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RESEARCH AND DEVELOPMENT OF MATERIEL

ENGINEERING DESIGN HANDBOOK

SERVOMECHANISMS SECTION 4, POWER ELEMENTS AND SYSTEM DESIGN

HEADQUARTERS, U. S. ARMY MATERIEL COMMAND AUGUST 1963

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AMCP 706-139, Servomechanisms, Section 4, Power Elements and System Design, forming part of the Army Materiel Command Engineering Design Handbook Series, is published for the information and guidance of all concerned,

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SERVOMECHANISMS

SECTION 4, POWER ELEMENTS AND SYSTEM DESIGN

CONSISTING OF

CHAPTERS 14–20

FOREWORD

INTRODUCTION

This is one of a group of handbooks covering the engineering information and quantitative data needed in the design and construction of ordnance equipment, which (as a group) constitutes the Ordnance Engineering Design Handbook Series.

PURPOSE OF HANDBOOK

The handbook on Servomechanisms has been prepared as an aid to designers of automatic control systems for Army Ordnance equipments, and as a guide to military and civilian personnel who are responsible for setting control-system specifications and ensuring their fulfillment.

SCOPE AND USE OF HANDBOOK

The publications are presented in handbook form rather than in the style of textbooks. Tables, charts, equations, and bibliographical references are used in abundance. Proofs and derivations are often omitted and only final results with interpretations are stated. Certain specific information that is always needed in carrying out design details has, of necessity, been omitted. Manufacturers' names, product serial numbers, technical specifications, and prices are subject to great variation and are more appropriately found in trade catalogs. It is essential that up-to-date catalogs be used by designers as supplements to this handbook.

To make effective use of the handbook during the design of a servo, the following procedure is suggested. The designer should turn first to Chapters 16 and 17 where design philosophy and methods are discussed. Implementation of the design procedure may require a review of certain theoretical concepts and methods which can be achieved through reference to Chapters 1 through 10. As the design proceeds, a stage will be reached at which the power capacity of the output member has been fixed. Reference to Chapters 14, 15, and 16 will then illustrate the salient features of output members having the required power capacity. After the designer has chosen the output member, he will find the information dealing with sensing elements and amplifiers (Chapters 11, 12, and 13) helpful in completing the design.

FEEDBACK CONTROL SYSTEMS AND SERVOMECHANISMS

Servomechanisms are part of a broad class of systems that operate on the principle of feedback. In a feedback control system, the output (response) signal is made to conform with the input (command) signal by feeding back to the input a signal that is a function of the output for the purpose of comparison. Should an error exist, a corrective action is automatically initiated to reduce the error toward zero. Thus, through feedback, output and input signals are made to conform essentially with each other.

In practice, the output signal of a feedback control system may be an electrical quantity such as a voltage or current, or any one of a variety of physical quantities such as a linear or angular displacement, velocity, pressure, or temperature. Similarly, the input signal may take any one of these forms. Moreover, in many applications, input signals belong to, one of these types, and the output to another. Suitable transducers or measuring devices must then be used. It is also common to find multiple feedback paths or loops in complicated feedback control systems. In these systems, the over-all system performance as characterized by stability, speed of response, or accuracy can be enhanced by feeding back signals from various points within the system to other points for comparison and initiation of correction signals at the comparison points.

At present, there is no standard definition of a servomechanism. Some engineers prefer to classify any system with a feedback loop as a servomechanism. According to this interpretation, an electronic amplifier with negative feedback is a servo. More frequently, however, the term servomechanism is reserved for a feedback control system containing a mechanical quantity. Thus, the IRE defines a servomechanism as "a feedback control system in which one or more of the system signals represents mechanical motion." Some would restrict the definition further by applying the term only to a special class of feedback control system in which the output is a mechanical position.

APPLICATION OF SERVOMECHANISMS TO ARMY EQUIPMENT

Servomechanisms are an important part of nearly every piece of modern mechanized Army equipment. They are used to automatically position gun mounts, missile launchers, and radar antennas. They aid in the control of the flight paths of jet-propelled rockets and ballistic missiles, and play an important role in the navigational systems of those vehicles. As instrument servos, they permit remote monitoring of physical and electrical quantities and facilitate mathematical operations in computers.

No single set of electrical and physical requirements can be stated for servomechanisms intended for these diverse military applications. The characteristics of each servomechanism are determined by the function it is to perform, by the characteristics of the other devices and equipments with which it is associated, and by the environment to which it is subjected. It will often be found that two or more servo-system configurations will meet a given set of performance specifications. Final choice of a system may then be determined by such factors as ability of the system to meet environmental specifications, availability of components, simplicity, reliability, ease of maintenance, ease of manufacture, and cost. Finally, the ability to translate any acceptable paper design into a piece of physical equipment that meets electrical and physical specifications and works reliably depends to a great extent upon the skill of the engineering and manufacturing groups responsible for building the system. The exercise of care and good judgment when specifying electrical, mechanical, and thermal tolerances on components and subsystems can contribute greatly to the successful implementation of servo-system design.

The handbook on Servomechanisms was prepared under the direction of the Ordnance Engineering Handbook Office, Duke University, under contract to the Office of Ordnance Research. The material for this pamphlet was prepared by Jackson & Moreland, Inc., Boston, Massachusetts, under subcontract to the Ordnance Engineering Handbook Office. Jackson & Moreland, Inc. was assisted in their work by consultants who are recognized authorities in the field of servomechanisms.

PREFACE

Section **4** of the handbook on Servomechanisms contains Chapters **14** through **20,** which discuss servo power elements and system design. The significant features of servo output members are presented in Chapter **14** (Power Elements Used in Controllers) and in Chapter **15** (Mechanical Auxiliaries Used in Controllers). Chapter **16** is devoted to a discussion of the typical procedure that can be used as a guide when designing a servo system; this is followed by descriptions in Chapter **17** of two representative servo systems from existing Army equipments. Supplementary design information is given in Chapter **18** (Auxiliaries Associated with Servomechanisms), Chapter **19** (Constructional Techniques) and Chapter **20** (Supplementary Tables, Formulas, and Charts).

For information on other servomechanism components and on feedback control theory, see one of the following applicable sections of this handbook:

ORDP 20-136 Section **1** Theory (Chapters **1-10)**

ORDP 20-137 Section **2** Measurement and Signal Converters (Chapters **11-12)**

ORDP 20-138 Section **3** Amplification (Chapter **13)**

An index for the material in all four sections is placed at the end of Section **4.**

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CHAPTER 14

POWER ELEMENTS USED IN CONTROLLERS*

14-1 INTRODUCTION

The following types of power elements or motors used in servomechanisms are discussed in this chapter: direct-current and alternating-current electrical motors, hydraulic and pneumatic motors, and magnetic-particle clutches. A comparison between a-c and d-c motors appears in Par. 14-2.1; problems of hydraulic motors, pneumatic motors, and magnetic clutches are listed in Pars. 14-4, 14-5, and 14-6, respectively. The choice of which output motor to use is discussed in Par. 16-3.4.

14-2 DIRECT-CURRENT MOTORS

14-2.1 USAGE OF D-C MOTORS

Of the two classes of motors available for servo applications, direct-current motors, in comparison with alternating-current motors, have the following advantages:

(a) Speed is easily controllable.

(b) By varying field excitation, high power gains are possible for speed control.

(c) High efficiency is obtained for motors larger than 100-watt rating.

(d) Dependable rotary electric amplifiers to drive larger size motors are readily available.

Direct-current motors have the following disadvantages:

(a) Commutator produces electrical noise.

(b) Brushes wear out (conditions of cold, dry air appear to accelerate wear).

(c) Brushes have an undesirable electrical characteristic of acting as a nonlinear resistance (an almost constant voltage drop occurs across the brush; this voltage drop must be exceeded when starting the motor before a proportional input-output relationship can be obtained).

(d) Brushes add coulomb friction (for small motors, 10 to 20 watts, this becomes of consequence).

(e) The added complexity of a demodulator is required if an a-c signal is used in the error detector and amplifier.

The types of d-c motors used in servo applications are the series motor, the separately excited shunt motor, .and the permanent-magnet motor. Series motors have high starting torque and poor speed regulation. Shunt motors have lower starting torque but better

By P.E. Smith, Jr.

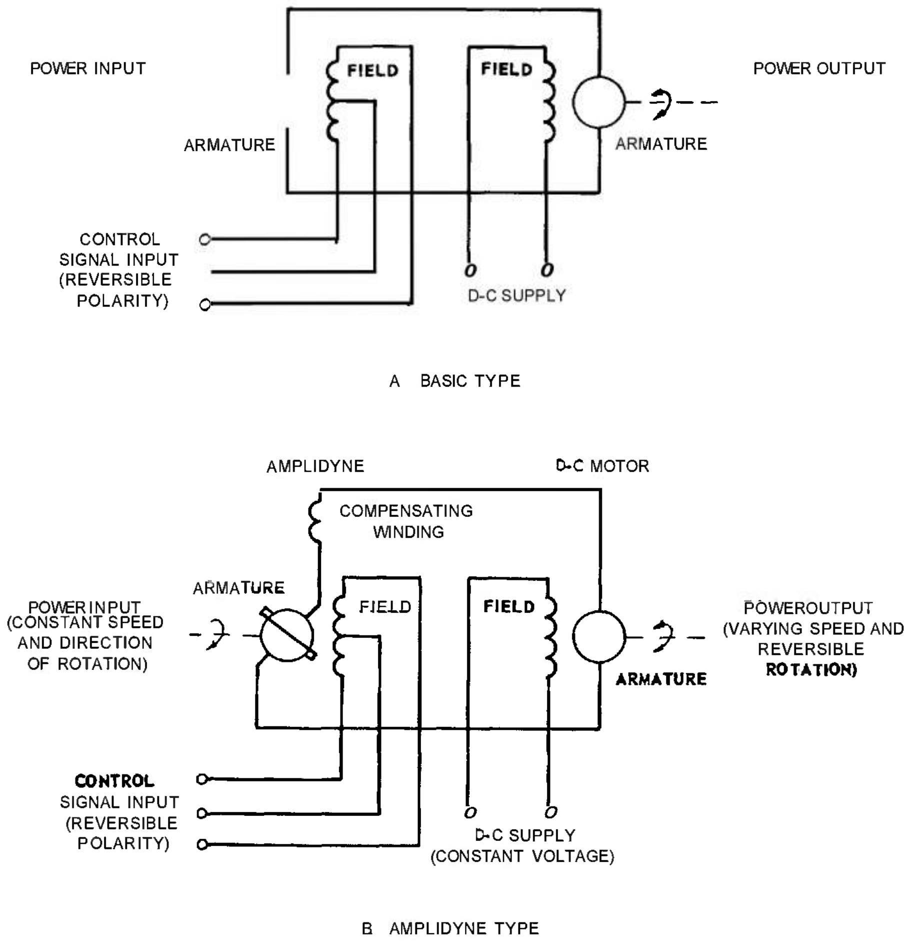


Fig. 14-1 Rotary electric amplifier circuits for control of d-c motor.

speed regulation than series motors. Permanent-magnet motors are easy to drive since no power is used for the field. A limitation is that the field may be demagnetized if the motor is badly overloaded. Conventional d-c motors are unidirectional devices. The armature rotation of series and shunt motors can be reversed by reversing the current flow through either the armature or field but not both simultaneously. Permanent-magnet motors are reversed by reversing the direction of armature current ; high torques on reversal are obtainable.

Many means are available for controlling the speed or armature position of d-c motors; they are

(a) Rotary electric amplifiers

(b) High-vacuum tube amplifiers

(c) Gas-filled tube amplifiers

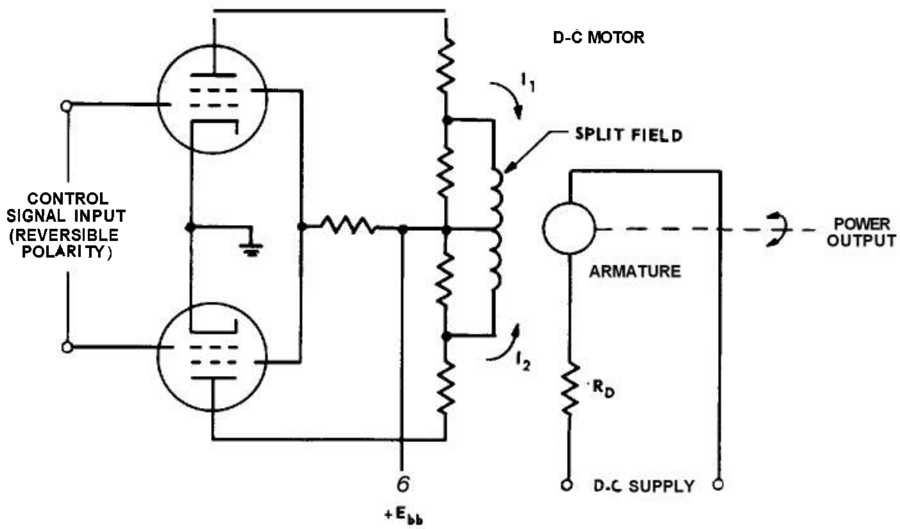
(d) Magnetic amplifiers

(e) Relay amplifiers

The generator, a basic rotary electric amplifier (Fig. 14-1A), operates at a constant speed and produces an output voltage that is proportional to the input signal applied to its field. The output voltage developed across the low impedance of the generator armature excites the d-c motor armature with comparatively low power losses. The amplifying action of the generator produces power gains up to 100. A variation of the basic rotary electric amplifier is the amplidyne (Fig. 14-1B). A short-circuited brush connection in the armature of the amplidyne makes it possible to obtain high power gains of 3000 to 10,000. By this means, it is feasible to control 5- to 10-hp motors from a source capable of delivering only 5 to 10 watts of input signal. The higher power gain of the amplidyne is achieved at the expense of a slower response. Rotary electric amplifiers (see Par. 13-4) are used when large motors are to be controlled; when drive motors less than 1/4-hp rating are to be controlled, this type of amplifier becomes impractical.

High-vacuum tube amplifiers (see Par. 13-1) are used to control d-c motors when the motor is small enough so that a rotary electric amplifier is impractical. The amplifier output may be used to excite either the armature or field of the motor. Armature control by high-vacuum tubes is limited to motors not larger than 1/20-hp rating because the entire motor output power is derived from the armature circuit. Field control (Fig. 14-2) may be used with motors whose ratings do not exceed 1/2 hp; this assumes an amplifier output of 20 watts and a motor field-to-armature power gain of about 25. To compensate for the normal characteristics of a field-excited motor to exhibit increased speed at decreased field excitation and reduced torque at high speed, a constant armature current is required. Partial compensation is obtained by using a current-limiting resistor in series with a high-voltage supply for the armature.[1] To prevent large induced voltages from being developed in the field windings when the field current is cut off for reversal, some protective device should be used to limit the voltage rise. For small motors, each field winding may be shunted by a fixed resistor.[2]

Gas-filled tube amplifiers (see Par. 13-1) can control motors about ten times larger than is feasible with high-vacuum tube amplifiers. Typical circuits using thyratrons with either a-c or d-c control signals are shown in Fig. 14-3.[3,4,5] The split-field series motor control circuit shown in Fig. 14-3A uses no plate transformer. In Fig. 14-3B, the plate transformer provides circuit isolation from the a-c supply and a better match of a-c supply voltage to tube and motor characteristics. In the circuit shown in Fig. 14-3C, two gas-filled diodes allow a common cathode input circuit to be used without a plate transformer. When $V1$ is conducting, armature current passes through it and diode $V4$. When $V2$ is made conducting, current passes through it and diode $V3$. Resistors $R1$ and $R2$ maintain the common cathode connection at a fixed potential to ground during the time neither triode is conducting. For gas-filled tube amplifiers, Burnett[6] states that the best system response may be obtained by the use



$$R_D \geq \frac{V_a}{I_R} - R_a$$

WHERE

R_D = CURRENT-LIMITING RESISTANCE

I_R = RATED ARMATURE CURRENT

V_a = D-C SUPPLY VOLTAGE

R_a = ARMATURE RESISTANCE

$I_1 > I_2$ LEADS TO CW ROTATION

$I_2 > I_1$ LEADS TO CCW ROTATION

Fig. 14-2 High-vacuum tube circuit for control of d-c motor.

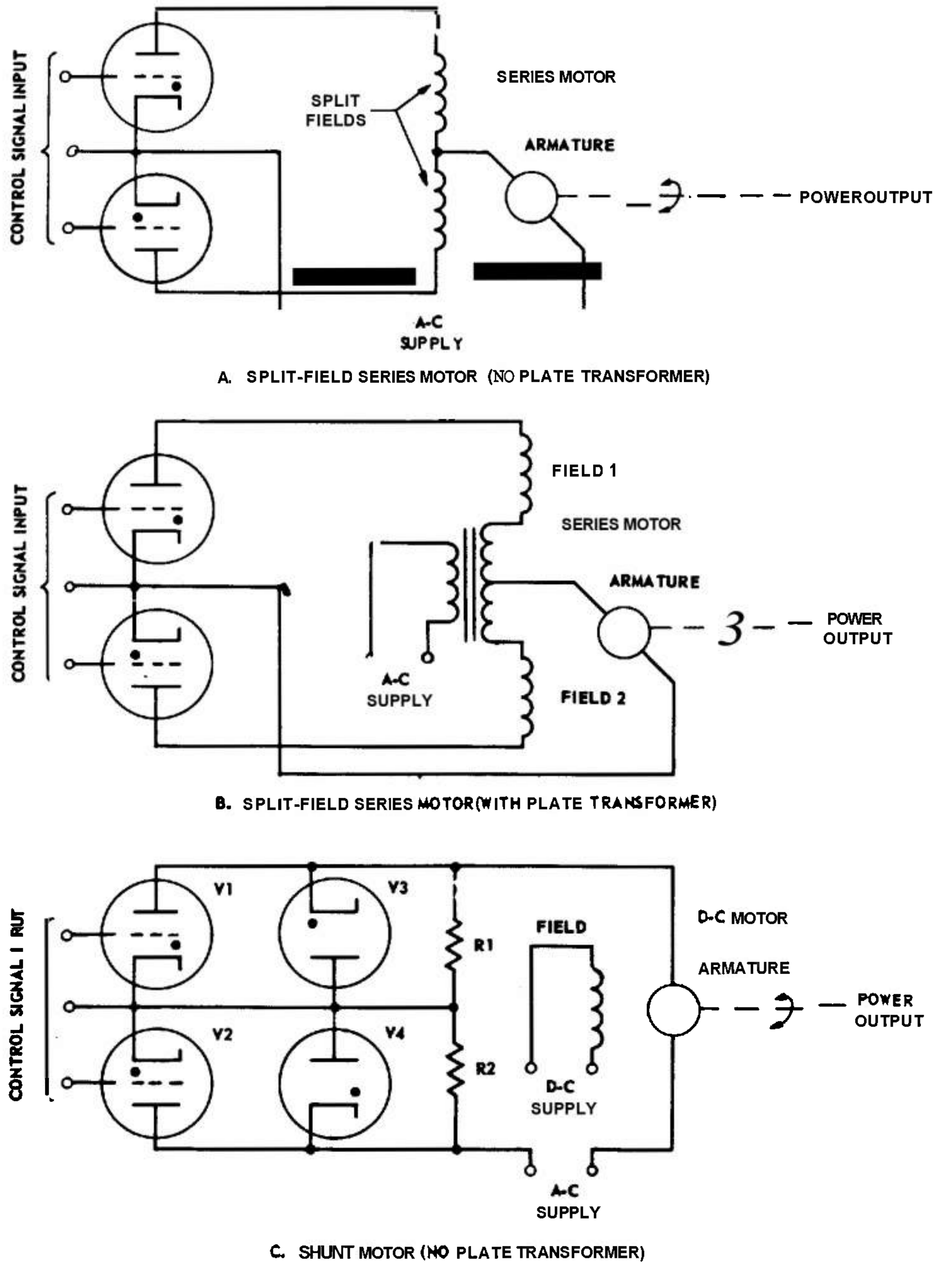


Fig. 74-3 Gas-filled tube circuits for control of d-c motors.

of armature control powered from a multiphase supply. He also states that, with 3-phase power, a 10-hp velocity servo with a frequency response flat to 15 cps is possible; position control with a flat response to 10 cps with the same motor is claimed.

Magnetic amplifiers (see Par. 13-3) provide more reliable control of d-c motors than do vacuum-tube or relay amplifiers. Figure 14-4 shows a typical magnetic amplifier with two cores for controlling the position of a series motor with split-field windings.[7] By reversing the polarity of the input signal, either direction of motor rotation can be obtained. If a separately excited motor with constant-voltage field supply were used, excessive circulating current would result when the motor operates in one direction.[8] Magnetic amplifiers can be used to control the speed of d-c motors by field control or armature control.[9] One circuit (Fig. 14-5) for unidirectional speed control uses a large inductance in series with the armature circuit to obtain sloping speed-torque characteristics very similar to the uncontrolled motor but with greater slope.[10] If compensation proportional to the armature current is added, nearly flat speed-torque characteristics can be achieved.

Relay amplifiers (see Par. 13-5) are used to control d-c motors when size and weight are primary considerations and reliability is secondary. A single-pole, double-throw polarized relay (Fig. 14-6A) provides a simple means for controlling a series motor with split-field windings. No motor damping is provided when the relay is not energized (contacts open). If motor damping is desired, two single-pole, double-throw relays which are alternately energized may be used (Fig. 14-6B). To minimize relay contact current, field control of a shunt motor can be used. In this case, the armature should be powered from a constant current source. The protection of relay contacts is discussed in Par. 13-5.

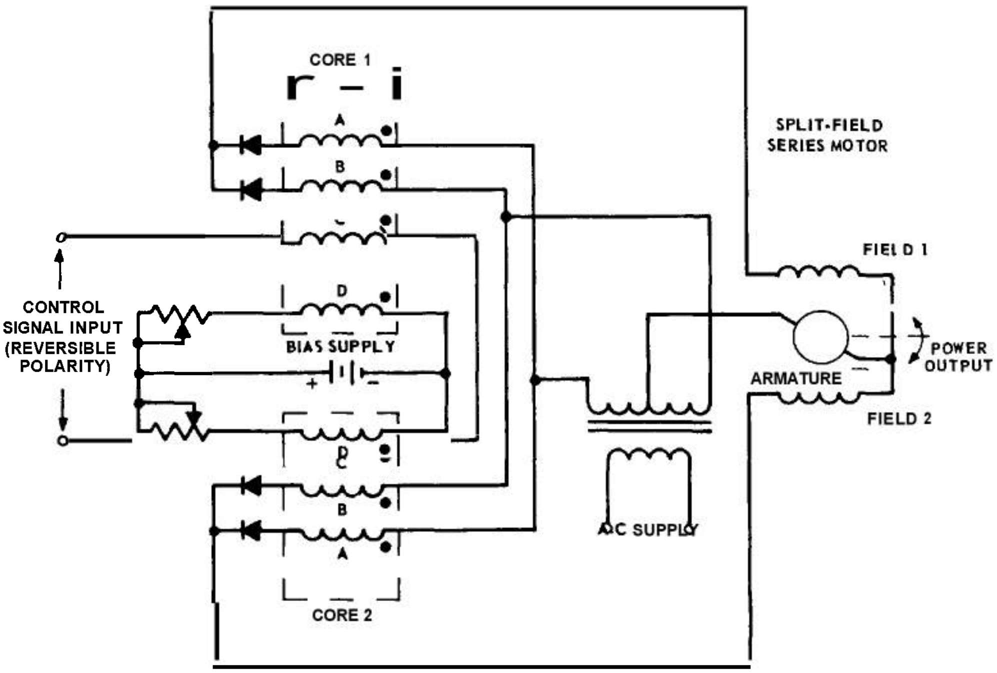


Fig 74-4 Magnetic amplifier circuit for control of series motor.

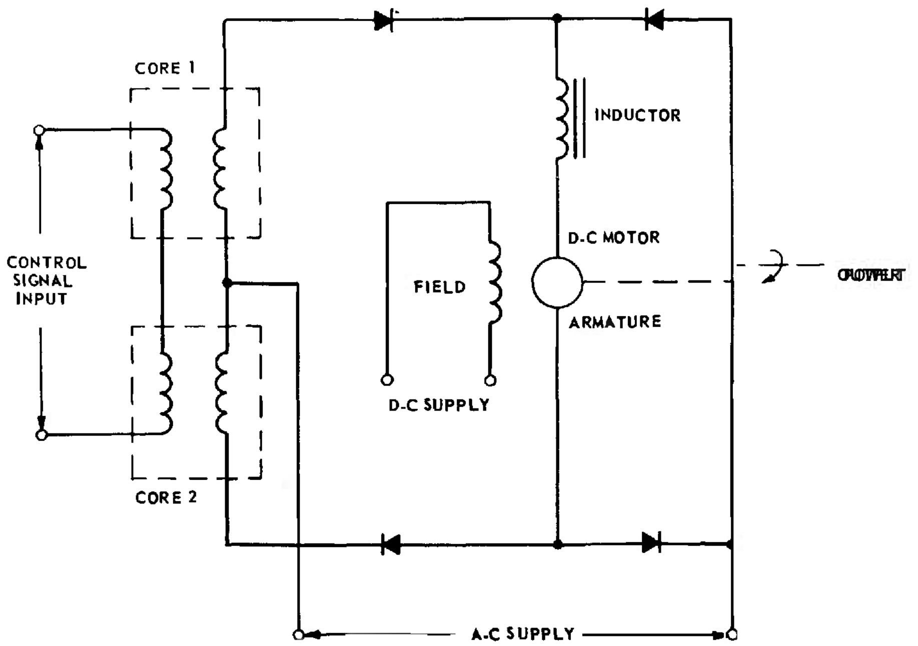


Fig. 14-5 Magnetic amplifier circuit for unidirectional control of shunt motor.

14-2.2 STATIC CHARACTERISTICS OF D-C MOTORS

In the general case for all types of d-c motor control, the following equations express the steady-state behavior of the motor:

$$V_t = I_a R_a + K I_f \omega \qquad (14\text{-}1)$$

$$T = K I_f I_a \qquad (14\text{-}2)$$

where

V_t = armature terminal voltage, in volts

I_a = armature current, in amperes

I_f = field current, in amperes

R_a = armature resistance, in ohms

K = a constant

T = torque produced by motor, in newton-meters

ω = speed of armature-shaft rotation, in radians/sec ($\omega = 2\pi \times$ rpm/60)

The above general equations may be modified for the following particular types of motor control:

(a) Armature control with constant field current

(b) Field control with constant armature current

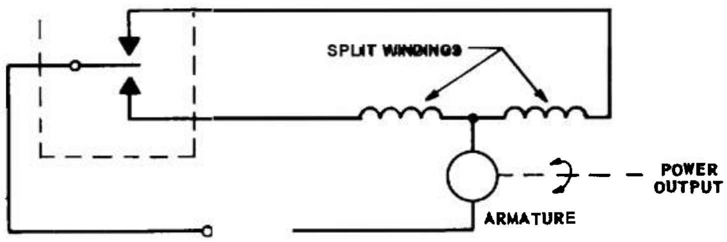
(c) Field control with constant armature voltage

(d) Series motor control

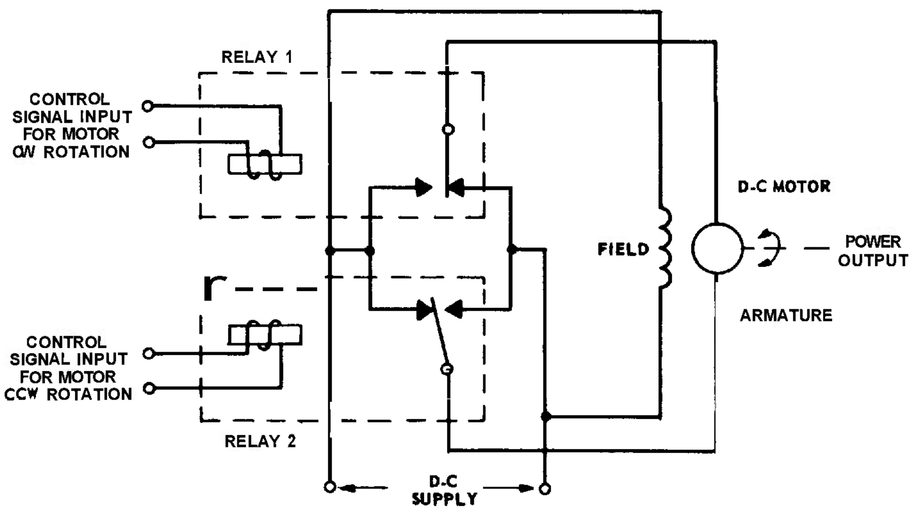
14-2.3 Armature Control with Constant Field Current

Since the field current is constant in this type of control, the term KI_f may be combined in a new term K_1, and substituted into Eqs. **(14-1)** and **(14-2)**. Then Eq. **(14-2)** is substituted into Eq. **(14-1)** which is solved for ω to obtain

$$\omega = \frac{V_t}{K_1} - T\frac{R_a}{K_1^2} \qquad (14\text{-}3)$$



A. SERIES MOTOR CONTROL (NO DAMPING)



B. SHUNT MOTOR CONTROL (WITH DAMPING)

Fig. 74-6 Relay amplifier circuits for control of d-c motors.

Solving **Eq. (14-3)** for T

$$T = \frac{K_1}{R_a} V_t - \frac{K_1^2}{R_a} \omega \qquad (14\text{-}4)$$

The stall torque T_M which occurs at the maximum armature terminal voltage V_{tM} is found from **Eq. (14-4)** when $T = T_M$, $V_t = V_{tM}$, and $\omega = 0$.

$$T_M = \frac{K_1}{R_a} V_{tM} \qquad (14\text{-}5)$$

Solving **Eq. (14-5)** for $\frac{K_1}{R_a}$

$$\frac{K_1}{R_a} = \frac{T_M}{V_{tM}} \qquad (14\text{-}6)$$

Substituting **Eq. (14-6)** into **Eq. (14-4)**

$$T = \frac{T_M}{V_{tM}} V_t - \frac{T_M}{V_{tM}} K_1 \omega \qquad (14\text{-}7)$$

The no-load speed ω_M at the maximum armature terminal voltage V_{tM} is found from **Eq. (14-3)** when $\omega = \omega_M$, $V_t = V_{tM}$, and $T = 0$.

$$\omega_M = \frac{V_{tM}}{K_1} \qquad (14\text{-}8)$$

Solving **Eq. (14-8)** for $\frac{K_1}{V_{tM}}$

$$\frac{K_1}{V_{tM}} = \frac{1}{\omega_M} \qquad (14\text{-}9)$$

Substituting **Eq. (14-9)** into **Eq. (14-7)**

$$T = \frac{T_M V_t}{V_{tM}} - \frac{T_M \omega}{\omega_M} \qquad (14\text{-}10)$$

Dividing **Eq. (14-10)** by T_M results in

$$\frac{T}{T_M} = \frac{V_t}{V_{tM}} - \frac{\omega}{\omega_M} \qquad (14\text{-}11)$$

A plot of T/T_M versus ω/ω_M characteristics with V_t/V_{tM} as a parameter is shown in Fig. **14-7.** The curves of this figure are nondimensionalized and based on Eq. **(14-11).**

If the manufacturer's data for a particular motor are known, the curves can be dimensionalized for that motor. For example, a motor has an armature resistance of **5** ohms and a no-load speed of **3000** rpm at a maximum armature terminal voltage of **90** volts.

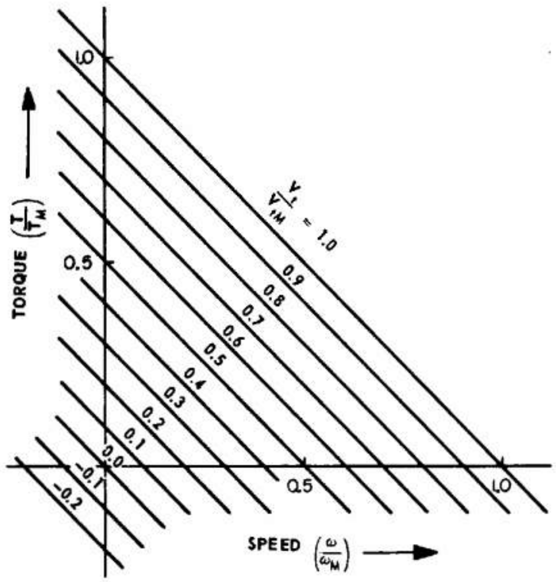


Fig. 14-7 Static torque-speed characteristics of armature-controlled d-c motor.

To dimensionalize a plot of T/T_M versus ω/ω_M characteristics, find the value of no-load speed ω_M in radians/sec and the value of stall torque T_M.

From **Eq. (14-1)**, ω is defined as $2\pi \times$ rpm/60. At no-load speed, $\omega = \omega_M$ which is

$$\omega_M = \frac{2\pi \times 3000}{60}$$

$$= 314 \text{ radians/sec}$$

At the point $\omega/\omega_M = 1$ on Fig. **14-7,** the no-load speed ω_M is **314** radians/sec. All other points on the ω/ω_M axis are dimensioned proportionally.

Substituting **314** for ω_M and **90** for V_{tM} in Eq. **(14-9)** and solving for K_1 gives

$$K_1 = \frac{V_{tM}}{\omega_M}$$

$$= \frac{90}{314}$$

$$= 0.286$$

From Eq. (14-5), the stall torque is

$$T_M = \frac{K_1}{R_a} V_{tM} \qquad (14\text{-}5)$$

$$= \frac{0.286}{5} \times 90$$

$$= 5.15 \text{ newton-meters}$$

At the point $T/T_M = 1$ on Fig. 14-7, the stall torque $T_M = T = 5.15$ newton-meters. All other points on the T/T_M axis are dimensioned proportionally. A value of 90 volts is assigned to curve $V_t/V_{tM} = 1$ (maximum armature terminal voltage), and the other curves are dimensioned proportionally.

14-2.4 Field Control with Constant Armature Current

The armature current must be maintained constant for linear operation of field-controlled motors. The torque is given by Eq. (14-2) which applies directly in this case.

$$T = K I_f I_a \qquad (14\text{-}2)$$

With the maximum field current I_{fM} applied, the maximum torque T_M which results is

$$T_M = K I_{fM} I_a \qquad (14\text{-}12)$$

Dividing Eq. (14-2) by Eq. (14-12) results in

$$\frac{T}{T_M} = \frac{I_f}{I_{fM}} \qquad (14\text{-}13)$$

Figure 14-8 is a plot of T/T_M versus ω/ω_M characteristics with I_f/I_{fM} as a parameter. Note that the figure and Eq. (14-13) are nondimensionalized and that the torque is independent of the motor speed.

14-2.5 Field Control with Constant Armature Voltage

Equations (14-1) and (14-2) express the steady-state behavior of this type of control. The nondimensionalized equation is*

$$\frac{T}{T_M} = \left(\frac{I_f}{I_{fM}}\right)\left(1 - \frac{I_f}{I_{fM}} \times \frac{\omega}{\omega_B}\right) \qquad (14\text{-}14)$$

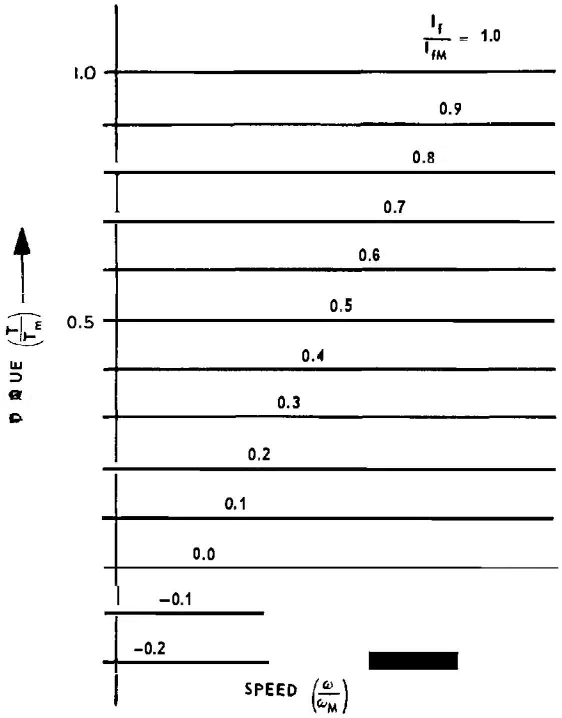


Fig. 14-8 Static torque-speed characteristics of field-controlled d-c motor, constant armature current.

where

I_{fm} = maximum field current

T_m = stall torque at $I_f = I_{fM}$

ω_B = no-load (zero-torque) speed for $I_f = I_{fM}$

A plot of Eq. (14-14) is shown in Fig. 14-9.

14-2.6 Series Motor Control

Since the armature current is the same as the field current in this type of control, the term I_f can be substituted into Eqs. (14-1) and (14-2) to obtain

$$V_t = I_a R_a + K I_a \omega \qquad (14\text{-}15)$$

$$T = K I_a^2 \qquad (14\text{-}16)$$

*See Appendix on page 14-24.

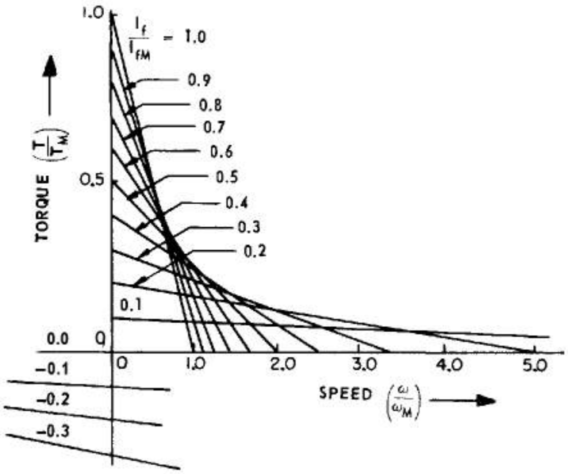


Fig. 14-9 Static torque-speed characteristics of field-controlled d-c motor, constant armature voltage.

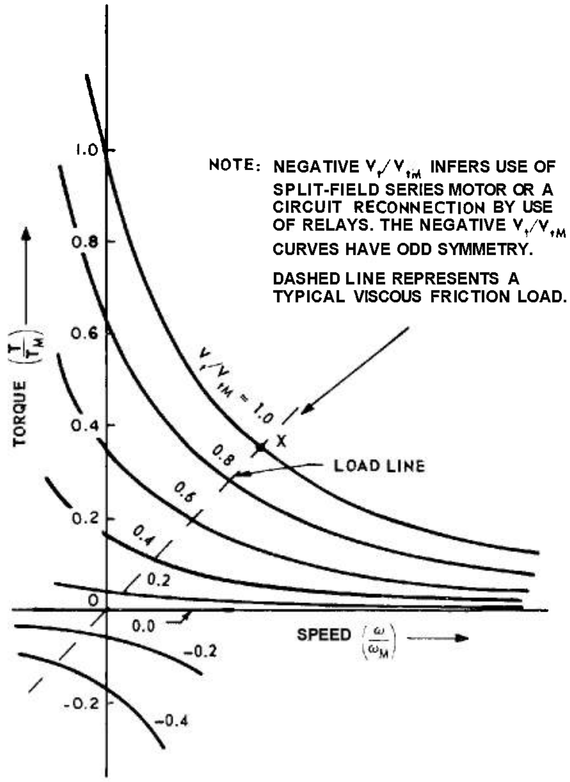


Fig. J4-10 Static torque-speed characteristics of series motor.

Solving **Eq. (14-16)** for $I_{,}$ and substituting into **Eq. (14-15)** gives

$$V_t = \left(\frac{T}{K}\right)^{1/2} \times \left(R_a + K\,\omega \right) \qquad \textbf{(14-17)}$$

By defining $\omega_o = R_a/K$ and T_M as the stall torque at V_{tM}, the maximum armature terminal voltage, it can be shown that

$$\frac{T}{T_M} = \frac{\left(\dfrac{V_t}{V_{tM}}\right)^2}{\left(1 + \dfrac{\omega}{\omega_o}\right)^2} \qquad \textbf{(14-18)}$$

A plot of T/T_M versus ω/ω_o characteristics with V_t/V_{tM} as a parameter is shown in Fig. **14-10.** The curves in this figure are non-dimensionalized and based on **Eq. (14-18).**

14-2.7 DYNAMIC CHARACTERISTICS OF D-C MOTORS

The dynamic characteristics of d-c motors involve additional parameters besides those that describe the static Characteristics. The effect of the armature or field inductance and the mechanical inertia and load must be considered. The dynamic characteristics of the following types of motor control are discussed :

(a) Armature control with constant field current

(b) Field control with constant armature current

(c) Field control with constant armature voltage

(d) Series motor control

(e) Rotary electric amplifier control

(f) Electronic amplifier control

14-2.8 Armature Control with Constant Field Current

The dynamic characteristics for the armature-controlled motor are expressed by the equations

$$V_t = I_a R_a + KI_f \omega + L_a s I_a \qquad (14\text{-}19)$$

$$T = T_{Lt} + Js\omega \qquad (14\text{-}20)$$

where

s = the complex frequency operator

L_a = armature inductance, in henries

J = inertia of motor, gear train, and load referred to the motor shaft, in kilogram-meters'

T_{Lt} =load, bearing, and motor windage torque referred to the motor, in newton-meters

If the load, bearing, and motor windage torque has a component which can be approximated as proportional to the speed ω, the load torque may be expressed as

$$T_{Lt} = T_L + B\omega \qquad (14\text{-}21)$$

where

Bo = viscous friction component of load torque

T_L = all effects of load torque except J and Bo

Equations (14-19), (14-20), and (14-21) are combined and expressed in block diagram form in Fig. 14-11. Block diagram formulation is explained in Par. 3-5.

14-2.9 Field Control with Constant Armature Current

The following equations describe the dynamic behavior of a field-controlled motor, assuming an ideal constant-current source for the armature:

$$V_f = I_f R_f + L_f s I_f \qquad (14\text{-}22)$$

$$T = T_L + Bo + Js\omega \qquad (14\text{-}23)$$

where

V_f = field terminal voltage, in volts

R_f = field resistance, in ohms

L_f = field inductance, in henries

Equations (14-22) and (14-23) are combined and expressed in block diagram form in Fig. 14-12.

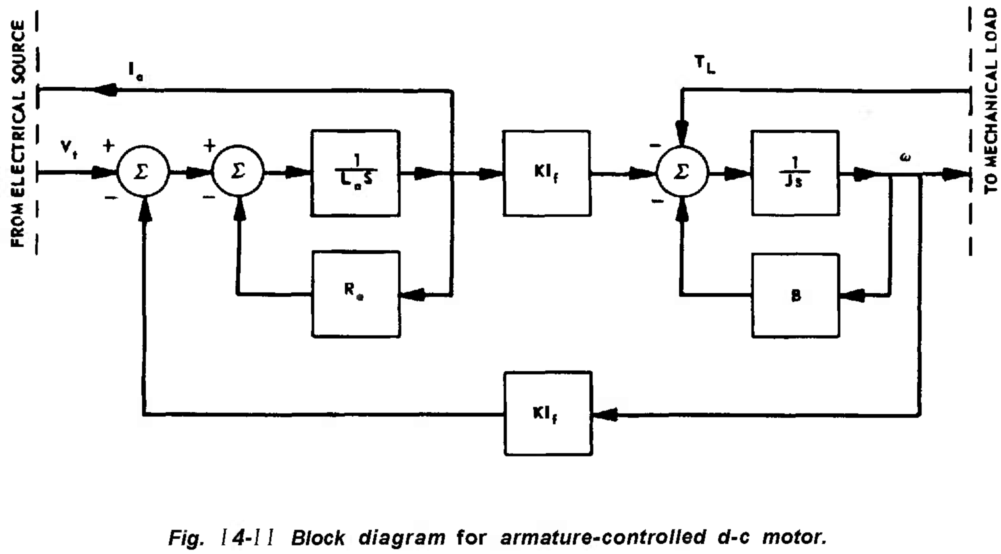


Fig. 14-11 Block diagram for armature-controlled d-c motor.

14-2.10 Field Control with Constant Armature Voltage

The dynamic behavior for the field-controlled motor with constant armature voltage is expressed by the equations

$$V_t = I_a R_a + KI_f\omega + L\frac{dI_a}{dt} \qquad (14\text{-}24)$$

$$T = KI_f I_a \qquad (14\text{-}2)$$

$$T = T_L + B\omega + J\frac{d\omega}{dt} \qquad (14\text{-}25)$$

The terms $KI_f\omega$ and $KI_f I_a$ contain the products of two variables, and introduce a nonlinearity. If the back-emf term $KI_f\omega$ is denoted as V_b, the variation in the quantities V_b and T can be written as

$$\Delta V_b = \frac{\partial V_b}{\partial I_f}\Delta I_f + \frac{\partial V_b}{\partial \omega}\Delta\omega \qquad (14\text{-}26)$$

$$\Delta T = \frac{\partial T}{\partial I_f}\Delta I_f + \frac{\partial T}{\partial I_a}\Delta I_a \qquad (14\text{-}27)$$

Add the average value V_{bo} to the variation ΔV_b; and add the average value T_o to the variation AT. Then

$$T = T_o + AT = T_o + \frac{\partial T}{\partial I_f}\Delta I_f + \frac{\partial T}{\partial I_a}\Delta I_a \qquad (14\text{-}29)$$

The partial derivatives become

$$\left.\begin{aligned} \frac{\partial V_b}{\partial I_f} &= K\omega \\ \frac{\partial V_b}{\partial \omega} &= KI_f \\ \frac{\partial T}{\partial I_f} &= KI_a \\ \frac{\partial T}{\partial I_a} &= KI_f \end{aligned}\right\} \qquad (14\text{-}30)$$

The average values of the various variables are: terminal voltage V_{to}; armature current I_{ao}; speed ω_o; torque T_o; and field current I_{fo}. In terms of average values only, Eqs. (14-24), (14-2), and (14-25) become

$$V_{to} = I_{ao}R_a + KI_{fo}\omega_o \qquad (14\text{-}31)$$

$$T_o = KI_{fo}I_{ao} \qquad (14\text{-}32)$$

$$T_o = T_{Lo} + B\omega_o \qquad (14\text{-}33)$$

$$V_b = V_{bo} + \Delta V_b = V_{bo} + \frac{\partial V_b}{\partial I_f}\Delta I_f + \frac{\partial V_b}{\partial \omega}\Delta\omega \qquad (14\text{-}28)$$

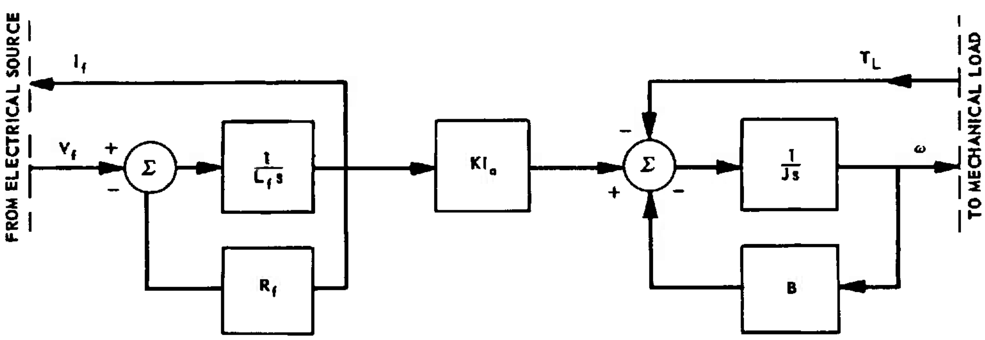


Fig. 14-12 Block diagram for field-controlled d-c motor, constant armature current.

The average field voltage is the product of the average field current and the field resistance; i.e., $V_{fo} = I_{fo}R_f$. For a position servo, ω_o can be considered as zero. In this case

$$V_{to} = I_{ao} R_a \tag{14-34}$$

$$T_o = KI_{fo}I_{ao} = T_{Lo} \tag{14-35}$$

Insert the expressions

$$I_a = I_{ao} + \Delta I_a$$

$$KI_f\omega = V_b = V_{bo} + \Delta V_b$$

$$\omega = \omega_o + \Delta\omega$$

$$I_f = I_{fo} + \Delta I_f$$

etc. into Eqs. (14-24), (14-2), and (14-25) after making use of Eqs. (14-28) through (14-30) to obtain

$$V_t = R_a(I_{ao} + \Delta I_a) + L\,\frac{d\Delta I_a}{dt} + KI_{fo}\omega_o + K(\omega_o + \Delta\omega)\Delta I_f + K(I_{fo} + \Delta I_f)\Delta\omega \tag{14-36}$$

$$T = KI_{fo}I_{ao} + K(I_{ao} + \Delta I_a)\,\Delta I_f + K(I_{fo} + \Delta I_f)\Delta I_a \tag{14-37}$$

$$T = J\,\frac{d(\Delta\omega)}{dt} + B\omega_o + B\Delta\omega + T_{Lo} + \Delta T_L \tag{14-38}$$

Substitute the average values given by Eqs. (14-34) and (14-35) into Eqs. (14-36) through (14-38) to obtain

$$V_t = I_{ao}R_a + \Delta I_aR_a + L\,\frac{d(\Delta I_a)}{dt} + 2K\Delta\omega\,\Delta I_f + KI_{fo}\,\Delta\omega \tag{14-39}$$

$$T_o = KI_{fo}I_{ao} + KI_{ao}\,\Delta I_f + 2K\Delta I_a\,\Delta I_f + KI_{fo}\,\Delta I_a = J\,\frac{d(\Delta\omega)}{dt} + B\omega_ot + B\Delta\omega + T_{Lo} + \Delta T_L \tag{14-40}$$

Subtract $V_{to} = I_{ao}R_a$ from Eq. (14-39) and $KI_{fo}I_{ao} = T_{Lo}$ *from* **Eq. (14-40) and** let **all** second-order differentials be zero (recall that $\omega_o = 0$) to obtain

$$\Delta V_t = 0 = \Delta I_aR_a + L\,\frac{d(\Delta I_a)}{dt} + KI_{fo}Ao \tag{14-41}$$

$$\Delta T = KI_{ao}\,\Delta I_f + KI_{fo}\,\Delta I_a$$

$$= J\,\frac{d(\Delta\omega)}{dt} + B\Delta\omega + AT, \qquad (14\text{-}42)$$

Solve Eq. **(14-41)** for *AI*, and use the Laplace potation to obtain

$$\Delta I_a = -\,\frac{KI_{fo}\Delta\omega}{R_a(1+\tau s)} \qquad (14\text{-}43)$$

where

$$\tau = L/R_a$$

Substitute Eq. **(14-43)** into Eq. **(14-42).**

$$AT = KI_{ao}\,\Delta I_f - \frac{(KI_{fo})^2\,Aw}{R_a(1+\tau s)}$$

$$= (Js + B)\,\Delta\omega + \Delta T_L \qquad (14\text{-}44)$$

From Eqs. **(14-34)** and **(14-35)**

$$\frac{(KI_{fo})^2}{R_a} = \frac{T_{Lo}^2}{I_{ao}^2 R_a} = \frac{T_{Lo}^2 R_a}{V_{to}^2} \qquad (14\text{-}45)$$

Substitute Eq. **(14-45)** into Eq. **(14-44).**

$$\Delta T = KI_{ao}\Delta I_f - \frac{\left(\dfrac{T_{Lo}^2 R_a}{V_{to}^2}\right)\Delta\omega}{R_a(1+\tau s)}$$

$$= (Js + B)\,\Delta\omega + \Delta T_L \qquad (14\text{-}46)$$

Equation **(14-46)** is shown in block diagram form in Fig. **14-13.** Note that in Fig. **14-13** all variables are increments; the *A* notation is dropped for convenience. If $T_{Lo} = 0$, Fig. **14-13** reduces to that of Fig. **14-12.**

14-2.11 Series Motor Control

The dynamic characteristics for the series-connected motor are expressed by the equations

$$V_t = I_a R_a + KI_a\omega + L_a s I_a \qquad (14\text{-}47)$$

$$T = T_L + B\omega + Js\omega \qquad (14\text{-}23)$$

The runaway speed of the series motor depends upon the load; for example, point X

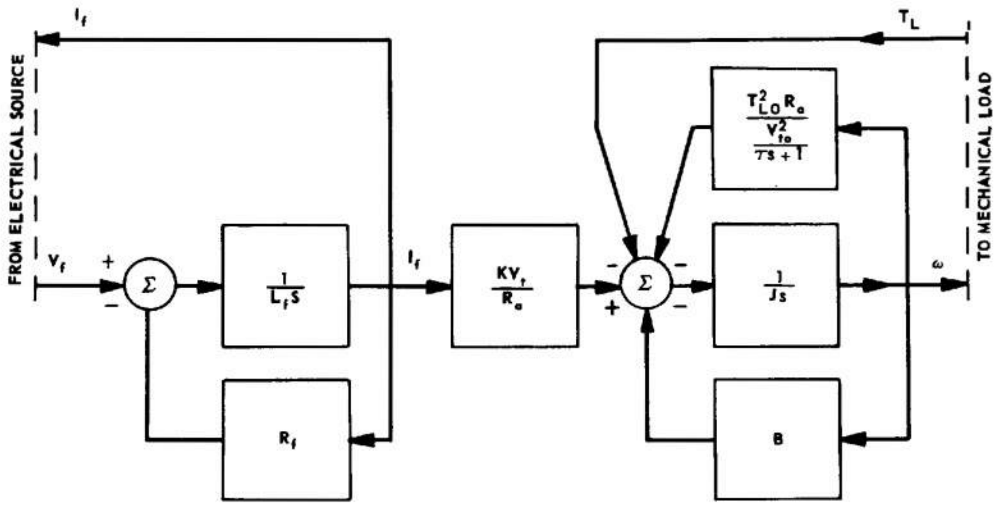


Fig. 14-13 Block diagram for field-controlled d-c motor, constant armature terminal voltage, incremental behavior.

shown in Fig. **14-10** indicates the runaway speed for a certain viscous load. By neglecting the armature inductance L_a and by assuming negligible dynamics throughout the rest of the system, it is possible to make a phase-plane solution of Eqs. **(14-47)** and **(14-23).** If the describing function method of solution to the problem is contemplated, computer solutions of Eqs. **(14-47)** and **(14-23)** for $\omega(t)$ will allow determination of an approximate transfer function. See Ch. **10** for an explanation of the describing function method and phase-plane solution of nonlinear problems.

