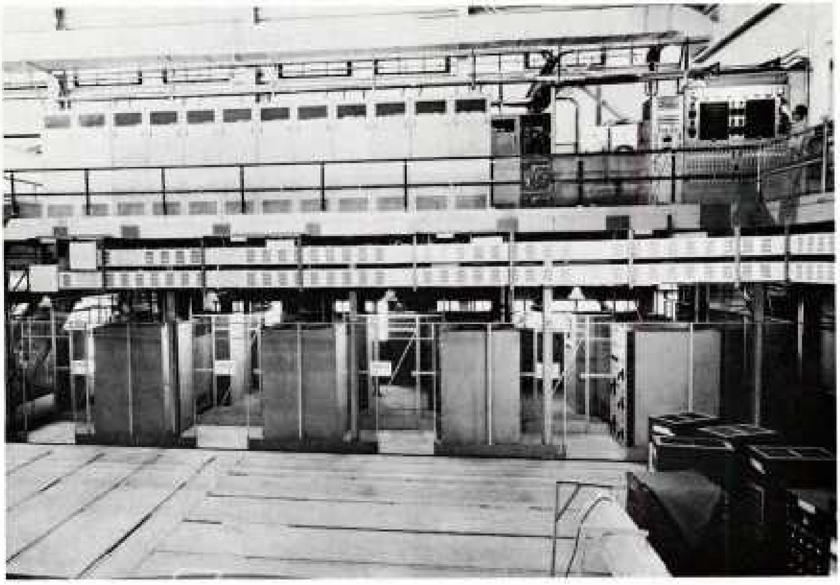

FIGURE 154.—Equipment for elevated-temperature test—programmers and saturable-core reactors (see also figs. 153 and 155).

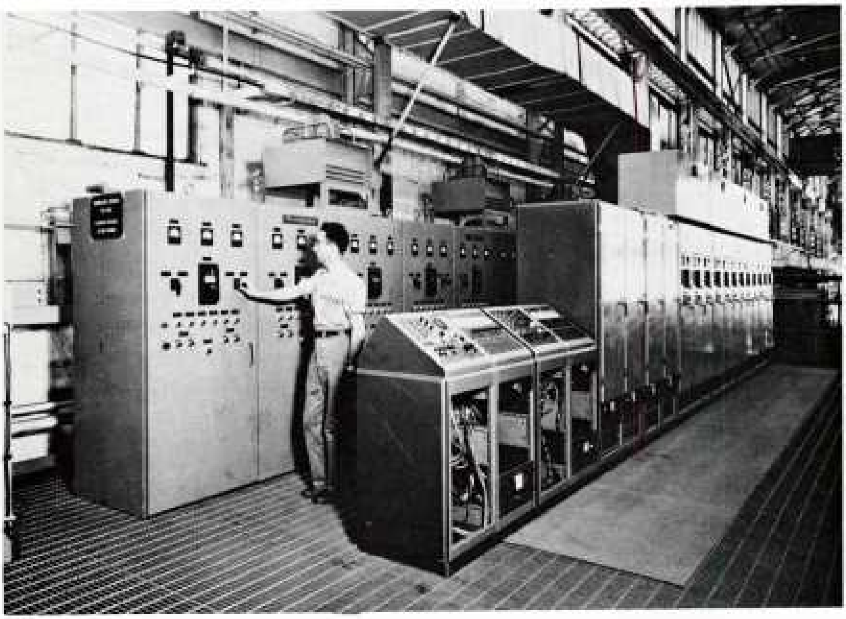

FIGURE 155.—Equipment for elevated-temperature test—programmers and ignitrons (see also figs. 153 and 154).

Inspection which requires disassembly must be considered, not only with regard to the interruption of testing time, but also with regard to possible effects of the disassembly and the reassembly on the structure. The schedule will often require revision as indications of change in deflection or of damage arise.

Procedures of inspection for fatigue damage are extremely difficult to plan. Visual inspection for cracks or excessive fretting where cracks may develop is the most widely useful procedure. Tools such as the borescope may aid visual inspection in some locations. Other aids may include the whole gamut of tools mentioned in the next chapter in regard to service inspection.

A particular concern in procedure is that of redesign and retrofit of items which fail unexpectedly early in the test. There are inevitably such items—many of which, if design and fabrication and assembly have been good, may be minor parts of the structure. Methods of crack stopping, local strengthening, and actual replacement have been and are being studied; procedures are varied in accordance with material, design and stress analysis, and accessibility. In connection with the test procedure, a question may arise as to the relative merits of fixing a cracked member or continuing the test to obtain further information on crack propagation characteristics. There is no unique answer to such a question, but recognition of the existence of the question is important.

When a full-scale test has proceeded to a significant length of projected service time, the interpretation of results becomes critically important. If no major failures have been observed, the validity of the apparent conclusion of demonstrated safe-life is open to question. An attempt to judge the dependability of such a conclusion requires consideration of the relevance of test loading and environment to anticipated service, and some speculation as to damage mechanisms that might be important. For example, a high load on a structural part may strengthen the part against fatigue from subsequent lower loads; hence, it is possible that the test conditions have been unrealistic in regard to the early occurrence of high loads. Another question is the possible influence of minor changes that may have been made. Still a third item for judgment is the range of variation possible for different vehicles of the same type, and for different missions and maintenance that individual aircraft may have in service—the whole question of statistical reliability of information from a single test.

The estimated validity of test results has some necessary uncertainty. One allowance for this is a somewhat arbitrarily conservative factor for projecting the test lifetime to a safe service life. Another valuable principle is that of test "lead time" over service time. This principle has been used extensively in regard to some components. An example is the development of retirement schedules for helicopter rotor blades. For a significantly new design a very conservative initial replacement time may be specified. Blades retired at this time may be tested (statically and in any feasible accelerated fatigue-testing program); when test results on used blades seem to warrant extending the lifetime, such extension may be tried. This procedure may be repeated until there is some indication of approaching a questionable limit.

FAIL-SAFE TESTING

Fail-safe testing may imply: (1) measurements of residual static strength after damage (often simulated by artificial cracks), (2) studies of rate of crack propagation, or (3) both.

To date, most full-scale testing for evaluation of safe-life characteristics has been of the first type. Reference *4* gives a summary of tests of a number of components with respect to fail-safe characteristics. For example, a structurally complete fuselage was tested under pressure and other loads, supplied by hydraulic jacks through tension straps, whiffle trees, and compression cradles, with 19 precuts and 19 dynamic cuts representing damage. Objectives were to find residual strength and especially the mode of failure (whether the damage would be localized with remaining load paths intact).

While most fail-safe acceptance testing is in the category of finding residual strength after simulated service damage, there is increasing interest in such fail-safe characteristics as material with slow fatigue-crack-propagation behavior, feasible crack-stopper techniques, and design to provide multiple load paths. Many of these characteristics can be studied in a structural fatigue test planned along safe-life objectives. In any full-scale fatigue test, inclusion of all feasible observations of crack-growth rates is, therefore, highly desirable.

CONCLUDING REMARKS

Despite techniques and details which vary with type of structure and expected loading, full-scale fatigue tests have in common the broad objectives mentioned earlier:

1. Finding any unanticipated weaknesses in a fully assembled structure or major component.
2. Studying load and stress distribution under simulated service conditions.
3. Estimating service life.

4. Identifying areas for inspection and recommending types of inspection and inspection intervals.
5. Calibration of fatigue-life monitoring instrumentation.

Detailed subobjectives may include such items as:

6. Designing and studying a "fix" for a potential weakness uncovered in the test.
7. Studying means of detection of fatigue damage.
8. Building improved engineering design and production methodology.

Experience to date has shown that a full-scale test with as complete representation of loadings and environmental conditions as feasible is extremely valuable. A growing problem is the increasing difficulty of representation of such conditions (for example, the time-temperature-load parameters in a supersonic transport or the complex loadings in rotary-wing or VTOL vehicles). Surmounting this difficulty, within tolerable limits of time and cost, will require continued development of techniques in full-scale fatigue tests.

REFERENCES FOR CHAPTER XIX

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3. Melcon, M. A., and McCullogh, A. J., "Simulation of Random Aircraft Service Loadings in Fatigue Tests," *Third AGARD–ICAF Symposium on Aeronautical Fatigue*, 1963.

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CHAPTER XX. CONSIDERATIONS DURING SERVICE

A number of considerations are pertinent to fatigue during the service lifetime of an aircraft. One is the recording of loads and environmental parameters in various flight- and ground-handling conditions. Another consideration is periodic inspection for possible incipient fatigue damage. Still another is reassessment of fatigue resistance after alterations, either in the structure or in mission assignment.

Fatigue considerations during service have various possible objectives. One is the continuing assessment of the reliability of a specific vehicle, culminating in a decision as to replacement of a major structural part or even as to retirement of the vehicle. Another is the adjustment of inspection periods and maintenance requirements. A long-range objective is the accumulation of service experience toward improvement of future methods of design and fabrication.

These varied objectives involve responsibilities for many groups: engineers, maintenance crews, pilots, and operations staffs. Moreover, variations in aircraft structural details, performance specifications, and operational requirements require vastly different procedures in recording, inspection, maintenance, and assessment of reliability against fatigue. Hence, an account of all considerations pertinent to fatigue during service could require a set of manuals for each type of vehicle. The following notes illustrate some approaches that have been taken and some factors to be considered.

RECORDING

Some Principles

From the academic design point of view, obtaining and analyzing a complete record of local strains at every fatigue-critical location in every flight vehicle would be desirable. This could involve assigning all payload to instrumentation, all crew to monitoring the instrumentation, and an immense computer facility to analysis of the records. On the other hand, from the point of view of systems operations, it would be helpful if no effort had to be expended in such recording and analysis and if no constraints were imposed on missions by considerations of structural fatigue. The clearly necessary compromise between such extremes is a challenge to engineering.

One approach to this challenge is sampling. A simplified sampling procedure might involve:

1. Extensive recording of loads and local strains on flight tests of one vehicle through conditions selected to cover all critical ranges of loading and environment expected in service.
2. Minimum recording (for example, one parameter such as c.g. vertical acceleration) for each vehicle of a fleet throughout its service.
3. Monitoring of fatigue damage indicators at periodic intervals.

Thus, the flight records might relate important loads and stress distribution to specific parameters (such as c.g. vertical acceleration). The fleet records would provide complete histories of the parameters. The combination of data from these service records together with design stress-analysis data and reasonable hand-logged information might afford local stress-time histories for fatigue evaluation.

A somewhat different, but not necessarily conflicting, approach may be called a "lead-the-fleet" method. In a simplified version of this approach, one early vehicle is followed with detailed care—both in flight records and in records of inspection and maintenance. The objective is to identify incipient trouble areas in all other vehicles of the same type.

An engineering-development procedure is to seek relations between the detailed loads and strains recordable with the extensive instrumentation used during checkout tests and the less complex signals which can be obtained from simpler instrumentation (which is more practical for continuous use on every vehicle). An example of this comparative procedure is the correlation of aerodynamic loads on wings with values of c.g. acceleration. Engineering development seeks a maximum of useful information from minimum records.

Some Pertinent Parameters

Recording ranges from minimal hand-logging to multichannel electronic recording of many parameters. A perpetual need is for a lightweight, wholly dependable, maintenance-free, multiparameter, continuous-recording unit.

Table 27 indicates some parameters that have been suggested for service recording to obtain information on loads and stresses in structural parts of a fixed-wing subsonic airplane.

Since some of the largest loads are in a vertical direction, the vertical acceleration (*g*) is of primary importance. Symmetrical air loads on

TABLE 27—*Some recordable parameters for structural loads*

Structure	Parameters often recorded [a]	Parameters of next importance	Other suggested parameters
Wing	W, V, H, G at c.g.	Other linear accelerations, rotational velocities, altitude.	Local strains.
Fuselage	Pressure differential, W, H, G at c.g.	Other linear accelerations, rotational velocities.	do.
Tail	V, H, G at c.g.	do	Positions of control surfaces, local strains.
Landing gear	W, V, G at c.g.	Attitude	Local strains.

[a] W = weight.
V = air speed.
H = altitude.
G = vertical accelerations.

the wing and on horizontal surfaces can be estimated from values of "g," of altitude, of air speed, of gross weight (from the log book), of weight distribution, of wing area, of lift coefficient, and of other factors generally obtainable from design specifications and test records.

Nonsymmetrical-air-load estimations require additional data. Presumably, data on linear accelerations in two other directions and on three rotational velocities (or accelerations) would go a long way toward permitting accurate determination of unsymmetrical loads on wing and tail surfaces. For estimating tail loads with comparable confidence, information on the attitude of the airplane is also needed.

Indications from sinking-speed measurements show at least statistical correlation with VGH⁽¹⁾ records. Hence, some information about landing-gear loads can be obtained from VGH measurements at the center of gravity. Nonsymmetrical-landing-load estimations require other data.

Changes in differential pressure in the fuselage are important in estimation of some loads on this part of the structure; useful data on cabin pressure are readily obtainable. However, defining the combined effects of differential pressure and loads from other sources, such as interactions with wing loads and tail loads, requires considerable analysis. Much of this analysis can be based upon flight-test studies, so that accelerometer readings and pressure recordings may supply enough parametric data.

Some of the records that have been obtained on fixed-wing aircraft and some of the methods that have been used are described in References *1* through *6*. Recording of structural-loading parameters in addition to c.g. vertical acceleration (together with airspeed and altitude) is difficult enough to be handled on a statistical-sampling basis. To date, eight-channel⁽²⁾ recorders have been used on a number of airplanes; with improvement in the recorder, more extensive use is anticipated.

Records required to document the loads and strains in critical parts of helicopters are relatively difficult to obtain. Reference *7* describes a survey made on two each of three models. Strain gages were used at several locations, and VGH data were recorded. A Navy survey of another helicopter's performance has involved oscillographic records of the following parameters: airspeed, altitude, c.g. accelerations, pilot-seat accelerations, instantaneous rotor rpm, rotor torque, collective stick position, longitudinal stick position, and angular-pitch accelerations. The minimum number of parameters that must be recorded for fatigue considerations on rotary-wing aircraft is large, and statistical records

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(1) VGH: velocity, vertical accelerations, and altitude.

