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SANDWICH PLATES HAVING
ENERGY DISSIPATIVE CORES,
AS VIBRATION ENERGY ABSORBERS

by

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DEFENSE RESEARCH LABORATORY



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Austin, Texas

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INTRODUCTION

The electronic equipment in a guided missile during flight is subjected to vibrations carried from the propulsion unit through the frame to the deck on which the equipment is mounted. It is desirable to keep the amplitude of these vibrations in the sensitive electronic units as low as possible. To accomplish this, vibration absorbers may be incorporated between the frame of the missile and the equipment deck, or the deck itself may be constructed so that it acts as a continuous vibration absorber. Such a deck may be made in the form of a sandwich plate, that is, a composite plate-like structure having two thin metal faces separated by a thicker core of some material which has dissipative properties. The core can be made of a visco-elastic material, such as polyethylene or rubber, or it can be made in such a way that upon bending, various parts of the core structure rub on one another, dissipating energy by means of Coulomb friction. In this report two types of sandwich plates having visco-elastic cores and one type of dry friction sandwich are studied. Some comparisons are made on the damping effectiveness of each kind of sandwich plate studied. It should be pointed out that there are some practical disadvantages of the visco-elastic core. As the interior of the missile is likely to experience wide temperature variations, the properties of the visco-elastic core material may change in a detrimental manner. The core may actually become too nearly fluid to be useful as a load carrying member. This disadvantage is not present in the dry friction type of core; however, the proportion of dissipated energy to total elastic energy is less for the dry friction type core for the cases studied in this report. Perhaps a modification of the dry friction core can be made to improve its energy dissipating qualities.

THE SANDWICH WITH A VISCO-ELASTIC CORE

An analysis is presented for the sandwich plate having a visco-elastic core. The equations describing the behavior of the core are as follows (see Fig. 1). Plane strain conditions are assumed.

Motion:

$$\left. \begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} &= \rho \frac{\partial v_x}{\partial t} \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} &= \rho \frac{\partial v_y}{\partial t} \end{aligned} \right\} \quad (1)$$

Continuity:

$$\left. \begin{aligned} \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} &= \frac{\partial}{\partial t} (\epsilon_x + \epsilon_y) \\ \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} &= \frac{\partial \gamma_{xy}}{\partial t} \end{aligned} \right\} \quad (2)$$

Compatibility:

$$\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} \quad (3)$$

This is the only one of the six equations of compatibility which is not satisfied identically.

Material:

$$\frac{E_A}{1-v^2} \left(\frac{\partial \epsilon_x}{\partial t} + v \frac{\partial \epsilon_y}{\partial t} \right) - \frac{\partial \sigma_x}{\partial t} = \beta \left[\sigma_x - \frac{E}{1-v^2} (\epsilon_x + v \epsilon_y) \right]$$

$$\frac{E_A}{1-v^2} \left(\frac{\partial \epsilon_y}{\partial t} + v \frac{\partial \epsilon_x}{\partial t} \right) - \frac{\partial \sigma_y}{\partial t} = \beta \left[\sigma_y - \frac{E}{1-v^2} (\epsilon_y + v \epsilon_x) \right] \quad (4)$$

$$G_A \frac{\partial \gamma_{xy}}{\partial t} - \frac{\partial \tau_{xy}}{\partial t} = \beta \left[\tau_{xy} - G \gamma_{xy} \right]$$

In the above equations the symbols used have the following meanings:

x, y = coordinates (in.)

t = time (sec.)

$\sigma_x, \sigma_y, \tau_{xy}$ = stresses (lb./in.²)

$\epsilon_x, \epsilon_y, \gamma_{xy}$ = strains (in./in.)

v_x, v_y = velocities (in./sec.)

ρ = density (lb.sec.²/in.⁴)

E_A, E = adiabatic and isothermal elastic moduli (lb./in.²)

G_A, G = adiabatic and isothermal shear moduli (lb./in.²)

$$\beta = \frac{E_A - E}{2\eta}$$

η = viscosity coefficient

In a previous report [Ref. (1)] equations similar to these were converted into equivalent equations in beam theory notation by performing certain integrations with respect to the thickness variable y . In Eqs. (1), the first equation is multiplied by $y dy$ and integrated between $y = -h$ and $y = +h$. The second is multiplied by dy and similarly integrated.

The operations on the other equations are of the same general type. The converted equations are shown below without the intermediate steps being shown. It is assumed that cross sections remain plane but not normal to the deformed mid-line, that v_y is independent of y , and that ϵ_y is zero.

