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# RESEARCH INVESTIGATION OF LASER LINE PROFILES

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United Aircraft Research Laboratories

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Reported By: D. A. Stetser  
D. A. Stetser  
Research Scientist

G. L. Lamb, Jr.  
G. L. Lamb, Jr.  
Senior Theoretical Physicist

C. M. Ferrar  
C. M. Ferrar  
Senior Research Scientist

A. J. DeMaria  
A. J. DeMaria  
Principal Scientist, Quantum Physics

Approved By: H. D. Taylor  
H. D. Taylor  
Manager, Physics Department

Date: August 28, 1967

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Research Investigation of Laser Line Profiles

ARPA Order No. 306, Project Code No. 6E3CK21

SUMMARY

The objective of this program was to conduct experimental and theoretical investigations of laser line profiles. The broadened homogeneous line width of the argon ion laser was studied as a function of pressure and excitation by means of the "Lamb Dip" technique. Broadened homogeneous line widths of 200 to 400 MHz were recorded. These line widths are much larger than the 100 MHz radiative line width. The most plausible source for the broadened line is nonradiative phase interruptions occurring from small-angle Coulomb scattering in ion-ion collisions. It is shown that the line width decreases with increasing pressure for a certain range of laser oscillation. A simple collision broadening approach is not applicable to the ion system because of the behavior of the ion density as a function of pressure and excitation. Preliminary experiments were also performed with a CO<sub>2</sub>-N<sub>2</sub>-He laser and no experimental indication of a Lamb Dip was found for this laser medium.

Some aspects of the theory of a gas laser recently developed by W. E. Lamb, Jr. have been recast in a form which more fully displays the role played by the particle dynamics. The Wigner distribution function has been used to derive kinetic equations which govern the external center of mass motion of the two-level systems as well as their internal dynamics. The effect of long range forces is discussed by treating the collision integral in a manner similar to that employed in plasma kinetic theory. A modification in the criterion for the existence of a dip in the output is shown. It is also shown that effects due to long range forces are most noticeable at long optical wavelengths and when there is a large difference between the lifetimes of the two laser levels.

## MEASUREMENTS OF AN ARGON ION LASER LINE PROFILE

## Introduction

In this section of the report are reported the experimental results of measurements of the broadened homogeneous line width of the  $4880\text{\AA}$  transition of an argon ion laser obtained by means of the "Lamb Dip" technique. Some of the more pertinent reasons why the argon ion laser was chosen for this investigation are: (1) ion lasers are the most intense sources of continuous coherent radiation in the visible region of the spectrum and, therefore, worthy of considerable study; (2) relatively little has been published on ion transitions when compared with the published literature on neutral transitions; (3) the ratio of the broadened homogeneous line width to the population inversion line width determines the percentage of the population inversion utilized in the laser action process; and (4) the broadened homogeneous line width determines whether the "Lamb Dip" can be utilized in laser frequency stabilization.

In this study, the broadened homogeneous line width of the argon ion laser was investigated as a function of pressure and excitation by means of the Lamb Dip technique. In addition, this investigation has provided basic information concerning the saturation behavior of a Doppler broadened transition as it interacts with an optical frequency field in the form of a standing wave. Specifically, the  $4880\text{\AA}$  of singly ionized argon was investigated for the range of parameters during which laser action occurred under conditions of a dc excited plasma in a 20 cm by 2-mm-diameter quartz capillary.

## Background Information

The thermal motion of ions in a gas gives rise to a Doppler width which is much wider in frequency than the radiative width of an optical atomic transition. Therefore, information pertaining to the natural line width is difficult to obtain from ordinary high resolution spectroscopy. While the center of the transition line shape is Gaussian as a result of the Doppler broadening, the tails of the line shape are Lorentzian and contain information about the radiative line width. However, in the tails of the line, the dependence of the shape upon frequency is rather slow and the influence of collisions on the line shape could be better determined in the presence of a larger Doppler response. In order to determine the radiative line shape

directly, high pressures are required to increase the natural breadth to a size comparable or larger than the Doppler width. However, at high pressures, other complicated effects such as one atom screening another atom, need to be considered. A method that would examine the radiation line width directly at moderate pressures is therefore desirable. Such a method has become available as a result of the advent of gas lasers.

In considering a derivation by Lamb<sup>(1)</sup> for the gain or attenuation of a monochromatic standing wave at frequencies within the Doppler width of an optical atomic transition, the gain is assumed Gaussian at the limit of low field intensities. At higher fields, the gain saturates and has a Lorentzian frequency dependence indicative of the natural line width. The gain or absorption is proportional to the imaginary part of the complex susceptibility. Lamb has shown the imaginary part of the susceptibility to be

$$\chi'' = \alpha \left\{ 1 - \beta E^2 \left[ 1 + \frac{\gamma_{ab}^2}{\gamma_{ab}^2 + (\omega - \omega_0)^2} \right] \right\} \exp \left[ - \left( \frac{\omega - \omega_0}{\Delta\omega} \right)^2 \right] , \quad (1)$$

where

$$\gamma = 1/2 \left[ 1/T_u + 1/T_l \right] . \quad (2)$$

$T_u$ ,  $T_l$  are the lifetimes of the upper and lower levels, respectfully;  $\omega$  is the frequency of the applied field;  $\omega_0$  is the center frequency of the atomic transition;  $\Delta\omega$  is the Doppler line width;  $\alpha$  is the unsaturated net gain,  $\beta$  is the saturation parameter which depends upon the matrix element connecting the two levels and the radiative lifetime of the levels; and  $\gamma_{ab}$  is the natural line width.

When the Doppler width  $\Delta\omega$  is much larger than  $\gamma_{ab}$ , the saturation term provides the major frequency dependence to the susceptibility. The natural line width could be determined by the measurement of the gain as a function of  $\omega$  about  $\omega_0$  of the Doppler broadened line. The frequency response of the saturation behavior is resonant about line center and indicates the frequency behavior of the individual ions which would ordinarily be clouded by the Doppler broadening of the whole ensemble of ions having random velocities.

Small pressure effects, which are indicative of broadening and distortions that are much smaller than the Doppler broadening, may be studied at low pressures.<sup>(2)</sup>

Close to the oscillation threshold, the output power is determined by the extent of saturation. As oscillation begins, saturation tends to decrease the gain, and the steady-state oscillation occurs at values of  $E^2$  for which the saturated gain equals the losses in the system. An obvious way to study the effect of saturation on the laser medium is to observe the power output of a laser as a function of frequency. If the laser is operating in a single mode within the bandwidth of the transition, direct information on the line shape and saturation behavior may be determined. For steady-state oscillation, the gain equals the losses in the system, or

$$1/Q = 4\pi X'', \quad (3)$$

where  $Q$  is the quality factor of the resonator. Substituting Eq. 1 into Eq. 3 yields

$$(4\pi\alpha\beta)E^2 = \frac{4\pi\alpha - 1/Q e^{-\left(\frac{\omega-\omega_0}{\Delta\omega}\right)^2}}{1 + \gamma_{00}^2 / [\gamma_{00}^2 + (\omega-\omega_0)^2]} \quad (4)$$

Since the power output,  $P_o$ , is proportional to  $E^2$ ,

$$P_o \propto E^2 = A \left\{ \frac{1 - B e^{-\left(\frac{\omega-\omega_0}{\Delta\omega}\right)^2}}{1 + \gamma_{00}^2 / [\gamma_{00}^2 + (\omega-\omega_0)^2]} \right\} \quad (5)$$

As a result of the relatively poor plasma stability of argon ion lasers, the major experimental problem in obtaining the saturation behavior of the Doppler broadened argon ion transition by means of observing the power output of a laser medium is the attainment of a stable, single-mode oscillation that can be controllably swept through line center,  $\omega = \omega_0$ . One solution to this problem is the internal suppression of all but the desired mode of oscillation. Equation 5 is valid at the limit of zero pressure. From this equation, a dip (i.e., the Lamb dip) will occur at  $\omega = \omega_0$ .

A physical interpretation of this decrease in output power about  $\omega_0$ , as described by Bennett,<sup>(3)</sup> is that the atomic transition is inhomogeneously broadened and a "hole" will be burned in the gain profile with a width that corresponds to the natural line width. In other words, the radiation field will only draw upon atoms that are within a natural line width of the oscillation frequency and that other atoms within the Doppler broadened transition will not contribute to this particular field. If the oscillation frequency is not at  $\omega = \omega_0$ , symmetric holes about  $\omega = \omega_0$  will be buried due to the fact that the gain profile is a distribution in velocities and that the standing wave nature of the cavity mode is really described by two running waves with equal positive and negative velocities. As the oscillation frequency is moved towards  $\omega = \omega_0$ , the holes will coincide at  $\omega = \omega_0$  and the number of atoms that are available to contribute to the field are decreased, thereby decreasing the power output at  $\omega = \omega_0$ .

Experimentally, the output curve has the general appearance of the form of Eq. 5. There is a deviation, however, which is quite pressure dependent. In the case of the He-Ne laser, the dependence on the pressure is such that the data, when extrapolated to the limit of zero pressure, is in agreement with the form given by Lamb's theory.

The effects of pressure may take many forms. Complete interruption due to hard collisions cause a broadening of the Lorentzian response of the natural shape of the line. This broadening may be included in Eq. 5 by assuming an increase in  $\gamma_{ab}$ . Other types of collisions will introduce a smearing of the response of an individual atom which causes an inhomogeneous type of broadening. For example, the interruption of phase of an atom may not be complete when it is caused by soft distant collisions. Also in small-angle scattering, the atom may continue to interact with the radiation field at a slightly displaced velocity, which is due to a Doppler effect. Javan<sup>(5)</sup> has accounted for this type of broadening by modifying the expression for  $\gamma_{ab}$ ,

$$\gamma_{ab} = 1/2 (1/T_0 + 1/T_b) + h \quad (6)$$



and

$$\gamma'_{ab} = \gamma_{ab} + s, \quad (7)$$

where  $h$  and  $s$  are the rates of collision due to hard and soft impacts and both are linearly dependent upon pressure.

#### Experimental Technique

For a cavity of very short length, the spacing of the resonances may exceed the Doppler-broadened line width and the laser will oscillate in a single mode. One expects oscillation at a single frequency to cause gain saturation of the medium only in the vicinity of this frequency. Two holes are "burned" in the gain curve due to the standing wave character of the optical fields in the cavity.<sup>(3)</sup> If the length of the cavity is tuned through the center of the Doppler profile, a "dip" is observed which is due to the fact that fewer atoms are contributing to the power output of the laser. For the specific case of the ionized argon gas laser, the gain profile is so wide that a short, single mode cavity could not be conveniently constructed and, therefore, the Lamb Dip could not be displayed by tuning the  $c/2L$  frequency through line center. If the output intensity of the argon laser was extremely stable (which it is not), it could be operated very close to threshold in one mode and the Lamb Dip could possibly be exhibited; however, no range of the excitation parameter would exist.