14-2.12 Rotary Electric Amplifier Control

To compute the dynamic response of a rotary electric amplifier, its transfer function must take the source into account. For the basic rotary electric amplifier (Fig. **14-1A),** the block diagram shown in Fig. **14-11** can be used to represent its operation if the generator armature inductance and resistance are combined with that of the driven d-c motor. Also, the generated voltage V_o is assumed to equal the motor terminal voltage V_t. By making term q of Eq. **(13-72)** equal to k and equal to unity, this equation may be rewritten as

$$V_o = \frac{\alpha_1 V_1(s)}{1+\tau_1 s} = \frac{\alpha_f V_f(s)}{1+\tau_f s} \qquad \textbf{(14-48)}$$

where the terms are as defined in Par. **13-4.**

The transfer function relating the motor speed ω to the generator field voltage V_f and the load torque T_L is given by the following equation :

$$\omega(s) = \frac{\left[\dfrac{K_o}{1+K_o}\right]\left[\dfrac{1}{KI_f}\right]\left[\dfrac{\alpha_f V_f(s)}{1+\tau_f s}\right] - \left[\dfrac{T_2 s+1}{B(1+K_o)}\right]\left[T_L(s)\right]}{(T_a s+1)(T_b s+1)} \qquad \textbf{(14-49)}$$

where

$K_o = \dfrac{(KI_f)^2}{R_T B_T}$

R_T = total armature resistance of generator and driven motor, in ohms

B_T = total viscous damping referred to motor shaft, in newton-meter-sec

$$T_a = \frac{T_1 + T_2 + \sqrt{(T_1+T_2)^2 - 4(1+K_o)(T_1 T_2)}}{1+K_o}$$

$$T_b = \frac{T_1 + T_2 - \sqrt{(T_1+T_2)^2 - 4(1+K_o)(T_1 T_2)}}{1+K_o}$$

$T_1 \quad = \frac{JT}{B_T}$

$T_2 \quad = \frac{L_T}{R_T}$

J_T = total inertia referred to motor shaft, in kilogram-meters^2

L_T = total armature inductance of generator and driven motor, in henries

KI_f = stall torque to armature current ratio of motor

For larger motors, it may occur that Eq. **(14-49)** yields complex values of T_a and T_b denoted as $(a + jb)$ and $(a - jb)$. In this case, **Eq. (14-49)** can be rewritten with a new denominator resulting in

$$\omega(s) = \frac{\left[\frac{K_o}{1+K_o}\right]\left[\frac{1}{KI_f}\right]\left[\frac{\alpha_f V_f(s)}{1+\tau_f s}\right] - \left[\frac{T_2 s+1}{B(1+K_o)}\right]\left[T_L s\right]}{\left(\frac{s}{\omega_n}\right)^2 + 2\frac{\zeta s}{\omega_n} + 1} \tag{14-50}$$

where

$$\omega_n = \frac{1}{\sqrt{a^2 + b^2}}$$

$\zeta \quad = a\omega_n$

For the case of the amplidyne type of rotary electric amplifier (Fig. **14-1B**), the expression for the generated voltage V, given by Eq. **(14-48)** must be modified. Refer to Par. **13-4.**

14-2.13 Electronic Amplifier Control

Two cases of electronic amplifier control are considered —one for the armature-driven motor, and 'the other for the field-driven motor. When an amplifier is used to drive an armature, and the armature is in the plate circuit, the amplifier can be represented by an equivalent voltage source $(V_r = \mu E_g)$ in series with an internal resistance R_s. **If** the internal resistance is added to the armature resistance, Eq. **(14-49)** will apply by letting τ_f be zero, V_f be E_g, **and** α_f be μ.

When an amplifier is used to drive a motor field (constant armature current), the transfer function that determines the dynamic response is found by the aid of Fig. **14-12** to be

$$\omega(s) = \frac{\frac{\mu E_g(s)}{R_T(T_c s+1)} - \frac{T_L(s)}{B}}{T_d s + 1} \tag{14-51}$$

where

$T_c \quad = \frac{L_f}{R_f + R_s}$

R_s = amplifier internal or source resistance, in ohms

$R_T \quad = R_f + R_s$

$T_d \quad = \frac{J}{B}$

μ = amplifier gain $= \frac{\Delta V_f}{\Delta E_g}$

14-2.14 MEASUREMENT OF D-C MOTOR PARAMETERS[1)]

The parameters of the d-c motor that are of interest in servomechanism applications are

R_a = armature resistance

L_a = armature inductance

R_f = field resistance

L_f = field inductance

K = motor constant

J = armature inertia

B = viscous damping

These parameters may include contributions from the source or from the load attached to the motor. In the measurements described in the paragraphs that follow, only the contributions of the motor itself to the parameters are considered.

14-2.15 Armature Resistance

If the manufacturer's data for armature resistance R_a are not available, it may be determined by the application of Ohm's Law, $R = E/I$. While keeping the armature blocked to prevent rotation, the applied armature voltage necessary to maintain rated armature current is measured. In the absence of armature rotation, there is no back-emf to oppose the applied armature voltage ; therefore, a much lower applied voltage than rated voltage will produce rated current. The applied voltage at various angular armature positions should be measured and averaged to calculate the resistance R_a. The applied voltage should be connected to the motor terminals so that the IR drops of the brushes at rated current are included. However, if some nonlinear representation of IR drop for brushes is used for computer studies, the brushes should be short-circuited during the measurements for R_a. For the more accurate representation, the IR drop of the brushes as a function of armature current I, should be measured and inserted in the applicable block diagram and computer representation.

14-2.16 Armature Inductance

Precise measurements of the armature inductance L_a are often unnecessary because this parameter does not affect appreciably the principal time constant of the motor. Probably the most convenient method of measuring L_a, and one of sufficient accuracy, requires an a-c voltmeter and an a-c ammeter. A 60-cps voltage is first applied to the armature which is blocked to prevent rotation; the field is excited at rated d-c field current. Measured values of the voltage E across the armature and the armature current I are substituted with the known value of armature resistance R_a in the equation

$$X = \sqrt{\left(\frac{E}{I}\right)^2 - R_a^2} \tag{14-52}$$

where

X = reactance of armature inductance, in ohms

Once the reactance is known, the armature inductance can be calculated from the equation

$$L_a = \frac{X}{377} \tag{14-53}$$

An alternate method of measuring the armature inductance is to observe the transient growth of current when the armature is excited by a step voltage.[8] The armature should be blocked to prevent rotation and the field should be excited at rated current. The step change in voltage is applied by closing a switch connected in series with the armature, a series resistance R_s, and a d-c source. The armature inductance can be calculated from the time-constant relationship of the voltage rise across the series resistance as observed by the aid of a cathode-ray oscilloscope ; that is,

$$L_a = (T^*)\ (R_a + R_s) \tag{14-54}$$

where

T^* = observed time constant of armature and series resistance R_s, in sec

14-2.17 Field Resistance

In the absence of manufacturer's data, the field resistance R_f can generally be determined by measurement with an ordinary ohmmeter. If the resistance is too low to be read accurately on an ohmmeter, a Wheatstone or Kelvin bridge can be used.

14-2.18 Field Inductance

The field inductance L_f may be determined by either of the methods described to measure the armature inductance. Rated d-c armature current should be used to obtain more accurate measurements of L_f. If the inductance is determined by finding its reactance, an a-c current whose rms value is about 1/2 the rated d-c current should be used. If the inductance is determined by the transient method, a step voltage that will result in a steady-state field current of 1/2 to 3/4 the rated value should be used.

14-2.19 Motor Constant

The motor constant K may be determined from a measurement of the stall torque at a known armature current and a known field current. By solving Eq. **(14-2)** for K, the motor constant is

$$K = \frac{T}{I_f I_a} \tag{14-55}$$

The stall torque should be measured at various angular positions of the armature, and an average of the maximum and minimum values should be used. If the motor is used with a constant field input, the value of normal field current should be used during the test.

A second method of determining the motor constant is to measure the armature current and speed for a given terminal voltage V_t. Knowing the armature resistance, the motor constant is then calculated from Eq. **(14-1)** by solving for K to obtain

$$K = \frac{V_t - I_a R_a}{I_f \omega} \tag{14-56}$$

14-2.20 Armature Inertia

If the armature can be easily removed from the motor, the armature inertia J can be measured by use of a piano-wire torsion pendulum. A small diameter wire, but one strong enough to support the armature, should be used. The longer the wire used, the lower the period of the pendulum, and the more accurate the measurements obtained. The pendulum should be calibrated first by measuring its period with a body of known (or calculable) inertia and simple geometry. Then, only the armature should be used ; from the resulting period, the inertia is calculated by

$$J = J_c \left(\frac{T_j}{T_c}\right)^2 \tag{14-57}$$

where

J = unknown inertia of the armature, in kilogram-meters^2

J_c = known inertia of the body used for calibration, in kilogram-meters^2

T_j = period when armature is used, in sec

T_c = period when body of known inertia is used, in sec

The armature inertia can be determined by a retardation test without removing the armature from the motor. This test consists of measuring : (1) the time T_1 for the armature to coast from full speed to half speed when the armature power is removed ; and (2) the time T_2, obtained in the same manner, but when an additional body of known inertia is coupled to the armature. Let the ratio T_1/T_2 be T_R. Then

$$T_R = \frac{T_1}{T_2} = \frac{J}{J + J_a} \tag{14-58}$$

where

J_a = known inertia of body added to armature, in kilogram-meters^2

Solving Eq. **(14-58)** for armature inertia J

$$J = J_a \left(\frac{T_R}{1 - T_R}\right) \tag{14-59}$$

If a tachometer is used to measure the motor speed during the retardation test, the tachometer inertia should be subtracted from the inertia found by Eq. **(14-59)** to obtain the true armature inertia. A satisfactory method of measuring motor speed without using the tachometer is with the d-c speed bridge.[12]

The armature inertia can be calculated from the following equation based on the inertia of a cylinder, if the armature dimensions and material are known :

$$J_{cyl} = 1/2\, M r^2 \tag{14-60}$$

where

J_{cyl} = inertia of the cylinder, in in.-lb-sec^2

$M = \frac{1}{g}$ = mass of the cylinder, in lb-sec^2/in.

$W = \pi\rho l r^2$

ρ = weight density, in lb/in.^{3}

l = cylinder length, in in.

r = cylinder radius, in in.

g = 386 in./sec^2

14-2.21 Viscous Damping

The viscous damping B should be determined when the motor is operating without a load and at rated field input. The armature current should be measured when only enough voltage is applied to the armature to obtain rated motor speed. The total armature input power is then

$$V_t I_a = I_a^2 R_a + B\omega^2 \quad (14\text{-}61)$$

Solving for the viscous damping results in

$$B = \frac{(V_t I_a - I_a^2 R_a)}{\omega^2} \quad (14\text{-}62)$$

where

B = viscous damping, in newton-meter-sec

The measurement should be repeated at 1/2 rated speed and the two results averaged to obtain a more accurate value for B.

14-2.22 Typical Parameter Values

The typical parameter values of eight d-c motors are listed in Table 14-1. This table can be used as a guide to indicate the approximate values that can be expected. The "stall torque/inertia" quantity is a Figure of Merit indicating the acceleration that can be expected from the motor when unloaded. The motor time constant $R_a J/K^2$ characterizes the motor when the viscous damping B, armature inductance L_a, and load inertia are neglected. This simplification is often valid and useful in block diagram representations.

14-2.23 D-C TORQUE MOTORS

D-c torque motors are special motors whose output is a rotary displacement that is proportional to the control-winding input. To achieve linear operation, the output displacement is usually limited to an over-all rotation of 10 to 20 degrees. As a result, torque motors are best adapted for the control of hydraulic control devices. In many applications, the rotary-displacement output is converted by a simple mechanical linkage into a translational displacement.

14-2.24 Description of Typical Torque Motors

The construction of a typical torque motor used to drive a hydraulic control valve is shown in Fig. 14-14A. The magnetic circuit for the polarizing flux ϕ_1 consists of permanent magnets *N-S*, magnetic yokes *Y-Y*, pole pieces *P-P*, rotor (armature) *R*, and the air gaps between the rotor and the pole pieces. Two control coils surround the rotor and are connected in push-pull (Fig. 14-14B) so that the magnetomotive forces of the control components i of the coil currents add to produce the control fluxes ϕ_c, but the magnetomotive forces of the quiescent current I_o cancel. When the coil currents are balanced (control components i are zero), the rotor is held in a neutral position by a spring. However, if the rotor is disturbed slightly or if the coil currents are unbalanced, the fluxes at one pair of diagonally opposite gaps increase, and the fluxes at the other pair of gaps decrease. The resultant force turns the rotor until equilibrium is reached against the spring or against stops. Without a spring, the rotor is unstable in the neutral position and, once disturbed, the rotor tends to clap immediately against the poles. Drive rods for hydraulic control valves are attached near the ends of the rotor at radius r. By using flexible drive rods, the rotational forces produced by the torque motor are converted into translational forces at the hydraulic control valves.

14-2.25 Static Characteristics

The torque developed by a typical torque motor (Fig. 14-14) is expressed by the equation

$$T = K_1 i + K_2 \theta_o \quad (14\text{-}63)$$

where

T = torque produced by motor, in lb-in.

i = control component of coil current, in amperes

TABLE 14-1 TYPICAL VALUES OF D-C MOTOR PARAMETERS

Motor No.	Type	Nominal Size (hp)	Rated Voltage (volts)	Motor Constant (newton-meters/ampere)	Armature Resistance (ohms)	Inertia (kilogram-meters^2)	Stall Torque (newton-meters)	Stall Torque/Inertia (sec^{-2})	Motor Time Constant $R_a J/\kappa^2$
1	shunt	0.008	115	0.23	280	1.1×10^{-5}	0.095	8600	0.061
2	permanent magnet	0.003	6	0.011	2.15	0.24×10^{-5}	0.025	10,200	0.043
3	permanent magnet	0.009	6	0.0091	0.82	0.37×10^{-5}	0.062	17,000	0.036
4	permanent magnet	0.022	6	0.0093	0.38	0.53×10^{-5}	0.141	27,000	0.023
5	permanent magnet	0.054	26	0.015	2.64	0.37×10^{-5}	0.135	37,000	0.043
6	permanent magnet	0.080	26	0.0143	1.85	0.23×10^{-5}	0.181	34,000	0.047
7	shunt	0.6	80* 100**	0.198	5.82	2.9×10^{-4}	2.72	9300	0.043
8	shunt	2.5	65* 110**	0.329	1.57	1×10^{-3}	13.6	13,200	0.016

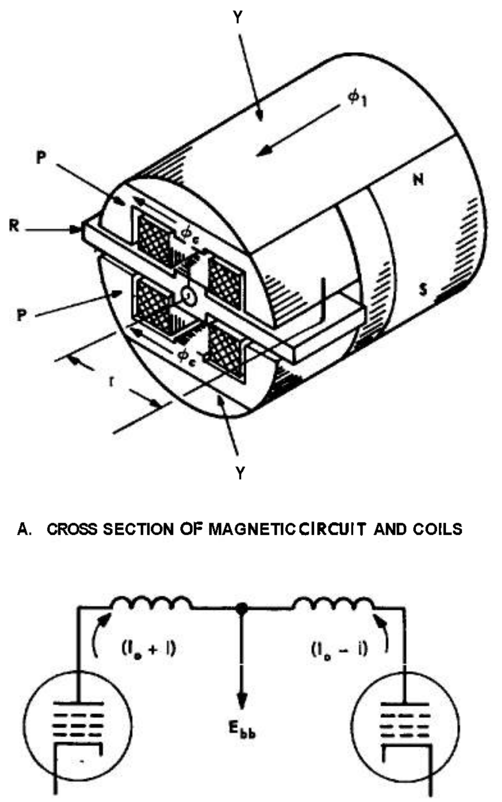
*Armature voltage
**Field voltage

θ_o = angular displacement of rotor, in radians

K_1 = constant (typical value is 200 lb-in./ampere)

K_2 = constant (typical value is 300 lb-in./radian)

The torque produced by the motor is opposed by a spring with constant K_s (typical value is 560 lb-in./radian). The static characteristic of the torque motor is shown in Fig. 14-15A, with the angular rotor deflection plotted as a function of the control current.



A. CROSS SECTION OF MAGNETIC CIRCUIT AND COILS

B. COIL CONNECTIONS

Fig. 74-74 D-c torque motor, permanent-magnet type.

Adapted from 'A Permanent-Magnet-Type Electric Actuator for Servovalves', by R. H. Frazier and R. D. Atchley. Dynamic Analysis and Control Laboratory. Report No. 66. June 1, 1952, Massachusetts Institute of Technology.

14-2.26 Dynamic Characteristics

The dynamic behavior of a typical torque motor (Fig. 14-14) is described by the equations

$$K_1 i = J_o s^2 \theta_o + B_o s \theta_o + (K_s - K_2)\theta_o \tag{14-64}$$

$$E_c = \mu e_1 = R_1 i + L_o s i + 0.1135\, K_1 s\, \theta_o \tag{14-65}$$

where

J_o = inertia of load, in lb-in.-sec^2

B_o = viscous damping of load, in lb-in.-sec

E_c = control-winding input voltage, in volts

μ = voltage gain of one tube in output stage of push-pull amplifier

e_1 = total voltage applied from grid-to-grid of amplifier output stage, in volts

$R_1 = 2r_p + R$ = total control-circuit resistance, in ohms

r_p = plate resistance of one tube in amplifier output stage, in ohms

R = total resistance of control winding, in ohms

L_o = self-inductance of control winding (two coils in series and rotor in neutral position), in henries

K_1 = torque per unit current, in lb-in./ampere

The dynamic behavior of the torque motor is summarized in terms of the frequency response as shown in Fig. 14-15B. An indication of the frequency response of other torque motors used to drive hydraulic control valves may be obtained from the information tabulated for hydraulic-amplifier dynamic reresponse in Par. 13-6.

14-2.27 Modified Servomotors

Most servomotors can be modified to serve as a torque motor by adding a gear train to provide the small motions usually required of the torque motor. The motor and gear train can be converted into a position source by opposing the motor torque with that of a

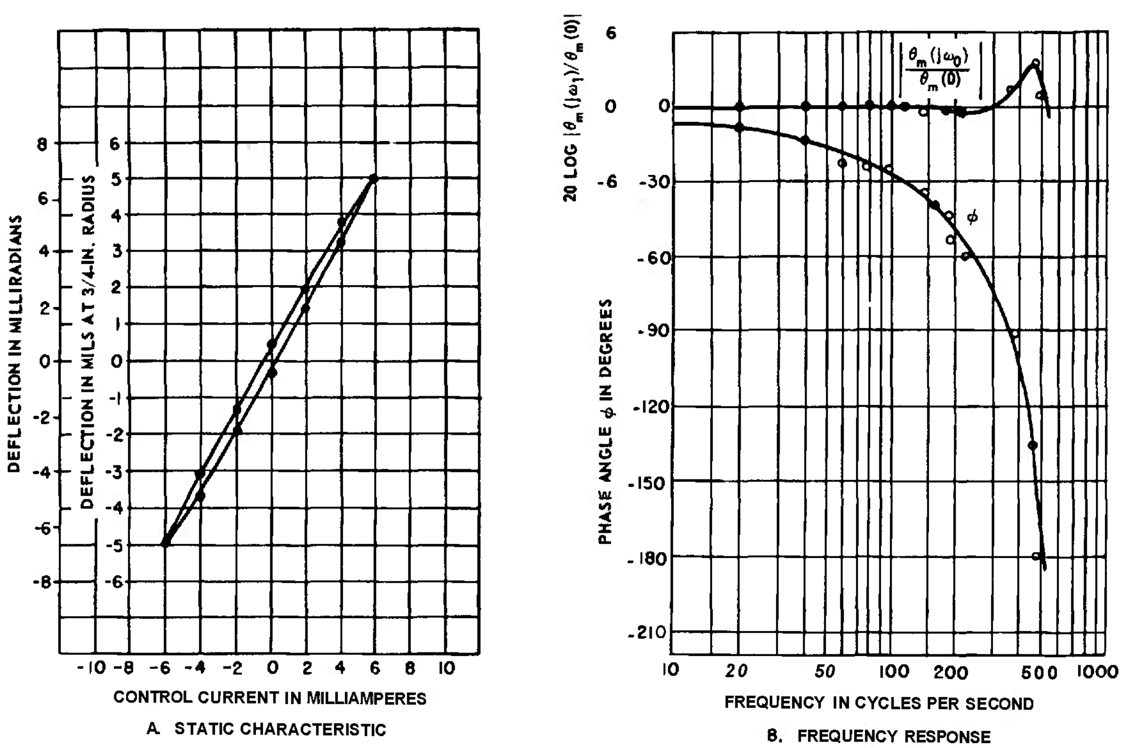


Fig. 14-15 Characteristics of d-c torque motor.

From 'A Permanent-Magnet-Type Electric Actuator for Servo-valves', by R. H. Frazier and R. D. Atchley, Dynamic Analysis and Control Laboratory, Report No. 66, June 1, 1952, Massachusetts Institute of Technology.

spring. In this case, the steady-state behavior is

$$\theta_o = \frac{R_g T_m}{K_s} \tag{14-66}$$

$$T_m = K_m E_c \tag{14-67}$$

where

θ_o = angular displacement of motor-spring combination, in radians

R_g = gear reduction ratio of gear train

T_m = torque produced by motor, in lb-in.

K_s = spring constant of opposing spring, in lb-in./radian

K_m = motor torque per volt constant, in lb-in./volt

E_c = control-winding input voltage, in volts

Equations (14-66) and (14-67) apply to a d-c motor with E_c applied to the armature and with a constant field input; to a d-c motor with E_c applied to the field and with a constant armature input; or to a 2-phase a-c servomotor.

14-2.28 CONVERSION FACTORS AND UNITS

The conversion factors and units commonly required when making calculations and measurements to determine the performance of d-c motors are listed in Table 14-2.

TABLE 14-2 CONVERSION FACTORS AND UNITS

Quantity	To Convert English Units	*To* Metric (Mks) Units	Multiply By
mass (M)	lb-sec^2/in.	kilograms	175
inertia *(J)*	lb-in.-sec^2	kilogram-meters^2	0.113
force *(F)*	lb	newtons	4.45
torque *(T)*	lb-in.	newton-meters	0.113
viscous damping *(B)*	lb-in.-sec	newton-meter-sec	0.113
spring constant *(K,)*	lb-in.	newton-meters	0.113

APPENDIX FOR PARAGRAPH 14-2.

Solve Eq. (14-1) and (14-2) for torque. There results

$$T = \frac{KI_fV_t}{R_a} - \frac{KI_fV_t}{R_a} \times \frac{KI_f\omega}{V_t} \qquad (1)$$

With zero speed and maximum current, I_{fM}, the stall torque T_M is seen to be $KI_{fM}V_t/R_a$. Divide the expression for T from Eq. (1) by T_M on the left and the equivalent expression on the right. There results.

$$\frac{T}{T_M} = \frac{I_f}{I_{fM}} - \frac{I_f}{I_{fM}} \times \frac{KI_f\omega}{V_t} \qquad (2)$$

Solve Eq. (1) for the no-load speed ω_B, at current I_{fM}, which is given by :

$$\frac{KI_{fM}\omega_B}{V_t} = 1 \qquad (3)$$

Rewrite Eq. (2) as follows :

$$\frac{T}{T_M} = \frac{I_f}{I_{fM}} \left(1 - \frac{KI_f}{I_{fM}} \times \frac{\omega_B I_{fM}}{V_t} \times \frac{\omega}{\omega_B}\right) \qquad (4)$$

Substitute Eq. (3) into Eq. (4) to arrive at the following [which is Eq. (14-14)]

$$\frac{T}{T_M} = \frac{I_f}{I_{fM}} \left(1 - \frac{I_f}{I_{fM}} \times \frac{\omega}{\omega_B}\right)$$

14-3 ALTERNATING-CURRENT MOTORS

14-3.1 TYPES OF A-C MOTORS USED IN SERVOMECHANISMS

The following three types of a-c motors are commonly used as servomotors (output members of servomechanisms) :

(a) 2-phase motors (commonly used in continuous linear servomechanisms)

(b) Single-phase motors (commonly used in relay servomechanisms)

(c) Torque motors with restricted rotation (commonly used to control the input to hydraulic amplifiers)

Amplification for driving servomotors may be furnished by magnetic, transistor, or vacuum-tube amplifiers (see Ch. 13). Typical schematic diagrams showing the use of servomotors and amplifiers are included in this paragraph.

14-3.2 Amplifier Control of Servomotors

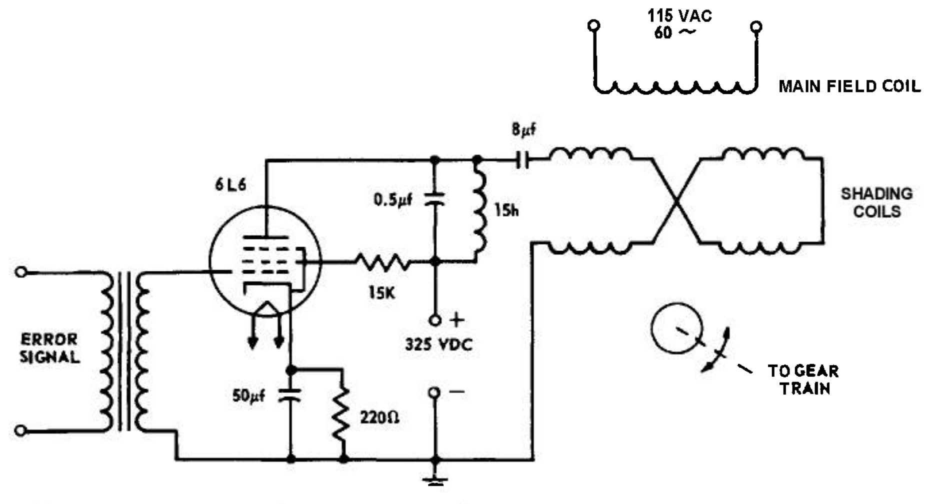
Figures 14-16 through 14-19 show the schematic diagrams of a few servomotor control circuits using amplifiers :

(a) Figure 14-16 shows a shaded-pole motor in a 60-cycle servo application. Wound shading coils are used, and the main winding is continuously excited.

(b) Figure 14-17 shows a 2-phase 60-cycle servomotor controlled by a vacuum-tube amplifier. A feedback loop is used to reduce the amplifier internal impedance presented to the motor winding, thus increasing the inherent motor damping. **A** high amplifier internal impedance would have the opposite effect ; in some cases, excessive impedance can reduce the damping to zero or even make it negative. (For 400-cycle operation, the size of amplifier coupling capacitors must be changed.)

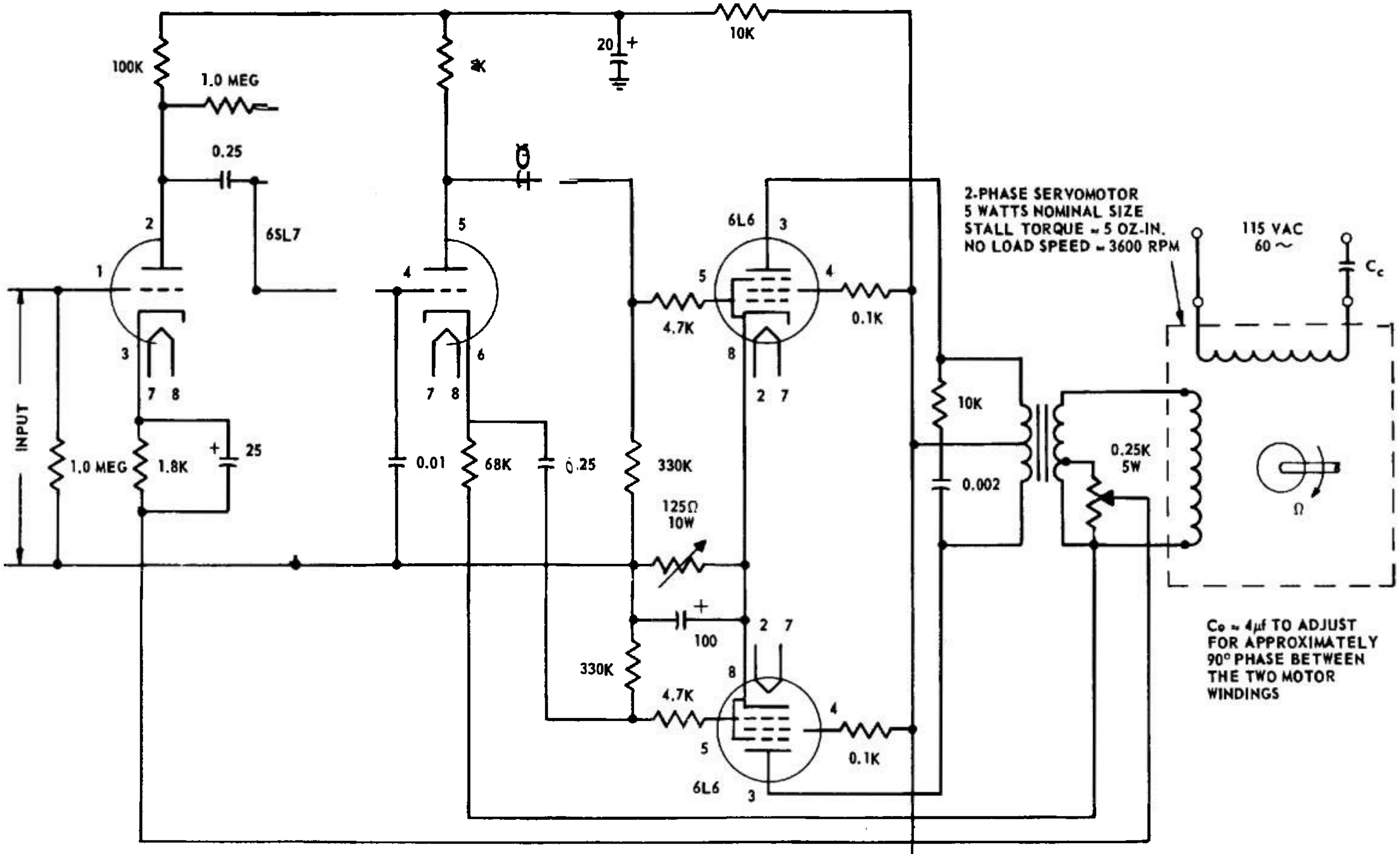
(c) Figure 14-18 shows a 2-phase servomotor controlled by power transistors.

(d) Figure 14-19 shows several magnetic amplifier circuits proposed by Geyger.[13] The



NOTE: ADDITIONAL AMPLIFICATION AND COMPENSATION MAY BE REQUIRED.

Fig. 14-76 Amplifier control circuit for shaded-pole motor.



NOTE: UNLESS OTHERWISE INDICATED
CAPACITORS ARE IN MICROFARADS
AND RESISTORS ARE IN OHMS.
K = 1000

Fig. 14-17 Amplifier control circuit for 2-phase servomotor.

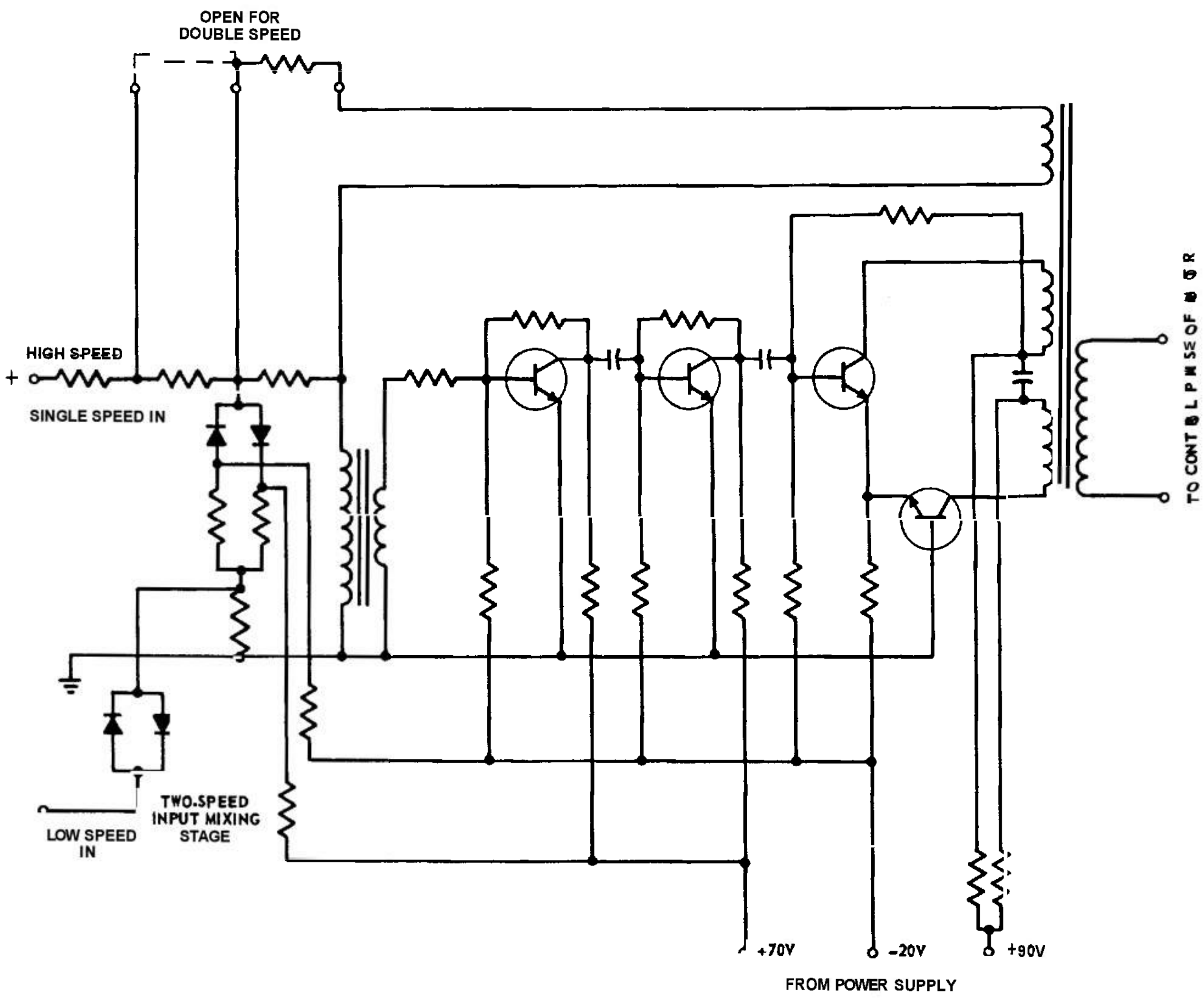


Fig. *14-18* Schematic of a transistor amplifier used with a 2-phase servomotor.

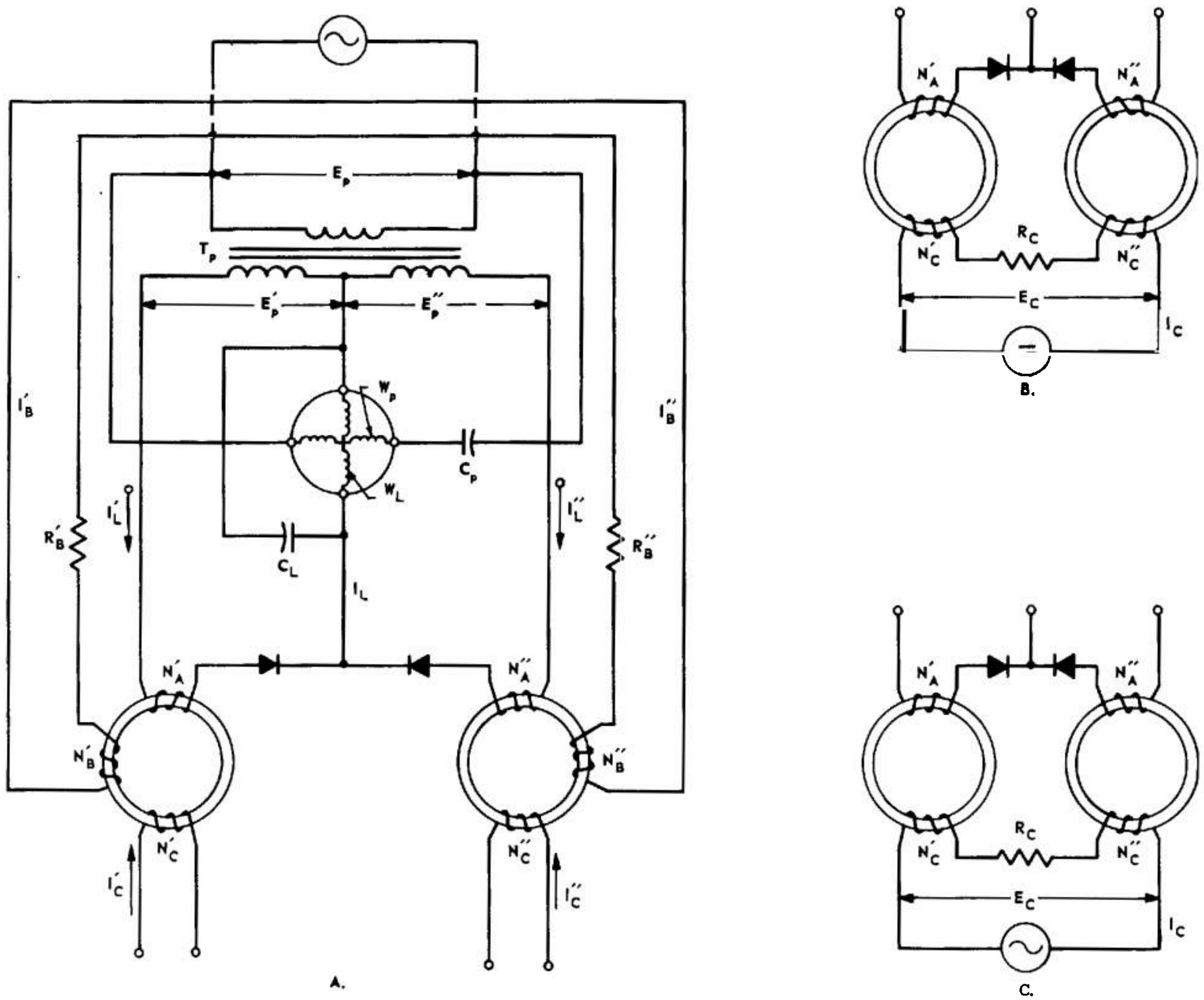


Fig. 14-19 Full-wave output-stage circuit of a magnetic servo amplifier having inherent dynamic-braking properties: (A) with two separate control winding elements N'_c, N''_c (B) with series-connected d-c control winding elements N'_c, N''_c (C) with series-connected a-c control winding elements N'_c, N''_c

inherent damping is greater than that of the motor alone because direct currents in the motor windings produce eddy-current damping.

Figures 14-20 and 14-21 show two circuits proposed by Storm,[14] both using a d-c control signal. Figure 14-20 shows a circuit with only one motor phase controlled, while Fig. 14-21 shows a circuit with both phases controlled. If both phases of the motor are controlled, the damping due to 2-phase action will be zero, with zero excitation at stall. If some direct current flows in the motor windings, eddy-current damping will result.

14-3.3 Types of a-c Motors Used in Relay Applications

The following types of a-c motors are suitable for relay applications:

(a) Split-phase, reluctance-type motors
(b) Series (universal) motors
(c) Torque motors
(d) 2-phase motors
(e) 3-phase motors, for high-power operation
(f) Shaded-pole motors

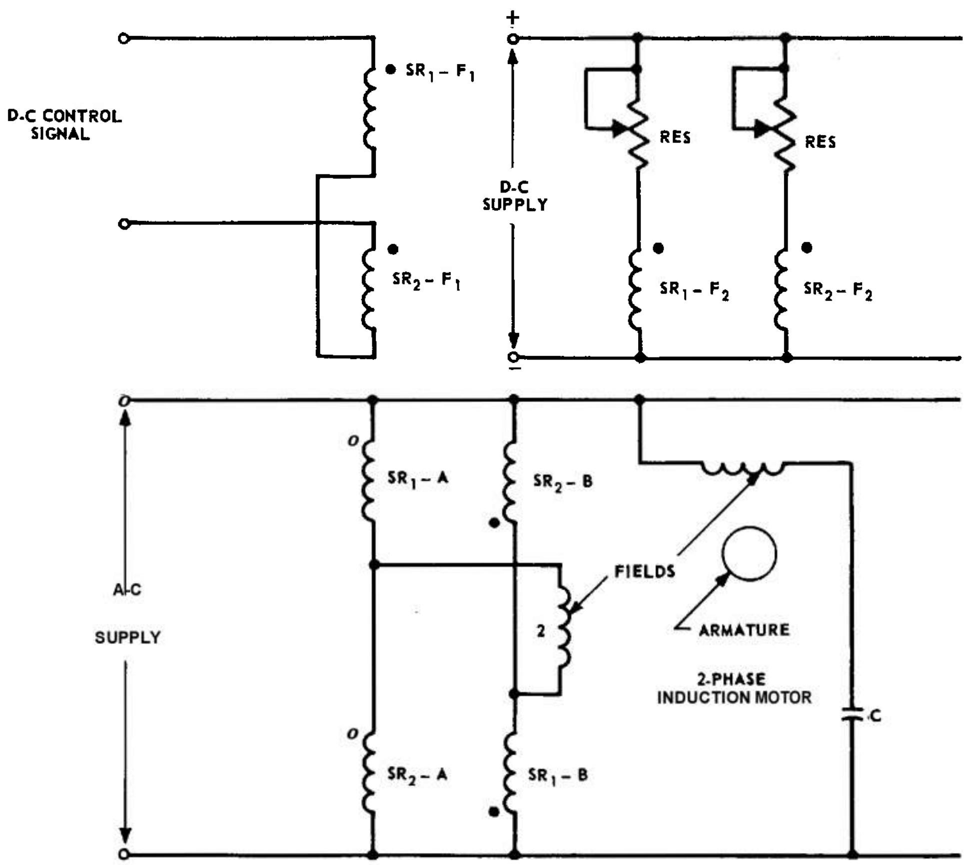


Fig. 74-20 A-c motor control (one phase fixed).

In the usual application, the motor must be reversible and have a starting torque about as large as the running torque or a little larger. High starting torque may be obtained from standard-design motors by increasing rotor resistance. This can be achieved by reducing the thickness of the rotor end piece. Shaded-pole motors can be used in pairs to obtain reversible operation —one motor for clockwise rotation, the other for counterclockwise rotation.

14-3.4 Typical relay-servo circuits. The series (universal) motor can be used with an a-c supply by employing the circuits described in Par. 14-2. Two-phase servomotors can be relay-controlled by using a circuit such as that shown in Fig. 14-22. The latter circuit is also usable with wound-shading-coil a-c motors. The only damping present is the inherent damping of the motor. Dynamic braking by eddy-current action can be provided as shown in Fig. 14-23. Proper choice of circuit elements will bring the motor to a dead stop from full speed within one revolution from the instant the relay contacts connect in the braking circuit. Capacitor C1 can be an electrolytic element. However, capacitor C2 should be an oil-filled unit, not an a-c electrolytic that is designed only for infrequent service such as starting. Choice of circuit-element values is discussed in detail in Par. 13-5.

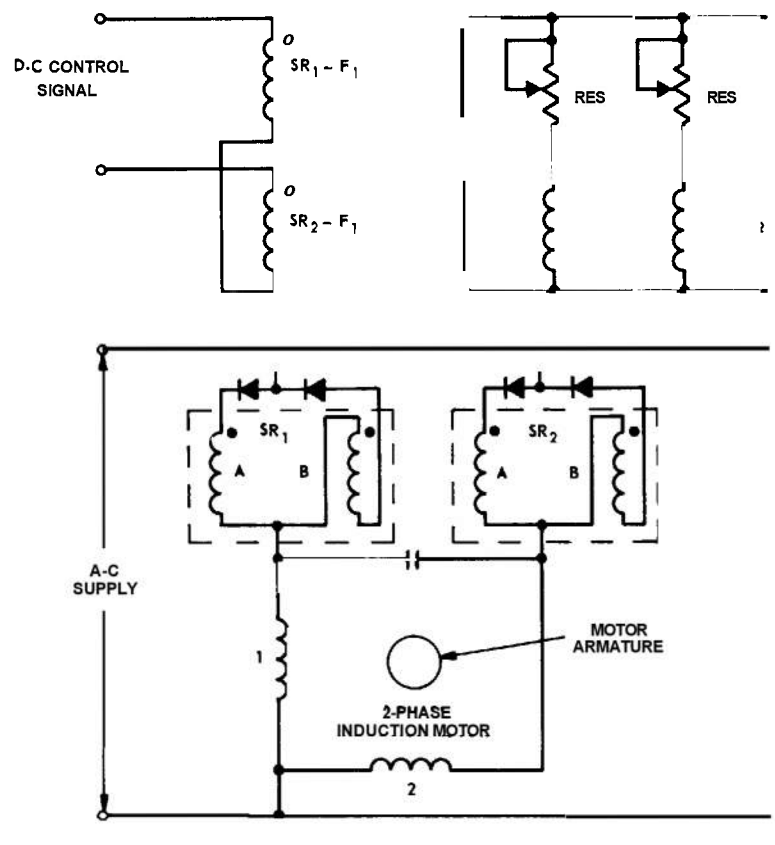


Fig. 74-27 A-c motor control (both phases variable).

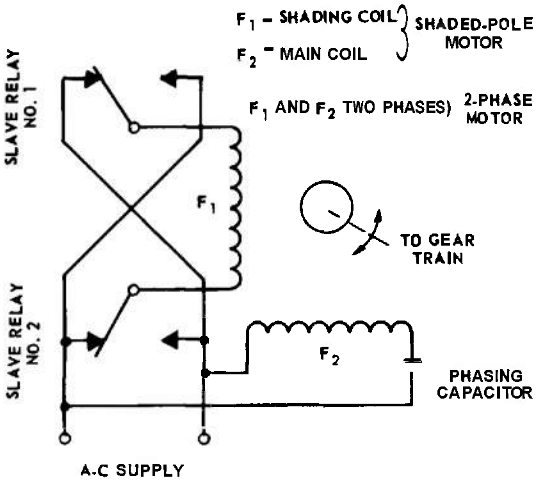


Fig. 14-22 Relay servo using wound-shading-coil or 2-phase motors.

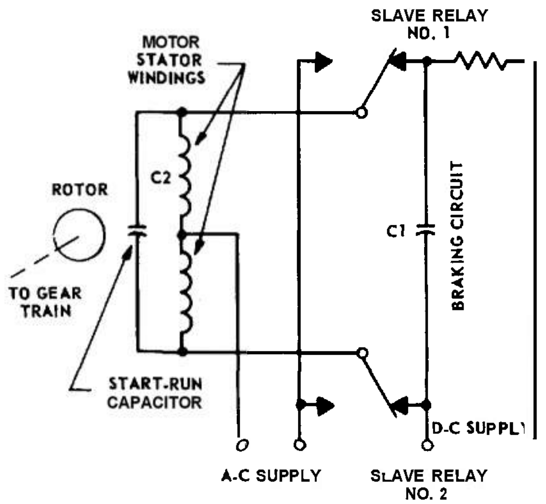


Fig. 14-23 Relay servo using capacitor-start-and-run motor.

14-3.5 STATIC CHARACTERISTICS OF A-C MOTORS

14-3.6 Typical 2-Phase Servomotor

The static torque-speed curves of a typical 2-phase servomotor are shown in Fig. 14-24.[15] These curves can be used to determine the motor parameters that influence dynamic performance.

