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"IMPLICIT MAP PROJECTIONS IN COMPUTER PRINT-OUTS"

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IMPLICIT MAP PROJECTIONS

IN

COMPUTER PRINT-OUTS

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The use of electronic computers to produce various geographical map projections is, of course, well known and established. If, for example, spherical coordinates of points (latitude and longitude) are given they may be converted to plane coordinates by whatever transformation is required to yield such combinations of desired properties as may be possible by appropriate computer programs. This is all straightforward. The computer does the mathematical manipulation of the values. And, if graphical output of the computed projection is needed then this too can be achieved by computer programs utilizing computer printers, scopes, or plotters.

In addition, if values for various phenomena are available at given locations these, too,can be manipulated in virtually limitless ways to produce an extraordinarily large variety of maps including ratio, density, and isoline types, for example, for various topics.

Within the Laboratory for Computer Graphics at Harvard University (under the direction of Professor Howard T. Fisher) there exists the "SYMAP" (Synagraphic Computer Mapping) Program for producing maps which depict quantitative and qualitative data graphically and which use standard computer printers to achieve these results. This program is a continually evolving one with many new options being currently developed. In its early versions a suitable source map of the geographic area to be studied was always a requirement for the user of the program. On source maps measurements were taken to obtain location coordinates for various purposes within the program. This option is still available to the user although now tabular data may be utilized directly, too, if available in this way.

Standard computer printers, properly set to print 8 rows per inch, produce output with characters (letters, numerals, etc.) located in rows one-eighth inch in height and in columns one-tenth inch in width. The standard output "grid" for mapping is thus composed of "character locations" or cells each measuring one-eighth inch vertically by one-tenth inch horizontally. All coordinate measurements are figured in terms of these cells (or their centers). A special SYMAP ruler, conveniently combining both units, is available to facilitate the measurement of coordinate locations on source maps in conformance with their positions on the maps to be produced, if the user chooses to work directly from a source map.

Current program versions still permit this type of source map determination of vertical and horizontal coordinates measured in the row and column units of the computer printer grid itself, of course, and also allow vertical and horizontal coordinates to be measured in the same scale (inches or centimeters, etc.) and expressed in decimal units and converted by a subroutine of the program. And, as noted above, tabular locational coordinates may also be used.

It is obvious that if measurements are made from a source map and retained directly without transformation within the program other than for linear scale, the map projection of the print-out will be the same as that of the source map itself, and the particular characteristics of the character space (i.e. one-eighth inch in height by one-tenth inch in width) is inconsequential except that the general problem problem of representing a point by any finite area remains to be considered. This, too, is a scale problem.

Let us examine the nature of the printer output grid more carefully, however, to see if its particular characteristics can be turned to advantage especially with regard to tabular data of latitude and longitude coordinates for locations. It should be noted here, too, that the one-eighth inch high by one-tenth inch wide character space, while standard for the SYMAP program,is not the only one available. For example, on the IBM 1401 printer for alphanumeric print-out, a simple turn of a knob allows one to have either eight rows to the inch or six rows to the inch. Thus, the character space could be one-sixth of an inch high by one-tenth of an inch wide if desired as well as one-eighth by one-tenth. In addition another adaptation allows the printing of ten rows to the inch so that the character space can be one-tenth of an inch by one-tenth of an inch. Consider now the ratio of width to height for each of the character spaces noted above. These width to height ratios are pure numbers and are exactly 0.60000, 0.80000, and 1.00000 respectively.

Suppose now that we regard the character spaces as bounded by straight lines in a grid. The family of lines in this grid running horizontally and separating the rows are, of course, at right angles to the family of lines running vertically and separating the columns. Regardless of the particular value of the above ratios, the cells in a grid are rectangles. The actual ratio involved determines the relative elongation of the cell.

Now consider these row lines and column lines as though they were actually selected latitude and longitude lines on the graphical portrayal of a map projection from a perfectly spherical earth to a plane. Thus, a difference in latitude (distance between row lines) would be taken along a column line and a difference in longitude (distance between columns) would be taken along a row line. Before we follow up these ideas,however, let us refresh our memories concerning a few of the major properties of the conventional latitude-longitude grid assumed to cover the assumed spherical earth. We shall not attempt to be exhaustive.

The family of latitude lines and the family of longitude lines on the earth intersect at right angles. Longitude lines run north and south and measure distances in angular terms to 180° east and west of a given zero longitude line. They are all of the same length with the maximum distance between any two occurring at the equator and the minimum (zero) occurring at the north and south poles. Thus longitude lines (sometimes called meridians) converge from the equator poleward in both the northern and the southern hemispheres. A given difference in longitude represents a varying over-the-earth distance depending upon where it is taken. Each longitude line is an arc of a great circle and the center of each of these great circles is the earth's center. The planes of this particular set of great circles intersect along the polar axis of the earth. These planes thus all contain the center of the earth as must the plane of any other great circle on the earth's surface.