An alternate solution which has been successfully applied during the course of this investigation is to use a three-mirror, variable reflectivity system at one end of the resonator.<sup>(4)</sup> A  $\lambda/2$  motion of this system will allow the two-cavity system to pass through alignment and will provide a resonance for one of the modes of the original cavity. By linearly driving one of the mirrors of the variable reflectivity cavity, the resonances may be scanned across the whole Doppler-broadened line and will exhibit the Lamb Dip as the particular oscillating modes are scanned in time. The Lamb Dip may now be plotted as a function of excitation and the area of the "hole" represents the relative power to be achieved by a single mode since the "hole" area represents the homogeneously broadened line width.

The experimental arrangement is shown pictorially in Fig. 1 and schematically in Fig. 2. A dc  $\text{Ar}^+$  discharge is contained in a quartz capillary which is 60 cm in length and has a 2-mm ID. The excitation can be varied from

3 to 10 amperes. An intracavity prism is incorporated into the design to enable the various atomic lines to be selected. The three-mirror, variable reflectivity system is located at one end of the cavity and has a multiwafer piezoelectric drive associated with mirror M which provides the scan of the short cavity relative to the long cavity.

Figure 2 illustrates one particular mode selection scheme first proposed by Smith.<sup>(4)</sup> A variable, reflectivity end reflector is formed by the three mirrors:  $M_2$ ,  $M_3$  and  $M_4$ . This end reflector has the property that incident radiation having a frequency equal to a resonance of the three-mirror cavity is reflected back into the laser tube while radiation having a frequency off resonance is reflected out of the cavity. Therefore, the feedback to the active medium can be discretely selected as a function of frequency. The separation between the resonances of the three-mirror, variable reflectivity end reflector is determined by the spacing,  $l_2 + l_3$ , and the width of the resonance is determined by the transmissivity of the mirrors:  $M_2$ ,  $M_3$  and  $M_4$ . The utilization of a resonant end reflector allows only one frequency to experience gain in the laser medium. By varying the spacing,  $l_2 + l_3$ , the operating frequency of the laser can be scanned and, therefore, the ion transition of the laser medium may also be scanned. The tabulation of the output power as a function of  $\omega$  will yield the saturation behavior as the resonance is scanned through line center. Figure 3 illustrates the several power output variations of the  $\text{Ar}^+$  laser as the operating frequency passes through line center. The Lamb Dip is clearly visible and demonstrates the suitability of the technique for a detailed investigation of the shape of the hole burned in the Doppler gain curve.

In the design of the mode selector,  $l_2 + l_3$  must be sufficiently short so that the spacing between the resonances is larger than the laser bandwidth (Doppler width of the ion transition) in order that only one mode will experience feedback through the laser medium. The curvatures of the mirrors must be properly chosen so as to match the beam waists of the cavities in order to have properly coupled cavities.

The mathematical analysis of the three-mirror end reflectors is as follows: For a wave,  $e^{i\omega t}$ , incident from the left on mirror  $M_2$ , the return wave traveling in the opposite direction will be

$$T_2^2 e^{i[\omega t + \frac{2\pi}{\lambda}(2l_2)]} \left[ 1 + R_2 T_2 e^{i\frac{2\pi}{\lambda}(2(l_2+l_3))} + R_2 T_2 e^{i\frac{2\pi}{\lambda}(2(l_3+l_2))} R_2 T_2 e^{i\frac{2\pi}{\lambda}(2(l_2+l_3))} + \dots \right]$$

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$$\begin{aligned}
 &= R_2^2 e^{i[\omega t + \frac{2\pi}{\lambda}(2l_2)]} \left[ 1 + R_2^2 e^{\frac{4\pi i}{\lambda}(l_2+l_3)} + R_2^4 e^{\frac{8\pi i}{\lambda}(l_2+l_3)} + \dots \right] \\
 &= R_2^2 e^{i[\omega t + \frac{4\pi}{\lambda}l_2]} \left[ 1 - R_2^2 e^{\frac{4\pi i}{\lambda}(l_2+l_3)} \right]^{-1},
 \end{aligned} \tag{8}$$

if we assume  $T_2 = R_2$  and  $R_3 = R_4 = 1$ .

The wave traveling to the left now passes through the active medium in the path length and experiences a voltage gain  $g$  so that the wave returning to the right can be expressed as

$$g R_2^2 e^{i[\omega t + \frac{4\pi}{\lambda}(l_1+l_2)]} \left[ 1 - R_2^2 e^{\frac{4\pi i}{\lambda}(l_2+l_3)} \right]^{-1} \tag{9}$$

and must be equal to  $e^{i\omega t}$  for self-consistency. Therefore,

$$g = \frac{1 - R_2^2 e^{\frac{4\pi i}{\lambda}(l_2+l_3)}}{R_2^2 e^{\frac{4\pi i}{\lambda}(l_1+l_2)}} \tag{10}$$

and since the gain,  $g$ , must be real, the resonance condition can be found.

$$g = \frac{1 - R_2^2 e^{[4\pi i/\lambda](l_2+l_3)}}{R_2^2 e^{4\pi i/\lambda(l_1+l_2)}} \tag{11}$$

If the imaginary part of the above expression is equated to zero, then we have

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the following expression

$$\begin{aligned} & -\sin(4\pi/\lambda)(\ell_1 + \ell_2) + R_2^2 \left[ \sin(4\pi/\lambda)(\ell_1 + \ell_2) \cos(4\pi/\lambda)(\ell_2 + \ell_3) \right. \\ & \left. - \sin(4\pi/\lambda)(\ell_2 + \ell_3) \cos(4\pi/\lambda)(\ell_1 - \ell_3) \right] = 0, \end{aligned} \quad (12)$$

or as simplified,

$$\sin[4\pi/\lambda(\ell_1 + \ell_2)] = R_2^2 \sin[4\pi/\lambda(\ell_1 - \ell_3)]. \quad (13)$$

The power gain, G, is given by

$$G = g g^* = \frac{[1 - R_2^2 e^{4\pi i/\lambda(\ell_2 + \ell_3)}][1 - R_2^2 e^{-4\pi i/\lambda(\ell_2 + \ell_3)}]}{[R_2^2 e^{4\pi i/\lambda(\ell_1 + \ell_2)}][R_2^2 e^{-4\pi i/\lambda(\ell_1 + \ell_2)}]}, \quad (14)$$

$$= [1 + R_2^2 - R_2^2 e^{4\pi i/\lambda(\ell_2 + \ell_3)} - R_2^2 e^{-4\pi i/\lambda(\ell_2 + \ell_3)}] / R_2^4,$$

or

$$G = \frac{(1 - R_2^2)^2 + 4R_2^2 \sin^2[(2\pi/\lambda)(\ell_2 + \ell_3)]}{R_2^4}. \quad (15)$$

For an oscillator with positive feedback and a loss, L,

$$G/(1 + LG) = 1 \quad (16)$$

and

$$L = 1 - 1/G$$

$$= 1 - \frac{R_2^4}{(1 - R_2^2)^2 + 4R_2^2 \sin^2[2\pi/\lambda(\ell_2 + \ell_3)]} \quad (17)$$

If the power reflectivity,  $R^2 = 0.5$  and  $\ell_2 + \ell_3 = \frac{m\lambda}{2}$ , the loss at a particular wavelength is given by

$$L = 1 - R_2^4 / (1 - R_2^2)^2$$

$$= 1 - 2R_2^2 + R_2^4 - R_2^4 = 0. \quad (18)$$

Therefore, at any one particular wavelength,  $\lambda$ , the losses may be zero and for other wavelengths,  $\lambda'$ , of the cavity formed by  $\ell_2 + \ell_3$ , the losses will be high. Since the length  $\ell_1 + \ell_2 \approx \ell_1 + \ell_3$ , the mode spacings are such that the  $c/2\ell$  mode spacings of the long cavity will coincide with only one of  $c/2\ell$  mode spacings of the short cavity in the frequency spectrum of the ion transition. The ion laser may now be operated in a single longitudinal mode at high output powers because of the high frequency discrimination of the three-mirror end cavity. Transverse mode selection is provided by means of an intracavity aperture. Figure 3 illustrates the frequency discrimination ability of this cavity arrangement.

The experimental apparatus (Fig. 2) consisted of a dc excited, quartz plasma tube, 60 cm in length with a 2-mm ID, water-cooled capillary. A filtered, 1000 volt, 10-ampere power supply was used in conjunction with a resistive ballast and a 30 milli-henry choke to provide discharge stability and to remove a 5-kilohertz spiking which was present in the laser output. A thermocouple vacuum gauge was used to record filling pressures. The relative mode amplitudes were observed with a Fairchild Dumont, model 6911 photomultiplier through mirror  $M_1$ . The sweeping of the 100 MHz modes was accomplished by driving mirror  $M_4$  through approximately 15 wavelengths of the 4880Å laser radiation by means of 10 stacked LTZ-24 Transducer Product annular transducers. The transducers were driven by a 50-watt, Krohr-Hite model DCA-50R amplifier and a Hewlett Packard model DCA-50R test oscillator. The bias supply for the transducer was a 0-1000 V, dc modified Dumont type 5314-A. The output of the photomultiplier and the sinusoidal driving voltage to the transducer were observed on a dual beam, Tektronix type 555 oscilloscope. Typical driving voltages for the transducer were 300 to 400 volts peak to peak with a 200 to 250 dc volts bias. The bias voltage was adjusted to have  $\omega_0$ , the center position

of the dip, coincide with the linear region of the sinusoidal sweep.

The sweep frequency for the transducer was approximately 200 Hz. In terms of the long cavity with a mode separation of 100 MHz, a mode spacing would be swept in approximately  $1/3$  ms. The laser would be oscillating for 20% of the time that it takes to sweep across two frequency modes since the width of the Fabry-Perot modes is about 20 MHz and the spacing is 100 MHz. Therefore, the "on" time for the laser in a single axial mode would be of the order of  $1/15$  ms. The recovery time of the argon ion laser was found to be of the order of  $10^{-5}$  to  $10^{-6}$  seconds (Fig. 4). The sweep frequency is therefore slow enough to allow the individual laser modes to come to full amplitude.

### Experimental Results

The  $4880\text{\AA}$  transition line width has been measured as a function of output power and filling pressure. The line width is a measure of the phase interruption lines of the excited species. The Lamb dips were assumed to be Lorentzian and were statistically constructed from approximately 10 pictures taken with identical pressure and average power output. These curves are shown in Figs. 5 to 13. The line width was determined by measuring the full width at half-maximum amplitude.

The measured broadened homogeneous Lorentzian widths greatly exceed the natural widths. The natural width is approximately 110 MHz.<sup>(6,7)</sup> The large experimental Lorentzian width indicates that the broadening is due to a nonradiative process. As proposed by Bennett,<sup>(8)</sup> the most important source of nonradiative line broadening for the ion transitions at low pressures appears to be the small-angle Coulomb scattering in ion-ion collisions.

In Fig. 14, the line width is plotted as a function of filling pressure. The widths decrease with increasing pressure for a certain range. This effect has been seen by other observers.<sup>(9)</sup> An explanation for this type of behavior is involved in the ion-ion interaction. As the pressure is increased, the number of neutral atoms in the plasma is increased and this causes a decrease in the electron temperature. The decreased electron temperature causes a corresponding decrease in the level of ionization and, consequently, fewer ions. The smaller number of ions involved in the ion-ion interaction will therefore give a smaller  $\gamma_{ab}$ , the line width of the interaction.

