AEDC-TR-67-156

EFFECTS OF ROLL ON THE FREE-FLIGHT MOTION OF STATICALLY STABLE BODIES

C. J. Welsh and R. M. Watt

ARO, Inc.

September 1967

VON KÁRMÁN GAS DYNAMICS FACILITY
ARNOLD ENGINEERING DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
ARNOLD AIR FORCE STATION, TENNESSEE

NOTICES

ERRATA

AEDC-TR-67-156, September 1967

EFFECTS OF ROLL ON THE FREE-FLIGHT MOTION OF STATICALLY STABLE BODIES

C. J. Welsh and R. M. Watt, ARO, Inc.

Arnold Engineering Development Center
Air Force Systems Command
Arnold Air Force Station, Tennessee

Please note the following revisions:

1. Pages v and 17: Change $|\phi|$'s to $|\phi'|$'s in title of Fig. 5.

2. Page 2, Eq. (2): Change ℓ to e (2 places).

3. Page 3, first line of first complete paragraph: Last word should be product

4. Page 3, second line of first complete paragraph: Change produce to product

5. Page 14, line 6: Change o's to ϕ''s

6. Page 14, Eq. (15):

   Change the radicals $\sqrt{a^2 + b^2 + a}$ and $\sqrt{a^2 + b^2 - a}$

   to $\sqrt{a^2 + b^2} + a$ and $\sqrt{a^2 + b^2} - a$

7. Page 14, equations at bottom of page:

   Change "m" to "M" (eight places).

EFFECTS OF ROLL ON THE FREE-FLIGHT MOTION OF STATICALLY STABLE BODIES

C. J. Welsh and R. M. Watt
ARO, Inc.

FOREWORD

The work reported herein was sponsored by the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), Arnold Air Force Station, Tennessee, under Program Element 65402234. The work was done by ARO, Inc. (a subsidiary of Sverdrup & Parcel and Associates, Inc.), contract operator of AEDC, AFSC, under ARO Project Nos. VG2706 and VT2727, and the manuscript was submitted for publication on July 5, 1967.

This technical report has been reviewed and is approved.

Donald H. Meyer
Major, USAF
AF Representative, VKF
Directorate of Test

Leonard T. Glaser
Colonel, USAF
Director of Test

ABSTRACT

The effects of roll on the free-flight yawing motion of statically stable test models, having linear force and moment characteristics, are discussed in more detail than in previous publications. The correspondence between the damping, frequency, and amplitude of the nutational and precessional vectors which define the motion is listed. Limitations in the use of simplified data reduction procedures when the model motion is monitored in only one plane are pointed out, and cases of pseudo-nonlinear characteristics being exhibited by the orthogonal components of the yawing motion of a rolling motion are demonstrated.

CONTENTS

ILLUSTRATIONS

NOMENCLATURE

C_D	Drag coefficient
C_{m_α}	Static-stability derivative
$C_{m_{p_\alpha}}$	Magnus-moment derivative
$C_{m_q} + C_{m_{\dot{\alpha}}}$	Damping-in-pitch derivatives, $\frac{\partial C_m}{\partial q\,(\ell/v)} + \frac{\partial C_m}{\partial \dot{\alpha}\,(\ell/v)}$
C_{N_α}	Normal-force derivative
H, M, P, T	Constants in Eq. (1)
I_x	Model moment of inertia (relative to the longitudinal axis)
I_y	Model moment of inertia (relative to a transverse axis)
K_1, K_2	Constants in Eq. (2)
k_1, k_2	Absolute values of K_1 and K_2, respectively
k_a	Radius of gyration relative to the longitudinal axis/ℓ
k_t	Radius of gyration relative to a transverse axis/ℓ
ℓ	Reference length
m	Model mass
p	Model roll rate (with respect to distance traveled)
S	Reference area
V	Model velocity
x	Distance traveled
α, β	Components of the complex yaw angle
δ	$\sqrt{\beta^2 + \alpha^2}$
θ_1, θ_2	Constants in Eq. (2)
μ_1, μ_2	Damping rates of the motion vectors in Eq. (2)
ξ	Complex yaw angle
ρ	Mass density of the range air
ϕ_1', ϕ_2'	Rates of rotation of the motion vectors in Eq. (2)
ψ	Precession angle, see Fig. 1

SUPERSCRIPTS

′	First derivative with respect to distance
″	Second derivative with respect to distance

SUBSCRIPTS

o	Corresponding to the start of a motion pattern
n	Nutational vector
p	Precessional vector

SECTION I
INTRODUCTION

In free-flight testing in either an aeroballistic range or in a wind tunnel, certain stability derivatives can be evaluated from the measured yawing motion of the model. For a nonrolling, axisymmetric model, the damping parameter and the frequency of each of the two orthogonal components of the yawing motion (α and β) are constant; further, both components have identical damping parameters and identical frequencies. As the roll velocity of the model increases from zero, the complexity of the motion of each orthogonal component increases and consists of two subcomponents having different damping parameters and different frequencies. It is apparent from the above that the analysis of the yawing motion of a rolling model can be appreciably more complicated.

Considerable emphasis is currently being directed to the free-flight testing (both aeroballistic ranges and wind tunnels) of aerodynamic configurations that are statically stable but subject to experience small roll rates. The small roll rates arise from disturbances during the launching of the free-flight models, from small asymmetries in nominally symmetric models, or from built-in asymmetries; this is in contrast to the large roll rates of spin-stabilized projectiles. It follows that a basic concern in free-flight testing (particularly in wind tunnels when motion is viewed in one plane) is related to how large a roll velocity can exist and it still be assumed zero in reducing stability data from the measured model motion.

The purpose of this report is to present a discussion of the effects of roll on the yawing motion of free-flight models experiencing small roll rates. In particular, the significance of roll effects in data reduction procedures is considered.

SECTION II
ANALYSIS OF FREE-FLIGHT RANGE DATA

The basic roll effects discussed in this report correspond to axisymmetric configurations. In discussing the effects of model roll on the yawing motion of a test model, it is useful to examine the conventional data analysis procedure as used in aeroballistic range testing. In range testing, roll rates from zero to large values consistent with spin-stabilized projectiles are experienced; hence, data reduction procedures permitting a nonzero roll velocity are necessarily required.

In range work the differential equation normally used in describing the general rolling-yawing motion of an axisymmetric model is

$$\xi'' + (H - iP)\,\xi' - (M + iPT)\,\xi = 0 \tag{1}$$

where

$$H = (\rho S/2m)\left[C_{L_\alpha} - C_D - k_t^{-2}\left(C_{m_q} + C_{m_{\dot\alpha}}\right)\right]$$

$$M = (\rho S/2m\ell)\, k_t^{-2}\, C_{m_\alpha}$$

$$T = (\rho S/2m)\left[C_{L_\alpha} + k_a^{-2}\, C_{m_{p_\alpha}}\right]$$

$$P = (I_x/I_y)\, p$$

Derivations of this equation (relative to a nonrolling axis system) have been presented previously, for example, Refs. 1 and 2. The solution of Eq. (1) for a model having linear variations of force and moment with yaw angle, and a constant p can be written

$$\xi = K_1 \ell^{\theta_1 x} + K_2 \ell^{\theta_2 x} \tag{2}$$

where

$$\theta_j = \mu_j + i\,\phi_j'$$

The θ's are roots of the auxiliary equation corresponding to the differential equation designated Eq. (1).

It should be noted that the damping of the motion corresponding to the θ_j root is defined by the real part of the root and that the frequency of the motion is defined by the imaginary part of the root. Hence, for oscillatory motion to exist the root must have a nonzero imaginary part.

The roots expressed as functions of the coefficients of Eq. (1) are

$$\mu_1 + i\,\phi_1' = 1/2\left[-H + iP + \sqrt{4M + H^2 - P^2 + i\,2P\,(2T - H)}\right] \tag{3}$$

and

$$\mu_2 + i\,\phi_2' = 1/2\left[-H + iP - \sqrt{4M + H^2 - P^2 + i\,2P\,(2T - H)}\right] \tag{4}$$

Values for the K's and θ's of Eq. (2) are obtained by fitting Eq. (2) to the measured variations of the α and β components of the angular motion of the model with distance traveled. This equation is nonlinear in terms of the θ's, and the use of an iterative, differential correction-type curve-fitting procedure is necessary. The desired H, M, P, and

T parameters can then be determined as functions of the evaluated θ's with use of the expressions for the sum and the product of $(\mu_1 + i\phi_1')$ and $(\mu_2 + i\phi_2)$ and can be written

$$H = -[\mu_1 + \mu_2] \tag{5}$$

$$P = \phi_1' + \phi_2' \tag{6}$$

$$T = -(\phi_2' \mu_1 + \phi_1' \mu_2)/P \tag{7}$$

$$M = \phi_1' \phi_2' - \mu_1 \mu_2 \tag{8}$$

$$\approx \phi_1' \phi_2'$$

The assumption in the approximate relationship for M, that the produc of the damping parameters is small relative to the produce of the frequencies, is quite reasonable for a typical aerodynamic configuration.

