NAEC-ASL 1 1 1 0

195

U. S. NAVAL AIR ENGINEERING CENTER

PHILADELPHIA, PENNSYLVANIA

AD656056

Distribution of this document
is unlimited

RECEIVED

AUG 1 5 1967

CFSTI

DISCLAIMER NOTICE

UNCLASSIFIED
Technical
Report

DEFENSE

TECHNICAL

INFORMATION

CENTER

UNCLASSIFIED
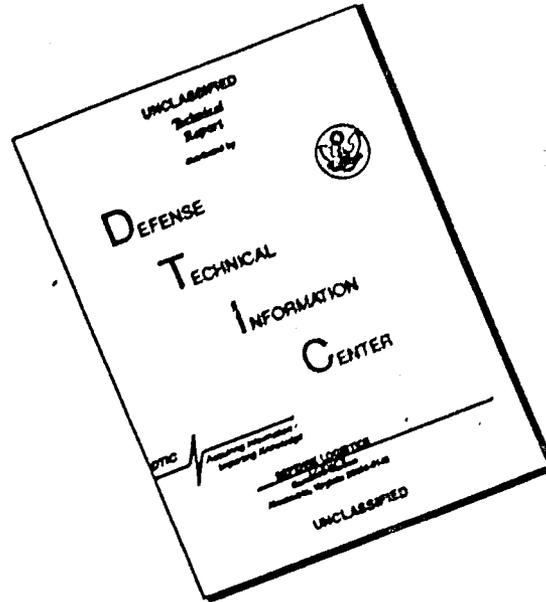
THIS DOCUMENT IS BEST QUALITY AVAILABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.

NOTICE

Reproduction of this document in any form by other than naval activities is not authorized except by special approval ot the Secretary of the Navy or the Chief of Naval Operations as appropriate.

The following Espionage notice can be disregarded unless this document is plainly marked CONFIDENTIAL or SECRET.

This document contains information affecting the national defense of the United States within the meaning of the Espionage Laws, Title 18, U.S.C., Sections 793 and 794. The transmission or the revelation of its contents in any manner to an unauthorized person is prohibited by law.

FOREWORD

This report is one of four reports to be prepared by Structural Mechanics Associates under Navy Contract No. N156-46654. This contract was initiated under Work Unit No. 530/07, "Development of Optimization Methods for the Design of Composite Structures Made from Anisotropic Material" (1-23-96) and was administered under the direction of the Aeronautical Structures Laboratory, Naval Air Engineering Center, with Messrs. R. Molella and A. Manno acting as Project Engineers. The reports resulting from this contract will be forwarded separately. Three reports are completed and cover work from 24 May 1965 to 31 December 1966. The title and approximate forwarding date for each report are as follows:

NAEC-ASL-1109, "Structural Optimization of Corrugated Core and Web Core Sandwich Panels Subjected to Uniaxial Compression," dated 15 May 1967. Forwarding date, June 1967.

NAEC-ASL-1110, "Structural Optimization of Flat, Corrugated Core and Web Core Sandwich Panels Under In-Plane Shear Loads and Combined Uniaxial Compression and In-Plane Shear Loads," dated 1 June 1967. Forwarding date, July 1967.

NAEC-ASL-1111, "A Method for Weight Optimization of Flat Truss Core Sandwich Panels Under Lateral Loads," dated 15 June 1967. Forwarding date, July 1967.

SUMMARY

This report presents the development of rational methods of structural optimization for flat, corrugated core (single truss core) and web-core sandwich panels under two loading conditions: in-plane shear loads, and combined uniaxial compression and in-plane shear loads. In the latter loading condition use is made of the methods developed in Reference 1 for these panels subjected to uniaxial compressive loads.

These methods provide a means by which minimum weight structures can be designed for a given load index, plate width, length, and face and core materials. Of equal importance is the fact that the methods developed can be used as a means of rational material selection by comparing weights of optimum construction for various material systems as a function of applied load index. The methods developed are sufficiently general to account for orthotropic or isotropic face and core materials and various boundary conditions.

In Chapter 1, methods of optimization are developed for flat triangulated core (truss core) sandwich panels subjected to in-plane shear loads. Chapter 2 provides methods of structural methods of optimization for web-core panels subjected to in-plane shear loads. In Chapter 3, a sample comparison is made between optimum construction of single-truss core, web-core, and hexagonal honeycomb core sandwich panels subjected to in-plane shear loads. Methods for the optimum honeycomb sandwich panels were developed in Reference 2 and the design procedures

presented in Reference 3.   Chapter 4 presents methods of
optimisation for triangulated core panels under combinations
of longitudinal uniaxial compressive and in-plane shear loads.
Chapter 5 treats web-core panels subjected to these combined
loadings.

In each chapter design procedures are given in detail
for the design engineer to use.

It should be noted that steady state temperature
effects are also easily accounted for by the methods developed
herein. In a panel at a given temperature condition it is
simply necessary to utilize the stress strain diagram or the
tangent modulus-stress diagram and other pertinent material
properties of the desired materials for the temperature of the
panel. Loads caused by thermal restraints must be included in
the load index along with the mechanically induced loadings.

TABLE OF CONTENTS

NOTATION

a   Panel dimension in the x direction, in.

$\overline{A}_c$ Area of the core per unit width of corrugation crossection parallel to the yz plane, in. (see Equations 1.1 and 2.1)

b   Panel dimension in the y direction, in.

d_f Dimension in the y direction over which face and core material are bonded or fastened together, in. (see Figure 4)

D_{qi} Transverse shear stiffness, per unit width, of a beam cut from the panel in the i direction (i = x,y), lbs./in. (see Equations 1.4, 1.5, 2.4, and 2.5)

D_i $\frac{1}{2} E_i t_f h_c^2$, lbs./in. (i = x,y)

E   Modulus of elasticity, lbs./in.^{2}

E_c Modulus of elasticity of corrugated core sheet material, lbs./in.^{2}

E_f Modulus of elasticity of face sheet material, lbs./in.^{2}

E_o Definitions given by Equation (4.9)

E_{si} $\left[E_{iy}^3 E_{ix}\right]^{1/4} \Big/ (1-\nu_{xyi}\,\nu_{yxi})$ $(i = c,f)$

$\overline{E}$ Reduced modulus of elasticity, lbs./in.^{2}

G_{xyi} Shear modulus, lbs./in.^{2} (i = c,f)

h_c Core depth, in.

$\bar{I}_c$ Moment of inertia of the core, per unit width of the corrugation crossection parallel to the yz plane taken about the centroidal axis of the corrugation crossection, in.^{3} (see Equations 1.2 and 2.2)

$\bar{I}_f$ Moment of inertia per unit width of the faces considered as membranes, with respect to the sandwich plate middle surface, in.^{3} (see Equations 1.8 and 2.6)

j Buckling coefficient

$\bar{J}$ Definition given by Equation (1.19)

K Buckling coefficient

K_M Definition given by Equation (1.10)

k_c Buckling coefficient for web element

k_f Buckling coefficient for face plate element

M_o Definition given by Equation (4.8)

N_x Compressive in-plane load in the x direction per unit panel width, lbs./in.

$N_{x_{cr}}$ Critical compressive load in the x direction per unit panel width, lbs./in.

N_{xy} Shear force per unit width, lbs./in. (Defined as critical shear load/inch in Chapters 1, 2, and 3, defined as applied shear load/inch in Chapters 4 and 5)

$N_{xy_{cr}}$ Critical shear load/inch in Chapters 4 and 5

S $\dfrac{h_c^2}{t_c^2} \; \sin^2\theta \; \cos\theta$

t_c Thickness of core web, in.

t_f Thickness of facing material, in.

V_i Core transverse shear flexibility parameter ($i = x,y$)

W Total weight per unit planform area of panel construction, lbs./in.^{2}

W_i Weight per unit planform area of core ($i = c$) or facing ($i = f$) materials

W_{ad} Weight of adhesive or other joining material between facing and core per unit planform area, lbs./in.^{2}

x Panel in-plane coordinate (see Figure 2)

y Panel in-plane coordinate (see Figure 2)

z Panel coordinate normal to mid-plane of panel (see Figures 1 and 4)

α $(N_{xy/b}) / (N_{x/b})$

ε_{xy} Shear strain (in./in.)

γ Definition given by Equation (5.4)

γ_i Shear angle (see Figure 3) ($i = c,f$)

η Plasticity reduction factor

ϕ Definition given by Equation (4.11)

θ Angle web material makes with a line normal to plane of faces

ν Poisson's ratio

ρ_i Density, lbs./in.^{3} (i = c,f)

σ Stress, psi

σ_M Principal stress, psi

τ_i Shear stress, psi   (i = c,f)

Ω_0 Definition given by Equation (4.10)

CHAPTER 1

METHODS OF STRUCTURAL OPTIMIZATION FOR FLAT, TRIANGULATED
(SINGLE TRUSS) CORE SANDWICH PANELS SUBJECTED TO IN-PLANE
SHEAR LOADS

A. INTRODUCTION

Consider a flat corrugated core sandwich panel
idealized by the following construction, shown in crossection
in Figure 1.
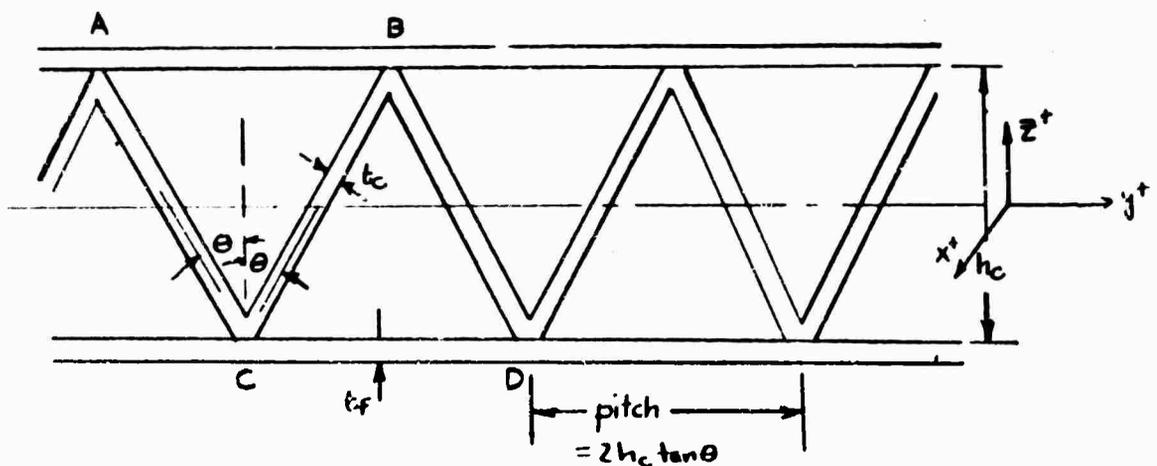
Figure 1

Triangulated Core Sandwich Panel Construction

There are four geometric variables with which to
optimize; namely, the core depth (h_c), the web thickness (t_c),

the face thickness (t_f), and the angle the web makes with a line normal to the faces (Θ).

The overall panel to be considered is shown in planform in Figure 2.
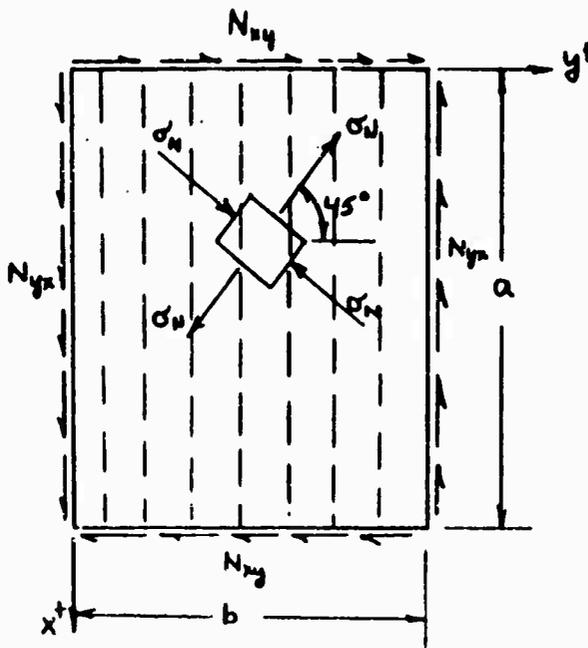
Figure 2

Planform View of the Panel

This panel of width b and length a is subjected to in-plane shear loads N_{xy} and N_{yx} (lb./in.) as shown in Figure 2.

This panel is considered to fail if any of the following instabilities occur: overall instability, local face shear instability, and shear instability in the web. Thus there are

- 2 -

three modes of instability, and four geometric variables with
which to optimize. To describe each instability, the
analytical expression used in the following is the best
available from the literature.

B. ELASTIC AND GEOMETRIC CONSTANTS ASSOCIATED WITH TRIANGULATED CORE CONSTRUCTION

The elastic and geometric constants for the triangulated core construction can be determined from those given in more general form by Libove and Hubka in Reference 4. In the core construction given in Figure 1, the following constants are obtained.

The area of the core per unit width of corrugation crossection parallel to the yz plane, $\overline{A}_c$, is given by

$$\overline{A}_c = \frac{t_c}{\sin \theta} \qquad (\text{in.}) \qquad (1.1)$$

where the symbols are defined in Figure 1.

The moment of inertia of the core per unit width of corrugation crossection parallel to the yz plane, taken about the centroidal axis of the corrugation crossection, $\overline{I}_c$, is seen to be,

$$\overline{I}_c = \frac{t_c h_c^2}{12 \sin \theta} - \frac{\overline{A}_c h_c^2}{12} \qquad (\text{in.}^3) \qquad (1.2)$$

The extensional stiffness of the plate in the x direction, EA_x, is given by

$$EA_x = E_c \overline{A}_c + 2 E_f t_f \qquad (\text{lbs./in.}) \qquad (1.3)$$

where E_c and E_f are the moduli of elasticity of the core and face material respectively.

- 4 -

The transverse shear stiffness, per unit width in the y direction, of an element of the sandwich cut by two xz planes, D_{qx}, is found to be

$$D_{qx} = \frac{G_c t_c \cos \theta}{\tan \theta} \qquad (\text{lb./in.} \qquad (1.4)$$

The transverse shear stiffness, per unit width in the x direction, of an element of the sandwich cut by two yz planes, D_{qy}, is given by

$$D_{qy} = \frac{Sh_c E_c}{(1-\nu_c^2)} \frac{t_c^3}{h_c^3} \qquad (\text{lb./in.}) \qquad (1.5)$$

where ν_c is the Poisson's ratio of the core material, and

$$S = \frac{h_c^2}{t_c^2} \sin^2 \theta \cos \theta \qquad (1.6)$$

Hence substituting (1.6) into (1.5) results in

$$D_{qy} = \frac{E_c t_c}{(1-\nu_c^2)} \cos^2 \theta \sin \theta \qquad (1.7)$$

This expression agrees with that determined by Anderson in Reference 5.

Lastly, the moment of inertia per unit width, $\overline{I}_f$, of the faces, considered as membranes with respect to the sandwich plate middle surface, is seen to be

$$\overline{I}_f = \frac{t_f h_c^2}{2} \qquad (\text{iu.}^3) \qquad (1.8)$$

C. GOVERNING EQUATIONS FOR PANELS WITH FACES AND CORES OF DIFFERENT ORTHOTROPIC MATERIALS

Since panels in which the faces and cores utilize different orthotropic materials is the most general materials system considered in this study, it is convenient to derive all expressions and subsequently perform the optimization for this case first.