(2) The eight parameters are: V, H, three linear c.g. accelerations, and three angular velocities.

are limited compared with those for gust loadings on fixed-wing aircraft.

However extensive load-monitoring may be, it provides, by itself, no indication of rate of accumulation of damage. Various approaches to instrumentation for indicating accumulating damage have been tried. One is the use of "fatigue-warning indicators"—coupons designed to fail before the structural part to which they are attached. Another is the recently developed "fatigue-life gage" which experiences an irreversible resistance change with cycles of stress. Such indicators have not yet been developed to a high degree of reliability in many conditions of service.

Extensive recording of many parameters is incompatible with other requirements of instrumentation such as low cost, lightness, compactness, reliability, and freedom from maintenance. Consequently, both the "sampling" approach and the "lead-the-fleet" approach must be followed. Meanwhile, research and development programs seek improved relations for estimation of local strains, in terms of parameters feasible to measure and analyze, for every vehicle throughout its lifetime.

Analysis of Recorded Data

Records made during service can be analyzed in several ways and with varied objectives such as:

1. Identifying any unusual deviations which might be significant.
2. Obtaining information, for a specific vehicle or model, concerning load and stress distribution in response to various operating conditions.
3. Providing statistical data with respect to distribution of various load and stress conditions throughout various types of service.

Analysis, particularly of the latter type, can be very time-consuming.

Considerable effort has been expended toward analysis of records such as VGH data on fixed-wing aircraft to provide statistical designation of occurrences of important loads. Quasimanual analyses of many records have been made at NASA-Langley (*1–5*). As many more data accumulate, provided by such instrumentation as the eight-channel recorder, analysis will require automation. Figures 156 and 157 show two views of part of a computer facility installed by the Air Force and the Navy at New York University for such analyses. This facility is an impressive indication of the magnitude of the problem of statistical analysis of load occurrences.

The complications inherent in utilizing service records in fatigue analysis include:

Figure 156.—One view of a computer facility for analyzing service-load records (see also fig. 157).

Figure 157.—Another view of the computer facility for analyzing service-load records (see also fig. 156).

1. Collection of *all* relevant data from varied sources: design- and flight-evaluation tests, pilots' logs, and automatic flight equipment.
2. Reduction to appropriate parameters for analysis.
3. Computation of loads and stresses.
4. Tabulation of loads and stresses in some manner suitable for statistical examination.
5. Summation of past experience in terms appropriate to cumulative-damage postulates for prediction of probable service life.

These complications result, in large part, from the criticality of local stresses in the development of fatigue failures. At present the most promising approach seems to be continued effort to separate out the most important items:

1. The critical location.
2. The loadings most likely to produce severe damage at each location.
3. The missions most likely to give rise to these loadings.
4. The recordable parameters that will reflect the salient features of the important loadings at critical locations.

A feasible number of records can then be made and analyzed to provide information for estimation of fatigue lifetime.

INSPECTION

Some Methods

An important part of regularly scheduled inspection of structural components is to detect incipient fatigue damage. This is clearly vital to the fail-safe design approach.

The highly localized nature of fatigue requires extremely careful examination for possible early detection. At present, there is no known certain way of finding evidence of damage prior to the development of cracks.(3) Observations that are feasible include (1) the detection of small cracks and (2) the detection of aggravations, such as corrosion pitting or excessive fretting, that may lead to fatigue cracking. In many situations a load or pressure application is essential for good visual inspection. Both are strongly dependent on the experience, skill, and care of the inspector. Further discussion in this chapter primarily relates to the detection of cracks.

Table 28 lists several methods of searching for fatigue cracks. The extreme difficulty and time-consuming nature of such searches require

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(3) Except, perhaps, in a few situations of academic interest; for example, at very high (electron microscope) magnification of spots in carefully surfaced pure metals.

TABLE 28.—*Some methods of inspection*

CLASSIFICATION	REMARKS
Nondestructive	
Commonly used:	
Visual	With such aids as dye penetrants, borescope, etc.
Electromagnetic	Magnaflux, eddy currents, etc.
Others:	
Ultrasonic	Inspection of special parts.
X-rays	Limited value for fatigue.
Destructive	
Metallographic:	
Surface	Primarily in failure analyses.
Fractographic	

(1) particular emphasis on locations judged to be critical and (2) searching first for any suspicious indications and later analyzing each indication for positive crack identification. In such procedures, there is no present method superior to visual inspection.

Visual inspection is frequently aided by the use of dye penetrants and "superpenetrants." These often afford a contrast enabling a crack to be spotted easily. A precaution in any sort of visual inspection is that various imperfections may be difficult to distinguish; a machine mark may look like a crack and vice versa.

Electromagnetic inspection includes such diverse methods as use of magnetic particles and eddy-current probes. The latter may be particularly helpful around discontinuities such as bolt holes.

Other methods of nondestructive inspection have particular usefulness in special situations, but less general applicability in the field than visual and electromagnetic methods. Ultrasonics can be used to locate very small flaws, including cracks, in parts of suitable geometry and such that good coupling to the surface is feasible. This method is more useful for specific components and under laboratory conditions than for general use in the field. X-rays may be used for radiography of spots not directly accessible. In the laboratory, with expert care, radiography can show up small fatigue cracks, but positive identification may be difficult or impossible under field conditions. Other means of locating cracks (thermal detection, electrical-resistance discontinuities, etc.) are at present solely of academic interest.

Destructive means of inspecting for early fatigue are limited to situations which warrant "surgery"—failure investigations and critical investigation of locations highly suspect on the basis of experience and of previous nondestructive examination. These are, however, important. The "metallographic" list might include both surface examination (destructive if much etching or surface removal or any sectioning is involved) and studies of the edges exposed by fracture. For either surface, of the part or of the crack, magnifications from small optical (5X to 50X) to large electron-microscopes (5,000X to 40,000X) may be used, depending on facilities, experience, and total background. Reference *8* summarizes some background available, and references *9* and *10* describe some current fractographic studies toward developing background.

The engineer concerned with fatigue ought to keep closely in touch with inspection procedures applicable to structures of interest to him. Inspection is an area full of compromises between expediency and technological possibilities—a continual challenge to engineering judgment.

Scheduling

Not only the method of inspection but also the scheduling of inspections is of concern in the prevention of fatigue. An illustration was given in chapter XVI, where figure 133 indicates, in regard to one aspect of fail-safe design, the interrelation between minimum detectable crack length and interval between inspections.

Minimum inspection intervals are set up by government agencies (including the FAA for civil aircraft) on the basis of many considerations. Included must be cost (in time and logistics as well as dollars), technical facilities and capabilities, and service experience (including that for engine parts and other components where time factors such as wear and corrosion are important). Fatigue considerations cannot be the only criteria.

Moreover, not all scheduled inspection intervals will be precisely the same throughout service lifetime. In addition to variations in some of the factors mentioned above, alteration and addition of parts and change of operational requirements may dictate change in the inspection schedule. For various reasons, there may be nonperiodic inspections.

In principle, every inspection that includes examination of any structural part can add to the accumulation of information pertinent to estimation of lifetime without fatigue failure. In practice, it is unrealistic to expect perfection in collecting and assessing all inspection data. However, the engineer concerned about fatigue should:

1. Seek to incorporate inspection results in a continually developing assessment of fatigue reliability—for a specific vehicle, for vehicles

of a fleet, and for air and aerospace vehicles in general.

2. Provide a proper input in recommending inspection intervals (as well as procedures) from the point of view of prevention of fatigue failure.

Inspection must be considered an important factor in the total objective of obtaining optimum reliability against fatigue.

ALTERATIONS

Alterations in Structure

Changes in structural parts may be made for many purposes:

1. Replacement by an improved design of a part found unsatisfactory in service.
2. Replacement by a new design to afford improved performance or capability for a change in mission.
3. Addition for some operational objective, etc.
4. Rework to improve fatigue resistance. Examples include: removal of damaged material in holes, improving a poor surface, increasing radii of necessary section discontinuities, shot-peening, redrilling holes and insertion of taper locks.

Whenever a change is made, effects on *fatigue resistance* of the part changed and of adjacent parts which might incur different loads *should be considered.* In the past such consideration has not always been adequate.

Current practice includes, to a varying extent:

1. Calculation of the effect of the alteration on fatigue resistance.
2. Laboratory testing of the changed or added part.
3. Flight testing of the vehicle with the alteration.
4. Reconsideration of subsequent inspection procedures and schedules.

One of the difficult decisions concerns the possible influence of a structural change upon other parts of the vehicle structure—including the potential effect of possible failure of the added or changed part.

After an airplane has received any major overhaul, it is commonly given a postoverhaul test flight. This aids in checking out the effectiveness of the overhaul and of any alterations. The extent to which it adds to insurance of fatigue reliability depends partly upon details of the test. Here is another point at which a compromise must be made between the effectiveness of the test flight or flights and the cost.

Alterations in Mission

Within the past decade, aircraft structures engineers have recognized the significant influence of variations in actual operation upon the fatigue lifetime of a vehicle or group of vehicles. Figure 16 in chapter I shows the great variation in loads that can exist for different operations of an airplane. Figure 158 shows some estimated variations in fatigue life with changes in operational procedure (*10*). In a paper some years ago, Parmley wrote: "If there is one aspect with the largest possible variance, which will affect the damage incurred by

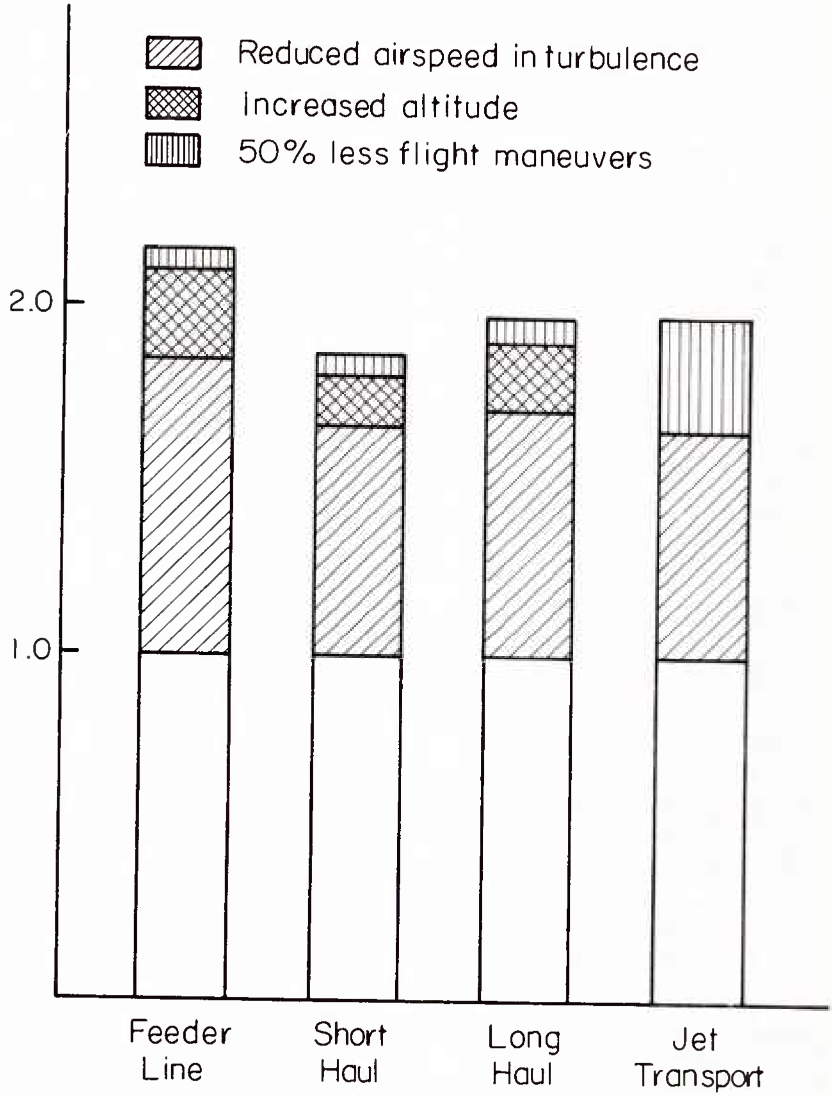

FIGURE 158.—Relative (fatigue) lifetime (shaded areas represent potential increases with extreme care in operations).

an individual aircraft or by the entire fleet of a given system, it undoubtedly will be the kind of missions and the location or environment in which these missions are flown. . . If any change to the mission planning or a shift of the system to another command is to be made, it is not only desirable but essential that the using command submit these to the cognizant engineering agency for the evaluation of its effect on the projected vehicle life" (*11*).

THE PURSUIT OF RELIABILITY

The overall objective of engineering is to provide increased reliability against failure (by any mode, including fatigue) during the service lifetime of any aerospace vehicle in any mission. The technology of reliability of a multicomponent system has received much attention during the past decade. Papers presented in a recent conference on "Aerospace Reliability and Maintainability" illustrate the problems involved in consideration of all aspects of the reliability of a complex system in a complex environment (*12*).

Much of the available statistical methodology, which was developed for small electronic components, is not applicable to analysis of the reliability of a flight vehicle against failure by fatigue. Use of the "mean time between failures" is not acceptable because the real concern is the time to the first failure. The statistics of very small samples has not been developed. Fatigue testing, which is both time-consuming and destructive, cannot be carried out on a large number of specimens —even of relatively simple components—to afford suitable values of parameters necessary in many reliability statistics. Moreover, assumptions of uncertain validity are required to utilize any data on materials or small components for estimation of vehicle reliability: assumptions concerning probable loads and environments, local distribution of such loads, cumulative local fatigue damage, and effects of local damage on performance and safety. Development of data to utilize many reliability approaches will require considerable effort over a long period of time, so that some approaches have little current engineering promise.

There are, however, several avenues promising progress toward better prediction and control of reliability.

