1

USE OF LAGRANGIAN COORDINATES FOR SHIP WAVE RESISTANCE (FIRST- AND SECOND-ORDER THIN-SHIP THEORY)

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41

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Abstract

The wave motion generated by the steady motion of a ship is formulated 'exactly' in Lagrangian coordinates. An approximation scheme is then developed based upon a method of iteration. The first iteration can be chosen so as to yield the Michell thin-ship theory. The second iteration produces more complicated formulas. However, if the smallness of draft/length is taken into account, the formula for the resistance simplifies to one similar to Michell's, but with the integral taken over a modified region and with a modified hull function, plus a line integral over the profile which represents a distribution of dipoles with strength proportional to the waterline slopes.

Since several derivations of Michell's integral, including Michell's own, are available, it seems appropriate to try to explain how any new one throws further light on the subject. The aim of the present one is not to introduce new techniques for solving the associated boundary-value problems, but instead to introduce both a new (in this problem) way of formulating the problem mathematically and a new approximation scheme. The first approximation recovers the classical results, so that any advantage here must be chiefly conceptual. The second approximation, however, already shows advantages of the present procedure. In order to weigh advantages against disadvantages, we shall first discuss some disadvantages of the formulation of free-surface problems in Eulerian coordinates, next how these are avoided in Lagrangian coordinates, and finally, show how some part of the supposed advantage of Lagrangian coordinates disappears in dealing with boundary conditions on rigid bodies. The necessity of dealing with this last situation led to a different approach to finding approximate solutions.

If one formulates without approximation the problem of flow with a free surface, the position of the free surface itself is generally one of the unknowns of the problem. Indeed, in most such problems, finding this position is the chief goal. If the problem has been formulated in Eulerian coordinates, this means that the domain of definition of the velocity and pressure field, i.e., the domain of validity of the equations of motion, is unknown. In one of the usual approximation schemes, one attempts to circumvent this difficulty by replacing boundary conditions on the free surface by boundary conditions on a known mean position. Since the mean surface does not usually lie wholly within the region occupied by fluid, this requires being able to extend the domain of definition of the field variables outside their original domain of definition in such a way that the equations of motion remain valid.

In problems of two-dimensional steady flow, one can avoid this step by using the stream function as one of the independent variables. In this case one seeks the field variables as functions of x and ψ (or of φ and ψ if there exists a velocity potential) and also y as a function of x and ψ (or both x and y as functions of φ and ψ). This effective device is unfortunatly not available in three-dimensional flows. However, another one is, namely, the use of Lagrangian variables in which each particle is labelled by a triple of numbers. This labelling can be done in such a way that the domain of the labelling variables is both known and convenient. The trajectories of particles, their velocities and pressures are then sought as functions of the labelling variables. This procedure is, of course, well known; the appropriate form of the equations of motion can be found in Lamb's Hydrodynamics, and is used there in the discussion of Gerstner's waves. The use of Lagrangian variables in water-wave problems has been especially exploited in recent years by J. Kravtchenko and his students [for some references see pp. 581-592 of Wehausen (1965)].

As will be shown later on, the advantage of Lagrangian variables mentioned above carries with it a concomitant disadvantage. If there is flow about a solid body (here a ship), the domain over which a boundary condition expressing the presence of the body is to be applied is not known, but has to be found as part of the solution (this is not the case for Eulerian variables). This would appear to put us back into a situation similar to that described for the Eulerian formulation of water-wave problems, except for one thing. The domain of application of the body boundary condition can be included in any approximation scheme without the necessity of extending the domain of definition of the various functions beyond their natural ones.

In this paper the 'exact' problem will first be formulated and subjected to certain manipulations. Thereafter an approximation scheme is proposed which is based upon iteration rather than a perturbation expansion. Because the starting point of the iteration is taken as a ship with zero beam, it is still appropriate to call the first two iterations 'thin-ship' approximations. The same method could be used with other starting points, but would not alter the matnematical difficulties inherent in these.

The first approximation, in the modified form chosen here, yields exactly the same expressions for wave resistance, trim and sinkage as one obtains with Eulerian variables. The second approximation will not in general agree with the second-order theories obtained from a perturbation expansion with Eulerian variables, although the differences should be of 'higher order'. This is partly because the second approximation here is not an irrotational flow, partly because the principle in treating certain product terms is different. Consequently, one cannot expect to be able to identify the second approximation here with results obtained in recent times by Eggers (1966), Maruo (1966), Sizov (1961) or the author (1963). However, a recent report of B. Yim (1966) is quite close in its approach. Yim uses as independent variables (x, β, z), in the notation introduced below. This choice yields one of the main advantages of the Lagrangian variables, namely, that the free surface is given by $\beta = 0$.

In a final section an attempt is made to simplify the formulas determining the sinkage and trim in the first approximation and the wave resistance in the second by making use of the fact that the draft/length ratio of normal ship forms is quite small.

Coordinate systems and notation.

We shall make use of two coordinate systems which coincide when the ship is at rest in its equilibrium position. The coordinate frame $O'x'y'z'$ will be taken as attached to the ship with the (y', z')-plane containing the midship section, the (z', x')-plane the water-plane section, and the (x', y')-plane the centerplane section. $O'y'$ is directed upwards, $O'x'$ toward the stern, and $O'z'$ toward port. The coordinate frame $Oxyz$

is taken so that the (z, x)-plane coincides with the mean water surface, and so that the two origins lie on a vertical line, the coordinates of O' being $(0, h, 0)$ in $Oxyz$. The center of gravity of the ship is at the point $G = (x'_G, y'_G, 0)$. A positive thrust T acts in the negative x' direction along a line intersecting the (y', z')-plane at $(0, e', 0)$; ordinarily $e' < 0$. Figure 1 shows schematically the two systems and defines the pitch angle φ. The two systems are related by the following equations:

$$
\begin{aligned}
x &= x' \cos\varphi - y' \sin\varphi, \\
y - h &= x' \sin\varphi + y' \cos\varphi, \\
z &= z'; \\
x' &= x \cos\varphi + (y-h)\sin\varphi, \\
y' &= -x \sin\varphi + (y-h)\cos\varphi, \\
z' &= z.
\end{aligned}
\tag{1}
$$

Whenever it seems notationally more convenient to do so, we shall replace x, y, z by x_1, x_2, x_3, u, v, w by u_1, u_2, u_3, etc. Derivatives will usually be indicated by subscripts, and furthermore, the summation convention for subscripts occurring twice will be followed. For example,

$$X_i X_{it} = X\frac{\partial X}{\partial t} + Y\frac{\partial Y}{\partial t} + Z\frac{\partial Z}{\partial t}.$$

When it seems convenient to do so, we shall mix the two notations. For example, in (20) we shall write

$$gY + U^2 X_\alpha + \frac{1}{2} U^2 X_{j\alpha} X_{j\alpha}$$

instead of

$$gX_2 + U^2 X_{1\alpha} + \frac{1}{2} U^2 X_{j\alpha} X_{j\alpha}.$$

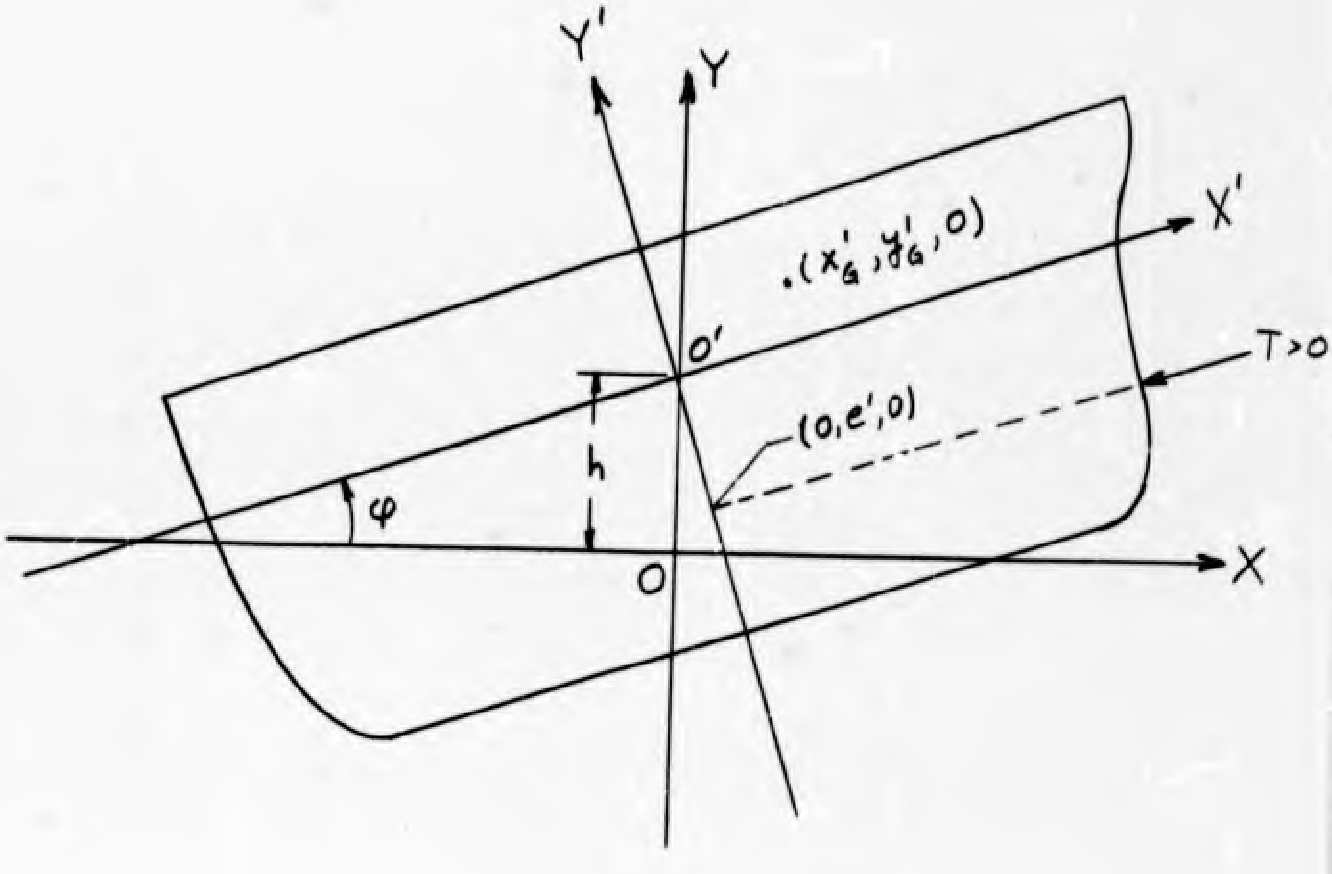

Figure 1

The equilibrium equations.