14-3.7 Relay Servos

Analysis of relay servos can be accomplished by either of two methods: the describing function approach; or the phase-plane method. In each case, the objective is to determine the motor-speed response from an arbitrary initial speed to full speed, or to a dead stop. Such responses may be calculated from the speed-torque curve of the motor used in the servo. Reference 15 provides typical characteristics of various single-phase a-c motors that can be used in relay servos. It also indicates the method of approximating the transient response of such motors.

14-3.8 DYNAMIC BEHAVIOR OF 2-PHASE SERVOMOTORS

At the present time, the 2-phase servomotor is the type used in servomechanisms more often than any other type. Therefore, its dynamic behavior will be considered. The effect of electrical transients caused by winding inductance and resistance is negligible in most applications. Where this effect is important, an additional time constant can be introduced to account for the effect. The break frequency associated with this time constant is in the order of 400 to 800 radians/sec for 60-cycle motors.

14-3.9 Equations

The curves in Fig. 14-24 may be used to evaluate the partial derivatives in the representation of the motor as a torque source having some associated damping. **The** equation is

$$dT = \left[\frac{\partial T/T_{max}}{\partial E/E_{max}}\right]\left[\frac{T_{max}}{E_{max}}\right] dE + \left[\frac{\partial T/T_{max}}{\partial \Omega/\Omega_{sync}}\right]\left[\frac{T_{max}}{\Omega_{sync}}\right] d\Omega \qquad (14\text{-}68)$$

where

$\frac{\partial T/T_{max}}{\partial E/E_{max}}$ = nondimensional torque gain

$\frac{\partial T/T_{max}}{\partial \Omega/\Omega_{sync}}$ = nondimensional damping

T_{max} = stall torque with full motor excitation

Ω_{sync} = synchronous speed

E_{max} = rated reference voltage

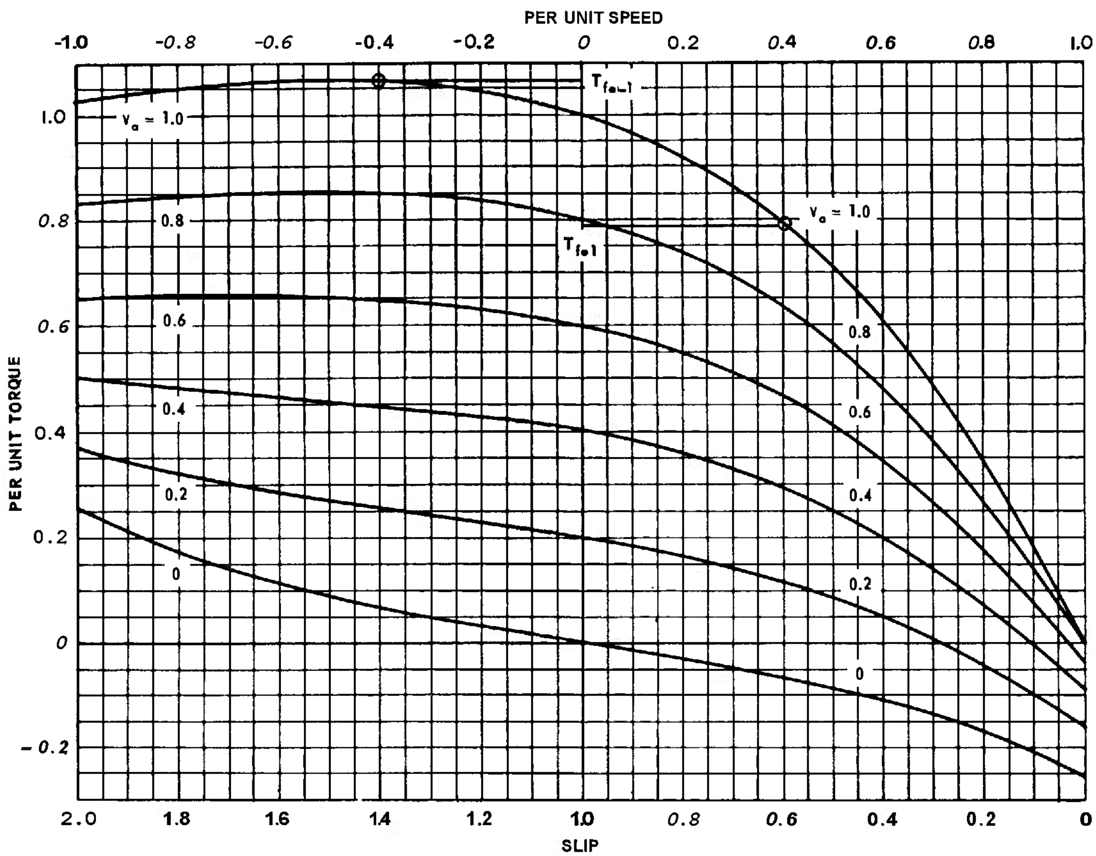


Fig. 74-24 Torque-speed curves of a 2-phase servomotor.

By permission from *Electric Machinery*. by A. E. Fitzgerald and Charles Kingsley, Jr., Copyright, 1952. McGraw-Hill Book Company, Inc.

The nondimensional gain and damping constants are plotted in **Figs. 14-25, 14-26,** and **14-27.** An approximate method of obtaining the damping constant $\partial T/\partial\Omega$ from the manufacturer's curve (which is almost universally made for full motor excitation) is to divide the damping by two (damping measured at stall and with rated motor excitation). The resultant damping is pertinent for low control-winding voltages. The torque gain can be derived from the manufacturer's curve by dividing the stall torque at full excitation by the full excitation voltage. Stall torque is theoretically and actually linear with applied voltage.

14-3.10 Alternatives. If more partial-coefficient information is desired, obtain the torque-speed curves for the motor to be used, measured at full and also at partial excitation. These curves can be obtained from the manufacturer or measured by the designer. The desired partial derivatives can be measured from these curves. **A** third method is to obtain the curves by calculation from the

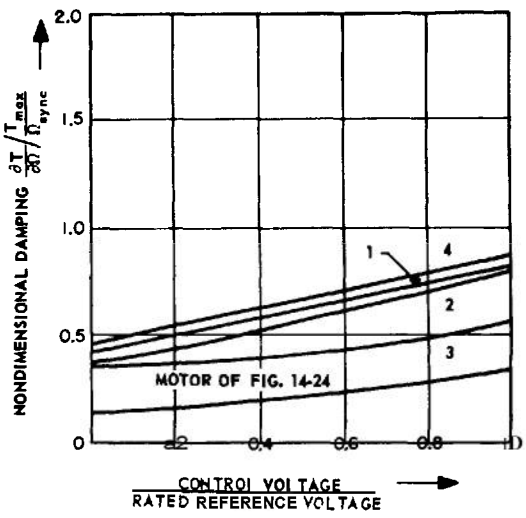


Fig. 14-26 Damping factors for some typical motors, taken with zero speed, Ω/Ω sync $= 0$.

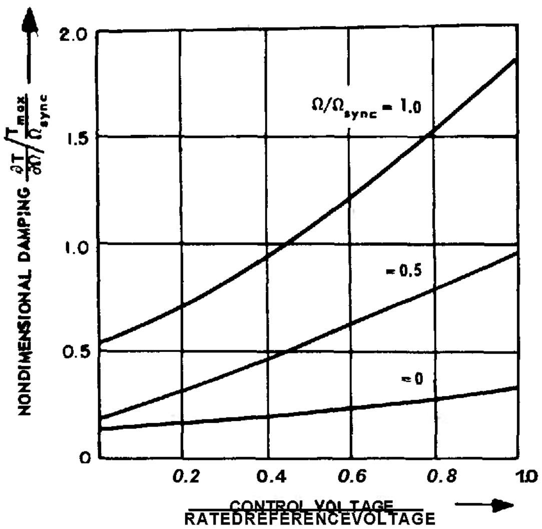


Fig. 74-25 Damping factor for motor having torque-speed curves shown in Fig. 74-24.

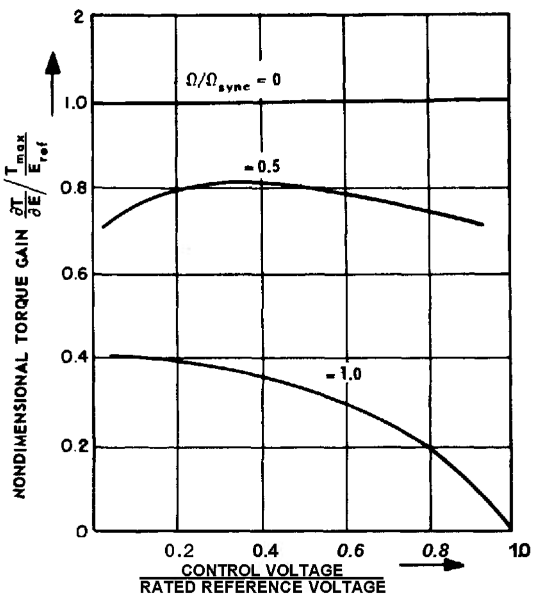


Fig. 74-27 Torque gain of typical 2-phase servomotor versus per unit control voltage at different speeds.

curve for full, rated excitation. This calculation can be performed by using symmetrical component voltages. It is necessary to have the torque-speed curve for negative as well as positive speeds. The manufacturer's curve for positive speeds can be extrapolated to negative speeds by assuming a power-series representation of the given nondimensional torque-speed curve:

$$T^* = a_0 + a_1\Omega^* + a_2\Omega^{*2} + a_3\Omega^{*3} + a_4\Omega^{*4} \tag{14-69}$$

where

$T^* = T/T_{max}$

$\Omega^* = \Omega/\Omega_{sync}$

The *"a"* coefficients can be evaluated by choosing five points on the given curve and finding a set of *a's* consistent with these five values. For less accurate results, the higher-power coefficients can be discarded.

14-3.11 Torque at partial excitation. The torque at a given speed, with less than full rated excitation, is found by subtracting the partial torque due to the voltage set which causes backward rotation from the torque due to the forward set. If the voltages V_b and V_f (Fig. **14-28)** are found by the equations in the figure, the torque at speed Ω_1^* is given by

$$T_1^* = V_f^2\,(T^*_{fe1}) - V_b^2\,(T^*_{fe-1}) \tag{14-70}$$

where

T^*_{fe1} = per unit torque $\dfrac{T_{fe1}}{T_{max}}$ evaluated at forward speed Ω_1^*

T^*_{fe-1} = per unit torque evaluated at backward speed Ω_1^*

The values of T^*_{fe1} and T^*_{fe-1} for Ω^* of **0.4** are shown in Fig. **14-24.** The relationship between stall-torque damping at zero control voltage and at full control voltage may thus be determined. This relationship permits calculation of the inherent motor damping for zero or small control-field voltages, using the full-excitation curve furnished by the manufacturer as a basis.

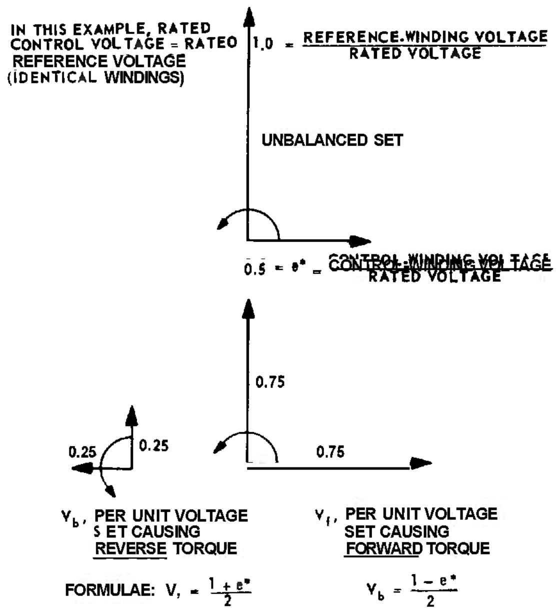


Fig. 14-28 Illustrating the forward and backward sets of voltage derived from an unbalanced excitation for a 2-phase motor.

14-3.12 Nondimensional damping. For $e^* = 0$ and $V_f = V_b = 0.5$, the nondimensional damping will be the total damping due to both the forward and backward components:

$$\frac{\partial T^*}{\partial\Omega^*} = \frac{\partial T_f^*}{\partial\Omega^*} + \frac{\partial T_b^*}{\partial\Omega^*} \tag{14-71}$$

$$= (0.5)^2\,\frac{\partial T^*_{fe0}}{\partial\Omega^*} + (0.5)^2\,\frac{\partial T^*_{fe-0}}{\partial\Omega^*} \tag{14-72}$$

$$= 0.5\,\frac{\partial T^*_{fe0}}{\partial\Omega^*} \tag{14-73}$$

where

T^*_{fe0} = full-excitation stall torque at zero speed

T^*_{fe-0} = full-excitation stall torque at negative zero speed

T^*_{fe0} $= T^*_{fe-0}$

$\dfrac{\partial T^*_{fe0}}{\partial \Omega^*}$ = full-excitation nondimensional coefficient measured at stall

The dimensionalized damping constants at zero excitation and full excitation are also related by a factor of 0.5.

14-3.13 Motor equations. The motor equations are

$$T = T_L + Js\omega_m + B\omega_m \tag{14-74}$$

$$K_mE - B_m\omega_m = T_L + Js\omega_m + B\omega_m \tag{14-75}$$

where

K_m = dimensionalized torque gain

B_m = dimensionalized damping

J = total inertia referred to the motor shaft

T_L = load torque referred to the motor shaft

B = viscous-friction constant associated with the load

T = motortorque

ω_m = motor-shaft speed

The factor E is the control-field voltage when the amplifier internal resistance is substantially lower than the motor impedance. Alternatively, when the motor partial derivatives have· been measured with the motor driven by its amplifier, E is the amplifier voltage input and K_m includes amplifier gain. Figure **14-29** shows the block diagrams corresponding to the equations.

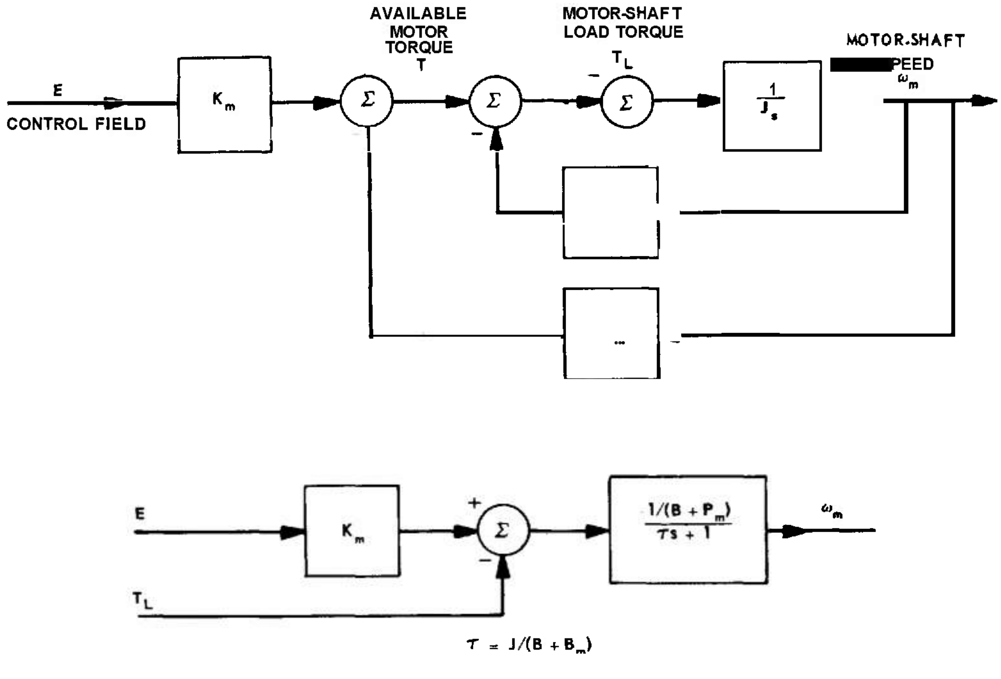


Fig. 74-29 Block diagrams of 2-phase motor and load.

TABLE 14-3 COMPARISON OF TYPICAL MOTORS

Motor	1"	2*	3*	4*	5**
Rated voltage {reference	115	115	110	115	115
Rated voltage {control	150–300	115	110	115	115
Frequency (cps)	400	400	400	400	60
Number of poles	8	8	4	4	2
Synchronous speed (rpm)	6000	6000	12,000	12,000	3600
Total weight (oz)	13.55	14.24	14.39	13.33	25
Rotor weight (oz)	0.91	—	1.13	0.89	—
Rotor inertia (gram-cm^2)	4.17	5.26	9.83	3.82	14.1
(oz-in^2)	0.0228	0.0288	0.0537	0.0209	0.077
Torque/inertia ratio (stall) (1/sec^2)	33,100	17,500	7630	46,000	27,600
Winding impedance (ohms) control {stall	500 + j1050	485 + j1130	870 + j745	300 + j370	720 + j330
control {no load	270 + j1240	212 + j1230	435 + j1490	170 + j550	430 + j860
ref {stall / no load	Z = 670	same as control	same as control	same as control	same as control
Max power output watts	2.6	1.7	3.1	6.1	5.5
at speed	3300	3200	6800	5400	2100
Rated control phase power at stall {watts	7.7	5.0	8.5	17.2	6
{milliamp	124	102	102	245	120
Stall torque (oz-in.) (at full excitation)	1.95	1.30	1.06	2.49	5.5
Inherent motor damping (see also Fig. 14–16) oz-in.-sec/rad (at full excitation)	23×10^{-4}	18×10^{-4}	4.8×10^{-4}	17×10^{-4}	72×10^{-4}
$\frac{\text{Torque-to-inertia ratio}}{\text{sync speed}}$ (1/sec)	52.7	27.9	12.1	36.6	73
$\frac{\text{Torque}^2}{\text{Inertia}}$ $\left(\frac{\text{oz-in}}{\text{sec}^2}\right)$	6.4×10^4	2.25×10^4	0.803×10^4	11.4×10^4	15.2×10^4
Figure of Merit*** (1/sec)	72	34.4	8.9	53.7	80

*From Kaiser[(11)]

**From manufacturer's data

***From Ferrell[(16)]

14-3.14 FIGURE OF MERIT

Several methods for determining a Figure of Merit have been proposed for servomotors. The three most common are:

(a) *Torque-to-inertia ratio.* In comparing motors, it is necessary to reduce motors to a common speed base by dividing by synchronous speed.

(b) *Gain-bandwidth.* This Figure of Merit[16] is the product of bandwidth and the square root of the power-transmission efficiency. The bandwidth is taken as $2f_m/J_m$, the power output P_o is $T_s^2/4f_m$, and the power input P_{IC} is the quotient of the control-winding voltage (squared) divided by the control-winding impedance (real part). The Figure of Merit becomes

$$\text{Figure of Merit} = \frac{2f_m}{J_m} \frac{P_o}{P_{IC}}$$

where

T_s = stall torque

f_m = damping factor = B_m

J_m = motor-rotor inertia

(c) *Torque-to-inertia ratio divided by base (synchronous) speed.* This Figure of Merit, proposed by Harris[17] and Newton,[18] uses the ratio of peak motor torque (squared) to inertia, divided by the base (synchronous) speed.

14-3.15 COMPARISON OF TYPICAL MOTORS

Table *14-3* is a summarization of a survey of some typical motor characteristics.[19] The report from which this data was taken is also the source of data used to plot Fig. *14-26.* An additional tabulation can be found in Ahrendt.[21]

14-4 HYDRAULIC MOTORS

14-4.1 INTRODUCTION

Hydraulic motors or actuators are of two general types: those producing a linear or translatory motion and those whose output is rotational. Either type may be supplied from a variable-output source. Sources are either control-valve amplifiers (see Par. 13-*6)* or variable-displacement pumps. A variable-displacement motor with a constant supply pressure source could also be used. Such a device would provide a torque source which, if used with an inertia load, would provide a system with an infinite velocity constant. Of course, suitable stabilization would be required in the hydraulic amplifier used to stroke the motor or in the stage preceding the hydraulic amplifier. Use of such a system is not wide-spread at the present time.

Linear hydraulic motors may be divided into single-ended (spring-opposed) and double-ended types. Rotary hydraulic motors used in servomechanisms are generally of the positive-displacement type. Piston motors of either radial or axial construction and ball-piston motors are used. Vane-type motors are used where relatively large leakage is tolerable. A single-vane rotary motor of limited travel is useful if restricted rotary motion is needed.

14-4.2 STATIC CHARACTERISTICS Of PISTON-TYPE ROTARY MOTORS

The static behavior of the piston-type rotary motor may be seen by simplifying the motor flow and torque equations. These equations, in Laplace notation, are

$$Q(s) = Q_l(s) + Q_c(s) + D_m\omega(s) \tag{14-76}$$

$$T(s) = D_mP(s) \tag{14-77}$$

$$Q_l(s) = L_mP(s) \tag{14-78}$$

$$Q_c(s) = \frac{V_m}{\beta} sP(s) \tag{14-79}$$

$$T(s) = T_L(s) + J_m s\omega(s) + B_m\omega(s) \tag{14-80}$$

where

Q = total flow to the motor, in in.³/sec

Q_l = leakage flow, in in.³/sec

Q_c = compressibility flow, in in.³/sec

ω = motor speed, in rad/sec

T = torque produced by motor, in in.-lb

P = differential motor pressure, in lb/in.²

T_L = load torque, not including viscous torque, in in.-lb

D_m = motor displacement, in in.³/rad

L_m = motor leakage coefficient, in in.⁵/lb-sec

B_m = viscous damping **of** motor rotor, in in.-lb-sec

V_m = volume of oil under compression in motor, in in.³

β = bulk modulus of fluid, in lb/in.²

J_m = motor-rotor inertia, in in.-lb-sec²

The steady-state equations are found **by** letting s be zero. Then

$$Q = L_m P + D_m\omega$$

$$= L_m \frac{T}{D_m} + D_m\omega$$

or

$$\omega = \frac{Q}{D_m} - \frac{L_m}{D_m^2} T \tag{14-81}$$

It is convenient to nondimensionalize **Eq. (14-81)** before plotting it. Thus

$$\omega = \left(\frac{Q}{Q_{MAX}}\right)\left(\frac{Q_{MAX}}{D_m}\right) - \frac{L_m}{D_m^2}\left(\frac{T}{T_{MAX}}\right) T_{MAX}$$

$$\frac{\omega}{\omega_{MAX}} = \left(\frac{Q}{Q_{MAX}}\right)\left(\frac{Q_{MAX}}{D_m\omega_{MAX}}\right) - \frac{L_m}{D_m^2}\left(\frac{T}{T_{MAX}}\right)\frac{T_{MAX}}{\omega_{MAX}}$$

or

$$\frac{T}{T_{MAX}} = \frac{Q}{Q_{MAX}} - \frac{\omega}{\omega_{MAX}} \tag{14-82}$$

since

$$\omega_{MAX} = \frac{Q_{MAX}}{D_m} \text{ (no-load flow vs speed relation)}$$

and

$$T_{MAX} = \frac{D_m^2}{L_m}\omega_{MAX} \text{ (the stall torque)}$$

A plot of Eq. **(14-82)** is shown in Fig. **14-30.**

The analogy between the constant-displacement motor and the electric shunt motor has been pointed out by Chu and Gould[23] and is shown in Table **14-4.** Although the analogies shown are not the ones most often used in elementary physical reasoning, they are proper and lead to an identical mathematical form for the block diagrams and equations of electrical and hydraulic transmissions. Compare the expression for speed in Eq. **(14-81)** with that in **Eq. (14-76).**

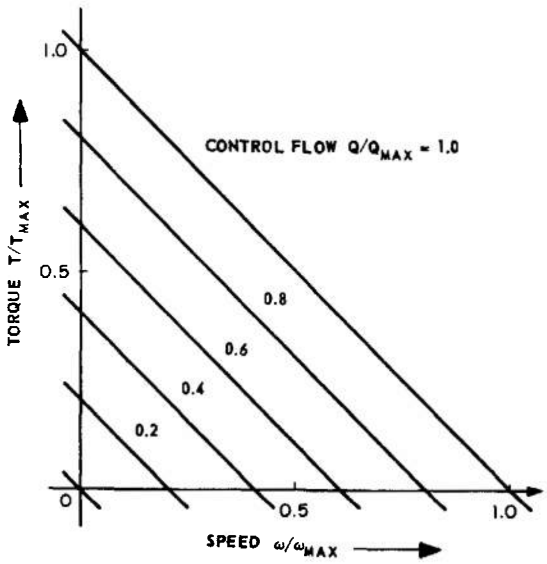


Fig. 74-30 Torque-speed curves, constant-displacement hydraulic motor.

TABLE 14-4 ANALOGOUS PARAMETERS OF CONSTANT-DISPLACEMENT HYDRAULIC MOTOR AND ELECTRIC SHUNT MOTOR

Hydraulic Motor	Electric Shunt Motor
Total flow, Q	Terminal voltage, V
Leakage flow, $L_m P$	Resistance drop, IR
Compressibility flow, $\frac{V}{\beta}\frac{dP}{dt}$	Inductance drop, $L\frac{dI}{dt}$
Pressure, P	Current, I
Displacement, D_m	Motor constant, K_m

14-4.3 STATIC EQUATIONS OF TRANSLATORY MOTOR (MOVING PISTON)

The equations pertinent to the piston are given in Par. 13-6. Transfer functions, relating input motion to the hydraulic amplifier to the output piston motion, are also presented in Par. 13-6. The static equations of the motor itself are

$$Q = L_P P + A_P V \tag{14-83}$$

$$F = A_P P \tag{14-84}$$

where

Q = total flow into cylinder

L_P = leakage coefficient (piston-cylinder wall leakage)

A_P = area of piston

V = velocity of piston motion

F = force available to do work

P = pressure difference across the piston load

Note that A_P is similar to D_m and that L_P, the piston leakage coefficient, is similar to L_m, the motor leakage coefficient. Thus

$$\frac{F}{F_{MAX}} = \frac{Q}{Q_{MAX}} - \frac{V}{V_{MAX}} \tag{14-85}$$

where

Q = total flow

Q_{MAX} = flow with zero load force.

V_{MAX} = maximum velocity = Q_{MAX}/A_P

$$F_{MAX} = \frac{A_P^2}{L_P} V_{MAX}$$

The plot of Eq. (14-85) is identical with that of Eq. (14-82) shown in Fig. 14-30 if torque is replaced by force and rotary speed is replaced by linear velocity.

14-4.4 DYNAMIC BEHAVIOR OF HYDRAULIC TRANSMISSIONS

The combination of rotary pump and motor, termed the *hydraulic transmission,* is discussed in Par. 13-6, where its equivalent hydraulic circuit, equations, and block diagram are given. Transfer functions available in Par. 13-6 are summarized in Table 14-5.

Typical dynamic and static performance data on transmissions[18] are summarized in Tables 14-6 through 14-10. Each transmission consists of a variable displacement hydraulic pump and a fixed displacement hydraulic motor. Weights and operating pressures are summarized in Table 14-6.

The compressibility effect in the case of the rotary motor is relatively constant as far as load motion is concerned, since quite a few motor revolutions occur for normal load travel and each revolution averages a number of cylinders. However, the compressibility effect in the case of the piston and cylinder is different since, from one extreme of piston movement to the other, the volume of oil under compression (V_m) varies from perhaps 0.1 to 0.9 of the total net cylinder volume. The result is a nonlinear equation, since $V_m = (X_0 \pm x) A$. To handle the problem analytically, an analog computer study may be made or else the equation may be linearized about some average value other than the centered one.

TABLE 14-5 REFERENCES TO TABLE LISTING HYDRAULIC TRANSMISSION TRANSFER FUNCTION

Source	Load	Refer to Table
Rotary pump	rotary motor	13–17(a)
Rotary pump	translatory motor (spring-opposed)	13–17(b)
Rotary pump	translatory motor (not spring-opposed)	13–17(a)
Spool valve	translatory motor (spring-opposed)	13–16(a)
Spool valve	translatory motor (not spring-opposed)	13–16(b)

TABLE 14-6 TRANSMISSION DATA — HYDRAULIC PUMPS

Item	Information	Units	Transmission I	Transmission II	Transmission III
1	approx power at {speed pressure stroke	hp rpm psi deg	4.5 4500 1250 25	2.2 3500 1200 10	8.5 1750 2000 30
2	weight dry	lb	27, double pump unit with auxiliaries (approx)	16.6, complete transmission (measured)	30 (catalog value)
3	maximum rated pressure†	psi	1000	1200	2000
4	maximum displacement at stroke	in.^{3}/radian deg	0.0509 25	0.0332 10	0.153 30
5	volume subject to high pressure	in.^{3}	0.62	only total oil vol known	1.71
6	minimum cylinder clearance	in.^{3}	0.0117 (estimated)	0.0399 (estimated)	0.019 (estimated)
7	total leakage††	in.^{3}/sec	0.22 at 1000 psi	only over-all leakage known	0.4 at 1000 psi 0.8 at 2000 psi

†Maximum rated pressure for continuous operation for normal life.

††Leakage at operating speed with fluid described in Table 14-8. All leakage data is approximate. Manufacturer's information used except for Transmission II where measurements on unit were made. All leakage data compensated for viscosity and temperature deviations. The breakdown of total leakage into direct and differential for Transmission I is estimated.

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TABLE 14-7 TRANSMISSION DATA — HYDRAULIC MOTORS

Item	Information	Units	Transmission I	Transmission II	Transmission III
1	approx power at {speed, pressure, stroke}	hp rpm psi deg	4.3 3600 1250 30	6.5 3600 1200 30	26 2200 200 30
2	weight dry	lb	2.5 (measured)	see pump	30 (catalog value)
3	maximum rated pressure†	psi	1000	unknown	2000
4	maximum rated speed†	rpm	3600	unknown	2200
5	displacement at stroke	in.^{3}/radian deg	0.0602 30	0.0954 30	0.384 30
6	volume subject to high pressure	in.^{3}	0.22	only total oil vol measured	2.52
7	total leakage††	in.^{3}/sec	0.22 at 1000 psi	only over-all leakage known	0.6 at 1000 psi 1.2 at 2000 psi
8	inertia	$\frac{\text{in.-lb-sec}^2}{\text{radian}}$	0.000669 (measured)	0.00144 (estimated)	0.0147 (estimated)

†Maximum rated speed and pressure for continuous operation for normal life.

††Leakage at operating speed with fluid described in Table 14-8. All leakage data is approximate. Manufacturer's information used except for Transmission II where measurements on unit were made. All leakage data compensated for viscosity and temperature deviations. The breakdown of total leakage into direct and differential for Transmission I is estimated.

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TABLE 14-8 TRANSMISSION DATA — FLUID

Item	Information	Units	Transmission I	Transmission II	Transmission III
1	viscosity	centistokes	7.5 at 158°F	10 at 150°F	23 at 140°F
2	specific gravity		0.84	0.84 (approx)	0.84 (approx)
3	compressibility†	in.^{2}/lb	4.0×10^{-6} at 0-2000 psi	4.0×10^{-6} at 0-2000 psi	4.0×10^{-6} at 0-2000 psi

†Reciprocal of bulk modulus.

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TABLE 14-9 TRANSMISSION DATA — GENERAL

Item	Information	Units	Transmission I	Transmission II	Transmission III
1	pump operating speed	rpm	4500	3500	3500
2	ambient temperature	°F	77	75 (approx)	68 (approx)
3	fluid temperature	°F	158	150 (approx)	140 (approx)
4	replenishing pressure	psi	50 (gauge)	35 (gauge)	50 (gauge, approx)
5	load inertia	in.-lb-sec^2	0.0159	0.0648	1.80
6	load damping	in.-lb-sec	negligible	negligible	negligible
7	total output-shaft inertia	in.-lb-sec^2	0.0166	0.0662	1.81
8	total damping	in.-lb-sec	0.0093	unknown	unknown
9	total volume of high-pressure oil†	in.^{3}	3.84	1.65	31.0
10	over-all leakage constant	in.^{5}/lb-sec	4.4×10^{-4}	3.67×10^{-4}	1.0×10^{-3}
11	approx max power	hp	4.3	2.2	8.5

†Slight differences in volumes (or lengths) of conduits, fittings, etc., communicating with the two pump connections have been averaged so that the data represents the mean volume (or length) of a high-pressure oil path.

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TABLE 14-10 COMPARISON OF PREDICTED AND MEASURED PARAMETERS

Parameter		Transmission I	Transmission II	Transmission III
$1/\omega_n$ (sec/rad)	predicted	5.27×10^{-2}	4.35×10^{-2}	2.45×10^{-1}
	measured	8.50×10^{-2}	7.27×10^{-2}	2.84×10^{-1}
ζ	predicted	0.153	0.226	0.173
	measured	1.14	0.234	0.332

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The effect of air entrainment is discussed later in this chapter. Pressure-control valves lessen the compressibility effect since they, in effect, provide a pressure at the piston independent of the flow to the piston which includes, of course, compressibility flow. If pressure-control valves are to be successful in this function, they must behave as ideal pressure sources, dynamically as well as statically.

14-4.5 APPROXIMATE DYNAMIC BEHAVIOR OF HYDRAULIC TRANSMISSION

It is usually possible to neglect line inertance, although this may not be the case if small diameter lines are used. (For the inertance formula, see Par. **13-6.27.**) If the load has negligible damping and line inertance is neglected, then

$$\omega(s) = \frac{\dfrac{d_P}{D_M} X(s) - \dfrac{L_{MP}}{D_M^2}\left(1 + \dfrac{C_{MP}}{L_{MP}} s\right) T(s)}{1 + 2\zeta \dfrac{s}{\omega_n} + \left(\dfrac{s}{\omega_n}\right)^2} \tag{14-86}$$

where

$$\omega_n = D_M \sqrt{\frac{B}{JV_{MP}}}$$

$$\zeta = \frac{1}{2} \frac{L}{D_M} \sqrt{\frac{BJ}{V_{MP}}}$$

L_{MP} = total leakage : pump and motor

C_{MP} = total compressibility : pump, motor, and line

V_{MP} = total oil volume: pump, motor, and line

Typical values of ζ and ω_n are about **0.3** and **20** rad/sec, respectively, if the load inertia is substantially greater than the motor inertia and air entrainment is absent. (See Table **14-10** for typical performance data.)

14-4.6 PARAMETER EVALUATION FOR ROTARY MOTOR

The essential parameters are leakage coefficient L_m, compressibility coefficient C_m, displacement coefficient D_m, and inertia J_m.

Leakage can be determined from static speed-torque curves once D_m is known. From Eq. **(14-81)**, the slope of the speed-torque curve is

$$\frac{\partial\omega}{\partial T} = \frac{L_m}{D_m^2} \tag{14-87}$$

or, solving **Eq. (14-87)** for L_m

$$L_m = D_m^2 \frac{\partial\omega}{\partial T} \tag{14-88}$$

The value of D_m is usually available from the manufacturer. If unavailable, it may be measured by causing the motor to act as a pump with zero terminal pressure drop. The flow Q at motor speed ω is measured. Then

$$D_m = \frac{Q}{\omega} \tag{14-89}$$

If facilities for driving the motor are unavailable, measure the motor torque T and the pressure drop P across the motor while running the motor at a convenient constant speed. Then

$$D_m = \frac{T}{P} \tag{14-90}$$

Leakage is a function of temperature, since it is inversely proportional to the viscosity, whose temperature dependence is shown in Par. **20-2.**

Compressibility depends upon the motor oil volume being compressed. This volume equals one half the displacement per revolution, d_m, for the motor alone. The volume of connecting lines must be taken into account. Thus

$$C_m = \frac{d_m}{2\beta} \tag{14-91}$$

where β is the bulk modulus of the fluid used. The effect of entrained air can be substantial, causing ζ to increase by a factor of **5** to **10.** (See Par. **13-6.27** for discussion.)

The motor inertia is usually available from the manufacturer. If not, it may be measured by a retardation run as described in Par. **14-2.**

The load and gear inertia are reflected to the motor shaft. The numerical value of the reflected inertia at the motor shaft is obtained by dividing the load and gear inertia by the square of the gear ratio between the motor and the point where the load or gear inertia occurs.

14-4.7 PROBLEMS ENCOUNTERED WITH HYDRAULIC MOTORS

The problems encountered with hydraulic motors are similar to those discussed under hydraulic amplifiers. Rotary-motor commutation causes a power loss proportional to the speed of rotation. The power is dependent upon position and gives rise to pressure pulsations observed as noise.[24]

Translatory motor or piston-end seals present a problem. High packing pressure to prevent leakage to the atmosphere is accompanied by coulomb friction which appears as a nonlinear component of F_L. Coulomb friction gives rise to a dead spot, since a certain pressure must be developed to overcome this friction before any motion can occur. If stiction occurs (high breakaway friction), this may cause unstable operation. Analysis using describing-function or phase-plane methods may be carried out.

The nonlinear effect of piston compressibility has been cited. Flexible hose, although convenient, is more elastic than steel tubing and may contribute to the total compressibility coefficient (see Par. **13-6** for formulae). Long lines between the motor and the pump should be avoided to prevent standing waves of pressure. At the very least, the large V_{MP} resulting causes both ζ and ω to decrease. Such effects are generally undesirable.

14-5 PNEUMATIC MOTORS

14-5.1 PRINCIPAL TYPES

A pneumatic motor is an air-operated device wherein a change in input air pressure produces a mechanical output motion. Pneumatic motors are of two basic kinds: translational and rotary. Current applications make wide use of the translational or linear motor. The principal types of translational motors are the capsule, bellows, diaphragm, ram, and piston. These types are shown functionally in Figs. **14-31** through **14-35.** In these figures, P_a and W_a are input load pressure and flow respectively controlled by a pneumatic valve, and Y is the motor-shaft displacement resulting from an input to the pneumatic valve. All of the motors illustrated, except the piston, are single acting; i.e., air pressure is applied on one side only, and motion is opposed (or assisted) by **a** spring. Since only one source of pressure is needed, the single-acting motor can be operated from a three-way valve. The piston is operated with air at each end (with or without springs) and hence requires the use of a four-way valve.

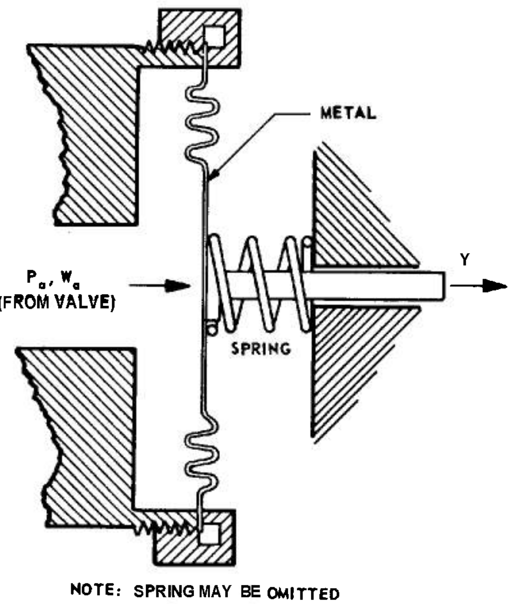

Fig. 14-31 Typical pneumatic capsule.

inherent damping is greater than that of the motor alone because direct currents in the motor windings produce eddy-current damping.

Figures 14-20 and 14-21 show two circuits proposed by Storm,[14] both using a d-c control signal. Figure 14-20 shows a circuit with only one motor phase controlled, while Fig. 14-21 shows a circuit with both phases controlled. If both phases of the motor are controlled, the damping due to 2-phase action will be zero, with zero excitation at stall. If some direct current flows in the motor windings, eddy-current damping will result.

14-3.3 Types of a-c Motors Used in Relay Applications

The following types of a-c motors are suitable for relay applications:

(a) Split-phase, reluctance-type motors

(b) Series (universal) motors

(c) Torque motors

(d) 2-phase motors

(e) 3-phase motors, for high-power operation

(f) Shaded-pole motors

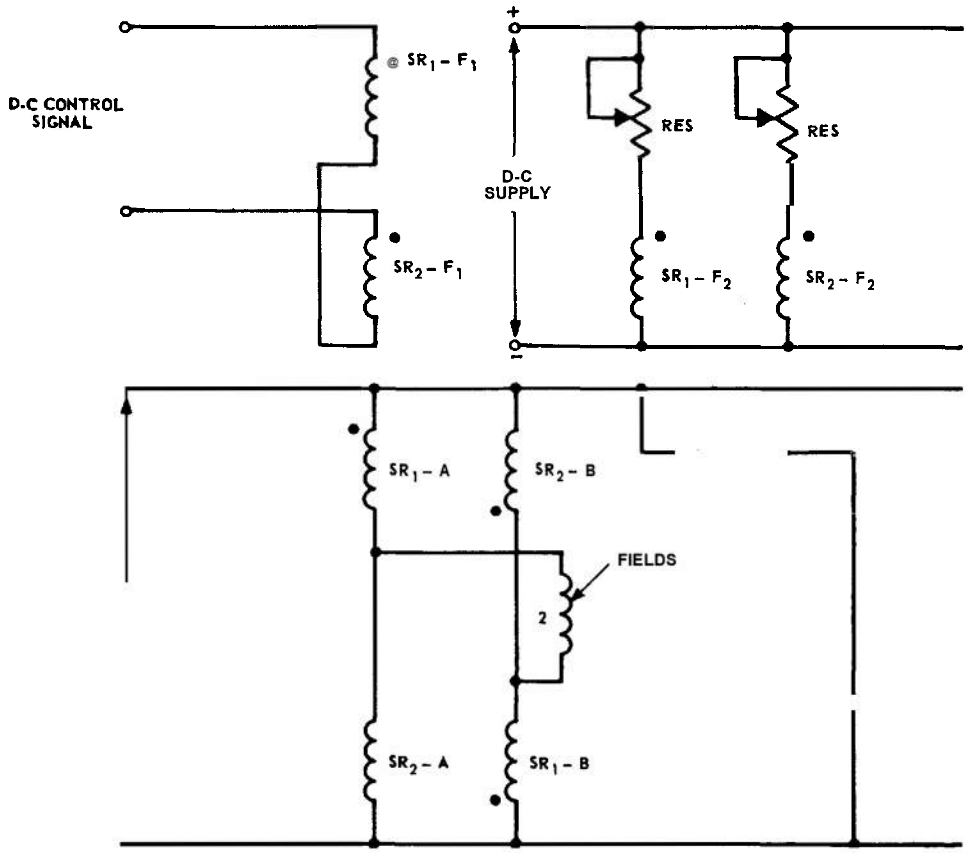

Fig. 74-20 A-c motor control (one phase fixed).'

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In the usual application, the motor must be reversible and have a starting torque about as large as the running torque or a little larger. High starting torque may be obtained from standard-design motors by increasing rotor resistance. This can be achieved by reducing the thickness of the rotor end piece. Shaded-pole motors can be used in pairs to obtain reversible operation —one motor for clockwise rotation, the other for counterclockwise rotation.

14-3.4 Typical relay-servo circuits. The series (universal) motor can be used with an a-c supply by employing the circuits described in Par. 14-2. Two-phase servomotors can be relay-controlled by using a circuit such as that shown in Fig. 14-22. The latter circuit is also usable with wound-shading-coil a-c motors. The only damping present is the inherent damping of the motor. Dynamic braking by eddy-current action can be provided as shown in Fig. 14-23. Proper choice of circuit elements will bring the motor to a dead stop from full speed within one revolution from the instant the relay contacts connect in the braking circuit. Capacitor C1 can be an electrolytic element. However, capacitor C2 should be an oil-filled unit, not an a-c electrolytic that is designed only for infrequent service such as starting. Choice of circuit-element values is discussed in detail in Par. **13-5.**

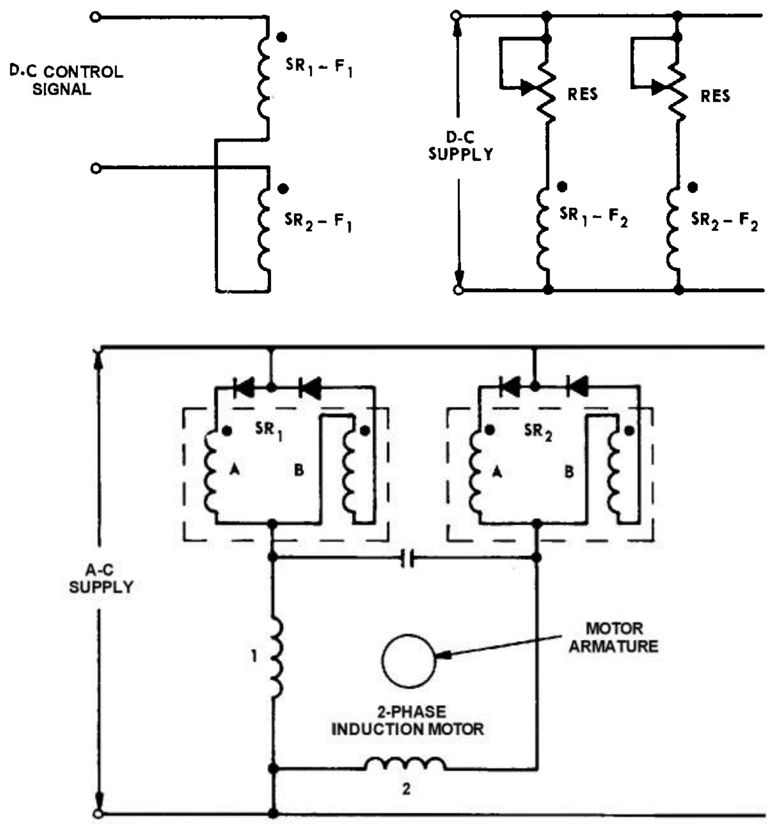

Fig. 74-27 A-c motor control (both phases variable).

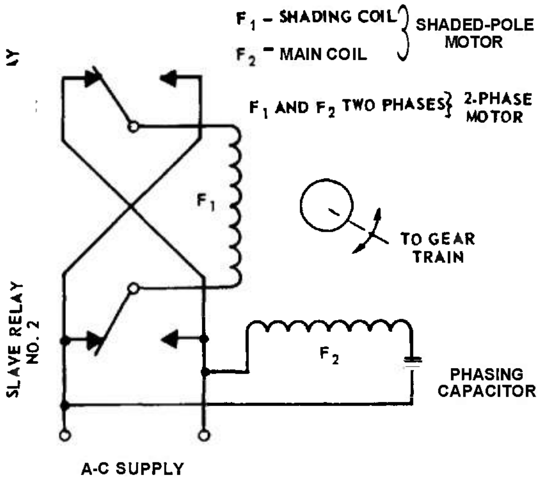

Fig. 74-22 Relay servo using wound-shading-coil or 2-phase motors.

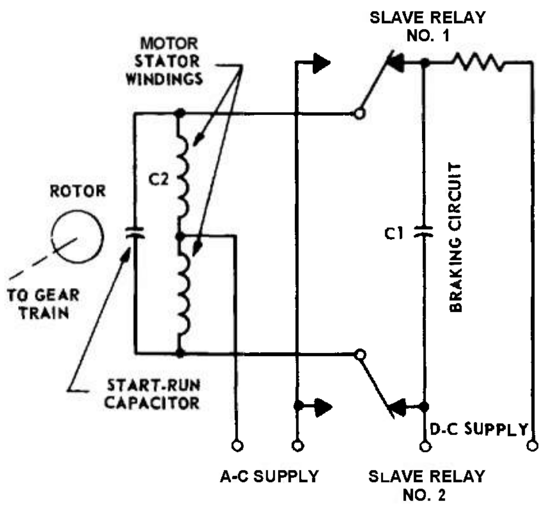

Fig. 14-23 Relay servo using capacitor-start-and-run motor.

14-3.5 STATIC CHARACTERISTICS OF A-C MOTORS

14-3.6 Typical 2-Phase Servomotor

The static torque-speed curves of a typical 2-phase servomotor are shown in Fig. 14-24.[15] These curves can be used to determine the motor parameters that influence dynamic performance.

14-3.7 Relay Servos

Analysis of relay servos can be accomplished by either of two methods: the describing function approach ; or the phase-plane method. In each case, the objective is to determine the motor-speed response from an arbitrary initial speed to full speed, or to a dead stop. Such responses may be calculated from the speed-torque curve of the motor used in the servo. Reference 15 provides typical characteristics of various single-phase a-c motors that can be used in relay servos. It also indicates the method of approximating the transient response of such motors.

14-3.8 DYNAMIC BEHAVIOR OF 2-PHASE SERVOMOTORS

At the present time, the 2-phase servomotor is the type used in servomechanisms more often than any other type. Therefore, its dynamic behavior will be considered. The effect of electrical transients caused by winding inductance and resistance is negligible in most applications. Where this effect is important, an additional time constant can be introduced to account for the effect. The break frequency associated with this time constant is in the order of 400 to 800 radians/sec for 60-cycle motors.

14-3.9 Equations

The curves in Fig. **14-24** may be used to evaluate the partial derivatives in the representation of the motor as a torque source having some associated damping. The equation is

$$dT = \left[\frac{\partial T/T_{max}}{\partial E/E_{max}}\right]\left[\frac{T_{max}}{E_{max}}\right] dE + \left[\frac{\partial T/T_{max}}{\partial \Omega/\Omega_{sync}}\right]\left[\frac{T_{max}}{\Omega_{sync}}\right] d\Omega \tag{14-68}$$

where

$\dfrac{\partial T/T_{max}}{\partial E/E_{max}}$ = nondimensional torque gain

$\dfrac{\partial T/T_{max}}{\partial \Omega/\Omega_{sync}}$ = nondimensional damping

T_{max} = stall torque with full motor excitation

Ω_{sync} = synchronous speed

E_{max} = rated reference voltage

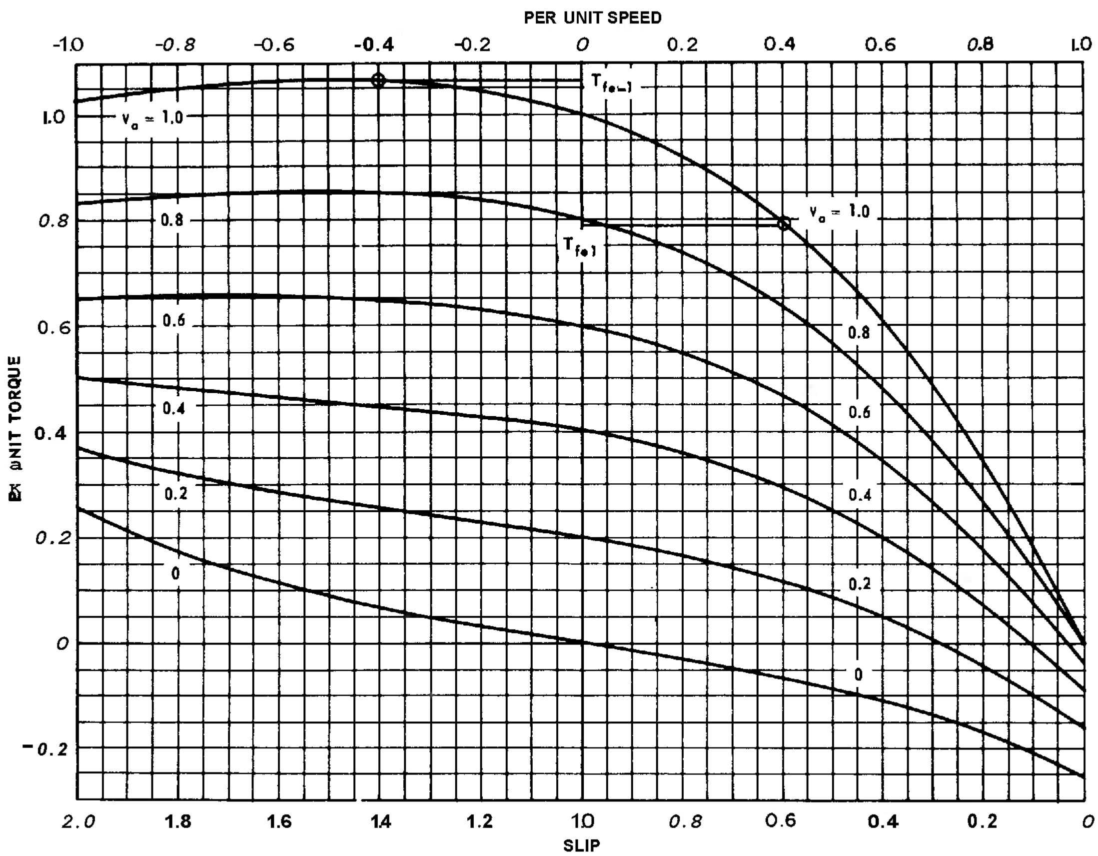

fig. 14-24 Torque-speed curves of a 2-phase servomotor.

