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BOOST PHASE TRAJECTORY ANALYSIS TECHNIQUES

by

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To: Distribution List of APL/JHU CF-2505

From: Editorial Project Supervisor, Technical Reports Group

Subject: APL/JHU CF-2505, "Boost Phase Analysis Techniques," by C. D. West, classification of and corrections to

The author of CF-2505 has informed this office that CF-2505 should have been published as unclassified instead of as Confidential since Figure 1, the only page marked Confidential was erroneously and inadvertently so marked.

The following corrections should also be made in the report:

Page 3; Eq. (2c) should be

$$m \dot{V}_m = T - 0.7\, C_{D_o}\, \rho M^2 S - mg \sin \theta_o$$

Appendix I; line 16 of text should be

"The values of X_t and $\dot{X}_t$ obtained . . ."

line 24 should be

"is used as the estimated value of $\ddot{X}_t$. The ..."

Appendix II; Eq. (6) should be

$$D(t) = 0.7\, C_D\, \rho M^2 S$$

It is requested that you mark your copy as indicated above

Paul E Clark

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PEC:jb

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CF-2505

BOOST PHASE TRAJECTORY ANALYSIS TECHNIQUES

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I. Introduction

The analysis of the performance of a rocket-boosted missile during the boost-phase portion of its flight generally requires that one solve a set of simultaneous differential equations with variable coefficients for a large variety of initial conditions. Analytical methods of solution are formidable if not impossible and consequently ways and means have been developed to accomplish the task with minimum possible effort while maintaining sufficient accuracy and reliability. This paper consists of a collection of four approaches to the problem and is presented with the hope that future analysts will not have to tread the same ground again.

The particular treatment outlined here is for a cruciform missile-booster combination, completely symmetrical with respect to any two normal planes passing through the longitudinal centerline. The extension of the methods to unsymmetrical configurations is simple since it merely adds one more equation. The additional volume of calculations is another matter.

Since the paper is concerned only with the boost phase performance, the word missile is used in the text to mean the missile-booster combination.

II. Equations of Motion

The equations of motion for a missile symmetrical to two planes which are normal to each other and which pass through the longitudinal centerline are (See Figure 1):

Normal to Flight Path*

$$m V_m \dot{\gamma} = T \sin(\alpha - \varepsilon) + D \sin\varepsilon - mg \cos\gamma + A\alpha + B\delta \qquad (1)$$

Parallel to Flight Path

$$m \dot{V}_m = T \cos(\alpha - \varepsilon) - D \cos\varepsilon - mg \sin\gamma \qquad (2)$$

Pitch About C.G. of Missile

$$I \ddot{\theta} = -C\alpha - C_1 |\alpha| - D_1 \dot{\theta} - D_2 \dot{\alpha} + E\delta + T \times L \qquad (3)$$

