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NUMBER 6

INVESTIGATIONS ON THE NONELASTIC BEHAVIOR OF THE UPPER MANTLE

1 November 1966 - 31 January 1967

Contract AF 61 (052)-861

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V E L A - U N I F O R M

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ABSTRACT

The spectrum of seismic body phases radiated from sources within the earth's crust is distorted by multiple reflections in the crustal layers. To estimate the crustal distortion crustal transfer functions for P-waves have been computed for sources having a simple radiation pattern in the infinite medium: the buried explosive point source standing for an underground explosion, and the vertical point source acting at the free surface of the crust and standing for an atmospheric explosion. The crustal model is taken from Finland.

Radiation patterns have been computed for P-waves in the frequency range from 0 to 0.5 cps. An attempt has been made to separate the effect of the free surface from that of the layered crust in case of the explosive source. The latter effect starts to be noticeable at frequencies above 0.3 cps. Using the model for the upper mantle model in Europe (Lehmann, 1959) the radiation patterns have been transformed into amplitude-distance functions. Implications for the interpretation of experimental amplitude-distance curves of P-waves are discussed.

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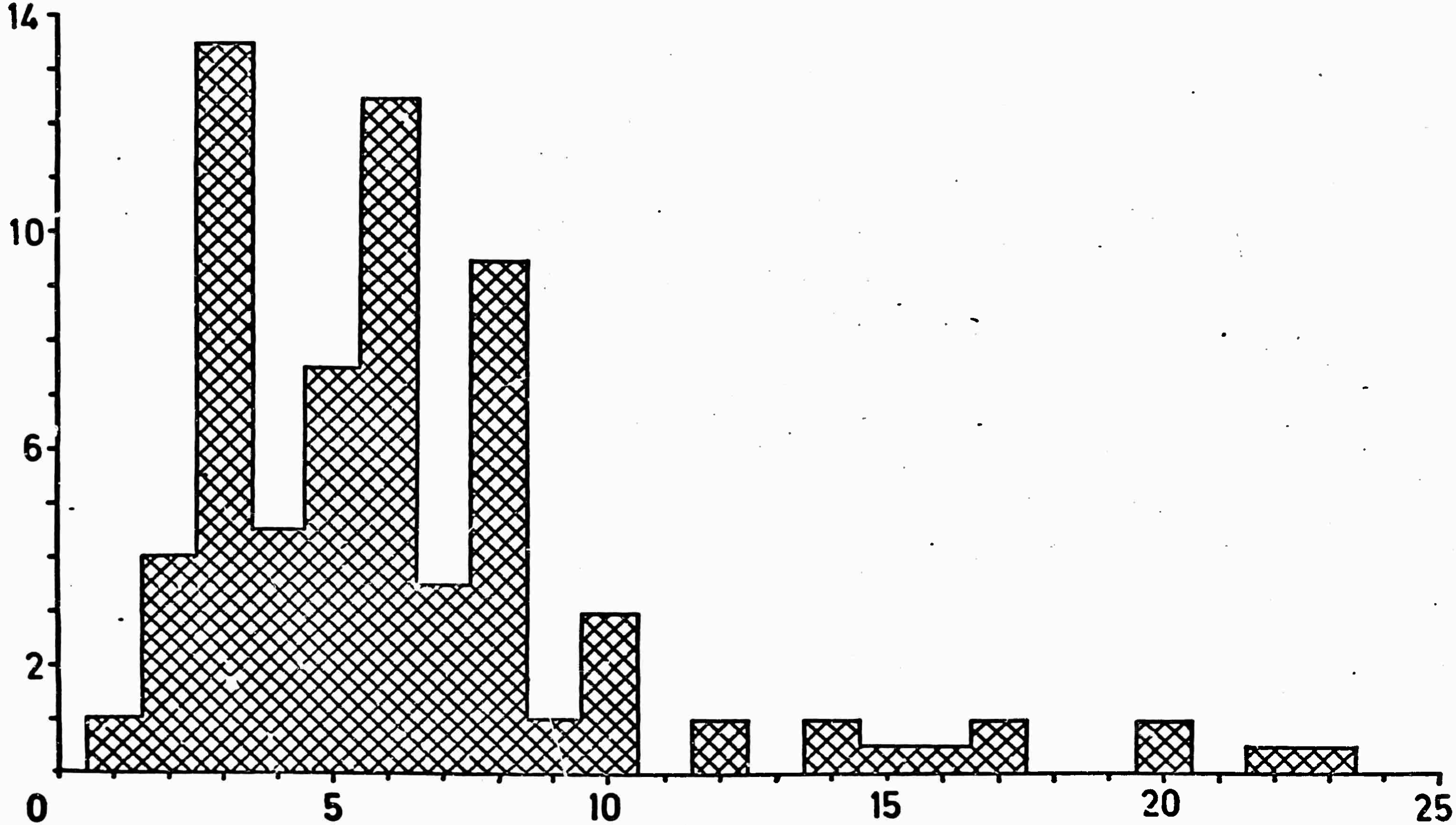
1. INTRODUCTION

For the study of the nonelastic behavior of the upper mantle we want to use body phases from explosive sources since their radiation pattern is comparatively simple. These explosive sources are located within or on top of the crust. Multiple reflections within the layered earth's crust are distorting the primary signal, originally radiated by the source. Therefore, the signal entering the upper mantle at the base of the "source" crust is different from the primary signal. Its spectrum will depend on the elastic parameters and the density of the layers within the crust, its thickness, the angle of incidence and the depth of the source. Also most earthquakes occur within the crust of the earth. As an example the depth distribution of hypocenters in South-Western Germany is given in Figure 1 (see Schneider, 1964). Therefore, we have to ask to what extent will the radiation pattern of shallow seismic sources be effected by the source crust. If necessary corrections for the effect of the source crust have to be applied.

The vertical point source acting on the free surface of the layered earth's crust will be used to estimate the influence of the crust on the spectrum of seismic body waves radiated by atmospheric explosions. The explosive point source buried at some depth within the crust is our model for underground explosions. After computing the radiation pattern for the far-field at different frequencies we will investigate how the normal



<u>Fig.1</u>: Depth distribution of earthquakes in S.W. Germany: number of hypocenters per kilometer of depth. (Data from Schneider, 1964)

amplitude-distance curve is effected by the distorted radiation pattern.

The formulas for the transfer function for P-waves for a system consisting of a point source within a layered medium over a homogeneous half-space have been derived by Fuchs (1966) using the theory of Harkrider (1964) for wave propagation from point sources in layered media. Those formulas are valid for the far-field within the half-space. The following assumptions regarding the model "point source in the crust" have been made:

(1) The actual source is approximated by a point source; i.e. the dimensions of the source region can be taken as small compared to the distance to the next boundary. With this limitation in mind the point source may be placed at any depth in a layered medium.

(2) The crust of the earth is approximated by a system of (n-1) homogeneous isotropic layers with plane parallel interfaces. The top layer is specified to be a free surface. The angle γ is the angle of incidence of P-waves in the mantle.

(3) The mantle, the n^{th} layer, will be regarded to be "relatively homogeneous". By this we mean, generally speaking, that the elastic properties of the mantle vary more gradually than they do in the crust. Therefore, we choose the n^{th} layer to be a homogeneous half-space.

(4) The solution becomes invalid for $\gamma = 90^{\circ}$ due to interference with the interface wave guided at the base of the crust.

2. THE TRANSFER FUNCTION FOR A VERTICAL POINT FORCE AT THE FREE SURFACE OF THE CRUST

The vertical point force at the free surface of the layered crust is taken as a first approximation of an atmospheric explosion. This point force is generating a Fourier transformed dilatational displacement potential $\bar{\phi}$ (r,z) at a point (r,z) in the half-space:

$$\bar{\phi}(r,z) = \frac{i\,\bar{\Sigma}}{2\pi k_{\alpha n}} \cos\gamma \cdot \bar{\Delta}_v \cdot \frac{e^{-i\,k_{\alpha n} R}}{R} \qquad (1)$$

This formula is taken from Fuchs (1966, equation (2.81)) with slight modifications.