On the theoretical side, development of Bayesian statistics holds some long-range promise. An illustration of this applied to a four-component system is given in reference *13*. The basic idea is to assume, on the best evidence available, prior component reliability distributions from which a system reliability can be computed (by assumption in regard to the mutual independence of component distributions) and subsequently modify these assumed component distributions by failure experience. However, the methodology has not been worked out to a degree suitable for the complexities involved in real aircraft structures under real operating conditions. Hence, this approach is currently academic, but not to be disregarded for future possibilities.

A different approach consists of attempting to construct some probabilistic model of strength degradation with time (because of fatigue, or creep, or corrosion, or a combination of these). (See, for example, reference *14*.) This procedure, which has seemed useful in some electronic reliability analyses, has not yet been developed suitably for dependability with respect to cumulative fatigue damage in structures.

Neither these nor other statistical procedures have reached a stage of practical application toward dependable prediction of probable service life. One current hope is that probabilistic considerations will at least help guide better experiments (*15*).

Meanwhile, practical assessment of reliability proceeds on an engineering basis that is frequently qualitative, always involves compromises, and is in continual development (*16*). Factors involved are: (1) defining the operational environment, (2) designing to withstand this environment, (3) testing, (4) inspection, and (5) monitoring.

RECAPITULATION

The objective of this chapter has been to emphasize the consideration of details of service experience in regard to reliability against fatigue. Recording in initial flight tests provides information about load distribution and actual strains at critical locations; recording of a limited number of parameters during long service provides information about actual environments and loads. Inspection for possible cracks and for damage that may initiate cracks provides not only a safety check but also a fund of evidence concerning cumulative damage. Alterations in structure or in mission assignment require reassessment of expected fatigue lifetime. Although much effort has been expended toward a probabilistic evaluation of reliability, practical determination of reliability against fatigue failure requires all inputs from flight testing and records, from inspection records, and from continual consideration of effects of alteration of structure or of mission assignment.

There are two kinds of output from service experience. One is the frequent recognition of some factor unanticipated in design: a manufacturing or assembly error, a complex load-structure response, an unanticipated load or environment factor. The other is a slowly developing body of information including statistics of load occurrences and of fatigue behavior of complex structures. Both are important. The engineer concerned with fatigue has a responsibility to keep continually in touch with actual service experience, both to provide maximum service toward the reliability of a particular vehicle type and to increase his skill in the "art" of prevention of fatigue in aerospace vehicles.

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APPENDIX A. FATIGUE UNDER COMBINED STRESS

In many structural parts, the service loads include combinations such as bending and torsion or axial loading and shear.

There has been much work over the past years toward establishing rules for design to prevent fatigue in such combined-stress situations. The objective of much of the work has been to find a rule for predicting fatigue strength under any system of stresses, when the fatigue strength has been determined by experiment under one (simple) stress condition. Good discussions of this work may be found in references *1*, *2*, and *3*. The following notes indicate some of the main suggestions that have been advanced.

FULLY REVERSED BIAXIAL STRESSES

Figure A–1 shows an element subject to tensile stresses, S_x and S_y, in two directions and to shear, S_{xy}. It can be shown that the stress normal to a plane at an angle θ to the x-axis will be maximum when

$$\tan 2\theta = \frac{2S_{xy}}{S_x - S_y}.$$

This is called the maximum principal stress, S_1. The minimum principal stress, S_2, is at 90 degrees to this direction; the maximum shearing stress, S_s, is at 45 degrees to S_1. Values of S_1, S_2, and S_s in terms of S_x, S_y, and S_{xy} are indicated on the figure. Also indicated is U_d, the elastic distortion energy.

Three suggestions that have been advanced are that fatigue occurs:

1. When the *maximum principal stress* reaches some critical value:

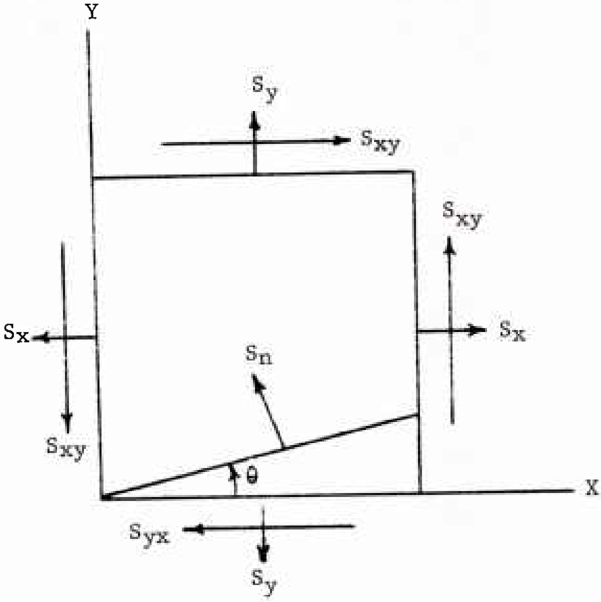
$$S_{cr} = S_1 = \frac{S_x + S_y}{2} + \left[\left(\frac{S_x - S_y}{2}\right)^2 + S_{xy}^2\right]^{1/2}.$$

Note that, for uniaxial stress, S_x:

$$S_{cr} = S_x,$$

while for pure torsion, S_{xy}:

$$S_{cr} = S_{xy}.$$



S_1 = PRINCIPAL STRESS

$$= (S_n)_{max} = \frac{S_x + S_y}{2} + \left[\left(\frac{S_x - S_y}{2}\right)^2 + S_{xy}^2\right]^{\frac{1}{2}}$$

S_2 = PRINCIPAL STRESS

$$= (S_n)_{min} = \frac{S_x + S_y}{2} - \left[\left(\frac{S_x - S_y}{2}\right)^2 + S_{xy}^2\right]^{\frac{1}{2}}$$

S_s = MAX SHEAR

$$= \left[\left(\frac{S_x - S_y}{2}\right)^2 + S_{xy}^2\right]^{\frac{1}{2}}$$

$$= S_1 - S_2$$

U_d = DISTORTION ENERGY

$$= \left(\frac{1+\nu}{E}\right)\left(S_1^2 \div S_2^2 - S_1 S_2\right)$$

$$= \left(\frac{1+\nu}{E}\right)\left(S_x^2 - S_x S_y + S_y^2 + 3\ S_{xy}\right)$$

FIGURE A-1.—Nomenclature for consideration of biaxial stresses.

Hence, this suggestion would imply a fatigue strength in reversed shear equal to that in reversed direct stress.

2. When the *maximum shear stress* reaches a critical value,

$$S_{cr}=S_1-S_2=\left[\left(\frac{S_x-S_y}{2}\right)^2+S_{xy}^{\ 2}\right]^{1/2}.$$

For uniaxial stress, S_x,

$$S_{cr}=\frac{S_x}{2}, \quad \text{or} \quad S_x=2S_{cr}.$$

For pure shear, S_{xy},

$$S_{cr} = S_{xy}.$$

Hence, this hypothesis implies a fatigue strength in reversed shear one-half that in reversed uniaxial stressing.

3. When the *elastic distortion energy*[1] reaches a critical value:

$$U_{d,\,cr}=\left(\frac{1+\nu}{E}\right)(S_1^{\ 2}-S_1S_2+S_2^{\ 2})$$

$$=\left(\frac{1+\nu}{E}\right)(S_x^{\ 2}-S_xS_y+S_y^{\ 2}+3S_{xy}^{\ 2}).$$

For uniaxial stress, S_x:

$$U_{d,\,cr}=\frac{1+\nu}{E}S_x \quad \text{or} \quad S_x=\sqrt{\frac{E\,U_{d,\,cr}}{1+\nu}}.$$

For pure shear, S_{xy}:

$$S_{xy}=\frac{1}{\sqrt{3}}\sqrt{\frac{E\,U_{d,\,cr}}{1+v}}.$$

This implies a fatigue strength in reversed shear equal to 0.577 $(=1/\sqrt{3})$ times that in reversed axial loading.

Stulen and Cummings (*4*), in an examination of 128 S–N curves, found a ratio of shear fatigue strength to axial fatigue strength from 0.48 to 0.91, with a median value of 0.62. Thus the effective distortion energy (or effective octahedral shear stress) fits a number of experimental results.

Some results, particularly in biaxial tension, do not fit any of the theories well. Gough (*5*) suggested an empirical relation

$$S_x^{\ 2}/S_{0x}^{\ 2} + S_y^{\ 2}/S_{0y}^{\ 2} = 1$$

---

[1] It is of considerable academic interest to note that essentially the same criterion results from quite different physical hypotheses: (1) a limiting shear stress on an octahedral plane, (2) a condition on the elastic stress invariants, (3) a critical shear stress on each of many randomly oriented crystal planes (see reference *2*).

where S_{0x} is the observed fatigue strength for uniaxial loading in the x-direction, and S_{0y} is the observed fatigue strength for uniaxial loading in the transverse y-direction.

There have been a number of attempts to modify one of the more basic hypotheses to include this effect of anisotropy of materials. One is the *modified-shear-stress* hypothesis:

$$S_{eff}=S_1-\lambda S_2=\frac{S_x+S_y}{2}(1-\lambda)+\left[\left(\frac{S_x-S_y}{2}\right)^2+S_{xy}^{\ 2}\right]^{1/2}(1+\lambda),$$

where λ is to be determined experimentally. The ratio $\frac{1}{1+\lambda}$ of shear fatigue strength to uniaxial fatigue strength is thus adjusted to fit observations. When this is done, the hypothesis apparently fits quite well results under other stress conditions. A modified octahedral stress hypothesis has also been advanced.

Effect of Mean Stresses on Biaxial Fatigue

There have been several suggestions for handling combined stresses when the mean stress is different from zero. Marin (*2*) suggests that the maximum octahedral stress and the mean octahedral stress be incorporated in a uniaxial constant-lifetime diagram. Sines (*1*) has proposed that the alternating octahedral shear stress be modified linearly by the mean hydrostatic stress invariant. Crossland (*6*) used the maximum hydrostatic stress; Findley (*8*) and Stulen (*4*) have recommended modifying the alternating shear stress by a normal stress component; Yokobori (*7*) has presented a "unified theory" in terms of dislocation pile-up near an obstacle.

To date, there is no widely accepted theory for combined-stress behavior when the loading is not fully reversed. Many observations suggest modification of either of the suggestions (critical shear stress or critical octahedral stress) that appear useful for fully reversed stressing.

OTHER COMBINED-STRESS SITUATIONS

While fatigue failures usually start at a surface where the stress condition is biaxial, subsurface failures, at locations of triaxial stresses, may occur. One example of this is the rolling load failure where subsurface shear stresses can be critical. The ideas already mentioned for biaxial stressing have been suggested for triaxial stress conditions: effective shear stress (unmodified or modified) or effective octahedral stress (unmodified or modified). Experimental data for definitive evaluation of such ideas are difficult to obtain in view of the many variables in rolling-ball experiments. At present the idea of an effective alternating octahedral stress is frequently used; this appears to be conservative, but the situation must be regarded as basically unknown.

The effect of combined stresses upon *notched specimens* has been explored. Peterson (*9*) suggests an effective stress-concentration factor based upon the critical octahedral shear criterion

$$K_t' = K_t[1 - c + c^2]^{1/2},$$

where $c = S_y/S_x$ at the root of a notch, and K_t is the theoretical stress-concentration factor defined in terms of maximum stress (in the x-direction). For some common materials, K_t' seems nearer to K_f (observed) than is K_t; however, the two stress-concentration factors may not differ much. Peterson has some discussion of combined-stress relationships in general.

Relatively little has been reported on *combined-stress* effects in low-cycle fatigue. The hypotheses suggested for long-lifetime fatigue have been tried to a limited extent without conclusive results. Halford and Morrow (*10*) have suggested using, in Coffin's relation (see ch. VII), octahedral shear strains instead of normal strains.

Cumulative fatigue damage under combined stresses has scarcely been investigated. Particularly in this connection, separation of the process of fatigue into crack initiation and crack propagation remains a field for study in combined-stress situations.

ENGINEERING DESIGN

The foregoing brief review indicates many uncertainties for handling design to prevent fatigue under combined stresses. This is hardly surprising in view of knowledge that the mechanism of fatigue is related to extremely localized and nonelastic reactions of a metal. However, there is a practical problem of choosing, for any specific situation, a reasonable approximate rule.

As in other fatigue design problems, the optimum rule for a particular situation depends greatly upon the available pertinent information. Data on components, when available under loading suitably close to that anticipated in service, "beg the question" by including many of the factors. When the only available data are from uniaxial stressing of a material coupon, some assumptions are needed. For long-lifetime fatigue of reasonably ductile metals, the distortion energy or octahedral shear stress principle seems reasonable. High mean stresses and/or a highly anisotropic material are conditions less well fitted by this; a modified (shear stress or octahedral stress) rule appears desirable if the data afford a value for a suitable parameter.

For many practical situations, allowance for combined stress can be made only on a speculative basis. For any real dependability, the speculation should be checked by experiment. The various hypotheses can be used to define a reasonably critical experiment.

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APPENDIX B. STATISTICS

INTRODUCTION

If many nominally identical specimens are tested in fatigue under nominally identical loading, they will fail at varied lifetimes. Table B–1 illustrates the more than 10-to-1 range of lifetimes that may be observed in careful tests. This "scatter" in observed lifetimes raises questions such as:

1. What is a good way to characterize the lifetimes of such a population?
2. From results of tests on n specimens, what prediction can be made about the expected lifetimes of other specimens (which might be parts of a structure)?
3. If two groups of specimens give distributions that overlap but show some difference in average lifetime, what significance can be attached to this difference?

Statistical analysis clarifies such questions and provides some answers.

SOME STATISTICAL CHARACTERIZATIONS

One way of characterizing such an array of lifetimes is to choose some measure of a central lifetime around which the observed values

Table B–1.—*Fatigue lifetimes, rotating-bending, unnotched specimens, single level of stress amplitude*

Spec. No.	Lifetime, N, kc.	Spec. No.	Lifetime, N, kc.	Spec. No.	Lifetime, N, kc.	Spec. No.	Lifetime, N, kc.
1	103	11	190	21	330	31	220
2	330	12	230	22	145	32	210
3	190	13	360	23	220	33	230
4	195	14	260	24	120	34	270
5	180	15	160	25	70	35	175
6	440	16	260	26	110	36	40
7	175	17	125	27	84	37	320
8	105	18	60	28	108	38	130
9	345	19	120	29	540	39	201
10	140	20	62	30	410	40	170

are grouped and some measure of the deviations from or scatter about this central value.