Let the equation of the ship's hull be given in the system $O'x'y'z'$ by

$$z' = \pm f(x', y'). \tag{2}$$

Then the unit normal vector into the ship is given in $O'x'y'z'$ by

$$\underset{\sim}{n}(x', y') = \frac{(f_1(x', y'),\ f_2(x', y'),\ \mp 1)}{[1 + f_1^2 + f_2^2]^{1/2}} \tag{3}$$

and in $Oxyz$ by

$$\underset{\sim}{n}(x, y) = \frac{(f_1(x', y')\cos\varphi - f_2 \sin\varphi,\ f_1 \sin\varphi + f_2 \cos\varphi,\ \mp 1)}{[1 + f_1^2 + f_2^2]^{1/2}}, \tag{4}$$

where

$$f_1 = \frac{\partial f}{\partial x'}, \quad f_2 = \frac{\partial f}{\partial y'},$$

and where in (4) one must use (1) to give x' and y' in terms of x and y.

Let p be the pressure at any point of the fluid and let S_w be the wetted surface of the ship. Then the force acting upon the ship because of the pressure is

$$\underset{\sim}{F} = \iint_{S_w} p\, \underset{\sim}{n}\, dS, \tag{5}$$

and the moment with respect to O is

$$\underset{\sim}{\mathcal{M}}_0 = \iint_{S_w} p\, \underset{\sim}{r} \times \underset{\sim}{n}\, dS, \tag{6}$$

where $\underset{\sim}{r}$ is the vector OP, P a point of S_w.

The force and moment may be resolved into components in either system. If we suppose the force to be resolved in the system $Oxyz$, the equations of static equilibrium are

$$F_x - T\cos\varphi = 0\,,\quad F_y - T\sin\varphi - Mg = 0\,,\quad F_z = 0. \tag{7}$$

For the moment we have

$$\mathcal{M}_{0x} = \mathcal{M}_{0y} = 0\,,\quad \mathcal{M}_{0z} + T(e' + h\cos\varphi) - Mg\,x_G = 0. \tag{8}$$

Equations governing the fluid.

Since we have assumed the motion to be steady, we may think of each streamline as a wire extending from $x = -\infty$ to $x = +\infty$ and identify it by its y and z coordinates at $x = -\infty$. Let them be β and γ, respectively. The trajectory of a particle may then be described by

$$x_i = x_i(t, \beta, \gamma)\,,\quad i = 1, 2, 3. \tag{9}$$

We shall use t, β, γ as the Lagrangian coordinates. That the time t can be used as one of the coordinates is, of course, a consequence of the assumed steadiness of the motion.

Since the motion must approach a uniform flow with velocity $(U, 0, 0)$ as $x \to -\infty$, the functions in (9) must satisfy

$$\lim_{t\to-\infty} x_t(t,\beta,\gamma) = U\,,\quad \lim_{t\to-\infty} y_t = 0\,,\quad \lim_{t\to-\infty} z_t = 0. \tag{10}$$

It will be convenient to express the functions $x_i(t, \beta, \gamma)$ in a somewhat different form. We shall take them as follows:

$$x = Ut + X(Ut,\beta,\gamma)\,,\quad y = \beta + Y(Ut,\beta,\gamma)\,,\quad z = \gamma + Z(Ut,\beta,\gamma). \tag{11}$$

The behavior as $x \to -\infty$ is then specified by the conditions

$$\lim_{t\to-\infty} X_i(Ut,\beta,\gamma) = 0, \quad i = 1,2,3. \tag{12}$$

The equation for the conservation of mass takes the following form, with $\alpha \equiv Ut$:

$$X_\alpha + Y_\beta + Z_\gamma + \frac{\partial(Y,Z)}{\partial(\beta,\gamma)} + \frac{\partial(Z,X)}{\partial(\gamma,\alpha)} + \frac{\partial(X,Y)}{\partial(\alpha,\beta)} + \frac{\partial(X,Y,Z)}{\partial(\alpha,\beta,\gamma)} = 0; \tag{13}$$

this may be manipulated into the equation

$$\begin{aligned} &X_\alpha + Y_\beta + Z_\gamma + \tfrac{1}{2}(X_\alpha + Y_\beta + Z_\gamma)^2 \\ &-\tfrac{1}{2}\left[X_\alpha^2 + X_\beta^2 + X_\gamma^2 + Y_\alpha^2 + Y_\beta^2 + Y_\gamma^2 + Z_\alpha^2 + Z_\beta^2 + Z_\gamma^2\right] \\ &+\tfrac{1}{2}\left[(Y_\gamma - Z_\beta)^2 + (Z_\alpha - X_\gamma)^2 + (X_\beta - Y_\alpha)^2\right] \\ &+ \frac{\partial(X,Y,Z)}{\partial(\alpha,\beta,\gamma)} = 0. \end{aligned} \tag{14}$$

If we add to (11) the following form for p ,

$$p(t,\beta,\gamma) = -\rho g \beta + P(Ut,\beta,\gamma), \tag{15}$$

then the momentum equations are

$$\begin{aligned} &\frac{1}{\rho}P_\alpha + gY_\alpha + U^2 X_{\alpha\alpha} + U^2 X_{j\alpha} X_{j\alpha\alpha} = 0, \\ &\frac{1}{\rho}P_\beta + gY_\beta + U^2 Y_{\alpha\alpha} + U^2 X_{j\beta} X_{j\alpha\alpha} = 0, \\ &\frac{1}{\rho}P_\gamma + gY_\gamma + U^2 Z_{\alpha\alpha} + U^2 X_{j\gamma} X_{j\alpha\alpha} = 0. \end{aligned} \tag{16}$$

It is easy to eliminate $\rho^{-1}P+gY$ in each pair of equations and to obtain the following:

$$X_{\alpha\alpha\beta} - Y_{\alpha\alpha\alpha} + X_{j\alpha\alpha\beta}X_{j\alpha} - X_{j\alpha\alpha\alpha}X_{j\beta} = 0, \tag{17}$$

$$Y_{\alpha\alpha\gamma} - Z_{\alpha\alpha\beta} + X_{j\alpha\alpha\gamma}X_{j\beta} - X_{j\alpha\alpha\beta}X_{j\gamma} = 0,$$

$$Z_{\alpha\alpha\alpha} - X_{\alpha\alpha\gamma} + X_{j\alpha\alpha\alpha}X_{j\gamma} - X_{j\alpha\alpha\gamma}X_{j\alpha} = 0.$$

These equations can each be integrated once with respect to α. Because of (12) the constants of integration will be zero. We then have

$$X_{\alpha\beta} - Y_{\alpha\alpha} + X_{j\alpha\beta}X_{j\alpha} - X_{j\alpha\alpha}X_{j\beta} = 0,$$

$$Y_{\alpha\gamma} - Z_{\alpha\beta} + X_{j\alpha\gamma}X_{j\beta} - X_{j\alpha\beta}X_{j\gamma} = 0, \tag{18}$$

$$Z_{\alpha\alpha} - X_{\alpha\gamma} + X_{j\alpha\alpha}X_{j\gamma} - X_{j\alpha\gamma}X_{j\alpha} = 0.$$

It is not difficult to verify that the equations (18) are not independent, and that from any pair of them the third can be derived.

By using equations (18) it is now possible to obtain an integral of the equations (16). It is

$$\frac{1}{\rho}P + gY + U^2X_\alpha + \frac{1}{2}X_{j\alpha}X_{j\alpha} = 0. \tag{19}$$

Here, without essential loss of generality, we have taken the atmospheric pressure as zero.

The free surface is defined by $\beta = 0$. The boundary condition $p(\alpha,0,\gamma)=0$ then takes the form

$$gY(\alpha,0,\gamma) + U^2X_\alpha + \frac{1}{2}U^2X_{j\alpha}X_{j\alpha} = 0. \tag{20}$$

The stream surface which includes the surface of the ship and plane of symmetry $z = 0$ outside the ship is defined by $\gamma = 0$. The functions X, Y, Z must also satisfy the following boundary condition:

$$
\begin{aligned}
Z(\alpha,\beta,\pm 0) &= \pm f_L(\alpha,\beta) \equiv \\
& f([\alpha + X(\alpha,\beta,0)]\cos\varphi + [\beta - h + Y]\sin\varphi, \\
& -[\alpha + X]\sin\varphi + [\beta - h + Y]\cos\varphi),
\end{aligned}
\tag{21}
$$

where we have extended the definition of f to be zero everywhere outside the centerplane section of the ship. For later developments it is necessary to be more specific about the region in the (α,β)-plane where $Z(\alpha,\beta,0) \neq 0$. Let us denote this region by S_L. It is evidently bounded above by $\beta = 0$, and the rest of the boundary will be a distortion of the submerged portion of the boundary of the centerplane section. The mapping

$$
x = \alpha + X(\alpha,\beta,0) \quad , \quad y = \beta + Y(\alpha,\beta,0) \tag{22}
$$

takes S_L onto the projection onto the centerplane section of the wetted hull, as described in the (x,y) coordinates; we denote this region by S_P. The mapping

$$
\begin{aligned}
x' &= [\alpha + X(\alpha,\beta,0)]\cos\varphi + [\beta - h + Y]\sin\varphi, \\
y' &= -[\alpha + X]\sin\varphi + [\beta - h + Y]\cos\varphi
\end{aligned}
\tag{23}
$$

takes S_L onto the same region as described in the (x',y') coordinates. The function $f_L(\alpha,\beta)$ defined by (21) vanishes outside S_L. Figure 2 shows schematically S_P and S_L together with some arrows mapping points from S_L to S_P.