Geometry

$$\theta = \alpha + \gamma - \varepsilon \qquad (4)$$

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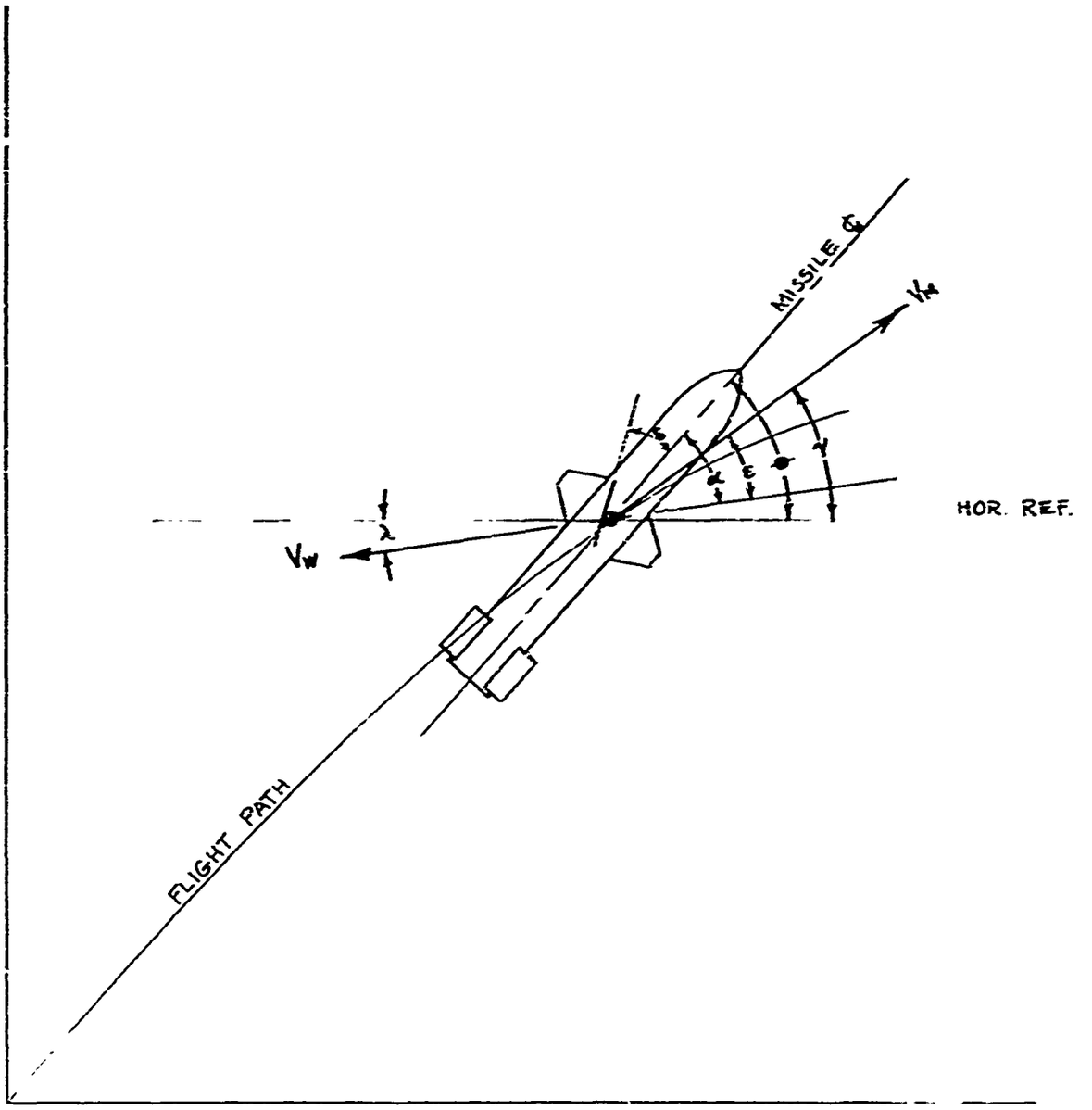

FIGURE 1

Wind Effects

$$\overline{V_R} = \overline{V_m} \rightarrow \overline{V_w} \tag{5}$$

$$\tan \mathcal{E} = \frac{V_w \sin (\gamma - \lambda)}{V_m + V_w \cos (\gamma - \lambda)} \tag{6}$$

*See list of symbols at end of paper

The assumptions underlying these equations are well known and may be found in any standard text on airplane stability (e.g., Reference 1, Chap. 10). One tacit assumption, however, is that the missile is roll controlled with no cross coupling between directional and roll maneuvers.

The simultaneous solution of Equations (1) through (6) would yield all the information one might desire about the flight characteristics of the missile. However, due to the non-linear nature of the aerodynamic coefficients, analytical solution is difficult if not impossible and one is forced to undertake numerical or analog procedures for solutions. The rest of this paper will outline two approaches to the complete solution of the equations and two approaches which by making several assumptions reduce the equations to special cases for specialized results. Only numerical procedures will be considered since analogs involve special techniques too involved for this limited paper.

III. Method of Analysis

A. "Exact" Analysis

It is admitted that any solution other than analytical will not be exact. However, by careful and refined numerical procedures one may attain answers which may be as close to the exact as may be desired for engineering purposes. In view of this, then, exact is used here to indicate that numerical solutions will be obtained from Equations (1) through (6) without making additional assumptions.

There are several numerical methods available (Reference 2) by which one may obtain solutions to differential equations. The simplest, and the one most often used in boost-phase analysis, is the trapezoidal rule, the details of which are given in Appendix I.