1. Overall Stability

The best analytical expression describing the overall instability of a triangulated core sandwich panel under in-plane shear loading is given by Equation (B), paragraph 4.2.2.1, page 84 of Reference 6. That equation, put in the terminology used in this study, is written as

$$\tau_F = \frac{(E_{fy}^3 E_{fx})^{1/4}}{(1 - \nu_{fy} \nu_{yxf})} \left(\frac{h_s}{b}\right)^2 \frac{1}{1 + 4\left[\frac{K_M(v=0)}{K_M} - 1\right]} \, j \tag{1.9}$$

where:

$$K_M = \frac{B_1 C_1 + 2B_2 C_2 + \frac{C_3}{B_1} + A\left[\frac{V_y}{C_4} + V_x\right]}{1 + \left(B_1 C_1 + B_3 C_2\right)\frac{V_y}{C_4} + \left(\frac{C_3}{B_1} + B_2 C_2\right)V_x + \frac{V_x V_y}{C_4} A} \tag{1.10}$$

$$A = C_1 C_3 - B_2^2 C_2^2 + B_3 C_2 \left(B_1 C_1 + 2 B_2 C_2 + \frac{C_3}{B_1}\right) \tag{1.11}$$

$$B_1 = \sqrt{\frac{E_{fy}}{E_{fx}}} \tag{1.12}$$

- 6 -

$$B_2 = \frac{2\, G_{xyf}\, (1 - \nu_{xyf}\, \nu_{xyf}) + E_{fy}\, \nu_{xyf}}{\sqrt{E_{fy}\, E_{fx}}} \tag{1.13}$$

$$B_3 = \frac{G_{xyf}\, (1 - \nu_{xyf}\, \nu_{yxf})}{\sqrt{E_{fy}\, E_{fx}}} \tag{1.14}$$

$$V_x = \frac{\pi^2}{b^2}\ \sqrt{\frac{D_x D_y}{D_{qx}}}\ -\ \frac{\pi^2}{2}\ \frac{\tan\theta}{\cos\theta}\ \left(\frac{t_f}{t_c}\right)\ \left(\frac{h_c}{b}\right)^2\ \frac{\sqrt{E_{fx} E_{fy}}}{G_{xyc}} \tag{1.15}$$

$$\left. \begin{aligned} V_y &= \frac{\pi^2}{b^2}\ \sqrt{\frac{D_x D_y}{D_{qy}}}\ -\ \frac{\pi^2}{2}\ \frac{(1 - \gamma_{xyc}\, \gamma_{yxc})}{(1 - \nu_{xyf}\, \gamma_{yxf})}\ \frac{\sqrt{E_{fx} E_{fy}}}{\sqrt{E_{cx} E_{cy}}}\ \times \\[2mm] &\quad \left(\frac{t_f}{t_c}\right)\ \left(\frac{h_c}{b}\right)^2\ \frac{1}{\cos^2\theta\, \sin\theta} \end{aligned} \right\} \tag{1.16}$$

C_1 through C_4 are given by Equations (7) through (10) in Reference 2 for various boundary conditions, where for shear loading n = 1 only in the expressions.

In the above subscripts c and f refer to the core and face respectively, and G_{xy} refers to the in-plane shear stiffness of the material. τ is the in-plane shear stress, and K_M (V=0) refers to the value of Equation 1.10 where $V_x = V_y = 0$.

The coefficient j is found by Figure 4-1, page 84, Reference 6 for orthotropic panels with simply supported edges whose axes of elastic symmetry are parallel to the edges. In this Figure, j is plotted as functions of B_2 above and $1/r$ where

$$\frac{1}{r} = \frac{b}{a} \left(\frac{E_{fx}}{E_{fy}}\right)^{1/4} \tag{1.17}$$

It is convenient to rewrite Equation 1.9 as

$$\tau_f = \frac{(E_{fy}^3 E_{fx})^{1/4}}{(1-\nu_{xyf}\nu_{yxf})} \left(\frac{h_c}{b}\right)^2 \overline{j} \tag{1.18}$$

where $\overline{j} = \dfrac{j}{1 + 4\left[\dfrac{K_M(V=0)}{K_M} - 1\right]}$ (1.19)

2. Face Plate Instability

Looking at Figure 1, it is seen that each plate element of the faces from A to B can buckle due to the applied shear loads N_{xy} and N_{yx}. Since the support condition of the plate element along the edges depicted by A and B are not known precisely, it is conservative to assume that they are simply supported edges. For such a case, the governing equation is given by Equation (9-29) of Reference 7, for an orthotropic plate whose axes of elastic symmetry are parallel to the edges. Placing the equation in the terminology used herein, and since $N_{xy} = \tau h$, where h is the plate thickness

$$\tau = \frac{k}{3} \quad \frac{\left[E_y^{\ 3} E_x \right]^{1/4}}{(1 - \nu_{xy} \nu_{yx})} \quad \left(\frac{h}{b} \right)^2 \tag{1.20}$$

In this expression k is a coefficient plotted in Reference 7, Figure 9-42, as a function of two parameters β and $\frac{1}{\theta}$. It is obvious that if $\beta = \frac{1}{r}$ and $\frac{1}{\theta} = B_2$, then Figure 9-42 of Reference 7 and Figure 4-1 of Reference 6 are identical. Hence, the k of Equation 1.20 is identical to the j of Equation (1.19).

From Figure 1, it is seen that for the face plate instability h = t_f and b = 2 h_c tan θ, hence, Equation (1.20) is written as

$$\tau_f = \frac{k_f}{12} \quad \frac{E_{fy}^{\ 3} E_{fx}^{\ 1/4}}{(1 - \nu_{xyf} \nu_{yxf})} \quad \frac{t_f^{\ 2}}{h_c^{\ 2} \tan^2 \theta} \tag{1.21}$$

where k_f is determined as j in Figure 4-1, Reference 6, in which for this plate element B_2 is given by Equation (1.13) and

$$\frac{1}{r} = \frac{2 h_c \tan \theta}{a} \quad \left(\frac{E_{fx}}{E_{fy}} \right)^{1/4} \tag{1.22}$$

3. Web Plate Instability

Likewise the local plate elements of the triangulated core can become unstable due to shear stresses induced into the core by the shearing of web faces. Again, the conservative assumption is made here that the web elements are simply supported

- 9 -

along the edges A and C depicted in Figure 1.

Referring to Equation (1.20) and the geometry of Figure 1, it is seen that $h = t_c$, $b = h_c/\cos \theta$, and the expression describing the web plate instability is

$$\tau_c = \frac{k_c}{3} \frac{\left[E_{yc}^3 E_{xc}\right]^{1/4}}{(1-\nu_{xyc}\nu_{yxc})} \frac{t_c^2}{h_c^2} \cos^2 \theta \qquad (1.23)$$

where k_c is found by Figure 4-1 of Reference 6, where

$$B_2 = \frac{2 G_{xyc} (1-\nu_{xyc}\nu_{yxc}) + E_{cy}\nu_{xyc}}{\sqrt{E_{cx}E_{cy}}} \qquad (1.24)$$

and

$$\frac{1}{r} = \frac{h_c}{a \cos \theta} \left(\frac{E_{cx}}{E_{cy}}\right)^{1/4}. \qquad (1.25)$$

4. Load-Stress Relationship

Looking at the construction shown in Figure 2 along the edges at $x = 0$ or $x = a$, the shear resultant N_{xy} is primarily resisted by the two faces. Even if the core elements are bonded to or otherwise fastened to some edge fixture through which the shear N_{xy} is transmitted little load will be introduced into the core web plates directly. The small "stiffness" of the web plates to loads in the y direction relative to the large "stiffness" of the facings results in the primary load path being the faces. Hence, the load stress relationship is taken to be

$$\frac{N_{yx}}{2t_f} = \frac{N_{xy}}{2t_f} = \tau_F \qquad (1.26)$$

- 10 -

This is not to imply that loads are not introduced into the core elements by the faces as the mechanics of this behavior described below shows.

5. Core Stress - Face Stress Relationship

Consider the repeated unit of the triangulated core construction shown in Figure 3.
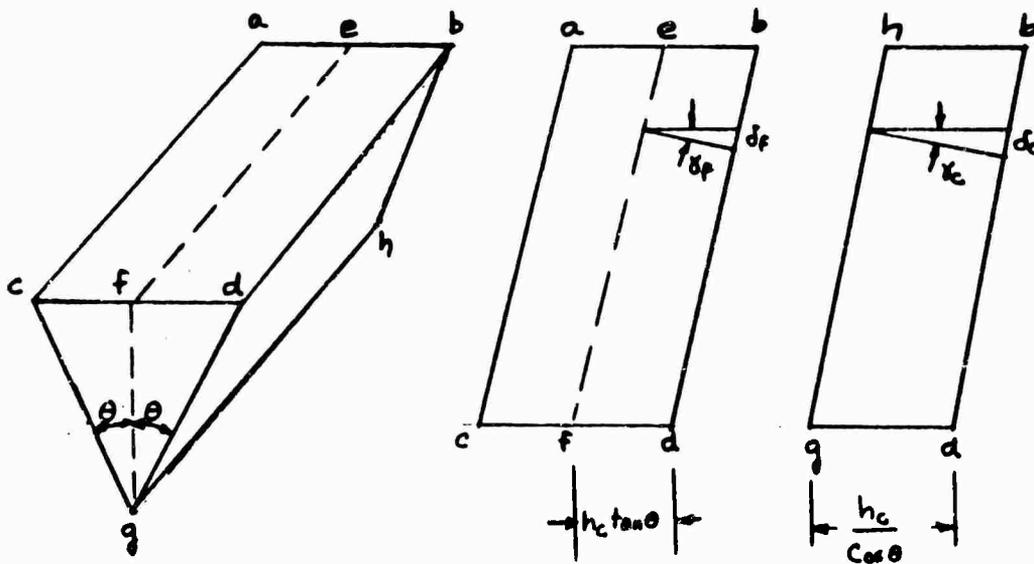
| Unit of Construction Prior to Deformation | Face Element Deformed (Planform) | Core Element Deformed (Planform) |

Figure 3

Due to the shearing deformations of the faces, shearing deformations occur in the core since the web element and the face are bonded or otherwise connected at their junctions along db, gh, etc.

From elementary elasticity the following relationships are valid where ε_{xy_i} (i=c,f) are the shearing strains and the other symbols are given in Figures 1, 2, and 3.

$$\varepsilon_{xyf} = \frac{1}{2G_{xyf}} \tau_f \qquad\qquad \varepsilon_{xyc} = \frac{1}{2G_{xyc}} \tau_c$$

$$2\varepsilon_{xyf} = \gamma_f \qquad\qquad 2\varepsilon_{xyc} = \gamma_c$$

$$\gamma_f = \frac{\delta_f}{h_c \tan\theta} \qquad\qquad \gamma_c = \frac{d_c}{h_c} \cos\theta$$

$$\therefore \delta_f = \frac{\tau_f h_c \tan\theta}{G_{xyf}} \qquad\qquad \therefore \delta_c = \frac{\tau_c h_c}{G_{xyc} \cos\theta}$$

But for compatability of deformations $\delta_c = \delta_f$, hence

$$\tau_c = \frac{G_{xyc}}{G_{xyf}} \tau_f \sin\theta \qquad\qquad (1.27)$$

It is obvious that if G_{xyc} or θ are zero there is no stress induced into the core element.

6. Weight Relationship

The weight relationship is seen to be, from Figure 1,

$$W = 2 \rho_f t_f + \rho_c \overline{A}_c + W_{ad} \quad \text{or}$$

$$W - W_{ad} = 2 \rho_f t_f + \frac{\rho_c t_c}{\sin \theta} \quad (1.28)$$

where ρ_f and ρ_c are the weight density of the face and core materials respectively;

W_{ad} is the weight in lbs./in.^{2} of planform area of the adhesive or any other material used to join face and core; W is weight in lbs./in^2 of planform area of the entire panel.

D. STRUCTURAL OPTIMIZATION OF PANELS WITH FACES AND CORE
OF DIFFERENT ORTHOTROPIC MATERIALS

The governing equations pertaining to this construc-
tion to be used in the optimization are given by Equations
(1.18), (1.21), (1.23), (1.26), (1.27), and (1.28), and are
repeated below for convenience:

$$\tau_f = \frac{(E_{fy}{}^3 E_{fx})^{1/4}}{(1-\nu_{xyf}\nu_{yx\cdot})} \left(\frac{h_c}{b}\right)^2 \; J \tag{1.29}$$

$$\epsilon_f = \frac{k_f}{12} \; \frac{(E_{fy}{}^3 E_{fx})^{1/4}}{(1-\nu_{xyf}\nu_{yxf})} \; \frac{t_f{}^2}{h_c{}^2 \; \tan \theta} \tag{1.30}$$

$$\tau_c = \frac{k_c}{3} \; \frac{(E_{cy}{}^3 E_{cx})^{1/4}}{(1-\nu_{xyc}\nu_{ycx})} \; \frac{t_c{}^2}{h_c{}^2} \; \cos^2 \theta \tag{1.31}$$

$$N_{xy} = N_{yx} = 2 \, \tau_f t_f \tag{1.32}$$

$$\tau_c = \frac{G_{xyc}}{G_{xyf}} \; \tau_f \; \sin \theta \tag{1.33}$$

$$\frac{W - W_{ad}}{b} = 2 \, \rho_f t_f + \frac{\rho_c t_c}{\sin \theta} \tag{1.34}$$

The philosophy of optimization is as follows: A
truly optimum structure is one which has a unique value for

each dependent variable within the class of structure being
- .udied (triangulated core sandwich panel, for example),
for each set of materials (7075-T6 clad aluminum, for
example), for each set of boundary conditions (simply
supported on all edges, for example), and is the minimum
possible weight for a specified set of design loads (in-
plane shear load per unit width, N_{xy}, for example), and
will maintain its structural integrity (no mode of failure
will occur at a load less than the design load). In this case
the optimum (minimum weight) structure will have the
characteristic that the panel will become unstable in all
three buckling modes simultaneously. If this is not the case
then one of the failure modes corresponding to a face stress,
say τ_1 , occurs at a higher value of stress than the other
two, say $\tau_2 = \tau_3 < \tau_1$. However, the panel will fail at a
load corresponding to the lower face stresses τ_2 and τ_3 ,
say $N_2 = N_3$. This in turn means that there exists material
(which of course has weight) in the structure which is not
being stressed or strained sufficiently for it to be used most
efficiently. Thus there are two alternatives available:
(1) Material can be removed until the failure mode originally
occurring at τ_1 is reduced to the critical stress $\tau_2 = \tau_3$,
in which case a lighter structure results for an applied load
$N_2 = N_3$. (2) Material can be rearranged, reducing the critical
stress value originally at τ_1 corresponding to the first mode
to some value τ_4, while raising the critical face stress values

- 15 -

associated with the other two failure modes originally occurring at τ_2 and τ_3 to a value $\tau_4 \left(\tau_1 > \tau_4 > \tau_2 = \tau_3 \right)$ Now the structure with the same weight as the original arrangement can withstand a load N_4 (corresponding to τ_4) where $N_4 > N_2 = N_3$, hence a greater load carrying ability for a given weight. Obviously both (1) and (2) can be performed simultaneously so that some material is removed and some rearranged, resulting in an optimum, minimum weight structure.

Returning to the Equations (1.29) through (1.34), the known or specified quantities are the applied shear load per inch N_{xy}, and the panel width b, which can be lumped together as the load index (N_{xy}/b); the material properties; and the panel boundary conditions. The buckling coefficients j, k_f and k_c are constants for any given set of variables and hence are constants for the optimum construction being sought.

The dependent variables in the set, with which to optimize the construction are the face thickness, t_f, the core depth, h_c, the web material thickness, t_c, the web angle, Θ, the face stress, τ_f, the core stress, τ_c, and the weight, $W - W_{ad}$.

It is seen that there are six equations and seven unknowns. The seventh equation is obtained by placing the weight equation in terms of one convenient variable, taking the derivative of the weight equation with respect to this variable, and equating it to zero to obtain the unique value of the

variable which results in a minimum weight structure.