Preliminary experiments have been performed to investigate the possibility of a Lamb dip in the output of a  $\text{CO}_2\text{-N}_2\text{-He}$  laser. The laser cavity was 1.5 meters long, giving a longitudinal mode separation of 100 mc.

This separation is substantially greater than the width (approx. 30 mc) of the gain profile of the active medium (a mixture of 1 Torr CO<sub>2</sub>, 3 Torr N<sub>2</sub>, and 5 Torr He) so that the laser can operate in no more than one mode for a given cavity length. Thus, by variation of the cavity length, the laser can be tuned successively through the various longitudinal modes. Cavity length variation was provided by mounting one mirror on a loudspeaker driven by a linear sweep voltage, while the laser output was monitored using a gold doped germanium detector. Under these conditions, if the Lamb dip is sufficiently pronounced, the laser output should exhibit this dip as each mode is tuned through the center of the gain profile.

However, in these experiments no such dip was observed as can be seen from Fig. 15. Here the observed laser power output is shown as a function of mirror position, each output peak corresponding to a mirror position for which an axial mode frequency coincides with the center frequency of the gain profile. The tentative conclusion is that collisional relaxation effects in the laser medium are so large that the entire gain profile is effectively homogeneously broadened. Since the width of the Lamb dip is determined by the width of the homogeneous broadening, it might be expected that if this broadening extends over the entire gain profile, the Lamb dip may be exhibited as a depression of the full profile rather than as a central dip. In this case, the observed absence of dips in the above experiments should be expected.

### Conclusions

The Lamb Dip has been observed and the line width measured in an argon ion laser system where the instability of the plasma and the large Doppler gain profile prevented the use of the standard technique of a short, stable tunable length cavity. The variable reflectivity resonator enables the line width to be determined not only close to threshold but over a range of excitation values.

The line width was found to actually decrease with increasing pressure over a range of pressure due to the indirect interaction of neutrals with the ion population through the electron temperature. As a function of excitation, the line width starts at 400 - 500 MHz and decreases to 200 - 300 MHz at increased power outputs. Even at elevated power levels, the line width is still quite broad compared to the radiative line width. We believe that this data indicates that the broad Lamb Dip in Ar<sup>+</sup> lasers is responsible for the high single mode power obtained with these lasers.

Preliminary experiments performed with CO<sub>2</sub>-N<sub>2</sub>-He lasers indicate no experimental evidence of a Lamb Dip in this laser medium.

## SUMMARY OF KINETIC THEORY FOR AN ION LASER

A theoretical model has been developed for discussing the effects of long range Coulomb collisions on the laser action in a gaseous plasma of two-level ions. It has been shown that the effects of long range collisions on the internal dynamics of the two-level systems can be treated by making only minor extensions in the standard plasma theory based on multiparticle distribution functions. The extension of the standard plasma theory which enables one to incorporate the internal dynamics of the two-level systems was carried out by using the Wigner distribution function. A detailed description of this work was given in United Aircraft Research Laboratories Semi-Annual Report No. F-920479-2 under the subject contract.<sup>(10)</sup> In order to avoid duplication, only an outline of this theoretical effort will be given here.

As in the classical plasma theory, the assumption of complete ionization leads to a tractable theory and the effects of Coulomb collisions may then be determined. Following the collisionless theory for a gas laser,<sup>(1)</sup> the starting point is the determination of a self-consistent polarization density  $\rho(x,t)$  for the laser medium which may act as a source term for the optical field satisfies the wave equation

$$\nabla \times \nabla \times E + \gamma \frac{\partial E}{\partial t} + \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = - \frac{4\pi}{c^2} \frac{\partial^2 \rho}{\partial t^2} . \quad (19)$$

The constant  $\gamma$  is a term proportional to a conductivity for the active medium and represents all loss mechanisms. In the present calculation,  $\rho$  is expressed in terms of an average over the polarization due to individual two-level systems. This average is carried out with the help of a Wigner distribution function.<sup>(11)</sup> This distribution provides a convenient means of evaluating averages for systems which possess both internal and external degrees of freedom. In terms of this distribution function, the polarization density is given by

$$\begin{aligned} \rho(Q_1, t) &= \int d^3P d^3p d^3q (-eq_1) f_N(P, Q, p, q, t) , \\ &= -e/V \int d^3P_1 d^3p_1 d^3q_1 q_1 f_1(P_1, Q_1, p_1, q_1, t) . \end{aligned} \quad (20)$$



where  $P$  refers to the external momentum variables of all particles  $(=P_1, \dots, P_n)$ ,  $Q$  refers to the external position variables, and  $p, q$  refer to the corresponding sets of internal variables.

The  $N$ -particle Wigner distribution function is defined in terms of an  $N$ -particle wave function for the system by the relation

$$f_N(P, Q, p, q, t) = \int \frac{d^{3N} \tau d^{3N} \tau'}{(2\pi)^{6N}} e^{-i(\tau \cdot p + \tau' \cdot P)} \psi_N(Q - \frac{\hbar}{2} \tau, q - \frac{\hbar}{2} \tau') \psi_N(Q + \frac{\hbar}{2} \tau, q + \frac{\hbar}{2} \tau'). \quad (21)$$

The wave function satisfies the  $N$ -particle Schrodinger equation for the system with the Hamiltonian

$$\mathcal{H} = \sum_{i=1}^N \left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial Q_i^2} + H_i + eE(Q_i, t) \cdot q_i \right] + \sum_{i < j} \frac{e^2}{|Q_i - Q_j|}, \quad (22)$$

where  $H_i$  is the Hamiltonian for the internal dynamics of an individual two-level system. The random fluctuations in the energy levels of an individual two-level system take place on a time scale which is much longer ( $10^{-13}$  sec) than that of the optical fields ( $10^{-14}$  sec) so that such variations in energy levels could be treated as an adiabatic variation. This aspect of the problem has not been treated here however, since it has been discussed elsewhere.<sup>(12)</sup>

With the above Hamiltonian, it is possible to develop a quantum Liouville equation for the function  $f_n$ . By integrating this equation over the phase space coordinates of all particle except one, an equation for the one-particle Wigner distribution function is developed which contains an integral over a two-particle function. This integral contains all effects associated with particle interactions and is exact. To treat the long range Coulomb forces that are present in the ionized gas laser, the collision integral is treated by methods employed in classical plasma physics.

As shown in detail in the semi-annual report, this kinetic equation may be further reduced by integrating over the internal coordinates. The results, then, the following set of equations for the functions which describe the translational motion:

$$\frac{\partial F_{00}}{\partial t} + \frac{P}{m} \cdot \frac{\partial F_{00}}{\partial Q} + \gamma_0 F_{00} - \frac{iE \cdot Q_{ab}}{\hbar} (F_{0b} - F_{0a}) - e \xi \cdot \frac{\partial F_{00}}{\partial p} = \lambda_0 \quad (23)$$

$$\frac{\partial F_{ab}}{\partial t} + \frac{P}{m} \cdot \frac{\partial F_{ab}}{\partial Q} + i \Omega_{ab} F_{ab} - \frac{i E \cdot P_{ab}}{\hbar} (F_{aa} - F_{bb}) - e \mathcal{E} \cdot \frac{\partial F_{aa}}{\partial P} = 0 \quad (24)$$

along with two similar equations that are obtained by interchanging a and b. In the above equation

$$\Omega_{ab} = \omega_{ab} - i \gamma_{ab} \quad , \quad (25)$$

where  $\omega_{ab}$  is the transition frequency of the two-level system and  $\gamma_{ab}$  is the arithmetic average of the spontaneous decay rates of the two levels. The constant  $P_{ab}$  is the dipole matrix element of levels a and b. The source term  $\lambda_a$  is related to the pumping rate and  $\mathcal{E}$  is given by

$$e \mathcal{E}(Q, t) = -n \int d^3 P' d^3 Q' \frac{\partial}{\partial Q} \left( \frac{e}{|Q - Q'|} \right) (F_{aa} + F_{bb} + F_{cc}) \quad , \quad (26)$$

where n is the total number density of charged particles and the distribution function  $F_{cc}$  refers to those ionized particles which are in the ground state. Equations 23 and 24 have been solved to third order in the electric field and a steady-state optical field determined. The result is an expression which shows the effect of long range collisions on the steady-state operating conditions of a laser. The result is

$$E^2 \sim \frac{1 - \frac{N_T}{N} e^{(\omega - \omega_{ab})^2 / (k v_0)^2}}{1 + \gamma_{ab}^2 \mathcal{L}(\omega - \omega_{ab}) + \Gamma} \quad , \quad (27)$$

where

$$\begin{aligned} N &= \text{excitation density} \\ N_T &= \text{threshold density at resonance} \\ \omega &= \text{frequency of optical field} \\ k &= \omega/c \\ v_0 &= \text{ion thermal velocity} \\ \mathcal{L}(X) &= 1/(X^2 + \gamma_{ab}^2) \\ \Gamma &= 2(\gamma_a - \gamma_b)^2 [(\gamma_a k L)^{-2} + (\gamma_b k L)^{-2}] \quad , \quad L = \text{Debye length.} \end{aligned}$$

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The constant  $\Gamma$  represents the correction term due to long range collisions. This equation contains a number of interesting results. It is seen that the effects of long range forces vanishes if the decay times of the upper and lower levels are the same. It is also seen that the effect of long range forces is more pronounced at the longer optical wavelengths.

The well known dip in the output of the laser as a function of detuning is merely displaced vertically when long range forces are included. This result, coupled with the fact that the experimental data naturally includes effects due to short range collisions as well, has made it impossible to gain experimental confirmation of the purely long range effects associated with the theory.