Actually, the exact analysis is seldom undertaken since experience has shown that two simple assumptions lead to a much easier and faster approach without significant loss of accuracy. The new approach will be discussed in the next section.

B. Semi-Exact Analysis

The semi-exact analysis is very much like the exact. However, based on experience and comparative calculations, it is assumed that oscillatory deviations from the flight path do not affect the velocity of the center of gravity along the flight path and that $\mathcal{E}$ approaches a negligible value quickly. These assumptions allow one to break the solution of the equations into two parts: the determination of the speed-time history alone

and the solution of the pitch and normal force equations using the speed-time history. This particular technique was used by the Germans in their V-2 program and has been verified numerous times since.

Assuming that oscillatory changes about the flight path do not affect the forward velocity we may rewrite equation (2) (neglecting the wind) as

$$m \dot{V}_m = T - D - mg \sin \gamma_o \tag{2a}$$

and since at $t = 0$, $\alpha = 0$ then

$$\gamma_o = \theta_o \tag{4a}$$

and

$$m \dot{V}_m = (T-D) - mg \sin \theta_o \tag{2b}$$

The procedure is to solve equation (2b) by the trapezoidal rule to obtain a speed-time history then substitute the values of V thus obtained in Equations (1) and (3) which, taken with (4), will yield a complete solution. The results are as accurate as the exact method provided care is exercised to the same degree.

C. McDonnell Speed-Time History

From the foregoing section it must be clear that an even faster or simpler method of obtaining the speed-time history would be desirable. McDonnell Aircraft Company has derived such a method in Reference 3 by making the additional assumption that speed is linear with time. A small digression is in order so that the method may be made a little more lucid.

Equation (2b) may be written fully as:

$$m V_m = T - 0.7 C_{D_o} pM^2 S - mg \sin \theta_o \tag{2c}$$

where the constant 0.7 is one-half the ratio of specific heats. Since, in this expression, C_{D_o} is a complicated function of Mach number a correct approach requires the trial and error procedure of Appendix I. But, recognizing that the speed-time history is almost linear and assuming that it is linear, one may compute the drag for any time. The solution of (2c) is then reduced to a summation process, and this is the heart of the McDonnell approach.

To apply the McDonnell method one estimates a separation speed and time. Then assuming that the speed varies linearly with time, one determines the altitude, pressure and speed of sound at each time. These are combined to obtain the drag-time history and Equation (2c) is completed. Details are contained in Appendix II.

The only difficulty in applying the McDonnell approach is in estimating the separation speed but a reliable method for this has been developed and will be discussed in the next section.

D. McCalley Method for Separation Speed

A reliable estimate for the separation speed is required for Section III-C and is quite often necessary in evaluating the performance of boosters. A simple iterative method has been derived by McCalley (Reference 4) based on an approach by Walker and Henke (Reference 5). The equation derived by McCalley is:

$$V_{n+1} = \frac{\frac{Tt}{m_c - \frac{1}{2} m_p} K_1 - g t_s \sin\theta}{1 + \frac{\rho C_D S (t_s - 1)}{4 (m_o - 2/3\, m_p)} K_2 V_n} \qquad (5)$$

where ρ and C_D are selected for the end point or separation condition. The solution of this equation is, then, merely an evaluation of the constants and iteration of V to convergence. The details underlying Equation (5) are contained in Appendix III.

IV. Computation Methods

There are, at APL, three methods of computation available for solution of these equations, namely, desk calculator, IBM and REAC. Each has its own peculiar advantages and disadvantages which will not be dwelt on here. Instead, a brief note as to which is adaptable to the analyses will be considered.

The REAC, an analog computer, could be used with the exact analysis if enough auxiliary equipment were available but since such side equipment is not normally available use of REAC is restricted to the semi-exact. That is, if speed histories are known, then the rest of the trajectory may be worked. It should be noted that even this requires a considerable quantity of extra equipment. Since the bulk of this paper is concerned with numerical procedures we shall let REAC lie on this brief note.

With regard to the numerical procedures it is true that IBM can do exactly the same work that can be done by desk calculators. This statement does not imply that it is true a priori that IBM is always the best. There is no general rule which may be invoked to determine when IBM is preferable to desk calculation except that when there is a large quantity of calculation IBM is essential. It should be fairly obvious the IBM machinery would be uncalled for on the McCalley analysis since it is so easily worked by hand. Otherwise, each case must be decided as it arises.