There are two frequently used measures of the central value. One is the *arithmetic mean,* defined by

$$\mu = \frac{\Sigma N}{n}.$$

For the data in table B–1, $\mu = 204$ kc. Another measure of the central value is the *median* lifetime, a value which is greater than half and less than half all observed lifetimes. Arranging the lifetimes in table B–1 in order of increasing N shows that the median is between 180 kc and 190 kc; in such a case, it is customary to use the mean of these numbers, 185 kc.

From the definition of μ, the sum of the deviations is zero. However, the sum of the squares of the deviations is not zero, and the average of this sum of squares of deviations from the mean provides a measure of scatter. This measure has the dimension of (lifetime)^2; hence, its square root, which has the dimension of lifetime, is more convenient in practice. A measure of scatter often used is the standard deviation

$$\sigma = \sqrt{\frac{\Sigma (N - \mu)^2}{n-1}}.$$

The value $(n - 1)$ rather than n can be justified theoretically. For the numerical example in table B–1, $\sigma = 36$ kc.

There are othei measures of the central value of the scatter, but discussion of these is somewhat beyond the scope of this note.

Distributions

More detailed consideration of the distribution of the values of N is helpful toward discussion of such questions as the latter two in the introductory paragraph.

A graphical picture can be obtained by dividing the total range of observed values of N into equal increments (say 50 kc.) and plotting a bar chart with the height of each bar representing the number of specimens with lifetimes in that increment. Table B-2 (pt. A) shows the data from table B-1 rearranged for this purpose, and figure B-1 (A) shows the chart or *histogram* resulting. Note that this figure has, as ordinates, the fractional number of failures rather than the actual number—this is convenient for many purposes. The dotted curve in Figure B–1 (A) represents the distribution which one might expect intuitively to be approximated by histogram of the lifetimes of an indefinitely large number of specimens.

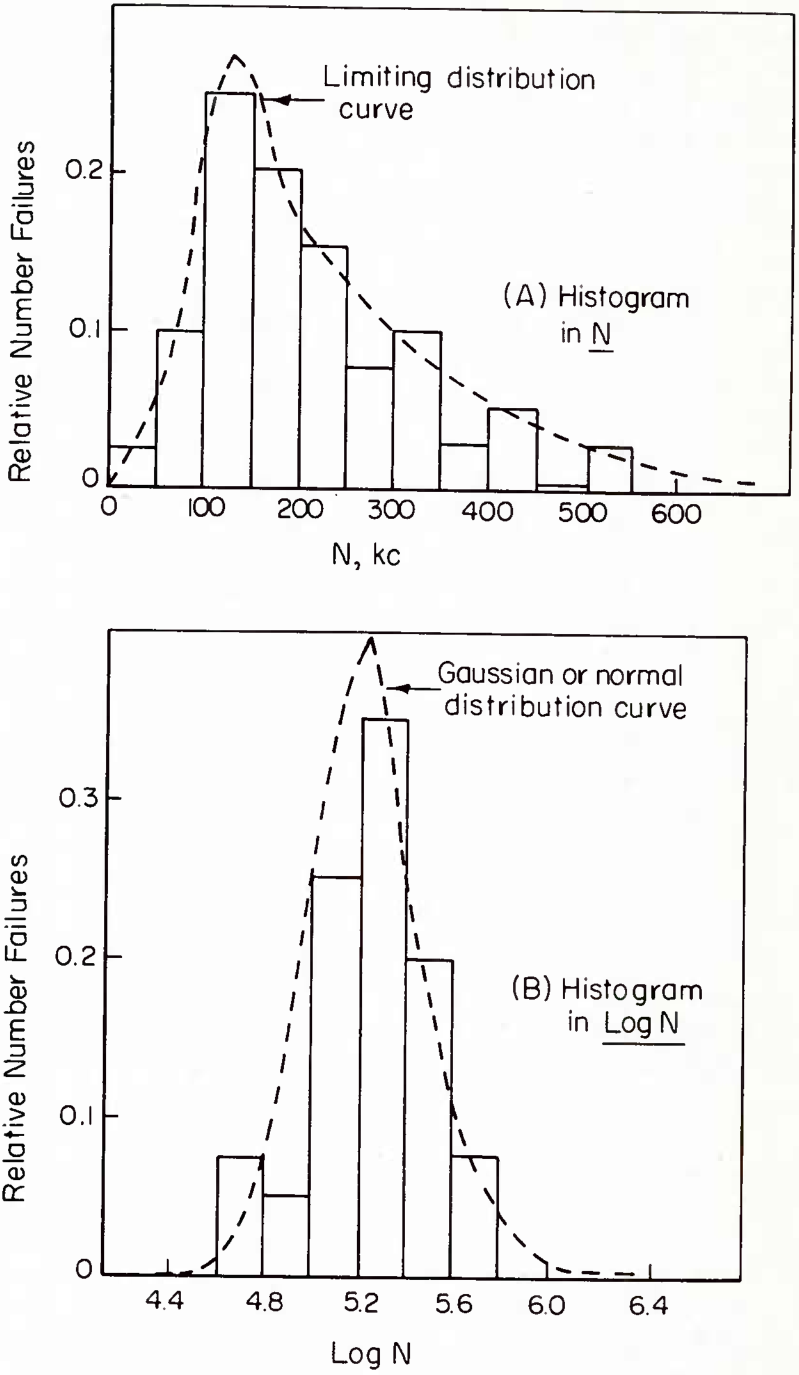

FIGURE B–1.—Histograms and frequency distributions for data in Table B–1.

TABLE B-2.—*Data from table B-1 (rearranged)*

Part A			Part B		
Lifetime interval, kc	Number of failures observed	Relative number	Log N interval	Number of failures observed	Relative number
0–49	1	0 025	4.60–4.79	3	0.075
50–99	4	.100	4.80–4.99	2	.050
100–149	10	.250	5.00–5.19	10	.250
150–199	8	.200	5.20–5.39	14	.350
200–249	6	.150	5.40–5.59	8	.200
250–299	3	.075	5.60–5.79	3	.075
300–349	4	.100			
350–399	1	.025			
400–449	2	.050			
450–499	0	.000			
500–549	1	.025			

Consider next another classification of the same data in increments of log N rather than N. Table B-2 shows the data appropriately rearranged and figure B-1 (B) is the resulting histogram. Clearly this histogram is more symmetrical than that in terms of N. In fact, the dotted line, which approximates the outline of the histogram in *log N*, is drawn to fit the relation

$${}_a\int^b f(\log N)\ d(\log N)$$

$$= \text{relative number of failures from } N = a \text{ to } N = b$$

$$= \frac{1}{2\pi}\ {}_a\int^b e^{-\frac{1}{2}\left(\frac{\log N - \overline{\log N}}{\sigma}\right)^2}\ d(\log N)\ ,$$

where log N and σ denote the mean and the standard deviation of log N. This is the widely used Gaussian or normal distribution of the variant log N. The approximate fit of the normal distribution curve to the histogram is an indication that the fatigue lifetimes are approximately "log-normal."

In the intermediate range of lifetimes, fatigue data often fit a log-normal distribution. At the fatigue limit, it is sometimes possible to define percentage of survivals to a specified lifetime in terms of a normal distribution with respect to stress level. There are few data for low-cycle fatigue.

There have been other distributions advocated for fatigue: in particular, "extreme-value" distributions (*1*) and the Weibull distribution (*2*); these are somewhat more complicated than the normal distribution. Several other distribution functions have been studied mathematically (*3*) but have not been advocated widely in fatigue of materials. Accordingly, the following illustrations of application of distribution functions will be limited to the normal or Gaussian distribution.

The Normal or Gaussian Distribution

From a set of data such as those in table B–1, an idea of the distribution can be obtained by observation of histograms such as those in figure B–1. A quick way of estimating whether or not a distribution is normal is to plot per cent failures on probability graph paper which is so designed that a normal distribution is represented by a straight line. Table B–3 represents the 40 lifetime observations in table B–1 arranged in increasing *N*. The number of each item is divided by 41 ($n + 1$) to represent the percentage of failures in the intervals up to that item. This percent is plotted against *N* in figure B–2 and against

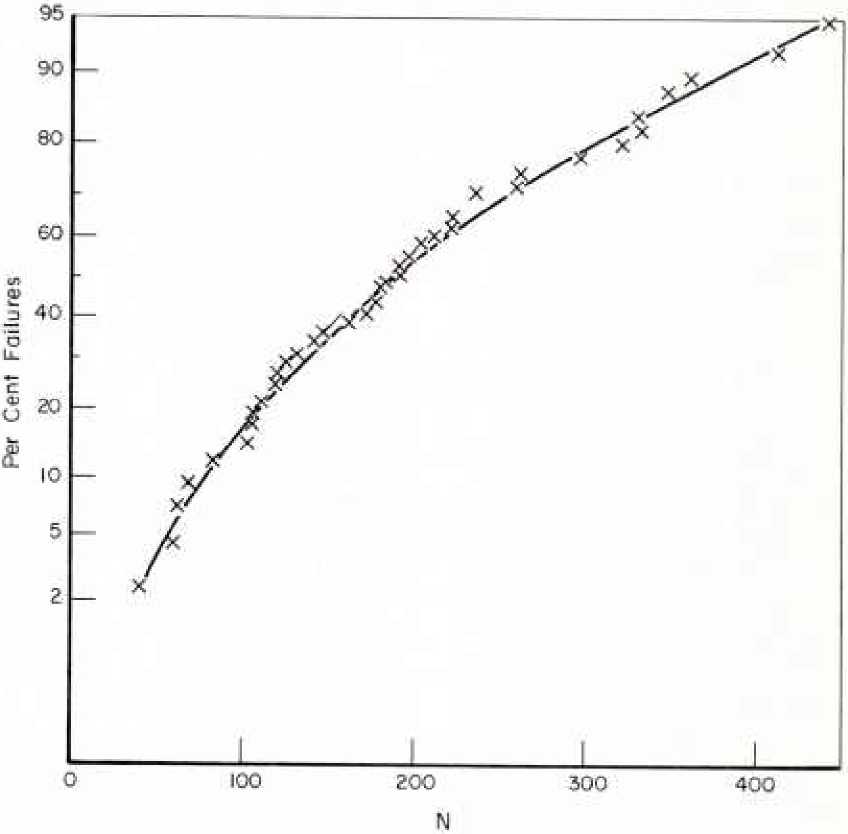

Figure B–2.—Probability plot of distribution in *N*.

Table B–3.—*Arrangement of data for use of probability paper*

Number of failure	Lifetime, kc	Percent failed
1	40	2.4
2	60	4.9
3	62	7.3
4	70	9.7
5	84	12.2
6	103	14.6
7	105	17.1
8	108	19.5
9	110	22.0
10	120	24.4
11	120	26.8
12	125	29.2
. .	. . .	. . .
. .	. . .	. . .
. .	. . .	. . .
. .	. . .	. . .
38	410	92.7
39	440	95.2
40	540	97.3

log *N* in figure B–3. The latter plot can be reasonably fitted by a straight line indicating that values of log *N* fit a normal distribution. In figure B–3 the value of *N* at which the straight line crosses the ordinate for 50-percent failure is the *median value of N* for the distribution; the logarithm of this value of *N* is log *N;* the slope of the line is a measure of the standard deviation (a steep line indicating a low σ or little scatter) .

Thus, values of log *N* and of σ for log *N* can be obtained by computation or by graphical methods. These are values for the sample of 40 specimens tested. If we assume that the distribution of the larger population from which the sample was drawn (randomly) is also log-normal, then tables available for the normal distribution provide information about the likely values of the mean and of the standard deviation of the whole population. More specifically, it is possible to compute an interval of log *N* about the sample mean such that the probability that this interval contains the population mean can be stated. An estimate of the standard deviation for the population can also be obtained (*4*) . With this information about the population, an estimate can be made concerning the probable lifetime of the next specimen or specimens drawn at random from this population. Thus,

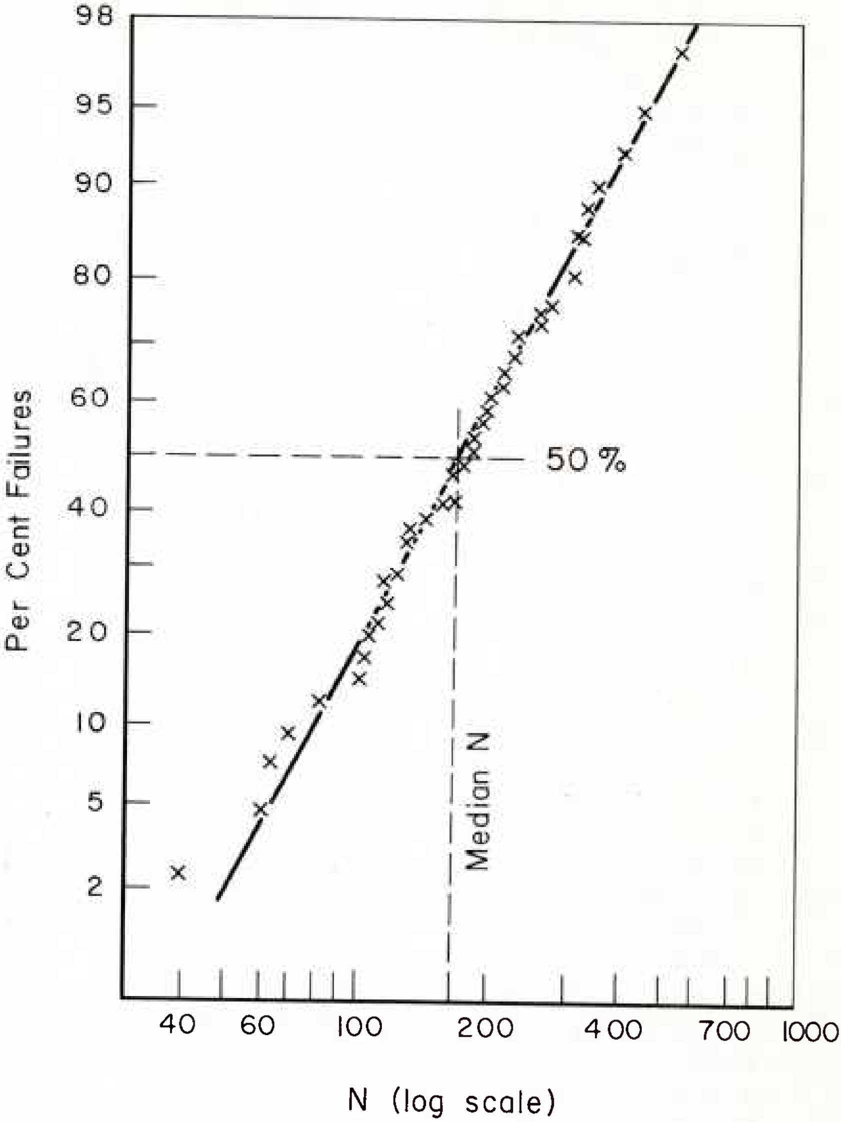

FIGURE B—3.—Probability plot of distribution in log *N*.

there is information concerning the second question mentioned in the introduction.

