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A REVIEW OF JET EFFLUX STUDIES APPLICABLE TO V/STOL AIRCRAFT

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ARO, Inc.

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ARNOLD ENGINEERING DEVELOPMENT CENTER
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NOTICES

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FOREWORD

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ABSTRACT

The state-of-the-art of jets exhausting into a subsonic crossflow is presented. These studies complement the current research effort in development of an analytical description of the flow field created by a V/STOL aircraft.

CONTENTS

ILLUSTRATIONS

NOMENCLATURE

A	Area
C_n	Force coefficient on an arrow wing
d	Jet exit diameter
G	Gas weight flow
h	Dimension of the jet cross section perpendicular to the freestream, see Fig. 1
M	Momentum
m	Mass flow rate in the jet

p	Static pressure
q	Dynamic pressure, $1/2\ \rho V^2$
R	Ratio of jet exit speed to uniform speed of crossflow (V_j/V_∞)
T_t	Total temperature
V	Velocity
x, y, z	Coordinates
γ	Specific weight
δ	Dimension of the jet cross section at the jet center line parallel to the freestream, see Fig. 1
ρ	Mass density
ℓ	Distance from jet exit measured along the jet axis

SUBSCRIPTS

c	Average conditions from mixing the jet with the freestream
j	Conditions at the jet exit
∞	Freestream properties

SUPERSCRIPT

$(\ )'$	Denotes the end of the potential core of the jet

SECTION I
INTRODUCTION

During the past year, studies have been conducted at the Arnold Engineering Development Center to evaluate the design parameters for a wind tunnel facility for testing Vertical/Short Takeoff and Landing (V/STOL) aircraft. An outgrowth of these studies was an investigation to determine a method of predicting the flow field created by a V/STOL aircraft in the hover and transition flight modes. A literature search was initiated to ascertain what had been accomplished, either analytically or experimentally, toward a description of the flow field due to a turbulent incompressible jet exhausting into a crosswind. Some references on the fan-in-wing configuration are included.

SECTION II
DISCUSSION

Various types of propulsion have been considered for providing the vertical thrust required by a V/STOL aircraft. Some of these are "pure" jet, fan-in-wing, tilt wing or tilt engine, stowed rotor, and deflected slipstreams. Because of the complexity of analysis of the flow field created by a propeller, this investigation was narrowed to the analysis of a pure jet. Obviously, the flow field of the pure jet would not be identical to a propeller slipstream, but the description of the flow field created by a pure jet exhausting into a crossflow could provide insight to a general V/STOL flow field solution. For the case of a fan-in-wing or a propeller of high disc loading, the pure jet flow would be a first approximation.

The content of most reports on this subject usually falls into one or more of three categories: the jet shape (cross section and trajectory), jet entrainment, and surface pressures on the jet exit plane. In keeping with this pattern, each of these facets will be discussed separately.

2.1 JET SHAPE

2.1.1 Cross Section

As an axisymmetric jet leaves the nozzle into a crossflow, the cross section of the jet is deformed into a kidney, or horseshoe, shape as shown in Fig. 1. To explain the formation of this shape, consider a circular cylinder in a uniform stream with the cylinder axis perpendicular to the stream. The pressure distribution around the cylinder, from potential theory, is given in Fig. 2 (Ref. 1). Next, consider the cylinder to have a plastic boundary and an internal pressure greater than that of the uniform stream. Because of the pressure distribution, the boundary will assume an oblong contour with its major dimension perpendicular to the freestream. Now, consider the interaction of the jet with the crossflow. As the jet emerges from the nozzle, a mixing layer is formed around the periphery of the jet. These particles on the periphery of the jet have less energy than those in the core of the jet and they are deflected more by the crossflow. Thus, the peripheral mixing and the pressure distribution on the boundary distort the jet into the shape of a kidney, or horseshoe. Additional explanation of the shape of the cross section is given by Abramovich (Ref. 2), and Keffer and Baines (Ref. 3).