V. Accuracy

When obtaining numerical solutions to differential equations there is always some loss of accuracy from the possible analytical solution. This is more especially true when using the trapezoidal rule which is, from a mathematical standpoint, rather crude. However, if care is exercised and incremental changes

kept small, the errors are negligible from an engineering standpoint. Therefore, it will be assumed that the solution to the "exact" analysis is exact and the other analyses compared with it.

The semi-exact analysis yields practically the same result as the exact and consequently the error is negligible. In fact, a check was made against data measured in an actual flight to check separation speed only and the difference between calculated and measured end speeds was minute. The details of this check are contained in Reference (6) for the interested reader.

An error analysis is very difficult to make on the McDonnell approach to speed-time histories. Numerical comparison has been made however, and the results show that the difference between the semi-exact and the McDonnell speed-time history is generally less than 2% when the estimated separation speed is between 2 and 4% in error (Reference 7). For most trajectory analyses the McDonnell speed-time history is sufficiently accurate since a small error in velocity at any given time does not appreciably affect the results.

The McCalley method of computing separation speeds produces errors up to 4% depending on the Mach number. It appears (Reference 7) that the error increases with increasing Mach number, which is not surprising since the method assumes time-linear drag. Hence, the higher the separation velocity (Mach number) in a given time the greater the drag deviates from a straight line and the greater the error.

VI. Application

Now that the various methods are in hand it would seem that a statement on the uses of each would be in order. Experience has indicated the area of usefulness to which each is suited and the types of problems which have been encountered.

As must be evident, the exact, or since the difference is small, the semi-exact analysis is called for whenever the interest is in the actual flight performance of the missile. Either of these will yield time histories of all of the variables in question which are essential in computing air-loads, inertia loads, dispersions, etc. This field of requirements is self-evident and usually no question will arise as to when to use the full analysis.

If one is interested only in the speed-time history then some conflict appears as to the better method. In fact, even when making the semi-exact analysis, the question arises as to whether an approximate speed-time history such as obtained from the McDonnell method is sufficient for trajectory analysis. It is the writer's opinion that it is. This opinion is based on the fact that the aerodynamic coefficients do not, throughout the largest portion of the boost-phase, change radically with Mach number. Thus if one has a small speed error at some step, it is obvious that the forces computed are not greatly different from the exact. The error generated thusly is more than compensated for by the saving of time and expense. However, one must judge for himself on this point.

There are times when the separation speed is of the utmost importance. For example, a certain minimum speed is required for engine starting and there may be a question as to whether the booster impulse is sufficient to produce this speed. In such a problem the semi-exact speed history is an absolute must.

Whenever the problem of booster design arises, there are several ways to attack it. First, by using the McCalley method one may determine a general area of impulse necessary to reach a given speed. The impulse may then be translated into more exact speeds by using one of the more refined methods. Furthermore, one may show the effects of changes in the booster and missile parameters by using the McCalley equation. Thus, in an initial design study the McCalley approach is extremely useful in setting approximate boundaries. All of this presupposes that the propellant characteristics are known or estimable as they relate to physical dimensions, gas pressures, etc. If one is starting from scratch then a more refined approach such as that developed by Hawley and Fenton in Reference 8 is required.

VII. Techniques

There are certain details which arise during the actual calculations which are sometimes confusing. This section will try to clarify a few of the more important ones.

A. First consideration will be given to wind effects and how these are included in the various analyses. When using the exact analysis one must account for the wind at every step. Obviously this amounts to solving Equations (1) through (6) without modification. It should be apparent that the relative velocity V_R is used in computing the aerodynamic terms. To illustrate, take Equation (2) and write in full:

$$m \dot{V}_m = T \cos(\alpha - \varepsilon) - C_D \frac{\rho V_R^2}{2} S \cos\varepsilon - mg \sin\gamma \qquad (6)$$

Thus, the $\dot{V}_m$ refers to acceleration relative to earth while the V_R refers to missile velocity relative to the air.