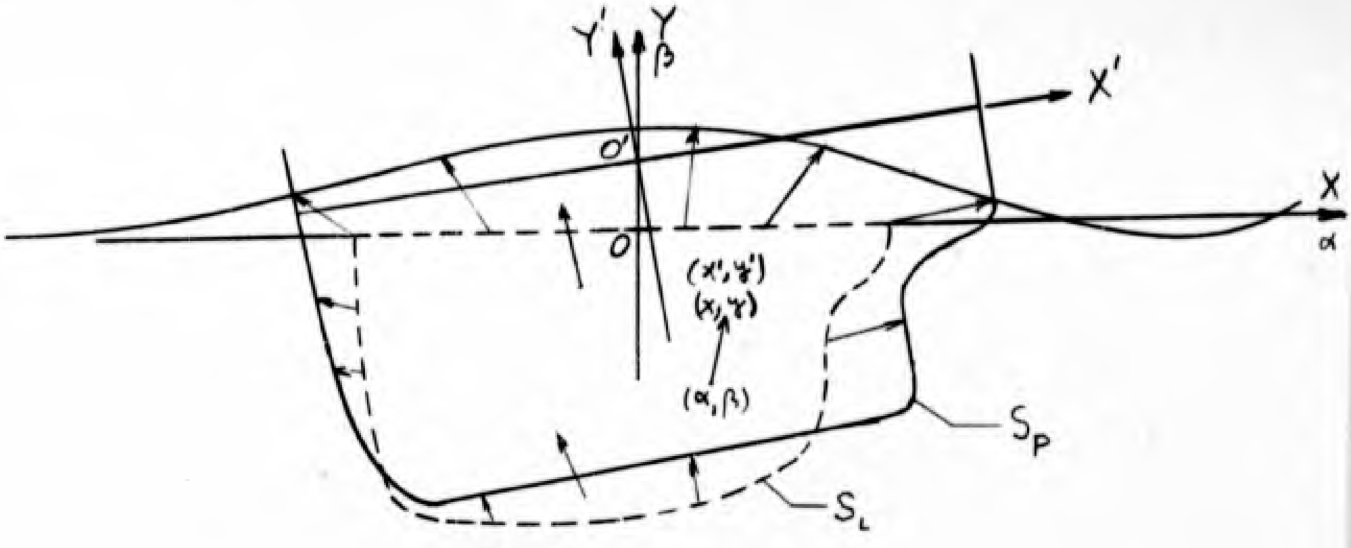

Figure 2

Next we introduce the Lagrangian notation into equations (7) and (8). In carrying out the integrations in the (α,β) plane, one must, of course, use the Jacobian transformation

$$dx\,dy = \frac{\partial(x,y)}{\partial(\alpha,\beta)}\,d\alpha\,d\beta. \tag{24}$$

The equations for F_x, F_y and $\mathcal{M}_{oz}$ take the following form:

$$\begin{aligned} 2\iint_{S_L} d\alpha\,d\beta\,&\{1+X_\alpha(\alpha,\beta,0)+Y_\beta+X_\alpha Y_\beta-Y_\alpha X_\beta\}\{-\rho g\beta+P(\alpha,\beta,+0)\}\times \\ &\{f_1([\alpha+X]\cos\varphi+[\beta-h+Y]\sin\varphi,\,-[\alpha+X]\sin\varphi+[\beta-h+Y]\cos\varphi)\cos\varphi \\ &-f_2\sin\varphi\} - T\cos\varphi = 0, \end{aligned} \tag{25a}$$

$$\begin{aligned} 2\iint_{S_L} d\alpha\,d\beta\,&\{1+X_\alpha+Y_\beta+X_\alpha Y_\beta-Y_\alpha X_\beta\}\{-\rho g\beta+P\}\times \\ &\{f_1\sin\varphi+f_2\cos\varphi\} - T\sin\varphi - Mg = 0, \end{aligned} \tag{25b}$$

$$2\iint_{S_L} d\alpha\, d\beta\,\{1+X_\alpha+Y_\beta+X_\alpha Y_\beta-Y_\alpha X_\beta\}\{-\rho g\beta+P\}$$
$$\{(\alpha+X)(f_1\sin\varphi+f_2\cos\varphi)-(\beta+Y)(f_1\cos\varphi-f_2\sin\varphi)\} \tag{25c}$$
$$+T(e'+h\cos\varphi)-Mg(x'_G\cos\varphi-y'_G\sin\varphi)=0.$$

The variables have been written out in several cases in the first equation. These indicate what they should be in appropriate situations elsewhere. Note that these equations are still 'exact' and that the integrals are taken over S_L, a region which is known only after X, Y, φ and h are known.

It will be convenient for future computations to make an integration by parts in (25 a, b, c). For this purpose we note that from (21)

$$\frac{\partial f_L}{\partial\alpha}=[f_1\cos\varphi-f_2\sin\varphi](1+X_\alpha)+[f_1\sin\varphi+f_2\cos\varphi]Y_\alpha,$$
$$\frac{\partial f_L}{\partial\beta}=[f_1\sin\varphi+f_2\cos\varphi](1+Y_\alpha)+[f_1\cos\varphi-f_2\sin\varphi]X_\beta. \tag{26}$$

From these equations follow

$$f_1\cos\varphi-f_2\sin\varphi=\frac{f_{L\alpha}(1+Y_\beta)-f_{L\beta}Y_\alpha}{(1+X_\alpha)(1+Y_\beta)-X_\beta Y_\alpha},$$

$$f_1\sin\varphi+f_2\cos\varphi=\frac{-f_{L\alpha}X_\beta+f_{L\beta}(1+X_\alpha)}{(1+X_\alpha)(1+Y_\beta)-X_\beta Y_\alpha}. \tag{27}$$

Substitution into (25 a, b, c) yields the following equations:

$$2\iint_{S_L} d\alpha\, d\beta \left\{-\rho g\beta + P(\alpha,\beta,0)\right\}\left\{f_{L\alpha}(1+Y_\beta) - f_{L\beta}Y_\alpha\right\} = T\cos\varphi,$$

$$2\iint_{S_L} d\alpha\, d\beta \left\{-\rho g\beta + P\right\}\left\{-f_{L\alpha}X_\beta + f_{L\beta}(1+X_\alpha)\right\} = Mg + T\sin\varphi,$$

$$\begin{aligned} 2\iint_{S_L} d\alpha\, d\beta \left\{-\rho g\beta + P\right\}\Big\{&(\alpha+X)\left[-f_{L\alpha}X_\beta + f_{L\beta}(1+X_\alpha)\right] \\ &-(\beta+Y)\left[f_{L\alpha}(1+Y_\beta) - f_{L\beta}Y_\alpha\right]\Big\} \\ &= -T(e'+h\cos\varphi) + Mg(x_G'\cos\varphi - y_G'\sin\varphi). \end{aligned} \tag{28}$$

Integrating by parts and making use of the fact that f_L vanishes on the parts of the boundary where $\beta < 0$ and that $-\rho g\beta + P$ vanishes for $\beta = 0$, one finds

$$2\iint_{S_L} d\alpha\, d\beta\, f_L(\alpha,\beta)\left\{-P_\alpha(1+Y_\beta) + (-\rho g + P_\beta)Y_\alpha\right\} = T\cos\varphi,$$

$$2\iint_{S_L} d\alpha\, d\beta\, f_L\left\{P_\alpha X_\beta - (-\rho g + P_\beta)(1+X_\alpha)\right\} = T\sin\varphi + Mg,$$

$$\begin{aligned} 2\iint_{S_L} d\alpha\, d\beta\, f_L\Big\{&(\alpha+X)\left[P_\alpha X_\beta - (-\rho g + P_\beta)(1+X_\alpha)\right] \\ &-(\beta+Y)\left[-P_\alpha(1+Y_\beta) + (-\rho g + P_\beta)Y_\alpha\right]\Big\} \\ &= -T(e'+h\cos\varphi) + Mg(x_G'\cos\varphi - y_G'\sin\varphi). \end{aligned} \tag{29}$$

If one now substitutes for P from (19) one obtains the following equations:

$$2\rho U^2 \iint_{S_L} d\alpha\, d\beta\, f_L \Big\{ [X_{\alpha\alpha} + X_{j\alpha} X_{j\alpha\alpha}](1+Y_\beta)$$

$$- [X_{\alpha\beta} + X_{j\alpha} X_{j\alpha\beta}] Y_\alpha \Big\} = T\cos\varphi ,$$

$$2 \iint_{S_L} d\alpha\, d\beta\, f_L \Big\{ [\rho g + \rho g Y_\beta + \rho U^2 X_{\alpha\beta} + \rho U^2 X_{j\alpha} X_{j\alpha\beta}](1+X_\alpha)$$

$$- [\rho g Y_\alpha + \rho U^2 X_{\alpha\alpha} + \rho U^2 X_{j\alpha} X_{j\alpha\alpha}] X_\beta = T\sin\varphi + Mg ,$$

$$2 \iint_{S_L} d\alpha\, d\beta\, f_L \Big\{ \big[(\rho g + \rho g Y_\beta + \rho U^2 X_{\alpha\beta} + \rho U^2 X_{j\alpha} X_{j\alpha\beta})(1+X_\alpha)$$

$$-(\rho g Y_\alpha + \rho U^2 X_{\alpha\alpha} + \rho U^2 X_{j\alpha} X_{j\alpha\alpha}) X_\beta \big] (\alpha + X)$$

$$- \big[(\rho g Y_\alpha + \rho U^2 X_{\alpha\alpha} + \rho U^2 X_{j\alpha} X_{j\alpha\alpha})(1+Y_\beta) \tag{30}$$