The following notation is used:

$\bar{\Sigma}(\omega)$	= Fourier transformed source function
$k_{\alpha n} = \frac{\omega}{\alpha_n}$	= dilatational wave number in the half-space
$\omega = 2\pi\nu$	= angular frequency
γ	= angle of incidence in the half-space
R	= distance from the plumb point of the source at the base of the crust to the receiver at (r,z).

The quantity $\bar{\Delta}_v$ contains the effect of the layered crust. It is a function of frequency, elastic parameters, thickness and density of the layers and the angle of incidence γ into the half-space. It is computed from the crustal matrix as given by Fuchs (1966).

From equation (1) the Fourier transformed total dilatational displacement $\bar{u}_T$ (r,z) at (r,z) is derived:

$$\bar{u}_T(r,z) = \frac{\bar{\Sigma}}{2\pi} \cdot \cos\gamma \cdot \bar{\Delta}_V \cdot \frac{e^{-ik_{\alpha n} \cdot R}}{R} \tag{2}$$

The factor: $$T(\omega,\gamma) = \frac{1}{2\pi} \cos\gamma \cdot \bar{\Delta}_V \tag{3}$$

may be regarded as the transfer function for the system vertical point source at the free surface of the layerd crust. $T(\omega,\gamma)$ has been computed on the IBM 7094 of the German Computer Center at Darmstadt. We have chosen the model NUR 1 from Finland listed in Table 1 (Penttilä, 1965) as an estimate for the crust under Novaja Zemlja.

Depth (km)	Thickness (km)	α (km/s)	β (km/s)	Density (g/cm^3)
	2.2	5.73	3.52	2.80
2.2	18.6	5.95	3.52	2.88
20.8	8.2	6.37	3.72	2.95
29.0		8.23	4.50	3.43

Table 1: Crustal Model NUR 1 - Finland (Penttilä, 1965)

Figure 2 depicts $T(\nu,\gamma)$ for various angle of incidence (γ = 20°, 30°, 40° and 50°) in the frequency range $0 \leq \nu \leq 0.5$ cps. Strong deformation of the radiation pattern is to be expected above 0.1 cps. The maxima of $T(\nu, 20^\circ)$ are nearly coinciding with the minima of $T(\nu, 50^\circ)$ and vice versa.

The radiation pattern for the frequencies ν = 0 to 0.5 cps are given in Figure 3 at increments of 0.05 cps. Only the right half sides of the radiation patterns have been

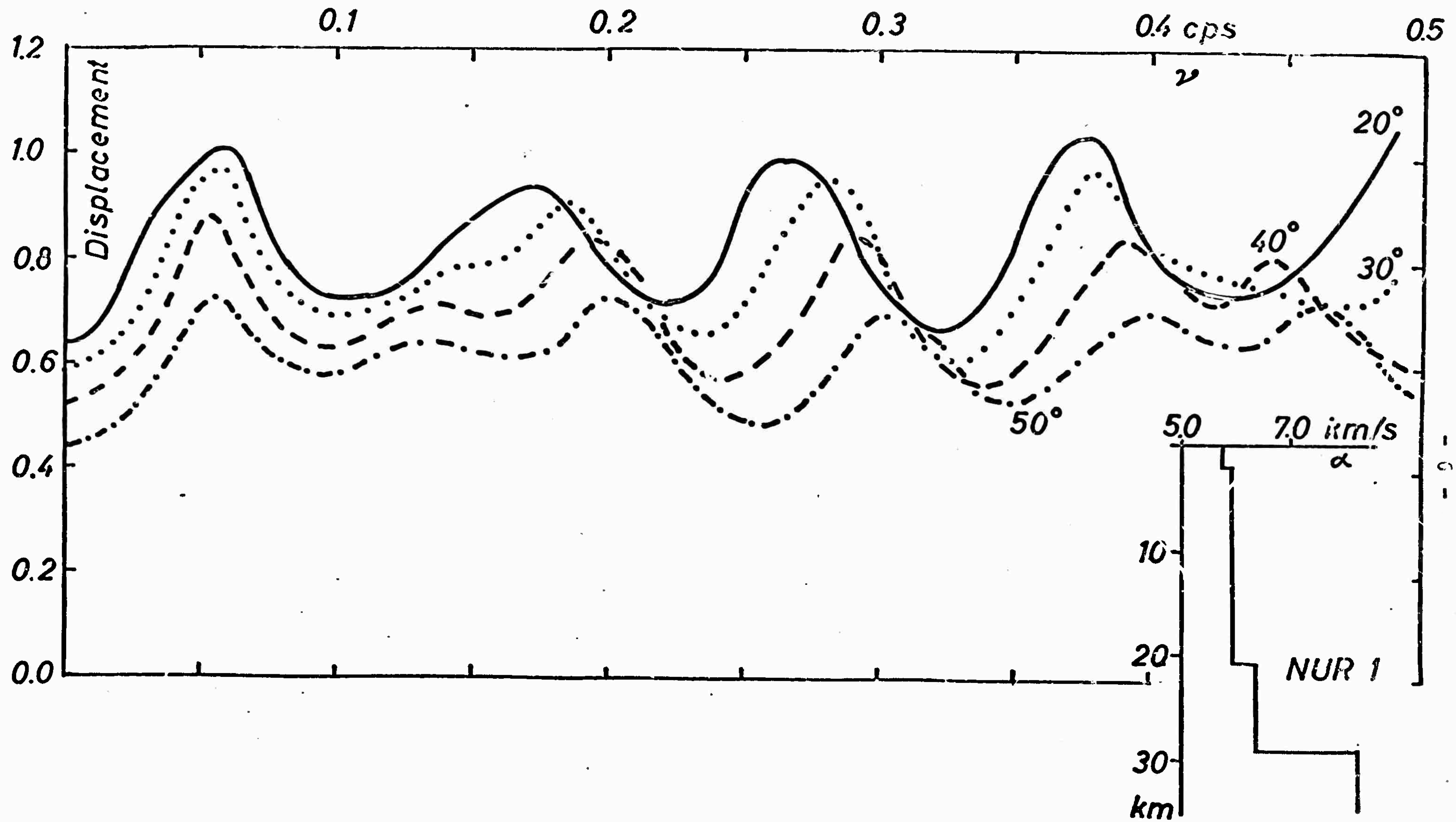


Fig.2: Transfer function T(ν,γ) for the vertical point source on top of crust NUR 1 (γ = 20°, 30°, 40° and 50°)