The above procedure, permitting large roll velocities, is very adequate in the analysis of yawing motion. It is difficult for one to appreciate the significance of the roll effects that the procedure accounts for; however considerable insight to the problem of roll effects can be obtained by examining the related equations. If P is set equal to zero,(implying a zero roll rate) in Eqs. (3) and (4), the corresponding μ's and ϕ's are

$$\mu + i\,\phi' = 1/2\left[-H \pm \sqrt{4M + H^2}\right] \tag{9}$$

Considering that oscillatory motion requires a complex or imaginary root and that H is a real number, the term $(4M + H^2)$ must be less than zero, or $4M < -H^2$. From the definition of M listed previously,

$$4M = 4\,(\rho S/2m\ell)\,k_t^{-2}\,C_{m_\alpha}$$

and it follows that C_{m_α} must be negative. An aerodynamic configuration is defined as being statically stable when it has a negative C_{m_α} value. Hence for oscillatory motion (nonrolling model) it is apparent from Eq. (9) that the model must be statically stable.

The two terms on the right side of Eq. (2) defining the complex yaw angle, ξ, correspond to rotating vectors in the (α, β) plane (see Fig. 1). The model yawing motion defined by the two vectors is obviously dependent on the values of the K's and θ's. Values of the θ's are functions primarily of the aerodynamic and inertia characteristics of the model and the rolling velocity; the K's are dependent on the initial disturbances of the model. For general rolling-yawing motion, K_1, K_2, θ_1, and θ_2 are general complex numbers; however, roll effects on the yawing motion of a model become more apparent from first examining the more restricted types of model motion.

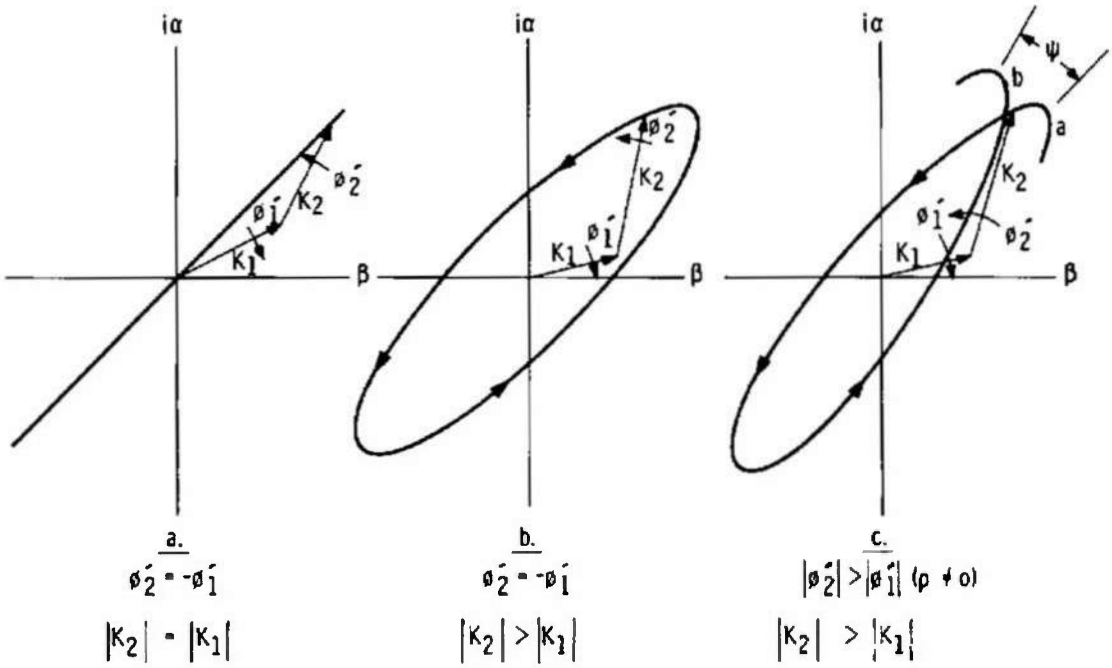


Fig. 1 Computed Motion Patterns as Functions of Initial Disturbances and Model Roll

The basic types of model motion corresponding to assigned restrictions on the K's and θ's of Eq. (2) are demonstrated in the following examples, in which zero damping is assumed:

a. If K_1 and K_2, and θ_1 and θ_2 are complex conjugates then the motion is planar and along the real axis. This type corresponds to the angular motion of a one-degree-of-freedom dynamic balance system as used in a wind tunnel. A basic point to be noted here is that for the θ's to be complex conjugates, the roll velocity, p, must necessarily be zero, which is apparent from Eqs. (3) and (4). It should be noted that the restriction on the θ's dictates that the angular frequencies, ϕ's, are equal in magnitude and have opposite signs.

b. Let the restriction of θ_1 and θ_2 being complex conjugates be retained (p = 0). If K_1 and K_2 are not required to be complex conjugates but $|K_1| = |K_2|$, then general planar motion exists. This is indicated in the sketch in Fig. 1a where the yawing motion is along a line displaced from the α and β axes.

c. If the restrictions of item (b) are retained except that $|K_1| \neq |K_2|$, then elliptic motion exists as indicated in Fig 1b. The limiting case here is circular motion corresponding to one of the K's being zero.

d. If K_1 and K_2 are general complex numbers as in item (c) and the restriction of θ_1 and θ_2 being complex conjugates is removed, the resulting motion is of the precessing elliptic type shown in Fig. 1c.

Removing the restriction of θ_1 and θ_2 being complex conjugates corresponds to a nonzero roll velocity (see Eqs. (3) and (4)). Hence, the precessing elliptic motion of item (d) is caused by model roll, and the motion patterns of items (a, b, c) are characteristic of nonrolling yawing motion.

The relationship between the roll velocity of a model and the direction and magnitude of the precession of the corresponding elliptic motion is obtained in the following derivation. In Fig. 1c the angle of precession, ψ, is defined as the angle between two adjacent peaks as indicated in the sketch. The peak at (a) corresponds to a point in flight where both vectors are aligned, and at peak (b) the vectors are aligned again after each vector has rotated approximately 360 deg. As previously noted, the two vectors rotate in opposite directions and as $|\phi_1'| \neq |\phi_2'|$, for ($p \neq o$), then ψ must have a nonzero value. The sign of ψ is defined consistent with rotating vector notation; hence, if the motion precesses counterclockwise ψ is positive. It is apparent from the above that the motion will precess in the direction of the rotation of the larger absolute frequency. From Eq. (6) it follows that the larger absolute frequency will have the same sign as the roll velocity of the model. Consistent with the notation of Fig. 1c,

$$(360^\circ + \psi) / \phi_2' = (-360^\circ + \psi) / \phi_1'$$

for

$$1/2 < | \phi_1' / \phi_2' | < 1$$

which corresponds to the roll rates of interest in this report. Hence,

$$(\phi_1' / \phi_2') = (\psi - 360^\circ) / (\psi + 360^\circ) \qquad (10)$$

With use of Eqs. (6) and (10) an expression for P can be written as

$$P = [2\psi / (\psi + 360)] \phi_2'$$

Hence,

$$p = (I_y/I_x) [2\psi / (\psi + 360)] \phi_2' \qquad (11)$$

and

$$\psi = 360^\circ p / [2(I_y/I_x) \phi_2' - p] \qquad (12)$$

For these expressions, ψ is in degrees; hence, p and the ϕ''s are in deg/ft units.

Equations (11) and (12) show explicitly the correspondence between the measured precessing elliptic motion of a test model, the inertia ratio (I_y/I_x), and the roll velocity of the model.

A further point of interest related to an (α, β) plot is roll resonance which can be of concern in testing an asymmetric configuration. Roll resonance occurs when $p \approx \phi_2'$, considering that the condition of resonance is slightly dependent on the damping of the system. The precession angle corresponding to resonance, ψ_{res}, can be obtained from Eq. (12):

$$\psi_{res} \approx 360° / [2(I_y/I_x) - 1]$$

In the above examples of model motion defined by Eq. (2), it should be noted that when $|K_1| \neq |K_2|$, the motion sketches are for the case where the vector having the larger amplitude corresponds to the larger absolute frequency. The correspondence between the magnitudes of frequency, damping, and amplitude of the vectors is discussed in detail in a later section of this report.