Manipulation of Equations (1.29) through (1.34) results in an expression for the weight equation involving only the dependent variable θ, as shown below.

$$\frac{W-W_{ad}}{b} = \frac{3^{k_4}}{2} \frac{P_f}{(k_f \bar{j})^{k_4}} \frac{(N_{xy}/b)^{1/2}}{E_{sf}^{1/2}}$$

$$X \left\{ \frac{4(\sin\theta)^{3/2} + \left(\frac{P_c}{P_f}\right)\left(\frac{G_{xyc}}{G_{xyf}}\right)^{1/2}\left(\frac{E_{sf}}{E_{sc}}\right)^{1/2}\left(\frac{k_f}{k_c}\right)^{1/2}}{(\sin\theta)(\cos\theta)^{1/2}} \right\} \qquad (1.35)$$

where $E_{si} = \left[E_{1y}^3 E_{1x}\right]^{1/4} / (1-\gamma_{xyi}\nu_{yxi})$, $(i = c,f)$. $\qquad (1.36)$

Taking the derivative of (1.35) with respect to θ, and equating it to zero results in a value of θ in terms of material properties and buckling coefficients which will result in minimum weight structure. This expression is:

$$2(\sin\theta)^{3/2}(\cos\theta)^2 + 2(\sin\theta)^{7/2}$$

$$- \left(\frac{P_c}{P_f}\right)\left(\frac{G_{xyc}}{G_{xyf}}\right)\left(\frac{E_{sf}}{E_{sc}}\right)^{1/2}\left(\frac{k_f}{k_c}\right)^{1/2}\left[\cos^2\theta - \frac{1}{2}\sin^2\theta\right] = 0 \qquad (1.37)$$

Note that the optimum web angle is independent of the load to which the panel is subjected.

A universal relationship" may be obtained from Equations (1.29) through (1.34) which relates the load index N_{xy}/b to a unique value of face shear stress τ_f for any set of material properties, which will result in minimum weight.

For a given load index N_{xy}/b, a panel designed which has a face stress τ_f <u>higher</u> or <u>lower</u> than the value given by the following relationship will result in a panel which has a weight greater than can be achieved if this universal relationship is used.

$$\frac{N_{xy}}{b} = \frac{4\sqrt{3}\ \tan\theta}{k_f^{1/2}\ \bar{J}^{1/2}} \quad \frac{\tau_f^2}{E_{sf}} \tag{1.38}$$

where θ is obtained from (1.37).

The remaining geometric variables t_f, t_c, and h_c, as well as the weight equation can now be expressed in terms of the optimum face stress τ_f obtained in (1.38) above.

$$\frac{t_f}{b} = \frac{2\sqrt{3}\ \tan\theta\ \tau_f}{k_f^{1/2}\bar{J}^{1/2}\ E_{sf}} \tag{1.39}$$

$$\frac{t_c}{b} = \frac{\sqrt{3}}{k_c^{1/2}\ \bar{J}^{1/2}} \frac{(\sin\theta)^{1/2}}{\cos\theta} \left(\frac{G_{xyc}}{G_{xyf}}\right)^{1/2} \frac{\tau_f}{E_{sc}^{1/2}E_{sf}^{1/2}} \tag{1.40}$$

$$\frac{h_c}{b} = \frac{1}{\bar{J}^{1/2}} \quad \frac{\tau_f^{1/2}}{E_{sf}^{1/2}} \tag{1.41}$$

$$\frac{W - W_{eff}}{b} = \frac{\sqrt{3}\, \rho_f\, \tau_f}{k_f^{1/2}\, j^{1/2}\, E_{sf}} \left\{ \frac{4(\sin\theta)^{3/2} + \left(\frac{\rho_c}{\rho_f}\right)\left(\frac{G_{xyc}}{G_{xyf}}\right)^{1/2}\left(\frac{E_{sf}}{E_{sc}}\right)^{1/2}\left(\frac{k_f}{k_c}\right)^{1/2}}{\cdot (\cos\theta)\,(\sin\theta)^{1/2}} \right\} \quad (1.42)$$

Of course Equations (1.39) through (1.41) can be expressed in terms of the load index by the proper substitution of the universal relationship Equation (1.38). However, the expressions are more complicated to use than those expressed in terms of the optimum face stress.

Another useful relationship is expressed below, using Equations (1.39) and (1.40).

$$\frac{t_c}{t_f} = \frac{1}{2}\left(\frac{G_{xyc}}{G_{xyf}}\right)^{1/2}\left(\frac{E_{sf}}{E_{sc}}\right)^{1/2}\left(\frac{k_f}{k_c}\right)^{1/2}\frac{1}{(\sin\theta)^{1/2}} \quad (1.43)$$

Detailed design procedures for this type of construction are given in summary in Section H of this chapter.

- 19 -

E. STRUCTURAL OPTIMIZATION OF PANELS WITH FACES AND CORE OF DIFFERENT ISOTROPIC MATERIALS

By making the following contractions of the results of the previous section, the optional relationships are obtained for panels with faces and cores of different isotropic materials.

$$\nu_{xyi} = \nu_{yxi} = \nu_i \qquad (i = c, f)$$

$$E_{ei} = \frac{E_i}{(1-\nu_c^2)} \qquad (i = c, f) \tag{1.44}$$

The expression yielding the optimum value of the web angle Θ is given by

$$2(\sin\Theta)^{3/2}\cos^2\Theta + 2(\sin\Theta)^{1/2} - \left(\frac{\rho_c}{\rho_f}\right)\left(\frac{1-\nu_c}{1-\nu_f}\right)^{1/2}\left(\frac{k_f}{k_c}\right)^{1/2}\left[\cos^2\Theta - \frac{1}{2}\sin^2\Theta\right] = 0 \tag{1.45}$$

Note that the optimum web angle is independent of the applied shear load.

The universal relationship is given by

$$\frac{N_{xy}}{b} = \frac{4\sqrt{3}\,(1-\nu^2)}{k_f^{1/2}\,\bar{j}^{1/2}}\quad \tan\Theta\quad \frac{\tau_f^2}{E_f} \tag{1.46}$$

The other geometric variables as well as the weight equation given in terms of the optimum stress are seen to be,

$$\frac{t_f}{b} = \frac{2\sqrt{3}\,(1-\nu_f^2)\,\tan\Theta\,\tau_f}{k_f^{1/2}\,\bar{j}^{1/2}\,E_f} \tag{1.47}$$

$$\frac{t_c}{b} = \frac{\sqrt{3}\left(1-v_c^2\right)^{1/2}\left(1-v_f^2\right)^{1/2}}{k_c^{1/2} \, J^{1/2} \, E_c^{1/2} \, E_f^{1/2}} \left(\frac{G_{xyc}}{G_{xyf}}\right)^{1/2} \frac{\left(\sin\theta\right)^{1/2}}{\cos\theta} \, \tau_F \tag{1.48}$$

$$\frac{h_c}{b} = \left[\frac{1-v_f^2}{E_f} \frac{\tau_F}{J}\right]^{1/2} \tag{1.49}$$

$$\frac{W - W_{eb}}{b} = \frac{\sqrt{3} \, \rho_F \left(1-v_f^2\right) \tau_F}{J^{1/2} \, k_f^{1/2} \, E_F} \left\{ \frac{4\left(\sin\theta\right)^{3/2} + \left(\frac{\rho_c}{\rho_f}\right)\left(\frac{1-v_c^2}{1-v_f^2}\right)^{1/2}\left(\frac{k_f}{k_c}\right)^{1/2}}{\left(\cos\theta\right)\left(\sin\theta\right)^{1/2}} \right\} \tag{1.50}$$

It is also seen that

$$\frac{t_c}{t_F} = \frac{1}{2}\left(\frac{k_F}{k_c}\right)^{1/2}\left(\frac{1-v_c}{1-v_F}\right)^{1/2}\frac{1}{\left(\sin\theta\right)^{1/2}} \tag{1.51}$$

Detailed design procedures for this type of construction are given in summary in Section H of this chapter.

F. STRUCTURAL OPTIMIZATION OF PANELS WITH FACES AND CORE OF THE SAME ORTHOTROPIC MATERIAL

By contracting the expressions of Section D for panels with faces and core of different orthotropic materials by the following substitution,

$$
\begin{Bmatrix} \nu_{xyc} \\ \nu_{yxc} \\ E_{sc} \end{Bmatrix} = \begin{Bmatrix} \nu_{xyf} \\ \nu_{yxf} \\ E_{sf} \end{Bmatrix} = \begin{Bmatrix} \nu_{xy} \\ \nu_{yx} \\ E_{s} \end{Bmatrix} \tag{1.52}
$$

the resulting expressions analogous to the previous two sections are found.

The optimum web angle Θ is determined by

$$
2 (\sin\theta)^{3/2} \cos^2\theta + 2(\sin\theta)^{7/2} - \left(\frac{k_F}{k_c}\right)^{1/2}\left[\cos^2\theta - \frac{1}{2}\sin^2\theta\right] = 0 \tag{1.53}
$$

Note that Θ is completely independent of all material properties as well as the applied shear load. In particular, if $k_f = k_c$, $\Theta = 28.40^\circ$.

The universal relationship is

$$
\frac{N_{xy}}{b} = \frac{4\sqrt{3} \, \tan\Theta}{k_F^{1/2} \, j^{1/2}} \, \frac{\tau_F^2}{E_s} \tag{1.54}
$$

The other important relationships are given by

$$
\frac{t_F}{b} = \frac{2\sqrt{3} \, \tan\Theta \, \tau_F}{k_F^{1/2} \, j^{1/2} \, \bar{E}_s} \tag{1.55}
$$

$$\frac{t_c}{b} = \frac{\sqrt{3}}{k_c^{1/2} \, \bar{j}^{1/2}} \, \frac{(\sin\theta)^{1/2}}{\cos\theta} \, \frac{\tau_F}{E_s} \tag{1.56}$$

$$\frac{h_c}{b} = \left[\frac{\tau_F}{E_s \, \bar{j}}\right]^{1/2} \tag{1.57}$$

$$\frac{W - W_{ad}}{b} = \frac{\sqrt{3}\,\rho\,\tau_F}{k_f^{1/2}\,\bar{j}^{1/2}\,E_s} \left\{ \frac{4(\sin\theta)^{3/2} + \left(\frac{k_f}{k_c}\right)^{1/2}}{\cos\theta \, (\sin\theta)^{1/2}} \right\} \tag{1.58}$$

$$\frac{t_c}{t_F} = \frac{1}{2}\left(\frac{k_f}{k_c}\right)^{1/2} \frac{1}{(\sin\theta)^{1/2}} \tag{1.59}$$

Simplifying the weight equation (1.35) for this class of material systems, the minimum weight panels for a given load index N_{xy}/b is given by

$$\frac{W - W_{ad}}{b} = \frac{3^{1/4}}{2} \frac{\rho}{k_f^{1/4}\,\bar{j}^{1/4}} \frac{(N_{xy}/b)^{1/2}}{E_s^{1/2}} \left\{ \frac{4(\sin\theta)^{3/2} + \left(\frac{k_f}{k_c}\right)^{1/2}}{\sin\theta \, (\cos\theta)^{1/2}} \right\} \tag{1.60}$$

Two conclusions are drawn. First, the best ortho-tropic material to use in such construction is the one with the highest ratio of $\dfrac{\left[E_y^3 \, E_x\right]^{1/8}}{\rho\left(1 - \nu_{xy}\nu_{yx}\right)^{1/4}}$.

Secondly, the ratio of face weights to core weight is

$$\frac{W_F}{W_c} = \frac{4\,(\sin\theta)^{3/2}}{(k_f/k_c)^{1/2}} . \tag{1.61}$$

In the case of $k_r = k_c$, $\theta = 28.4^\circ$ and $W_r/W_c = 1.316$.

Detailed design procedures for this type of construction are given in summary in Section H of this chapter.

0. STRUCTURAL OPTIMIZATION OF PANELS WITH FACES AND CORE OF THE SAME ISOTROPIC MATERIAL

Analogous to the previous sections it is easily seen that the important optional relationships for this class of material systems are as follows.

The optimum web angle θ is determined from the equation

$$2\left(\sin\theta\right)^{3/2}\cos^2\theta + 2\left(\sin\theta\right)^{1/2} + \left(\frac{k_f}{k_c}\right)^{1/2}\left[\frac{1}{2}\sin^2\theta - \cos^2\theta\right] \tag{1.62}$$

Again, the optimum web angle θ is independent of the material used as well as the applied load. For $k_c = k_f$, the optimum web angle $\theta = 28.40°$.

Having found the optimum web angle θ, the universal relationship is given by

$$\frac{N_{xy}}{b} = \frac{4\sqrt{3}\left(1-\nu^2\right)\tau_f^2\tan\theta}{k_f^{1/2}\,j^{1/2}\,E} \tag{1.63}$$

The other relations defining the optimum construction are:

$$\frac{t_F}{b} = \frac{2\sqrt{3}\left(1-\nu^2\right)\tan\theta\,\tau_f}{k_f^{1/2}\,j^{1/2}\,E} \tag{1.64}$$

$$\frac{t_c}{b} = \frac{\sqrt{3}\left(1-\nu^2\right)\left(\sin\theta\right)^{1/2}\tau_f}{k_c^{1/2}\,j^{1/2}\,E\,\cos\theta} \tag{1.65}$$

$$\frac{h_c}{b} = \left[\frac{\left(1-\nu^2\right)\tau_f}{E\,j}\right]^{1/2} \tag{1.66}$$

- 24 -

$$\frac{W-W_{ad}}{b} = \frac{\sqrt{3}\,\rho\,\tau_F}{k_f^{1/2}\,j^{-1/2}}\,\frac{1-\nu^2}{E}\left\{\frac{4\,(\sin\theta)^{3/2}+\left(\frac{k_f}{k_c}\right)^{1/2}}{\cos\theta\,(\sin\theta)^{1/2}}\right\} \tag{1.67}$$

$$\frac{t_c}{t_f} = \frac{1}{2}\left(\frac{k_f}{k_c}\right)^{1/2}\frac{1}{(\sin\theta)^{1/2}} \tag{1.68}$$

Simplifying the weight equation given by (1.35) for this class of material systems, the weight equation in terms of the load index is seen to be:

$$\frac{W-W_{ad}}{b} = \frac{(3)^{1/4}(1-\nu^2)^{1/2}\rho\left(N_{xy}/b\right)^{1/2}}{2\,E^{1/2}\,j^{-1/4}\,k_f^{1/4}}\left\{\frac{4(\sin\theta)^{3/2}+\left(k_f/k_c\right)^{1/2}}{\sin\theta\,(\cos\theta)^{1/2}}\right\} \tag{1.69}$$

Similar to the previous section, it is seen that the best material to use in this type of construction for these loading conditions is the material with the highest value of $E^{1/2}/\rho(1-\nu^2)^{1/2}$. Secondly, the ratio of face to core weight is

$$\frac{W_F}{W_c} = \frac{4\,(\sin\theta)^{3/2}}{(k_f/k_c)^{1/2}} \tag{1.70}$$

If $k_f = k_c$, $\theta = 28.4°$, and $\dfrac{W_f}{W_c} = 1.316$.

Detailed design procedures for this type of construction are given in summary in Section H of this chapter.

H. DESIGN PROCEDURES FOR PANELS OF OPTIMUM TRIANGULATED CORE (SINGLE TRUSS CORE) CONSTRUCTION UNDER IN-PLANE SHEAR LOADS

Prior to discussing the detailed design procedures, it is advantageous to discuss certain characteristics of the coefficients $\bar{J}$, k_f, and k_c which result in significant simplifications to the design procedures. From Equation (1.19) it is seen that

$$\bar{J} = \frac{1}{1 + 4\left[\dfrac{K_M(V=0)}{K_M} - 1\right]} \tag{1.71}$$

The coefficient j is determined in a straight forward manner from Figure 4-1 of Reference 6, in which it is plotted as a function of B_2, given by Equation (1.13). (Note $B_2 = 1$ for an isotropic material), and $1/r = (b/a) (E_{fc}/E_{fy})^{1/4}$.