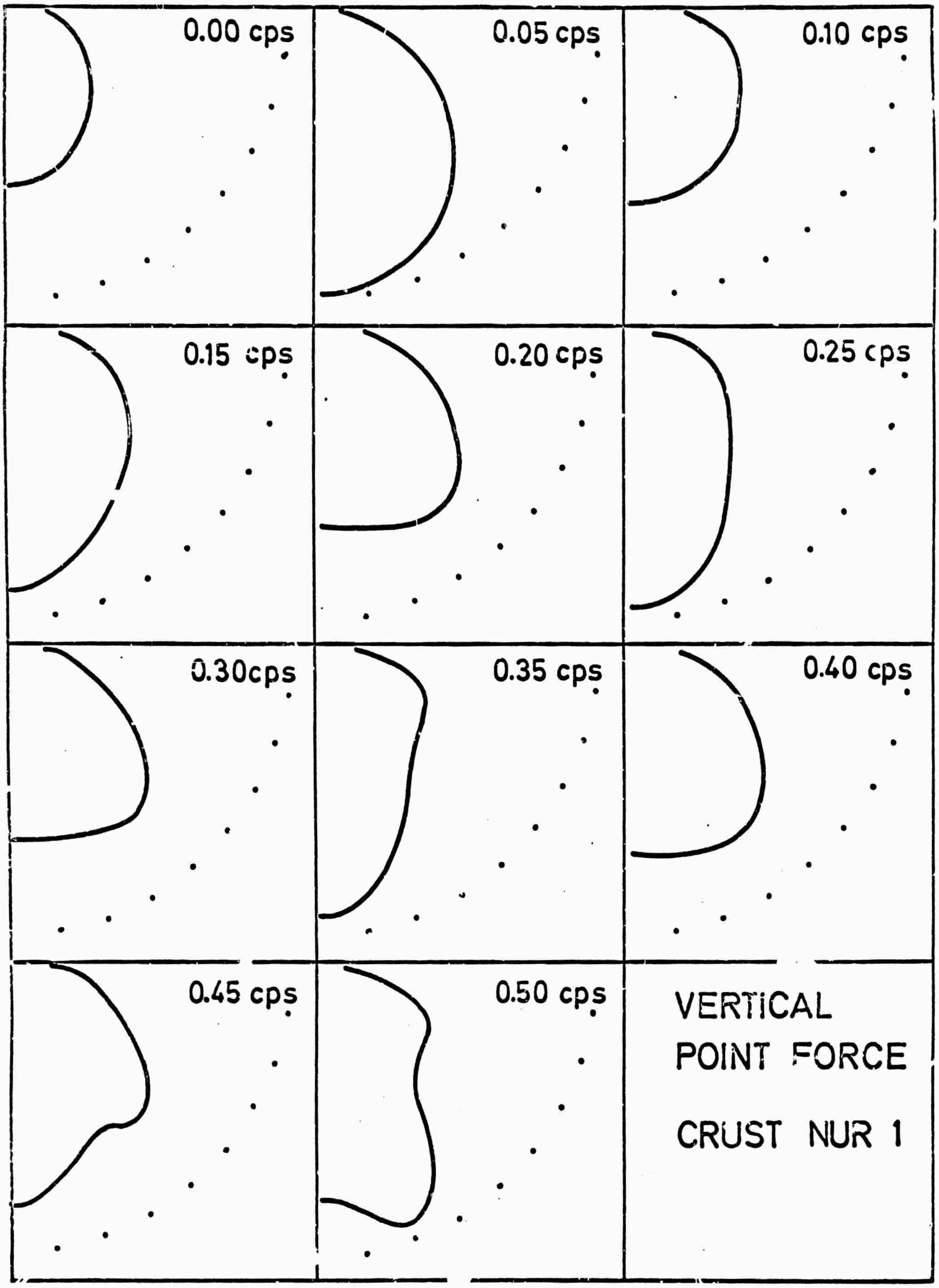


Fig.3: Radiation patterns for the vertical point source on top of crust NUR 1 (see Table 1) (ν = 0.0 to 0.5 cps).

drawn. The left half sides are symmetrical. At 0.0 cps the radiation pattern corresponds to that of the vertical point force acting at the free surface of a homogeneous half-space of subcrustal material. As the frequency is increasing the radiation pattern is gradually distorted. At some frequencies (e.g. 0.10, 0.20, 0.30 cps) the maximal energy is no longer radiated at $\gamma = 0^{\circ}$, that is vertically into the half-space, but at angles γ between 20° and 40°. The effect on the amplitude-distance function will be discussed in section 4.

3. THE TRANSFER FUNCTION FOR AN EXPLOSIVE POINT SOURCE AT DEPTH IN THE CRUST

The transfer function for an explosive point source buried at some depth in the crust has also been derived by Fuchs (1966). This transfer function $T_1(\omega,\gamma)$ is defined as the ratio of the displacement caused by the explosive point source in the crust to the displacement that would have been caused if the same source had been located in an infinite homogeneous medium of subcrustal material at the plumb point of the source at the base of the crust. It is again a function of frequency, elastic parameters, thickness and density of the crustal layers and the angle of incidence in the half-space.

As an example of an underground explosion the transfer function for an explosive point source at a depth of 500 m within the crustal model NUR 1 has been computed. The radiation pattern for the frequencies ν = 0.0 to 0.5 cps at 0.05 cps increments are given in Figure 4.

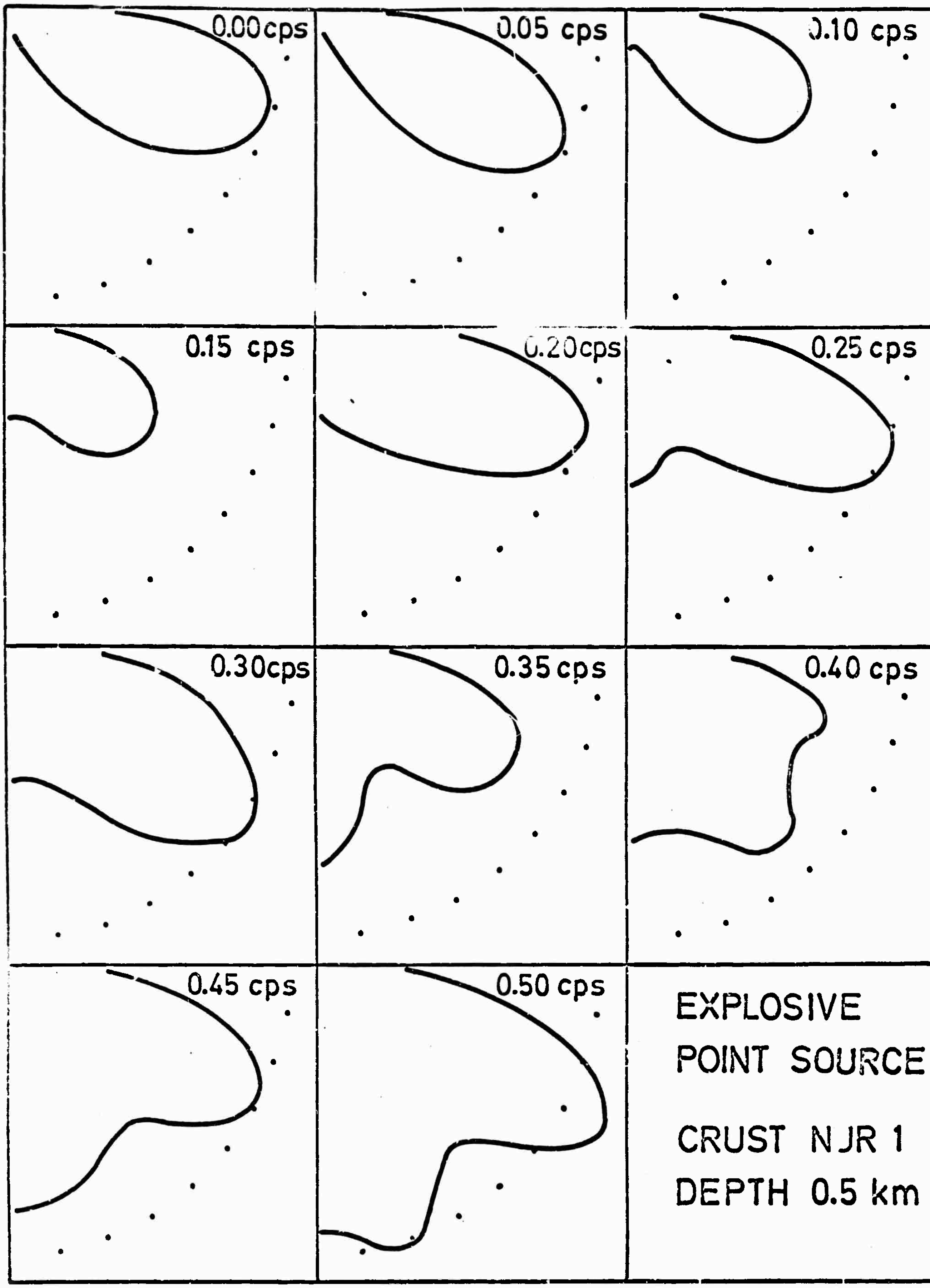


Fig.4: Radiation patterns for the buried explosive point source in the crust NJR 1 (Table 1) at depth 500 m (v = 0.0 to 0.5 cps)

The radiation pattern of the explosive source differs significantly from that of the vertical point force. In the case of the buried explosive source a strong primary reflection is generated at the free surface of the crust. Its interference with the primary wave is dominating the radiation pattern. This strong reflection is absent in the case of the vertical point source at the free surface.

