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MEASUREMENT OF TRANSISTOR HYBRID PARAMETERS AS A FUNCTION OF FREQUENCY

by

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Measurement of Transistor Hybrid Parameters As A Function of Frequency

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Introduction

A complete set of information useful in the design of transistor circuits should include data on the frequency dependence of the parameters of the transistor to be used. This information is of particular importance in determining the stability of such circuits. This report describes the methods used to measure the following parameters:

1) $|h_{11}|$ vs frequency
2) $|h_{12}|$ vs frequency
3) $|h_{21}|$ vs frequency
4) $|h_{22}|$ vs frequency
5) C_c

A complete treatment should also include measurements of the phase response of each of these parameters. However, these measurements are quite difficult and will not be included here.

Measurement of h Parameters

Of the six practical ways of describing the behavior of a two terminal pair, the hybrid, or h, parameter set has been recognized as the one which lends itself to easy and accurate measurement.

A dynamic low frequency h meter has been designed which allows the measurement of h_{11}, h_{12}, $1 + h_{21}$, and h_{22} at a frequency of 270 cps. This instrument has been described in CF-2212, "A Dynamic Method for Measuring the Hybrid Parameters of Junction Transistors." A discussion of the theory involved in such measurements is included in that report. With the low frequency instrument, a means exists for checking any other method of h parameter measurement. This has proven valuable in the methods to be described below.

1) <u>Measurement of h_{11}</u>. h_{11} is the common base input impedance with the collector short circuited for a.c. signals. The circuit of Figure 1 has been successfully employed for this measurement.

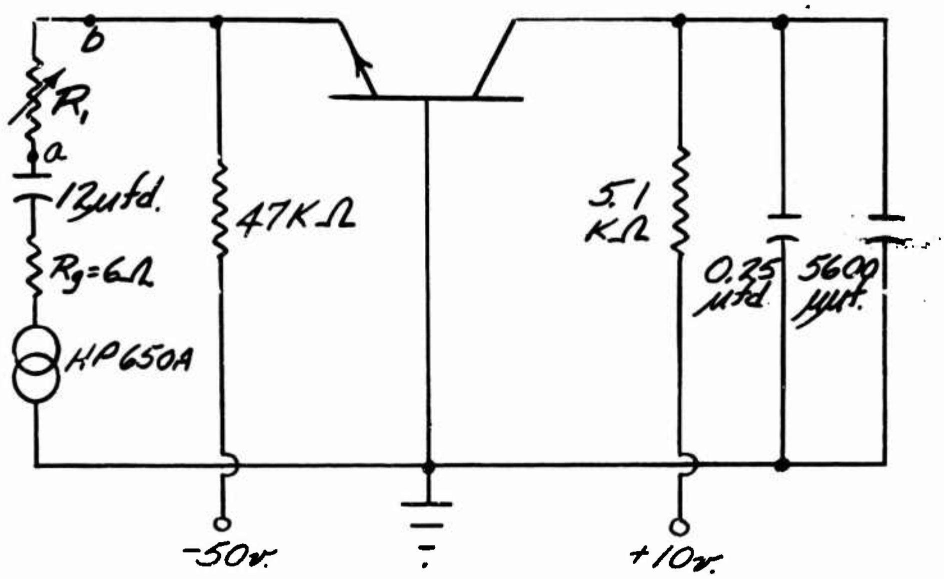

Figure 1

The 0.25 μfd capacitor provides the required a.c. short circuit across the collector, while the mica capacitor provides additional bypass at high frequencies. The emitter bias resistor is large enough to have little influence on the value of h_{11} even at high frequencies. Measurements are made by setting the required oscillator frequency and then adjusting the carbon potentiometer R_1 until the voltage at b is one half that at a. The value of R_1 is then measured, and this is equal to $|h_{11}|$.

2) <u>Measurement of h_{12}</u>. h_{12} is the reverse voltage gain with the input open circuited. The circuit of Figure 2 is useful for this measurement.

