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THE DAMPING OF BEAM VIBRATION BY ROTATIONAL DAMPING AT THE SUPPORTS

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FOREWORD

This report was prepared by Dr.D.J.Mead and Mrs.J.F.Wilby of the University of Southampton, Institute of Sound and Vibration Research, Southampton, England and USAF Contract AF 61(052)-504. The contract was initiated under Project No. 7351, "Metallic Materials", Task No.735106, "Behavior of Metals" and was administered by the European Office, Office of Aerospace Research. The work was monitored by the AF Materials Laboratory Research and Technology Division, Air Force Systems Command, Wright-Patterson Air Force Base under the direction of Mr.W.J.Trapp.

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ABSTRACT

If the edges of a plate are embedded in a visco-elastic material, flexural vibration of the plate is damped by virtue of the damping forces and couples exerted at its boundaries. This paper analyses and assesses the effectiveness of this form of artificial damping when applied to a uniform beam and compares it with the effectiveness of homogeneous damping layers applied throughout the length of the beam.

First, the theory is developed for the linear flexural response of the uniform beam to uniform harmonic loading. Transverse displacements of the beam are prevented altogether while rotation is opposed by the linear elastic and damping couples from the embedding material. Explicit expressions are derived for the amplitudes of curvature at the centre of the beam and from these it is shown that there exist optimum values of the end constraint damping properties which will minimise the beam resonant response. Methods of estimating these optimum values are d discussed. It is shown that different optimum values are required to give the maximum effective flexural loss factors of the beam. This greatest value may be of the order of 0.33.

Comparison with the effectiveness of homogeneous layers shows that the edge-constraint damping mechanism is more effective than thin homogeneous layers, but much less effective than thick layers.

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CONTENTS

ILLUSTRATIONS

NOTATION

a, b	dimensions of damping material
D	flexural rigidity of the plate
EI	flexural stiffness of the beam
E_c	Young's modulus of damping material (real part)
$f_c(x)$, $f_e(x)$	non-dimensional modes of beam displacement
F_2, F_6 etc.	functions of λ defined in the text
h_1	thickness of the plate
$h = \eta_c k$	constraint damping
$h^* = \frac{h\ell}{EI}$	non-dimensional constraint damping
H	generalised hysteretic damping coefficient
k	constraint stiffness
2ℓ	length of the beam
m	mass of the beam per unit length
$\bar{M}$	amplitude of harmonic moment at the end of the beam
p	loading on the beam per unit length
q	generalised displacement
$\bar{q}$	amplitude of harmonic generalised displacement
w	transverse displacement of the beam
$\bar{w}$	amplitude of harmonic transverse displacement of the beam
x	space co-ordinate
$\alpha = \left(\frac{d^2\bar{w}}{dx^2}\right)_{x=o} \cdot \frac{EI}{p\ell^2}$	non-dimensional curvature at beam centre
η	beam loss factor
η_c	constraint loss factor

η_t	loss factor of beam after treatment with a layer of damping material
$\bar{\theta}$	amplitude of harmonic rotation at the end of the beam
$\kappa = \frac{K\ell}{EI}$	non-dimensional constraint stiffness
λ	frequency parameter
λ_e	frequency parameter defined by equation 15
ν_c	Poisson's ratio for damping material
$\sigma EI(1 + i\eta_t)$	complex stiffness of beam after treatment with a layer of damping material
τ	(thickness of treatment) ÷ (thickness of plate) for unconstrained layer
or	(thickness of core) ÷ (thickness of face-plate) for sandwich plate
ω	angular frequency

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1. INTRODUCTION

Plate vibration problems are often alleviated by applying damping treatments to the plate. Usually the whole or a large part of the surface is covered with a layer of a visco-elastic material which dissipates energy as the plate bends. An alternative method considered in this paper is to embed the edges of the plate in a visco-elastic material so that energy is dissipated as the edges rotate under the bending displacements. It is the purpose of this paper to compare the effectiveness of these methods of damping.

This has been done by considering the resonant response of a simply-supported beam (rather than a plate, for simplicity) to a uniform harmonic loading. To represent the embedding visco-elastic material, rotational constraints are introduced at the supports which exert both elastic and damping couples. The response of the whole system has been computed for several different values of the stiffness and damping rates of the constraints. This is compared with the response of a similar beam damped only by (a) an unconstrained layer type of treatment (e.g. Aquaplas) and (b) a sandwiched damping layer.

Simple reasoning suggests that there must be optimum value of the edge constraint damping to minimise the beam resonant response. Suppose that the constraint stiffness remains constant while the damping rate varies between zero and infinity. With zero damping, the beam vibrates as an undamped beam with infinite response at resonance. With infinite constraint damping, rotation at the supports is altogether prevented so that the beam is effectively fully clamped at its end. Since the ends do not rotate, no energy is dissipated there and the beam motion is effectively undamped, again yielding infinite response at resonance. Between these two extremes rotation of the ends must occur and energy must be dissipated as the beam vibrates. This necessarily means that the resonant response is finite. Since the resonant response is infinite for the two extreme values of the constraint damping, and finite for intermediate values, it follows that there must be one (or more) particular "optimum" value(s) of the constraint damping at which the resonant response has a minimum value. Explicit expressions are derived in this paper by two different methods for the magnitude of this optimum damping.

2. RESPONSE OF BEAM WITH ROTATIONAL END CONSTRAINTS

Consider a uniform beam of length 2ℓ, initially simply-supported at its ends. Equal rotational constraints are then applied to each end such that a couple of magnitude $K + ih = K(1 + i\eta_c)$ is exerted on the end per unit harmonic rotation. K is the elastic restoring moment per unit rotation and ih is the hysteretic damping moment per unit harmonic rotation. A uniform harmonic loading of $p\, e^{i\omega t}$ per unit length acts on the beam. In the absence of internal damping within the beam itself, the equation of motion of the beam is

$$EI \frac{\partial^4 w}{\partial x^4} = - m \frac{\partial^2 w}{\partial t^2} + p\, e^{i\omega t} \qquad (1)$$

where w is the transverse displacement of the beam, m is the mass per unit length and EI is the flexural stiffness.

Let the longitudinal co-ordinate x have its origin at the beam centre. Putting $w = \bar{w}\, e^{i\omega t}$, and considering only the solution of the differential equation which is symmetrical about the beam centre, we have

$$\bar{w} = A \cosh \frac{\lambda x}{\ell} + B \cos \frac{\lambda x}{\ell} - P/m\omega^2 \tag{2}$$

where

$$\lambda^4 = \frac{m\omega^2}{EI} \quad \ell^4 \tag{3}$$

At the right hand support $(x = \ell)$ the displacement $\bar{w}$ is zero, and the bending moment in the beam is equal to the complex moment from the constraint i.e.

$$EI \frac{d^2\bar{w}}{dx^2} = - K(1 + i\eta_c) \frac{d\bar{w}}{dx} \tag{4}$$

From these boundary conditions, the constants A and B are found.

From now on, we shall consider the effect of the damped constraints on the curvature at the centre of the beam. This, being proportional to the bending stress in the beam, is the most important response quantity in connection with the fatigue life of a vibrating beam. Accordingly, we differentiate equation (2) twice with respect to x, and using the derived expressions for A and B, we find

$$\left(\frac{d^2\bar{w}}{dx^2}\right)_{x=0} \cdot \frac{EI}{p\ell^2} = \alpha = \frac{1}{\lambda^2} \cdot \frac{(\cos\lambda - \cosh\lambda) + K(1+i\eta_c)(\sin\lambda - \sinh\lambda)\ell/EI}{2\cos\lambda \cosh\lambda + K(1+i\eta_c)(\sin\lambda\cosh\lambda + \cos\lambda \sinh\lambda)\ell/EI}$$

We now write this in the notation of Bishop (Ref. 1, p. 359), using

$$F_2 = \cos\lambda \quad \cosh\lambda$$

$$F_6 = \cos\lambda \quad \sinh\lambda + \sin\lambda \quad \cosh\lambda$$

$$F_8 = \sin\lambda \quad -\sinh\lambda$$

$$F_{10} = \cos\lambda \quad -\cosh\lambda$$

Hence, with some further re-arrangement, we find

$$\left(\frac{d^2\bar{w}}{dx^2}\right)_{x=0} \cdot \frac{EI}{p\ell^2} = \alpha = \frac{(F_{10} + KF_8/\lambda) + i\eta_c KF_8/\lambda}{(2\lambda^2 F_2 + K\lambda F_6) + i\eta_c K F_6} \tag{5}$$

In this, $\mathcal{K} = \frac{K\ell}{EI}$ (6)

and is therefore a non-dimensional measure of the end constraint stiffness.