At 0.0 cps, where the crust is ignored, the radiation pattern is that of an explosive dipole in an infinite medium, which radiates no energy in the direction of the dipole axis. As the frequency increases the finite distance of the source from the free surface allows energy also to be emitted at $\gamma = 0^{\circ}$. The direction of the radiation of maximal energy is a function of frequency.

It is important to get an estimate of that part of the transfer function which is caused by the free surface alone and that part which is caused by the layering of the crust. While the first part is the same for all crustal models - provided the source is located at a corresponding depth - the second part will vary with the crustal model. For comparison we have placed the explosive point source in a homogeneous half-space of the subcrustal material of the NUR 1 model, but at a depth of 715 m. This depth has been chosen in such a way that the travel time for the P-wave from the source to the free surface remains the same at normal incidence as in the case of the source in the NUR 1 crust at a depth of 500 m. The radiation patterns are drawn in Figure 5. Especially for frequencies larger than 0.3 cps the radiation patterns for the explosive source buried in the homogeneous half-space are much smoother than for the same source buried in the layered crust.

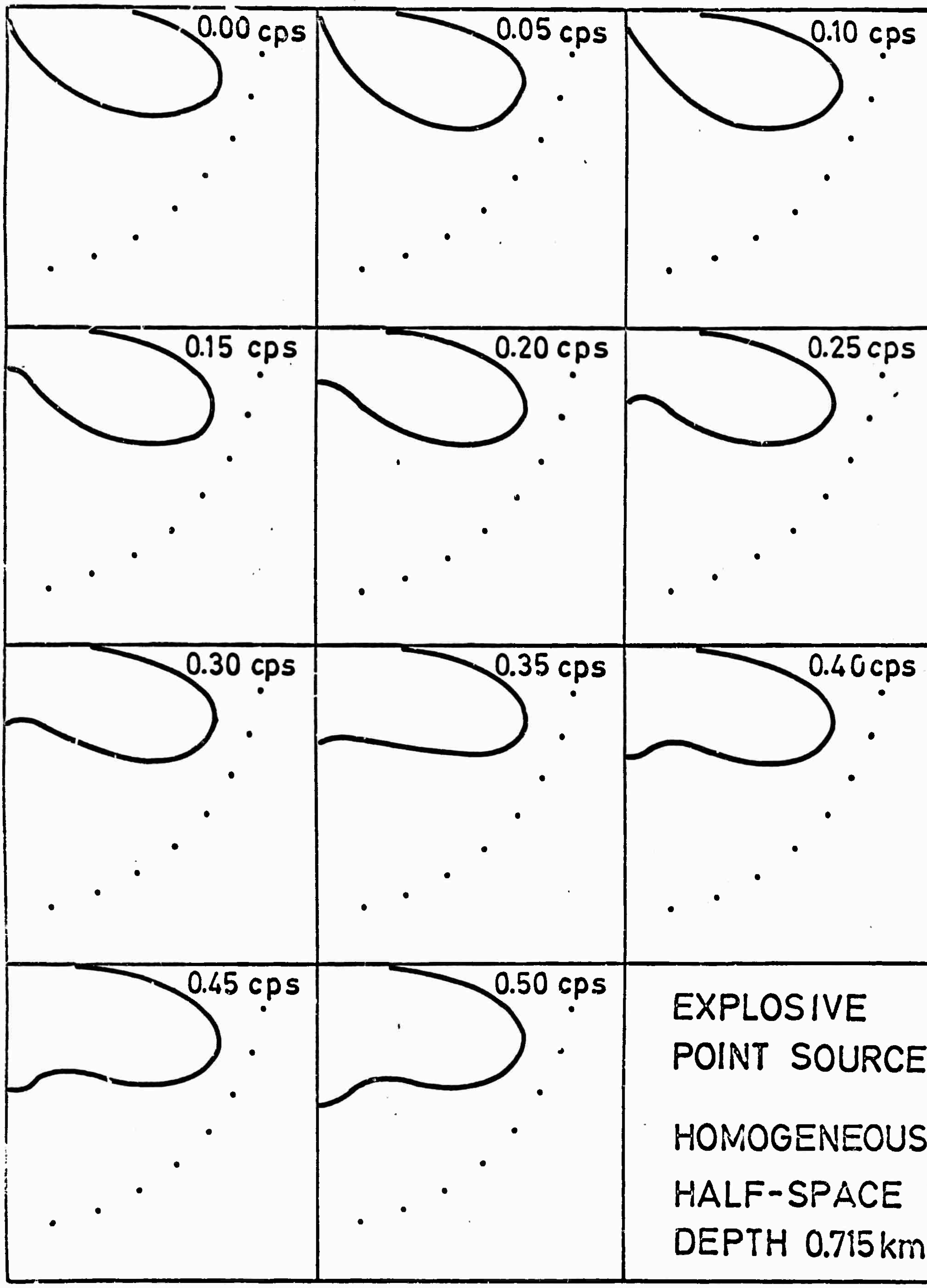


Fig.5: Radiation patterns for the buried explosive point source in the homogeneous half-space at depth 715 m (ν = 0.0 to 0.5 cps)

To study the effect of the source depth the explosive source has also been placed at a depth of 25 km in the crust NUR 1. The radiation patterns are given in Figure 6. Now strong energy is radiated vertically into the half-space ($\gamma = 0^{o}$) at comparatively low frequencies of about 0.05 cps. Here the direct wave from the source and the reflection from the free surface are interfering constructively. To estimate the effect of the free surface alone, the radiation pattern of the explosive source in a homogeneous half-space of subcrustal material is depicted in Figure 7. The depth of the source is 32.4 km with equal travel time to the surface at normal incidence.

A comparison of the radiation patterns of Figure 6 and 7 shows that there are considerable differences. Only in the range from 0 to 0.1 cps the patterns show similar shape. For larger frequencies the influence of the crust causes appreciable deformation of the radiation patterns.