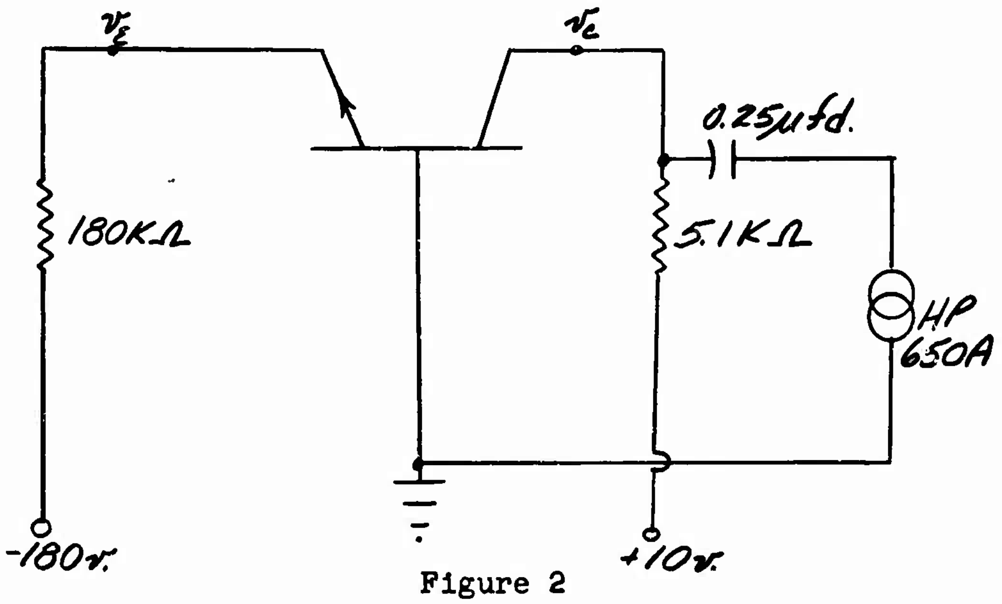

Figure 2

The 180KΩ emitter bias resistor provides an a.c. open circuit if the shunt capacity across it is low. v_c and v_ε are measured with a low capacity cathode follower probe feeding a Hewlett Packard 400 C voltmeter. The probe capacity placed between emitter and ground in measuring v_ε is small enough so that a reasonably good open emitter circuit is maintained up to a frequency of several megacycles. At frequencies up to about 50 Kc, $|h_{12}|$ is small, and the measurement of v_ε is a problem because of stray noise pickup. A tuned voltmeter, such as the General Radio Type 736 Wave Analyzer, can be used advantageously to fill in the information in the region up to 16 Kc.

3) Measurement of h_{21}. h_{21} is the negative short circuit current gain, or $-\alpha$, of the transistor. A circuit has been developed which will measure α with reasonable accuracy up to a frequency of approximately 10 megacycles. For frequencies greater than this, neutralization must be employed, and a sweep method of doing this is being investigated at present. The circuit for the low frequency α measurement set is shown in Figure 3.

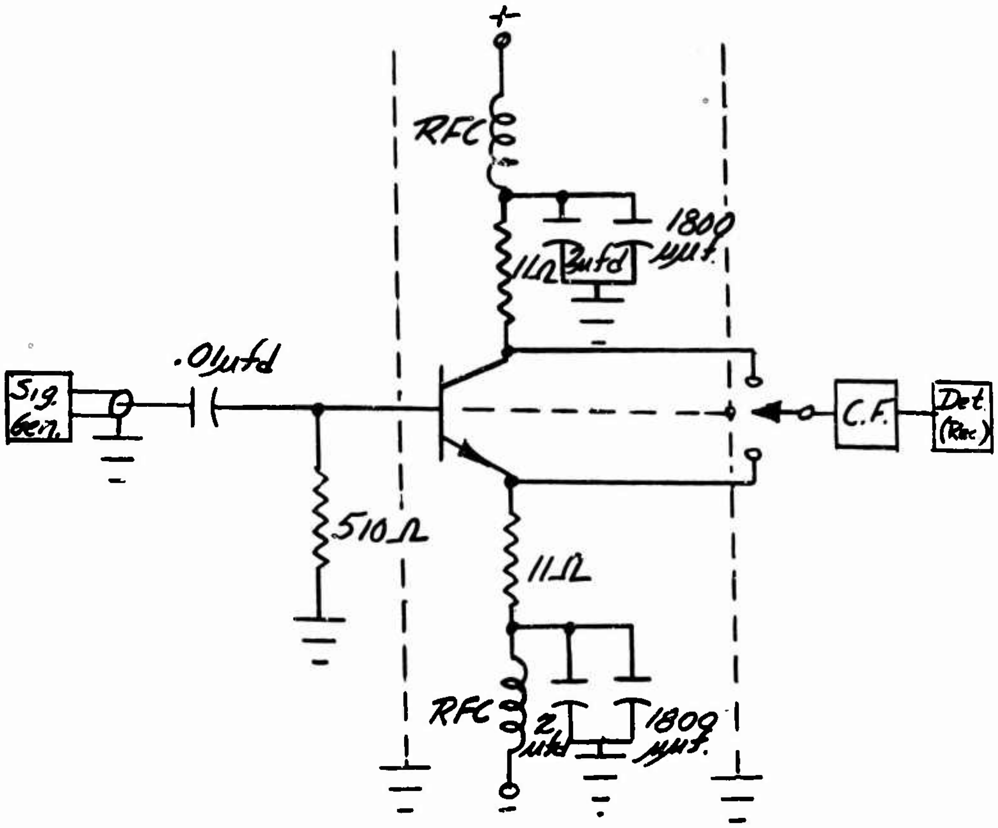

Figure 3

The signal is fed to the base, and voltages proportional to the emitter and collector currents are measured across the 11 ohm resistors in those two leads. A simple ratio then gives the α of the transistor.

4) <u>Measurement of h_{22}.</u> h_{22} is the reciprocal of the open circuit output impedance. A circuit for measuring this parameter is shown in Figure 4.

