

IITRI Project M6103  
Final Report

DEBRIS FORMATION AND TRANSLATION

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Prepared for  
Department of the Army  
Office of the Secretary of the Army  
Office of Civil Defense  
OCD-PS-64-201 Work Unit 3322B

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IIT Research Institute  
Technology Center  
Chicago, Illinois 60616

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SUMMARY

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by  
Ralph L. Barnett  
James F. Costello  
David I. Feinstein

Prepared for  
Department of the Army  
Office of the Secretary of the Army  
Office of Civil Defense

OCD-PS-64-201  
Work Unit 3322B

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## DEBRIS FORMATION AND TRANSLATION SUMMARY

### INTRODUCTION

This report represents a systematic effort to examine the physical basis for predicting the final location of blast-initiated debris. There are three principal sources of this debris:

- Frangible structural elements, such as masonry wall panels.
- Nonfrangible structural elements, such as building frames and wood or metal siding and roofing.
- Building contents.

The first two categories require a method to predict the loads at which they will come apart and the kinds of pieces into which they will break, or more generally, a method of failure prediction. Chapters Two and Three of this report deal with this problem.

Presuming a knowledge of the failure modes, the important question from a postattack point of view is: how much of these elements end up obstructing the adjacent roadway? More particularly, there is interest in the weight-size-composition, height, and total volume of matter in the desired right-of-way. Chapter Four is concerned with the construction of a computer-oriented model to predict the distribution of "loose particles", that is, structural fragments and building contents. Also in Chapter Four, assorted loose ends are tied up concerning the finer points associated with the transport model.

A summary of the state-of-the-art in debris prediction is shown in Table 1, an examination of which will show that with the results given in this report, the theoretical basis for debris prediction is pretty well covered. However, a few holes still exist. The most noticeable is the restriction of the fragmentation model to homogeneous wall panels. Further modification will be required to be able to handle nonhomogeneous wall panels.

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Since a great number of walls (including those made of brick), are in this category, such an extension would be desirable.

Table 1  
SUMMARY OF DEBRIS-PREDICTION

Debris Source	Method of Failure Prediction	Method of Final Location Prediction
Frangible Structural Elements	Fragmentation Theory	Transport Model
Nonfrangible Structural Elements	Limited Plasticity	Continuity (Frames) Transport Model (Siding and Roofing)
Building Contents	Not Applicable	Transport Model (Plus overturning and sliding analysis for diffraction- sensitive items.)

#### NONFRANGIBLE STRUCTURAL ELEMENTS

Debris resulting from the effects of blast on nonfrangible structural elements, such as beams and columns, seems worthy of consideration in any attempt to provide meaningful inputs for postattack recovery planning. This follows from the fact that while elements of this sort have a smaller volume of potential debris than frangible ones, the resulting "particles" will be larger, more cumbersome, and hence, more demanding, pound for pound, in any clean-up effort. With this motivation, we have striven to develop an analytical procedure capable of predicting the size and weight distribution of the debris deposited, in a nuclear blast environment, by elements which have some ductility. Such elements will be denoted as nonfrangible to distinguish them from frangible (or brittle) ones, such as

unreinforced wall panels, which have no capacity to absorb energy beyond their yield points.

For all practical purposes, the load-response behavior of nonfrangible structural elements can be divided into two categories, based on the plastic regions of their stress-strain diagrams. The response is either sufficiently ductile to allow the use of an elastic-perfectly plastic model or the amount of strain that can be accommodated is limited, requiring a "limited plasticity" model. The former case, which has been thoroughly investigated over the last twenty years, is generally applicable to steel-framed structures. The latter case, which is appropriate for reinforced-concrete structures, was considered and the effect of the limited ductility was demonstrated.

Finally, a small series of experiments on model frames was devised to check the validity of the limited-plasticity model and verify the hypothesis that any energy supplied to a frame in excess of that necessary to cause collapse is taken up by rotations of the plastic hinges to the extent of their capacities and acceleration of the mechanism, rather than in secondary damage between hinges. The information gained from this series of experiments was qualitative in nature.

Some conclusions about the utility of the theories and techniques demonstrated are:

- The limited-plasticity theory provides a realistic approach for predicting blast-induced debris from nonfrangible structural elements in a manner which is consistent with, and indeed an extension of, design procedures.
- Recourse to modern computer-oriented analysis techniques overcomes the prohibitive computational complexity which heretofore has inhibited applications of limited plasticity.

- Models of reinforced-concrete structures, constructed at low cost from inexpensive materials, can be used to provide meaningful answers to questions about debris production which characteristically involve gross behavior such as the collapse mode.

### FRAGMENTATION OF FRANGIBLE STRUCTURAL ELEMENTS

The frangible plate structure represents a significant debris producing element in the form of wall panels and a vital source of dangerous missiles in the form of plate glass. The fragmentation characteristics of such structures are studied in this section using a pragmatic approach which blends results from statistical fracture theory with those recently obtained by IITRI on an experimental study of dynamically-loaded plaster plates (Ref. 1). The work extends the considerations of two previous investigations on beam fragmentation to the plate (Ref. 2 and 3).

The general fragmentation algorithm consists of four steps:

- Determine the maximum dynamic stresses throughout the plate.
- Compute the probability of fracture initiation throughout the plate.
- Divide the plate into appropriate regions based on crack propagation.
- Compute the distribution of fragment "sizes."

Three computational procedures are described for determining the distribution of fragment sizes. Each of these methods begins by dividing a plate into regions or strips formed by the principal stress trajectories. These strips independently fracture or remain intact and the combination of fracture and

nonfracture determines the geometry and number of fragments. The first computation scheme, the combination method, considers individually each of the possible  $2^n$  combinations of failure and nonfailure of the strips where  $n$  is the total number of strips. This method provides the specific description and quantity of every possible fragment, and in addition, it details the various possible mixtures of large and small fragments. It unfortunately, is very time consuming even with the aid of very large computers.

If we are not interested in how the various fragments are mixed together, we can adopt a very efficient procedure called the fragment group method, for calculating the total number of every possible type of fragment. Here, there are only  $(n/2)(n+1)$  combinations of fragment groups to be considered. Although the increased efficiency of the method of fragment groups is considerable, an even faster method can be used if we again settle for less information. The final method, called the method of runs, determines the number of identical contiguous nonfractured strips. It will not furnish information about fragment geometry; only fragment weights.

#### TRANSPORT MODEL

In order to represent the effect of debris transport and subsequent distribution, it is necessary to move from a problem space consisting of the real world to a more abstract mathematical model. This abstraction consists of representing the initial condition of possible debris as a series of lumped masses at levels above ground. Each lumped mass is characterized by a unique particle size distribution. The particle size, in turn, has weight and shape attributes associated with it. The trajectory model assumes two ideal initial conditions. These are:

- Zero failure time of fragmented elements.
- An initial particle velocity of zero.

These assumptions were made, initially, due to a lack of knowledge concerning any other possible values.

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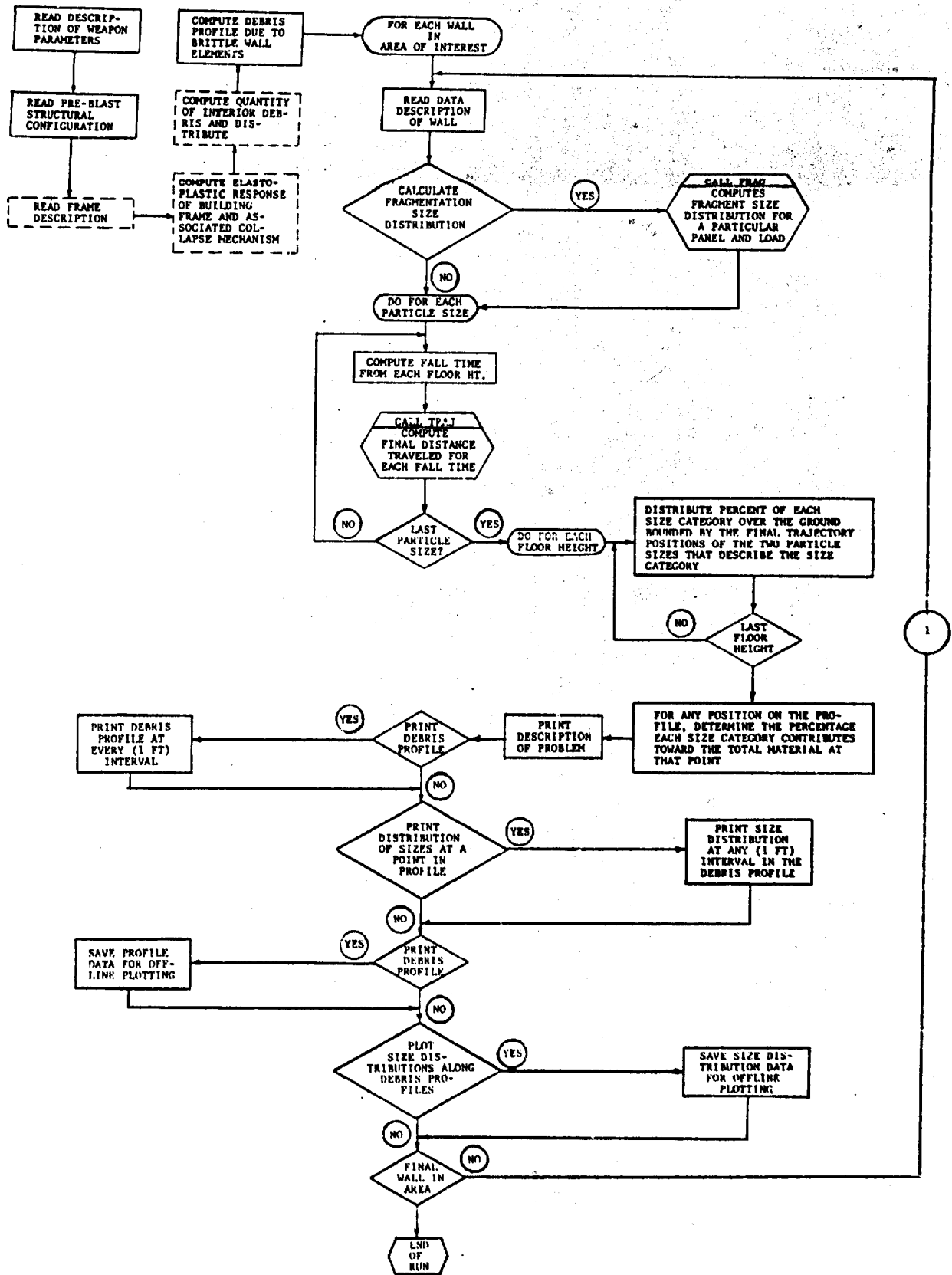


A study concerning these parameters has since been made and is reported. The result of this study indicates that the initial assumptions are well grounded.

SINBAD (Simulation Investigation of Nuclear Blast Associated Debris) is a problem-oriented computer language that deals with the problem of postattack structural debris. In a previous investigation ( Ref.3 ) debris profile curves (i.e., height of debris versus distance thrown) were developed for a free-standing masonry panel wall. Several analyses, both manual and computerized, were utilized to predict the profile of a single wall. The present study is a refinement of the previous techniques and is extended to include any grouping of walls subjected to a frontal shock. It is now also possible to determine the size distribution and a measure of the momentum of the debris at any point in the profile. The language is expandable and in its entirety will include frame response as well as the interior contents of the structure. The flow diagram indicates the general computational scheme. The boxes that are now dotted are components that will be added to the system at a later time.

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1. Liber, T., Experimental Study of Fragmentation of Structural Wall Panels, for OCD, Contract No. OCD-PS-64-50, October 1966.
2. Ahlers, E. B., Debris Clearance Study, OCD Contract No. OCD-OS-62-202, Subtask 3322A; IIT Research Institute Project No. M264, September 1963.
3. Feinstein, D. I., Debris Distribution, Task 3322B for Office of Civil Defense, Washington, D.C., August 1965.



COMPUTATIONAL FLOW GRAPH FOR SINBAD

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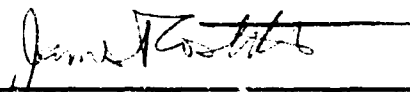
FOREWORD


This is the Final Report on the research performed under Subcontract No. B-70942(4949 A-34)-US, "Debris Formation and Translation". The major topics investigated were:

- Debris generated by nonfrangible structural elements.
- Fragmentation of plate-type elements.
- Trajectory of debris particles.

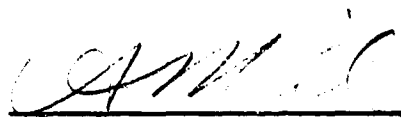
Limited investigations were also performed on selected topics relevant to debris prediction.

Respectfully submitted,

  
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\_\_\_\_\_  
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APPROVED:

  
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Assistant Director of Research  
Mechanics Research Division

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ABSTRACT

A comprehensive view is taken of the physical models required to estimate volumes and heights of blast-initiated debris. Particular emphasis and development is directed toward three areas: the fragmentation of frangible elements, the failure of elements with limited ductility, and the transport of debris particles by blast winds. Computer programs to handle the computations involved in these three models have been written.

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CHAPTER ONE  
INTRODUCTION

1.1 PERSPECTIVE OF DEBRIS PREDICTION

This report represents a systematic effort to examine the physical basis for predicting the final location of blast-initiated debris. There are three principal sources of this debris:

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The first two categories require a method to predict the loads at which they will come apart and the kinds of pieces into which they will break, or more generally, a method of failure prediction. Chapters Two and Three of this report deal with this problem.

Presuming a knowledge of the failure modes, the important question from a postattack point of view is: how much of these elements end up obstructing the adjacent roadway? More particularly, there is interest in the weight-size-composition, height, and total volume of matter in the desired right-of-way. Chapter Four is concerned with the construction of a computer-oriented model to predict the distribution of "loose particles", that is, structural fragments and building contents. Also in Chapter Four, assorted loose ends are tied up concerning the finer points associated with the transport model.

A summary of the state-of-the-art in debris prediction is shown in Table 1, an examination of which will show that with the results given in this report, the theoretical basis for

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Building Contents	Not Applicable	Transport Model (Plus overturning and sliding analysis for diffraction- sensitive items.)

debris prediction is pretty well covered. However, a few holes still exist. The most noticeable is the restriction of the fragmentation model to homogeneous wall panels. Further modification will be required to be able to handle nonhomogeneous walls. Since there are a great number of walls in this category, which includes those made of brick, such an extension would be desirable.

CHAPTER TWO  
DEBRIS GENERATED BY NONFRANGIBLE STRUCTURAL ELEMENTS

2.1 INTRODUCTION

Debris resulting from the effects of blast on nonfrangible structural elements, such as beams and columns, seems worthy of consideration in any attempt to provide meaningful inputs for postattack recovery planning. This follows from the fact that while elements of this sort have a smaller volume of potential debris than frangible ones, the resulting "particles" will be larger, more cumbersome, and hence, more demanding, pound for pound, in any clean-up effort. With this motivation, we have striven to develop an analytical procedure capable of predicting the size and weight distribution of the debris deposited, in a nuclear blast environment, by elements which have some ductility. Such elements will be denoted as nonfrangible to distinguish them from frangible (or brittle) ones, such as unreinforced wall panels, which have no capacity to absorb energy beyond their yield points.

For all practical purposes, the load-response behavior of nonfrangible structural elements can be divided into two categories, based on the plastic regions of their stress-strain diagrams. This distinction is shown in Fig. 1 for a bending member where moment corresponds to stress and rotation to strain. The response is either sufficiently ductile to allow the use of an elastic-perfectly plastic model or the amount of strain that can be accommodated is limited, requiring a "limited-plasticity" model. The former case, although rather thoroughly investigated over the last 20 years, is of little interest for debris-prediction purposes. The latter model, however, has considerable applicability (Ref. 1). In the first place, the removal of both metal and wooden siding from building frames can be formulated as a limited plasticity problem since the mode of failure involves both rupture at connections and tearing apart of the

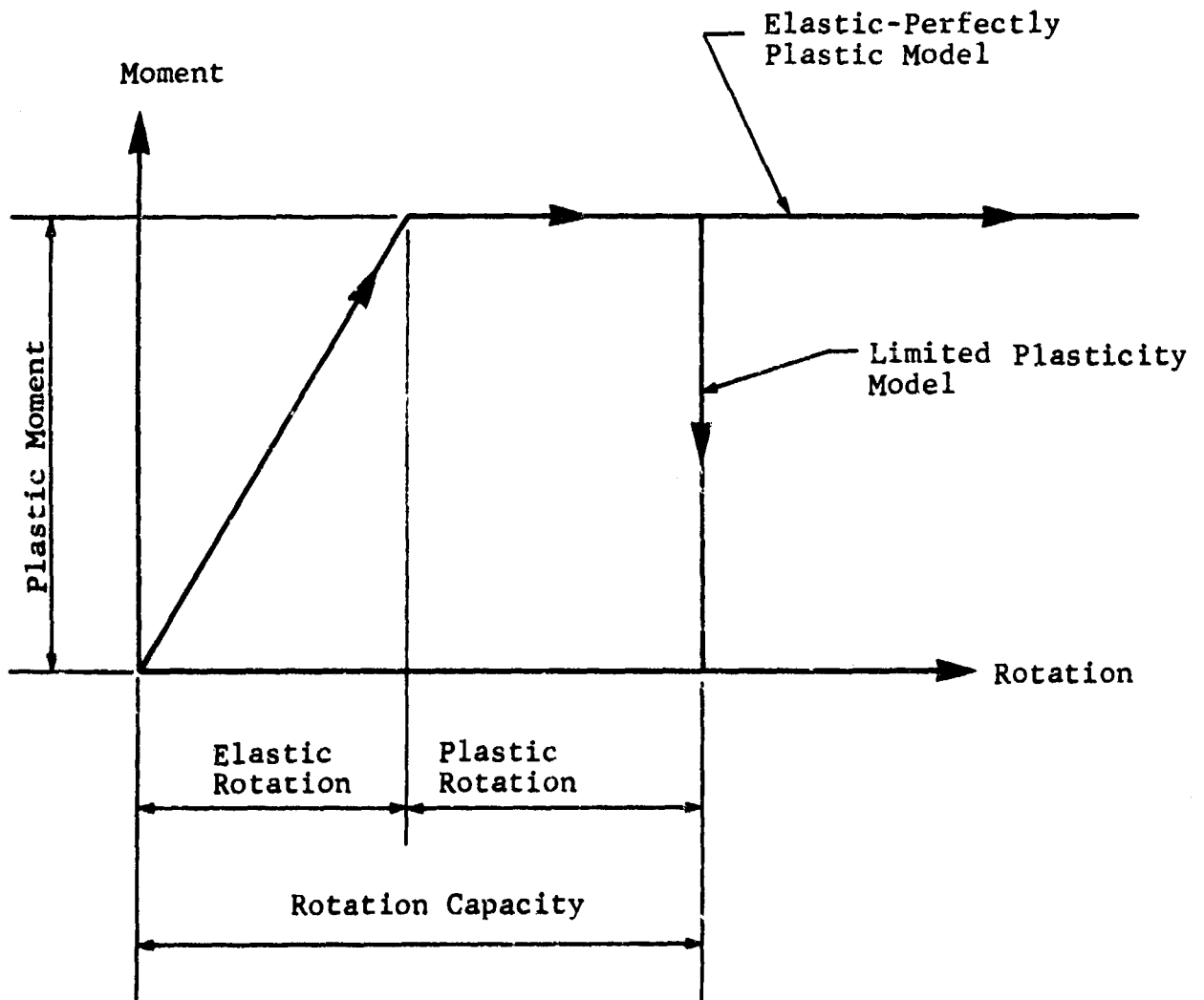


Fig. 1 ELASTIC-PERFECTLY PLASTIC AND LIMITED ROTATION MODELS

panels after large deformations. This aspect was not treated in detail in this portion of the research effort, but rather, attention was focused on the more difficult application to the collapse of reinforced concrete frames. However, the goal of this study was an ability to predict the locations on a reinforced concrete frame at which the inelastic rotations will be sufficiently large for the reinforcing steel to be exposed. The steel can be cut at these points and the structure dismantled. This corresponds to a first (or ready-made) level of debris clearance. Attempts to break the structure into smaller pieces will require chipping away the concrete in order to sever the reinforcement.

## 2.2 LIMITED PLASTICITY OF FRAMES

The behavior of reinforced concrete frames is still quite a controversial subject (Ref. 2). One point of view insists that for multistory frames in particular, the loss of stiffness due to the beam-column effect must be considered in ultimate load calculations and is supported by experiments on model frameworks (Ref. 3 through 6). However, an analysis of this sort neglects the support given by walls and floors and is bound to be overly conservative when applied to complete buildings. Even in an examination of blast-load effects on framed buildings, where the walls are considered to have been removed by the diffraction loading, beam-column effects are important only in tall, slender frames. We will concentrate on the simpler theory which is applicable to the great majority of buildings. Another basic item of contention is the choice of a model to represent the flexural behavior of reinforced concrete. One side demands that "strain-softening" (i.e., resistance increasing to a maximum and then decreasing smoothly as the deformation increases) be included in the model. (Consult the papers by Barnard and Rosenblueth in Ref. 2). The other, and preponderant, viewpoint expressed in the paper of Baker and Amarakone, also in Ref. 2, adopts a limited-plastic model of

the type shown in Fig. 1. We have gone along with the majority in using the straight-line model for limited plasticity. The reasoning on our part was simple since we are concerned with the plastic moment, rotation capacity, and energy absorption (which is the area under the curve). Thus, whatever the exact shape of the moment-rotation curve may be, a la Bernard and Rosenbleuth, we can pick three straight-line segments which will match those salient characteristics. (This is done at the expense of accuracy in the "elastic rotation" which we do not care about.)

Once the moment at a section becomes equal to the plastic moment, a "plastic hinge" is formed and the rotation increases at constant moment. If we postulate a limited rotation capacity, the behavior beyond that amount is like a "real hinge" and rotation increases, but no moment is transmitted across the section. Clearly then, if the rotation at a point in a loaded structure exceeds the capacity at that point, the ultimate load which can be carried will be the same as would be indicated by an analysis of a modified structure that had a real hinge at the point in question. Moreover, and of greater interest from the debris removal aspect, the hinge pattern will differ in general from that found under an assumption of unlimited rotation capacity.

In order to have the ability to assess rapidly the magnitudes of the inelastic rotations encountered in a large framed structure, a computational method, first suggested by Wang (Ref. 7) as a limit analysis procedure, was programmed for IIT Research Institute (IITRI) 7094 computer. Basically, the approach is to perform a sequence of elastic analyses on the structure. Consider a given structure and loading pattern. If an elastic analysis is performed, and the location of maximum moment determined, the load factor can be adjusted to cause a plastic hinge to form at that point. Then, after adjusting the moments at all nodes in accordance with this load factor, the remaining moment resistances can be found. Next, an elastic analysis

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performed on a structure which is identical with the original one (except for a pin inserted at the location of the plastic hinge) will indicate the node having the maximum moment. A load factor can then be found which will induce a moment at that point equal to its remaining moment resistance, implying the formation of a plastic hinge. This process is repeated until a collapse mechanism is formed. The sum of the load factors for all cycles is the ultimate load factor.

This method may seem roundabout, and perhaps it is, but it is well suited for the exceptionally efficient computer solutions utilizing matrix algebra, since the modifications can be performed automatically during the analysis. Also, the inelastic rotations can be computed at each stage, permitting inclusion of the effect of limited rotation capacity. The basis for the calculations is the well-known deflection method, where, using matrix notation, the member end-rotations,  $\{\phi\}$ , and moments  $\{M\}$ , are vectors related by the stiffness matrix  $S$ ,

$$\{M\} = S\{\phi\} \quad (1)$$

The external forces,  $\{P\}$ , are related to the end moments by the beam and bent equations,

$$\{P\} = A\{M\}. \quad (2)$$

It follows that the external displacements,  $\{X\}$ , are related to the end rotations by

$$\{\phi\} = A^T\{X\}, \text{ where } A^T \text{ is the transpose of } A. \quad (3)$$

The procedure for solution is to use Eqs. (1), (2), and (3) to solve for the displacements,

$$\{X\} = [A S A^T]^{-1} \{P\}, \quad (4)$$

and then to compute the end moments by

$$\{M\} = [S A^T] \{X\}. \quad (5)$$

If the stiffness matrix of the original structure is designated as  $S_0$ , the end rotations at the end of the first cycle and corresponding to the formation of the first plastic hinge, computed by either Eq. (1) or (3) as

$$\{\phi\} = [S_0]^{-1} \{M\} \quad (6)$$

will be identical. After subsequent cycles, during which the stiffness matrix of the structure has been modified, the inelastic rotation at the nodes H will be given by the difference

$$\{H\} = [S_0]^{-1} \{M\} - A^T \{X\}. \quad (7)$$

A listing of the FORTRAN IV computer program is given in Appendix A.

### 2.3 RESULTS OF FRAME STUDIES

A limited-plasticity analysis of a framed structure can give results which will differ from those of a standard limit analysis in three areas:

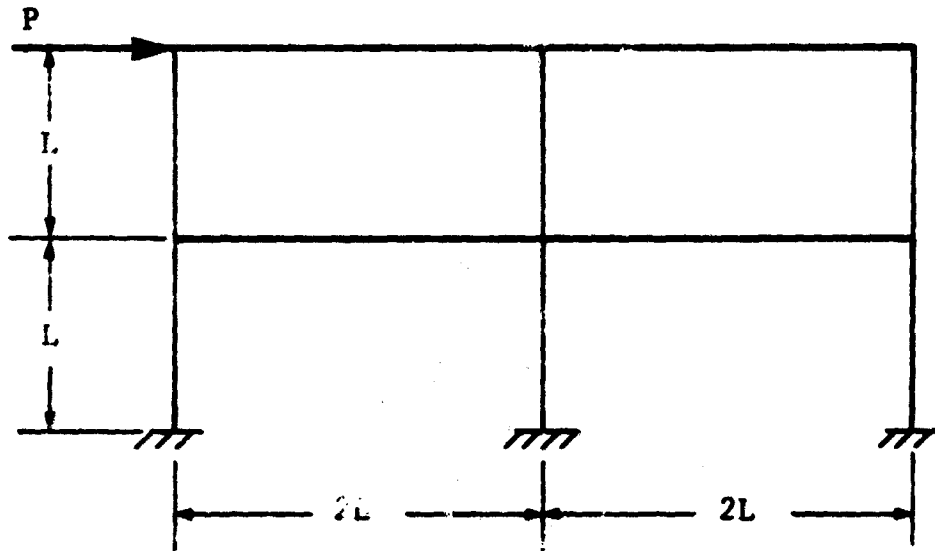
- the ultimate load carried,
- the total energy absorbed, and
- the final collapse mode.

Two example problems were run to demonstrate these disparities. The first example demonstrates the reductions in both ultimate load and energy-absorption due to limited rotation capacity. The frame analyzed is shown in Fig. 2 along with the notation consistent with the computer program. As can be seen in Fig. 3, an elastic-perfectly plastic (i.e., "limit") analysis will indicate collapse at a load factor of 3000. Now, for purposes of

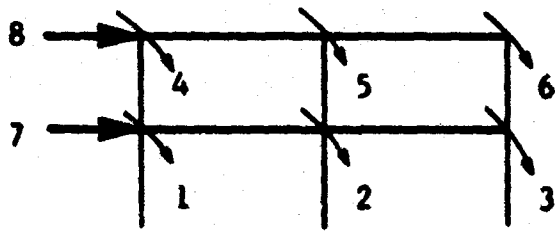
$L = 10$

All members have:  $M_p = 6 \times 10^3$

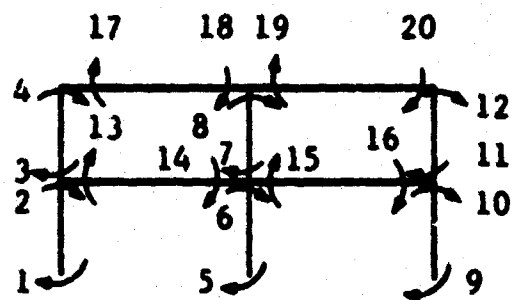
$EI = 18 \times 10^5$



(a) Dimensions and Loading



(b) External Force -  
Displacement  
Notation



(c) Internal Moment -  
Rotation  
Notation

FIG. 2 FRAME USED IN SAMPLE PROBLEM

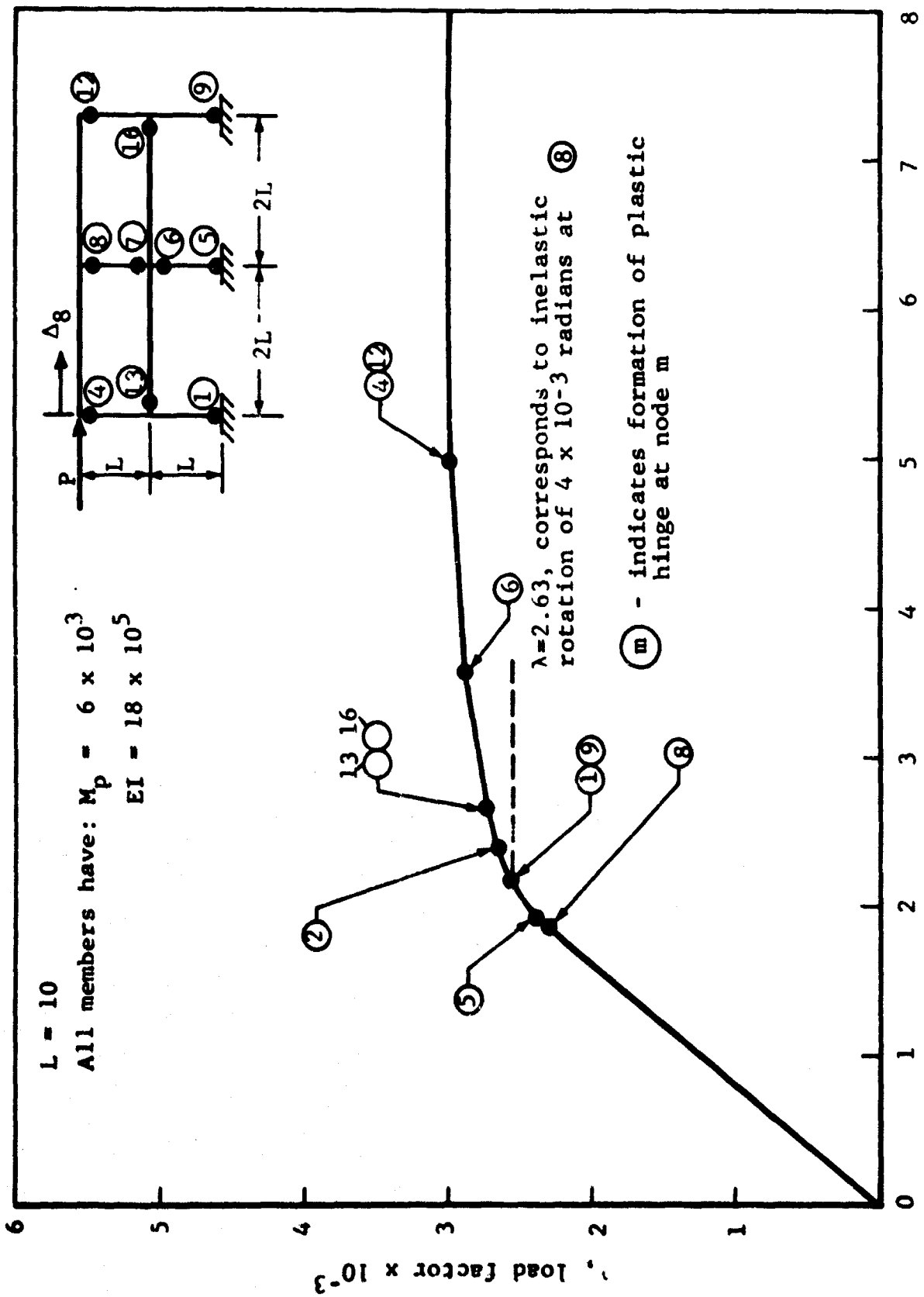


Fig. 3 ELASTIC-PLASTIC ANALYSIS OF ORIGINAL FRAME

illustration, say that the inelastic rotation capacity at all nodes is 0.004 radian. An examination of the computer output for this problem, which is displayed as Appendix B, will show that with this restriction the inelastic rotation at node 8 becomes critical. A linear interpolation between load factors and hinge rotations allows us to fix the load factor consistent with the rotation constraint at 2630.

To find the energy absorbing capacity of the frame, we consider the load which can be supported by the frame under imposed deformation. When the rotation capacity at node 8 is exceeded, the load which can be sustained is that associated with the same deformation in a frame, identical with the original frame except for a real hinge at node 8. An elastic-perfectly plastic analysis can be run on such a frame and the effects of the rotation constraints found. In this manner, a series of modified frames can be considered and the solid curve shown in Fig. 4 constructed. The area under this curve is a measure of the energy which can be absorbed by the frame in question.

To illustrate the possibility of restrictions on rotation capacity leading to a different collapse mode, the frame shown in Fig. 5, 6, and 7 was analyzed. Since node 16 proved to be critical in this case, it was assumed that its rotation capacity would be exceeded while that of all other nodes would not. The structure was then analyzed with a real hinge inserted at node 16. The results are shown in Fig. 8 and 9. It can be seen that not only is the collapse load lowered, but also the mode of collapse differs, since dead loads are included. In the previous example, since only side-on loads acted, the collapse had to be in a side sway mode.

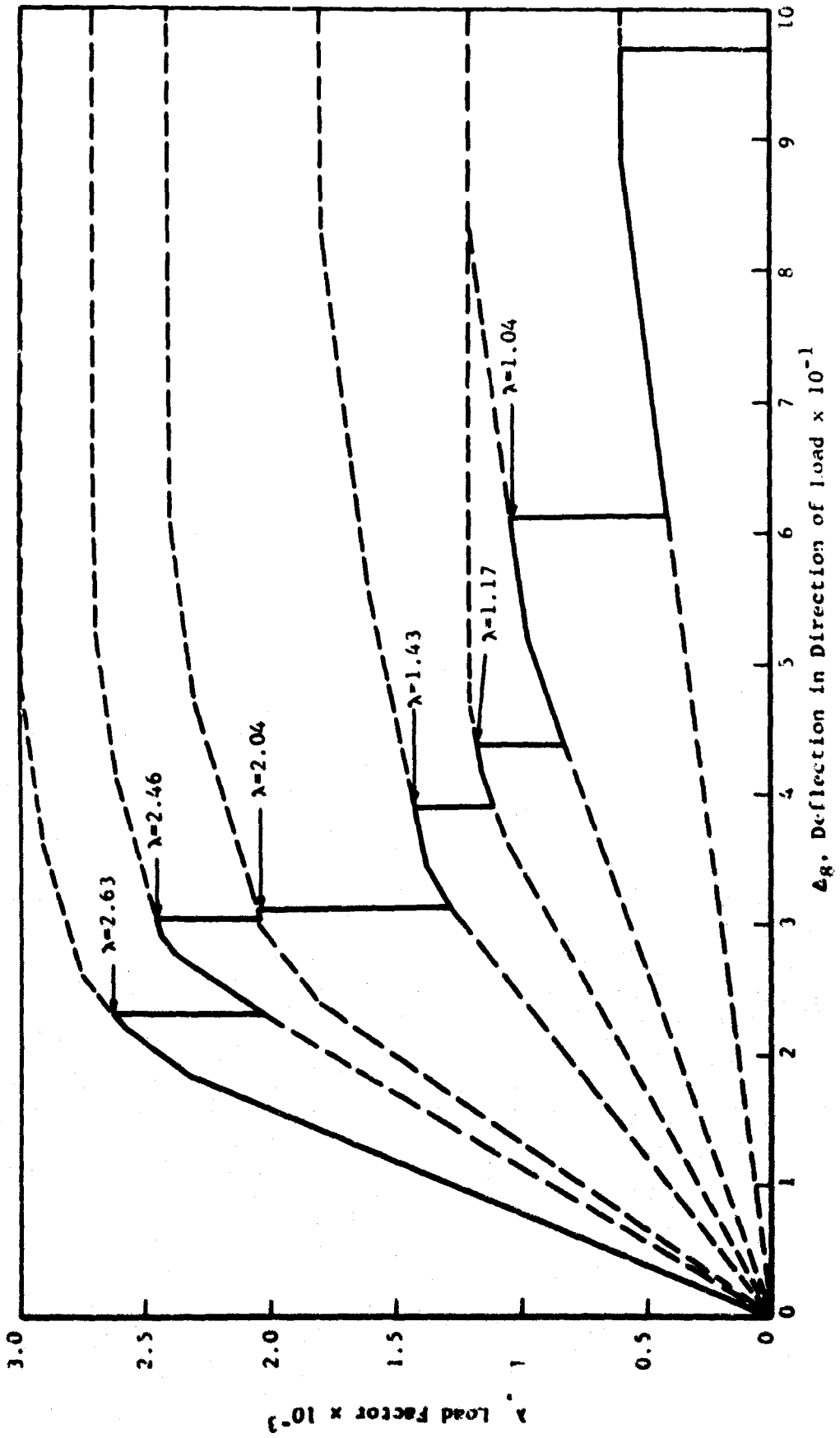
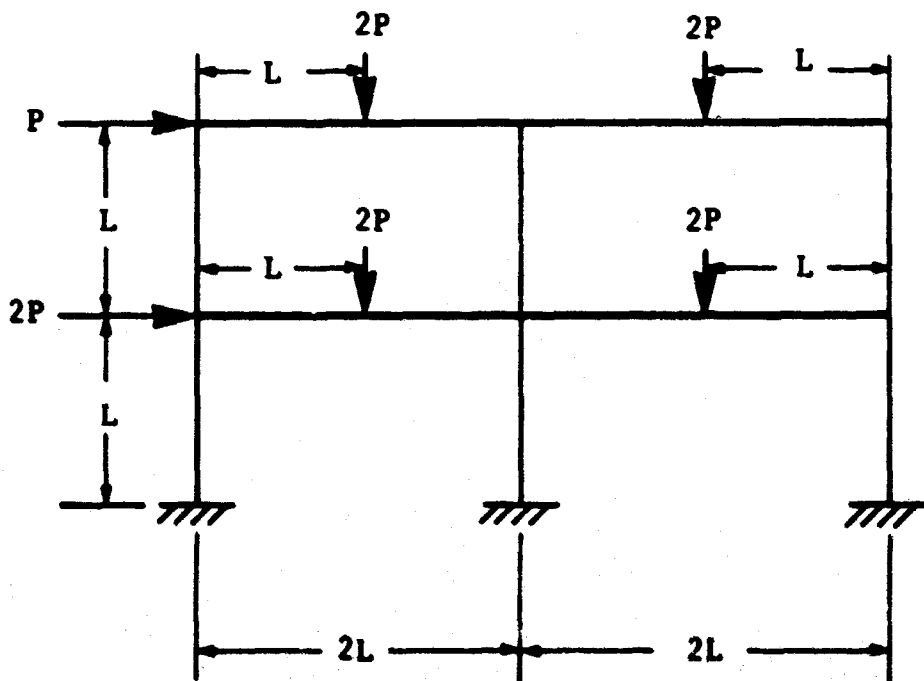


FIG. 4 ENERGY-ABSORBING CAPACITY OF FRAME



All members have the same stiffness,  $EI$ , and plastic moment,  $PM$ .

Fig. 5 LOADING AND GEOMETRY FOR SAMPLE PROBLEM

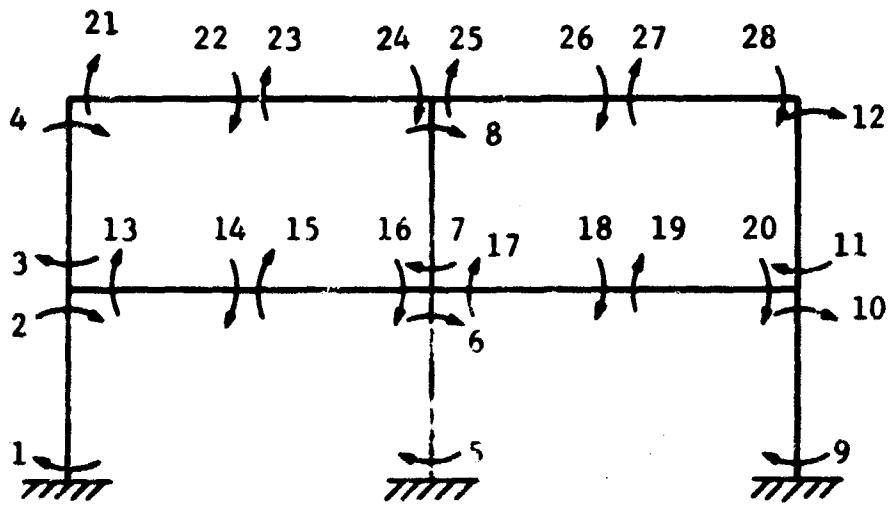


Fig. 6 POSITIVE DIRECTIONS FOR END MOMENTS AND ROTATIONS

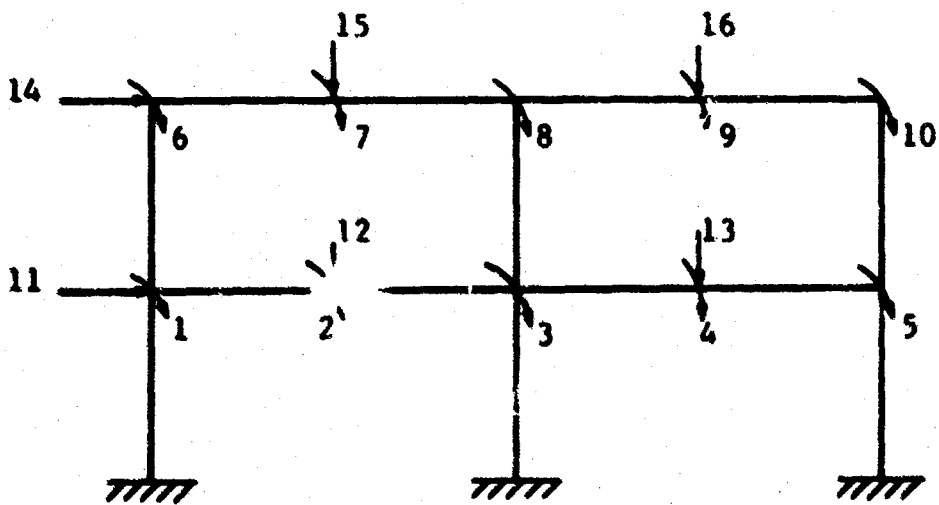


Fig. 7 POSITIVE DIRECTIONS FOR EXTERNAL FORCES AND DISPLACEMENTS



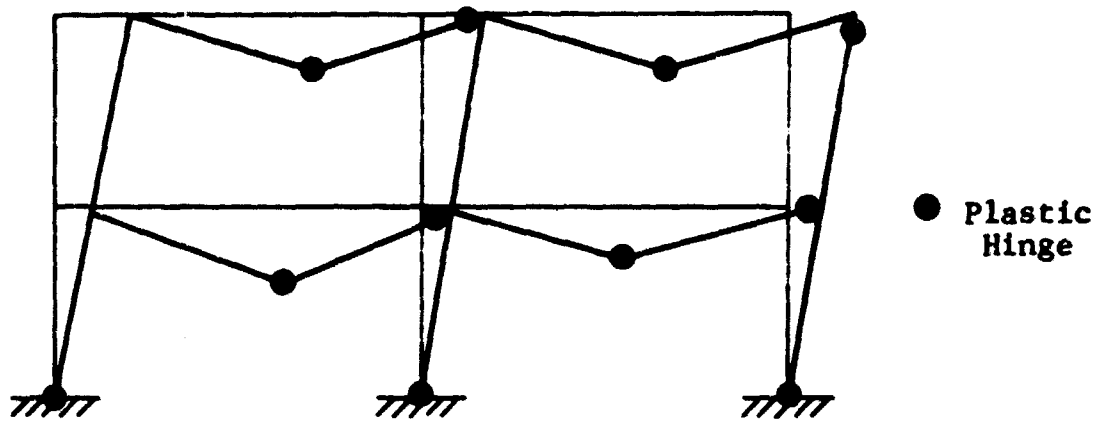


Fig. 8 COLLAPSE MECHANISM FOR CASE NO. 1,  
 $P_u = 950$

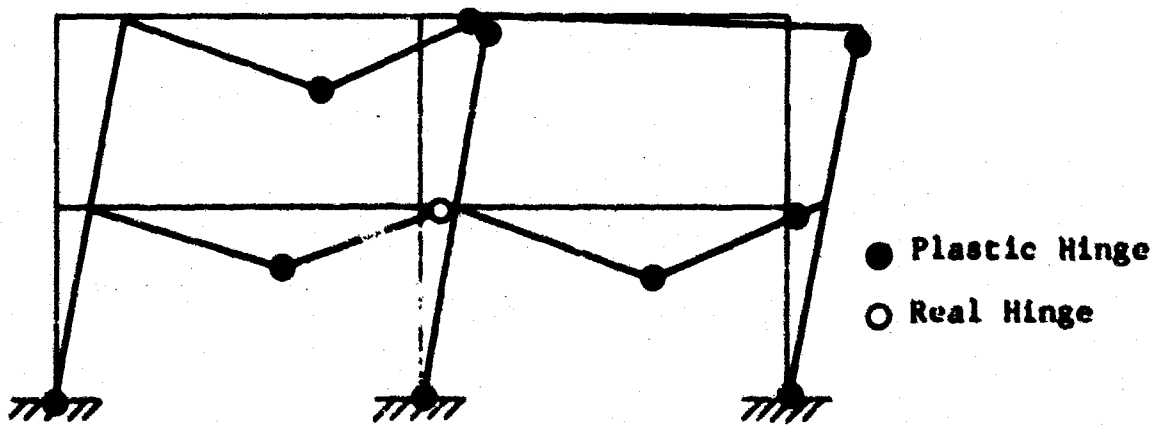


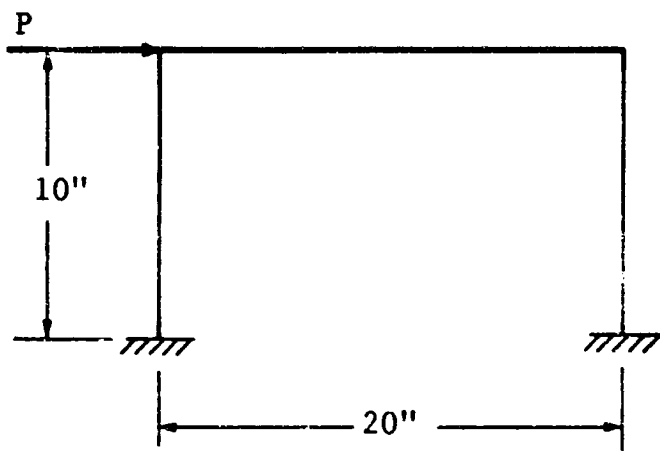
Fig. 9 COLLAPSE MECHANISM FOR CASE NO. 2,  
 $P_u = 840$

## 2.4 QUALITATIVE FRAME EXPERIMENTS

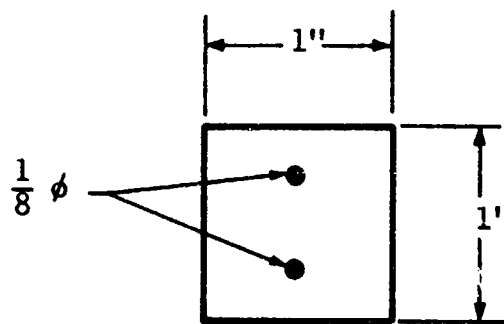
A small series of experiments on model frames was devised to check the validity of the limited-plasticity model and verify the hypothesis that any energy supplied to a frame in excess of that necessary to cause collapse is taken up by rotations of the plastic hinges to the extent of their capacities and acceleration of the mechanism rather than in secondary damage between hinges. The information to be gained from this series of experiments was qualitative in nature.

The geometry of the frames tested is shown in Fig. 10. The materials used were Hydrostone plaster and a soft wire reinforcement. The mold used to cast the frames is displayed in Fig. 11. Due to the small percentage of reinforcement, about 1 percent, the behavior of the frames was governed almost entirely by the reinforcement. Static collapse load predictions are shown in Fig. 12 and the observed collapse loads in Fig. 13 and 14. Since the objectives were qualitative in nature, the static collapse tests were performed in a Riehle testing machine for ease of load application. The fact that the load scale on this machine only permitted readings to the nearest 10 lb was still sufficient to show satisfactory agreement between prediction and observation. Further verification of the limited-plasticity theory was gained from the static collapse test on the single-story frame. The history of the failure was as follows:

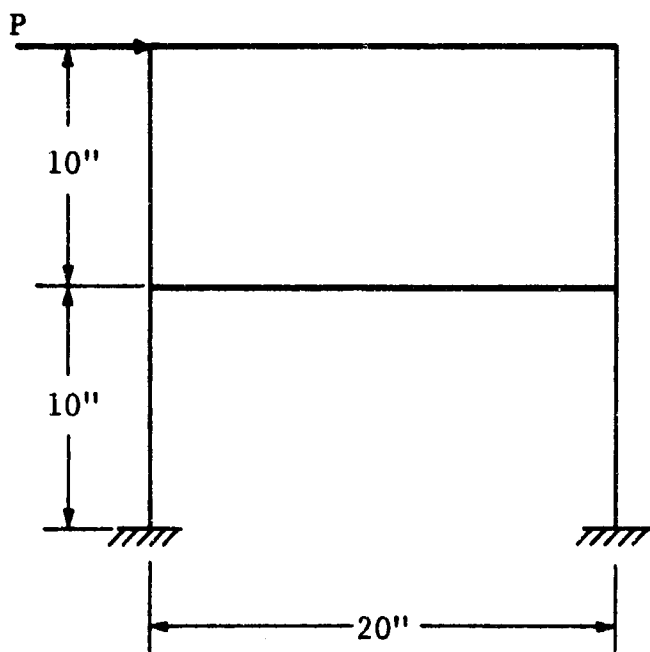
- At a load somewhat below 100 lb, cracks became visible at the column bases.
- Deformation continued without increase in load at about 100 lb.
- As deformation increased, the load fell suddenly to about 50 lb.
- After further deformation at this level, the load fell to zero.