Expression 5 has been used to calculate the complex curvature for several different values of $\mathcal{K}$ and η_c over the non-dimensional frequency range $\lambda^2 = 0$ to $6{\cdot}6$. This range covers the resonant portion of the response curves of both simply-supported and clamped beams. Figure 1 shows the modulus of the curvature plotted against λ^2 for several different values of $h^*(= \eta_c\mathcal{K} = h\ell/EI)$, and with $\mathcal{K} = 0$, i.e. η_c has an infinite value, but $\eta_c\mathcal{K}$ is finite. The end constraint therefore consists solely of a hysteretic damper. In Figure 2, $\mathcal{K}$ has the value of 1. Figure 3 shows the curvature modulus plotted against λ^2 for different values of $\mathcal{K}$ but with $\eta_c = 1$.

Figure 1 and 2 illustrate the effect described in the introduction, viz. that the resonant response approaches infinity when the constraint damping approaches zero or infinity, and that a minimum resonant response occurs at an intermediate value of the damping. Further, it is seen that with zero damping the resonant frequency is that of the elastically constrained, simply-supported beam, but with infinite damping it is that of a clamped beam. Indeed, the whole curve for $\eta_c = \infty$ is identical with that of an undamped clamped beam, as can be seen from equation 5 by letting $\mathcal{K}$ approach ∞, (clamped beam) or letting $\eta_c \to \infty$ (infinite constraint damping).

Of particular interest in each of Figures 1 and 2 is the point through which all the curves pass, at $\lambda^2 = 3{\cdot}95$ for $\mathcal{K} = 0$ and at $\lambda^2 = 4{\cdot}32$ for $\mathcal{K} = 1$. The curvature represented by the point is obviously the minimum resonant curvature that can be achieved for the given value of $\mathcal{K}$. These points may be located without reference to the constraint damping at all, for they are also the intersection points of the undamped response curves of the fully clamped beam and of the beam with the purely elastic constraint, $\mathcal{K}$. Using equation 5, it can therefore be shown that the frequency of the common intersection point is given by the solution to

$$\frac{F_{10} + \mathcal{K}F_8/\lambda}{2\lambda^2 F_2 + \mathcal{K}\lambda F_6} = \pm \frac{F_8}{\lambda^2 F_6} \qquad (7)$$

and the corresponding curvature amplitude is given by

$$\left(\left|\frac{d^2\bar{w}}{dx^2}\right|\right)_{x=0} \cdot \frac{EI}{p\ell^2} = \left|\frac{F_8}{\lambda^2 F_6}\right| \qquad (8)$$

evaluated at the appropriate value of λ. For the first intersection point, using equation (7) with the negative sign, $\lambda^2 = 3{\cdot}95$.

3. OPTIMUM DAMPING FOR GIVEN CONSTRAINT STIFFNESS

The optimum value of the constraint damping rate h , is the value which gives the minimum resonant response. It may be estimated for different values of K using sets of curves such as those of Figures 1 and 2. Plotting the resonant response against η_c, curves of the type shown in Figure 4 are obtained. From these, the optimum values of η_c, and hence of h^* are easily estimated.

This method of estimating h^*_{opt} is obviously rather tedious. Two alternative, more rapid methods, are therefore presented.

3.1 The Method Involving the Known Resonant Frequency

Figures 1 and 2 have shown that the minimum resonant response occurs at a resonant frequency given by the common intersection point of all the curves. For a given value of K, therefore, we know from equation 7 the resonant frequency, λ_o^2, of the system with optimum damping.

Now at the resonant frequency of a single degree of freedom system with hysteretic damping, the displacement response is exactly in quadrature with the exciting force, i.e. the real part of the response is zero. In the analysis which follows, it will therefore be assumed that the real part of the beam curvature is also zero at the known resonant frequency. This leads to a simple equation for an approximate value of $\eta_{c_{opt}}$.

Equating to zero the real part of the curvature given by equation 5, we find

$$(F_{10} + KF_8/\lambda)(2\lambda^2F_2 + K\lambda F_6) + \eta_c^2K^2F_6F_8 = 0 \qquad (9)$$

Combining this with equation 7 (which is the frequency equation for the common intersection point) yields

$$\eta_cK = h^* = \left| 2\lambda_oF_2/F_6 + K \right| \qquad (10)$$

When this is evaluated at the appropriate, known value of λ_o it yields the required approximation for $\eta_{c_{opt}}$.

It must be emphasised that the assumption leading to this is only an approximation. Some justification for it is afforded by examining the vector response diagrams of Figure 5. These show in the usual way, the real part of the curvature plotted against the imaginary part for different values of the frequency parameter λ . Each circle has been computed for the same value of $K(= 0)$ but with different values of h^*. It is evident that the value of λ for resonance (maximum response) differs slightly from that for zero real component. The difference, however, is not large enough to introduce serious errors in the calculated value of h^*_{opt}. From examination of Figure 5 in conjunction with Figures 1 and 2, it may be deduced that the approximate value of h^*_{opt} must give a frequency response curve which has its peak at a frequency slightly higher than that of the common intersection point. The corresponding curvature amplitude will however differ negligibly from the minimum possible value.

Table I shows the values of $\eta_{c_{opt}}$ calculated from equation 10, and compares them with the optimum values derived from the minima of Figure 4.

3.2 <u>The Method of Intersecting Asymptotes</u>

Figure 4 shows that at very high or very low values of η_c, the resonant curvature behaves asymptotically. The two asymptotes of a given curve intersect at a value of η_c which is evidently very close to the optimum value. The equations to the asymptotes are quite easily found from energy considerations, so that the optimum value of η_c (or h^*) may be determined.

When η_c is very small, the mode of beam displacement at resonance is virtually the same as the principal mode of the undamped beam with the same elastic end constraint, K. Denote this non-dimensionalised mode by $f_e(x)$ and its derivative by $f'_e(x)$. The absolute displacement in this mode is given by

$$w = q\, f_e(x)$$

where q is the generalised displacement co-ordinate. The rotation of each end of the beam is $q\, f'_e(\ell) = \theta$, and the curvature at the centre of the beam is $q\, f''_e(0)$.

If the generalised hysteretic damping coefficient in this mode is H, then the amplitude of $q (= \bar{q})$ at resonance is given by

$$\bar{q} = \text{Generalised Force Amplitude} \div H \tag{11}$$

H is given by the energy relationship:

Energy dissipated in the system in the harmonic cycle $\bar{q}e^{i\omega t}$

$$= \pi H \bar{q}^2 \tag{12}$$

and the generalised force amplitude by $\int_{-\ell}^{\ell} p\, f_e(x)\, dx$

Now the energy dissipated by the damper h at the end of the beam per cycle of rotation of amplitude $\bar{\theta}$ is $\pi h \bar{\theta}^2$. Assuming that the damping of the beam derives solely from the end dampers, we then have

$$\pi H \bar{q}^2 = 2\pi h \bar{\theta}^2$$

Hence

$$H = 2\, h \left[f'_e(1)\right]^2$$

so that the amplitude of the beam curvature at resonance becomes

$$\left|\frac{d^2\bar{w}}{dx^2}\right|_{x=o} = \bar{q}\, f''_e(0) = \frac{p\int f_e(x)\, dx.\; f_e''(0)}{2h\left[f'_e(\ell)\right]^2} \tag{13}$$

Using the appropriate functions of the F's for $f_e(x)$, etc. and introducing the non-dimensional parameters η_c and K, we can re-write equation 13 in the form

$$\left|\frac{d^2\bar{w}}{dx^2}\right|_{x=o} \cdot \frac{EI}{p\ell^2} = -\frac{1}{\eta_c K}\left\{\frac{F_{10}}{\lambda_e F_6} + \frac{KF_8}{\lambda_e^2 F_6}\right\} \tag{14}$$

This is the equation for the low-η_c asymptote to the curves of Figure 4.

λ_e is the root of the frequency equation

$$2\lambda F_2 + KF_6 = 0 \tag{15}$$

which is the frequency equation for the elastically restrained beam. Each of the F's in equation 14 must be evaluated for $\lambda = \lambda_e$.

The high η_c asymptote is found in a generally similar way. The mode of beam displacement is now virtually that of the fully-clamped beam, $f_c(x)$. The rotation of the ends is vanishingly small, whereas the magnitude of the hysteretic damper is becoming indefinitely large. However, with a given displacement amplitude, $\bar{q}$, the amplitude of the moment acting at the end approaches the constant value $\bar{q}.EI.f''_c(1) = \bar{M}$; this is the end moment in the undamped, fully-clamped beam. For the energy dissipated per cycle by each end damper we now use the alternative expression

$$\frac{\pi\bar{M}^2}{h}\left(\frac{\eta_c^2}{1+\eta_c^2}\right) \rightarrow \frac{\pi\bar{M}^2}{h} \quad \text{as} \quad \eta_c \rightarrow \infty \tag{16}$$

Hence $$\pi H\bar{q}^2 = \frac{2\pi\bar{M}^2}{h}$$

and $$H = \frac{2}{h}\left[EI.f''_c(\ell)\right]^2$$

The amplitude of beam curvature at resonance now becomes

$$\left|\frac{d^2\bar{w}}{dx^2}\right|_{x=o} = \frac{p\int f_c(x)dx.f''_c(0)}{(2/h)\left[EI.f_c''(\ell)\right]^2} \qquad (17)$$

which can be re-arranged into the form

$$\left|\frac{d^2\bar{w}}{dx^2}\right|_{x=o} \cdot \frac{EI}{p\ell^2} = \eta_c K. \frac{F_8}{2\lambda_c^3 F_2} \qquad (18)$$

This is the equation for the high-η_c asymptote to the curves of Figure 4.