4. THE INFLUENCE OF THE RADIATION PATTERN ON THE AMPLITUDE-DISTANCE FUNCTION

Radiation patterns cannot be observed directly. What we do observe are amplitudes as a function of epicentral distances. To understand how the amplitude distance function is effected by the radiation pattern, especially

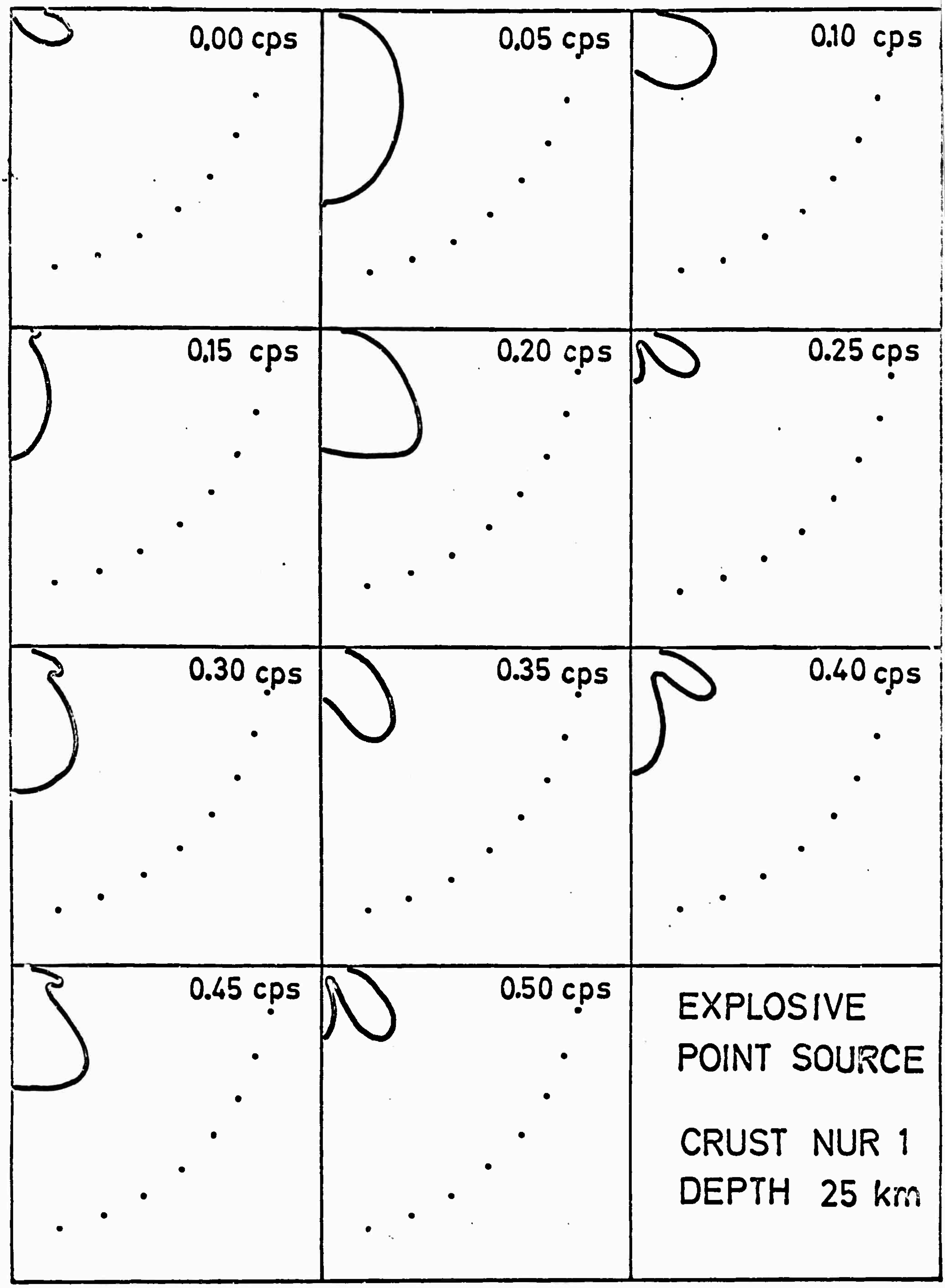


Fig.6: Radiation patterns for the buried explosive point source in the crust NUR 1 (Table 1) at depth 25 km (ν = 0.0 to 0.5 cps)

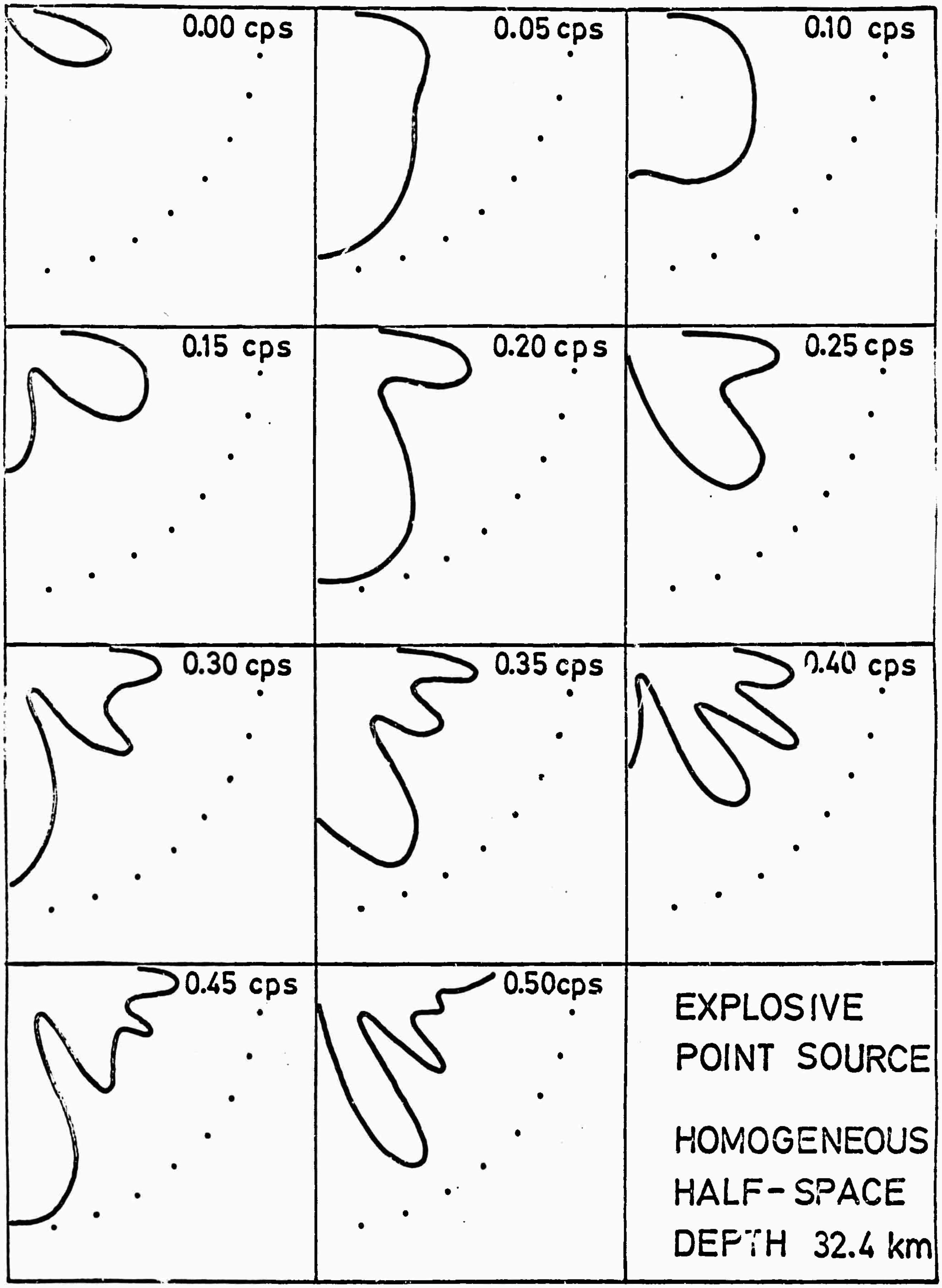


Fig.7: Radiation patterns for the buried explosive point source in the homogeneous half-space at depth 32.4 km (ν = 0.0 to 0.5 cps)

for waves penetrating the upper mantle, we have converted the angle of incidence γ at the base of the crust into epicentral distances Δ. The best model known for the upper mantle in Europe is the one derived by Lehmann (1959). It is listed in Table 2 and depicted in Figure 8. The same figure also gives the branches of the $T - \Delta$ - diagram of the refracted and overcritically reflected P-waves for this model. Travel times have been reduced with $\Delta/6.0$(km/(km/sec)). The branch A corresponds to the waves refracted between the M-discontinuity at a depth of 35 km and the first order discontinuity at 220 km. Branch B is the overcritical reflection from this discontinuity at a depth of 220 km. Branch C corresponds to rays penetrating the upper mantle at depths larger than 220 kms.