$$-(\rho g + \rho g Y_\beta + \rho U^2 X_{\alpha\beta} + \rho U^2 X_{j\alpha} X_{j\alpha\beta}) Y_\alpha \big] (\beta + Y) \Big\}$$

$$= -T(c' + h\cos\varphi) + Mg(x'_G \cos\varphi - y'_G \sin\varphi) .$$

Finally, we make one further change in the second and third equations in which the purely hydrostatic term in ρg or $\rho g \alpha$ is expressed in (x', y') coordinates. Since

$$\iint_{S_p} dx'\, dy'\, f(x', y') = \iint_{S_L} d\alpha\, d\beta\, f_L(\alpha,\beta)\,[1 + X_\alpha + Y_\beta + X_\alpha Y_\beta - X_\beta Y_\alpha],$$

$$\iint_{S_p} dx'\, dy'\, f(x', y')\, x' = \iint_{S_L} d\alpha\, d\beta\, f_L(\alpha,\beta)\,[1 + X_\alpha + Y_\beta + X_\alpha Y_\beta - X_\beta Y_\alpha]\,(\alpha + X), \tag{31}$$

the second and third equations may be manipulated into the following form:

$$2\rho g \iint_{S_p} dx'dy'\, f(x',y') + 2\rho U^2 \iint_{S_L} d\alpha\, d\beta\, f_L \{[X_{\alpha\beta} + X_{j\alpha}X_{j\alpha\beta}](1+X_\alpha)$$

$$-[X_{\alpha\alpha} + X_{j\alpha}X_{j\alpha\alpha}]X_\beta\} = T\sin\varphi + Mg,$$

$$2\rho g \iint_{S_p} dx'dy'\, f(x',y')(x'\cos\varphi - y'\sin\varphi)$$

$$+ 2\rho U^2 \iint_{S_L} d\alpha\, d\beta\, f_L \{[(X_{\alpha\beta} + X_{j\alpha}X_{j\alpha\beta})(1+X_\alpha)$$

$$-(X_{\alpha\alpha} + X_{j\alpha}X_{j\alpha\alpha})X_\beta](\alpha+X)$$

$$-[(X_{\alpha\alpha} + X_{j\alpha}X_{j\alpha\alpha})(1+Y_\beta)$$

$$-(X_{\alpha\beta} + X_{j\alpha}X_{j\alpha\beta})Y_\alpha](\beta+Y)\} \tag{32}$$

$$= -T(e' + h\cos\varphi) + Mg(x_G'\cos\varphi - y_G'\sin\varphi).$$

Finally we need conditions at infinity. We have already assumed that $X_i \to 0$ as $\alpha \to -\infty$. A stronger statement than this is necessary in order to guarantee that the ship's wave pattern follows the ship. We assume the following:

$$\lim_{\alpha\to-\infty} (\alpha^2+\gamma^2)^{1/4} X_i = 0. \tag{33}$$

For convenience we shall also assume an infinitely deep fluid, so that

$$\lim_{\beta\to-\infty} X_i = 0. \tag{34}$$

From the geometry of the problem it is evident that X, Y, Z have the following symmetry properties:

$$X(\alpha,\beta,-\gamma)=X(\alpha,\beta,\gamma),\ Y(\alpha,\beta,-\gamma)=Y(\alpha,\beta,\gamma),\ Z(\alpha,\beta,-\gamma)=-Z(\alpha,\beta,\gamma). \quad (35)$$

As a result, there will be no loss in generality if we imagine the (x, y)-plane replaced by a wall along which half of the ship slides.

The problem before us is then to find the functions $X_i(\alpha,\beta,\gamma)$ satisfying the equations (14) and (18) and the various boundary conditions expressed by (7), (8), (20), (21), (30), (31), and (32). If the problem can be solved, one will have found the trim angle φ, the sinkage h, the necessary thrust T, and the form of the free surface, as well as the velocity and pressure field in the fluid. The complicated way in which unknown quantities are entangled with each other in the boundary conditions makes an explicit solution as unlikely here as in the Eulerian formulation of the problem.

An approximation scheme.

In view of the evident impossibility of obtaining an explicit solution to the exact problem, we shall attempt to develop a method for finding approximate solutions. Instead of introducing a perturbation parameter and assuming formal power-series expansions in it, as is customary in the Eulerian formulation of the problem, we shall use a method of iteration. Since the starting point will be a ship of zero beam-to-length ratio, the approximation solutions may also be considered "thin-ship" approximations. In fact, with some modification, the first iteration yields exactly the same results as one obtains from a perturbation expansion in Eulerian variables.

Let us start the iteration by taking

$$X^{(0)}=Y^{(0)}=Z^{(0)}=0,\quad \varphi^{(0)}=0,\quad h^{(0)}=0. \quad (36)$$

We further define S_0 as the region S_p when the ship is at rest in its equilibrium position.

Let us now suppose that $X^{(n)}$, $Y^{(n)}$, $Z^{(n)}$, $\varphi^{(n)}$, $h^{(n)}$, $T^{(n)}$ have been found and write out the prescription for finding the functions with superscript $n+1$. First we define S_n as the region in the (α,β) plane where $\beta \leq 0$ and

$$f^{(n)}(\alpha,\beta) \equiv f([\alpha+X^{(n)}(\alpha,\beta,0)]\cos\varphi^{(n)} + [\beta-h^{(n)}+Y^{(n)}]\sin\varphi^{(n)},$$
$$-[\alpha+X^{(n)}]\sin\varphi^{(n)}+[\beta-h^{(n)}+Y^{(n)}]\cos\varphi^{(n)}) \geq 0. \tag{37}$$

We define $f^{(n)} = 0$ for (α,β) outside of S_n. The mapping

$$x = \alpha + X^{(n)}(\alpha,\beta,0), \quad y = \beta + Y^{(n)}(\alpha,\beta,0) \tag{38}$$

takes S_n onto S_{pn} and the mapping

$$x' = [\alpha + X^{(n)}(\alpha,\beta,0)]\cos\varphi^{(n)} + [\beta-h^{(n)}+Y^{(n)}]\sin\varphi^{(n)},$$
$$y' = -[\alpha+X^{(n)}]\sin\varphi^{(n)} + [\beta-h^{(n)}+Y^{(n)}]\cos\varphi^{(n)} \tag{39}$$

takes S_n onto S_{pn} described in (x',y') coordinates. In the (x',y')-plane the boundaries of S_p and S_{pn} will coincide where they are both submerged. However, the free boundaries will differ since that of S_{pn} is determined by (39) with $\beta=0$, which is an approximation to the exact free boundary. Since the relation between the (x,y) and (x',y') coordinates is determined by φ and h and not $\varphi^{(n)}$ and $h^{(n)}$, the under-water boundaries of S_{pn} and S_p will not necessarily coincide, but will be congruent. If we may assume that $X^{(n)} \to X$, $\ldots$, $h^{(n)} \to h$, then also $f^{(n)} \to f_L$, $S_n \to S_L$; and $S_{pn} \to S_p$.

The functions $X^{(n+1)}$, $Y^{(n+1)}$, $Z^{(n+1)}$ are to be solutions of the partial differential equations

$$X_\alpha^{(n+1)} + Y_\beta^{(n+1)} + Z_\gamma^{(n+1)} + \frac{\partial(Y^{(n)}, Z^{(n)})}{\partial(\beta,\gamma)} + \frac{\partial(Z^{(n)}, X^{(n)})}{\partial(\gamma,\alpha)}$$

$$+ \frac{\partial(X^{(n)}, Y^{(n)})}{\partial(\alpha,\beta)} + \frac{\partial(X^{(n)}, Y^{(n)}, Z^{(n)})}{\partial(\alpha,\beta,\gamma)} = 0; \tag{40}$$

$$X_{\alpha\beta}^{(n+1)} - Y_{\alpha\alpha}^{(n+1)} = X_{j\alpha\alpha}^{(n)} X_{j\beta}^{(n)} - X_{j\alpha\beta}^{(n)} X_{j\alpha}^{(n)},$$

$$Y_{\alpha\gamma}^{(n+1)} - Z_{\alpha\beta}^{(n+1)} = X_{j\alpha\beta}^{(n)} X_{j\gamma}^{(n)} - X_{j\alpha\gamma}^{(n)} X_{j\beta}^{(n)},$$

$$Z_{\alpha\alpha}^{(n+1)} - X_{\alpha\gamma}^{(n+1)} = X_{j\alpha\gamma}^{(n)} X_{j\alpha}^{(n)} - X_{j\alpha\alpha}^{(n)} X_{j\gamma}^{(n)}. \tag{41}$$

They are also to satisfy the boundary conditions

$$g\,Y^{(n+1)}(\alpha,0,\gamma) + U^2 X_\alpha^{(n+1)} = -\frac{1}{2} U^2 X_{j\alpha}^{(n)} X_{j\alpha}^{(n)}, \tag{42}$$

$$Z^{(n+1)}(\alpha,\beta,\pm 0) = \begin{cases} \pm f^{(n)}(\alpha,\beta) & ,(\alpha,\beta) \in S_n, \\ 0 & ,(\alpha,\beta) \notin S_n. \end{cases} \tag{43}$$