Perhaps equally important is that, if mean values and values of standard deviation are known for two samples, tables allow tests of the significance of any difference between the two means, and inferences as to whether or not the populations from which the two samples were drawn have been shown to be different. This is information in regard to the third question.

NONPARAMETRIC METHODS

Some significance tests, less efficient than those for situations where a distribution function is assumed, have been formulated for situations which do not involve estimation of the parameters (such as mean and standard deviation) of a distribution function.

Nonparametric significance tests depend upon such assumptions as these: (1) a continuous distribution function exists, even though its parameters are not determined, (2) the sample of test specimens has been drawn at random from the total population. Then, from the observed distribution of specimen lifetimes in the sample, the probability of the lifetime of another specimen falling within some interval can be estimated. Freudenthal and Gumbel (*1*) have discussed the application to fatigue of the statistics of extreme values. Examples of nonparametric methods for fatigue data may be found in reference *4*.

Usefulness of Statistics in Engineering to Prevent Fatigue Failures

It is difficult to suggest the extent to which an engineer should attempt to apply mathematical statistics to practical problems.

Very seldom are sufficient data available to utilize extensively the statistical tables so well developed for large samples. Distribution functions are not well determined for structural materials under all the repeated stress conditions of interest. Moreover, the interaction of stresses is such that parameters (for example, values of standard deviation) obtained from laboratory tests of simple coupons may not be applicable to the distribution of lifetimes of components; component testing is so expensive that a small statistical sample is too large to be practical.

On the other hand, the considerations of statistical reasoning quite properly question the significance of some observations and the extrapolation from limited data to estimation of the fatigue life of an aircraft structure.

The following recommendations are appropriate:

1. The engineer should be cognizant of statistical measures of significance. In general the summary in reference *4* indicates the current status of application of statistics to laboratory data.
2. Often, help in planning experiments and in analyzing data can be obtained from a statistician.

3. The formalism of mathematical statistics should not be used as a substitute for consideration of all of the physical factors that may influence the behavior of a structure.

The last item is particularly important, since there may be systematic errors in an experiment (for example, a fixed error in a load calibration) which may not show in the distribution of observed values.

Finally, the application of statistics to fatigue data is a developing area. As more data are obtained, better ideas of distribution functions may be expected to develop. As more theoretical analyses are completed, it may be hoped that justifiable simplifications may appear. In any specific situation, the engineering challenge is to use the most appropriate analysis, avoiding extreme consideration of details of one kind (for example, variability of material) when other factors (for example, stress distribution) may be very inadequately known, balancing all against economics and acceptable risks.

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APPENDIX C. FATIGUE PROPERTIES OF SOME AIRCRAFT MATERIALS

Results of a great many laboratory fatigue tests on a large number of engineering materials have been reported. The task of collecting these data, resolving discrepancies, and collating all useful information is forbidding; moreover, such a task should be continuous, since data-containing reports are appearing at a high rate.

Some recent books (for example, reference *1*) contain rather extensive tables of data. The Aircraft Industries Association has collected a number of S–N curves (*2*). The Balfour Co. has a fairly extensive amount of data in a punchcard file from which various collections can be retrieved readily (*3*). MIL-Handbook 5 contains fatigue data on several aircraft materials (*4*). From these and other sources (ASTM (*5*), manufacturers' literature, and company data books), it is possible to accumulate several volumes of data on fatigue properties of materials.

This appendix contains information on "typical" fatigue properties of a few materials commonly used in aircraft structures. Proper use of this, or any tabulation of fatigue properties, requires some appreciation of the metallurgical variables concerned, and of the interpolation (and sometimes extrapolation) involved in graphical or tabular presentation of values derived from tests.

METALLURGICAL FACTORS INFLUENCING FATIGUE STRENGTH

Metals and alloys show fatigue behavior characteristic of their metallurgical structure. For example, differences in the fatigue properties of two lots of the same composition, but with different heat treatments, may be anticipated. Castings and forgings, even of the same composition and static strength, may show different fatigue strengths. Casting produces differences between core and skin (which may not always be completely machined off), characteristic residual stresses (perhaps diminished, but often not completely removed, by subsequent heat treatment), and voids and inclusions dependent on details of material and casting practice. Forging, rolling, and extrusion may break up and disperse some voids and certain types of inclusion, and may strain-harden some alloys and introduce other distributions of residual stresses.

Different groups of alloys respond differently to fabrication processes such as casting, forging, rolling, extrusion, and machining. Each group has its own characteristics in response to heat treatment and to hot- or cold-working (note app. D). Hence, the fatigue properties of metals and alloys should be considered with regard to the metallurgical group to which each belongs.

Carbon and low-alloy steels show a rough correlation between long-life fatigue strength and ultimate tensile strength. Sometimes the fully reversed bending-fatigue limit of such a steel is assumed at about 0.50 times the ultimate tensile strength. This is very approximate because (1) a great deal of scatter is observed in this ratio, and (2) the notch sensitivity is usually observed to increase with increase of tensile strength. The fatigue properties of the widely used steels appear to be relatively insensitive to composition(1), but quite sensitive to metallurgical structure and to cleanliness or homogeneity.

Austenitic stainless steels are characterized in long-lifetime fatigue by their resistance to corrosion fatigue and by good notch properties. Their low fatigue-notch sensitivity appears most commonly, however, in the annealed state. For example, work-hardened 18–8 may have about as high fatigue-notch sensitivity as a low-alloy steel at moderate hardness.

Aluminum alloys show two points of contrast with low-alloy steels: (1) they do not have as clear a fatigue limit, and (2) their fatigue limit(2) does not increase linearly with increase in ultimate tensile strength. In fact, some compositions and heat treatments that afford relatively high yield strength and rather high tensile strength may show no increase in long-life fatigue strength over other lower strength alloys.

Metallurgical nature can affect comparative fatigue evaluations in still another significant way: fabrication and finishing procedures vary with alloy type. Machining and heat-treatment variations that influence fatigue are different for aluminum alloys, low-alloy steels, and stainless steels. Surfacing may include such diverse items as anodizing or cladding for aluminum alloys, and carburizing or chrome-

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(1) Except as this influences heat-treatment response. Note also that other factors (machinability, forgability, corrosion resistance) may be sensitive to composition.

(2) Arbitrarily defined as fatigue strength at some long lifetime (specified, for example, as 5×10^8 cycles).

plating for steels. Fatigue cracks usually start at or near a surface, so that fatigue strength is very dependent upon surface conditions.

These comments on the sensitivity of fatigue to metallurgical details imply several precautions:

1. In making fatigue tests for evaluation of a material, both plan and record details of fabrication, heat treatment, and surfacing of test specimens.
2. In interpretation and application of collected data, take due regard for such details.
3. In judging the applicability of a material for use in a structural part, take cognizance of the metallurgical details of the available data in relation to conditions for the structural part. Quality to be expected in production lots, or sensitivity of the material to normal shop practices in fabrication, heat treatment, and surfacing, may drastically affect the relevance of laboratory data to production items.

While these implications are almost obvious in view of the known sensitivity of fatigue to quite localized weaknesses, their neglect has been costly in the past.

ASSEMBLING FATIGUE-STRENGTH DATA

Information on the fatigue behavior of several commonly used aircraft materials is provided in subsequent diagrams (figs. C–2 through C–15).

Before looking at these diagrams, it is worthwhile to consider a few points about the collecting of information on the fatigue properties of materials. Figure C–1 shows some data collected by one laboratory. The values are from tests on several heats (although all taken in a single laboratory and with considerable care—both in metallurgy and in testing). The dotted lines indicate a scatter band suggested as representative; the solid line is that from which values were taken for the constant-lifetime diagram in figure C–9. Only occasionally will data be found for which statistical analysis has been made or for which testing conditions are such as to make statistical analysis straightforward. The results in the succeeding figures are typical in the sense that they are derived by estimation of median values from data considered to be carefully taken, with reasonable documentation as to the materials involved.

Figures C–2 through C–8 are constant-lifetime diagrams for AISI 4340 steel. Tests were on round-bar specimens under axial loading. Unnotched specimens were hand-polished to 10 μ-in. RMS; notches (60-degree grooves) were machined to the same finish. For specimens at

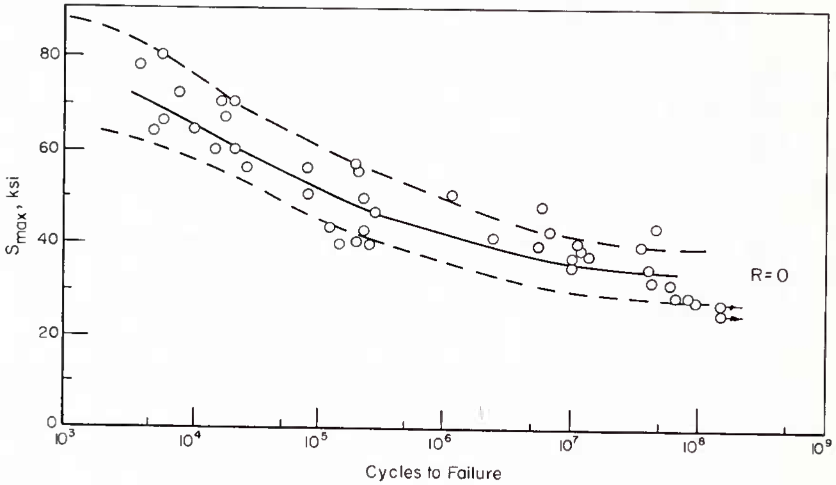

Figure C–1.—Some fatigue data (see text and Figure C–9).

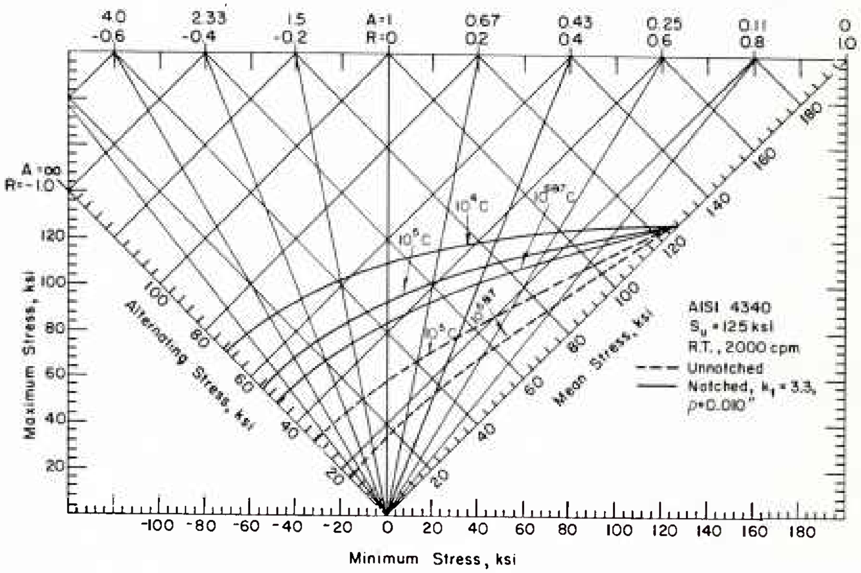

Figure C–2.—Typical fatigue properties, AISI–4340 steel at 125 ksi tensile ultimate.

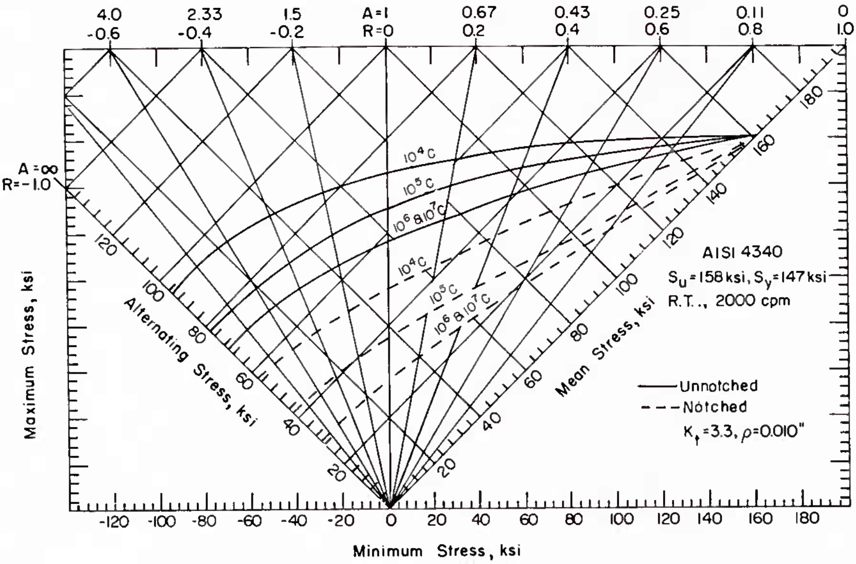

FIGURE C-3.—Typical fatigue properties, AISI-4340 steel at 158 ksi tensile ultimate.