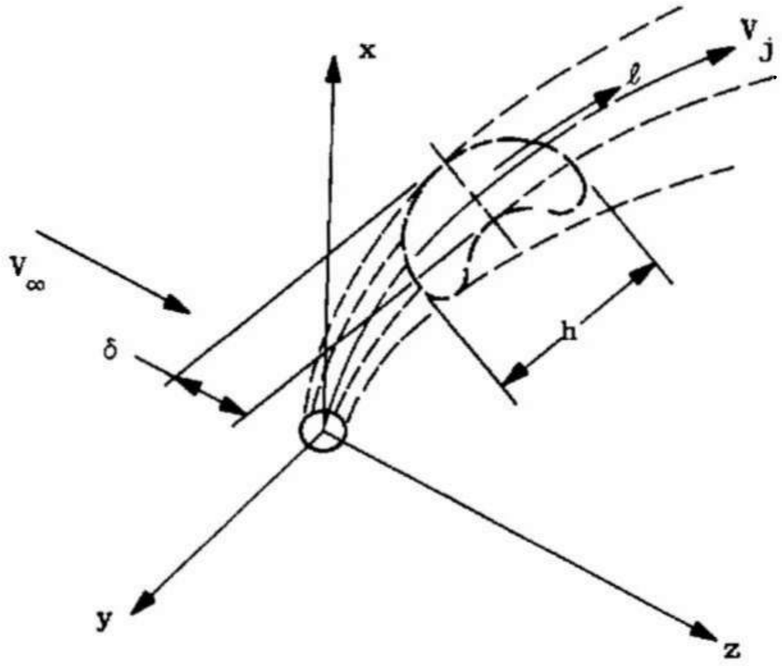

Fig. 1 Diagram of a Jet Exhausting into a Cross Wind

The horseshoe cross-section shape has been observed experimentally by Jordinson (Ref. 4). Abramovich states that the circular jet begins its deformation to the horseshoe shape at $\ell/d \approx 1.5$. Also, from Jordinson's data it is seen that the cross section of the jet increases as the jet leaves the nozzle. Abramovich approximates the growth of the jet by the following semiempirical equations:

$$
\begin{aligned}
h &= 2.25\,d + 0.22\,\ell \\
\delta &= 0.45\,d + 0.22\,\ell
\end{aligned}
\tag{1}
$$

where h is the dimension perpendicular to the freestream and δ is the dimension parallel to the freestream at the jet centerline; these symbols are defined graphically in Fig. 1.

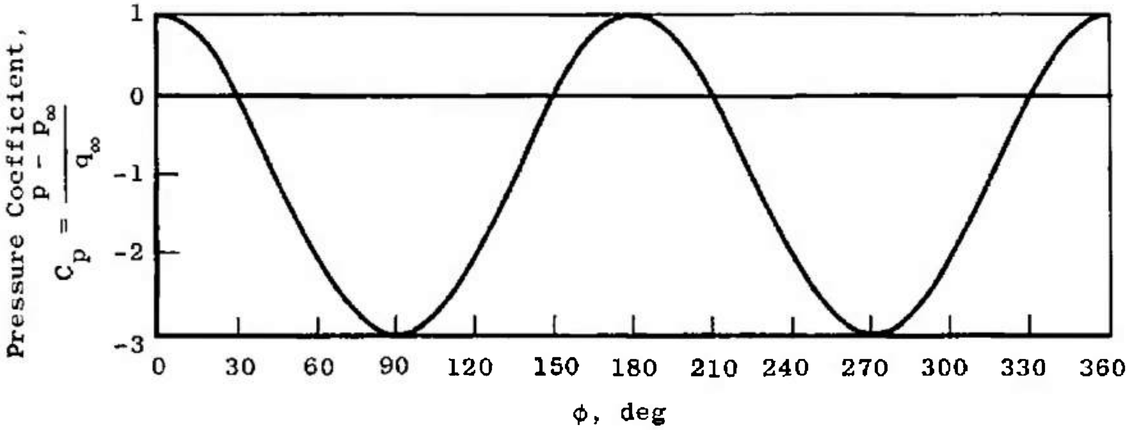

Fig. 2 Pressure Distribution around a Right Circular Cylinder in an Inviscid, Uniform Stream (ϕ Measured from the Stagnation Point)

2.1.2 Jet Trajectory

The trajectory of the centerline of the jet, which is considered as the line of maximum velocity, has been described by several empirical equations. Some of these equations are given below; they have been altered to conform to the coordinate system of Fig. 1.

Abramovich presents the results of two different experiments. From one of these, in the range $2 \leq q_j/q_\infty \leq 22$,

$$\frac{z}{d} = \frac{q_\infty}{q_j} \left(\frac{x}{d}\right)^{2.55} \qquad (2)$$

In this experiment the total temperature ratio was varied over the range $1 \leq T_{t_\infty}/T_{t_j} \leq 3$. Abramovich states that the ratio q_∞/q_j, in Eq. (2), takes into account the influence of the temperature ratio on the trajectory of the jet. From the other experiment, in the range $12 < q_j/q_\infty < 1000$,

$$\frac{z}{d} = \left(\frac{q_\infty}{q_j}\right)^{1.3} \left(\frac{x}{d}\right)^{3} \qquad (3)$$

In addition, Abramovich presents a method for calculating the influence of the finite channel dimensions on the curvature of a jet. The average velocity obtained from mixing the jet with freestream gas,

$$V_c = \frac{G_\infty + G_O}{\gamma_c \quad A_\infty}$$

is used to compute $q_c = \gamma_c \,(V_c^2/2)$. Then q_c is substituted into the above trajectory equations for q_∞, where A_∞ is the channel cross section, γc is the specific weight of the mixture, and G is gas weight flow.