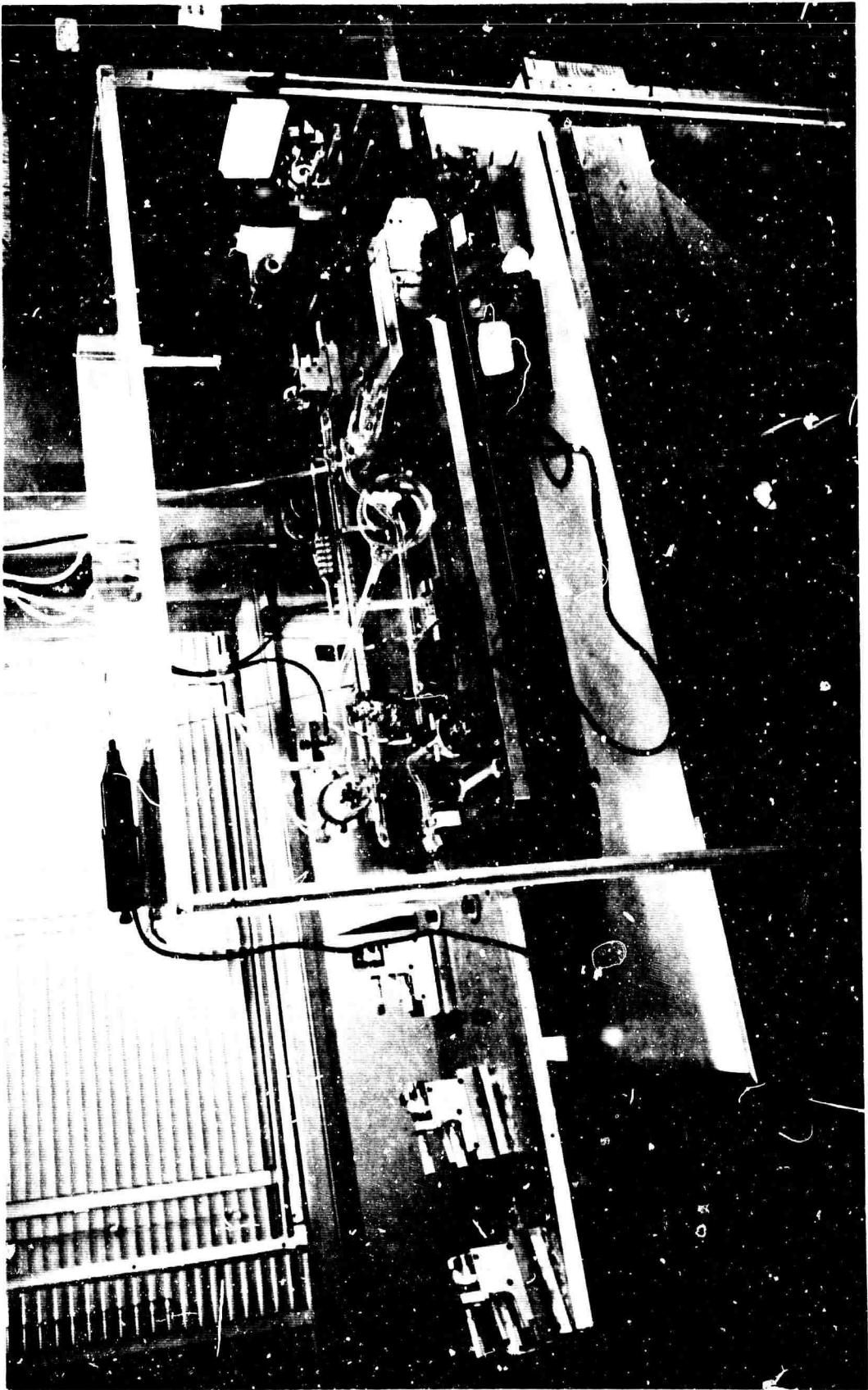
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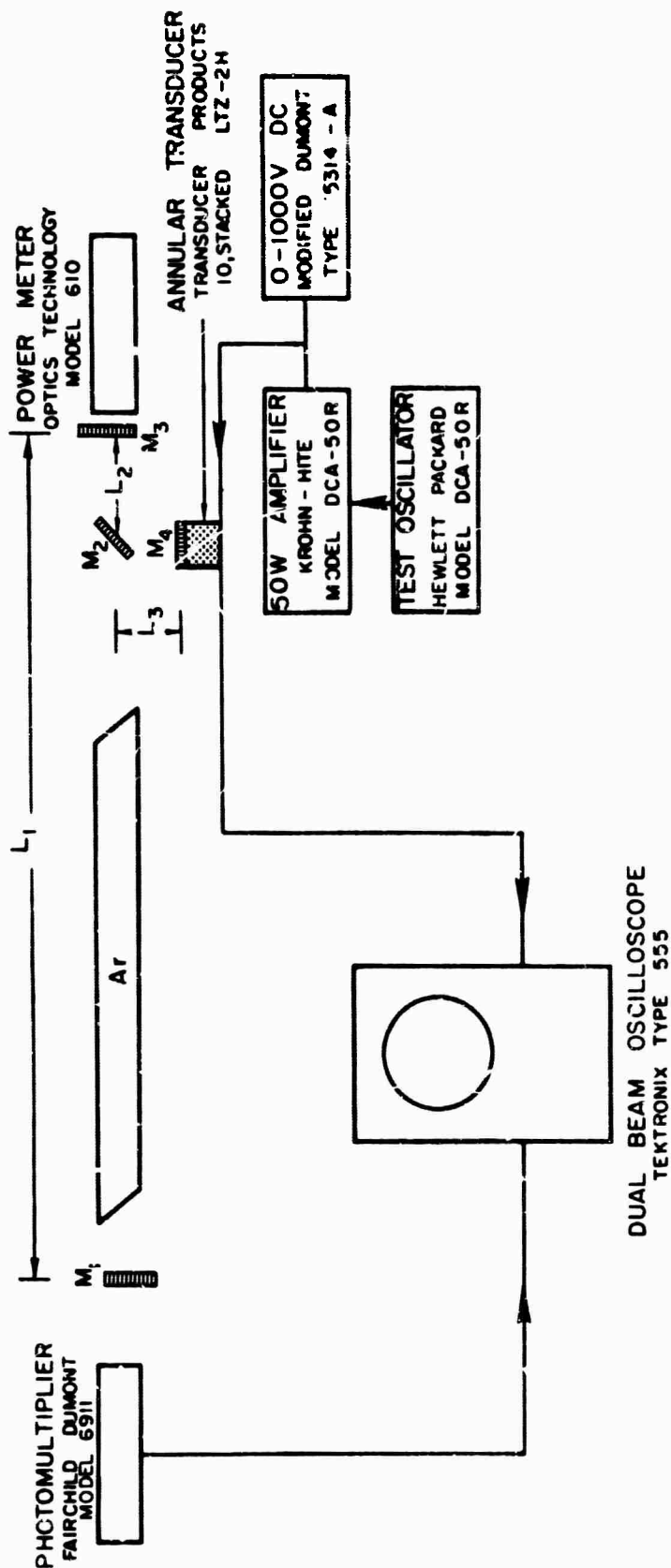
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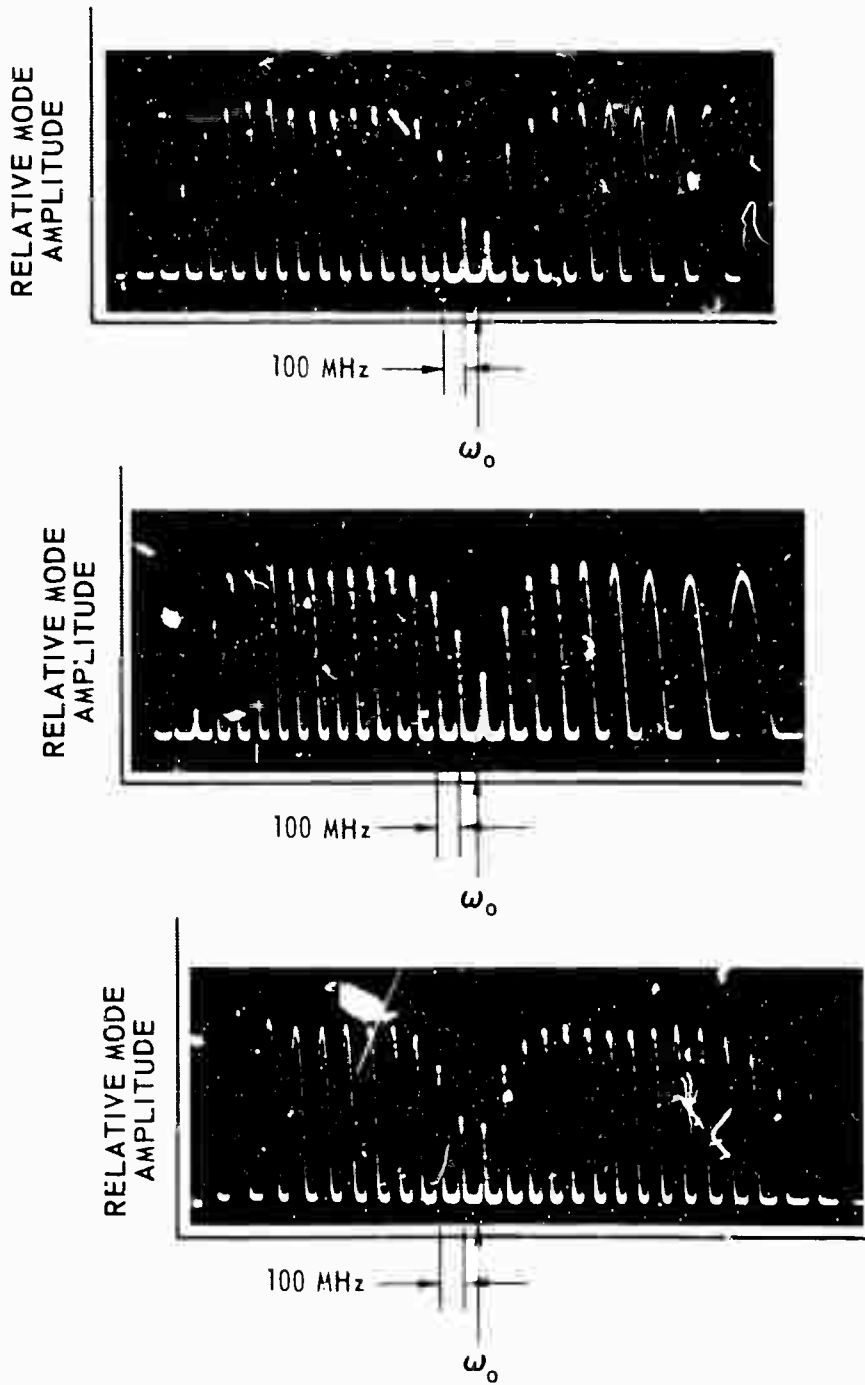
PHOTOGRAPH OF THE Ar<sup>+</sup> LASER "LAMB DIP" EXPERIMENT ARRANGEMENT