Single-Story Frame



Cross Section  
(uniform)



Two-Story Frame

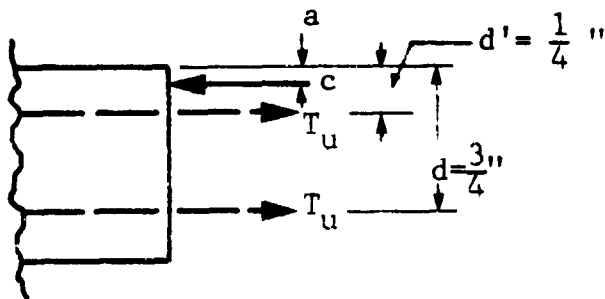
Fig. 10 TEST FRAMES



Fig. 11 MOLD FOR MODEL FRAMES

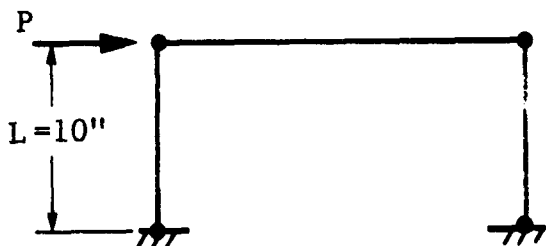
$$M_p = T_u(d + d') - ca = T_u(d' + d)$$

Since the moment due to the compressive force is negligible.

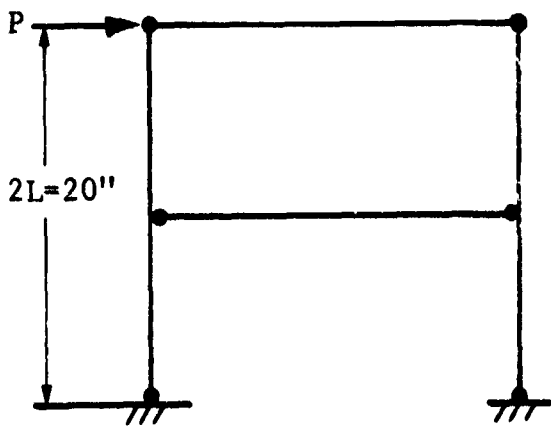


Test data indicate  $T_u = 240\#$

$$\therefore M_p = 240 \left( \frac{1}{4} + \frac{3}{4} \right) = 240 \text{ in.}\#$$



$$P_u = \frac{4M_p}{L} = \frac{4(240)}{10} = 96\#$$



$$P_u = \frac{3M_p}{L} = \frac{3(240)}{10} = 72\#$$

Fig. 12 STATIC COLLAPSE LOAD PREDICTION

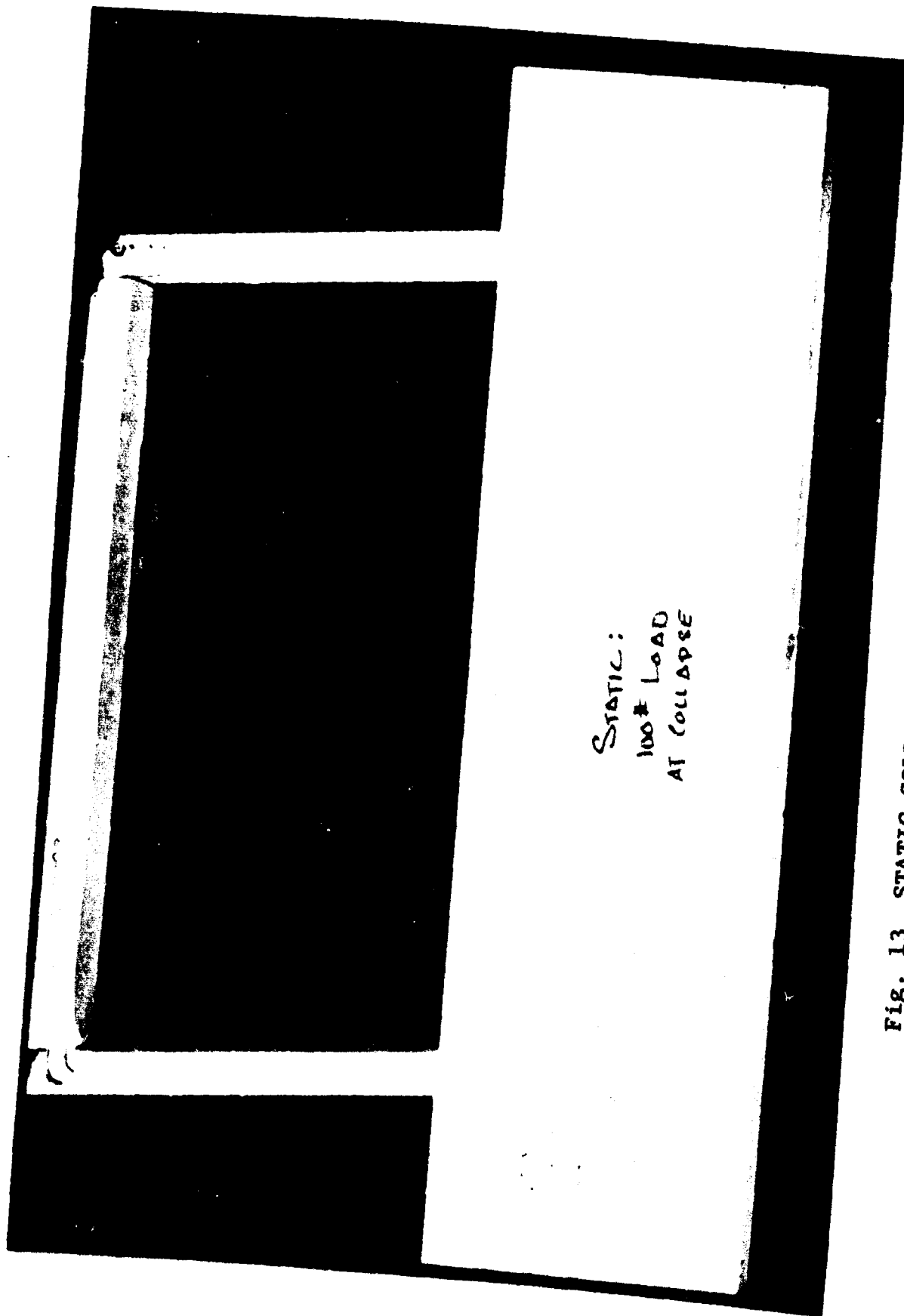


FIG. 13 STATIC COLLAPSE, SINGLE-STORY FRAME

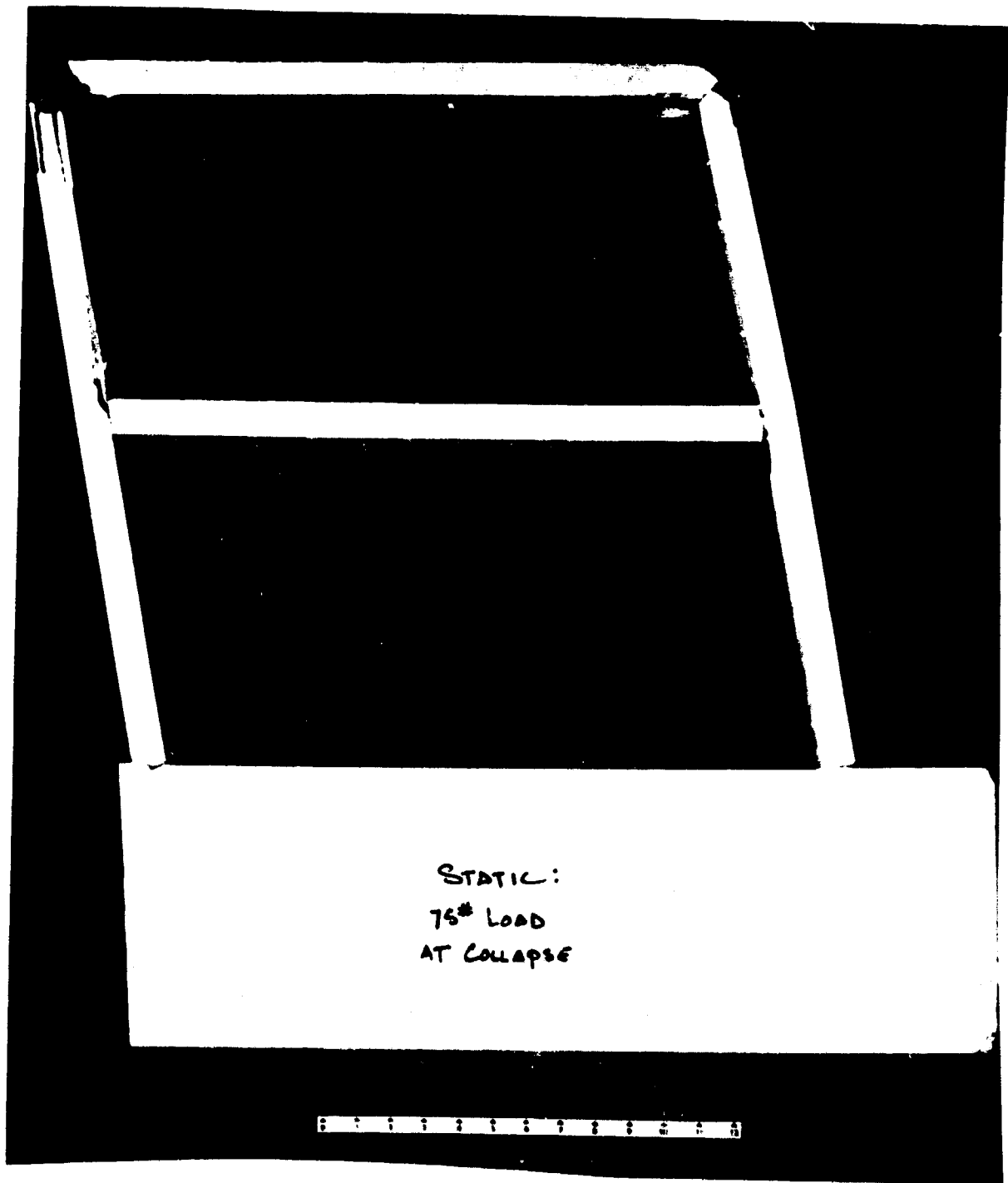


Fig. 14 STATIC COLLAPSE, TWO-STORY FRAME

This sequence of events is consistent with a limited-plasticity formulation shown in Fig. 15.

Finally, dynamic collapse tests were performed on both single-story and two-story frames. Since the behavior of frames in a high-yield blast environment is, for all intents and purposes, solely dependent on response to drag loadings of durations much greater than the natural period of the structure, a dynamic loading fixture was devised to produce a load pulse as shown in Fig. 16. Loads, both slightly greater than the observed static collapse loads and more than twice as much, were applied in this fashion. The collapse modes and amounts of damage at the hinges were comparable in all cases, as was predicted. The responses of the four frames tested under impact are shown in Fig. 17 through 20.

## 2.5 CONCLUSIONS

Some conclusions about the utility of the theories and techniques demonstrated in this chapter are appropriate:

- The limited-plasticity theory provides a realistic approach for predicting blast-induced debris from nonfrangible structural elements in a manner which is consistent with, and indeed an extension of, design procedures.
- Recourse to modern computer-oriented analysis techniques overcomes the prohibitive computational complexity which heretofore has inhibited applications of limited plasticity.
- Models of reinforced-concrete structures, constructed at low cost from inexpensive materials, can be used to provide meaningful answers to questions about debris production which characteristically involve gross behavior such as the collapse mode.

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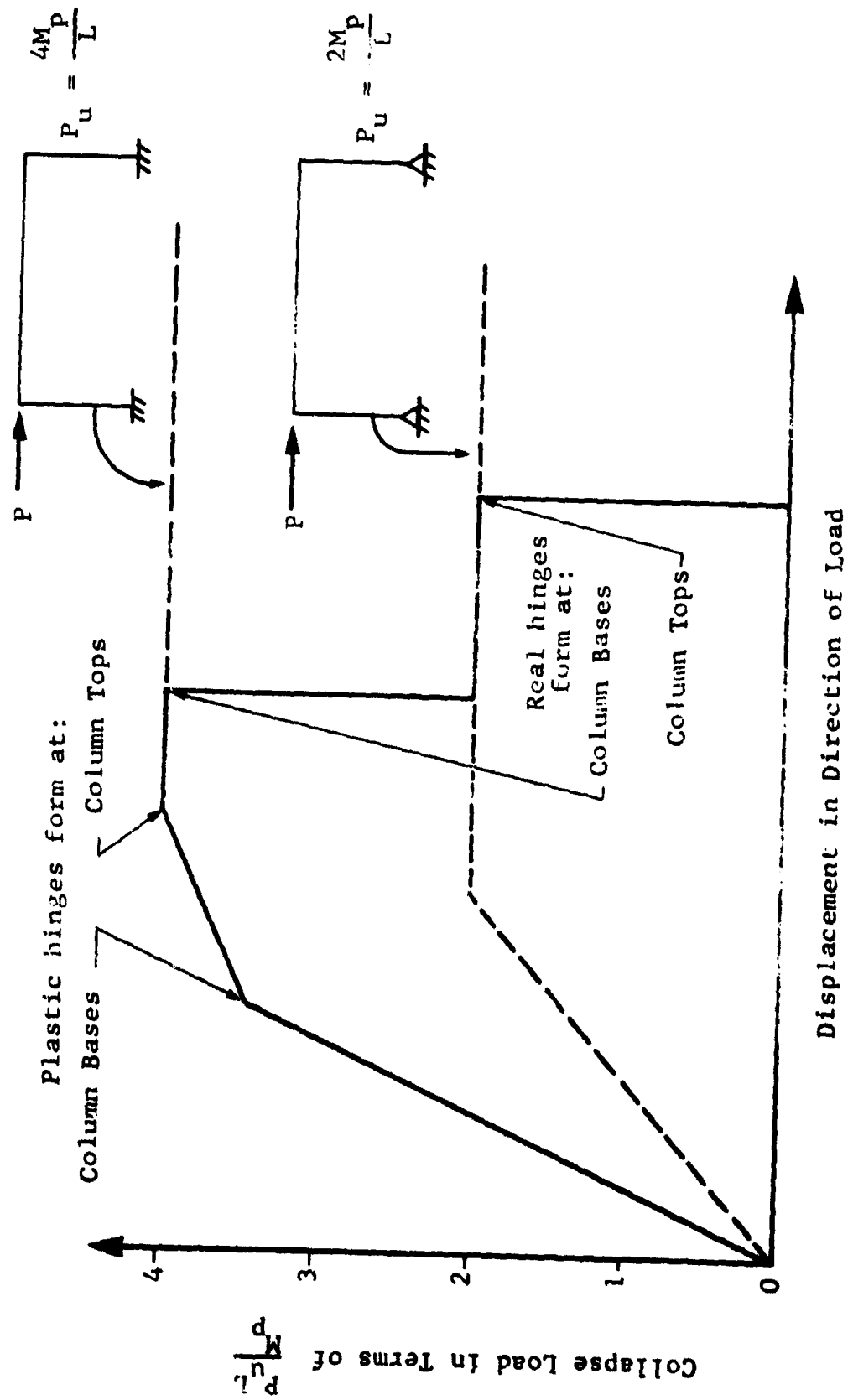


Fig. 15 LIMITED-PLASTICITY BEHAVIOR OF TEST FRAME

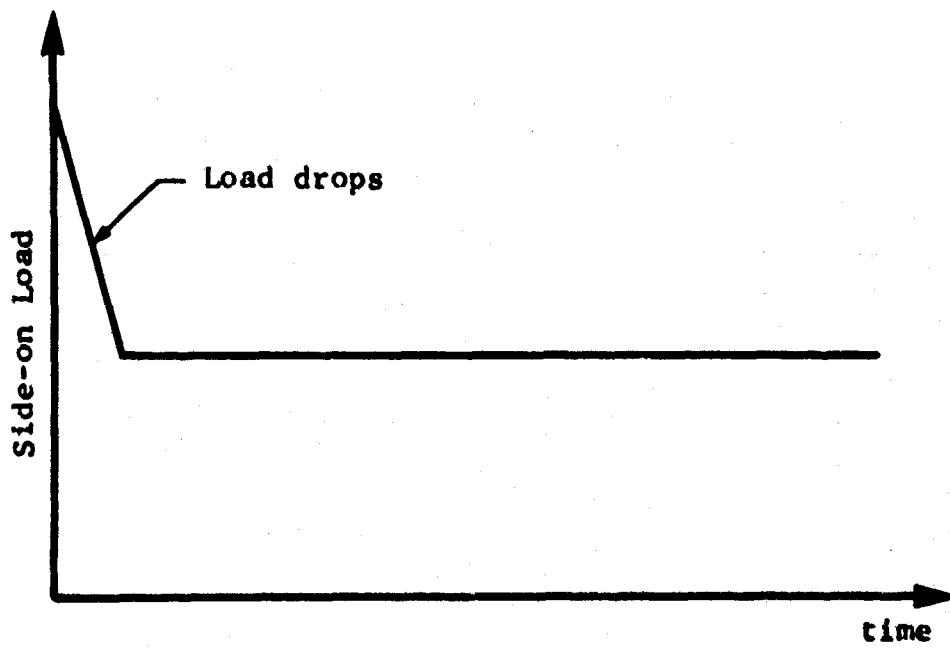
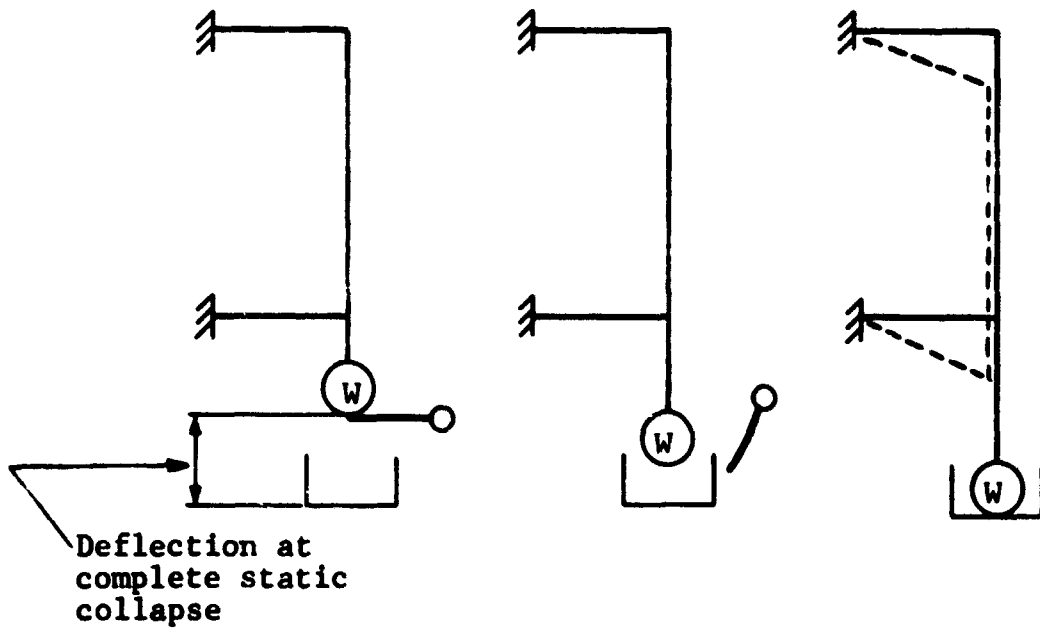


Fig. 16 DYNAMIC LOADING OF TEST FRAMES

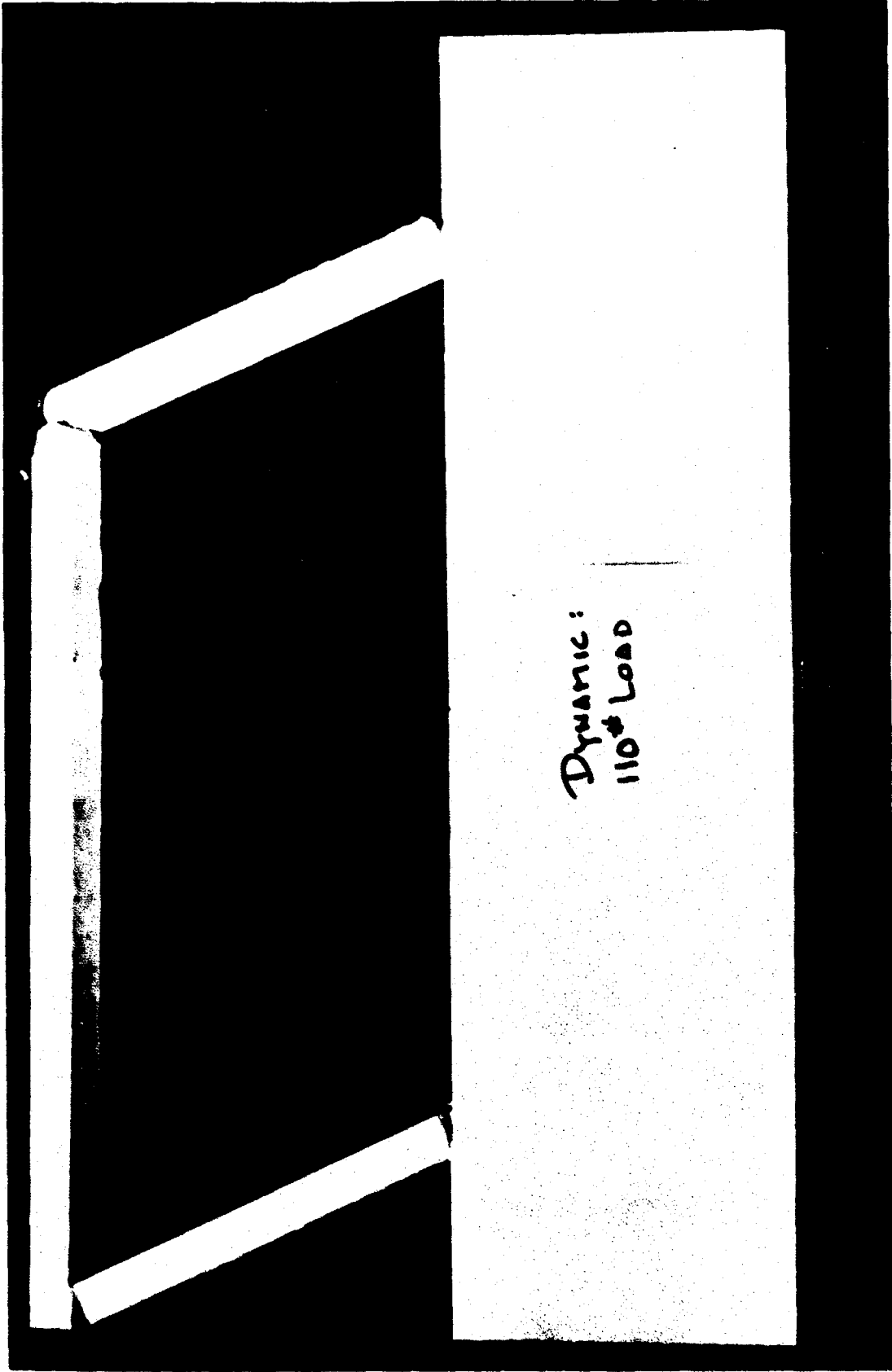


FIG. 17 DYNAMIC COLLAPSE, SINGLE-STORY FRAME

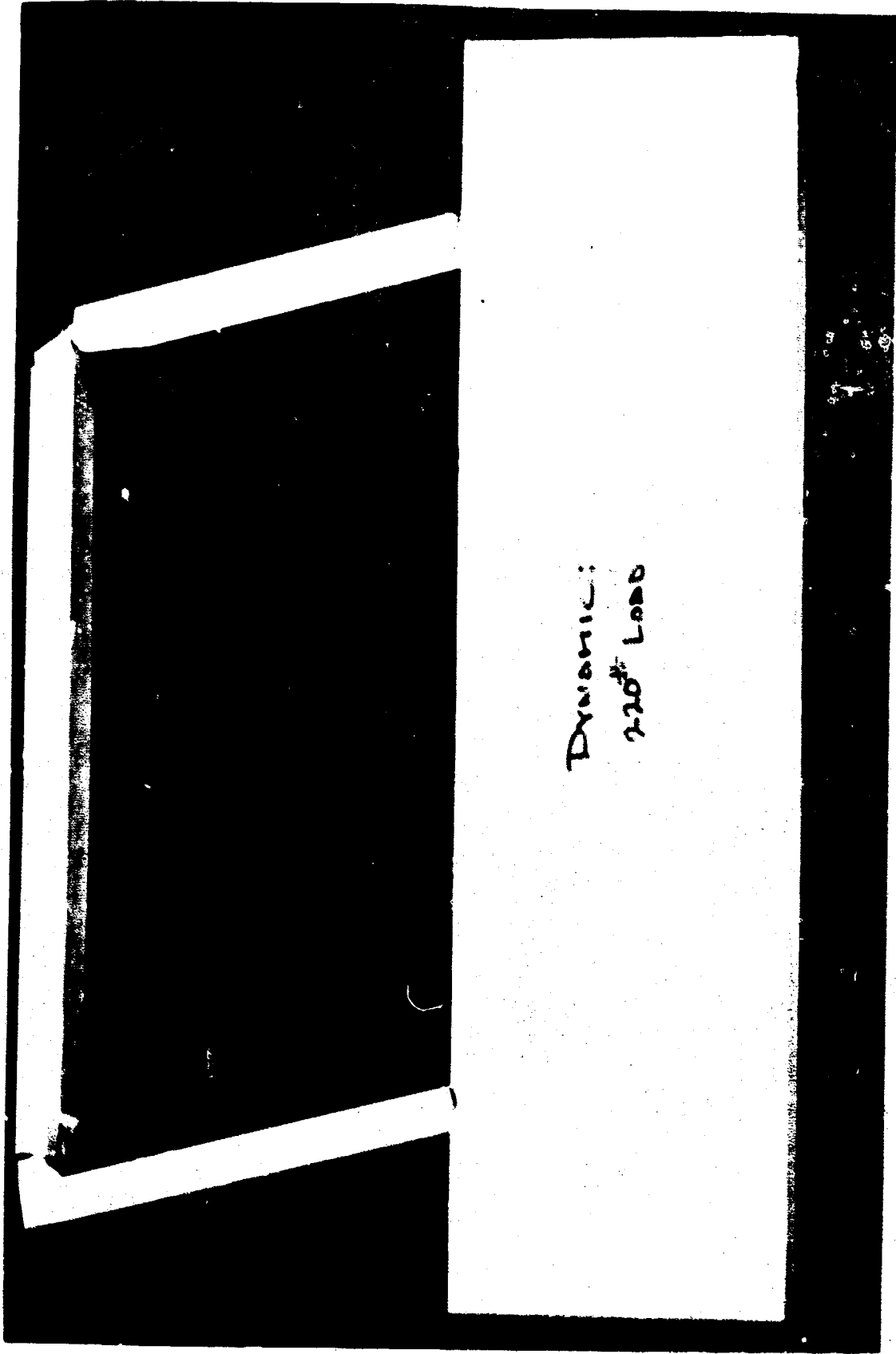


FIG. 18 DYNAMIC COLLAPSE, SINGLE-STORY FRAME

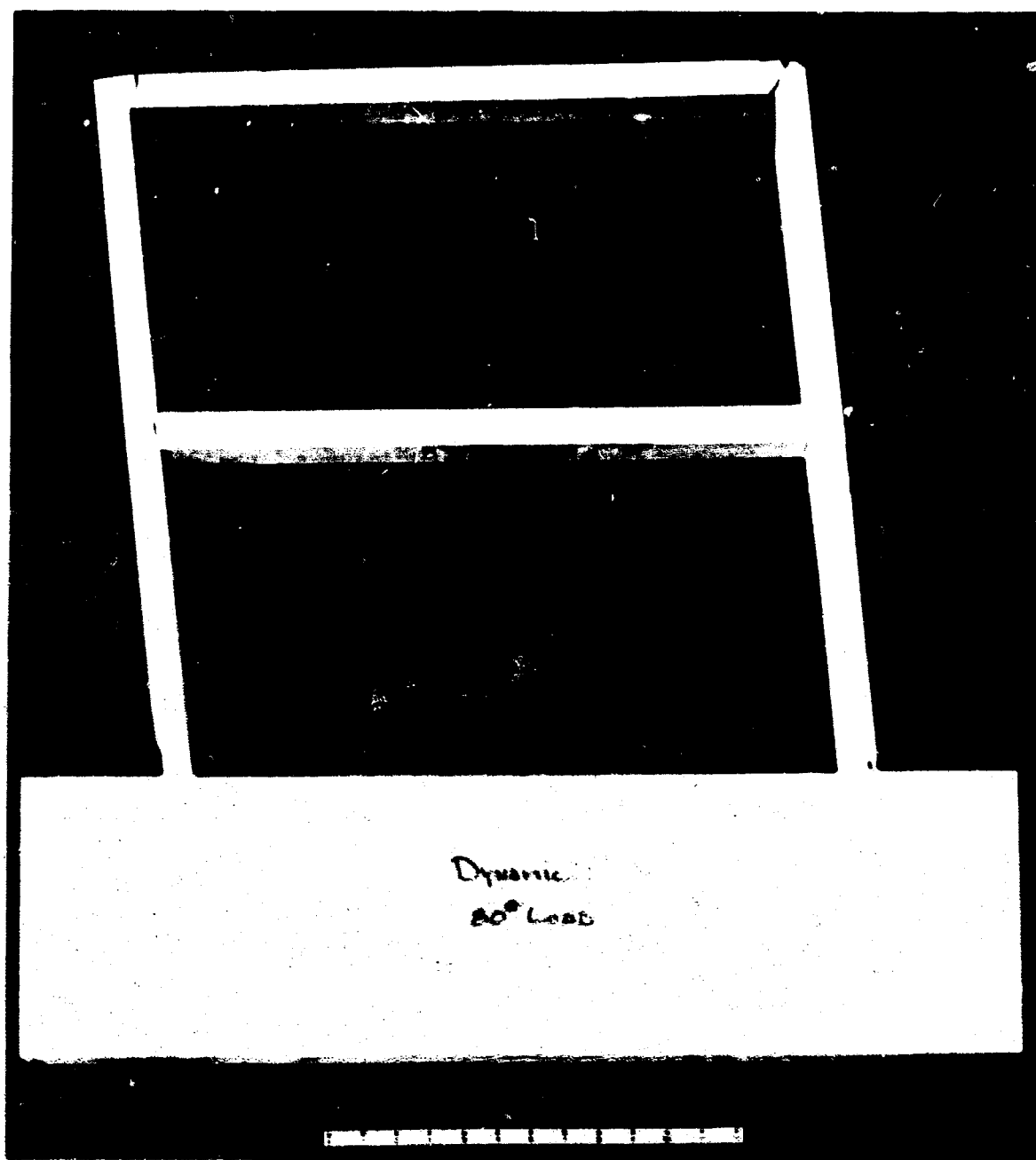


Fig. 19 DYNAMIC COLLAPSE, TWO-STORY FRAME

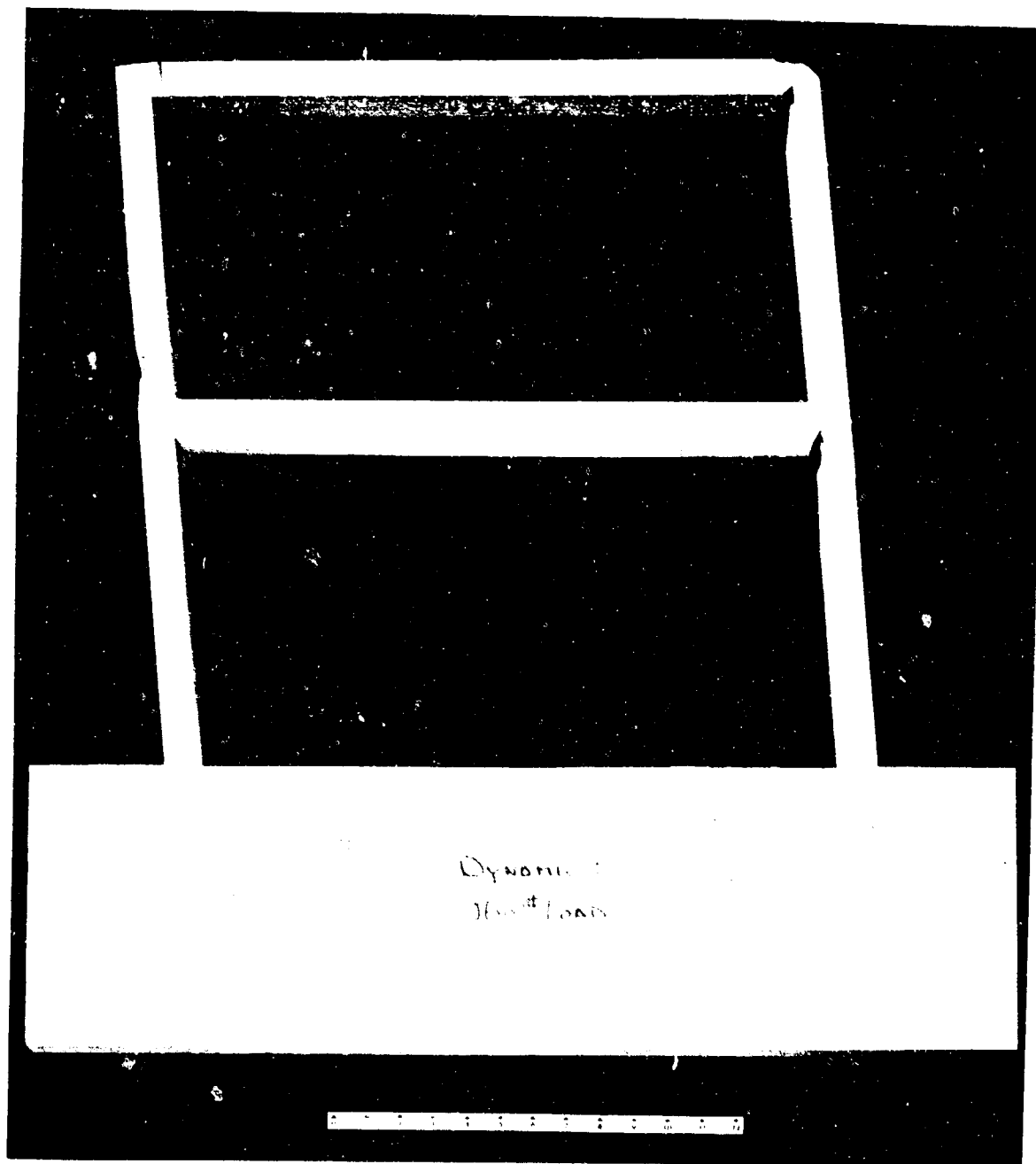


Fig. 20 DYNAMIC COLLAPSE, TWO-STORY FRAME

## CHAPTER THREE

### PLATE FRAGMENTATION

#### 3.1 INTRODUCTION

The frangible plate structure represents a significant debris producing element in the form of wall panels and a vital source of dangerous missiles in the form of plate glass. The fragmentation characteristics of such structures are studied in this section using a pragmatic approach which blends results from statistical fracture theory with those recently obtained by IITRI on an experimental study of dynamically-loaded plaster plates (Ref. 8). The work we shall describe extends the considerations of two previous programs on beam fragmentation to the plate (Ref. 9 and 10).

In the first of these programs, the statistical nature of the problem is established together with the physical assumptions underlying the basic computational scheme. Essentially, the method considers separately every possible combination of crack patterns. As such, it provides a description of the distribution of fragment shapes and masses, and in addition, it can be used to characterize the mixture of different fragments. Unfortunately, the computational time for this program is very great even for large computers. In the second beam fragmentation program, a very efficient and rapid computation method related to the theory of runs was proposed which described only the fragment size distribution - the original locations of the fragments cannot be determined nor are they required for beam response. As we shall see, this additional information may be useful for describing the fragmentation of plates.

The general fragmentation algorithm consists of four steps:

- Determine the maximum dynamic stresses throughout the plate.

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- Compute the probability of fracture initiation throughout the plate.
- Divide the plate into appropriate regions based on crack propagation.
- Compute the distribution of fragment "sizes."

Each of these steps is discussed in the following subsections.

### 3.2 DYNAMIC STRESS ANALYSIS

To decide whether or not fracture will initiate at a point in a dynamically loaded plate, we must first know the "worst" stress state that can occur at the point. This is a straightforward determination when no fractures occur throughout the load history. If, on the other hand, fractures do develop during the loading process, the problem is considerably more complicated. Even for a material with a deterministic strength we would have to consider changing boundary conditions, the speed of crack growth, and the direction of crack propagation. For a brittle material with statistically distributed strength, the number of combinations requiring analysis would truly be enormous.

To extricate ourselves from this forbidding prospect, we have introduced the assumption that the maximum dynamic stresses are independent of the fracture characteristics of the structure. The following comments are relevant to this approximation:

1. No experimental evidence has been sought to examine the validity of this assumption for different types of dynamic loading.
2. The unloading that accompanies the first fracture of a slowly loaded statically determinate beam usually precludes a second fracture.



3. Multiple fractures invariably occur on a rapidly loaded statically determinate beam.
4. The more severe the dynamic loading the smaller the fragment size and the greater the number of fragments.
5. Under such an assumption the various possible fracture patterns are stochastically independent.
6. Crack velocity is substantially below the velocity of elastic disturbances.
7. The actual stress magnitudes in a structure will usually be equal to or lower than those computed for a dynamically equivalent plate with infinite strength. This implies that we will experience fewer crack initiations and larger pieces than we might predict.

Consistent with our principal assumption of independence, i.e. 5 (above), we shall proceed to calculate the maximum stresses occurring in a rectangular simply-supported plate subject to uniform load across its surface but varying in time. The coordinate system and plate dimensions are shown in Fig. 21. Conventional small deformation theory is used and the plate is assumed to be homogeneous and isotropic.

As a specific example, we have chosen a simply-supported rectangular plate with an exponentially decaying load,  $q = p_0 \exp[-\mu t]$ . The initial velocities and displacements are taken to be zero. The deflection for such a plate is described in Section 9.5 of Ref. 11 where their general deflection expression, Eq. (8), can be specialized by taking  $q = p_0 \exp(-\mu t)$  and  $f = g = 0$ . Then, using Eq. (10) of this reference, we obtain after a simple integration the required plate deflection:

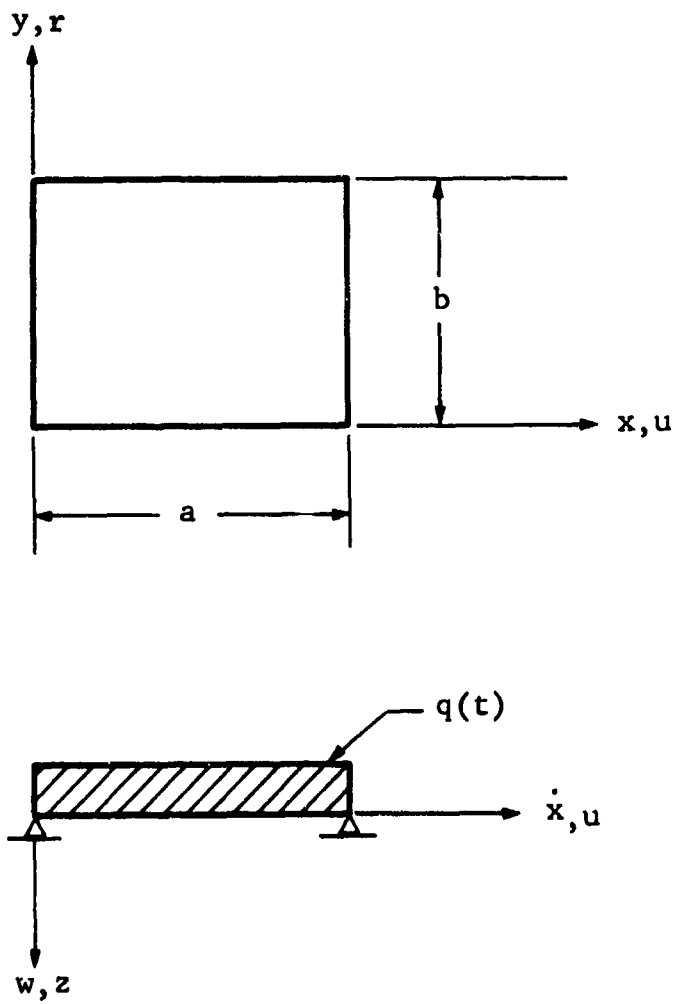


Fig. 21 PLATE COORDINATES AND DIMENSIONS

$$w(x,y,t) = \left( \frac{16 p_0 g}{a b h \gamma} \right) \sum_{m=1,3,\dots} \sum_{n=1,3,\dots} \frac{1}{\beta_m \alpha_n (w_{nm}^2 + \mu^2)} \left\{ \sin(\alpha_n x) \sin(\beta_m y) \left[ \exp(-\mu t) + \frac{\mu}{w_{mn}} \sin(w_{mn} t) - \cos(w_{mn} t) \right] \right\} \quad (8)$$

where

$$\alpha_n = \frac{n\pi}{a}$$

$$\beta_m = \frac{m\pi}{b}$$

$$w_{nm}^2 = \left[ \alpha_n^2 + \beta_m^2 \right]^2 \frac{Eh^2 g}{12(1-\nu^2)\gamma}$$

h = plate thickness

E = Young's modulus

$\nu$  = Poisson's Ratio

$\gamma$  = weight density

g = acceleration of gravity.

The resulting moments can be found by substituting Eq. (8) into the following which relate moments to deflections.

$$M_{xx} = \frac{-Eh^3}{12(1-\nu^2)} \left[ \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right] \quad (a)$$

$$M_{yy} = \frac{-Eh^3}{12(1-\nu^2)} \left[ \frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right] \quad (b) \quad (9)$$

$$M_{xy} = \frac{-Eh^3(1-\nu)}{12(1-\nu^2)} \frac{\partial^2 w}{\partial x \partial y} \quad (c)$$

It is then possible to find the principal moments from:

$$M_1, M_2 = 1/2 \left[ M_{xx} + M_{yy} \right] \pm \sqrt{\left[ \frac{M_{xx} - M_{yy}}{2} \right]^2 + M_{xy}^2} \quad (10)$$

Since the principal stresses are related to the principal moments by

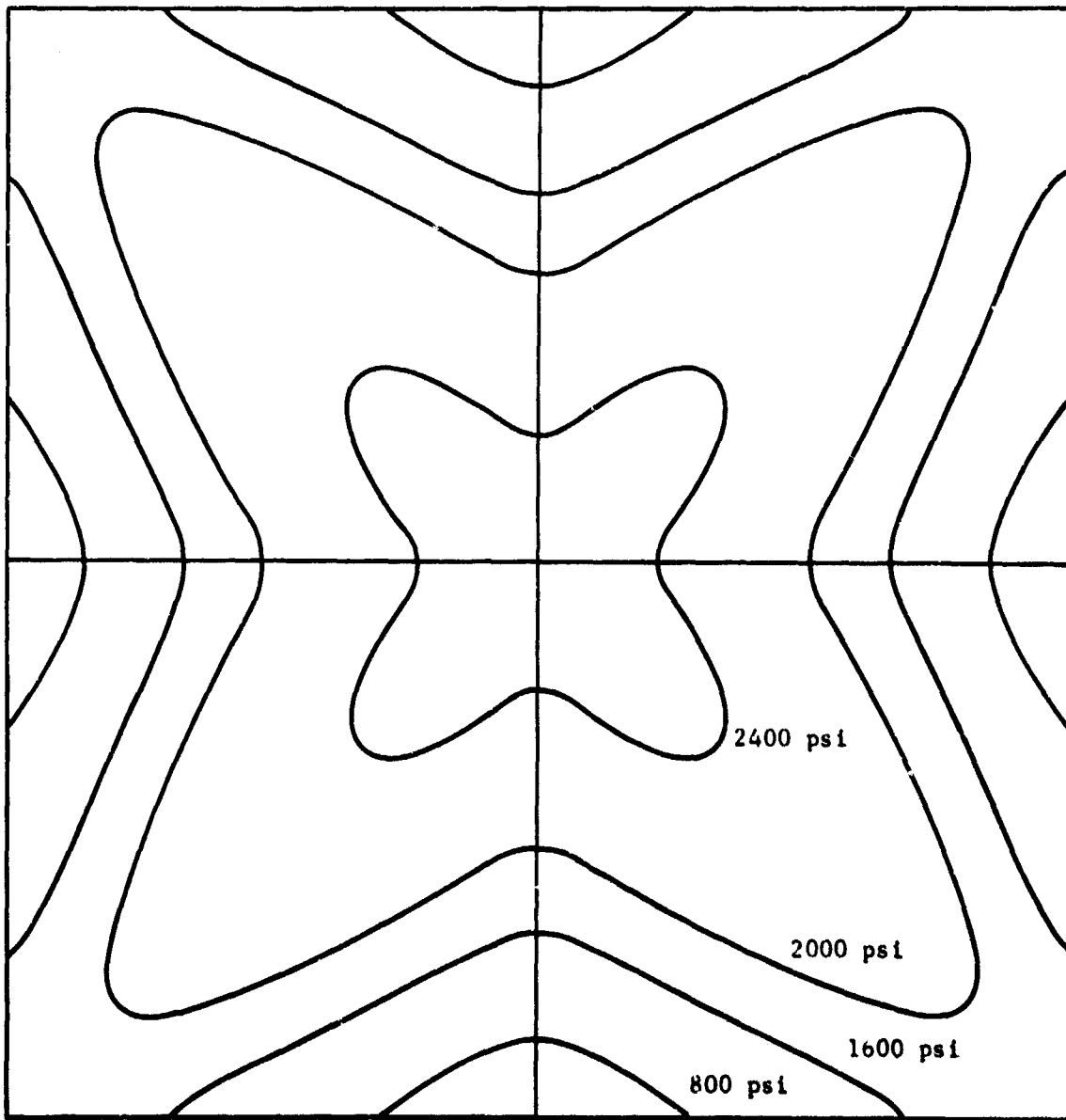
$$S_{1,2} = \frac{6}{h^2} M_{1,2} , \quad (11)$$

we can find the magnitudes and directions of the principal stresses in the plate at any time.

Because of the arduous summations involved, Eqs. (8) through (11) were programmed for the IBM 7094 digital computer. A particular problem, that of a square plate 15 in. on a side and 1/2 in. thick, was run and the resulting contours of maximum principal stress are shown in Fig. 22 for  $P_0 = 5$  psi and  $\mu = 2 \text{ sec}^{-1}$ . The curves are the contours at the time when the stress at the center of the plate (which, of course, is the maximum stress in a simply-supported plate) is a maximum, i.e.,  $t = 0.001958$  sec. The maximum stresses are very closely approximated by the stresses associated with the contour lines in Fig. 22 because the plate deflects predominantly in the first mode.

### 3.3 PROBABILITY OF FRACTURE INITIATION

Using the principal stresses calculated by the methods of the previous subsection, we shall address ourselves to the problem of establishing the probability that fracture will initiate in a typical subdivision of the plate shown in Fig. 23. These subdivisions are identified by the integers running from 1 to 120 and their associated bending moments are calculated at their centroids. Figure 24 shows a subdivision from which we have extracted a slice which is subjected to the principal stresses  $(S_1, S_2, 0)$ . Before we can establish its reliability, it is necessary that a theory be developed for multiaxial stress fields.



Note:  $a = b = 15$  inches  
 $h = 1/2$  inch

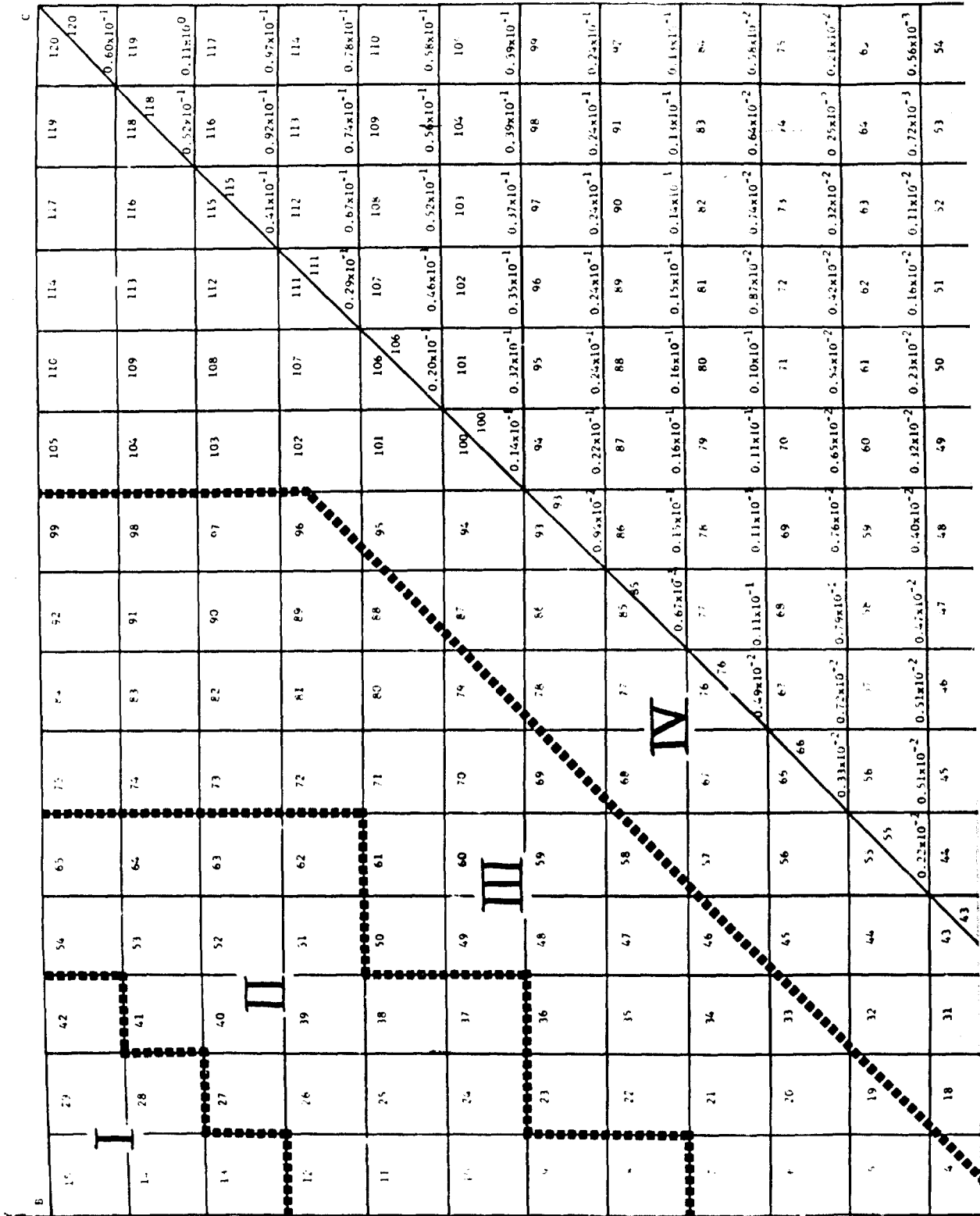
Fig. 22 LINES OF CONSTANT MAXIMUM STRESS

### 3.3.1 Combined Stress Theory

In his classic paper of 1939 (Ref. 12), Weibull developed an expression for the fracture probability of a brittle material under a polyaxial stress state. Using a different point of view, we shall expand on his brief statistical treatment of this combined stress problem and extend our results to cases of varying mechanical and thermal loading, and to materials which cannot be represented by the Weibull distribution function.