Motion:

$$\left. \begin{aligned} \frac{\partial M}{\partial x} - Q &= \rho I \frac{\partial^2 u}{\partial t^2} \\ \frac{\partial Q}{\partial x} &= \rho A \frac{\partial v}{\partial t} \end{aligned} \right\} \quad (5)$$

Continuity:

$$\left. \begin{aligned} \frac{\partial K}{\partial t} &= \frac{\partial m}{\partial x} \\ \frac{\partial \gamma}{\partial t} &= \frac{\partial v}{\partial x} + \omega \end{aligned} \right\} \quad (6)$$

Compatibility: Satisfied identically because of the assumption of plane sections.

Material:

$$\begin{aligned} E_A I \frac{\partial K}{\partial t} - \frac{\partial M}{\partial t} &= \beta [M - EIK] \\ G_{A_s} A_s \frac{\partial \gamma}{\partial t} - \frac{\partial Q}{\partial t} &= \beta [Q - G_{A_s} \gamma] \end{aligned} \quad (7)$$

In the above equations the symbols are defined as follows:

$$M = \int_{-h}^h \sigma_x y \, dy = \text{moment (in.-lb.)}$$

$$Q = \int_{-h}^h \tau_{xy} \, dy = \text{shear force (lb.)}$$

$$K = \frac{\epsilon_x}{y} = \text{curvature (in.}^{-1}\text{)}$$

$$\gamma = (\gamma_{xy})_{y=0} = \text{shear strain at mid-line}$$

$$\omega = \frac{v_x}{y} = \text{angular velocity (rad./sec.)}$$

$$v = (v_y)_{y=0} = \text{transverse velocity at mid-line (in./sec.)}$$

$$I = \text{section moment of inertia (in.}^4\text{)}$$

$$A = \text{section area (in.}^2\text{)}$$

$$A_s = \text{equivalent shear area (in.}^2\text{)}$$

Two extreme cases are considered for the behavior of the core. In one case the core is assumed to have deformations of the pure shear type ($K = 0$, $\omega = 0$), and in the other, it is assumed that pure bending occurs ($\gamma = 0$). The first case is an approximation to what might occur for a sandwich having a low shear rigidity core and rather stiff faces. The second case could be approximated by incorporating spikes on the faces, the spikes penetrating into the core forcing it to deform with the spikes. These two extremes are shown in Fig. 2.

Pure Shear Case ($K = 0$, $\omega = 0$)

For the pure shear case the equations which are needed for analysis are the second of each of Eqs. (5), (6), (7). Upon eliminating Q and γ from these equations, a single third-order equation in v results.

It is as follows:

$$G_A A_s \frac{\partial^3 v}{\partial x^2 \partial t} - \rho A \frac{\partial^3 v}{\partial t^3} - \beta \rho A \frac{\partial^2 v}{\partial t^2} + \beta G A_s \frac{\partial^2 v}{\partial x^2} = 0 \quad (8)$$

The only solution of Eq. (8) studied in this report is that for damped sinusoidal oscillations. It is assumed that

$$v = v_0 \sin kx e^{\alpha t} \quad (9)$$

where

$$k = \frac{2\pi}{\lambda}, \quad \lambda = \text{wave length.}$$

The quantity α is determined from the cubic equation:

$$\rho A \alpha^3 + \beta \rho A \alpha^2 + G_{A/S} k^2 \alpha + \beta G_{A/S} k^2 = 0, \quad (10)$$

obtained when Eq. (9) is substituted into Eq. (8). Upon letting

$$(c_Q)_A^2 = \frac{G_{A/S}}{\rho A} = \text{square of adiabatic shear wave velocity}$$

$$(c_Q)_S^2 = \frac{G_A}{\rho A} = \text{square of isothermal shear wave velocity}$$

In the above cubic, a simpler looking equation results:

$$\alpha^3 + \beta \alpha^2 + (c_Q)_A^2 k^2 \alpha + \beta (c_Q)_S^2 k^2 = 0 \quad (11)$$

Of the three roots of this cubic, one is real and negative, the other two are complex conjugates, with a negative real part. The latter pair of complex roots are the only ones of interest, since they represent in Eq. (9) damped oscillations varying sinusoidally with time.

Pure Bending Case ($\gamma = 0$)

For the pure bending case (with spikes) all six of Eqs. (5), (6), and (7) must be used with $\gamma = 0$ in the second of Eqs. (6) and (7). Upon elimination of all but one variable, say ω , there results a single fifth-order partial differential equation:

$$\begin{aligned} \rho I \frac{\partial^5 \omega}{\partial x^2 \partial t^3} - \rho A \frac{\partial^3 \omega}{\partial t^3} + \beta \rho I \frac{\partial^4 \omega}{\partial x^2 \partial t^2} - \beta \rho A \frac{\partial^2 \omega}{\partial t^2} \\ - E_A I \frac{\partial^5 \omega}{\partial x^4 \partial t} - \beta E I \frac{\partial^4 \omega}{\partial x^4} = 0 \end{aligned} \quad (12)$$