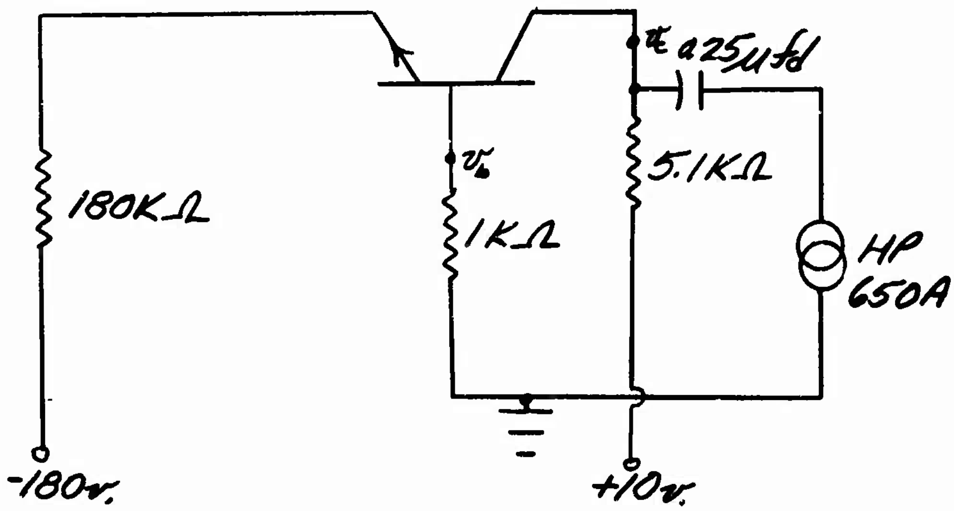

Figure 4

Since the emitter is open circuited, $i_b = i_c$, and so the voltage across the 1000 ohm base resistor is proportional to i_c. Here again, the low capacity probe was used, since it is important that the impedance in the base lead remain relatively constant up to a frequency of several megacycles.

As an example of the type of data obtained using the methods described above, complete h parameter measurements on a Western Electric Type M1859 n-p-n grown junction transistor are included in this report. These data appear in Figures 5 through 8.

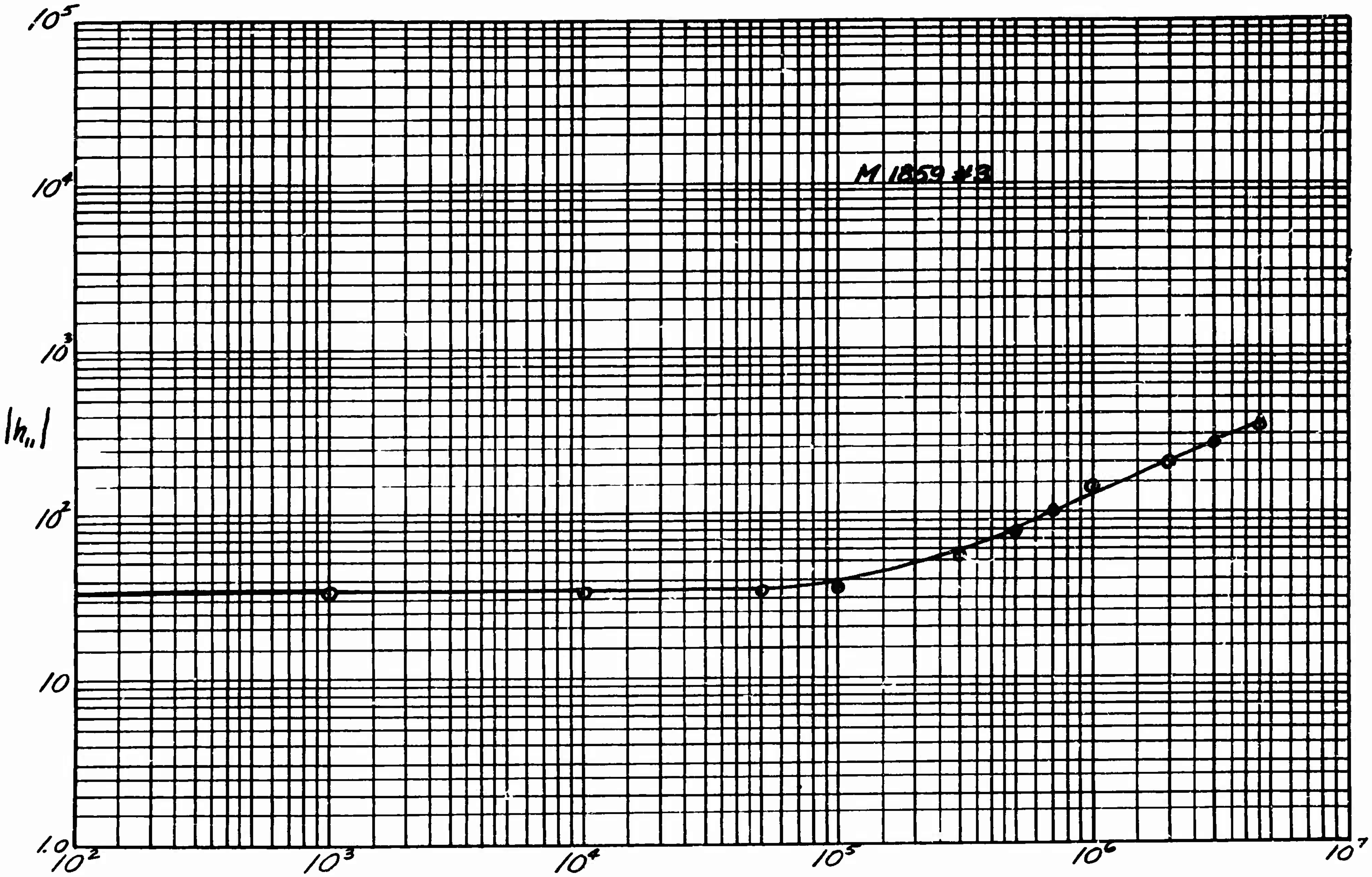
M 1859 #3
$|h_{11}|$
10^5
10^4
10^3
10^2
10
1.0
10^2
10^3
10^4
10^5
10^6
10^7
FREQUENCY (cps)