The semi-exact method may fortunately be handled a little differently. Since the speed of the missile becomes much greater than the wind in a very short time then one is justified in neglection of the angle ε. Therefore, the only things one must consider is the change in angle of attack which is defined by

$$\tan\alpha = \frac{V_w \sin(\theta - \lambda)}{V_m + V_w \cos(\theta - \lambda)} \qquad (7)$$

where positive V_w is a headwind in this scalar equation.

In the McDonnell analysis one must assume a velocity history based on an average acceleration. To compute the drag the relative velocity is used so that one must add at every time the longitudinal component of the wind to the speed-time history computed from the average acceleration.

The McCalley equation makes no provision for wind but it may be considered that the equation always yields the relative velocity as the solution. That is, the V obtained by iteration already contains a wind component.

B. A technique often used by McDonnell is that of calculating everything in the so-called "slant plane". This amounts to rotating the coordinate axes through the angle θ_o or the original launch angle. There is some simplification achieved in this way and in fact by neglecting gravity one may determine a sort of "universal" trajectory. The principal advantage is in the relative size of the numbers handled. The details are not important since all that is required is rotation of the axes.

C. In setting up the McDonnell analysis for solution on IBM equipment, one encounters the difficulty of non-linear pressure-altitude or density-altitude relations. If, however, these relationships are expanded in a Taylor series about the altitude of the launching site then by taking the first two terms the pressure (or density) may be determined approximately in linear form. It has been found that no substantial error is introduced up to an altitude of about 5000 ft above the launching site. The same sort of approximation may be made for the speed of sound.

D. In working the speed histories using the semi-exact analysis and the trapezoidal rule it has been found that a sizeable error may be allowed in the estimated value of V (or as notated in Appendix I, $\ddot{X}$). The reason for this is apparent through examination of the following:

Let $\ddot{X}_t$ be the correct value of the acceleration at the end of the next time increment and $\ddot{X}_E$ be the estimated value. Then

$$\dot{X}_t = \frac{\ddot{X}_t + \ddot{X}_{t-1}}{2} \Delta t + \dot{X}_{t-1}$$

$$= \frac{\ddot{X}_t + \ddot{X}_E + R}{2} \Delta t + \dot{X}_{t-1}$$

$$= \frac{\ddot{X}_t + \ddot{X}_E}{2} \Delta t + \dot{X}_{t-1} + \frac{R}{2} \Delta t$$

$$X_t = \frac{\dot{X}_t + \dot{X}_{t-1}}{2} \Delta t + X_{t-1}$$

$$= \frac{\frac{\ddot{X}_t + \ddot{X}_E}{2} \Delta t + 2\dot{X}_{t-1} + \frac{R}{2} \Delta t}{2} \Delta t + X_{t-1}$$

$$= \frac{\ddot{X}_t + \ddot{X}_E}{4} \Delta t^2 + X_{t-1} \Delta t + X_{t-1} + \frac{R}{4} \Delta t^2$$

This shows that the value of X_t is in error by the amount of $\frac{R}{4} \Delta t^2$. If $\Delta t = 0.1$ sec and $R = 100$ ft/sec^2 then the error in X_t is only 0.25 ft with an even smaller change in altitude. The error in the

speed $\dot{L}_t$ is $\frac{a}{2}\Delta t$ which, using the same numbers as before, is 5 ft/sec. The combination of the small change in Mach number and altitude shows that a large error in $\dot{V}_g$ is not serious if small time increments are used.

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APPENDIX I

Numerical Integration by Trapezoid Rule

Assume the following equation is to be solved

$$\ddot{X} = b\dot{X} + eX + d \tag{1}$$

where the b, e, and d are, in general, functions of X and t, and that we have reached the solution of the equation up to some time $t = \tau$.