# BLOCK DIAGRAM OF EXPERIMENTAL APPARATUS



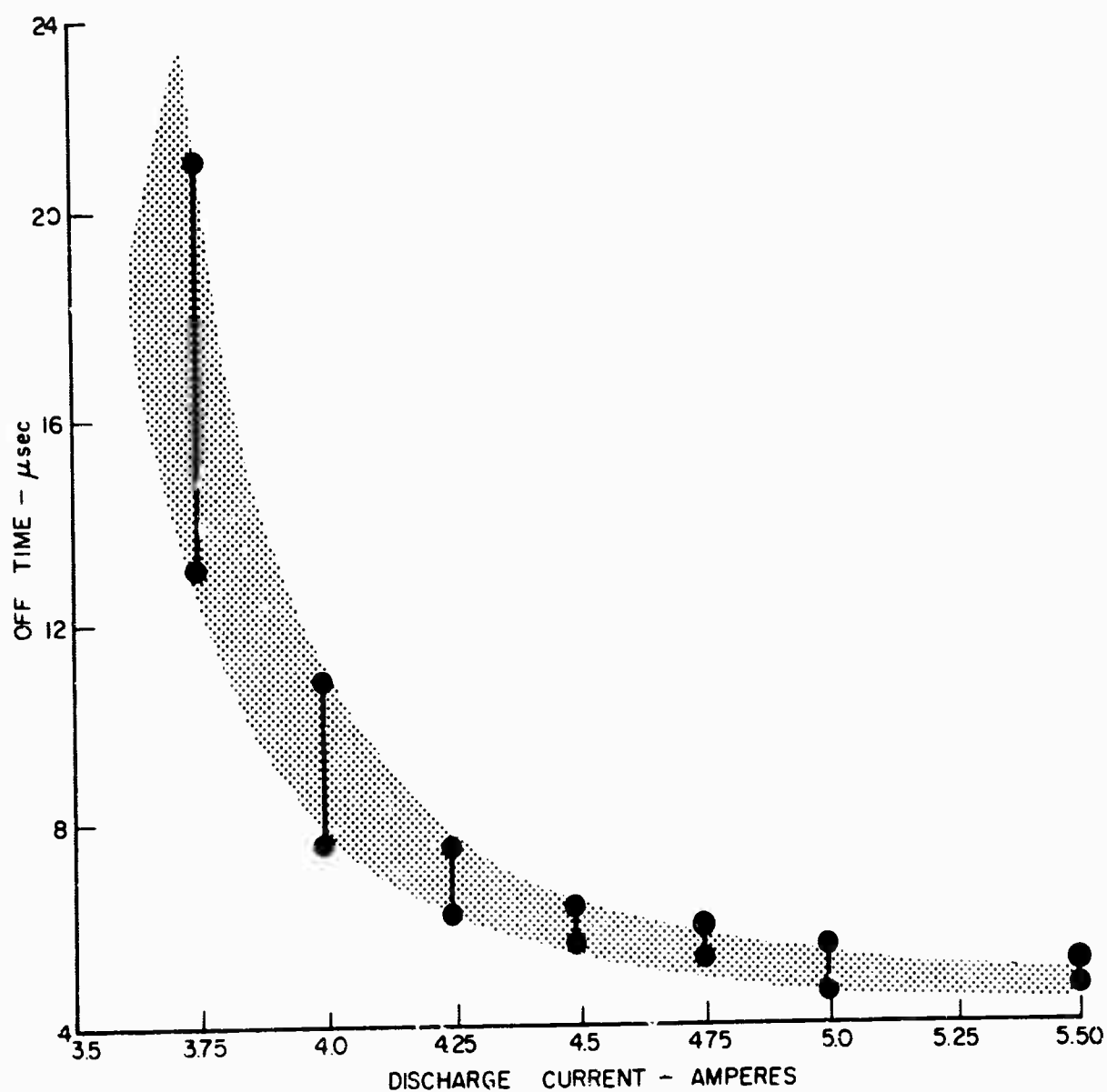
REPRESENTATIVE LAMB DIPS



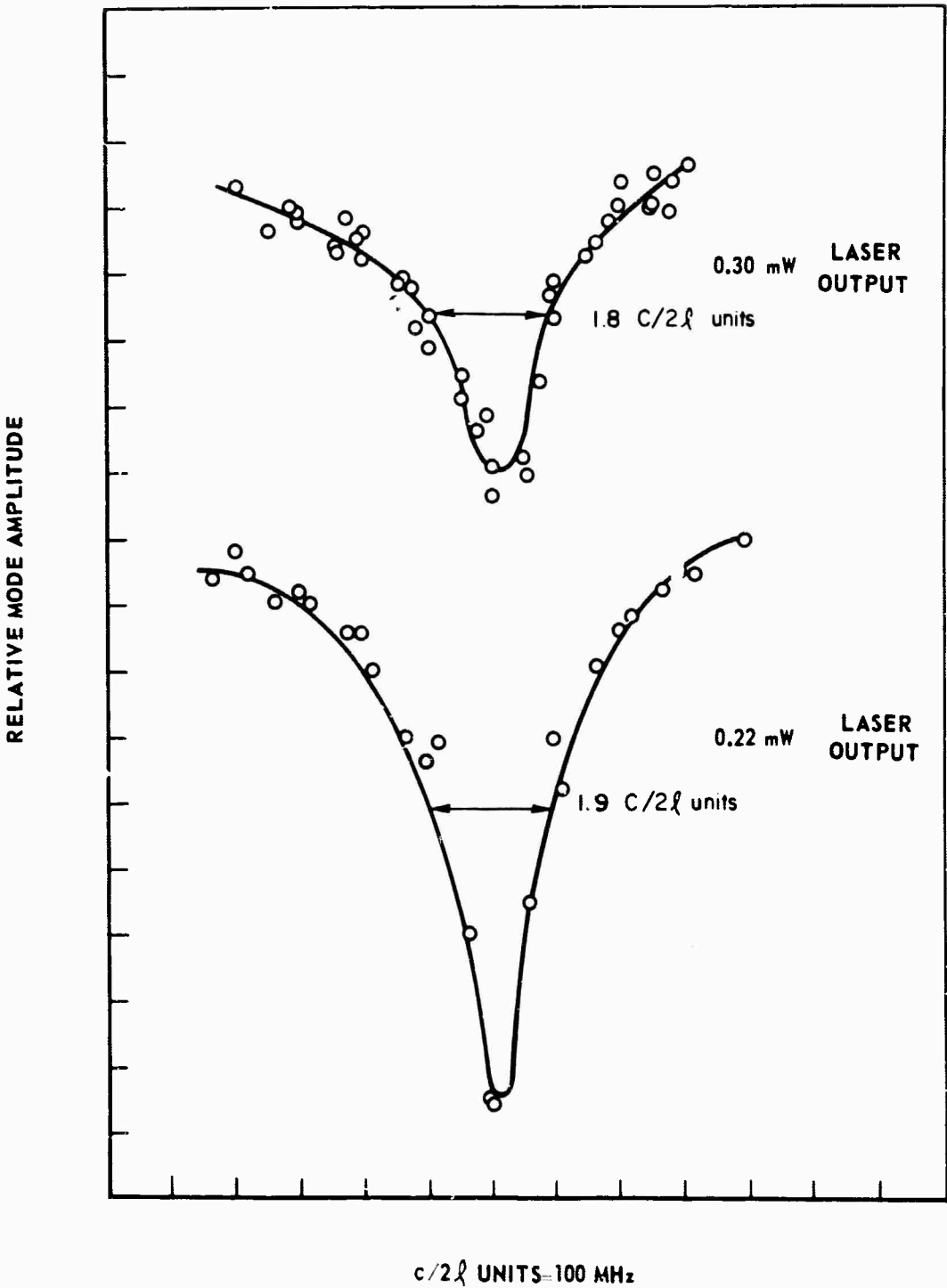


# TRANSIENT RECOVERY TIME OF ARGON LASER TO 4.8 $\mu$ SEC TURN OFF PULSE

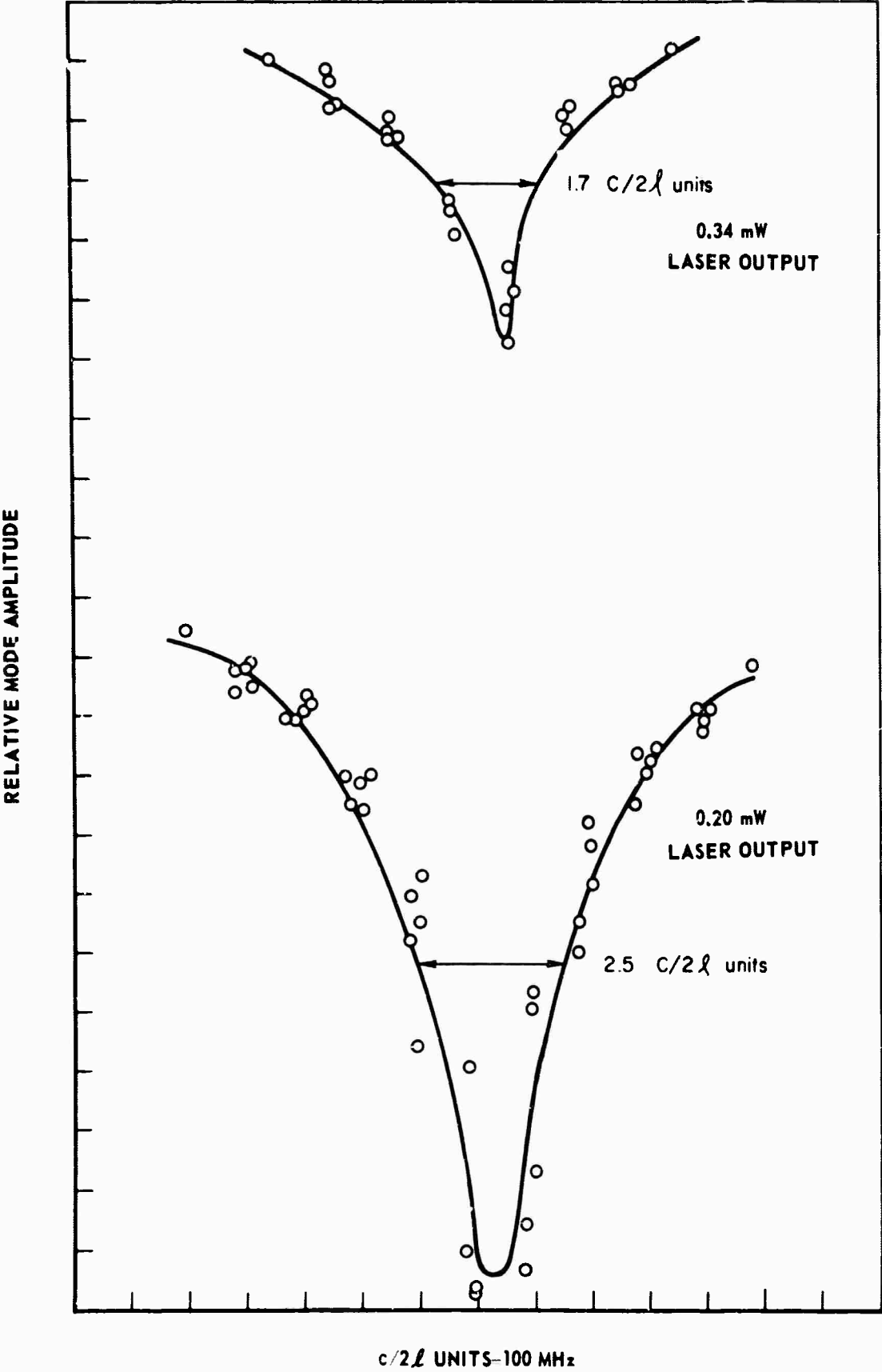