By permission from *Electric Machinery*, by A. E. Fitzgerald and Charles Kingsley, Jr., Copyright. 1952. McGraw-Hill Book Company, Inc.

The nondimensional gain and damping constants are plotted in Figs. **14-25, 14-26,** and **14-27.** An approximate method of obtaining the damping constant $\partial T/\partial \Omega$ from the manufacturer's curve (which is almost universally made for full motor excitation) is to divide the damping by two (damping measured at stall and with rated motor excitation). The resultant damping is pertinent for low control-winding voltages. The torque gain can be derived from the manufacturer's curve **by** dividing the stall torque at full excitation by the full excitation voltage. Stall torque is theoretically and actually linear with applied voltage.

14-3.10 Alternatives. If more partial-coefficient information is desired, obtain the torque-speed curves for the motor to be used, measured at full and also at partial excitation. These curves can be obtained from the manufacturer or measured by the designer. The desired partial derivatives can be measured from these curves. A third method is to obtain the curves by calculation from the

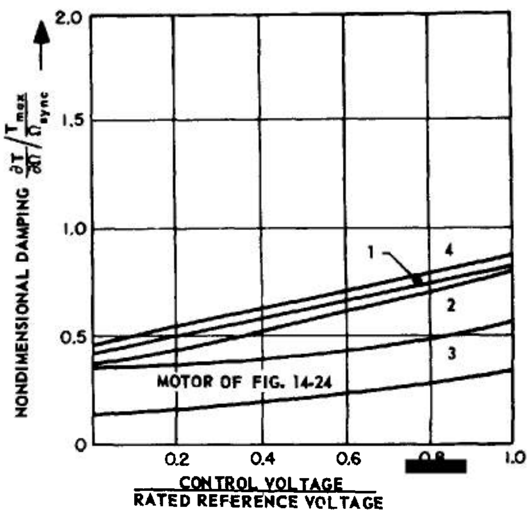

Fig. 74-26 Damping factors for some typical motors, taken with zero speed, Ω/Ω sync $= 0$.

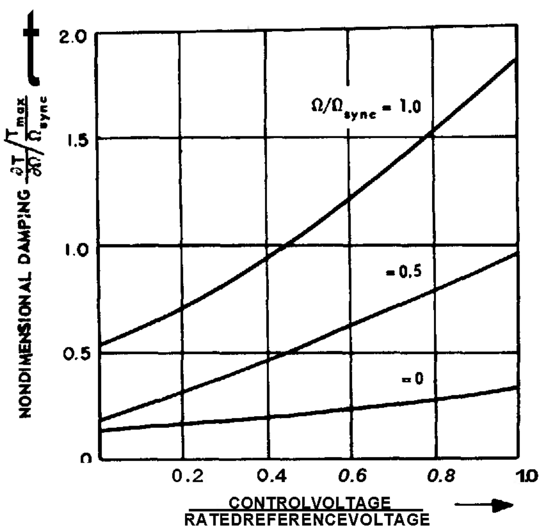

Fig. 74-25 Damping factor for motor having torque-speed curves shown in Fig. 74-24.

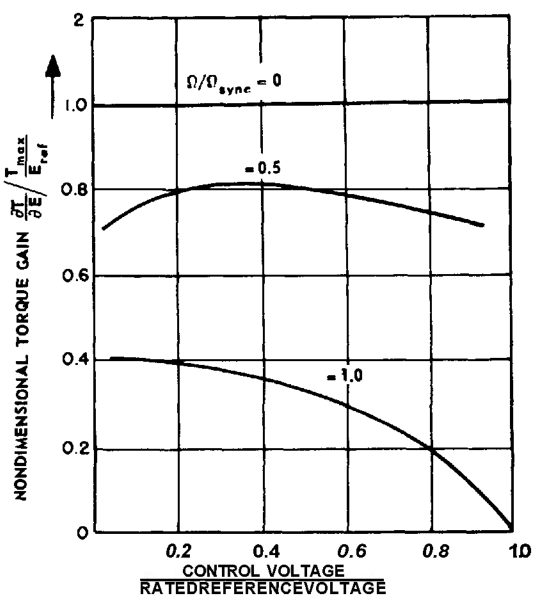

Fig. 74-27 Torque gain of typical 2-phase servomotor versus per unit control voltage at different speeds.

curve for full, rated excitation. This calculation can be performed by using symmetrical component voltages. It is necessary to have the torque-speed curve for negative as well as positive speeds. The manufacturer's curve for positive speeds can be extrapolated to negative speeds by assuming a power-series representation of the given nondimensional torque-speed curve:

$$T^* = a_0 + a_1\Omega^* + a_2\Omega^{*2} + a_3\Omega^{*3} + a_4\Omega^{*4} \tag{14-69}$$

where

$T^* = T/T_{max}$

$\Omega^* = \Omega/\Omega_{sync}$

The *"a"* coefficients can be evaluated by choosing five points on the given curve and finding **a** set of *a's* consistent with these five values. For less accurate results, the higher-power coefficients can be discarded.

14-3.11 Torque at partial excitation. The torque at a given speed, with less than full rated excitation, is found by subtracting the partial torque due to the voltage set which causes backward rotation from the torque due to the forward set. If the voltages V_b and V_f (Fig. **14-28)** are found by the equations in the figure, the torque at speed Ω_1^* is given by

$$T^* = V_f^2\,(T^*_{fe1}) - V_b^2\,(T^*_{fe-1}) \tag{14-70}$$

where

T^*_{fe1} = per unit torque $\dfrac{T_{fe1}}{T_{max}}$ evaluated at forward speed Ω_1^*

T^*_{fe-1} = per unit torque evaluated at backward speed Ω_1^*

The values of T^*_{fe1} and T^*_{e-1} for Ω^* of **0.4** are shown in Fig. **14-24.** The relationship between stall-torque damping at zero control voltage and at full control voltage may thus be determined. This relationship permits calculation of the inherent motor damping for zero or small control-field voltages, using the full-excitation curve furnished by the manufacturer as a basis.

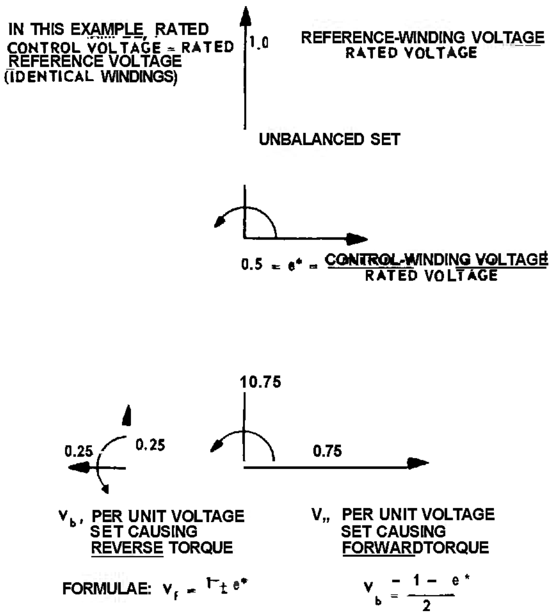

Fig. 14-28 Illustrating the forward and backward sets of voltage derived from an unbalanced excitation for a 2-phase motor.

14-3.12 Nondimensional damping. For $e^* = 0$ and $V_f = V_b = 0.5$, the nondimensional damping will be the total damping due to both the forward and backward components:

$$\frac{\partial T^*}{\partial\Omega^*} = \frac{\partial T_f^*}{\partial\Omega^*} + \frac{\partial T_b^*}{\partial\Omega^*} \tag{14-71}$$

$$= (0.5)^2\,\frac{\partial T^*_{fe0}}{\partial\Omega^*} + (0.5)^2\,\frac{\partial T^*_{fe-0}}{\partial\Omega^*} \tag{14-72}$$

$$= 0.5\,\frac{\partial T^*_{fe0}}{\partial\Omega^*} \tag{14-73}$$

where

T^*_{fe0} = full-excitation stall torque at zero speed

T^*_{fe-0} = full-excitation stall torque at negative zero speed

$T^*_{fe0} = T^*_{fe-0}$

$\dfrac{\partial T^*_{fe0}}{\partial \Omega^*}$ = full-excitation nondimensional coefficient measured at stall

The dimensionalized damping constants at zero excitation and full excitation are also related by a factor of **0.5.**

14-3.13 Motor equations. The motor equations are

$$T = T_L + Js\omega_m + B\omega_m \qquad (14\text{-}74)$$

$$K_m E - B_m\omega_m = T_L + Js\omega_m + B\omega_m \qquad (14\text{-}75)$$

where

K_m = dimensionalized torque gain

B_m = dimensionalized damping

J = total inertia referred to the motor shaft

T_L = load torque referred to the motor shaft

B = viscous-friction constant associated with the load

T = motortorque

ω_m = motor-shaft speed

The factor E is the control-field voltage when the amplifier internal resistance is substantially lower than the motor impedance. Alternatively, when the motor partial derivatives have been measured with the motor driven by its amplifier, E is the amplifier voltage input and K_m includes amplifier gain. Figure **14-29** shows the block diagrams corresponding to the equations.

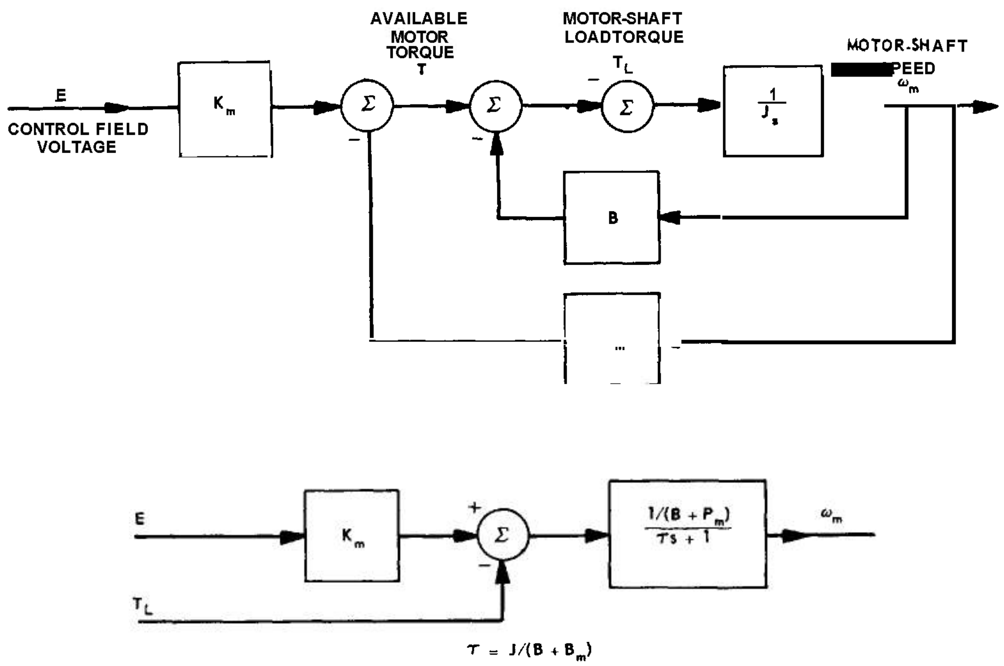

Fig. 74-29 Block diagrams of 2-phase motor and load.

TABLE 14-3 COMPARISON OF TYPICAL MOTORS

Motor	1*	2*	3*	4*	5**
Rated voltage {reference {control	115 150–300	115 115	110 110	115 115	115 115
Frequency (cps)	400	400	400	400	60
Number of poles	8	8	4	4	2
Synchronous speed (rpm)	6000	6000	12,000	12,000	3600
Total weight (oz)	13.55	14.24	14.39	13.33	25
Rotor weight (oz)	0.91	—	1.13	0.89	—
Rotor inertia (gram-cm^2) (oz-in^2)	4.17 0.0228	5.26 0.0288	9.83 0.0537	3.82 0.0209	14.1 0.077
Torque/inertia ratio (stall) (1/sec^2)	33,100	17,500	7630	46,000	27,600
Winding impedance (ohms) stall {no load	500 + j1050 270 + j1240	485 + j1130 212 + j1230	870 + j745 435 + j1490	300 + j370 170 + j550	720 + j330 430 + j860
ref stall {no load	Z = 670	same as control	same as control	same as control	same as control
Max power output watts at speed	2.6 3300	1.7 3200	3.1 6800	6.1 5400	5.5 2100
Rated control phase power at stall {watts {milliamp	7.7 124	5.0 102	8.5 102	17.2 245	6 120
Stall torque (oz-in.) (at full excitation)	1.95	1.30	1.06	2.49	5.5
Inherent motor damping (see also Fig. 14–16) oz-in.-sec/rad (at full excitation)	23×10^{-4}	18×10^{-4}	48×10^{-4}	17×10^{-4}	72×10^{-4}
$\frac{\text{Torque-to-inertia ratio}}{\text{sync speed}}$ (1/sec)	52.7	27.9	12.1	36.6	73
[illegible] $\left(\frac{\text{oz-in}}{\text{sec}^2}\right)$	6.4×10^4	2.25×10^4	0.803×10^4	11.4×10^4	15.2×10^4
Figure of Merit*** (1/sec)	72	34.4	3.9	53.7	80

*From Kaiser[15]

**From manufacturer's data

***From Ferrell[16]

14-3.14 FIGURE OF MERIT

Several methods for determining a Figure of Merit have been proposed for servomotors. The three most common are:

(a) *Torque-to-inertia ratio.* In comparing motors, it is necessary to reduce motors to a common speed base by dividing by synchronous speed.

(b) *Gain-bandwidth.* This Figure of Merit[16] is the product of bandwidth and the square root of the power-transmission efficiency. The bandwidth is taken as $2f_m/J_m$, the power output P_o is $T_s^2/4f_m$, and the power input P_{IC} is the quotient of the control-winding voltage (squared) divided by the control-winding impedance (real part). The Figure of Merit becomes

$$\text{Figure of Merit} = \frac{2f_m}{Jm}\frac{P_o}{P_{IC}}$$

where

T_s = stall torque

f_m = damping factor $= B_m$

J_m = motor-rotor inertia

(c) *Torque-to-inertia ratio divided by base (synchronous) speed.* This Figure of Merit, proposed by Harris[17] and Newton,[18] uses the ratio of peak motor torque (squared) to inertia, divided by the base (synchronous) speed.

14-3.15 COMPARISON OF TYPICAL MOTORS

Table *14-3* is a summarization of a survey of some typical motor characteristics.[19] The report from which this data was taken is also the source of data used to plot Fig. *14-26.* An additional tabulation can be found in Ahrendt.[21]

14-4 HYDRAULIC MOTORS

14-4.1 INTRODUCTION

Hydraulic motors or actuators are of two general types: those producing a linear or translatory motion and those whose output is rotational. Either type may be supplied from a variable-output source. Sources are either control-valve amplifiers (see Par. 13-*6)* or variable-displacement pumps. A variable-displacement motor with a constant supply pressure source could also be used. Such a device would provide a torque source which, if used with an inertia load, would provide a system with an infinite velocity constant. Of course, suitable stabilization would be required in the hydraulic amplifier used to stroke the motor or in the stage preceding the hydraulic amplifier. Use of such a system is not wide-spread at the present time.

Linear hydraulic motors may be divided into single-ended (spring-opposed) and double-ended types. Rotary hydraulic motors used in servomechanisms are generally of the positive-displacement type. Piston motors of either radial or axial construction and ball-piston motors are used. Vane-type motors are used where relatively large leakage is tolerable. A single-vane rotary motor of limited travel is useful if restricted rotary motion is needed.

14-4.2 STATIC CHARACTERISTICS OF PISTON-TYPE ROTARY MOTORS

The static behavior of the piston-type rotary motor may be seen by simplifying the motor flow and torque equations. These equations, in Laplace notation, are

$$Q(s) = Q_l(s) + Q_c(s) + D_m\omega(s) \tag{14-76}$$

$$T(s) = D_m P(s) \tag{14-77}$$

$$Q_l(s) = L_m P(s) \tag{14-78}$$

$$Q_c(s) = \frac{V_m}{e} sP(s) \tag{14-79}$$

$$T(s) = T_L(s) + J_m s\omega(s) + B_m\omega(s) \tag{14-80}$$

where

Q = total flow to the motor, in in.^{3}/sec

Q_l = leakage flow, in in.^{3}/sec

Q_c = compressibility flow, in in.^{3}/sec

ω = motor speed, in rad/sec

T = torque produced by motor, in in.-lb

P = differential motor pressure, in lb/in.^{2}

T_L = load torque, not including viscous torque, in in.-lb

D_m = motor displacement, in in.^{3}/rad

L_m = motor leakage coefficient, in in.^{5}/lb-sec

B_m = viscous damping of motor rotor, in in.-lb-sec

V_m = volume of oil under compression in motor, in in.^{3}

β = bulk modulus of fluid, in lb/in.^{2}

J_m = motor-rotor inertia, in in.-lb-sec^2

The steady-state equations are found by letting s be zero. Then

$$Q = L_m P + D_m\omega$$

$$= L_m \frac{T}{D_m} + D_m\omega$$

or

$$\omega = \frac{Q}{D_m} - \frac{L_m}{D_m^2} T \tag{14-81}$$

It is convenient to nondimensionalize Eq. (14-81) before plotting it. Thus

$$\omega = \left(\frac{Q}{Q_{MAX}}\right)\left(\frac{Q_{MAX}}{D_m}\right) - \frac{L_m}{D_m^2}\left(\frac{T}{T_{MAX}}\right) T_{MAX}$$

$$\frac{\omega}{\omega_{MAX}} = \left(\frac{Q}{Q_{MAX}}\right)\left(\frac{Q_{MAX}}{D_m\omega_{MAX}}\right) - \frac{L_m}{D_m^2}\left(\frac{T}{T_{MAX}}\right)\frac{T_{MAX}}{\omega_{MAX}}$$

or

$$\frac{T}{T_{MAX}} = \frac{Q}{Q_{MAX}} - \frac{\omega}{\omega_{MAX}} \tag{14-82}$$

since

$$\omega_{MAX} = \frac{Q_{MAX}}{D_m} \text{ (no-load flow vs speed relation)}$$

and

$$T_{MAX} = \frac{D_m^2}{L_m}\omega_{MAX} \text{ (the stall torque)}$$

A plot of Eq. (14-82) is shown In Fig. 14-30.

The analogy between the constant-displacement motor and the electric shunt motor has been pointed out by Chu and Gould[23] and is shown in Table 14-4. Although the analogies shown are not the ones most often used in elementary physical reasoning, they are proper and lead to an identical mathematical form for the block diagrams and equations of electrical and hydraulic transmissions. Compare the expression for speed in Eq. (14-81) with that in Eq. (14-76).

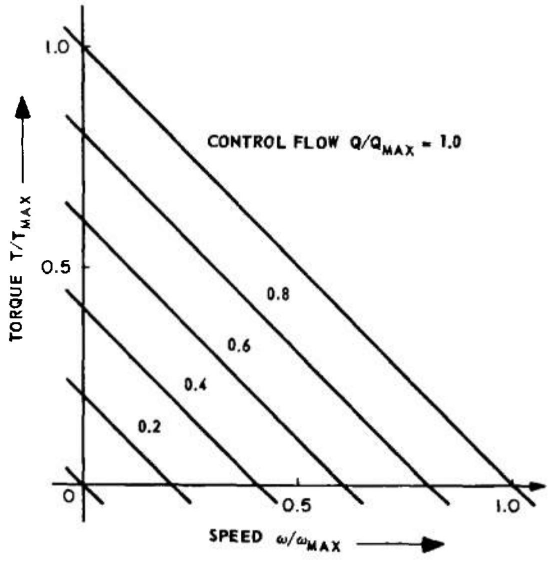

fig. 14-30 Torque-speed curves, constant-displacement hydraulic motor.

TABLE 14-4 ANALOGOUS PARAMETERS OF CONSTANT-DISPLACEMENT HYDRAULIC MOTOR AND ELECTRIC SHUNT MOTOR

Hydraulic Motor	Electric Shunt Motor
Total flow, Q	Terminal voltage, V
Leakage flow, $L_m P$	Resistance drop, IR
Compressibility flow, $\frac{V}{\beta}\frac{dP}{dt}$	Inductance drop, $L\frac{dI}{dt}$
Pressure, P	Current, I
Displacement, D_m	Motor constant, K_m

14-4.3 STATIC EQUATIONS OF TRANSLATORY MOTOR (MOVING PISTON)

The equations pertinent to the piston are given in Par. 13-6. Transfer functions, relating input motion to the hydraulic amplifier to the output piston motion, are also presented in Par. 13-6. The static equations of the motor itself are

$$Q = L_P P + A_P V \tag{14-83}$$

$$F = A_P P \tag{14-84}$$

where

Q = total flow into cylinder

L_P = leakage coefficient (piston-cylinder wall leakage)

A_P = area of piston

V = velocity of piston motion

F = force available to do work

P = pressure difference across the piston load

Note that A_P is similar to D_m and that L_P, the piston leakage coefficient, is similar to L_m, the motor leakage coefficient. Thus

$$\frac{F}{F_{MAX}} = \frac{Q}{Q_{MAX}} - \frac{V}{V_{MAX}} \tag{14-85}$$

where

Q = total flow

Q_{max} = flow with zero load force.

V_{MAX} = maximum velocity = Q_{MAX}/A_P

$$F_{MAX} = \frac{A_P^2}{L_P} V_{MAX}$$

The plot of Eq. **(14-85)** is identical with that of **Eq. (14-82)** shown in Fig. **14-30** if torque is replaced by force and rotary speed is replaced by linear velocity.

14-4.4 DYNAMIC BEHAVIOR OF HYDRAULIC TRANSMISSIONS

The combination of rotary pump and motor, termed the *hydraulic transmission*, is discussed in Par. **13-6,** where its equivalent hydraulic circuit, equations, and block diagram are given. Transfer functions available in Par. **13-6** are summarized in Table **14-5.**

Typical dynamic and static performance data on transmissions[18] are summarized in Tables **14-6** through **14-10.** Each transmission consists of a variable displacement hydraulic pump and a fixed displacement hydraulic motor. Weights and operating pressures are summarized in Table **14-6.**

The compressibility effect in the case of the rotary motor is relatively constant as far as load motion is concerned, since quite a few motor revolutions occur for normal load travel and each revolution averages a number of cylinders. However, the compressibility effect in the case of the piston and cylinder is different since, from one extreme of piston movement to the other, the volume of oil under compression (V_m) varies from perhaps **0.1** to 0.9 of the total net cylinder volume. The result is a nonlinear equation, since $V_m = (X_0 \pm x)\, A$. To handle the problem analytically, an analog computer study may be made or else the equation may be linearized about some average value other than the centered one.

TABLE 14-5 REFERENCES TO TABLE LISTING HYDRAULIC TRANSMISSION TRANSFER FUNCTION

Source	Load	Refer to Table
Rotary pump	rotary motor	13–17(a)
Rotary pump	translatory motor (spring-opposed)	13–17(b)
Rotary pump	translatory motor (not spring-opposed)	13–17(a)
Spool valve	translatory motor (spring-opposed)	13–16(a)
Spool valve	translatory motor (not spring-opposed)	13–16(b)

TABLE 14-6 TRANSMISSION DATA—HYDRAULIC PUMPS

Item	Information	Units	Transmission I	Transmission II	Transmission III
1	approx power at speed at pressure at stroke	hp rpm psi deg	4.5 4500 1250 25	2.2 3500 1200 10	8.5 1750 2000 30
2	weight dry	lb	27, double pump unit with auxiliaries (approx)	16.6, complete transmission (measured)	30 (catalog value)
3	maximum rated pressure†	psi	1000	1200	2000
4	maximum displacement at stroke	in.^{3}/radian deg	0.0509 25	0.0332 10	0.153 30
5	volume subject to high pressure	in.^{3}	0.62	only total oil vol known	1.71
6	minimum cylinder clearance	in.^{3}	0.0117 (estimated)	0.0399 (estimated)	0.019 (estimated)
7	total leakage††	in.^{3}/sec	0.22 at 1000 psi	only over-all leakage known	0.4 at 1000 psi 0.8 at 2000 psi

†Maximum rated pressure for continuous operation for normal life.

††Leakage at operating speed with fluid described in Table 14-8. All leakage data is approximate. Manufacturer's information used except for Transmission II where measurements on unit were made. All leakage data compensated for viscosity and temperature deviations. The breakdown of total leakage into direct and differential for Transmission 1 is estimated.

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TABLE 14-7 TRANSMISSION DATA — HYDRAULIC MOTORS

Item	Information	Units	Transmission I	Transmission II	Transmission III
1	approx power at speed at pressure at stroke	hp rpm psi deg	4.3 3600 1250 30	6.5 3600 1200 30	26 2200 200 30
2	weight dry	lb	2.5 (measured)	see pump	30 (catalog value)
3	maximum rated pressure†	psi	1000	unknown	2000
4	maximum rated speed†	rpm	3600	unknown	2200
5	displacement at stroke	in.^{3}/radian deg	0.0602 30	0.0954 30	0.384 30
6	volume subject to high pressure	in.^{3}	0.22	only total oil vol measured	2.52
7	total leakage††	in.^{3}/sec	0.22 at 1000 psi	only over-all leakage known	0.6 at 1000 psi 1.2 at 2000 psi
8	inertia	$\frac{\text{in.-lb-sec}^2}{\text{radian}}$	0.000669 (measured)	0.00144 (estimated)	0.0147 (estimated)

†Maximum rated speed and pressure for continuous operation for normal life.

††Leakage at operating speed with fluid described in Table 14-8. All leakage data is approximate. Manufacturer's information used except for Transmission II where measurements on unit were made. All leakage data compensated for viscosity and temperature deviations. The breakdown of total leakage into direct and differential for Transmission I is estimated.

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TABLE 14-8 TRANSMISSION DATA — FLUID

Item	Information	Units	Transmission I	Transmission II	Transmission III
1	viscosity	centistokes	7.5 at 158°F	10 at 150°F	23 at 140°F
2	specific gravity		0.84	0.84 (approx)	0.84 (approx)
3	compressibility†	in.^{2}/lb	4.0×10^{-6} at 0-2000 psi	4.0×10^{-6} at 0-2000 psi	4.0×10^{-6} at 0-2000 psi

Adapted by permission from the *Journal* of The *Franklin Institute*. Volume 243, No. 6, June. 1947. from article entitled 'Hydraulic Variable-Speed Transmissions as Servomotors', by C. C. Newton, Jr.

TABLE 14-9 TRANSMISSION DATA — GENERAL

Item	Information	Units	Transmission I	Transmission II	Transmission III
1	pump operating speed	rpm	4500	3500	3500
2	ambient temperature	"F	77	75 (approx)	68 (approx)
3	fluid temperature	"F	158	150 (approx)	140 (approx)
4	replenishing pressure	psi	50 (gauge)	35 (gauge)	50 (gauge, approx)
5	load inertia	in.-lb-sec^2	0.0159	0.0648	1.80
6	load damping	in.-lb-sec	negligible	negligible	negligible
7	total output-shaft inertia	in.-lb-sec^2	0.0166	0.0662	1.81
8	total damping	in.-lb-sec	0.0093	unknown	unknown
9	total volume of high-pressure oil†	in."	3.84	1.65	31.0
10	over-all leakage constant	in.^{5}/lb-sec	4.4×10^{-4}	3.67×10^{-4}	1.0×10^{-3}
11	approx max power	hp	4.3	2.2	8.5

†Slight differences in volumes (or lengths) of conduits, fittings, etc., communicating with the two pump connections have been averaged so that the data represents the mean volume (or length) of a high-pressure oil path.

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TABLE 14-10 COMPARISON OF PREDICTED AND MEASURED PARAMETERS

Parameter		Transmission I	Transmission II	Transmission III
$1/\omega_n$ (sec/rad)	predicted measured	5.27×10^{-2} 8.50×10^{-2}	4.35×10^{-2} 7.27×10^{-2}	2.45×10^{-1} 2.84×10^{-1}
ζ	predicted measured	0.153 1.14	0.226 0.234	0.173 0.332

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The effect of air entrainment is discussed later in this chapter. Pressure-control valves lessen the compressibility effect since they, in effect, provide a pressure at the piston independent of the flow to the piston which includes, of course, compressibility flow. If pressure-control valves are to be successful in this function, they must behave as ideal pressure sources, dynamically as well as statically.

14-4.5 APPROXIMATE DYNAMIC BEHAVIOR OF HYDRAULIC TRANSMISSION

It is usually possible to neglect line inertance, although this may not be the case if small diameter lines are used. (For the inertance formula, see Par. **13-6.27.**) If the load has negligible damping and line inertance is neglected, then

$$\omega(s) = \frac{\dfrac{d_P}{D_M} X(s) - \dfrac{L_{MP}}{D_M^2}\left(1 + \dfrac{C_{MP}}{L_{MP}} s\right) T(s)}{1 + 2\zeta \dfrac{s}{\omega_n} + \left(\dfrac{s}{\omega_n}\right)^2} \tag{14-86}$$

where

$$\omega_n = D_M \sqrt{\frac{B}{JV_{MP}}}$$

$$\zeta = \frac{1}{2} \frac{L}{D_M} \sqrt{\frac{BJ}{V_{MP}}}$$

L_{MP} = total leakage : pump and motor

C_{MP} = total compressibility : pump, motor, and line

V_{MP} = total oil volume: pump, motor, and line

Typical values of ζ and ω_n are about **0.3** and **20** rad/sec, respectively, if the load inertia is substantially greater than the motor inertia and air entrainment is absent. (See Table **14-10** for typical performance data.)

14-4.6 PARAMETER EVALUATION FOR ROTARY MOTOR

The essential parameters are leakage coefficient L_m, compressibility coefficient C_m, displacement coefficient D_m, and inertia J_m.

Leakage can be determined from static speed-torque curves once D_m is known. From Eq. **(14-81),** the slope of the speed-torque curve is

$$\frac{\partial \omega}{\partial T} = \frac{L_m}{D_m^2} \tag{14-87}$$

or, solving **Eq. (14-87)** for L_m

$$L_m = D_m^2 \frac{\partial \omega}{\partial T} \tag{14-88}$$

The value of D_m is usually available from the manufacturer. If unavailable, it may be measured by causing the motor to act as a pump with zero terminal pressure drop. The flow Q at motor speed ω is measured. Then

$$D_m = \frac{Q}{\omega} \tag{14-89}$$

If facilities for driving the motor are unavailable, measure the motor torque T and the pressure drop P across the motor while running the motor at a convenient constant speed. Then

$$D_m = \frac{T}{P} \tag{14-90}$$

Leakage is a function of temperature, since it is inversely proportional to the viscosity, whose temperature dependence is shown in Par. 20-2.

Compressibility depends upon the motor oil volume being compressed. This volume equals one half the displacement per revolution, d_m, for the motor alone. The volume of connecting lines must be taken into account.

Thus

$$C_m = \frac{d_m}{2\beta} \tag{14-91}$$

where β is the bulk modulus of the fluid used. The effect of entrained air can be substantial, causing ζ to increase by a factor of **5** to 10. (See Par. **13-6.27** for discussion.)

The motor inertia is usually available from the manufacturer. If not, it may be measured by a retardation run as described in Par. **14-2.**

The load and gear inertia are reflected to the motor shaft. The numerical value of the reflected inertia at the motor shaft is obtained by dividing the load and gear inertia by the square of the gear ratio between the motor and the point where the load or gear inertia occurs.

14-4.7 PROBLEMS ENCOUNTERED WITH HYDRAULIC MOTORS

The problems encountered with hydraulic motors are similar to those discussed under hydraulic amplifiers. Rotary-motor commutation causes a power loss proportional to the speed of rotation. The power is dependent upon position and gives rise to pressure pulsations observed as noise.[24]

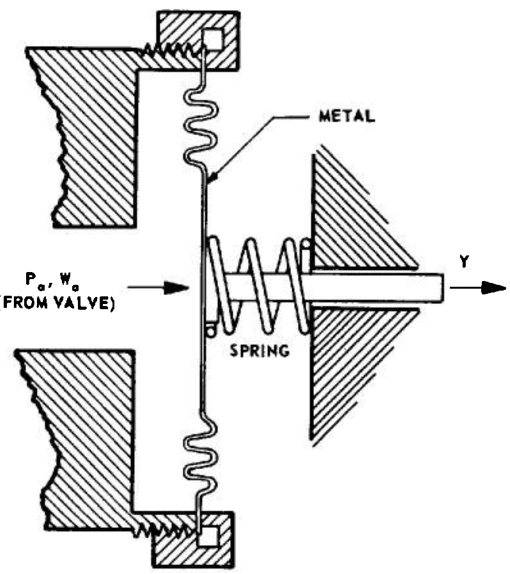
Translatory motor or piston-end seals present a problem. High packing pressure to prevent leakage to the atmosphere is accompanied by coulomb friction which appears as a nonlinear component of F_L. Coulomb friction gives rise to a dead spot, since **a** certain pressure must be developed to overcome this friction before any motion can occur. If stiction occurs (high breakaway friction), this may cause unstable operation. Analysis using describing-function or phase-plane methods may be carried out.

The nonlinear effect of piston compressibility has been cited. Flexible hose, although convenient, is more elastic than steel tubing and may contribute to the total compressibility coefficient (see Par. **13-6** for formulae). Long lines between the motor and the pump should be avoided to prevent standing waves of pressure. At the very least, the large V_{MP} resulting causes both ζ and ω to decrease. Such effects are generally undesirable.

14-5 PNEUMATIC MOTORS

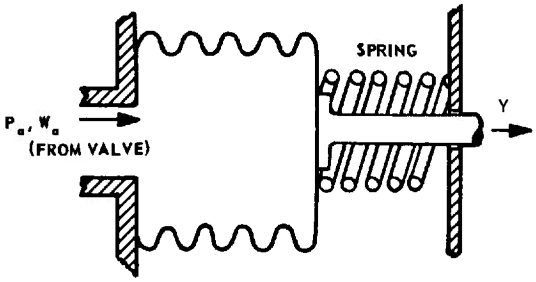
14-5.1 PRINCIPAL TYPES

A pneumatic motor is an air-operated device wherein a change in input air pressure produces a mechanical output motion. Pneumatic motors are of two basic kinds: translational and rotary. Current applications make wide use of the translational or linear motor. The principal types of translational motors are the capsule, bellows, diaphragm, ram, and piston. These types are shown functionally in Figs. **14-31** through **14-35.** In these figures, P_a, and W_a, are input load pressure and flow respectively controlled by a pneumatic valve, and Y is the motor-shaft displacement resulting from an input to the pneumatic valve. All of the motors illustrated, except the piston, are single acting; i.e., air pressure is applied on one side only, and motion is opposed (or assisted) by **a** spring. Since only one source of pressure is needed, the single-acting motor can be operated from a three-way valve. The piston is operated with air at each end (with or without springs) and hence requires the use of a four-way valve.



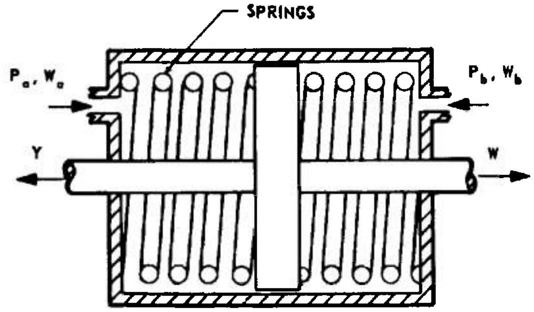
NOTE: SPRING MAY BE OMITTED

Fig. 14-31 Typical pneumatic capsule.



NOTE: SPRING MAY BE OMITTED

Fig. 74-32 Typical pneumatic bellows.



NOTE: SPRINGS MAY BE OMITTED

Fig. 74-35 Typical pneumatic piston.

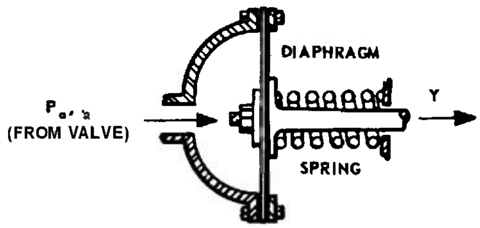


Fig. 74-33 Typical pneumatic diaphragm motor.

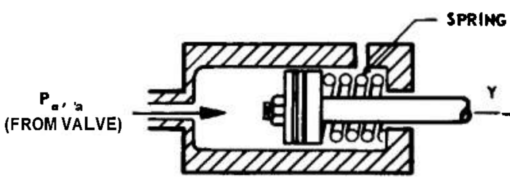


Fig. 74-34 Typical pneumatic ram.

Capsules and bellows are used at the lower force levels. For highest sensitivity, the opposing spring is eliminated and the spring effect of the capsule or bellows provides the only opposition to the applied pressure. At the higher force levels, diaphragms, rams, and pistons are used.

14-5.2 STATIC BEHAVIOR

The static behavior of pneumatic motors is unaffected by the compressibility of air. The static behavior is identical with the static performance of translational hydraulic motors (see Par. **14-4.3**).

14-5.3 DYNAMIC BEHAVIOR

The dynamic behavior of three-way valves combined with various translational motors is discussed in Par. **13-7.10.** For small displacements about an operating point where the load (motor) pressure has some average value, or for small step changes, it is possible to linearize the valve and motor behavior.

The compressibility effect associated with the volume of air within the motor is nonlinear. It can be represented by the compressibility flow W_c defined as

$$W_c = C \frac{dP}{dt} \qquad (14\text{-}92)$$

where $C = \rho g \left(\frac{V}{P}\right)$ or $\rho g \left(\frac{V}{1.4P}\right)$, as explained in Par. **13-7.13** and the symbols correspond to those used in Par. 13-7. The compressed volume V varies with output displacement, and the pressure P varies with loading and input. The result is that **Eq. (14-92)** is nonlinear. For investigation of transient behavior associated with large changes, Eq. **(14-92)** together with the orifice equations should be represented on an analog computer.

Stabilization of the valve-motor combination for the case of ram and piston motors can be aided by the use of tanks connected to each end of the cylinder by means of capillary tubes. This method has been reported by Shearer[25] and is shown in Fig. **14-36.**

14-5.4 DIFFICULTIES ENCOUNTERED WITH PNEUMATIC MOTORS

The following difficulties are encountered in the use of pneumatic motors:

(a) Capsules and bellows used without opposing springs are subject to the same limitations as linear springs, namely saturation (nonlinear performance) and hysteresis. In addition, their spring constant is temperature sensitive. All of these effects are more serious in open-loop devices, such as measuring instruments, than in closed-loop positioning systems.

(b) Rams and pistons exhibit air leakage to atmosphere. The principal disadvantage of this leakage is the inherent inefficiency involved. Various packing and sealing schemes are used to limit air leakage, but all produce a load force tending to oppose ram or piston motion. An approximation of the force required to prevent air leakage may be found as follows:

Consider a piston rod with the elementary packing gland shown functionally in Fig. **14-37** and assume the packing to be a fluid. In order to prevent leakage, the gland must be tightened until the pressure in the packing is at least equal to or greater than $(P_2 - P_1)$. The packing will exert this same pressure on the shaft. The packing-material area that bears on the shaft is πdL. The bearing force F is

$$F = (P_2 - P_1)\pi dL\, c_f \qquad (14\text{-}93)$$

where

F = packing-gland force opposing ram or piston motion, in lb

P_2 = cylinder pressure, in psi

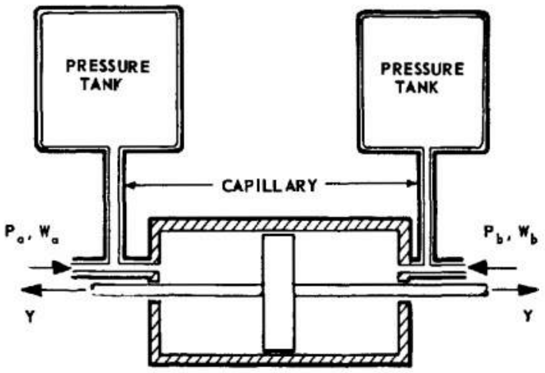


Fig. 74-36 Typical piston and cylinder with stabilizing capillaries and tanks.

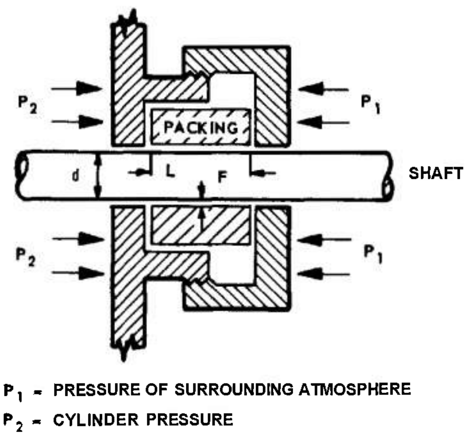


Fig. 14-37 Simplified packing gland.

P_1 = pressure of surrounding atmosphere, in psi

d = diameter of shaft, in in.

L = length of packing along axis of shaft, in in.

c_f = coefficient of friction of packing material, either sliding or static (values of c_f can be found in a standard mechanical engineers' handbook or from manufacturers' data)

(Teflon and teflon-asbestos mixtures are good packing materials. They provide dimensional and thermal stability and have a low coefficient of friction — about **0.1** as compared with **0.4** to **0.6** for many common packing materials.)

(c) The problem of maintaining seals is more difficult with air than with oil because air is a poorer lubricant and is more difficult to contain.

(d) Manufacturing imperfections can result in rams and pistons being out of round, crowned, or tilted. All of these defects must be guarded against as they tend to cause excessive leakage or friction or both.

(e) Differential thermal expansion may cause difficulty if it occurs between parts having close tolerances.

14-6 MAGNETIC-PARTICLE CLUTCHES

14-6.1 DESCRIPTION

A magnetic-particle clutch uses a dry or liquid mixture of powdered iron particles and one or more other materials to transfer the torque from the driven member of the clutch to the load member. Silicon oil is often used as a vehicle for the particles. The particles are magnetized by an electromagnet coil that is usually imbedded in the load member (stator) of the clutch. The electromagnet coil is energized by a d-c control signal. When magnetized, the particles adhere to each other, forming a bond between the driven member (rotor) and the load member (stator) of the clutch. Examples of magnetic-particle clutches are shown in Figs. **14-38** through **14-41.**

14-6.2 METHODS OF USE

Magnetic-particle clutches are reversible and can be used with almost any type of a-c or d-c drive motor. Where a nonreversible motor is used, output rotation in either direction can be achieved by using the dual-clutch arrangement shown in **Fig. 14-41. A** linear relationship between clutch-coil current and the transmitted torque can be obtained by adjusting the bias current and avoiding operation near saturation.

14-6.3 ADVANTAGES

Magnetic-particle clutches have a very small inertia and therefore provide extremely large torque-to-inertia ratios in the order of 500,000 radians/sec^2. Magnetic-particle clutches are small and are adaptable to applications where several controlled motions are required in a restricted space, since one motor can drive many clutches. In addition, a magnetic-particle clutch can be used as a brake, or in a relay servo application, where it provides excellent performance. An additional clutch should be used as a brake in the relay-servo dead zone.

14-6.4 DISADVANTAGES

The disadvantages of the magnetic-particle clutch are as follows:

(a) Relatively short life expectancy when operated continuously at rated load.

(b) Particle separation.

(c) Lubricant breakdown (some magnetic-particle clutches are designed to be operated without additive lubricants).

(d) Particle oxidation (ratings of clutch must be held low enough to retard oxidation of iron particles which results in reduced torque/current gain).

(e) Contamination of bearings by iron particles.

(f) Deterioration of seals (various additives have been tried to overcome tendency for seals to deteriorate).

The clutch is a dissipative device; i.e., control is achieved at the expense of **a** substantial heat loss caused by clutch slippage. For **a**

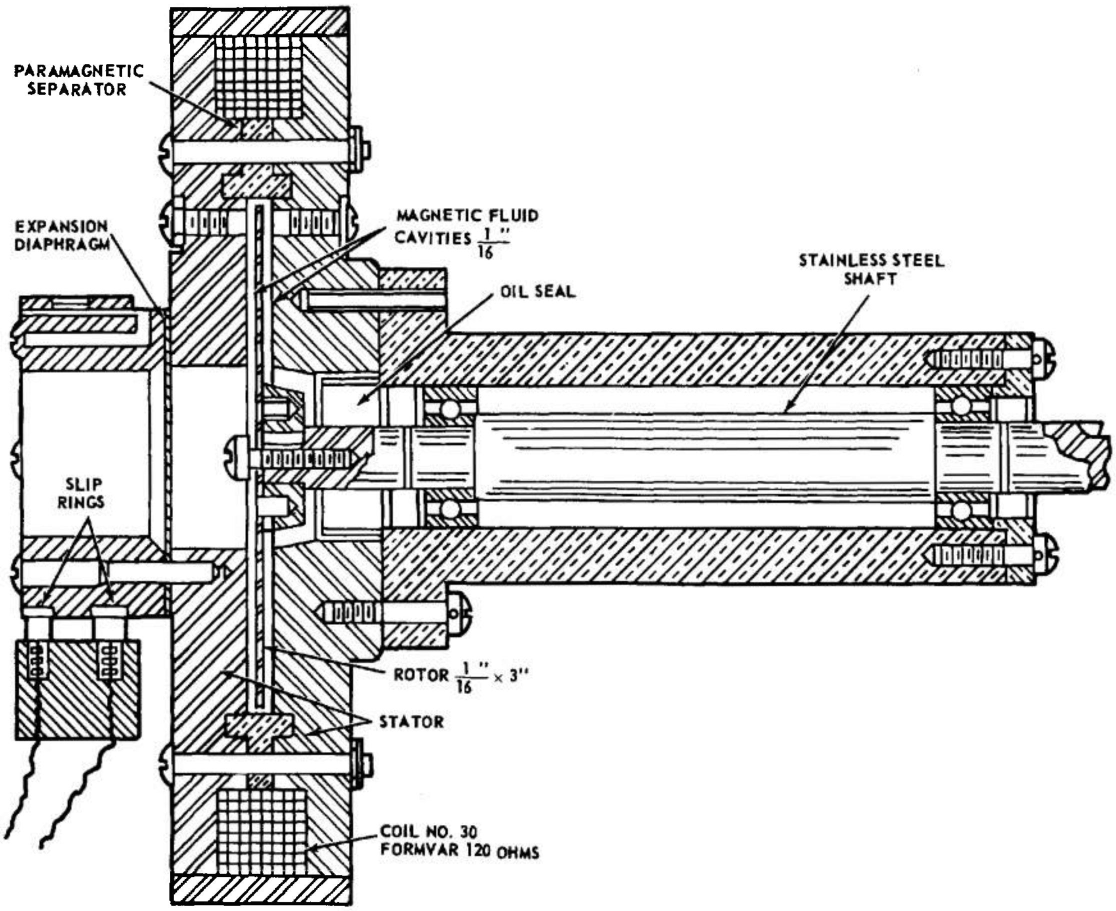


Fig. 14-38 Cross section of magnetic-particle clutch.

By permission from *Electronics*, Vol. 22, November, 1949, from article entitled 'Magnetic Fluid Clutch in Servo Applications', by G. R. Nelson.

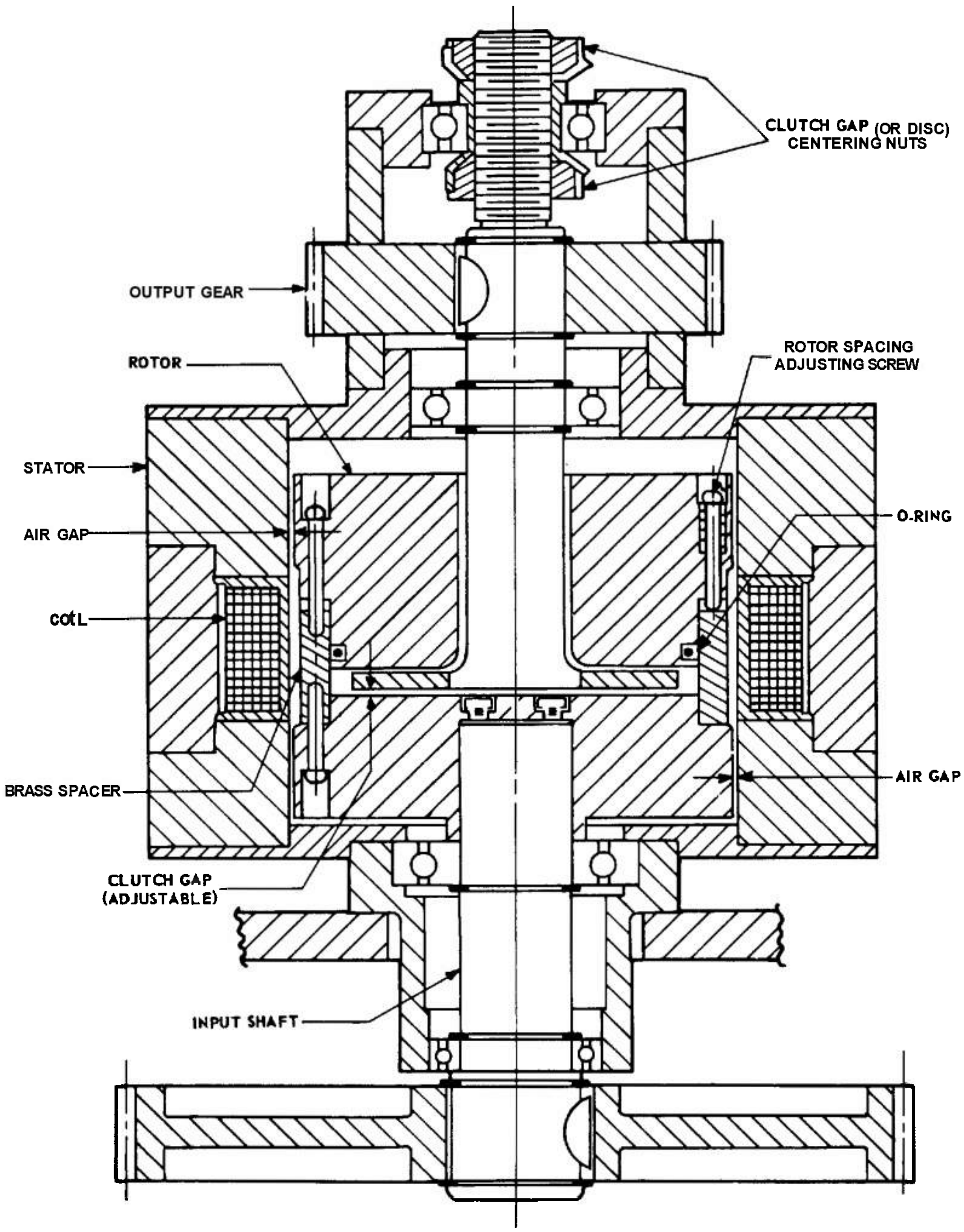


Fig. 14-39 Cross section of magnetic-particle clutch.