The solution of this equation which is considered in this report is of the form:

$$\omega = \omega_0 \sin kx e^{\alpha t} \quad (13)$$

Substitution of Eq. (13) into Eq. (12) yields a cubic equation for α as follows:

$$\rho(Ik^2 + A) \alpha^3 + \beta \rho(Ik^2 + A) \alpha^2 + E_A Ik^4 \alpha - \beta E Ik^4 = 0 \quad (14)$$

Another form of Eq. (14) is:

$$\alpha^3 + \beta \alpha^2 + (c_M)_A^2 \frac{Ik^4}{Ik^2 + A} \alpha + \beta (c_M)_0^2 \frac{Ik^4}{Ik^2 + A} = 0 \quad (15)$$

where

$$(c_M)_A^2 = \frac{E_A}{\rho} = \text{square of adiabatic bending wave velocity}$$

$$(c_M)_0^2 = \frac{E}{\rho} = \text{square of isothermal bending wave velocity.}$$

Numerical studies of the above cases are made in Section 4 of this report.

SANDWICH WITH COULOMB FRICTION CORE

A third type of sandwich is discussed here. In this (see Fig. 3) the "core" is made up of prongs fastened alternately to the upper and lower faces. Each prong rubs on the face opposite from that to which it is fastened. During any kind of vibratory motion, energy is dissipated by friction whenever there is any bending of the structure. Because of the non-linear nature of the Coulomb friction vibration problem, this type of sandwich will be analyzed for a quasi-static case. For concave upward bending the relation between moment and curvature will be derived. The damping effectiveness of the structure will be estimated by finding the proportion of the energy which is lost through friction during the slow non-reversing bending of the structure.

In Fig. 4 is shown one of the repeating patterns of the sandwich of Fig 3. Assuming equilibrium of moments with respect to the center of curvature, the following difference equation is obtained:

$$(\Delta T) (R + h) = 2\mu Ph$$

Since $h \ll R$ in most practical cases, the above equation is approximately given by:

$$\Delta T = \frac{2\mu Ph}{R}$$

The moment difference, for the complete sandwich plate, between stations on opposite ends of one repeating element is therefore:

$$\Delta M = 2h\Delta T = \frac{4\mu Ph^2}{R} \quad (16)$$

This can be converted to a differential equation by letting:

$$P = p w dx$$

where p = pressure between faces per unit area

w = width of face

h_f = face thickness

Then

$$dM = \frac{4\mu pwh^2}{R} dx \quad (17)$$

Also,

$$M = \frac{EI}{R} \quad (18)$$

where

$$I = 2h_f wh^2$$

Differentiation of Eq. (18) yields

$$dM = -\frac{EI}{R^2} dR \quad (19)$$

On eliminating dM between Eqs. (17) and (19), a relation between $\frac{1}{R}$ and x is found:

$$EI \frac{dR}{R} + 4\mu h^2 p w dx = 0 \quad (20)$$

This can be integrated to the following:

$$R = R_0 e^{-(4\mu pwh^2/EI)x} \quad (21)$$

where R_0 = radius of curvature at $x = 0$.

The curvature K , which is the reciprocal of R , is approximately:

$$K = \frac{1}{R} \approx \frac{d^2y}{dx^2} = K_0 e^{(4\mu pwh^2/EI)x} \quad (22)$$

The energy lost by friction is given by

$$W_F = \int_0^L dM \left(\frac{dy}{dx} \right)$$

$$\begin{aligned}
 &= \int_0^L \frac{4\mu pwh^2}{R} \frac{dy}{dx} dx \quad [\text{using Eq. (17)}] \\
 &\doteq \int_0^L 4\mu pwh^2 \left(\frac{d^2y}{dx^2} \right) \left(\frac{dy}{dx} \right) dx \quad [\text{using Eq. (22)}] \quad (23)
 \end{aligned}$$

On integrating, the value of W_F is found to be:

$$W_F \doteq \frac{M_o^2}{8\mu pwh^2} \left[e^{(4\mu pwh^2 L/EI)} - 1 \right]^2 \quad (24)$$

where

$$M_o \doteq EI \left(\frac{d^2y}{dx^2} \right)_{x=0}$$

The elastic energy, assumed to be stored only in the two faces, is given by

$$W_E = \frac{1}{2} \int_0^L M d\theta \quad (25)$$

where $\theta \doteq \frac{dy}{dx}$ = slope of center line. Then,

$$W_E \doteq \frac{1}{2} \int_0^L M \frac{d^2y}{dx^2} dx \quad (26)$$

On using $M \doteq EI \frac{d^2y}{dx^2}$, together with the expression for $\frac{d^2y}{dx^2}$ given by