From (41) and (43) we can derive boundary conditions to be satisfied by $X^{(n+1)}$ and $Y^{(n+1)}$ on S_n :

$$X_{\alpha\gamma}^{(n+1)}(\alpha,\beta,\pm 0) = \pm f_{\alpha\alpha}^{(n)}(\alpha,\beta) + X_{\alpha\alpha}^{(n)}(\alpha,\beta,0)X_{\gamma}^{(n)}(\alpha,\beta,\pm 0) + Y_{\alpha\alpha}^{(n)}(\alpha,\beta,0)Y_{\gamma}^{(n)}(\alpha,\beta,\pm 0)$$
$$+ Z_{\alpha\alpha}^{(n)}(\alpha,\beta,\pm 0)Z_{\gamma}^{(n)}(\alpha,\beta,0) - X_{\alpha\gamma}^{(n)}(\alpha,\beta,\pm 0)X_{\alpha}^{(n)}(\alpha,\beta,0)$$
$$- Y_{\alpha\gamma}^{(n)}(\alpha,\beta,\pm 0)Y_{\alpha}^{(n)}(\alpha,\beta,0) - Z_{\alpha\gamma}^{(n)}(\alpha,\beta,0)Z_{\alpha}^{(n)}(\alpha,\beta,\pm 0), \tag{43'}$$

$$Y_{\alpha\gamma}^{(n+1)}(\alpha,\beta,\pm 0) = \pm f_{\beta}^{(n)}(\alpha,\beta) + X_{\alpha\beta}^{(n)}(\alpha,\beta,0)X_{\gamma}^{(n)}(\alpha,\beta,\pm 0) + Y_{\alpha\beta}^{(n)}(\alpha,\beta,0)Y_{\gamma}^{(n)}(\alpha,\beta,\pm 0)$$
$$+ Z_{\alpha\beta}^{(n)}(\alpha,\beta,\pm 0)Z_{\gamma}^{(n)}(\alpha,\beta,0) - X_{\alpha\gamma}^{(n)}(\alpha,\beta,\pm 0)X_{\beta}^{(n)}(\alpha,\beta,0)$$
$$- Y_{\alpha\gamma}^{(n)}(\alpha,\beta,\pm 0)Y_{\beta}^{(n)}(\alpha,\beta,0) - Z_{\alpha\gamma}^{(n)}(\alpha,\beta,0)Z_{\beta}^{(n)}(\alpha,\beta,\pm 0).$$

The $X_i^{(n)}$ themselves were required to satisfy boundary conditions on S_{n-1}. However, since $X_\gamma^{(n)}$, $Y_\gamma^{(n)}$, $Z^{(n)}$ vanish outside S_{n-1}, the contribution from the product terms in (43') is non-zero only in the intersection of S_n and S_{n-1}.

The boundary conditions at ∞ are

$$\lim_{\alpha\to-\infty}(\alpha^2+\gamma^2)^{1/4}X_i^{(n+1)} = 0, \quad \lim_{\beta\to-\infty} X_i^{(n+1)} = 0. \tag{44}$$

The quantities $T^{(n+1)}$ and $\varphi^{(n+1)}$ and $h^{(n+1)}$, which enter into the definition of S_n, are determined from the equations

$$T^{(n+1)}\cos\varphi^{(n)} = 2\rho U^2 \iint_{S_n} d\alpha\, d\beta\, f^{(n)}\Big\{X_{\alpha\alpha}^{(n+1)} + X_{j\alpha}^{(n+1)}X_{j\alpha\alpha}^{(n+1)}(1+Y_\beta^{(n+1)})$$
$$-(X_{\alpha\beta}^{(n+1)} + X_{j\alpha}^{(n+1)}X_{j\alpha\beta}^{(n+1)})Y_\alpha^{(n+1)}\Big\}, \tag{45}$$

$$
\begin{aligned}
T^{(n+1)}\sin\varphi^{(n)} + Mg &= 2\rho g \iint_{S_{P,n+1}} dx'dy'\, f(x',y') \\
&\quad + 2\rho U^2 \iint_{S_n} d\alpha\, d\beta\, f^{(n)} \Big\{ [X_{\alpha\beta}^{(n+1)} + X_{j\alpha}^{(n+1)} X_{j\alpha\beta}^{(n+1)}](1+X_\alpha^{(n+1)}) \\
&\qquad - [X_{\alpha\alpha}^{(n+1)} + X_{j\alpha}^{(n+1)} X_{j\alpha\alpha}^{(n+1)}] X_\beta^{(n+1)} \Big\},
\end{aligned}
\tag{46}
$$

$$
\begin{aligned}
-T^{(n+1)}(e' + h^{(n)}\cos\varphi^{(n)}) &+ Mg\,(x'_G \cos\varphi^{(n+1)} - y'_G \sin\varphi^{(n+1)}) = \\
&= 2\rho g \iint_{S_{P,n+1}} dx'dy'\, f(x',y')\,(x'\cos\varphi^{(n+1)} - y'\sin\varphi^{(n+1)}) \\
&\quad + 2\rho U^2 \iint_{S_n} d\alpha\, d\beta\, f^{(n)}(\alpha,\beta) \Big\{ [(X_{\alpha\beta}^{(n+1)} + X_{j\alpha}^{(n+1)} X_{j\alpha\beta}^{(n+1)})(1+X_\alpha^{(n+1)}) \\
&\qquad - (X_{\alpha\alpha}^{(n+1)} + X_{j\alpha}^{(n+1)} X_{j\alpha\alpha}^{(n+1)}) X_\beta^{(n+1)}](\alpha + X^{(n+1)}) \\
&\qquad - [(X_{\alpha\alpha}^{(n+1)} + X_{j\alpha}^{(n+1)} X_{j\alpha\alpha}^{(n+1)})(1+Y_\beta^{(n+1)}) \\
&\qquad - (X_{\alpha\beta}^{(n+1)} + X_{j\alpha}^{(n+1)} X_{j\alpha\beta}^{(n+1)}) Y_\alpha^{(n+1)}](\beta + Y^{(n+1)}) .
\end{aligned}
\tag{47}
$$

In equations (45) to (47) no effort has been made to arrange the iteration scheme so that all associated terms are 'of the same order', as is usual in perturbation expansions. It would be possible to do this, but the equations would become even more unwieldy, for the domains of integration must be matched to any alteration of the superscripts in a given cluster of functions. On the other hand, there is no reason why we should not make some modifications for the first and second iterations, the only ones with which we shall be concerned in detail, and, in fact, we shall do this. It will not be necessary to correct explicitly for such modifications in later iterations; the result is simply that a somewhat different set of functions is fed into the following iteration.

One should note especially that the domain of integration changes as one proceeds from one step to the next in the iteration process. This is different from the situation in the usual perturbation expansions, where at each step one solves a boundary-value problem with boundary values assigned on the same domain; in the present problem, for example, the boundary conditions (43) would be replaced by conditions to be satisfied on S_0 , and all integrals in (45) to (47) would be taken over S_0 . As was shown some years ago by Lighthill (1958) for the relatively simple case of a thin symmetrical wing, the perturbation expansion diverges in the neighborhood of the leading and trailing edges. The procedure used here seems to avoid this difficulty and to effect in a natural way the successive straining of the coordinate system proposed by Lighthill.

Equation (45) determines $T^{(n+1)}$. Equations (46) and (47) together determine $\varphi^{(n+1)}$ and $h^{(n+1)}$. These quantities occur both explicitly and also implicitly through the first integrals over $S_{P,n+1}$ on the right-hand sides. Let us examine these integrals in more detail.

First we note that the wave profile along the ship will be given parametrically in (x', y') coordinates by the following equations:

$$
\begin{aligned}
x' &= [\alpha + X(\alpha,0,0)]\cos\varphi + [-h + Y(\alpha,0,0)]\sin\varphi, \\
y' &= -[\alpha + X(\alpha,0,0)]\sin\varphi + [-h + Y(\alpha,0,0)]\cos\varphi.
\end{aligned}
\tag{48}
$$

We denote the explicit dependence of y' upon x' by $y' = \eta(x')$. If we write

$$
\begin{aligned}
\alpha &= x'\sec\varphi - X(\alpha,0,0) - [-h + Y(\alpha,0,0)]\tan\varphi, \\
y' &= -x'\tan\varphi - h\sec\varphi + \sec\varphi\, Y(\alpha,0,0),
\end{aligned}
\tag{49}
$$

it is not difficult to develop a method of successive approximations for determining $\eta(x')$, assuming X , Y , h and φ are small. We carry this out through the second order. Write

$$\begin{aligned} \alpha &= x'\sec\varphi - X(x'\sec\varphi - X(\alpha,0,0) - [-h+Y]\tan\varphi, 0, 0) \\ &\quad - [-h + Y(x'\sec\varphi - X(\alpha,0,0) - [-h+Y]\tan\varphi, 0, 0)]\tan\varphi \\ &= x'\sec\varphi - X(x'\sec\varphi, 0, 0) - [-h + Y(x'\sec\varphi, 0, 0)]\tan\varphi \\ &\quad + \cdots . \end{aligned} \tag{50}$$

Then

$$\begin{aligned} y' &= -x'\tan\varphi - h\sec\varphi \\ &\quad + \sec\varphi \left\{ Y(x'\sec\varphi, 0, 0) - Y_\alpha \cdot [X + (-h+Y)\tan\varphi] + \cdots \right\} . \end{aligned} \tag{51}$$

As part of a general scheme, we may define

$$\begin{aligned} y' &= \eta^{(1)}(x') = -x'\tan\varphi^{(1)} - h^{(1)}\sec\varphi^{(1)} + \sec\varphi^{(1)} Y^{(1)}(x'\sec\varphi^{(1)}, 0, 0), \\ y' &= \eta^{(2)}(x') = -x'\tan\varphi^{(2)} - h^{(2)}\sec\varphi^{(2)} + \sec\varphi^{(2)} \{ Y^{(2)}(x'\sec\varphi^{(2)}, 0, 0) \\ &\quad - Y_\alpha^{(2)}(x'\sec\varphi^{(2)}, 0, 0) X^{(2)}(x'\sec\varphi^{(2)}, 0, 0) \} , \end{aligned} \tag{52}$$

etc.

Because we are going to modify the first two approximations anyway, we shall modify (52) also as follows:

$$\begin{aligned} y' &= \eta^{(1)}(x') = -x'\varphi^{(1)} - h^{(1)} + Y^{(1)}(x', 0, 0), \\ y' &= \eta^{(2)}(x') = -x'\varphi^{(2)} - h^{(2)} + Y^{(2)}(x', 0, 0) - Y_\alpha^{(2)} X^{(2)} . \end{aligned} \tag{53}$$

For $r^{(3)}(x')$ it would be simpler to go back to the general scheme of (51) rather than contend with series expansions of the several trigonometric functions.