FIGURE 5

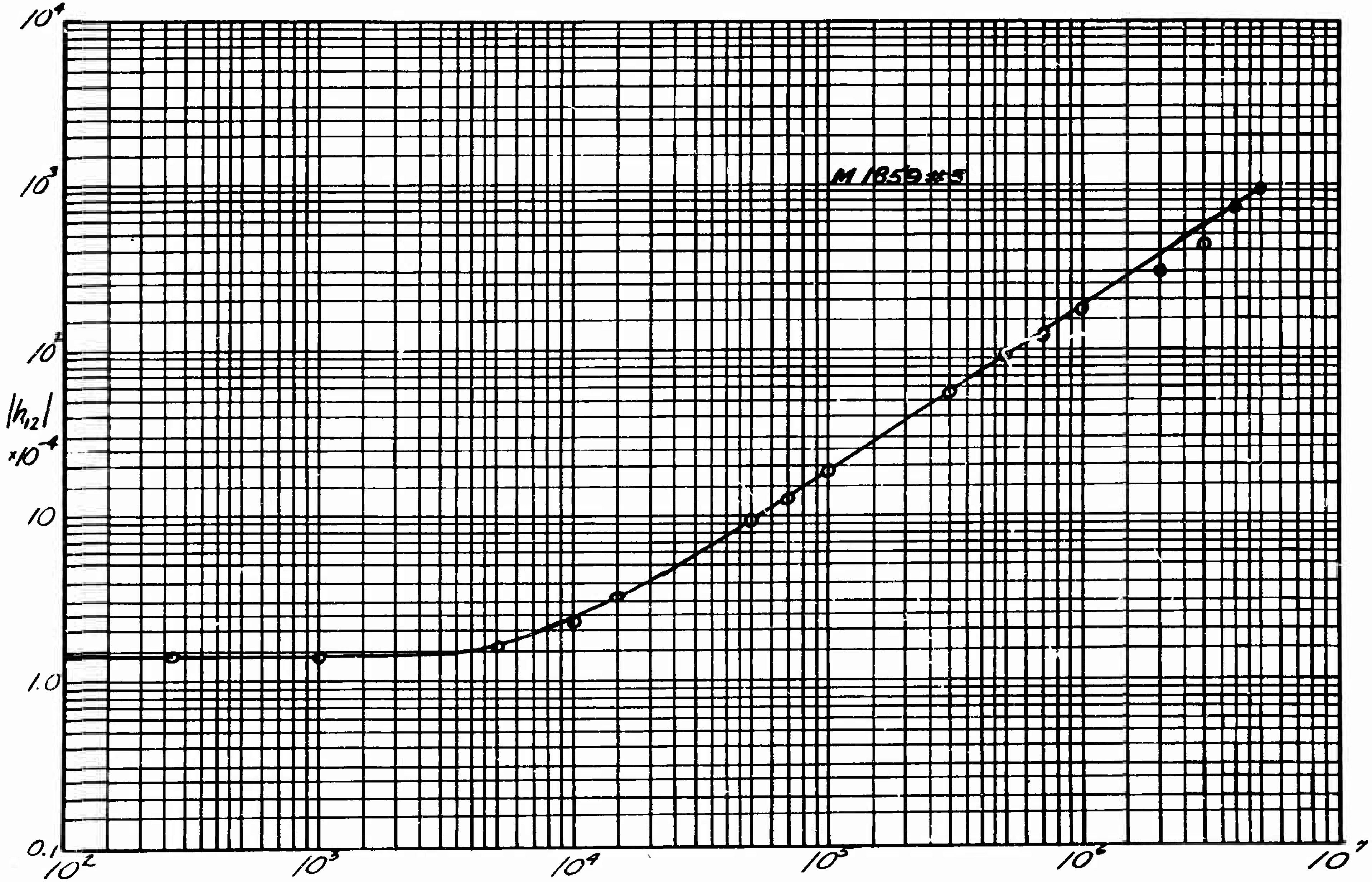
M 1859 #5
$|h_{12}|$ $\times 10^{-4}$
10^4
10^3
10^2
10
1.0
0.1
10^2
10^3
10^4
10^5
10^6
10^7
FREQUENCY (cps.)