Eq. (22), the following result is obtained:

$$W_E \doteq \frac{M_o^2}{16\mu pwh^2} \left[e^{(8\mu pwh^2 L/EI)} - 1 \right] \quad (27)$$

The ratio of frictional energy dissipated to stored elastic energy is:

$$\frac{W_F}{W_E} = 2 \frac{(e^\varphi - 1)^2}{(e^{2\varphi} - 1)} \quad (28)$$

where $\varphi = 4\mu pwh^2L/EI$. The quantity φ is small compared to unity; hence, the fraction on the right can be easily approximated. Expansions of e^φ and $e^{2\varphi}$ are given here:

$$e^\varphi = 1 + \varphi + \frac{\varphi^2}{2} + \dots$$

$$e^{2\varphi} = 1 + 2\varphi + 2\varphi^2 + \dots$$

The fraction

$$\begin{aligned} \frac{(e^\varphi - 1)^2}{e^{2\varphi} - 1} &= \frac{e^{2\varphi} - 2e^\varphi + 1}{e^{2\varphi} - 1} = 1 - 2 \frac{e^\varphi - 1}{e^{2\varphi} - 1} \\ &= 1 - 2 \frac{\varphi + \frac{\varphi^2}{2} + \dots}{2\varphi + 2\varphi^2 + \dots} \\ &= 1 - \frac{1 + \frac{\varphi}{2}}{1 + \varphi} \doteq \frac{\varphi}{2} \text{ for } \varphi \ll 1. \end{aligned}$$

Therefore,

$$\frac{W_F}{W_E} \doteq \varphi = 4\mu pwh^2L/EI \quad (29)$$

This is, roughly, the energy ratio per cycle in a vibrating plate. The damping factor per cycle for amplitudes is approximately equal to half of this energy ratio. The form of φ as given by Eq. (29) is changed to a simpler one in the following. The value of I is given by

$$I = 2\omega h^2$$

Therefore,

$$\varphi = \frac{W_f}{W_E} = \frac{2\mu p L}{E h_f} \quad (30)$$

The damping factor, i.e., amplitude decay per unit time

$$\delta = \frac{\mu p L}{E h_f} \times \text{frequency} \quad (31)$$

The damping is proportional to the coefficient of friction μ , to the ratio of pressure between faces to face modulus p/E , and to the length-face thickness ratio L/h_f .

NUMERICAL EXAMPLES OF ALL THREE TYPES OF SANDWICH PLATES

To make comparisons among the three types of sandwich plates discussed in the foregoing articles some numerical values will be assumed which are rough approximations to values found in real materials. For the sandwich with the visco-elastic core, the solutions of the cubic equations, Eq. (11) and Eq. (15), are compared for equal values of k and material constants. For the dry friction type of sandwich, comparisons are made for certain frequencies with corresponding results for the visco-elastic sandwich.

Visco-elastic Sandwich-Pure Shear Case

It is convenient at this point to rewrite Eq. (11) in dimensionless form by making the following substitutions. Barred quantities are dimensionless.

$$\left. \begin{aligned} \alpha &= \frac{c}{b} \bar{\alpha} \\ \beta &= \frac{c}{b} \bar{\beta} \\ k &= \frac{1}{b} \bar{k} \end{aligned} \right\} \quad (32)$$

The cubic equation then becomes:

$$\bar{\alpha}^3 + \bar{\beta} \bar{\alpha}^2 + \frac{(c_Q)^2}{(c_Q)^2} \bar{k}^2 \bar{\alpha} + \bar{\beta} \bar{k}^2 = c \quad (33)$$

Computations for the solution of Eq. (33) have been made for a range of values of \bar{k} and $\bar{\beta}$. Roots of the cubic consist of one negative real root, and two conjugate complex roots with a negative real part, of the form $-\bar{\delta} \pm i\omega$. Only the latter pair is physically significant in the problem being discussed. The values of $\bar{\delta}$ are plotted in Fig. 5

for a range of values of \bar{k} and $\bar{\beta}$ with $\frac{(c_Q)^2}{(c_Q)^2} = 1.10$.