The $\gamma - \Delta$ - curve for this model of the European upper mantle has been computed for a surface focus and is shown in Figure 9. The branches have again been labelled A,B and C using the notation of Figure 7.

This $\gamma - \Delta$ - curve has now been used to convert the radiation patterns $T(\gamma,\nu)$ into an amplitude-distance function $T(\Delta,\nu)$ of epicentral distance Δ and frequency ν. $T(\Delta,\nu)$ does not contain a correction for geometrical spreading and amplitude splitting at first order discontinuities. It is so to speak a projection of the radiation pattern on the free surface of the spherical earth following the ray paths in the upper mantle.

The amplitude-distance function $T(\Delta,\nu)$ has been plotted in Figure 10 for the explosive point source at 500 m depth in the crustal model NUR 1 and for the vertical point force acting on the same crust in Figure 11. The frequency range is 0.0 to 0.5 cps in increments of 0.05 cps.

Depth (km)	Radius (km)	P-velocity (km/sec)
0	6370	6.2
35	6335	6.2
35	6335	8.0
55	6315	8.12
95	6275	8.12
159	6211	8.12
220	6150	8.12
220	6150	8.40
286	6084	8.68
349	6021	8.95
412	5958	9.24
476	5894	9.54
539	5831	9.86
602	5768	10.18
666	5704	10.53
729	5641	10.89
793	5577	11.26

Table 2:

P-wave velocities of the upper mantle in Europe (Lehmann, 1959)

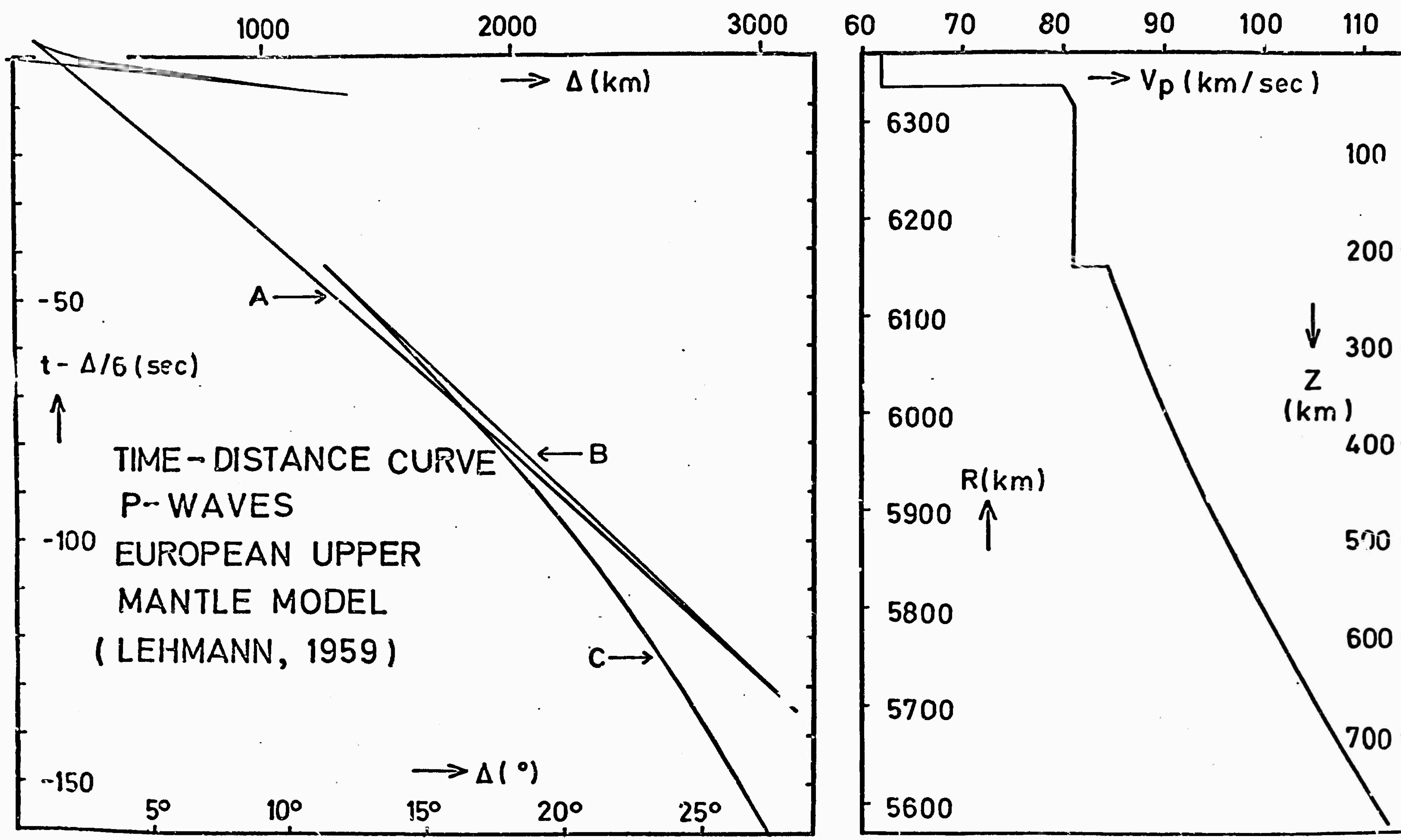


Fig.6: European upper mantle model (Lehmann, 1959) for P-waves and corresponding time-distance curve