By permission from *Transactions of the AIEE*, Volume 69, Part I. 1950, from article entitled 'Characteristics of Some Magnetic-Fluid Clutch Servomechanism', by A. J. Parziale and P. D. Tilton.

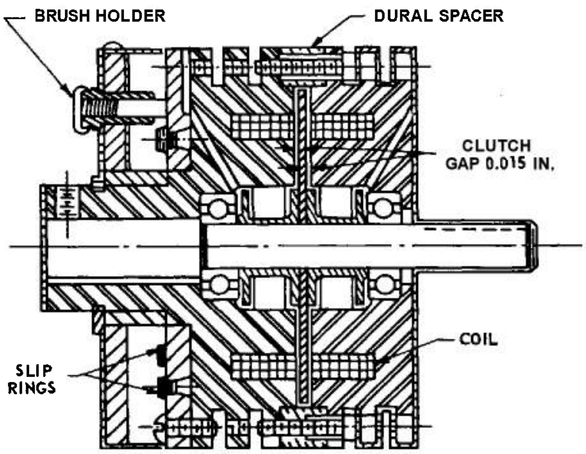


Fig. 14-40 **Cross** *section* **of** *magnetic-particle clutch.*

By permission from *Transactions* of *the AIEE,* Volume 69, Part I, 1950, from article entitled 'Characteristics of Some Magnetic-Fluid Clutch Servomechanisms', by A. J. Parziale and P. D. Tilton.

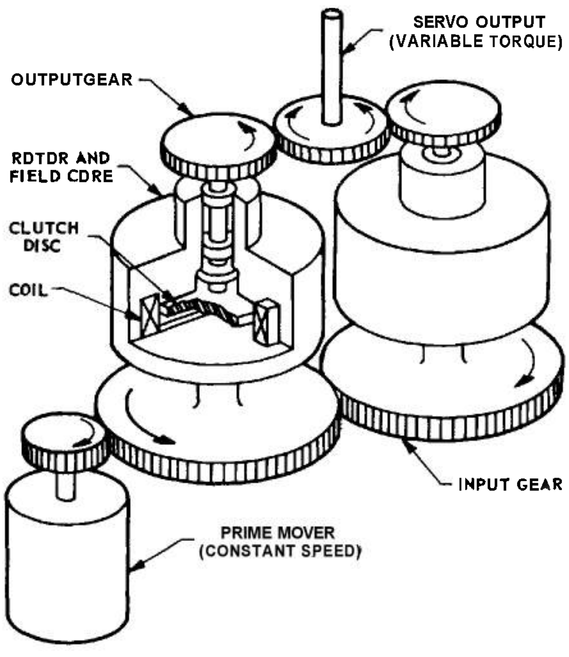


Fig. 14-41 Pictorial diagram of push-pull arrangement of dual-clutch servomechanism.

By permission from *Transactions* of *the AIEE,* Volume 69, Part I, 1960, from article entitled 'Characteristics of Some Magnetic-Fluid Clutch Servomechanisms', by A. J. Parziale and P. D. Tilton.

torque T_L transmitted to the load at half the prime-mover speed, the torque-speed at the prime mover is $T_L\Omega_{PM}$, in the load it is $T_L\Omega_{PM}/2$, and in the clutch it is $T_L\Omega_{PM}/2$, the last amount appearing as heat.

14-6.5 STATIC BEHAVIOR

Figure **14-42** shows the torque-speed curves for a typical magnetic-particle clutch ; Fig. **14-43** shows the torque-versus-coil current characteristic.[26] The use of high-nickel steel, such as Allegheny **4750,** is recommended for minimizing hysteresis.[27]

A list of static properties of a line of commercial clutches is given in Table **14-11.**

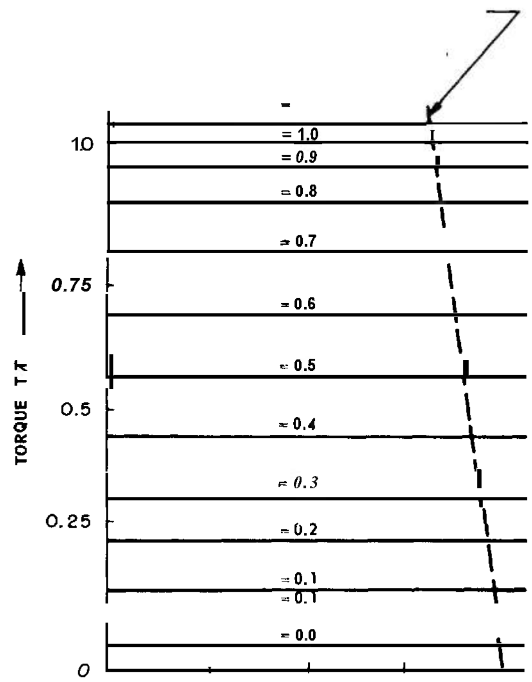
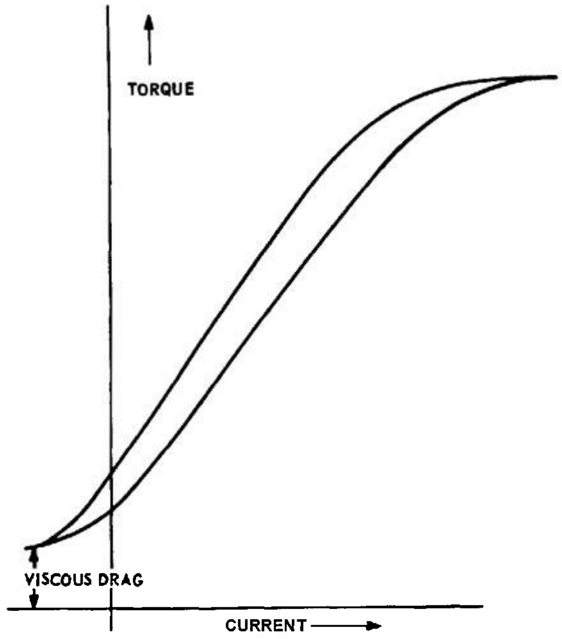


Fig. 14-42 Typical static torque-speed curves **for** *magnetic-particle clutch.*



Fig, 14-43 Torque vs coil current of single magnetic-particle clutch.

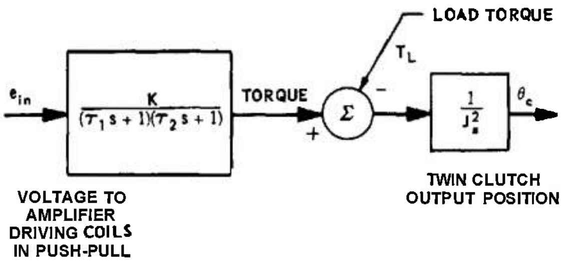


Fig. 14-44 Block diagram for magnetic-particle clutch and preamplifier, inertia load.

14-6.6 DYNAMIC BEHAVIOR

The steady-state torque shown in Fig. 14-42 does not develop suddenly following a change in coil current. Instead, a finite length of time is required. Torque build-up can be approximated by a first-order time lag, the value of which depends upon whether current is increasing or decreasing. Table 14-12 lists typical time constants of some magnetic-particle clutches as determined by several independent workers.[26,27,28]

14-6.7 Voltage-to-Position Relationship

If two magnetic-particle clutches are used to drive an inertia load, as is the normal case, the voltage-to-position relationship will be as shown in block diagram form in Fig. 14-44. The two time constants are generated by the existence of the primary coils, plus a loosely coupled shorting coil formed by the rotor structure. The torque build-up time constant attributed to the shorted turn is due to eddy currents in the clutch structure.[28] This constant is proportional to μ/ρ for the steel used in the rotor. A small time constant can be achieved by using 2.5 percent silicon iron. Magnetic clutches with a total time constant of 12 to 18 milliseconds (from the instant the signal is applied to the grids of the amplifier output stage to the instant actual torque is produced by the clutch) are now claimed by clutch manufacturers. The existence of the double integration indicates the possibility of achieving an infinite velocity constant and also the necessity for providing stabilization ;e.g., a tachometer or a cascade lead network.

14-6.8 LIFE EXPECTANCY

Manufacturers' claims and experiments by independent workers[28,29] indicate that a life from several hundred to a thousand hours can be expected. Table 14-13 is a summary of results obtained with different magnetic-particle mixtures.[29]

TABLE 14-11 PROPERTIES OF A TYPICAL LINE OF MAGNETIC-PARTICLE CLUTCHES

Clutch No.	Torque Constant (lb-in./amp-turn)	Production Unit Coil Turns (wire size)	Coil Resistance (ohms)	Normal Operating Torque (in.-lb)	Maximum Torque (in.-lb)
1	0.016	1200 (36)	80	0-0.5	1.5
2	0.04	3500 (39)	700	0-2	3.75
3	0.08	15,000 (43)	10,000	0-8	12
4	0.3	3000 (38)	400	0-2	4
5	0.5	10,000 (34)	2000	0-120	160
6	2.0	1650 (25)	80	0-1000	2000

Clutch No.	Maximum Rec. Speed (rpm)	Torque-Inertia Ratio (clutch only) (rad/sec²)	Excitation Power for Max. Torque (watts)	Max. Allowable Heat Dissipation Due to Slippage (watts)
1	2000	350,000	0.5	5
2	2000	440,000	0.7	10
3	1500	200,000	1	15
4	2000	440,000	0.7	10
5	1500	200,000	10	60
6	1000	20,000	13	500

Clutch No.	Drag Torque At Zero Excitation (in.-lb)	Inertia of Clutch Output Shaft (lb-in.-sec²)	Weight of Clutch (lb)
1	0.2	4.5×10^{-6}	0.34
2	0.2	9×10^{-6}	0.45
3	0.2	5.7×10^{-5}	0.95
4	0.2	9×10^{-6}	10
5	1.5	8.5×10^{-4}	7.5
6	20	0.1	80

TABLE 14-12 TYPICAL TIME CONSTANTS

Source of Data	Clutch	Time Constant (millisec)
Reference (26)	clutch A, increasing current	120
	clutch A, decreasing current	80
	clutch B, increasing current	40
	clutch B, decreasing current	20
Reference (27)	increasing current to 1/3 rated	20*
	increasing current to 1/2 rated	15*
	increasing current to full rated	7*
Reference (28)	soft-iron core of variable size	40-70

*Refers to T_D, and equivalent dead time.(27)

TABLE 14-13 SUMMARY OF CLUTCHES USING VARIOUS MAGNETIC-PARTICLE MIXTURES

Mixture	Cake Forma-tion	Clutching Action		Torque,** (oz-in.)			Duration? of Run (hr)	Remarks
		Initial	Final	Initial	Average	Final		
Iron and acetylene black	none	poor	poor	156	95	171	41	Mixture packed causing sintering.
Iron and boron nitride	soft	good	good	153	110	113	210	
Iron and cadmium sulfide	none	poor	poor	120	95	72	30	Loss of mixture. Rough operation.
Iron and copper tin phthalocyanine	soft	poor	poor	174	70	30	97	Loss of mixture. Rough operation.
Iron and graphite	none	good	good	138	95	48	48	Mixture did not remain in gap.
Iron and hi sil (silicon dioxide)	soft	good	good	107	108	116	228	Uniform and smooth torque.
Iron and lamp black	none	poor	poor	146	95	55.5	122	Rough clutching, gradual torque loss.
Iron and lead oxide	none	poor	poor	156	60	27	88	Continued torque loss.
Iron and molybdenum sulfide	soft	good	good	125	94	103	193	
Iron and oil	hard	good	poor	120	102	78	12.5	Mixture was dry and packed when clutch was disassembled.
Iron and santocel (silicon dioxide)	none	good	good	133	90	77	234	Gradual torque loss stabilized at low value.
Iron and silicon dioxide	none	good	good	141	130	110	327	
Iron silver coated	none	good	poor	146	110	100	197	Coated by Brashear method. Coating wore off during operation.
Iron and sodium nitrite	hard	poor		168				Extreme initial packing of mixture.

Iron sodium nitrite and molybdenum sulfide	soft	poor	poor	123	63	3	18	Loss of mixture.
Iron sodium nitrite and zinc oxide	none	poor	poor	192		192	1	Extreme packing.
Iron and spheron number 6 (carbon black)	none	poor	poor	117	55	52	200	Clutching action rough.
Iron and stearic acid	hard	poor	poor	132	40	18	90	Additive melted, adhering to clutch surfaces.
Iron and talc	hard	poor	poor	103	55	54	37	Rough clutching action.
Iron and vapor phase inhibitor 260 (oxidation inhibitor)	hard	poor	poor	120	88	56	3	Loss of mixture. Volatilization of vapor phase inhibitor.
Iron and zinc oxide	none	good	good	133	118	123	429	
Iron and zinc stearate	soft	poor	poor	165	35	8	65	Additive melted, adhering to clutch surfaces.
Iron and zinc sulfide	soft	good	good	142	105	77	204	Continued torque loss.

**Measured at 0.8 ampere. †36 oz-in. torque at 1700 rpm.

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CHAPTER 15

MECHANBCAL AUXILIARIES USED IN CONTROLLERS*

15-1 GEAR TRAINS

15-1.1 PURPOSE

Gear trains are most frequently used in controllers to transmit motion from the servomotor to the load with an appropriate change in speed. The transmitted power can range from a small fraction of a horsepower for instrument-type servomechanisms to several horsepower for large gun or launcher drives.

Gear trains are also used to transmit input, feedback, and error-signal information, when an advantage will be gained by this use of gear trains, as in a completely mechanical-hydraulic servomechanism. In most cases, such gear trains transmit very small amounts of power.

The discussion of gear trains will be confined to gears with involute tooth form, because this tooth form is used almost exclusively for engineering purposes. Other tooth forms which result in rolling action between meshing teeth have been made, but these special tooth forms are confined to very special applications, and cutters for their manufacture are not normally available.

15-1.2 DEFINITIONS

The most important parts of a gear are defined in the following subparagraphs (Fig. **15-1**) :

*By J. O Silvey

(a) The *p tch line* **or** *pitch circle* of a gear is that circle which has the same tangential velocity as the pitch circle of a second gear, when the two gears are properly meshed and rotated together. The pitch circle is located slightly closer to the tops of the gear teeth than it is to the bottoms of the spaces between teeth.

(b) The *pitch diameter* of a gear is the diameter of the pitch circle. In bevel gears, it is the largest diameter of the pitch cone.

(c) The *circular pitch* is the distance (normally expressed in inches) on the circumference of the pitch circle between corresponding points of adjacent teeth. Circular pitch is' equal to π times the pitch diameter divided by the number of teeth on the gear.

(d) The *diametric pitch* is equal to the number of teeth on the gear divided by the pitch diameter.

(e) The *addendum* is the radial distance between the pitch circle and the tops of the teeth.

(f) The *dedendum* is the radial distance between the pitch circle and the bottoms of the spaces between the gear teeth. The dedendum of a gear is always slightly larger than the addendum to provide clearance for the cutter of generated gear teeth, and for the tops of the teeth of a mating gear.

(g) The *face width* is the length of the teeth along the axis of the gear.

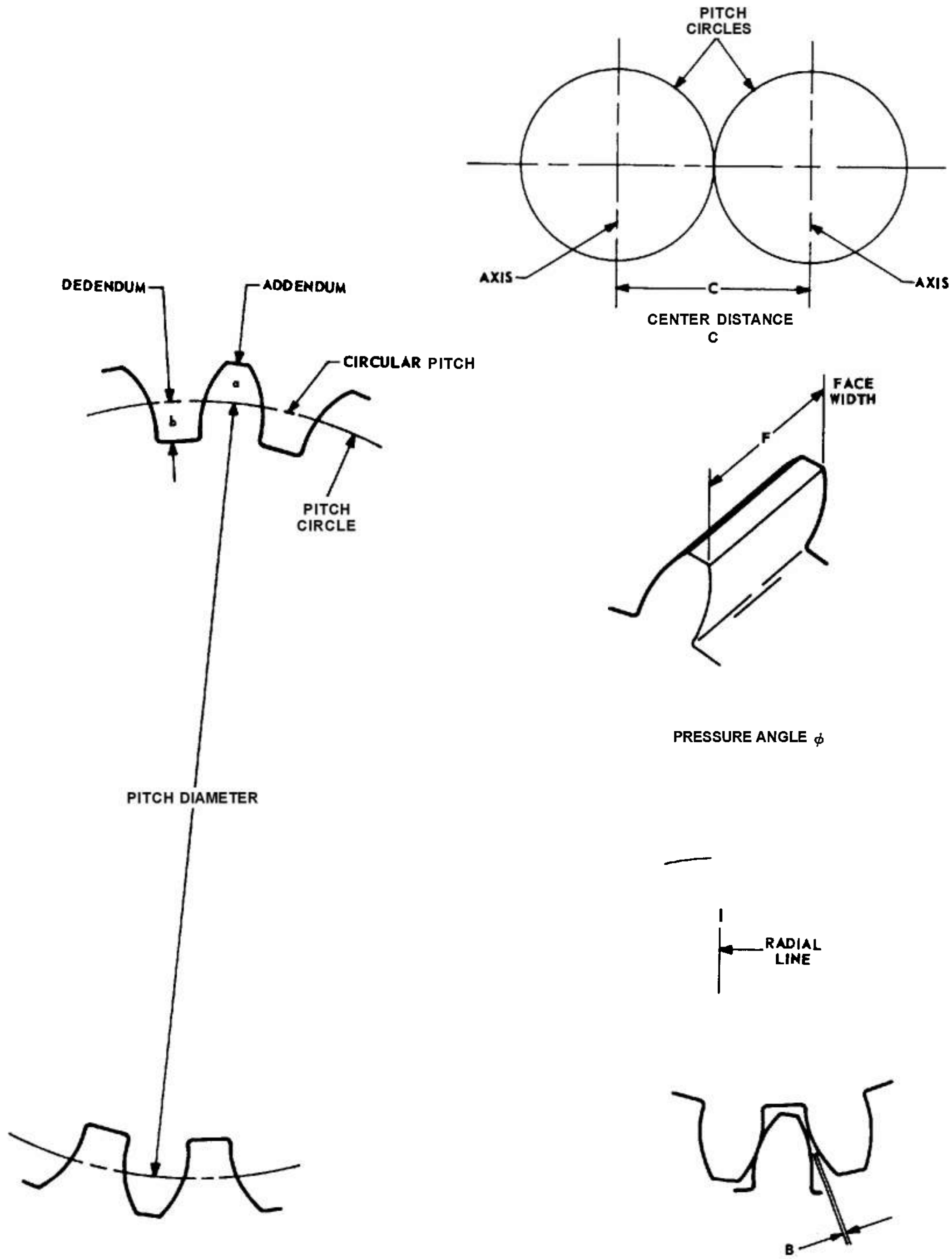

Fig. 15-1 Most important parts of a gear.

(h) The ***pressure angle*** is the angle between a tangent to the tooth profile at the pitch circle and a radius to this pitch point. Pressure angle is also the angle of inclination of the sides of the teeth in the basic rack.

(i) The ***center distance*** is the shortest distance between gear axes of a pair of mating gears. For gear pairs with parallel shafts, the center distance is the distance between axes.

(j) The ***backlash*** is the play between mating teeth or the shortest distance between the nondriving surfaces of adjacent teeth.

(k) The ***gear ratio*** of a pair of mating gears is the ratio of the number of teeth on the gear divided by the number of teeth on the pinion.

15-1.3 GEAR TYPES

The following five types of gears are in common use (Fig. 15-2) :

(a) ***Spur gears*** (Fig. 15-2A) are used more in control work than any other type gear. They are the least expensive to manufacture, have high mechanical efficiency, and, except where the gear ratio is high, can be operated to either increase or decrease speed. Spur gear teeth are parallel to the gear axis, producing negligible thrust along their shafts when driven. Four spur-gear systems will satisfy most requirements. They are classified by pressure angle and tooth form, and have been standardized by the American Standards Association.[1] These four systems are as follows :

(1) ***14-1/2-deg composite.*** Gears of this system are machined by milling one tooth space at a time, a series of eight cutters being required to cover the range between a 12-tooth gear and rack. Gears of the other three systems are machined by a hob or shaper.

(2) ***14-1/2-deg full-depth tooth.*** Gears of this system are usually produced by hobbing and have the same strength as gears of the composite system. However, full-depth pinions of standard proportions but with less than 32 teeth will have the teeth undercut, and the undercut may become excessive for smooth operation of gears with less than 22 teeth.

(3) ***20-deg full-depth tooth.*** This system produces stronger gear teeth than the 14-1/2 degree system. Teeth are produced by hobbing or shaping and are not undercut in gears with more than 18 teeth. Undercut may be excessive in gears with less than 14 teeth.

(4) ***20-deg stub tooth.*** This system produces the strongest teeth, which are hobbed or shaped. It also produces pinions with the least number of teeth without undercut. By increasing the diameter of the pinion above that specified by standard gear proportions, the undercut of small pinions can be reduced. The center distance of the gear pair must be increased to accommodate the larger-diameter pinion.

(b) ***Helical gears.*** The teeth of helical gears (Fig. 15-2B) are cut in the form of a helix on the pitch cylinder, the helix angle variable between widely separated limits. Some end thrust is developed when helical gears are driven. Helical gears operate quieter, and produce smoother action than spur gears of equal precision. However, helical gears are more expensive to produce. When helical gears are mounted with their shafts at right angles to each other, they are called spiral gears, or crossed helical gears. Two side-by-side helical gears, with equal pitch diameters and equal but opposite helical angles, comprise a herringbone gear. Such a gear can be cut in a single piece of stock, or it can comprise two separate gears mounted together (side-by-side as a unit) on the same shaft. Herringbone gears transmit no thrust to their bearings because the thrusts of the two helical gears are equal but act in opposite directions.

(c) ***Internal gears.*** The primary uses for internal gears (Fig. 15-2C) are as clutches, splines, and as components of planetary-gear systems.

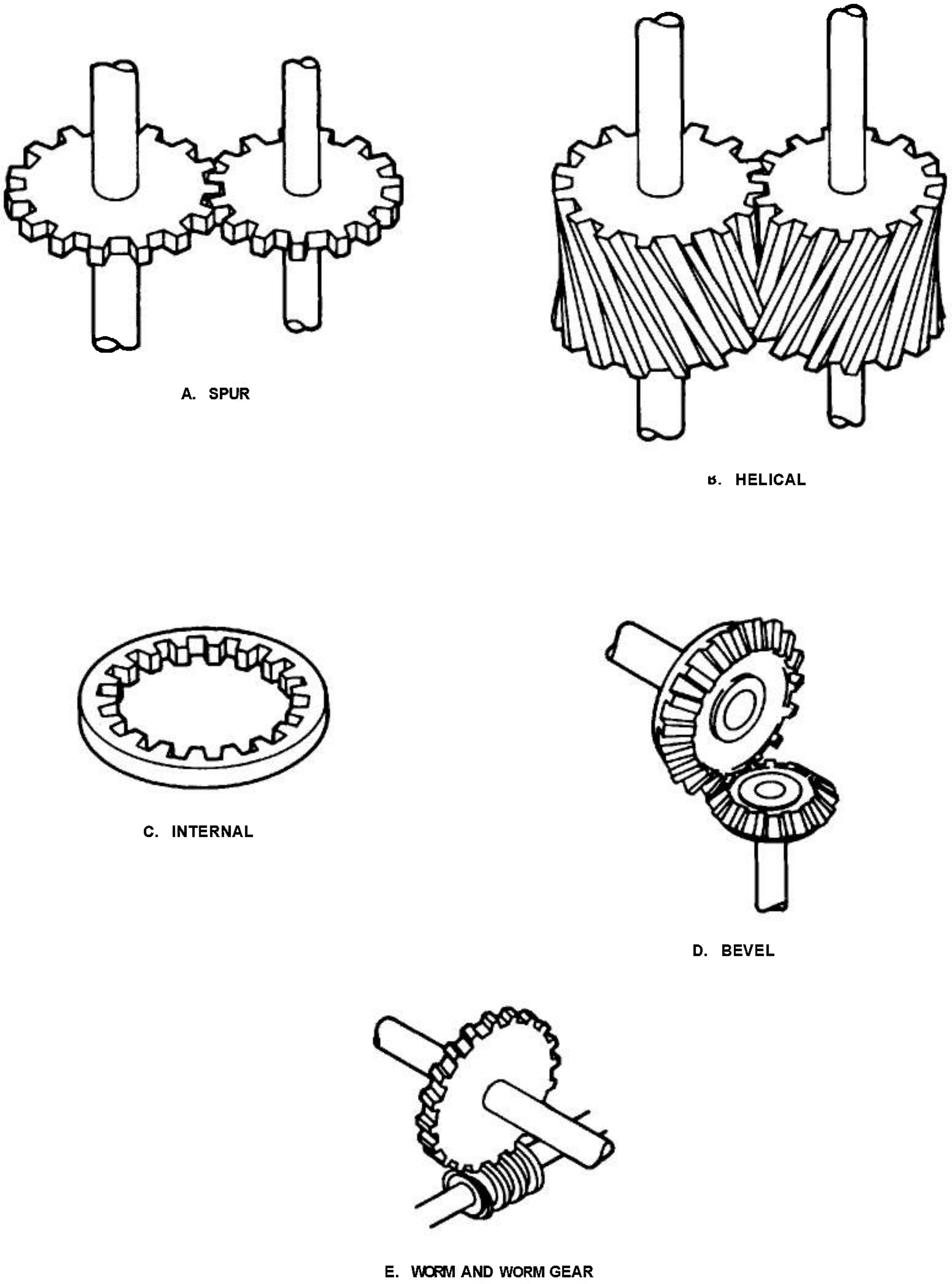

Fig. 15-2 Types of gears.

(d) *Bevel geum.* These gears (Fig. **15-2D**) are mounted with their shafts at an angle to each other. When the gear ratio is unity, and the shafts are at an angle of 90°, bevel gears are called mitre gears. The teeth of bevel gears may be straight or helical, either type always producing end thrust when driven. To change the gear ratio of bevel gears, both gears of a pair must be changed.

(e) *Worms and worm gears.* Worms and worm gears (Fig. 15-2E) are mounted with their shafts at right angles to each other. The worm may have a single thread or several threads. Worm-and-gear systems are often used to obtain a large speed reduction in a small space, but the mechanical efficiency is low because the action between teeth is of a sliding nature, instead of rolling. Normally, worm drives can only be driven by rotating the worm, not by rotating the gear, because of the low helix angle of the worm.

15-1.4 DESIGN FUNDAMENTALS[3]

The subsequent paragraphs cover in some detail the following design fundamentals: backlash, dynamic load, gear accuracy, beam strength of teeth, Y factor, margin of safety, and limit load for wear. Equations and tables are supplied to supplement the descriptive text. The presented method of computing permissible gear load is adequate for the great majority of gearing problems encountered in control systems. If extremely small size or high speed is required in a gear train, other methods, such as those employed by Almen,[27] McFarland,[28] and others should be used. In general, the design of such gears requires a great deal of experience and should be done by a gear expert.

15-1.5 Backlash

Backlash between meshed gears is the gap or clearance required between nondriving surfaces of adjacent teeth to prevent binding. High-speed gearing requires more backlash than low-speed gearing. If appreciable heat is generated by the gears, enough additional backlash must be provided to permit thermal expansion of the gear material. In any gear-train design, the backlash between gear pairs must also be great enough to prevent binding due to eccentricities of the bearings, shafts, and gear-pitch circles. Backlash standards for general-purpose spur gearing have been established by the American Standards Association,[2,3,4] and a method of specifying backlash in fine-pitch gearing has also been adopted.[5]

The required amount of backlash can be obtained by cutting the teeth thinner than the theoretical optimum dimensions. This is achieved by cutting the teeth deeper than normal. In most cases, the teeth of each gear of a mating pair are cut thinner by an amount equal to half the required backlash to retain as much strength as possible in each gear. However, where small pinions are used, all of the backlash should be obtained by cutting the teeth of the mating gear thinner by an amount equal to the total required backlash. The excess depth of cut X to provide the required backlash when both gears are cut deeper is

$$X = \frac{B}{4 \sin \phi} \text{ (for both gears)} \qquad (15\text{-}1)$$

and, when only one gear is cut deeper

$$X = \frac{B}{2 \sin \phi} \text{ (one gear only)} \qquad (15\text{-}2)$$

where

B = amount of backlash, in in.

X = excess depth of cut to provide backlash, in in.

4 = pressure angle of generating tool

15-1.6 Dynamic load

The maximum momentary load set up by the operating or dynamic conditions is called the dynamic load and is given by

$$W_d = \frac{0.05V\,(FC + W)}{0.05V + \sqrt{FC + W}} + W \qquad (15\text{-}3)$$

where

$$W = \frac{33{,}000\,HP}{V} = \text{total applied load, in lb} \qquad (15\text{-}4)$$

W_d = total dynamic load, in lb

V = pitch-line velocity, in fpm

HP = number of horsepower transmitted

C = deformation factor (see Table 15-1)

F = face width of gears, in in.

For planetary and differential gear trains, the velocity of actual tooth engagement must be used to determine the amount of dynamic load. For. materials not listed in Table 15-1, the value of deformation factor C may be determined as follows:

$$C = \frac{0.107e}{\frac{1}{E_1} + \frac{1}{E_2}} \quad \text{(for 14-1/2-deg tooth form)} \tag{15-5}$$

$$C = \frac{0.111e}{\frac{1}{E_1} + \frac{1}{E_2}} \quad \text{(for 20-deg full depth form)} \tag{15-6}$$

$$C = \frac{0.115e}{\frac{1}{E_1} + \frac{1}{E_2}} \quad \text{(for 20-deg stub tooth form)} \tag{15-7}$$

where

e = error in action, in in.

E, and E, = modulus of elasticity of materials

To solve Eqs. (15-5) through (15-7) the error in action e of the gears must be known. Judging from test results, the error in action on well-cut commercial gears ranges from about 0.002 inch on gears of 6 dp (diametric pitch) and finer, to about 0.004 inch on gears of 2 dp. When the gears are cut with great care, these errors are reduced. On ground gears made with extreme care, as well as on cut gears where every effort is made to achieve the utmost accuracy, the errors are

TABLE 15-1 VALUES OF DEFORMATION FACTOR C

Material	Tooth Form (deg)	Error in Action (in.)					
		0.0005	0.001	0.002	0.003	0.004	0.005
Cast iron and cast iron	14-1/2	400	800	1600	2400	3200	4000
Cast iron and steel	14-1/2	550	1100	2200	3300	4400	5500
Steel and steel	14-1/2	800	1600	3200	4800	6400	8000
Cast iron and cast iron	20 (full depth)	415	830	1660	2490	3320	4150
Cast iron and steel	20 (fulldepth)	570	1140	2280	3420	4560	5700
Steel and steel	20 (fulldepth)	830	1660	3320	4980	6640	8300
Cast iron and cast iron	20 (stub)	430	860	1720	2580	3440	4300
Cast iron and steel	20 (stub)	590	1180	2360	3540	4720	5900
Steel and steel	20 (stub)	860	1720	3440	5160	6880	8600

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reduced to even smaller amounts. The foregoing conditions of error in action are listed in Table 15-2 and are designated as Classes 1, 2, and 3.

15-1.7 Gear Accuracy

The noise of gear operation is usually a very good indication of gear accuracy. Table 15-3 gives some idea of the order of accuracy required at different pitch-line velocities, and should serve as a guide for selection of the proper class of gear to meet specified speed conditions. Where extreme quietness of gear operation is required, a higher order of accuracy than shown in the table will be required. However, the values in the table should keep operating noise and the intensity of the dynamic load within reasonable limits.

TABLE 15-2 MAXIMUM ERROR IN ACTION BETWEEN GEARS AS A FUNCTION OF CLASS

Diametric Pitch	Class 1	Class 2	Class 3
1	0.0048	0.0024	0.0012
2	0.0040	0.0020	0.0010
3	0.0032	0.0016	0.0008
4	0.0026	0.0013	0.0007
5	0.0022	0.0011	0.0006
6 and finer	0.0020	0.0010	0.0005

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TABLE 15-3 MAXIMUM ERROR IN ACTION BETWEEN GEARS AS A FUNCTION OF PITCH LINE VELOCITY

V (ft/min)	Error (in.)	V (ft/min)	Error (in.)	V (ft/min)	Error (in.)	V (ft/min)	Error (in.)
250	0.0037	1500	0.0019	2750	0.0010	4500	0.0006
500	0.0032	1750	0.0017	3000	0.0009	5000 and over	0.0005
750	0.0028	2000	0.0015	3250	0.0008		
1000	0.0024	2250	0.0013	3500	0.0007		
1250	0.0021	2500	0.0012	4000	0.0006		

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15-1.8 Beam Strength of Teeth

To calculate the beam strength of the gear teeth, the Lewis equation can be used to determine the safe static strength.

$$W_b = s_t p F y \qquad (15\text{-}8)$$

where

W_b = safe static beam load on teeth, in lb

p = circular pitch, in in.

F = face width, in in.

s_t = safe static bending stress for materials, in psi

y = tooth form factor

Values of y for various tooth forms are given in Table **15-4.** The flexural endurance limit of a gear material will give a satisfactory value for the safe static bending stress s_t. Fatigue tests on steel indicate that this endurance limit follows the Brinnell hardness number quite closely. Endurance limits are listed in Table 15-5. The static beam strength of the gear tooth should always be greater than the dynamic load; thus

$$W_b = 1.25\, W_d \text{ for steady loads} \qquad (15\text{-}9)$$

$$W_b = 1.35\, W_d \text{ for pulsating loads} \qquad (15\text{-}10)$$

$$W_b = 1.50\, W_d \text{ for shock loads} \qquad (15\text{-}11)$$

where

W_b = safe static beam load on teeth, in lb

W_d = total dynamic load, in lb

These values should be modified when found desirable by experience.

15-1.9 limit load for Wear

The limit load for wear depends upon the surface endurance limits of the materials, the radii of the curvature of mating profiles, and upon the relative hardness of the mating surfaces. A harder material in the pinion will cold-work the surface of the softer and more malleable material in the mating gear, and will thus materially increase the surface endurance limit of the gear. The value of the limiting static load for wear should be equal to or greater than the value of the dynamic load, and is given by

$$W_w = DFKQ \qquad (15\text{-}12)$$

where

W_w = limiting static load for wear, in lb

D = pitch diameter of pinion, in in.

F = face width, in in.

K = load-stress factor

Q = ratio factor

For spur gears

$$Q = \frac{2N_g}{N_p + N_g} \qquad (15\text{-}13)$$

For internal gears

$$Q = \frac{2N_g}{N_g - N_p} \qquad (15\text{-}14)$$

$$K = \frac{s_c^2 \sin\phi}{1.400}\left(\frac{1}{E_1} + \frac{1}{E_2}\right) \qquad (15\text{-}15)$$

where

N_g = number of teeth in gear

N_p = number of teeth in pinion

ϕ = pressure angle of gears

s_c = surface endurance limit of material, in psi

E_1 and E_2 = modulus of elasticity of materials

Values of the load-stress factor K are given in Tables 15-6 and 15-7 for various combinations of materials, taking into consideration the cold-working received in operation. The classification "cast iron" also includes the ordinary semi-steel. Some of the high-test and semi-steels, and other special alloys of cast iron, have a greater modulus of elasticity than cast iron. For these materials, the value of K and the value of the dynamic load W_d for such materials must be calculated directly

TABLE 15-4 VALUES OF TOOTH FORM FACTOR *y* FOR VARIOUS TOOTH FORMS

Number of Teeth	14-1/2° Composite and Involute Form	20° Full Depth Involute System	20° Stub Tooth System
12	0.067	0.078	0.099
13	0.071	0.083	0.103
14	0.075	0.088	0.108
15	0.078	0.092	0.111
16	0.081	0.094	0.115
17	0.084	0.096	0.117
18	0.086	0.098	0.120
19	0.088	0.100	0.123
20	0.090	0.102	0.125
21	0.092	0.104	0.127
22	0.093	0.105	0.129
24	0.095	0.107	0.132
26	0.098	0.110	0.135
28	0.100	0.112	0.137
30	0.101	0.114	0.139
34	0.104	0.118	0.142
38	0.106	0.122	0.145
43	0.108	0.126	0.147
50	0.110	0.130	0.151
60	0.113	0.134	0.154
75	0.115	0.138	0.158
100	0.117	0.142	0.161
150	0.119	0.146	0.165
300	0.122	0.150	0.170
Rack	0.124	0.154	0.175

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TABLE 15-5 VALUES OF SAFE STATIC BENDING STRESS s_t (psi)

Material	Brinnell Hardness Number	s_t
Gray iron	160	12,000
Semi-steel	190	18,000
Phos. bronze	100	24,000
Steel	150	36,000
Steel	200	50,000
Steel	240	60,000
Steel	280	70,000
Steel	320	80,000
Steel	360	90,000
Steel	400	100,000

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from their own specific properties. The surface endurance limit of gear material is a measure of the load limit in psi that the material will tolerate before deformation occurs. The values of surface endurance limit s_c for steel appear to vary quite consistently with the Brinnell hardness number up to about 400 Brinnel hardness. Values of s_c for hardness numbers from 150 to 400 are listed in Table 15-6. When the Brinnell hardness number is much higher than 400, the steel does not appear to have a definite endurance limit. The values given in Table 15-7 are suggested for use with steels harder than 400 Brinnell hardness. Table 15-7 also gives values for the load-stress factor K for these harder steels. The load-carrying ability of a pair of metal spur gears, for example, may be limited by either the beam strength of the gear tooth, or by the surface endurance limit of the material. The lower of these two values should be used to establish the load-carrying ability of any given pair of gears.

Spur gears designed to conform to the preceding requirements can be expected to operate successfully with average loads. However, if heavy inertia loads are to be coupled rigidly to the gears, the dynamic load should be calculated in accordance with the method described by Buckingham.[3,8]

Other types of gears such as internal, helical, bevel, and worm gears are used so infrequently in control work that the particulars of their design are not included here. If use of any of these types is contemplated, the follpwing references should be consulted :

(a) Internal gears — Buckingham[3] and Kent[4]

(b) Helical gears — Kent[4] and Buckingham[6]

(c) Bevel gears — Jones[7]

(d) Worm gears — Kent,[4] Buckingham,[6] and Jones

TABLE 15-6 VALUES OF LOAD-STRESS FACTOR *K* FOR VARIOUS MATERIALS

Material in Pinion	Brinnell Number	Material in Gear	Brinnell Number	s_c	*K* (14-1/2°)	*K* (20°)
Steel	150	Steel	150	50,000	30	41
Steel	200	Steel	150	60,000	43	58
Steel	250	Steel	150	70,000	58	79
Steel	200	Steel	200	70,000	58	79
Steel	250	Steel	200	80,000	76	103
Steel	300	Steel	200	90,000	96	131
Steel	250	Steel	250	90,000	96	131
Steel	300	Steel	250	100,000	119	162
Steel	350	Steel	250	110,000	144	196
Steel	300	Steel	300	110,000	144	196
Steel	350	Steel	300	120,000	171	233
Steel	400	Steel	300	125,000	186	254
Steel	350	Steel	350	130,000	201	275
Steel	400	Steel	350	140,000	233	318
Steel	400	Steel	400	150,000	268	366
Steel	150	Cast iron		50,000	44	60
Steel	200	Cast iron		70,000	87	119
Steel	250	Cast iron		90,000	144	196
Steel	150	Ph. bronze		50,000	46	62
Steel	200	Ph. bronze		70,000	91	124
Steel	250	Ph. bronze		85,000	135	204
Cast iron		Cast iron		90,000	193	284

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TABLE 15-7 VALUES OF LOAD-STRESS FACTOR *K* FOR HARDENED STEEL

10,000,000 Repetitions of Stress				20,000,000 Repetitions of Stress			
Brinnell Number	s_c	K (14-1/2°)	K (20°)	Brinnell Number	s_c	K (14-1/2°)	K (20°)
450	188,000	421	575	450	170,000	344	470
500	210,000	525	718	500	190,000	430	588
550	233,000	647	884	550	210,000	525	718
50,000,000 Repetitions of Stress				**100,000,000 Repetitions of Stress**			
450	147,000	257	351	450	132,000	208	284
500	165,000	324	443	500	148,000	261	356
550	182,000	394	544	550	163,000	316	432
600	200,000	476	651	600	179,000	382	522

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15-1.10 NONIDEAL CHARACTERISTICS OF GEARS AND GEAR TRAINS

In many control applications, the nonideal characteristics of gears are of extreme importance. In some applications, for example, the choice of gear trains is actually dictated by these characteristics. The subsequent paragraphs cover the following nonideal characteristics: inaccuracies, friction, inertia, backlash, and compliance.

15-1.11 Inaccuracies

Three types of inaccuracies are present to some extent in all gears.[5] These inaccuracies are:

(a) *Departure of tooth form from the true involute.* The error in action figures given in Table 15-2 indicate the deviation from the true involute form that may be expected in actual production gears. Tooth errors of less than 0.0002 inch at the pitch line are difficult to achieve, and are obtained only by a high degree of accuracy in the machine on which the gears are cut. Often, however, such highly accurate gears are obtained from a routine production run by careful selection. In general, the more-precise profiles are obtained only at increased cost. Most gears for use in control work can be obtained with tooth-profile errors of less than 0.001 inch, and gears of 20 diametric pitch or finer can be produced with tooth errors of less than 0.0005 inch at little additional cost.

(b) *Errors in tooth spacing.* Tooth-spacing errors are the result of indexing errors, and usually are a result of the inaccuracy of the machine on which the gear is cut. Gears with a maximum tooth-spacing error below 15 seconds of arc are seldom produced, the usual maximum accumulated spacing errors being about 3 minutes of arc.

(c) *Eccentricity of the gear pitch circle.* Eccentricity is produced primarily by errors in holding the gear blank in exact alignment while the teeth are being cut. Eccentricities can be held as low as 0.00025 inch on suitably designed gears, but larger values should be specified as permissible if they are compatible with other design values. It should be remembered that bearing and shaft eccentricities also contribute to the total pitch-line eccentricity of a gear when it is installed in a gear train.

In the majority of control system applications, gear inaccuracies become calibration errors. Two examples of applications where this occurs are: the gears driving the feedback synchros of a gun mount ; and the gears driving the synchros of a gun director. Unless extreme accuracy is required, gear inaccuracies in themselves contribute little to control system inaccuracies. However, eccentricities of the gears may necessitate an excessive amount of backlash in the gear train, and this may produce system instability or low system gain. The designer must therefore consider the accuracy requirements of a system when specifying permissible tolerances in gear accuracy.

15-1.12 Friction[29, 30]

Spur, internal, and bevel gearings usually do not have sufficient friction to warrant consideration. Spiral gears or worms with helix angles below 10°, however, are extremely inefficient because of friction. In general, spiral gears or worms with helix angles below 20° are not reversible; i.e., the worm cannot be rotated by applying torque to the worm gear. In control systems, nonreversible gearing should not be used between the servomotor and the feedback device if the load has any inertia or other source of overdriving torques. When the load tends to overhaul the servomotor, the worm or helical gear locks, and the result is extremely poor servo performance. Improperly designed planetary-gear drives sometime exhibit the same characteristics because high stresses on the gear teeth produce excessive deformations and consequent high friction.

15-1.13 Inertia

Gearing inertia is often an important portion of the load on a servomotor. The inertia of any gear in a gear train, referred to the driving point, is

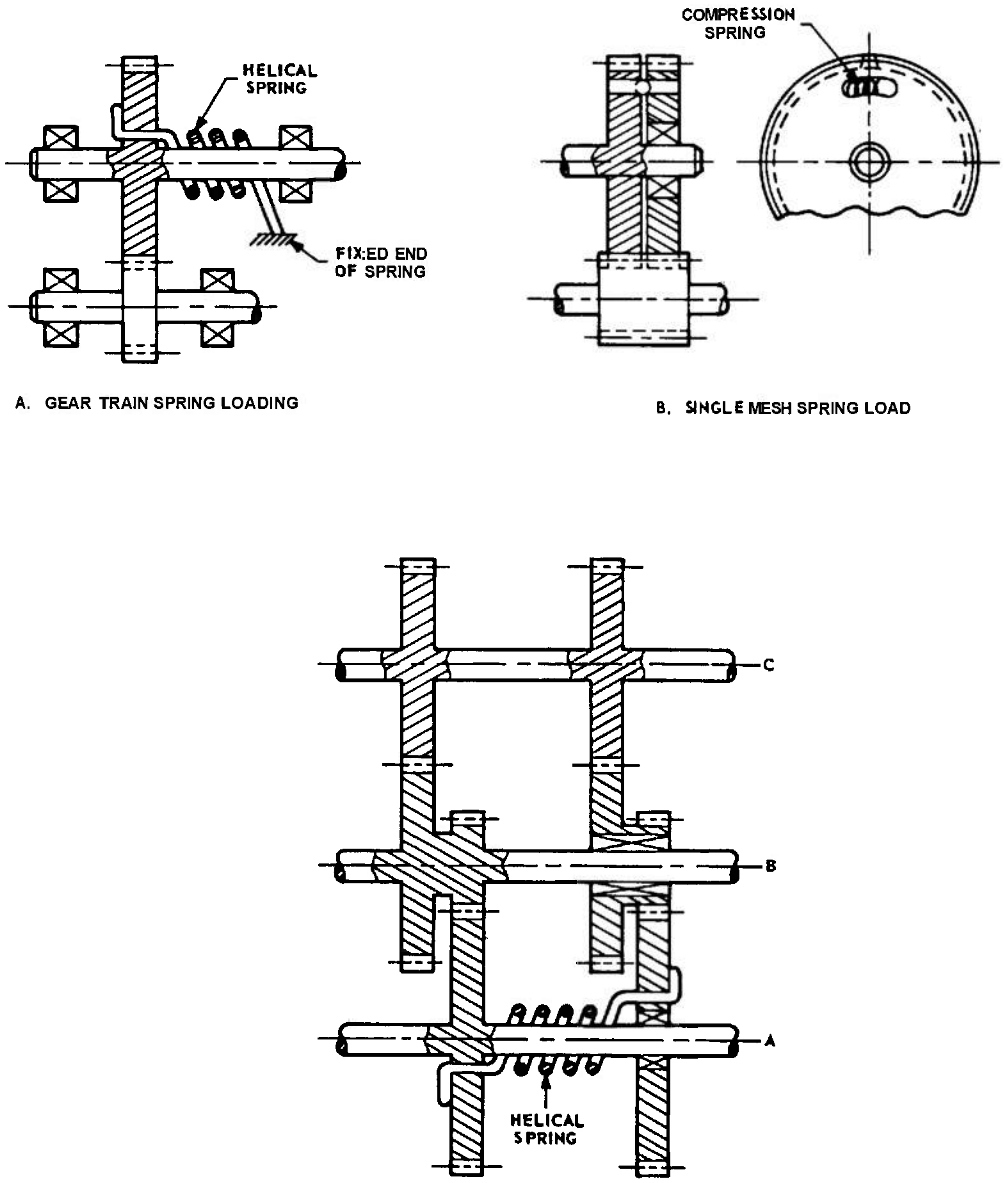
$$J_r = \frac{J}{N^2} \tag{15-16}$$

where J is the actual gear inertia, and N is the gear ratio between the reference point and the gear being considered. If the considered gear rotates slower than the reference point, N is greater than unity. If the considered gear rotates faster than the reference point, N is less than unity. The total moment of inertia of the gear train is the sum of all the reflected inertias. The inertia of small-diameter gears is obviously lower than the inertia of larger gears, and the size of the gears in a train can be kept small by mating them with pinions having a minimum number of teeth. In practice, therefore, the total reflected inertia of a gear train can be reduced by using several gear pairs with ratios of 2:1 or less, instead of a single pair with a large ratio. It is often necessary to compromise between reflected inertia and the backlash introduced by a large number of gear meshes.

15-1.14 Backlash

Backlash results in a region of input motion within which no variation of the output can be detected. When placed within a control-system closed loop, this type of "dead zone" may produce system instability, and usually causes the system to oscillate at a low frequency, through an angle of one to five times the backlash angle. Backlash can be reduced by making the driving gear large (to reduce the angle of backlash) or by using gears with small backlash. Backlash can be eliminated by spring loading the gears.

Three methods of eliminating backlash by spring loading are shown in Fig. 15-3. The method shown in Fig. **15-3A** uses a spring



A. GEAR TRAIN SPRING LOADING

B. SINGLE MESH SPRING LOAD

C. SPRING-LOADED DOUBLE GEAR TRAIN

Fig. 15-3 Methods of eliminating backlash.

on the output shaft to exert enough continuous torque to maintain a load on one side of the gear teeth. This method can be used only when there is ample torque available to drive the load, and only when the angle of rotation of the gear is limited to a few revolutions. In Fig. **15-3B,** the gear is split in half, forming two identical gears. One gear is attached rigidly to the shaft, and the other is free to rotate on the shaft when driven by expansion of the spring. When the gears are to be meshed with a pinion, the free gear is rotated manually on the shaft to compress the spring. After meshing, the free gear is released and the spring presses the teeth on the two halves of the split gear against opposite sides of the pinion teeth, thus maintaining bidirectional contact between pinion and gear teeth. The method shown in Fig. **15-3C** is similar to the split-gear method, except that it can be used to simultaneously load several gear meshes. In this method, the pinion on shaft *A* is split and spring loaded, thereby loading the entire train. The gear trains shown in Fig. **15-3B** and **15-3C** are capable of revolving continuously and therefore have a constant loading torque. Any of the three spring-loaded gear-train systems shown in Fig. **15-3** adds to the torque required to drive the gear train because it adds to the load on the bearings and to deformation of the gear teeth.