Monical (Ref. 5) curve fitted the data of Jordinson and obtained the following expression for $\rho_j = \rho_\infty$:

$$\frac{z}{d} = 1.58 \left(\frac{V_\infty}{V_j}\right)^{2.5} \left(\frac{x}{d}\right)^{278} \tag{4}$$

Jordinson's data were for values of $V_j/V_\infty = 4$, 6, and 8.

By curve fitting the data from their experiment, Callaghan and Ruggeri (Ref. 5) found that

$$\frac{z}{d} = 0.1183 \left(\frac{\rho_\infty \; V_\infty}{\rho_j \; V_j}\right)^{2} \left(\frac{x}{d}\right)^{2.3} \tag{5}$$

for the range $2 \leq (\rho_j \; V_j)/\rho_\infty \; V_\infty) \leq 7$, and $V_\infty = 260$ and 360 fps. In subsequent experiments, Callaghan, et al. (Refs. 6, 7, and 8) compared the flow coefficients of circular, square, and elliptical orifices and the depth of penetration of jets issuing from these orifices perpendicularly into an airstream. They found that the elliptical orifice yielded the highest flow coefficients for a particular value of pressure ratio and jet total temperature. In addition, for orifices of equal area the square orifice, with two sides parallel to the freestream, had better penetration than the circular orifice. The penetration of the elliptical orifices was dependent on the axis ratio and axis orientation with respect to the freestream. With the major axis parallel to the freestream, greater penetration was obtained by increasing the axis ratio.

Bradbury and Wood (Ref. 9) present the following trajectory equations for $p_j = \rho_\infty$:

$$\frac{z}{d} = 2.3 \left(\frac{V_\infty}{V_j}\right)^{3} \left(\frac{x}{d}\right)^{3} \tag{6}$$

Keffer and Baines (Ref. 3) show that for their experiments the trajectories of the jet can be expressed as $z/d = f\left[(V_j/V_\infty)(x/d)\right]$ with $\rho_j = \rho_\infty$. They show further that by dividing the coordinates by R^2, where $R = V_j/V_\infty$, the jet centerline may be expressed by a single function independent of velocity ratio. This relationship is shown in Fig. 3. Note that this curve is plotted as

$$\frac{\ell - \ell'}{dR^2} \quad \text{versus} \quad \frac{x - x'}{dR^2}$$

where x' and ℓ' denote the end of the potential core of the jet. The potential core is defined in Section 2.2.