Briefly, it is our objective to establish a fracture surface, i.e., to find a relationship among the strengths achieved under various stress states. The usual approach to this problem in either brittle or ductile materials is to find a property common to all stress states that will indicate failure or non-failure. In ductile materials the distortion energy represents such a property, since incipient flow occurs in any stress state in which the distortion energy is equal to the distortion energy obtained in a tension specimen at yield. Stated in another way, we can correlate yielding under any stress state with the distortion energy. Our approach for brittle materials is completely analogous - we shall try to find a property that will correlate with the reliabilities associated with the various possible combined stress conditions.

To avoid the "size effect" problem observed in the strength of brittle elements, (i.e., increasing fracture stress with decreasing volume) we shall begin our study by considering a finite unit volume  $\Delta V$  of fixed size. We assume that both the material and the stress state in this unit volume are homogeneous and that the materials used in all the unit volumes to be considered have been drawn from the same population. In addition, we shall restrict the study to brittle materials that are statistically isotropic, i.e., the distribution of strengths obtained from an indefinitely large number of unit volumes will be identical in every direction.







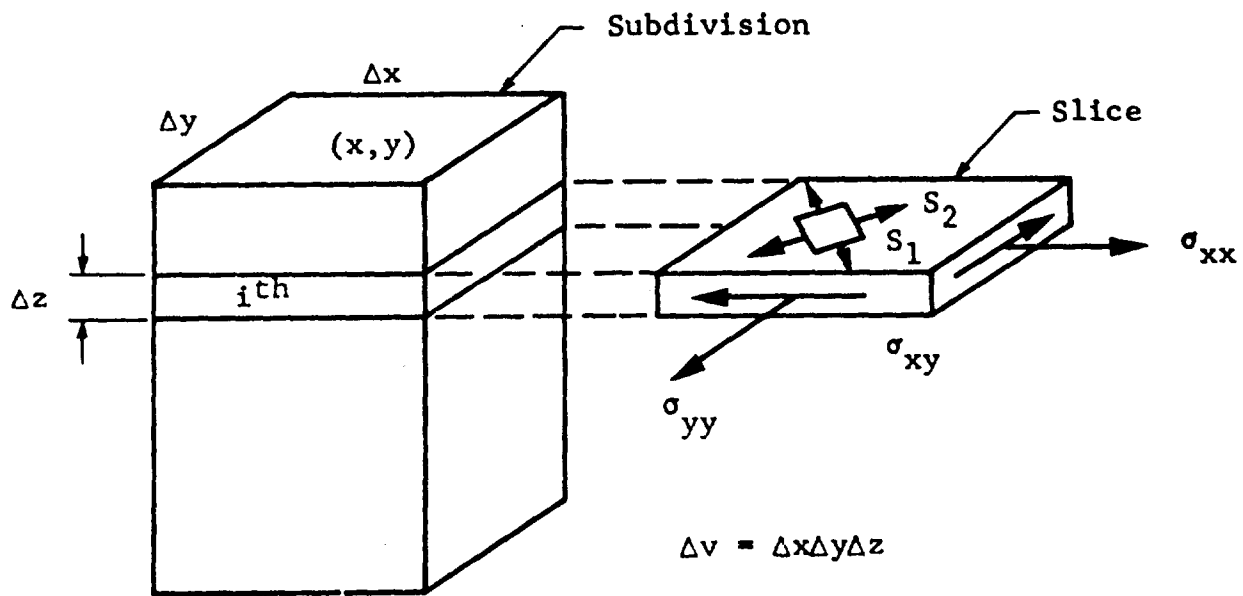


Fig. 24 TYPICAL PLATE SUBDIVISION

We shall assume that the principal stresses  $S_1, S_2, S_3$  which act on a basic unit volume are proportional to a load factor  $S$ , that is,

$$\begin{aligned} S_1 &= \alpha S \\ S_2 &= \beta S \\ S_3 &= \gamma S \end{aligned} \tag{12}$$

where  $\alpha, \beta, \gamma$  are constants which define the stress state. The strength of a basic element will be taken as the maximum load factor that it can equilibrate. Failure of the unit element is represented by its inability to equilibrate the applied loading. It is important to point out that it is possible for cracks to initiate and propagate within the unit volume without causing failure of the element. Materials in which cracks can be arrested or which provide alternative load paths when local failures occur are classified as parallel or series-parallel materials. If a local failure necessarily leads to overall failure, the associated material is called a series or "weakest link" material. One can advantageously adopt an infinitesimal unit volume for the series material and, as we shall subsequently discuss, combined stress testing is greatly simplified in this case.

Only the tensile or cohesive mode of failure will be considered in this investigation. We shall assume that neither compressive nor shear stresses influence the strength of a brittle material. The potential usefulness of this tension criterion is a consequence of two observations; first, that the shear strength of brittle materials is usually an order of magnitude greater than the tensile strength, and, second, that it is extremely difficult to eliminate tensile stresses from prototype or laboratory elements. Almost every structural failure of a brittle component can be attributed to the presence of some distribution of tensile stresses.

### 3.3.2 Two-Dimensional Theory Heuristic Development

When we attempt to describe the statistical fracture strength of a unit volume of material under a uniaxial stress state, the axial stress (strain) is the only reasonable choice for the statistical variate. Taking a general form for any cumulation distribution function, we can write the fracture probability  $F$  for the uniaxial stress state as

$$F(\sigma) = 1 - \exp \left[ -\frac{\Delta V}{v} g(\sigma) \right] \quad (13)$$

where  $\Delta V$  is the specified volume of the basic unit element,  $v$  is a volume of unity, and  $\sigma$  is the axial stress. The delineation of the constant  $\Delta V/v$  does not affect the generality of this expression and in the special case of a series material it provides a convenient representation. If we examine the strength of a unit volume of an isotropic material under a general homogeneous stress state, it follows that failure will depend only on the three principal stresses acting on the unit. Thus, the probability of failure of the unit volume can be designated as  $F(S_1, S_2, S_3)$  where the three principal stresses are taken as the statistical variates. For this case we shall take Eq. (13) in the form

$$\frac{-\ln [1 - F(S_1, S_2, S_3)]}{\Delta V/v} = g(S_1, S_2, S_3) \quad (14)$$

For a specified reliability  $(1-F)$ , we note that Eq. (14) becomes  $g(S_1, S_2, S_3) = \text{constant}$ , which defines our fracture surface.

On the basis that failure is caused only by tensile stresses, it seems reasonable to look for the function  $g$  within the collection of all possible tensile stresses which can occur at any point in the unit volume. In the plane stress problem we can relate the normal stress  $\sigma_n$  acting in any direction to the principal stresses through the expression

$$\sigma_n = S_1 \cos^2 \theta + S_2 \sin^2 \theta \quad (15)$$

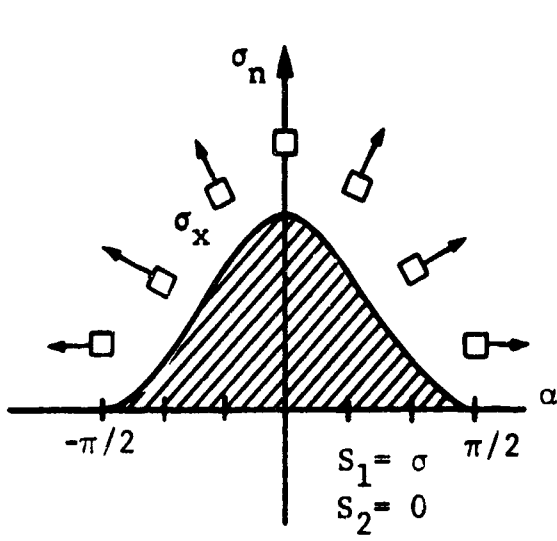
where  $\theta$  is the angle between  $\sigma_n$  and  $S_1$ . As  $\theta$  sweeps through all values from  $-\pi/2$  to  $\pi/2$ , Eq. (15) describes every possible normal stress acting at a point. The normal stresses associated with the various directions described by  $\theta$  are shown in Fig. 25 for several different stress states. The question, now, is what are the distinguishing features of these figures that will reflect the differences they cause in a material's response?

The most obvious first guess is to differentiate among these stress states by comparing the areas associated with the tensile normal stresses. However, this approach does not reflect the possibility that the magnitude of the stresses may have a different influence than their extent or distribution. For example, hydrostatic and pure tension stress states are depicted in Fig. 26 that lead to the same area but where one peak stress is twice the other. Experience indicates that the pure tension state is the more critical. This suggests that we "weight" the ordinates in these figures and then compare the areas of the weighted normal stress-theta diagrams.

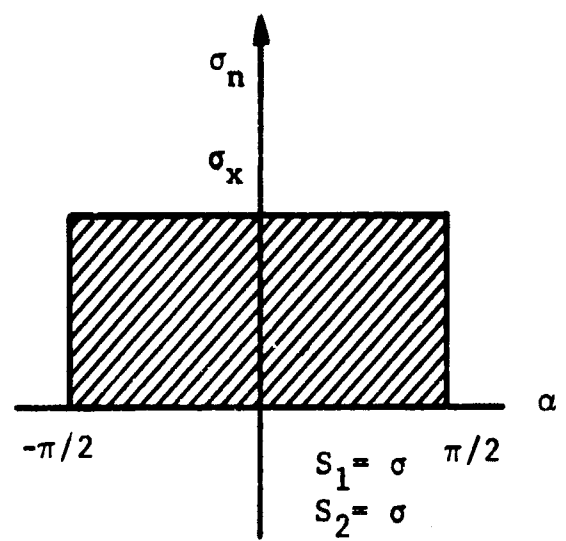
Assuming a statistically isotropic material, the weighting should be independent of the orientation  $\theta$  of the normal stress. We might use for example a power function to modify the normal stresses, i.e.,  $D\sigma_n^k$  where  $D$  and  $k$  are constants. This alteration results in the dashed curve shown on the left side of Fig. 27. If the normal stress distribution for several stress states were weighted in this fashion, we could compare the areas of the resulting curves, that is,

$$g(S_1, S_2) = \text{Area} = D \int_{\sigma_n \geq 0} \sigma_n^k d\theta \quad (16)$$

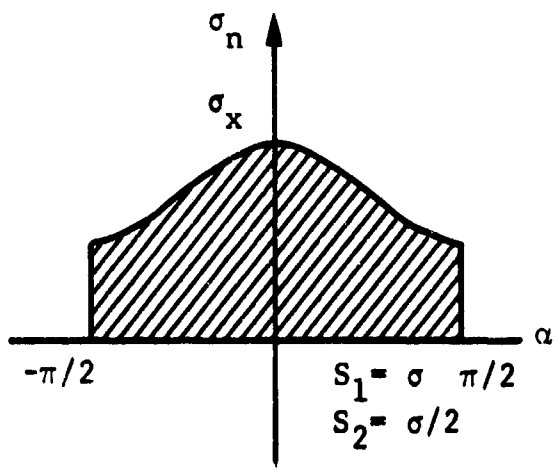
where the integration extends over those values of  $\theta$  where the normal stress is non-negative. Because of symmetry we need consider only the positive normal stresses in the interval zero to  $\pi/2$ . To account for the possibility that tensile stresses below a certain magnitude  $\sigma_\ell$  may not cause failure, we may choose to weight the difference  $(\sigma_n - \sigma_\ell)$  as shown in the right half of Fig. 27.



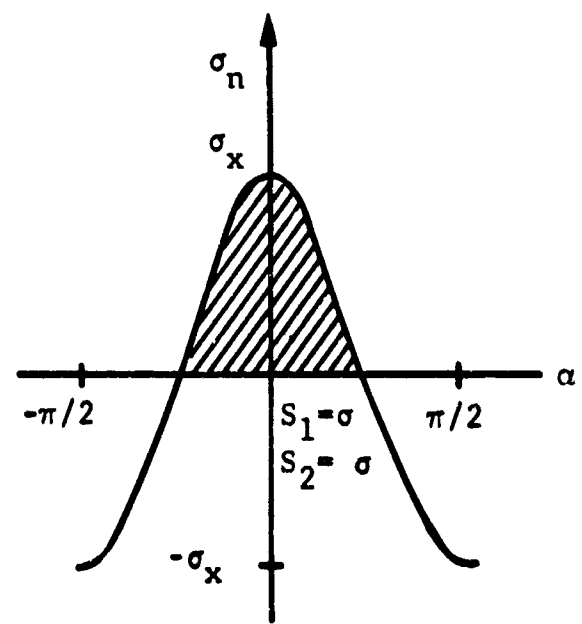
Pure Tension



Hydrostatic Tension



Biaxial Tension



Pure Shear

Fig. 25 NORMAL STRESS DISTRIBUTIONS FOR VARIOUS STRESS STATES

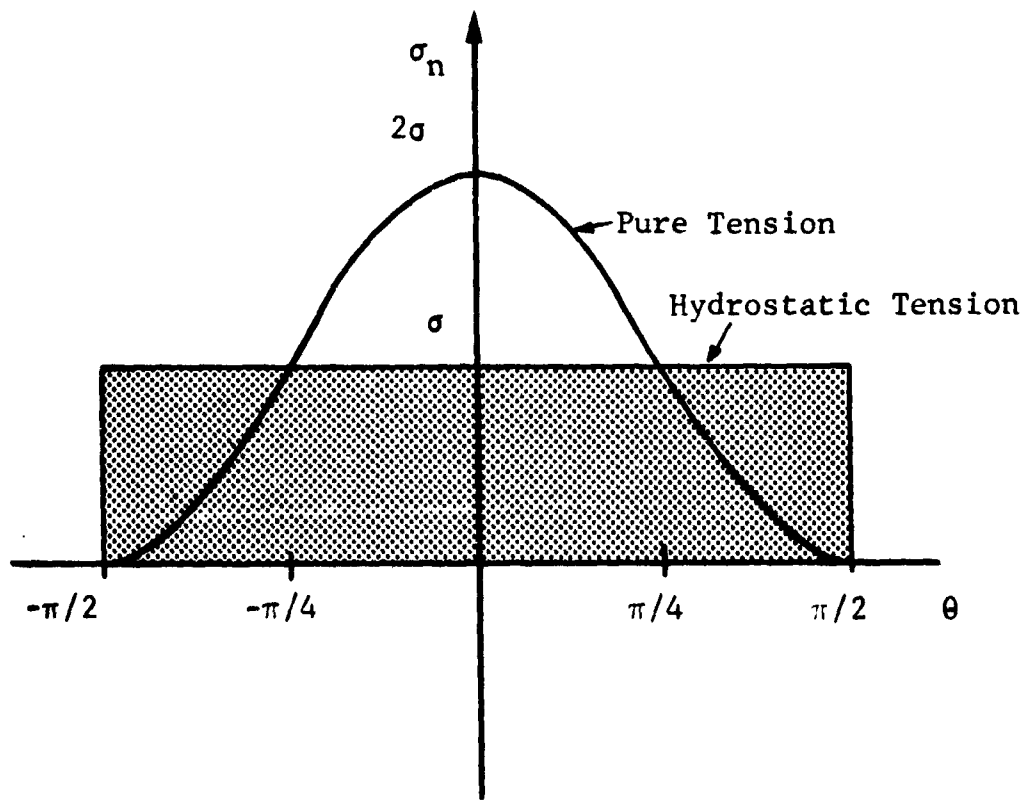


Fig. 26 EQUAL AREAS, UNEQUAL MAXIMUM STRESSES

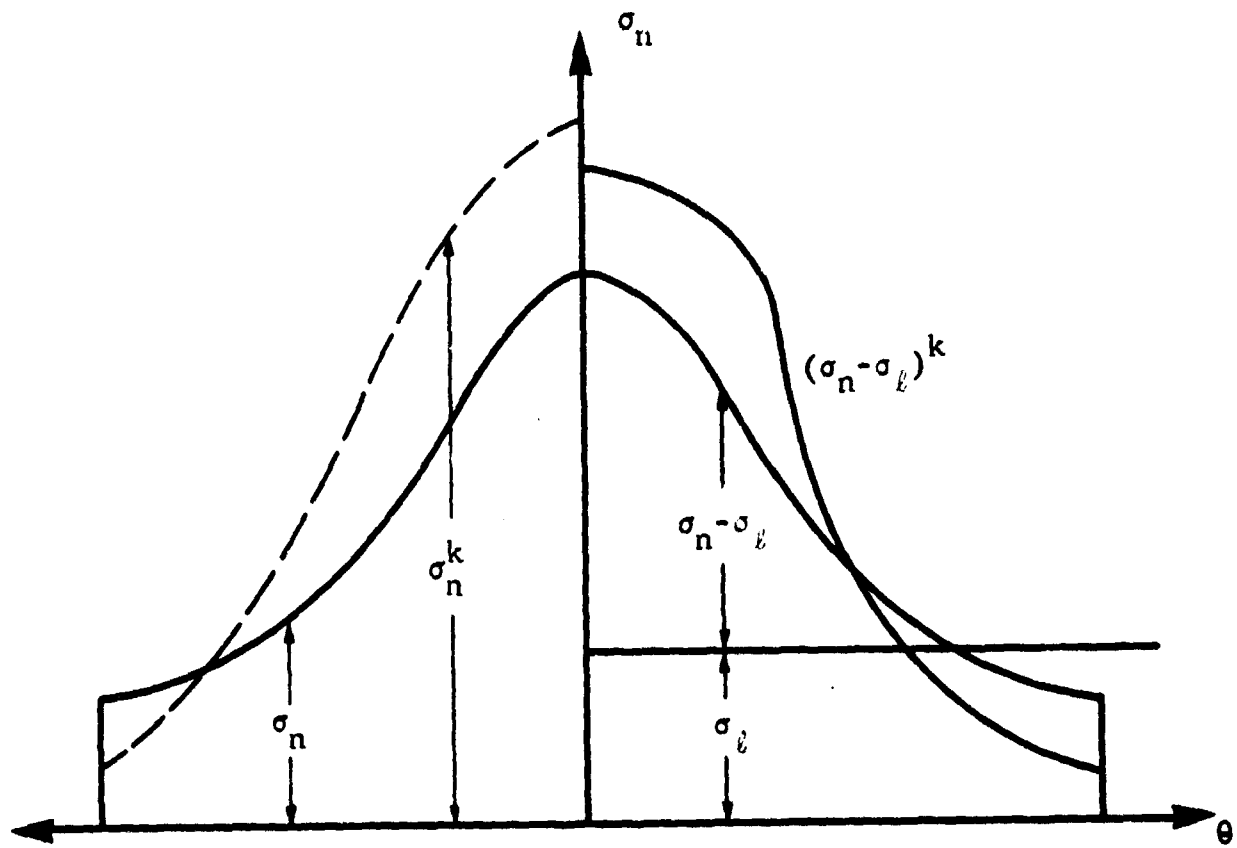


Fig. 27 "WEIGHTED" NORMAL STRESS DIAGRAM

The associated area is given by

$$g(S_1, S_2) = \text{Area} = D \int_{\sigma_n \geq \sigma_\ell} (\sigma_n - \sigma_\ell)^k d\theta. \quad (17)$$

Certainly, the use of a power function to weight the normal stress-theta diagrams is completely arbitrary and there are many other ways of manipulating and distorting such curves. Our problem is to find a weighting function that will reflect the influence of stress state on the reliability of a unit volume. Denoting the weighting function by  $f$ , the fracture probability becomes

$$F(S_1, S_2) = 1 - \exp \left[ - \frac{\Delta V}{V} \int_{\sigma_n \geq \sigma_\ell} f(\sigma_n - \sigma_\ell) d\theta \right]. \quad (18)$$

We are now in a position to describe certain guidelines for the selection of  $f$ . First, to account for the possible existence of a zero fracture probability stress  $\sigma_\ell$ , we must take

$$\begin{aligned} f &= f(\sigma_n - \sigma_\ell) & \sigma_n \geq \sigma_\ell \geq 0 \\ f &= 0 & \sigma_n \leq \sigma_\ell. \end{aligned}$$

The latter condition implies that both  $S_1 \leq \sigma_\ell$  and  $S_2 \leq \sigma_\ell$ , and that in such cases  $F = 0$ . At the other extreme, we expect that fracture is a certainty when either  $S_1$  or  $S_2$  is positive and unbounded; hence,  $F = 1$  implies that

$$f \rightarrow \infty \text{ when } S_1 \rightarrow + \infty.$$



Furthermore, we would expect on physical grounds that the failure probability would increase continuously with increasing principal stresses, thus,

$f$  ... continuous and monotone increasing.

Finally,  $f$  must be chosen in such a way that the associated  $F(S_1, S_2)$  fits the cumulative distribution curve obtained from fracture tests conducted using various stress states. In particular, it is necessary that fracture data obtained under pure tension be represented by  $F(S_1, 0)$ .

Typical examples of admissible forms for the weighting function  $f$  are the following:

$$f = \left( \frac{\sigma_n - \sigma_\ell}{\sigma_0} \right)^k \quad (19)$$

$$f = \exp \left[ a(\sigma_n - \sigma_\ell) \right] - 1 \quad (20)$$

$$f = \exp \left\{ \exp \left[ a(\sigma_n - \sigma_\ell) \right] - 1 \right\} - 1 \quad (21)$$

$$f = A(\sigma_n - \sigma_\ell) + B(\sigma_n - \sigma_\ell)^2 + C(\sigma_n - \sigma_\ell)^3 + \dots \quad (22)$$

$$A \geq 0, B \geq 0, C \geq 0$$

where  $a$ ,  $k$ ,  $A$ ,  $B$ ,  $C$ ,  $\sigma_0$ , and  $\sigma_\ell$  are constants of the material.

### 3.3.3 Three-Dimensional Theory

The extension of our theory given in Eq. (18) to three dimensions requires that we appropriately distort the surface formed by the normal stress vector in three dimensions. This vector is given in the polar coordinates as

$$\sigma_n = \cos^2 \phi (S_1 \cos^2 \psi + S_2 \sin^2 \psi) + S_3 \sin^2 \phi \quad (23)$$

where the angles  $\phi$  and  $\psi$  are defined in Fig. 28a. A typical surface representing the focus of normal stress vectors is shown in Fig. 28b for a biaxial tension field. A weighted surface is formed by  $f(\sigma_n - \sigma_l)$  and its volume can be introduced into the general distribution function to give

$$F(S_1, S_2, S_3) = 1 - \exp \left[ - \frac{\Delta V}{V} \iiint_{\sigma_n \geq \sigma_l} f(\sigma_n - \sigma_l) dV \right]. \quad (24)$$

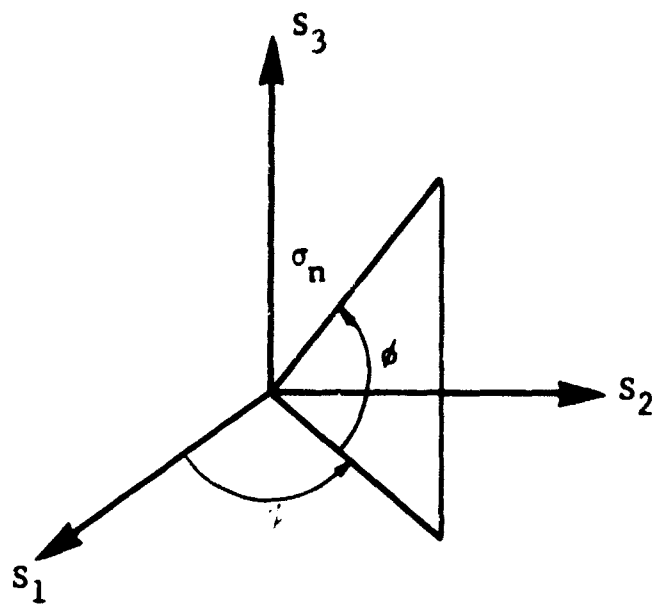
Specializing to the form of  $f$  given in Eq. (19) and using polar coordinates, the failure probability  $F$  is given by

$$F = 1 - \exp \left[ - \frac{1}{3} \frac{\Delta V}{V} \int_0^{\pi/2} d\psi \int_{\phi_L}^{\phi_U} \cos \phi d\phi \left( \frac{\sigma_n - \sigma_l}{\sigma_0} \right)^{3k} \right] \quad (25)$$

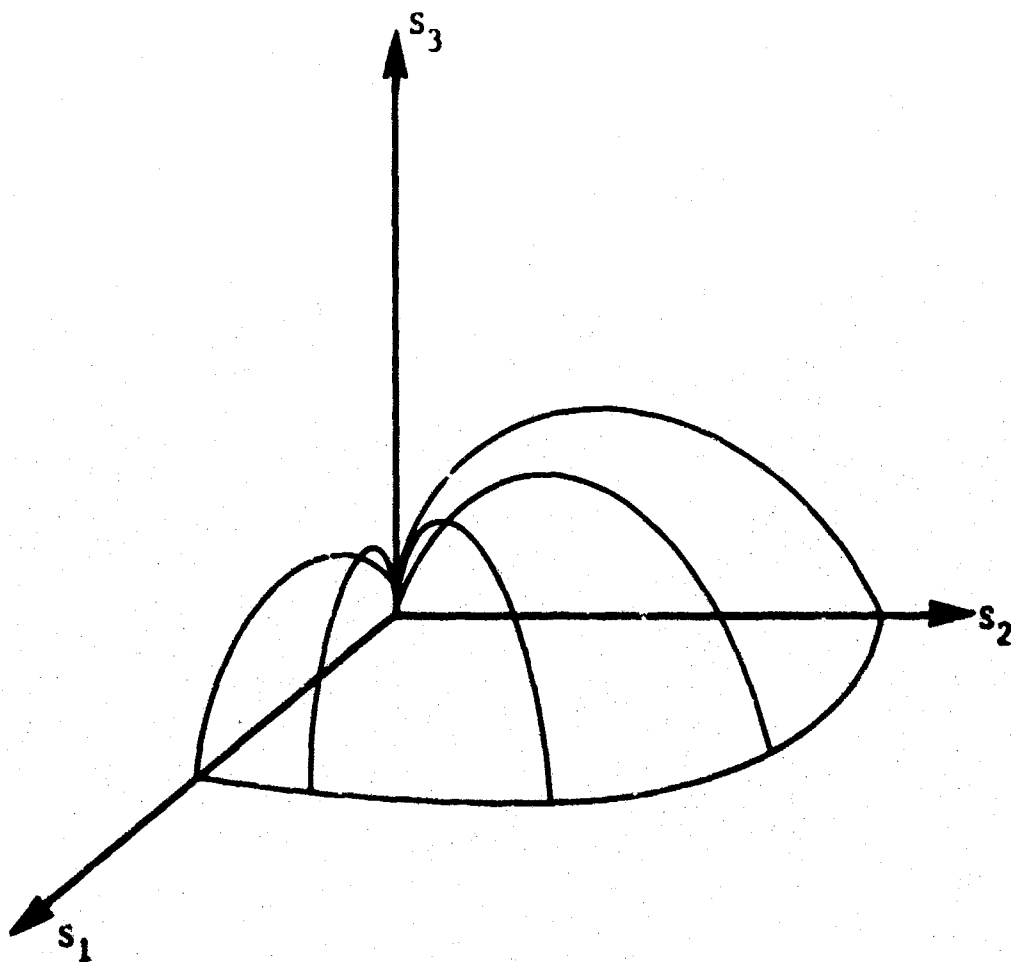
where we find three cases:

$$1) \quad S_1 \geq S_2 \geq \sigma_l$$

$$\phi_L = 0; \quad \phi_U = \cos^{-1} \sqrt{\frac{\sigma_l}{S_1 \cos^2 \psi + S_2 \sin^2 \psi}}$$



a) Coordinate System



b) General Biaxial Tension

Fig. 28 NORMAL STRESS SURFACE IN THREE DIMENSIONS

$$2) S_1 \geq \sigma_l, S_2 \leq \sigma_l$$

$$\phi_L = \cos^{-1} \sqrt{\frac{S_1 - \sigma_l}{S_1 - S_2 \cos^2 \psi}}; \phi_U = \pi/2$$

$$3) S_1 \leq \sigma_l, S_2 \leq \sigma_l$$

$$\phi_L = 0, \phi_U = 0 \quad (F = 0) .$$

Equation (25) can be written in the form

$$1 - F = e^{-B} \quad (26)$$

where the "risk of rupture" B is given by the negative of the term within the square brackets of Eq. (25). The risk of rupture B was evaluated numerically for each slice of every plate subdivision indicated in Fig. 23. Specifically, the following data was used:

Plate size:	15 x 15 x 1/2 in.
Overpressure:	$P_o = 5$ psi
Pressure decay:	$\mu = 2$ sec <sup>-1</sup>
Statistical parameters:	$k = 3$
	$\sigma_o = 1500$ psi
	$\sigma_l = 50$ psi

Now, a value of  $B_i$  for the  $i$ th slice shown in Fig. 24 enables us through Eq. (26) to establish the probability that no fracture will initiate in the slice,  $(1-F_i)$ . The probability that no fracture will initiate in the entire subdivision  $(1-F_S)$ , requires the simultaneous survival of each slice, thus,

$$(1-F_S) = (1-F_1)(1-F_2)\dots(1-F_n) = \prod_{i=1}^n (1-F_i) \quad (27)$$

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where  $n$  is the total number of slices. Substituting Eq. (26) into this equation we obtain

$$(1-F_S) = \exp(-B_S) = \exp\left(-\sum_{i=1}^n B_i\right) . \quad (28)$$

Therefore, the risk of rupture of a "big piece" is equal to the sum of the risk of ruptures of its component "small pieces". The sum of the slice risk of ruptures for each plate subdivision is tabulated in Table 2, together with its centroid coordinates and maximum principal bending stresses. These risk of rupture values are displayed in Fig. 23 by the lower number in each subdivision.

#### 3.4 PLATE EXPERIMENTS

One of the most difficult aspects of the plate fragmentation problem concerns the question of crack propagation. Crack initiation was the concern of the previous two subsections. In the beam problem, when a crack initiated within the beam volume this always resulted in a fracture surface which was roughly perpendicular to the beam axis. When a crack initiates within a plate, its direction of travel is not obvious. Furthermore, we meet a new problem when many cracks are propagating because one crack crossing the path of a second crack will generally arrest the second crack. We are faced, therefore, with the "who got there first" problem. In the face of these complications, we examined the results of experiments conducted with Hydrostone plaster plates under dynamic loadings. The experiments conducted at IITRI (Ref. 8), were supposed to demonstrate characteristic crack patterns that would provide the needed propagation information for our fragmentation analysis. If no patterns were obtained, our analysis procedure would have to be abandoned, and indeed, the hope of developing a rational prediction scheme would be pretty gloomy. Fortunately, patterns did emerge from these tests and we shall very briefly summarize the findings which are described in detail in Ref. 8.

### 3.4.1 Description of Drop Test

It has been shown in Ref. 8 that the response of a plate under any uniform time-dependent loading can be made identical to that achieved in a drop test when the appropriate support deceleration is imposed. To produce the dynamic load in our drop test facility, the plate support was mounted on the drop table as shown in Fig. 29. The idea was to drop the table and suddenly decelerate it, which would load the plate mounted on the supports (as shown in Fig. 30) with downward acting inertia body forces. To increase the downward loading, sand was piled onto the plate and held in place by the box device in Fig. 31. The results of a typical drop test are illustrated in Fig. 32 where the fragments are held intact by masking tape on their upper surface.

To check out the symmetry of the drop test loading, two plastic plates were stress coated and dropped from different heights. As can be observed from Fig. 33 and 34, the loading is excellent and a pattern of principal directions is obtained which is not unlike that obtained for the pressure loading  $q = p_0 \exp[-ut]$  as shown in Fig. 22.

### 3.4.2 Results and Conclusions

Typical examples of the crack outlines obtained for five different size Hydrostone plaster plates are shown in Fig. 35. We first observe that these cracks form a pattern. Second, by comparing the crack pattern on the square plates to the stress coat patterns of Fig. 33 and 34 we see that for the most part the cracks propagate along the principal direction trajectories. Further examination of the square plates indicates that the central pattern forms first. In all of the cases, cracks occur along 45 deg lines at the corners.

On the basis of these observations, we shall postulate the formation of the primary fracture mode shown in Fig. 36a and the secondary fracture mode shown in Fig. 36c. The strips in the secondary mode are intended to approximate the principal stress trajectories.

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Table 2

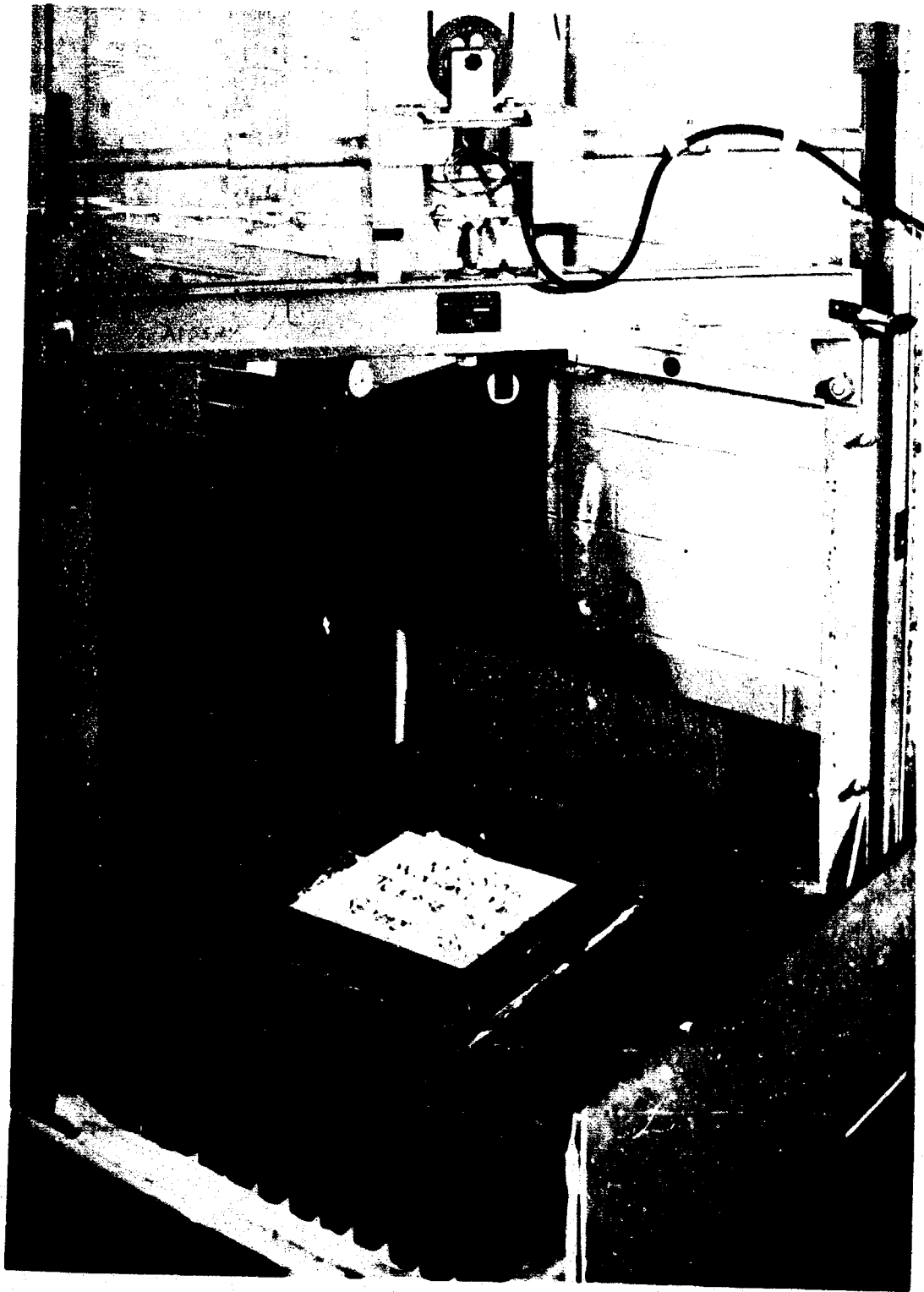
PRINCIPAL STRESSES AND RISKS OF RUPTURE  
( $P_0 = 5$  psi;  $\mu = 2$  sec<sup>-1</sup>)

i	X(i)	Y(i)	S(i)	Z(i)	BV(i)
1	0.33333333	0.14666667	0.18117125	-0.16171255	0.92048060
2	0.74999999	0.25000000	0.18413438	-0.17527666	0.15775269
3	0.12500000	0.25000000	0.18018105	-0.16447019	0.11747227
4	0.12500000	0.25000000	0.17364095	-0.15400735	0.77354835
5	0.22500000	0.25000000	0.16441553	-0.14366426	0.44899248
6	0.27500000	0.25000000	0.15433889	-0.13008348	0.22951261
7	0.32500000	0.25000000	0.14234622	-0.11571118	0.10341446
8	0.37500000	0.25000000	0.12944717	-0.10089843	0.40989199
9	0.42499999	0.25000000	0.11591054	-0.85777081	0.14132072
10	0.47499999	0.25000000	0.10182966	-0.70272936	0.40844197
11	0.52499999	0.25000000	0.87190876	-0.54482000	0.92633029
12	0.57499999	0.25000000	0.72127242	-0.38586947	0.15020885
13	0.62499999	0.25000000	0.56314604	-0.22444656	0.14636680
14	0.67499999	0.25000000	0.41279781	-0.66573882	0.62334323
15	0.72499999	0.25000000	0.25734122	0.90746545	0.49608781
16	0.83333333	0.64666666	0.18958777	-0.16422518	0.78262611
17	0.12500000	0.75000000	0.18069799	-0.15131230	0.14671994
18	0.17500000	0.75000000	0.18747648	-0.13688098	0.12818827
19	0.22500000	0.75000000	0.18251732	-0.12128430	0.92227943
20	0.27500000	0.75000000	0.17317588	-0.10560759	0.68344243
21	0.32500000	0.75000000	0.16584436	-0.88573476	0.41738033
22	0.37500000	0.75000000	0.15549376	-0.72197435	0.22813417
23	0.42499999	0.75000000	0.14341238	-0.55770200	0.11256317
24	0.47499999	0.75000000	0.13106676	-0.39115191	0.49765560
25	0.52499999	0.75000000	0.11771576	-0.22455278	0.19027528
26	0.57499999	0.75000000	0.10384200	-0.60504936	0.61078177
27	0.62499999	0.75000000	0.89220366	0.10188642	0.16678019
28	0.67499999	0.75000000	0.74957175	0.26065458	0.35215909
29	0.72499999	0.75000000	0.61113629	0.40472312	0.69224218
30	0.13333333	0.11666667	0.19496411	-0.13473393	0.91593651
31	0.17500000	0.12500000	0.19720676	-0.11687198	0.16968844
32	0.22500000	0.12500000	0.19465316	-0.98416479	0.12672371
33	0.27500000	0.12500000	0.18201586	-0.79873373	0.85087321
34	0.32500000	0.12500000	0.18546199	-0.61773271	0.26718809
35	0.37500000	0.12500000	0.17705390	-0.44596298	0.52516809
36	0.42499999	0.12500000	0.16743149	-0.26693905	0.29964329
37	0.47499999	0.12500000	0.15888742	-0.91304420	0.15950005
38	0.52499999	0.12500000	0.14325193	-0.81271392	0.17323342
39	0.57499999	0.12500000	0.13267167	-0.24774379	0.31068400
40	0.62499999	0.12500000	0.11959053	0.41008268	0.12277633
41	0.67499999	0.12500000	0.10627782	0.58067312	0.31068400
42	0.72499999	0.12500000	0.84346334	0.69410158	0.35809191
43	0.18333333	0.16666667	0.20304975	-0.96224950	0.13962859
44	0.22500000	0.17500000	0.20376119	-0.75333245	0.32807431
45	0.27500000	0.17500000	0.20995695	-0.55007759	0.32857061
46	0.32500000	0.17500000	0.20114272	-0.35643891	0.28746090
47	0.37500000	0.17500000	0.19322337	-0.17041312	0.22623475
48	0.42499999	0.17500000	0.18780000	0.12175735	0.16506640
49	0.47499999	0.17500000	0.17923908	0.19455238	0.11171958
50	0.52499999	0.17500000	0.15924400	0.37051952	0.74029474
51	0.57499999	0.17500000	0.15284200	0.53719437	0.43121718
52	0.62499999	0.17500000	0.14856572	0.69800039	0.23985600
53	0.67499999	0.17500000	0.13460344	0.84753608	0.13247059
54	0.72499999	0.17500000	0.12427802	0.96363689	0.85202475
55	0.23333333	0.21666667	0.21148770	-0.52733936	0.22134321
56	0.27500000	0.22500000	0.21394200	-0.31031530	0.51475524
57	0.32500000	0.22500000	0.21290175	-0.10741397	0.51400328
58	0.37500000	0.22500000	0.20930562	0.85626457	0.47708762
59	0.42499999	0.22500000	0.20421898	0.27520680	0.39984915
60	0.47499999	0.22500000	0.19749118	0.46199524	0.31586935
61	0.52499999	0.22500000	0.19146324	0.61475981	0.13247059
62	0.57499999	0.17500000	0.18442592	0.76363689	0.85202475
63	0.62499999	0.17500000	0.17800000	0.96363689	0.85202475
64	0.67499999	0.17500000	0.17200000	0.96363689	0.85202475
65	0.72499999	0.17500000	0.16600000	0.96363689	0.85202475

56	0.27500000	01	0.22500000	01	0.2193223	04	0.8672590	03	0.1324705	02
57	0.32500000	01	0.22500000	01	0.2193223	04	0.1242780	03	0.9636369	03
58	0.37500000	01	0.22500000	01	0.2193223	04	0.2148779	04	0.5273398	03
59	0.42499999	01	0.22500000	01	0.2193223	04	0.2139340	04	-0.3103150	03
60	0.47499999	01	0.22500000	01	0.2193223	04	0.2093852	04	-0.1074130	03
61	0.52499999	01	0.22500000	01	0.2193223	04	0.2021898	04	0.8542645	02
62	0.57499999	01	0.22500000	01	0.2193223	04	0.1976919	04	0.2732080	02
63	0.62499999	01	0.22500000	01	0.2193223	04	0.1895462	04	0.4619954	02
64	0.67499999	01	0.22500000	01	0.2193223	04	0.1807313	04	0.6389782	03
65	0.72499999	01	0.22500000	01	0.2193223	04	0.1698687	04	0.8043434	03
66	0.77499999	01	0.22500000	01	0.2193223	04	0.1592461	04	0.9624644	02
67	0.82499999	01	0.22500000	01	0.2193223	04	0.1502549	04	0.1105350	04
68	0.87499999	01	0.22500000	01	0.2193223	04	0.2108487	04	0.1212182	04
69	0.92499999	01	0.22500000	01	0.2193223	04	0.2195081	04	-0.8541446	02
70	0.97499999	01	0.22500000	01	0.2193223	04	0.2259386	04	0.1251935	03
71	0.02500000	01	0.22500000	01	0.2193223	04	0.2195081	04	0.3243892	03
72	0.07500000	01	0.22500000	01	0.2193223	04	0.2159227	04	0.7062968	02
73	0.12500000	01	0.22500000	01	0.2193223	04	0.2150582	04	0.7388797	02
74	0.17500000	01	0.22500000	01	0.2193223	04	0.1918114	04	0.6536602	02
75	0.22500000	01	0.22500000	01	0.2193223	04	0.1718717	04	0.8385899	02
76	0.27500000	01	0.22500000	01	0.2193223	04	0.2257651	04	0.4192212	02
77	0.32500000	01	0.22500000	01	0.2193223	04	0.2285876	04	0.2139724	02
78	0.37500000	01	0.22500000	01	0.2193223	04	0.2251131	04	0.4856631	02
79	0.42499999	01	0.22500000	01	0.2193223	04	0.2251814	04	0.1022359	01
80	0.47499999	01	0.22500000	01	0.2193223	04	0.2226179	04	0.1134134	01
81	0.52499999	01	0.22500000	01	0.2193223	04	0.2211363	04	0.1104922	01
82	0.57499999	01	0.22500000	01	0.2193223	04	0.2191752	04	0.1004847	01
83	0.62499999	01	0.22500000	01	0.2193223	04	0.2166408	04	0.8678305	02
84	0.67499999	01	0.22500000	01	0.2193223	04	0.1948912	04	0.7416480	02
85	0.72499999	01	0.22500000	01	0.2193223	04	0.1893459	04	0.6383377	02
86	0.77499999	01	0.22500000	01	0.2193223	04	0.2282082	04	0.5817730	02
87	0.82499999	01	0.22500000	01	0.2193223	04	0.2307419	04	0.6680184	02
88	0.87499999	01	0.22500000	01	0.2193223	04	0.2314513	04	0.1527220	02
89	0.92499999	01	0.22500000	01	0.2193223	04	0.2272754	04	0.1623979	02
90	0.97499999	01	0.22500000	01	0.2193223	04	0.2284231	04	0.1613301	02
91	0.02500000	01	0.22500000	01	0.2193223	04	0.2100741	04	0.1524879	01
92	0.07500000	01	0.22500000	01	0.2193223	04	0.2036479	04	0.1422120	01
93	0.12500000	01	0.22500000	01	0.2193223	04	0.2339570	04	0.1263040	01
94	0.17500000	01	0.22500000	01	0.2193223	04	0.2353681	04	0.9402068	02
95	0.22500000	01	0.22500000	01	0.2193223	04	0.2344277	04	0.2181914	02
96	0.27500000	01	0.22500000	01	0.2193223	04	0.2315394	04	0.2354147	01
97	0.32500000	01	0.22500000	01	0.2193223	04	0.2271857	04	0.2420926	01
98	0.37500000	01	0.22500000	01	0.2193223	04	0.2215214	04	0.2394691	02
99	0.42499999	01	0.22500000	01	0.2193223	04	0.2384077	04	0.2394691	02
100	0.47499999	01	0.22500000	01	0.2193223	04	0.2387522	04	0.2367687	01
101	0.52499999	01	0.22500000	01	0.2193223	04	0.2357582	04	0.1360313	01
102	0.57499999	01	0.22500000	01	0.2193223	04	0.2327558	04	0.3181347	01
103	0.62499999	01	0.22500000	01	0.2193223	04	0.2309368	04	0.3742315	01
104	0.67499999	01	0.22500000	01	0.2193223	04	0.2261122	04	0.3882489	01
105	0.72499999	01	0.22500000	01	0.2193223	04	0.2261122	04	0.3941122	01
106	0.77499999	01	0.22500000	01	0.2193223	04	0.2417578	04	0.1578780	01
107	0.82499999	01	0.22500000	01	0.2193223	04	0.2422142	04	0.4649703	01
108	0.87499999	01	0.22500000	01	0.2193223	04	0.2409636	04	0.5236824	01
109	0.92499999	01	0.22500000	01	0.2193223	04	0.2397750	04	0.5633313	01
110	0.97499999	01	0.22500000	01	0.2193223	04	0.2407551	04	0.6726710	02
111	0.02500000	01	0.22500000	01	0.2193223	04	0.2443062	04	0.5834304	01
112	0.07500000	01	0.22500000	01	0.2193223	04	0.2428522	04	0.2861496	02
113	0.12500000	01	0.22500000	01	0.2193223	04	0.2399277	04	0.6726710	02
114	0.17500000	01	0.22500000	01	0.2193223	04	0.2461280	04	0.7819216	01
115	0.22500000	01	0.22500000	01	0.2193223	04	0.2461280	04	0.4053983	01
116	0.27500000	01	0.22500000	01	0.2193223	04	0.2461280	04	0.9183681	01
117	0.32500000	01	0.22500000	01	0.2193223	04	0.2461280	04	0.9739236	01
118	0.37500000	01	0.22500000	01	0.2193223	04	0.2473833	04	0.5240438	01
119	0.42499999	01	0.22500000	01	0.2193223	04	0.2476819	04	0.1121471	01
120	0.47499999	01	0.22500000	01	0.2193223	04	0.2479579	04	0.5993295	02



Figure 35 provides typical fracture patterns of rectangular panels with length-to-width ratios of approximately 2, 3, and 4. The fracture patterns are generally what would be expected. As the length to width ratio of the plate increases, the performance of the plate appears to approach that of one supported on the two long sides only. The "square" center section of a square plate associated with the primary failure mode apparently rather rapidly degenerates from a square through a rectangular phase and into essentially a line. Figure 35, for example, tends to indicate that for even a length-to-width ratio of 2, the center section has almost entirely degenerated. Thus, the prediction of the primary fracture mode for rectangular plates may be simpler than for square plates. It would appear to follow from the degeneration of the plate's center region to a line that debris fragment sizes might be derived on the basis of the procedures for the secondary fracture mode alone.