SECTION III
SIMPLIFIED DATA ANALYSIS

As previously noted, the components β and α of the yawing motion of a nonrolling model are defined by damped sinusoidal curves having identical frequencies and damping parameters. It follows that in free-flight testing in wind tunnels, where the model motion is monitored in only one plane, it is particularly significant to be able to assume that the model has nonrolling motion. However, some model roll arising from the launching of the model or from small model asymmetries that are either accidental or designed into the model may exist, and the concern in this type of testing is the sensitivity of the components of the yawing motion to model roll. To aid in examining this problem, the β and α components of Eq. (2) can be separated and written as

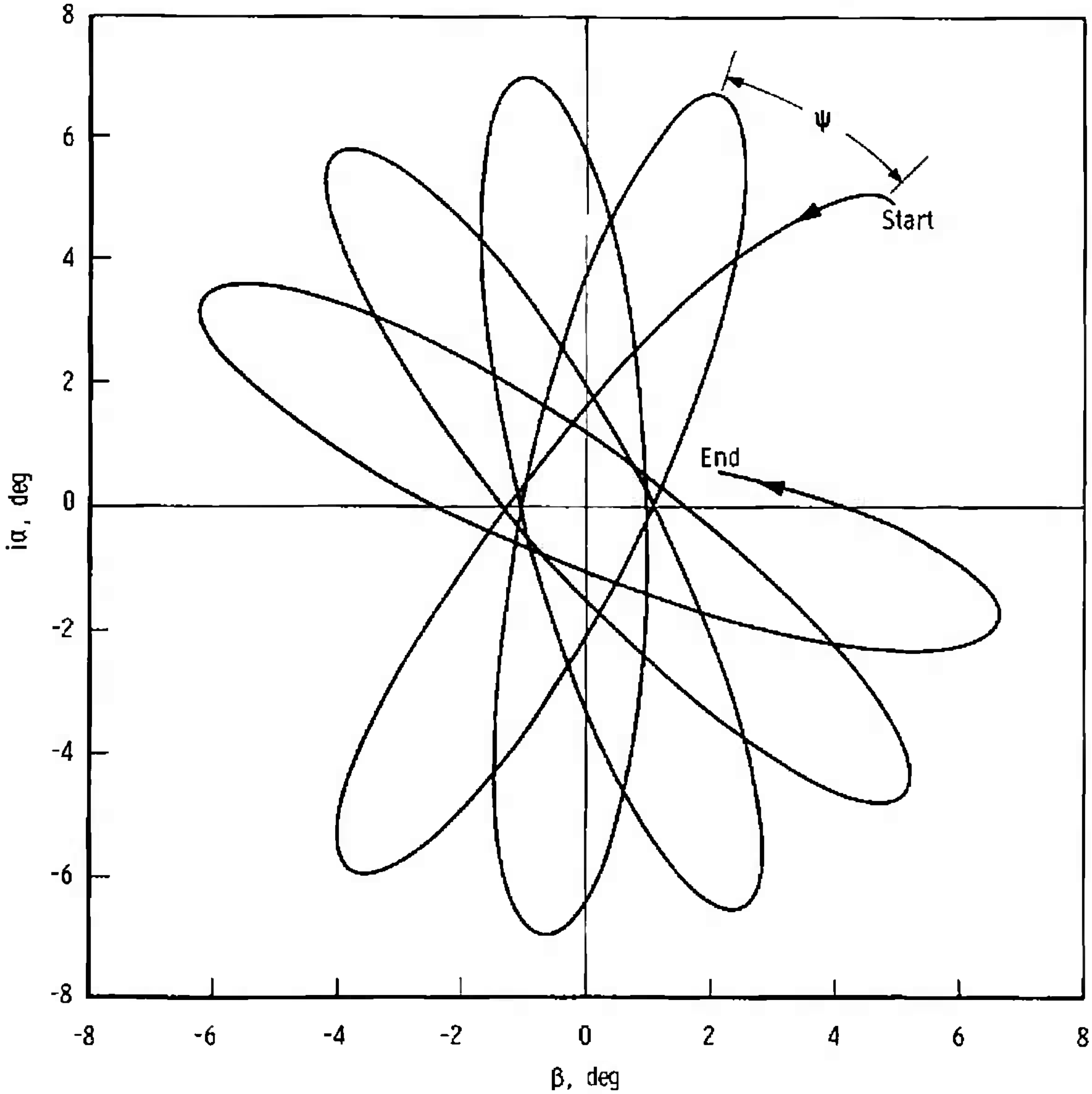
$$\beta = (a \cos \phi_1' x - b \sin \phi_1' x) \exp(\mu_1 x) + (c \cos \phi_2' x - d \sin \phi_2' x) \exp(\mu_2 x) \quad (13)$$

and

$$\alpha = (b \cos \phi_1' x + a \sin \phi_1' x) \exp(\mu_1 x) + (d \cos \phi_2' x + c \sin \phi_2' x) \exp(\mu_2 x) \quad (14)$$

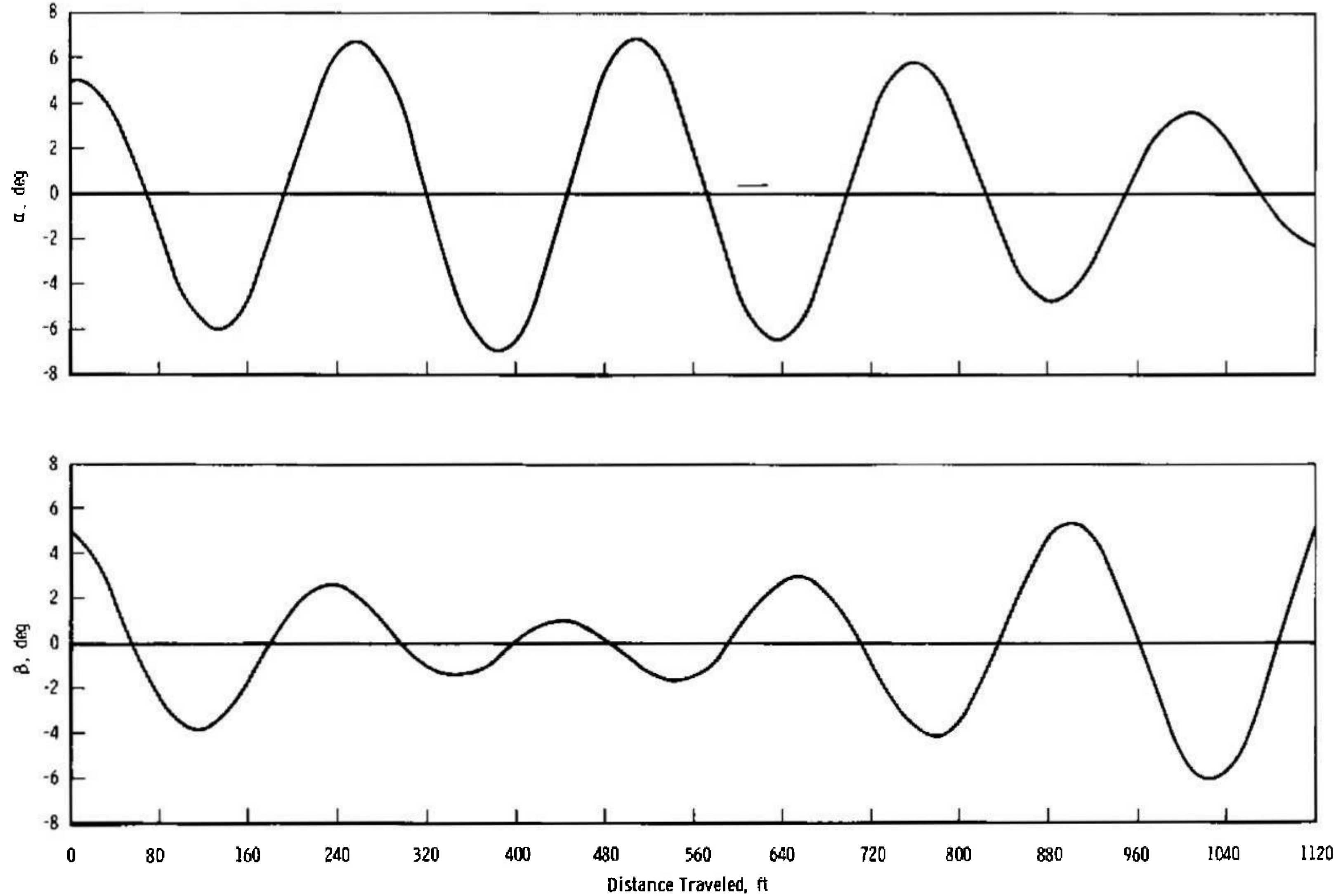
where a, b, c, and d are real constants.