FIGURE 6

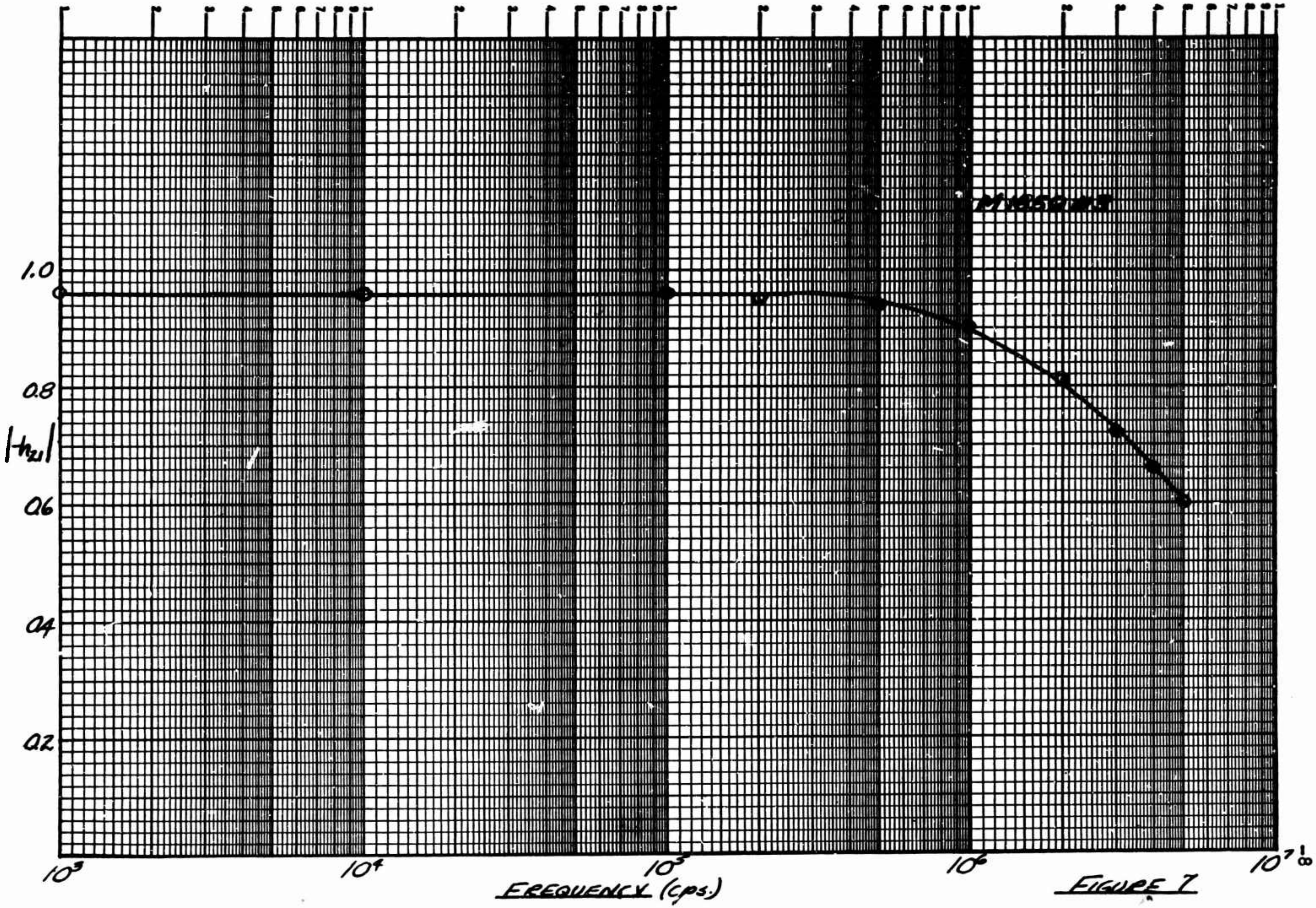

FIGURE 7

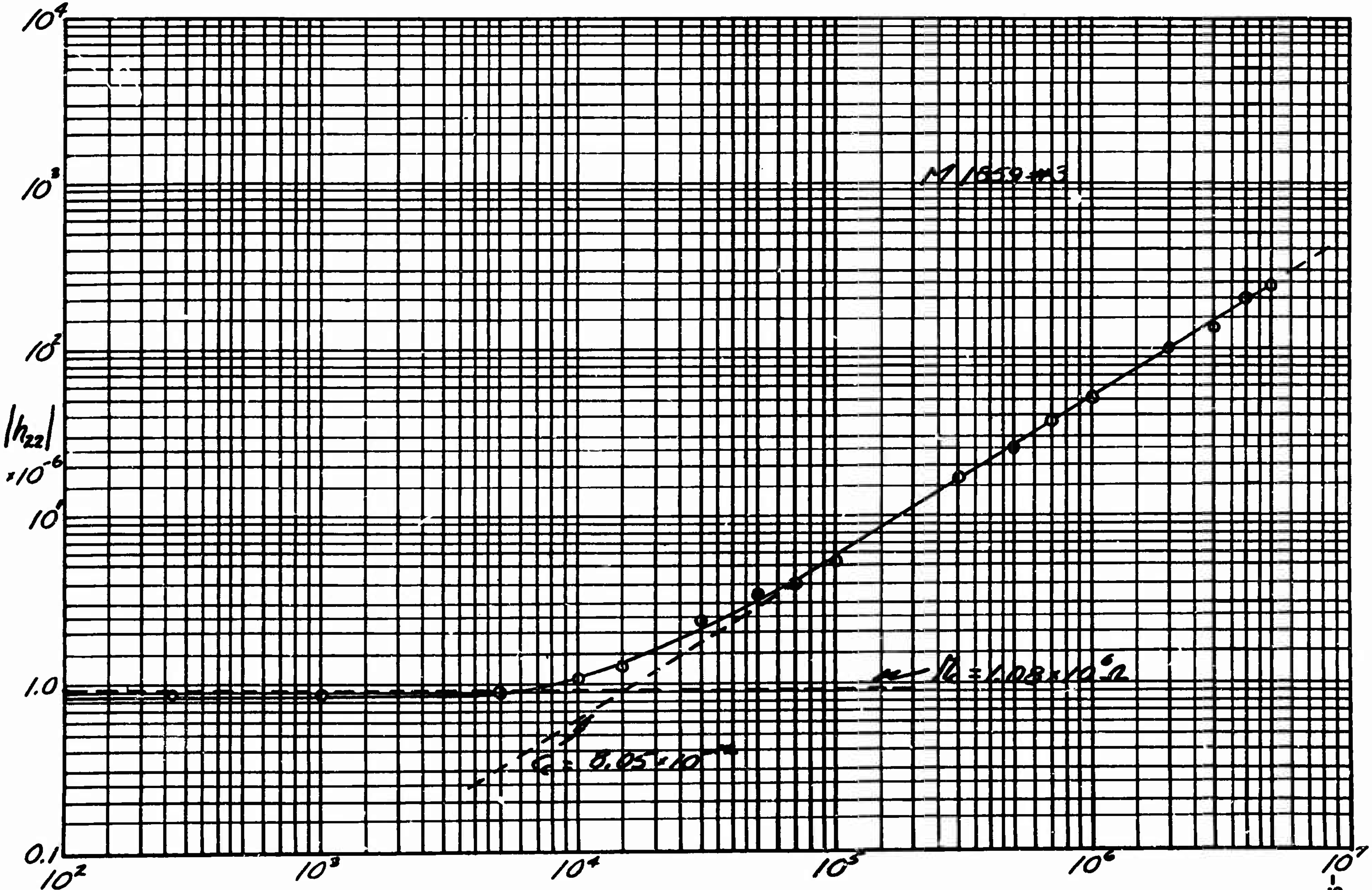
M 1859 #3
$|h_{22}|$
$\times 10^{-6}$
10^4
10^3
10^2
10^1
1.0
0.1
10^2
10^3
10^4
10^5
10^6
10^7
$R_c = 1.08 \times 10^6 \Omega$
$C_c = 8.05 \times 10$
FREQUENCY (cps)

FIGURE 8

The low frequency h meter measurements on this particular unit yielded the following values at $V_c = +5.0$ volts and $I_{\varepsilon} = -1.0$ ma.;

$h_{11} = 32$
$h_{12} = 1.32 \times 10^{-4}$
$1-\alpha = .0375 (\alpha = .9625)$
$h_{22} = .93 \times 10^{-6} (r_{22} = 1.08 \times 10^{-6})$

In addition, the I_{co} was 4.2 microamperes, and $f_{c\alpha}$ was 3.9 megacycles.

5) Measurement of Collector Capacity, C_c. The slope of the h_{22} or output admittance plot of Figure 8 at high frequencies closely approximates that of a capacitive reactance. In fact, the curve $h_{22} = \omega r_c C_c$ for $r_c = 1.08 \times 10^6$ and $C_c = 8.05$ μμf., which is plotted as a dotted line in Figure 8, coincides with the measured value of h_{22} over a considerable frequency range. This coincidence indicates a method for the measurement of C_c. If the capacitive portion of the open circuit output admittance can be measured, this should be approximately C_c. The output admittance with the emitter open circuited can be calculated using the circuit of Figure 9.