Let us now consider the first integral on the right-hand side of (46). If the stem and stern of the ship are vertical in a sufficiently large region near the waterline, this integral may be split as follows

$$2\rho g \iint_{S_0} dx'dy' f(x',y') + 2\rho g \int dx' \int_0^{\eta^{(n+1)}(x')} dy' f(x',y')$$
$$= \rho g V + 2\rho g \int dx' \int_0^{\eta^{(n+1)}} dy' f(x',y'), \tag{54}$$

where V is the volume displacement of the ship. The first integral in (47) may be split similarly to yield

$$\rho g V [x_B' \cos\varphi^{(n+1)} - y_B' \sin\varphi^{(n+1)}]$$
$$+ 2\rho g \int dx' \int_0^{\eta^{(n+1)}} dy' f(x',y')(x'\cos\varphi^{(n+1)} - y'\sin\varphi^{(n+1)}), \tag{55}$$

where $(x_B', y_B', 0)$ is the center of buoyancy. The other integrals on the right-hand sides of (46) and (47) will not involve either $\varphi^{(n+1)}$ or $h^{(n+1)}$.

<u>The first approximation</u>.

We now write out explicitly the equations for the first approximation, but with certain modifications in (61) to (63):

$$X_\alpha^{(1)} + Y_\beta^{(1)} + Z_\gamma^{(1)} = 0, \tag{56}$$

$$X_\beta^{(1)} - Y_\alpha^{(1)} = Y_\gamma^{(1)} - Z_\beta^{(1)} = Z_\alpha^{(1)} - X_\gamma^{(1)} = 0, \tag{57}$$

$$g Y^{(1)}(\alpha,0,\gamma) + U^2 X_\alpha^{(1)} = 0, \tag{58}$$

$$Z^{(1)}(\alpha,\beta,\pm 0) = \begin{cases} \pm f(\alpha,\beta), & (\alpha,\beta) \in S_0, \\ 0, & (\alpha,\beta) \notin S_0, \end{cases} \tag{59}$$

$$\lim_{\alpha\to-\infty} (\alpha^2+\gamma^2)^{1/4} X_i^{(1)} = 0, \quad \lim_{\beta\to-\infty} X_i^{(1)} = 0, \tag{60}$$

$$T^{(1)} = 2\rho U^2 \iint_{S_0} d\alpha d\beta\, f(\alpha,\beta)\, X_{\alpha\alpha}^{(1)}(\alpha,\beta,0), \tag{61}$$

$$\begin{aligned} Mg &= \rho g V + 2\rho g \int dx' [-x'\varphi^{(1)} - h^{(1)} + Y^{(1)}(x',0,0)] f(x',0) \\ &\quad + 2\rho U^2 \iint_{S_0} d\alpha d\beta\, f(\alpha,\beta) X_{\alpha\beta}^{(1)}(\alpha,\beta,0), \end{aligned} \tag{62}$$

$$\begin{aligned} -T^{(1)}e' + Mg\,(x_G' - y_G'\varphi^{(1)}) &= \rho g V\,(x_B' - y_B'\varphi^{(1)}) \\ &\quad + 2\rho g \int dx' [-x'\varphi^{(1)} - h^{(1)} + Y^{(1)}(x',0,0)](x' + y'\varphi^{(1)}) f(x',0) \\ &\quad + 2\rho U^2 \iint_{S_0} d\alpha d\beta\, f(\alpha,\beta)[X_{\alpha\beta}^{(1)}(\alpha,\beta,0)\alpha - X_{\alpha\alpha}^{(1)}\beta]\ . \end{aligned} \tag{63}$$

When the ship is at rest in its equilibrium position, it is evident that (62) and (63) yield

$$Mg = \rho g V \quad , \quad x_G' = x_B' , \tag{64}$$

the usual hydrostatic equilibrium equations.

Define

$$\begin{aligned} A &= 2\int dx'\, f(x',0)\ , \quad x_A' = \frac{2}{A}\int dx'\, f(x',0)\,x', \\ H_p &= \frac{2}{V}\int dx'\, f(x',0)\,x'^2 + y_B' - y_G' , \end{aligned} \tag{65}$$

i.e., A is the waterplane area, (x_A' , 0) its center of area, and H_P the metacentric height in pitching. (62) and (63) may then be rewritten as follows:

$$x_A' \varphi^{(1)} + h^{(1)} = \frac{2}{A}\int dx' f(x',0)\, Y^{(1)}(x',0,0) + \frac{2U^2}{Ag}\iint_{S_0} d\alpha\, d\beta\, f(\alpha,\beta) X^{(1)}_{\alpha\beta}(\alpha,\beta,0), \tag{66}$$

$$\frac{V}{A} H_P \varphi^{(1)} + x_A' h^{(1)} = \frac{2}{A} T^{(1)} e' + \frac{2}{A}\int dx' f(x',0)\, Y^{(1)}(x',0,0)\, x' + \frac{2U^2}{Ag}\iint_{S_0} d\alpha\, d\beta\, f(\alpha,\beta)\left[\alpha X^{(1)}_{\alpha\beta}(\alpha,\beta,0) - \beta X^{(1)}_{\alpha\alpha}\right].$$

It is not hard to see that the problem formulated above leads to the same solution as the first-order problem in Eulerian coordinates. From (56) and (57) one finds easily

$$X^{(1)}_{\alpha\alpha} + X^{(1)}_{\beta\beta} + X^{(1)}_{\gamma\gamma} = 0. \tag{67}$$

From (57) and (59) one finds

$$X^{(1)}_{\gamma}(\alpha,\beta,\pm 0) = \pm f_{\alpha}(\alpha,\beta), \quad (\alpha,\beta) \in S_0. \tag{68}$$

Equations (57) and (58) yield

$$g X^{(1)}_{\beta}(\alpha,0,\gamma) + U^2 X^{(1)}_{\alpha\alpha} = 0. \tag{69}$$

Equations (67) to (69) and (60) are the same equations which are used to find the velocity potential in Eulerian coordinates.

As far as (61) and (66) are concerned, it suffices to have found $X^{(1)}$, for $Y^{(1)}(x', 0, 0)$ can be determined from (57).

Because we shall make use of the technique of solution for the second approximation, we briefly review it here. Let $G(\alpha,\beta,\gamma;\alpha',\beta',\gamma')$ be a potential function with a singularity of the type r^{-1} which satisfies (60), (67), (68), and (69). Such a function may be written as follows:

$$G(\alpha,\beta,\gamma;\alpha',\beta',\gamma') = -[(\alpha-\alpha')^2+(\beta-\beta')^2+(\gamma-\gamma')^2]^{-1/2}+[(\alpha-\alpha')^2+(\beta+\beta')^2+(\gamma-\gamma')^2]^{-1/2}$$

$$+\frac{4}{\pi}k_0\int_0^{\frac{1}{2}\pi}d\theta\,\sec^2\theta \oint_0^{\infty} dk\, e^{k(\beta+\beta')}\frac{\cos[k(\alpha-\alpha')\cos\theta]\cos[k(\gamma-\gamma')\sin\theta]}{k-k_0\sec^2\theta}$$

$$-4k_0\int_0^{\frac{1}{2}\pi}d\theta\,\sec^2\theta\, e^{k_0(\beta+\beta')\sec^2\theta}\sin[k_0(\alpha-\alpha')\sec\theta]\cos[k_0(\gamma-\gamma')\sin\theta\sec^2\theta], \tag{70}$$

where $k_0 = g/U^2$.

$X^{(1)}$ is now given by [see Kellogg (1929), p. 164]

$$X^{(1)}(\alpha,\beta,\gamma) = \frac{1}{2\pi}\iint_{S_0} d\alpha' d\beta'\, G(\alpha,\beta,\gamma;\alpha',\beta',0)\, f_1(\alpha',\beta')$$

$$= -\frac{1}{2\pi}\iint_{S_0} d\alpha' d\beta'\, G_{\alpha'} f = \frac{1}{2\pi}\iint_{S_0} d\alpha' d\beta'\, G_{\alpha} f. \tag{71}$$

We can immediately find $Y^{(1)}$ and $Z^{(1)}$ from (57) and (12):

$$Y^{(1)}(\alpha,\beta,\gamma) = \frac{1}{2\pi}\iint_{S_0} d\alpha' d\beta'\, G_{\beta} f,$$

$$Z^{(1)}(\alpha,\beta,\gamma) = \frac{1}{2\pi}\iint_{S_0} d\alpha' d\beta'\, G_{\gamma} f. \tag{72}$$

Substitution of $X^{(1)}$ into (61) yields

$$T^{(1)} = \frac{\rho U^2}{\pi} \iint_{S_0} d\alpha\, d\beta \iint_{S_0} d\alpha' d\beta'\, G_{\alpha\alpha\alpha}(\alpha,\beta,0;\alpha',\beta',0)\, f(\alpha,\beta)\, f(\alpha',\beta'). \tag{73}$$

If one now notes that the terms in $G_{\alpha\alpha\alpha}$ which are odd in $\alpha-\alpha'$ do not contribute to the integral, one obtains

$$\begin{aligned} T^{(1)} = \frac{4}{\pi} \rho g k_0^3 \iint_{S_0} d\alpha\, d\beta \iint_{S_0} d\alpha' d\beta'\, f(\alpha,\beta) f(\alpha',\beta') \cdot \\ \int_0^{\frac{1}{2}\pi} d\theta \sec^5\theta \exp[k_0(\beta+\beta')\sec^2\theta] \sin[k_0(\alpha-\alpha')\sec\theta], \end{aligned} \tag{74}$$

the classical result of Michell. Unfortunately, the symmetry considerations which simplify (73) do not simplify (66), so that we shall not write out the equations with the expression for $X^{(1)}$ substituted into them.

The second approximation.