TUBE DIMENSION 2mm I.D. AND 61cm LENGTH

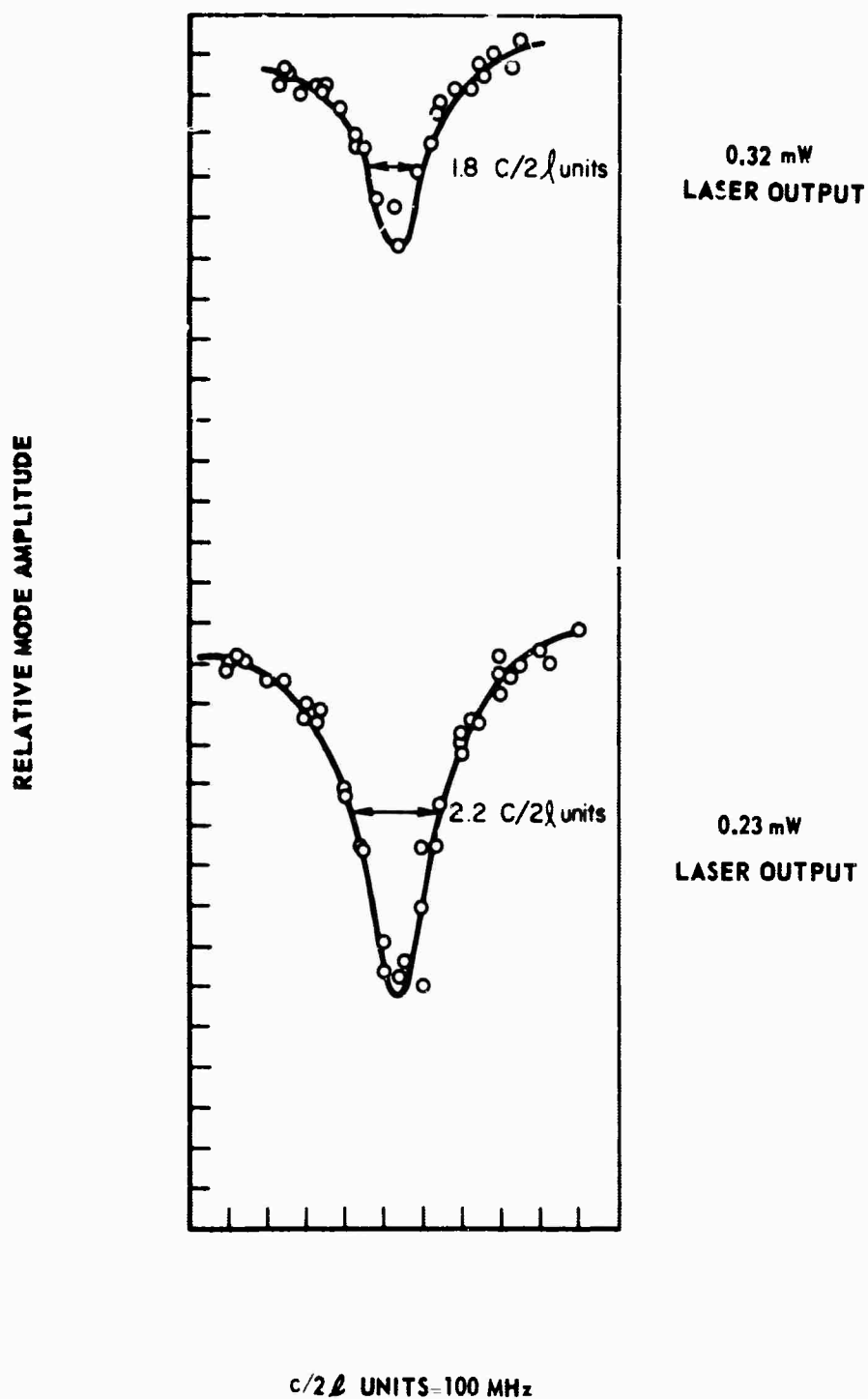


Ar<sup>+</sup> LASER SATURATION BEHAVIOR AT 0.225 mm Hg

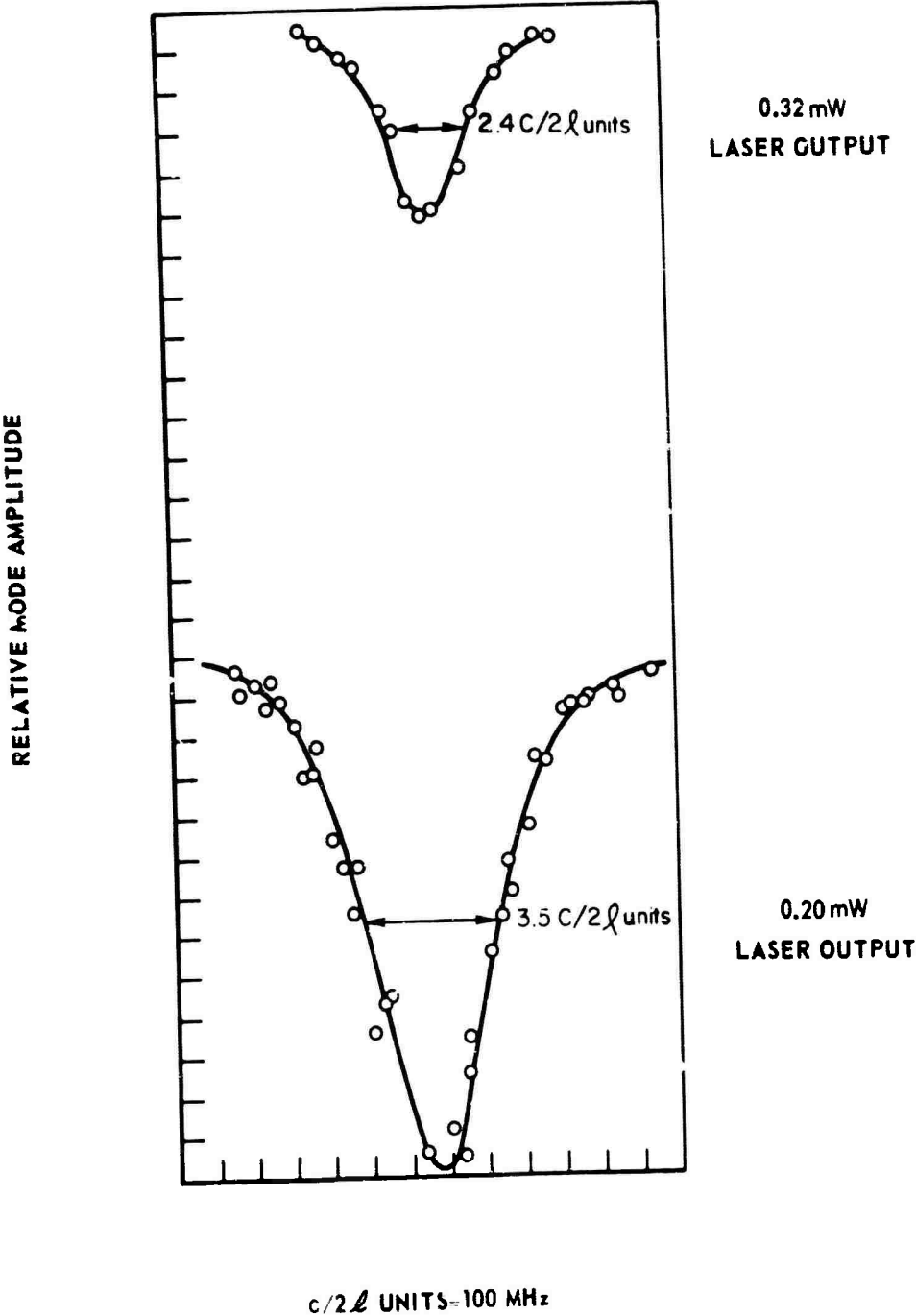


Ar<sup>+</sup> LASER SATURATION BEHAVIOR AT 0.30u mm Hg