**Fig. 29 DROP TABLE**

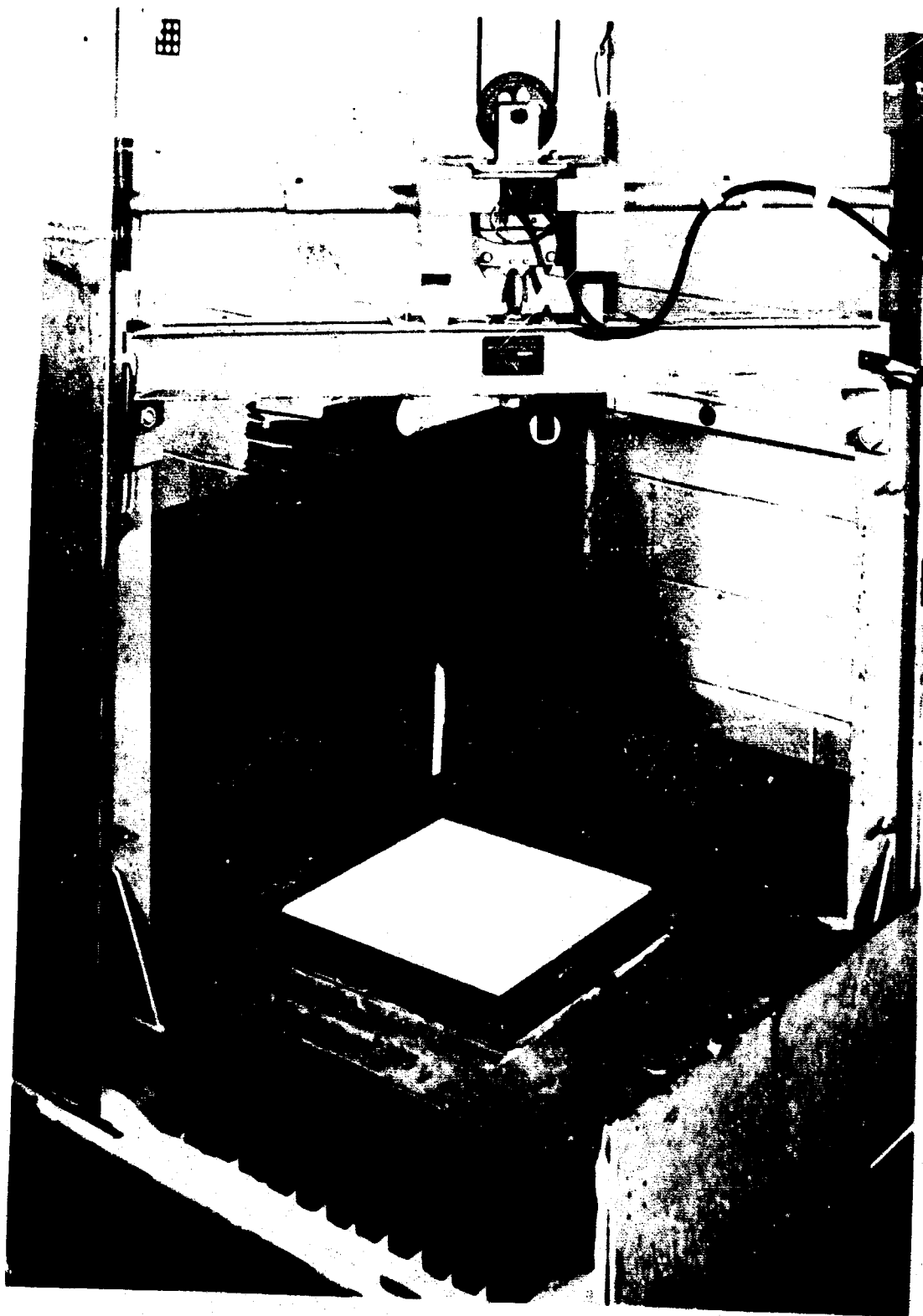


Fig. 30 HYDROSTONE PLASTER PLATE MOUNTED ON DROP TABLE



**Fig. 31 PLASTER PLATE TEST WITH SAND OVERBURDEN**



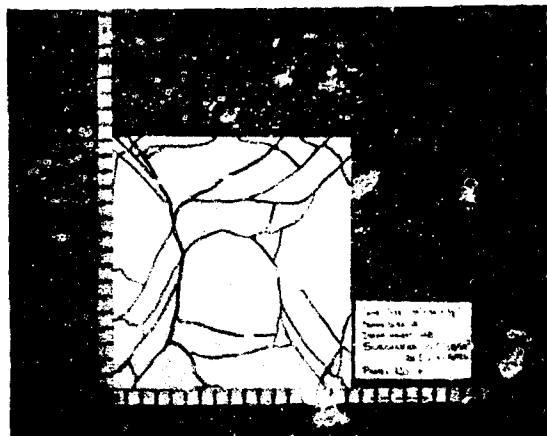
**Fig. 32 TYPICAL PLATE FRAGMENTATION**



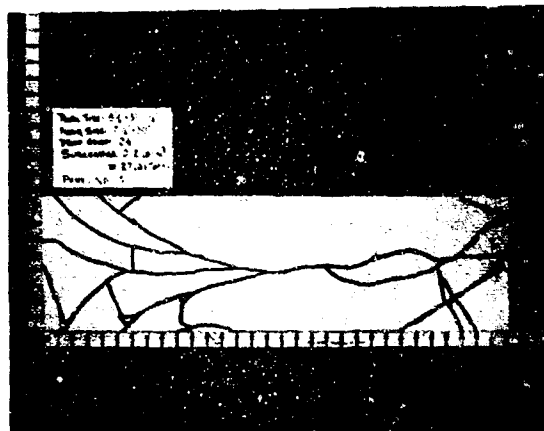
**Fig. 33 STRESS COAT PATTERN (Drop Height 36 in.,  
Total Uniform Sand Load 40 lbs)**



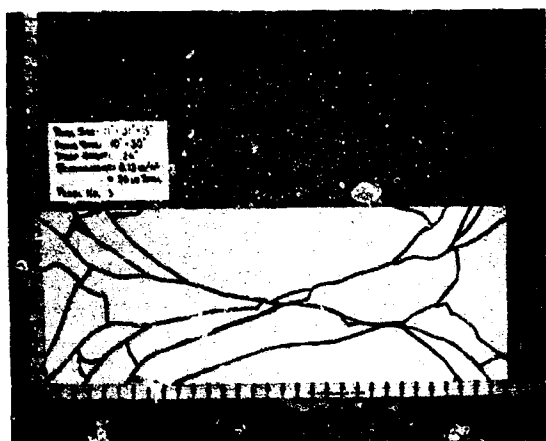
**Fig. 34 STRESS COAT PATTERN (Drop Height 18 in.,  
Total Uniform Sand Load 40 lbs)**



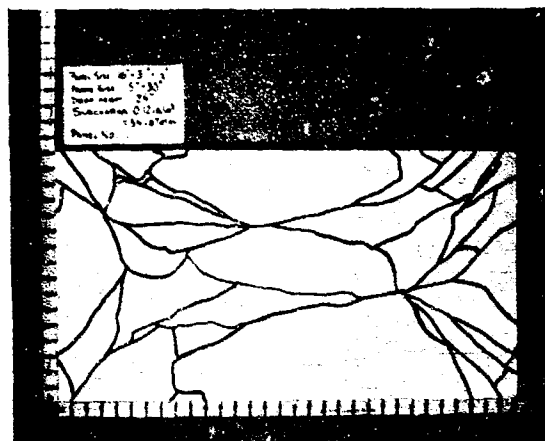
16 x 16 (1:1)



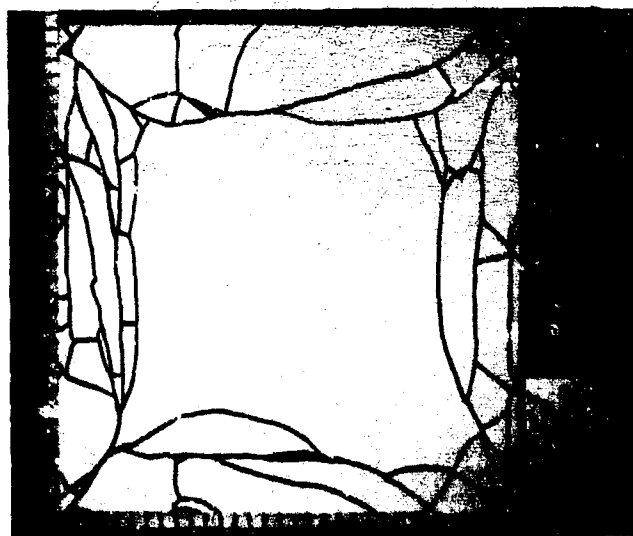
8.5 x 31 (~1:4)



11 x 31 (~1:3)



16 x 31 (~1:2)



31 x 31 (1:1)

Fig. 35 TYPICAL FRACTURE PATTERNS FOR PLASTER PLATES



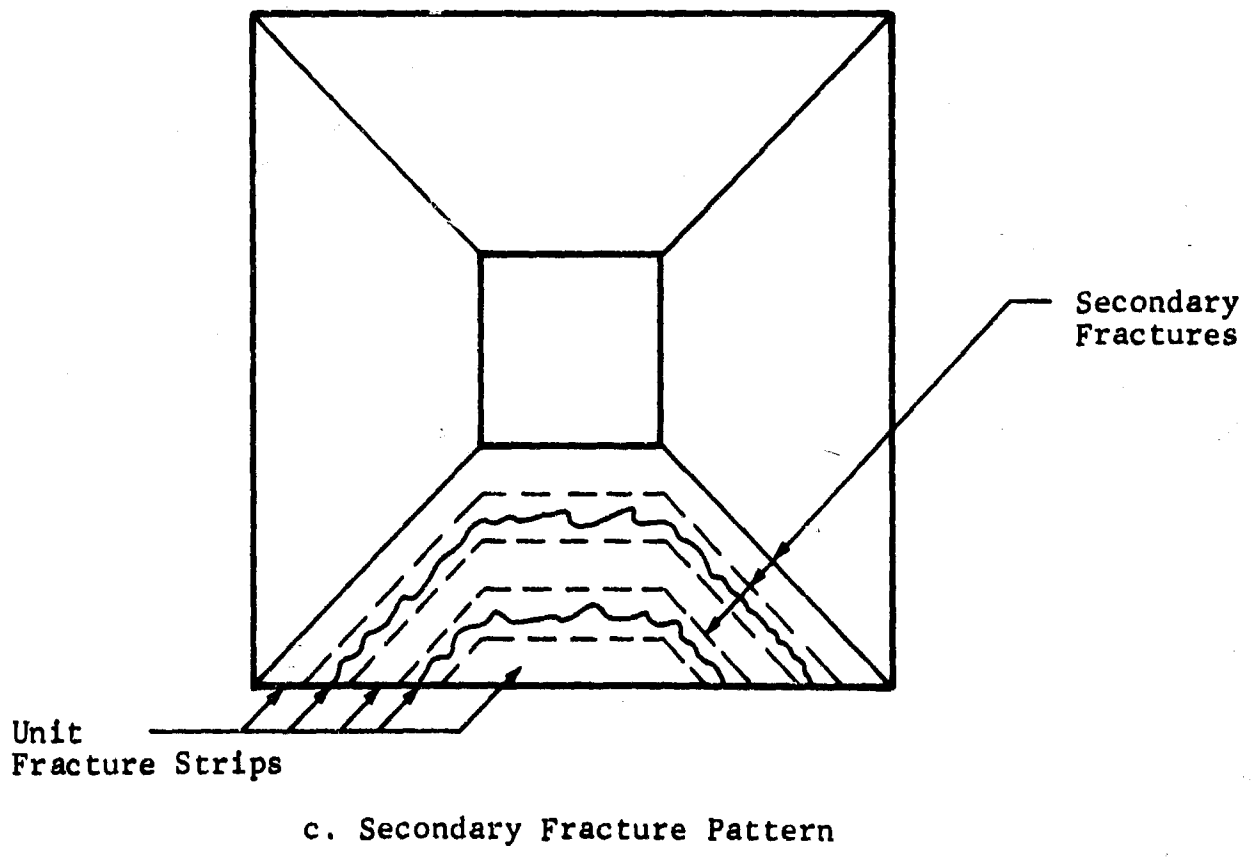
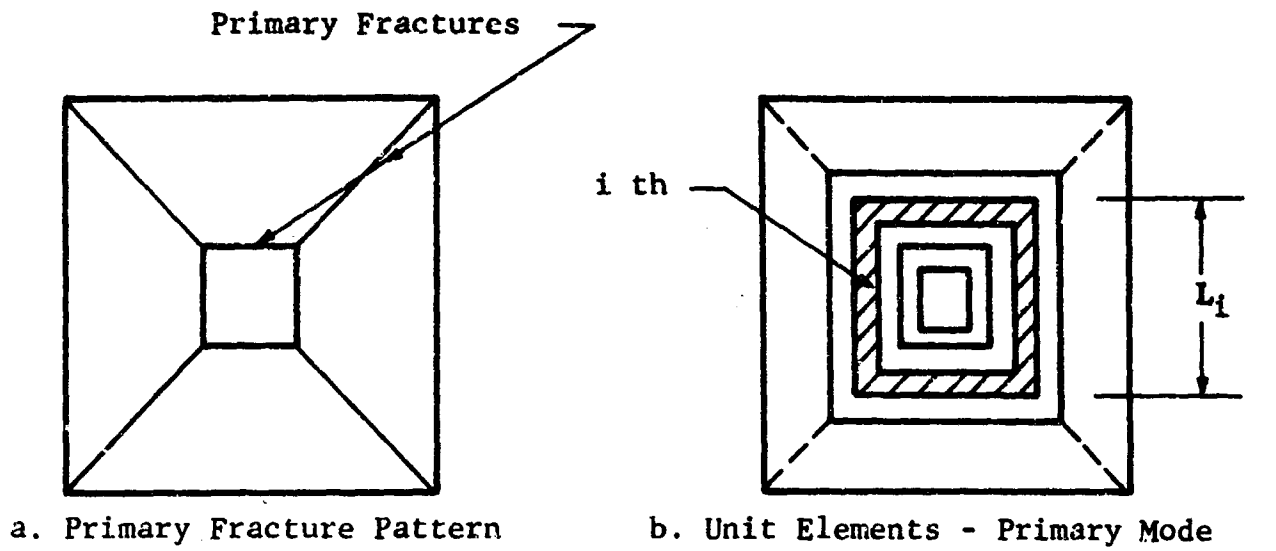


Fig. 36 FRACTURE PATTERNS FOR A SQUARE PLATE

### 3.5 FRAGMENTATION ANALYSIS

Before we describe the methods for defining primary and secondary fragments we must first establish the probability that a crack will initiate within an arbitrary region of a plate. Consider the triangular area enclosed by OBC for the square plate of Fig. 23. Assume that the principal stress trajectories suggest that the area be subdivided into the four strips identified by the Roman numerals shown in the figure. Note that strip II contains the plate subdivisions, 8, 9, 10, 11, 12, 24, 25, 26, 27, 37, 38, 39, 40, 41, 51, 52, 53, 54, 62, 63, 64 and 65. The risk of rupture for strip II,  $B_{II}$ , is equal to the sum of the risks of rupture associated with the preceding sequence of subdivisions, i.e.,

$$B_{II} = B_8 + B_9 + \dots + B_{64} + B_{65} = 0.00558$$

Now, the probability that a crack will initiate in strip II is simply

$$F_{II} = 1 - e^{-B_{II}} = 1 - e^{-0.00558}$$

We may now consider the primary mode.

#### 3.5.1 Primary Fracture Mode

To establish the size of the central square fracture pattern we will divide the central region of the plate into the imaginary square strips shown in Fig. 36b. The failure probability  $F_i$  of each of the strips will be computed and the length  $L_i$  associated with the largest  $F_i$  or  $B_i$  will be taken as the size of the square pattern.

In Fig. 37 we have computed one-eighth of the risk of rupture for each of the square strips shown in Fig. 36b. The maximum occurs in the strip containing the subdivisions 111, 112, 113, and 114. Clearly then, this defines the primary fracture mode. We note in passing that the maximum stresses decrease as we move away from the center and that the primary strip volumes increase as we move from the center. This explains why we find a relative maximum between the center and the edges.

$B_{120} = 0$

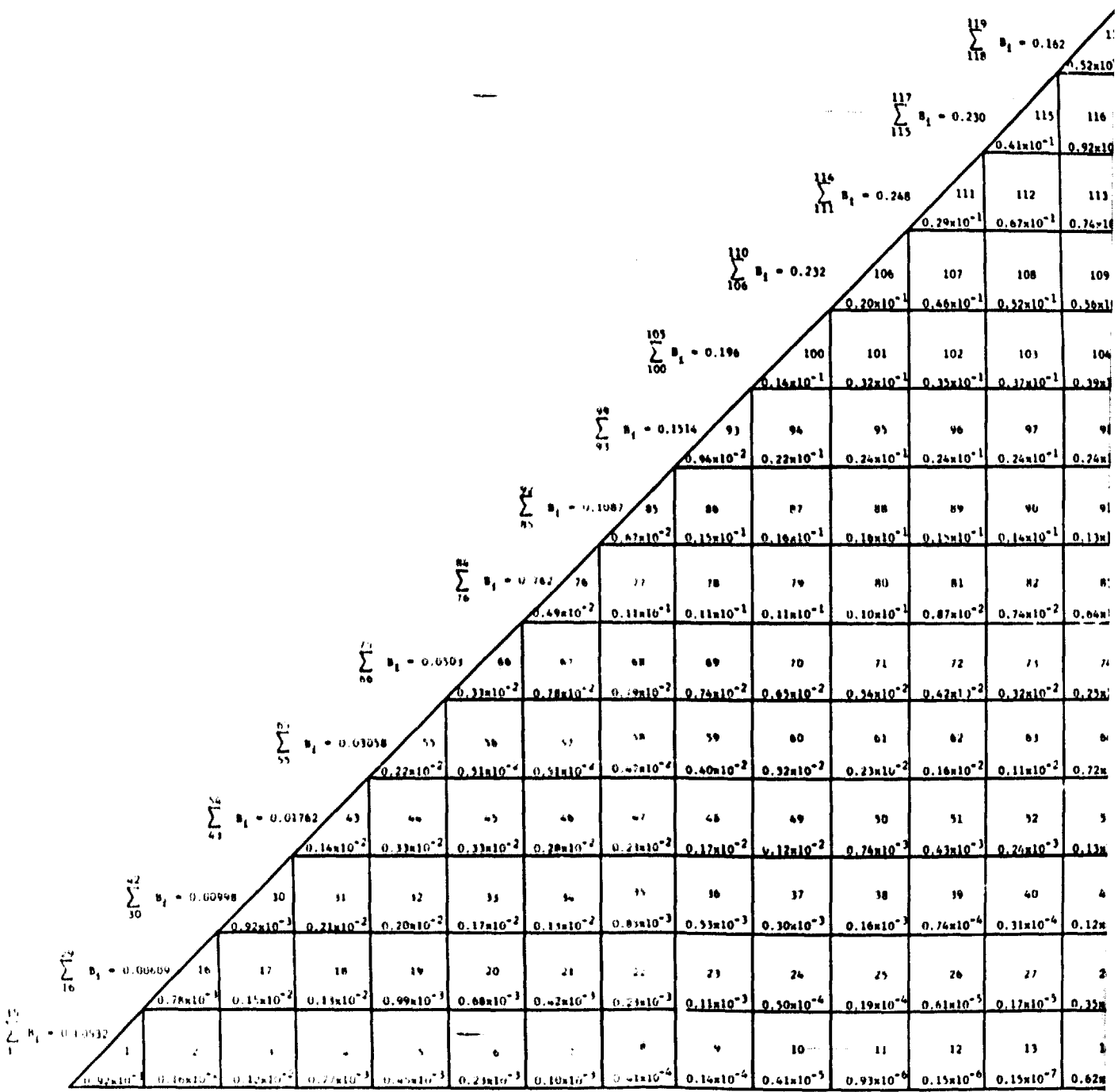


Fig. 37 RISKS OF RUPTURE ALONG HORIZONTAL STRIPS, PRIMARY MODE

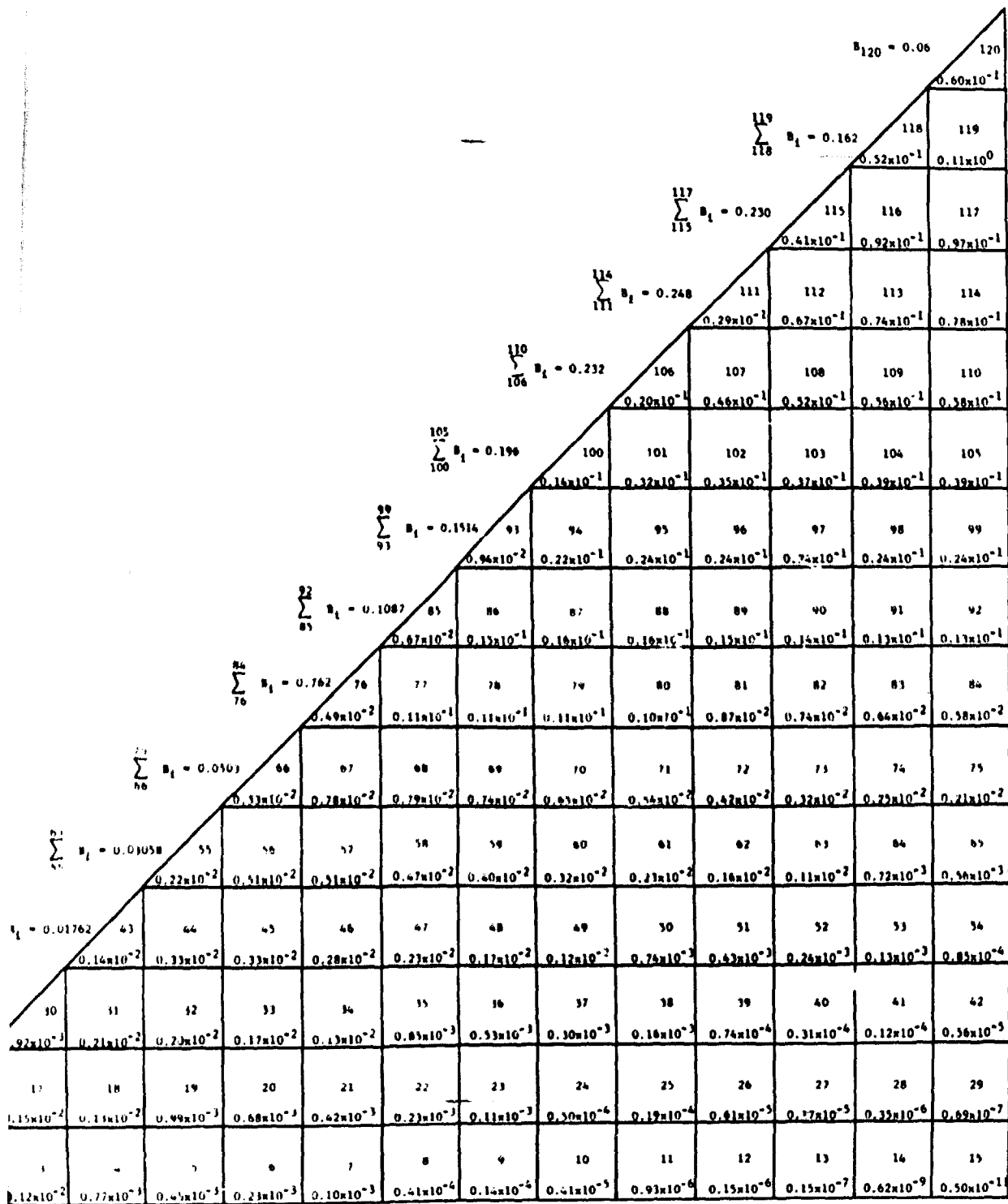


Fig. 37 RISKS OF RUPTURE ALONG HORIZONTAL STRIPS, PRIMARY MODE

We observed from the various replications of drop tests on square plates that the size of the center square remained fairly constant. If large variations would have occurred the probability of getting a size  $L_i$  is simply  $F_i$ .

### 3.5.2 Secondary Fracture Mode

Using the hypothesis that the secondary cracks will follow the principal stress trajectories, we can divide the trapezoidal regions formed by the primary cracks, into the fracture strips shown in Fig. 38. Each of these strips will independently fracture or remain intact in exactly the same manner previously described for beam fragmentation. Specifically, the simply supported plate gives rise to the same problem solved for the simply supported beam.

There are three methods available for dealing with secondary plate fractures. We begin each method by numbering the strips as shown in Fig. 38. Then, using the same technique employed to find the probability of fracture initiation in strip II of Fig. 23, we can find the fracture probability of each of the  $n$  strips indicated in Fig. 36,  $F_m$ . Finally, we observe that the periphery ABCD is a free boundary. We may now consider each method separately.

#### 3.5.2.1 Combination Method

This method, which is described in Ref. 9, considers individually each of the possible  $2^n$  combinations of failure and nonfailure of the strips. If  $F_i$  is the fracture probability of the  $i$ th strip and  $S_i$  the associated survival probability (note:  $S_i = 1 - F_i$ ), then the following combinations of fracture and survival tabulated in Table 3 are possible in a four-strip plate.

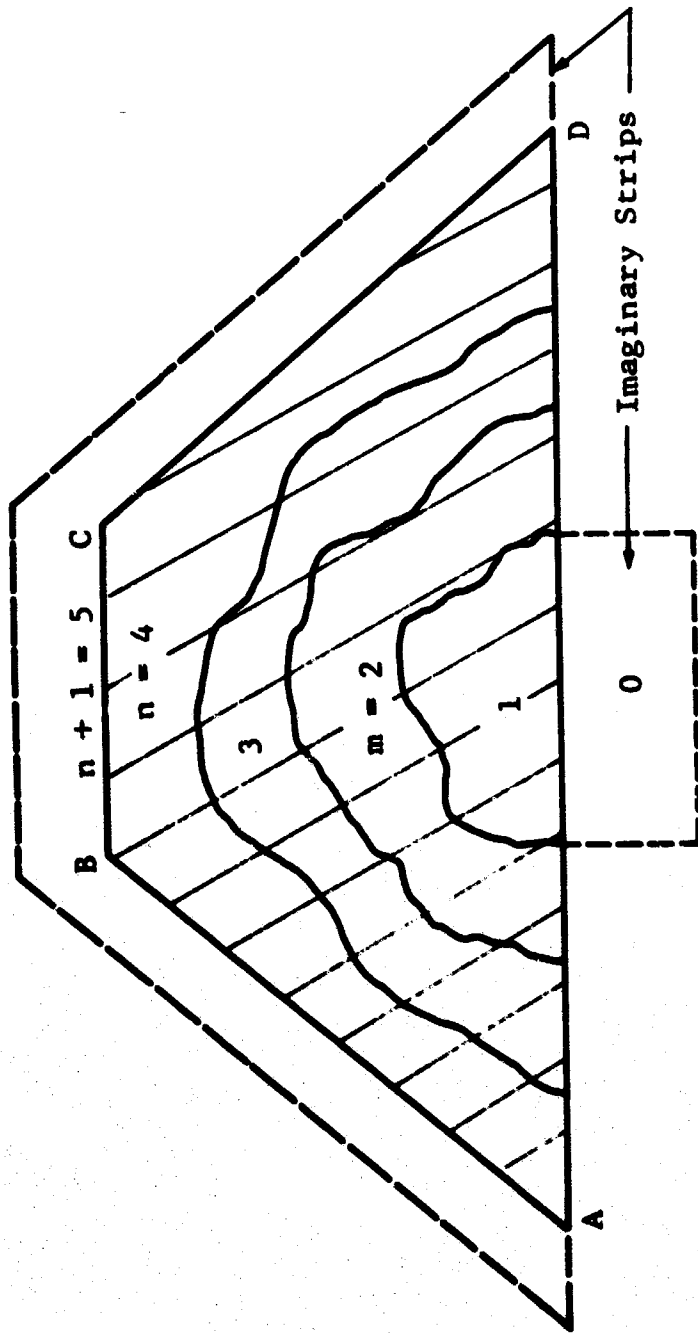


Fig. 38 NUMBERING SYSTEM FOR PLATE STRIPS

Table 3  
POSSIBLE COMBINATIONS IN A FOUR-STRIP TRAPEZOID

Combinations			
$S_1 S_2 S_3 S_4$	$S_1 S_2 S_3 F_4$	$S_1 F_2 F_3 S_4$	$F_1 F_2 S_3 F_4$
$F_1 S_2 S_3 S_4$	$F_1 F_2 S_3 S_4$	$S_1 F_2 S_3 F_4$	$F_1 S_2 F_3 F_4$
<u><math>S_1 F_2 S_3 S_4</math></u>	$F_1 S_2 F_3 S_4$	$S_1 S_3 F_3 F_4$	$S_1 F_2 F_3 F_4$
$S_1 S_2 F_3 S_4$	$F_1 S_2 S_3 F_4$	$F_1 F_2 F_3 S_4$	$F_1 F_2 F_3 F_4$

Each of these products represent the probability that the represented combination will occur. The sum of these probabilities will, of course, equal unity.

Now, let us examine a typical combination, say the underlined one, and describe its significance to the fragmentation problem. First, if  $n$  plates are dynamically loaded,  $4n$  trapezoids will give rise to secondary fractures. Consequently, the number of times the underlined combination will occur is  $4n(S_1 F_2 S_3 S_4)$ . Associated with this particular combination is the mixture of the two fragments (strip 1) and (strips 3 + 4). An examination of Table 3 indicates that these two fragments can arise from other combinations; for example, strip (3 + 4) is formed by both  $S_1 F_2 S_3 F_4$  and  $F_1 F_2 S_3 S_4$ . It is a simple matter of bookkeeping to accumulate the number of times each possible fragment occurs. On the other hand, it is very time consuming to consider each of the  $2^n$  possible combinations which generate the various fragments.

The type and efficiency of debris removal equipment will be influenced in a significant way by the composition of the debris. By studying the more frequently occurring combinations of fracture and nonfracture, it is possible to estimate the character of a mixture of fragments. The combination which appears most frequently is associated with the following probability.

$$P_{\max} = \prod_{i=1}^n \max(F_i, S_i) \quad (29)$$

If  $F_1 \neq 0.5$  this combination is unique.

### 3.5.2.2 Fragment Group Method

If we are not interested in how the various fragments are mixed together, we can adopt a very efficient procedure for calculating the total number of every possible type of fragment. In an  $n$ -strip plate segment there are  $(n/2)(n+1)$  possible combinations of contiguous strips. Each of these combinations represent a possible fragment. We can easily display these combinations as shown in Table 4 for a four-strip trapezoid. The fragments are designated by the numbers of the strips contained in the fragment. For example, fragment 2,3 is composed of the strips 2 and 3 in Fig. 38. To obtain this fragment, it is clear from this figure that strips 1 and 4 must fracture, and strips 2 and 3 must not fracture. The probability of this happening is represented as the probability of simultaneously getting fracture in strips 1 and 4 and getting no fracture in strips 2 and 3, i.e.,  $F_1 S_2 S_3 F_4$ .

As another example, we see that fragment 1,2 can be realized by survival of strips 1 and 2 followed by fracture in strip 3. It does not matter whether strip 4 fractures or not. Thus, the probability of obtaining fragment 1,2 in a trapezoid is simply  $(S_1 S_2 F_3 \cdot 1)$ . The total number of fragments "1,2" realized from  $n$  plate experiments is  $4n(S_1 S_2 F_3)$ .

If a fragment is composed of strips  $k, k+1, \dots, k+l$ , the probability of its occurrence in a trapezoid,  $P_{k,k+1,\dots,k+l}$  is given by

$$P_{k,k+1,\dots,k+l} = F_{k-1} F_{k+l+1} \prod_{i=k}^{i=k+l} (1-F_i) \quad (30)$$

where the fracture probabilities  $F_0$  and  $F_{n+1}$  represent the imaginary strips shown in Fig. 38.



For a trapezoid with a free boundary,  $F_0 = F_{n+1} = 1$ , a fixed boundary condition is represented by  $F_0 = 1$ . Until the fracture pattern is known, we cannot comment on the shape, location or behavior of the  $n+1$  strip in the fixed boundary plate.

Table 4  
NUMBER AND TYPE OF FRAGMENTS IN FOUR-STRIP TRAPEZOID

Fragment Designation	Probability of Occurrence-Trapezoid
1	$S_1F_2$
2	$F_1S_2F_3$
3	$F_2S_3F_4$
4	$F_3S_4$
1,2	$S_1S_2F_3$
2,3	$F_1S_2S_3F_4$
3,4	$F_2S_3S_4$
1,2,3	$S_1S_2S_3S_4$
2,3,4	$F_1S_2S_3S_4$
1,2,3,4	$S_1S_2S_3S_4$

### 3.5.2.3 Method of Runs

By considering every one of the possible  $2^n$  distinct fracture patterns, the method of combinations provides the specific description and quantity of every possible fragment, and in addition, it details the various possible mixtures of large and small fragments. The method of fragment groups sacrifices this latter information, but it increases the computational efficiency enormously. For example, if the number of strips  $n$  is equal to 20, the combination method considers  $2^n = 1,048,576$  distinct fracture patterns; the method of fragment groups will consider at most all of the possible fragment combinations,  $(n/2)(n+1)=2010$ .

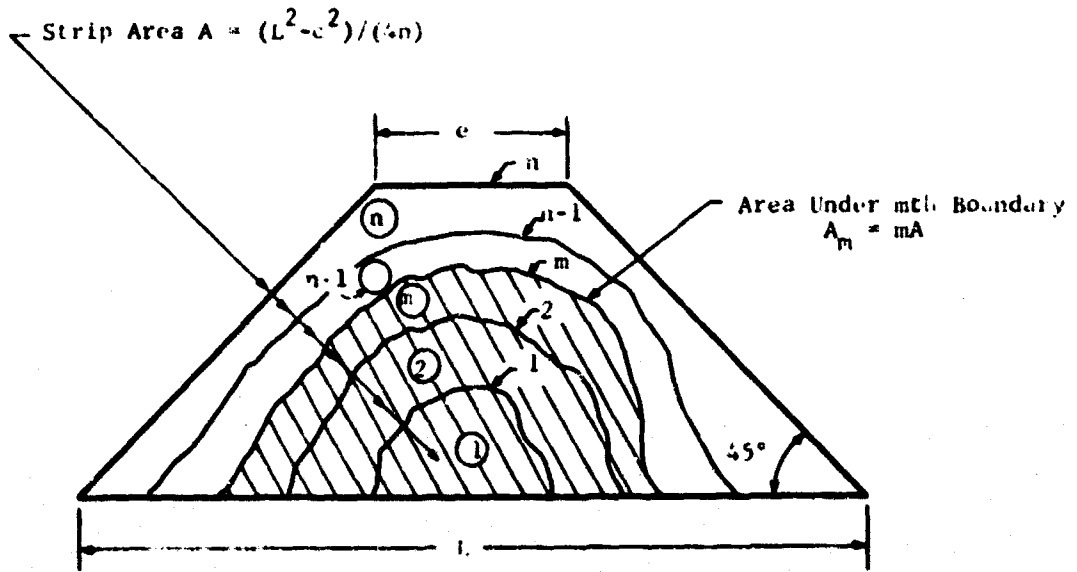
Although the increased efficiency of the method of fragment groups is considerable, an even faster method can be used if we settle for less information. This method, called the method of runs, was described in Ref. 7 for the fragmentation of beams. The procedure as developed is not directly applicable to the plate problem. To see this we shall consider the general problem of describing the fragments resulting from fractures in strips 1 and 3. For the beam we would say that we had a one-unit piece between units 1 and 3 and between unit 3 and the support; that is, units 2 and 4 remain intact. Therefore, for this combination we would have recorded two "one-unit" pieces. In the plate, a glance at Fig. 38 indicates that strips 2 and 4 are different and we cannot claim generally that we have two one-unit strips.

With this in mind, we shall begin our approach by selecting strips with equal areas as shown in general in Fig. 39a or in particular in Fig. 39b. Now, every one-strip, two-strip, and r-strip fragment (or run) has the same mass.

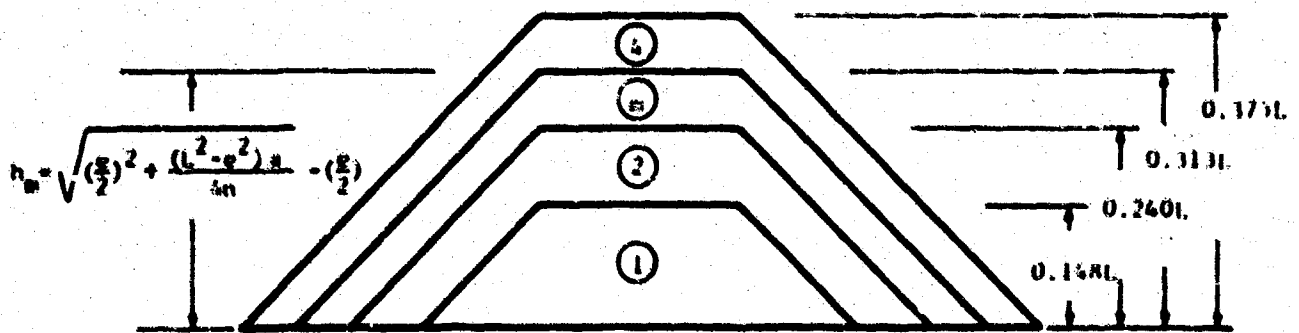
This choice of equal strip areas reduces the fragmentation problem exactly to that described for beams in Ref. 10. For example, to find the total number of two-unit runs in the four strips shown in Fig. 39b, we observe that a two-unit strip can occur in two ways: two nonfailure strips followed by a failure at either end or two nonfailure preceded and followed by failures. The probability that these events will take place is given by:

$(1-F_1)(1-F_2)F_3$	two-unit fragment, bottom
$(1-F_4)(1-F_3)F_2$	two-unit fragment, top
$F_1(1-F_2)(1-F_3)F_4$	two-unit fragment, middle

The sum of these individual probabilities is the total probability of obtaining a two-unit fragment from one trapezoid in the plate.



a) Arbitrary Strip Geometry



b) Specific Strip Geometry

Fig. 39 GEOMETRIC PROPERTIES FOR EQUAL STRIP AREAS

The general formula for calculating the probability that a run of  $r$  equal area strips will occur in a trapezoid is given by

$$P_{(r)} = \sum_{k=1}^{n-\ell} P_{k,k+1,\dots,k+\ell} \quad (31)$$

or

$$P_{(r)} = \sum_{k=1}^{n-r+1} F_{k-1} F_{k+r} \prod_{i=k}^{k+r-1} (1-F_i) \quad (32)$$

where  $F_0$  and  $F_{n+1}$  are the fracture probabilities in the two imaginary strips shown in Fig. 38. Here,  $F_0 = F_{n+1} = 1$ . Computing the fragmentation from this formula is very rapid and inexpensive; however, we know only the weight characteristics of the fragments, not their geometry or their mixture.

As a final comment we should note that the propagation of a crack is at best a temperamental and sensitive phenomenon. One should not be surprised if a single crack branches into two cracks, or if an occasional crack propagates across the principal stress trajectories. These peculiarities will produce a larger number of small fragments and a smaller number of large fragments than predicted.

CHAPTER FOUR  
TRAJECTORY OF DEBRIS PARTICLES

4.1 DESCRIPTION OF THE PHYSICAL MODEL

In order to represent the effect of debris transport and subsequent distribution, it is necessary to move from a problem space consisting of the real world to a more abstract mathematical model. This abstraction consists of representing the initial condition of possible debris as a series of lumped masses at levels above ground. Each lumped mass is characterized by a unique particle size distribution. The particle size, in turn, has weight and shape attributes associated with it. The trajectory model assumes two ideal initial conditions. These are:

- Zero failure time of fragmented elements.
- An initial particle velocity of zero.

These assumptions were made, initially, due to a lack of knowledge concerning any other possible values. A study concerning these parameters has since been made and is reported at the conclusion of this chapter. The result of this study indicates that the initial assumptions are well grounded.

4.2 INTRODUCTION TO SINBAD

SINBAD (Simulation Investigation of Nuclear Blast Associated Debris) is a problem-oriented computer language that deals with the problem of postattack structural debris. In a previous investigation (Ref.10) debris profile curves (i.e., height of debris versus distance thrown) were developed for a free-standing masonry panel wall. Several analyses, both manual and computerized, were utilized to predict the profile of a single wall. The present study is a refinement of the previous techniques and is extended to include any grouping of walls subjected to a frontal shock. It is now also possible to determine the size distribution and a measure of the momentum of the debris at any point in the profile. The language is expandable and in its entirety will

include frame response as well as the interior contents of the structure. The flow diagram illustrated in Fig. 40 indicates the general computational scheme. The boxes that are now dotted are components that will be added to the system at a later time. The remaining sections of this chapter describe the input language and sample problems run on the program.

#### 4.2.1 Input Language

The form of the input to the SINBAD processor differs significantly from most other computer programs. Format and ordering of card input have been almost eliminated; they have been replaced by a set of commands consistent with postattack terminology. The fact that a group of characters starts with a letter is sufficient to recognize a word. Similarly, a number indicates numerical data; a decimal point distinguishes a decimal number from an integer; and a blank or a comma after a group of characters indicates the end of the group.

The input commands may be data descriptors, data to be stored, or more generally information about the input process. A data descriptor (e.g., YIELD or OVERPRESSURE) communicates to the system that the number that follows is to be associated with that command. Data to be stored consist of the numerical data associated with data descriptors. Commands such as WEAPON PARAMETERS, PREBLAST STRUCTURAL CONFIGURATION and SOLVE actually control the internal flow of the program. Table 5 contains the dictionary of available commands. Each command occupies a separate input card in the data and a card may be continued by placing a dollar sign (\$) in the first column of the following cards. Each input card is printed on the system output before the solution phase of the processor takes over. It is possible to put comment cards into the input phase simply by placing an asterisk (\*) in column 1 of the card. This card is simply echo printed, but otherwise ignored. Table 6 illustrates a set of commands that is sufficient to describe a debris problem. Once the problem has been initially described for one wall and solved, it is necessary to change only those parameters which one wishes to vary in any subsequent wall or problem.

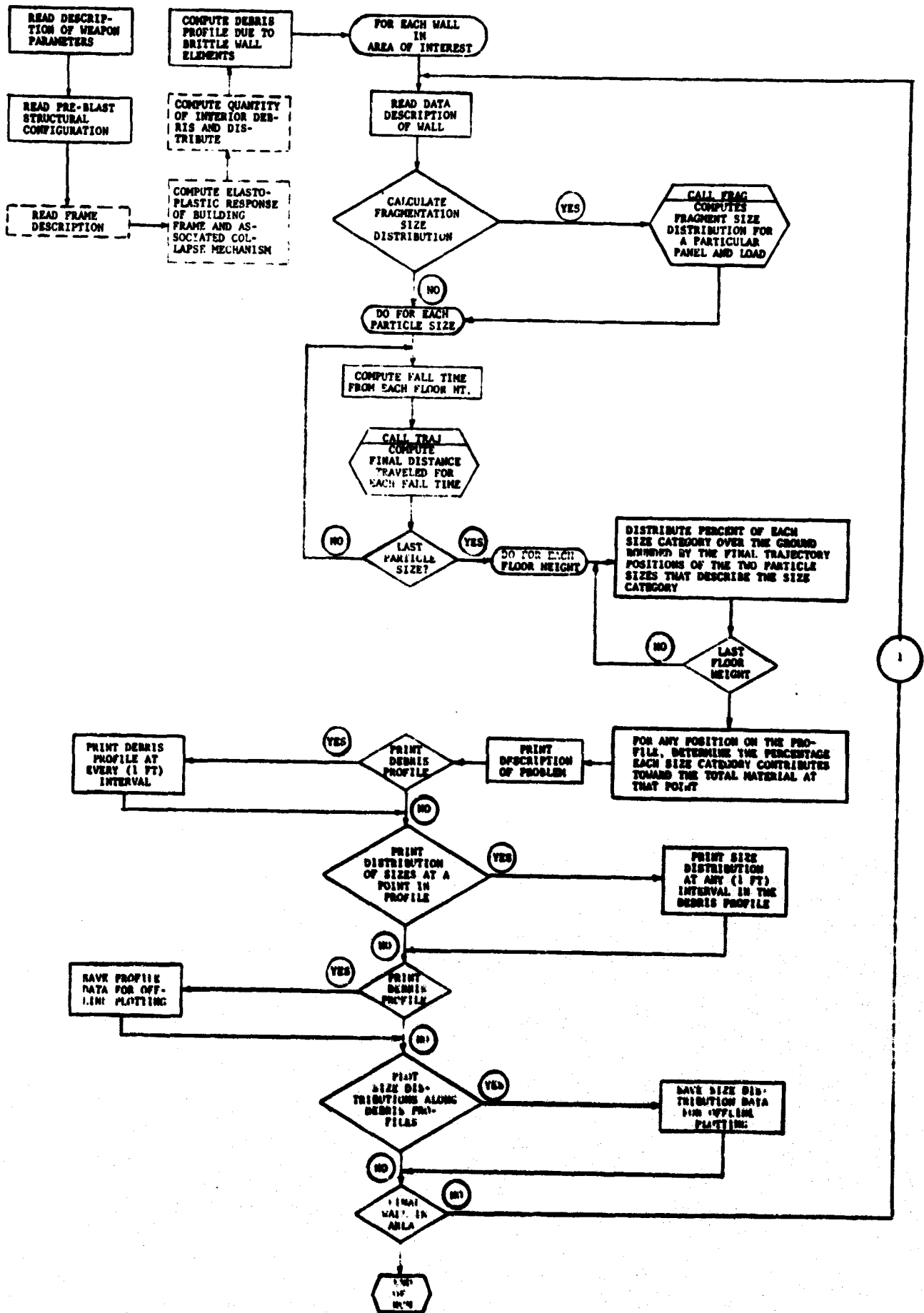


Fig. 40 COMPUTATIONAL FLOW GRAPH FOR SINBAD

Table 5  
 DICTIONARY OF PROCESS COMMANDS AND DATA DESCRIPTORS

Process Commands	Data Descriptor
WEAPON PARAMETERS	YIELD
	OVERPRESSURE
	GROUND ZERO DISTANCE
PREBLAST STRUCTURAL CONFIGURATION	WALL HEIGHT
	HEIGHT BETWEEN FLOORS
	SPACE BETWEEN WALLS
	NORMALIZING FACTOR
FRAGMENTATION CHARACTERISTICS	NUMBER OF PARTICLE SIZES
	PARTICLE SIZES
	PERCENTAGE BY SIZE
	ACCELERATION COEFFICIENT
COMPUTE FRAGMENTATION CHARACTERISTICS	BEAM REPRESENTATION
	LENGTH
	DEPTH
	WIDTH
	STRESSO
	STRESSU
OUTPUT	PROFILE DISTRIBUTION
	LOCATIONS
	DISTANCES FROM FIRST WALL
	VELOCITY DESCRIPTION
	DEBRIS PROFILE PLOT
SOLVE	



**Table 6**  
**A SUFFICIENT SET OF COMMANDS AND INPUT TO**  
**SPECIFY A DEBRIS PROBLEM TO SINBAD**

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**WEAPON PARAMETERS**

**YIELD 5000 KT**

**OVERPRESSURE 10 PSI**

**PREBLAST STRUCTURAL CONFIGURATION**

**WALL HEIGHT 40 FLOORS**

**HEIGHT BETWEEN FLOORS 10 FEET**

**DISTANCE OF WALL FROM INITIAL WALL 50 FEET**

**NORMALIZING FACTOR 1.0**

**FRAGMENTATION CHARACTERISTICS**

**NUMBER OF PARTICLE SIZES 5**

**PARTICLE SIZES 2.0, 4.0, 6.0, 8.0, 10.0 INCHES**

**PERCENTAGE BY SIZE 0.05, 0.32, 0.16, 0.32, 0.05**

**ACCELERATION COEFFICIENT 0.0**

**OUTPUT**

**PROFILE DISTRIBUTION**

**DISTRIBUTION OF SIZES**

**LOCATIONS 3**

**DISTANCES FROM FIRST WALL 50, 150, 300 FEET**

**DEBRIS PROFILE PLOT**

**SIZE DISTRIBUTION PLOT**

**SOLVE**

**WEAPON PARAMETERS**

**OVERPRESSURE = 20.0 PSI**

**SOLVE**

---

The command SOLVE terminates the input phase of the processor and transfers control to the computational section. When the specified problem is solved and the answer printed, control is automatically returned to the input phase. Each of the data descriptors will now be discussed in detail.

- The process command WEAPON PARAMETERS has three data descriptors: YIELD, OVERPRESSURE, and GROUND ZERO DISTANCE. The YIELD is the weapon size in kilotons, and is used in conjunction with either the OVERPRESSURE (psi) or GROUND ZERO DISTANCE (ft) to specify an overpressure-distance relationship. This relationship is presently based on a mach region surface burst assumption, however, as the overall system is modular in concept, airburst and regular reflection capabilities could be included with only some additional effort. Again, it is only necessary to specify either the OVERPRESSURE or the GROUND ZERO DISTANCE. Knowledge of one of these parameters, along with YIELD, is sufficient to determine the other.
- PREBLAST STRUCTURAL CONFIGURATION consists of four data descriptors that describe the wall under investigation. WALL HEIGHT gives the total number of floors (i.e., panels) in the wall. HEIGHT BETWEEN FLOORS is the panel height in feet. SPACE BETWEEN WALLS is the distance in feet of the wall presently being investigated from the last previously investigated wall. If only one wall, or the initial wall in a multiwall configuration is being investigated, this descriptor is unnecessary. Finally, a NORMALIZING FACTOR descriptor is included to account for the normalization of the debris profile curve.

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This normalization has been explained in the previous report (Ref.10) and it suffices to say that this descriptor is usually the product of an individual panel's length and thickness. If the NORMALIZING FACTOR is unity, then the subsequent debris profile will be normalized by a unit width volume (i.e., the product of the length and thickness, sq ft, of an individual panel).

- The process commands FRAGMENTATION CHARACTERISTICS of COMPUTE FRAGMENTATION CHARACTERISTICS describe the type of particles that result due to panel fragmentation. This report will only include a description of the FRAGMENTATION CHARACTERISTICS process command since the computational model of panel fragmentation is only in a formative stage at present. The panel fragmentation model, based on a beam analogy that was developed in the previous report, has been included in the present system but has not been utilized. This was done because its use was considered marginal in light of the work done on panel fragmentation as discussed in Chapter Three. Thus, the data descriptors listed under COMPUTE FRAGMENTATION CHARACTERISTICS are consistent with the input necessary for that previous fragmentation analysis. The data descriptor NUMBER OF PARTICLE SIZES indicates the number of different size particles resulting from panel fragmentation. PARTICLE SIZES is the descriptor of an array of the individual particle sizes in inch units and each is separated by a comma. This array is listed in descending order of size. In a similar manner, PERCENTAGE BY SIZE is a corresponding array of the percentages of an individual panel falling into each of the previously described particle sizes. ACCELERATION COEFFICIENT describes the shape and orientation in flight of an individual debris projectile.

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Under normal usage this parameter is set equal to zero and the program assumes a sphere of an equivalent volume radius. If it is desired to investigate other shapes with several orientations, then ACCELERATION is equal to  $2 \times \text{mass}/\text{projected area}$  in units of lb/sq in.