The equations for the second approximation will be taken as shown below, again a modification of the general iteration scheme in which certain higher-order terms are discarded.

$$X_\alpha^{(2)} + Y_\beta^{(2)} + Z_\gamma^{(2)} = \tfrac{1}{2}[X_\alpha^{(1)2} + X_\beta^{(1)2} + \dots + Z_\beta^{(1)2} + Z_\gamma^{(1)2}]; \tag{75}$$

$$\begin{aligned} X_{\alpha\beta}^{(2)} - Y_{\alpha\alpha}^{(2)} &= X_{j\alpha\alpha}^{(1)} X_{j\beta}^{(1)} - X_{j\alpha\beta}^{(1)} X_{j\alpha}^{(1)}, \\ Y_{\alpha\gamma}^{(2)} - Z_{\alpha\beta}^{(2)} &= X_{j\alpha\beta}^{(1)} X_{j\gamma}^{(1)} - X_{j\alpha\gamma}^{(1)} X_{j\beta}^{(1)}, \\ Z_{\alpha\alpha}^{(2)} - X_{\alpha\gamma}^{(2)} &= X_{j\alpha\gamma}^{(1)} X_{j\alpha}^{(1)} - X_{j\alpha\alpha}^{(1)} X_{j\gamma}^{(1)}; \end{aligned} \tag{76}$$

$$g\, Y^{(2)}(\alpha,0,\gamma) + U^2 X_\alpha^{(2)} = -\tfrac{1}{2} U^2 X_{j\alpha}^{(1)} X_{j\alpha}^{(1)}, \tag{77}$$

$$Z^{(2)}(\alpha,\beta,\pm 0) = \pm f^{(1)}(\alpha,\beta), \quad (\alpha,\beta) \in S_1; \tag{78}$$

$$\lim_{\alpha\to-\infty} (\alpha^2+\gamma^2)^{1/4} X_i^{(2)} = 0\,, \quad \lim_{\beta\to-\infty} X_i^{(2)} = 0\,, \quad i=1,2,3\,; \tag{79}$$

$$T^{(2)} = 2\rho U^2 \iint_{S_1} d\alpha\, d\beta\, f^{(1)} \left\{ X_{\alpha\alpha}^{(2)} + X_{j\alpha}^{(2)} X_{j\alpha\alpha}^{(2)} + Y_\beta^{(2)} X_{\alpha\alpha}^{(1)} - Y_\alpha^{(2)} X_{\alpha\beta}^{(2)} \right\}, \tag{80}$$

$$T^{(2)}\varphi^{(1)} = 2\rho g \int dx' \int_0^{\eta^{(2)}(x')} dy'\, f(x',y')$$

$$+\, 2\rho U^2 \iint_{S_1} d\alpha\, d\beta\, f^{(1)} \left\{ X_{\alpha\beta}^{(2)} + X_{j\alpha}^{(2)} X_{j\alpha\beta}^{(2)} + X_\alpha^{(1)} X_{\alpha\beta}^{(2)} - X_\beta^{(2)} X_{\alpha\alpha}^{(2)} \right\}, \tag{81}$$

$$-T^{(2)}(e'+h^{(1)}) + \rho g V (y_B' - y_G')\varphi^{(2)} =$$

$$= 2\rho g \int dx' \int_0^{\eta^{(2)}} dy'\, f(x',y')(x' + y'\varphi^{(2)})$$

$$+\, 2\rho U^2 \iint_{S_1} d\alpha\, d\beta\, f^{(1)} \Big\{ \left[X_{\alpha\beta}^{(2)} + X_{j\alpha}^{(2)} X_{j\alpha\beta}^{(2)} + X_\alpha^{(2)} X_{\alpha\beta}^{(2)} - X_\beta^{(2)} X_{\alpha\alpha}^{(2)} \right]\alpha \tag{82}$$

$$- \left[X_{\alpha\alpha}^{(2)} + X_{j\alpha}^{(2)} X_{j\alpha\alpha}^{(2)} + Y_\beta^{(2)} X_{\alpha\alpha}^{(2)} - Y_\alpha^{(2)} X_{\alpha\beta}^{(2)} \right]\beta$$

$$+\, X_{\alpha\beta}^{(2)} X^{(2)} - X_{\alpha\alpha}^{(2)} Y^{(2)} \Big\}.$$

Let us next differentiate (75) twice with respect to α and substitute from equations (76). After some simplification one obtains the following equation which must be satisfied by $X_\alpha^{(2)}$:

$$\Delta X_{\alpha}^{(2)} = 2\{ X_{\alpha\alpha\alpha}^{(1)} X_{\alpha}^{(1)} + X_{\alpha\alpha\beta}^{(1)} X_{\beta}^{(1)} + X_{\alpha\alpha\gamma}^{(1)} X_{\gamma}^{(1)}$$

$$+ Y_{\alpha\alpha\alpha}^{(1)} Y_{\alpha}^{(1)} + Y_{\alpha\alpha\beta}^{(1)} Y_{\beta}^{(1)} + Y_{\alpha\alpha\gamma}^{(1)} Y_{\gamma}^{(1)} \tag{83}$$

$$+ Z_{\alpha\alpha\alpha}^{(1)} Z_{\alpha}^{(1)} + Z_{\alpha\alpha\beta}^{(1)} Z_{\beta}^{(1)} + Z_{\alpha\alpha\gamma}^{(1)} Z_{\gamma}^{(1)} \}.$$

Similarly one can derive equations for $Y_{\alpha}^{(2)}$ and $Z_{\alpha}^{(2)}$. The three equations can be presented in unified form as follows:

$$\Delta X_{i\alpha}^{(2)} = 2 X_{j\alpha\alpha_i\alpha_k}^{(1)} X_{j\alpha_k}^{(1)} \equiv H_i(\alpha,\beta,\gamma) \; , i = 1,2,3. \tag{84}$$

Each one of $X_{\alpha}^{(2)}$, $Y_{\alpha}^{(2)}$, $Z_{\alpha}^{(2)}$ satisfies Poisson's rather than Laplace's equation.

In order to find the free-surface condition for $X_{\alpha}^{(2)}$, differentiate (77) twice with respect to α and substitute for $Y_{\alpha\alpha}^{(2)}$ from (76). For $Z_{\alpha}^{(2)}$ one differentiates twice with respect to α and γ and again uses (77). For $Y_{\alpha}^{(2)}$ one uses the equation just derived for $X_{\alpha}^{(2)}$, and uses (77) again to replace $X_{\alpha}^{(2)}$ by $Y^{(2)}$. The three boundary conditions can be written together as follows:

$$k_0 X_{i\alpha\beta}^{(2)}(\alpha,0,\gamma) + X_{i\alpha\alpha\alpha}^{(2)} = -\frac{\partial}{\partial\alpha}\left(X_{j\alpha\alpha}^{(1)} X_{j\alpha_i}^{(1)} \right) +$$

$$+ k_0 \left(X_{j\alpha\alpha_i}^{(1)} X_{j\beta}^{(1)} - X_{j\alpha\beta}^{(1)} X_{j\alpha_i}^{(1)} \right) \equiv \frac{1}{\rho U} \Pi_{i\alpha}(\alpha,\gamma), \; i = 1,2,3. \tag{85}$$

Equation (78) already gives a boundary condition on S_1 for $Z^{(2)}$, and hence for $Z_{\alpha}^{(2)}$ by differentiating. Boundary conditions for $X_{\alpha}^{(2)}$ and $Y_{\alpha}^{(2)}$ are given directly by (43'). If one makes use of the conditions already satisfied by the $X_i^{(1)}$, one finds the following boundary conditions on S_1 :

$$X^{(2)}_{\alpha\gamma}(\alpha,\beta,\pm 0) = \pm \Big\{ f^{(1)}_{\alpha\alpha}(\alpha,\beta) + [X^{(1)}_{\alpha\alpha}(\alpha,\beta,0) - Z^{(1)}_{\alpha\gamma}] f_\alpha + Y^{(1)}_{\alpha\alpha} f_\beta$$

$$- [X^{(1)}_{\alpha} - Z^{(1)}_{\gamma}] f_{\alpha\alpha} - Y^{(1)}_{\alpha} f_{\alpha\beta} \Big\} \equiv \pm F_1(\alpha,\beta),$$

$$Y^{(2)}_{\alpha\gamma}(\alpha,\beta,\pm 0) = \pm \Big\{ f^{(1)}_{\alpha\beta}(\alpha,\beta) + X^{(1)}_{\alpha\beta}(\alpha,\beta,0) f_\alpha + [Y^{(1)}_{\alpha\beta} - Z^{(1)}_{\alpha\gamma}] f_\beta$$

$$- X^{(1)}_{\beta} f_{\alpha\alpha} + [Y^{(1)}_{\beta} - Z^{(1)}_{\gamma}] f_{\alpha\beta} \Big\} \equiv \pm F_2(\alpha,\beta),$$

$$Z^{(2)}_{\alpha}(\alpha,\beta,\pm 0) = \pm f^{(1)}_{\alpha}(\alpha,\beta). \tag{86}$$

We recall that $f(\alpha,\beta)$ is defined to be zero outside S_0, so that the contributions from the terms involving the $X^{(1)}_i$ vanish outside the intersection of S_0 and S_1. It is a kind of aesthetic blemish on the approximation scheme that the right-hand sides of the first two equations in (86) do not consist of known functions defined 'naturally' upon S_1. Unfortunately, this seems unavoidable if at each step in the approximation one is to have a linear problem to solve.