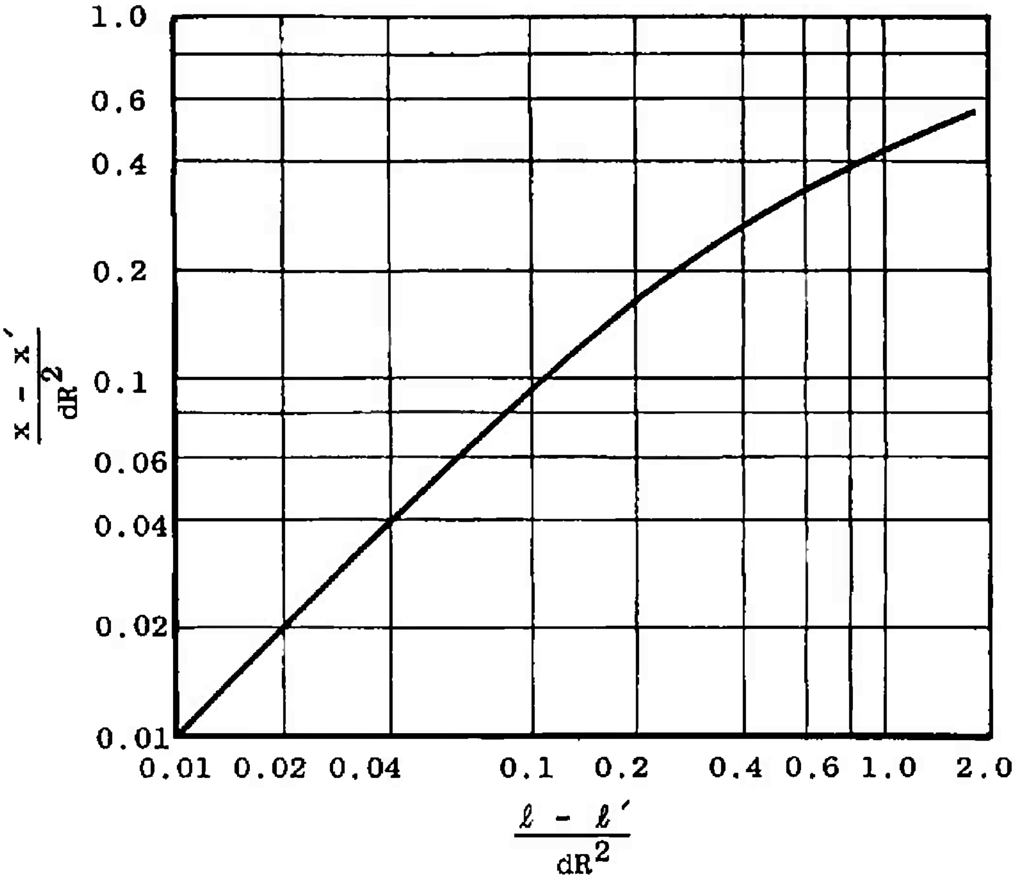

Fig. 3 Jet Trajectory as a Function of (V_j/V_∞) with $(\rho_j = \rho_\infty)$

Heyson (Ref. 10) presents the following trajectory curve for a jet exhausting perpendicularly into the freestream:

$$\frac{z}{d} = \frac{1}{4}\left(\frac{V_\infty}{V_j}\right)^2\left(\frac{x}{d}\right)^3 \tag{7}$$

In addition to the empirical equations, there have been several attempts to describe the jet trajectory theoretically. Abramovich (Ref. 2)* presents the derivation for the following equation:

$$\frac{z}{d} = \left[\frac{39}{C_n}\,\frac{\rho_j\, V_j^2}{\rho_\infty\, V_\infty^2}\right]^{1/3} \ln\left[1 + 0.1\,\frac{x}{d}\left(1 + \sqrt{1 + 20\,\frac{d}{x}}\right)\right] \tag{8}$$

---

*Much of the information on this topic which Abramovich presents in Ref. 2 is from Russian investigators. Consequently, many of his references could not be checked.

where C_n is the force coefficient on an arrow wing. He uses $C_n = 3$ for a jet issuing from a circular nozzle. In deriving this equation, Eqs. (1) were used for the growth of the jet.

In their original form, some of these trajectory equations contained terms which described a jet exhausting at an angle (other than 90 deg) with the freestream. However, they have been written here for a jet exhausting perpendicular to the freestream.

2.2 ENTRAINMENT

It has been observed that the mass flow in a jet increases as the jet leaves the nozzle. The additional mass flow is drawn from the atmosphere into which the jet is exhausting. This process is known as entrainment.

First, consider the case of a circular jet exhausting into a quiescent atmosphere; a schematic representation of the flow is given in Fig. 4. As the jet emerges from the nozzle, it separates at the nozzle lip and a mixing layer develops at the jet boundary. The mixing layer increases in thickness until it intersects the jet centerline; this point is defined as the beginning of the zone of established flow. Between this point and the nozzle exit the mixing layer surrounds a conical portion of the pure jet flow which is described as the potential core. The length of the potential core is approximately six jet diameters, depending upon the Reynolds number and the turbulence of the initial jet (Ref. 11). Analytical investigation of this problem can be found in Refs. 1 and 12.