Visco-elastic Sandwich-Pure Bending Case

As was done for Eq. (11), Eq. (15) is also modified by making it dimensionless. The following substitutions are made. Starred quantities are dimensionless.

$$\begin{aligned}
 \alpha &= \frac{(c_M)_0}{h} \alpha^* \\
 \beta &= \frac{(c_M)_0}{h} \beta^* \\
 k &= \frac{1}{h} k^* \quad \left[\text{Note } k^* = \bar{k} \text{ of Eq. (32)} \right] \\
 A &= h^2 A^* \\
 I &= h^4 I^*
 \end{aligned}
 \tag{34}$$

The cubic then becomes:

$$(\alpha^*)^3 + \beta^* (\alpha^*) + \frac{(c_M)_0^2 A}{(c_M)_0^2} \wedge (k^*)^4 \alpha^* + \beta^* \wedge (k^*)^4 = 0
 \tag{35}$$

where

$$\wedge = \frac{I^*}{I^* (k^*)^2 + A^*}
 \tag{36}$$

The quantity \wedge for wave lengths long compared to h , reduces approximately to

$$\wedge \approx \frac{I^*}{A^*} = (r^*)^2 = \text{square of dimensionless radius of gyration}
 \tag{37}$$

As does Eq. (33), Eq. (35) has three roots. One root is real and negative; the other two are conjugate complex with a negative real part of the form $-\delta^* + i\omega$. In Fig. 6 are plotted some values of δ^* , the damping factor per unit dimensionless time, for a small range of values of k^* and β^* . In all these calculations, the approximate value of \wedge given in Eq. (37) is used. For a rectangular section $\wedge = (r^*)^2 = 1/3$.

To compare the damping effectiveness of the two kinds of visco-elastic sandwich plates, use can be made of Figs. 5 and 6. To compare cores of similar materials, β must be equal for each core. Since $(c_M)_0$ is roughly twice c_Q for many materials, it is seen from the definitions of $\bar{\beta}$ and β^* that for the same value of β , β^* is roughly one-half of $\bar{\beta}$. Also, since $-\bar{\delta}$ is the real part of $\bar{\alpha}$, and $-\delta^*$ is the real part of α^* , it can also be concluded from the definitions of $\bar{\alpha}$ and α^* that

$$\frac{\alpha^*}{\bar{\alpha}} \doteq \frac{1}{2} \frac{\alpha \text{ for pure bending}}{\alpha \text{ for pure shear}}$$

or

$$\frac{\delta \text{ for pure bending}}{\delta \text{ for pure shear}} \doteq 2 \left(\frac{\delta^*}{\bar{\delta}} \right) \quad \beta = \text{const.} \quad (38)$$

for $\bar{k} = 2$, for instance, the curves in Fig. 6 for $\beta^* = 0.01$ and $\beta^* = 0.1$ have the same heights as the curves in Fig. 5 for $\bar{\beta} = 0.01$ and $\bar{\beta} = 0.1$. Thus, the δ values for pure bending and for pure shear are equal. For $\bar{\beta} = 1.0$ in Fig. 5, $\bar{\delta} = 0.0380$; for $\beta^* = 0.5$, $\delta^* = 0.0245$. From Eq. (38),

$$\frac{\delta(\text{pure bending})}{\delta(\text{pure shear})} = 2 \frac{0.0245}{0.0380} = 1.30$$

That is, for $\bar{k} = 2$, or $k = \frac{2}{h} = \frac{2\pi}{\lambda}$, i.e., for $\lambda = \pi h$, a 30 percent improvement in damping can be expected when spikes are incorporated in the faces of the sandwich. A similar comparison for $\bar{k} = 1$ yields a gain in damping effectiveness of about 75 percent. The spiked sandwich is more effective than the unspiked in the shorter wave length region, and the effect is most pronounced for the larger values of β^* or $\bar{\beta}$, that is, for materials for which the viscosity coefficient is low, or where $E_A - E$ is large compared to what it is in a metal.

The Friction-Type Sandwich

For the particular case studied in the previous paragraph, i.e., $\lambda = \pi h$, the value of δ from Eq. (31) is

$$\delta = \frac{\mu p L}{E h_j} \frac{(c_M)_0}{\pi h} = \frac{(c_M)_0}{h} \frac{\mu p L}{E h_f \pi}$$

Thus, the quantity $\frac{\mu p L}{E h_f \pi}$ is equivalent to δ^* for the visco-elastic sandwich with spikes. Assuming $\mu = 0.6$, $p = 10^3$ psi, $E = 10^7$ psi, $L/h_f = 10^3$, the above ratio is

$$\frac{\mu p L}{E h_f \pi} = 0.019$$

Although it is less, this result is roughly in the general neighborhood of the result of damping produced by the visco-elastic type of sandwich. As the dry friction core has advantages over the visco-elastic type insofar as its behavior with changes in temperature is concerned, it seems that such a device is certainly worth trying as an electronic equipment deck for a guided missile.

This report has been distributed in accordance with the list for Aerodynamics contained in APL/JHU, TG 8 - 11, dated November, 1953.