Latitude lines run east and west and measure angular distance to 90° north and south of the equator as the zero line. Latitude lines are equally spaced throughout for a given difference in latitude. Thus latitude lines are small circles save for the equator which is a great circle. Latitude lines are sometimes referred to as

Parallels of Latitude. This is ambiguous for although any two latitude lines remain equidistant, neither of these lines may, in general, be regarded as a "straight" line (great circle). In fact, at most only one latitude line, the equator, is a straight line. The planes of the latitude lines are parallel however, and intersect the planes of the meridians at right angles. Any two straight lines on a sphere must intersect not only once, but twice and the points of intersection must be antipodal.

It is to be stressed that whereas a unit of latitude, say a degree, is everywhere of constant length (60 nautical miles or approximately 69 statute miles) on a spherical earth having the radius it does, a degree of longitude is equivalent in length to a degree of latitude only at the equator. Everywhere else a degree of longitude is shorter. Assuming a degree of longitude to be unit "over the earth" length when taken along the equator, then the length of a degree of longitude is, for example, approximately 0.86603 times unit length in latitude 30° north and south, exactly 0.50000 times in latitude 60°, and exactly 0.00000 times at the geographical North and South Poles. These particular values are only specific instances of the fact that the "over the earth" length of a degree of longitude varies as the cosine of the angle of the latitude in which it is taken.

Let us return to the rows and columns of the computer printer output grid. As noted earlier, they do intersect at right angles as do latitude and longitude on the earth's surface. A given difference between rows is everywhere a constant distance as is a given latitude difference on the earth. On the computer print-out a given column difference is everywhere a constant distance unlike, however, the varying distances this constant angular difference represents on the earth.

Consider again the various width to height ratios for the character spaces of the print-out. The ratios mentioned previously were 1.00000, 0.80000 and 0.60000. Regard these ratios now as cosines of angles. For an angle of 0° the cosine is 1.00000 and thus on the earth's surface the length of a degree of longitude is equal to that of a degree of latitude when taken at the equator. The length of a degree of longitude is 0.80000 times that of a degree of latitude in latitudes 36°52'+ N. and S. and is 0.60000 in latitudes 53°07'+ N. and S.

Suppose that we wished to map the entire earth under the assumption that each print-out character space is to represent an area on the earth's surface one degree of latitude by one degree of longitude in extent. We establish the same proportionality between number of columns and degrees of longitude as between number of rows and degrees of latitude. In this case both ratios have the value, one. That is to say, for example, every area on the earth's surface 20° of latitude by 20° of longitude in extent will be represented by 400 character spaces within the print-out space of 20 rows by 20 columns, respectively. The actual size of such a bounded area on the earth's surface varies systematically with latitude. On the print-out map it is, of course, constant. (What this area is depends on the height-width ratio and is to be discussed subsequently.) Thus, to map the entire earth with its full extent of 180° of latitude and 360° of longitude would require a print-out

180 rows and 360 columns if the ratios established above are to be preserved. This, of course, would require a multiple-width print-out as the width of one strip is 130 columns. In the SYMAP program this presents no particular problem. Multiple-width maps are easily accomplished. Continuity across strips is preserved with a one-column overlap. Thus, to map the world, as above, would require two full strips of 130 columns each and 102 columns on a third one. The map would then measure 36 inches in its longitudinal extent assuming that it is mounted with the overlaps and thus reduced to 360 columns. (Note that each of the character spaces noted earlier has a width of one-tenth of an inch.)

The 180 rows of latitudinal extent occasions no particular problem either as the print-out may be as long as required in one continuous strip. Paper folds occur every eleven inches and it may be desired to skip rows at these palces. This is possible, but let us assume that such an option is not exercised. The actual measure for this latitudinal extent would, of course, depend upon the particular character space used. If the space having as its height one-sixth of an inch were used, the map would measure 30 inches for its latitudinal extent; for a height of one-eighth of an inch, the map would measure 22 1/2 inches; and for a height of one-tenth of an inch the map would measure 18 inches.

Each of these three maps represents a different linear scale relationship with the earth. We will discuss this in more detail subsequently. For the time being, however, let us consider certain general characteristics of these maps in terms of a class of map projections, namely, cylindrical projections.

Cylindrical projections, themselves, may be regarded as limiting cases of conical projections when the so-called constant of the cone, n, is equal to zero, n being defined for conical projections as the ratio of the angle on the map produced by two given meridians (straight lines converging to a vertex) to the angle which corresponding terrestrial meridians make at the geographical poles. The "simple conical projection" has latitude lines represented by concentric circles with their center as the above noted vertex.