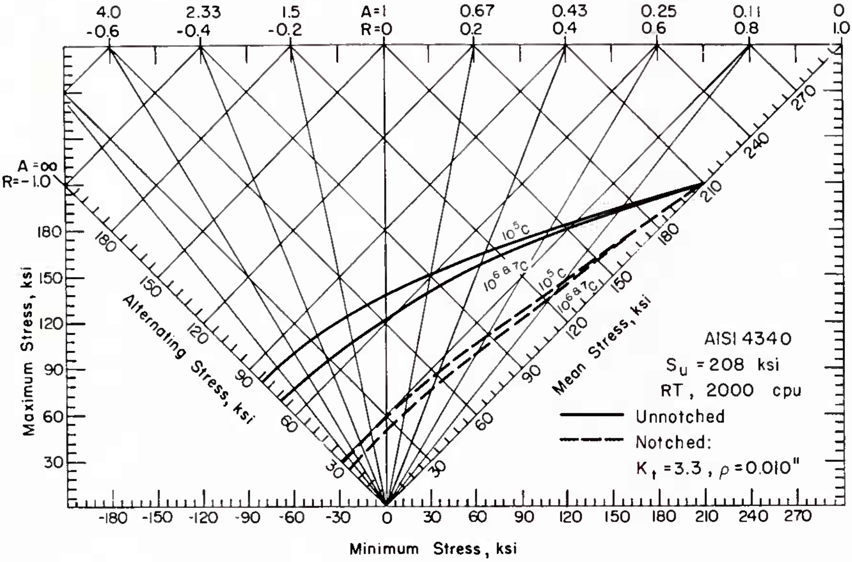

FIGURE C-4.—Typical fatigue properties, AISI-4340 steel at 208 ksi tensile ultimate.

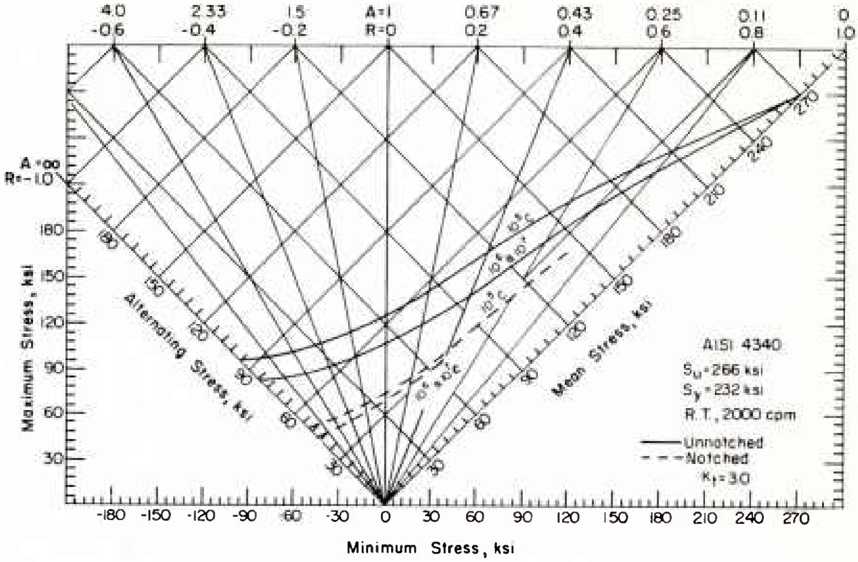

FIGURE C-5.—Typical fatigue properties, AISI-4340 steel at 266 ksi tensile ultimate.

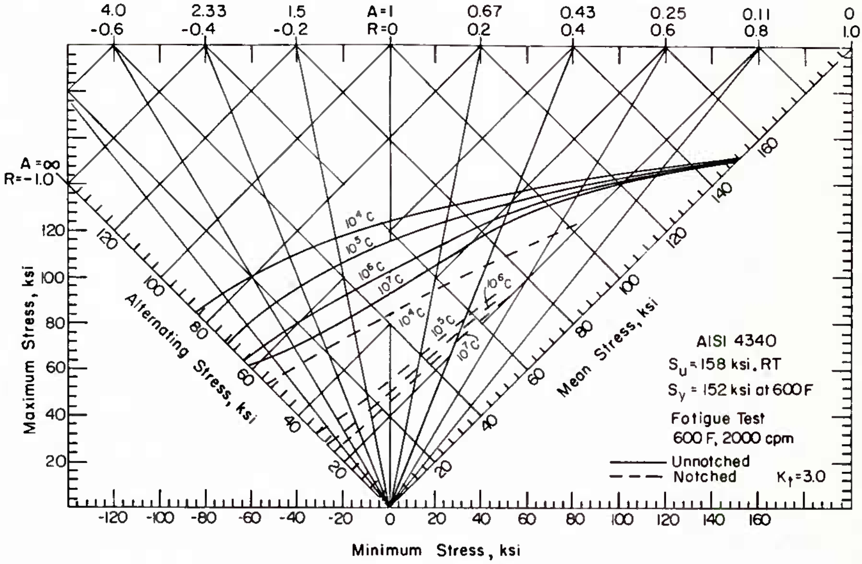

FIGURE C-6.—Typical fatigue properties, AISI-4340 steel, 158 ksi tensile ultimate at room temperature, tested at 600° F.

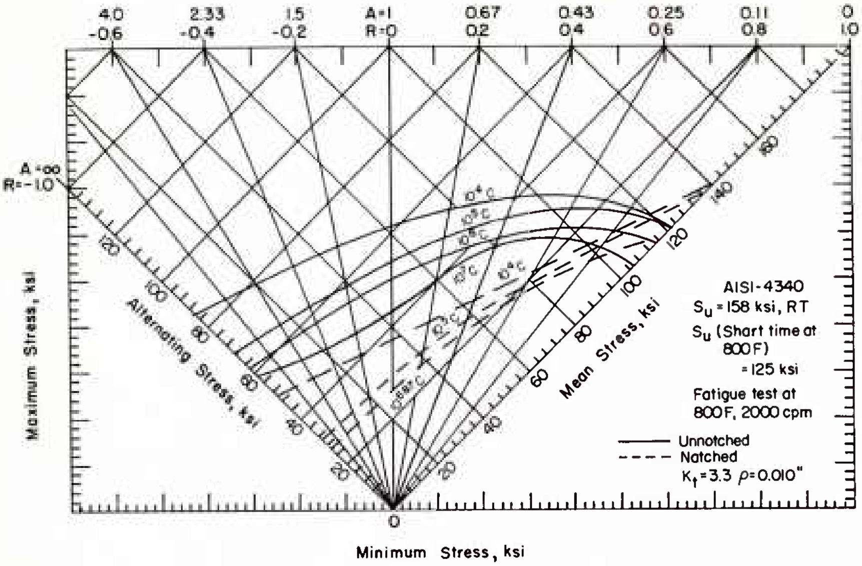

Figure C–7.—Typical fatigue properties, AISI–4340 steel, 158 ksi tensile ultimate at room temperature, tested at 800° F.

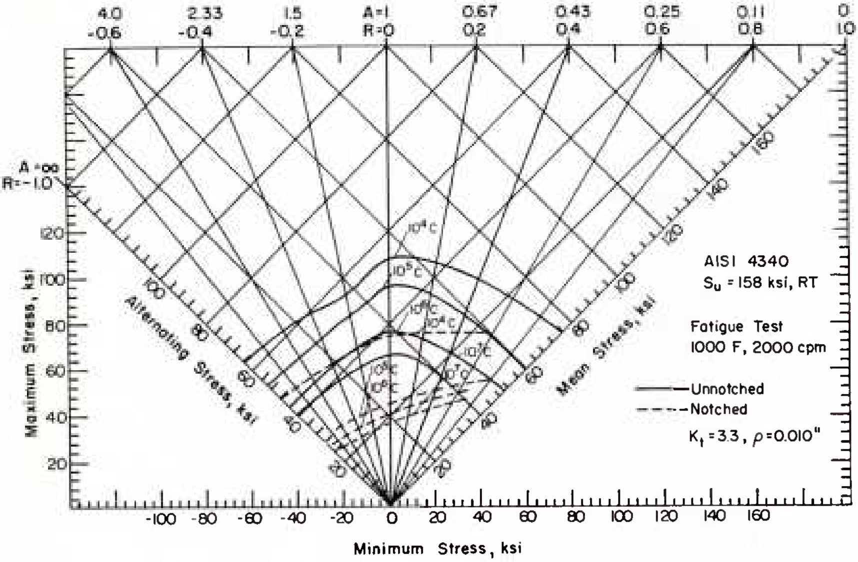

Figure C–8.—Typical fatigue properties, AISI–4340 steel, 158 ksi tensile ultimate at room temperature, tested at 1000° F.

elevated temperatures, values at $S_m = 0$ were obtained from creep-rupture curves for the appropriate time (cycles divided by test frequency).

Figures C–9 through C–12 show constant-lifetime diagrams for various wrought forms of four commonly used aluminum alloys. The results are all for tests under axial loading at room temperature. Unnotched specimens were generally polished with 900 grit; notched specimens were as-machined.

Figures C–13 through C–15 are constant-lifetime diagrams for a titanium alloy tested in sheet form under axial loading.

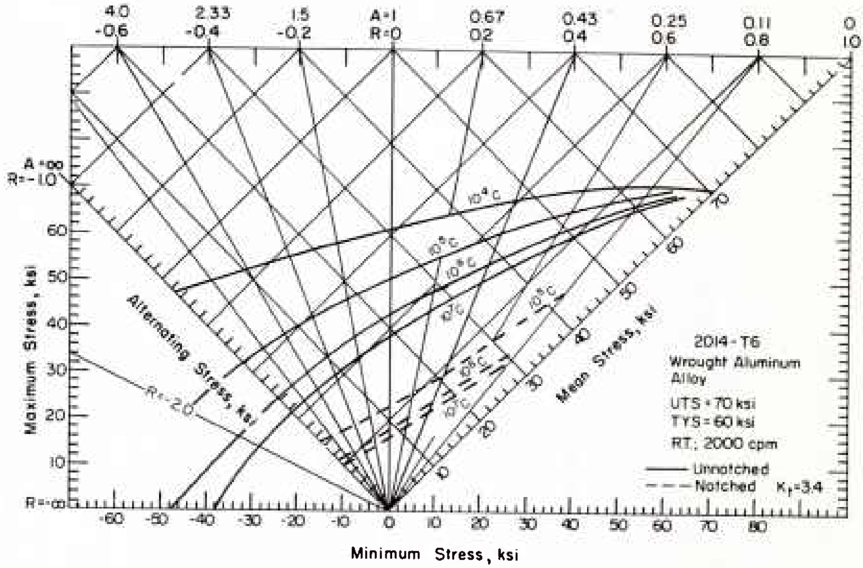

FIGURE C–9.—Typical fatigue properties, wrought 2014–T6 aluminum alloy at room temperature.

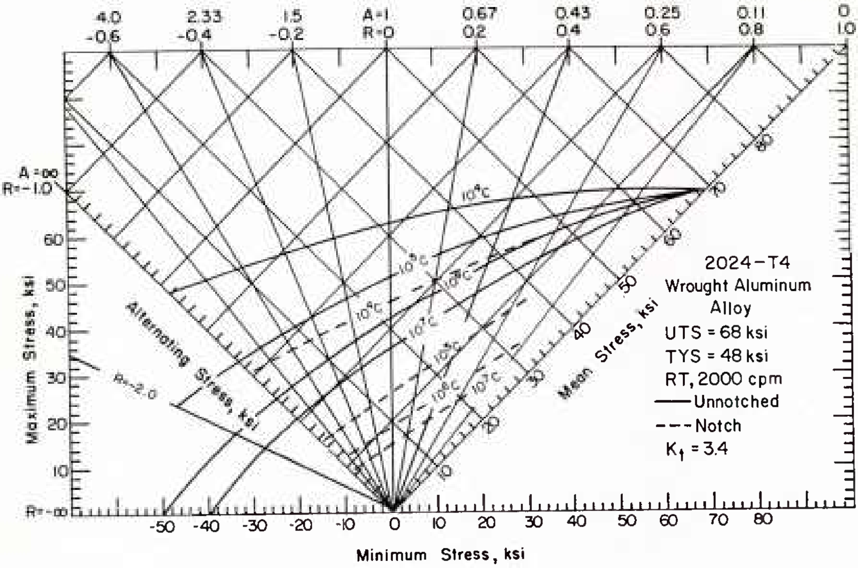

Figure C–10.—Typical fatigue properties, wrought 2024–T4 aluminum alloy at room temperature.

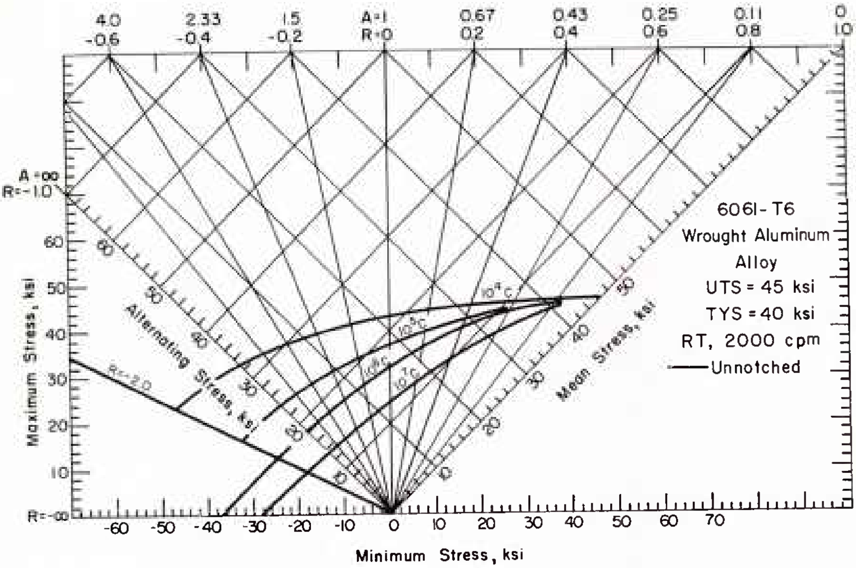

Figure C–11.—Typical fatigue properties, wrought 6061–T6 aluminum alloy at room temperature.