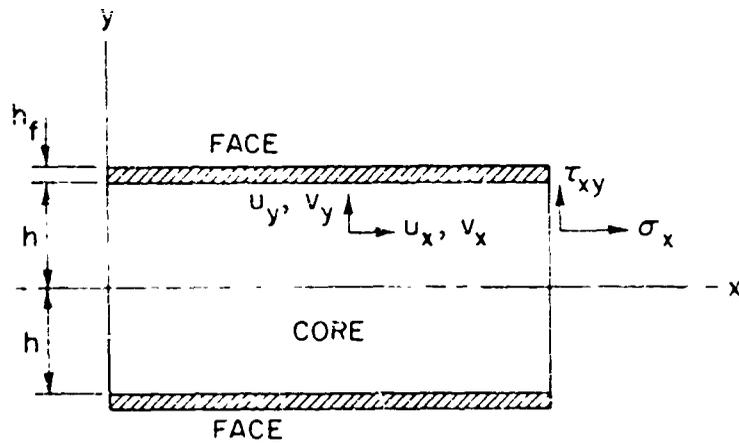


FIG 1 - STRESSES, DISPLACEMENTS AND VELOCITIES IN SANDWICH CORE

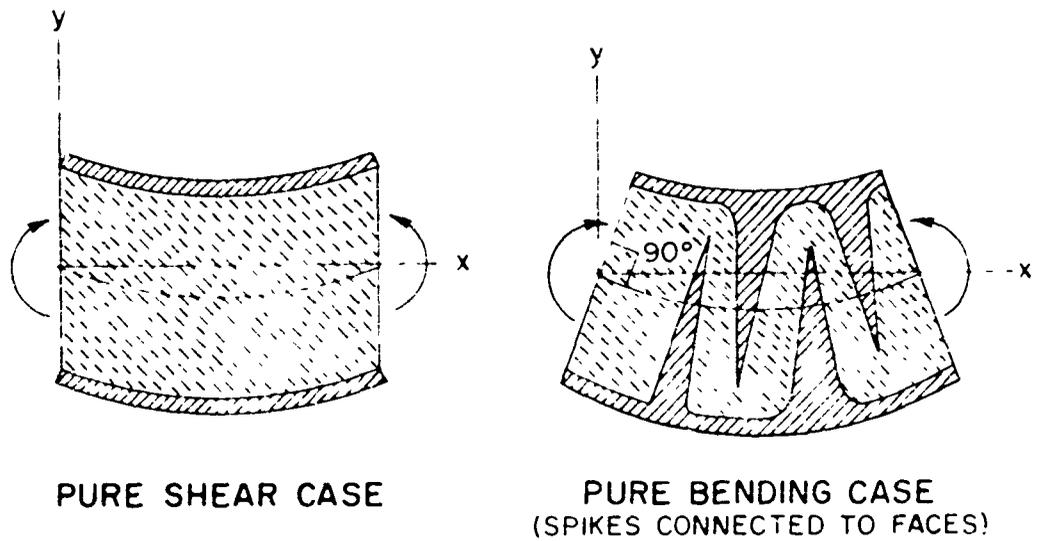


FIG. 2 - TWO EXTREME CASES OF CORE DEFORMATION IN A SANDWICH