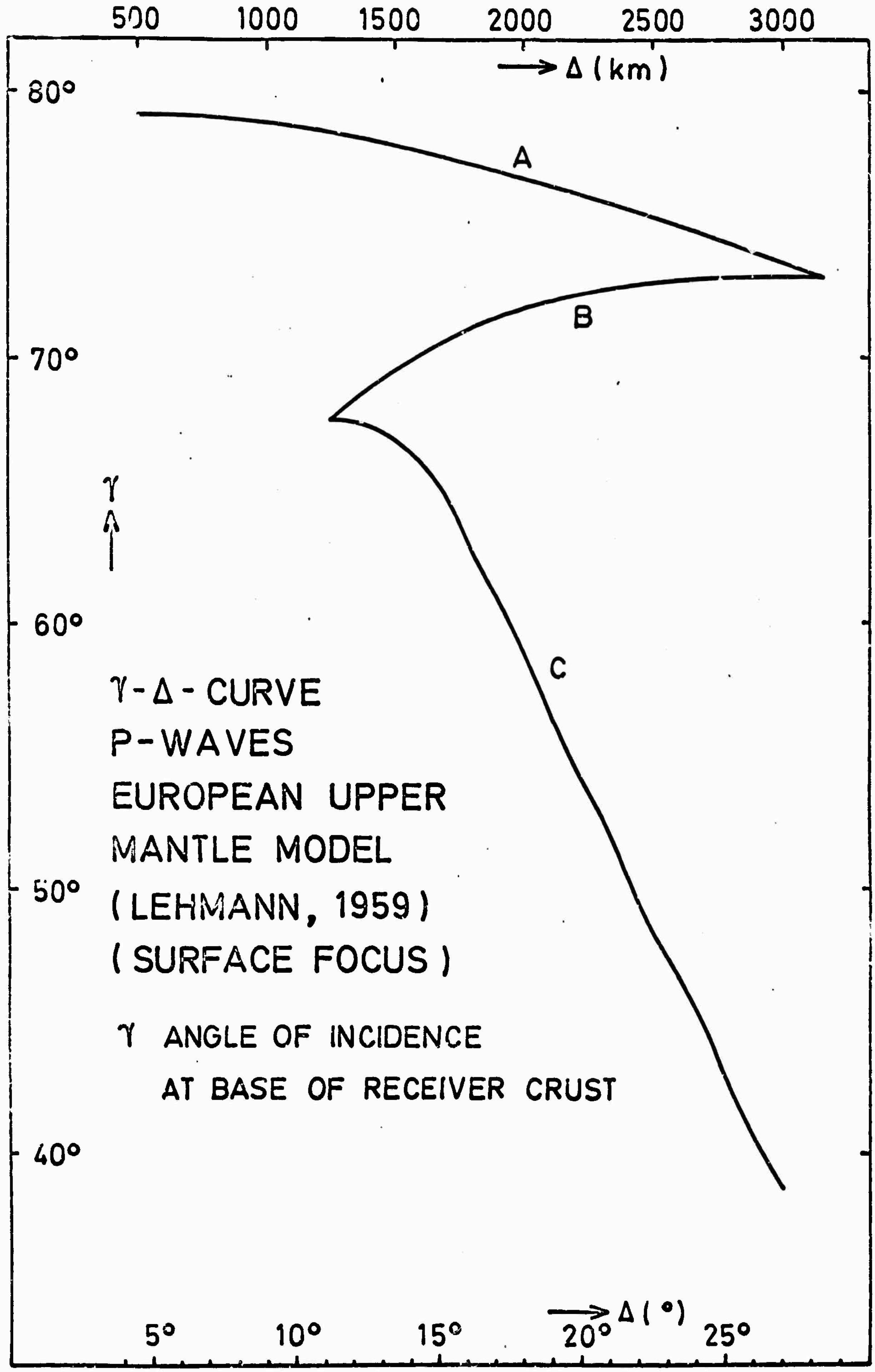


Fig.9: Angle of incidence γ at base of crust to distance Δ relation for European upper mantle model (see figure 8).

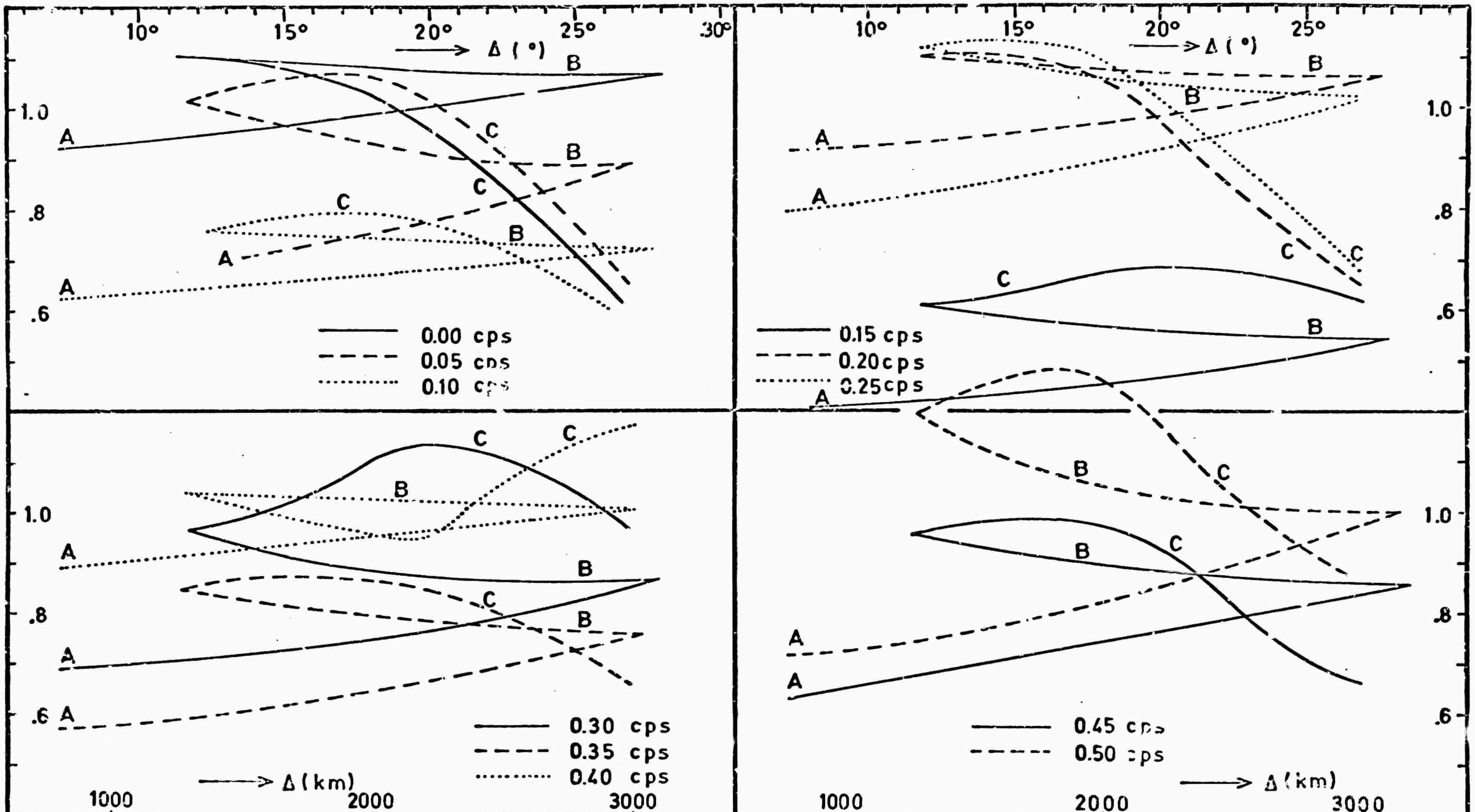


Fig.10: Amplitude-distance function $T(\Delta,\nu)$ from radiation patterns of explosive point source at depth 500 m in crust NUR 1 (compare Figure 4)