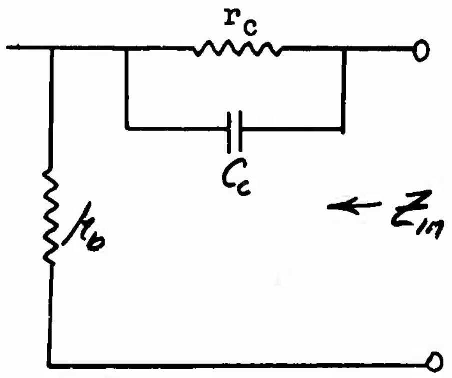

Figure 9

The capacitive component of this admittance is given by

$$C_{in} = \frac{r_c^{\,2} C_c}{(r_b + r_c)^2 + \omega^2 r_b r_c C_c}$$

If a measurement frequency is chosen such that $\omega^2 r_b C_{in} << r_c$, then $C_{in} \cong C_c$. For example, for the particular 1859 used to obtain the data included here, $C_c = 7.60$ μμf at 100Kc, for $V_c = +5.0$ volts and $I_\varepsilon = -1.0$ ma., measured with the Wayne Kerr bridge as shown in Figure 10.

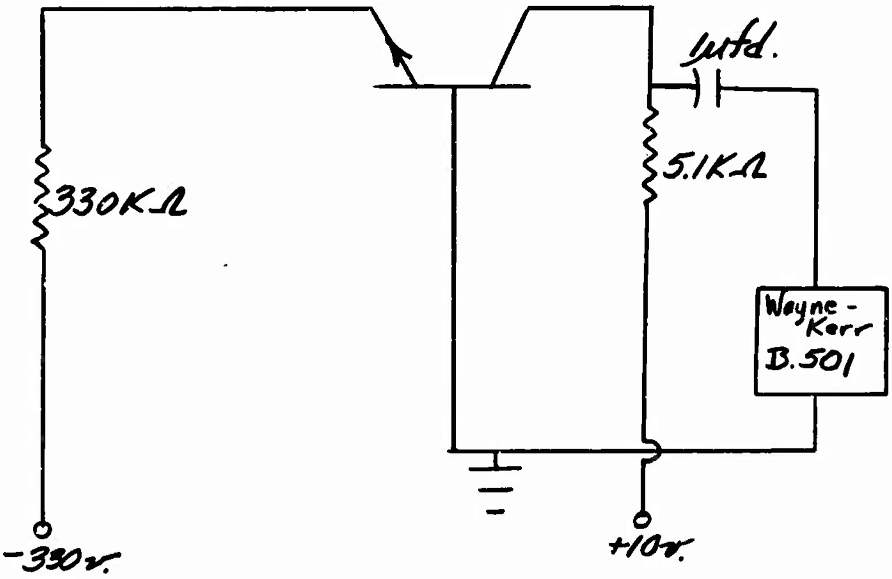

Figure 10

As another example of the usefulness of this method of determining C_c, the graph of Figure 11 has been included. Here collector capacity, measured at a frequency of 100 Kc with the bias condition, $I_\varepsilon = -1.0$ ma., is plotted against collector voltage.