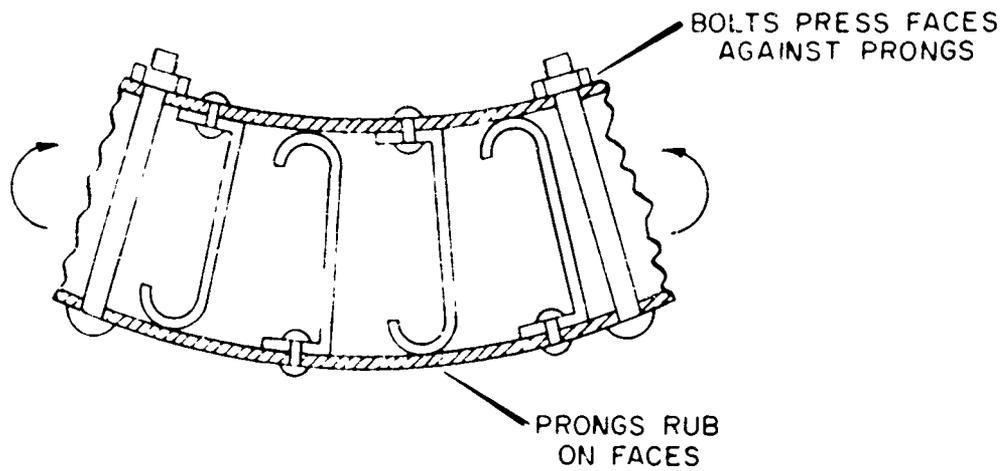


FIG. 3 · ELEMENT OF DRY FRICTION SANDWICH PLATE

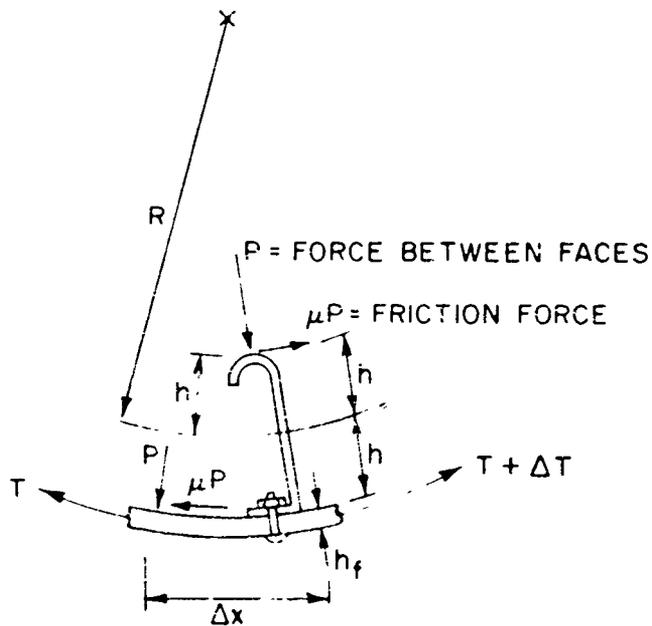
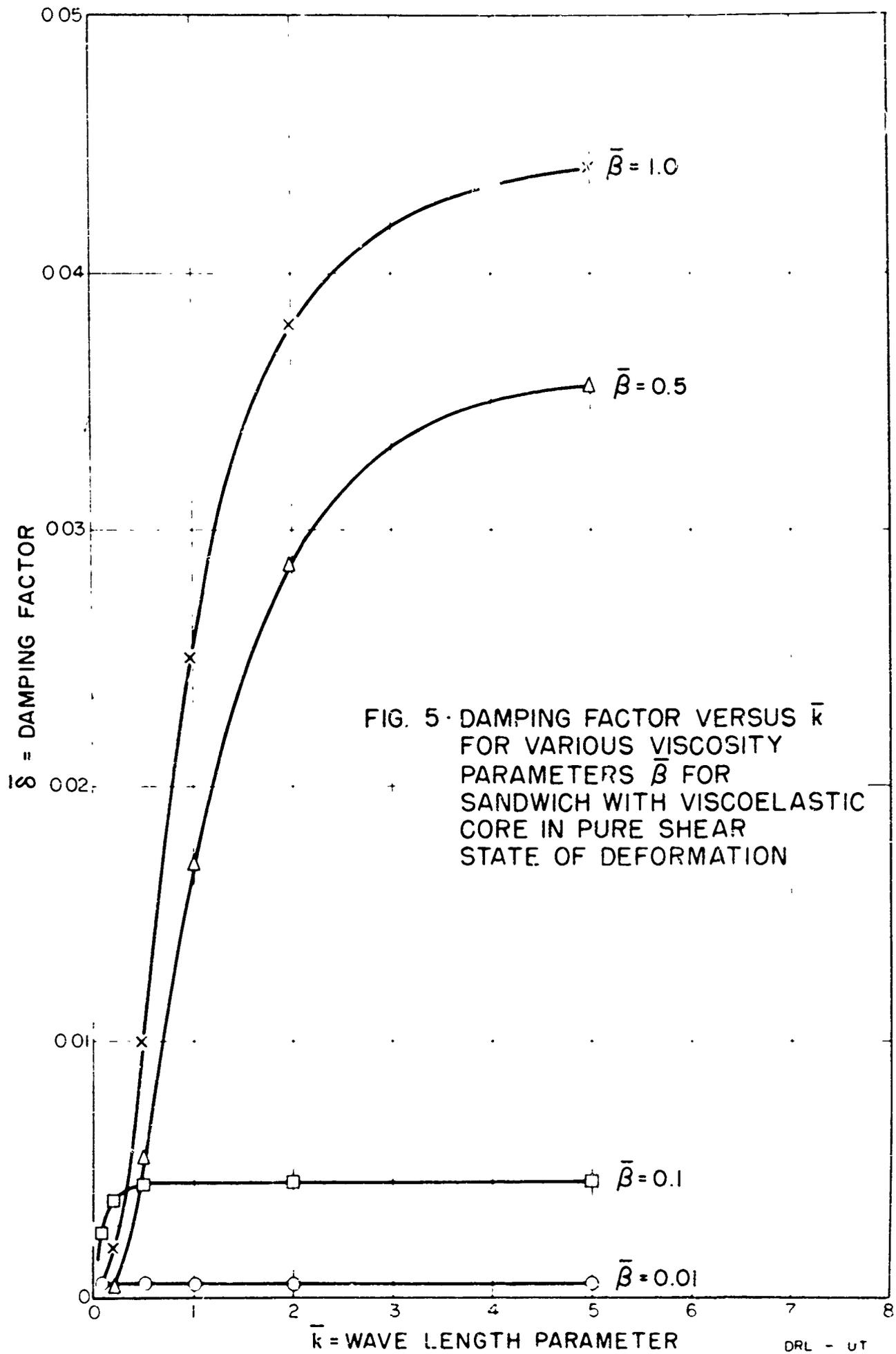