In (1.71), K_M is a function of the core transverse shear flexibility parameters, V_x and V_y, given by Equations (1.15) and (1.16). It is found that in many material systems and a broad spectrum of values of (N_{xy}/b), V_x and V_y are very small. For instance, for a square panel composed of 7075 - T6 clad aluminum alloy, for a load as high as that corresponding to $\tau_f = 40,000$ psi (2000 psi below the ultimate shear stress), $V_x = 1.10 \times 10^{-3}$ and $V_y = 1.8 \times 10^{-3}$.

Hence, in most numerical calculations $K_M (V=0) \approx K_M$, resulting in

$$\bar{J} = J \tag{1.72}$$

This can be determined at the outset of a design and hence an iteration is not needed to design the panel.

Similarly k_f is determined as the coefficient j in Figure 4-1 of Reference 6 plotted as a function of B_2 given by Equation (1.13) and $1/r = (2 h_c \tan \theta/a) (E_{fx}/E_{fy})^{1/4}$. Since in most panels $h_c/a \ll 1$, $1/r \approx 0$. In this case k_f can be obtained, without iterating, from Figure 4-1, Reference 6 at the outset. Since k_f is nearly constant for $0 < 1/r < 0.1$, it would appear that only a very unusual combination of materials and geometry would require an iteration.

Likewise k_c is determined as the coefficient j in Figure 4-1 of Reference 6, plotted as a function of B_2 given by Equation (1.24), and $1/r = (h_c/a \cos \theta) (E_{cx}/E_{cy})^{1/4}$. As above $h_c/a \ll 1$ so that $1/r \approx 0$. Hence, k_c can be determined in most cases at the outset for $1/r = 0$.

Note also that as a result of the above discussion usually assuming $k_f/k_c = 1$ is valid.

Turning to the design procedures, utilizing the expressions derived in the previous sections, there are several ways to proceed to design a panel. However, to save time and effort in developing design curves for panels of this type of construction subjected to in-plane shear, the following procedure is suggested. Since there is considerable duplication in procedures for each of the various material systems, the procedures below are presented in a unified fashion.

- 27 -

1. Known quantities: a, b, N_{xy}/b

2. Select the material system and obtain the material properties: E_{cx}, E_{cy}, E_{fx}, E_{fy}, ν_{xyf}, ν_{yxf}, ν_{xyc}, ν_{yxc}, ρ_c, ρ_f, and the ultimate shear stress.

3. From Figure 4-1, Reference 6 obtain j utilizing the facts that

Orthotropic Face Material:	Isotropic Face Material
$\beta_2 = \dfrac{2G_{xyf}(1-\nu_{xyf}\nu_{yxf}) + E_{fy}\nu_{xyf}}{\sqrt{E_{fx}E_{fy}}}$	$\beta_2 = 1$
$\dfrac{1}{r} = \dfrac{b}{a}\left(\dfrac{E_{fx}}{E_{fy}}\right)^{1/4}$	$\dfrac{1}{r} = \dfrac{b}{a}$

   At the outset assume $j = \bar{j}$.

4. From Figure 4-1, Reference 6 obtain k_f initially for

Orthotropic Face Material	Isotropic Face Material
$\beta_2 = \dfrac{2G_{xyf}(1-\nu_{xyf}\nu_{yxf}) + E_{fy}\nu_{xyf}}{\sqrt{E_{fx}E_{fy}}}$	$\beta_2 = 1$
$\dfrac{1}{r} = 0$	$\dfrac{1}{r} = 0$

5. From Figure 4-1, Reference 6 obtain k_c initially for:

Orthotropic Core Material	Isotropic Core Material
$\beta_2 = \dfrac{2G_{xyc}(1-\nu_{xyc}\nu_{yxc}) + E_{cy}\nu_{xyc}}{\sqrt{E_{cx}E_{cy}}}$	$\beta_2 = 1$
$\dfrac{1}{r} = 0$	$\dfrac{1}{r} = 0$

   Note: For initial calculation $k_f/k_c = 1$.

6. Determine the web angle θ for the minimum weight structure through the following equation:

Faces and core of different orthotropic materials:

$$2(\sin\theta)^{3/2}(\cos\theta)^2 + 2(\sin\theta)^{7/2}$$

$$- \left(\frac{\rho_c}{\rho_f}\right)\left(\frac{G_{xyc}}{G_{yf}}\right)\left(\frac{E_{sf}}{E_{sc}}\right)^{1/2}\left(\frac{k_f}{k_c}\right)^{1/2}\left[\cos^2\theta - \frac{1}{2}\sin^2\theta\right] = 0$$

Faces and core of different isotropic materials:

$$2(\sin\theta)^{3/2}(\cos\theta)^2 + 2(\sin\theta)^{7/2} - \left(\frac{\rho_c}{\rho_f}\right)\left(\frac{1-\nu_c}{1-\nu_f}\right)^{1/2}\left(\frac{k_f}{k_c}\right)^{1/2}\left[\cos^2\theta - \frac{1}{2}\sin^2\theta\right] = 0$$

Faces and core of the same orthotropic or
isotropic materials:

$$2(\sin\theta)^{3/2}(\cos\theta)^2 + 2(\sin\theta)^{7/2} - \left(\frac{k_f}{k_c}\right)^{1/2}\left[\cos^2\theta - \frac{1}{2}\sin^2\theta\right] = 0$$

For initial calculation $\theta = 28.4°$ (i.e. where

$k_f/k_c = 1$).

7. For the calculated value of θ and the load index,
   determine τ_f from the "universal relationship".
   Faces and core of different orthotropic materials:

$$\frac{N_{xy}}{b} = \frac{4\sqrt{3}\,\tan\theta}{k_f^{1/2}\,\overline{J}^{1/2}}\,\frac{\tau_f^2}{E_{sf}}$$

Faces and core of different isotropic materials:

$$\frac{N_{xy}}{b} = \frac{4\sqrt{3}\,(1-\nu_f^2)\,\tan\theta}{k_f^{1/2}\,\overline{J}^{1/2}}\,\frac{\tau_f^2}{E_f}$$

Faces and core of the same orthotropic materials:

$$\frac{N_{xy}}{b} = \frac{4\sqrt{3}\,\tan\theta}{k_f^{1/2}\,\overline{J}^{1/2}}\,\frac{\tau_f^2}{E_s}$$

- 29 -

Faces and core of the same isotropic materials:

$$\frac{N_{xy}}{b} = \frac{4\sqrt{3}\ \tan\theta\ (1-\nu^2)\ \tau_f^{\ 2}}{k_f^{\ 1/2}\ \bar{j}^{-1/2}\ E}$$

8. Determine the optimum face thickness by

$$\frac{t_f}{b} = \frac{N_{xy}/b}{2\ \tau_f}$$

9. Determine the optimum web core thickness by:

Faces and core of different orthotropic materials:

$$\frac{t_c}{t_f} = \frac{1}{2}\left(\frac{G_{xy_c}}{G_{xy_f}}\right)^{1/2}\left(\frac{E_{sf}}{E_{sc}}\right)^{1/2}\left(\frac{k_f}{k_c}\right)^{1/2}\frac{1}{(\sin\theta)^{1/2}}$$

Faces and core of different isotropic materials:

$$\frac{t_c}{t_f} = \frac{1}{2}\left(\frac{k_f}{k_c}\right)^{1/2}\left(\frac{1-\nu_c}{1-\nu_f}\right)^{1/2}\frac{1}{(\sin\theta)^{1/2}}$$

Faces and core of same orthotropic or isotropic materials:

$$\frac{t_c}{t_f} = \frac{1}{2}\left(\frac{k_f}{k_c}\right)^{1/2}\frac{1}{(\sin\theta)^{1/2}}\ .$$

10. Determine the optimum depth of core by:

Faces and core of different orthotropic materials:

$$\frac{h_c}{b} = \left[\frac{\tau_F}{E_{sF}\ \bar{j}}\right]^{1/2}$$

Faces and core of different isotropic materials:

$$\frac{h_c}{b} = \left[\frac{(1-\nu_f^2)\ \tau_F}{E_f\ \bar{j}}\right]^{1/2}$$

- 30 -

Faces and core of same orthotropic material:

$$\frac{h_c}{b} = \left[\frac{\tau_f}{E_s \bar{j}}\right]^{1/2}$$

Faces and core of same isotropic material:

$$\frac{h_c}{b} = \left[\frac{(1-\nu^2)\,\tau_f}{E\,\bar{j}}\right]^{1/2}$$

11. Determine the weight of the optimum construction by:

Faces and core of different materials:

$$\frac{W-W_{ad}}{b} = 2\rho_f\left(\frac{t_f}{b}\right) + \frac{\rho_c\,(t_c/b)}{(\sin\theta)}$$

Faces and core of the same material:

$$\frac{W-W_{ad}}{b} = \rho\left[2\left(\frac{t_f}{b}\right) + \frac{(t_c/b)}{\sin\theta}\right]$$

12. The initially calculated values of $\bar{J}$, k_f and k_c
can now be checked.

For the optimum configuration calculated in
steps 6 through 11, V_x and V_y can now be
calculated using Equations (1.15) and (1.16).
Then K_M can be calculated using Equation (1.10)
and K_M ($V = 0$) can be calculated using (1.10)
in which $V_x = V_y = 0$. Hence $\bar{J}$ can be calculated
from (1.19) in which j is the number calculated
in Step 3 above.

A new value of k_f can be calculated from
Figure 4-1 of Reference 6, in which the B_2
value calculated in Step 4 above is used
with the actual value of $1/r$, which is

$$\frac{1}{r} = \frac{2 \, h_c \, \tan \theta}{a} \, (\frac{E_{fx}}{E_{fy}})^{1/4}$$

A new value of k_c can be calculated from
Figure 4-1 of Reference 6, in which the B_2
value calculated in Step 5 above is used in
conjunction with the actual value of $1/r$, which
is

$$\frac{1}{r} = \frac{h_c}{a \, \text{Cos} \, \theta} \, (\frac{E_{cx}}{E_{cy}})^{1/4}$$

The new values of $\bar{J}$, k_f, and k_c can be
compared with the initially calculated values
to determine if an iteration should be made.
It is worthwhile to note that in the construc-
tions involving the same face and core
materials, be the material orthotropic or iso-
tropic, the optimum weight given by Equations
(1.60) and (1.69) varies explicitly as the
inverse of $(\bar{J} \, k_f)^{1/4}$, although the ratio
$(k_f/k_c)^{1/2}$ is involved both explicitly in the
weight equation as well as in determining θ.
However, there should be a sizeable difference

- 32 -

in the values calculated here compared with those
calculated initially to merit an iteration in
the calculation, since the weight is so
insensitive to the values of $\bar{j}$, k_f, and k_c.

13. Finally, since in any elastic body in a two-
dimensional stress field, depicted in Figure 2,
subjected to pure shear,

$$\sigma_N = \tau = \frac{N_{xy}}{2t_f}$$

it is necessary that τ_f and τ_c remain at a
value at or below a stress value corresponding
to the proportional limit of the material for
the relations in this optimization to be valid,
since they depend upon Hooke's Law.

Hence for the optimum construction, calculate

$$\tau_c = \frac{G_{cx}}{G_{fx}} \, \tau_f \, Sin \, \theta$$

τ_f must be equal to or below the proportional
limit of the face material and τ_c must be
equal to or below the proportional limit of the
core material.

CHAPTER 2

METHODS OF STRUCTURAL OPTIMIZATION FOR FLAT, WEB CORE SANDWICH PANELS SUBJECTED TO IN-PLANE SHEAR LOADS

A. **INTRODUCTION**

Consider a flat, generalized web core sandwich panel shown in crossection in Figure 4 below. The term "generalized" refers to the fact that at the outset, the angle θ is not taken as zero.
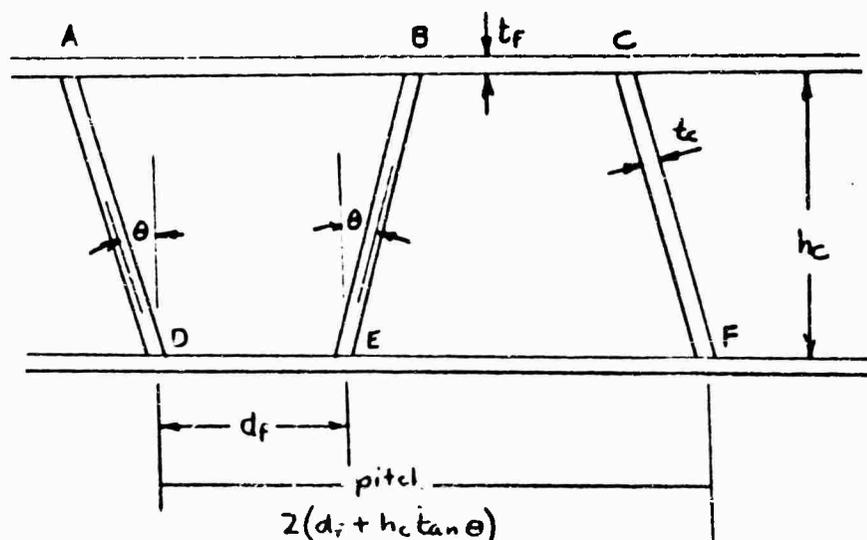
Figure 4

Generalized Web-Core Sandwich Panel Construction

There are five geometric variables with which to optimize; namely, the core depth (h_c), the web thickness (t_c), the face thickness (t_f), the angle the web makes with a line normal to the faces (θ), and the distance between webs (d_f).

The overall panel to be considered is shown in planform in Figure 2. As in Chapter 1, the panel of width b and length a is subjected to in-plane shear loads. N_{xy} and N_{yx}.

The panel of _generalized_ web core construction is considered to fail if any of the following instabilities occur: overall instability, local face shear instability between A and B (Figure 4), local face shear instability between B and C (Figure 4), and shear instability in the web.

Thus, for the generalized construction there are four modes of instability N failure, and five geometric variables with which to optimize. To describe each failure mode, the analytical expression used in the following is the best available in the literature.

B. ELASTIC AND GEOMETRIC CONSTANTS ASSOCIATED WITH GENERALIZED WEB-CORE CONSTRUCTION

The elastic and geometric constants for generalized web-core construction can be determined from those given in more general form by Libove and Hubka in Reference 4. For the core construction shown in Figure 4, the following constants are obtained.

The area of the core per unit width of web core crossection parallel to the yz plane, $\overline{A}_c$ is given by

$$\overline{A}_c = \frac{t_c h_c}{(d_f + h_c \tan \theta) \cos \theta} \quad \text{(in.)} \quad (2.1)$$

where the symbols are defined in Figure 4.

The moment of inertia of the core per unit width of web-core crossection parallel to the yz plane, taken about the centroidal axis of the core crossection, $\overline{I}_c$, is seen to be

$$\overline{I}_c = \frac{t_c h_c^3}{12(d_f + h_c \tan \theta) \cos \theta} = \frac{\overline{A}_c h_c^2}{12} \quad \text{(in.}^3) \quad (2.2)$$

The extensional stiffness of the plate in the x direction EA_x, is given by

$$EA_x = E_c \overline{A}_c + 2 E_f t_f \quad \text{(lbs./in.)} \quad (2.3)$$

where E_c and E_f are the moduli of elasticity of the core and face material respectively.

The transverse shear stiffness, per unit width in the y direction, of an element of the sandwich cut by two xz planes, D_{qx}, is found to be

$$D_{qx} = \frac{G_c t_c h_c \cos \theta}{(d_f + h_c \tan \theta)} \qquad \text{(lbs./in.)} \qquad (2.4)$$

The transverse shear stiffness, per unit width in the x direction, of an element cut by two yz planes, D_{qy}, because of the discontinuity of the core, is seen to be

$$D_{qy} = 0 \qquad\qquad (2.5)$$

Lastly, the moment of inertia per unit width, $\overline{I}_f$, of the faces, considered as membranes with respect to the sandwich plate middle surface, is seen to be,

$$\overline{I}_f = \frac{t_f h_c^2}{2} \qquad \text{(in.}^3) \qquad\qquad (2.6)$$

C. **GOVERNING EQUATIONS FOR PANELS WITH FACES AND CORE OF**
DIFFERENT ORTHOTROPIC MATERIALS

Since panels in which faces and core utilize
different orthotropic materials in the most general
materials system considered in this study, it is con-
venient to derive all expressions and subsequently perform
the optimization for this case first.