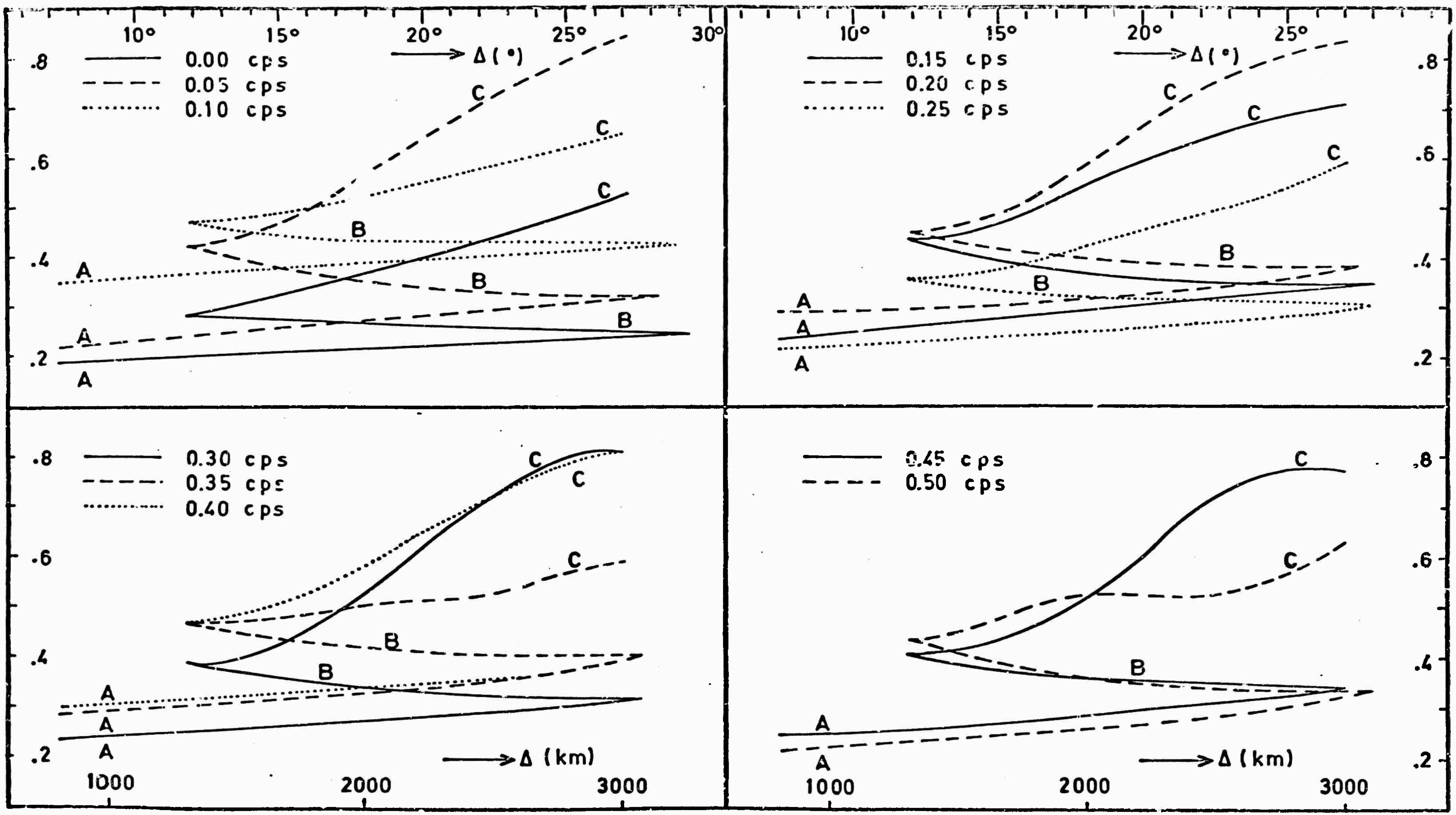


Fig.11: Amplitude-distance function $T(\Delta,\nu)$ from radiation patterns of vertical point source on top of crust NUR 1 (compare Figure 3).

The following points deserve special attention in the case of the explosive source (Figure 9):

1.) Within the frequency range 0 to 0.5 cps the radiation pattern of the explosive source is causing a relative increase of amplitudes with distance along the refracted branch A. This implies that any observed amplitude attenuation along branch A should be even stronger if corrected for the effect of the radiation pattern.

2.) Amplitudes along the branch B are slightly decreased with distance by the radiation pattern.

3.) The most significant effect of the radiation pattern is to be observed along the branch C. The maximum of the radiation pattern occurs on this branch at epicentral distances Δ between 11° and 21°. For most frequencies amplitudes are strongly decreasing beyond this maximum. This is especially true at the following frequencies: 0.0, 0.05, 0.1, 0.2, 0.25, 0.35, 0.45 and 0.5 cps. Only at 0.4 cps amplitudes are increasing with distance. In the case of 0.0, 0.05, 0.2 and 0.25 cps the amplitude decrease amounts to about 40 % between distances $\Delta = 11^{\circ}$ and 27°.

It should be noted that the distance range of this strong decrease of amplitudes coincides with a minimum in experimental amplitude distance curves for P_n and P-waves by Gutenberg, Richter (1956), de Bremaecker (1955) and Vanek, Stelzner (1962). It is possible that this minimum is not caused by geometrical spreading of the ray bundle or non-elastic behavior of the penetrated region but simply by the radiation pattern of the source in the crust. The amplitude-distance function for P-waves radiated from the vertical point source (Figure 10) differs considerably from that of the explosive source. While the behavior of the amplitudes along branches A,B has the same tendency as in the case of the explosive point source - an apparent increase of amplitudes with distance caused by the radiation pattern - the amplitudes along branch C display a strong increase of about 100 % in the distance range Δ from 11° to 27°. Therefore, experimentally determined amplitude-distance function from underground explosions should show stronger attenuation than corresponding functions from atmospheric explosions.

We are planning to compare experimental data from both types of explosions to study the theoretical predicted effect. We shall also investigate how variations of the crustal parameters will influence the amplitude distance function.

5. REFERENCES

DE BREMAECKER, J.Cl., "Use of amplitudes, Part I: P_n from 3° to 23°." Bull.Seism.Soc.Amer., 45, 219-244, 1955.

FUCHS, K., "The transfer function for P-waves for a system consisting of a point source in a layered medium." Bull.Seism.Soc.Amer., 56, 75-108, 1966.

GUTENBERG, B. and C.F. RICHTER, "Magnitude and energy of earthquakes." Ann. Geofisica, 9, 1-15, 1956.

HARKRIDER, D.G., "Surface waves in multilayered elastic media. I. Rayleigh and Love waves from buried sources in a multilayered elastic half-space." Bull. Seism.Soc.Amer., 54, 2, 627-679, 1964.

LEHMANN, I., "Velocities of longitudinal waves in the upper part of the earth's mantle." Ann. de Geophys., 15, 93-118, 1959.

PENTTILÄ, E., "On seismological investigation of crustal structure of Finland." Publ. No. 60, Institute of Seismology, Univ. of Helsinki, 1965.

SCHNEIDER, G., "Die Erdbeben in Baden-Württemberg 1955 - 1962. Veröffentlichung d. Landeserdbebendienstes Baden-Württemberg, Stuttgart, 46 p., 1964.

VANEK, J. and J. STELZNER, "Amplitudenkurven der seismischen Raumwellen." Gerlands Beitr. zur Geophys., 7, 105-119, 1962.

6. APPENDIX

Administrative Information

a) Technical Status

Equipment for the digitization of paper and film seismograms has been installed at our institute during December 1966. At present we are testing the data on the punched paper tape and the transfer to our programs for spectral analysis at the IBM 7094 of the German Computer Center in Darmstadt. The general overhaul of the various short-period seismometers at the Graefenberg Observatory (GGGR) has continued.

b) Major Accomplishments

Our efforts concentrated on a study of the distortion of the spectrum of seismic body waves by the crust in the neighborhood of the source. Radiation patterns and amplitude-distance functions have been computed for the buried explosive source and the vertical point source at the free surface of the crust.

The routine maintenance at the technical installations of the Graefenberg Observatory continued. The systematic measurements of seismic background noise at various places in Germany are still in progress.

c) Problems encountered

The GEOTECH low-speed tape recorders are still out of order. They will be returned to Dallas (Texas) for repair.

d) Actions required by AFOSR and EOAR

No immediate action of AFOSR and EOAR is required at this time.

e) Future plans

Work will continue along the lines of our original project outline. The special study of the effect of radiation patterns on amplitude distance functions will be carried on.

f) Personal and Organization

Since September 1, 1966 Mrs. Bleisch is employed as a half-day secretary at the Graefenberg Observatory.

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13. ABSTRACT

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Radiation patterns have been computed for P-waves in the frequency range from 0 to 0.5 cps. An attempt has been made to separate the effect of the free surface from that of the layered crust in case of the explosive source. The latter effect starts to be noticeable at frequencies above 0.3 cps. Using the model for the upper mantle model in Europe (Lehmann, 1959) the radiation patterns have been transformed into amplitude-distance functions. Implications for the interpretation of experimental amplitude-distance curves of P-waves are discussed.

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Radiation Pattern of Vertical Point Force						
Spectral Behavior of the Source Crust						
Crustal and Upper Mantle Structure in Europe						
Travel Times of Body Waves in Europe						
Angle of Incidence - Epicentral Distance Relation in Europe						
Amplitude-Distance Curves of P-waves						

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