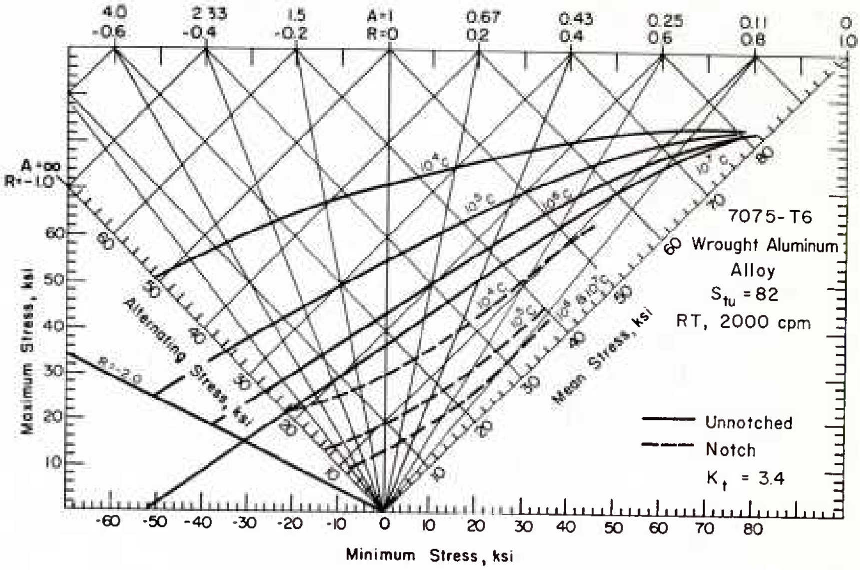

Figure C-12.—Typical fatigue properties, wrought 7075-T6 aluminum alloy at room temperature.

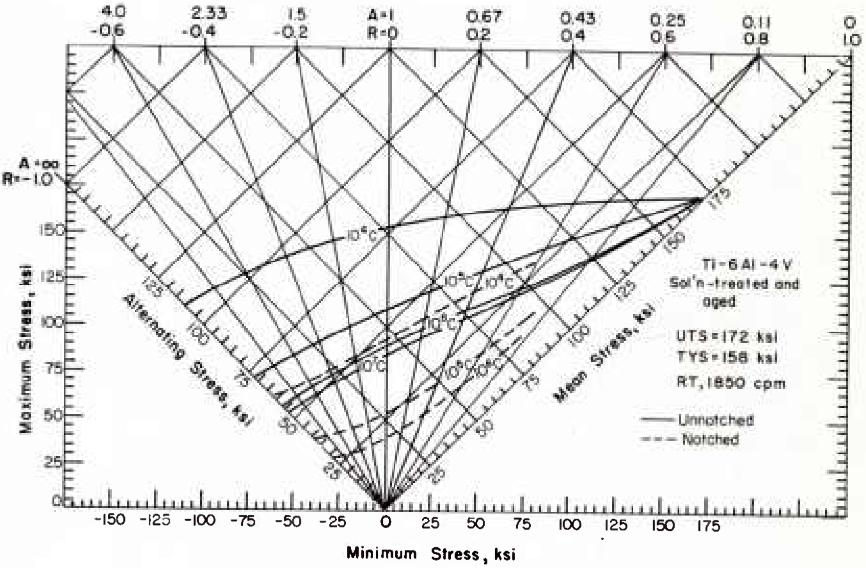

Figure C-13.—Typical fatigue properties, Ti-6Al-4V alloy, solution-treated and aged, tested at room temperature.

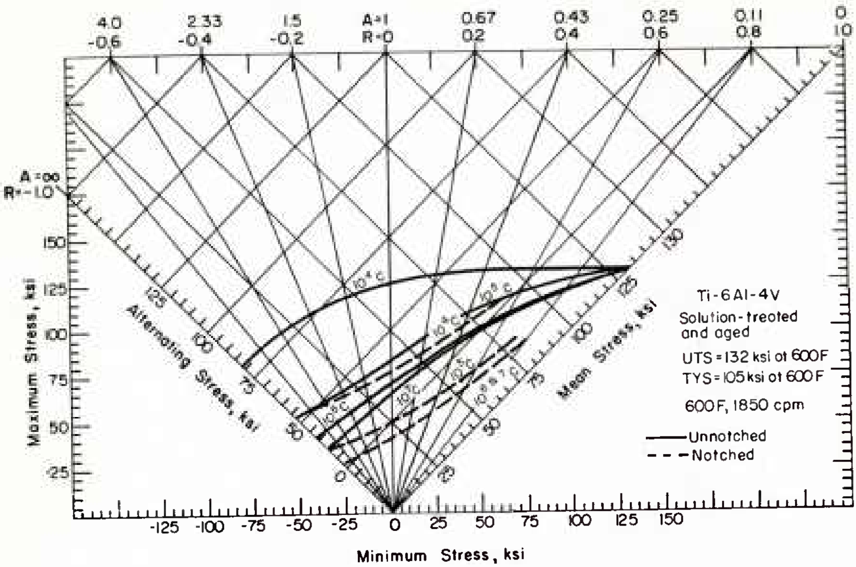

FIGURE C-14.—Typical fatigue properties, Ti-6Al-4V alloy, solution-treated and aged, tested at 600° F.

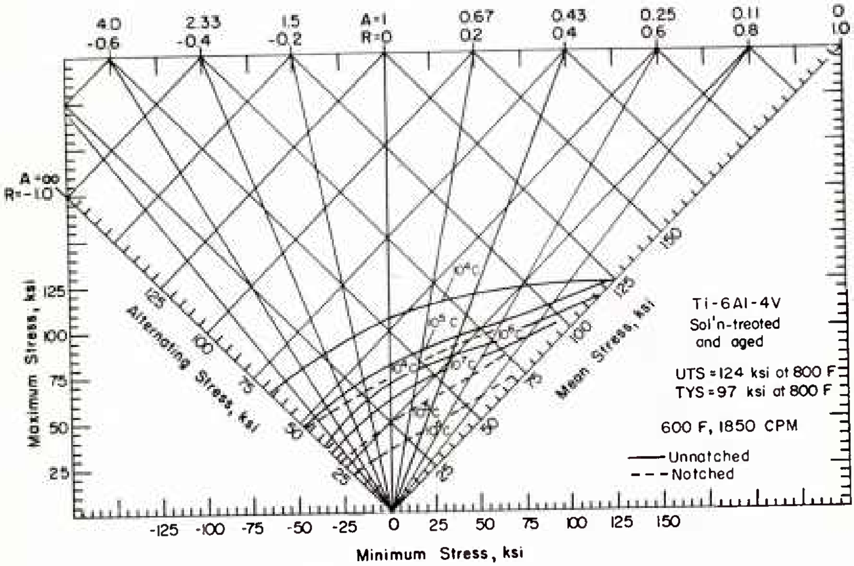

FIGURE C-15.—Typical fatigue properties, Ti-6Al-4V alloy, solution-treated and aged, tested at 800° F.

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APPENDIX D. SOME FACTORS IN PROCESSING METALS

The previous appendix lists "typical" fatigue strengths of a number of alloys used in aircraft structures. Fatigue strengths actually obtained are greatly influenced by processing procedures all the way from melting practice, through forging or extrusion or rolling, to surface finishing of a manufactured component. The influences are qualitatively understandable in view of the known criticality of small discontinuities in the response of a material to repreated stressing. However, the potential importance of each processing step poses difficult questions in engineering.

The following account of some processes of concern in aircraft structures is intended to:

1. Suggest a check list of items worth consideration.
2. Provide suggestions concerning magnitudes of reported effects on fatigue strength.
3. Add references for further information.

In most situations, quality control of a process is so important that extrapolation of an effect reported from some laboratory experiments to shop production can be misleading. However, many known trends are important and should be considered in engineering.

MELTING PRACTICE AND INHOMOGENEITIES

Industrial metals and alloys are not entirely uniform in composition or density. Among the variations in the ingot stage are *porosity, segregation* of chemical elements, and *impurities.* The latter may be present *in solution* or as mechanically held particles called *inclusions.*

Fatigue behavior is sensitive to almost any inhomogeneity, but particular attention has been given to inclusions. The kinds of inclusions in commercial alloys vary with the nature of the alloy as well as with the melting practice used. The size, shape, and distribution of inclusions vary with melting practice and also with forming procedures.

The effect of inclusions upon fatigue strength would be expected to vary with many factors.[1] Cummings, Stulen, and Schulte (*1*) show results of rotating-bending fatigue tests on an aircraft-quality SAE 4340 steel—part of which are reproduced in figure D–1. The large effect possible (note curve 3) is evident. Detailed observations implied the impossibility of a widely applicable simple relation between just inclusion size and fatigue strength. Atkinson provides additional data on several steels and suggests allowance for shape and distribution as well as size (*2*).

Undoubtedly the variation in inhomogeneities resulting from melting practices contributes a large part to observed "scatter" in fatigue-strength properties of laboratory specimens of any alloy. Since there is no positive way to eliminate all inhomogeneities, a precaution is to use the cleanest material possible, especially when resistance to fatigue is important.

FORGING, ROLLING, AND EXTRUSION

While there are a few castings used in particular applications, most alloys used in aircraft structures are in some kind of a wrought condition. Forging, rolling, or extrusion of an ingot tends to break up inclusions and to smooth out inhomogeneities.

Wrought materials usually have somewhat *higher fatigue strengths* than *cast* metals and alloys of comparable static strength. Evans, Ebert, and Briggs report, for the ratios of rotating-bending fatigue limits to ultimate tensile strength, the following values for steels (*3*):

Type	Fatigue ratio	
	Cast	Wrought
Plain carbon	0.40	0.48
Low-alloy	0.42–0.50	0.55–0.60

However, the "notched fatigue limit for the cast and for the wrought steel for any given series and heat treatment was about the same . . .".

The mechanical working of an ingot tends to orient, as well as to break up, inclusions and also to give a preferred orientation to

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[1] For example, the elastic modulus and the yield strength of the inclusions with respect to the surrounding metal matrix, the average size and shape of the inclusions, their orientation with respect to the applied stress, and their spatial distribution.

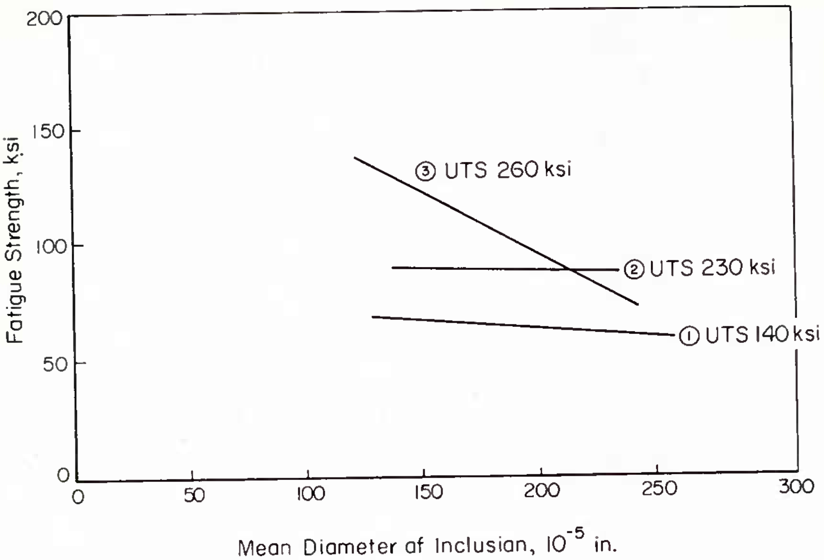

FIGURE D–1.—Rotating-bending fatigue strengths of SAE 4340 steels with different spheroidal inclusions (see reference *1*) .

grains. These two factors produce a directionality in mechanical properties. There are numerous reports of the effect of directionality in fatigue strength of steels (for example, references *3* and *4*) . The long-life fatigue strength transverse to the direction of rolling may be 5 to 30 percent lower than the fatigue strength in the rolling direction—for unnotched specimens. Fatigue strengths of notched steel specimens usually show less directionality. Most aluminum alloys apparently show less effect from directionality than the low-alloy steels; for other materials there are relatively few data.

HEAT TREATMENT AND HARDENING

Transformation Hardening—Steels

Carbon and low-alloy steels are strengthened by a phase transformation from austenite to mixtures of ferrite and carbide. Metallurgical structures (pearlite and bainite) formed under certain conditions of cooling have been generally established as having less desirable mechanical strengths (including fatigue) than the hard martensite obtained by rapid quenching. Consequently, strength is obtained by suitable heat treatments to produce proper forms and distribution of carbides. There are a number of procedures which may be employed, all starting with a high temperature to form austenite and followed by:

1. Rapid *quenching* to a low temperature to form martensite, *and* tempering of the martensite.
2. *Martempering*: Quenching to the highest temperature at which martensite will form, holding for temperature equalization, cooling to a lower temperature to stabilize, and tempering the formed and stabilized martensite.
3. *Austempering*: Quenching to a temperature above that for forming martensite, holding isothermally, quenching to room temperature and forming martensite, and tempering the martensite.
4. *Maraging*: Relatively slow cooling to martensitic transformation, again forming the martensite at relatively high temperature.
5. *Marstraining*: Quenching and tempering, straining the martensite, and then retempering.
6. *Ausforming*: Straining the steel in the bainite range (above the temperature for the formation of martensite), quenching to martensite, and final tempering.

Not all processes are suitable for all steels (particularly in useful section sizes). Ausforming, for example, requires heat treatment and some forming in one final operation. Most of the special treatments (rapid quenching and tempering being considered conventional) require particular care to be successful.

The objective of choosing a heat treatment, within limitations of cost and facilities, is usually to obtain high tensile strength with good ductility and toughness. For the martensitic steels, fatigue strength (for unnotched specimens) seems to increase with increasing tensile strength. Thus, an ausformed H–11 steel in comparison with a quenched and tempered SAE 5160 steel has been reported as showing:

	SAE 5160, quenched and tempered	H–11, ausformed
Tensile strength	250 ksi.	380 ksi.
Fatigue limit	130 ksi.	175 ksi.
Ratio	52 percent.	46 percent.