- The process descriptor OUTPUT controls the type of printed and computed output that can be obtained from the system. PROFILE DESCRIPTION indicates that a record of debris height as a function of distance from an initial wall is desired. The next two data descriptors are utilized to obtain the percent by size range at each desired location in the debris profile. LOCATIONS is the number of points in the debris profile where a size distribution breakdown is wanted. DISTANCES FROM FIRST WALL is the array of distances in feet from the initial wall to the points in the debris profile where a size distribution is desired. VELOCITY DESCRIPTION generates three output relationships: cumulative debris momentum, minimum debris momentum, and maximum debris momentum as a function of distance from the initial wall in feet. These relationships are normalized by the mass of an individual panel and are actually momentum per unit length along the debris profile. This will be further developed in the following section. The data descriptor DEBRIS PROFILE PLOT results in the machine plotting of the different relations previously outputted in printed form.
- The process command SOLVE transfers control from the input phase to the computational mode.

The process commands and data descriptors described above and in Table 5 can be inputted in any order, however, the order outlined in Table 6 seems to be logical.

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Once a problem has been initially defined it is only necessary to re-specify those data descriptors that change in any subsequent problem. This is also illustrated in Table 6 by the change in overpressure from 10 to 20 psi.

#### 4.3 MOMENTUM ANALYSIS

One of the primary effects of nuclear associated structural debris is the tertiary effect it has on the unsheltered population. It has been shown (Ref. 13) that whereas one may survive from free-field prompt effects of a nuclear explosion, (i.e., blast, thermal and radiation) he may still be highly vulnerable to high-speed flying debris projectiles. In order to measure the effectiveness of this type of phenomenology the projectile's mass as well as its speed must be included. This is accomplished by describing the projectile's momentum per unit length over the debris profile.

The trajectory analysis that was utilized to find the final position of flying debris also yields the projectile's final speed. Figure 41 illustrates how a normalized momentum per unit length is determined. A size range is specified by two projectile sizes. Each of these sizes has a final speed associated with it as well as a final horizontal displacement from its original position. The normalized momentum per unit length is determined by:

$$M = \frac{\left( z_i \frac{V_j + V_{j+1}}{2} \right)}{x_j - x_{j+1}} \quad (33)$$

where

$M$  is the normalized debris momentum per unit length

$z_i$  is the percent of debris falling into size range  $i$  which is composed of particle sizes falling between sizes  $j$  and  $j+1$

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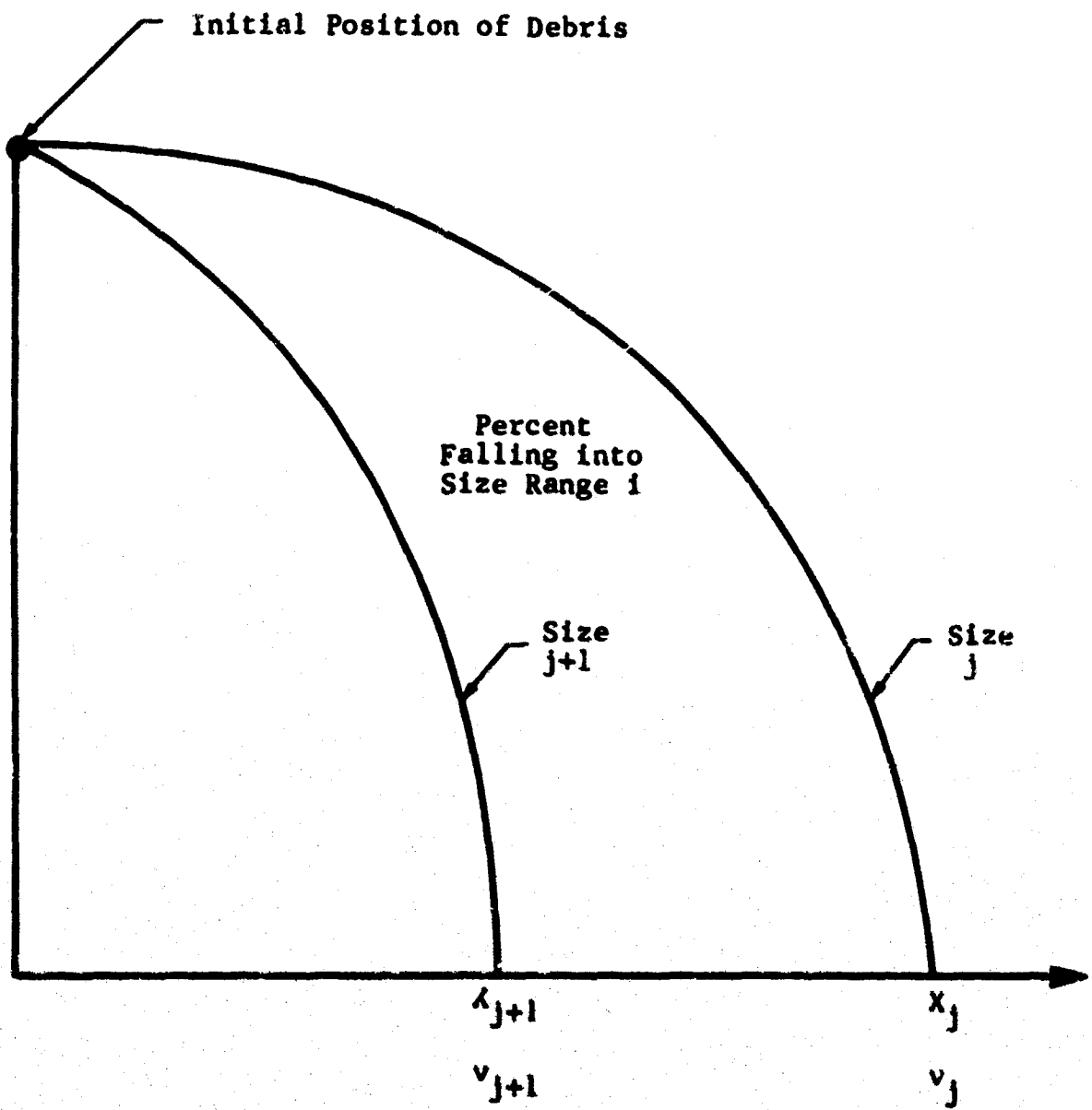


Fig. 41 FORMULATION OF MASS NORMALIZED MOMENTUM PER UNIT LENGTH

$V_j, V_{j+1}$  are the speeds (i.e., magnitude of velocities) of projectile size  $j$  and  $j+1$  respectively

$x_j, x_{j+1}$  are the final displacements of the debris particles from their initial position.

If  $\bar{M}$  is multiplied by the mass of one panel then the actual momentum per unit length may be obtained. However,  $\bar{M}$  is presently left in a mass normalized condition because this allows for window openings in the panel and variation in material properties. Presently  $\bar{M}$  is utilized to form three different relationships:

- Mass normalized cumulative debris momentum per unit length along the profile.
- Mass normalized minimum debris momentum per unit length along the profile.
- Mass normalized maximum debris momentum per unit length along the profile.

Once the value of  $\bar{M}$  has been determined, it is applied along the length of profile determined by  $x_j$  and  $x_{j+1}$ . In the case of cumulative momentum, all the individual  $\bar{M}$  for all size ranges and for all floor heights are accumulated along  $x$ . This gives an indication of the amount of projectiles and their total effect along the debris profile. The minimum momentum relationship along the profile consists of the minimum momentum of any single size range at each  $x$  location (i.e., every foot) along the debris profile. The maximum momentum is likewise the maximum effect of any single size range acting along the debris profile. The maximum and minimum momentum relationships along the debris profile establish bounds on the individual projectile's momentum. Whereas the maximum and minimum momentum bounds give the effect of individual projectiles, the cumulative momentum is some indication of the effect of many projectiles landing at any one spot along the debris profile. These relationships, when coupled with available biological data as to impact, are sufficient to estimate the casualties caused by flying structural debris.

#### 4.4 SAMPLE PROBLEMS

Two example problems were run on SINBAD to illustrate the system versatility. The results of these problems are only presented to demonstrate the problem solving capability of the system. They are not meant to illustrate actual debris situations. The first problem is illustrated in Fig. 42 and includes four free-standing frangible walls all of the same length. No shielding of one wall by another is assumed to take place since the example is designed to show the superposition procedure alone. The time to fragmentation of all walls is assumed to be zero as is the initial velocity of all fragments. In this problem all calculated parameters (i.e., debris profile, size distributions at selected points in the profile, and momentum) were printed first for one wall, then two, three and finally all four walls. The input to the problem is also printed and both it and the output are displayed as Appendix C. Plots of the profiles resulting from the different wall combinations are illustrated in Fig. 43 through 46. All profile distances are relative to the first wall and the remaining walls are located down wind of the first wall. It may be seen that this example illustrates that multiple wall configurations may be studied and that the walls can have different structural configurations.

The second sample problem which is independent of the first example demonstrates how a variation of parameters study on the aerodynamic properties of a single brick may be carried out conveniently with the system. A free-standing wall, 40 floors at 400 ft high, consisting of only a single size particle (i.e., a masonry brick with nominal dimensions of 2-1/4 x 3-3/4 x 8 in.) is exposed to a 1 MT weapon. The brick has essentially three orientations: side-on, face-on and end-on. The aerodynamic properties of these three orientations have been documented previously (Ref. 14). Five separate cases were run on SINBAD. These included:

- A volume equivalent sphere.
- Side-on orientation.
- Face-on orientation.
- End-on orientation.
- The numerical average of cases 2, 3 and 4.



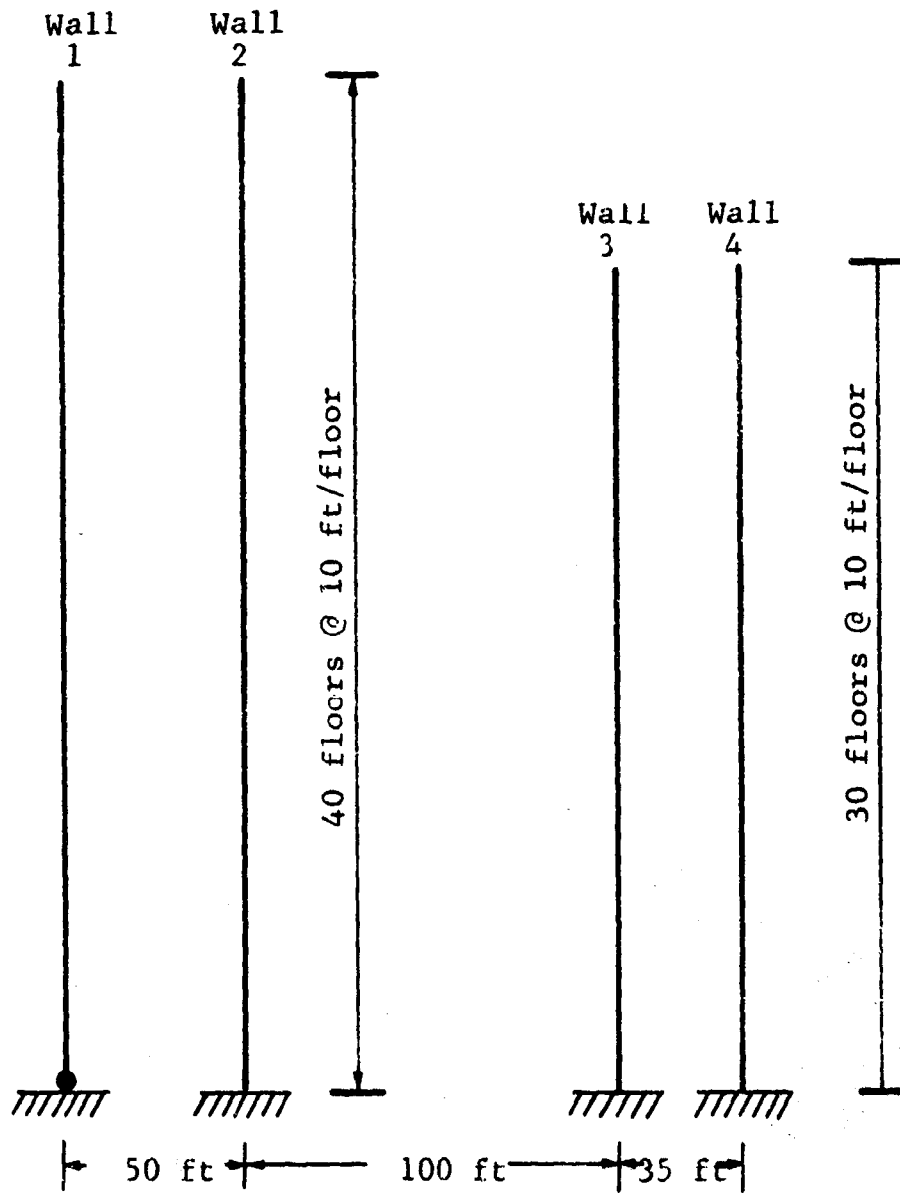


Fig. 42 STRUCTURAL CONFIGURATION FOR SAMPLE PROBLEM 1

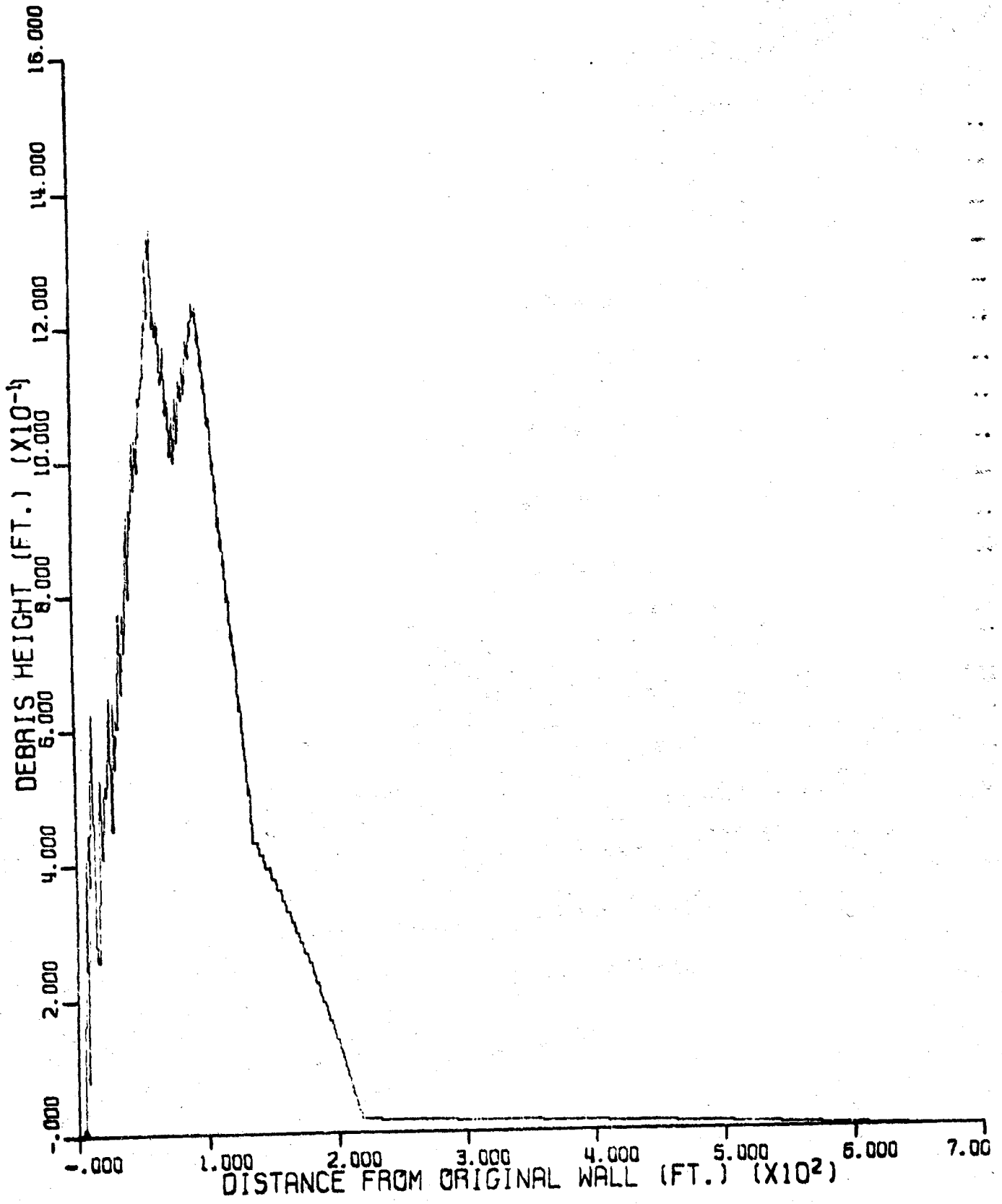


Fig. 43 DEBRIS HEIGHT OF WALL 1

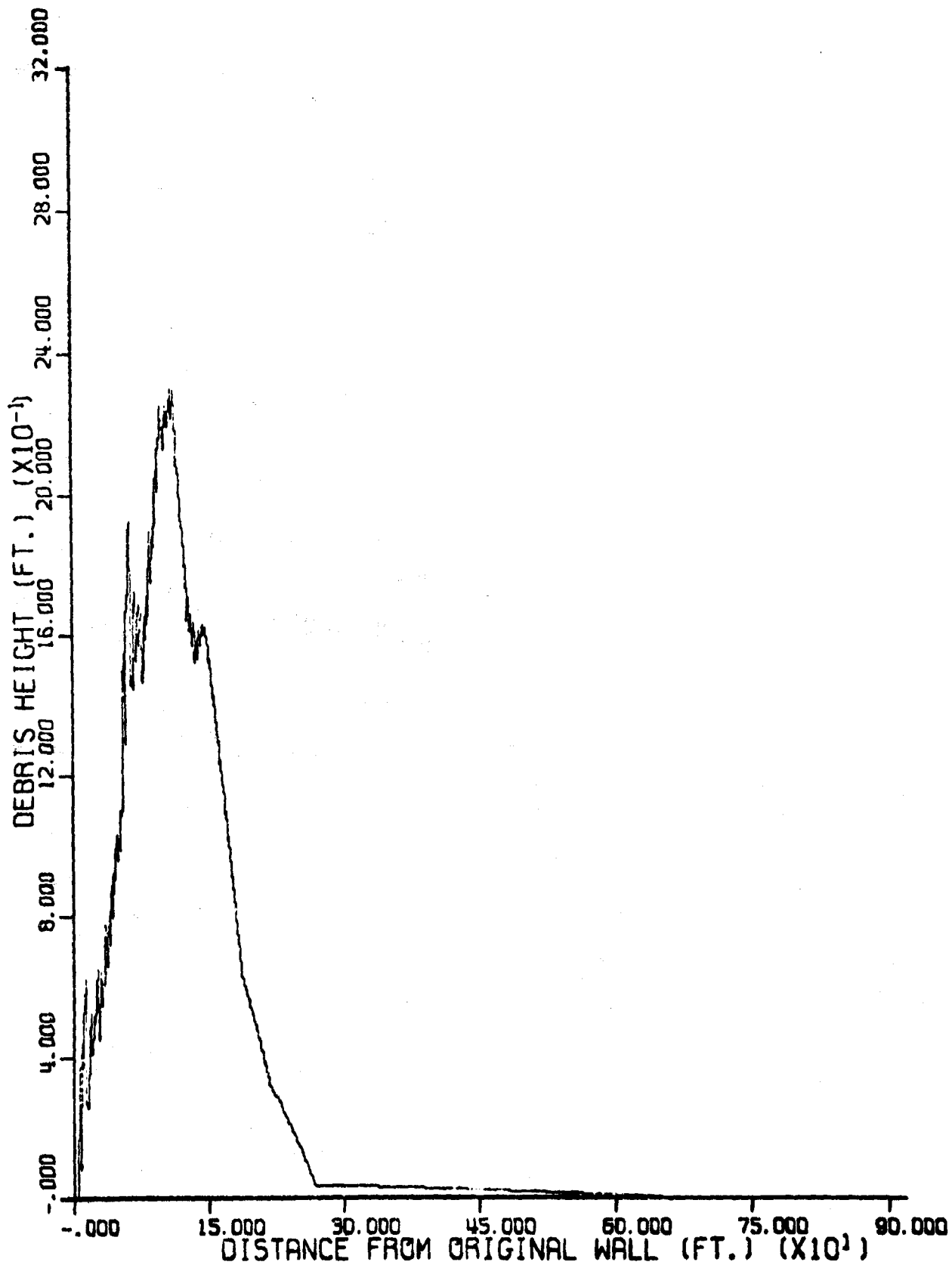


Fig.44 DEBRIS HEIGHT OF WALLS 1 AND 2

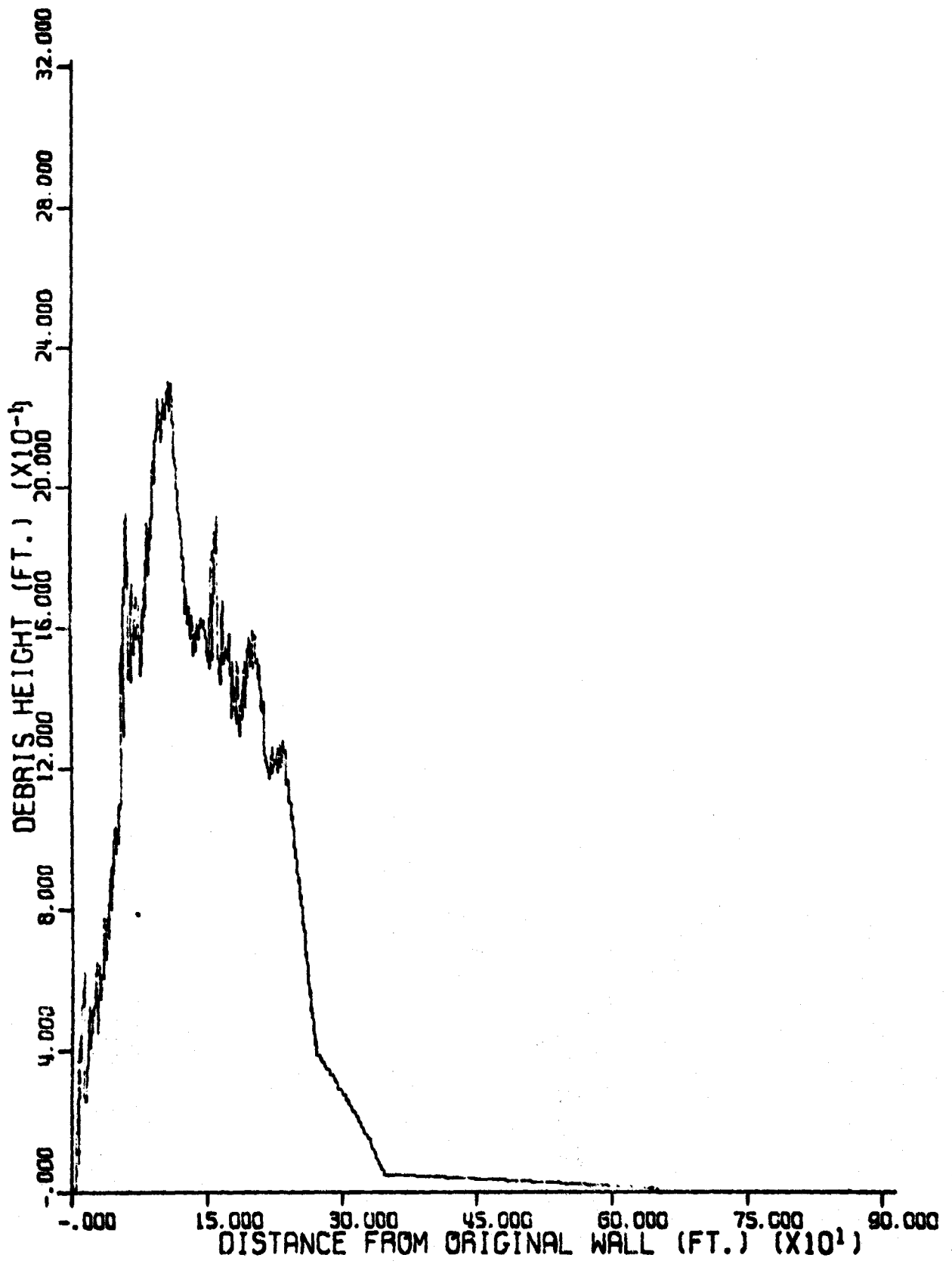


Fig. 45 DEBRIS HEIGHT OF WALLS 1, 2, AND 3

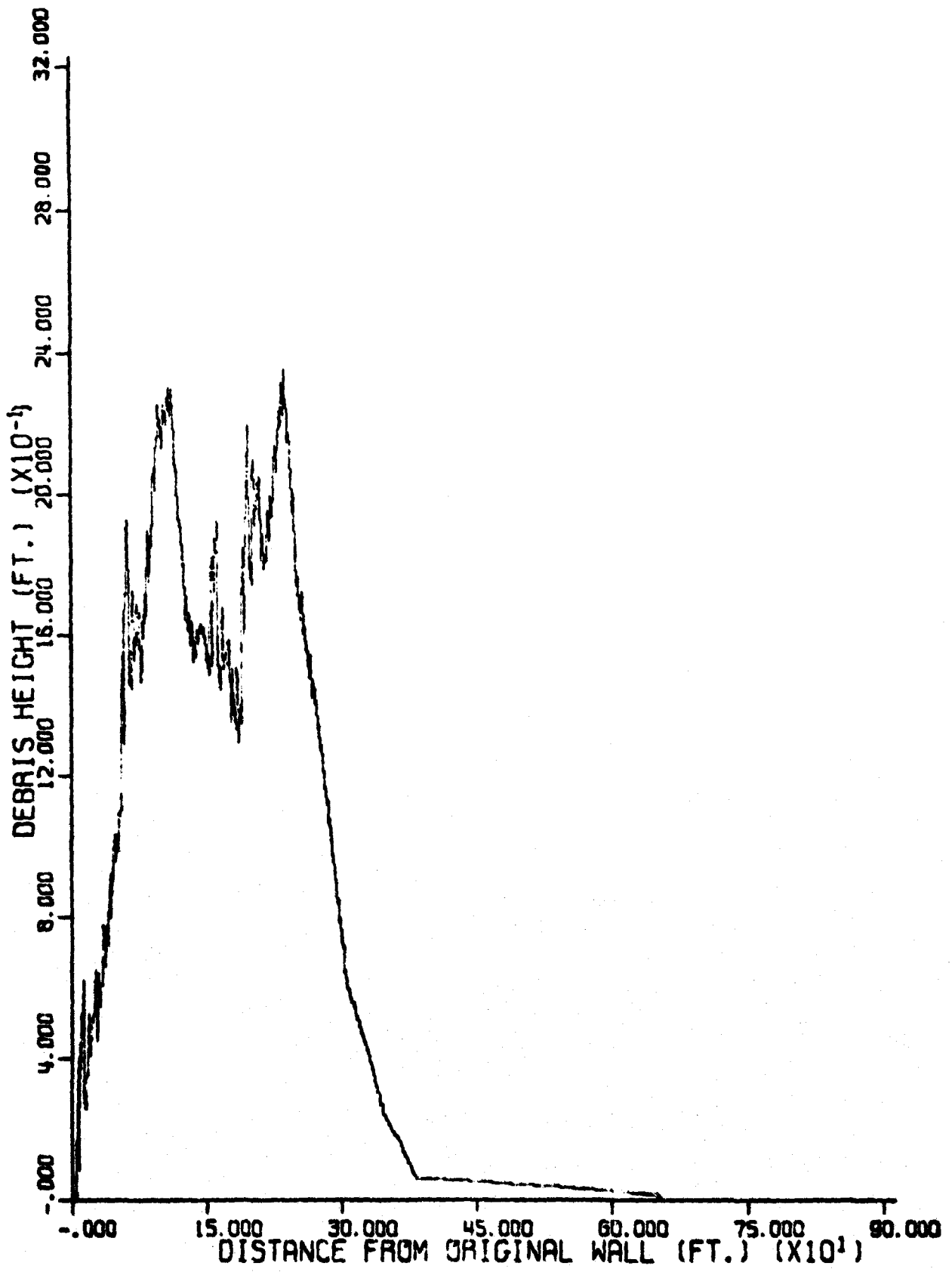


Fig. 46 DEBRIS HEIGHT OF WALLS 1, 2, 3 AND 4

The results of the analysis are summarized in Fig. 47 through 51 for debris profiles, Fig. 52 through 56 for normalized cumulative debris momentum, and Fig. 57 through 61 for maximum and minimum bounds on normalized debris.

It is perhaps interesting to note the almost exact correspondence between cases 1 and 5 of this problem. This is to be expected since an object with the dimensions of a brick is not very different in shape from a spherical object when an average orientation is assumed. Larger objects with more extreme dimensional variation will probably not display this similarity.

The two examples presented were to show the versatility of SINBAD. Thus it is difficult to draw specific conclusions as to debris dispersal from these two problems. The second example however, does illustrate that maximum cumulative debris occurs at the same point down range as maximum debris depth. This fact is substantiated by Table 7. Intuitive reasoning would lead to this same conclusion since the point of maximum debris height is more than likely the point where the most individual particles fall. The maximum momentum of an individual particle falls much closer to the wall than the maximum cumulative momentum.

Table 7  
SUMMARY OF RESULTS OF EXAMPLE PROBLEM 2

Particle Type	Maximum Height	Momentum	
		Cumulative	Distance
Sphere (Fig. 47)	0.72	210	380
Side-on (Fig. 48)	0.60	250	480
Face-on (Fig. 49)	0.52	290	560
End-on (Fig. 50)	1.25	110	220
Average (Fig. 51)	0.73	190	370
Particle Type	Maximum Height	Distance @ Maximum Height	Maximum Distance
Sphere (Fig. 52)	0.72	210	380
Side-on (Fig. 53)	0.60	250	480
Face-on (Fig. 54)	0.52	290	560
End-on (Fig. 55)	1.25	110	220
Average (Fig. 56)	0.73	190	370

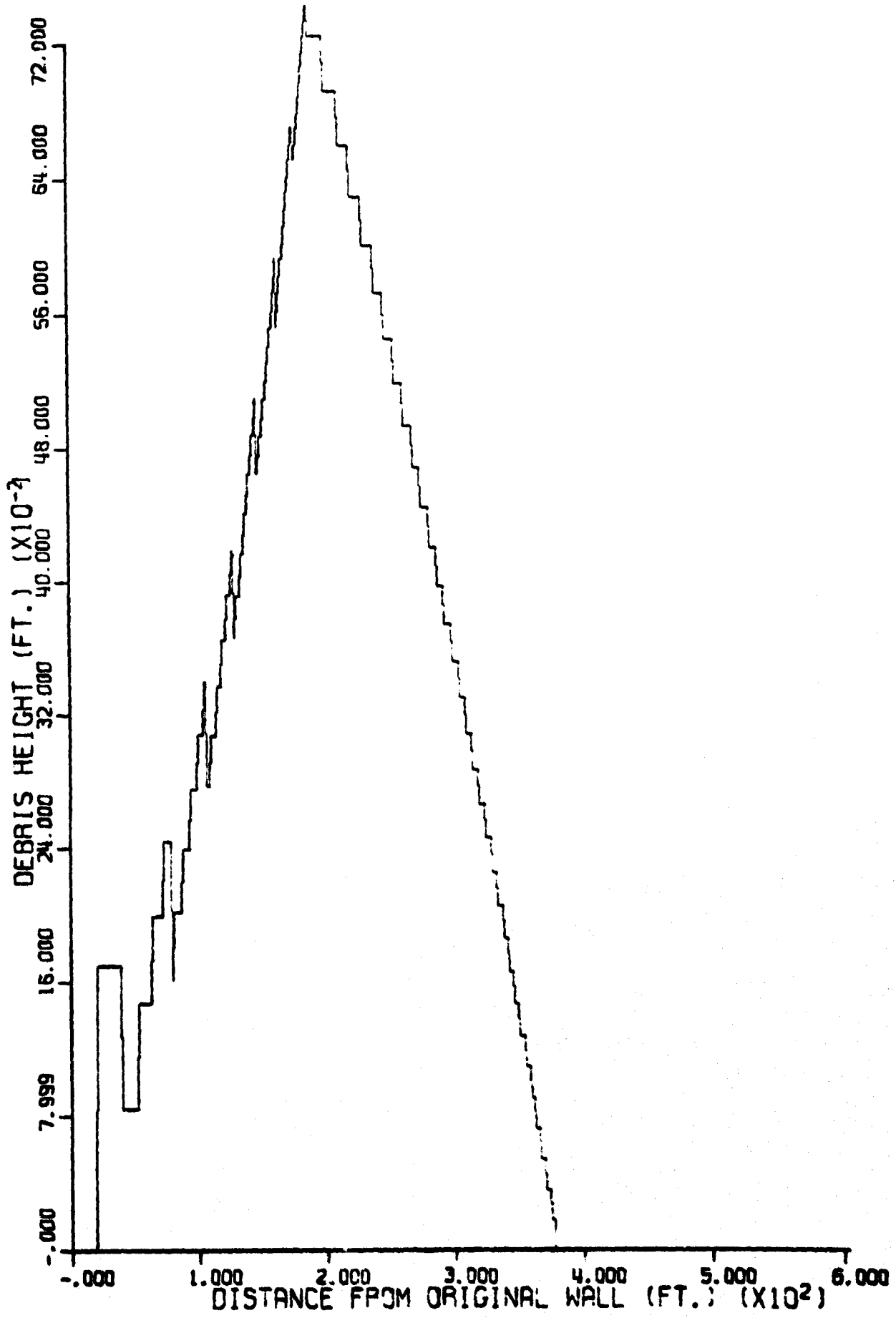
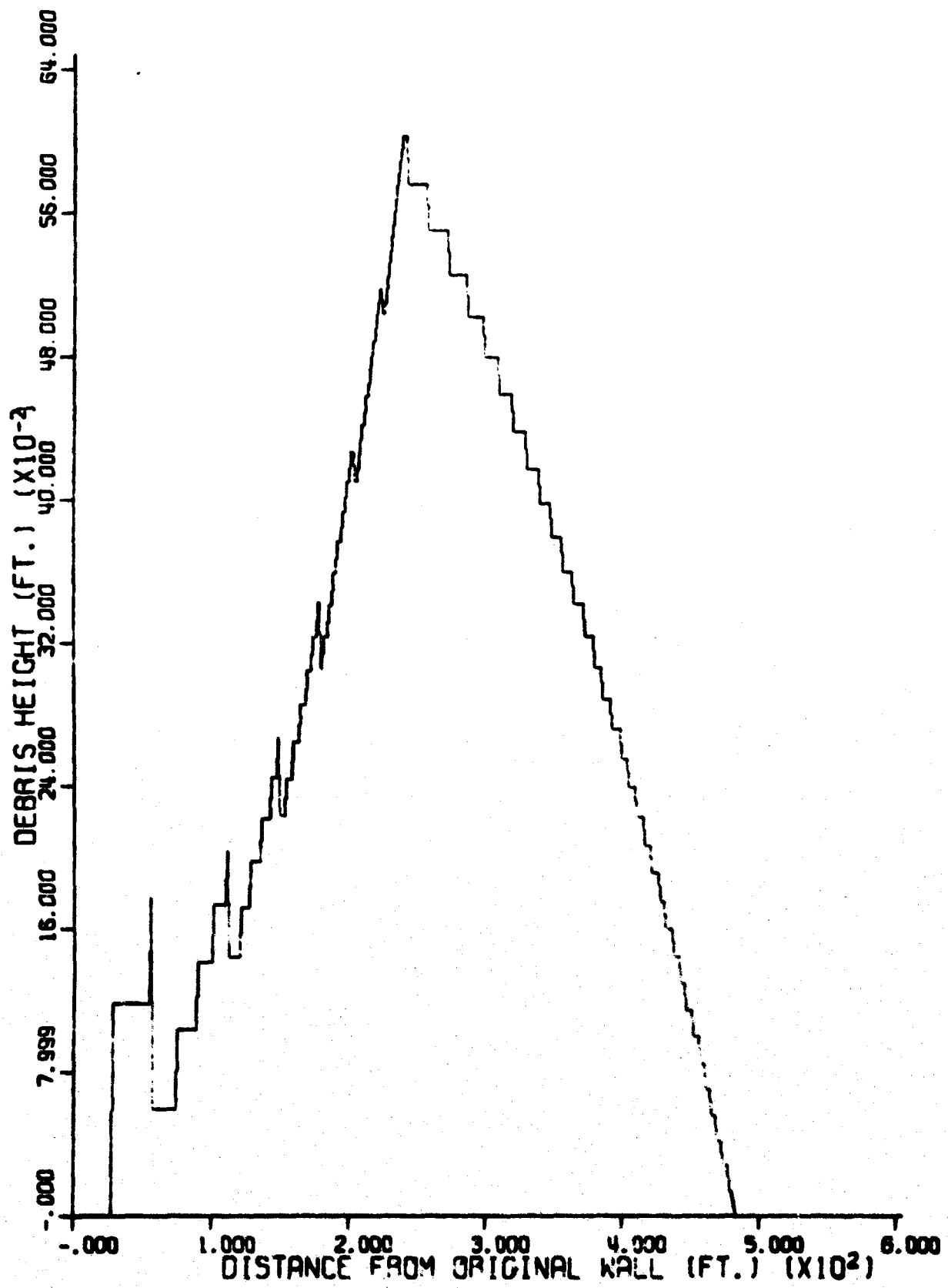