The optimum amount of spring loading applied to a gear train can sometimes be determined only by trial-and-error. Theoretically, the spring load should be at least equal to the maximum torque the gears are required to transmit. In practice, however, the spring load can often be reduced below this value, with a resulting decrease in friction in the train. Such decreased loading torques are practicable in applications where the load is primarily inertia, and errors during acceleration are permissible.

15-1.1 5 Compliance

All gear trains have an effective spring constant as the result of elasticity of the materials. In the presence of heavy inertia or coulomb-friction loads, the compliance (reciprocal of spring constant) of the output gear train of a servomechanism may result in a low system gain, or a highly undesirable resonant frequency. Gear-train compliance comes from the following three sources:

(a) Elastic deformation of the gear teeth

(b) Torsion and bending of the shafts

(c) Deformation of the bearings and housing.

Normally, torsion and bending of the shafts are the only significant factors. The total compliance of a gear train can be computed by adding the compliances of each gear shaft after referring them to a common point. The referred compliance of a shaft is

$$C_r = N^2C \qquad \textbf{(15-17)}$$

where

C = actual compliance of the shaft

N = gear ratio between shaft and reference

C_r = referred compliance

N is greater than **1.0** when the actual shaft rotates slower than the reference shaft, and is less than **1.0** when the actual shaft rotates faster than the reference.

15-2 MECHANICAL DIFFERENTIALS

15-2.1 PURPOSE

Mechanical differentials are used to obtain the algebraic sums of two or more motions. When more than two motions are to be summed, two or more differentials are employed to achieve the required result. The following discussion covers geared differentials and differential linkages.

15-2.2 GEARED DIFFERENTIALS

The geared differential is the conventional type of mechanical differential. The most common configurations are shown in Fig. **15-4.** Differentials constructed entirely of spur gears are made. However, most differentials incorporate bevel-gear meshes. Bevel-gear differentials are shown in Figs. **15-2A** and B. In both types of differentials, the relation between the rotation of the three shafts is

$$2\theta_3 = \theta_1 + \theta_2 \qquad (15\text{-}18)$$

where θ_1, θ_2, and θ_3 are the angles shown in Fig. **15-4.**

The number of teeth on meshing gears of a differential is usually selected so that several revolutions of the gears will be made before any particular mesh of gear teeth is repeated. This design reduces and distributes gear wear. Both the good and bad characteristics inherent in any other type of gear train are also existent in geared differentials. Because of the large number of gear meshes in a differential, the amount of backlash is

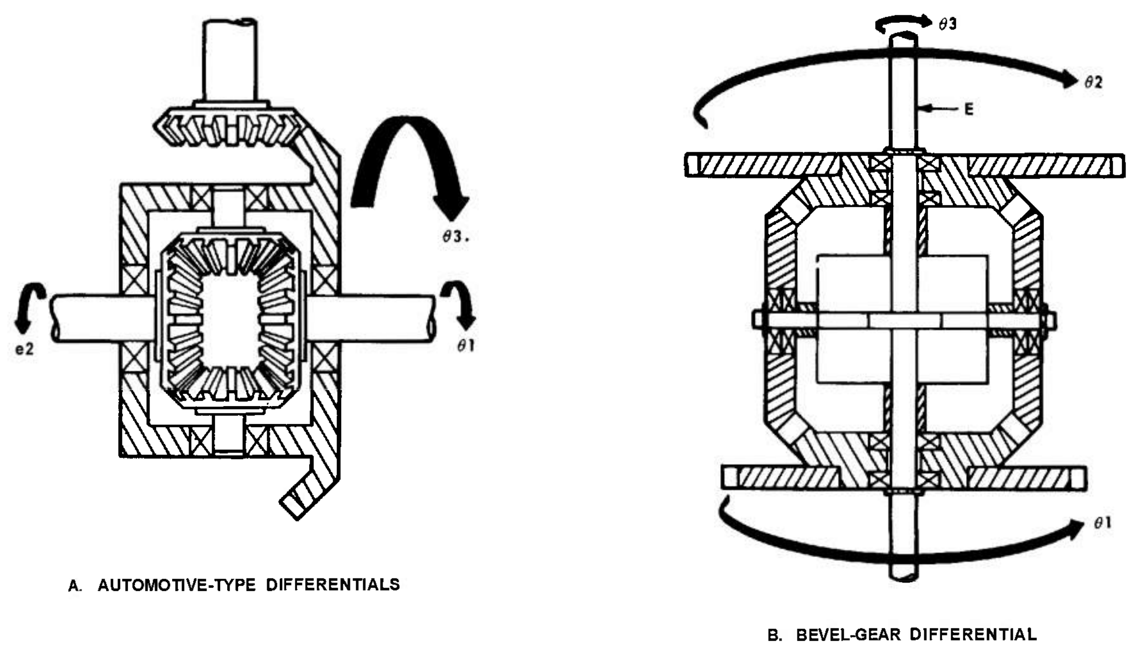


Fig. 15-4 Geared differentials.

sometimes undesirably high. Some servomechanisms use geared differentials to obtain the system error from input-shaft and output-shaft rotations. Since motion of the error-motion-driven device (potentiometer, pilot valve, etc.) is usually limited, a torque-limiting device such as a slip clutch must be used in the system to avoid damage to the gearing or to the driven device when large errors occur. Commercial gear differentials are listed by Michalec.[16]

15-2.3 DIFFERENTIAL LINKAGES

Differeiitial linkages are sometimes used to add two rectilinear motions (Fig. 15-5). In this type of linkage, one motion must be transmitted to the main link through one pivot, to prevent collapse of the linkage, and the other two motions must pass through auxiliary links to prevent binding. An exact relation between the motions is complicated, and depends upon the lengths of the auxiliary links as well as upon the spacing of pivots in the main link. For small motions of the linkage from the position in which the links are perpendicular to each other, an approximate relation between motions (Fig. 15-5) is

$$(A+B)Y = BX + AZ \tag{15-19}$$

Differential-linkage motions are seldom used when the angle between the main link and the line-of-motion of inputs and outputs becomes less than 75°

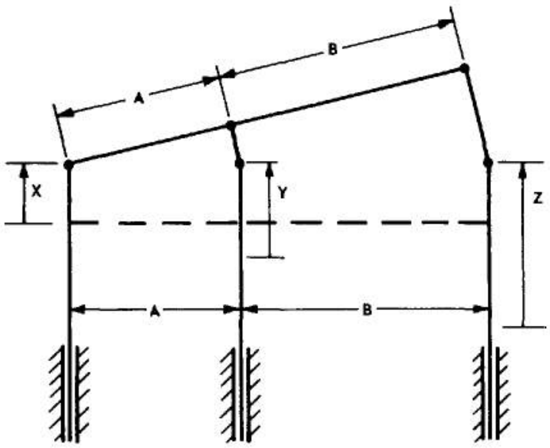


Fig. 15-5 **Differential lever.**

15-3 LINKAGES AND LEVERS

15-3.1 BASIC PURPOSE

Linkages and levers are used for the following purposes :

(a) To obtain the sum or difference **of** two motions.

(b) To transmit motion in such a way that the ratio of output-to-input motions is essentially constant over the range used.

(c) To transmit motion in such a way that the ratio of output-to-input motions varies with input position.

(d) To sum forces or torques. This usually involves the use of springs as well as linkages.

15-3.2 PRACTICAL APPLICATIONS

Linkages have been used in servomechanisms for the following purposes :

(a) To elevate or depress a gun, using the rectilinear motion produced by a hydraulic cylinder.

(b) To obtain **a** nonlinear scale for a servo-driven instrument.

(c) To operate a low-level hydraulic amplifier.

(d) To introduce time lags from spring and dashpot systems.

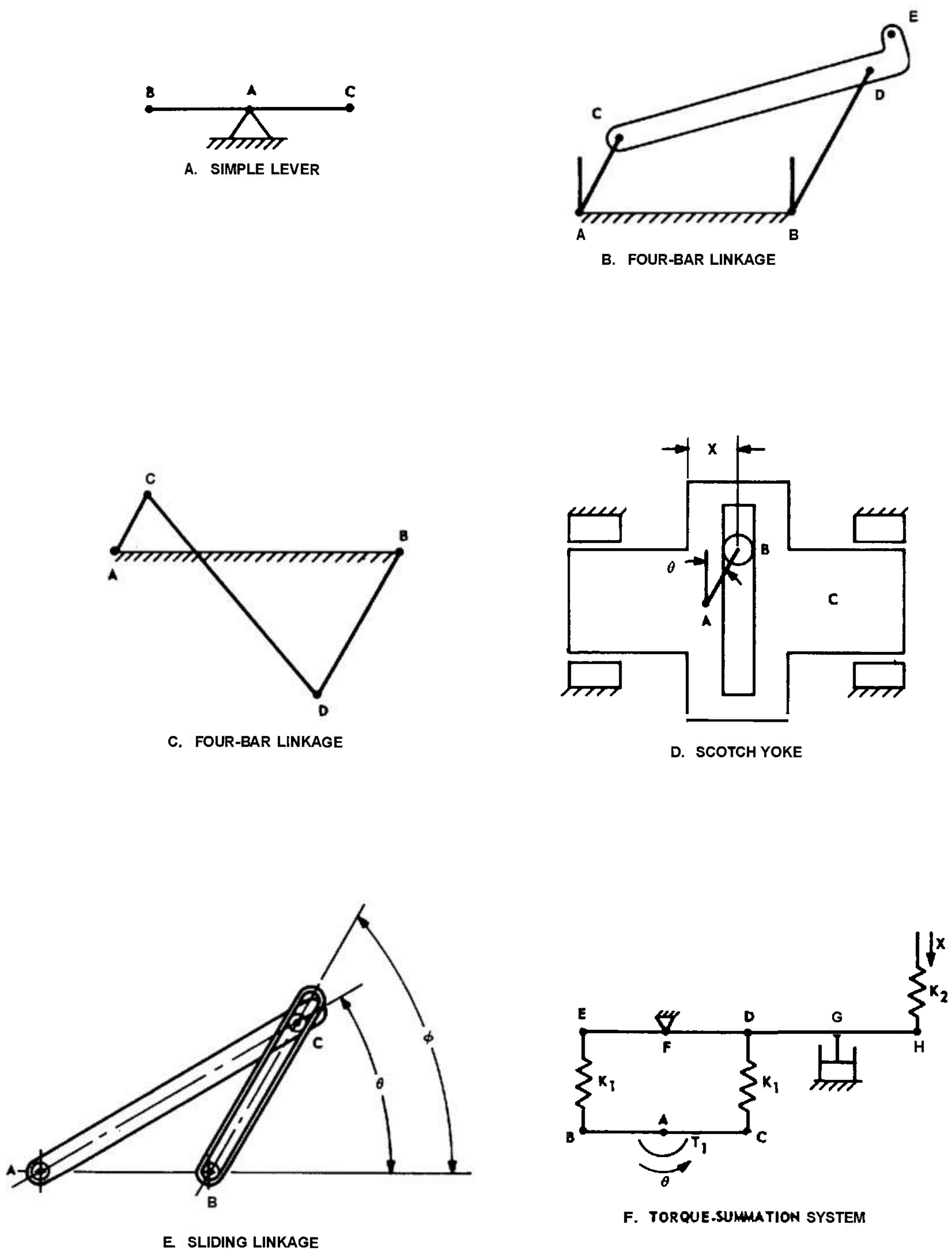


Fig. 15-6 Typical linkages.

15-3.3 EXAMPLES

Figure 15-6 shows typical arrangements of linkages and levers. A simple first-class lever is shown in Fig. 15-6A. The simple levers—first, second, or third class—are normally used to obtain an essentially linear characteristic between input and output motions.

Two configurations of four-bar linkages are shown in Figs. 15-6B and C. In these linkages, two cranks *AC* and BD are pivoted on a fixed base *AB,* and connected by a floating link CD. If *AC* is the driving crank, output motion may be taken either from crank BD or from some point on the connecting link *CD,* such as point *E* of Fig. 15-6B. Some properties of the four-bar linkage and the resulting motions are

(a) If $(AB + AC) < (BD + CD)$, and $(AB - AC) < (CD - BD)$, an oscillating motion of crank *BD* is produced by continuous motion of crank *AC.*

(b) If $AC \ll AB, BD, CD$, and the angle $ACD = 90°$ when angle $BDC = 90°$, the rotation of crank *BD* is approximately proportional to the sine of the angular position of crank *AC. AC* can rotate continuously.

(c) If $AC = BD$, and $AB = CD$ (in Fig. 15-6B only), the angles through which the two cranks rotate are equal. The angle of rotation of the cranks is limited to 180°, unless the output has appreciable inertia to carry it through the dead-center position.

Four-bar linkages have been in use for many years. Consequently, they have received considerable study. Hrones and Nelson(17) show the path of point *E* of Fig. 15-6B for five-degree increments of the position of crank *AC* for various linkage proportions that permit continuous rotation of *AC.* Svoboda(18) shows methods of determining linkage proportions that will produce a motion of crank *BD* that satisfies a desired analytical function. Most analyses of four-bar linkages are performed graphically.(19)

The scotch yoke shown in Fig. 15-6D is sometimes used to convert a rotary motion into a sinusoidal motion. In this device, rotary input crank *AB* is fitted with a roller at *B,* which fits into a vertical slot in slide C. As crank *AB* rotates around point *A,* it imparts a motion to slide *C* that is proportional to the sine of the angle of the input crank.

Sliding linkages similar to that shown in Fig. 15-6E are sometimes used to obtain a nonlinear relation between input and output. A roller or slide on input crank *AC* fits into the slot in the arm pivoted at *B.* The relation between the angles for this linkage is

$$\tan \phi = \frac{\sin \theta}{\cos \theta - \dfrac{AB}{AC}} \tag{15-20}$$

The torque-summation system shown in Fig. 15-6F is occasionally used at low torque levels. In this system, a torque motor at *A* produces a torque T_1, which tends to rotate arm *BC* against the restraining springs K_1, producing a torque on lever *EH.* Motion *X* compresses spring K_2, producing a second torque on *EH.* When *EH* is turning about F, the dashpot at G will produce a resisting force and hence a torque on *EH.* By the proper selection of springs, lever ratios, and dashpot, the position of arm *BC* can be changed instantaneously by changing torque T_1 and, with a time lag, by changing motion *X.*

15-3.4 NONIDEAL CHARACTERISTICS

15-3.5 Mass of Links and Cranks

Linkage mass is usually small enough to contribute a negligible lag to servomechanism response. However, in the case of systems subject to high extraneous accelerations, such as gunfire or road shocks, the mass of a linkage in a servo error system may produce spurious input signals. In addition to making the linkage light in weight, the following procedures will reduce the effects of acceleration :

(a) Eliminate all possible coupling by mounting the linkage with its pivot axes at right angles to the axis of angular acceleration or parallel to the direction of linear acceleration.

(b) Balance the cranks by placing their center of gravity on their rotation axis to eliminate the effect of linear acceleration. This increases the effect of angular acceleration because it increases the total moment of inertia.

(c) Mount the linkages close to the center of rotation to make centrifugal forces small. Linkage systems with floating links, such as the four-bar linkage, cannot be completely balanced in all linkage positions against either linear or angular accelerations.

15-3.6 Compliance

Linkage compliance is a result of the following:

(a) Torsion of crank shafts

(b) Bending of cranks

(c) Stretch or compression of links

(d) Torsion of cranks (in some systems)

These are functions of the particular linkage design and load. The calculations of the effect of these factors are covered in many of the standard texts on strength of materials. To refer the compliance to a particular point in the linkage, use the method of gearing outlined in Par. 15-1.

15-3.7 Backlash in Pivots and Slides

When linkages have only very small motions, backlash can be eliminated by using flexure or spring pivots that have no free 'play. Many linkages can also be spring loaded to eliminate free play at the pivots. For example, the four-bar linkage shown in Fig. **15-6B** can be loaded by connecting a spring between cranks *AC* and BD. If the linkage can be made larger without increasing pivot clearance, the resulting angular free play will be reduced. Backlash of a sliding pivot can also be eliminated by spring loading.

15-4 SHEAVES AND TAPES

15-4.1 PURPOSE

Metal tapes can be used to transmit rotary motion (through limited angles) in much the same way that belts and pulleys are used (Fig. **15-7**). With careful design, very accurate transmission of motion can be achieved. The two active sheaves shown in Fig. **15-7** are *A* and *D*; *B* and *C* are idlers. Tape tension is maintained by the expansion spring attached to *B*. The tape is securely fastened to sheaves *A* and *D* to prevent relative motion (slippage) between the tape and the sheaves; motion is thus limited to 180°, or one half a revolution of *A*.

15-4.2 STRESS

The stress of the outer fibers of the tape (due to bending only) is

$$S_b = \frac{c}{R} E \qquad (15\text{-}21)$$

where

S_b = tensile stress due to bending, in psi

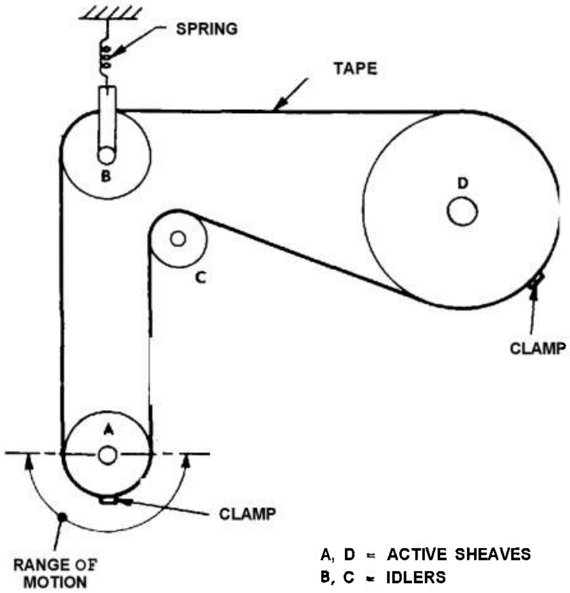


Fig. 15-7 Metal tape and sheaves.

c = one half of tape thickness, in in.

R = sheave radius, in in.

E = modulus of elasticity, in psi

15-4.3 TENSION

Because the tape can support no load in compression, a tension force is required for successful operation of the system. The total stress due to both bending and tension is

$$S = S_b + S_T = \frac{c}{R} E + \frac{P}{A} \qquad (15\text{-}22)$$

where

S = total maximum stress, in psi

S_T = stress due to tensions, in psi

P = applied tension, in lb

A = tape cross-sectional area, in in.^2

15-4.4 SHEAVE SIZES

Values of sheave diameters that can be used for different tape materials of various thicknesses are given in Table **15-8.** The assumed bending stresses are approximately **16** percent of the yield stress of the material. If heavy friction or inertia loads are to be driven by these tapes, the sheave diameters given in Table **15-8** should be increased to make the combined stress under load less than the yield point of the material.

15-4.5 COMPLIANCE

When using tape-and-sheave systems, the compliance of the system should be computed to be sure that adequate stiffness is provided. Care must also be taken to ensure that thermal effects do not alter the desired relative positions of input and output sheaves.

TABLE 15-8 SHEAVE DIAMETERS (in.)

	Tape Material			
Tape Material Thickness (in.)	steel (AISI 1020) and (Type 304) stainless $E = 29 \times 10^6$ $S_b = 5000$	steel (AISI 1090) (tempered) $E = 29 \times 10^6$ $S_b = 104$	chrome or nickel steel (tempered) $E = 29 \times 10^6$ $S_b = 2 \times 104$	phosphor bronze $E = 15 \times 10^6$ $S_b = 1.25 \times 104$
0.001	2.9	1.43	0.73	0.06
0.002	5.8	2.90	1.45	0.12
0.004	11.6	5.80	2.90	0.24
0.010	29.0	14.50	7.24	0.60
0.015	43.6	21.80	10.80	0.90
1/32	90.6	45.20	22.60	1.87
1/16	181.0	90.60	43.20	3.74
1/8	362.0	181.00	90.60	7.50
1/4	724.0	362.00	181.00	15.00
1/2	1450.0	724.00	362.00	30.00

15-5 MECHANICAL COUPLING DEVICES

15-5.1 COUPLINGS

The shafts of two rotating devices are often connected together with flexible couplings. To be adequate for most applications, a coupling must permit some shaft misalignment in each of the following ways:

(a) Angular misalignment of shafts (shaft centers intersect at an angle)

(b) Lateral misalignment (shaft centers do not intersect)

(c) Shaft end play

In addition, it is often convenient to use couplings that can be disconnected easily so that either the coupling or the devices to which it is attached can be serviced. Many couplings that are completely satisfactory when driving devices at constant speed are unsatisfactory in some control applications because of backlash, compliance, and motion irregularities introduced by coupling action. Because the required accuracy as well as the permissible backlash and compliance vary widely between particular systems, no definite rules can be made. Figure 15-8 illustrates a few commercially available couplings. Many other equally satisfactory couplings are also available. The manufacturer should be consulted for torque and speed ratings.

The universal joint shown in Fig. 15-8A is seldom used because of backlash and output-speed variation. A double unit is required to accommodate lateral shaft misalignment. The spring universal joint shown in Fig. 15-8B accommodates angular misalignment only and produces output-speed variations. There is no backlash from this coupling, and little compliance. This coupling can be used at both high and low power levels. The double-spring universal coupling shown in Fig. 15-8C is one of the best couplings for all power levels, as it accommodates angular and lateral misalignments and produces uniform output. The bellows coupling shown in Fig. 15-8D accommodates angular and lateral misalignments, but care is required in selecting bellows material to avoid hysteresis effects. This coupling is normally used in instrument systems. The sleeve coupling shown in Fig. 15-8E accommodates no misalignment and must be very accurately machined and installed for satisfactory operation. The parallel-face coupling shown in Fig. '15-8F is usable for constant-speed drives, but has too much backlash for use in most control loops. Lateral misalignment causes variation in output speed. This type is not usually available at instrument levels. The ribbon coupling shown in Fig. 15-8G is suitable for use in control loops if the ribbon provides an effective spring force which eliminates backlash. This type is not generally suitable for instrument systems. The spring-loaded pin coupling shown in Fig. 15-8H is sometimes used in precision instrument systems; however, for accurate motion transmission, the shafts must be exactly in line. The Oldham coupling of Fig. 15-81 probably requires less space than any other coupling, as the slots can be machined directly into the shafts to be coupled. It accommodates both lateral and angular shaft misalignments; it produces output-speed variations as a result of lateral misalignment; and it has some backlash. Because lateral misalignment produces sliding between components of the Oldham coupling, a judicious choice of materials and, in some instances, lubrication is required when this coupling is used to drive heavy loads. The rubber coupling of Fig. 15-85 consists of a rubber cylinder vulcanized to two metal end pieces. For some applications, it can accommodate both lateral and angular shaft misalignment without producing excessive output-speed variations. Although the rubber

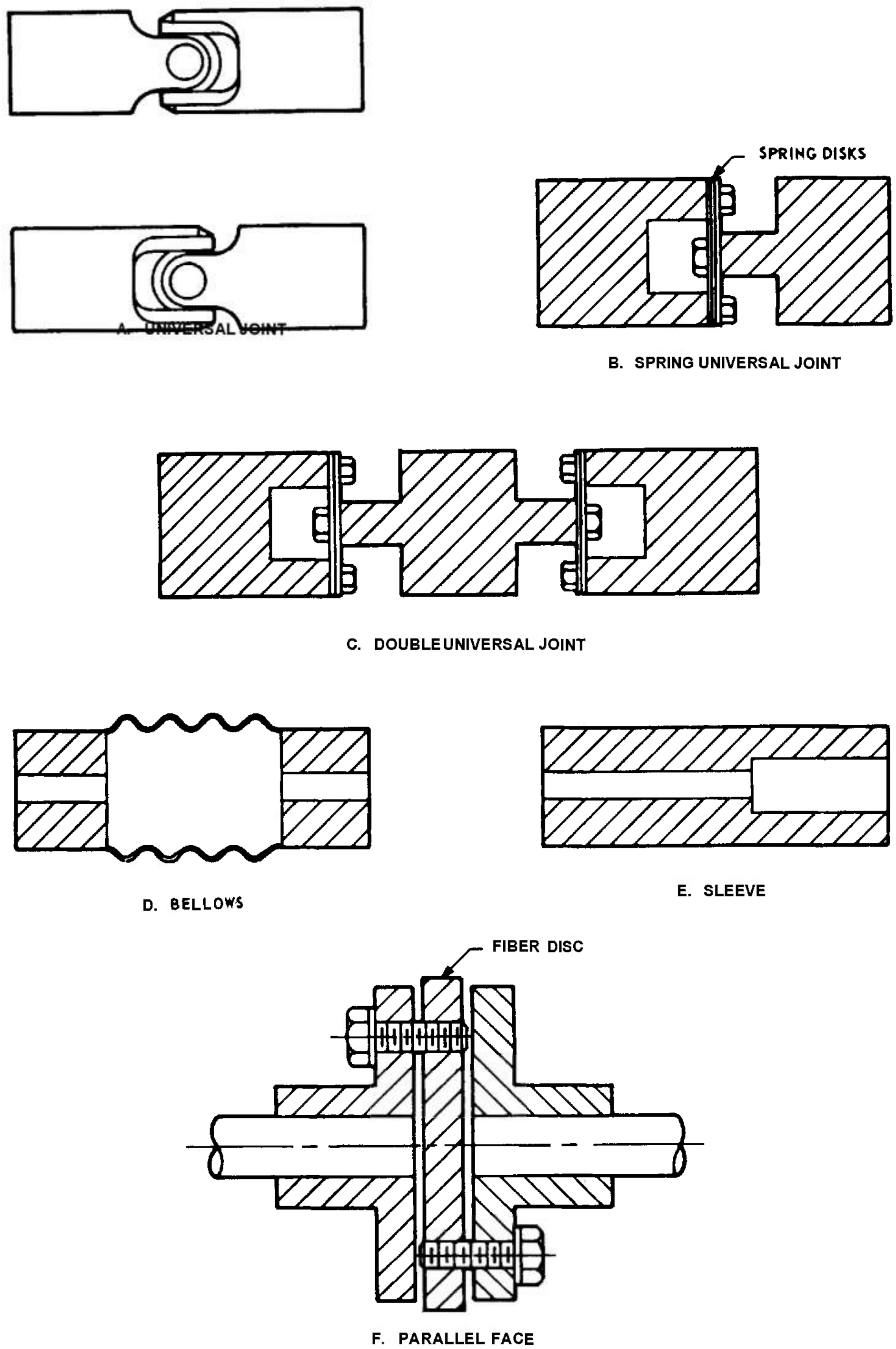
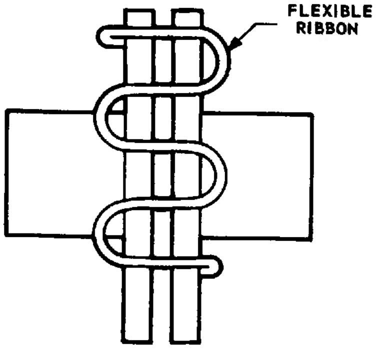
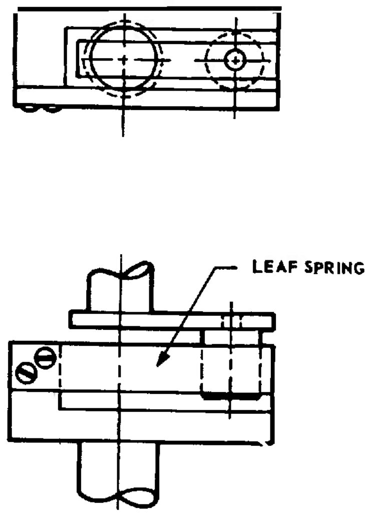


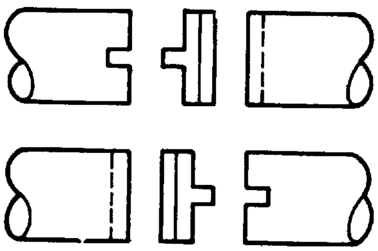
Fig. 15-8 Coupling types. (Sheet 1 of 2)



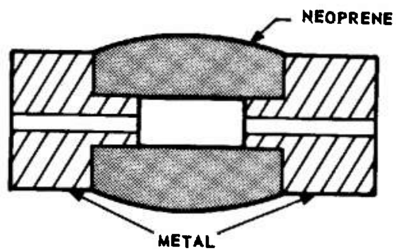
G. RIBBON



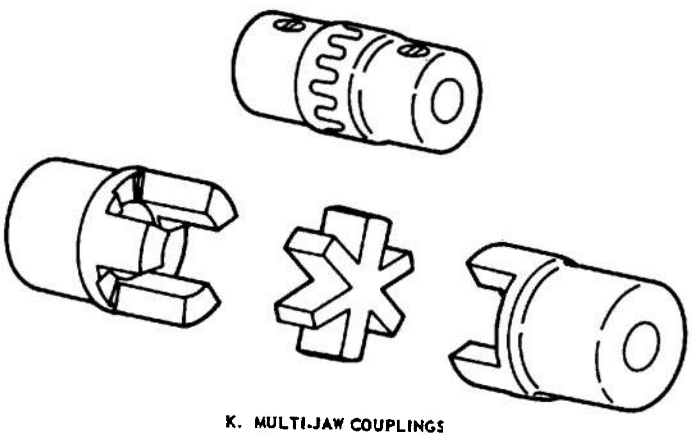
H. SPRING LOADED PIN



I. OLDHAM COUPLING



J. RUBBER COUPLING



K. MULTI-JAW COUPLINGS

Fig. 15–8 Coupling types. (Sheet 2.of 2) exhibits compliance, it also exhibits damping and hysteresis. The coupling can consequently be used for tachometer drives if rapid transmission of speed changes is not required. The multi-jaw couplings of Fig. 15-8K have appreciable backlash and consequently are seldom used when reversal of shaft rotation occurs.

15-5.2 KEYS AND SPLINES

Keys and splines are used to prevent relative motion (slippage) between couplings and the shafts to which they are attached.

15-5.3 Keys

Many types of keys have been used.[4,21,23] Keyways are usually broached into the coupling hubs, and keyseats are milled into the shafts. Dimensions for some standard sizes of square-parallel keys (Fig. 15-9) are given in Table 15-9. Woodruff keys (Fig. 15-10) are circular on the key-seat side to prevent movement of the key along the shaft when the coupling hub is installed or removed. Dimensions of some of the smaller sizes of Woodruff keys are listed in Table 15-10.[4,21,23,25]

15-5.4 Splines[22]

The simplest form of spline is a taper pin (Fig. 15-11), often used to transmit light loads. Taper pins have the advantage of zero backlash when properly fitted. The tapered hole into which the pin fits is formed by taper-reaming the two members while they are clamped together. Dimensions of taper pins are given in Table 15-11.

A system of square-sided splines having 4, 6, 10, and 16 splines has been standardized by the SAE (a four-spline hub is shown in Fig. 15-12). These splines are intended to be used with soft, broached hubs. Tolerances can be adjusted so that contact between hub and shaft occurs at either the large diameter,

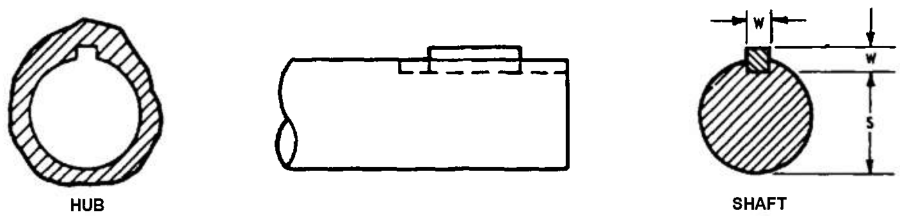


Fig. 15-9 Square-parallel stock key.

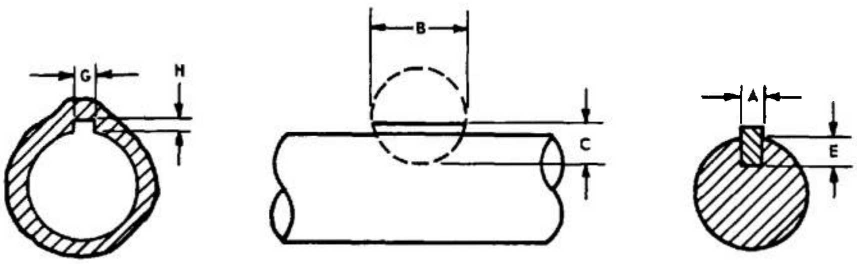


Fig. 15-70 Woodruff key — SA€ standard.

TABLE *15-9* DIMENSIONS FOR SQUARE-PARALLEL STOCK KEYS

Shaft Diameter (in.)	Width and Thickness of Key W (in. ± 0.002 in.)	Bottom of Keyseat to Opposite Side of Shaft 5 (in.)
1/2	1/8	0.430
9/16	1/8	0.493
5/8	3/16	0.517
11/16	3/16	0.581
3/4	3/16	0.644
13/16	3/16	0.708
7/8	3/16	0.771
15/16	1/4	0.796
1	1/4	0.859
1-1/16	1/4	0.923
1-1/8	1/4	0.983
1-3/16	1/4	1.049
1-1/4	1/4	1.112
1-5/16	5/16	1.137
1-3/8	5/16	1.201
1-7/16	3/8	1.225
1-1/2	3/8	1.289
1-9/16	3/8	1.352
1-5/8	3/8	1.416
1-11/16	3/8	1.479
1-3/4	3/8	1.542

TABLE 15-10 DIMENSIONS OF SOME WOODRUFF KEYS, KEYSLOTS, AND KEYWAYS

SAE Nominal Size (in.)	Key			Keyway		Keyslot		
	A +0.001 −0.000 (in.)	B +0.000 −0.010 (in.)	C +0.000 −0.005 (in.)	G +0.002 −0.000 (in.)	H +0.005 −0.000 (in.)	A min (in.)	B min (in.)	E +0.005 −0.000 (in.)
1/16× 1/4	0.0625	0.250	0.109	0.0635	0.0372	0.0615	0.250	0.0728
1/16× 5/16	0.0625	0.312	0.140	0.0635	0.0372	0.0615	0.312	0.1038
3/32× 5/16	0.0938	0.312	0.140	0.0948	0.0529	0.0928	0.312	0.0882
1/16× 3/8	0.0625	0.375	0.172	0.0635	0.0372	0.0615	0.375	0.1358
3/32× 3/8	0.0938	0.375	0.172	0.0948	0.0529	0.0928	0.375	0.1202
1/8× 3/8	0.125	0.375	0.172	0.1260	0.0685	0.1240	0.375	0.1045
1/16× 1/2	0.0625	0.500	0.203	0.0635	0.0372	0.0615	0.500	0.1668
3/32× 1/2	0.0938	0.500	0.203	0.0948	0.0529	0.0928	0.500	0.1511
1/8× 1/2	0.125	0.500	0.203	0.126	0.3685	0.1240	0.500	0.1355
1/8× 5/8	0.125	0.625	0.250	0.126	0.0685	0.1240	0.625	0.1825
3/16× 5/8	0.1875	0.625	0.250	0.1885	0.0997	0.1863	0.625	0.1513
3/16× 3/4	0.1875	0.750	0.313	0.1885	0.0997	0.1863	0.750	0.2143
3/16× 7/8	0.1875	0.875	0.375	0.1885	0.0997	0.1863	0.875	0.2763
1/4× 1	0.250	1.000	0.438	0.2510	0.1310	0.2487	1.000	0.3080
7/32× 1-1/8	0.2188	1.125	0.484	0.2198	0.1153	0.2175	1.125	0.3697
3/16× 1-1/4	0.1875	1.250	0.547	0.1885	0.0997	0.1863	1.250	0.4483
3/8× 1-1/4	0.375	1.250	0.547	0.3760	0.1935	0.3735	1.250	0.3545

Adapted by permission from *SAE Handbook*, 1956, from Tables 1a and 2a. Society of Automotive Engineers, Inc.

TABLE 15-11 DIMENSIONS OF TAPER PINS
(Taper: 1/4 in. per 12 in.)

Number	Diameter of Large End (in.)	Maximum Length (in.)	Number	Diameter of Large End (in.)	Maximum Length (in.)
7/0	0.0625	3/4	5	0.289	4
6/0	0.0780	1	6	0.341	5
5/0	0.0940	1	7	0.409	5
4/0	0.1090	1	8	0.490	5
3/0	0.1250	1	9	0.591	6
2/0	0.1410	2-1/2	10	0.706	6
0	0.1560	3	11	0.860	9
1	0.1720	3	12	1.032	9
2	0.1930	3-1/2	13	1.241	11
3	0.2190	3-1/2	14	1.523	13
4	0.2500	4			

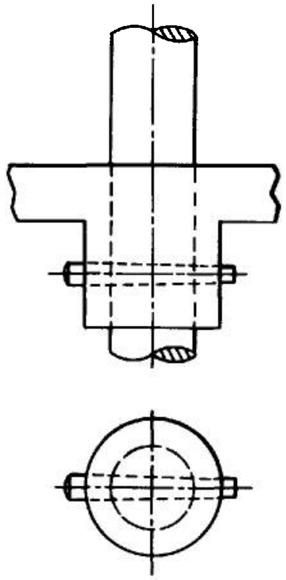


Fig. 75-77 Taper pin.

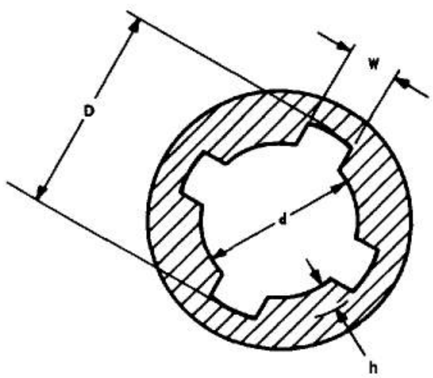


Fig. 75-72 SAE square-spline hub.

the small diameter, or on the sides of the splines. The torque available from a spline of this type, for a compression stress of 1000 psi at the spline side, is

$$T = 1000 n r h L \tag{15-23}$$

where

T = torque, in in.-lb

n = number of splines

r = mean radius, in in.

h = spline depth, in in.

L = spline length, in in.

15-5.5 Involute Splines

A single system of involute splines has been adopted by the SAE, ASA, and AGMA. These splines consist of two 30-degree pressure angle involute gears, one external and one internal, fitted together in the desired manner. The nomenclature of involute splines is identical with that of involute gears, except that "major diameter" and "minor diameter" are substituted for "outside diameter" and "root diameter". As in the case of square splines, the internal spline is usually broached, and the dimensions of the external spline are adjusted to produce the desired fit.[22,24]

15-6 BEARINGS

15-6.1 ROLLER BEARINGS

Roller bearings consist of an outer race, an inner race, and a number of rollers to provide rolling action when either or both races are rotated in relation to each other. The rollers are usually separated by a retainer, or spacer, to prevent the rollers from rubbing against each other. The rollers and races are usually made of hardened alloy steel. The generally used types are shown in Fig. 15-13.

The straight roller bearing shown in Fig. 15-13A can sustain no end thrust at all; in fact, a very light force will cause the bearing to come apart. Therefore, some type of thrust bearing must be used with straight roller bearings on all applications.

The solid roller bearing shown in Fig. 15-13B is made with lips on the races that extend around the ends of the rollers, thereby preventing inadvertent disassembly. These bearings will sustain a light end-thrust load and, for such conditions, need no additional thrust bearings.

Needle bearings (Fig. 15-13C) use rollers with a large length-to-diameter ratio. They are made in two types — one with only an outer race, the shaft being used for the inner race (as shown) ; the second with both inner and outer races. Needle bearings are generally used where space is limited, or for high-speed operation. They are usually made with a full complement of rollers and can sustain no end thrust.

The rollers and races of a tapered-roller bearing (Fig. 15-13D) are made in the form of sections of right circular cones. These bearings can sustain large end thrusts and radial loads. They are frequently used in opposed pairs, with a thrust preload between them, to obtain a shaft with zero end play.

Several other types of roller bearings are manufactured. All types range in size from a minimum of approximately 1.5-in. inside diameter (bore) to any desired maximum.

15-6.2 BALL BEARINGS

In control systems, ball bearings are used much more frequently than any other type of bearing because of their small size, low radial and axial play, rigidity, low friction, precise dimensions, and low maintenance requirements. Usually, the load-carrying capacity of

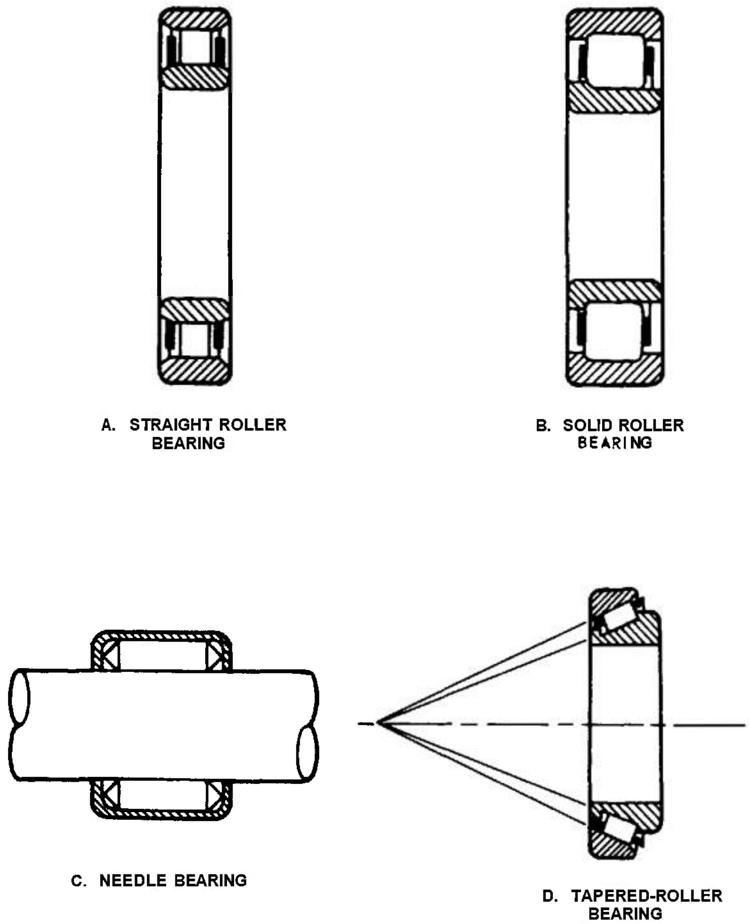


Fig. 15-13 Roller bearings.

the bearing far exceeds the applied load.

Two types of single-row ball bearings are shown in Figs. **15-14** and **15-15.** The grooves in the races, the thickness of the races, and the number of balls are varied to produce bearings with different radial and end-thrust load ratings. Ball retainers are used for high-speed operation, but are omitted for slow-speed operation to permit the use of the maximum number of balls for heavy loading of the bearing.

Double-row ball bearings (Fig. **15-16)** have two grooves in both the inner and outer races, each groove containing a full complement of balls. Separable bearings are made by finishing one side of the inner or outer race to the depth of the groove. Self-aligning bearings are made by making the outer race spherical and without grooves.

Standard metric bearings are manufactured in a wide variety of sizes and types by many companies, but will not be discussed here. Very special types, used on tank turrets, require extensive design and are beyond the scope of this publication.

Inch-type ball bearings are used extensively in servomechanisms. Some commercially available inch-type models are similar to those listed in Air Force —Navy Aeronautical Standards AN200 through AN208. Table

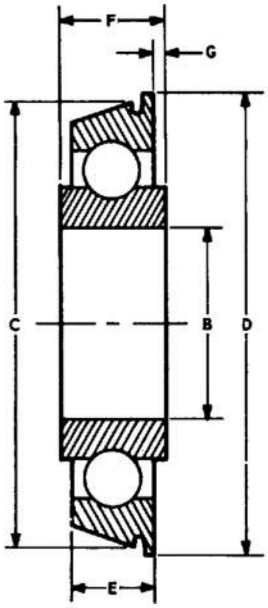


Fig. 15-14 Flange-type ball bearing.

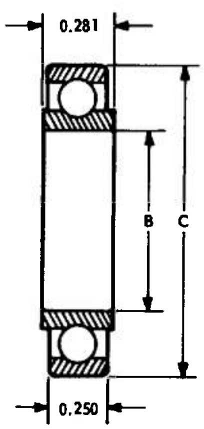


Fig. 15-15 Torque-tube bearing.

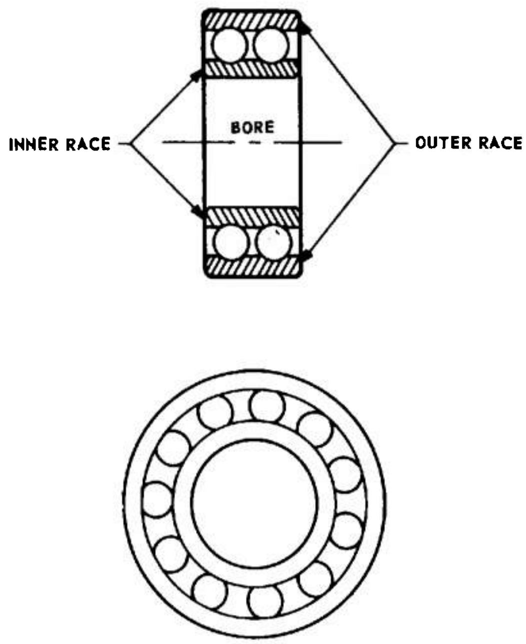


Fig. 15-16 Double-row ball bearing.

15-12 lists the dimensions of some of the smaller commercial, unshielded, inch-type bearings with retainers.

Additional load capacity can be realized on some types of bearings without affecting dimensions by using bearings without retainers. Shielded bearings are available in some types, but with an increased width. It is sometimes desirable to use flange bearings (Fig. **15-14**), the flange on the outer race replacing **a** retainer ring in many applications. The dimensions of some commercial bearings of this type (Fig. **15-14**) are given in Table **15-13.**

TABLE 15-12 DIMENSIONS OF COMMERCIAL UNSHIELDED INCH-TYPE BALL BEARINGS WITH' RETAINERS

Bore (in.)	OD (in.)	Width (in.)
0.0469	0.1562	0.0625
0.0781	0.2500	0.0937
0.0937	0.3125	0.1094
0.1250	0.2500	0.0937
0.1250	0.5000	0.1719
0.1562	0.3125	0.1094
0.1875	0.3125	0.1094
0.1875	0.5000	0.1562
0.2500	0.6250	0.1960
0.3750	0.8750	0.2188
0.5000	1.1250	0.2500
0.6250	1.3750	0.2812
0.7500	1.6250	0.3125
0.8750	1.8750	0.3750
1.0000	2.0000	0.3750
1.1250	2.1250	0.3750
1.2500	2.2500	0.3750
1.3750	2.5000	0.4375

For some applications, a thin bearing with a large-diameter bore, known as a "torque-tube" bearing (Fig. **15-15**), is desirable. Dimensions of a few of these bearings are given in Table **15-14.** Torque-tube bearings are designed primarily for heavy radial loads and oscillating motion, but are not satisfactory for continuous high-speed operation. **As** the races are thin and flexible, dependence is placed upon the rigidity of the bearing housing and the shaft to maintain the circular shape of the races.

15-6.3 LUBRICATION

For **a** reasonable life expectancy, ball and roller bearings should be lubricated, although in most applications they need very little lubricant. For low and moderate speeds, grease is adequate; at high speeds, light oil is preferable because friction between the lubricant and the balls or rollers generates less heat. For ordnance material that is to be operated at low temperatures, standard ordnance lubrication procedures should be followed.,

15-6.4 FRICTION

Little has been published about ball-bearing friction. For a ball or roller bearing with standard proportions, the coefficient of friction μ can be defined as

$$\mu = \frac{T}{RP} \qquad \textbf{(15-24)}$$

where

T = friction torque, in lb-in.

R = bearing bore radius, in in.

P = radial load, in lb

An average value of μ for ball bearings is approximately **0.0025** and ranges between **0.003** and 0.001[4,25]. An empirical equation for the friction torque produced by small, lightly-loaded ball bearings is given in reference **29.** Ball bearing friction is also discussed in reference **31.**

TABLE 15-13 DIMENSIONS OF COMMERICAL UNSHIELDED FLANGE-TYPE BEARINGS

Bore	Race O.D.	Flange O.D.	Outside Width	Inside Width	Flange Width	
B (in.)	C (in.)	D (in.)	E (in.)	F (in.)	G (in.)	Taper of Outside Diameter (in. per ft)
0.0550	0.1875	15/64	0.0781	0.0781	0.023	0
0.0781	0.2500	19/64	0.0937	0.0937	0.023	0
0.0937	0.3125	23/64	0.1094	0.1094	0.023	0
0.1250	0.3125	23/64	0.1094	0.1094	0.023	0
0.1562	0.3125	23/64	0.1094	0.1094	0.023	0
0.1875	0.3125	23/64	0.1094	0.1094	0.023	0
0.1878	0.4382	0.500	0.1630	0.188	0.041	0.068
0.2503	0.6257	0.6875	0.2260	0.250	0.042	0.068

TABLE 15-14 DIMENSIONS OF TORQUE-TUBE BEARINGS

B (in.)	C (in.)
0.6250	1.0625
0.7500	1.1875
0.8750	1.3125
1.3125	1.7500
1.5625	2.0000
1.8125	2.2500
2.0625	2.6250
2.3125	2.8750

15-6.5 SLEEVE BEARINGS

Sleeve bearings have two primary disadvantages that make them inappropriate for ordnance servomechanisms: (1) they have a decided tendency to greatly increase their friction level when they stop; and (2) they require periodic maintenance. For these reasons, sleeve bearings are seldom used, unless the loads are very light and the lubrication is either available in the bearing itself or the bearing is flooded with lubricant.

Linkage pivots that operate under oil, or that can be drip-lubricated or spray-lubricated while the equipment is running, have been successfully used. In this type of equipment, the pins are hardened to approximately 45 Rockwell C, and the links are approximately 35 Rockwell C. Pin diameters of 1/8 to 1/4 inch can be used.

Bearings requiring no lubrication have been successfully used on the low-speed shafts of instrument-type servomechanisms. These bearings are made by press-moulding powdered metal to make it porous, and then the pores are filled with oil. Care must be exercised in both the selection and installation of these bearings if satisfactory operational results are to be realized.

A great amount of theoretical and testing work has been done on sleeve bearings for continuously rotating shafts. If a sleeve bearing must be designed, consult standard mechanical engineers' handbooks and the Transactions of the ASME.

15-6.6 MISCELLANEOUS BEARINGS

Many other types of bearings have been made, such as: fluid, air, knife-edge, jeweled pivot, ball-bearing pivot, jeweled journal, flexure, and ligament. These bearings are used occasionally in control systems.

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CHAPTER 16

TYPICAL PROCEDURE*

16-1 INTRODUCTION

The design of servo systems to meet a set of specifications can be carried out by either a trial-and-error or an optimization procedure. The method .used for a particular design depends to a large extent upon the nature of the specifications and the manner in which they are stated. In many cases, the way the specifications are stated is controlled by the techniques that are most familiar to the specifier.