From Eqs. (13) and (14) it is apparent that each component, for the case of a rolling model, is the sum of trigonometric terms containing different frequencies; hence, the resulting curve for the simplified case of zero damping will necessarily have an apparently varying frequency and a varying amplitude. Examples of such motion are shown in Figs. 2, 3, and 4. The motion plots can be deceiving in that apparent nonlinearities can be indicated. For large roll rates, Fig. 2, these apparent nonlinear characteristics in a component of the motion are easily detected; hence, the primary concern in experiments is related to possible effects on the yawing motion existing at small roll rates where the apparent nonlinearities are not obvious from the motion plots when the model motion is monitored in only one plane. The model motion plots in both Figs. 3 and 4 correspond to zero damping and a precession angle of 5 deg and are representative of the case where each individual component of the motion appears to be a sinusoidal trace with a constant damping factor.



a. (Alpha, Beta) Variation

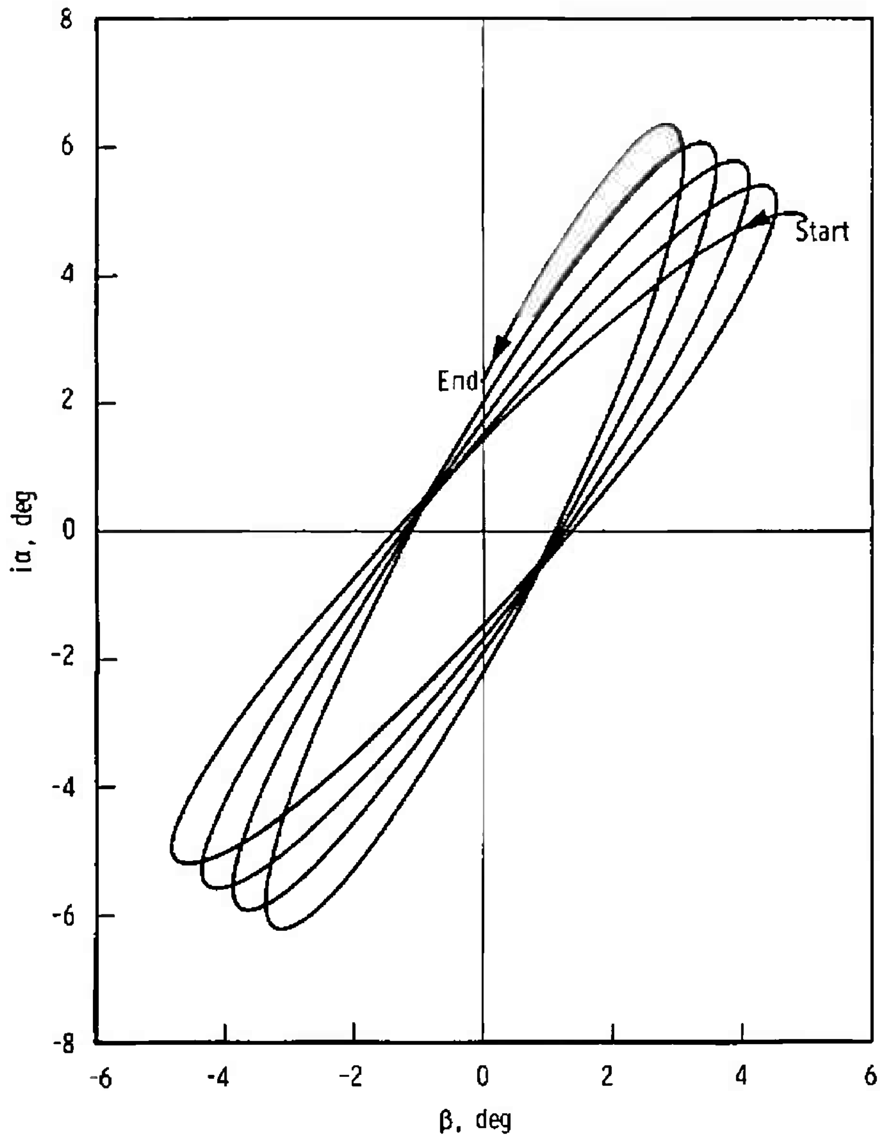
Fig. 2 Computed Motion for a Statically Stable Configuration, ψ = 26.6 deg



b. Individual Motion Components

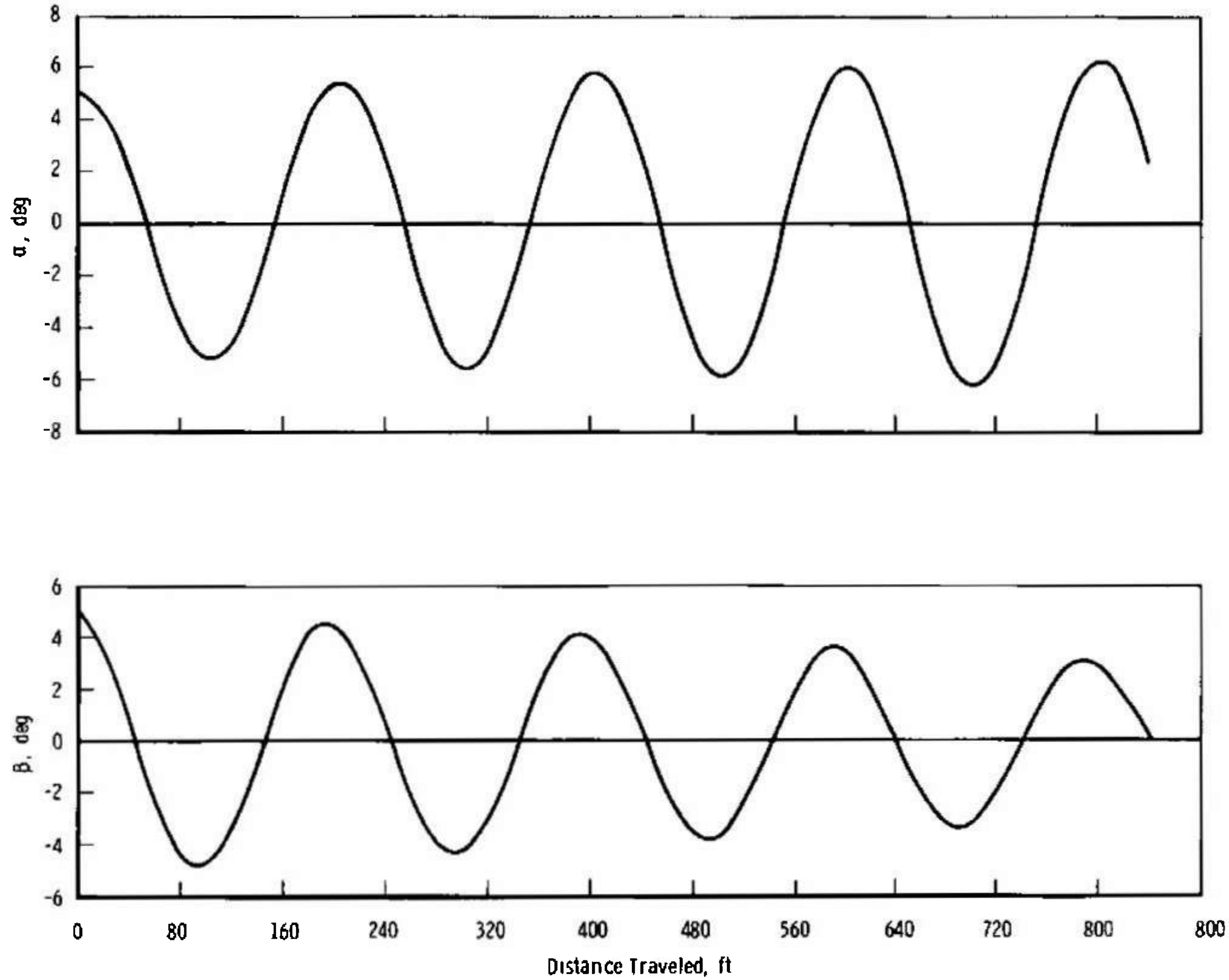
Fig. 2 Concluded

As can be observed from the corresponding (α, β) plot, in Fig. 3, the initial disturbance of the oscillatory motion (ξ_0) is in a plane oriented at 45 deg relative to the α and β axes, and in Fig. 4, ξ_0 is in the plane corresponding to the α axis. Precession angles well above 5 deg have been measured on axisymmetric models launched in aeroballistic ranges using unrifled launch tubes; hence, precession angles on the order of 5 deg could be expected in wind tunnel tests, particularly with asymmetric test models.