Fig. 47 DEBRIS PROFILE OF EQUIVALENT SPHERICAL BRICK PARTICLE



**Fig. 48 DEBRIS PROFILE OF BRICK PARTICLE IN SIDE-ON ORIENTATION**



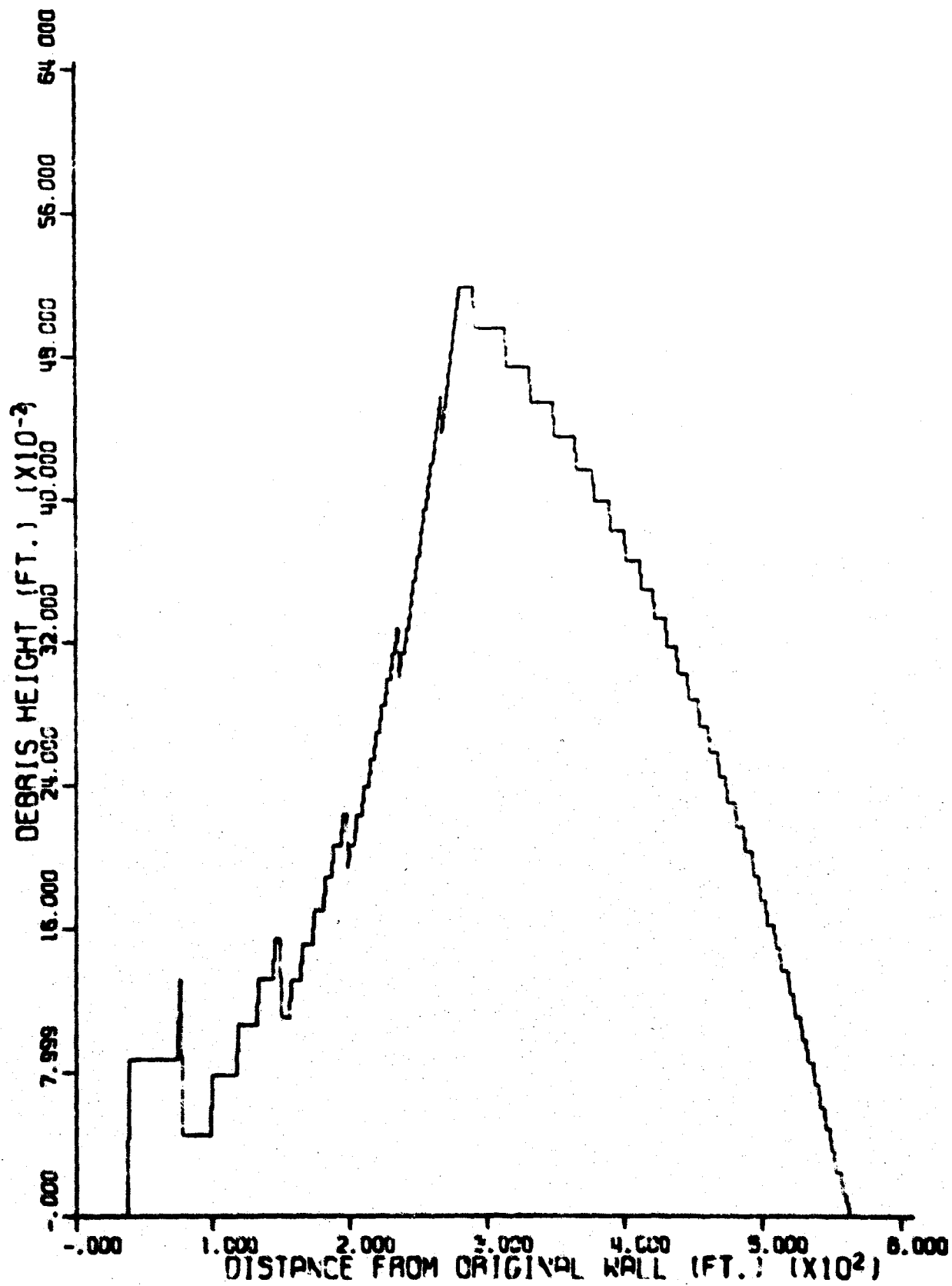


Fig. 49 DEBRIS PROFILE OF BRICK PARTICLE IN FACE-ON ORIENTATION

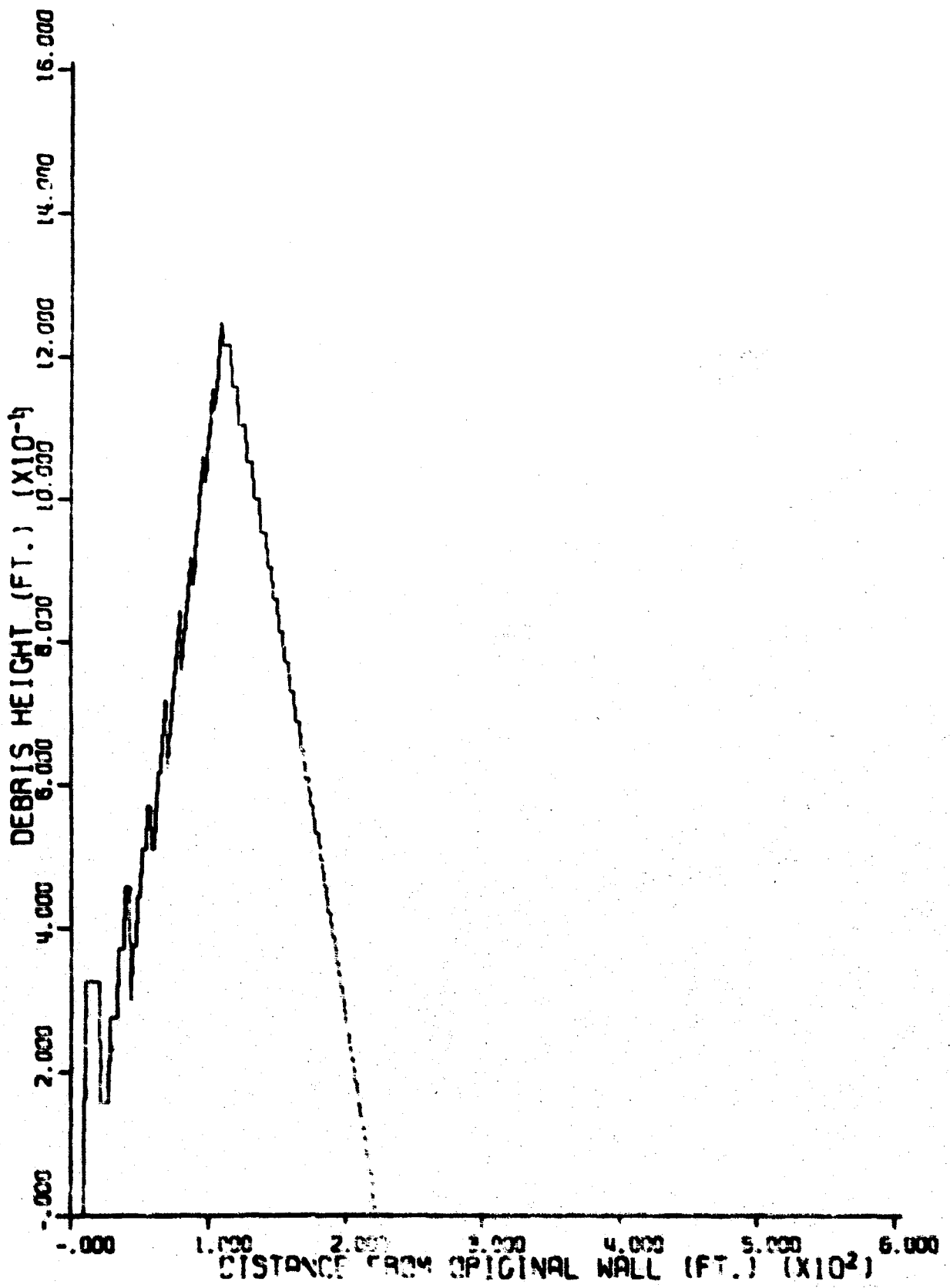


Fig. 50 DEBRIS PROFILE OF BRICK PARTICLE IN END-ON ORIENTATION

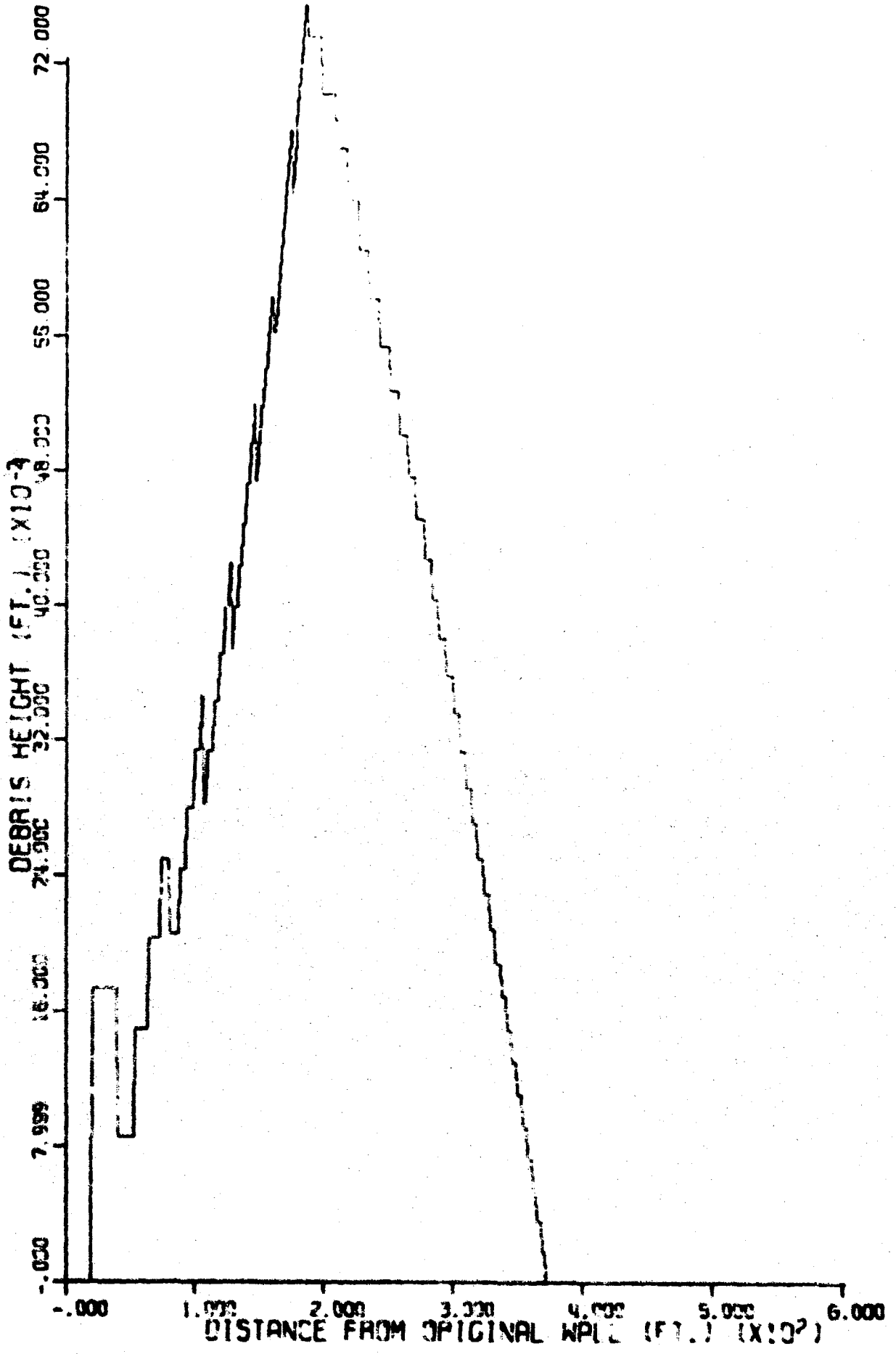


Fig. 51 DEBRIS PROFILE OF BRICK PARTICLE IN AVERAGE ORIENTATION

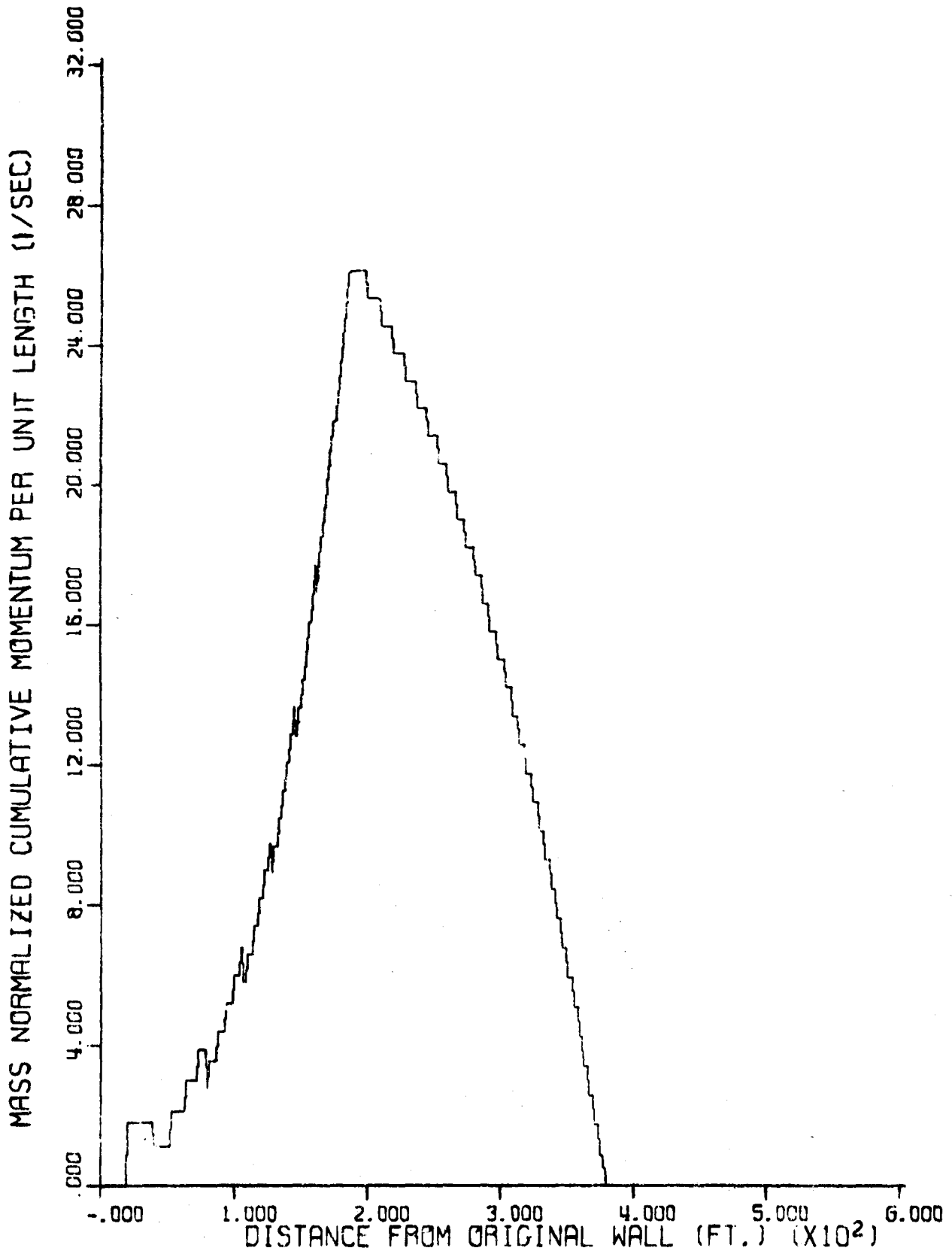


Fig. 52 CUMULATIVE MOMENTUM ALONG DEBRIS PROFILE FOR EQUIVALENT SPHERICAL PARTICLE

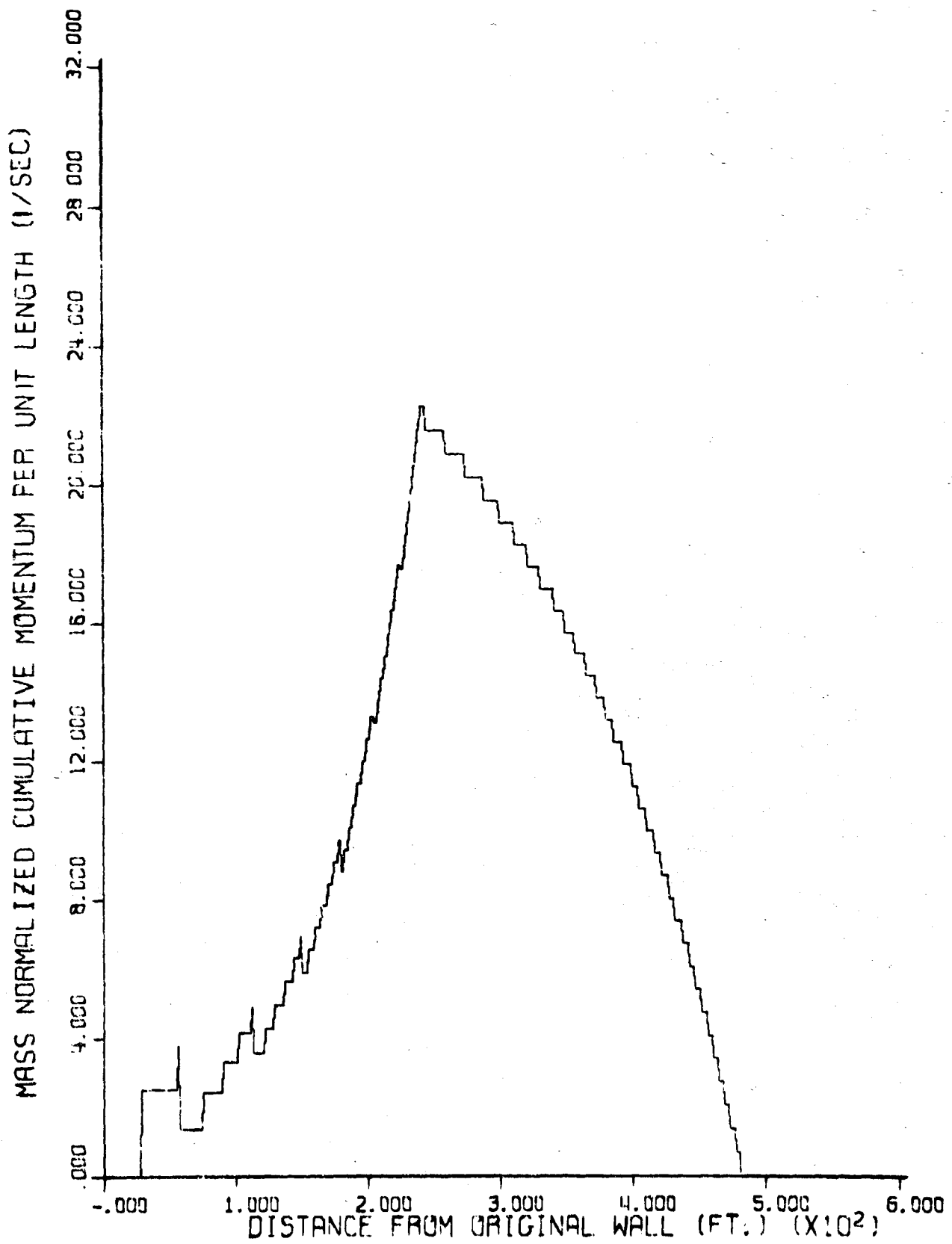


Fig. 53 CUMULATIVE MOMENTUM ALONG DEBRIS PROFILE FOR SIDE-ON ORIENTATION

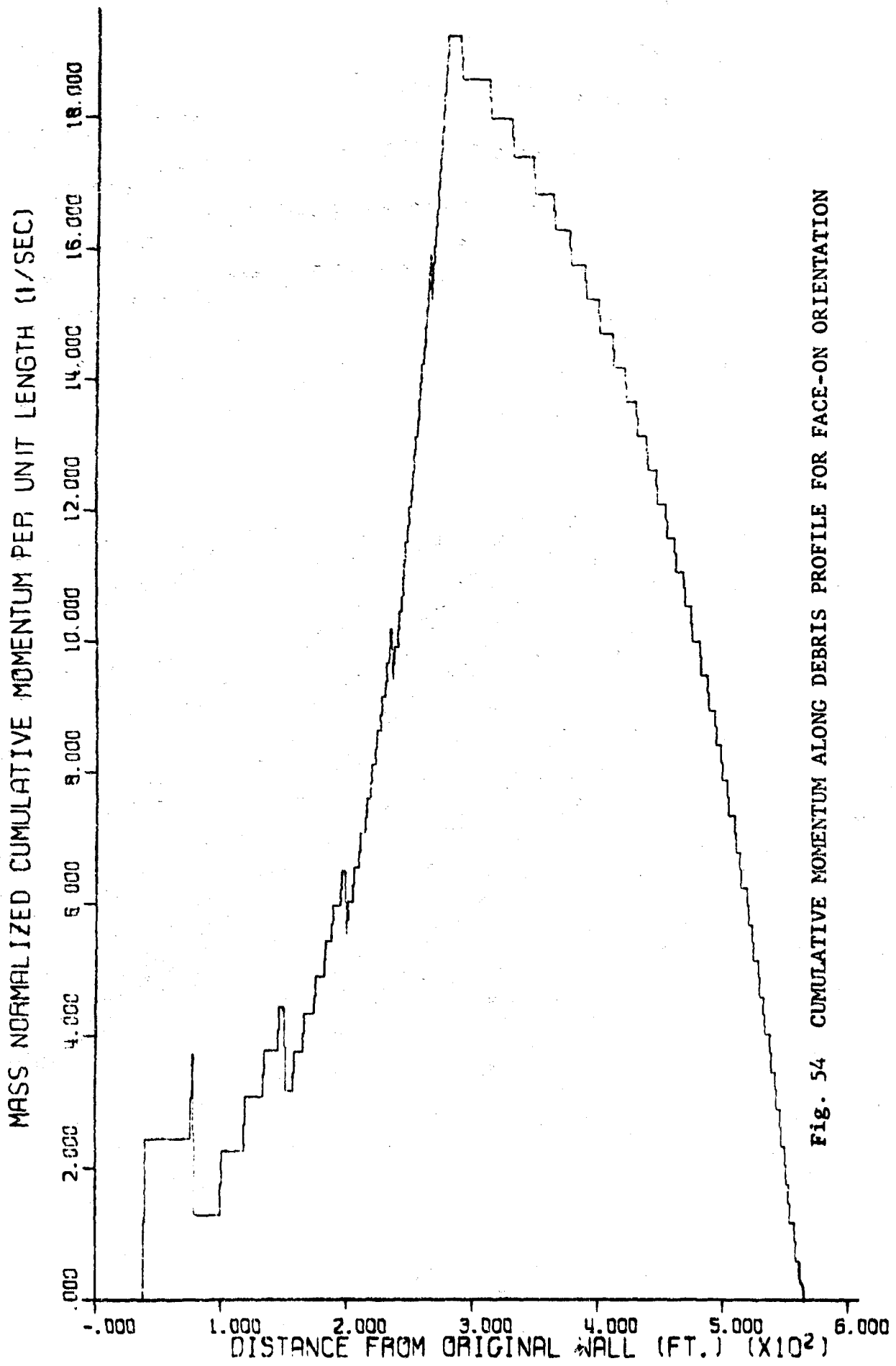


Fig. 54 CUMULATIVE MOMENTUM ALONG DEBRIS PROFILE FOR FACE-ON ORIENTATION

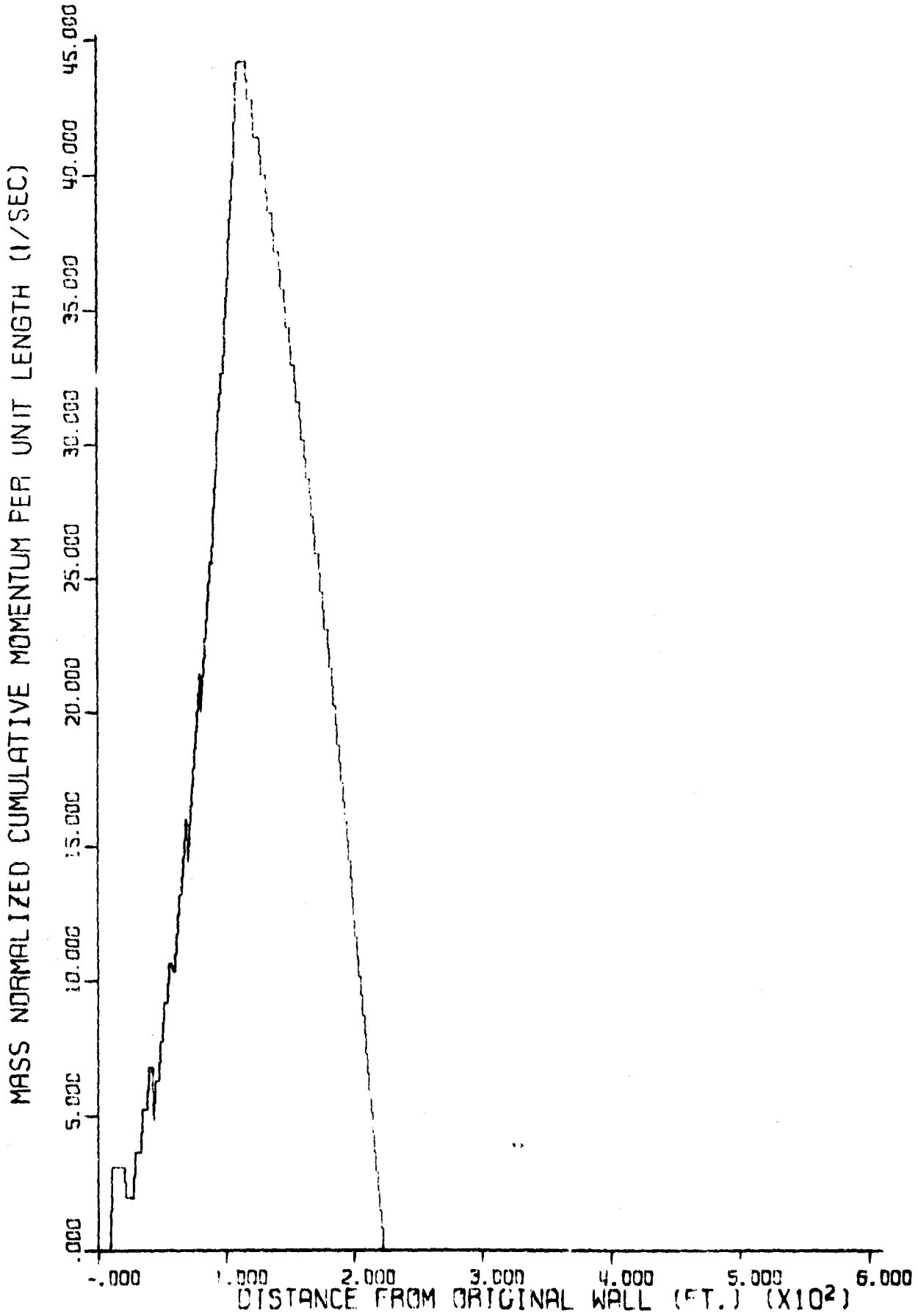


Fig. 55 CUMULATIVE MOMENTUM ALONG DEBRIS PROFILE FOR END-ON ORIENTATION

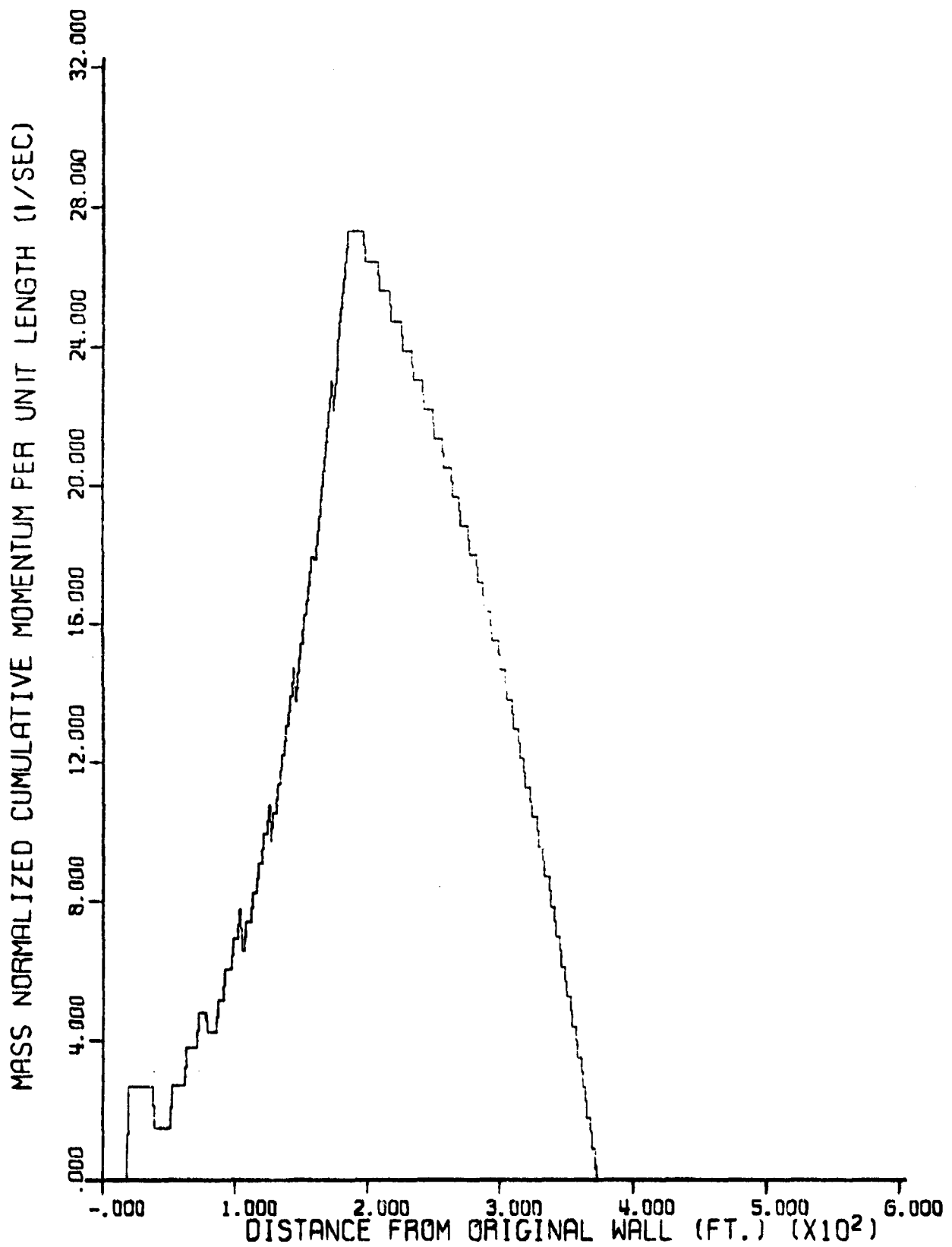


Fig. 56 CUMULATIVE MOMENTUM ALONG DEBRIS PROFILE FOR AVERAGE ORIENTATION



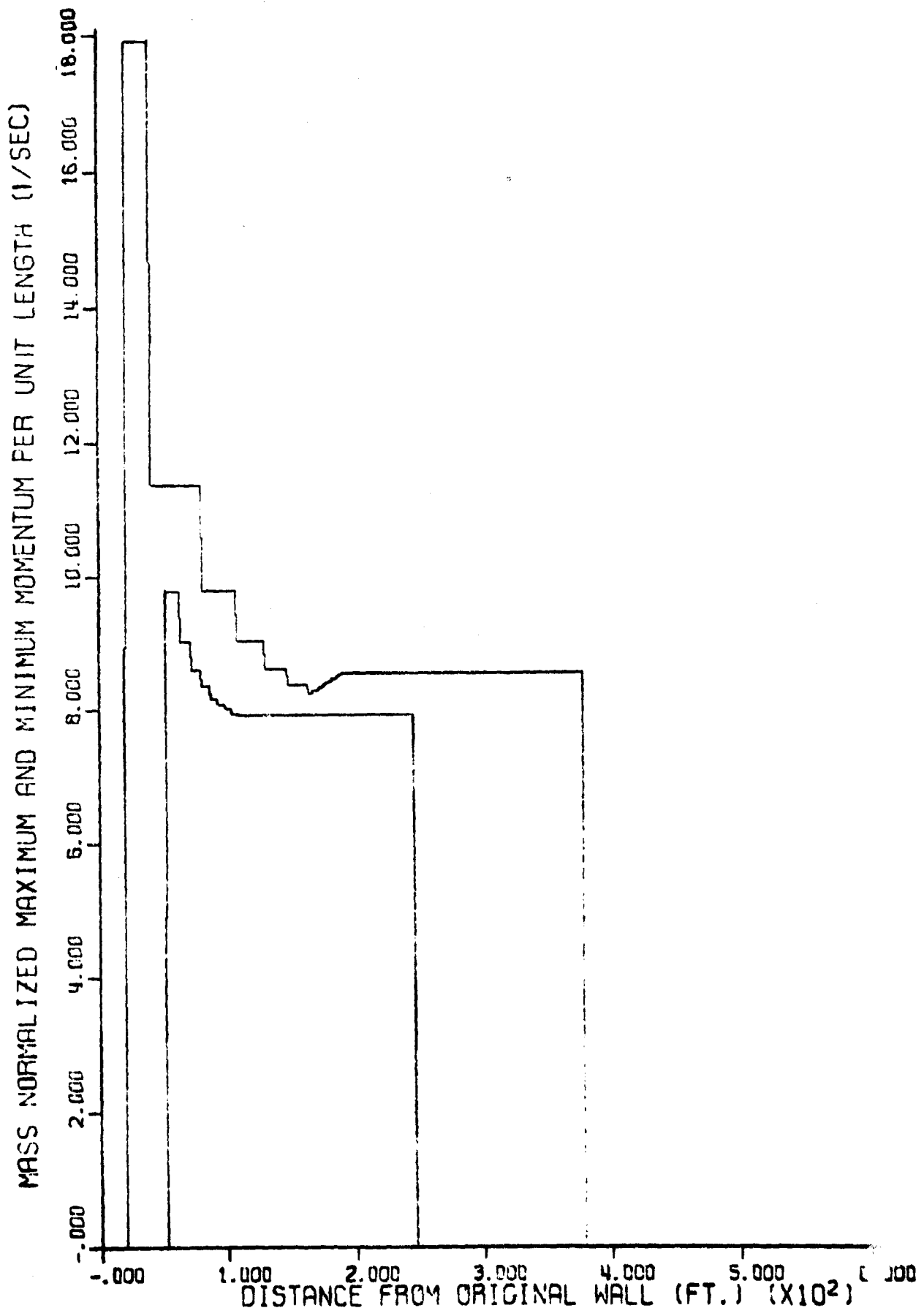
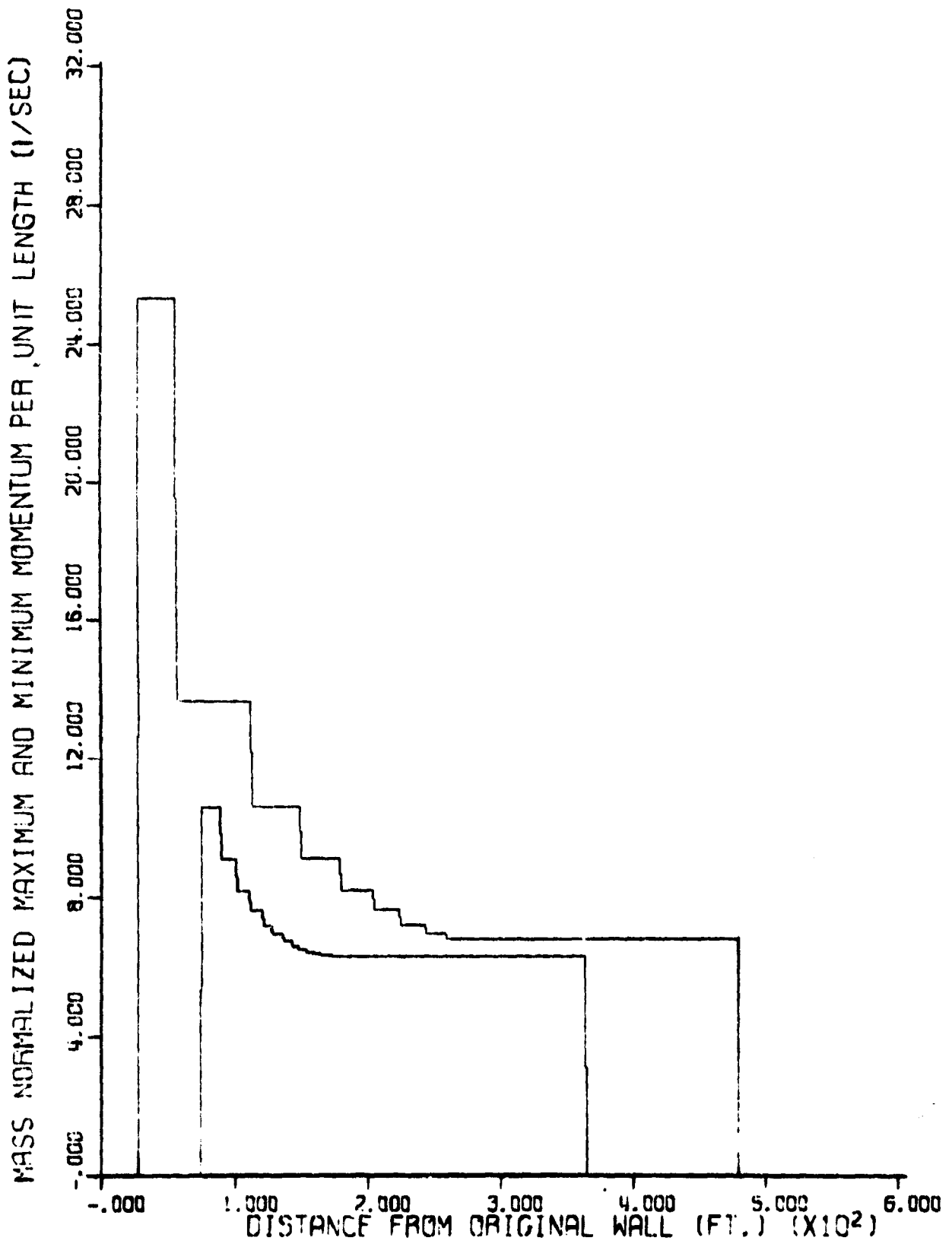


Fig. 57 MAXIMUM AND MINIMUM MOMENTUM ALONG DEBRIS PROFILE FOR EQUIVALENT SPHERICAL PARTICLE



**Fig. 58 MAXIMUM AND MINIMUM MOMENTUM ALONG DEBRIS PROFILE FOR SIDE-ON ORIENTATION**

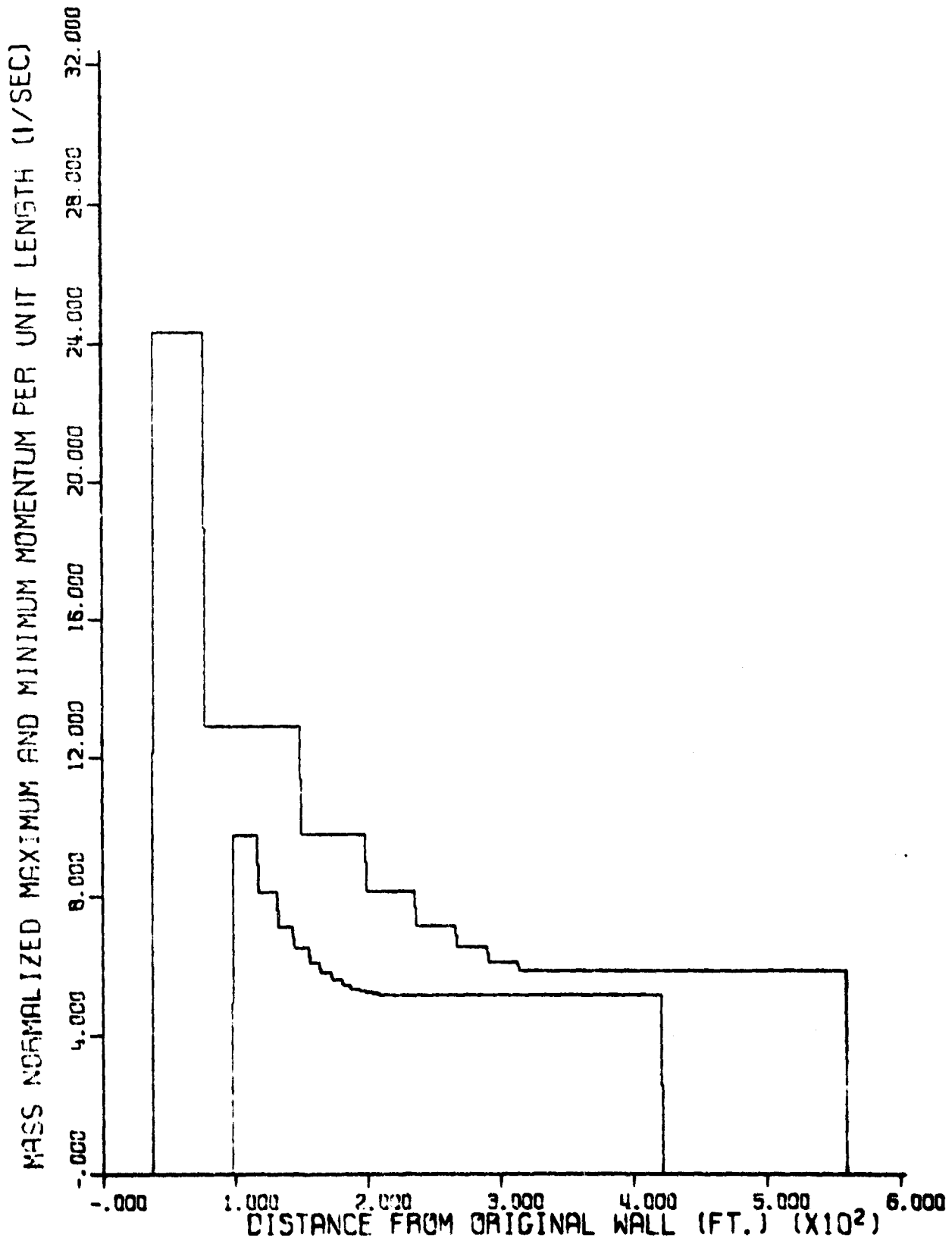
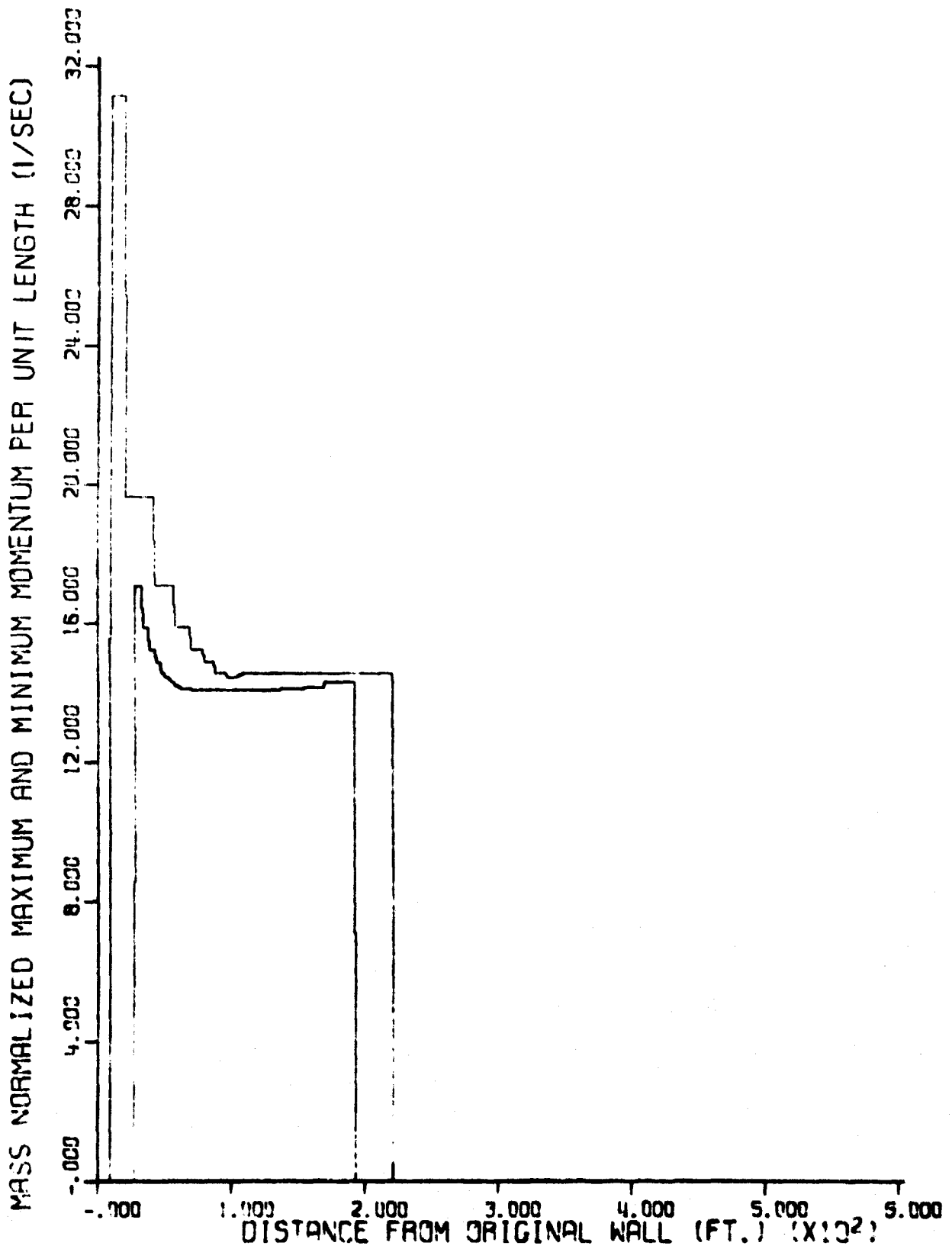


Fig. 59 MAXIMUM AND MINIMUM MOMENTUM ALONG DEBRIS PROFILE FOR FACE-ON ORIENTATION



**Fig. 60 MAXIMUM AND MINIMUM MOMENTUM ALONG DEBRIS PROFILE FOR END-ON ORIENTATION**

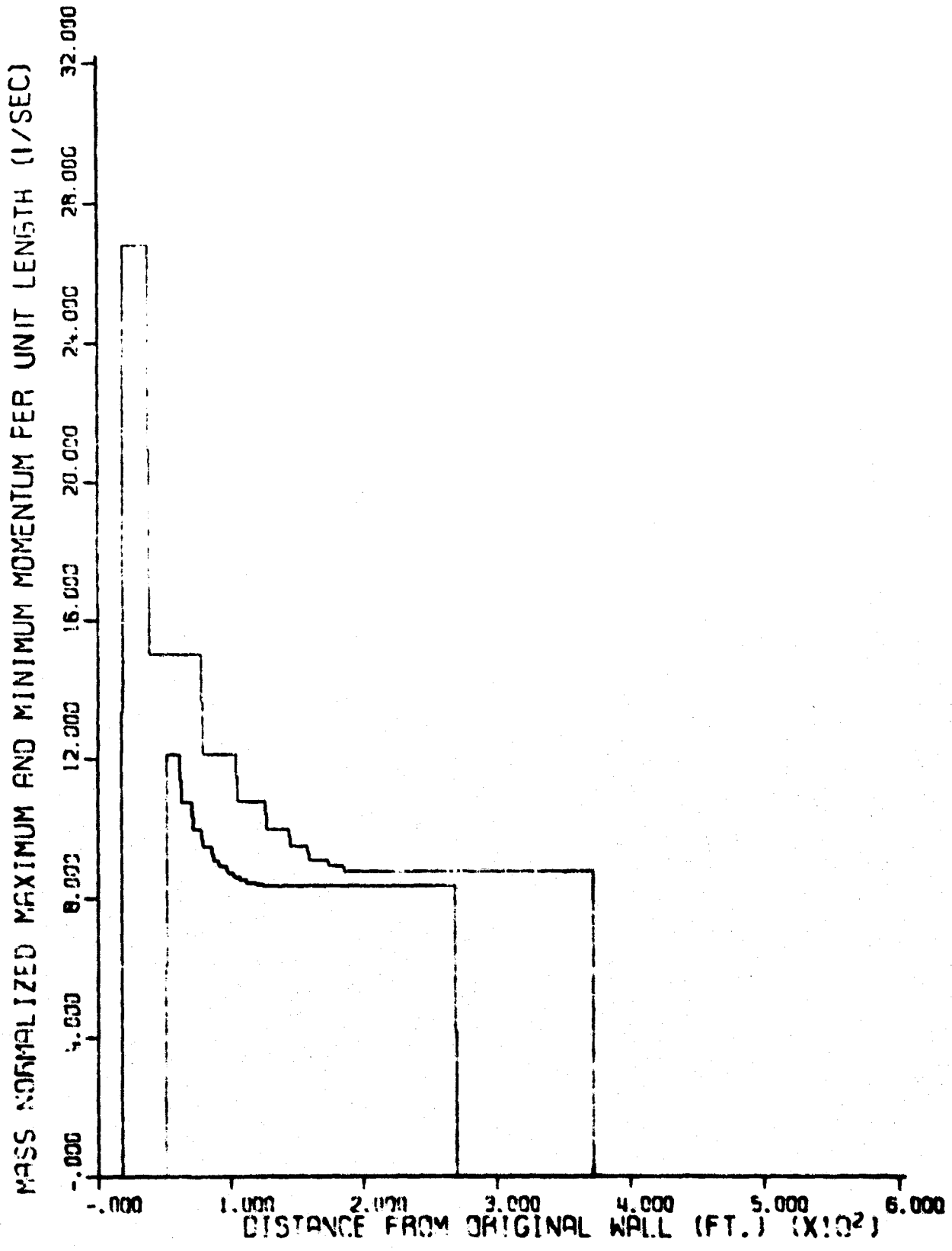


Fig. 61 MAXIMUM AND MINIMUM MOMENTUM ALONG DEBRIS PROFILE FOR AVERAGE ORIENTATION

#### 4.5 FRAGMENTATION DELAY TIMES AND INITIAL VELOCITIES

As previously mentioned, the trajectory analysis used in the SINBAD code is based on the assumptions of zero initial velocity and zero fragmentation time. The trajectory analysis is essentially a numerical solution of a complex differential equation. Since this numerical solution can have arbitrary initial conditions (i.e., delay time and initial velocity), a study was made to see how changes in the fragmentation delay times might affect the final transport position of a particle. The results of that study are summarized in Fig. 62. The figure illustrates the influence of delay time on the final distance a projectile travels. Case A is for a particle initially at 271 ft above ground surface, while Case B is for a particle at 31 ft above ground surface. As delay time is increased, the total distance a particle travels decreases. This decrease however, is insignificant for delay times which are physically meaningful (i.e., up to 0.1 sec) for frangible panels commonly found in structures. The delay time variation was made again with a zero initial velocity. Increasing initial velocity will tend to offset the delay time effect. As fragmentation time for an element increases, the strain energy within the element builds up. This strain energy is likely to impart some kinetic energy to the particle when it is free to fly. Therefore, an increase in fragmentation delay time tends to be counteracted by a corresponding increase in initial velocity and the entire effect on total particle displacement is negligible.

#### 4.6 MODIFICATION OF BLAST LOADING DUE TO LOCAL SHIELDING

One companion problem associated with debris estimation is an accurate description of the blast loading. Most estimates of blast loading on structures are developed under the assumption that there are no obstructions between ground zero and the point of load application. In the real world problem this is far from true; the blast wave must interact with a variety of obstructions in its path to the structure of interest. This phenomenon is

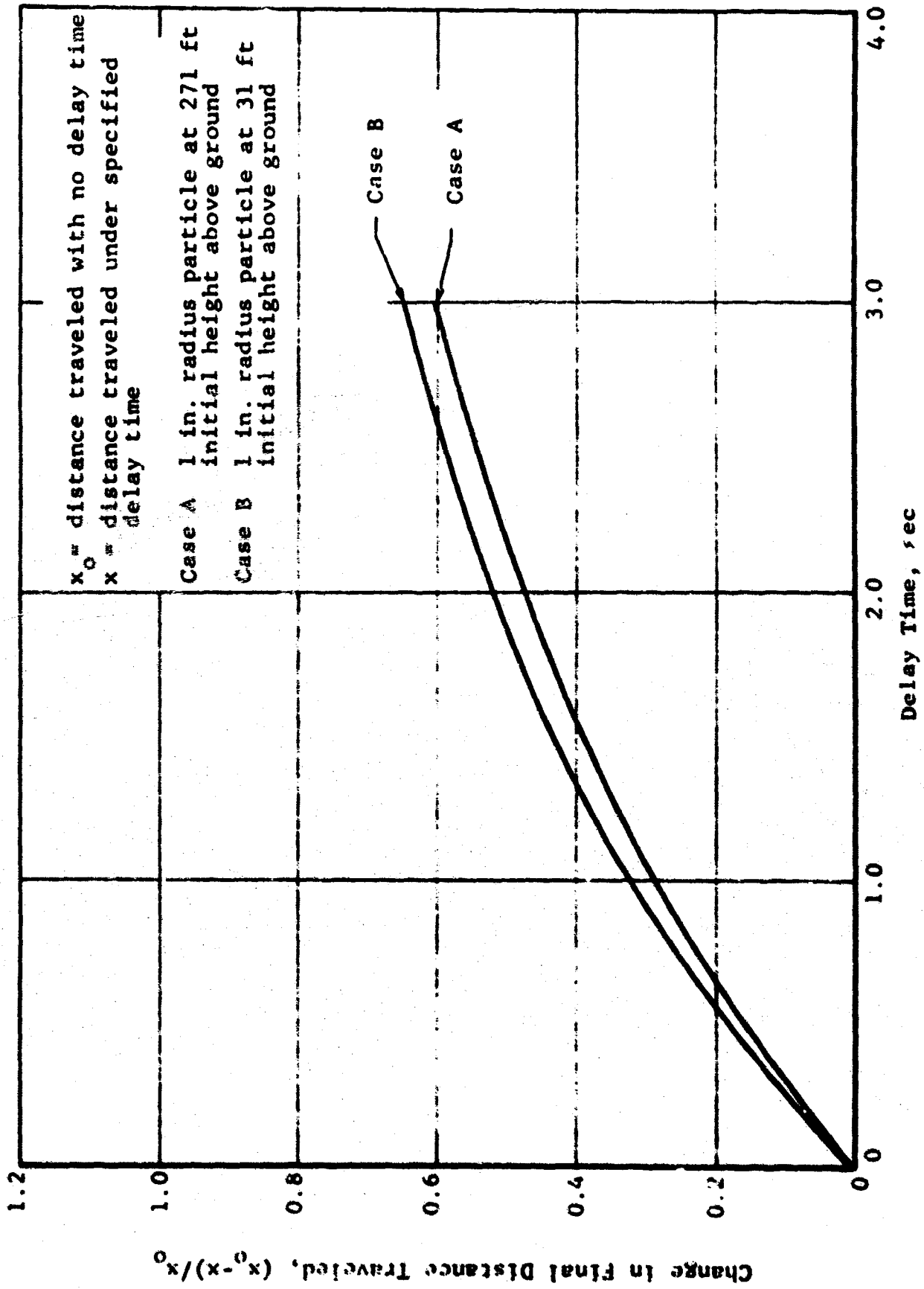


FIG. 62 INFLUENCE OF FRAGMENTATION DELAY TIME ON FINAL TRANSPORT DISTANCE

known as blast shielding. Blast shielding is accounted for in the SINBAD model by attenuating the free-field overpressure by a factor which is an empirical function of building height, length, and spacing from contiguous structures. This factor was determined in a previous experimental program (Ref. 15) and Fig. 63 illustrates the applied results. The three curves represent different ratios of exposed length to height for the structures investigated. The separation ratio is determined from the spacing between neighboring structures and the height of the structure. These curves, Fig. 63, are based on previous model studies and are the most appropriate data which could be found on attenuation due to structural shielding.

#### 4.7 IMPINGEMENT OF DEBRIS FROM ONE STRUCTURE ON ANOTHER

Although it is conceivable that under the right set of circumstances the debris from one structure might collide in midflight with another structure, this result has not been observed in problems run to date. Such a result, in any case, is difficult to observe and still more difficult to analyze. This phenomenon has not been incorporated into SINBAD and can only be detected from intermediate results.

#### 4.8 INTERIOR BUILDING CONTENTS AS POTENTIAL DEBRIS

After the blast wave interacts with the exterior walls of a structure, it enters the interior of the building. During the transition from the outside to the inside of the structure the blast overpressure undergoes still another attenuation. This attenuation is, again, determined from empirical results obtained from an experimental investigation (Ref. 16). As discussed previously, the SINBAD Code operates on an idealized space consisting of lumped particles at discrete initial heights.



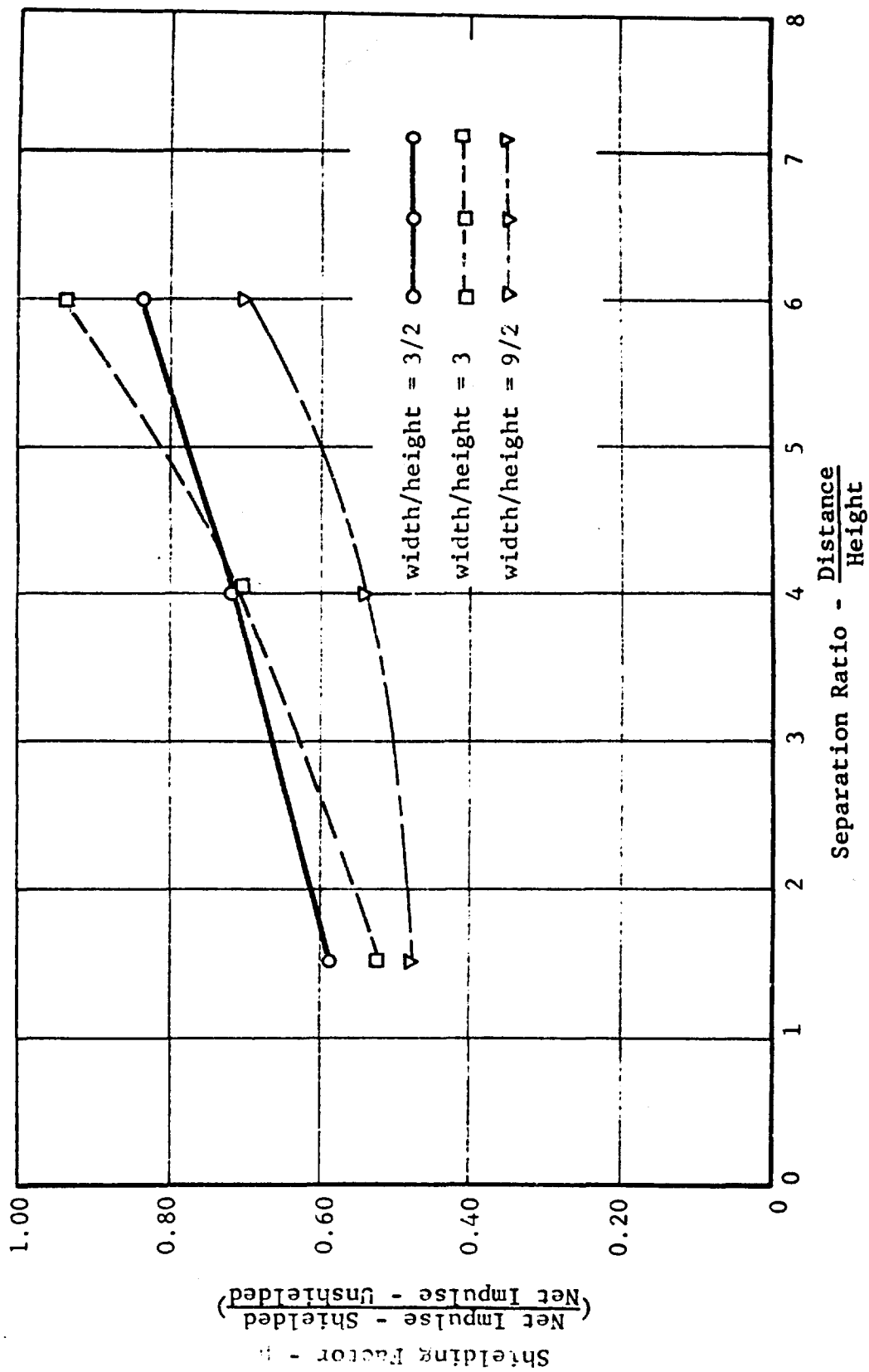


Fig. 63 SHIELDING FROM CONTIGUOUS STRUCTURES

These particles may be any size; however, at some finite size the model may not yield plausible results. For interior items such as furniture, ice boxes, etc., SINBAD will yield good results, however, large bulky objects such as might be found in a warehouse are another story. These objects are highly sensitive to diffraction loading and must gain some inertia before they can be picked up by drag loading. A more meaningful analysis for this type of interior debris item might include a sliding overturning study that would establish whether the debris can start moving or not. When it is established that the debris moves, the SINBAD analysis may be utilized.