The trial-and-error design procedure is widely used and has the advantage of being based on well-established and familiar techniques. An optimization procedure, as described in Ch. 8, is tied very closely to the performance index that is being optimized. If the performance index is representative of actual conditions, the optimization procedure is able to determine once and for all whether the specifications are compatible and if so, whether they can be met. No such certainty is possible if a trial-and-error method is used. The major disadvantage of the optimization procedure is that the performance indices which are easily handled by analytical techniques do not correspond too closely to the behavior that is significant to the specifier. Most performance indices that are close approximations to significant behavior are thus far practically impossible to treat by analytical methods, although analog computers can be used in some cases. An idealization of a practical performance index can be made, however, so that the idealization is amenable to analysis. An example is the use of the integral-square error criterion as an approximation to the peak error criterion for transient inputs. This is justified since systems designed to minimize the integral-square error will probably also minimize the peak error. An example of a practical performance index significant to the specifier and still analytically convenient for the designer is the use of the mean-square error criterion in applications where noise is present at the input or where the input is best represented as a stochastic signal.

Because of the difficulties involved in the use of optimization procedures in practical cases, a compromise is needed. One possibility is to use idealized performance indices in an optimization procedure and then use the resulting optimum system as a guide in the trial-and-error method.

For those cases where the specifications are direct descriptions of the desired closed-loop transfer function (bandwidth, settling time, peak overshoot, etc.) , the design procedure is reduced to one of merely synthesizing a transfer function having the desired characteristics. Since the direct specification of the system transfer function is a statement that omits inputs and disturbances, the freedom of the designer is severely limited. The specifier should be sure to specify inputs representative of the actual inputs that the servo will encounter during operation.

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16-2 GATHERING OF SPECIFICATIONS

The first step in the design of a servomechanism is to define what the system is to do and how well it is to do it. As pointed out in Ch. 1, a servomechanism may perform any or all of the following functions:

(a) Bring about a change in the actual value of the output so that it is in conformity with a desired value at all times.

(b) Minimize the effect of disturbances; i.e., variables other than the desired value of the output.

(c) Minimize the effect of varying component performance on the output.

The designer must first determine which one, or combination, of these functions is to be achieved.

The degree of excellence to which the servomechanism must carry out the above functions is established by the servo performance specifications. These specifications, furnished by the system designer, are based on a set of system specifications established by the ultimate user of the system.

Servo performance specifications must be given in terms of the desired value of the output of the servomechanism for a given value of the input signal to the servomechanism. The input signal encountered may be one of the following types :

(a) Aperiodic, noise free

(b) Aperiodic, with noise

(c) Periodic, noise free

(d) Periodic, with noise

(e) Stochastic, noise free

(f) Stochastic, with noise

Noise is regarded as any input-signal variation that is not a measure of the information carried by the input. The desired value of the output will be expressed in terms of a signal consistent with the input signal and will therefore be one of the types listed.

Some typical specifications for the six types of input signals listed above are as follows:

(a) Aperiodic, noise free (step input).

(1) The system error shall neither exceed a specified maximum value at any time nor exceed a specified value after the transient is over ; or, alternatively

(2) The integral-square system error shall not exceed a specified value; or, alternatively

(3) The integral of the time-multiplied absolute value of the system error shall not exceed a specified value.

In addition to one of the preceding requirements, it may also be specified that, after the transient due to the step input has died out, the system error shall not exceed a specified amount in the presence of a specified load (disturbance) that occurs at some specified point in the system. Other possible error specifications are listed in Ch. 7.

(b) Aperiodic, with noise (step input).

(1) The square root of the sum of the squares of the two components of system error shall not exceed a specified value. The first component of the system error is that found for an input step with zero noise; the second component is the value of the output when the input is the noise alone.

Other aperiodic signals commonly considered are ramps, impulse, pulse, and various inputs chosen to simulate target behavior, such as arc-tangent curves or an input expressed as a power series in time t. For inputs of the latter class, it may sometimes be specified that the error coefficients or the error constants, such as the velocity constant or the torque constant, shall have certain maximum or minimum values, respectively. These constants, in a sense, are derived specifications for they actually are chosen with the idea of meeting a certain maximum allowable value of the system error.

(c) Periodic, noise free (only fundamental frequency present).

(1) The output/input frequency response shall be characterized by a specified M_p occurring at a frequency of ω_r radians/sec ; or, alternatively

(2) The output/input magnitude ratio shall be within a band of some specified number of decibels over a specified frequency range, and the phase shift shall not exceed a specified number of degrees over this same frequency range.

(d) Periodic, with noise.

(1) The square root of the sum of the squares of the two components of the system error shall not exceed a specified value. The first component of system error is the magnitude of the error due to a sinusoidal input of specified amplitude and frequency. The second component is the rms value of the output when the input is the noise alone.

(e) *Stochastic, noise free.*

(1) The system error shall not exceed a specified rms value when the input autocorrelation function has a given value.

(f) *Stochastic, with noise.*

(1) The system error shall not exceed a specified rms value when the input-signal autocorrelation function, the input-noise autocorrelation function, and the signal-to-noise crosscorrelation function are given.

System performance in response to a given load, disturbance, or combination of disturbances occurring at points different from the input must also be specified. The load or disturbance can be classed as aperiodic, periodic, or stochastic. Typical load specifications, given in terms of the load signal, are as follows:

(a) *Aperiodic load.*

(1) The output shall not deviate from the desired value by more than a specified amount in the presence of a step change in load of a given value; and

(2) The output shall not deviate from the desired value by more than a specified amount for a steady load of specified value; and

(3) The output shall recover to within a specified value of deviation before the elapse of a given time.

(b) *Stochastic load or disturbance.*

(1) The output due to a stochastic load or disturbance of given autocorrelation function shall not have an rms value in excess of a specified value.

In addition to the specifications on system performance discussed above, the servomechanism design is influenced by environmental conditions. The designer must know the allowable values of such physical properties as over-all servo weight and size. Characteristics of the available source of power must also be known. Important among these characteristics are:

(a) *Form,* such as 60-cps or 400-cps alternating current, hydraulic oil at 3000 psi, compressed air at 10 psi, 440-volt direct current, etc.

(b) *Capacity,* such as 115 volts, 15 amperes, 3000 psi at a maximum of 2 ft^3/min, etc.

(c) *Regulation,* such as 115 $\pm$ 1 volt at 400 $\pm$ 2 cps.

The ambient temperature range is significant since it may necessitate the choice of components having special temperature characteristics. Other ambient conditions such as pressure and humidity may be significant. Applicable military specifications for fulfilling environmental conditions must, of course, be followed in the production model. The constraints imposed by safety requirements should also be known; e.g., flammable hydraulic oil may be forbidden in certain applications. Finally, cost may be an important consideration, especially if large numbers of units are to be constructed.

It may often happen that the specifications given to the servo designer are either incomplete, incompatible, or incomprehensible. Incompatibility has been discussed in the beginning of this chapter. Incomplete specifications are those that fail to limit the designer to any particular servo performance. Incomprehensible specifications must be clarified through consultation with the specifier. In the event that the specifier does not have information about the input signal or disturbance, it may be necessary to take sufficient measurements in order to determine an autocorrelation function. Radar noise may have to be measured and its autocorrelation function found. Typical target paths may have to be decided upon and translated into input-signal variations. In general, designer and specifier have to work together to resolve the difficulties raised by incomplete specifications. As a last resort, where formulation of precise and complete specifications is impossible, estimation based on past experience or on the experience of similar servomechanisms must be used.

As a rule, it is more convenient to carry out the trial design in terms of frequency response as discussed in Chs. 5 and **6.** Since the input-output specifications may be in terms of transient behavior, some means of conversion is useful. Once the synthesis has been completed, analytical determination of the exact transient response will have to be carried out. For example, if transient rise time and output overshoot are specified, the approximate equivalent frequency-response specifications may be deduced through use of the methods outlined in Ch. **7. If** the input is given in terms of a specified time function, and if the maximum allowable system error is specified, it may be feasible to determine the various rates of change of input, estimate the error coefficients that result in a total error within specifications, and then proceed with the design to achieve these error coefficients as outlined in Ch. **7.**

If the performance specifications do not include a degree of stability in terms of overshoot or frequency response, the designer may arbitrarily pick a value of M_p to be used in the trial-and-error method employing frequency-response techniques. **A** value of **1.5** decilogs is a reasonable start for many systems. The effect of varying the M_p on the different performance indices should be assessed.

16-3 CHOICE OF TRIAL COMPONENTS*

16-3.1 GENERAL

Once the servo performance specifications have been assembled, a list of suitable power elements, sensing elements, error detectors, and amplifiers can be drawn up. If the load imposed on the servomechanism is known, the size of the output member or power member may be determined. The procedure suggested by Newton[1] may be followed. The method that leads to a choice of motor and gear train (gear ratio) is summarized in this section. Newton's method is applicable to any sort of power element. (See Chapter **14** for descriptions of various sorts of power elements). It is necessary to characterize the power element in terms of its peak available torque, T_{MP}, its maximum allowable velocity, v_{MP}, and the inertia associated with its output shaft, J_M. Other procedures for choice of output motor can be found in Chestnut and Mayer[5] and Ahrendt.[6]

*See **Appendix** on **page 16-8** for equation derivations.

16-3.2 DETERMINATION OF MOTOR SIZE

To determine motor size, the following trial-and-error procedure is used :

(a) On the basis of the application specifications, compute the load peak power P_{LP} and the load time constant τ_{LP} by means of the following equations :

$$P_{LP} \stackrel{\Delta}{=} v_{LP}\,(J_L a_{LP} + T_{LP}) \qquad (16\text{-}1)$$

$$\tau_{LP} \stackrel{\Delta}{=} \frac{v_{LP}}{a_{LP}} \qquad (16\text{-}2)$$

where

P_{LP} = load peak power

v_{LP} = load peak velocity

J_L = load inertia

a_{LP} = load peak acceleration

T_{LP} = load peak torque other than inertial

τ_{LP} = load time constant

v_{LP}, T_{LP}, a_{LP} are assumed to act at random, and a consistent set of units are used so that τ_{LP} has a dimension of seconds. One consistent set of units might be the following: v_{LP} in radians/sec, T_{LP} in ft-lb, a_{LP} in radians/sec², J_L in ft-lb-sec², and P_{LP} in ft-lb/sec. Other equally correct sets could be used if desired.

(b) Estimate a preliminary figure for the required motor peak power P_{MP}. (This first guess may be taken to be the load peak power multiplied by a factor equal to one plus a fraction indicated by the designer's experience. A good number is **1.3.**)

(c) Referring to data tables for the line of motors under consideration, tentatively choose the motor with the next greater peak power than the estimated figure. Denote this motor peak power by $P_{MP\text{-}ACT}$.

(d) Compute the ratio of the motor time constant τ_{MP} to the load time constant τ_{LP} by use of Eq. **(16-3)** and the value previously found in Eq. **(16-2)**.

$$\tau_{MP} \triangleq \frac{v_{MP}\, T_{MP}}{\left(\dfrac{T_{MP}^2}{J_M}\right)} \triangleq \frac{P_{MP}}{\left(\dfrac{T_{MP}^2}{J_M}\right)} \tag{16-3}$$

where

τ_{MP} = motor time constant

v_{MP} = motor peak velocity available for the motor chosen in (c)

T_{MP} = motor peak torque available for the motor chosen in (c)

J_M = motor inertia for the motor chosen in (c)

P_{MP} = motor peak power $= v_{MP}\, T_{MP}$

and a consistent set of units are used so that τ_{MP} has a dimension of seconds.

(e) Find the minimum motor-to-load peak power ratio P_{MP}/P_{LP} for this time-constant ratio by means of the appropriate relation

$$\frac{P_{MP}}{P_{LP}} \geqq \frac{1}{1-\dfrac{\tau_{MP}}{\tau_{LP}}}, \text{ if } \frac{\tau_{MP}}{\tau_{LP}} \leqq \frac{1}{2}$$

or

$$\frac{P_{MP}}{P_{LP}} \geqq 4\,\frac{\tau_{MP}}{\tau_{MP}}, \text{ if } \frac{\tau_{LP}}{\tau_{LP}} \geqq \frac{1}{2}$$

(f) Using this minimum power ratio and the value of P_{LP} from **Eq. (16-1),** compute the minimum allowable motor peak power $P_{MP\text{-}MIN}$. Find $P_{MP\text{-}ACT}/P_{MP\text{-}MIN}$, which is defined as an "over-power" factor F_o. F_o must be equal to, or greater than, unity. If it is not, return to step (c), pick a larger motor, and repeat steps (d) to (f). Continue until a motor with $F_o \geqq 1$ is found.

16-3.3 DETERMINATION OF GEAR RATIO

The gear ratio R between motor and load can be found from the following equations, which yield minimum and maximum values of R:

$$\left.\frac{R}{\left(\dfrac{v_{MP}}{v_{LP}}\right)}\right|_{\min} = \frac{1}{2}\times\frac{1}{\left(\dfrac{\tau_{MP}}{\tau_{LP}}\right)}\left\{1-\sqrt{1-4\left[\frac{\left(\dfrac{\tau_{MP}}{\tau_{LP}}\right)-\left(\dfrac{\tau_{MP}}{\tau_{LP}}\right)^2}{F_o}\right]}\right\} \tag{16-4}$$

$$\text{for } \frac{\tau_{MP}}{\tau_{LP}} \leqq \frac{1}{2}$$

$$\left.\frac{R}{\left(\frac{v_{MP}}{v_{LP}}\right)}\right|_{min} = \frac{1}{2} \times \frac{1}{\left(\frac{\tau_{MP}}{\tau_{LP}}\right)} \left[1 - \sqrt{1 - \frac{1}{F_o}}\right] \tag{16-5}$$

$$\text{for } \frac{\tau_{MP}}{\tau_{LP}} \geqq \frac{1}{2}$$

$$\left.\frac{R}{\left(\frac{v_{MP}}{v_{LP}}\right)}\right|_{max} = 1 \qquad \text{for } \frac{\tau_{MP}}{\tau_{LP}} \leqq \frac{1 + \sqrt{1 - \frac{1}{F_o}}}{2} \tag{16-6}$$

$$\left.\frac{R}{\left(\frac{v_{MP}}{v_{LP}}\right)}\right|_{max} = \frac{1}{2} \times \frac{1}{\left(\frac{\tau_{MP}}{\tau_{LP}}\right)} \left[1 + \sqrt{1 - \frac{1}{F_o}}\right] \tag{16-7}$$

$$\text{for } \frac{\tau_{MP}}{\tau_{LP}} \geqq \frac{1 + \sqrt{1 - \frac{1}{F_o}}}{2}$$

16-3.4 SELECTION OF OUTPUT MEMBER AND REMAINING ELEMENTS

The proper size motor may be available in either hydraulic or electric form. The choice between the various types of output members must be made on the basis of available power supply and on the relative advantages and disadvantages of the various output members cited in Ch. **14.** Although it is possible to find some hydraulic or electric motor to position any given load, it may be found that the weight of the hydraulic motor is substantially less than that of the electric motor in certain cases (see Newton[1]). However, the total weight of the servo including amplifiers and auxiliaries must be considered; it is possible that the weight of power supplies may lessen the comparative advantage of the hydraulic motor in some applications. Ready availability of energy in convenient form elsewhere in a system may vitally influence the choice of which type of servomotor to use. Ease of maintenance and trouble-free operation are factors which must be considered in the light of expected ambient conditions. ***Up-to-date*** manufacturers' data should be considered for these items as well as in matters of size, weight, and cost of modern component parts. A general comparison **of** motor types which serves to illustrate some of the significant relative factors appears in Table **16-1.** Final choice must be guided by up-to-date manufacturers' information.

Having chosen the output member, the designer must now choose the remaining elements. The set selected must be compatible; e.g., the output member may require force to cause its output to vary. Hence, a source of force (such as a hydraulic amplifier or an electromechanical transducer) must be chosen based on availability and capabiiity of the various types. An amplifier with an output of proper form and capability must then be chosen to drive the force source. The dynamic behavior of the various components chosen for the trial set should be reasonably compatible. For example, if a hydraulic valve and piston are chosen as an output member in order to achieve a servo natural frequency of **30** cps, it would be poor practice to stroke the valve from a synchro differential having a 2-cps natural frequency. On the other hand, if a servo with a narrow bandwidth is desired, the use of expensive hydraulic components with wide frequency response is not recommended. The measuring and error-sensing device (including the reference) must be chosen with particular attention given to accuracy, since the accuracy of the servomechanism is based on that of its measuring device.

TABLE 16-1 COMPARISON OF SERVOMOTORS

	D-C Servomotors	A-C Servomotors	Pneumatic Actuators	Hydraulic Motors	Magnetic-Particle Clutches
Efficiency	High	Low	High	High	Low
Commutators and brushes	Problem	None	None	None	None
Filter required	Electrical Radio Noise Suppression	None	Remove foreign particles	Remove foreign particles	None
Flammability	Low	Low	None	High (unless special oils used)	Low
Explosionhazard	High	Medium	Low	Low	Medium
Coulomb friction, stiction,dead-spot	High	Low	Medium	Medium	Medium
Lifetime	Medium	Long	Medium	Medium	Short

APPENDIX FOR PARAGRAPH 16-3

DERIVATION OF EQUATIONS PERTAINING TO CHOICE OF MOTOR AND GEAR TRAIN

Assumptions

Motor has capability of delivering peak torque T_{MP}

Motor can operate at peak velocity v_{MP}

Motor has an inertia J_M

Load requires a speed-independent torque T_{LP}

Load requires a peak velocity v_{LP}

Load requires a peak acceleration a_{LP}

Load inertia is J_L

Gear-train reduction ratio from motor to load is R.

Units. Any self-consistent set can be used. Two examples:

Torque	Velocity	Acceleration	Inertia
newton-meters	radians/sec	radians/sec^2	kg-meter^2
foot-lbs	radians/sec	radians/sec^2	lb-ft-sec^2

Derivation

Peak motor torque must at least equal total load torque referred to motor; thus:

$$T_{MP} \geqq \frac{J_L a_{LP} + T_{LP}}{R} + RJ_M a_{LP} \tag{1}$$

Peak motor speed must at least equal required load speed referred to motor; thus:

$$v_{MP} \geqq Rv_{LP} \tag{2}$$

From Eq. (2), the gear ratio must be as follows:

$$R \leqq \frac{v_{MP}}{v_{LP}} \tag{3}$$

Eq. (1) sets upper and lower limits on R. Equation (3) sets an independent upper limit on R. In order for the motor to drive the load, the minimum ratio allowed by Eq. (1) must be less than or equal to the maximum value allowed by Eq. (3). Solve Eq. (1) for minimum value of R; thus:

$$R_{min} = \frac{T_{MP}}{2J_M a_{LP}} - \sqrt{\frac{T_{MP}^2}{4J_M^2 a^2_{LP}} - \frac{J_L a_{LP} + T_{LP}}{J_M a_{LP}}} \tag{4}$$

Combine the limits from Eqs. (3) and (4) ; thus:

$$\frac{v_{MP}}{v_{LP}} \geqq \frac{T_{MP}}{2J_M\, a_{LP}} \left\{ 1 - \sqrt{1 - \frac{4a_{LP}\,(J_L\, a_{LP} + T_{LP})}{\frac{T_{MP}^2}{J_M}}} \right\} \tag{5}$$

Note that Eq. (5) involves motor capabilities and load requirements independent of gear train. Two conditions can be found from Eq. (5). First, the radical must be real ; therefore :

$$\frac{T_{MP}^2}{J_M} \geqq 4a_{LP}\,(J_L\, a_{LP} + T_{LP}) \tag{6}$$

To find another condition, rewrite Eq. (5) as follows:

$$\sqrt{1 - \frac{4a_{LP}\,(J_L\, a_{LP} + T_{LP})}{(T_{MP}^2/J_M)}} \geqq 1 - 2\left\{\frac{a_{LP}}{v_{LP}}\right\} \frac{v_{MP}\, T_{MP}}{\frac{T_{MP}^2}{JM}} \tag{7}$$

Two cases arise. *Case I.* If

$$\frac{2a_{LP}}{v_{LP}} \times \frac{v_{MP}\, T_{MP}}{(T_{MP}^2/J_M)} \geqq 1$$

then the right side of Eq. (7) will be less than zero, and so long as the radical is real, Eq. (7) will be satisfied. Thus, for Case I, no additional constraint is found from Eq. (7).

Case II. If

$$\frac{2a_{LP}}{v_{LP}} \times \frac{v_{MP}\, T_{MP}}{T_{MP}^2/J_M} \leqq 1$$

when Eq. (7) is satisfied, then Eq. (6) will automatically be satisfied. To find the conditions for satisfying Eq. (7), square both sides. There results :

$$\frac{a_{LP}}{v_{LP}} \cdot \frac{v_{MP}\, T_{MP}}{T_{MP}^2/J_M} \left\{ 1 - \frac{a_{LP}}{v_{LP}} \times \frac{v_{MP}\, T_{MP}}{\frac{T_{MP}^2}{JM}} \right\} \geqq \frac{a_{LP}\, J_L\, a_{LP} + T_{LP}}{T_{MP}^2/J_M} \tag{8}$$

To summarize :

Case I. If

$$\frac{a_{LP}}{v_{LP}} \times \frac{v_{MP}\, T_{MP}}{T_{MP}^2/J_M} \leqq \frac{1}{2}$$

then

$$(v_{MP}\, T_{MP}) \left\{ 1 - \left(\frac{a_{LP}}{v_{LP}}\right) \frac{v_{MP}\, T_{MP}}{T_{MP}^2/J_M} \right\} \geqq v_{LP}\,(J_L\, a_{LP} + T_{LP}) \tag{9}$$

Case 11. If

$$\frac{a_{LP}}{v_{LP}} \times \frac{v_{MP}\, T_{MP}}{T_{MP}^2/J_M} \geqq \frac{1}{2} \tag{10}$$

then T_{MP}^2/J_M must be greater than $4a_{LP}\,(J_L\, a_{LP} + T_{LP})$.

The requirements for Cases I and II can be rewritten by multiplying the equations respectively by

$$\frac{1}{v_{LP}\left(J_L\, a_{LP} + T_{PL}\left[1 - \frac{a_{LP}}{v_{LP}} \times \frac{v_{MP}\, T_{MP}}{T_{MP}^2/J_M}\right]\right)} \tag{11}$$

and

$$\frac{v_{MP}\, T_{MP}}{w_{LP}\,(J_L\, a_{LP} + T_{LP})\left(\frac{T_{MP}^2}{J_M}\right)} \tag{12}$$

Now, define certain characteristic times or time constants. Thus:

$$\tau_{MP} = \frac{v_{MP}\, T_{MP}}{T_{MP}^2/J_M}$$

$$\tau_{LP} = v_{LP}/a_{LP}$$

Also define peak-load and peak-motor powers as:

$$v_{LP}\,(J_L\, a_{LP} + T_{LP}) = P_{LP}$$

$$v_{MP} \times T_{MP} = P_{MP}$$

In terms of these constants, the rewritten Cases I and II become:

Case I.

$$\frac{P_{MP}}{P_{LP}} \geqq \frac{1}{1 - \frac{\tau_{MP}}{\tau_{LP}}}, \text{ if } \frac{\tau_{MP}}{\tau_{LP}} \leqq \frac{1}{2} \tag{13}$$

Case II.

$$\frac{P_{MP}}{P_{LP}} \geqq 4\,\frac{\tau_{MP}}{\tau_{LP}}, \text{ if } \frac{\tau_{MP}}{\tau_{LP}} \geqq \frac{1}{2} \tag{14}$$

The results are used in the manner described in Par. **16-3.2.**

Gear Ratio

The following derivation is for the equations used in the choice of a gear ratio. As mentioned in the chapter, it is always possible to find a motor to drive any load as specified. Then it is only necessary to find a gear ratio. The procedure ensures that once the motor is found, a realizable gear ratio will result.

Multiply Eq. (1) by the factor

$$\frac{\left(\frac{v_{MP}}{P_{MP}}\right)(R)}{\left(\frac{\tau_{MP}}{\tau_{LP}}\right)\left(\frac{v_{MP}}{v_{LP}}\right)}$$

and rearrange Eq. **(2).** Introduce the time constants.

The result is :

$$\left(\frac{1}{\frac{\tau_{MP}}{\tau_{LP}}}\right)\left(\frac{R}{\frac{v_{MP}}{v_{LP}}}\right) \geqq \frac{1}{\left(\frac{\tau_{MP}}{\tau_{LP}}\right)} \times \frac{1}{\left(\frac{P_{MP}}{P_{LP}}\right)} + \left(\frac{R}{\frac{v_{MP}}{v_{LP}}}\right)^2 \tag{15}$$

and

$$\frac{R}{v_{MP}/v_{LP}} \leq 1 \tag{16}$$

Consider $\frac{R}{v_{MP}/v_{LP}}$ a nondimensional gear ratio and combine Eqs. **(15)** and **(16)** so that the result is:

$$\frac{1-\sqrt{1-\frac{4\,(\tau_{MP}/\tau_{LP})}{P_{MP}/P_{LP}}}}{2\left(\frac{\tau_{MP}}{\tau_{LP}}\right)} \leqq \left\{\frac{R}{v_{MP}/v_{LP}}\right\} \leqq \left|\begin{matrix} 1 \\ \text{or} \\ \frac{1+\sqrt{1-\frac{4\left(\frac{\tau_{MP}}{\tau_{LP}}\right)}{P_{MP}/P_{LP}}}}{2\,(\tau_{MP}/\tau_{LP})} \end{matrix}\right| \tag{17}$$

The lower value of the two upper limits sets the upper bound on R. Define a motor over power factor as

$$F_o = \frac{P_{MP\text{-}ACT}}{P_{LP}} \times \frac{1}{\left.\frac{P_{MP}}{P_{LP}}\right|_{min}} \tag{18}$$

where $\left.\frac{P_{MP}}{P_{LP}}\right|_{min}$ is the minimum allowable power ratio and $\frac{P_{MP\text{-}ACT}}{P_{LP}}$ is the actual power ratio for the motor chosen. From Eqs. **(13)** and **(14)**, it is seen that

$$\left.\frac{P_{MP}}{P_{LP}}\right|_{min} = \frac{1}{1 \quad \frac{\tau_{MP}}{\tau_{LP}}}$$

for the case where

$$\frac{\tau_{MP}}{\tau_{LP}} \leqq \frac{1}{2}$$

and

$$\left.\frac{P_{MP}}{P_{LP}}\right|_{min} = 4\,\frac{\tau_{MP}}{\tau_{LP}}$$

for the case where

$$(\tau_{MP}/\tau_{LP}) \geqq \frac{1}{2}$$

Solve Eq. **(18)** for the actual value of P_{PM}/P_{LP}; thus :

$$\frac{P_{MP\text{-}ACT}}{P_{LP}} = F_o \frac{P_{MP}}{P_{LP}}\bigg|_{min}$$

Substitute the values for $\frac{P_{MP}}{P_{LP}}$ into **Eq. (19)** to find :

$$\frac{P_{MP\text{-}ACT}}{P_{LP}} = \frac{F_o}{1 - \frac{\tau_{MP}}{\tau_{LP}}}, \text{ if } \frac{\tau_{MP}}{\tau_{LP}} \leqq \frac{1}{2}$$

and

$$\frac{P_{MP\text{-}ACT}}{P_{LP}} = 4\,F_o\,\frac{\tau_{MP}}{\tau_{LP}}, \text{ if } \frac{\tau_{MP}}{\tau_{LP}} \geqq \frac{1}{2} \qquad \textbf{(20)}$$

Substitute Eq. **(20)** into Eq. **(17).** The two minimum values of $R/(v_{MP}/v_{LP})$ are found directly from the left-hand side of Eq. **(17).** The maximum values of $R/(v_{MP}/v_{LP})$ are found from either the upper or lower inequality of the right-hand side of Eq. **(17).** The condition as to whether the upper or lower inequality of Eq. **(17)** holds depends upon which of the two is the greater. These results all appear as Eqs. **(16-4)** through **(16-7).**

16-4 ANALYSIS OF TRIAL SYSTEM

When the trail system has been chosen, analysis of its performance is the next step in the design procedure. As outlined in Ch. *6,* the trial-and-error design method starts with the simplest form of compensation, namely, a pure gain. Stability is checked. If the system is unstable, suitable stabilization networks are added, using either cascade or parallel compensation. Then the gain is adjusted to meet the degree-of-stability specification. At this point, the system and its compensation are defined and the performance can be analyzed to verify whether the error specifications are met. If the error specifications are not met, the compensation is modified so as to permit an increase in gain while maintaining the specified degree of stability. Again, the performance is analyzed and the error specifications are checked. This procedure is continued until all specifications have been met. Since there is no guarantee that the trial system can be forced to meet the specifications, the designer should become suspicious if unduly complex compensation functions do not produce satisfactory performance. This is so because the idealizations required to obtain a mathematical model for the physical hardware neglect secondary effects that become primary when excessively complex compensation is attempted.

16-5 MODIFICATION OR REDESIGN OF TRIAL SYSTEM

If, after trying a few simple compensation functions, the designer finds that the performance of the trial system is unsatisfactory, he may choose one of two possible directions. The trial system may be modified by means of new components that give promise of better performance than those used in the initial trial. An example of this is the replacement of an electric drive by a hydraulic drive to obtain a faster output member. Alternatively, the designer may suspect that the specifications are either incompatible or cannot be met by any known system. Since there is no way the incompatibility can be disclosed **by** the trial-and-error method, the designer is forced, in practice, to try various new components. If no available components lead to a system where the specifications are met, the designer can only conclude that the specifications are too stringent and ask for their revision. Should this be deemed impossible, two courses of action remain: (1) present the problem to a different design group in the hope that a fresh approach will lead to success; or (2) design and build new components that will enable specifications to be met. Because of the time, cost, effort, and nonguarantee of success, this final approach should be used only as a last resort. It is conceivable, **of** course, that the present state of the art is such that it may be impossible to meet a certain set of specifications.

16-6 CONSTRUCTION AND TEST OF EXPERIMENTAL EQUIPMENT

Once the design has been completed and the specifications apparently met, the next step is to obtain a set of the proposed components and then proceed to assemble and test. Construction of experimental equipment may be facilitated by the use of commercially available clamps, motor brackets, and various prefabricated parts such as base plates, bearing supports, standard shafting, gears, etc.

Dynamic and static testing should be carried out under conditions as similar as possible to those to be met in the field. Environmental testing should also be performed.

16-7 TRANSLATION OF EXPERIMENTAL EQUIPMENT INTO PRODUCTION MODEL

When the experimental model has been tested and the designer is satisfied that the specifications have been met, the specifications for a prototype should be drawn based on the components used in the experimental model. The prototype will differ from the experimental model in that production-type construction will be employed wherever possible. It should be designed so that the production model can be made essentially identical. Standard components meeting military specifications should be used, and the prototype should be subjected to performance and environmental tests. Shock testing should also be carried out and accelerated life tests made. It is important that every effort be put forth to expose component weaknesses at this point. Deterioration of performance with aging should be such that the specified performance is still available.

When the designer is satisfied that the prototype meets all specifications, he may draw specifications for the production model and let contracts. Servos taken from actual production should be rigorously tested to ensure that all specifications are met. Acceptance tests should be made in accordance with accepted practice.

16-8 ILLUSTRATIVE EXAMPLE — DESIGN OF A SERVO DATA REPEATER

16-8.1 NATURE OF THE MEASUREMENT PROBLEM

Accuracy assessment of modern automatic fire-control systems has posed a number of difficult instrumentation problems. In the assessment method commonly used for manned aircraft at the present time, data for analysis are taken during nonfiring engagements, and it is necessary to measure and record a number of physical angles in the fire-control system under test very accurately and with a high degree of resolution.

A typical example will serve to illustrate the nature of the requirements. Suppose it is desired to sample the azimuth and elevation angles of a moving radar antenna a number of times per second, with an accuracy of ±0.5 milliradian throughout a total angular coverage of 120° in each axis. An over-all dynamic accuracy of ±**0.03°** and a resolution of approximately one part in **4,000** are therefore required in a process which includes: measuring; recording in the aircraft; and later, reading on the ground for data analysis. Recording is the most critical part in this process since there are chances for error in transferring measurements to the storage medium, in the medium itself, and in reading out of the medium.

Of the available recording methods, analog recording by means of direct-writing oscillographs or physical displacement of images on photographic film is out of the question where such accuracy and resolution are concerned. The alternative is a numerical, or digital, representation which permits storage and recovery without loss of accuracy. One common method of digital recording is to display the desired information numerically on a dial where it can be photographed and later recovered without loss of accuracy. In most present applications, fire-control instrumentation takes this form. Since the dial can seldom be connected directly to the device to be measured, there remains the problem **of** driving a remote indicating dial so that it repeats accurately the position of the device being measured.

Environmental conditions for airborne equipment are severe and variable. They include large temperature changes under different operating conditions, continuous vibration, acceleration loadings, and unstable voltage and frequency of power sources. An analysis of simple synchro transmitter-motor links to drive the indicating dials under these conditions quickly led to their rejection. The problem is not with the electrical accuracy of the synchro motors, but rather with the low torque gradient available to position the rotor. As a result, large errors occur if shaft friction or disturbing forces exist. Also, the low torque that is available limits the acceleration capability of the rotor and therefore the frequency response of the indicator.

If a synchro is used only as the error detector in a feedback-control loop, the error signals can be used to control any level of torque desired, and the torque gradient can be made almost infinite compared with the synchro motor. This section describes the design of a servo repeater to drive the dials indicating antenna azimuth and elevation angles.

16-8.2 DESIGN OF THE SERVO

16-8.3 Accuracy Determinations

16-8.4 Antenna characteristics. The radar antenna whose position is to be indicated has a maximum velocity of **120** deg/sec and a maximum acceleration of 600 deg/sec^2. The performance of the servo repeater should equal **or** exceed these values. Also, the antenna servo-drive bandwidth (defined as the frequency at which antenna-drive phase lag is **60")** is 5 cps. If the repeater is to follow antenna motions up to 5 cps faithfully, no amplitude peaking and negligible phase shift (less than 5") in this band are desired. The repeater must therefore have a bandwidth several times that of the antenna.

16-8.5 Synchro accuracies. The antenna is equipped with 27-speed and 1-speed synchro transmitters. These are 400-cps types with a maximum error of ± 7 minutes of arc for each synchro. If the gear errors in the antenna and in the repeater are neglected, the greatest synchro error, when a 27-speed transmitter is used at the antenna and a 27-speed control transformer is used at the repeater, is

$$\frac{2 \times 7}{27} \sim \frac{1}{2} \text{ minute}$$

$$\approx 0.008''$$

referred to the 1-speed shaft. This is well above the required **0.03"** system accuracy and hence no special synchro selection is necessary.

16-8.6 Dials. In order to rear the repeater output without introducing excessive errors, two dials are used. For ease in reading, one turns at 36 times antenna speed (10' of antenna motion/dial revolution), permitting direct division of the dial at 0.1' intervals. A vernier then permits accurate readings to 0.01°, again well above the required system accuracy, provided that the dials themselves are absolutely accurate. Although even higher speed dials would make reading easier, gear errors between the finest dial and the 27-speed synchro would increase, and the system inertia would become excessive, since the inertia of each dial must be multiplied by the square of the dial gear ratio.

16-8.7 Friction and drift. Friction levels in a mechanism are difficult to estimate in advance. Knowledge of the friction ratings for the synchros and the servomotor and estimates for the gear train indicate that angular errors are negligible if a very high torque gradient is obtained in the servo. Hence, a tolerance of 0.005" is tentatively established for errors due to static or running friction. Drift is also considered a serious problem because of the environmental conditions, and its reduction becomes a major factor in selecting the method of servo compensation.

16-8.8 Dynamic errors. The greatest sum of the forementioned static errors is 0.008' + 0.01° + *0.005"* = 0.023°, leaving 0.007° for dynamic errors caused by constant velocity inputs and input accelerations. Maximum possible error is used as a criterion, rather than an rms combination of errors, to allow a margin of safety. Since the highest input velocity for which accurate following is desired is 20 deg/sec, the required velocity constant is

$$K_v = \frac{20\ \mathrm{deg/sec}}{0.007''} \approx 2860\ \mathrm{sec}^{-1}$$

Since no information is available on sustained antenna accelerations likely during airborne tests, it is therefore specified only that repeater acceleration capability exceed that of the antenna.

16-8.9 Choice of Servo Compensation Method

In order to meet the requirements of extreme accuracy and stability in the repeater design, the obvious choice is an all a-c system with synchro data transmission. D-c compensating networks are discarded because of drift in the associated modulators and demodulators. Carrier compensation is also considered a poor choice because the a-c supply frequency in aircraft is not held to close limits. Three remaining possibilities are: tachometer feedback, viscous damping, and viscous-coupled inertia damping. The form of compensation chosen for this application after a complete study of the requirements was the viscous-coupled inertia damper.[2]

A simple damper consists of an inertia slug coupled viscously to the motor shaft. In one practical form shown in Fig. 16-1, the heavy inertia slug is mounted on ball bearings riding on the motor shaft and is completely surrounded by a light fluid-tight shell pinned directly to the motor shaft. The space between the shell and the inertia slug is filled with a silicone damping fluid. A practical way of looking at the damper is that, for small high-frequency oscillation, the damper acts like a direct viscous drag to stabilize the

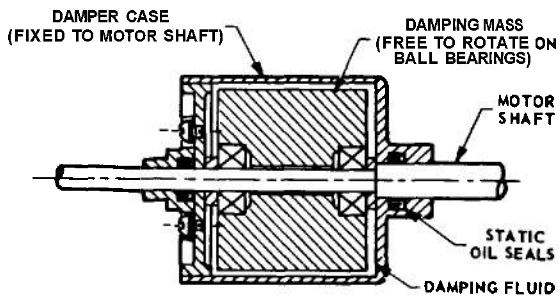


Fig. 76-1 Viscous-coupled inertia damper.

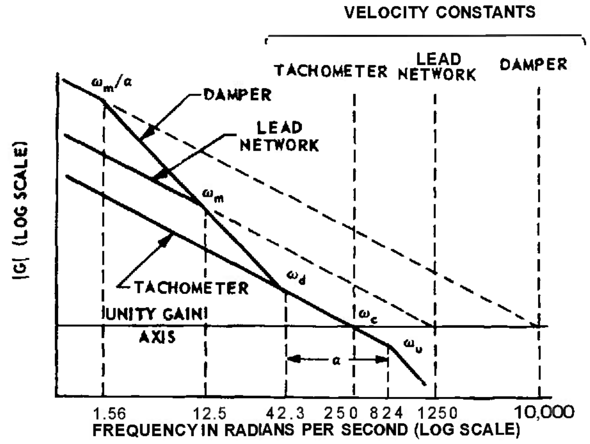
From *1954 National Telemetering Conference Record,* from article entitled "A Damper Stabilized Servo Data Repeater", by J. E. Ward.

servo. At sustained motor velocities, however, the slug eventually attains the same speed as the motor. This property overcomes the major disadvantage of direct viscous friction damping because no power is dissipated at constant velocities.

Because the damper is a load compensation device, rather than a cascade or feedback device, the amplifier need provide only a-c gain from the synchro to the a-c control motor. This feature minimizes any possibility of drift, which was one of the design goals.

The transfer function for a damper has the same form as that for a conventional d-c lead network and is derived in Par. 16-8.10. For equal servo bandwidths, however, the damper-compensated servo has considerable advantage over both lead-network compensation and tachometer compensation. A lead network provides a high velocity constant and a relatively low torque constant, whereas tachometer feedback provides a high torque constant and a relatively low velocity constant. A damper provides both a high torque constant and a very high velocity constant. Figure 16-2 compares the magnitudes of typical loop transfer functions for all three types of compensation, with the gains adjusted for the same servo bandwidths, and indicates the relative velocity constants that are obtained. Because of the large increase in motor-shaft inertia, the damper provides a transfer function that can only be duplicated by either a combination of tachometer feedback and an integral network or a tachometer feedback with a high-pass filter, both of which would require d-c networks. Figure 16-3 compares the magnitudes of the torque-error transfer functions and indicates that the lead network provides a smaller torque constant than a damper or tachometer except at very high frequencies. A high torque constant at zero frequency is desirable to combat friction errors.

The damper has an additional advantage in that it directly attenuates any system noise that may have reached the motor shaft.



ω_m = MOTOR BREAK FREQUENCY

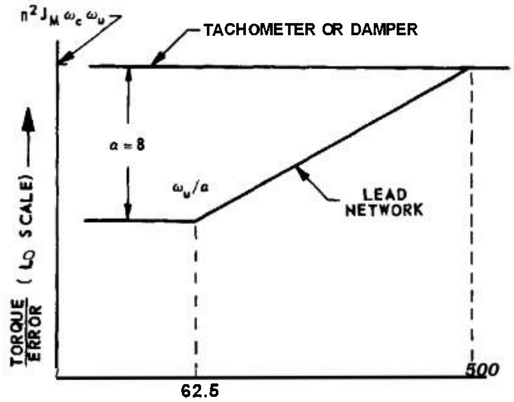
ω_d = LOWER NETWORK BREAK FREQUENCY

ω_u = UPPER NETWORK BREAK FREQUENCY

ω_c = GAIN-CROSSOVER FREQUENCY

a = ATTENUATION RATIO OF NETWORK

Fig. 76-2 Comparison of loop transfer functions.



ω_c = GAIN-CROSSOVER FREQUENCY

a = ATTENUATION RATIO OF NETWORK

n = GEAR RATIO

J_M = TOTAL MOTOR-SHAFT INERTIA

ω_M = UPPER NETWORK BREAK FREQUENCY

Fig. 76-3 Comparison of torque constants.

An outstanding feature of a damper-compensated servo is extreme smoothness of operation. Also, because of the noise attenuation, a damper servo is much more tolerant than any other servo type with regard to backlash between the motor and the feedback synchro. The penalty which must be paid is that the additional inertia added to the system seriously limits the accelerating capabilities of the servo motor and, in order to maintain load accelerations, the gear ratio between load and servomotor must be kept quite low compared with normal practice. However, this is not a serious objection in instrument servos, which usually do not require high torque levels at the output shaft. Also, the damper attenuates the effects of motor cogging and other motor-torque irregularities that are usually objectionable at low gear ratios.

The damper-compensated servo is insensitive to power-supply voltage changes, since a $\pm$ 2:1 change in loop gain is possible with negligible change in performance. Also, because there are no frequency-sensitive elements such as lead networks, the servo is insensitive to changes in the prime power frequency which excites the synchros and the motor. The servo operates without noticeable change for power frequencies of 400 $\pm$ 100 cps. The high torque constant makes the repeater insensitive to vibration. The only remaining problem, therefore, is the effect of temperature on the viscous damping fluid. Silicone fluid has excellent temperature characteristics, and a variation of $\pm$ 50°F results in negligible performance change in the servo. For airborne applications, a heater and thermostat are generally used to keep the damper at 160 $\pm$ 25°F.

16-8.10 Design of the Damper-Stabilized Servo

16-8.11 Detailed analysis. The functional block diagram of the servo data repeater is shown in Fig. 16-4. A fine dial diameter of

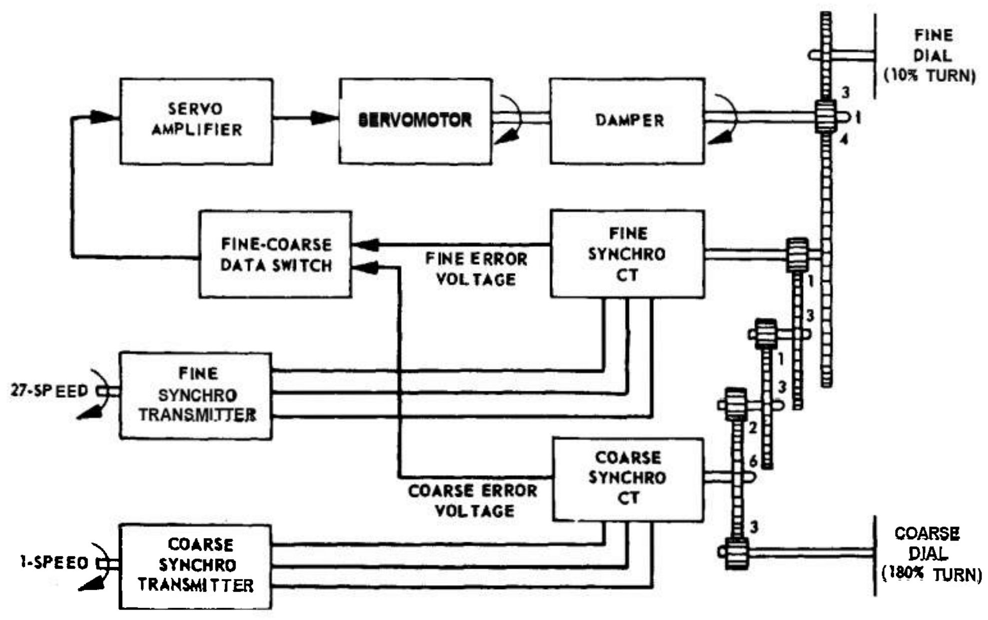


Fig. 16-4 Functional block diagram of servo data repeater.

From *1954 National Telemetering Conference Record,* from article entitled "A Damper Stabilized Servo Data Repeater", by J. E. Ward.

2 inches is chosen **for** readability with a vernier to **1** part in **1000,** and, since the dial turns at 36-speed, fine dial inertia becomes the predominant factor in choosing the motor gear ratio. It was later found that **a** gear ratio of 3:1 from motor to dial, giving a motor speed of 108 times antenna speed, maximized the output acceleration. In the following, output refers to the 1-speed synchro shaft of the repeater, which corresponds to antenna speed. Gear ratios are indicated in Fig. 16-4. The first step in analyzing the system is to derive the transfer function for the motor and damper. **(A** pictorial representation of the damper-motor is shown **in** Fig. 16-5.) The following nomenclature is used in the derivation:

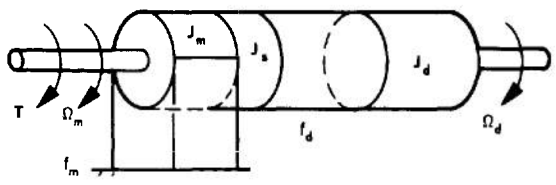


fig. 76-5 Pictorial diagram of *damper-mofor.*

Adapted by permission from *Transaction8* of *the ASME,* Volume 79, April, 1967, from article entitled 'A Dual-Mode Damper-Stabilized Servo', by J. Jursik, J. F. Kaiser, and J. E. Ward.

a_o = maximum output acceleration, in rad/sec^2

f_d = damping constant of damper, in in.-oz-sec/rad

f_m = internal damping of motor, in in.-oz-sec/rad

n = gear ratio from motor shaft to output

J_d = inertia of damper slug, in in.-oz-sec^2/rad

J_m = inertia of motor rotor, gears, and load in in.-oz-sec^2/rad

J_M = total motor-shaft inertia $= J_m + J_s$

J_s = inertia of damper shell, in in.-oz-sec^2/rad

K_1 = 1/gear ratio from fine synchro to output

K_2 = synchro output constant, in volts/deg

K_3 = amplifier gain

K_4 = conversion factor from radians to degrees = $180/\pi$

K_t = system torque constant (output torque per deg error), in in.-oz/deg

K_T = motor torque constant, in in.-oz/volt

K_v = system velocity constant, in sec^{-1}

s = Laplace operator

T = internal motor torque, in in.-oz

T_s = stall torque of motor at rated control voltage, in in.-oz

V = applied motor voltage, in volts

α = attenuation ratio defined by Eq. (16-15)

ϵ = servo error, in deg

ϵ_s = saturation error, in deg

θ_i = input (antenna) position, in deg

θ_m = motor angular position, in rad

θ_o = output position, in deg

T_d = major velocity build-up time constant, in sec

Ω_d = damper angular velocity, in rad/sec

Ω_m = motor angular velocity, in rad/sec

Ω_s = maximum velocity of motor, in rad/sec

Ω_o = maximum output velocity, in deg/sec

ω_m = motor break frequency, in rad/sec

ω_d = damper break frequency, in rad/sec

ω_c = gain-crossover frequency, in rad/sec

From Fig. **16-5,** the transfer function for the motor-damper is derived as follows:

$$T = K_T V \tag{16-8}$$

$$T = (J_m + J_s)s\Omega_m + f_d(\Omega_m - \Omega_d) + f_m\Omega_m \tag{16-9}$$

$$f_d(\Omega_m - \Omega_d) = J_d s \Omega_d \qquad (16\text{-}10)$$

Eliminating Ω_d gives

$$\frac{\Omega_m}{T} = \frac{s + \dfrac{f_d}{J_d}}{(J_m + J_s)\left[s^2 + s\left(\dfrac{f_d}{J_m + J_s} + \dfrac{f_d}{J_d} + \dfrac{f_m}{J_m + J_s}\right) + \dfrac{f_m f_d}{(J_m + J_s) J_d}\right]} \qquad (16\text{-}11)$$

Eq. (16-11) can be written more simply in the approximate form

$$\frac{\Omega_m}{T} = \frac{(s + \omega_d)}{(J_m + J_s)\left(s + \dfrac{\omega_m}{\alpha}\right)(s + \alpha\omega_d)} \qquad (16\text{-}12)$$

where

$$\omega_d = \frac{f_d}{J_d} \qquad (16\text{-}13)$$

$$\omega_m = \frac{f_m}{J_m + J_s} \qquad (16\text{-}14)$$

$$a = \frac{J_m + J_s + J_d}{J_m + J_s} \qquad (16\text{-}15)$$

Further

$$\frac{\theta_m}{V} = \frac{K_T}{s} \times \frac{\Omega_m}{T} \qquad (16\text{-}16)$$

The motor-damper transfer function is therefore

$$\frac{\theta_m}{V} = \frac{K_T(s + \omega_d)}{s(J_m + J_s)\left(s + \dfrac{\omega_m}{\alpha}\right)(s + \alpha\omega_d)} \qquad (16\text{-}17)$$

The operational block diagram for the complete system is shown in Fig. 16-6. The constants have the following values:

$K_1 = 27$
$K_2 = 0.4$ volt/deg
$K_3 = 375$
$K_4 = 57.3$ deg/rad
$\omega = 2\pi f$
$n = 108$
$K_T = 0.016$ in.-oz/volt
$J_m = 0.72 \times 10^{-4}$ in.-oz-sec²/rad
$J_s = 1.41 \times 10^{-4}$ in.-oz-sec²/rad
$J_d = 1.69 \times 10^{-3}$ in.-oz-sec²/rad
$f_m = 1.23 \times 10^{-3}$ in.-oz-sec/rad
$f_d = 0.156$ in.-oz-sec/rad
$T_s = 2.4$ in.-oz
$\Omega_s = 600$ rad/sec

Using these values, the damper-motor transfer function of Eq. (16-17) is found to be

$$\frac{\theta_m}{V} = \frac{75(s + 92.3)}{s(s + 0.65)(s + 824)} \qquad (16\text{-}18)$$

The remaining constants are

$$\frac{K_1 K_2 K_3 K_4}{n} = 2.15 \times 10^3 \text{ motor volts/motor radian}$$

so that the complete loop transfer function is

$$\frac{\theta_o}{E} = \frac{161.25 \times 10^3 (s + 92.3)}{s(s + 0.65)(s + 824)} \qquad (16\text{-}19)$$

The velocity constant is found by multiplying through by s, and then letting s go to zero:

$$K_v = \frac{s\theta_o}{\varepsilon} = \frac{\Omega_o}{E} \qquad (16\text{-}20)$$

$$= \frac{(161.25)(92.3) \times 10^3}{(0.65)(824)}$$

$$\approx 27{,}800 \text{ sec}^{-1}$$

However, this value depends greatly upon the assumptions made concerning the motor damping f_m and upon friction effects in the gearing. In practice, the K_v for this servo is difficult to measure, but is found to exceed

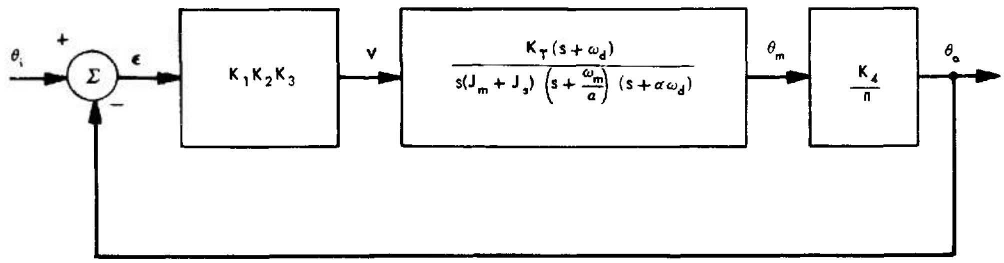


Fig. 16-6 Operational block diagram for servo data repeater.