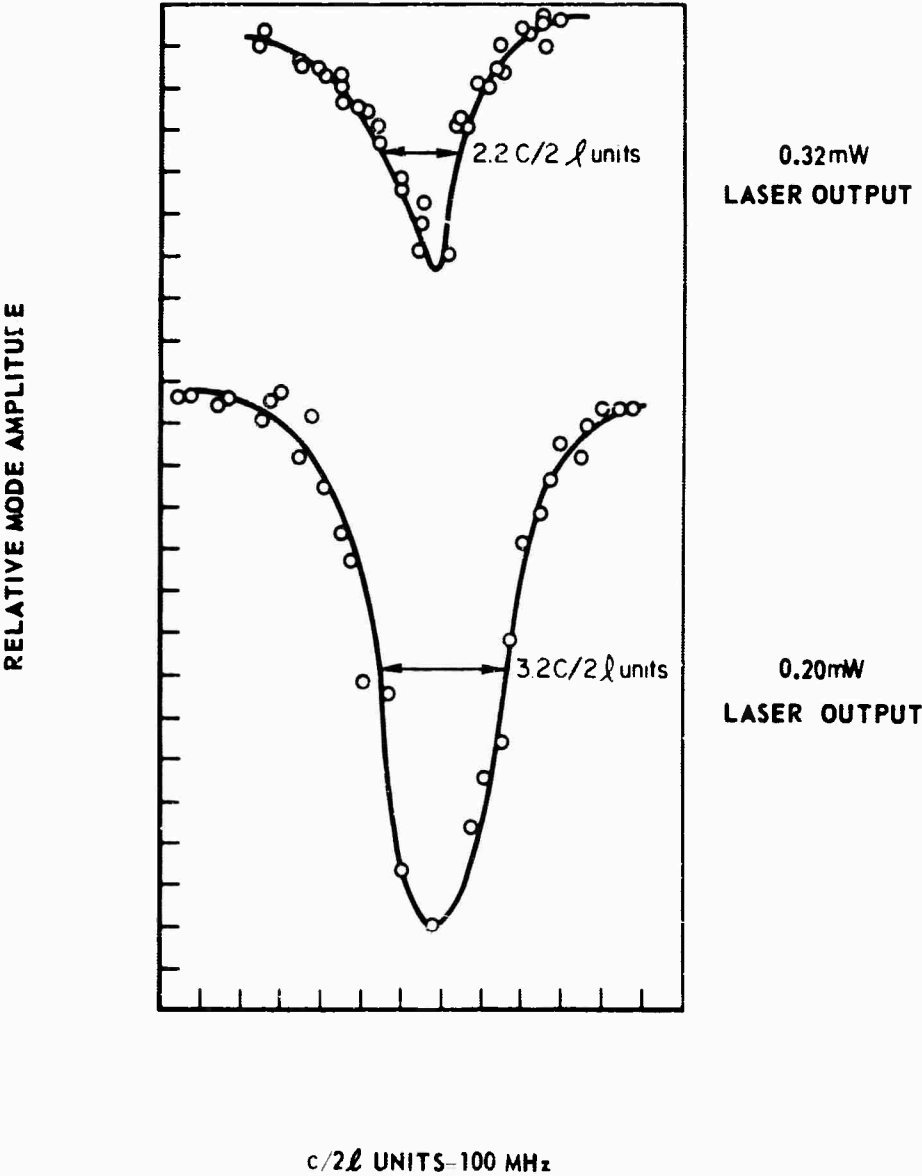


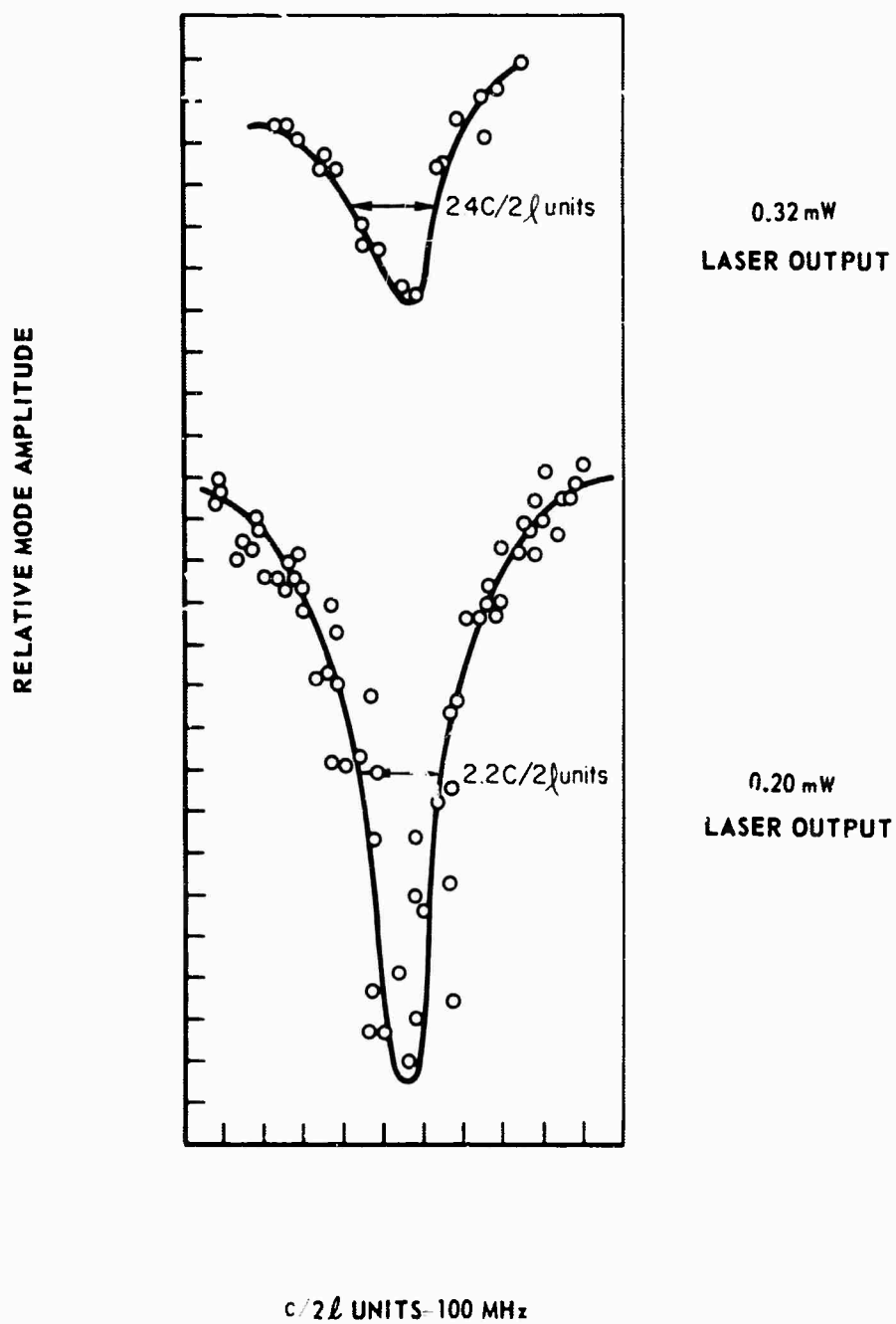
**Ar<sup>+</sup> LASER SATURATION BEHAVIOR AT 0.375 mm Hg**

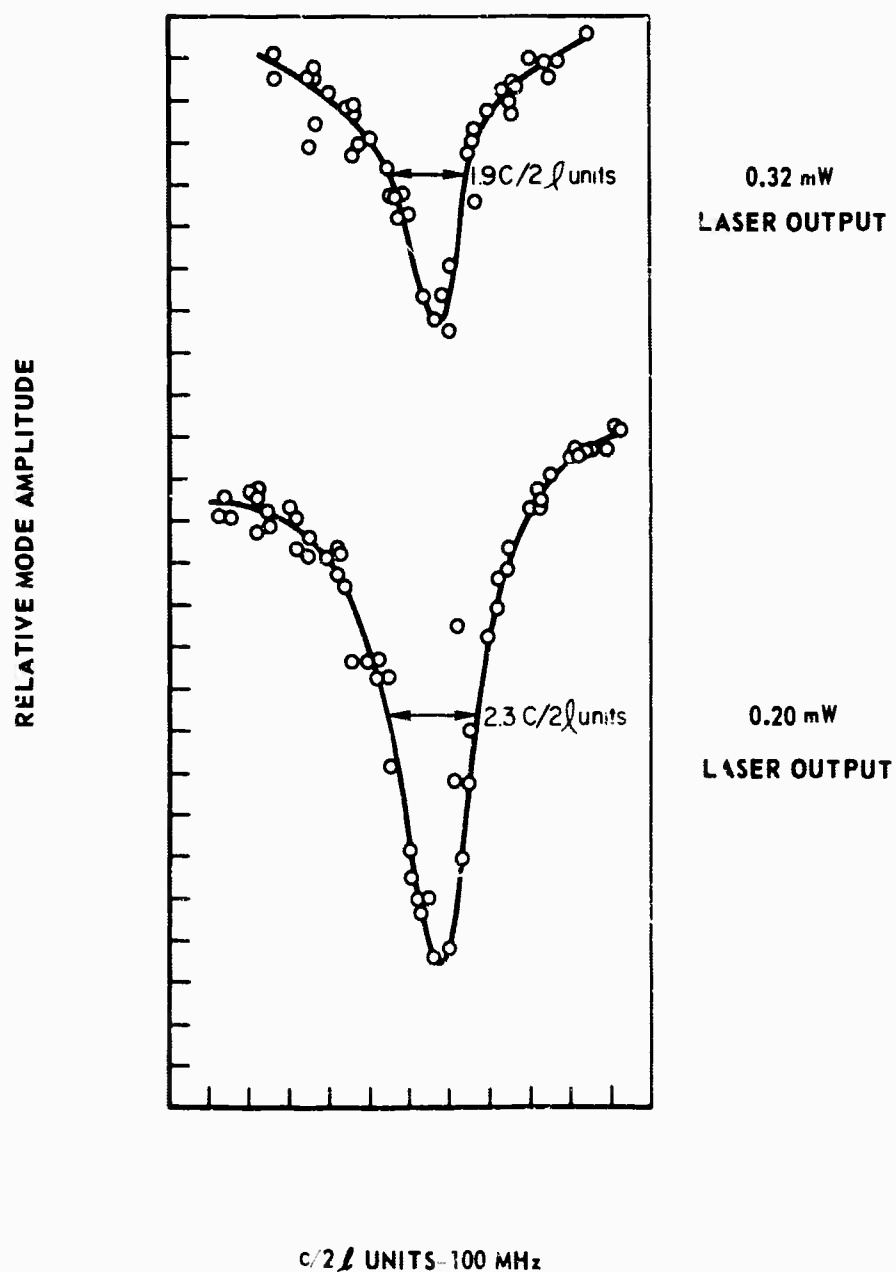
Ar<sup>+</sup> LASER SATURATION BEHAVIOR AT 0.450 mm Hg