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APPENDIX A

COMPUTER PROGRAM FOR LIMITED  
ROTATION ANALYSIS

```

C      LIMIT ANALYSIS OF FRAMES
      DIMENSION A(20,30),S(60),ASAT(20,20),INDEX(20),P(20)
      DIMENSION SATX(30),PM(30),ALF(30),CX(20)
      DIMENSION CM(30),D(60),H(30)

C
C      INPUT DATA- NP IS TOTAL DEGREES OF FREEDOM
C                    NF IS TWICE THE NUMBER OF MEMBERS
C                    A IS THE INTERNAL-EXTERNAL FORCE MATRIX
C                    P IS THE LOAD VECTOR
C                    PM IS THE PLASTIC MOMENT AT EACH NODE
C                    S IS THE SLOPE-DELECTION STIFFNESS MATRIX (IN VECTOR FORM)
C
1 READ (5,2) JJ
2 FORMAT(1I5)
  IF (JJ) 4,4,3
3 READ (5,101) NP,NF
101 FORMAT (2I5)
  READ (5,102) ((A(I,J),J=1,NF),I=1,NP)
102 FORMAT (8F10.4)
  NPT2=NF*2
  READ (5,102) (S(I),I=1,NPT2)
  NP1=NP+1
  READ (5,102) (P(I),I=1,NP)
  READ (5,102) (PM(I),I=1,NF)
  WRITE (6,103)
103 FORMAT (31#LIMIT ANALYSIS OF RIGID FRAMES//)
  WRITE(6,5) JJ
  5 FORMAT(8#ICASE NO,73)
  WRITE (6,104)
104 FORMAT (13#THE MATRIX A)
  DO 105 I=1,NP
105 WRITE (6,106) I,(A(I,J),J=1,NF)
106 FORMAT (4#ROW,13,1X,1P4F16.7/(8X,1P4E16.7))
  WRITE (6,107)
107 FORMAT (13#THE MATRIX S)
  DO 108 I=1,NF
  I1=(I-1)/2*2+1
  I2=(I+1)/2*2
  I3=2*I-1
  I4=2*I
108 WRITE (6,109) I,I1,S(I3),I2,S(I4)
109 FORMAT (4#ROW,13,5X,3#COL,13,1PE16.7,5X,3#COL,13,1PE16.7)
  WRITE (6,110)
110 FORMAT (13#THE MATRIX P)
  DO 221 I=1,NP
221 WRITE (6,106) I,P(I)
  WRITE (6,111)

```

```

111 FORMAT(14H0THE MATRIX PM)
DO 222 I=1,NP
222 WRITE (6,106) I,PM(I)
DO 112 I=1,NF

```

C  
C

C           INVERT THE STIFFNESS MATRIX . D=(S\*\*1)

C  
C

```

IF (I/2*2-1) 113,112,112
113 N1=2*I-1
N2=2*I
IP1=I+1
N3=2*IP1-1
N4=2*IP1
IF (S(N1)) 711,712,711
712 D(N4)=1./S(N4)
D(N1)=0.
D(N2)=0.
D(N3)=0.
GO TO 112
711 IF(S(N4)) 710,713,710
713 D(N1)=1./S(N1)
D(N2)=0.
D(N3)=0.
D(N4)=0.
GO TO 112
710 TEMP=1./(S(N1)*S(N4)-S(N2)*S(N3))
D(N1)=S(N4)*TEMP
D(N4)=S(N1)*TEMP
D(N2)=-S(N2)*TEMP
D(N3)=D(N2)
112 CONTINUE

```

C  
C  
C  
C  
C

          PLASTIC ANALYSIS FOR UNIT LOADS

```

500 NCYCL=0
CLF=0.
DO 24 I=1,NP
24 CX(I)=0.
DO 26 I=1,NF
26 CM(I)=0.
15 DO 116 I=1,NP
DO 116 J=1,NP
ASAT(I,J)=0.
DO 116 K=1,NF
K1=(K-1)/2*2+1
K2=(K+1)/2*2
K3=2*K-1
K4=2*K

```

```

116 ASAT(I,J)=ASAT(I,J)+A(I,K)*(S(K3)*A(J,K1)+S(K4)*A(J,K2))
    DO 151 I=1,NP
151 ASAT(I,NP1)=P(I)
    DO 117 I=1,NP
117 INDEX(I)=0
118 AMAX=-1.
    DO 119 I=1,NP
    IF (INDEX(I)) 119,120,119
120 TEMP=ABS(ASAT(I,I))
    IF (TEMP-AMAX) 119,119,121

```

```

121 IROW=I
    AMAX=TEMP
119 CONTINUE

```

C  
C  
C  
C  
C  
C

CHECK FOR ZERO IN PIVOT ELEMENT OR EXCESSIVE DEFLECTION  
THESE CONDITIONS ARE INSTABILITY CHECKS

```

    IF (AMAX) 122,147,124
124 INDEX(IROW)=1
    PIVOT=1./(ASAT(IROW,IROW))
    DO 125 J=1,NP1
125 ASAT(IROW,J)=ASAT(IROW,J)*PIVOT
    DO 126 I=1,NP
    IF (I-IROW) 127,126,127
127 TEMP=ASAT(I,IROW)
    DO 128 J=1,NP1
128 ASAT(I,J)=ASAT(I,J)-ASAT(IROW,J)*TEMP
126 CONTINUE
    GO TO 118
147 WRITE (6,347)
347 FORMAT(24H0ZFRO PIVOT IN INVERSION)
    GO TO 47
122 DO 311 I=1,NP
    IF (ABS(ASAT(I,NP1))-1.E+10) 311,647,647
311 CONTINUE
    GO TO 303
647 WRITE (6,647)
647 FORMAT(21H0DEFLECTION TOO LARGE)
    GO TO 47

```

C  
C  
C  
C

COMPUTE THE MOMENTS

```

303 DO 131 I=1,NP
    I1=(I-1)/2*2+1
    I2=(I+1)/2*2
    I3=2*I-1
    I4=2*I

```

```
SATX(I)=0.  
DO 131 K=1,NP  
131 SATX(I)=SATX(I)+ASAT(K,NP1)*(S(I3)*A(K,I1)+S(I4)*A(K,I2))
```

C  
C  
C  
C  
C  
C

FIND ADDITIONAL LOAD FACTOR REQUIRED TO BRING JOINT WITH LARGEST  
MOMENT UNDER UNIT LOAD UP TO PLASTIC MOMENT  
CHECK THAT MOMENT IS INCREASING IN MAGNITUDE UNDER UNIT LOAD

```
DO 201 I=1,NF  
IF (ABS(SATX(I))-1.E-04) 202,202,203  
202 ALF(I)=1.E20  
GO TO 201  
203 ALF(I)=(PM(I)-ABS(CM(I)))/ABS(SATX(I))  
201 CONTINUE
```

```
3ALF=1.E20  
DO 204 I=1,NF  
TEST = CM(I)*SATX(I)  
IF (TEST) 204,205,205  
205 IF (ALF(I)-SALF) 1206,204,204  
1206 SALF = ALF(I)  
NPH=I  
204 CONTINUE
```

C  
C  
C  
C  
C

IF THERE IS NO INCREASE IN LOAD FACTOR, COLLAPSE MECHANISM EXISTS

```
IF (SALF-1.E-07) 247,247,302  
247 WRITE (6,447)  
447 FORMAT(22HLOAD FACTOR TOO SMALL)  
GO TO 47
```

C  
C  
C  
C  
C

COMPUTE MOMENTS UNDER CURRENT LOAD FACTOR

```
302 DO 207 I=1,NF  
SATX(I)=SALF*SATX(I)  
207 CM(I)=CM(I)+SATX(I)
```

C  
C  
C  
C  
C

DOUBLE-CHECK ADMISSIBILITY OF SOLUTION, MOMENT, LE, PM AT ALL JOINTS

```
DO 314 I=1,NF  
IF (PM(I)-ABS(CM(I))>1.E-03) 547,314,314  
314 CONTINUE  
GO TO 304  
547 WRITE (6,747)  
747 FORMAT(24HPLASTIC MOMENT EXCEEDED)  
GO TO 47
```

C  
C



C  
C  
C  
WRITE MOMENTS AND DEFLECTIONS UNDER CURRENT LOAD FACTOR

```
304 CLF=CLF+SALF
    DO 206 I=1,NP
      ASAT(I,NP1)=SALF*ASAT(I,NP1)
206  CX(I)=CX(I)+ASAT(I,NP1)
      NCYCL=NCYCL+1
      WRITE (6,401) NCYCL,NPH
401  FORMAT (18H1PLASTIC HINGE NO.,I3,2X,15HFORMED AT POINT,I3)
      WRITE (6,402)
402  FORMAT(12HCLOAD FACTOR,3X,10HADDITIONAL,9X,10HCUMULATIVE)
      WRITE (6,403) NCYCL,SALF,CLF
403  FORMAT(7H0STAGE(,I3,1H),1PE18.7,1PE19.7)
      WRITE (6,404)
404  FORMAT(11H0DEFLECTION,4X,10HADDITIONAL,9X,10HCUMULATIVE/)
      DO 208 I=1,NP

208  WRITE (6,405) I,ASAT(I,NP1),CX(I)
405  FORMAT(3H X(,I3,1H),1PE22.7,1PE19.7)
      WRITE (6,406)
406  FORMAT(7HMOMENTAX,10HADDITIONAL,9X10HCUMULATIVE10X,8HPLAS MOM/)
      DO 209 I=1,NF
209  WRITE (6,407) I,SATX(I),CM(I),PM(I)
407  FORMAT(3H M(,I3,1H),F18.4,2F19.4)
```

C  
C  
C  
C  
C  
COMPUTE INELASTIC HINGE ROTATIONS UNDER CURRENT LOAD FACTOR

```
DO 933 I=1,NF
  I1=(I-1)/2*2+1
  I2=(I+1)/2*2
  I3=2*I-1
  I4=2*I
  H(I)=D(I3)*CM(I1)+D(I4)*CM(I2)
  DO 933 K=1,NP
933  H(I)=H(I)-A(K,I)*CX(K)
  DO 501 I=1,NF
501  IF(ABS(H(I)).LT.(1.E-07))H(I)=0.0
      WRITE (6,138)
138  FORMAT(140,14X,15HHINGE ROTATIONS/)
      DO 939 I=1,NF
939  WRITE (6,140) I,H(I)
140  FORMAT(10H AT POINT(,I3,1H),1PE15.7)
```

C  
C

C  
C  
C

MODIFY STIFFNESS MATRIX TO INCLUDE PLASTIC HINGE

```
IF ((NPH/2*2)-NPH) 211,210,210
211 N1=2*NPH-1
    N2=2*NPH
    NPH1=NPH+1
    N3=2*NPH1-1
    N4=2*NPH1
    S(N4)=S(N4)*(1.-S(N2)*S(N3)/(S(N1)*S(N4)))
    S(N1)=0.
    S(N2)=0.
    S(N3)=0.
    GO TO 212
210 NPHM1=NPH-1
    N1=2*NPHM1-1
    N2=2*NPHM1
    N3=2*NPH-1
    N4=2*NPH
    S(N1)=S(N1)*(1.-S(N2)*S(N3)/(S(N1)*S(N4)))
    S(N2)=0.
    S(N3)=0.
    S(N4)=0.
212 GO TO 15
```

C  
C

```
47 WRITE (6,408)
```

```
408 FORMAT(36H0COLLAPSE MECHANISM HAS BEEN REACHED)
GO TO 1
4 STOP
END
```

APPENDIX B

RESULTS OF SAMPLE PROBLEM

CASE NO 2

THE MATRIX A

ROW 1	-0.0000000E-39	1.0000000E 00	1.0000000E 00	-0.0000000E-39
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
	1.0000000E 00	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
ROW 2	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
	-0.0000000E-39	1.0000000E 00	1.0000000E 00	-0.0000000E-39
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
	-0.0000000E-39	1.0000000E 00	1.0000000E 00	-0.0000000E-39
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
ROW 3	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
	-0.0000000E-39	1.0000000E 00	1.0000000E 00	-0.0000000E-39
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	1.0000000E 00
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
ROW 4	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	1.0000000E 00
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
	1.0000000E 00	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
ROW 5	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	1.0000000E 00
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
	-0.0000000E-39	1.0000000E 00	1.0000000E 00	-0.0000000E-39
ROW 6	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	1.0000000E 00
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	1.0000000E 00
ROW 7	1.0000000E-01	1.0000000E-01	-1.0000000E-01	-1.0000000E-01
	1.0000000E-01	1.0000000E-01	-1.0000000E-01	-1.0000000E-01
	1.0000000E-01	1.0000000E-01	-1.0000000E-01	-1.0000000E-01
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
ROW 8	-0.0000000E-39	-0.0000000E-39	1.0000000E-01	1.0000000E-01
	-0.0000000E-39	-0.0000000E-39	1.0000000E-01	1.0000000E-01
	-0.0000000E-39	-0.0000000E-39	1.0000000E-01	1.0000000E-01
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39
	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39	-0.0000000E-39

THE MATRIX S

ROW 1	COL 1	7.2000000E 05	COL 2	3.6000000E 05
ROW 2	COL 1	3.6000000E 05	COL 2	7.2000000E 05
ROW 3	COL 3	7.2000000E 05	COL 4	3.6000000E 05
ROW 4	COL 3	3.6000000E 05	COL 4	7.2000000E 05
ROW 5	COL 5	7.2000000E 05	COL 6	3.6000000E 05
ROW 6	COL 5	3.6000000E 05	COL 6	7.2000000E 05
ROW 7	COL 7	7.2000000E 05	COL 8	3.6000000E 05
ROW 8	COL 7	3.6000000E 05	COL 8	7.2000000E 05

ROW 9	COL 9	7.200000E 05	COL 10	3.600000E 05
ROW 10	COL 9	3.600000E 05	COL 10	7.200000E 05
ROW 11	COL 11	7.200000E 05	COL 12	3.600000E 05
ROW 12	COL 11	3.600000E 05	COL 12	7.200000E 05
ROW 13	COL 13	3.600000E 05	COL 14	1.800000E 05
ROW 14	COL 13	1.800000E 05	COL 14	3.600000E 05
ROW 15	COL 15	3.600000E 05	COL 16	1.800000E 05
ROW 16	COL 15	1.800000E 05	COL 16	3.600000E 05
ROW 17	COL 17	3.600000E 05	COL 18	1.800000E 05
ROW 18	COL 17	1.800000E 05	COL 18	3.600000E 05
ROW 19	COL 19	3.600000E 05	COL 20	1.800000E 05
ROW 20	COL 19	1.800000E 05	COL 20	3.600000E 05

THE MATRIX P

ROW 1	-0.000000E-39
ROW 2	-0.000000E-39
ROW 3	-0.000000E-39
ROW 4	-0.000000E-39
ROW 5	-0.000000E-39
ROW 6	-0.000000E-39
ROW 7	-0.000000E-39
ROW 8	1.000000E 00

THE MATRIX PM

ROW 1	6.000000E 03
ROW 2	6.000000E 03
ROW 3	6.000000E 03
ROW 4	6.000000E 03
ROW 5	6.000000E 03
ROW 6	6.000000E 03
ROW 7	6.000000E 03
ROW 8	6.000000E 03

PLASTIC HINGE NO. 1 FORMED AT POINT 8

LOAD FACTOR	ADDITIONAL	CUMULATIVE
STAGE( 1)	2.3483869E 03	2.3483869E 03
DEFLECTION	ADDITIONAL	CUMULATIVE
X( 1)	-9.2293903E-03	-9.2293903E-03
X( 2)	-7.1684583E-03	-7.1684583E-03
X( 3)	-9.2293903E-03	-9.2293903E-03
X( 4)	-7.2281954E-03	-7.2281954E-03
X( 5)	-4.7192351E-03	-4.7192351E-03
X( 6)	-7.2281957E-03	-7.2281957E-03
X( 7)	7.8952607E-02	7.8952607E-02
X( 8)	1.8986459E-01	1.8986459E-01

MOMENT	ADDITIONAL	CUMULATIVE	PLAS MOM
M( 1)	5204.3008	5204.3008	6000.0000
M( 2)	1881.7204	1881.7204	6000.0000
M( 3)	2731.1828	2731.1828	6000.0000
M( 4)	3451.6130	3451.6130	6000.0000
M( 5)	5946.2365	5946.2365	6000.0000
M( 6)	3365.5915	3365.5915	6000.0000
M( 7)	5118.2797	5118.2797	6000.0000
M( 8)	5999.9999	5999.9999	6000.0000
M( 9)	5204.3008	5204.3008	6000.0000
M( 10)	1881.7204	1881.7204	6000.0000
M( 11)	2731.1828	2731.1828	6000.0000
M( 12)	3451.6128	3451.6128	6000.0000
M( 13)	-4612.9030	-4612.9030	6000.0000
M( 14)	-4241.9352	-4241.9352	6000.0000
M( 15)	-4241.9352	-4241.9352	6000.0000
M( 16)	-4612.9030	-4612.9030	6000.0000
M( 17)	-3451.6126	-3451.6126	6000.0000
M( 18)	-2999.9998	-2999.9998	6000.0000
M( 19)	-2999.9998	-2999.9998	6000.0000
M( 20)	-3451.6127	-3451.6127	6000.0000

HINGE ROTATIONS	
AT POINT( 1)	0.0000000E-39
AT POINT( 2)	0.0000000E-39
AT POINT( 3)	0.0000000E-39
AT POINT( 4)	0.0000000E-39
AT POINT( 5)	0.0000000E-39
AT POINT( 6)	0.0000000E-39
AT POINT( 7)	0.0000000E-39
AT POINT( 8)	0.0000000E-39
AT POINT( 9)	0.0000000E-39
AT POINT( 10)	0.0000000E-39
AT POINT( 11)	0.0000000E-39
AT POINT( 12)	0.0000000E-39
AT POINT( 13)	0.0000000E-39
AT POINT( 14)	0.0000000E-39
AT POINT( 15)	0.0000000E-39
AT POINT( 16)	0.0000000E-39
AT POINT( 17)	0.0000000E-39
AT POINT( 18)	0.0000000E-39
AT POINT( 19)	0.0000000E-39
AT POINT( 20)	0.0000000E-39

PLASTIC HINGE NO. 2 FORMED AT POINT 5

LOAD FACTOR	ADDITIONAL	CUMULATIVE
STAGE( 2)	1.8046505E 01	2.3664334E 03

DEFLECTION	ADDITIONAL	CUMULATIVE
X( 1)	-9.4618827E-05	-9.3240091E-03
X( 2)	-5.7648556E-05	-7.2261068E-03
X( 3)	-9.4618829E-05	-9.3240091E-03

X( 4)	-1.1446163E-04	-7.3426570E-03
X( 5)	5.7230812E-05	-4.6620043E-03
X( 6)	-1.1446162E-04	-7.3426573E-03
X( 7)	6.8997251E-04	7.9642579E-02
X( 8)	2.0546002E-03	1.9191919E-01

MOMENT	ADDITIONAL	CUMULATIVE	PLAS MOM
M( 1)	40.4543	5244.7551	6000.0000
M( 2)	6.3915	1888.1118	6000.0000
M( 3)	38.0481	2769.2308	6000.0000
M( 4)	30.9046	3482.5177	6000.0000
M( 5)	53.7635	5999.9999	6000.0000
M( 6)	33.0101	3398.6015	6000.0000
M( 7)	42.5597	5160.8394	6000.0000
M( 8)	0.0000	5999.9999	6000.0000
M( 9)	40.4543	5244.7551	6000.0000
M( 10)	6.3915	1888.1118	6000.0000
M( 11)	38.0480	2769.2308	6000.0000
M( 12)	30.9046	3482.5174	6000.0000
M( 13)	-44.4395	-4657.3425	6000.0000
M( 14)	-37.7849	-4279.7200	6000.0000
M( 15)	-37.7849	-4279.7200	6000.0000
M( 16)	-44.4395	-4657.3425	6000.0000
M( 17)	-30.9046	-3482.5172	6000.0000
M( 18)	-0.0000	-2999.9998	6000.0000
M( 19)	-0.0000	-2999.9998	6000.0000
M( 20)	-30.9046	-3482.5174	6000.0000

HINGE ROTATIONS	
AT POINT( 1)	0.0000000E-39
AT POINT( 2)	0.0000000E-39
AT POINT( 3)	0.0000000E-39
AT POINT( 4)	0.0000000E-39
AT POINT( 5)	0.0000000E-39
AT POINT( 6)	0.0000000E-39
AT POINT( 7)	0.0000000E-39
AT POINT( 8)	-2.3310143E-04
AT POINT( 9)	0.0000000E-39
AT POINT( 10)	0.0000000E-39
AT POINT( 11)	0.0000000E-39
AT POINT( 12)	0.0000000E-39
AT POINT( 13)	0.0000000E-39
AT POINT( 14)	0.0000000E-39
AT POINT( 15)	0.0000000E-39
AT POINT( 16)	0.0000000E-39
AT POINT( 17)	0.0000000E-39
AT POINT( 18)	0.0000000E-39
AT POINT( 19)	0.0000000E-39
AT POINT( 20)	0.0000000E-39

PLASTIC HINGE NO. 3 FORMED AT POINT 9

LOAD FACTOR      ADDITIONAL      CUMULATIVE  
 STAGE ( 3)      2.2768410E 02      2.5941175E 03

DEFLECTION      ADDITIONAL      CUMULATIVE  
 X( 1)      -1.4603046E-03      -1.0784314E-02  
 X( 2)      -6.1703011E-04      -7.8431369E-03  
 X( 3)      -1.4603046E-03      -1.0784314E-02  
 X( 4)      -1.4808722E-03      -8.8235291E-03  
 X( 5)      7.4043609E-04      -3.9215682E-03  
 X( 6)      -1.4808722E-03      -8.8235295E-03  
 X( 7)      1.1860691E-02      9.1503263E-02  
 X( 8)      3.0303036E-02      2.2222223E-01

MOMENT	ADDITIONAL	CUMULATIVE	PLAS MOM
M( 1)	755.2449	5999.9999	6000.0000
M( 2)	229.5353	2117.6471	6000.0000
M( 3)	407.2399	3176.4707	6000.0000
M( 4)	399.8356	3882.3532	6000.0000
M( 5)	-0.0000	5999.9999	6000.0000
M( 6)	307.2810	3705.8825	6000.0000
M( 7)	662.6904	5823.5297	6000.0000
M( 8)	0.0000	5999.9999	6000.0000
M( 9)	755.2449	5999.9999	6000.0000
M( 10)	229.5353	2117.6471	6000.0000
M( 11)	407.2400	3176.4707	6000.0000
M( 12)	399.8356	3882.3530	6000.0000
M( 13)	-636.7751	-5294.1176	6000.0000
M( 14)	-484.9857	-4764.7057	6000.0000
M( 15)	-484.9857	-4764.7057	6000.0000
M( 16)	-636.7751	-5294.1175	6000.0000
M( 17)	-399.8355	-3882.3527	6000.0000
M( 18)	-0.0000	-2999.9998	6000.0000
M( 19)	-0.0000	-2999.9998	6000.0000
M( 20)	-399.8355	-3882.3528	6000.0000

HINGE ROTATIONS

AT POINT( 1) 0.0000000E-39  
 AT POINT( 2) 0.0000000E-39  
 AT POINT( 3) 0.0000000E-39  
 AT POINT( 4) 0.0000000E-39  
 AT POINT( 5) -1.4705887E-03  
 AT POINT( 6) 0.0000000E-39  
 AT POINT( 7) 0.0000000E-39  
 AT POINT( 8) -3.4313742E-03  
 AT POINT( 9) 0.0000000E-39  
 AT POINT( 10) 0.0000000E-39  
 AT POINT( 11) 0.0000000E-39  
 AT POINT( 12) 0.0000000E-39  
 AT POINT( 13) 0.0000000E-39  
 AT POINT( 14) 0.0000000E-39  
 AT POINT( 15) 0.0000000E-39  
 AT POINT( 16) 0.0000000E-39  
 AT POINT( 17) 0.0000000E-39  
 AT POINT( 18) 0.0000000E-39  
 AT POINT( 19) 0.0000000E-39  
 AT POINT( 20) 0.0000000E-39



PLASTIC HINGE NO. 4 FORMED AT POINT 1

LOAD FACTOR	ADDITIONAL	CUMULATIVE
STAGE( 4)	1.2065213E-05	2.5941175E 03

DEFLECTION	ADDITIONAL	CUMULATIVE
X( 1)	-9.5428928E-11	-1.0784314E-02
X( 2)	-4.0570425E-11	-7.8431369E-03
X( 3)	-7.5083876E-11	-1.0784314E-02
X( 4)	-7.8787207E-11	-8.8235291E-03
X( 5)	4.1089025E-11	-3.9215681E-03
X( 6)	-8.5568894E-11	-8.8235295E-03
X( 7)	8.8323677E-10	9.1503269E-02
X( 8)	1.9207236E-09	2.2222223E-01

MOMENT	ADDITIONAL	CUMULATIVE	PLAS MOM
M( 1)	0.0001	5999.9999	6000.0000
M( 2)	0.0000	2117.6471	6000.0000
M( 3)	0.0000	3176.4707	6000.0000
M( 4)	0.0000	3882.3532	6000.0000
M( 5)	-0.0000	5999.9999	6000.0000
M( 6)	0.0000	3705.8825	6000.0000
M( 7)	0.0000	5823.5297	6000.0000
M( 8)	0.0000	5999.9999	6000.0000
M( 9)	-0.0000	5999.9999	6000.0000
M( 10)	0.0000	2117.6471	6000.0000
M( 11)	0.0000	3176.4707	6000.0000
M( 12)	0.0000	3882.3530	6000.0000
M( 13)	-0.0000	-5294.1176	6000.0000
M( 14)	-0.0000	-4764.7057	6000.0000
M( 15)	-0.0000	-4764.7057	6000.0000
M( 16)	-0.0000	-5294.1175	6000.0000
M( 17)	-0.0000	-3882.3527	6000.0000
M( 18)	0.0000	-2999.9997	6000.0000
M( 19)	-0.0000	-2999.9998	6000.0000
M( 20)	-0.0000	-3882.3528	6000.0000

HINGE ROTATIONS	
AT POINT( 1)	0.0000000E-39
AT POINT( 2)	0.0000000E-39
AT POINT( 3)	0.0000000E-39
AT POINT( 4)	0.0000000E-39
AT POINT( 5)	-1.4705887E-03
AT POINT( 6)	0.0000000E-39
AT POINT( 7)	0.0000000E-39
AT POINT( 8)	-3.4313744E-03
AT POINT( 9)	0.0000000E-39
AT POINT( 10)	0.0000000E-39
AT POINT( 11)	0.0000000E-39
AT POINT( 12)	0.0000000E-39
AT POINT( 13)	0.0000000E-39
AT POINT( 14)	0.0000000E-39
AT POINT( 15)	0.0000000E-39
AT POINT( 16)	0.0000000E-39
AT POINT( 17)	0.0000000E-39
AT POINT( 18)	0.0000000E-39
AT POINT( 19)	0.0000000E-39
AT POINT( 20)	0.0000000E-39

PLASTIC HINGE NO. 5 FORMED AT POINT 7

LOAD FACTOR	ADDITIONAL	CUMULATIVE
STAGE( 5)	6.8895929E 01	2.6630134E 03

DEFLECTION	ADDITIONAL	CUMULATIVE
X( 1)	-6.3121024E-04	-1.1415524E-02
X( 2)	-3.7604015E-04	-8.2191770E-03
X( 3)	-6.3121024E-04	-1.1415524E-02
X( 4)	-5.3720015E-04	-9.3607293E-03
X( 5)	2.6860008E-04	-3.6529680E-03
X( 6)	-5.3720015E-04	-9.3607296E-03
X( 7)	9.7143713E-03	1.0121764E-01
X( 8)	1.6742741E-02	2.3896497E-01

MOMENT	ADDITIONAL	CUMULATIVE	PLAS MOM
M( 1)	-0.0000	5999.9999	6000.0000
M( 2)	183.7225	2301.3696	6000.0000
M( 3)	111.2005	3287.6711	6000.0000
M( 4)	145.0441	4027.3973	6000.0000
M( 5)	-0.0000	5999.9999	6000.0000
M( 6)	321.5144	4027.3969	6000.0000
M( 7)	176.4703	5999.9999	6000.0000
M( 8)	0.0000	5999.9999	6000.0000
M( 9)	-0.0000	5999.9999	6000.0000
M( 10)	183.7225	2301.3696	6000.0000
M( 11)	111.2005	3287.6712	6000.0000
M( 12)	145.0441	4027.3971	6000.0000
M( 13)	-294.9229	-5589.0405	6000.0000
M( 14)	-248.9923	-5013.6979	6000.0000
M( 15)	-248.9923	-5013.6979	6000.0000
M( 16)	-294.9229	-5589.0404	6000.0000
M( 17)	-145.044	-4027.3968	6000.0000
M( 18)	-0.0000	-2999.9997	6000.0000
M( 19)	-0.0000	-2999.9998	6000.0000
M( 20)	-145.0440	-4027.3969	6000.0000

HINGE ROTATIONS		
AT POINT( 1)		-1.1415508E-03
AT POINT( 2)		0.0000000E-39
AT POINT( 3)		0.0000000E-39
AT POINT( 4)		0.0000000E-39
AT POINT( 5)		-2.7397243E-03
AT POINT( 6)		0.0000000E-39
AT POINT( 7)		0.0000000E-39
AT POINT( 8)		-4.5662099E-03
AT POINT( 9)		-1.1415508E-03
AT POINT( 10)		0.0000000E-39
AT POINT( 11)		0.0000000E-39
AT POINT( 12)		0.0000000E-39
AT POINT( 13)		0.0000000E-39
AT POINT( 14)		0.0000000E-39
AT POINT( 15)		0.0000000E-39
AT POINT( 16)		0.0000000E-39
AT POINT( 17)		0.0000000E-39
AT POINT( 18)		0.0000000E-39
AT POINT( 19)		0.0000000E-39
AT POINT( 20)		0.0000000E-39

PLASTIC HINGE NO. 6 FORMED AT POINT 13

LOAD FACTOR	ADDITIONAL	CUMULATIVE
STAGE( 6)	9.4433264E 01	2.7574467E 03

DEFLECTION	ADDITIONAL	CUMULATIVE
X( 1)	-9.9582392E-04	-1.2411348E-02
X( 2)	-2.9146067E-04	-8.5106377E-03
X( 3)	-9.9582391E-04	-1.2411348E-02
X( 4)	-9.2295874E-04	-1.0283688E-02
X( 5)	4.6147938E-04	-3.1914886E-03
X( 6)	-9.2295874E-04	-1.0283688E-02
X( 7)	1.3439576E-02	1.1465721E-01
X( 8)	2.5219445E-02	2.6418442E-01

MOMENT	ADDITIONAL	CUMULATIVE	PLAS MOM
M( 1)	-0.0000	5999.9999	6000.0000
M( 2)	187.9922	2489.3617	6000.0000
M( 3)	222.9674	3510.6385	6000.0000
M( 4)	249.1989	4276.5962	6000.0000
M( 5)	-0.0000	5999.9999	6000.0000
M( 6)	568.3483	4595.7452	6000.0000
M( 7)	0.0000	5999.9999	6000.0000
M( 8)	0.0000	5999.9999	6000.0000
M( 9)	-0.0000	5999.9999	6000.0000
M( 10)	187.9922	2489.3617	6000.0000
M( 11)	222.9674	3510.6386	6000.0000
M( 12)	249.1989	4276.5959	6000.0000
M( 13)	-410.9595	-5999.9999	6000.0000
M( 14)	-284.1741	-5297.8721	6000.0000
M( 15)	-284.1741	-5297.8721	6000.0000
M( 16)	-410.9595	-5999.9999	6000.0000
M( 17)	-249.1989	-4276.5956	6000.0000
M( 18)	0.0000	-2999.9997	6000.0000
M( 19)	0.0000	-2999.9998	6000.0000
M( 20)	-249.1989	-4276.5957	6000.0000

HINGE ROTATIONS	
AT POINT( 1)	-2.6595752E-03
AT POINT( 2)	0.0000000E-39
AT POINT( 3)	0.0000000E-39
AT POINT( 4)	0.0000000E-39
AT POINT( 5)	-4.6099302E-03
AT POINT( 6)	0.0000000E-39
AT POINT( 7)	-8.8652689E-04
AT POINT( 8)	-6.2056759E-03
AT POINT( 9)	-2.6595752E-03
AT POINT( 10)	0.0000000E-39
AT POINT( 11)	0.0000000E-39
AT POINT( 12)	0.0000000E-39
AT POINT( 13)	0.0000000E-39
AT POINT( 14)	0.0000000E-39
AT POINT( 15)	0.0000000E-39
AT POINT( 16)	0.0000000E-39
AT POINT( 17)	0.0000000E-39
AT POINT( 18)	0.0000000E-39
AT POINT( 19)	0.0000000E-39
AT POINT( 20)	0.0000000E-39

PLASTIC HINGE NO. 7 FORMED AT POINT 16

LOAD FACTOR	ADDITIONAL	CUMULATIVE	
STAGE( 7)	1.9151234E-05	2.7574467E 03	
DEFLECTION			
X( 1)	-3.7942877E-10	-1.2411348E-02	
X( 2)	-1.3346932E-10	-2.5106378E-03	
X( 3)	-2.7234954E-10	-1.2411348E-02	
X( 4)	-2.1890485E-10	-1.0283688E-02	
X( 5)	1.1837570E-10	-3.1914885E-03	
X( 6)	-2.5459793E-10	-1.0283689E-02	
X( 7)	3.7996672E-09	1.1465722E-01	
X( 8)	7.0561856E-09	2.6418442E-01	
MOMENT			
M( 1)	-0.0000	5999.9999	6000.0000
M( 2)	0.0000	2489.3617	6000.0000
M( 3)	-0.0000	3510.6385	6000.0000
M( 4)	0.0001	4276.5962	6000.0000
M( 5)	-0.0000	5999.9999	6000.0000
M( 6)	0.0001	4595.7453	6000.0000
M( 7)	0.0000	5999.9999	6000.0000
M( 8)	0.0000	5999.9999	6000.0000
M( 9)	-0.0000	5999.9999	6000.0000
M( 10)	0.0001	2489.3618	6000.0000
M( 11)	0.0001	3510.6386	6000.0000
M( 12)	0.0001	4276.5960	6000.0000
M( 13)	-0.0000	-5999.9999	6000.0000
M( 14)	-0.0000	-5297.8721	6000.0000
M( 15)	-0.0001	-5297.8721	6000.0000
M( 16)	-0.0001	-5999.9999	6000.0000
M( 17)	-0.0001	-4276.5956	6000.0000
M( 18)	0.0000	-2999.9997	6000.0000
M( 19)	-0.0000	-2999.9998	6000.0000
M( 20)	-0.0001	-4276.5958	6000.0000

HINGE ROTATIONS	
AT POINT( 1)	-2.6595755E-03
AT POINT( 2)	0.0000000E-39
AT POINT( 3)	0.0000000E-39
AT POINT( 4)	0.0000000E-39
AT POINT( 5)	-4.6099306E-03
AT POINT( 6)	0.0000000E-39
AT POINT( 7)	-8.8652689E-04
AT POINT( 8)	-6.2056761E-03
AT POINT( 9)	-2.6595755E-03
AT POINT( 10)	0.0000000E-39
AT POINT( 11)	0.0000000E-39
AT POINT( 12)	0.0000000E-39
AT POINT( 13)	0.0000000E-39
AT POINT( 14)	0.0000000E-39
AT POINT( 15)	0.0000000E-39
AT POINT( 16)	0.0000000E-39
AT POINT( 17)	0.0000000E-39
AT POINT( 18)	0.0000000E-39
AT POINT( 19)	0.0000000E-39
AT POINT( 20)	0.0000000E-39

PLASTIC HINGE NO. 8 FORMED AT POINT 6

LOAD FACTOR	ADDITIONAL	CUMULATIVE
STAGE( 8)	1.4781631E 02	2.9052630E 03

DEFLECTION	ADDITIONAL	CUMULATIVE
X( 1)	-5.1325095E-03	-1.7543857E-02
X( 2)	-2.6004716E-03	-1.1111109E-02
X( 3)	-5.1325095E-03	-1.7543857E-02
X( 4)	-2.8742050E-03	-1.3157893E-02
X( 5)	1.4371025E-03	-1.7543859E-03
X( 6)	-2.8742050E-03	-1.3157894E-02
X( 7)	5.2009434E-02	1.6666665E-01
X( 8)	9.5464680E-02	3.5964910E-01

MOMENT	ADDITIONAL	CUMULATIVE	PLAS MOM
M( 1)	0.0000	5999.9999	6000.0000
M( 2)	36.9543	2526.3160	6000.0000
M( 3)	-36.9541	3473.6844	6000.0000
M( 4)	776.0355	5052.6317	6000.0000
M( 5)	0.0000	5999.9999	6000.0000
M( 6)	1404.2547	5999.9999	6000.0000
M( 7)	0.0000	5999.9999	6000.0000
M( 8)	0.0000	5999.9999	6000.0000
M( 9)	0.0000	5999.9999	6000.0000
M( 10)	36.9543	2526.3160	6000.0000
M( 11)	-36.9541	3473.6845	6000.0000
M( 12)	776.0355	5052.6315	6000.0000
M( 13)	0.0000	-5999.9999	6000.0000
M( 14)	-702.1273	-5999.9994	6000.0000
M( 15)	-702.1273	-5999.9995	6000.0000
M( 16)	0.0000	-5999.9999	6000.0000
M( 17)	-776.0353	-5052.6309	6000.0000
M( 18)	0.0000	-2999.9996	6000.0000
M( 19)	0.0000	-2999.9998	6000.0000
M( 20)	-776.0353	-5052.6311	6000.0000

HINGE ROTATIONS	
AT POINT( 1)	-7.8947357E-03
AT POINT( 2)	0.0000000E-39
AT POINT( 3)	0.0000000E-39
AT POINT( 4)	0.0000000E-39
AT POINT( 5)	-1.1111110E-02
AT POINT( 6)	0.0000000E-39
AT POINT( 7)	-2.6315800E-03
AT POINT( 8)	-1.1988303E-02
AT POINT( 9)	-7.8947357E-03
AT POINT( 10)	0.0000000E-39
AT POINT( 11)	0.0000000E-39
AT POINT( 12)	0.0000000E-39
AT POINT( 13)	6.4327457E-03
AT POINT( 14)	0.0000000E-39
AT POINT( 15)	0.0000000E-39
AT POINT( 16)	6.4327456E-03
AT POINT( 17)	0.0000000E-39
AT POINT( 18)	0.0000000E-39
AT POINT( 19)	0.0000000E-39
AT POINT( 20)	0.0000000E-39

PLASTIC HINGE NO. 9 FORMED AT POINT 4

LOAD FACTOR	ADDITIONAL	CUMULATIVE	
STAGE( 9)	9.4736889E 01	2.9999998E 03	
DEFLECTION			
X( 1)	-7.4561370E-03	-2.4999994E-02	
X( 2)	-0.0000000E-39	-1.1111109E-02	
X( 3)	-7.4561370E-03	-2.4999994E-02	
X( 4)	-3.5087701E-03	-1.6666663E-02	
X( 5)	1.7543851E-03	-8.4401108E-10	
X( 6)	-3.5087701E-03	-1.6666664E-02	
X( 7)	8.3333299E-02	2.4999995E-01	
X( 8)	1.4035082E-01	4.9999992E-01	
MOMENT			
M( 1)	-0.0000	5999.9999	6000.0000
M( 2)	473.6841	3000.0001	6000.0000
M( 3)	-473.6837	3000.0007	6000.0000
M( 4)	947.3683	5999.9999	6000.0000
M( 5)	-0.0000	5999.9999	6000.0000
M( 6)	-0.0000	5999.9999	6000.0000
M( 7)	0.0000	5999.9999	6000.0000
M( 8)	0.0000	5999.9999	6000.0000
M( 9)	-0.0000	5999.9999	6000.0000
M( 10)	473.6841	3000.0001	6000.0000
M( 11)	-473.6837	3000.0008	6000.0000
M( 12)	947.3683	5999.9998	6000.0000
M( 13)	-0.0000	-5999.9999	6000.0000
M( 14)	-0.0000	-5999.9994	6000.0000
M( 15)	-0.0000	-5999.9995	6000.0000
M( 16)	-0.0000	-5999.9999	6000.0000
M( 17)	-947.3679	-5999.9988	6000.0000
M( 18)	0.0000	-2999.9996	6000.0000
M( 19)	0.0000	-2999.9998	6000.0000
M( 20)	-947.3679	-5999.9990	6000.0000

HINGE ROTATIONS	
AT POINT( 1)	-1.6666662E-02
AT POINT( 2)	0.0000000E-39
AT POINT( 3)	0.0000000E-39
AT POINT( 4)	0.0000000E-39
AT POINT( 5)	-1.9444439E-02
AT POINT( 6)	-8.3333300E-03
AT POINT( 7)	-8.3333319E-03
AT POINT( 8)	-1.9444440E-02
AT POINT( 9)	-1.6666662E-02
AT POINT( 10)	0.0000000E-39
AT POINT( 11)	0.0000000E-39
AT POINT( 12)	0.0000000E-39
AT POINT( 13)	1.3888883E-02
AT POINT( 14)	0.0000000E-39
AT POINT( 15)	0.0000000E-39
AT POINT( 16)	1.3888882E-02
AT POINT( 17)	0.0000000E-39
AT POINT( 18)	0.0000000E-39
AT POINT( 19)	0.0000000E-39
AT POINT( 20)	0.0000000E-39

PLASTIC HINGE NO. 10 FORMED AT POINT 12

LOAD FACTOR	ADDITIONAL	CUMULATIVE
STAGE( 10)	1.2207035E-05	2.9999998E 03

DEFLECTION	ADDITIONAL	CUMULATIVE
X( 1)	-1.4411074E-09	-2.4999996E-02
X( 2)	0.0000000E-39	-1.1111109E-02
X( 3)	-1.5541353E-09	-2.4999995E-02
X( 4)	-1.1302803E-10	-1.6666663E-02
X( 5)	2.2605605E-10	-6.1795503E-10
X( 6)	-7.9119615E-10	-1.6666664E-02
X( 7)	1.6106494E-08	2.4999996E-01
X( 8)	2.8822148E-08	4.9999994E-01

MOMENT	ADDITIONAL	CUMULATIVE	PLAS MOM
M( 1)	-0.0000	5999.9999	6000.0000
M( 2)	0.0001	3000.0002	6000.0000
M( 3)	-0.0001	3000.0006	6000.0000
M( 4)	0.0000	5999.9999	6000.0000
M( 5)	-0.0000	5999.9999	6000.0000
M( 6)	-0.0000	5999.9999	6000.0000
M( 7)	0.0000	5999.9999	6000.0000
M( 8)	0.0000	5999.9999	6000.0000
M( 9)	-0.0000	5999.9999	6000.0000
M( 10)	0.0000	3000.0002	6000.0000
M( 11)	-0.0000	3000.0008	6000.0000
M( 12)	0.0002	5999.9999	6000.0000
M( 13)	-0.0000	-5999.9999	6000.0000
M( 14)	-0.0000	-5999.9994	6000.0000
M( 15)	-0.0000	-5999.9995	6000.0000
M( 16)	-0.0000	-5999.9999	6000.0000
M( 17)	-0.0000	-5999.9988	6000.0000
M( 18)	0.0001	-2999.9995	6000.0000
M( 19)	-0.0001	-2999.9998	6000.0000
M( 20)	-0.0002	-5999.9992	6000.0000

AT POINT	HINGE ROTATIONS
( 1)	-1.6666663E-02
( 2)	0.0000000E-39
( 3)	0.0000000E-39
( 4)	0.0000000E-39
( 5)	-1.9444441E-02
( 6)	-8.3333314E-03
( 7)	-8.3333333E-03
( 8)	-1.9444442E-02
( 9)	-1.6666663E-02
( 10)	0.0000000E-39
( 11)	0.0000000E-39
( 12)	0.0000000E-39
( 13)	1.3888884E-02
( 14)	0.0000000E-39
( 15)	0.0000000E-39
( 16)	1.3888884E-02
( 17)	0.0000000E-39
( 18)	0.0000000E-39
( 19)	0.0000000E-39
( 20)	0.0000000E-39

APPENDIX C

COMPUTER INPUT AND OUTPUT LISTINGS  
FOR SAMPLE PROBLEM 1



\* SAMPLE PROBLEM I DEBRIS CHARACTERISTICS OF FOUR CONTIGUOUS WALLS

\* FIRST WALL (ALL DISTRIBUTIONS ARE TAKEN RELATIVE TO THE POSITION

\* OF THIS WALL)

WEAPON PARAMETERS

YIELD 1000. KILG-TONS

OVERPRESSURE 10. PSI

PREBLAST STRUCTURAL CONFIGURATION

WALL HEIGHT 40 FEET

HEIGHT BETWEEN WALLS 10. FEET

NORMALIZING FACTOR 3.333

FRAGMENTATION CHARACTERISTICS

NUMBER OF PARTICLE SIZES 5

PARTICLE SIZES 10.0, 8.0, 6.0, 4.0, 2.0 INCHES EQUIVALENT RADIUS

PERCENTAGE BY SIZE 0.13, 0.05, 0.45, 0.32, 0.05

ACCELERATION COEFFICIENT 0.0

OUTPUT

PROFILE DISTRIBUTION 1

DISTRIBUTION OF SIZES 1

LOCATIONS 3

DISTANCES FROM FIRST WALL 50., 150., 300. FEET

VELOCITY DESCRIPTION 1

DEBRIS PROFILE PLCT 1

SOLVE

5 PARTICLE SIZES

4 STORY BUILDING

10,000 CFT. BETWEEN FLOORS

YIELD = 1000 CFT.

UNFRESHNESS = 10,000

PARTICLE RADII PERCENT OF PANEL

10,000 FT.	C. 12 X100
5,000 FT.	C. 05 X100
6,000 FT.	C. 45 X100
4,000 FT.	C. 32 X100
2,000 FT.	C. 05 X100

DISTANCE OF CURRENT WALL FROM STARTING WALL C. FT.

SIZE RANGE 1	0.00 IN.	TC	10.00 IN.
SIZE RANGE 2	6.00 IN.	TC	8.00 IN.
SIZE RANGE 3	4.00 IN.	TC	6.00 IN.
SIZE RANGE 4	2.00 IN.	TC	4.00 IN.
SIZE RANGE 5	C.	IN.	TC 2.00 IN.