We can use the following standard notation following Hinks (1921) for the simple conical projection with one standard latitude, ϕ_o:

$$n = \sin \phi_o,\ r_o = R \cot \phi_o, \text{ and}$$

$$r = R[\cot \phi_o - (\phi - \phi_o)];$$

when n is as defined above, R is the radius of the Earth (assumed spherical) expressed in scale units of the intended map, and r_o is the radius of the standard latitude taken as the length of the tangent drawn from the polar axis to touch the sphere at the standard latitude, and r the radius for a latitude line, ϕ.

Let λ be the longitude of a point on the Earth. Then, the length of an element of the standard latitude is:

$$R \cos \phi_o \cdot \Delta\lambda$$

If we take θ as the angle between two radii representing meridians whose difference of longitude is $\Delta\lambda$, we have $r_o \cdot \theta$ as an alternative expression for the length of an element of the standard latitude.

Thus:

$$R \cos \phi_o \cdot \Delta\lambda = R \cot \phi_o \cdot \theta$$

$$\text{and } n = \theta/\Delta\lambda = \sin \phi_o.$$

The lengths along the meridians are true and the general expression for the radius of any latitude line is:

$$r = r_0 - R(\phi-\phi_0)$$
$$= R[\cot\phi_0-(\phi-\phi_0)]$$

as given above.

To produce such a map graphically one could draw the standard latitude as an arc with radius $R \cot \phi_0$ and length $2\pi R\cos\phi_0$. The central meridian is then laid off and marked with the true to scale distances to other latitude lines which can then be drawn in. To obtain other meridians, the standard latitude may then also be divided truly as required, and the points so obtained can then be joined to the pole of the projection to obtain these meridians.

All of the meridians are their true lengths to scale by construction; the expression for this constant scale along the meridians is obtained by differentiation. It is:

$$\frac{dr}{Rd\phi} = -1$$

As ϕ increases, r decreases and the sign is thus negative.

Along any line of latitude ϕ, the scale is:

$$\frac{rd\theta}{R\cos\phi d\lambda} = \frac{r \sin\phi_0}{R\cos\phi} =$$

$$[\cos\phi_0-(\phi-\phi_0)\sin\phi_0]\sec\phi.$$

The scale along the standard latitude is, of course, unity, but along every other one it is greater than unity, increasing with distance from the standard latitude. The area scale of the map varies as the linear scale along the latitude lines since linear scale along the meridians is constant.

Now, if, as suggested above, we deliberately make n equal to zero, ϕ_0 also is zero and the standard latitude is the equator. The radius r_0 of this standard latitude becomes infinite but $r_0 - r = R\phi$ and is finite. Such a map projection may be regarded as a simple cylindrical projection and as one of the limiting cases of the simple conic projections. (The other would be a plane zenithal projection.) Though not a perspective projection the simple cylindrical may be regarded as derived from a circumscribing cylinder tangent to a sphere along its equator and with a normal polar axis for the sphere.

The longitude and latitude lines become sets of equidistant straight lines intersecting at right angles and forming a series of squares. The length of degrees of longitude (variable on the Earth) are shown on the map as everywhere equal to each other and everywhere equal in length to degrees of latitude (constant on Earth). Distances along the meridians and along the equator are correct. Distances along the other latitude lines are shown at changing scale, so that both the equator, a line of approximately 25,000 miles in length on the Earth, and the North Pole, a point, are shown as lines of the same length on the map.

This simple cylindrical map described above is obviously the map produced by the direct identification of the rows and the columns of the computer printer with latitude and longitude respectively and by the assumption of the same proportions between the pairs of units when the character space size opted is that with the width to height ratio equal to 1.00000.

This projection composed, as it is, of squares has been called by the Germans, "quadratische Plattkarte," by the French, "projection plate caree," and by the English, "plain charts." It is a simple conventional projection that serves very well indeed in the Tropics where the appropriate cosines for the angles of these low latitudes vary little from 1.00000. For example, the value does not become less than 0.90000until nearly 26° N. and S., and is still as large as 0.80902 outside the Tropics at 36° N. and S. (This latitude is a central one for the Mediterranean Sea, for example.) In low latitudes the actual system of spherical polar coordinate specification of latitude and longitude is very closely approximated by Cartesian plane coordinates.

It is interesting to note that the terms, latitude (Latin, latitudo, breadth) and longitude (Latin, longitudo, length), were conceived originally with respect to the Mediterranean Sea and its surrounding oikoumene (Greek) or known "habitable" world.

In the Geographia of Claudius Ptolemaeus (c. 90-168 A.D.) we find:

> Not unreasonably we call the distance extending from the setting to the rising sun the longitude, and the distance from the north pole to the south pole the latitude, when we mark the parallels in the vault of the heavens. Moreover, the greater distance we call longitude, which is accepted by all, for the extent of our habitable earth from west to east all concede is much greater than its extent from the north to the south.

Ptolemy did, of course, regard the earth as spherical and recognized latitude and longitude as spherical polar coordinates rather than as plane coordinates. The conception of the earth as a sphere probably dates from the late fourth or early fifth century B.C.