a. (Alpha, Beta) Variation

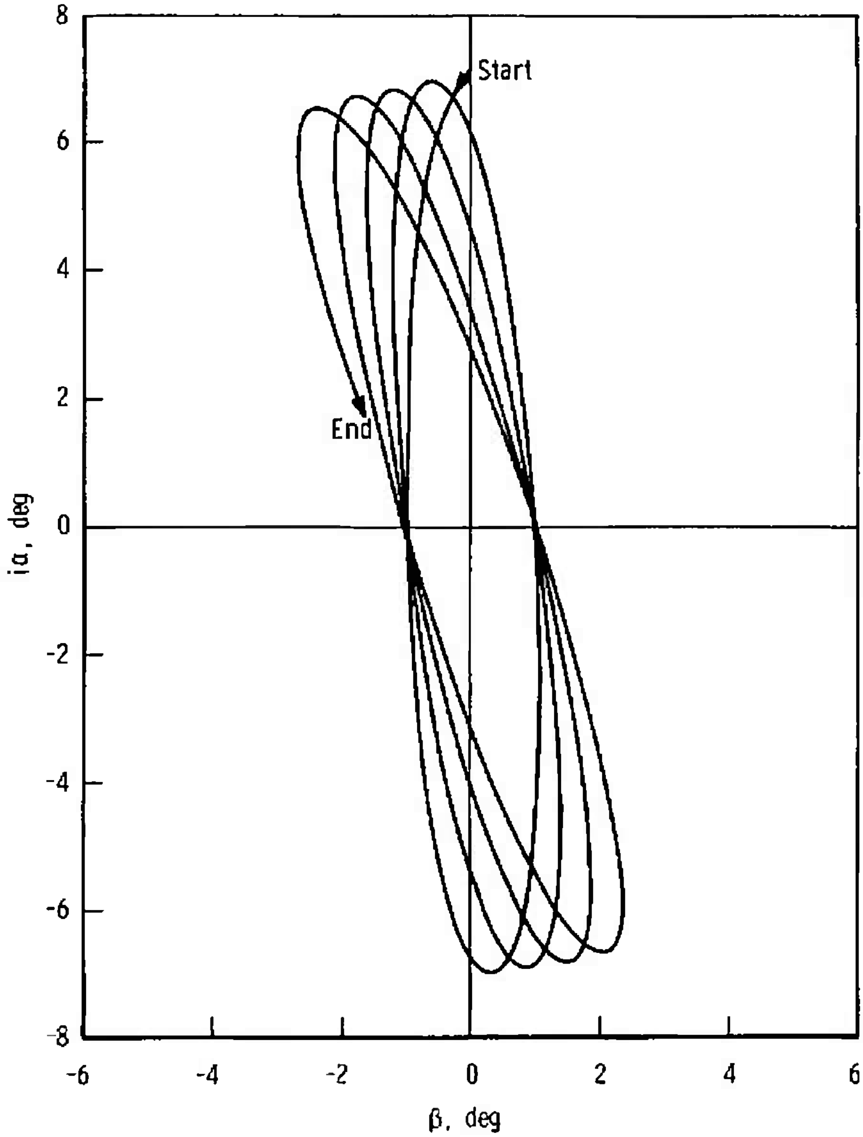
Fig. 3 Computed Motion for a Statically Stable Configuration, ψ = 5 deg, ξ_o Displaced 45 deg from the α-Axis



b. Individual Motion Components

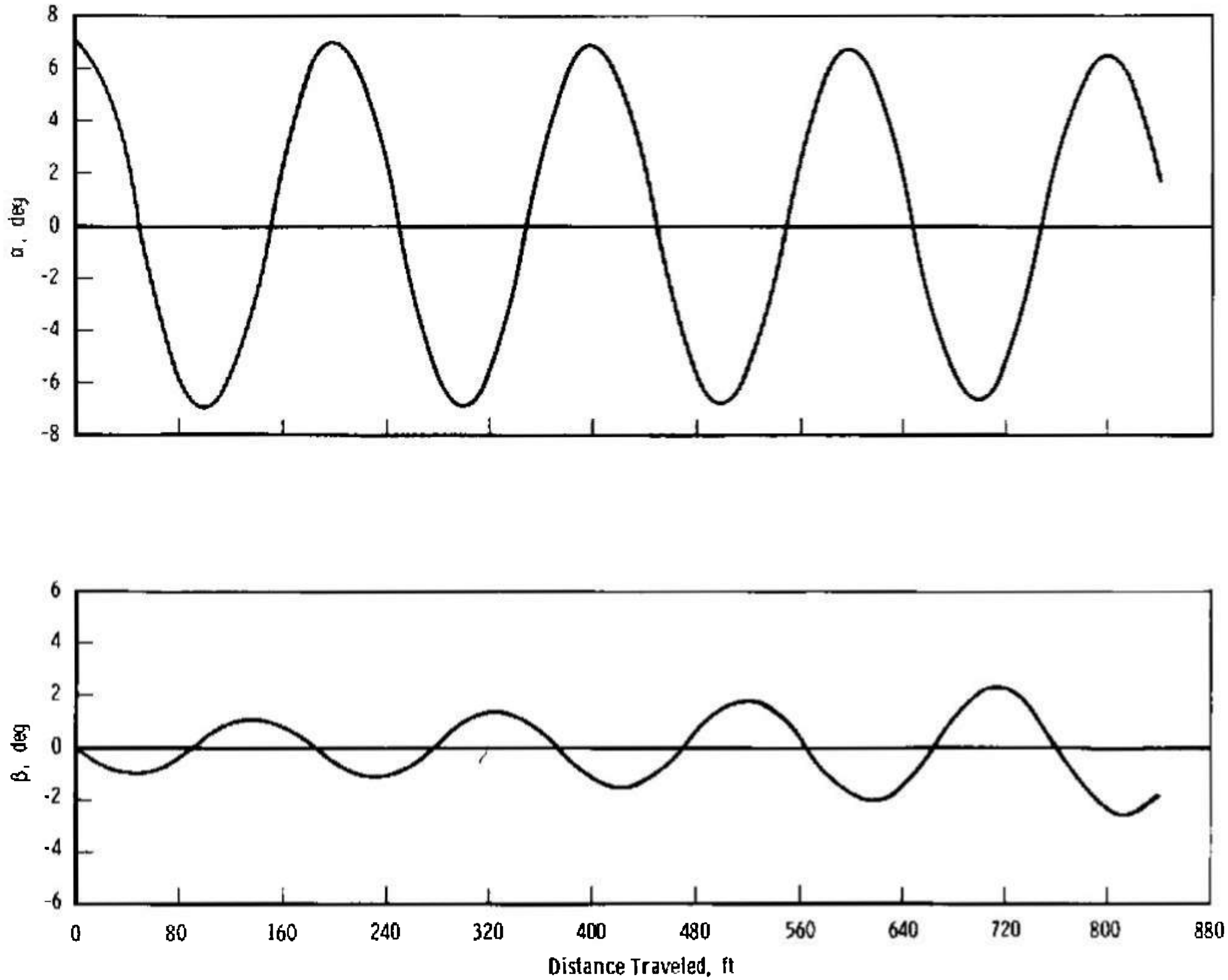
Fig. 3 Concluded

The variation of the α component in Fig. 4 is more representative of the case confronted in wind tunnel testing; this follows from noting that the initial angle disturbance in wind tunnel tests can be primarily restricted to the plane in which the yawing motion is being monitored. In examining the α component (Fig. 4), presuming roll effects are negligible, one obtains a decay of the α component in four cycles of about 0.4 deg, although the computed model motion corresponds to zero aerodynamic damping. An amplitude decay error of 0.4 deg corresponds to an error of about twenty percent in the measured damping factor of a system, presuming that the damping of the system causes an amplitude decay in the four-cycle interval of about 0.3 of the initial amplitude of 7 deg, which is quite reasonable. There was no measureable difference in the frequency of the α component of the motion of Fig. 4 from the exact nonrolling frequency.



a. (Alpha, Beta) Variation

Fig. 4 Computed Motion for a Statically Stable Configuration, ψ = 5 deg, ξ_o Aligned with the α-Axis



b. Individual Motion Components

Fig. 4 Concluded

The above analysis indicates that the wind tunnel data reduction procedure in which only one component of the yawing motion is examined on the basis of presumed negligible roll effects in some cases can lead to significant errors in the measured damping; hence, particular care should be exercised in this type of an analysis to ensure that roll effects are, in fact, negligible.

It should be noted that other approximate procedures proposed for the analysis of free-flight data and dependent on measured motion decay (e.g., see Ref. 3) must be used cautiously; this will become more evident in a later section of this report. Further, a point of particular interest concerns the curve fitting of typical motion patterns in the α-β plane. In the discussed range procedure, the measured values of both α and β are fitted simultaneously, and hence, various types of motion from planar to patterns exhibiting large precession can be fitted adequately. It was noted in Ref. 4 that all of the equation constants in the expression for the complex yaw angle, ξ, are present in each component of ξ, (α or β). Further, it was stated, in view of this, that the unknown equation constants could be determined by fitting either the α or β values separately or both simultaneously. That the equation constants corresponding to general motion patterns can be evaluated adequately by fitting one component of the motion is believed to be optimistic. The case where the motion pattern of an (α, β) plot can be adequately fitted using only α values, for example, implies that the motion pattern is uniquely defined by only the α values. This condition can exist for model motion for which large precession angles exist (usually associated with large roll rates); for planar motion, it is apparent that the motion pattern is not uniquely defined by only one of the components. Hence, there will be a transition region between planar motion (that can not be adequately fitted) and motion having large precession, and in this region difficulty in curve fitting can be expected. Unfortunately, a large portion of current stability testing falls in this transition region.

SECTION IV
CORRESPONDENCE BETWEEN ϕ''s, μ's, K's, and P

There are pronounced variations that can exist in the basic precessing elliptic motion patterns shown previously. From data reduction considerations, recognition of these variations can be particularly significant (e.g., see Ref. 5) as an aid in confirming whether or not the measured motion is basically linear. The possible variations of the yawing motion of a rolling body are associated with the correspondence between the ϕ''s, μ's, and K's in Eq. (2). The two vectors in Eq. (2) have been identified in the past as the nutational vector (having the larger absolute frequency) and the precessional vector. However, from previous publications it is difficult to identify which damping or which amplitude (as to the smaller or larger value) corresponds to the nutational or precessional vector. The purpose of the following discussion is to clarify the correspondence between the μ's, o's, and K's and to serve as an aid in recognizing various possible variations in the motion patterns.