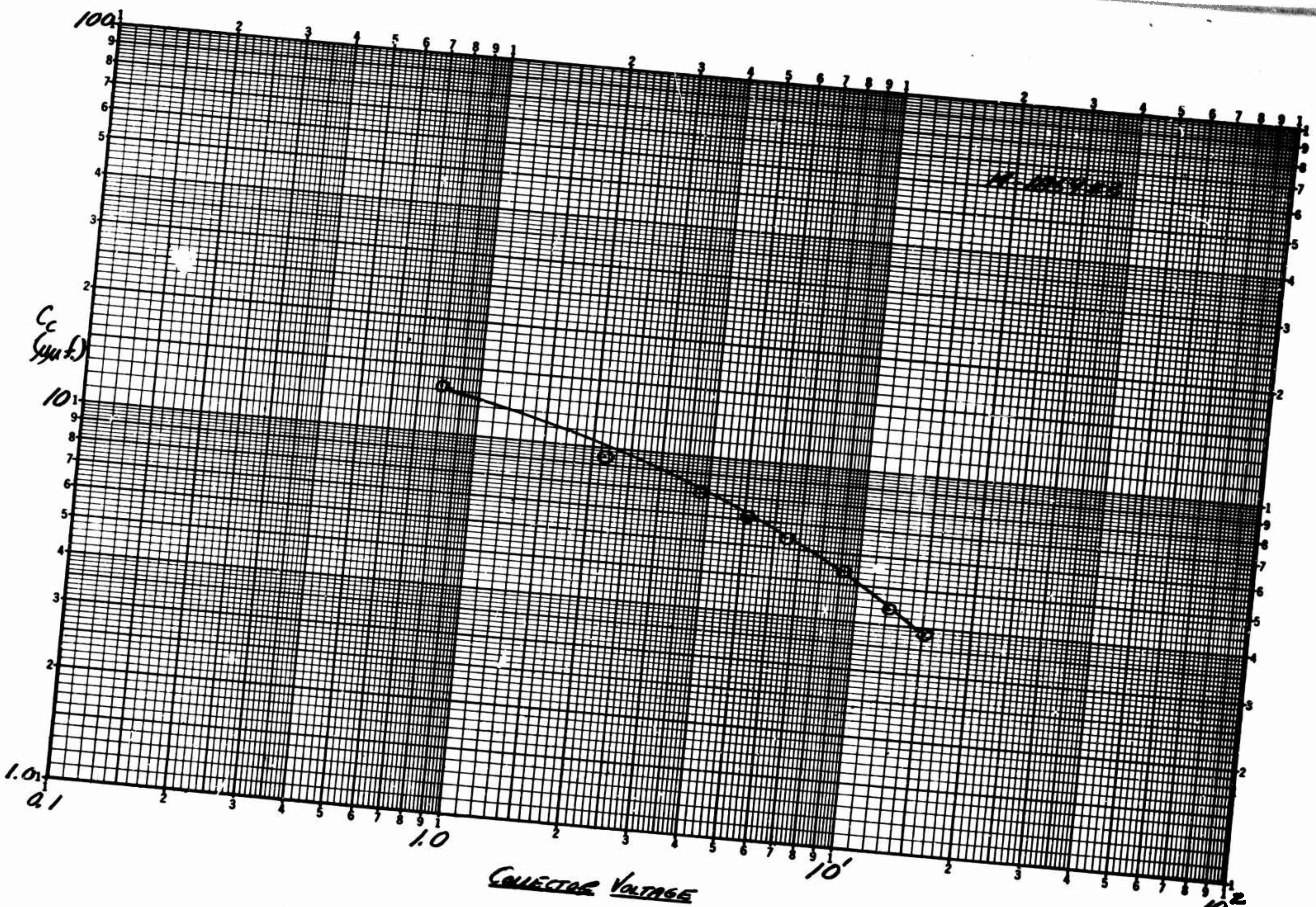

Figure 11

The rapid variation of collector capacity with collector voltage indicates that reasonable care should be taken in duplicating bias conditions throughout a given series of tests. This is especially true where high frequency effects are important.

<u>Analytical Expressions for the h Parameters.</u>

The data obtained by the methods outlined above can be used for circuit analysis, provided frequency dependent expressions can be written for each of the four parameters. For example, consider Figure 8, the curve for h_{22}. At low frequencies, $h_{22} = 1/r_c$, and at high frequencies, according to the graph of Figure 8, $h_{22} = \omega r_c C_c$. Furthermore, the curve of Figure 8 closely approximates the amplitude factor for a minimum phase network. Therefore, h_{22} can be expressed in the following form.

$$h_{22} = 1/r_c + j\omega C_c$$

$$\text{or, } h_{22} = (1 + j\omega\tau)/r_c, \text{ where } \tau = r_c C_c$$

An equation of similar form can be obtained for the h_{12} plot of Figure 6. At low frequencies, $h_{12} = r_b/r_c$, and at high frequencies, $h_{12} = \omega r_{b1} C_c$. r_{b1} is the number which allows the fitting of the capacitive reactance curve to the measured curve at high frequencies.

Again, the curve of Figure 6 approximates the amplitude factor of a minimum phase network, and therefore h_{12} can be represented by

$$h_{12} = \frac{r_{b1}}{r_c}\left(\frac{r_b}{r_{b1}} + j\omega\tau\right)$$

The α curve of Figure 7 is fitted quite closely by the formula

$$\alpha = \frac{\alpha_o}{1 + j\frac{f}{f_{c\alpha}}}$$

up to about the alpha cutoff frequency, $f_{c\alpha}$. Therefore, the following formulae represent the h parameters for frequencies up to approximately $f_{c\alpha}$.

$$h_{11} = r_\varepsilon + \frac{h_{12}}{h_{22}}(1 + h_{21})$$

$$h_{12} = \frac{r_{b1}}{r_c}\left(\frac{r_b}{r_{b1}} + j\omega\tau\right)$$

$$h_{21} = \frac{-\alpha_o}{1 + j\frac{\omega}{\omega_{c\alpha}}}$$

$$h_{22} = \frac{1 + j\omega\tau}{r_c}$$

where $\tau = r_c C_c$

For any particular transistor, the unknown quantities in the above equations can be determined by the method outlined below.

1. Measure the low frequency h parameters from these, r_ε, r_b, r_c, and α_o can be determined.
2. Measure C_c by the bridge method as outlined above, at as low a frequency as possible.

3. Measure $|h_{21}|$ vs frequency, and pick the value of $f_{c\alpha}$ off this curve.

4. Measure $|h_{12}|$ at a frequency such that $\omega r_c C_c >> r_b / r_c$. From this, r_{b1} may be calculated, since at this frequency,

   $r_{b1} = |h_{12}| / \omega C_c$.

h_{22} can be determined from steps 1 and 2.

h_{21} can be determined from steps 1 through 3.

h_{12} can be determined from steps 1 through 4.

h_{11} is determined by the other three h parameters.

Conclusions

Reasonably accurate measurements of the h parameters for junction transistors can be made up to frequencies of several megacycles if precautions are taken to reduce stray capacity and circuit noise. With the information thus obtained, analytic expressions for the h parameters can be found which will allow the accurate design of high frequency transistor circuits.