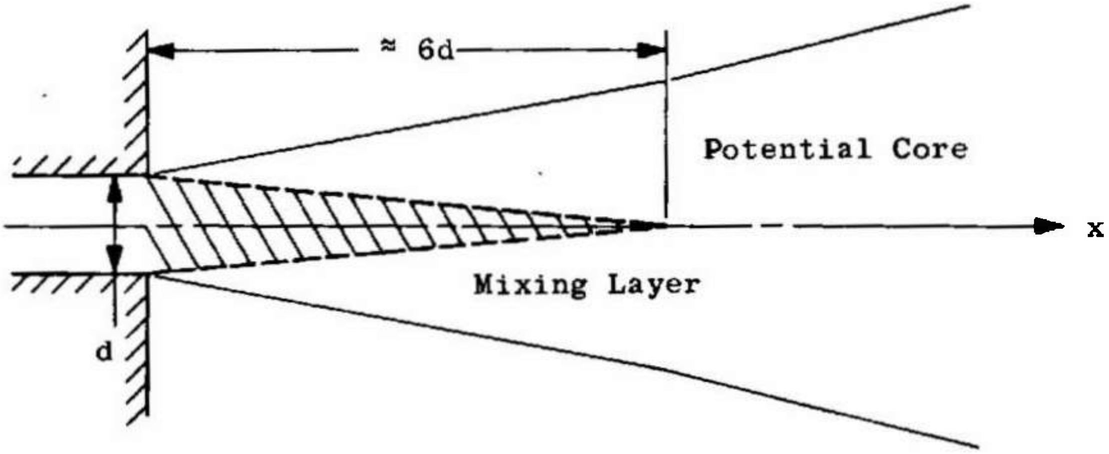

Fig. 4 Schematic Representation of a Jet Exhausting into a Quiescent Atmosphere

Ricou and Spalding (Ref. 13) conducted an experiment to determine the amount of mass flow entrained by the jet exhausting into a static region. They found that

$$\frac{m}{m_j} = 0.32 \left(\frac{\rho_\infty}{\rho_j}\right)^{1/2} \frac{x}{d} \tag{9}$$

In an alternate form

$$m = 0.282\, M_j^{1/2}\, \rho_\infty^{1/2}\, x \tag{10}$$

where M_j is the excess momentum of the jet.

Bradbury and Wood (Ref. 9) altered Eq. (8) to read

$$\frac{m}{m_j} = 0.32 \left(\frac{\rho_\infty}{\rho_j}\right)^{1/2} \left(\frac{x}{d} - \frac{x'}{d}\right) \text{ for } x \leq x' \tag{11}$$

where x′ is the end of the potential core. In addition, they state that

$$\frac{m}{m_j} = 1 + 0.1316 \frac{x}{d} + 0.02384 \left(\frac{x}{d}\right)^2 \text{ for } x \leq x' \tag{12}$$

Wygnanski (Ref. 14) states that

$$m = 0.1005\, d^2 \rho_j V_j \left(\frac{x}{d}\right) \text{ for } x \leq x' \tag{13}$$

and

$$m = 0.358\, d^2 \rho_j V_j \left(\frac{x}{d}\right) \text{ for } x \geq x' \tag{14}$$

Keffer and Baines (Ref. 3) conducted an experiment of a circular jet directed normal to a uniform, steady crosswind. They state that the potential core is deflected and is approximately one-half as long as for the case of no crosswind. It is their opinion that the entrainment is proportional to the difference between jet velocity and crosswind velocity rather than the component of the crosswind velocity parallel to the jet. To support this argument they develop several dimensionless functional relations which correlate with their test data.

2.3 SURFACE PRESSURES

There are probably more experimental data available for the surface pressures on the jet exit plane than any other aspect of a jet exhausting into a crosswind. This is understandable since data of this nature are relatively easy to obtain from a test. References 9, 15, and 16 are three of the better sources for these data. These references present the pressure contours on the exit plant for various velocity ratios and plate configurations.

2.4 JET WAKE

Separation of the crossflow from the jet boundary occurs just behind the midsection of the jet, as in the analogous case of a right circular cylinder. Thus, a pair of contrarotating vortices is formed in the wake of the jet. Both Abramovich (Ref. 2) and Jordinson (Ref. 4) present data which show that the velocity vectors behind the jet have components perpendicular to the axis of the jet. This indicates a circulatory, entraining flow in the wake of the jet.

It is the opinion of Keffer and Baines (Ref. 3) that the vortices in the wake are the dominant mixing agents and exert a major effect on the flow. In addition, they state that the vortices cause an internal circulation and large-scale mixing within the jet.

2.5 FAN-IN-WING

In recent months, the fan-in-wing propulsion system has been popular with designers of V/STOL aircraft. Consequently, there have been several studies made, both analytical and experimental, of the flow field produced by a fan-in-wing and the parameters affecting its performance. References 17 through 26 present some of these studies.

SECTION III
CONCLUDING REMARKS

The description of a jet exhausting into a subsonic crossflow has been presented along with a semiempirical equation to compute the growth of the jet cross section and several semiempirical equations for calculating the jet trajectory and entrainment. It is apparent that experimental techniques have exerted a strong influence upon the form and/or constants involved for the equation developed from any particular experiment. There is no information available to this author which would aid in the selection of one equation over another. It is suggested that the sources of the information be consulted before any of the equations are used. In addition, references are given which present experimental data for exit plane surface pressures.

References 17 through 26 relate to the fan-in-wing type of V/STOL propulsion.

As was stated in Section I, this report presents the results of a literature search. It is solely intended to assist one who is initiating a jet efflux study; Refs. 2, 3, 4, and 13 are highly recommended for this purpose also.

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13. ABSTRACT

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