1. Overall Stability

The best analytical expression describing the overall
instability of the generalized web core sandwich panel under
in-plane shear loading is given by Equation (1.9). From
Equation (1.16) it is seen that due to $D_{qy} = 0$, V_y for this
construction is

$$V_y = \infty \tag{2.7}$$

Under this condition, the value of K_M given by
Equation (1.10) can be determined for this construction by
dividing both numerator and denominator by V_y, then setting
$V_y = \infty$. The result is

$$K_M = \frac{A}{B_1 C_1 + B_3 C_2 + V_x A} \tag{2.8}$$

As before,

$$K_M (V = 0) = B_1 C_1 + 2 B_2 C_2 + \frac{C_3}{B_1} \tag{2.9}$$

The final relations for overall buckling to be used in the optimization below are given in Equations (1.18) and (1.19), in which Equations (2.8) and (2.9) are utilized.

2. <u>Face Plate Instability (Region A to B in Figure 4)</u>

Proceeding as in Chapter 1, Equation (1.20) is utilized where for this region it is seen that $h = t_f$ and $b = d_f + 2 h_c \tan \theta$. The final expression is therefore

$$\mathcal{T}_f = \frac{k_{f1}}{3} \frac{\left[E_{fy}{}^3 E_{fx}\right]^{1/4}}{(1 - \nu_{xyf} \nu_{yxf})} \frac{t_f{}^2}{(d_f + 2 h_c \tan \theta)^2} \qquad (2.10)$$

where k_f is determined as j in Figure 4-1, Reference 6, in which for this plate element B_2 is given by Equation (1.13) and

$$\frac{1}{r} = \frac{d_f + 2 h_c \tan \theta}{a} \left(\frac{E_{fx}}{E_{fy}}\right)^{1/4} \qquad (2.11)$$

3. <u>Face Plate Instability (Region B to C in Figure 4)</u>

Again Equation (1.20) is utilized where for this region it is seen that $h = t_f$ and $b = d_f$. The final expression is therefore

$$\mathcal{T}_f = \frac{k_{f2}}{3} \frac{\left[E_{fy}{}^3 E_{fx}\right]^{1/4}}{(1 - \nu_{xyf} \nu_{yxf})} \frac{t_f{}^2}{d_f{}^2} \qquad (2.12)$$

where k_f is determined as j in Figure 4-1, Reference 6 in which, for this plate element, B_2 is given in Equation (1.13) and

$$\frac{1}{r} = \frac{d_f}{a} \left(\frac{E_{fx}}{E_{fy}}\right)^{1/4} \qquad (2.13)$$

4. Web Plate Instability

For the web plate instability in the generalized web-core construction, the expression used is Equation (1.20) in which $h = t_c$ and $b = h_c / \cos \theta$, with the result that

$$\tau_c = \frac{k_c}{3} \quad \frac{\left[E_{cy}^{3} E_{cx} \right]^{1/4}}{(1 - \nu_{xyc} \nu_{yxc})} \quad \frac{t_c^{2} \cos^2 \theta}{h_c^{2}} \tag{2.14}$$

in which k_c is determined as j in Figure 4-1, Reference 6, utilizing B_2 given by Equation 1.24 and $1/r$ given by Equation (1.25).

5. Load-Stress Relationship

As in the case of triangulated core construction, the load stress relationship is given by Equation (1.26).

6. Core Stress - Face Stress Relationship

It is seen that the mechanics of shear deformation compatability between face and core for the generalized web core construction is identical to that of the triangulated core construction since d_f does not enter into consideration. Hence the core stress - face stress relationship for the generalized web core construction is given by Equation (1.27).

7. Weight Relationship

The weight of the generalized web core panel is given by

$$W - W_{ad} = 2 \rho_f t_f + \rho_c \bar{A}_c$$

Utilizing Equation (2.1), the result is

$$W - W_{ad} = 2 \rho_f t_f + \frac{\rho_c t_c h_c}{(d_f + h_c \tan \theta) \cos \theta} \tag{2.15}$$

D. **STRUCTURAL OPTIMIZATION OF PANELS WITH FACES AND CORE OF DIFFERENT ORTHOTROFIC MATERIALS**

The governing equations pertaining to the __generalized__ web core construction of Figure 4 to be used in the optimization are given by Equations (1.18), (2.10), (2.12), (2.14), (1.26), (1.27), and (2.15), repeated below for convenience.

$$\tau_f = \frac{\left[E_{fy}^3 E_{fx}\right]^{1/4}}{\left(1-\nu_{xyf}\nu_{yxf}\right)} \frac{h_c^2}{b^2} \bar{j} \tag{2.16}$$

$$\tau_f = \frac{k_{f_1}}{3} \frac{\left[E_{fy}^3 E_{fx}\right]^{1/4}}{\left(1-\nu_{xyf}\nu_{yxf}\right)} \frac{t_f^2}{\left(d_f + 2h_c\tan\theta\right)^2} \tag{2.17}$$

$$\tau_f = \frac{k_{f_2}}{3} \frac{\left[E_{fy}^3 E_{fx}\right]^{1/4}}{\left(1-\nu_{xyf}\nu_{yxf}\right)} \frac{t_f^2}{d_f^2} \tag{2.18}$$

$$\tau_c = \frac{k_c}{3} \frac{\left[E_{cy}^3 E_{cx}\right]^{1/4}}{\left(1-\nu_{xyc}\nu_{yxc}\right)} \frac{t_c^2}{h_c^2} \cos^2\theta \tag{2.19}$$

$$N_{xy} = N_{yx} = 2t_f\tau_f \tag{2.20}$$

- 42 -

NAEC-ASL-1110

$$\tau_c = \frac{G_{xyc}}{G_{xyf}} \tau_f \sin\theta \qquad (2.21)$$

$$W - W_{ad} = 2\rho_f t_f + \frac{\rho_c t_c h_c}{(d_f + h_c \tan\theta)\cos\theta} \qquad (2.22)$$

The philosophy of optimization is identical to that
given in Section D, Chapter 1.

The known or specified quantities in the set of
Equations (2.16) through (2.22) are the applied shear load
per inch N_{xy}, and the panel width b, which can be lumped
together as the load index N_{xy}/b; the material properties;
and the panel boundary conditions. The buckling coefficients
j, k_{f1}, k_{f2} and k_c are constants for any given set of variables
and hence are constants for the optimum construction being
sought.

The dependent variables in the set with which to
optimize the construction are the face thickness, t_f; the core
depth, h_c; the web material thickness, t_c; the web angle, θ;
the distance between webs, d_f, the face shear stress, τ_f;
the core shear stress, τ_c; and the weight, $W - W_{ad}$.

Hence, for the generalized construction there are
seven equations and eight unknowns. However equating Equations
(2.17) and (2.18) it is easily determined that for the optimum
web core construction,

$$\theta = 0° \qquad (2.23)$$

- 43 -

However, it is then seen that with $\theta = 0$, Equation (2.21)
shows that

$$\tau_c = 0 \qquad\qquad (2.24)$$

Hence it is seen that for the optimum web core
construction, in-plane shear stresses applied to the edges of
the panel induce no stress into the core, since $\theta = 0^\circ$. These
results are intuitively obvious. This in turn means that the
Equation (2.19) is no longer a governing equation since the
core can never buckle because it remains unloaded.

Utilizing (2.23) and (2.24) the governing equations
now are seen to be:

$$\tau_F = \frac{\left[t_{fy}^3 E_{fx} \right]^{1/4}}{(1 - \nu_{xyf} \nu_{yxf})} \frac{h_c^2}{b^2} \bar{j} \qquad\qquad (2.25)$$

$$\tau_{\cdot} = \frac{k_F}{3} \frac{\left[E_{fy}^3 E_{fx} \right]^{1/4}}{(1 - \nu_{xyf} \nu_{yxf})} \frac{t_f^2}{d_f^2} \qquad\qquad (2.26)$$

where $k_f = k_{f2}$

$$N_{xy} = 2 t_f \tau_f \qquad\qquad (2.27)$$

$$(W - W_{ad}) = 2 \rho_f t_F + \frac{\rho_c t_c l_c}{d_f} \qquad\qquad (2.28)$$

Hence, there are four equations and six unknowns, namely t_f,
t_c, h_c, d_f, and $W - W_{ad}$, remaining with which to optimize.

Through algebraic manipulation of the above, the weight
equation can be placed in the following convenient form.

$$\frac{W - W_{ad}}{b} = \frac{\rho_f \left(N_{xy}/b\right)}{\tau_f} + \frac{\rho_c \left(1 - \nu_{xy f} \nu_{yx f}\right)\left(t_c/b\right) 2\sqrt{3}\, \tau_f^2}{k_f^{1/2} j^{-1/2} \left[E_{fy}^3 E_{fx}\right]^{1/4} \left(N_{xy}/b\right)} \tag{2.29}$$

The weight equation is then expressed in terms of the
face stress τ_f and the core thickness t_c, which are the only
unknowns. It is seen that obviously minimum weight occurs
where $t_c = 0$, however that violates the type of construction
assumed. Also t_c cannot be determined by buckling requirements
since the core is not stressed. For the present t_c will be
considered as a constant quantity to be specified later by
another criterion.

If the derivative of the weight equation with respect
to the remaining variable τ_f is taken and equated to zero
the following relationship results which provides that value
of τ_f for a minimum weight structure.

$$\frac{N_{xy}}{b} = \frac{2(3)^{1/4}}{k_f^{1/4} j^{-1/4}} \left(\frac{\rho_c}{\rho_f}\right)^{1/2} \frac{\left(1 - \nu_{xy f} \nu_{yx f}\right)^{1/2}}{\left[E_{fy}^3 E_{fx}\right]^{1/8}} \left(t_c/b\right)^{1/2} \tau_F^{3/2} \tag{2.30}$$

This is a "universal relationship" relating load index to
optimum face stress for a specified (t_c/b), as yet unknown.
This relationship can be rearranged to give the following:

- 45 -

$$\tau_F = \frac{k_f^{\frac{1}{6}} j^{-\frac{1}{6}}}{2^{\frac{2}{3}} (3)^{\frac{1}{6}}} \left(\frac{\rho_r}{\rho_c}\right)^{\frac{1}{3}} \frac{\left[E_{f_j}{}^3 E_{f_x}\right]^{\frac{1}{12}}}{\left(1 - \nu_{xyf}\, \nu_{yxf}\right)^{\frac{1}{3}}} \frac{\left(N_{ry}/b\right)^{\frac{2}{3}}}{(t_c/b)^{\frac{1}{3}}} \tag{2.31}$$

Substituting (2.31) into (2.29) gives the weight of the optimum structure in terms of all known quantities except the unknown t_c.

$$\frac{W - W_{ad}}{b} = \frac{3^{\frac{1}{6}}}{2^{\frac{1}{3}}} \frac{\rho_c^{\frac{1}{3}} \rho_f^{\frac{2}{3}}}{k_f^{\frac{1}{6}} j^{\frac{1}{6}}} \frac{\left(1 - \nu_{xyf}\, \nu_{yxf}\right)^{\frac{1}{3}}}{\left[E_{f_j}{}^3 E_{f_x}\right]^{\frac{1}{12}}} (t_c/b)^{\frac{1}{3}} (N_{xy}/b)^{\frac{1}{3}} \tag{2.32}$$

It is quite clear that the smaller the value of t_c, the lower the weight for a panel with given materials and a given load. From the "universal relationship" (2.30), for a given material system and load index, (t_c/b) decreases as τ_f increases. Therefore, it can be concluded that from Equation (2.32) the weight is minimum for the highest allowable τ_f that can be tolerated.

This differs from all optimizations previously performed in References 1, 2, and 3, and in Chapter 1 of this report. The reason is that in all previous cases t_c was determined by buckling criteria. Here, t_c is determined by strength considerations, not by buckling requirements, since the core is unstressed under this loading condition. Hence for minimum weight,

$$\tau_f = \tau_{f_{all}}. \tag{2.33}$$

where $\tau_{Fall.}$ is the maximum allowable shear stress for the face material. For isotropic face materials, $\tau_{Fall.}$ can be taken as the ultimate shear stress of the material. Since the shear stress τ_F is equal in value to the maximum principal stress σ_N (see Equation (1.51), Reference 2), for stresses above the proportional limit reduced values of the elastic modulus can be used, as discussed in Equation (1.25) of Reference 1. For orthotropic face materials, $\tau_{Fall.}$ is the proportional limit of the material for all relationships in this optimization to be valid.

Substituting (2.33) into (2.30) (t_c/b) is now clearly determined for a given material system and load index (N_{xy}/b). All other variables are also determined. The results are summarized below:

$$\theta = \tau_c = 0 \tag{2.34}$$

$$\frac{t_F}{b} = \frac{(N_{xy}/b)}{2\,\tau_{Fall.}} \tag{2.35}$$

$$\frac{t_c}{b} = \left(\frac{\rho_F}{\rho_c}\right)\frac{k_f^{1/2}\,\overline{j}^{-1/2}\,E_{Sf}\,(N_{xy}/b)^2}{4\sqrt{3}\;\tau_{Fall}^{3}} \tag{2.36}$$

$$\left(\frac{h_c}{b}\right) = \left[\frac{\tau_{Fall.}}{\overline{j}\,E_{Sf}}\right]^{1/2} \tag{2.37}$$

$$\left(\frac{d_F}{b}\right) = \frac{k_f^{1/2}}{2\sqrt{3}}\,\frac{E_{Sf}^{1/2}\,(N_{xy}/b)}{\tau_{Fall}^{3/2}} \tag{2.38}$$

- 47 -

$$\frac{W - W_{al}}{b} = \frac{3\rho_f \left(N_{xy}/b\right)}{2\, \tau_{face}} \tag{2.39}$$

In addition, there are two useful relationships for the optimum construction

$$\frac{W_f}{W_c} = 2 \tag{2.40}$$

$$\frac{h_c}{d_f} = \frac{\rho_f}{\rho_c}\left(\frac{t_f}{t_c}\right) \tag{2.41}$$

Several points are worthy of note here, which are also valid for this and subsequent sections dealing with isotropic materials, or for faces and core of the same material.

Again as for optimum web core construction under uniaxial compression , the optimum web core construction under in-plane shear has the characteristics that $\theta = 0^{\circ}$, $(h_c/d_f) = (\rho_f/\rho_c)\,(t_f/t_c)$ and $W_f/W_c = 2$. However, other parameters of the geometry differ.

It is interesting to note that, as shown in (2.35), the face thickness for a given load index is determined only by the allowable shear stress of the face material. Also from (2.37) the core depth is, for all practical purposes, independent of the load applied. Also, from (2.39) the weight of the optimum structure is independent of all material properties except the allowable shear stress and the density of the face material and varies linearly as the load index. Thus the best material for this construction is the one with the highest ratio of (τ_{face}/ρ_f). Detailed design procedures for this type of construction are given in Section H of this chapter.

E. STRUCTURAL OPTIMIZATION OF PANELS WITH FACES AND CORE OF DIFFERENT ISOTROPIC MATERIALS

By making the following contractions of the results of Section D, the optimal relationships are obtained for panels with faces and cores of different isotropic materials.

$$\nu_{xyi} = \nu_{yxi} = \nu_i \qquad (i = c,f)$$

$$E_{si} = \frac{E_i}{(1 - \nu_i^2)} \qquad (i = c,f) \qquad (2.42)$$

$E_f = \bar{E}_f$ for stresses above the proportional limit.