λ_c is the root of the frequency equation

$$F_6 = 0 \qquad (19)$$

which is the frequency equation for the fully clamped beam. We shall consider only the lowest root, at which value the right hand side of equation 17 becomes $\eta_c K/22.35$. The optimum value of η_c is found by equating the curvatures of equations 14 and 18, from which we obtain

$$(\eta_c K)^2_{opt} = h^{*2}_{opt} = \left\{\frac{F_{10}}{\lambda_e F_6} + K\frac{F_8}{\lambda_e^2 F_6}\right\} 22.35 \qquad (20)$$

The optimum values calculated from equation 18 are shown in Table I.

4. OPTIMUM STIFFNESS FOR GIVEN CONSTRAINT LOSS FACTOR

Although the previous section gives insight into the method of optimizing the constraint properties, it is more realistic to suppose that the constraint loss factor η_c remains at a constant value, while the constraint stiffness is varied to obtain the minimum response. This is so because the likely approach to a practical problem is to choose for the constraint a high damping material having a known loss factor. The variable which remains is the thickness of the material which must be optimized to minimise the response of the beam. Varying the thickness of the material will change the stiffness, K, but η_c will remain constant.

The method of § 3.2 is readily adapted to find the optimum stiffness. In the first place, Figure 6 shows curves of resonant curvature plotted against K for different values of η_c. These were obtained from sets of curves such as those of Figure 3. The equations to the asymptotes may be found in the same way as in §3.2 and are

$$\left|\frac{d^2\bar{w}}{dx^2}\right|_{x=o} \cdot \frac{EI}{p\ell^2} = -\frac{1}{\eta_c K} \cdot \frac{F_{10}}{\lambda F_6} \quad \text{for small } K \tag{21}$$

at a frequency given by $F_2 = 0$ (22)

and

$$\left|\frac{d^2\bar{w}}{dx^2}\right|_{x=o} \cdot \frac{EI}{p\ell^2} = K\frac{(1+\eta_c^2)}{\eta_c} \cdot \frac{F_8}{2\lambda^3 F_2} \quad \text{for large } K \tag{23}$$

at a frequency given by $F_6 = 0$ (24)

(Note that in deriving equation 23, the general expression of 16 is used for the energy dissipation and not the limiting expression for large η_c.)

Equating 21 and 23 and using the appropriate values of λ in the two expressions leads to

$$K_{opt} \doteqdot 3{\cdot}78\,(1+\eta_c^2)^{-1/2} \tag{25}$$

Table II compares the values of K_{opt} given by equation 25 with the values measured from the minima of the curves of Figure 6.

5. LOSS FACTOR OF DAMPED BEAM

The loss factor η of a system which is vibrating in a given mode is a non-dimensional measure of the generalised hysteretic damping and is convenient for making superficial comparisons of the effectiveness of different damping methods (ref. 2). It can be defined by

$$\eta = \frac{\text{Generalised Hysteretic Damping Coefficient } H}{\text{Generalised Stiffness Coefficient } K} \tag{26a}$$

$$= \frac{1}{2\pi}\,\frac{\text{Energy dissipated per cycle}}{\text{Maximum energy stored during the cycle}} \tag{26b}$$

If the system is lightly damped and vibrates in a principal mode, the loss factor may be related to the bandwidth of the frequency response curve by the approximate expression

$$\eta \doteqdot \frac{\Delta\omega}{\omega_r} \tag{27}$$

ω_r is the resonant frequency of the mode, and $\Delta\omega$ is the difference between the two frequencies at which the response is $1/\sqrt{2}$ times the resonant response.

Equation 27 has been used to find the effective loss factor of the constrained beam by using values of $\Delta\omega$ and ω_r measured from the computed curves of Figures 1, 2, 3, etc. Figures 7 and 8 show the values of η calculated in this way. Figure 7 shows the effect of varying η_c, keeping K constant, whereas Figure 8 shows the effect of varying K, keeping η_c constant. The maximum values of η that are indicated are not high enough to invalidate the use of the approximate equation 27, but it must be borne in mind that the errors involved in the approximation do increase as η increases.

The asymptotic nature of the curves of Figures 7 and 8 can be utilised, as in section 3.2, to arrive at the optimum values of η_c (or K) for maximum loss factor. The equations of the asymptotes can easily be found using the energy definition of η (equation 26b). The intersection points of the asymptotes can then be located. In this way, the optimum value of η_c for a given value of K is found to be given by

$$\eta_{c_{opt}}^2 = \frac{1}{2}\left(\frac{1}{\cos^2\lambda_e} + \frac{1}{\cosh^2\lambda_e}\right) + \frac{1}{K} \qquad (28)$$

Likewise, the optimum value of K for a given value of η_c is found to be

$$K_{opt} = 2{\cdot}22\,(1 + \eta_c^2)^{-1/2}$$

It will be noticed that these optimum values differ from those required for minimum curvature (equations 20 and 25). This is a feature which has previously been observed in the optimisation of damped sandwich plates (ref. 3) and stems from the fact that as the properties of the damping material are changed, so also is the stiffness of the whole system, as well as its damping.

Tables III and IV show values of $\eta_{c_{opt}}$ and K_{opt} respectively, calculated from the above expressions and from examination of Figures 7 and 8. Some corresponding values of η for the beam are also given; these were found from the peaks of figures 7 and 8. Now damping materials are available have material loss factors η_c, of up to about 1·0. From Table IV it is seen that with such materials it should be possible to obtain overall effective loss factors of 0·33 with the optimum constraint stiffness. This is close to the loss factor obtainable from a damped sandwich plate with a thin core having a material loss factor of 1·0 (ref. 3).

6. NUMERICAL EXAMPLE USING GIVEN MATERIAL

Suppose a long aluminium plate, 0·04 in. thick and 8 in. wide is to be damped by constraining its long edges. This can be achieved by gripping the edges between two rigid surfaces, A and B, (see Figure 9) which enclose the damping material. Since the plate is long compared with its width, for the present purpose it may be represented by a width-wise strip of unit length, i.e., by a beam, 8 in. long and 1 in. wide. This must have the flexural rigidity, $D = Eh_1^3/12(1-\nu^2)$ which is equivalent to the EI of the foregoing theory. It is required to determine the dimensions a and b (Figure 9) of the damping layer to minimise the resonant curvature of the plate. The material to be used has a complex Young's Modulus, $E_c(1+i\eta_c)$, of $10^3(1 + i.\ 1{\cdot}0)$ lb.in.$^{-2}$ E for aluminium is $10{\cdot}5 \times 10^6$ lb.in^{-2}.

Now when the edge of the plate-beam element rotates harmonically as a rigid body about point O, the restoring moment per unit rotation exerted by the two layers of damping material can be shown to be

$$K + ih = E_c(1 + i\eta_c)\,\frac{a}{6(1-\nu_c^2)b}\left[a^2 + \frac{3}{2}h_1^2(1-\nu_c)\right] \qquad (30)$$

per unit length of plate. ν_c is Poisson's Ratio for the damping material. η_c for the constraint (h/k) is identical to η_c for the material, so that no distinguishing symbol need be used. As its value has been specified, equation 25 may be used to determine the optimum real part of the constraint stiffness, k, for minimum resonant curvature. Equating this to the real part of the righthand side of equation 30 leads to the following equation for a and b:

$$\frac{a}{b}\left[a^2 + \frac{3}{2}h_1^2(1-\nu_c)\right] = 3{\cdot}78\,(1+\eta_c^2)^{-\frac{1}{2}}\frac{D}{l}\;\frac{6(1-\nu_c^2)}{E_c} \qquad (31)$$

Inserting the appropriate values into this, some typical pairs of values for a and b are found to be:

a	b
0·473 in.	0·473 in.
0·333 in.	0·166 in.
0·234 in.	0·058 in.
0·1443in.	0·0144in.

7. COMPARISON WITH OTHER DAMPING TECHNIQUES

The damping effectiveness of the end-constrained beam will now be compared with that of the unconstrained layer type of treatment (e.g. Aquaplas) and also with that of the sandwich plate with a damped core.

Suppose the beam to be treated is rigidly clamped at each end. Its flexural stiffness before the treatment is applied is the same as that of the beam of section 2, viz. EI. When the treatment is added, EI is both increased and made complex so that the new flexural stiffness may be written in the form $\sigma EI(1 + i\eta_t)$. It will be assumed that the treatment is uniformly applied, so that σ and η_t are constants. It will be assumed that the damping due to the treatment is much greater than that of all other sources.