These pertinent ideas apparently were lost to Europeans, however, with the decline of the Roman Empire and had to be invented anew. During the interval, however, the terms, latitude and longitude did not lie unused. For example, in the fourteenth century, Bradwardine of Paris and his followers in Merton College, Oxford, used the terms in a non-geographical but abstract geometrical sense in connection with the idea of functional relationships among natural phenomena.

When applied to giving quantitative expression to changes of quality, the problem of intension or latitudo formarum (the latitude of forms), the ideas refined at Oxford involved conceptions of amounts by which a quality or "form" varied numerically with regard to a fixed scale in relation to a scaled extension or longitudo formarum.

This conception of the relationship between intensions and extensions gave rise in the fourteenth century to geometrical methods and graphical portrayals. The Greeks and Arabs had used algebra in connection with geometry and the idea of plotting a "position" of a "point" in a coordinate system had been familiar to geographers in classical times. Geographical latitude and longitude are, of course, both measures of extension and the idea of conceiving and depicting graphically (i.e. cartographically) the geographical variation in the intensity of some phenomenon was not achieved until Halley's isolining of terrestrial magnetic variation in the latter part of the seventeenth century.

Sometime well before the great era of geographical exploration in the sixteenth century, the use of geographical coordinates for locations had again come into general use. The plane charts employed for coastal and Mediterranean navigations contained such latitude-longitude grids. Within this framework the plane chart had proved sufficient. However, the utter inadequacy of these plane charts as maps for world-wide geography and navigation required new ones and led to the graphical development of a more suitable chart by Gerardus Mercator in 1569 and to the mathematical statement of this "Mercator" projection by Edward Wright in 1599. Discussion of this important part of cartographic history, however, lies beyond the scope and intent of this paper.

Let us return now to the implications of two other available character space sizes for modern computer print-out, i.e. with width to height ratios of 0.8 and 0.6 respectively.

It is possible to have a conical projection with two standard latitudes of correct length and with true meridians. Between the standard latitude lines, the scale is too small along the latitude lines and beyond them it is too large. The differences, however, do not increase nearly so rapidly as in the case of one standard latitude line. It should be noted that in general the standard latitudes are in the same hemisphere. Rigorous conditions can be imposed to determine the selection of the appropriate latitudes for given areas and given map requirements concerning the distribution of "errors."

In the ordinary conical projection with two standard latitude lines, ϕ_1 and ϕ_2, the usual notation is:

$$n = \frac{\cos \phi_1 - \cos \phi_2}{\phi_2 - \phi_1}, \quad r_1 = \frac{R(\phi_2 - \phi_1)\cos \phi_1}{\cos \phi_1 - \cos \phi_2},$$

and $r_1 - r = R(\phi - \phi_1)$.

For the cylindrical case we make n equal to zero and then $\phi_1 = -\phi_2$ which makes r_1 infinite. However, $r_1 - r$ remains finite.

Now, the two standard latitudes are of the same numerical designation but one is north of the equator and the other is south of it.

The computer print-out having the character space with the width to height ratio of 0.80000 (assuming again the same proportionality for columns and longitude as for rows and latitude) may be regarded as representing graphically a cylindrical projection with 36°52'+ North and 36°52'+ South as two standard latitude lines. (35°52'+ is the angle for which 0.80000 is the cosine.)

For the character space that is 0.6 times as wide as it is high, the implicit cylindrical projection with two standard latitudes has 53°07'+ North and 53°07'+ South as the standard latitudes.

In each of these two projections, the latitude and longitude lines form assemblages of rectangles rather than squares. On each of these projections the respective rectangles are everywhere constant in size although, of course, the size differs as to the projection. Maps of this kind have been designated by the French as "projection plate parallélogrammatique" and by the Germans as "rechteckige Plattkarte." Hinks noted that there seemed to be no English name for it since "the projection has obviously no serious value." He added that, as a consequence, it would "not be considered further" by him. Perhaps now though it is interesting and instructive enough to "consider further."

Consider again the projection with 36°52'+ N. and S. as the standard latitudes. What actually exists there on the earth's surface that this might be an appropriate projeciton for mapping?

Well, this particular latitude line is virtually the mid-latitude for the conterminous United States. Moreover, the conterminous United States has a considerably larger east-west extent than it has a north and south one. The range of latitude is from approximately 25°N. to 49°N.

On the map in question the distances are correct along the meridians and the same linear scale that applies to the meridians also applies along 36°52' N. latitude. The extreme linear scale differences for such a map of the United States would occur, of course, along the latitude lines that are the most northerly and the most southerly ones for the United States as noted above. To the north of 36°52' N. the linear scale error may be regarded as positive, that is to say, such latitude lines are shown as too long. Latitude lines south of this standard latitude may be regarded as having a negative linear scale error. The portrayal lines are shown as too short.