The resulting expressions can be written as

$$\theta = \tau_c = 0 \tag{2.43}$$

$$\left(\frac{t_f}{b}\right) = \frac{(N_{xy}/b)}{2 \, \tau_{F_{all}}} \tag{2.44}$$

$$\left(\frac{t_c}{b}\right) = \left(\frac{\rho_F}{\rho_c}\right) \frac{k_F^{1/2} \, j^{-1/2} \, \bar{E}_F \, (N_{xy}/b)^2}{4\sqrt{3} \, (1-\nu_f^2) \, \tau_{F_{all}}^3} \tag{2.45}$$

$$\left(\frac{h_c}{b}\right) = \frac{(1-\nu_f^2)^{1/2} \, \tau_{F_{all}}^{1/2}}{j^{1/2} \, \bar{E}_f^{1/2}} \tag{2.46}$$

$$\frac{d_f}{b} = \frac{k_F^{1/2}}{2\sqrt{3}} \frac{\bar{E}_F^{1/2} \, (N_{xy}/b)}{(1-\nu_f^2) \, (\tau_{F_{all}})^{3/2}} \tag{2.47}$$

$$\frac{W - W_{od}}{b} = \frac{3\rho_F \left(N_{x_g}/b \right)}{2 \, \tau_{f \, all}} \tag{2.48}$$

$$\frac{W_F}{N_c} = 2 \tag{2.49}$$

$$\frac{h_c}{d_r} = \left(\frac{\rho_F}{\rho_c} \right) \left(\frac{t_F}{t_c} \right) \tag{2.50}$$

Detailed design procedures for this type of construction are given in summary in Section B of this chapter.

F. STRUCTURAL OPTIMIZATION OF PANELS WITH FACES AND CORE OF THE SAME ORTHOTROPIC MATERIAL

By contracting the expressions of Section for panels with faces and core of different orthotropic materials by the following substitutions,

$$\nu_{xyc} = \gamma_{xyf} = \gamma_{xy}$$

$$\nu_{yxc} = \nu_{yxc} = \nu_{xy} \tag{2.51}$$

$$E_{sc} = E_{sf} = E_s$$

the following equations apply to optimum construction in which the face and core are of the same orthotropic material.

$$\theta = \tau_c = 0 \tag{2.52}$$

$$\frac{t_F}{b} = \frac{(N_{xy}/b)}{2\,\tau_{Fall.}} \tag{2.53}$$

$$\frac{t_c}{b} = \frac{\rho_F}{\rho_c}\frac{k_f^{1/2}\,j^{-1/2}\,E_s\,(N_{xy}/b)^2}{4\sqrt{3}\,\tau_{Fall}^3} \tag{2.54}$$

$$\frac{h_c}{b} = \left[\frac{\tau_{Fall}}{E_s\,j}\right]^{1/2} \tag{2.55}$$

$$\frac{d_F}{b} = \frac{k_f^{1/2}\,E_s^{1/2}\,(N_{xy}/b)}{2\sqrt{3}\,(\tau_{Fall})^{1/2}} \tag{2.56}$$

- 51 -

$$\frac{W - W_{ad}}{b} = \frac{3 \rho \left(N_{xy} h \right)}{2 \tau_{Fall}} \tag{2.57}$$

$$\frac{W_f}{W_c} = 2 \tag{2.58}$$

$$\frac{h_c}{d_f} = \frac{t_f}{t_c} \tag{2.59}$$

Detailed design procedures for this type of construction are given in summary in Section H of this chapter.

G. STRUCTURAL OPTIMIZATION OF PANELS WITH FACES AND CORE OF THE SAME ISOTROPIC MATERIALS

From Section E, simply removing the subscripts associated with each material property provides the resulting governing equations. Also from (2.50) it is seen that, as in the construction involving face and core of the same orthotropic material,

$$\frac{a_c}{d_f} = \frac{t_f}{t_c} \tag{2.60}$$

H. DESIGN PROCEDURES FOR PANELS OF OPTIMUM WEB-CORE CONSTRUCTION UNDER IN-PLANE SHEAR LOADING

Prior to discussing detailed design procedures, it is advantageous to discuss certain characteristics of the coefficients $\bar{j}$ and k_f, which can result in significant simplifications to the design procedures. From Equation (1.19) it is seen that

$$\bar{j} = \frac{j}{1 + 4\left[\dfrac{k_M(V=0)}{k_M} - 1\right]} \qquad (2.61)$$

The coefficient j is determined in a straight-forward manner from Figure 4-1 of Reference 6, where it is plotted as a function of B_2, given by Equation (1.13) (Note $B_2 = 1$ for an isotropic face material), and $1/r = (b/a)\,(E_{fx}/E_{fy})^{1/4}$.

In Equation (2.61), K_M given by Equation (2.8) for this type of construction is a function of the core transverse shear flexibility parameter V_x.

From Reference 6, V_x is seen to be

$$V_x = \frac{\pi^2}{b^2}\frac{\sqrt{D_x D_4}}{D_{qx}} \qquad (2.62)$$

In terms of this construction, utilizing (2.4) for D_{qx},

$$V_x = \frac{\pi^2}{2}\frac{\sqrt{E_{fx} E_{fy}}}{G_{xyc}}\left(\frac{t_f}{t_s}\right)\left(\frac{h_c}{b}\right)\left(\frac{d_f}{b}\right) \qquad (2.63)$$

- 54 -

Now for the most general materials system, namely faces and core of different orthotropic materials, substitution of (2.35) through (2.38) into (2.65) results in

$$V_x = \frac{\pi^2}{2 \sqrt{J} k_f^{1/6}} \left(\frac{E_{fx}}{E_{fy}}\right)^{1/4} \left(\frac{\rho_c}{\rho_F}\right) \left(1 - \sqrt{\nu_{xy_f} \nu_{yx_f}}\right) \frac{\tau_{F_{all}}^{1/2}}{G_{x_{yc}}} \qquad (2.64)$$

Hence, for almost all material systems $V_x \ll 1$

Hence since in Equation (2.8) all other quantities are usually of order one

$$K_M \approx \frac{A}{B_1 C_1 + B_3 C_2} \qquad (2.65)$$

Therefore $\bar{J}$ can be determined at the outset by Equations (2.61), (2.9), and (2.65), and only occasionally will an iteration be necessary.

The coefficient k_f can be also closely approximated from Figure 4-1, Reference 6, by taking $1/r \sim 0$, since in Equation (2.13) for most cases $d_f/a \ll 1$, and k_f is almost constant for a given B_2 in the range $0 < 1/r < 0.1$. This philosophy is discussed more in detail in Section H of Chapter 1.

Note also that the minimum weight expression is independent of $\bar{J}$ and k_f. They are needed to proportion the panel only.

Turning now to design procedures, utilizing the expressions derived in the previous sections, there are several ways to proceed to design for minimum weight. However, to save

time and effort in developing _design curves_ for web-core
panels subjected to in-plane shear, the following procedure is
suggested. Since there is considerable duplication in
procedures for each of the various material systems, the
procedures below are presented in a unified fashion.

1. Known quantities: a, b, N_{xy}/b

2. Select the material system and obtain the
   material properties: E_{cx}, E_{cy}, E_{fx}, ν_{xyf},
   ν_{yxf}, ν_{xyc}, ν_{yxc}, ρ_c, ρ_f, τ_{Fall}.

3. From Figure 4-1, Reference 6, obtain j
   utilizing the following calculated values.

   Orthotropic Face Material   **Isotropic Face Material**

   $$B_2 = \frac{2G_{xyf}(1-\nu_{xyf}\nu_{yxf})+E_{fy}\nu_{xyf}}{\sqrt{E_{fx}E_{fy}}} \qquad B_2 = 1$$

   $$\frac{1}{r} = \frac{b}{a}\left(\frac{E_{fx}}{E_{fy}}\right)^{1/4} \qquad\qquad \frac{1}{r} = \frac{b}{a}$$

4. Calculate an initial value of $\bar{j}$ from

   $$\bar{j} = \frac{j}{1+4\left[\dfrac{K_M(V=0)}{K_M}-1\right]}$$

   where

   $$K_M(V=0) = B_1 C_1 + 2B_2 C_2 + \frac{C_3}{B_1}$$

   $$K_M \approx \frac{A}{B_1 C_1 + B_2 C_2}$$

   and the constants A, B_1, and B_2 are given in
   Equations (1.11), (1.12), and (1.13). The
   values are given in Equations (7) through (10)
   in Reference 2 for various boundary conditions,
   where for shear loading n = 1 only.

5. From Figure 4-1, Reference 6, obtain initial value of x_f by using B_2 of step 3 above, and $1/r = 0$.

6. Determine the optimum weight for any material system by

$$\frac{W - W_{ad}}{b} = \frac{3 \rho_F \left(N_{xy}/b \right)}{2 \tau_{F_{all}}}$$

7. Determine the optimum face thickness for any material system by

$$\frac{t_F}{b} = \frac{N_{xy}/b}{2 \tau_{F_{all}}}$$

8. Determine the optimum web core thickness by

Faces and core of different orthotropic material

$$\frac{t_c}{b} = \frac{\rho_F}{\rho_c} \frac{k_f^{1/2} \, j^{-1/2} \, E_{SF} \left(N_{xy}/b \right)^2}{4 \sqrt{3} \, \tau_{F_{all}}^3}$$

Faces and core of the same orthotropic material

$$\frac{t_c}{b} = \frac{k_f^{1/2} \, j^{-1/2} \, E_S \left(N_{xy}/b \right)^2}{4 \sqrt{3} \, \tau_{all}^3}$$

Faces and core of different isotropic materials

$$\frac{t_c}{b} = \frac{\rho_F}{\rho_c} \frac{k_f^{1/2} \, j^{-1/2} \, E_F \left(N_{xy}/b \right)^2}{4 \sqrt{3} \, (1 - \nu_F^2) \, \tau_{F_{all}}^3}$$

Faces and core of the same isotropic material

$$\frac{t_c}{b} = \frac{k_f^{1/2} \, j^{-1/2} \, E \left(N_{xy}/b \right)^2}{4 \sqrt{3} \, (1 - \nu^2) \, \tau_{all}^3}$$

9. Determine the optimum core depth by

$$\frac{h_c}{b} = \left[\frac{\tau_{F_{all}}}{\bar{j} \, \bar{E}} \right]^{1/2}$$

where $\widetilde{E} = \begin{cases} E_{sf} & \text{for faces and core of different or the same orthotropic material} \\ \dfrac{E_f}{(1-V_f^2)} & \text{for faces and core of different or the same isotropic materials} \end{cases}$

10. Determine the optimum web spacing by

$$\frac{d_f}{b} = \frac{k_f^{1/2} \; \widetilde{E}^{1/2} \; (N_{xy}/b)}{2\sqrt{3} \; \tau_{F_{all}}^{3/2}}$$

where $\widetilde{E}$ is defined above.

11. The initially calculated values of $\bar{J}$ and k_f can be checked. For the optimum configuration above, V_x can now be calculated using Equation (2.64) or its simplifications for isotropic materials, or when core and face materials are the same. Then K_M can be calculated using Equation (2.8). Finally the actual $\bar{J}$ can be calculated using Equation (2.61) and compared to the assumed value of step 4 above to see if an iteration is required.

Next with the optimized geometry, the actual value of $1/r$ can be calculated using Equation (2.13). Looking at Figure 4-1, Reference 6, the actual k_f can be compared with that obtained in step 5 above to see if an iteration is required. If an iteration is worthwhile, then with the new values of $\bar{J}$ and k_f, steps 6 through 11 can be repeated.

CHAPTER 3

COMPARISON OF OPTIMIZED FLAT SANDWICH PANELS UNDER IN-PLANE
SHEAR LOADS FOR THREE CORE GEOMETRIES

7075-T6 clad aluminum is chosen as the typical
isotropic material with which to make a comparison between
the subject types of construction to determine relative
merits. Also, in studies of ten material systems listed and
studied in References 1 and 3 which include metals and re-
inforced plastics, as well as isotropic and orthotropic
materials, the results showed that with the exception of
beryllium, 7075-T6 aluminum is significantly better than the
other materials for honeycomb panels subjected to in-plane
shear loads. A square panel (a/b = 1) is chosen with a
hexagonal cell honeycomb core. The constants for honeycomb
cores as determined by Kaechele (Reference 9), the precedures
given in Reference 2, and the methods developed in Reference 1
are used herein. For the optimum web core construction an
allowable stress of 40,000 psi is used. The results are
presented in Table 1.

Table 1

Comparison Between Optimum Triangulated, Corrugated
Core Optimum Web Core and Optimum Hexagonal Honeycomb
Core Square Panel Subjec.e' to In-Plane Shear Loads
(7075-T6 Clad Aluminum Faces and Core) Honeycomb Core
Constants by Kaechele

τ_f(psi)	N_{xy}/b(psi) Honeycomb	$(W-W_{ad})/b$ (lb/in^3) Honeycomb	N_{xy}/b(psi) Truss-Core	$(W-W_{ad})/b$ (lb/in^3) Tru -Core	$(W-W_{ad})/b$ (lb/in^3) Web Core
12,000	4.52	0.379×10^{-4}	2.66	0.395×10^{-4}	0.163×10^{-4}
16,000	8.14	0.517×10^{-4}	4.80	0.534×10^{-4}	0.294×10^{-4}
18,000	10.52	0.594×10^{-4}	6.20	0.613×10^{-4}	0.380×10^{-4}
20,000	13.04	0.661×10^{-4}	7.68	0.684×10^{-4}	0.470×10^{-4}
30,000	29.4	1.00×10^{-4}	17.3	1.015×10^{-4}	1.061×10^{-4}
40,000	52.2	1.338×10^{-4}	30.7	1.37×10^{-4}	1.885×10^{-4}

It is seen that for the same face stress, the truss-core construction carries much less load and yet weighs more than the honeycomb core construction. However the comparison can best be made by plotting the weight as a function of the load, as is seen in Figure 5. The percentage figures shown refer to the percent overweight the optimum triangulated core construction is compared to the optimum honeycomb construction.