The correspondence between the μ's and ϕ''s can be better defined by evaluating the complex radical in Eqs. (3) and (4). In Ref. 1, the corresponding radical was evaluated approximately by using a binomial expansion. The approximate equations obtained for the μ's and ϕ''s in Ref. 1 indicate that the larger damping value will always correspond to the nutational vector. The use of approximate solutions for similar complex radicals has continued in recent publications (e.g., Ref. 6). However, the evaluation of such a complex radical can be obtained precisely with use of DeMoivre's theorem and can be written as

$$\sqrt{a + ib} = \pm \left[\sqrt{\frac{\sqrt{a^2 + b^2} + a}{2}} \pm i \sqrt{\frac{\sqrt{a^2 + b^2} - a}{2}} \right] \tag{15}$$

The sign of the imaginary term is positive if (b) (the imaginary part of the radical $\sqrt{a + ib}$) is positive, and is negative if (b) is negative. From Eqs. (3) and (4), $a = 4M + H^2 - P^2$ and $b = 2P(2T - H)$.

In evaluating the complex radical of Eqs. (3) and (4), it should be noted that the term (2T - H), in general, can be expected to be negative; however, it is quite possible for the term to be positive for certain configurations. It follows then that the sign of (b) will be the same as the sign of P for $(2T - H) > 0$ and opposite the sign of P for $(2T - H) < 0$. With use of Eq. (15), the following expressions can be written for the μ's and ϕ''s:

$$\mu_1 = 1/2 \left[-H + \sqrt{\frac{\sqrt{(4m + H^2 - P^2)^2 + [2P(2T - H)]^2} + (4m + H^2 - P^2)}{2}} \right]$$

$$\phi_1' = 1/2 \left[P + (\pm) \sqrt{\frac{\sqrt{(4m + H^2 - P^2)^2 + [2P(2T - H)]^2} - (4m + H^2 - P^2)}{2}} \right]$$

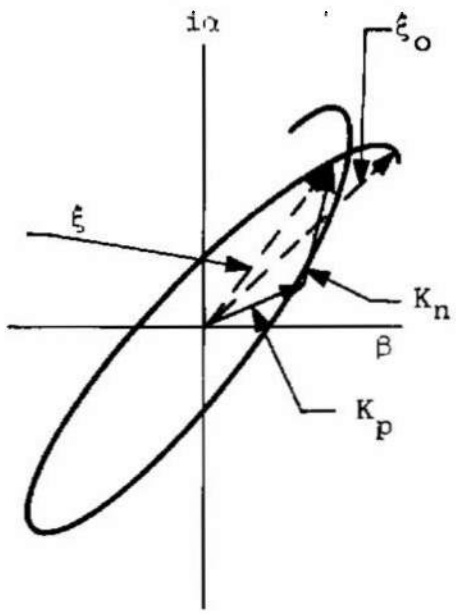
$$\mu_2 = 1/2 \left[-H - \sqrt{\frac{\sqrt{(4m + H^2 - P^2)^2 + [2P(2T - H)]^2} + (4m + H^2 - P^2)}{2}} \right]$$

$$\phi_2' = 1/2 \left[P - (\pm) \sqrt{\frac{\sqrt{(4m + H^2 - P^2)^2 + [2P(2T - H)]^2} - (4m + H^2 - P^2)}{2}} \right]$$

Consistent with above comments, for the case $(2T - H) < 0$ the positive sign preceding the radical in the expressions for the ϕ's is used for $p < 0$ and the negative sign is used for $p > 0$. The above expressions indicate that the larger damping value corresponds to the nutational frequency when $(2T - H) < 0$, however, for configurations where $(2T - H) > 0$, the larger damping value corresponds to the precessional frequency.

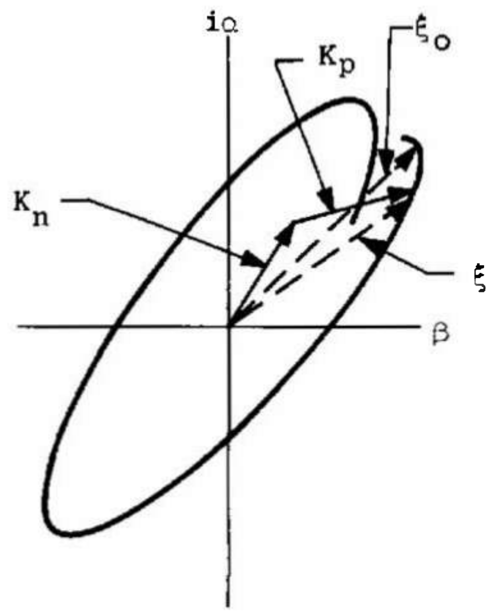
Although the (±) sign on the right side of Eq. (15) precedes the imaginary term, it should be pointed out that Eq. (15) would hold equally as well if the (±) sign preceded the real term and was used consistent with the restriction on the sign of (b). Such a sign change would not affect the above comments on the correspondence of the μ's and ϕ's.

The relationship between the frequency of a vector and its amplitude is dependent on the initial disturbances of the model and can be examined with use of the following sketches in which the yawing motion has the same initial displacement, ξ_o:



Pattern When Nutational Vector Has the Larger Amplitude

Sketch a



Pattern When Precessional Vector Has the Larger Amplitude

Sketch b

Note that the ξ vector in Sketch (a) is rotating counterclockwise and is rotating clockwise in Sketch (b). It is apparent that the ξ vector will rotate (for small p values) in a direction consistent with the rotation direction of the motion vector having the larger amplitude, (K_n or K_p). As previously discussed, the elliptic motion will precess in the direction of the rotation of the nutational vector (direction of the model roll); hence, the nutational vector will have the larger amplitude only if the ξ vector rotation is in the same direction as that of the nutational vector. The direction of rotation of the ξ vector and the ratio of the amplitudes of the two vectors corresponding to the elliptic motion is defined by the initial velocity disturbance of the ξ vector, ($\dot{\xi}_O$).

Some summary comments concerning the nutational and precessional vectors which follow from the above discussion of precessing elliptic motion are:

a. The angular direction of precessing elliptic motion will be the same as the rotation direction of the nutational vector; the nutational vector will always have the same rotation direction as that of the model roll.

b. The nutational vector will have the larger damping value if $(2T - H) < 0$, and conversely if $(2T - H) > 0$.

c. The nutational vector will have the larger amplitude only if the rotation of the ξ vector has the same direction as the rotation direction of the nutational vector. The direction of the rotation of the ξ vector and the ratio of the amplitudes of the two motion vectors are defined by the initial velocity disturbance of the model, $\dot{\xi}_O$.

d. In consideration of item (c), it follows that the decay of the amplitude peaks on an (α, β) plot is dependent on the initial disturbances of the model.

Figure 5 furnishes an indication of the variations in the damping parameters and the frequencies of the motion vectors as functions of the model roll (precession angle) for assigned aerodynamic and inertia parameters representative of a 5-deg semiangle cone model.

From the correspondence between the μ's and $\dot{\phi}$'s and between the $\dot{\phi}$'s and the amplitudes of the vectors, it follows that a model can be disturbed such that the larger amplitude vector can have either the larger or smaller damping value; this is significant in that it implies that the variation in the amplitude of the yawing motion of a rolling model is a function of the initial disturbances of the model. In Ref. 3 an approximate data reduction procedure is proposed in which it is assumed that a curve of the logarithm of the absolute value of ξ, (δ), corresponding to the peaks of the (α, β) plot as a function of distance traveled is linear and that the slope of this curve is proportional to the damping of the configuration. Values of the damping factor (μ_δ) evaluated consistent with the procedure of Ref. 3 have been obtained from computed model motion patterns corresponding to a 5-deg semiangle cone configuration.