Along latitude 49° N. the magnitude of the error is nearly 18 percent, the scale being too large for the standard while along latitude 25° N. the scale is too small for the standard by about 13 percent. Along the equator, the negative scale error reaches its maximum, being 25 percent south of the equator the negative error again declines becoming non-existent along latitude 36°52' S. at which line the scale error becomes positive again. It is interesting to note that a positive error of 25 percent occurs, for this projection, at about 53°07' N. and S. Hence there exists a range of latitude of slightly more than 106° within which the linear scale difference does not exceed 25% from that for the standard latitudes. Moreover, this maximum "error" is realized only when distances are measured at right angles to the meridians, the linear scale being everywhere constant and correct along the meridians, themselves.

The area scale of the map varies as the linear scale along the latitude lines and hence as a relationship among the cosines of the angles of the latitudes. This projection, of course, is not an equal area, that is to say, equivalent, projection. It follows from what has been said before that the area scale may be regarded as virtually correct in the vicinity of latitudes 36°52' N. and S. The following table shows how a 5° so-called spherical trapezoid (i.e. a portion of the earth's surface having a span of latitude of 5° and a span of longitude of 5°) varies as to area taken within the latitude range of the conterminous United States.

One trapezoid of 5° (lat. x long.) at latitude:	Area in square miles
25°-30°	105,606
30°-35°	100,514
35°-40°	94,653
40°-45°	88,064
45°-50°	80,790

On this projection, of course, all such trapezoids are portrayed as of equal size. Subsequently we shall discuss the actual linear scales and their variations over the maps on these and other projections.

Conformality is that property which if it exists in a given projection renders a map such that all angles on the earth are properly portrayed. Thus, shapes, too, will be correct, but only for infinitesimal areas. Since linear scale must vary over the map, shapes of large features must therefore be distorted, even though angles are correctly maintained at all points. In the "cylindrical" projections described above, the latitude and longitude lines do intersect at right angles on the map as they do on the earth. This is a necessary condition for conformality on the map, but it is not sufficient. Intervening angles must also necessarily be maintained. This can occur only if the linear scale is the same (though not necessarily correct) in all directions around any given point. Of course the linear scale may (must) vary from point to point, but whatever it may be at a given point it must be independent of direction taken around that point. On the maps described above this condition does obtain for the standard latitudes where the linear scale along the latitude line and the longitude lines are identical. Moreover, they are not only identical, but also correct.

All of the above remarks about equivalence and conformality serve to demonstrate the general usefulness of a rectangular projection for the neighborhood of its standard lines of latitude.

In the case of the computer print-out with the character space having a width to height ratio of 1.00000 the use of a constant ratio for columns to longitude, and the assumption that each character space represents an area bounded by one degree of latitude and one degree of longitude on the earth yields a map print-out having a width of 36 inches and a length of 18 inches. (North at the top). The nominal linear scale of the map would be approximately 1/44,000,000 (nearly 700 miles to the inch) and would be correct along the equator and along all meridians, varying elsewhere as described above. For the character space with a width to height ratio of 0.80000 the map would again measure 36 inches in width but its meridians would be 22.5 inches long. The now larger nominal linear scale would be about 1/35,200,000 (roughly 550 miles to the inch) again true along the meridians, but true now along latitudes 36°52'+ N. and S. The character space ratio 0.60000 yields a map also 36 inches wide, but now 30 inches from the north pole to the south pole along the meridians and true along latitudes 53°07'+ N. and S. and, of course, the meridians. The still larger nominal linear scale is about 1/26,400,000 or nearly 420 miles to the inch.

It is obvious how these scales would be affected if the constant proportionality of columns to longitude and rows to latitude were taken as some value other than one--say two. The linear dimensions of all three kinds of print-out would be halved as would the nominal scales in each case.

In our definition of cylindrical projections we noted that they are characterized in the conventional case by the representation of the meridians equally-spaced straight lines of the same length. (Transverse and oblique projections are possible in which the cylinder is assumed to be tangent to or intersecting the sphere not along the latitude lines but rather along some meridian for transverse projections and other than latitudes or a meridian for the oblique case.) Let us continue to assume the conventional case for the longitude arrangement, but now permit the latitude spacing to be varied over the map to achieve desired properties in the resulting projection. We still maintain latitude lines as parallel straight lines, however unevenly spaced, and thus continue to make use of the rectilinearity of the form the standard printer put out. A series of illustrations is given here to demonstrate the variety.