It is seen that at least for this example the optimum honeycomb construction is significantly better than the optimum triangulated corrugated core construction. It is believed that the same comparison will hold for square panels in which faces and core are composed of any other isotropic material. In

FIGURE 5

WEIGHT AS A FUNCTION OF LOAD INDEX UNDER IN-PLANE SHEAR LOADS; 7075-T6 CLAD ALUMINUM FACES AND CORE; $a/b = 1$.
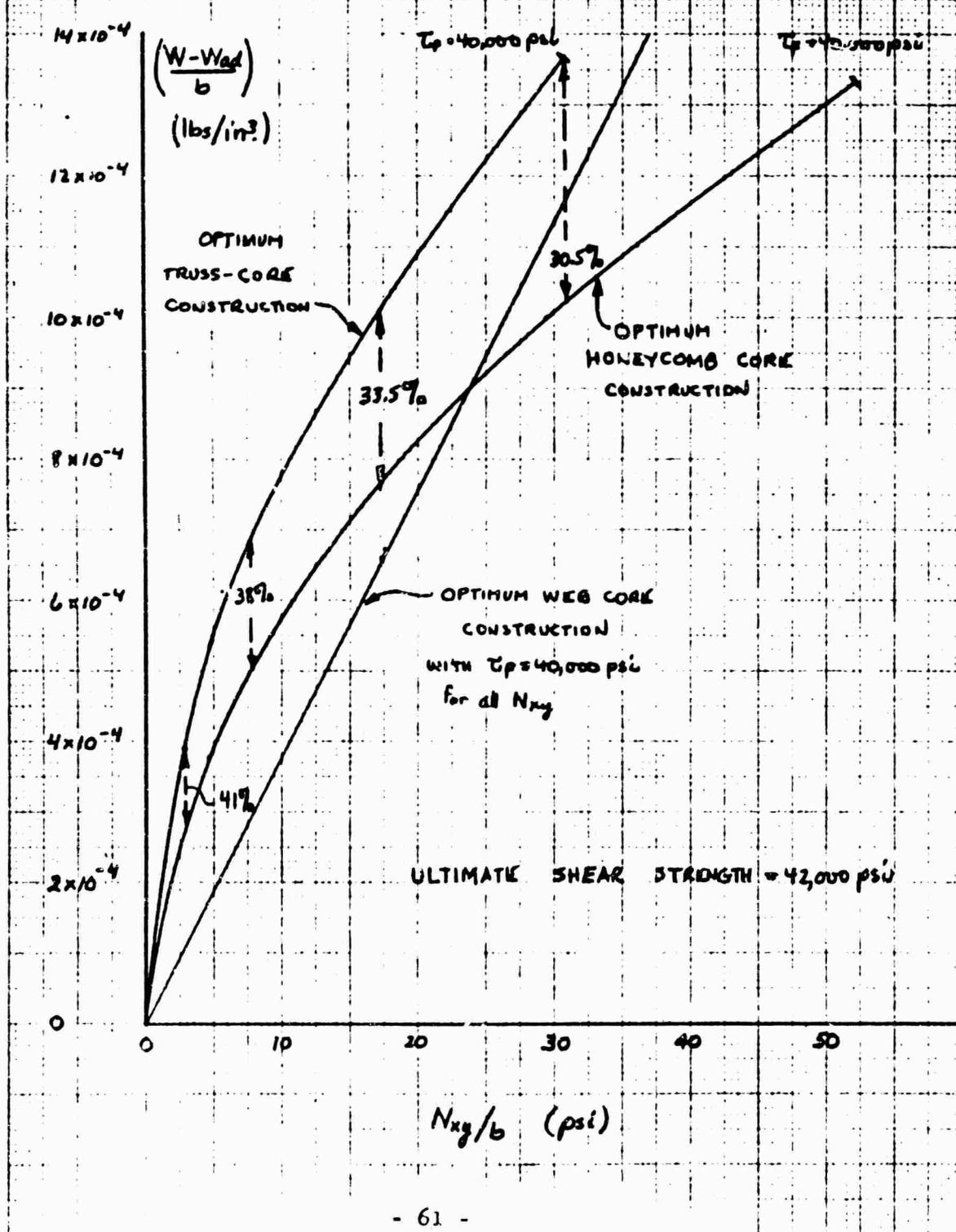
addition it is felt that any changes in a/b ratio should not alter the comparison to such an extent that truss core construction would be competitive. Nor is it likely that the use of orthotropic materials could alter the comparison so significantly that truss core construction would be favorably competitive with the same materials used in honeycomb core panels for in-plane shear loadings.

It is seen that the optimum web core construction results in considerable weight savings over optimum honeycomb core construction in the low load index range. However, in the higher load index range, it does not compare favorably with honeycomb core construction. This _trend_ will exist in other material systems and a/b values because in optimum honeycomb sandwich construction the weight varies as $(N_{xy}/b)^{1/2}$ while in optimum web core construction the weight varies as (N_{xy}/b). Note also that in the optimum web core construction the face stress τ_f is constant over the entire range of load index. If τ_{Fall} is reduced for some other factor the slope of the curve in Figure 5 increases.

CHAPTER 4

METHODS OF STRUCTURAL OPTIMIZATION FOR FLAT TRIANGULATED
(SINGLE TRUSS) CORE SANDWICH PANELS SUBJECTED TO BOTH
UNIAXIAL COMPRESSION AND IN-PLANE SHEAR LOADS

Methods of analysis for the structural optimization
of truss-core sandwich panels subjected to uniaxial compressive
loads were derived in Chapter 1 of Reference 1. It was seen
in subsequent numerical examples that under uniaxial com-
pressive loads optimum flat panels of single truss core con-
struction were a few percent greater in weight than optimum
honeycomb core sandwich panel construction.

Methods of structural optimization for truss-core
sandwich panel construction subjected to in-plane shear loads
were developed in Chapter 1 of this report. In Figure 5 it
is seen that in the numerical example performed in Chapter 3
truss core construction is significantly heavier than optimum
honeycomb core construction.

It can be surmised that if single truss core panels
are to be used at all for combined uniaxial compressive loads
and in-plane shear loads it will very likely be for combina-
tions in which the in-plane shear load index N_{xy}/b is smaller
than the uniaxial compressive load index, N_x/b. Otherwise
the construction could very probably be inefficient compared
to other alternatives.

It can also be seen from the expressions for optimum construction that the optimum truss core construction for uniaxial compressive loads differs in proportion from that for optimum construction for in-plane shear loads. For instance, when face and core materials are the same, $\theta = 32.4^\circ$ for the uniaxial compressive loads, and $\theta = 28.4^\circ$ for in-plane shear loads. It is not possible therefore to have all failure modes in compression and all failure modes associated with in-plane shear loads occur simultaneously.

Therefore in the case of combined loads in which the axial compression load index predominates (which is the more desirable ratio of N_x/N_{xy} to use in this construction) it appears logical to use the optimum construction due to uniaxial compression only, and modify the construction to account for the smaller in-plane shear loads.

Under combined loads of uniaxial compression and in-plane shear, Reference 8 gives the following relationship for a stability equation for a flat plate,

$$\frac{\sigma_f}{\sigma_{f_{cr}}} + \left(\frac{\tau_f}{\tau_{f_{cr}}}\right)^2 = 1 \tag{4.1}$$

where σ_f applied compressive stress

           $\sigma_{f_{cr}}$ critical compressive stress

           τ_f applied in-plane shear stress

           $\tau_{f_{cr}}$ critical in-plane shear stress.

Subscript f refers to face.

- 64 -

This same relationship has been used in Section 4.2.4.1 of Reference 6 for a sandwich panel.

In the following the subscript cr refers to critical buckling stress which has been omitted in previous chapters. Also, in the following, stresses or load indices without subscript refers to applied stresses or loads.

From Equation (1.18) of Reference 1 it is seen that

$$\sigma_f = \frac{N_x}{\left[\dfrac{E_c}{E_f}\dfrac{t_c}{\sin\theta} + 2t_f\right]} \tag{4.2}$$

$$\sigma_{f_{cr}} = \frac{N_{x_{cr}}}{\left[\dfrac{E_c}{E_f}\dfrac{t_c}{\sin\theta} + 2t_f\right]} \tag{4.3}$$

From Equation (1.26) it is seen that

$$\tau_f = \frac{N_{xy}}{2t_f} \tag{4.4}$$

and

$$\tau_{f_{cr}} = \frac{N_{xy_{cr}}}{2t_f} \tag{4.5}$$

Thus, utilizing (4.2) through (4.5), (4.1) can be written as

$$\frac{(N_x/b)}{(N_{x_{cr}}/b)} + \frac{(N_{xy}/b)^2}{(N_{xy_{cr}}/b)^2} = 1 \tag{4.6}$$

Using the structure proportioned optimally for axially compressive loads acting only, when $N_{x_{cr}}/b$ is achieved, there will be simultaneous failure of the panel in overall

- 65 -

buckling, face plate buckling, and web plate buckling.
However with the panel optimally proportioned for uniaxial
compression, when shear loads increase, the three failure
modes due to in-plane shear will not occur simultaneously,
because the panel is not optimum for shear loads. It is
therefore necessary to determine which failure mode will occur
due to shear loads, at the lowest shear stress
for a panel that is optimum for compressive loads.

A truss core panel proportioned optimally for
uniaxial compression, for the most general material system is
specified by Equations (1.86), (1.87), and (1.88) of Refer-
ence 1. The optimum web angle θ is given by Equation (1.81)
of Reference 1. Substituting these values into the three
expressions for stability for the panel subjected to in-plane
shear loads, given by (1.29), (1.30), and (1.31), it is found
that the panel proportioned optimally for compressive leads
will have the lowest critical load in face plate buckling for
shear loads.

Thus substituting Equations (1.86) and (1.88) of
Reference 1 into Equation (1.30), and utilizing (4.5) above
results in the following:

$$\frac{N_{xy_{cr}}}{b} = \frac{4 k_f}{\pi^2 M_o} \frac{\left[E_{f_y}^3 E_{f_x}\right]^{1/4}}{E_{of}} \sin^2\theta \left(\frac{N_{x_{cr}}}{b}\right) \qquad (4.7)$$

where

$$M_o = \left[\left(\frac{E_{cx}}{E_{fx}}\right)^{3/2} \left(\frac{E_{of}}{E_{cc}}\right)^{1/2} \Omega_o + 4 \sin^2\theta\right] \qquad (4.8)$$

- 66 -

$$E_{0i} = \frac{1}{2}\left[\left(E_{ix}E_{iy}\right)^{1/2} + \nu_{xy_i}E_{ix} + 2G_{xyi}\left(1-\nu_{xy_i}\nu_{yx_i}\right)\right] \qquad (4.9)$$

and

$$\Omega_o = \left[\frac{1-\nu_{xy_c}\nu_{yx_c}}{1-\nu_{xy_f}\nu_{yx_f}}\right]^{1/2} \qquad (4.10)$$

and Θ is given by Equation (1.81), Reference 1.

Notice that Equation (4.7) provides a key relationship between $\left(N_{xy_{cr}}/b\right)$ and $\left(N_{x_{cr}}/b\right)$.

It is now convenient to define ϕ as

$$\phi = \frac{4k_f}{\pi^2 M_o}\frac{\left[E_{fy}^3 E_{fx}\right]^{1/4}}{E_{0f}}\sin^2\Theta \qquad (4.11)$$

Hence,

$$\left(\frac{N_{xy_{cr}}}{b}\right) = \phi\left(\frac{N_{x_{cr}}}{b}\right) \qquad (4.12)$$

It is further convenient to define α as

$$\alpha = \frac{\left(N_{xy}/b\right)}{\left(N_x/b\right)} \qquad (4.13)$$

Substituting (4.12) and (4.13) into (4.6), the result can be written as

$$\left(N_{x_{cr}}/b\right)^2 - \left(N_x/b\right)\left(N_{x_{cr}}/b\right) - \frac{\alpha^2}{\phi^2}\left(N_x/b\right)^2 = 0 \qquad (4.14)$$

The solution of this equation is then

$$\frac{N_{x_{cr}}}{b} = \frac{1}{2}\left(\frac{N_x}{b}\right)\left\{1 + \left(1 + 4\frac{\alpha^2}{\phi^2}\right)^{1/2}\right\} \qquad (4.15)$$

Note that if no shear loads are applied, then $\alpha = 0$, and (4.15) would become $\dfrac{N_{xcr}}{b} = \dfrac{N_x}{b}$, or the compressive load applied would be identical to the critical load, and since the panel is optimum for compressive loads, then when $\dfrac{N_{xcr}}{b}$ is applied, simultaneous failure would occur in all three modes of failure. Where $\alpha > 0$, then the panel must be designed for a critical load $\dfrac{N_{x_{cr}}}{b}$ greater than $\dfrac{N_x}{b}$, the applied axial loading.

Looking now at the functional relationships for the structure proportioned to be optimum for axial compression as given in Chapter 1 of Reference 1 for various material systems it is seen that t_f/b, t_c/b, and $(W-W_{ad}/b)$ are proportional to $(N_{x_{cr}}/b)^{1/2}$. (h_c/b) is proportional to $(N_{x_{cr}}/b)^{1/4}$. Therefore, it can be seen that

$$\begin{bmatrix} \left(\dfrac{t_f}{b}\right) \\[2mm] \left(\dfrac{t_c}{b}\right) \\[2mm] \left(\dfrac{W-W_{ad}}{b}\right) \end{bmatrix}_{\substack{N_x \text{ given} \\ N_{xy} \text{ given}}} = \begin{bmatrix} \left(\dfrac{t_f}{b}\right) \\[2mm] \left(\dfrac{t_c}{b}\right) \\[2mm] \left(\dfrac{W-W_{ad}}{b}\right) \end{bmatrix}_{\substack{N_x \text{ given} \\ N_{xy}=0}} \left(\frac{1}{2}\left[1+\left(1+\frac{4\alpha^2}{\phi^2}\right)^{1/2}\right]\right)^{1/2} \quad (4.16)$$

$$\left(h_c/b\right)_{\substack{N_x \text{ given} \\ N_{xy} \text{ given}}} = \left(h_c/b\right)_{\substack{N_x \text{ given} \\ N_{xy}=0}} \left\{\frac{1}{2}\left[1+\left(1+\frac{4\alpha^2}{\phi^2}\right)^{1/2}\right]\right\}^{1/4} \quad (4.17)$$

The following design procedures are therefore given for single truss core panels subjected to combined loads of uniaxial compression and in-plane shear loading, where the uniaxial compressive loads predominate (which is the situation in which this type of construction appears more favorable).

1. Given: N_x, N_{xy}, a, and b
2. Select material system and obtain all material properties needed.
3. Letting N_x above be considered as the critical load when no shear forces are present, use the procedures of Chapter 1, Reference 1, to determine the optimum configuration, in the absence of shear stresses, thereby determining Θ as well as

$$\left\{ t_f/b, \quad t_c/b, \quad (\frac{W-W_{sd}}{b}), \quad h_c/b, \right\}_{\substack{N_x \text{ given} \\ N_{xy} = 0}}$$

4. Determine ϕ
   For faces and core of different orthotropic materials:

$$\phi = \frac{4 k_f}{\pi^2 M_o} \frac{\left[E_{f_y}{}^3 E_{fx} \right]^{1/4}}{E_{of}} \sin^2 \Theta$$

- 69 -

For faces and core of different isotropic
materials:

$$\phi = \frac{4k_f}{\pi^2} \frac{\sin^2\theta}{\left[\left(\frac{E_c}{E_f}\right)\left(\frac{1-\nu_c^2}{1-\nu_f^2}\right)^{1/2} + 4\sin^2\theta\right]}$$

For faces and core of the same orthotropic
material ($\sin^2\theta = 2/7$):

$$\phi = \frac{8}{15} \frac{k_f}{\pi^2} \frac{\left[E_{fy}^3 E_{fx}\right]^{1/4}}{E_{of}}$$

For faces and core of the same isotropic
material ($\sin^2\theta = 2/7$):

$$\phi = \frac{8}{15} \frac{k_f}{\pi^2}$$

5. Calculate

$$\alpha = \frac{(N_{xy}/b)}{(N_x/b)}$$

6. Determine the required panel parameters and the
weight by Equations (4.16) and (4.17).

Consider a construction determined above for faces
and core of the same isotropic material. It can be shown
that for optimum construction in this case $k_f = 13.17$ (Figure
4-1, Reference 6). Then $\phi = (0.712)$. Thus for various

values of α it is of interest to determine the value of

$$\left\{ \frac{1}{2} \left[1 + \left(1 + \frac{4\alpha^2}{\phi^2} \right)^{1/2} \right] \right\}^{1/2}$$

in Equation (4.16) to determine the weight penalty caused when in-plane shear loads are present.

α	$\left\{ \frac{1}{2} \left[1 + \left(1 + \frac{4\alpha^2}{\phi^2} \right)^{1/2} \right] \right\}^{1/2}$
0	1.000
0.2	1.036
0.4	1.12
0.6	1.218
0.8	1.315
1.0	1.413

CHAPTER 5

METHODS OF STRUCTURAL OPTIMIZATION FOR FLAT, WEB-CORE
SANDWICH PANELS SUBJECTED TO BOTH UNIAXIAL COMPRESSION
AND IN-PLANE SHEAR LOADS

Methods of analysis for the structural optimization
of web-core sandwich panels subjected to uniaxial compressive
loads were derived in Chapter 2 of Reference 1. It was seen
in calculations that optimum   web-core construction did
not compare favorably with optimum honeycomb sandwich core
construction for use in applications involving only uniaxial
compressive loadings.