By extrapolating the past history, or by any means available, one estimates the value of $\ddot{X}$ after some small time increment, $(t - \tau)$, as $\ddot{X}_t$. If the time increment is small enough and the functions behaving smoothly then an average value of $\ddot{X}$ may be calculated for the interval as:

$$\ddot{X}_{avg} = \frac{\ddot{X}_t + \ddot{X}_\tau}{2} \tag{2}$$

Then the value of $\dot{X} = \dot{X}_t$ at the end of the interval is approximately

$$\begin{aligned} \dot{X}_t &= \ddot{X}_{avg}(t - \tau) + \dot{X}_\tau \\ &= \frac{\ddot{X}_t + \ddot{X}_\tau}{2} (t - \tau) + \dot{X}_\tau \end{aligned} \tag{3}$$

and the value of $X = X_t$ is

$$\begin{aligned} X_t &= \dot{X}_{avg} (t - \tau) + X_\tau \\ &= \frac{\dot{X}_t + \dot{X}_\tau}{2} (t - \tau) + X_\tau \end{aligned} \tag{4}$$

The values of $\dot{X}_t$ and X_t obtained from Equations (3) and (4) are then multiplied by b and e, which are selected for $t = t$ and $x = x_t$ and added to the value of d selected the same way. The sum thus obtained called the <u>calculated</u> value of $\ddot{X}_t$ is compared to the estimated value of $\ddot{X}_t$ and if agreement is attained one proceeds to the next time increment. If agreement is not realized then $\ddot{X}_t$ is estimated again until the estimated and calculated values of $\ddot{X}_t$ are very nearly the same.

If in Equation (1) b, e and d are constants or are functions of t only then the process becomes iterative, i.e., the value of $\ddot{X}_t$ obtained from calculation is used as the estimated value of $\ddot{X}_t$. The process is usually convergent. Obviously, if b, e and d are constants then an analytical solution is straightforward.

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APPENDIX II

McDonnell Method of Computing Speed-Time Histories

Assume the following:

1. The speed-time history is linear
2. The separation speed, V_s
3. The separation time, t_s

With these assumptions one proceeds by calculating an average acceleration as:

$$a_{avg} = \frac{V_s}{t_s} \tag{1}$$

Then compute the speed

$$V(t) = a_{avg}\, t + V_o \tag{2}$$

and the slant range

$$S(t) = \tfrac{1}{2}\, a_{avg}\, t^2 + V_o t + S_o$$

The altitude h, is then calculated from S(t) and the speed of sound and pressure determined as functions of h as follows:

$$h = S(t) \sin \theta_o \tag{3}$$
$$c = f_1(h) \tag{4}$$
$$p = f_2(h) \tag{5}$$

The drag, D, as a function of time is then easily found by the following steps:

1. Compute the Mach number $M(t) = \frac{V(t)}{c}$
2. Select the drag coefficient corresponding to M(t) at each time interval.
3. Complete the drag calculation by the following equation:

$$D(t) = 0.7\, C_D\, M^2 S \tag{6}$$

where the 0.7 is one-half the ratio of specific heats for air.and S is the reference area.

The remainder of the speed history is just a question of evaluating the following equation and applying the trapezoid rule.

$$a(t) = \frac{T(t)}{M(t)} - \frac{D(t)}{M(t)} - g \sin \theta \tag{7}$$

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APPENDIX III

McCalley Equation

The McCalley equation is based on a method developed by Walker and Henke wherein the drag is assumed to be linear with time. Under this assumption plus the additional one that the mass varies linearly with time, Walker obtains the following equation:

$$V = \frac{G}{2} + \frac{G}{2}\sqrt{1 + \left(\frac{4}{G}\right)\left[\frac{Tt_s}{M_F}\ln\left(1 - \frac{M_F}{M_o}\right) + gt_s \sin\theta\right]} \tag{1}$$

where

$$G = \frac{2 M_o \left(\frac{M_F}{M_o}\right)^2}{\rho C_D S t_s \left[\left(\frac{M_F}{M_o}\right) + \ln\left(1 - \frac{M_F}{M_o}\right)\right]} \tag{2}$$

McCalley's contribution comes about by restating the Walker equation as

$$V_{n+1} = \frac{\frac{-T t_s}{M_F}\ln\left(1 - \frac{M_F}{M_o}\right) - g t_s \sin\theta}{1 - \frac{\rho S C_D t_s M_o}{2 M_F^2}\left[\left(\frac{M_F}{M_o}\right) + \ln\left(1 - \frac{M_F}{M_o}\right)\right] V_n} \tag{3}$$

and then expanding the logarithm terms in a series. Taking only the first two (linear) terms and applying correction factors McCalley derives the following expression:

$$V_{n+1} = \frac{\frac{T t_s}{M_o - \frac{1}{2} M_F} K_1 - g t_s \sin\theta}{1 + \frac{\rho C_D S t_s}{4\left(M_o - \frac{2}{3} M_F\right)} K_2 V_n} \tag{4}$$