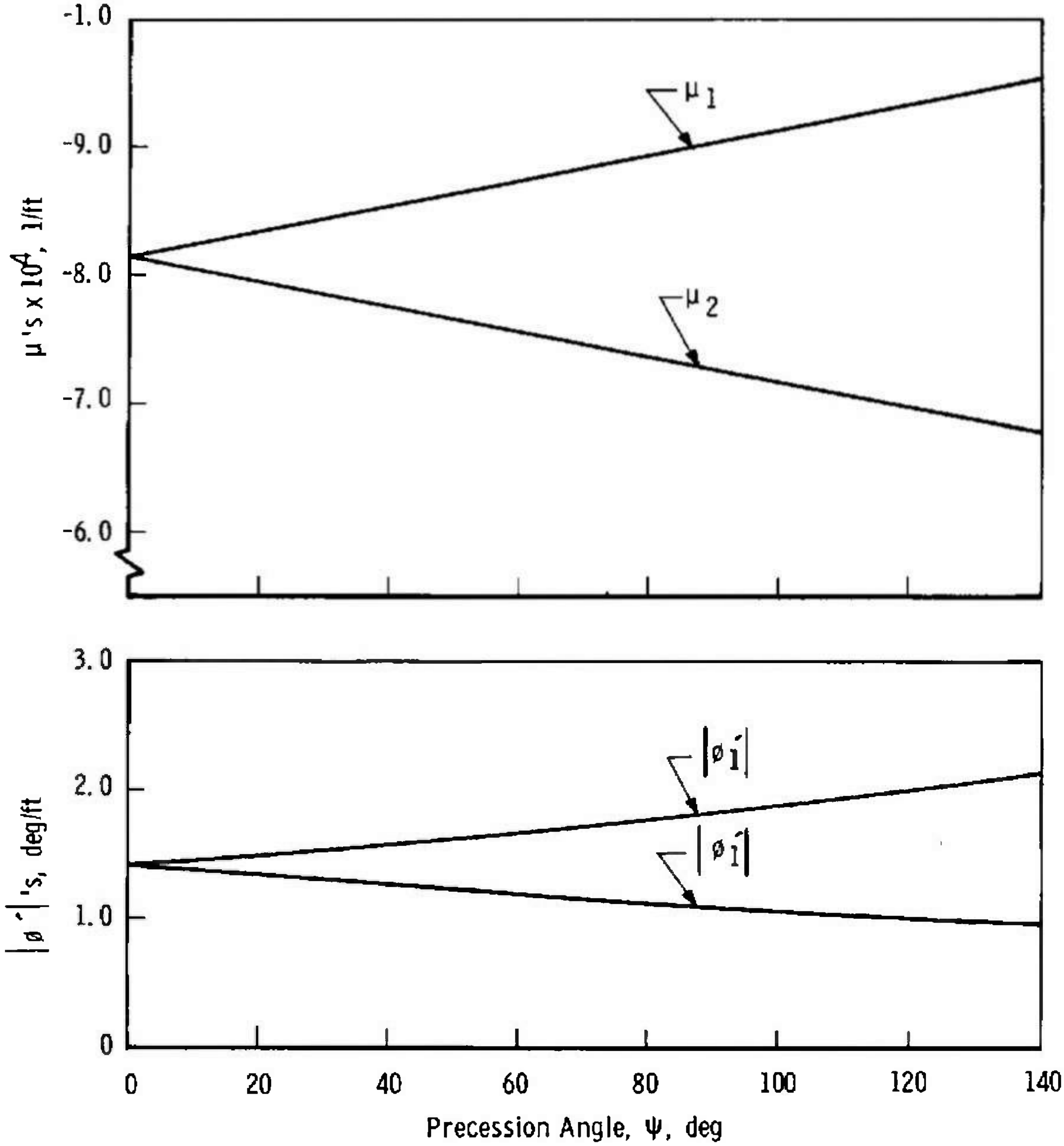


Fig. 5 Variation of μ's and φ́'s as Functions of Precessional Angle for a Simulated 5-deg, Semiangle Cone Model

The μ_δ values obtained for two different initial disturbance conditions are shown in Fig. 6 and indicate that the damping evaluated as a function of the measured amplitude is dependent on the initial disturbances of a rolling model. Deviations in the measured damping can be appreciable at the larger roll velocities.

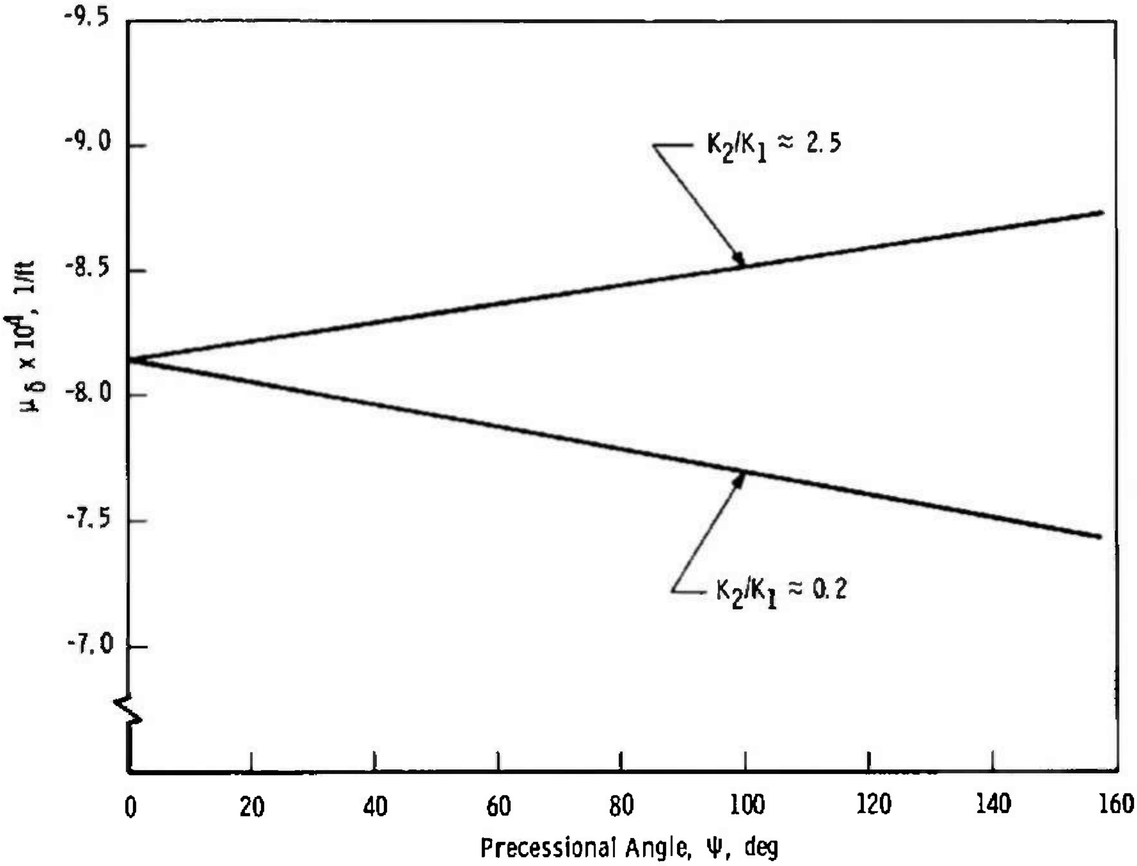
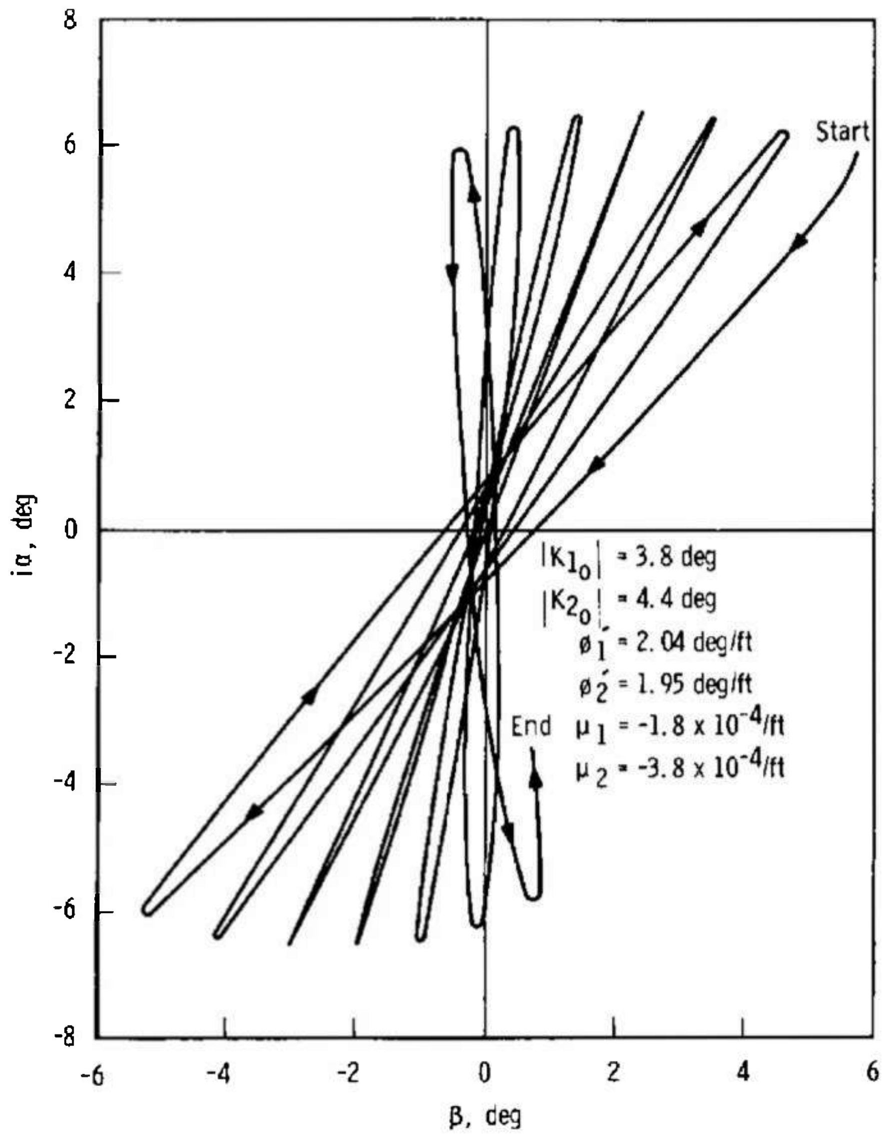