One may prove the following properties of the functions H_i and Π_i by using the symmetry properties of X, Y, Z given in (35):

$$H_i(\alpha,\beta,-\gamma) = H_i(\alpha,\beta,\gamma),\ i = 1,2, \quad H_3(\alpha,\beta,-\gamma) = -H_3(\alpha,\beta,\gamma),$$

$$\Pi_i(\alpha,-\gamma) = \Pi_i(\alpha,\gamma),\ i = 1,2, \quad \Pi_3(\alpha,-\gamma) = -\Pi_3(\alpha,\gamma). \tag{87}$$

We are now ready to construct solutions to equations (84) with boundary conditions (79), (85), and (86). They will be represented as sums of three functions. Let G be as in (70) and define

$$A_i(\alpha,\beta,\gamma) = \frac{1}{4\pi}\int_{-\infty}^{\infty}d\alpha'\int_{-\infty}^{0}d\beta'\int_{-\infty}^{\infty}d\gamma'\ H_i(\alpha',\beta',\gamma')\,G(\alpha,\beta,\gamma;\alpha',\beta',\gamma'). \tag{88}$$

Then [Kellogg (1929), p. 156]

$$\begin{aligned} &\Delta A_i = H_i\,, \\ &k_0 A_{i\beta}(\alpha,0,\gamma) + A_{i\alpha\alpha\alpha}\,, \\ &A_{1\gamma}(\alpha,\beta,\pm 0) = A_{2\gamma} = A_3 = 0. \end{aligned} \tag{89}$$

Furthermore, (79) is satisfied. Next let

$$\begin{aligned} &B_i(\alpha,\beta,\gamma) = \frac{1}{2\pi}\iint_{S_1} d\alpha'd\beta'\,G(\alpha,\beta,\gamma;\alpha',\beta',0)\,F_i(\alpha',\beta')\,,\quad i=1,2, \\ &B_3(\alpha,\beta,\gamma) = \frac{1}{2\pi}\iint_{S_1} d\alpha'd\beta'\,G_\gamma(\alpha,\beta,\gamma;\alpha',\beta',0)\,f_\alpha'''(\alpha',\beta'). \end{aligned} \tag{90}$$

These functions satisfy (79) and [see Kellogg (1929), pp. 164, 167]

$$\begin{aligned} &\Delta B_i = 0 \\ &k_0 B_{i\beta}(\alpha,0,\gamma) + B_{i\alpha\alpha} = 0 \\ &B_{i\gamma}(\alpha,\beta,\pm 0) = \pm F_i\,,\ i=1,2,\quad B_3(\alpha,\beta,\pm 0) = \pm f_\alpha'''. \end{aligned} \tag{91}$$

Finally, let

$$C_i(\alpha,\beta,\gamma) = -\frac{1}{4\pi}\iint_{-\infty}^{\infty} d\alpha'd\gamma'\,G_\alpha(\alpha,\beta,\gamma;\alpha',0,\gamma')\,\Pi_i(\alpha',\gamma'). \tag{92}$$

Then [see Wehausen and Laitone (1960), sect. 21] the C_i satisfy (79) and

$$\Delta C_i = 0,$$

$$k_0 C_{i\beta}(\alpha,0,\gamma) + C_{i\alpha\alpha} = \frac{1}{\rho U}\Pi_{i\alpha}(\alpha,\gamma), \tag{93}$$

$$C_{i\gamma}(\alpha,\beta,\pm 0) = 0,\ i=1,2,\quad C_3(\alpha,\beta,\pm 0) = 0.$$

It is now evident that

$$X_{i\alpha}^{(2)} = A_i + B_i + C_i, \tag{94}$$

is a solution to the problem.

Effect of small draft/length ratio

Since the iteration scheme used above started with the functions (36), stopping after one or two steps is tantamount to assuming that the beam/length ratio of the ship is small. For most practical ships this ratio is small compared with 1. However, for most ships the draft/length ratio is also small, in fact, smaller than the beam/length ratio. It should be possible to exploit this fact to simplify some of the formulas which have been deduced above.

In order to estimate the effect of this ratio,

$$\delta = H/L, \tag{95}$$

it will be convenient to introduce dimensionless coordinates. This will be done by measuring lengths in terms of L, velocities in terms of $\sqrt{gL}$, and forces in terms of $\rho g L^3$. Thus we may introduce the dimensionless coordinates

$$\hat{\alpha}_i = \alpha_i/L,\ \hat{X}_i = X_i/L,\ \hat{h} = h/L,\ \hat{\varphi} = \varphi,$$

$$\hat{f}(\hat{\alpha},\hat{\beta}) = f(L\hat{\alpha},L\hat{\beta})/L,\ \hat{G} = LG,\ \hat{T} = T/\rho g L^3. \tag{96}$$

Only minor modifications are necessary to render dimensionless the various formulas which have been derived. We shall not rewrite them, but only describe the modifications. The combination $k_0 = g/U^2$ becomes $k_0 L = gL/U^2$; in force equations, factors ρg disappear and factors ρU^2 become U^2/gL. Henceforth, keeping these changes in mind, we shall write the dimensionless variables without the circumflex.

Consider first the equations (71) and (72). The integrals extend over a length 1 in α' and a length δ in β'. Both G and f are independent of δ. Hence we may conclude that

$$X_i^{(1)} = O(\delta). \tag{97}$$

Consider next (74). Again δ enters only though the region of integration, and here evidently

$$T^{(1)} = O(\delta^2). \tag{98}$$

Let us now turn to (66). On the right-hand sides the single integrals will be $O(\delta)$ because of the $Y^{(1)}$, but the double integrals will be $O(\delta^2)$ because of the domain of integration. Hence, if we retain only the terms of order δ, we may replace (66) by the simpler equations

$$\begin{aligned} x_A'\varphi^{(1)} + h^{(1)} &= \frac{2}{A}\int dx'\, f(x',0)\, Y^{(1)}(x',0,0), \\ \frac{V}{A} H_p \varphi^{(1)} + x_A' h^{(1)} &= \frac{2}{A}\int dx'\, f(x',0)\, Y^{(1)}(x',0,0)\, x'. \end{aligned} \tag{99}$$

Both $\varphi^{(1)}$ and $h^{(1)}$ are $O(\delta)$. Note that in this approximation both $\varphi^{(1)}$ and $h^{(1)}$ are determined hydrostatically.

Let us turn now to the second approximation. First we note that the functions H_i and Π_i are all $O(\delta^2)$ from their defimitions in terms of the $X_j^{(1)}$. It is then evident from (89) and (92) that A_i and C_i are also $O(\delta^2)$. The functions F_i defined in (86) and $f^{(1)}$ are all $O(1)$. The domain of integration in (90) is S_1 whose vertical dimension is still $O(\delta)$. Hence B_i is $O(\delta)$. Furthermore, in the expressions defining F_1 and F_2 the terms involving the $X_i^{(1)}$ are all $O(\delta)$. Hence

$$B_1 = \frac{1}{2\pi} \iint_{S_1} d\alpha' d\beta' \, G \, f_{\alpha\alpha}^{(1)} + O(\delta^2),$$
$$B_2 = \frac{1}{2\pi} \iint_{S_1} d\alpha' d\beta' \, G \, f_{\alpha\beta}^{(1)} + O(\delta^2). \tag{100}$$

Putting these various results together, we see that

$$X_\alpha^{(2)} = \frac{1}{2\pi} \iint_{S_1} d\alpha' d\beta' \, G(\alpha,\beta,\gamma;\alpha',\beta',0) \, f_{\alpha\alpha}^{(1)}(\alpha',\beta') + O(\delta^2),$$

$$Y_\alpha^{(2)} = \frac{1}{2\pi} \iint_{S_1} d\alpha' d\beta' \, G(\alpha,\beta,\gamma;\alpha',\beta',0) \, f_{\alpha\beta}^{(1)}(\alpha',\beta') + O(\delta^2), \tag{101}$$

$$Z_\alpha^{(2)} = \frac{1}{2\pi} \iint_{S_1} d\alpha' d\beta' \, G_\gamma(\alpha,\beta,\gamma;\alpha',\beta',0) \, f_\alpha^{(1)}(\alpha',\beta') + O(\delta^2).$$

Consider now the thrust $T^{(2)}$. From (80) it is clear that the first term in the curly brackets is $O(\delta^2)$ and the remaining ones $O(\delta^2)$. Hence, it follows that $T^{(2)} = O(\delta^2)$, as one expects since $T^{(1)} = O(\delta^2)$, and that

$$T^{(2)} = 2\rho U^2 \iint_{S_1} d\alpha d\beta \, f^{(1)}(\alpha,\beta) X_{\alpha\alpha}^{(2)} + O(\delta^3). \tag{102}$$

(In (102)-(104) the variables are again dimensional.) If we now substitute from (101), we find

$$T^{(2)} = \frac{\rho U^2}{\pi} \iint_{S_1} d\alpha d\beta \iint_{S_1} d\alpha' d\beta' \, f^{(1)}(\alpha,\beta) \, f_{\alpha\alpha}^{(1)}(\alpha',\beta') \, G_\alpha(\alpha,\beta,0;\alpha',\beta',0) + O(\delta^3). \tag{103}$$

After integrating twice by parts and using the property $G_{\alpha'} = -G_{\alpha}$, we may also write this as follows:

$$T^{(2)} = \frac{\rho U^2}{\pi} \iint_{S_1} d\alpha\, d\beta\, f^{(1)}(\alpha,\beta) \int d\beta'\, f^{(1)}_{\alpha}(\alpha',\beta')\, G_{\alpha}(\alpha,\beta,0;\alpha',\beta',0)\Big|_{\text{stem}}^{\text{stern}} \tag{104}$$

$$+ \frac{\rho U^2}{\pi} \iint_{S_1} d\alpha\, d\beta \iint_{S_1} d\alpha' d\beta'\, f^{(1)}(\alpha,\beta)\, f^{(1)}(\alpha',\beta')\, G_{\alpha\alpha\alpha}(\alpha,\beta,0;\alpha',\beta',0) + O(\delta^3).$$

The second integral is of exactly the same form as Michell's integral, but with S_0 and $f(\alpha,\beta)$ replaced by S_1 and $f^{(1)}(\alpha,\beta)$. The first integral results from a dipole distribution along the profile of S_1 with strength proportional to the waterline slope of $f^{(1)}$.

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The wave motion generated by the steady motion of a ship is formulated 'exactly' in Lagrangian coordinates. An approximation scheme is then developed based upon a method of iteration. The first iteration can be chosen so as to yield the Mitchell thin-ship theory. The second iteration produces more complicated formulas. However, if the smallness of draft/length is taken into account, the formula for the resistance simplifies to one similar to Michell's, but with the integral taken over a modified region and with a modified hull function, plus a line integral over the profile which represents a distribution of dipoles with strength proportional to the waterline slopes.

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