Observation of drag calculations made under more exact conditions has led to the conclusion that the drag is linear not from t = 0 but from t = 1 sec. This fact in turn leads to a modification of Equation (4) which is more nearly in keeping with the physical situation:

$$V_{n+1} = \frac{\frac{T\ t_s}{M_o - \frac{1}{2} M_P} K_1 - g\ t_s \sin\theta}{1 + \frac{\rho C_D S (t_s - 1)}{4 (M_o - \frac{2}{3} M_W)} K_2 V_n}$$

It should be noted that the value of T shown here is an average one which will give the proper impulse between $t = 0$ and $t = t_s$.

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SYMBOLS

A = Aerodynamic Coefficient

a = Acceleration, ft/sec^2

B = Aerodynamic Coefficient

C = Aerodynamic Coefficient

C_1 = Aerodynamic Coefficient

c = Speed of sound, ft/sec

D = Drag force, lbs

D_1 = Aerodynamic Coefficient

D_2 = Aerodynamic Coefficient

d = General Coefficient

E = Aerodynamic Coefficient

e = General Coefficient

G = Defined in Equation (2) of Appendix III

g = Gravitational Constant, ft/sec^2

h = Altitude, ft

K_1 = f_1 (M_F/M_o), Correction Constant

K_2 = f_2 (M_F/M_o), Correction Constant

L = Perpendicular distance from center of gravity of missile to action line of thrust, ft

M = Mach Number = V/C

m = Mass, slugs

M_o = Total Mass at Launch, slugs

M_F = Mass of useable propellant, slugs

p = Atmospheric pressure, lb/ft^2

R = Difference between assumed and true acceleration, ft/sec^2

S = Reference Area, ft^2

S = Slant Range, ft

T = Thrust Force, lbs

t = Time

V = Velocity, ft/sec

X = Distance measured along launching line, ft

C_{D_o} = Zero angle of attack drag coefficient

α = Angle of Attack

γ = Flight Path Angle

δ = Surface deflection either by control, deformation or manufacturing difficulties

ε = Angle between relative wind and flight path

θ = Angle between missile centerline and reference plane

λ = Angle between reference plane and wind vector

ρ = Air Density, slugs/ft^3

Subscripts

E = Estimated Condition

n = Iteration Number

m = Refers to Properties of Missile

0 = Initial Conditions

R = Refers to Conditions relative to Missile

s = Separation Conditions

t = Conditions at Time t

t-1 = Conditions at Preceding Time Considered (not necessarily a unit increment of time such as 1 sec)

Superscripts

Dots - Refer to derivatives with respect to time e.g., $\dot{X} = \frac{dx}{dt}$

Bars - Refer to vector quantities

BIBLIOGRAPHY

1. Perkins, Courtland D., and Hage, Robert E. - Airplane Performance Stability and Control, New York: John Wiley and Sons, Inc., 1949

2. Scarborough, J. B. - Numerical Mathematical Analysis, Second Edition, Baltimore: The Johns Hopkins University Press, 1950

3. McDonnell Aircraft Corporation Memo No. 4712, AWM-83 - "Preliminary Boost-Phase Performance of the TALOS 6bW1 Weapon", R. C. Brown, December 8, 1954

4. APL/JHU Engineering Memorandum No. 1572, "A Simplified Method of Making Approximate Boost Velocity Calculations", R. B. McCalley, September 3, 1953

5. Letter from J. H. Walker and J. C. Henke to J. F. R. Floyd:
Subject: Nomograph to Determine Booster Terminal Velocity
Dated 27 May 1949

6. APL/JHU Engineering Memorandum No. 2390, "Analytical Determination of S/N 48 Boost-Phase Separation Speed", C. D. West, December 29, 1955

7. APL/JHU Engineering Memorandum No. 2158, "Comparison of Methods for Computing Separation Velocities", C. D. West, February 21, 1955

8. APL/JHU CF-2350, "A Method for Determining Booster Rocket Requirements", W. W. Hawley and W. H. Genton, March 1, 1955.

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