FIG. 4 · ELEMENT OF FRICTION TYPE SANDWICH PLATE SHOWING FORCES BETWEEN CORE AND FACES



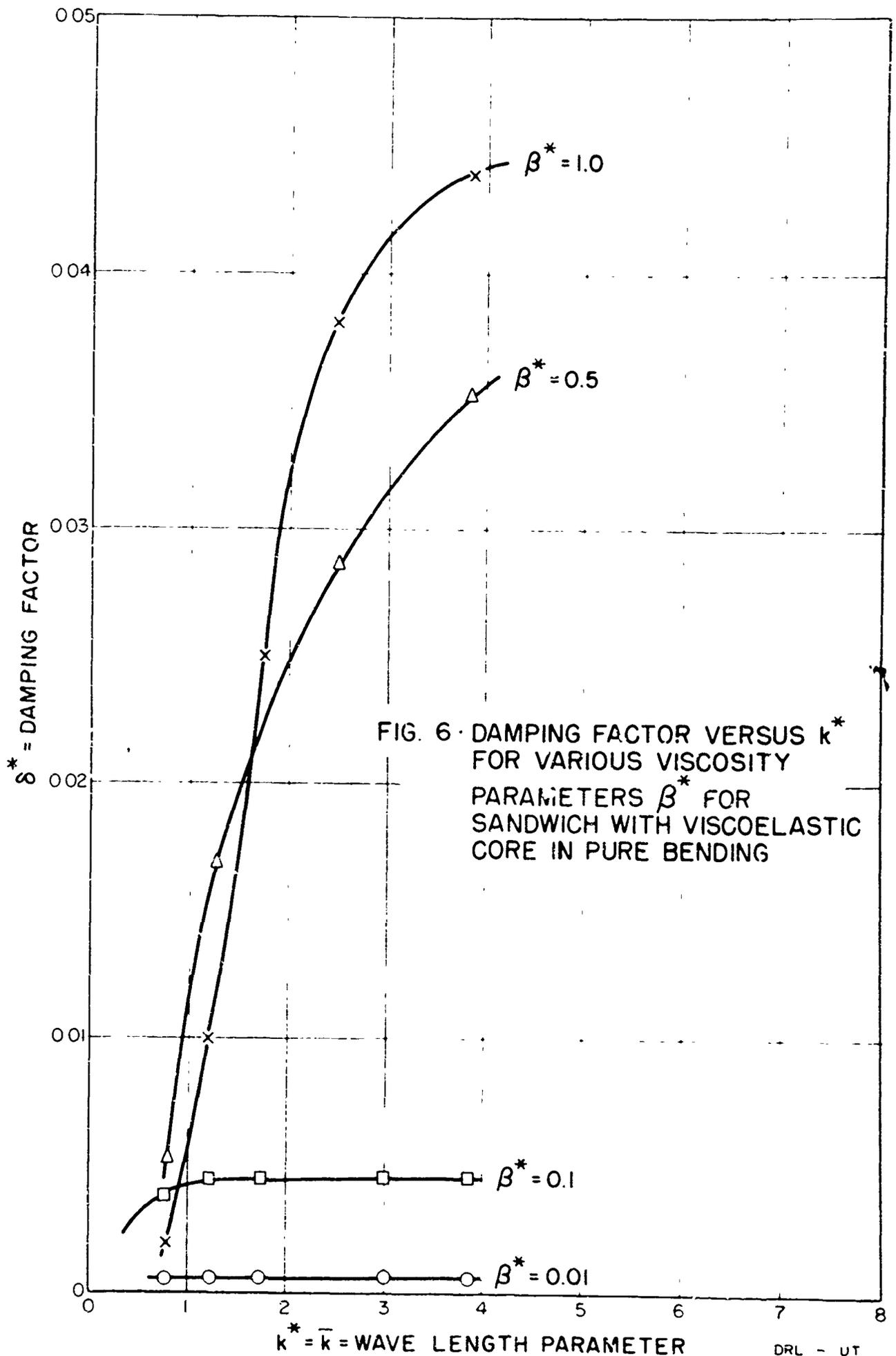


FIG. 6 · DAMPING FACTOR VERSUS k^* FOR VARIOUS VISCOSITY PARAMETERS β^* FOR SANDWICH WITH VISCOELASTIC CORE IN PURE BENDING