The beam is to be excited by a uniform harmonic loading $pe^{i\omega t}$ at the resonant frequency ω_r of the fundamental bending mode. As an approximation, it will be assumed that the resultant mode of forced vibration is identical with the corresponding undamped principal mode, $f_c(x)$. The generalised displacement, q, in this mode and at the frequency ω_r is then found to be

$$q = \frac{\int pf_c(x)dx.e^{i\omega_r t}}{i\int_{-\ell}^{\ell} \sigma\eta_t EI(f_c''(x))^2 dx} \tag{32}$$

The amplitude of curvature at the centre of the beam is $\bar{q}f''_c(0)$, i.e.

$$\bar{q}f_c''(0) = \left|\frac{d^2\bar{w}}{dx^2}\right|_{x=0} = \frac{p\int f_c(x)dx.f_c''(0)}{\sigma\eta_t EI\int [f_c''(x)]^2\, dx} \tag{33}$$

Evaluating the integrals in terms of the F functions, this equation may be written in the form equivalent to equation 5, viz.

$$\left|\frac{d^2\bar{w}}{dx^2}\right|_{x=0} \cdot \frac{EI}{p\ell^2} = \frac{1}{\sigma\eta t}\left[\frac{2F_8}{\lambda^3 F_2}\right] \tag{34}$$

which must be evaluated with the argument $\lambda = 2{\cdot}365$ in the F functions. The right hand side of equation 34 then becomes

$$0{\cdot}179/\sigma\eta_t \tag{35}$$

Now for a given damping material on a given plate (beam) the product $\sigma\eta_t$ is a function only of the quotient (thickness of treatment) ÷ (thickness of plate), or (Weight of treatment) ÷ (Weight of plate). $\sigma\eta_t$ has been evaluated previously (ref. 2) for two commercial treatments having the following properties, and applied to an aluminium plate:

	Young's Modulus. $lb.in^{-2}$ (real part)	Loss factor	Specific Gravity
Treatment A	860,000	0·193	1·20
Treatment B	1,080,000	0·33	1·68

The values of $\sigma\eta_t$ from reference 2, have been used to evaluate the resonant curvature expression of equations 34 and 35 for a range of values of τ = (thickness of treatment) ÷ (thickness of plate). The results have been plotted on Figure 10. Also indicated on this figure for comparison purposes are the minimum resonant curvatures obtained with optimised end constraint damping, as in section 4; these are not to be interpreted as being functions of the abscissa co-ordinates τ.

The corresponding analysis for the curvature at the centre of a damped sandwich plate, clamped at each end, is much more complicated and no accurate solution has yet been published. However, curves of $\sigma\eta_t$ for simply-supported plates have been presented (ref. 3), and those values corresponding to a core loss factor of 1·0 have been used in equation 35 to obtain first approximations to the resonant curvatures of the clamped sandwich plate. These approximate values are also shown on Figure 10 as a function of τ = (thickness of core) ÷ (thickness of face-plate).

Now suppose, as in section 5, that η_c = 1·0 is the highest value that can be achieved for a practicable constraint material. Comparing the sandwich plate and unconstrained layer curves of Figure 10 with the η_c = 1·0 line shows that the end constraint damping is more effective than the other damping methods when their material thickness ratios are less than 0·2 and 0·75 respectively. With greater thickness ratios than these, the sandwich and unconstrained layer methods become much more effective than the end constraint damping.

Now the calculations and curves of this paper have all been carried out for the fundamental mode of vibration of the damped beam. If a higher mode of vibration is excited (by a point force, say) the effectiveness of the end constraint damping will not be as great as it is for the fundamental mode. In contrast to this, the effectiveness of the unconstrained layer damping method is not affected at all by the mode in which the plate vibrates. The sandwich damping effectiveness does depend on the mode of vibration, but probably not to the same extent as that of the end constrained beam. Accordingly, it may be concluded that the edge-constraint method of damping a vibrating plate is superior to the other methods only if the other methods are limited to very thin layers of damping treatment and if vibration in the fundamental mode alone is to be remedied.

8. CONCLUSIONS

For the edge-constraint method of damping a beam or plate to be most effective, there must be an optimum relationship between the rotational stiffness and damping of the constraint. Explicit relationships have been derived for the optimum constraint damping for a given stiffness and vice-versa. These relationships depend on the length and stiffness of the beam to be damped. Of the methods used to derive them, the method of intersecting asymptotes is the most accurate, and is the most readily applicable to the problem of finding the optimum constraint damping to minimise any vibration response quantity of the beam.

If the constraint material has a loss factor of 1·0, the maximum possible effective loss factor of the beam is about 0·33 when the beam vibrates in the fundamental mode. The corresponding optimum constraint stiffness is about 60% of that required to minimise the resonant curvature of the beam.

When the edge-constraint has the optimum stiffness and a loss factor of 1·0, the beam resonant curvatures are lower than those of beams of plates with conventional thin unconstrained or sandwiched damping layers. However, if these layers are thick, the conventional methods of damping are much superior to the edge constraint method. Furthermore, when the beam vibrates in a higher mode the conventional methods do not lose their effectiveness to the same extent as the edge constraint method.

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TABLE I

Optimum Constraint Loss Factor η_c

for Minimum Resonant Response

K	$\eta_{c\,opt}$ Figure 4	$\eta_{c\,opt}$ Equation 10	$\eta_{c\,opt}$ Equation 20
·25		13·1	15·9
·5	8·5	7·1	8·52
1	4·85	4·1	4·82
2	2·9	2·59	2·92
3	2·3	2·04	2·28
4	2·0	1·8	1·96
10	1·4	1·31	1·39

TABLE II

Optimum Constraint Stiffness K

for Minimum Resonant Response

η_c	K_{opt} Figure 6	K_{opt} Equation 25
·25		3·66
·5	3·50	3·38
1	2·70	2·67
2	1·75	1·69
4	·95	·917
10	·38	·376

TABLE III

Optimum Constraint Loss Factor η_c

for Maximum Loss Factor of the Beam

K	$\eta_{c_{opt}}$ Equation 28	$\eta_{c_{opt}}$ Figure 7	Beam Loss Factor
·25	10·2		
·5	5·7		
·1	3·39	3·5	.60
2	2·25		
3	1·86		
4	1·67	1·7	·315
10	1·27	1·25	·162

TABLE IV

Optimum Constraint Stiffness K

for Maximum Loss Factor of the Beam

η_c	K_{opt} Equation 29	K_{opt} Figure 8	Beam Loss Factor
·25	2·15		
·5	1·985	2·1	·18
1	1·57	1·7	·33
2	·994	·96	·51
4	·539	·52	·69
10	·221		

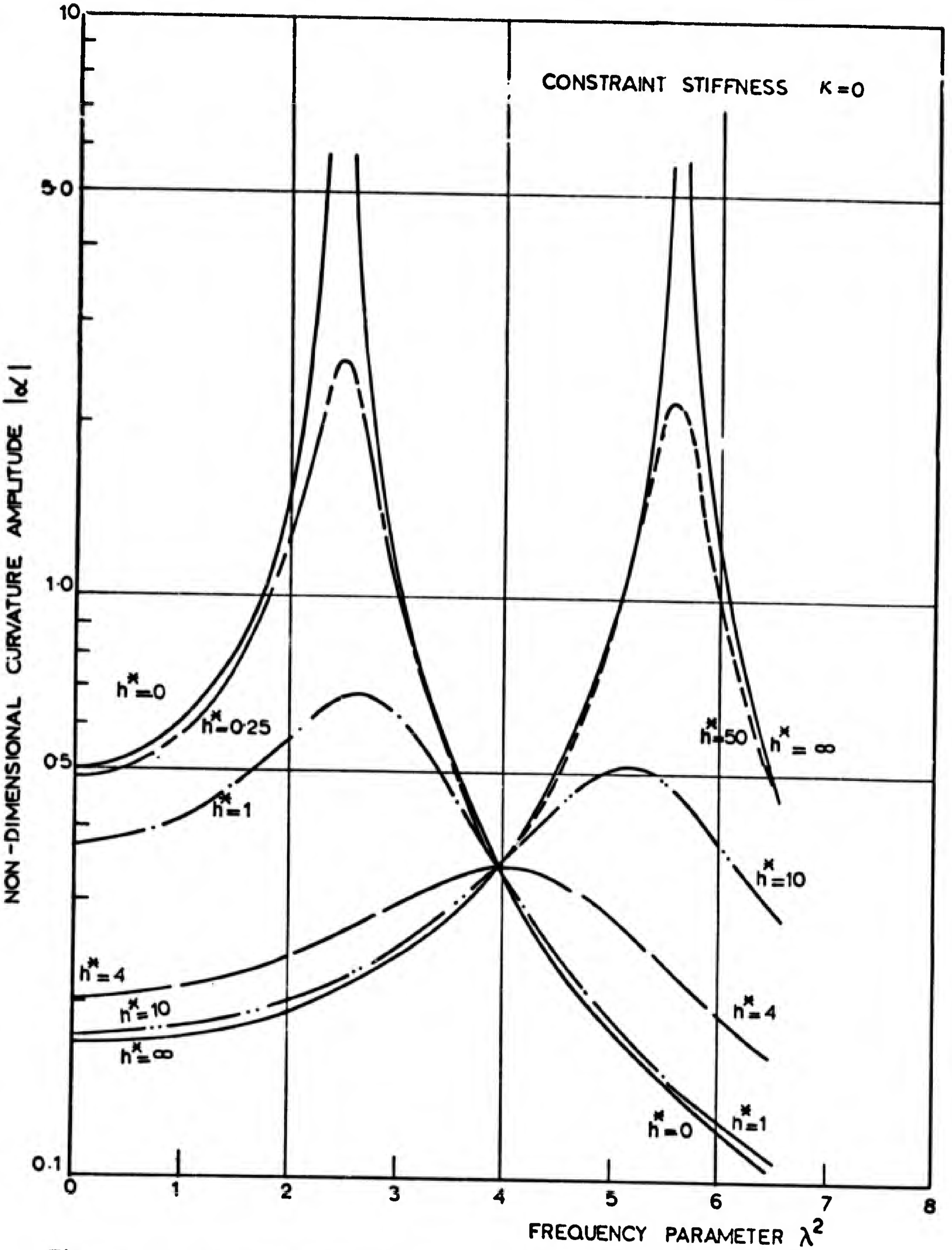

Figure 1. Curvature amplitude at beam centre $|\alpha|$ plotted against frequency parameter λ^2. Constraint stiffness $\kappa = 0$