Methods of structural optimization for truss-core
sandwich panel construction subjected to in-plane shear loads
were developed in Chapter 2 of this report. In Figure 5, it
is indicated that web-core construction appears most favorable
in the lower range of N_{xy}/b.

Under in-plane shear loads only, the web thickness t_c
(which is not stressed) is determined only by the requirement
that the face stress τ_F be maintained within the material
allowables for any given N_{xy}/b. However, when uniaxial
compressive loads are present then t_c/b must be sufficient to
prevent buckling of the web due to compressive loads. This
t_c/b requirement may therefore lower τ_F that is feasible to
satisfy the universal relationship given by Equation (2.30),
when t_c/b is specified. Since the weight of the panel in shear

varies inversely as τ_f , if τ_f is reduced from the material allowable due to some requirement placed on t_c/b such as resisting in-plane compressive loads, then in Figure 5, the slope increases, and thus web-core construction appears favorable over a lower band of N_{xy}/b.

So as in Chapter 4, again it may be that if web-core construction is to appear favorable for combined loads of uniaxial compression and in-plane shear it will very likely do so under low shear loadings, where axial compressive loads dominate the behavior.

It can also be seen from the expressions in Chapter 2 of Reference 1, and Chapter 2 of this report that the geometric variables associated with optimum construction for uniaxial compression and the optimum construction for in-plane shear loading differ. It is not possible therefore to have all failure modes in compression occur simultaneously with all failure modes due to shear.

Therefore, in the case of combined loads in which the axial load predominates, it appears logical to utilize the web-core construction proportioned to be optimum or minimum weight for axial compression alone, and simply modify this construction to account for the smaller in-plane shear loads.

Identical to Chapter 4, it is found that the governing relation for the structural integrity of the panel under these combined loads is given by

$$
\frac{N_x/b}{(N_{x_{cr}}/b)} + \frac{(N_{xy}/b)^2}{(N_{xy_{cr}}/b)^2} = 1 \tag{5.1}
$$

Using the web core construction proportioned
optimally for compressive loads only, when $N_x/b = N_{x_{cr}}/b$,
there will be simultaneous failure of the panel in overall
buckling, face plate buckling, and web plate buckling.
However with the panel optimally preportioned for uniaxial
compression as τ_F is increased, the failure modes will not
occur simultaneously. It is necessary to determine which
mode will occur at the lower face stress; and to determine
that lower buckling stress as $\tau_{F_{cr}}$, in order to determine
$N_{xy_{cr}}$.

The preportions for the optimum web-core panel for
uniaxial compressive loads are given in Chapter 2, Reference 1,
for each of the material systems considered. Substituting
these values in terms of $(N_{x_{cr}}/b)$ (given as N_{xy}/b in
Reference 1) into the expressions for stability of the web
core panel due to shear loads given by Equations (2.25) and
(2.26) it is found that for the panel preportioned to be
optimum for uniaxial compression, the lower critical stress
occurs for face plate buckling due to shear loads.

In fact the ratio of critical shear stresses for the
panel preportioned to be optimal for compressive loads is,
for the most general materials systems,

$$\frac{\tau_{F_1}}{\tau_{F_2}} = \frac{2\bar{j}}{kK}\left(\frac{\rho_F}{\rho_c}\right)\left(\frac{E_{cx}}{E_{F_x}}\right)\frac{E_{oF}}{[E_{F_x}E_{F_y}]^{k}}\frac{(N_x/b)^{1/4}}{\left\{1+2\left(\frac{\rho_c}{\rho_F}\right)\left(\frac{E_{F_x}}{E_{cx}}\right)\right\}^{1/8}} \tag{5.2}$$

- 74 -

where τ_{f_1} = critical stress for overall buckling

τ_{f_2} = critical stress for face plate buckling.

It can be shown that for most combinations of materials the ratio given by (5.2) is greater than unity, remembering that we are generally discussing combined loads in which compressive loads predominate.

Thus, substituting the equations for t_f/b and d_f/b given in Chapter 2 , Reference 1, for the most general material system into Equation (2.26), and utilizing (4.5) above, the result can be written as

$$\frac{N_{xy_{cr}}}{b} = \frac{2k_f}{\pi^2} \frac{\left[E_{r_y}^3 E_{rx}\right]^{1/4}}{E_{oF}} \left(\frac{E_{fx}}{E_{cx}}\right)\left(\frac{\rho_c}{\rho_f}\right) \frac{N_{xcr}/b}{\left\{1 + 2\left(\frac{\rho_c}{\rho_f}\right)\left(\frac{E_{fx}}{E_{cx}}\right)\right\}} \tag{5.3}$$

This equation therefore provides a key relationship between the two critical loads.

It is now convenient to define the dimensionless quantity γ , such that

$$\gamma \equiv \frac{2k}{\pi^2} \frac{\left[E_{r_y}^3 E_{Fx}\right]^{1/4}}{E_{oF}} \left(\frac{E_{fx}}{E_{cx}}\right)\left(\frac{\rho_c}{\rho_f}\right) \frac{1}{\left\{1 + 2\left(\frac{\rho_c}{\rho_f}\right)\left(\frac{E_{fx}}{E_{cx}}\right)\right\}} \tag{5.4}$$

Hence,

$$\left(\frac{N_{xy_{cr}}}{b}\right) = \gamma\left(\frac{N_{xcr}}{b}\right) \tag{5.5}$$

Substituting (5.5) and (4.13) into (5.1) results in the equation

$$\left(N_{xcc}/b\right)^2 - \left(N_x/b\right)\left(N_{xcr}/b\right) - \frac{\alpha^2}{\gamma^2}\left(N_x/b\right)^2 = 0 \tag{5.6}$$

The solution for this equation is

$$\left(\frac{N_{xcr}}{b}\right) = \left(\frac{N_x}{b}\right)\left\{\frac{1}{2}\left[1 + \left(1 + \frac{4\alpha^2}{\gamma^2}\right)^{1/2}\right]\right\} \tag{5.7}$$

Therefore analogous to the methods of Chapter 4,

$$\begin{bmatrix} \left(\frac{t_f}{b}\right) \\ \left(\frac{t_c}{b}\right) \\ \left(\frac{w-w_{od}}{b}\right) \end{bmatrix}_{\substack{N_x\,given \\ N_{xy}\,given}} = \begin{bmatrix} \left(\frac{t_f}{b}\right) \\ \left(\frac{t_c}{b}\right) \\ \left(\frac{w-w_{od}}{b}\right) \end{bmatrix}_{\substack{N_x\,given \\ N_{xy}=0}} \left(\frac{1}{2}\left[1 + \left(1 + \frac{4\alpha^2}{\gamma^2}\right)^{1/2}\right]\right)^{1/2} \tag{5.8}$$

$$\begin{bmatrix} \left(\frac{h_c}{b}\right) \\ \left(\frac{d_f}{b}\right) \end{bmatrix}_{\substack{N_x\,given \\ N_{xy}\,given}} = \begin{bmatrix} \left(\frac{h_c}{b}\right) \\ \left(\frac{d_f}{b}\right) \end{bmatrix}_{\substack{N_x\,given \\ N_{xy}=0}} \left(\frac{1}{2}\left[1 + \left(1 + \frac{4\alpha^2}{\gamma^2}\right)^{1/2}\right]\right)^{1/4} \tag{5.9}$$

The following design procedures are therefore given for web-core sandwich panels subjected to combined loads of uniaxial compression and in-plane shear loading where the uniaxial compressive loads predominate (which is the condition for which this type of construction appears more favorable).

    1. Given: N_x, N_{xy}, a, and b

    2. Select material system and obtain all material properties needed.

3. Letting N_x given in step 1 be considered as the
    critical compressive load when no shear faces are
    present, use the procedures of Chapter 2,
    Reference 1, to determine the optimum configura-
    tion, in the absence of shear loads.

    Thus $\left\{ \dfrac{t_f}{b} , \dfrac{t_c}{b} , \dfrac{h_c}{b} , \dfrac{d_f}{b} , \dfrac{(W-W_{ad})}{b} \right\} N_x$ given

    $\qquad\qquad\qquad\qquad\qquad\qquad N_{xy} = 0$

    are determined.

4. Determine γ :

    For faces and core of different orthotropic
    materials:

    $$\gamma = \frac{2 k_f}{\pi^2} \frac{\left[E_{f_y}{}^3 E_{f_x}\right]^{1/4}}{E_{of}} \left(\frac{E_{fx}}{E_{cx}}\right)\left(\frac{\rho_c}{\rho_f}\right) \frac{1}{\left\{1 + 2\left(\frac{\rho_c}{\rho_f}\right)\left(\frac{E_{fx}}{E_{cx}}\right)\right\}}$$

    For faces and core of different isotropic
    materials:

    $$\gamma = \frac{2 k_f}{\pi^2} \left(\frac{E_f}{E_c}\right)\left(\frac{\rho_c}{\rho_f}\right) \frac{1}{\left\{1 + 2\left(\frac{\rho_c}{\rho_f}\right)\left(\frac{E_f}{E_c}\right)\right\}}$$

    For faces and core of the same orthotropic
    material:

    $$\gamma = \frac{2 k_f}{3\pi^2} \frac{\left[E_y{}^3 E_x\right]^{1/4}}{E_o}$$

    For faces and core of the same isotropic material:

    $$\gamma = \frac{2 k_f}{3\pi^2}$$

5. Calculate α :

$$\alpha = \frac{(N_{xy}/b)}{(N_x/b)}$$

6. Determine the required panel parameters and the weight by Equations (5.8) and (5.9).

CHAPTER 6

CONCLUSIONS

Methods have been derived to design minimum weight for flat sandwich panels using triangulated (single truss) core construction or web-core construction. Panels having cores and faces of differing or the same orthotropic materials, as well as differing or the same isotropic materials, have been treated. The panel loadings are: (1) in-plane shear loads and (2) combined uniaxial compression and in-plane shear loads.

For the triangulated core construction under in-plane shear loads, where the face and core are composed of the same isotropic or orthotropic materials, the angle $\theta = 28.4^\circ$ for the optimum construction when the edge restraint coefficients are equal for both web and face elements. Similarly the weight ratio for faces to core per unit planform area is 1.316. For isotropic triangulated core construction in which both faces and core are of the same isotropic material, that material which has the highest ratio of $E^{1/2}/\rho(1-\nu^2)^{1/2}$ will result in the least weight panel.

For web core construction under in-plane shear loads $\theta = 0^\circ$ will result in minimum weight construction regardless of the materials used. It is found that the optimum face thickness for a given load index is determined entirely by the allowable shear stress of the face material from a strength

- 79 -

viewpoint. The optimum core depth is independent of the load index. Also the weight of the optimum (minimum weight) panel is independent of all material properties except the allowable face shear stress and the density of the face material. Thus the best material for this type of construction is the one having the highest value of (τ_{face}/ρ_f).

From the example comparison of Figure 5, page 61, it is seen that for the typical example the optimum honeycomb core construction is significantly better than the optimum triangulated corrugated core construction, and is capable of carrying a considerably greater shear load.

The optimum web core construction results in considerable weight savings over the optimum honeycomb sandwich construction in the low load index range. However, it does not compare favorably in the higher load range.

Combined loadings are treated in Chapters 4 and 5. One benefit derived by the development of methods of analysis for optimum (minimum weight design) structures, other than the obvious benefit, is that it enables the designer to compare the absolute minimum weight construction employing commercially available sizes that approximate the actual optimum dimensions. In this way he can more rationally assess the following considerations: the weight penalty of using commercially available sizes or the cost penalty of using non-commercially available sizes to obtain minimum weight. Obviously, the decision rests on the specific application, but it can be made rationally.

It is also recommended that a test program be designed and executed to evaluate the optimization procedures developed herein.

REFERENCES

1. Vinson, J. R., and Shore, S., "Structural Optimization of Flat, Corrugated Core and Web-Core Sandwich Panels Under Uniaxial Compressive Loads", Structural Mechanics Associates Report No. 040266, July 1966.

2. Vinson, J. R., and Shore, S., "Methods of Structural Optimization for Flat Sandwich Panels", U. S. Naval Air Engineering Center Report No. NAEC - ASL - 1083, 15 April 1965. [*]

3. Vinson, J. R., and Shore, S., "Design Procedures for the Structural Optimization of Flat Sandwich Panels", U. S. Naval Air Engineering Center Report No. NAEC - ASL - 1084, 15 April 1965. [*]

4. Libove, C. and Hubka, R. E., "Elastic Constants for Corrugated-Core Sandwich Plates", NACA TN 2289, February 1951.

5. Anderson, Melvin S., "Optimum Proportions of Truss Core and Web-Core Sandwich Plates Loaded in Compression", NASA TN D-98, September 1959.

---

[*] Also, presented at the 5th U. S. National Congress of Applied Mechanics, Minneapolis, Minnesota, June 1966, and abstracts appearing in the Transactions of the Congress.

6. "Sandwich Construction for Aircraft, Part II", ANC-23
   Second Edition 1955, issued by the Department of
   the Air Force Air Research and Development Command;
   Department of the Navy, Bureau of Aeronautics; and
   the Department of Commerce, Civil Aeronautics
   Administration.

7. Timoshenko, S. and Gere, J. M., "Theory of Elastic
   Stability", Second Edition. McGraw-Hill Book Co.,
   Inc., New York 1961.

8. Gerard, George and Becker, Herbert, "Handbook of
   Structural Stability, Part III, Buckling of
   Curved Plates and Shells", NACA TN 3783,
   August 1957.

9. Kaechele, L. E., "Minimum Weight Design of Sandwich
   Panels", U. S. Air Force Project Rand Research
   Memorandum, RM 1895. The Rand Corporation,
   Armed Services Information Agency, Accession
   No. AD 133011, March 1957.

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY *(Corporate author)*	2a REPORT SECURITY CLASSIFICATION
Structural Mechanics Associates	Unclassified
1433 Sandy Circle	**2b GROUP**
Narberth, Pa.	None

3 REPORT TITLE

STRUCTURAL OPTIMIZATION OF FLAT, CORRUGATED CORE AND WEB-CORE SANDWICH PANELS UNDER IN-PLANE SHEAR LOADS AND COMBINED UNIAXIAL COMPRESSION AND IN-PLANE SHEAR LOADS

4 DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Final report (24 May 1965 to 31 December 1966)

5 AUTHOR(S) *(Last name, first name, initial)*

Vinson, Jack R.
Shore, Sidney

6 REPORT DATE	7a TOTAL NO. OF PAGES	7b NO OF REFS
1 June 1967	99	9

8a CONTRACT OR GRANT NO	9a ORIGINATOR'S REPORT NUMBER(S)
N156-46654	
b. PROJECT NO	NAEC-ASL-1110
Work Unit 530/07 (P.A. 1-23-96)	
c	**9b. OTHER REPORT NO(S)** *(Any other numbers that may be assigned this report)*
d	

10 AVAILABILITY/LIMITATION NOTICES

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Air Engineering Center
	Aeronautical Structures Laboratory
	Philadelphia, Pa. 19112

13 ABSTRACT

In this report is presented the development of rational methods of structural optimization for flat, corrugated core (single truss core) and web-core sandwich panels under two loading conditions: in-plane shear loads, and combined uniaxial compression and in-plane shear loads.

These methods provide a means by which minimum weight structures can be designed for a given load index, plate width, length, and face and core materials. The methods developed can be used as a means of rational material selection by comparing weights of optimum construction for various material systems as a function of applied load index. The methods are sufficiently general to account for orthotropic or isotropic face and core materials and various boundary conditions. Design procedures are given in detail for the design engineer to use.

DD FORM 1473
1 JAN 64

023558

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Optimization						
Corrugated Core						
Truss Core						
Sandwich Panels						
Buckling						

INSTRUCTIONS

I. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (corporate author) issuing the report.

2a. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7a. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8a. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers (either by the originator or by the sponsor), also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through

_____ ."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through

_____ ."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through

_____ ."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring (paying for) the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TS), (S), (C), or (U)

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.