There are two points about fatigue worth emphasis in the continuing search for higher strength steels. First is that the *notched fatigue strength does not increase as rapidly* with increasing tensile strength as does the unnotched strength. This is illustrated in figure D–2 (adapted from reference *4*). Second, most of the nonconventional heat

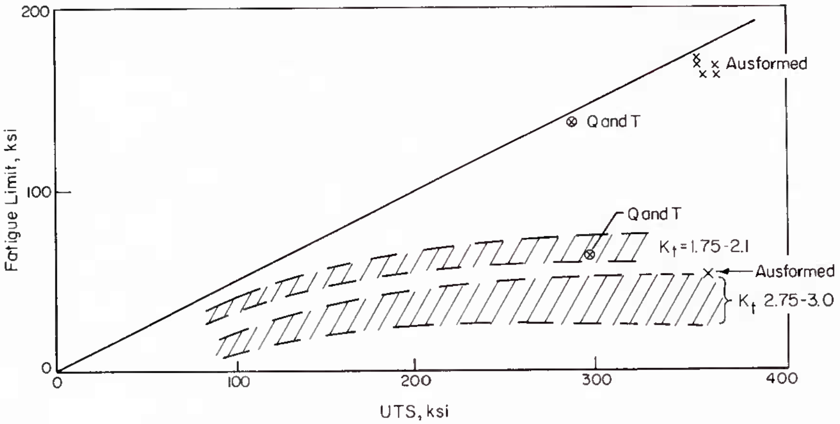

FIGURE D-2.—Some effects of heat treatment of steel on rotating-bending fatigue strength.

treatments of steel require special care (in precise control of temperature, time, and heat distribution) to be effective. Proper heat treatment of steels for effectiveness against fatigue failure is a concern increasingly important in the trend toward higher design strengths.

Precipitation Hardening—Alluminum Alloys, Titanium Alloys

In many nonferrous metals (aluminum alloys and titanium alloys being examples of particular interest in aircraft and aerospace structures), hardening is produced by precipitation of small particles (compounds of alloying elements) so dispersed as to inhibit deformation processes under stressing. In one sense, this works opposite to the quenching of steels, in that rapid cooling preserves the soft, formable solid solution. Hence, the first step is often rapid cooling from a relatively high temperature—a *solution heat treatment.* For some materials, more standing at room temperature produces the hardening by *aging.* In some instances elevated temperature accelerates the aging—this is a *precipitation heat treatment.* Overaging and going beyond the maximum yield strength for a particular composition are possible. For some alloys, stretching a bit in the solution-treated condition, before reheating for precipitation or between reheatings, modifies the properties.

Details of composition and heat-treating practices for the precipitation-hardenable materials used in aerospace structures are many and require consultation with an experienced metallurgist. However, there are a few generalities that warrant emphasis here:

TABLE D–1.—*Examples of effects of precipitation hardening on fatigue strength*

Material and condition	Ultimate tensile strength, ksi	Fatigue strength,⁽ᵃ⁾ ksi	Ratio
Aluminum alloy 2024:			
O (annealed)	27	13	0.48
T_3 (precipitation hardened)	70	20	.29
Titanium—6 Al–4 V:			
Annealed	137	72	.42
1,750° F, *WQ*, *T* at 900° F	171	86	.37

(a) Rotating-bending at 5×10^8 cycles for aluminum alloy. Reversed axial loading at 10^7 cycles for titanium alloy.

1. As for other kinds of hardening, treatments that increase static-strength properties do not usually produce a proportional increase in fatigue strength. For examples see table D–1. Notch-fatigue strength may increase little.
2. At least in some instances, there have been indications that an alloy precipitation hardened to a high value of static tensile ultimate may "over-age" under cyclic straining in fatigue loading.
3. Fabrication processes such as local heating in welding or local mechanical straining may produce local changes in hardness. The details vary with the characteristics of each alloy.

Strain Hardening—Austenitic Stainless Steels, Etc.

Some high-chromium steels with the addition of nickel retain the austenitic structure on quenching, but the crystalline structure of the soft austenite can be realigned by plastic deformation to result in a harder material—this is called strain-hardening. An especially common example is the 18 percent Cr—8 percent Ni steel.

A work-hardening material such as 18:8 stainless steel has some interesting characteristics pertinent to fatigue. Most fabrication procedures work the material so that a part made from annealed or partly hardened stock may be locally hard at a spot which has been extensively strained locally during the fabrication. Conversely, welding tends to leave an area of annealed material in a piece which was previously highly strain-hardened; such an annealed area cannot be rehardened by heat treatment and has a high susceptibility to strain-hardening under repeated stressing. This suggests an important precaution: watch the order of heat treating (including welding) and machining or other fabrication.

SURFACING TREATMENTS

Some Metallurgical Surfacing Processes—Case-Hardening Steels

Steels are often surface hardened by such processes as *nitriding, case-carburizing,* or *flame* or *induction hardening.* Nitriding consists of heating in a nitrogen-containing medium (an ammonia atmosphere or a cyanide-salt bath) to form nitrides with alloying elements; the nitrides form in a surface layer and harden it. Carburizing involves heating in a carbonaceous medium and subsequently quenching to form hardened surface material. Flame hardening or induction hardening essentially heat-treats the surface to a hardness intentionally greater than that of the core. All three procedures involve not only a hardened surface but also residual stresses (that may reach high compressive values at the surface); both factors contribute to increased fatigue strength. This implies a precaution: after surface hardening, do not machine as deep as the hardened layer.

Since these treatments harden the case compared to the core, the increase in fatigue strength may be large in cyclic loading (such as rotating-bending) that produces highest stresses at the surface compared to axial loading that produces essentially uniform stress across the section. For example, Sutton reported a two percent increase in axial-loading fatigue strength and a twenty percent increase in rotating-bending fatigue strength due to nitriding a particular steel (*5*). Forrest gives several examples of increases in fatigue strengths of steels by these three types of surfacing (*6*).

The suitability of a case-hardening procedure for a particular steel part depends on the type of steel, the depth of case required, the amount of distortion allowable, the type of service stressing, and the service environment (including temperature). For any method, there is a danger of cracks being formed during the hardening, or of locally penetrating the case in any any necessary subsequent machining or grinding. Nevertheless, with care, case-hardening can improve the fatigue strength of many steel parts; in some instances, local hardening near stress raisers such as bolt holes, grooves, or threads has been advantageous in improving fatigue strength.

For nonferrous metals and alloys, there are few surface treatments analagous to the case-hardening of steels. Aluminum alloys are often given a surface coating of pure aluminum ("Alclad") to improve corrosion resistance; this cladding reduces the fatigue strength of unnotched laboratory test specimens but has relatively little effect on the fatigue strength of joints.

Chemical Surface Processes—Pickling and Etching, Surface Deposition

The preparation of aluminum alloys for spot-welding or for adhesive bonding commonly includes some sort of acid-cleaning or etching. Harris (7) quotes the following values for the rotating-bending fatigue strength (at 10^8 cycles) of the British alloy DTD 683 (akin to the U.S. alloy 7075).

Condition	Fatigue strength, ksi
Mechanically polished	24
Pickled in 15 percent H_2SO_4–5 percent CrO_3 at 60°C. for 20 minutes	19
Pickled, same conditions, but 120 minutes	13
Pickled in 1 percent sodium fluoride–10 percent sulphuric acid, 3 minutes	22

Thus, the influence of chemical pickling can vary, within extremes of suggested processes, from negligible to severe reduction of the fatigue strength of a sensitive aluminum alloy.

An extensive use of chemical surface reactions is *chemical* or *electrochemical machining,* sometimes called "contour etching." Figure D–3 illustrates some points involved:

1. The as-rolled plate can contain favorable residual stresses which are removed by the etching.
2. In addition, chemical etching usually produces pits which may be fairly uniformly distributed and hence provide a roughness effect.
3. Vapor blasting (or other mechanical surface treatment) can round pits and give a shallow surface compressive stress that restores or enhances the original fatigue strength.

The practical implication is that etching, even to the extent of chemical machining of thick plates, can be carried out with good resultant fatigue characteristics, IF adequate precautions are taken. These include: (1) prevention of hydrogen embrittlement (taken for granted in the examples above), (2) specification of conditions to minimize differential etching that may produce pitting, and (3) restoration of favorable surface stress conditions by additional mechanical treatment.

Electroplating has been discussed briefly in chapter IX; further discussion of some specific processes in aircraft applications may be found in reference 7. The use of *electrodeposited* metal for *build-up* in certain situations of repair or adjustment is subject to the same precautions as for surfaces originally plated. These include mainly:

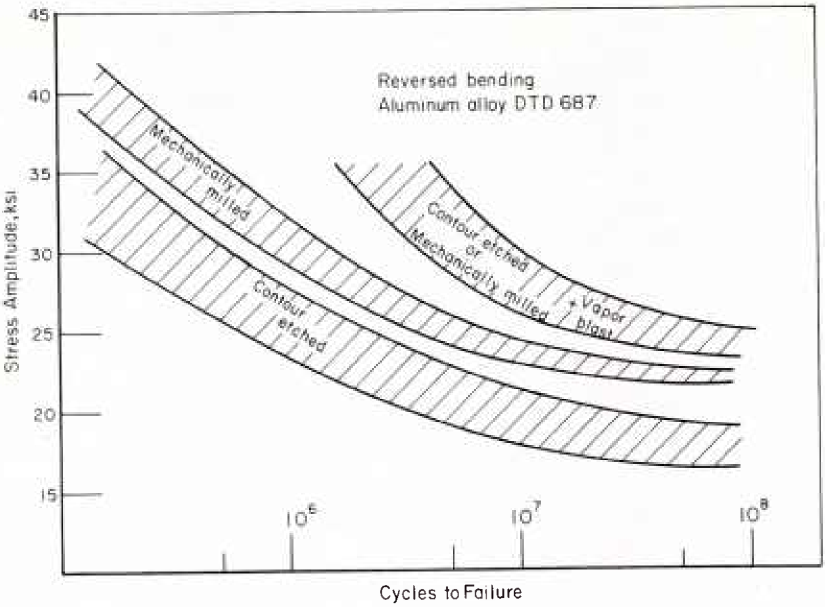

FIGURE D-3.—Example of effect of contour etching.

(1) avoidance of hydrogen embrittlement in sensitive metals, (2) complete coverage and avoidance of cracks, especially in hard coatings, and (3) avoidance of residual tension stresses that are particularly likely to exist in hard deposits. Some indication of the range of possible effects is shown by the following values of rotating-bending fatigue limit for a Ni-Cr steel:

1. Unplated—as machined	83 ksi.
2. Plated 0.010-inch Cr	48 ksi.
3. Plated 0.010-inch Cr—stress-relieved for 1 hour at 300° F.	28 ksi.
4. Plated 0.010-inch Cr—stress-relieved for 1 hour at 850° F.	37 ksi.
5. Plated 0.010-inch Cr—stress-relieved for 1 hour at 850° F. then final ground to 0.005 inch	56 ksi.

Thus, reduction of long-life fatigue strength by electroplating may be as high as 50 percent.

Mechanical Surface Treatment—Peening, Rolling, Etc.

Mechanically stressing the surface of a part generally produces (1) stain-hardening in alloys capable of such response, and (2) surface compressive residual stresses. Both increase the fatigue strength under many types of loading, and to separate out the relative contribution of each factor is extremely difficult. However, surface stressing is helpful in increasing the fatigue resistance of most commonly used alloys.

Peening has been widely used for steels, and a number of reviews have been published concerning the effects of peening on fatigue strength (*8*). Variants include: *shot-peening, grit-blasting,* and *vapor-blasting.* The increases in fatigue strength are larger for the heavier shot, and less for the lighter particles in the latter two processes; however, controlled heavy shot peening is sometimes inapplicable, particularly in regard to penetration of sharp notches and in smoothness of the peened surface. In some applications, *strain-peening* (that is, peening while under strain from an applied load) has been used to enhance the beneficial value of the compressive stress. The quantitative improvement in fatigue strength from surface peening has been generally identifiable with the favorable residual stress imparted. However, this depends on very many factors: the original hardness of the surface, the intensity of peening, the reaction of the metal to local (peening) deformation, the completeness of surface coverage, the relation of the residual stress to the fatigue-inducing stress. Consequently, the quantitative improvement(2) is not easily predictable and must be found by test. Depending upon peening to obtain a specific design strength can be dangerous.

Surface *rolling* is another method of improving fatigue strength. Horger obtained increases in fatigue strength of steel shafts ranging from 20 to 80 percent by rolling (*9*). For other examples see reference *6*. The comment about lack of quantitative predictability made for peening applies also for rolling.

Perhaps the most useful application of surface working is under situations where the additional strengthening is not counted upon in design but is considered to provide a kind of extra safety. Peening or rolling may be a valuable additional precaution in the region of a necessary stress raiser such as a gear tooth or a screw thread. Coining and burnishing have been useful around bolt holes (*10*).

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(2) Very few cases have been reported of overpeening giving slight damage by surface roughening or, in exceptional circumstances, opening up surface cracks. Several cases of negligible improvement have been reported. Situations that would be expected to show relatively little improvement include: (1) materials too hard to be effectively deformed by the peening practice employed, (2) pieces loaded in tension where the uniform stress across the section may add to a subsurface residual tension, (3) loadings high enough to dissipate the favorable peening stresses.

OTHER FACTORS

Any processing from the melt to the final assembly that affects the homogeneity of metal, the metallurgical structure, the residual stresses, or the surface condition of a part may significantly influence the fatigue strength. *Identification marking* (by stamping, electric etching, or chemical etching) can lower the fatigue strength disastrously (7). *Assembly practices* to be watched include: forcing fits, removing metal injudiciously, undertorquing bolts, etc. All possibilities of putting unfavorable residual stresses or items of stress concentration in a part require constant vigilance to prevent possible loss in resistance to fatigue.

REFERENCES FOR APPENDIX D

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