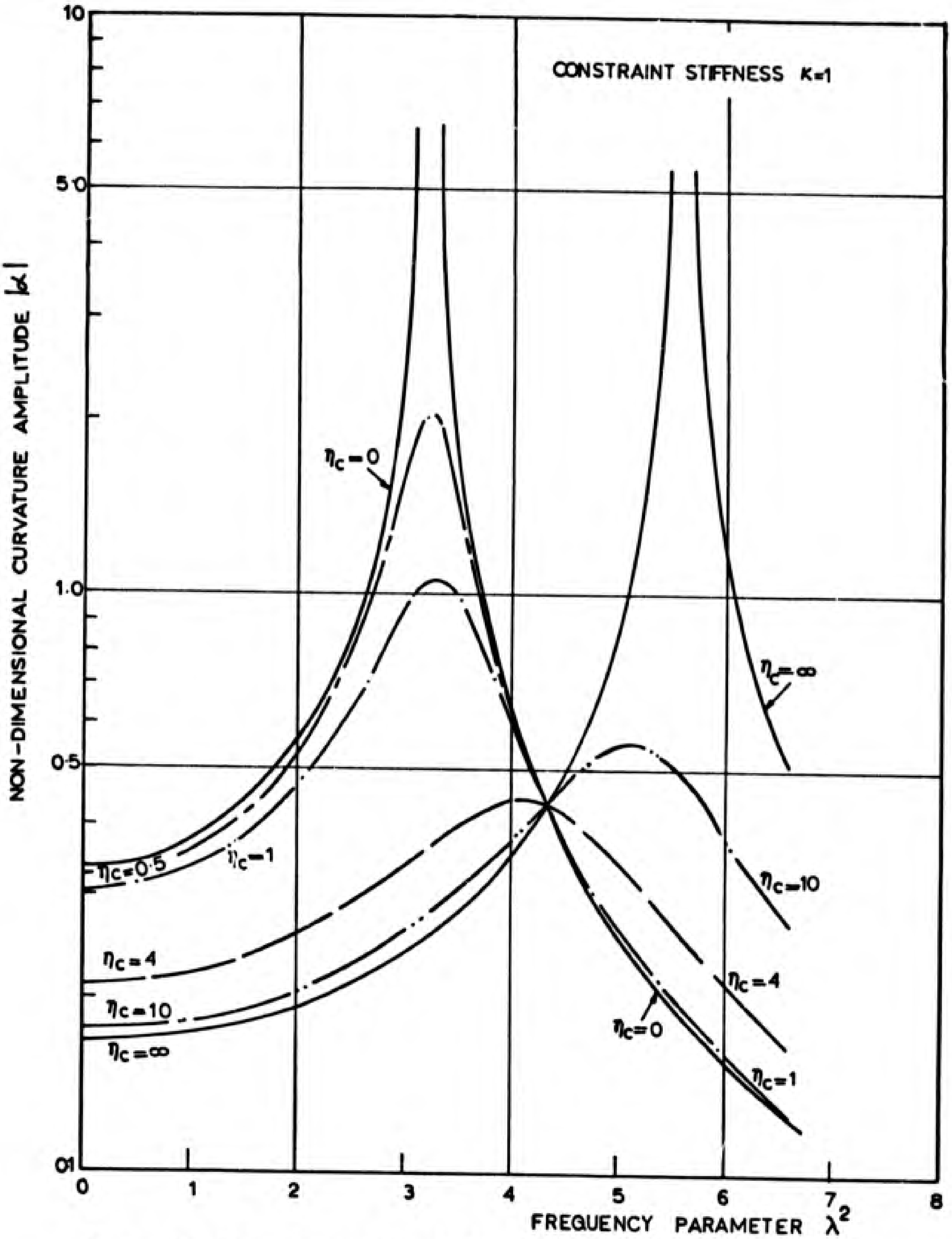

Figure 2. Curvature amplitude at beam centre $|\alpha|$ plotted against frequency parameter λ^2. Constraint stiffness $\kappa=1$

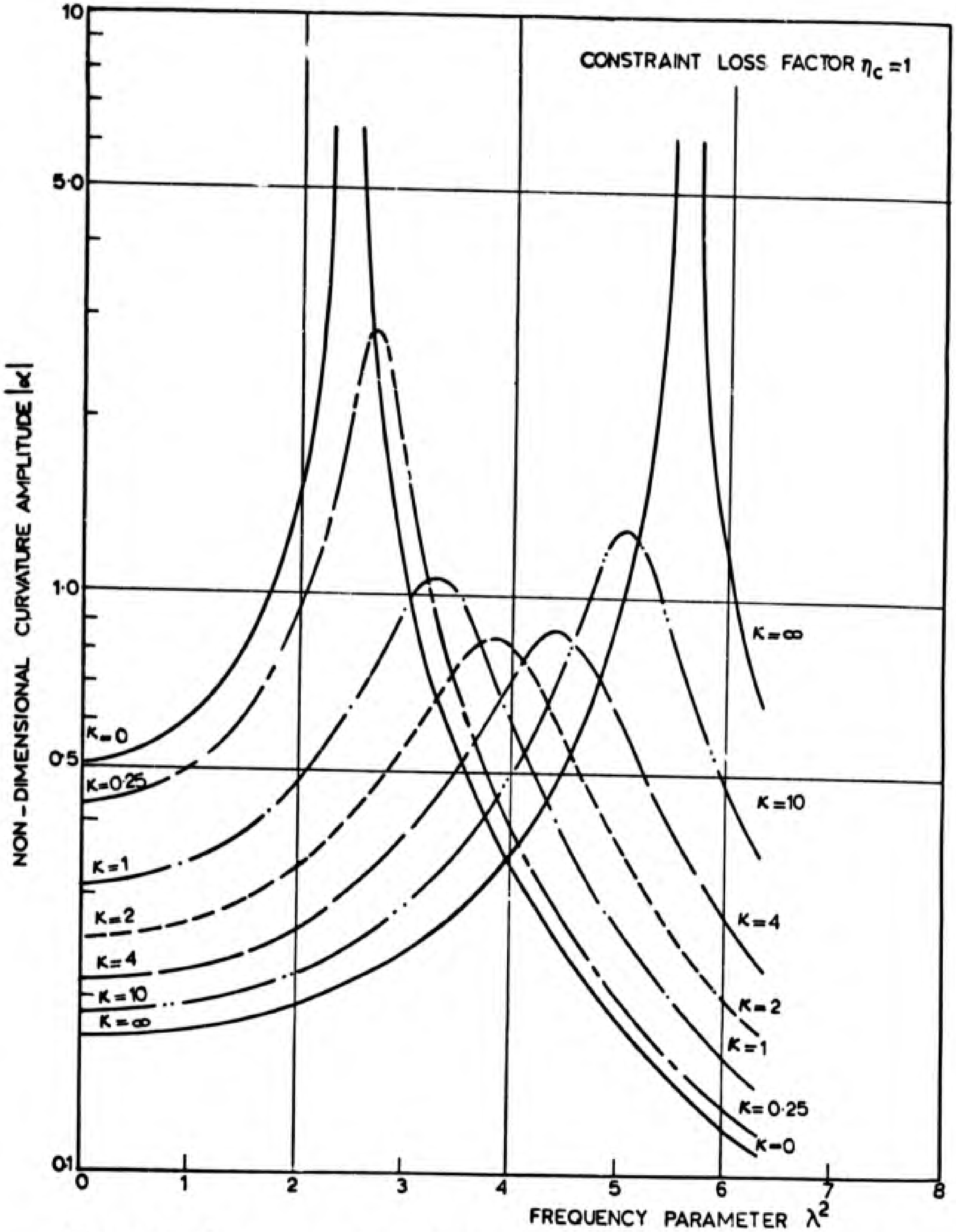

Figure 3. Curvature amplitude at beam centre $|\alpha|$ plotted against frequency parameter λ^2. Constraint loss factor $\eta_c = 1$