Fig. 6 Damping Evaluated from the Amplitude Variation of the Computed Motion for a Simulated 5-deg, Semiangle Cone Model, $|\mu_2| > |\mu_1|$

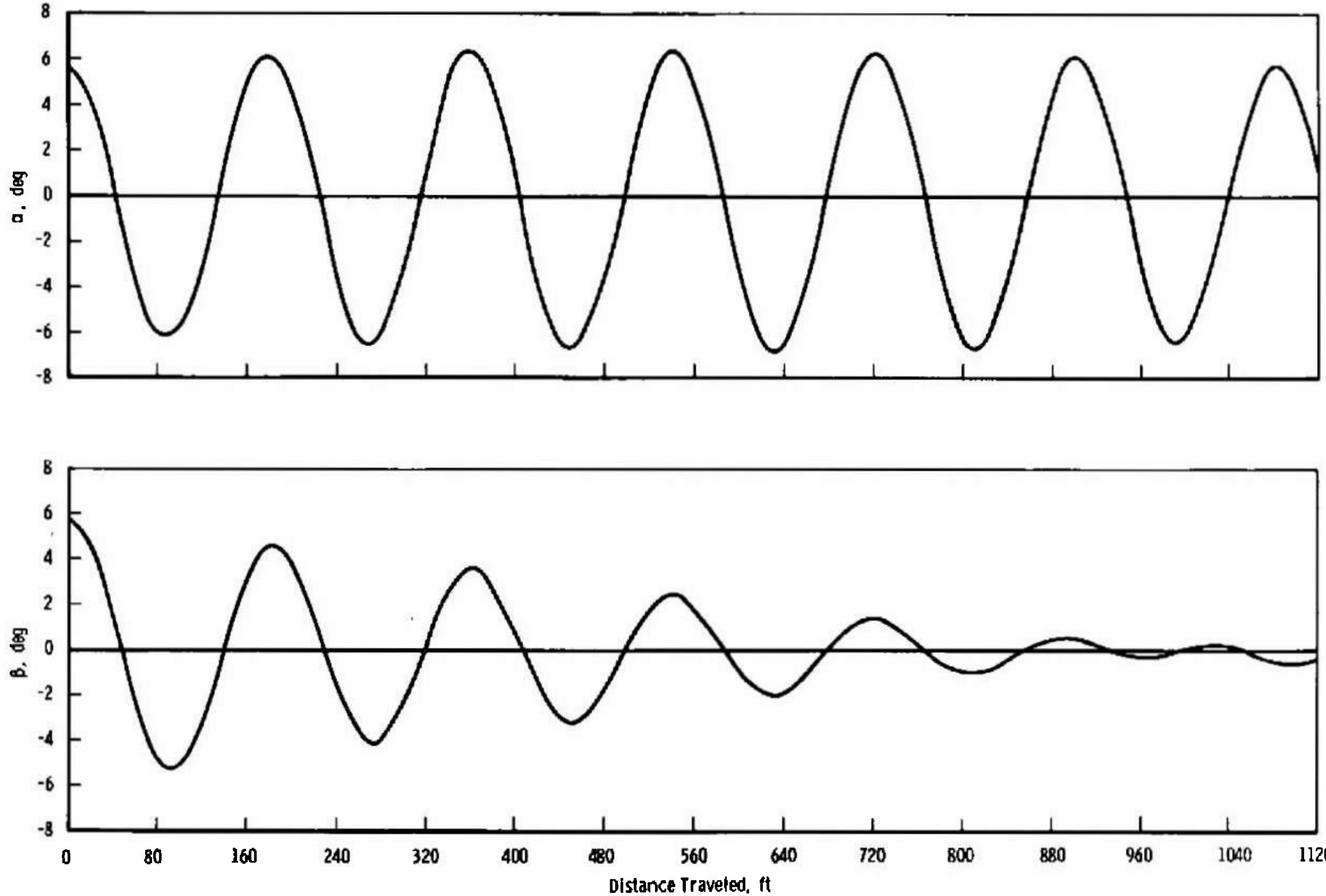
Note from the correspondence between μ's, ϕ''s, and K's that the ξ vector rotates in a direction consistent with the rotation of the motion vector having the larger amplitude (for small roll rates); it follows when the larger amplitude vector initially has the larger damping, the motion (ξ vector) can have a reversal. This type of (α, β) plot for computed motion is shown in Fig. 7 where the precessional vector, k_p, initially has the larger damping and the slightly larger amplitude. Later in the flight the precessional vector has damped such that it has the smaller amplitude and a motion reversal occurs.

The motion patterns in Fig. 7 and Fig. 2 are of particular significance in conjunction with comments made in Ref. 5. In Ref. 5 motion patterns exhibiting converging-diverging oscillations and motion reversals were attributed to probable nonlinearities in the model motion. Although the test configurations of Ref. 5 may have had nonlinear characteristics, it should be pointed out, in consideration of the above discussion, that measured converging-diverging oscillations and motion reversals are not necessarily indicative of nonlinear model motion.



a. (Alpha, Beta) Variation

Fig. 7 Computed Motion for a Simulated Linear Configuration to Demonstrate Motion Reversal



b. Individual Motion Components

Fig. 7 Concluded

SECTION V
CONCLUDING REMARKS

The effects of roll on the free-flight yawing motion of statically stable test models having linear force and moment characteristics are discussed in more detail than in previous publications.

The zero-damping motion patterns defined by the basic linear equation of motion for an axisymmetric model are described with emphasis on the effects of model roll and initial model disturbances. It is shown that for a rolling model, particular care must be exercised in reducing stability data from free-flight tests in a wind tunnel when the motion is monitored in only one plane. This follows as the orthogonal components of the yawing motion of a rolling model can differ appreciably and can exhibit pseudo-nonlinear characteristics. The relationships between the frequency, damping, and amplitude of the motion vectors are discussed in relation to the possible variations in motion patterns, which should be an aid in alleviating some apparent misconceptions that have arisen from recent free-flight testing in wind tunnels.

REFERENCES

1. Nicolaides, J. D. "On the Free Flight Motion of Missiles Having Slight Configurational Asymmetries." Ballistic Research Laboratories Report 858, June 1953.

2. Murphy, C. H. "Advances in the Dynamic Analysis of Range Data." Ballistic Research Laboratories BRL-MR-1270, May 1960.

3. Waldron, H. F., McMahon, H. M., and Letarte, M. "Free Flight Facilities and Aerodynamic Studies at Canadian Armament Research and Development Establishment." AIAA Preprint 60-90, June 1960.

4. Nicolaides, J. D. and Elkenberry, R. S. "Dynamic Wind Tunnel Testing Techniques." Aerodynamics Testing Conference, Los Angeles, California, September 1966, AIAA paper No. 66-752.

5. Levy, Lionel L., Jr. and Fletcher, Leroy S. "Free-Flight Aerodynamics of a Blunt-Faced Re-entry Shape with and without Ablation." AIAA Preprint 66-61, January 1966.

6. Nicolaides, J. D. "Missile Flight and Aerodynamics." Bureau of Weapons, Department of the Navy, TN 100A, 1961.

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Arnold Engineering Development Center ARO, Inc., Operating Contractor Arnold Air Force Station, Tennessee	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP N/A

3. REPORT TITLE

EFFECTS OF ROLL ON THE FREE-FLIGHT MOTION OF STATICALLY STABLE BODIES

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Final Report

5. AUTHOR(S) (First name, middle initial, last name)

C. J. Welsh and R. M. Watt, ARO, Inc.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
September 1967	29	6

8a. CONTRACT OR GRANT NO. AF40(600)-1200 b. PROJECT NO. c. Program Element 65402234 d.	9a. ORIGINATOR'S REPORT NUMBER(S) AEDC-TR-67-156 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) N/A

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
Available in DDC.	Arnold Engineering Development Center, Air Force Systems Command, Arnold AF Station, Tennessee

13. ABSTRACT

The effects of roll on the free-flight yawing motion of statically stable test models, having linear force and moment characteristics, are discussed in more detail than in previous publications. The correspondence between the damping, frequency, and amplitude of the nutational and precessional vectors which define the motion is listed. Limitations in the use of simplified data reduction procedures when the model motion is monitored in only one plane are pointed out, and cases of pseudo-nonlinear characteristics being exhibited by the orthogonal components of the yawing motion of a rolling motion are demonstrated.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
rolling motion						
yawing motion						
damping						
frequency						
amplitude						
free-flight testing						
static stability						
aeroballistics ranges						
wind tunnels						
free-flight models						
data reduction procedures						

1. Missiles -- Rolling effects
2 " -- yawing
3 Projectiles -- Rolling effects
4 " -- 1-2