Ar<sup>+</sup> LASER SATURATION BEHAVIOR AT 0.525 mm Hg

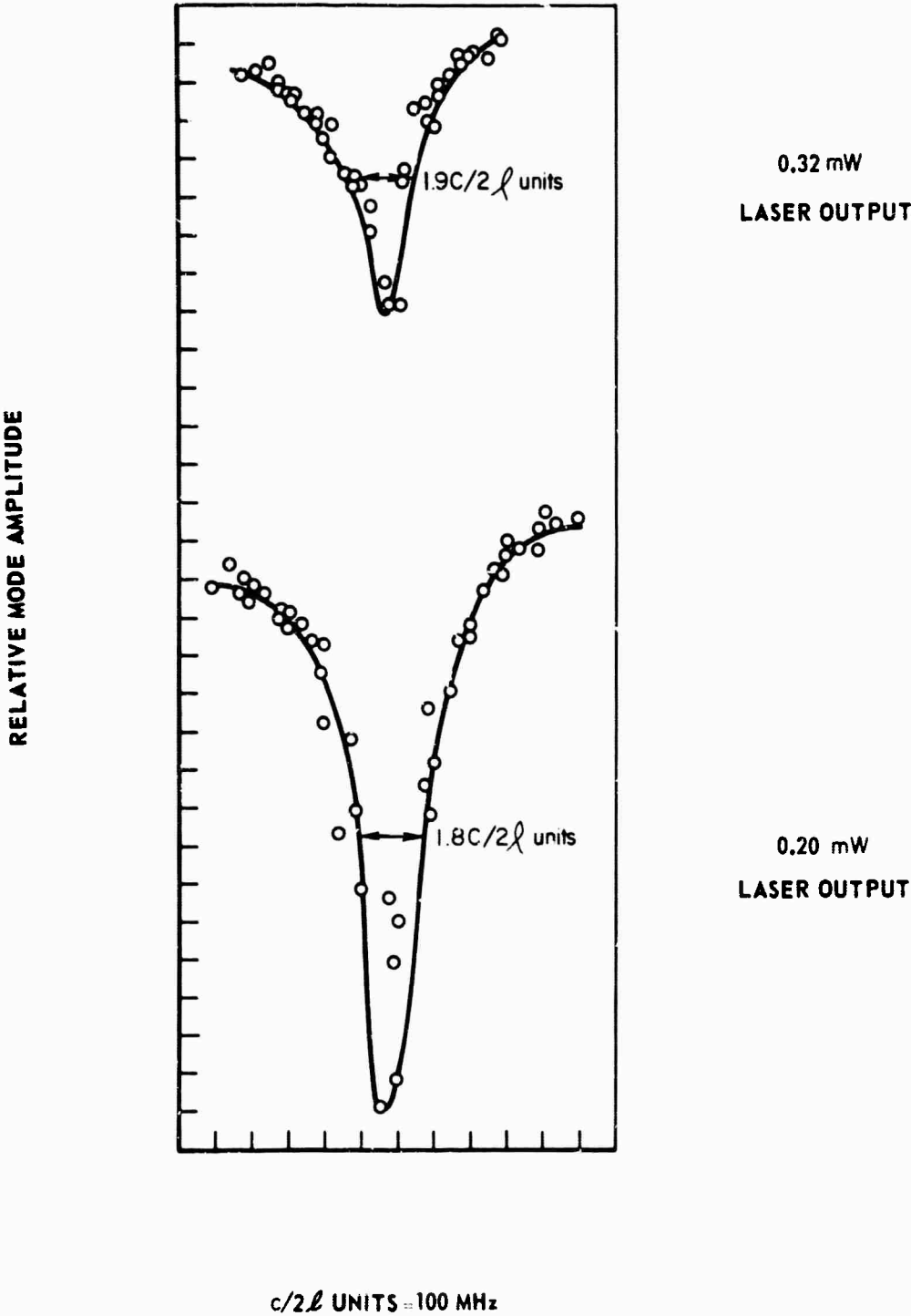


Ar<sup>+</sup> LASER SATURATION BEHAVIOR AT 0.600 min Hg

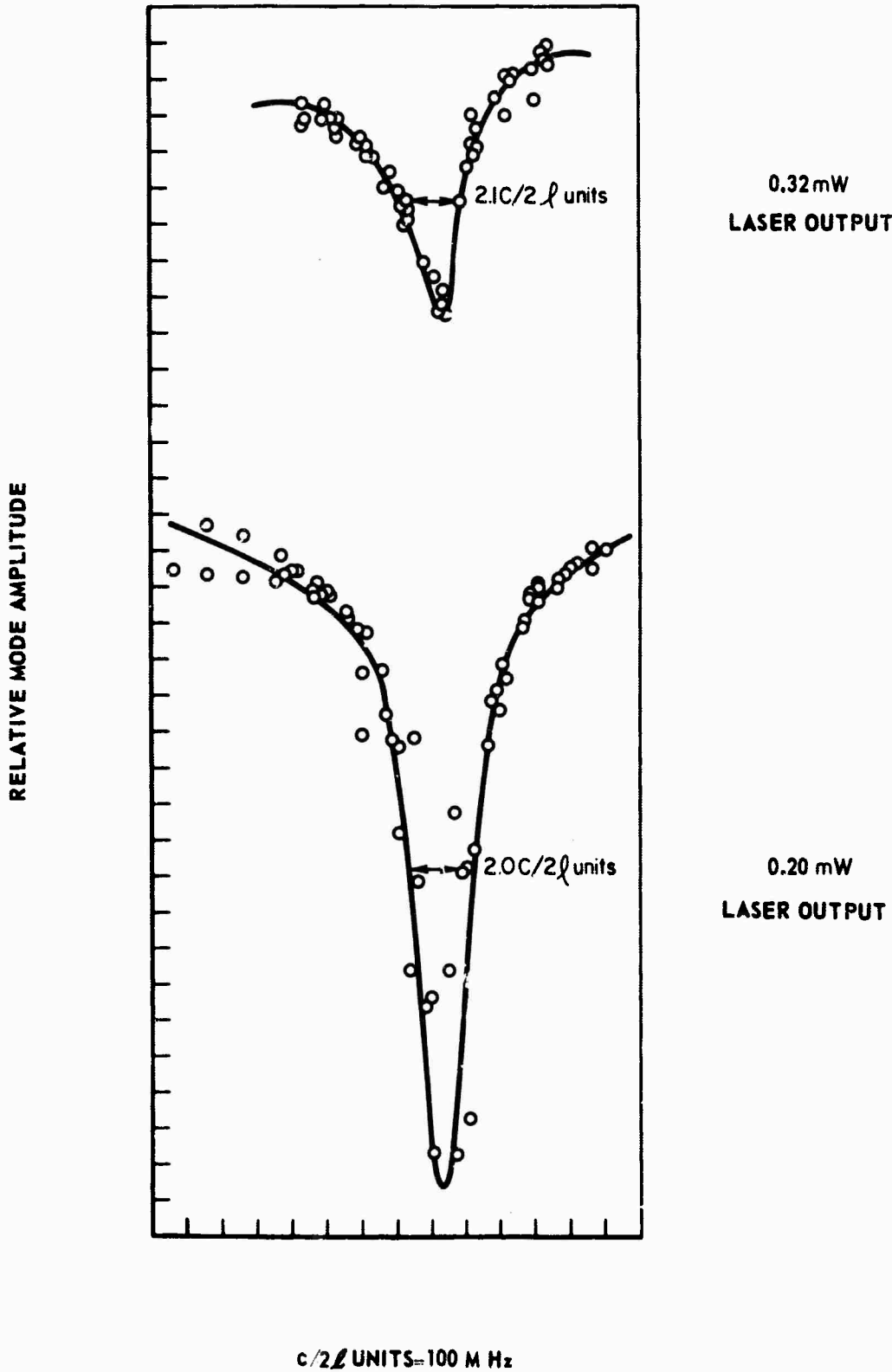
Ar<sup>+</sup> LASER SATURATION BEHAVIOR AT 0.675 mm Hg



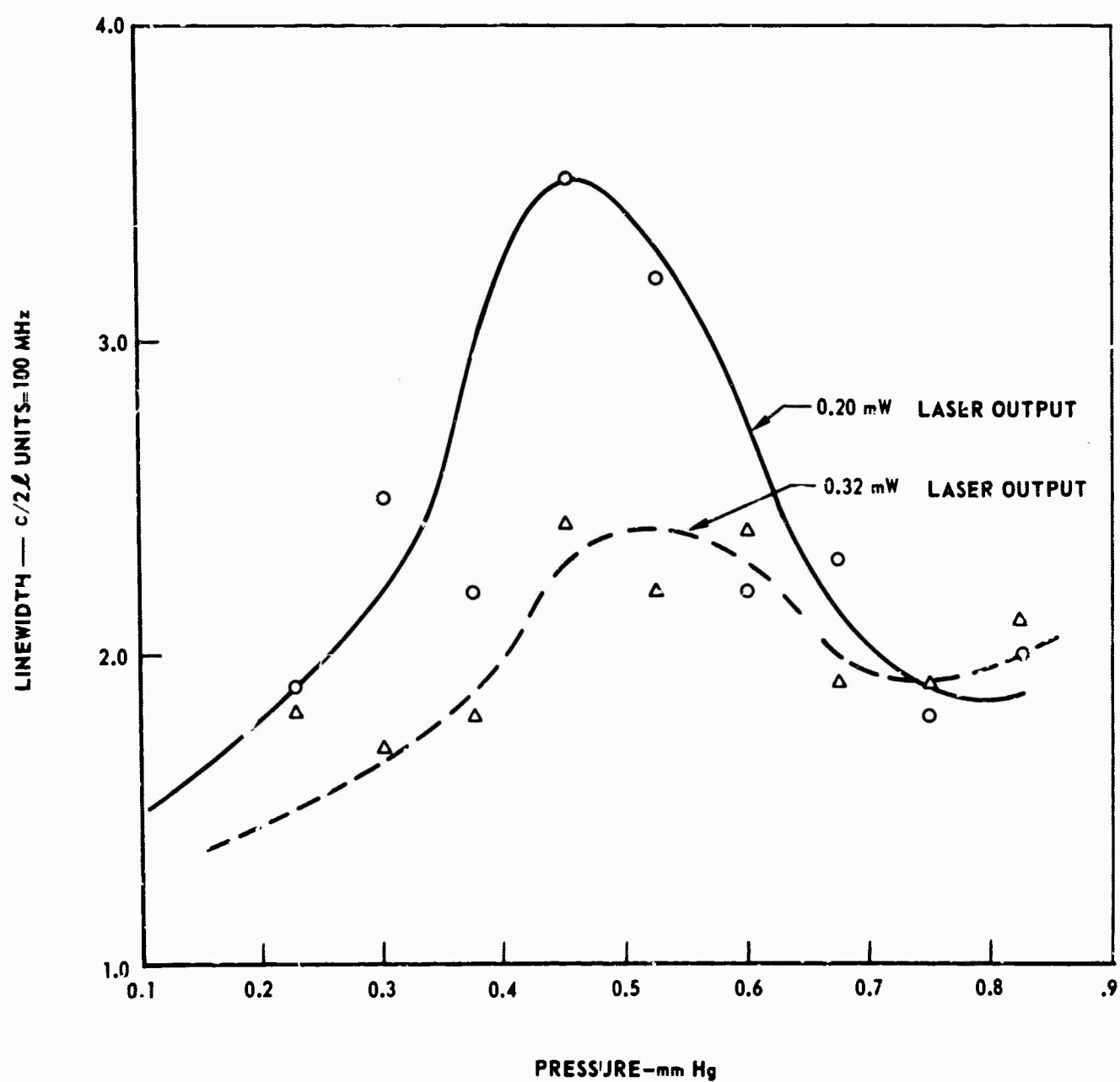
Ar<sup>+</sup> LASER SATURATION BEHAVIOR AT 0.750 mm Hg



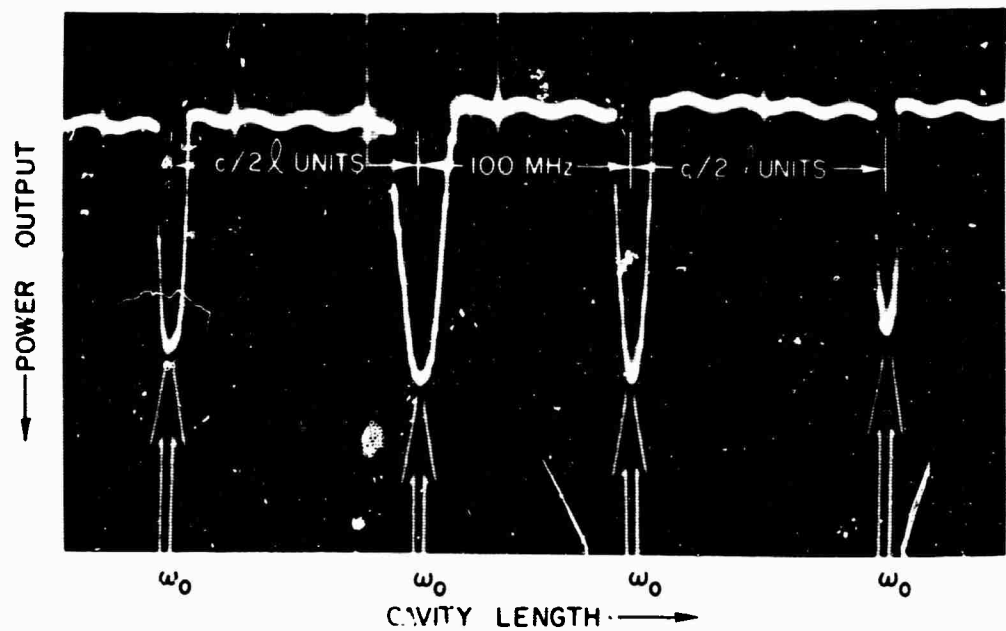
Ar<sup>+</sup> LASER SATURATION BEHAVIOR AT 0.825 mm Hg



## ARGON ION LINEWIDTH VERSUS FILLING PRESSURE



POWER OUTPUT VS FREQUENCY FOR THE CO<sub>2</sub> LASER.



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13. ABSTRACT

The objective of this program was to conduct experimental and theoretical investigations of laser line profiles. The broadened homogeneous line width of an argon ion laser was studied as a function of pressure and excitation by means of the "Lamb Dip" technique. Broadened homogeneous line widths of 200 to 400 MHz were recorded. These line widths are much larger than the 100 MHz radiative line width. The most plausible source for the broadened line is nonradiative phase interruptions occurring from small-angle Coulomb scattering in ion-ion collisions. It is shown that the line width decreases with increasing pressure for a certain range of laser oscillation. A simple collision broadening approach is not applicable to the ion system because of the behavior of the ion density as a function of pressure and excitation. Preliminary experiments were also performed with a CO<sub>2</sub>-N<sub>2</sub>-He laser and no experimental indication of a Lamb Dip was found for this laser medium.

Some aspects of the theory of a gas laser recently developed by W. E. Lamb, Jr. have been recast in a form which more fully displays the role played by the particle dynamics. It is shown that effects due to long range forces are most noticeable at long optical wavelengths and when there is a large difference between the lifetimes of the two-laser levels.

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