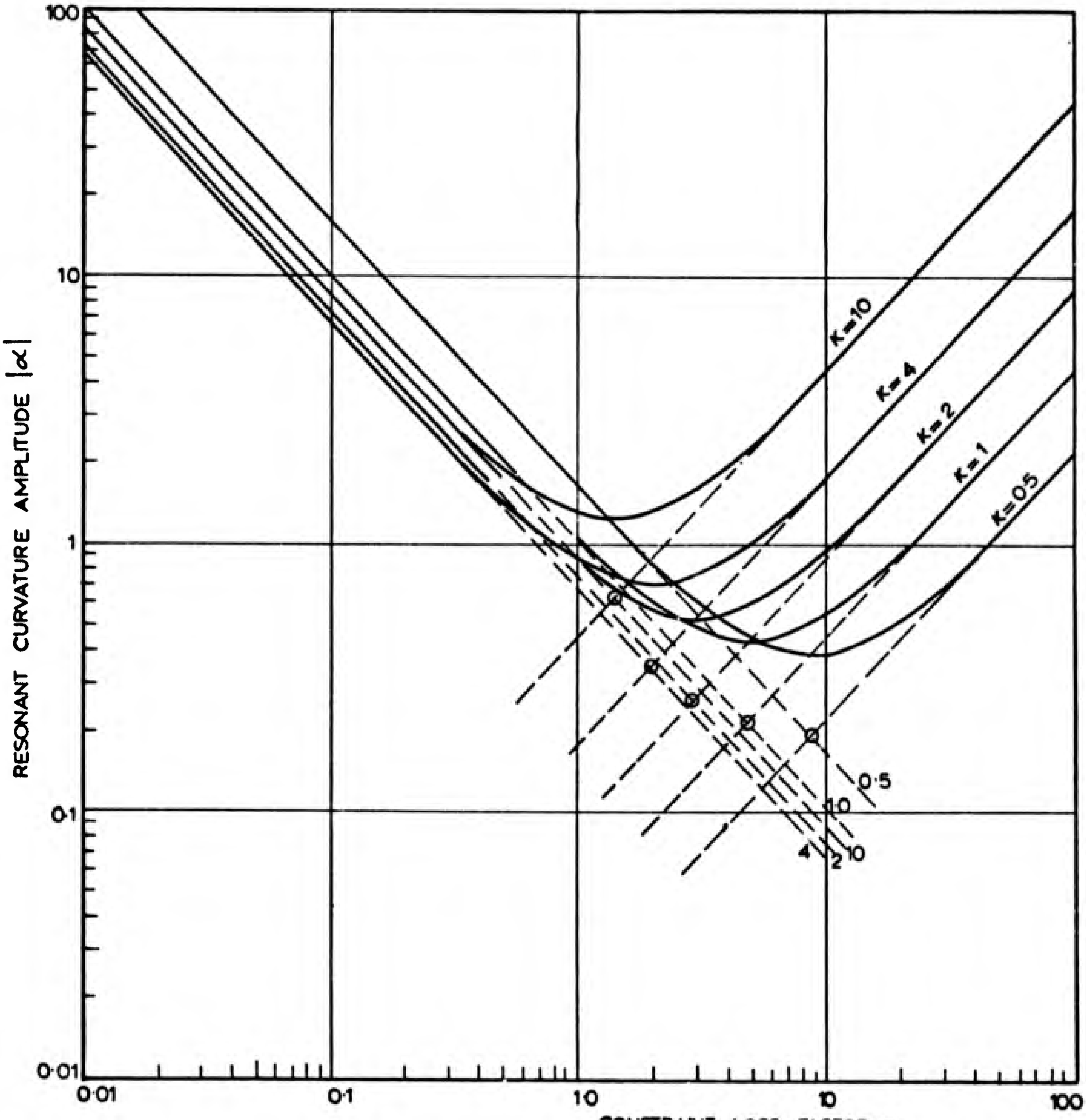

Figure 4. Resonant curvatures of beam for given constraint stiffnesses κ

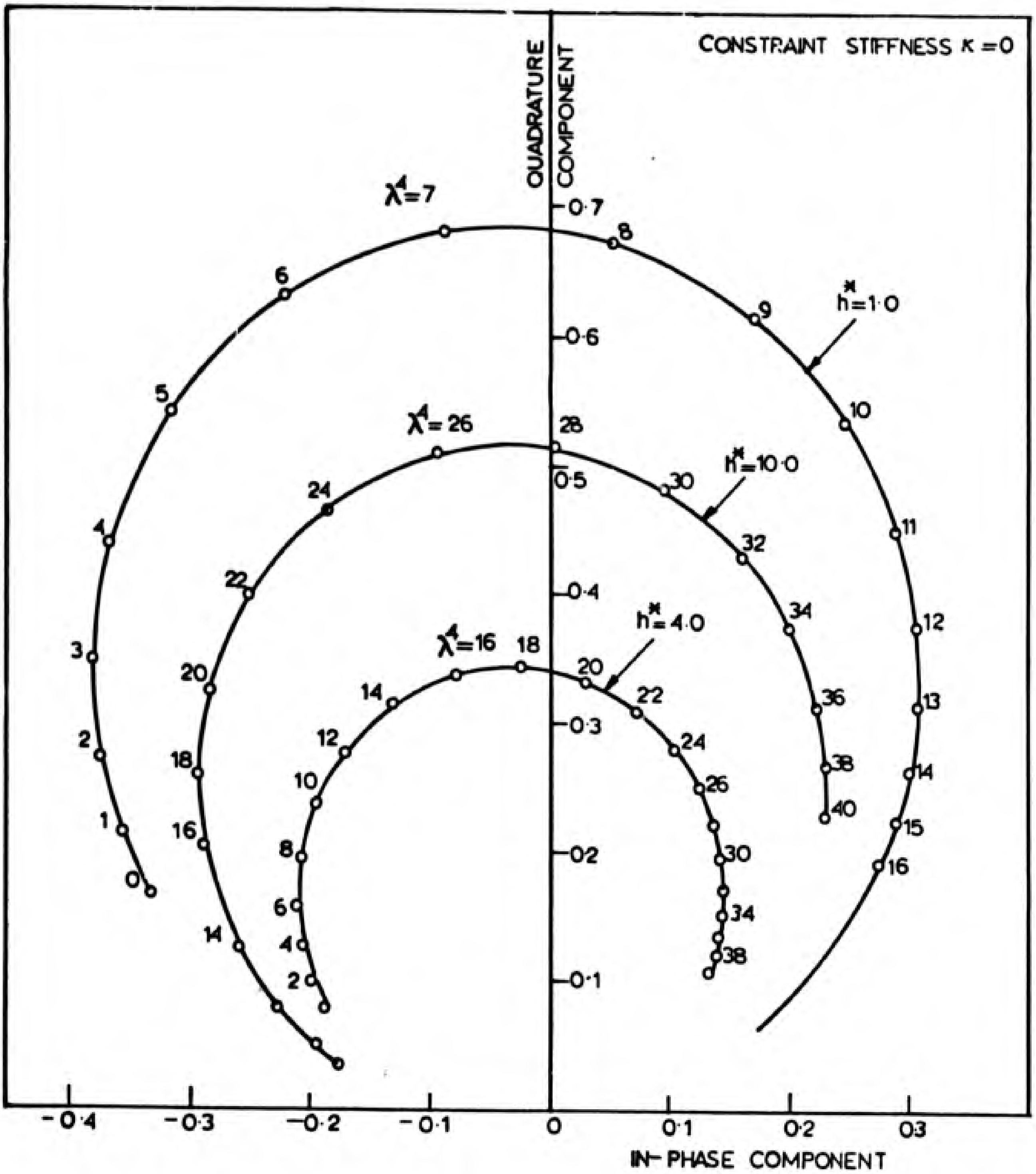

Figure 5. Vector response diagram. Variation with (frequency)²

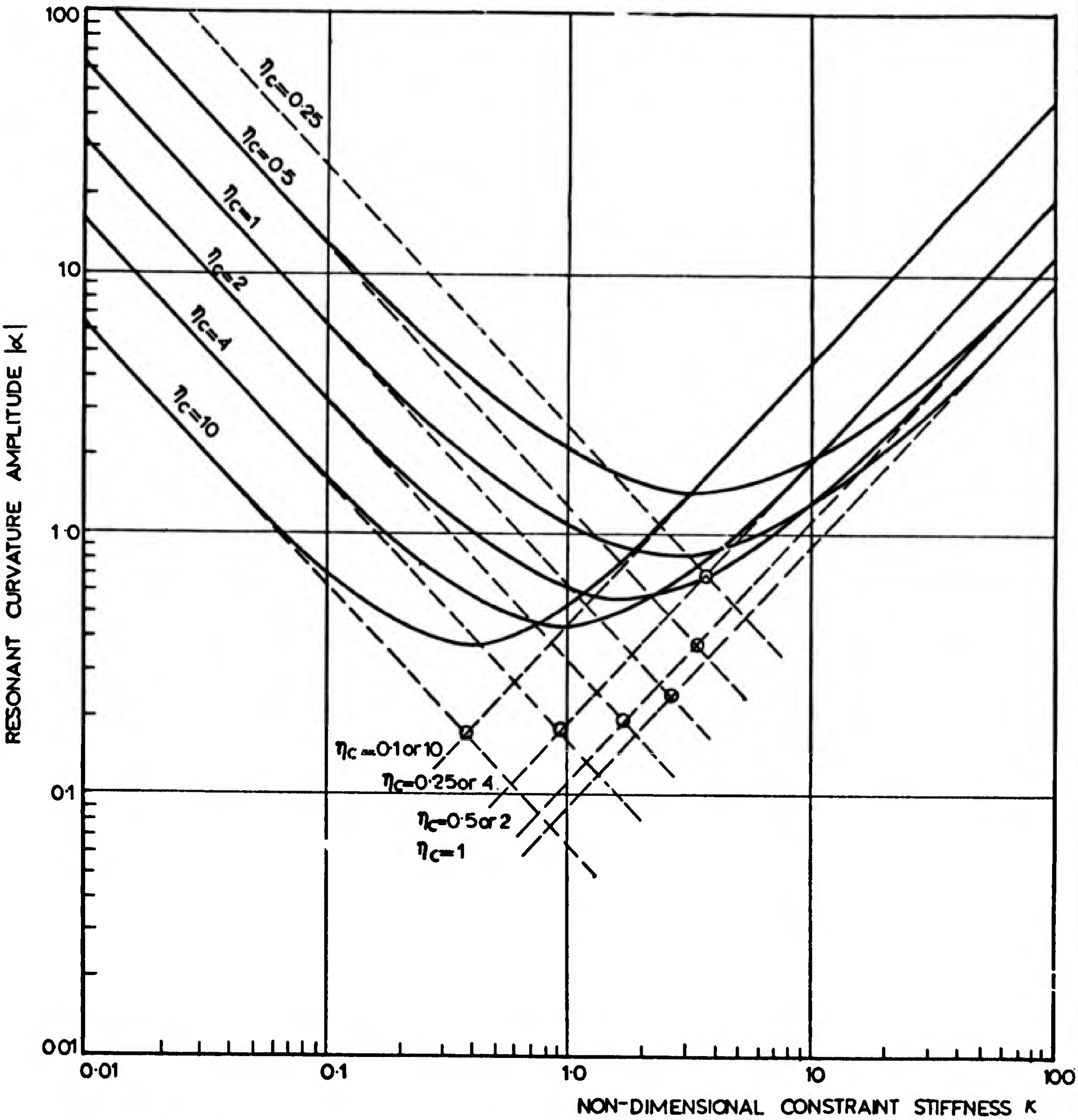

Figure 6. Resonant curvatures of beam for given constraint loss factors η_c

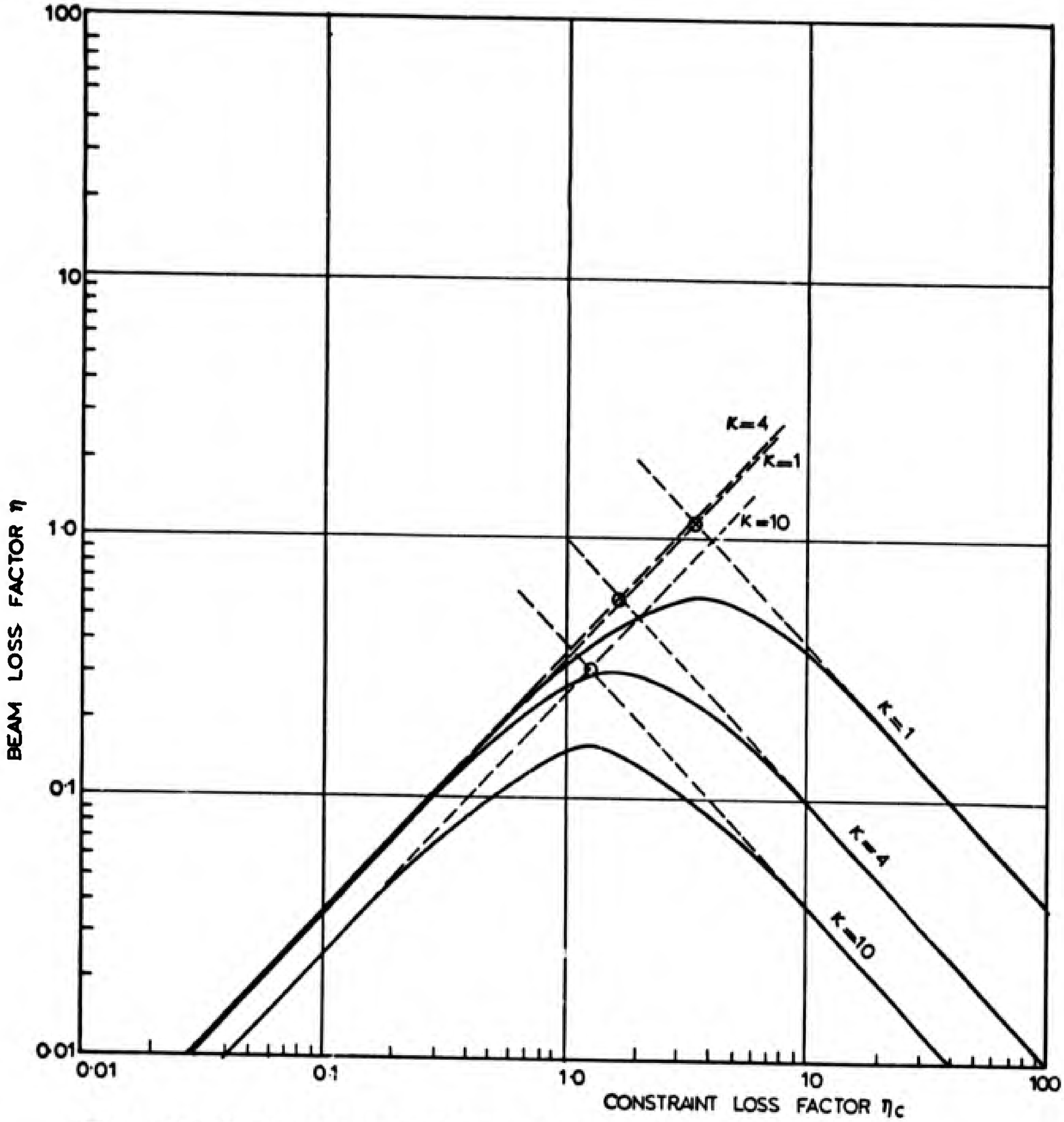

Figure 7. Beam loss factors for given constraint stiffnesses κ

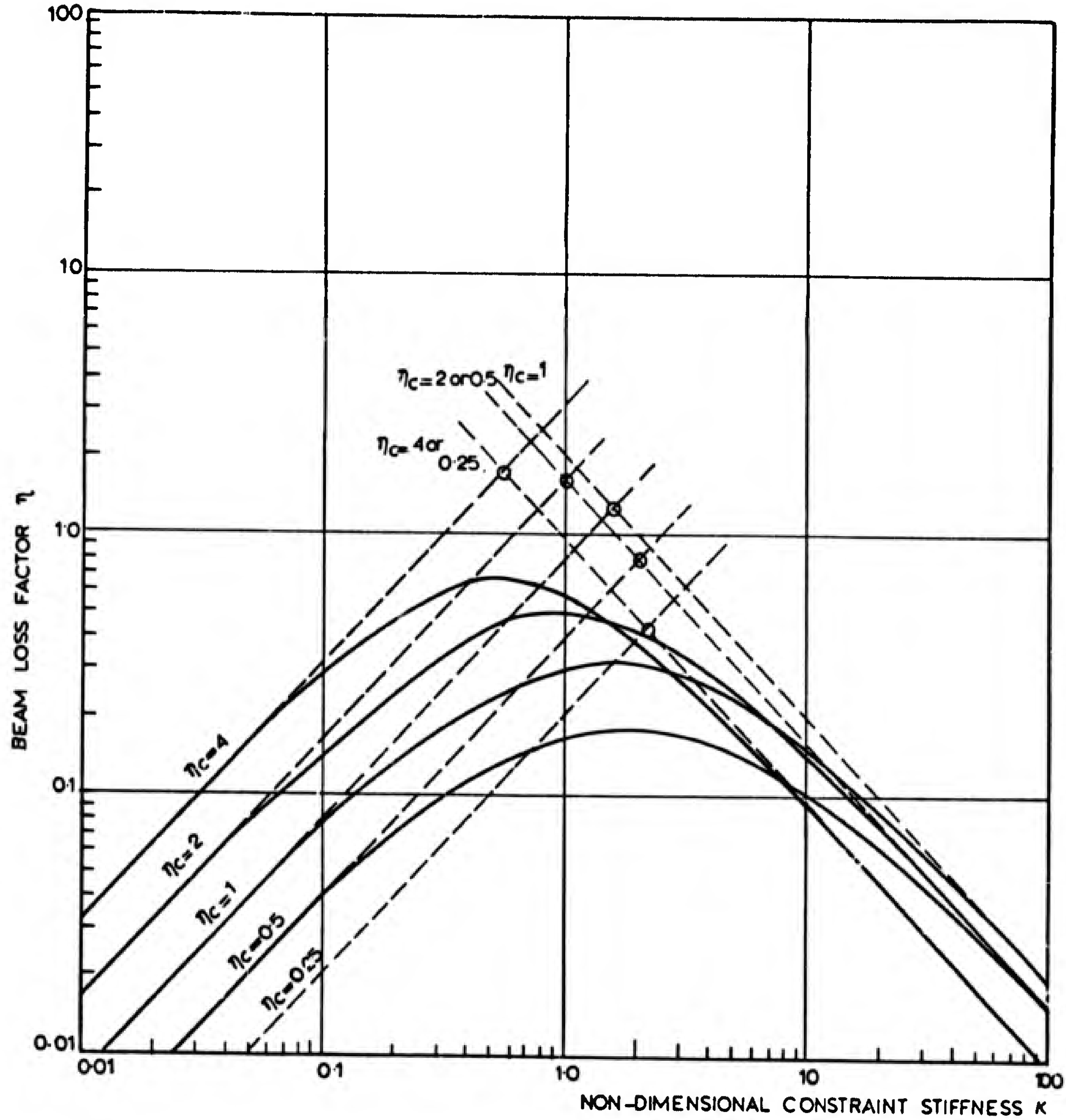

Figure 8. Beam loss factors for given constraint loss factors

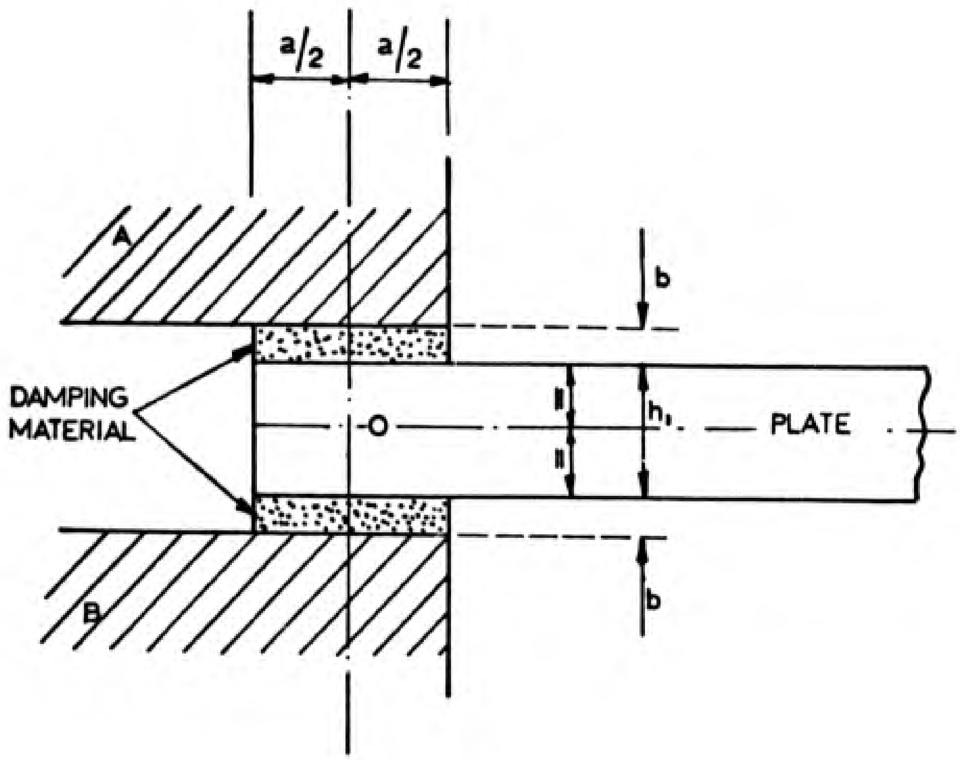

Figure 9. Diagram showing dimensions of root constraining layers

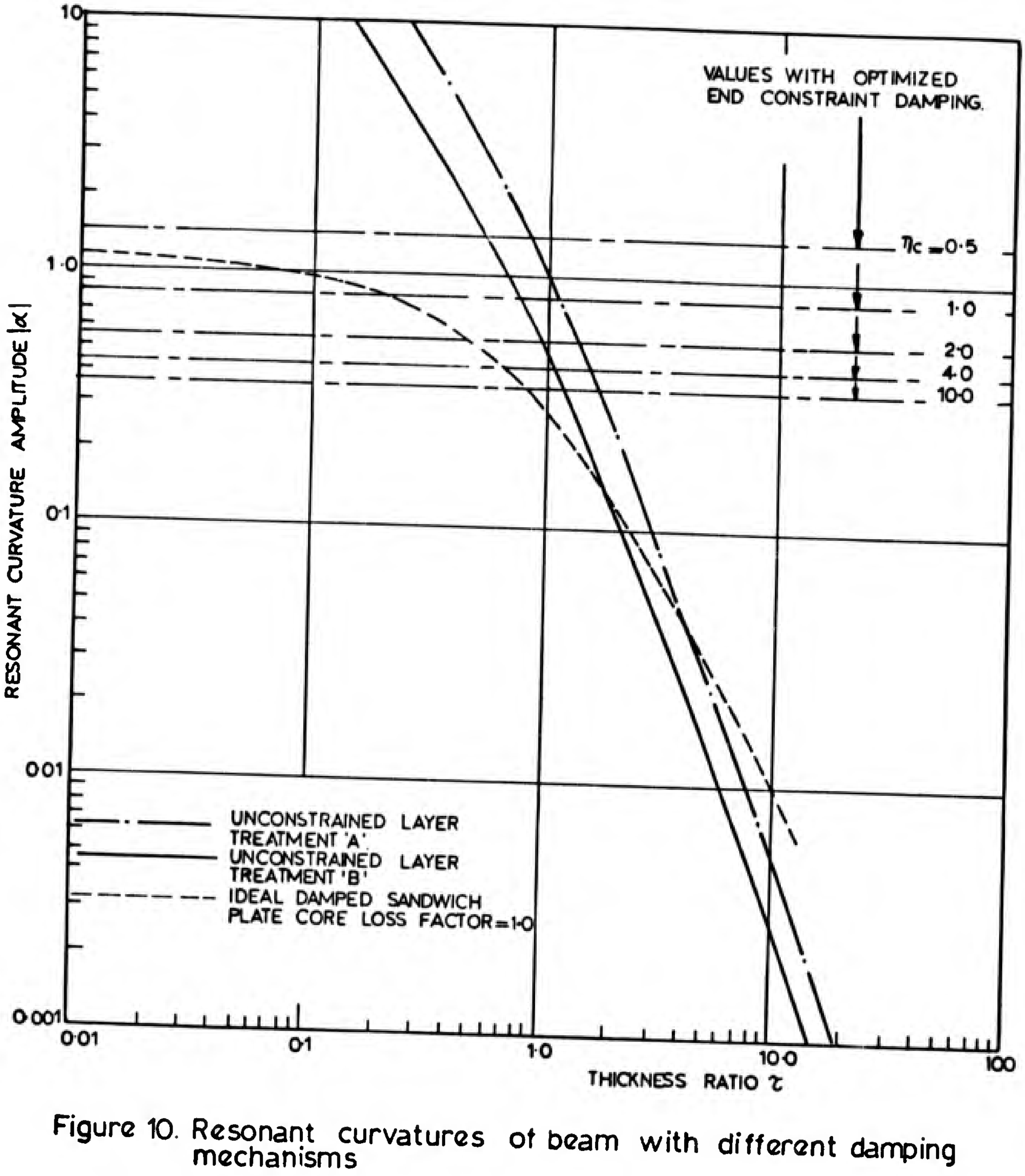

Figure 10. Resonant curvatures of beam with different damping mechanisms

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If the edges of a plate are embedded in a visco-elastic material, flexural vibration of the plate is damped by virtue of the damping forces and couples exerted at its boundaries. This paper analyses and assesses the effectiveness of this form of artificial damping when applied to a uniform beam and compares it with the effectiveness of homogeneous damping layers applied throughout the length of the beam.

Firstly, the theory is developed for the linear flexural response of the uniform beam to uniform harmonic loading. Transverse displacements of the beam are prevented altogether while rotation is opposed by the linear elastic and damping couples from the embedding material. Explicity expressions are derived for the amplitudes of curvature at the centre of the beam and from these it is shown that there exist optimum values of the end constraint damping properties which will minimise the beam resonant response. Methods of estimating these optimum values are discussed. It is shown that different optimum values are required to give the maximum effective flexural loss factors of the beam. This greatest value may be of the order of 0·33.

Comparison with the effectiveness of homogeneous layers shows that the edge-constraint damping mechanism is more effective than thin homogeneous layers, but much less effective than thick layers.

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