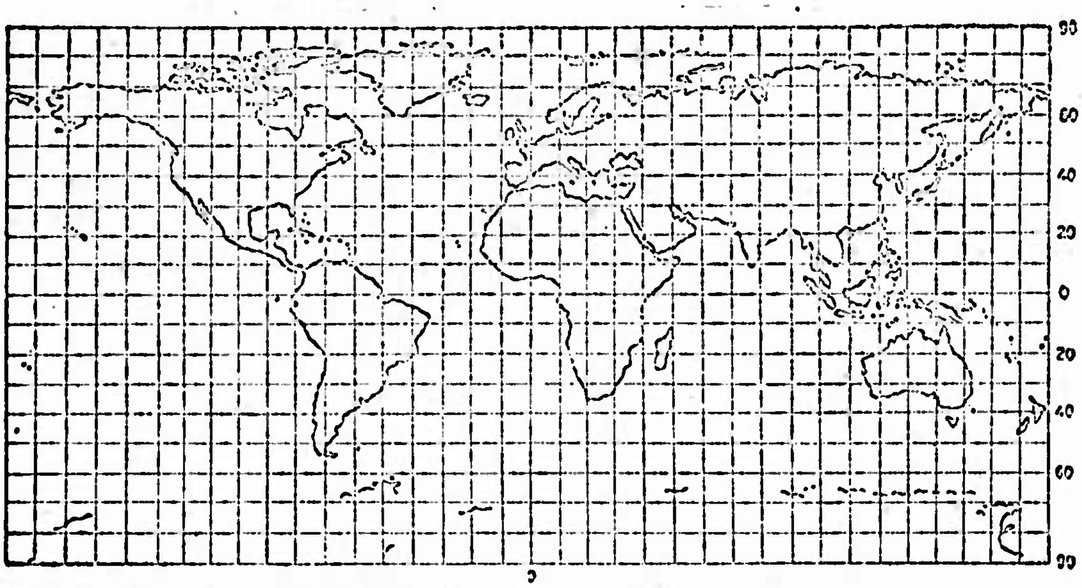

Fig. 1--Cylindrical equal-spaced projection

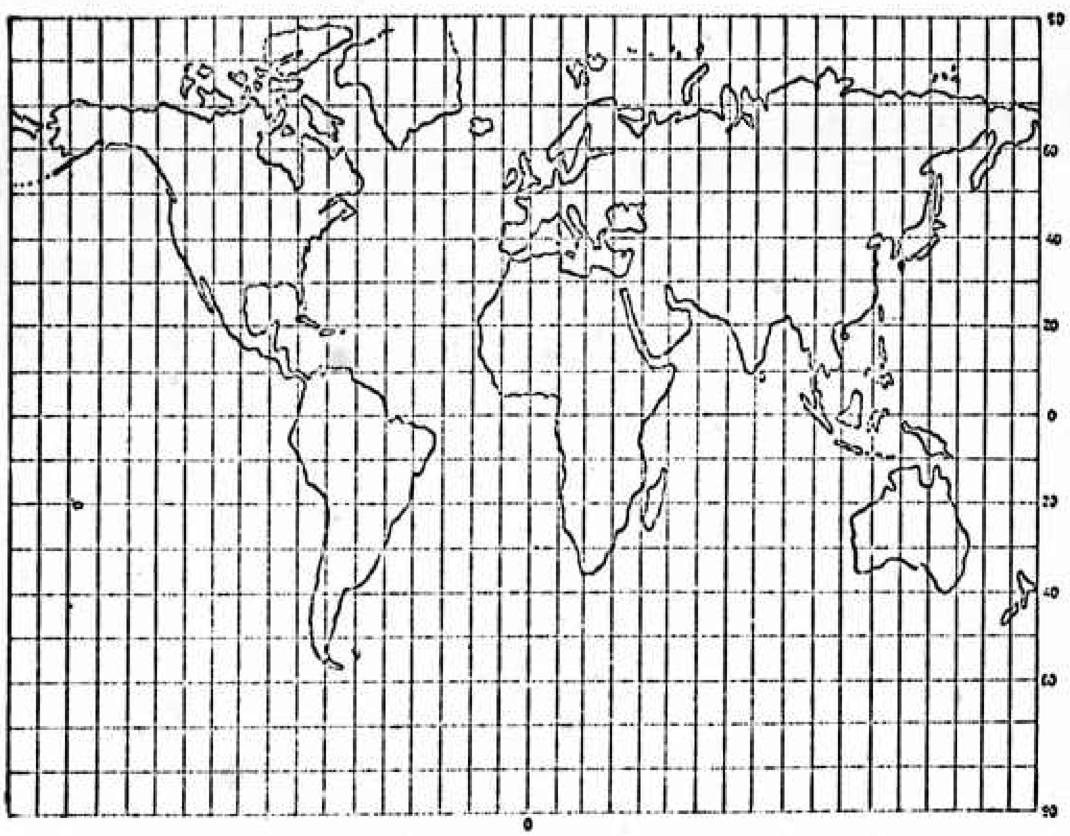

Fig. 2--Modified cylindrical equal-spaced projection

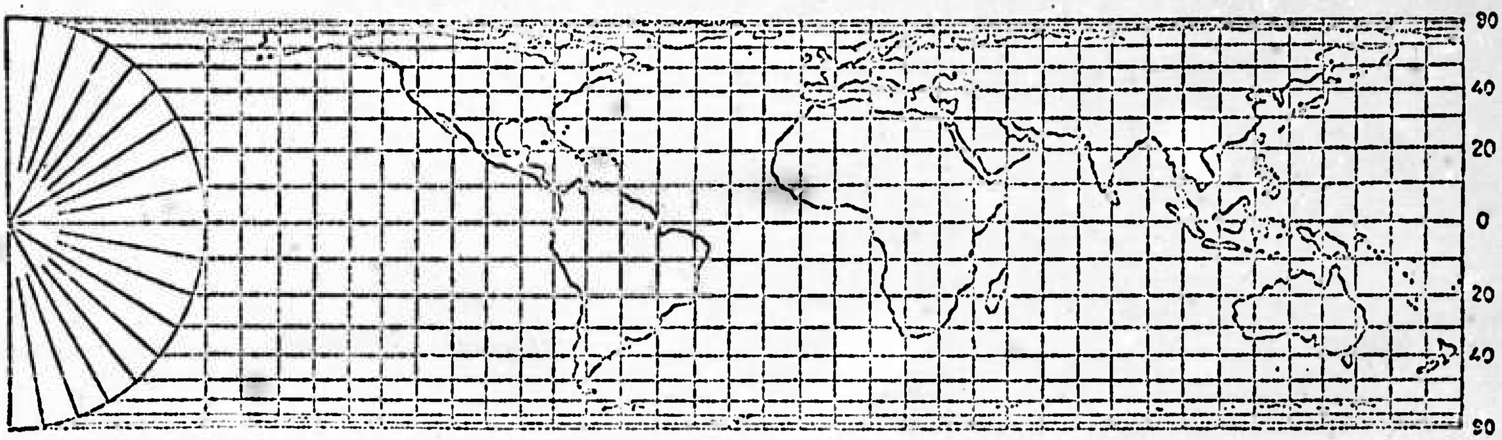

Fig. 3--Cylindrical equal-area projection

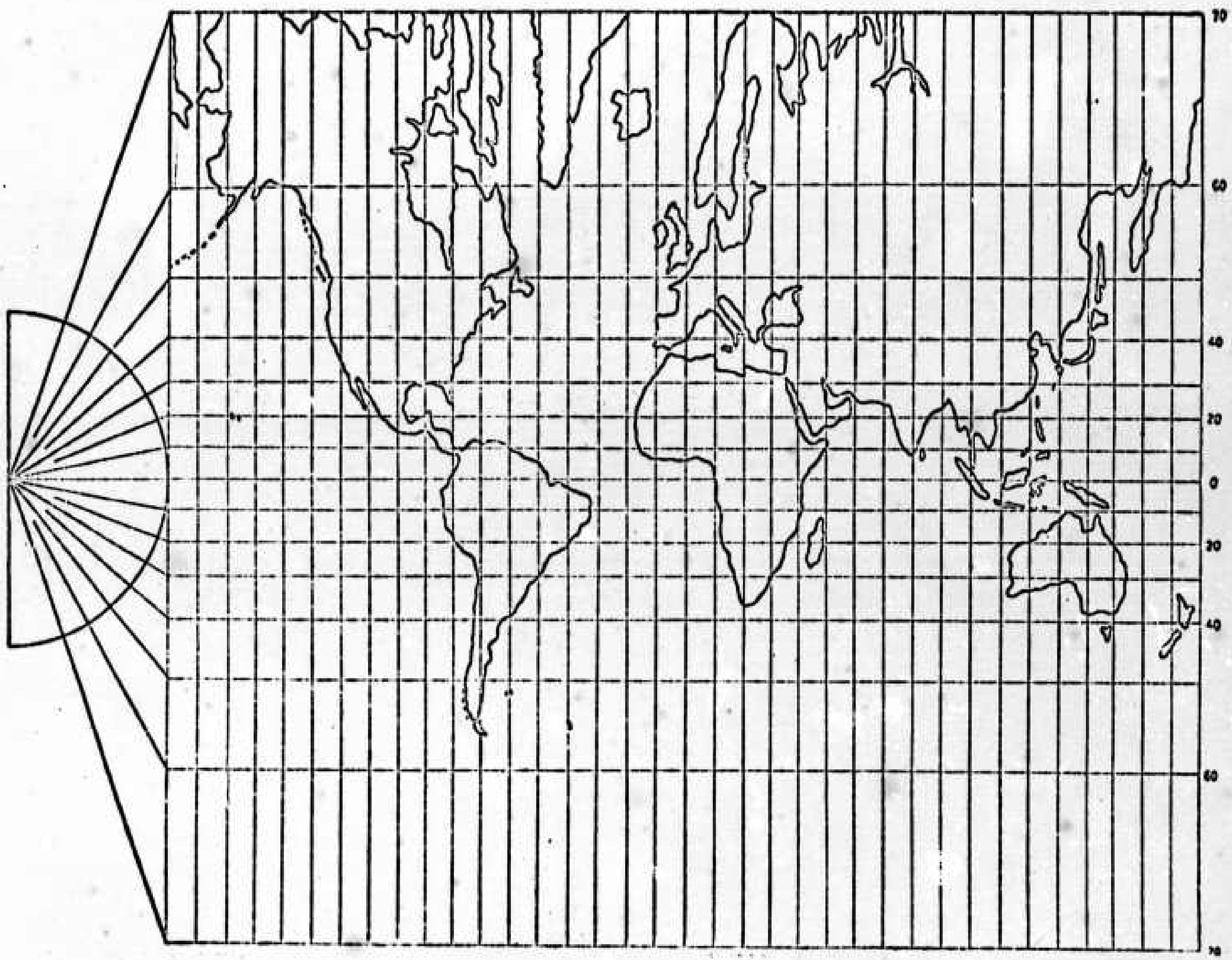

Fig. 4--Perspective projection upon a tangent cylinder

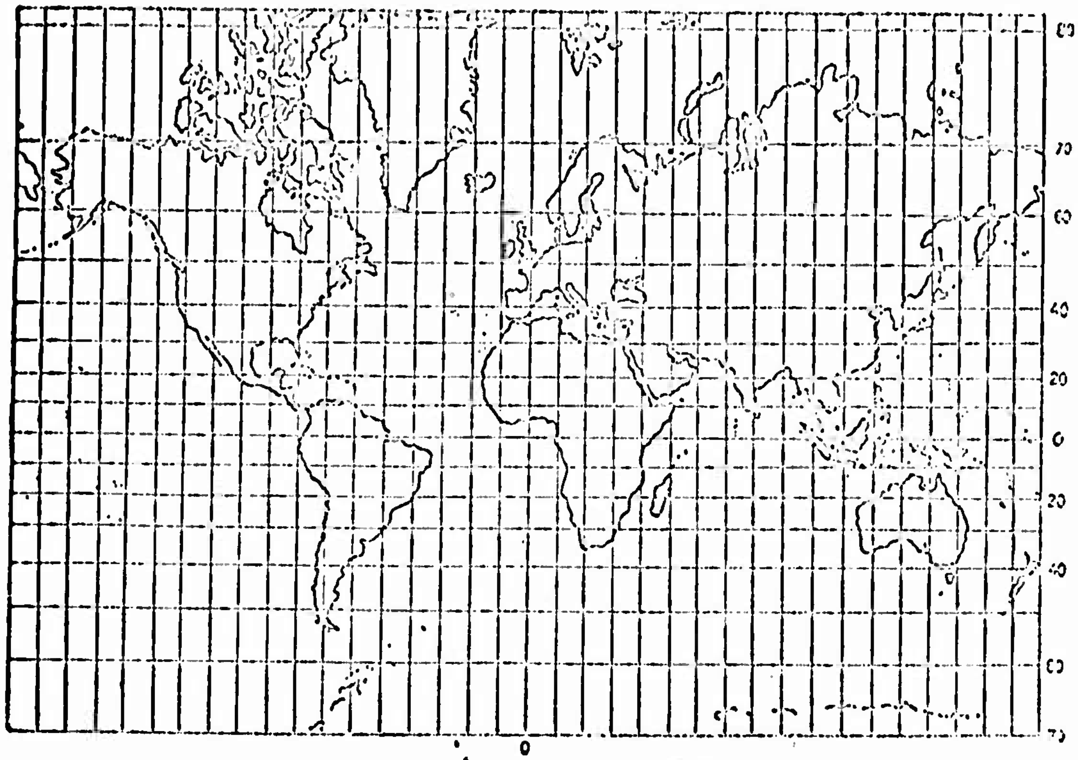

Fig. 5--Mercator projection

Such projections, as indicated, would include the general perspective cylindrical, the equal area cylindrical and the Mercator projection which is a conformal projection and has the added property of permitting all constant heading, i.e. loxodromic curves (rhumb lines) from the earth to be portrayed as straight lines and vice versa. When the rectilinearity constraint is relaxed, the variety achievable becomes virtually infinite.

When, however, as immediately above we begin to add additional requirements to the determination of the latitude-longitude spacing, we lose the beautiful simplicity of the implicit projections and their virtually effortless achievement in the form of the standard computer print-out. We stress again that the 0.80000 option for the width to height ratio for the print-out character space, an exceedingly fine general purpose projection for the conterminous United States can be easily achieved.

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13. ABSTRACT

The use of high-speed electronic digital computers for the computation and the use of their printers, plotters, and scopes for the graphical portrayal of various kinds of map projections is now well established. In this paper, the presence of implicit or built-in, but hitherto ignored, projections in the various sizes of character space print-outs for the standard computer printers is considered.