DEPTH HEIGHT (FT.) AT IFT. INTERVALS FROM ORIGINAL POSITION

1 FT.	0.	C.	C. 07EC72	C.	0.445253	0.	0.367180
6 FT.	0.330084	0.408156	C.625690	C.	0.296915	0.	0.255358
11 FT.	0.528454	C.528454	C.43E196	C.	0.527403	0.	0.407687
16 FT.	C.255358	0.255358	C.517660	C.	0.500055	0.	0.539420
21 FT.	0.407687	C.493983	C.445815	C.	0.445815	0.	0.644539
26 FT.	0.582019	0.652413	C.604127	C.	0.604127		
31 FT.	0.542003	0.618976					

36FT.	0.775632	0.554074	C.731747	C.775645	0.716664
41FT.	0.777205	0.645109	C.920992	C.796856	0.929227
46FT.	0.902784	0.955474	1.031642	C.957461	1.034885
51FT.	1.006507	0.533730	1.097313	1.085667	1.113363
56FT.	1.113492	1.144866	1.208325	1.197526	1.315751
61FT.	1.213945	1.343850	1.302046	1.354180	1.199628
66FT.	1.226740	1.184411	1.151174	1.195054	1.152172
71FT.	1.115949	1.141784	1.132452	1.093362	1.094257
76FT.	1.087330	1.007652	1.014116	1.070256	0.997177
81FT.	1.043305	1.057927	1.027304	1.072162	1.125356
86FT.	1.100669	1.092013	1.152479	1.102091	1.135814
91FT.	1.186171	1.162974	1.154855	1.212355	1.189963
96FT.	1.238439	1.217505	1.222616	1.232005	1.179748
101FT.	1.195582	1.144402	1.160035	1.109746	1.125154
106FT.	1.076632	1.053151	1.068346	1.034585	0.987387
111FT.	1.002234	0.955386	C.970810	C.902486	0.917035
116FT.	0.871504	0.825902	C.841042	C.856015	0.790615
121FT.	0.804747	0.775089	C.731536	C.745836	0.703192
126FT.	0.717745	0.690334	C.627358	C.640041	0.613229
131FT.	0.573033	0.566995	C.539925	C.499895	0.513123
136FT.	0.487279	0.429209	C.429209	C.429209	0.429209
141FT.	0.429800	0.411519	C.411519	C.411519	0.410360
146FT.	0.392554	0.392554	C.392554	C.392554	0.393111
151FT.	0.375742	0.375742	C.375742	C.376284	0.359279
156FT.	0.359279	0.359279	C.359279	C.343141	0.343141
161FT.	0.343658	0.321273	C.327273	C.327273	0.327778
166FT.	0.311690	0.311690	C.312185	C.296346	0.296346
171FT.	0.296832	0.281200	C.281200	C.281677	0.266269
176FT.	0.266269	0.266269	C.266738	C.251542	0.251542
181FT.	0.252004	0.236986	C.237441	0.222594	0.222594
186FT.	0.223042	0.208357	C.208800	C.194250	0.194250
191FT.	0.194686	0.180288	C.180719	C.165151	0.165151
196FT.	0.165576	0.151444	C.151864	C.137848	0.137848
201FT.	0.138264	0.124361	C.124772	C.110575	0.111386
206FT.	0.097699	0.058101	C.084518	C.084516	0.071417
211FT.	0.071811	0.058410	C.058801	C.045475	0.045866
216FT.	0.032638	C.033071	C.019869	C.019869	0.019869
221FT.	0.019869	0.019869	C.019869	C.019869	0.019869
226FT.	0.019869	0.019869	C.019869	C.019869	0.019869
231FT.	0.018790	0.019869	C.018790	C.018790	0.018790
236FT.	0.018790	0.018790	C.018790	C.018790	0.018790
241FT.	0.018790	0.018790	C.018790	C.018790	0.018790

246FI.	0.018790	C.018790	C.018790
251FI.	0.018790	C.018790	C.018790
256FI.	C.018790	C.018790	C.018790
261FI.	0.018790	C.018790	C.018790
266FI.	0.017846	C.017846	C.017846
271FI.	0.017846	C.017846	C.017846
276FI.	0.017846	C.017846	C.017846
281FI.	0.017846	C.017846	C.017846
286FI.	C.017846	C.017846	C.017846
291FI.	0.017846	C.017846	C.017846
296FI.	0.016987	C.016987	C.016987
301FI.	0.016987	C.016987	C.016987
306FI.	0.016987	C.016987	C.016987
311FI.	0.016987	C.016987	C.016987
316FI.	0.016987	C.016987	C.016987
321FI.	0.016199	C.016199	C.016199
326FI.	0.016199	C.016199	C.016199
331FI.	0.016199	C.016199	C.016199
336FI.	0.016199	C.016199	C.016199
341FI.	0.015458	C.015458	C.015458
346FI.	0.015458	C.015458	C.015458
351FI.	0.015458	C.015458	C.015458
356FI.	0.015458	C.015458	C.015458
361FI.	0.014759	C.014759	C.014759
366FI.	0.014759	C.014759	C.014759
371FI.	0.014759	C.014759	C.014759
376FI.	0.014759	C.014759	C.014759
381FI.	0.014095	C.014095	C.014095
386FI.	0.014095	C.014095	C.014095
391FI.	0.014095	C.014095	C.014095
396FI.	0.013459	C.013459	C.013459
401FI.	0.013459	C.013459	C.013459
406FI.	0.013459	C.013459	C.013459
411FI.	0.012847	C.012847	C.012847
416FI.	0.012847	C.012847	C.012847
421FI.	0.012847	C.012847	C.012847
426FI.	0.012255	C.012255	C.012255
431FI.	0.012255	C.012255	C.012255
436FI.	0.012255	C.012255	C.012255
441FI.	0.011682	C.011682	C.011682
446FI.	0.011682	C.011682	C.011682



DISTRIBUTION OF SIZES AT 50 FT. FROM ORIGINAL POSITION

SIZE RANGE	PERCENT
1	40.75
2	8.56
3	40.84
4	9.35
5	0.50

DISTRIBUTION OF SIZES AT 150 FT. FROM ORIGINAL POSITION

SIZE RANGE	PERCENT
1	0.
2	0.
3	0.
4	97.45
5	2.55

DISTRIBUTION OF SIZES AT 300 FT. FROM ORIGINAL POSITION

SIZE RANGE	PERCENT
1	0.
2	0.
3	0.
4	0.
5	100.00

CUMULATIVE DEBRIS MCMENTUP (FT./SEC.) AT 1FT INTERVALS FROM ORIGINAL POSITION  
(MULTIPLY BY MASS CF CNE PANEL)

1FT.	0.	C.	C.	C.	C.
6FT.	2.386594	C.63E248	4.335165	4.335165	3.696522
11FT.	5.405084	6.862768	3.60C571	3.60C571	3.454173
16FT.	3.454173	5.79E788	7.248E43	7.248E43	5.687479
21FT.	5.687479	7.514506	7.505C77	7.505C77	8.083C16
26FT.	9.117875	7.945C13	7.945C13	7.945C13	11.618105
31FT.	9.805254	11.555785	11.555785	11.555785	15.136837
36FT.	15.474083	15.56C373	16.977215	16.977215	15.565096
41FT.	17.333539	21.14523C	19.269737	19.269737	22.789735
46FT.	22.465785	26.381796	25.212E5C	25.212E5C	27.601C27
51FT.	27.880796	31.1C5674	31.147294	31.147294	32.102242
56FT.	33.655144	37.162220	37.211C14	37.211C14	41.381230
61FT.	39.334664	42.97E363	44.184946	44.184946	40.295345
66FT.	40.936779	4C.12E47	35.206264	35.206264	38.120965
71FT.	36.656732	3E.1CC864	34.90766E	34.90766E	35.564526
76FT.	34.988850	32.882643	34.86CC74	34.86CC74	32.782650
81FT.	34.587931	34.497C25	36.3175C3	36.3175C3	38.325420
86FT.	38.068248	37.7C7194	39.272143	39.272143	40.763350
91FT.	42.803449	42.566010	44.622287	44.622287	44.391C76
96FT.	46.462622	46.250708	47.213742	47.213742	45.641564
101FT.	46.200190	44.436951	45.0CC592	45.0CC592	43.794174
106FT.	42.032731	41.463365	42.054354	42.054354	39.054883
111FT.	39.634661	37.835953	3E.437224	3E.437224	36.677984
116FT.	34.862829	35.455564	33.635C86	33.635C86	31.883058
121FT.	32.484025	31.255995	29.415133	29.415133	28.176798
126FT.	28.807395	27.037483	25.176584	25.176584	24.558319
131FT.	22.700323	22.796663	21.55C848	21.55C848	20.300566
136FT.	19.067093	16.634137	16.634137	16.634137	16.634137
141FT.	16.651051	16.115095	16.115095	16.115095	16.106304
146FT.	15.568192	15.56E192	15.56E192	15.56E192	15.585120
151FT.	15.044865	15.044865	15.044E65	15.044E65	14.518121
156FT.	14.518121	14.518121	14.535141	14.535141	13.988090
161FT.	14.005191	13.453710	12.453710	12.453710	13.470839
166FT.	12.916302	12.916302	12.933496	12.933496	12.37487C
171FT.	12.392164	11.828523	11.828523	11.828523	11.278487
176FT.	11.278487	11.279487	11.295E76	11.295E76	10.724887
181FT.	10.742355	10.166888	1C.184429	9.0C4C5C	9.604650
186FT.	9.622257	9.0C3H321	5.056C27	P.467152	8.467152

191FI.	8.484913	7.692177	7.91CC24	7.29C5CE	7.290908
196FI.	7.308801	6.707835	6.725805	6.12C487	6.120487
201FI.	6.138529	5.528992	5.547102	4.933465	4.951643
206FI.	4.334030	4.352262	3.73C780	3.7491C1	3.123066
211FI.	3.141440	2.511749	2.53C204	1.896184	1.914685
216FI.	1.277201	1.295776	C.654164	C.654164	0.654164
221FI.	0.654164	0.654164	C.654164	C.654164	0.654164
226FI.	0.654164	0.654164	C.654164	C.654164	0.654164
231FI.	0.634352	0.634352	C.634352	C.634352	0.634352
236FI.	0.634352	0.634352	C.634352	C.634352	0.634352
241FI.	0.634352	0.634352	C.634352	C.634352	0.634352
246FI.	0.634352	0.634352	C.634352	C.634352	0.634352
251FI.	0.634352	0.634352	C.634352	C.634352	0.634352
256FI.	0.634352	0.634352	C.634352	C.634352	0.634352
261FI.	0.634352	0.634352	C.634352	C.634352	0.634352
266FY.	0.615674	0.615674	C.615674	C.615674	0.615674
271FI.	0.615674	0.615674	C.615674	C.615674	0.615674
276FI.	0.615674	0.615674	C.615674	C.615674	0.615674
281FI.	0.615674	0.615674	C.615674	C.615674	0.615674
286FI.	0.615674	0.615674	C.615674	C.615674	0.615674
291FI.	0.615674	0.597603	C.597603	C.597603	0.597603
296FY.	0.597603	0.597603	C.597603	C.597603	0.597603
301FI.	0.597603	0.597603	C.597603	C.597603	0.597603
306FI.	0.597603	0.597603	C.597603	C.597603	0.597603
311FI.	0.597603	0.597603	C.597603	C.597603	0.597603
316FI.	0.597603	0.597603	C.597603	C.597603	0.597603
321FI.	0.580064	0.580064	C.580064	C.580064	0.580064
326FI.	0.580064	0.580064	C.580064	C.580064	0.580064
331FI.	0.580064	0.580064	C.580064	C.580064	0.580064
336FY.	0.580064	0.580064	C.580064	C.580064	0.580064
341FI.	0.562728	0.562728	C.562728	C.562728	0.562728
346FI.	0.562728	0.562728	C.562728	C.562728	0.562728
351FI.	0.562728	0.562728	C.562728	C.562728	0.562728
356FI.	0.562728	0.562728	C.562728	C.562728	0.562728
361FI.	0.545585	0.545585	C.545585	C.545585	0.545585
366FI.	0.545585	0.545585	C.545585	C.545585	0.545585
371FI.	0.545585	0.545585	C.545585	C.545585	0.545585
376FY.	0.545585	0.545585	C.545585	C.545585	0.545585
381FI.	0.528621	0.528621	C.528621	C.528621	0.528621
386FY.	0.528621	0.528621	C.528621	C.528621	0.528621
391FI.	0.528621	0.528621	C.528621	C.511689	0.511689



396FT.	0.511689	C.511689	0.511689
401FT.	0.511689	C.511689	0.511689
406FT.	0.511689	C.494794	0.494794
411FT.	0.494794	C.494794	0.494794
416FT.	0.494794	C.477880	0.477880
421FT.	0.494794	C.477880	0.477880
426FT.	0.477880	C.477880	0.477880
431FT.	0.477880	C.477880	0.477880
436FT.	0.477880	C.460956	0.460956
441FT.	0.460956	C.460956	0.460956
446FT.	0.460956	C.460956	0.460956
451FT.	0.444028	C.444028	0.444028
456FT.	0.444028	C.444028	0.444028
461FT.	0.444028	C.427049	0.427049
466FT.	0.427049	C.427049	0.427049
471FT.	0.427049	C.427049	0.427049
476FT.	0.410029	C.410029	0.410029
481FT.	0.410029	C.410029	0.410029
486FT.	0.392928	C.392928	0.392928
491FT.	0.392928	C.392928	0.392928
496FT.	0.375800	C.375800	0.375800
501FT.	0.375800	C.375800	0.375800
506FT.	0.358606	C.358606	0.358606
511FT.	0.358606	C.358606	0.358606
516FT.	0.341312	C.341312	0.341312
521FT.	0.341312	C.341312	0.341312
526FT.	0.323967	C.323967	0.323967
531FT.	0.323967	C.323967	0.323967
536FT.	0.306578	C.306578	0.306578
541FT.	0.306578	C.289110	0.289110
546FT.	0.289110	C.289110	0.289110
551FT.	0.271569	C.271569	0.271569
556FT.	0.271569	C.271569	0.271569
561FT.	0.253962	C.253962	0.253962
566FT.	0.236256	C.236256	0.236256
571FT.	0.236256	C.236256	0.236256
576FT.	0.218496	C.218496	0.218496
581FT.	0.200648	C.200648	0.200648
586FT.	0.200648	C.200648	0.200648
591FT.	0.182755	C.182755	0.182755
596FT.	0.164785	C.164785	0.164785
601FT.	0.164785	C.146743	0.146743

606FT.  
611FT.  
616FT.  
621FT.  
626FT.  
631FT.  
636FT.  
641FT.  
646FT.

0.146743  
0.128633  
0.128633  
0.110460  
0.092227  
0.073905  
0.055531  
0.037076  
0.037076

0.146743  
0.128633  
0.110460  
0.110460  
0.092227  
0.073905  
0.055531  
0.037076  
0.037076

C.146743  
C.128633  
C.110460  
C.092227  
C.092227  
C.073905  
C.055531  
C.037076  
C.018575

C.146743  
C.128633  
0.110460  
C.092227  
C.073905  
C.073905  
C.055531  
C.037076  
C.018575

0.128633  
0.128633  
0.110460  
0.092227  
0.073905  
0.055531  
0.055531  
C.037076  
0.018575

MINIMUM DEBRIS MOMENTUM (FT./SEC.) AT 1FT. INTERVALS FROM ORIGINAL POSITION  
(MULTIPLY BY MASS OF CASE PANEL)

1FT.	0.	0.	0.	0.	0.
6FT.	0.	0.	0.	0.	0.
11FT.	1.708162	1.708162	1.45E684	C.434124	C.434124
16FT.	0.434124	0.434124	C.434124	C.385408	C.385408
21FT.	0.385408	0.385408	C.365219	C.041708	C.369219
26FT.	0.365219	0.358099	C.358099	C.358099	C.353427
31FT.	0.353427	0.347978	0.347978	C.347978	0.347439
36FT.	0.347439	0.346072	C.346072	0.346072	0.344207
41FT.	0.344207	0.344207	C.344207	C.344207	0.344207
46FT.	0.344207	0.344207	C.025715	C.025715	0.025715
51FT.	0.025715	0.025715	C.025715	C.025715	0.025715
56FT.	0.025715	0.025715	C.025715	C.025715	0.025715
61FT.	0.025715	0.025715	C.025715	C.021756	0.021756
66FT.	0.021756	0.021756	C.021756	C.021756	0.021756
71FT.	0.021756	0.021756	C.021756	C.021756	0.021756
76FT.	0.021756	0.019812	C.019812	C.019812	0.019812
81FT.	0.019812	0.019812	C.019812	C.019812	0.019812
86FT.	0.019812	0.019812	C.018678	C.018678	0.018678
91FT.	0.018678	0.018678	C.018678	C.018678	0.018678
96FT.	0.018678	0.018071	C.018071	C.018071	0.018071
101FT.	0.018071	0.018071	C.018071	C.018071	0.018071
106FT.	0.017539	0.017539	C.017539	C.017539	0.017539
111FT.	0.017539	0.017539	C.017539	C.017539	0.017539
116FT.	0.017336	0.017336	C.017336	C.017336	0.017336
121FT.	0.017143	0.017143	C.017143	C.017143	0.017143
126FT.	0.016964	0.016964	C.016964	C.016964	0.016964
131FT.	0.016932	0.016932	C.016932	C.016932	0.016932
136FT.	0.016895	0.016895	C.016895	C.016895	0.016895
141FT.	0.016895	0.016895	C.016895	C.016895	0.016895
146FT.	0.016895	0.016895	C.016895	C.016895	0.016895
151FT.	0.016895	0.016895	C.016895	C.016895	0.016895
156FT.	0.016895	0.016895	C.016895	C.016895	0.016895
161FT.	0.016895	0.016895	C.016895	C.016895	0.016895
166FT.	0.016895	0.016895	C.016895	C.016895	0.016895
171FT.	0.016895	0.016895	C.016895	C.016895	0.016895
176FT.	0.016895	0.016895	C.016895	C.016895	0.016895
181FT.	0.016895	0.016895	C.016895	C.016895	0.016895
186FT.	0.016895	0.016895	C.016895	C.016895	0.016895

191FI.	0.016895	C.016895	0.016895
196FI.	0.016895	C.016895	0.016895
201FI.	0.016895	C.016895	0.016895
206FI.	0.016895	C.016895	0.016895
211FI.	0.016895	C.016895	0.016895
216FI.	0.016895	C.016895	0.016895
221FI.	0.016895	C.016895	0.016895
226FI.	0.016895	C.016895	0.016895
231FI.	0.016895	C.016895	0.016895
236FI.	0.016895	C.016895	0.016895
241FI.	0.016895	C.016895	0.016895
246FI.	0.016895	C.016895	0.016895
251FI.	0.016895	C.016895	0.016895
256FI.	0.016895	C.016895	0.016895
261FI.	0.016895	C.016895	0.016895
266FI.	0.016895	C.016895	0.016895
271FI.	0.016895	C.016895	0.016895
276FI.	0.016895	C.016895	0.016895
281FI.	0.016895	C.016895	0.016895
286FI.	0.016895	C.016895	0.016895
291FI.	0.016895	C.016895	0.016895
296FI.	0.016895	C.016895	0.016895
301FI.	0.016895	C.016895	0.016895
306FI.	0.016895	C.016895	0.016895
311FI.	0.016895	C.016895	0.016895
316FI.	0.016895	C.016895	0.016895
321FI.	0.016895	C.016895	0.016895
326FI.	0.016895	C.016895	0.016895
331FI.	0.016895	C.016895	0.016895
336FI.	0.016895	C.016895	0.016895
341FI.	0.016895	C.016895	0.016895
346FI.	0.016895	C.016895	0.016895
351FI.	0.016895	C.016895	0.016895
356FI.	0.016895	C.016895	0.016895
361FI.	0.016895	C.016895	0.016895
366FI.	0.016895	C.016895	0.016895
371FI.	0.016895	C.016895	0.016895
376FI.	0.016895	C.016895	0.016895
381FI.	0.016895	C.016895	0.016895
386FI.	0.016895	C.016895	0.016895
391FI.	0.016895	C.016895	0.016895

MAXIMUM GEBRIS MOMENTUM (FT./SEC.) AT 1FT. INTERVALS FROM ORIGINAL POSITION  
 (MULTIPLY BY MASS OF ONE PANEL)

1FT.	0.	0.	0.	0.	0.
6FT.	2.386594	C.63E248	C.696922	3.696922	3.696922
11FT.	3.696922	3.696922	1.708162	1.708162	1.561364
16FT.	1.561364	2.344616	2.344616	2.344616	2.344616
21FT.	2.344616	2.344616	2.344616	2.344616	2.344616
26FT.	2.344616	2.036624	2.036624	2.036624	2.036624
31FT.	2.036624	2.036624	2.036624	2.036624	2.036624
36FT.	2.036624	1.896301	1.896301	1.896301	1.896301
41FT.	1.896301	1.896301	1.896301	1.896301	1.818809
46FT.	1.818809	1.818809	1.818809	1.818809	1.818809
51FT.	1.782281	1.782281	1.782281	1.782281	1.782281
56FT.	1.747065	1.747065	1.747065	1.747065	1.747065
61FT.	1.739187	1.739187	1.739187	1.739187	1.739187
66FT.	1.743933	1.752017	1.752017	1.752017	1.752017
71FT.	1.756222	1.772800	1.772800	1.772800	1.772800
76FT.	1.778982	1.791403	1.791403	1.791403	1.791403
81FT.	1.805281	1.815155	1.815155	1.815155	1.815155
86FT.	1.833348	1.840862	1.840862	1.840862	1.840862
91FT.	1.860499	1.866183	1.866183	1.866183	1.866183
96FT.	1.887766	1.891980	1.891980	1.891980	1.891980
101FT.	1.899187	1.899187	1.899187	1.899187	1.899187
106FT.	1.899187	1.899187	1.899187	1.899187	1.899187
111FT.	1.899187	1.899187	1.899187	1.899187	1.899187
116FT.	1.899187	1.899187	1.899187	1.899187	1.899187
121FT.	1.899187	1.899187	1.899187	1.899187	1.899187
126FT.	1.899187	1.899187	1.899187	1.899187	1.899187
131FT.	1.899187	1.899187	1.899187	1.899187	1.899187
136FT.	1.899187	0.641611	0.641611	0.641611	0.641611
141FT.	0.641611	0.641611	0.641611	0.641611	0.641611
146FT.	0.641611	0.641611	0.641611	0.641611	0.641611
151FT.	0.641611	0.641611	0.641611	0.641611	0.641611
156FT.	0.641611	0.641611	0.641611	0.641611	0.641611
161FT.	0.641611	0.641611	0.641611	0.641611	0.641611
166FT.	0.641611	0.641611	0.641611	0.641611	0.641611
171FT.	0.641611	0.641611	0.641611	0.641611	0.641611
176FT.	0.641611	0.641611	0.641611	0.641611	0.641611
181FT.	0.641611	0.641611	0.641611	0.641611	0.641611
186FT.	0.641611	0.641611	0.641611	0.641611	0.641611

191FT.	0.641611	C.641611	0.641611	C.641611
196FT.	0.641611	C.641611	0.641611	C.641611
201FT.	0.641611	C.641611	0.641611	C.641611
206FT.	0.641611	C.641611	0.641611	C.641611
211FT.	0.641611	C.641611	0.641611	C.641611
216FT.	0.641611	C.641611	0.641611	C.641611
221FT.	0.019812	C.019812	0.019812	C.019812
226FT.	0.019812	C.019812	0.019812	C.019812
231FT.	0.019812	C.019812	0.019812	C.019812
236FT.	0.018678	C.018678	0.018678	C.018678
241FT.	0.018678	C.018678	0.018678	C.018678
246FT.	0.018678	C.018678	0.018678	C.018678
251FT.	0.018678	C.018678	0.018678	C.018678
256FT.	0.018678	C.018678	0.018678	C.018678
261FT.	0.018678	C.018678	0.018678	C.018678
266FT.	0.018575	C.018575	0.018575	C.018575
271FT.	0.018575	C.018575	0.018575	C.018575
276FT.	0.018575	C.018575	0.018575	C.018575
281FT.	0.018575	C.018575	0.018575	C.018575
286FT.	0.018575	C.018575	0.018575	C.018575
291FT.	0.018575	C.018575	0.018575	C.018575
296FT.	0.018575	C.018575	0.018575	C.018575
301FT.	0.018575	C.018575	0.018575	C.018575
306FT.	0.018575	C.018575	0.018575	C.018575
311FT.	0.018575	C.018575	0.018575	C.018575
316FT.	0.018575	C.018575	0.018575	C.018575
321FT.	0.018575	C.018575	0.018575	C.018575
326FT.	0.018575	C.018575	0.018575	C.018575
331FT.	0.018575	C.018575	0.018575	C.018575
336FT.	0.018575	C.018575	0.018575	C.018575
341FT.	0.018575	C.018575	0.018575	C.018575
346FT.	0.018575	C.018575	0.018575	C.018575
351FT.	0.018575	C.018575	0.018575	C.018575
356FT.	0.018575	C.018575	0.018575	C.018575
361FT.	0.018575	C.018575	0.018575	C.018575
366FT.	0.018575	C.018575	0.018575	C.018575
371FT.	0.018575	C.018575	0.018575	C.018575
376FT.	0.018575	C.018575	0.018575	C.018575
381FT.	0.018575	C.018575	0.018575	C.018575
386FT.	0.018575	C.018575	0.018575	C.018575
391FT.	0.018575	C.018575	0.018575	C.018575



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- WALL 2 SUPERIMPOSED ON WALL 1
  - THERE ARE 50 FT. BETWEEN WALLS 1 AND 2
- PRECAST STRUCTURAL CONFIGURATION  
SPACE BETWEEN WALLS 50. FT.

SCLVE

PARTICIP SIZE.  
 4CSTERY BUILDING  
 10-CCFT. BETWEEN FLEERS  
 VIFLD= 1000.CKT.  
 (VMPRESSURE= 10.0PSI

PARTICLE RADII                      PERCENT CF PANEL

10-00 FT.                              C.12 X100  
 8-00 FT.                                C.05 X100  
 6-00 FT.                                C.45 X100  
 4-00 FT.                                C.32 X100  
 2-00 FT.                                C.05 X100

DISTANCE CF CLARENT WALL FROM STARTING WALL      50.0 FT.

SIZE RANGE      1      8.00 IN.      TC      10.00 IN.  
 SIZE RANGE      2      6.00 IN.      TC      8.00 IN.  
 SIZE RANGE      3      4.00 IN.      TC      6.00 IN.  
 SIZE RANGE      4      2.00 IN.      TC      4.00 IN.  
 SIZE RANGE      5      C.      IN.      TC      2.00 IN.

LEERS HEIGHT (FT.) AT 1FT. INTERVALS FROM CRITICAL POSITION

1FT.	C.	0.330084	C.	0.072072	C.	0.445251	C.	0.367180
6FT.	0.	0.528454	0.	0.625040	0.	0.296511	0.	0.255358
11FT.	0.	0.255358	0.	0.428196	0.	0.527403	0.	0.407687
16FT.	0.	0.407687	0.	0.517660	0.	0.500055	0.	0.539420
21FT.	0.	0.582019	0.	0.440915	0.	0.445815	0.	0.644539
26FT.	0.	0.542703	0.	0.604127	0.	0.404127	0.	0.773387
31FT.								

36FI.	0.775632	0.654074	C.731747	C.775645	0.716664
41FI.	0.777205	0.849109	C.920592	C.796856	0.929227
46FI.	0.902788	0.555474	1.031642	C.957461	1.034885
51FI.	1.006507	0.983730	1.097313	1.085667	1.113363
56FI.	1.443976	1.553040	1.286397	1.643178	1.682931
61FI.	1.742439	1.872304	1.926735	1.651055	1.454586
66FI.	1.482098	1.443774	1.652369	1.726457	1.559858
71FI.	1.524636	1.415767	1.651112	1.565416	1.633877
76FI.	1.669349	1.600665	1.420531	1.520071	1.641716
81FI.	1.586208	1.716903	1.631431	1.672250	1.898743
86FI.	1.876301	1.746087	1.885226	1.877741	1.852478
91FI.	1.963375	2.012082	2.075847	2.009211	2.119190
96FI.	2.141227	2.172979	2.255258	2.189470	2.214633
101FI.	2.202095	2.128132	2.257347	2.195413	2.238518
106FI.	2.190524	2.158035	2.276671	2.232511	2.303137
111FI.	2.216219	2.299235	2.273856	2.256666	2.116663
116FI.	2.098244	2.074317	2.056216	2.055068	1.942787
121FI.	1.920696	1.916873	1.905388	1.815200	1.797449
126FI.	1.804778	1.676685	1.641474	1.711197	1.610406
131FI.	1.616337	1.664727	1.567229	1.572058	1.638480
136FI.	1.587948	1.521222	1.582687	1.531300	1.565023
141FI.	1.615971	1.574492	1.566374	1.623873	1.600322
146FI.	1.630994	1.610059	1.616170	1.624564	1.572859
151FI.	1.571330	1.520144	1.535777	1.486030	1.484434
156FI.	1.435911	1.412430	1.428154	1.378126	1.330527
161FI.	1.745892	1.292658	1.298087	1.229758	1.244813
166FI.	1.183193	1.197591	1.152227	1.152360	1.086961
171FI.	1.101579	1.056288	1.012136	1.027516	0.969461
176FI.	0.983917	0.935303	C.894096	C.822483	0.864772
181FI.	0.825037	0.803982	C.777367	C.722485	0.735717
186FI.	0.710320	0.637566	C.636008	C.623455	0.623459
191FI.	0.624487	0.591807	C.592238	C.576669	0.575510
196FI.	0.558130	0.543998	C.544418	C.530403	0.530959
201FI.	0.514006	0.500104	C.500514	0.487263	0.470665
206FI.	0.456978	0.457380	C.444326	0.428057	0.414558
211FI.	0.415469	0.385683	C.386073	C.372752	0.373644
216FI.	0.344327	0.344711	C.332053	C.316214	0.316214
221FI.	0.316701	0.301068	C.301068	0.301546	0.286137
226FI.	0.286137	0.286137	C.286606	C.271411	0.271411
231FI.	0.271873	0.256855	C.256232	C.241384	0.241384
236FI.	0.241832	0.227148	C.227590	C.213041	0.213041
241FI.	0.213677	0.194079	C.155505	C.183541	0.183541

246FI.	0.184366	0.170234	0.170654	0.156639	0.156639
251FI.	0.157054	0.143152	0.143562	C.12977C	0.130176
256FI.	0.116490	0.116892	C.10330S	C.1037C7	0.090208
261FI.	0.090602	0.077201	C.077591	C.06427C	0.064657
266FI.	0.050483	0.050867	C.037714	C.037714	0.037714
271FI.	0.037714	0.037714	C.037714	C.037714	0.037714
276FI.	0.037714	0.037714	C.037714	C.037714	0.037714
281FI.	0.036636	0.036636	C.036636	C.036636	0.036636
286FI.	0.036636	0.036636	C.036636	C.036636	0.036636
291FI.	0.035778	0.035778	C.035778	C.035778	0.035778
296FI.	0.035778	0.035778	C.035778	C.035778	0.035778
301FI.	0.035778	0.035778	C.035778	C.035778	0.035778
306FI.	0.035778	0.035778	C.035778	C.035778	0.035778
311FI.	0.034833	0.034833	C.034833	C.034833	0.034833
316FI.	0.034044	0.034044	C.034044	C.034044	0.034044
321FI.	0.034044	0.034044	C.034044	C.034044	0.034044
326FI.	0.034044	0.034044	C.034044	C.034044	0.034044
331FI.	0.034044	0.034044	C.034044	C.034044	0.034044
336FI.	0.034044	0.034044	C.034044	C.034044	0.034044
341FI.	0.033304	0.033304	C.033304	C.033304	0.033304
346FI.	0.032446	0.032446	C.032446	C.032446	0.032446
351FI.	0.032446	0.032446	C.032446	C.032446	0.032446
356FI.	0.032446	0.032446	C.032446	C.032446	0.032446
361FI.	0.031746	0.031746	C.031746	C.031746	0.031746
366FI.	0.031746	0.031746	C.031746	C.031746	0.031746
371FI.	0.030958	0.030958	C.030958	C.030958	0.030958
376FI.	0.030958	0.030958	C.030958	C.030958	0.030958
381FI.	0.030294	0.030294	C.030294	C.030294	0.030294
386FI.	0.030294	0.030294	C.030294	C.030294	0.030294
391FI.	0.029554	0.029554	C.029554	C.029554	0.029554
396FI.	0.028917	0.028917	C.028917	C.028917	0.028917
401FI.	0.028917	0.028917	C.028917	C.028917	0.028917
406FI.	0.028917	0.028917	C.028917	C.028917	0.028917
411FI.	0.027606	0.027606	C.027606	C.027606	0.027606
416FI.	0.027606	0.027606	C.027606	C.027606	0.027606
421FI.	0.027606	0.027606	C.027606	C.027606	0.027606
426FI.	0.027014	0.027014	C.027014	C.027014	0.027014
431FI.	0.026350	0.026350	C.026350	C.026350	0.026350
436FI.	0.026350	0.026350	C.026350	C.026350	0.026350
441FI.	0.025777	0.025777	C.025777	C.025777	0.025777
446FI.	0.025141	0.025141	C.025141	C.025141	0.025141

451FI.	C.024585	0.024585	0.024585	0.024585	0.024585
456FI.	0.024585	0.024585	C.024585	C.023973	0.023973
461FI.	0.023431	0.023431	C.023431	C.023431	0.023431
466FI.	0.023431	0.023431	C.023431	C.023431	0.023431
471FI.	0.023431	0.023431	C.023431	C.022311	0.022311
476FI.	0.022311	0.022311	C.022311	C.022311	0.022311
481FI.	0.022311	0.022311	C.022311	C.021793	0.021793
486FI.	0.021793	0.021793	C.021793	C.021220	0.021220
491FI.	0.021220	0.021220	C.021220	C.021220	0.021220
496FI.	0.020715	0.020715	C.020715	C.020715	0.020715
501FI.	0.020159	0.020159	C.020159	C.020159	0.020159
506FI.	0.019664	0.019664	C.019664	C.019664	0.019664
511FI.	0.019664	0.019122	C.019122	C.019122	0.018635
516FI.	0.018635	0.018635	C.018635	C.018635	0.018635
521FI.	0.018635	0.018635	C.018635	C.017629	0.017629
526FI.	0.017629	0.017629	C.017629	C.017629	0.017629
531FI.	0.017629	0.017629	C.017629	C.016643	0.016643
536FI.	0.016643	0.016643	C.016643	C.016182	0.016182
541FI.	0.016643	0.016643	C.016643	C.015676	0.015676
546FI.	0.015676	0.015676	C.015676	C.015222	0.015222
551FI.	0.015222	0.015222	C.015222	C.014279	0.014279
556FI.	0.014279	0.014279	C.014279	C.014279	0.013792
561FI.	0.014279	0.014279	C.014279	C.013350	0.013350
566FI.	0.013350	0.013350	C.013350	C.012436	0.012436
571FI.	0.013350	0.012436	C.012436	C.012436	0.012436
576FI.	0.012436	0.012005	C.012005	C.011536	0.011536
581FI.	0.012005	0.011536	C.011536	C.011112	0.011112
586FI.	0.011536	0.011112	C.011112	C.010650	0.010650
591FI.	0.011112	0.010650	C.010650	C.010230	0.010230
596FI.	0.010650	0.009360	C.009360	C.009360	0.009360
601FI.	0.009360	0.008502	C.008502	C.008502	0.008502
606FI.	0.008502	0.008502	C.008502	C.007653	0.007653
611FI.	0.008502	0.007653	C.007653	C.006815	0.006815
616FI.	0.008053	0.007653	C.007653	C.006417	0.006417
621FI.	0.007653	0.006815	C.006815	C.005986	0.005986
626FI.	0.006815	0.005986	C.005986	C.005168	0.005168
631FI.	0.005986	0.005592	C.005592	C.004777	0.004777
636FI.	0.005592	0.004777	C.004777	C.003970	0.003970
641FI.	0.004777	0.004357	C.004357	C.003172	0.003172
646FI.	0.004357	0.003970	C.003970	C.003172	0.002751
651FI.	0.003970	0.003172	C.003172	C.003172	
656FI.	0.003172	0.003172	C.003172		

661FT.	0.002761	0.002761	C.002761	0.002761
666FT.	0.002761	0.002355	C.002355	0.002355
671FT.	0.002355	0.002355	C.001553	0.001553
676FT.	0.001953	0.001953	C.001953	0.001555
681FT.	0.001555	0.001555	C.001555	0.001161
686FT.	0.001161	0.001161	C.001161	0.001161
691FT.	0.000770	0.000770	C.000770	0.000770
696FT.	0.000770	0.000383	C.000383	0.000383

DISTRIBUTION OF SIZES AT 50 FT. FROM ORIGINAL POSITION

SIZE RANGE	PERCENT
1	40.75
2	8.56
3	40.84
4	9.35
5	0.50

DISTRIBUTION OF SIZES AT 150 FT. FROM ORIGINAL POSITION

SIZE RANGE	PERCENT
1	0.
2	0.
3	59.07
4	39.92
5	1.01

DISTRIBUTION OF SIZES AT 300 FT. FROM ORIGINAL POSITION

SIZE RANGE	PERCENT
1	0.
2	0.
3	0.
4	0.
5	100.00

CUMULATIVE DEBRIS MOMENTUM (FT./SEC.) AT IFT INTERVALS FROM ORIGINAL POSITION  
(MULTIPLY BY MASS OF ONE PANEL)

1FT.	0.	C.	C.	O.
6FT.	2.386594	4.335169	6.382248	3.024842
11FT.	5.405084	3.600971	6.862768	5.405084
16FT.	3.454173	7.248843	5.798788	3.454173
21FT.	5.687479	7.505077	7.514506	7.145287
26FT.	9.117875	7.949013	7.949013	10.327536
31FT.	9.805254	11.555785	11.555785	11.565352
36FT.	15.474083	16.977215	15.566373	13.783531
41FT.	17.333539	19.269737	21.145230	19.091831
46FT.	22.465785	25.212890	26.381796	24.255568
51FT.	27.880796	31.147294	31.147294	27.571948
56FT.	35.941738	41.546183	37.800467	38.067154
61FT.	44.739747	47.785516	45.842131	49.825205
66FT.	44.390951	46.455106	45.926836	42.535864
71FT.	42.344210	42.412744	45.615389	44.449048
76FT.	44.106723	42.805084	40.831654	42.401595
81FT.	44.393182	47.873285	46.052807	48.147382
86FT.	53.542328	56.249356	55.672360	51.490723
91FT.	60.136985	63.892022	62.345534	61.657838
96FT.	68.928402	72.610670	72.595532	70.506271
101FT.	74.080979	74.374080	76.106259	72.008894
106FT.	75.587868	78.057352	79.216564	76.525670
111FT.	78.969316	80.436105	81.415579	82.256065
116FT.	75.799600	76.973321	73.762126	74.937249
121FT.	69.140748	73.455845	67.516010	68.559748
126FT.	63.796240	64.932331	58.055622	59.111539
131FT.	57.289247	57.340965	56.047869	59.378694
136FT.	57.135338	58.625983	56.741124	54.341329
141FT.	59.454496	57.397484	58.315399	58.681102
146FT.	62.030810	60.497377	62.781930	61.818896
151FT.	61.245051	61.226681	60.045454	59.481812
156FT.	56.550849	58.312292	56.585491	56.001483
161FT.	53.635849	53.042569	51.890531	51.289660
166FT.	47.779128	50.148819	46.568579	48.371863
171FT.	44.876186	44.257925	41.242653	43.084515
176FT.	40.085879	39.455282	36.472857	38.315960
181FT.	33.442675	35.283204	31.735274	32.563549
186FT.	28.689348	29.905214	25.6500162	25.672457
		25.101287		

191FT.	25.135961	24.CC727C	24.025117	23.406CC2	23.397211
196FT.	22.876992	22.276025	22.292395	21.68667E	21.705606
201FT.	21.183393	20.573856	20.591966	19.995311	19.469763
206FT.	18.852150	18.870383	18.265920	17.737191	17.111156
211FT.	17.146631	15.965459	15.982914	15.349854	15.385523
216FT.	14.193503	14.212078	12.58766C	13.025C34	13.029C34
221FT.	13.046328	12.482687	12.482687	12.50CC32	11.932651
226FT.	11.932651	11.932651	11.95CC40	11.374C51	11.379051
231FT.	11.39652C	10.821052	1C.818781	1C.239C02	10.239C02
236FT.	10.25661C	9.672674	9.69C375	9.1015C4	9.101504
241FT.	9.119265	8.526529	8.544377	7.92526C	7.925260
246FT.	7.943154	7.342187	7.36C157	6.754839	6.754839
251FT.	6.772882	6.163345	6.181454	5.567E22	5.585995
256FT.	4.968382	4.986615	4.365132	4.383454	3.757418
261FT.	3.775792	3.146101	3.164556	2.53C537	2.549037
266FT.	1.892874	1.911449	1.269838	1.26983E	1.269838
271FT.	1.269838	1.269838	1.269838	1.269838	1.269838
276FT.	1.269838	1.269838	1.269838	1.26983E	1.269838
281FT.	1.269838	1.269838	1.25C026	1.25C026	1.250026
286FT.	1.250026	1.250026	1.25C026	1.25C026	1.250026
291FT.	1.250026	1.231955	1.231955	1.231955	1.231955
296FT.	1.231955	1.231955	1.231955	1.231955	1.231955
301FT.	1.231955	1.231955	1.231955	1.231955	1.231955
306FT.	1.231955	1.231955	1.231955	1.231955	1.231955
311FT.	1.231955	1.231955	1.231955	1.231955	1.231955
316FT.	1.213276	1.213276	1.195737	1.195737	1.195737
321FT.	1.195737	1.195737	1.195737	1.195737	1.195737
326FT.	1.195737	1.195737	1.195737	1.195737	1.195737
331FT.	1.195737	1.195737	1.195737	1.195737	1.195737
336FT.	1.195737	1.195737	1.195737	1.1784C2	1.178402
341FT.	1.178402	1.160330	1.16C330	1.16C33C	1.160330
346FT.	1.16C330	1.160330	1.16C330	1.16C33C	1.160330
351FT.	1.160330	1.160330	1.16C330	1.16C33C	1.160330
356FT.	1.160330	1.160330	1.142187	1.143187	1.143187
361FT.	1.143187	1.143187	1.142187	1.143187	1.143187
366FT.	1.143187	1.143187	1.125648	1.12564E	1.125648
371FT.	1.125648	1.125648	1.125648	1.12564E	1.125648
376FT.	1.125648	1.125648	1.1CE684	1.108684	1.108684
381FT.	1.108684	1.108684	1.1CE684	1.108684	1.108684
386FT.	1.108684	1.108684	1.1CE684	1.09134E	1.091348
391FT.	1.091348	1.091348	1.09134E	1.074416	1.074416





6C6FI.	0.418312	C.41E312	C.40C7C4	0.382595
611FI.	0.382595	C.382595	C.382595	0.382595
616FI.	0.346716	C.346716	C.346716	0.346716
621FI.	0.346716	C.52E483	C.31C722	0.310722
626FI.	0.310722	C.31C722	C.29240C	0.29240C
631FI.	0.274553	C.274553	C.274553	0.256179
636FI.	0.256179	C.256179	C.238286	0.238286
641FI.	0.219830	C.219830	C.219830	0.219830
646FI.	0.201860	C.183360	C.183360	0.183360
651FI.	0.183360	C.146743	C.146743	0.146743
656FI.	0.146743	C.146743	C.146743	0.128633
661FI.	0.128633	C.12E633	C.128633	0.128633
666FI.	0.110460	C.11C460	C.11C460	0.110460
671FI.	0.110460	C.092227	C.092227	0.092227
676FI.	0.092227	C.092227	C.073905	0.073905
681FI.	0.073905	C.073905	C.073905	0.055531
686FI.	0.055531	C.055531	0.055531	0.055531
691FI.	0.037076	C.037076	C.037076	0.037076
696FI.	0.037076	C.01E575	C.01E575	0.01E575

MINIMUM DEBRIS MOMENTUM (FT./SEC.) AT 1FT. INTERVALS FROM ORIGINAL POSITION  
(MULTIPLY BY MASS OF ONE PANEL)

1FT.	0.	0.	0.	0.	0.	0.	0.	0.	0.
6FT.	0.	0.	0.	0.	0.	0.	0.	0.	0.
11FT.	1.708172	1.708162	1.708124	1.458684	1.458684	1.458684	0.434124	0.434124	0.434124
16FT.	0.434124	0.434124	0.434124	0.389408	0.389408	0.389408	0.389408	0.389408	0.389408
21FT.	0.389408	0.389408	0.389408	0.365219	0.365219	0.365219	0.365219	0.365219	0.365219
26FT.	0.365219	0.365219	0.365219	0.358099	0.358099	0.358099	0.358099	0.358099	0.358099
31FT.	0.358099	0.358099	0.358099	0.347578	0.347578	0.347578	0.347578	0.347578	0.347578
36FT.	0.347578	0.347578	0.347578	0.346072	0.346072	0.346072	0.346072	0.346072	0.346072
41FT.	0.346072	0.346072	0.346072	0.344207	0.344207	0.344207	0.344207	0.344207	0.344207
46FT.	0.344207	0.344207	0.344207	0.344207	0.344207	0.344207	0.344207	0.344207	0.344207
51FT.	0.344207	0.344207	0.344207	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
56FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
61FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
66FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
71FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
76FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
81FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
86FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
91FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
96FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
101FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
106FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
111FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
116FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
121FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
126FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
131FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
136FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
141FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
146FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
151FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
156FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
161FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
166FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
171FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
176FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
181FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715
186FT.	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715	0.025715



396FT.	0.016895	C.016895	C.016895	C.016895
401FT.	0.016895	C.016895	C.016895	C.016895
406FT.	0.016895	C.016895	C.016895	C.016895
411FT.	0.016895	C.016895	C.016895	C.016895
416FT.	0.016895	C.016895	C.016895	C.016895
421FT.	0.016895	C.016895	C.016895	C.016895
426FT.	0.016895	C.016895	C.016895	C.016895
431FT.	0.016895	C.016895	C.016895	C.016895
436FT.	0.016895	C.016895	C.016895	C.016895
441FT.	0.016895	C.016895	C.016895	C.016895
446FT.	0.016895	C.016895	C.016895	C.016895
451FT.	0.016895	C.016895	C.016895	C.016895
456FT.	0.016895	C.016895	C.016895	C.016895
461FT.	0.016914	C.016914	C.016914	C.016914
466FT.	0.016914	C.016914	C.016914	C.016914
471FT.	0.016914	C.016914	C.016914	C.016914
476FT.	0.016924	C.016924	C.016924	C.016924
481FT.	0.016924	C.016924	C.016924	C.016924
486FT.	0.016924	C.016924	C.016924	C.016924
491FT.	0.016928	C.016928	C.016928	C.016928
496FT.	0.016928	C.016928	C.016928	C.016928
501FT.	0.016978	C.016978	C.016978	C.016978
506FT.	0.016978	C.016978	C.016978	C.016978
511FT.	0.016978	C.016978	C.016978	C.016978
516FT.	0.017020	C.017020	C.017020	C.017020
521FT.	0.017020	C.017020	C.017020	C.017020
526FT.	0.017020	C.017020	C.017020	C.017020
531FT.	0.017101	C.017101	C.017101	C.017101
536FT.	0.017101	C.017101	C.017101	C.017101
541FT.	0.017128	C.017128	C.017128	C.017128
546FT.	0.017128	C.017128	C.017128	C.017128
551FT.	0.017194	C.017194	C.017194	C.017194
556FT.	0.017194	C.017194	C.017194	C.017194
561FT.	0.017294	C.017294	C.017294	C.017294
566FT.	0.017294	C.017294	C.017294	C.017294
571FT.	0.017345	C.017345	C.017345	C.017345
576FT.	0.017345	C.017345	C.017345	C.017345
581FT.	0.017389	C.017389	C.017389	C.017389
586FT.	0.017389	C.017389	C.017389	C.017389
591FT.	0.017468	C.017468	C.017468	C.017468
596FT.	0.017468	C.017468	C.017468	C.017468
601FT.	0.017541	C.017541	C.017541	C.017541
606FT.	0.017541	C.017541	C.017541	C.017541
611FT.	0.017607	C.017607	C.017607	C.017607

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0.017848  
0.017848  
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021F1.  
025F1.  
031F1.  
036F1.  
041F1.  
046F1.  
051F1.  
056F1.  
061F1.  
066F1.  
071F1.  
076F1.  
081F1.  
086F1.  
091F1.  
096F1.

MAXIMUM DEBRIS MOMENTUM (FT./SEC.) AT 1FT. INTERVALS FROM ORIGINAL POSITION  
(MULTIPLY BY MASS OF ONE PANEL)

1FT.	O.	C.	C.	C.
6FT.	2.386594	3.696522	3.696522	3.696522
11FT.	3.696922	1.708162	1.708162	1.561364
16FT.	1.561364	2.344616	2.344616	2.344616
21FT.	2.344616	2.344616	2.344616	2.344616
26FT.	2.344616	2.036624	2.036624	2.036624
31FT.	2.036624	2.036624	2.036624	2.036624
36FT.	2.036624	1.896301	1.896301	1.896301
41FT.	1.896301	1.896301	1.818809	1.818809
46FT.	1.818809	1.818809	1.818809	1.818809
51FT.	1.782281	1.782281	1.782281	1.782281
56FT.	2.386594	1.747005	3.696522	3.696522
61FT.	3.696922	3.696922	1.739187	1.738668
66FT.	1.743933	2.344616	2.344616	2.344616
71FT.	2.344616	2.344616	2.344616	2.344616
76FT.	2.344616	2.036624	2.036624	2.036624
81FT.	2.036624	2.036624	2.036624	2.036624
86FT.	2.036624	1.896301	1.896301	1.896301
91FT.	1.896301	1.896301	1.874929	1.879836
96FT.	1.877766	1.891980	1.895187	1.899187
101FT.	1.899187	1.899187	1.895187	1.899187
106FT.	1.899187	1.895187	1.895187	1.899187
111FT.	1.899187	1.895187	1.895187	1.899187
116FT.	1.899187	1.899187	1.895187	1.899187
121FT.	1.895187	1.899187	1.895187	1.899187
126FT.	1.895187	1.899187	1.895187	1.899187
131FT.	1.899187	1.899187	1.895187	1.899187
136FT.	1.899187	1.899187	1.895187	1.899187
141FT.	1.899187	1.899187	1.895187	1.899187
146FT.	1.899187	1.899187	1.895187	1.899187
151FT.	1.899187	1.899187	1.895187	1.899187
156FT.	1.899187	1.899187	1.895187	1.899187
161FT.	1.899187	1.899187	1.895187	1.899187
166FT.	1.899187	1.899187	1.895187	1.899187
171FT.	1.899187	1.899187	1.895187	1.899187
176FT.	1.899187	1.899187	1.895187	1.899187
181FT.	1.899187	1.899187	1.895187	1.899187
186FT.	1.899187	0.641611	C.641611	0.641611







666FT.	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575
611FT.	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575
616FT.	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575
621FT.	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575
626FT.	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575
631FT.	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575
636FT.	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575
641FT.	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575
646FT.	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575
651FT.	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575
656FT.	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575
661FT.	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575
666FT.	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575
671FT.	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575
676FT.	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575
681FT.	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575
686FT.	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575
691FT.	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575
696FT.	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575	0.018575

- WALL 3 SUPERIMPOSED ON WALLS 1 AND 2
- THERE ARE 100 FT. BETWEEN WALLS 2 AND 3
- WALL 3 IS ONLY 30 FLOORS HIGH

PRECAST STRUCTURAL CONFIGURATION

WALL HEIGHT 30 FLOORS

SPACE BETWEEN WALLS 100. FEET

SOLVE

SPARTICLE SIZES

3CSTORY BUILDING

10.CCFT. BETWEEN FLCCRS

YIELD= 1000.CKT.

OVERPRESSURE= 10.OPSI

PARTICLE RADIUS PERCENT OF PANEL

10.00 FT. C.13 X100  
 8.00 FT. C.05 X100  
 6.00 FT. C.40 X100  
 4.00 FT. C.32 X100  
 2.00 FT. C.05 X100

DISTANCE OF CURRENT WALL FROM STARTING WALL 150.0 FT.

SIZE RANGE 1 8.00 IN. TO 10.00 IN.  
 SIZE RANGE 2 6.00 IN. TO 8.00 IN.  
 SIZE RANGE 3 4.00 IN. TO 6.00 IN.  
 SIZE RANGE 4 2.00 IN. TO 4.00 IN.  
 SIZE RANGE 5 C. IN. TO 2.00 IN.

PERIS HEIGHT (FT.) AT 1FT. INTERVALS FROM ORIGINAL POSITION

1FT.	0.	C.078C72	0.	C.445252	0.	0.367180
6FT.	0.330084	C.078C72	0.408156	C.445252	0.445252	0.367180
11FT.	0.528454	C.625690	0.528454	C.296515	C.296515	0.255358
16FT.	C.255358	C.438196	C.255358	C.527403	C.527403	0.407687
21FT.	0.407687	C.51766C	0.453983	C.500C55	C.500C55	0.539420
26FT.	0.582019	C.445815	0.652413	C.445815	C.445815	0.644539
31FT.	0.542903	C.604127	0.618976	C.604127	C.604127	0.773387

- WALL 4 SUPERIMPOSED ON WALLS 1, 2, AND 3
- THERE ARE 35 FT. BETWEEN WALLS 3 AND 4
- WALL 4 IS THE SAME HEIGHT AS WALL 3

PRECAST STRUCTURAL CONFIGURATION

SPACE BETWEEN WALLS 35. FEET

SOLVE

5 PARTICLE SIZES

30 STORY BUILDING

10.00 FT. BETWEEN FLOORS

YIELD= 1000.000

OVERPRESSURE= 10.00 PSI

PARTICLE RADII PERCENT OF PANEL

- 10.00 FT. C.13 X100
- 8.00 FT. C.05 X100
- 6.00 FT. C.45 X100
- 4.00 FT. C.32 X100
- 2.00 FT. C.05 X100

DISTANCE OF CURRENT WALL FROM STARTING WALL 185.0 FT.

SIZE RANGE	1	2	3	4	5
10.00 IN.	TC	10.00	IN.		
8.00 IN.	TC	8.00	IN.		
6.00 IN.	TC	6.00	IN.		
4.00 IN.	TC	4.00	IN.		
2.00 IN.	TC	2.00	IN.		

HEIGHTS BETWEEN WALLS AT 10.00 FT. INTERVALS FROM ORIGINAL POSITION

1 FT.	0.	0.430084	0.	C.078072	0.	C.445253
6 FT.	0.528454	0.528454	0.401156	C.625690	0.367180	C.445253
11 FT.	0.255358	0.255358	0.528454	C.438196	0.255358	C.296915
16 FT.	0.407687	0.407687	0.255358	C.517660	0.407687	C.527402
21 FT.	0.582019	0.582019	0.407687	C.445815	0.539420	C.500055
26 FT.	0.542903	0.542903	0.582019	C.604127	0.644539	C.445815
31 FT.	0.775632	0.775632	0.542903	C.731747	0.773387	C.604127
36 FT.			0.775632		0.716664	C.775649

- SAMPLE PROBLEM II VARIATION OF AERODYNAMIC COEFFICIENTS FOR A
- SINGLE MASCARY BRICK WITH 2.25 X 3.75 X 8 INCH NOMINAL DIMENSIONS
- CASE I EQUIVALENT SPHERICAL RADIUS = 2.53 INCHES

WEAPON PARAMETERS

YIELD 1000. KILOGRAMS

OVERPRESSURE 10. PSI

PREBLAST STRUCTURAL CONFIGURATION

WALL HEIGHT 40 FEET

HEIGHT BETWEEN FLOORS 10. FEET

SPACE BETWEEN WALLS 6.0 FEET

NORMALIZING FACTOR 0.333

FRAGMENTATION CHARACTERISTICS

NUMBER OF PARTICLE SIZES 1

PARTICLE SIZE 2.53

PERCENTAGE BY SIZE 1.0

ACCELERATION COEFFICIENT 0.0

CUTPLY

PROFILE DISTRIBUTION 1

DISTRIBUTION OF SIZES 1

LOCATIONS 3

DISTANCES FROM FIRST WALL SC..60..7C..PC..ICC. FEET

VELOCITY DESCRIPTION 1

DEBRIS PROFILE PLCT 1

SOLVE



UNCLASSIFIED

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13 ABSTRACT  A comprehensive view is taken of the physical models required to estimate volumes and heights of blast-initiated debris. Particular emphasis and development is directed toward three areas: the fragmentation of frangible elements, the failure of elements with limited ductility, and the transport of debris particles by blast winds. Computer programs to handle the computations involved in these three models have been written.		