By permission from *Transactions* of *the ASME*, Volume 79, April, 1957, from article entitled 'A Dual-Mode Damper-Stabilized Servo', by J. Jursik, J. F. Kaiser, and J. E. Ward.

10,000 sec^{-1}. Thus, error for the maximum input velocity of 120°/sec is only 0.012°. The torque constant at the output shaft is

$$K_t = K_T K_1 K_2 K_3 (n) \tag{16-21}$$

$$= (0.016)(27)(0.4)(375)(108)$$

$$\approx 7000 \text{ in.-oz/deg}$$

This value is so large that friction errors in the servo are indeed negligible.

The linear range of the servo is found by determining the error ε_s at which the amplifier output voltage saturates (reaches maximum value). For the amplifier used, the saturation voltage V_s is 150 volts. Therefore

$$\varepsilon_s = \pm \frac{V_s}{K_1 K_2 K_3} \tag{16-22}$$

$$= \pm \frac{150}{(27)(0.4)(375)}$$

$$= \pm 0.037'$$

This corresponds to a motor-shaft angle of only 20.037 × 108, or ±4°.

Perhaps the best measure of the system bandwidth is the gain-crossover frequency ω_c at which the magnitude of the loop transfer function θ_o/ε is equal to unity. This value depends upon the loop-gain setting and is found to be approximately

$$\omega_c = 250 \text{ rad/sec} \approx 40 \text{ cps}$$

for best performance. The acceleration constant of the system is equal to

$$K_A = \omega_c \omega_d \tag{16-23}$$

$$= (250)(92.3)$$

$$\approx 23{,}000 \text{ rad/sec}^2\text{/rad error}$$

Finally, the maximum output velocity is

$$\Omega_o = \frac{\Omega_s}{n} \tag{16-24}$$

$$= \frac{600 \text{ rad/sec}}{108}$$

$$= 5.56 \text{ rad/sec} = 320 \text{ deg/sec}$$

and the maximum output acceleration is

$$a_o = \frac{T_s}{J_m + J_s + J_d} \times \frac{1}{n} \tag{16-25}$$

$$= \frac{2.4}{(0.72 + 1.41 + 16.9) \times 10^{-4} \times (108)}$$

$$= \frac{24{,}000}{(19.03)(108)}$$

$$= 11.7 \text{ rad/sec}^2 \approx 670 \text{ deg/sec}^2$$

Both of these figures are in excess of those required.

16-8.12 Performance of the completed servo. The servo data repeater is shown in Fig. **16-7.** The mechanism is packaged in **a** 3-inch diameter aircraft instrument-type case, requires a 9-inch space behind the panel, and

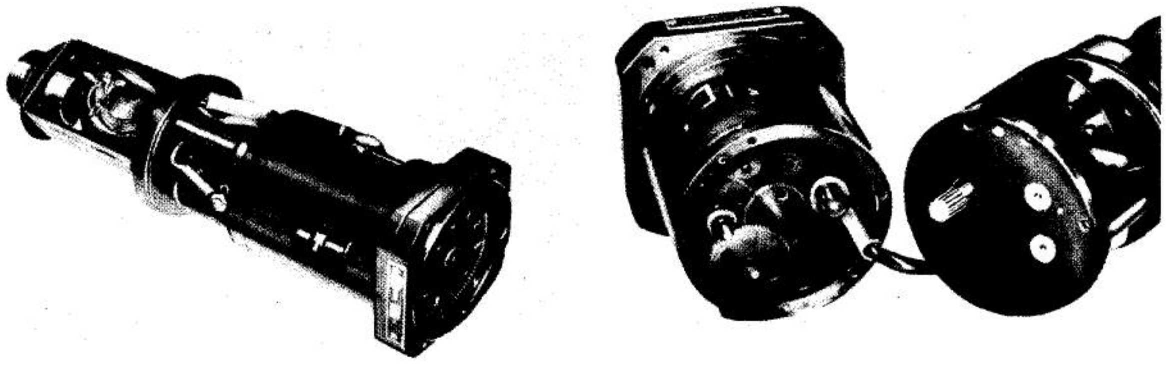


Fig. 16-7 Servo data repeater.

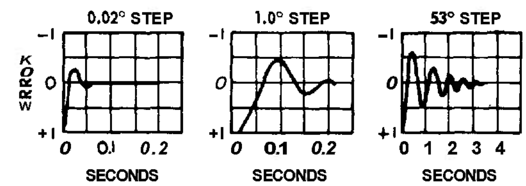
From *1954 National Telemetering Conference Record,* from article entitled "A Damper Stabilized Servo Data Repeater", by J. E. Ward.

weighs 3.5 pounds. The unit has a fine dial that indicates 10° per revolution in 0.1' increments, and a coarse dial that indicates 120° in 10° increments (total coarse dial travel is 180'). The vernier permits fine dial readings to 0.01'. Reference to Fig. 16-4 shows that there are only two important gear meshes in the repeater —that between the motor and the 27-speed synchro, and that between the motor and the fine dial. Backlash in these meshes represents an error of less than 0.005' in the dial indication. Dial engraving errors are about 0.01° at the worst point.

A conventional a-c amplifier with transformer output is used to drive the repeater. A gain of 375 is required, and a fine-coarse switching circuit is provided for 1-speed synchronization. The amplifier is about 4 $\times$ 5 $\times$ 6 inches in size. Magnetic amplifiers can also be used, but servo bandwidth must be reduced as a result of time lags in the magnetic amplifier. Also, magnetic amplifiers usually have a d-c drift, which introduces error.

The transient response of the repeater to a small step within the linear range is shown in Fig. 16-8. Also shown are the responses to larger steps, which indicate the deterioration that occurs with increasing saturation in the system. The long settling time for large input steps, such as are encountered in initial synchronization, is not critical for this application.

The repeater has been used successfully for the original antenna follow-up application and for many others. A later variation in the design substitutes a 10-turn potentiometer for the synchros as the feedback element, and an accuracy of 1 part in 4000 has been achieved in measuring voltage ratios.



Adapted by permission from *Transactions of the ASME,* Volume 79, April, 1957, from article entitled 'A Dual-Mode Damper-Stabilized Servo', by J. Jursik, J. F. Kaiser, and J. E. Ward.

Fig. 16-8 Transient response of servo data repeater to various steps.

16-8.13 Improvement by using dual-mode operation. The large-step response of the repeater is rather poor because kinetic energy stored in the damper slug during slew conditions cannot be dissipated rapidly by the limited motor torque available. One solution is to use a separate mode of operation for large transients, in which case the damper is temporarily disconnected to prevent energy storage. An error-actuated clutch is used to couple the damper to the motor.

A cross-sectional view of the dual-mode damper package attached to a servomotor is shown in Fig. 16-9. The damper consists of a sintered-tungsten slug mounted on bearings within a spun-aluminum case filled with a silicone damping fluid. The aluminum damper case is supported by two bearings — one fixed in an extension of the motor housing, and one running on the motor shaft. The clutch is composed of a small collet attached to the damper case. The collet can be expanded into a drum attached to the motor shaft.

Normally, the clutch is kept engaged by a spring that acts on a push rod so as to expand the collet. The means for controlling this clutch is provided by the solenoid, which releases the collet by acting on the push rod in opposition to the spring. A 4-pound force is required to release the clutch, and the necessary push-rod travel is 0.005 inch. The solenoid is constructed very much like a telephone receiver. It has a circular armature, which is pulled down toward the center post when the coil is energized. The return spring for the armature is provided by the thin diaphragm attached to the armature at its center. The clutch can be disengaged by applying approximately 2.5 watts to the solenoid coil. The operating time for the clutch

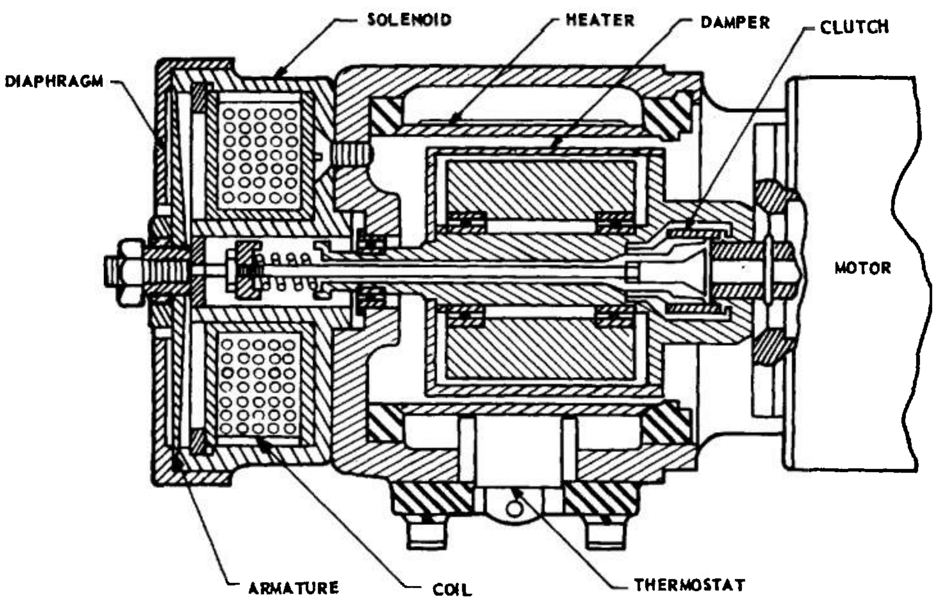


Fig. 16-9 Cross *section* of *clutch-damper.*

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is less than 10 milliseconds. Adjustments are provided for setting the maximum slip torque of the clutch and for setting the free play between the solenoid armature and the push rod. The total armature travel is established by grinding the center post during manufacture. The entire assembly is **2-3/8** in. long by **2** in. in diameter and has been designed to operate with a Mk 8 400-cps servomotor. **A** view of the complete dual-mode package is shown in Fig. 16-10. The functional block diagram of the dual-mode servo is shown in Fig. 16-11.

As stated previously, the transient performance of a damper-stabilized servo for large signals, such as those encountered in synchronizing to a new position, is sometimes undesirable. A typical large-signal transient

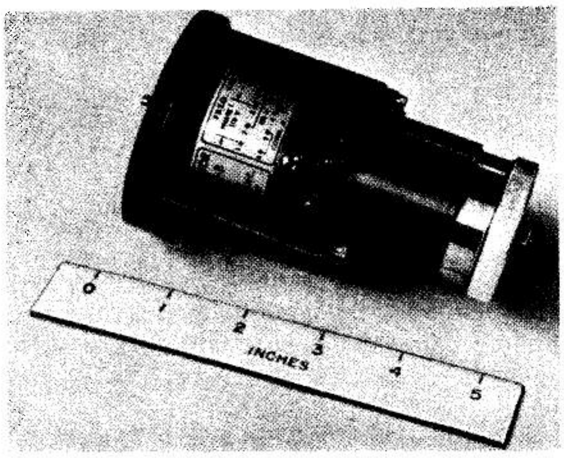

Fig. 16-10 Dual-mode package.

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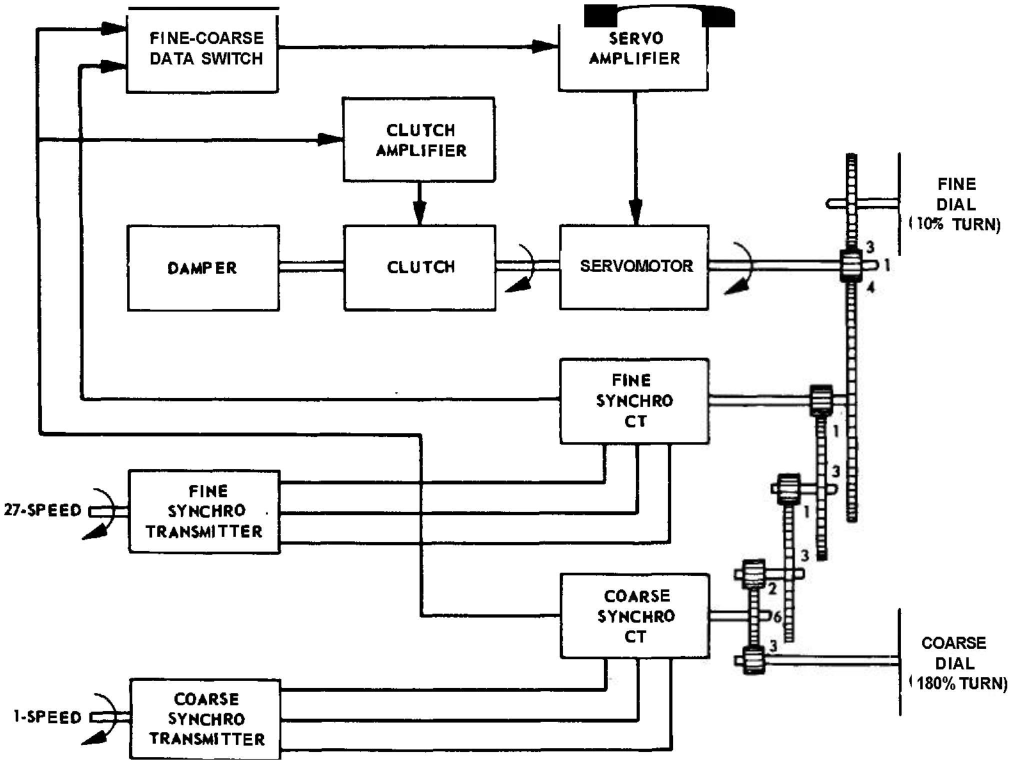

Fig. 16-11 Functional block diagram of dual-mode servo data repeater.

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for single-mode operation of the damper system is shown in the upper part of Fig. 16-12. Note that the repeater output has an 80 percent initial overshoot and requires at least 6 complete cycles to reach and stay within the linear range of operation.

With dual-mode operation, the transient shown in the lower part of Fig. 16-12 is obtained. In this mode, the repeater initially reaches the desired position more rapidly because of the reduced system inertia with the damper disconnected. Reconnection of the damper just before the repeater reaches the desired position results in an exceedingly high rate of deceleration which prevents an overshoot of more than about 1 percent. Once within the linear range, the synchronization is completed rapidly because of the wide bandwidth of the normal servo loop. The repeater servo completely settles in less time than is taken to reach the first overshoot in single-mode operation.

The optimum response for large steps is obtained with the clutch adjusted to operate at an error of about 4". Dual-mode operation is therefore obtained only for inputs that are larger than 4". .

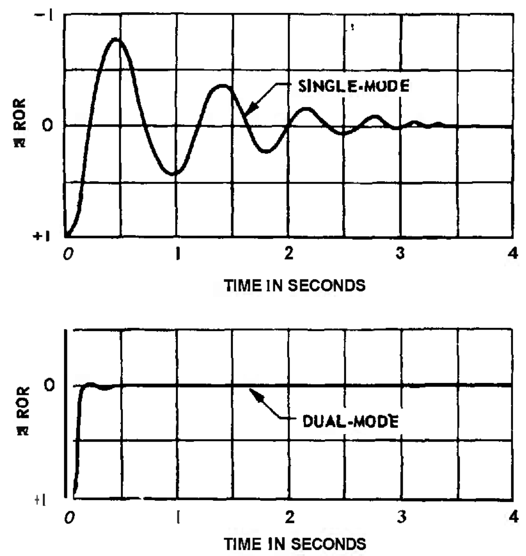

Fig. 76-72 Comparison of large-step (53°) servo response for single-mode and dual-mode operation.

Adapted by permission from *Transactions of the ASME*, Volume 79, April, 1957, from article entitled 'A Dual-Mode Damper-Stabilized Servo', by J. Jursik, J. F. Kaiser, and J. E. Ward.

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CHAPTER 17

REPRESENTATIVE DESIGNS*

17-1 INTRODUCTION

The examples of servomechanism systems described in this chapter have been selected from Army equipments which now carry an unclassified designation. The objectives of presenting these designs is to illustrate the analytical procedures discussed in the earlier chapters and to show how various components may be integrated to meet a set of over-all requirements. It should be borne in mind that each new requirement necessitates, in general, a fresh approach to system synthesis in order to achieve best results, and that no single type of system organization can be regarded as the best design for all purposes. The user of the handbook is therefore cautioned against applying the representative designs in this chapter to new situations.

17-2 A SERVO SYSTEM FOR A TRACKING-RADAR ANTENNA

17-2.1 GENERAL

The M33 Antiaircraft Fire-Control System employs electrical servos for driving the tracking-radar antenna in azimuth and elevation during the tracking of a target. A functional block diagram showing the interconnection of the units is drawn in Fig. 17-1. Only the elevation-control channel is illustrated since both channels are identical with the exception that the azimuth power servo amplifier drives four motors whereas the elevation servo amplifier drives a single motor. The units shown are those used in the automatic-tracking mode.

17-2.2 PURPOSE

The primary purpose of the azimuth and elevation servos is to steer the tracking-radar antenna so that an airborne target may be tracked automatically in two coordinates as it moves through space. Another purpose of the servos is to provide for rapid slewing of the tracking antenna to the angular position of the associated acquisition-radar antenna. Still another purpose is to provide alignment (in elevation) of the optical system with the tracking-radar antenna. The two servos also supply angular-position data to a computer and to remote indicators.

17-2.3 OPERATION

An 1800-rpm conical-scan drive motor attached to an off-axis antenna feed (see Fig. 17-1) spins the beam in space and, in so doing, places 30-cps amplitude modulation on the chain of echo signals received from a target. After amplification and detection by the radar receiver, the 30-cps amplitude-modulated video echo pulses are applied to a pulse demodulator where the modulation signal is recovered from the video pulses. The 30-cps signal is then applied to an elevation-angle

*By J. F. Reintjes and L. A. Gould

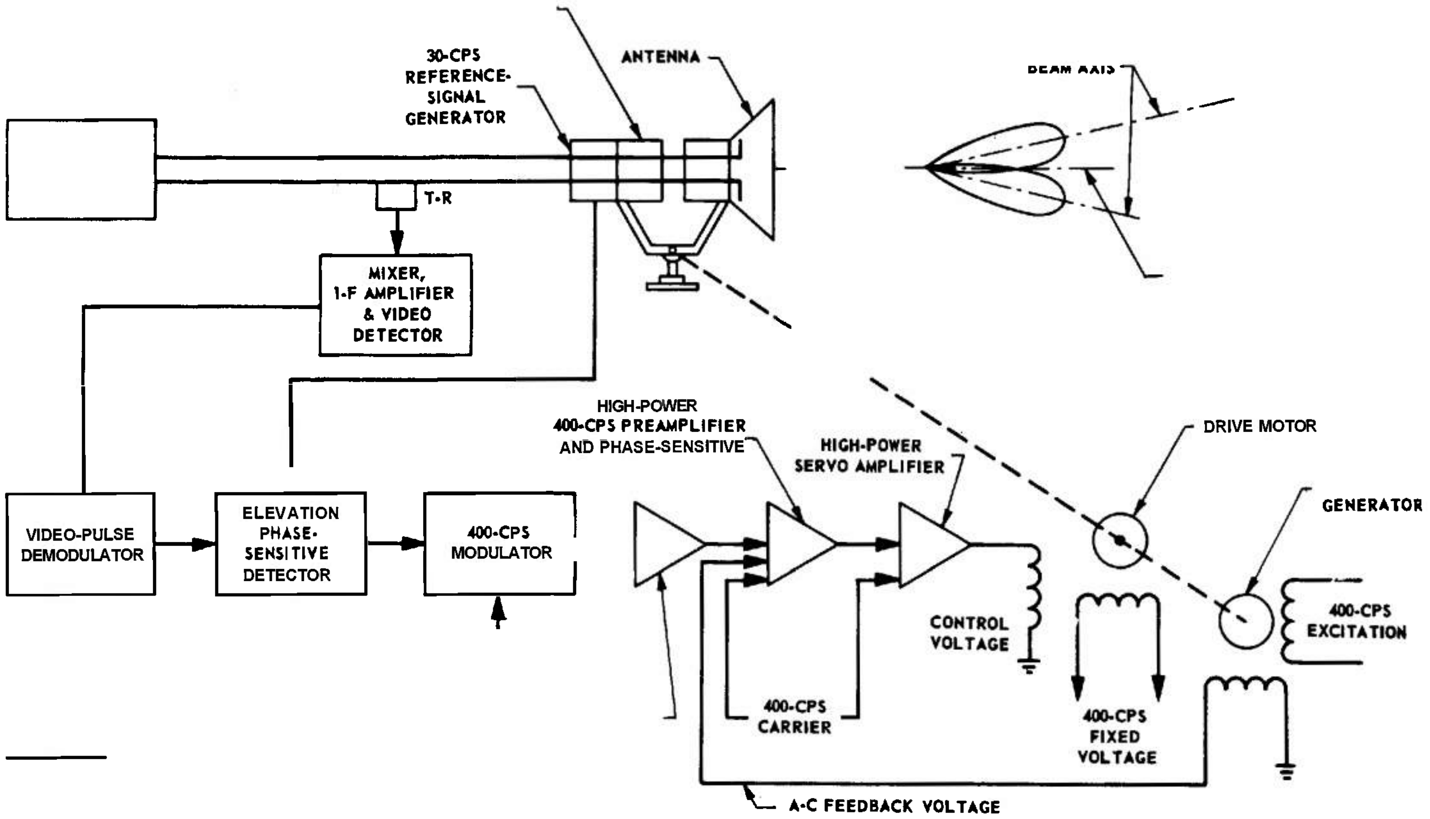

Fig. 17-1 Simplified functional block diagram of servo system for controlling M33 tracking-radar antenna in elevation.

phase-sensitive detector (see Par. 12-4) together with a 30-cps signal from a reference-signal generator coupled to the conical-scan drive motor at the antenna. The reference signal serves as a phase reference for the elevation angle of the antenna-beam axis measured with respect to the parabolic-reflector axis. If the antenna is on target, no error signal is generated in the phase-sensitive detector. If the antenna is off target in elevation, there is generated an error signal whose magnitude is a measure of the amount the antenna-beam axis is off target and whose 'polarity with respect to the elevation reference signal depends upon whether the beam axis is above or below target. Yence, as the antenna tracks a moving target, the output of the phase-sensitive detector is a slowly varying positive or negative voltage, the magnitude and polarity of which are a measure of the elevation tracking error.

The error signal from the phase-sensitive detector modulates a 400-cps carrier voltage, and the modulated carrier is then amplified in a low-power electronic servo amplifier. This amplifier consists of two stages of voltage amplification, a phase inverter, and a push-pull output stage (see Par. 13-1). The phase inverter provides two signals, 180" out of phase, for driving the push-pull output stage. The output of the low-power servo amplifier is then applied to the input of a 400-cps high-power servo preamplifier. Also applied to the preamplifier input is a 400-cps feedback voltage from an a-c tachometer generator (see Par. 11-5) that is coupled to the elevation drive shaft of the antenna. An a-c velocity feedback voltage is thus provided in the servo system. Because the high-power servo-amplifier stage requires a push-pull d-c voltage input signal, the preamplifier also includes a demodulator. The input signal to the preamplifier is therefore a modulated 400-cps carrier signal and the output is the modulation envelope.

The high-power servo amplifier controls the 400-cps 2-phase elevation drive motor and consists of a push-pull d-c amplifier driving a 400-cps magnetic modulator (see Par. **12-3**). The output of the magnetic modulator is **a** 400-cps voltage whose amplitude is proportional to the difference between the output currents in each tube of the push-pull amplifier and whose phase is equal or opposite to that of the 400-cps excitation voltage of the magnetic modulator, depending upon which of the two tube currents is greater. The output signal is applied to the control winding of the 2-phase motor.

17-2.4 OPERATIONAL BLOCK DIAGRAM

The operational block diagram of the servo system for controlling the tracking-radar antenna in elevation is shown in Fig. 17-2. In this figure

r = line-of-sight elevation angle

c = antenna elevation angle

e = actuating signal of servo system

n = radar noise

θ_m = angle of servomotor shaft

b = tachometer-generator feedback signal

U = load-torque disturbance at antenna reflector

$G_1(s)$ = transfer function of radar set, elevation phase-sensitive detector, 400-cps modulator, and low-power 400-cps amplifier

$G_2(s)$ = transfer function of high-power 400-cps preamplifier and phase-sensitive demodulator, high-power servo amplifier, and servomotor control field

$G_3(s)$ = transfer function of servomotor

$G_4(s)$ = transfer function of antenna mechanical assembly

$H_1(s)$ = transfer function of tachometer generator

$H_2(s)$ = transfer function due to imperfect coupling between antenna reflector and servomotor shaft

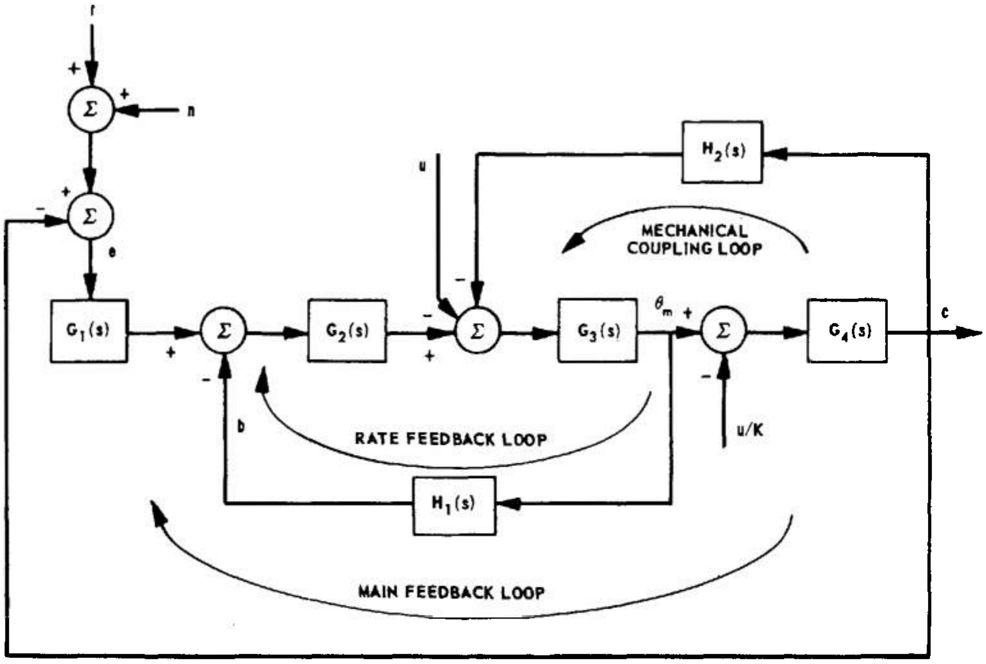

Fig. 17-2 Operational block diagram of servo system for controlling M33 tracking-radar antenna in elevation.

K = stiffness of antenna structure

The rate feedback supplied by the a-c tachometer generator [$H_1(s)$] is imperfect because the antenna structure is not absolutely rigid. The rate feedback therefore tends to improve the performance of the servomotor but is not completely effective in reducing the mechanical resonance of the antenna structure. The dynamic behavior of the servomotor and the antenna structure cannot be separated in the system, as indicated by the mechanical coupling loop. However, if the resonant frequency of the antenna structure is very high, only the inertia of the structure is important; the rate feedback is then a direct measure of the elevation angular velocity of the antenna reflector.

17-2.5 NOISE

The presence of radar noise in the line-of-sight signal prevents the actuating signal e from being a true measure of the system error, $y_e = r\text{-}c$. In order to minimize the effect of noise, the bandwidth of the system has been kept at a low value so that the servomechanism acts as a low-pass filter.

17-2.6 DESIGN CHARACTERISTICS

The following is a list of characteristics of the elevation servo system :

Maximum power output	120 w
Maximum velocity	500 mils/sec
Maximum acceleration	1000 mils/sec^2
Maximum static error	0.04 mil
Main power requirements	360 va
Control power requirements	360 va
Resonant frequency	1 cps
M_p	1.5 db

17-3 A POWER CONTROL SYSTEM FOR THE M-38 FIRE-CONTROL SYSTEM

17-3.1 GENERAL

The M-38 Fire-Control System† is a mobile **75mm** antiaircraft weapon designated as the ***Skysweeper.*** The power control system for this weapon (see Fig. 17-3) is an electrically controlled hydromechanical drive mounted on the carriage of the gun mount, and serves to position the gun in azimuth and elevation in accordance with command signals from the fire-control computer. The azimuth and elevation servo systems are essentially identical.

17-3.2 OPERATION

The fire-control computer provides either present or future target position data, depending upon the mode of computer operation. These data are compared with positional data from the gun. If the desired and actual positions do not correspond, there is generated an error-signal voltage whose amplitude is proportional to the difference between the desired and actual gun positions and whose phase is a measure of whether the gun lags or leads the desired position. The error signal is then amplified in a servo amplifier to a power level sufficient to drive a stroke-control servomotor. This servomotor, in turn, is geared and linked mechanically to the follow-valve control (pilot valve) of a power piston that regulates the oil delivery rate of a hydraulic pump and therefore the output velocity of the hydraulic motor connected to the output gearing.

As indicated in Fig. 17-3, voltages from the computer are supplied to synchro control transformers as **1**:1 coarse data and **16**:1 fine data. The rotors of the synchros are connected mechanically to the output gearing. The resultant error-signal voltages, after being combined with secondary signals to be discussed below, are amplified and applied to the power amplifier. For large error-signal voltages, the coarse error signal predominates and serves to swing the gun approximately on target. For smaller angular errors, the fine error signal predominates and serves to keep the gun precisely on target.

The coarse synchro control transformer rotor is geared to make one revolution for each revolution of the gun, whereas the fine synchro control transformer rotor makes sixteen revolutions for each gun revolution. **As** a result, the envelope of the coarse synchro signal undergoes two phase reversals for each revolution of the gun, while the envelope of the fine synchro signal undergoes thirty-two phase reversals. At the 3200-mil azimuth gun position, therefore, the coarse and fine voltage envelopes are exactly 180" out of phase. Because of this 180" phase relationship, it is possible to synchronize the gun exactly 180° away from its correct position.

To avoid this false synchronization, a small a-c synchronizing voltage, called an *antistick-off* voltage, is placed in series with the rotor voltage of the coarse synchro control transformer. By proper adjustment of the phase angle and amplitude of this auxiliary voltage, it is possible to bring the coarse and fine synchro voltage envelopes into phase agreement in the region of possible false synchronization (see Par. 11-3).

The error signals from the coarse and fine synchro control transformers are in the form **of** sinusoidal voltages. The error voltage from the fine synchro control transformer is passed through an amplitude limiter ***so*** as to limit the maximum error it can correct to approximately 25 mils. The coarse error-voltage waveform, on the other hand, is reshaped so that its amplitude is essentially zero for signals corresponding to an error of 25 mils or less. Hence, the coarse error signal effectively controls circuit operation for tracking

†*Formerly called the T-38 Fire-Control System.*

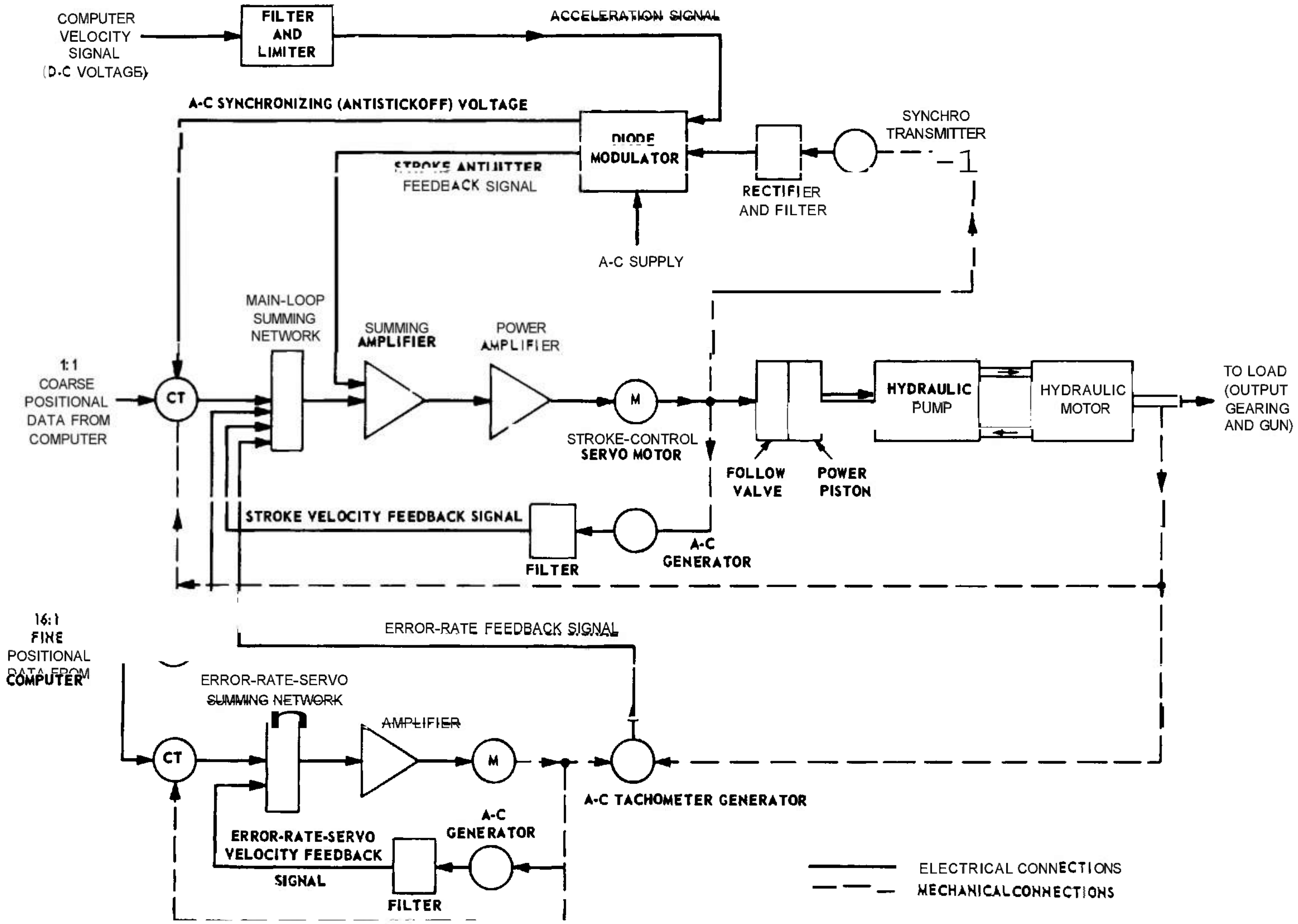

Fig. 77-3 Power control system for M-38 Fire-Control System.

errors greater than **25** mils and the fine error signal controls circuit operation for errors less than 25 mils.

The summing amplifier in Fig. 17-3 is similar to that described in Par. 13-1; the power amplifier consists of a phase inverter and a push-pull amplifier. The output of the latter stage drives the control winding of a 2-phase stroke-control servomotor, the direction and speed of motor rotation being dependent upon the amplitude and phase of the amplifier output signal. The servomotor controls the position of a follow valve in a hydraulic servo cylinder (see Par. **14-4)** and thus regulates the volume and direction of fluid delivery to the fixed-displacement hydraulic motor geared to the gun. Controlling fluid delivery controls the speed and direction of the hydraulic motor and hence the speed and direction of gun travel.

The coarse and fine positional-signal-control channel described above contains several internal feedback loops. In addition, an auxiliary error-rate servo with its own error and feedback voltages, mixing circuit, amplifier, and servomotor is included in the over-all power control system. To reduce overshoot and to eliminate hunting of the stroke-control servomotor, a stroke velocity signal is fed back to the summing amplifier through a phase-correcting network. The feedback voltage is obtained from an a-c eddy-current generator attached directly to the shaft of the stroke-control servomotor. The amplitude of the generator output voltage is proportional to the speed of shaft rotation of the servomotor, and the phase of the voltage is dependent upon the direction of rotation.

A second feedback path from stroke-control servomotor to summing amplifier serves to suppress small oscillations of the servomotor output shaft that would otherwise be transmitted to the gun. The stroke antijitter voltage is obtained from a modified synchro control transmitter that is friction-coupled to the stroke-control servomotor shaft. The transmitter rotor is excited from a potentiometer connected to a 6.3-volt a-c source. The potentiometer is adjusted to provide the minimum voltage consistent with the elimination of gun jitter. The output of the transmitter is taken from two series-connected stator windings, and thus the magnitude of the output signal is a function of the displacement of the rotor. The rotor is limited by means of stops to ±30° displacement. The output voltage of the transmitter is then rectified, filtered, and differentiated in a rate circuit. All steady voltages are therefore blocked, but transient signals are passed on to the diode modulator circuit.

The diode modulator circuit supplies the summing amplifier with square-wave signals that are in phase with the coarse and fine error signals and that vary in amplitude in proportion to the sum of the stroke antijitter signal and an acceleration signal. The latter signal provides a means of anticipating sudden changes in speed or direction of the gun and is obtained from a velocity signal in the computer. The velocity signal is supplied as a d-c voltage from a permanent-magnet generator. After filtering to remove commutator ripple, the velocity signal is differentiated in order to obtain a voltage that is proportional to acceleration. The acceleration signal is then combined with the stroke antijitter signal as indicated above.

The error-rate servo in Fig. **17-3** receives its input signals from the 16:1 speed transmitter in the computer. These signals are amplified to drive the rotor of an a-c tachometer generator, the stator of which is driven by the gun gearing. The error voltage thus generated is applied, after being phase-synchronized and mixed with the other feedback signals, to the main summing amplifier. As long as the gun turns in synchronism with the fine-speed data from the computer, there is no relative motion between rotor and stator of the a-c tachometer generator and no signal voltage is generated. Should the gun begin to lead or lag the point of synchronization, the phase and magnitude of the error-rate signal will accelerate or decelerate the gun and thus bring the gun velocity into agreement with the velocity of the computer synchro-transmitter rotor.

17-3.3 OPERATIONAL BLOCK DIAGRAM

The detailed operation block diagram of the fine-speed portion of the power control system for the **M-38** Fire-Control System is shown in Fig. **17-4.** The nomenclature for this diagram is presented in Table 17-1. If the dynamic lags (G_8, G_9, G_{10}) associated with the motors and their driving members are neglected, and only low-frequency behavior is considered, the block diagram can be reduced to the form shown in Fig. 17-5, where

ω_1 = magnitude crossover frequency of error-rate servo

ω_2 = resonant frequency of main control loop

T_1 = gain factor associated with error-rate stabilization

T_2 = gain factor associated with anticipatory acceleration signal

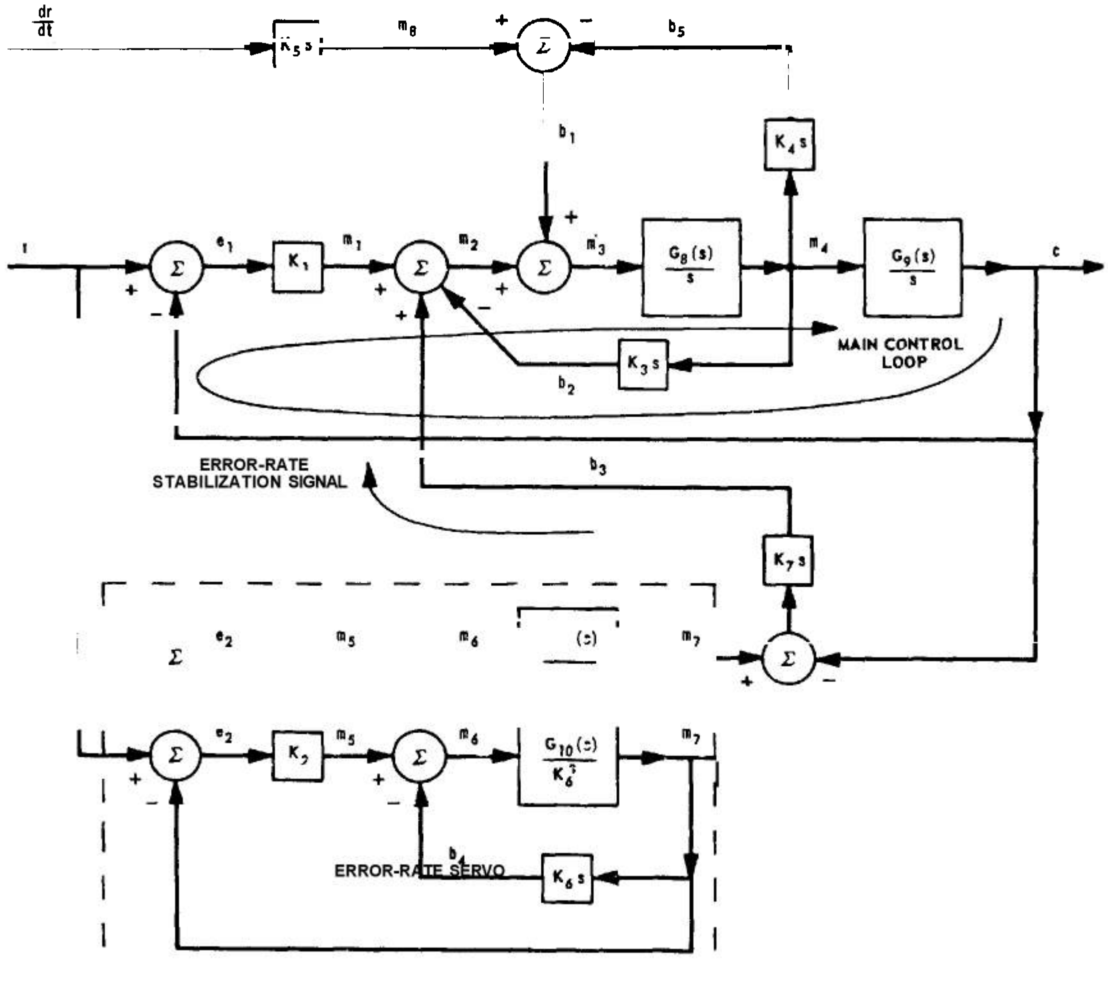

Fig. 17-4 Operational block diagram of fine-speed portion of power control system for **M-38** *Fire-Control System.*

It is evident that the main control loop of this system is inherently unstable because of the presence of two integrations, one due to the stroke-control servomotor and one due to the hydraulic motor. The error-rate stabilization signal is therefore introduced to supply the damping that is necessary for stable operation of the main control loop. If the magnitude crossover frequency ω_1 of the error-rate servo is high compared with the resonant frequency ω_2 of the main control loop, the diagram in Fig. **17-5** can be reduced to the form shown in Fig. **17-6.** In this form, it is evident that the gain factor T_1 associated with the error-rate stabilization directly controls the damping of the main control loop. Furthermore, the addition of the acceleration signal raises the possibility of cancelling the transfer function of the main control loop (by setting $T_2 = 1/\omega_2$) and thus producing an extremely rapid response for the system. Because of the approximations made in the successive block diagram reductions, such a cancellation will not actually occur, but the response of the system can be effectively speeded up. This improvement, of course, is limited by the dynamic lags that have been neglected and by the restriction of linear operation that is implicit in the analysis.

17-3.4 DESIGN CHARACTERISTICS

The over-all azimuth servo characteristics are as follows:

Maximum power output	5 hp
Maximum velocity	**1050** mils/sec
Maximum acceleration	**2000** mils/sec^2
Maximum static error	**0.75** mil
Main power requirements	**12** kw
Control power requirements	200 w
Resonant frequency	0.5 and **3** cps

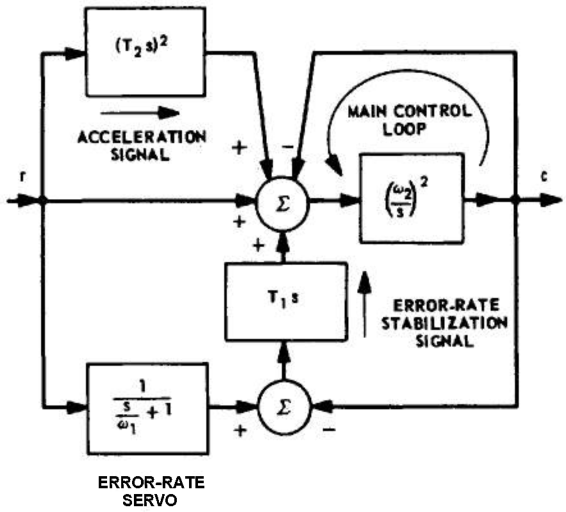

Fig. 17-5 Simplified block diagram of power control system for M-38 Fire-Control System.

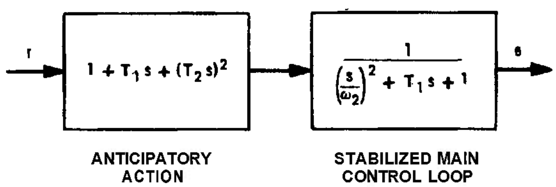

Fig. 17-6 Final block diagram of power control system for M-38 Fire-Control System.

TABLE 17-1 NOMENCLATURE FOR M-38 POWER CONTROL SYSTEM

r	= fire-control-computer target position data (reference input)
$\frac{dr}{dt}$	= fire-control-computer velocity data
c	= gun position (controlled variable)
e_1	= difference between reference input r and gun position c (main-loop actuating signal)
e_2	= difference between reference input r and error-rate-servo output position m_7 (error-rate-servo actuating signal)
m_1	= output signal of main-loop fine-speed control transformer
m_2	= output signal of main-loop summing network
m_3	= sum of stroke antijitter feedback signal b_1 and output signal m_2 of main-loop summing network
m_4	= stroke-control servomotor position
m_5	= output signal of error-rate-servo control transformer
m_6	= output signal of error-rate-servo summing network
m_7	= error-rate-servo output position
m_8	= acceleration signal
b_1	= stroke antijitter feedback signal
b_2	= stroke velocity feedback signal
b_3	= error-rate feedback signal
b_4	= error-rate-servo velocity feedback signal
b_5	= smoothed and differentiated synchro-transmitter signal
K_1	= main-loop control-transformer sensitivity
K_2	= error-rate-servo control-transformer sensitivity
K_3	= gain factor relating stroke velocity feedback signal b_2 to stroke-control servomotor velocity dm_4/dt
K_4	= gain factor relating differentiated synchro-transmitter signal b_5 to stroke-control servomotor velocity dm_4/dt
K_5	= gain factor relating acceleration signal m_8 to rate of change $d/dt(dr/dt)$ of computer velocity data
K_6	= gain factor relating error-rate-servo velocity feedback signal b_4 to error-rate-servo output velocity dm_7/dt
K_7	= gain factor relating error-rate feedback signal b_3 to difference between error-rate-servo output velocity dm_7/dt and gun velocity dc/dt
$G_8(s)$	= transfer function relating stroke-control servomotor velocity dm_4/dt to sum of stroke antijitter feedback signal b_1 and output m_2 of main-loop summing network
$G_9(s)$	= transfer function relating gun velocity dc/dt to stroke-control servomotor position m_4
$G_{10}(s)$	= transfer function relating error-rate-servo output velocity dm_7/dt to output m_6 of error-rate-servo summing network

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7 *Notes on Development Type Materiel — Fire Control System, AA, T33, Contract W30-069-ORD-4490, Vol. IV, Maintenance Instructions* (prepared by Western Electric Company), Dept. of the Army, FCDD-261, May 1, 1950.

8 *Notes on Development Type Materiel — Fire Control System, AA, T33, Contract W30-069-ORD-4490, Vol. II, Wiring Diagrams* (prepared by Western Electric Company), Dept. of the Army, FCDD-261, May 1, 1950.

9 *Notes on Development Type Materiel — Fire Control System, AA, T33, Contract W30-069-ORD-4490, Vol. III, Schematic Diagrams* (prepared by Western Electric Company), Dept. of the Army, FCDD-261, May 1, 1950.

10 *Notes on Materiel — Fire Control System T38 for 75-MM Gun T83E1 and 75-MM AA Gun Mount T69, Vol. 1, Skysweeper System and Power Control T21E1* (prepared by Sperry Gyroscope Company), Dept. of the Army, FCDD-263, Sperry Publication No. 14-8093, February 1, 1951.

11 *Notes on Materiel — Fire Control System T38 for 75-MM Gun T83E1 and 75-MM AA Gun Mount T69, Vol. III, Computer T27E2, etc.* (prepared by Sperry Gyroscope Company), Dept. of the Army, FCDD-263, Sperry Publication No. 14-8093, February 1, 1951.

12 *Notes on Development Type Materiel — Ranging and Sighting Equipment for 76-MM Gun Tank T41E1, 90-MM Gun Tanks T42 and M47, and 120-MM Gun Tank T43,* Ordnance Corps, FCDD-274, April, 1952.

13 *Notes on Materiel — Short Range AA Weapon (Stinger), Vol. I, Description, Theory, and Operation* (prepared by

Sperry Gyroscope Company), Dept. of the Army, FCDD-291, Sperry Publication No. 14-8094, November 1, 1953 (confidential).

14 *Notes on Materiel —Short Range AA Weapon (Stinger), Vol. II, Maintenance Instructions* (prepared by Sperry Gyroscope Company), Dept. of the Army, FCDD-291, Sperry Publication No. 14-8094, November **1,** 1953 (confidential).

15 *Notes on Materiel — Short Range AA Weapon (Stinger),* ***Vol.*** *III, Schematics, Servo Loop Diagrams, and Wiring Lists* (prepared **by** Sperry Gyroscope Company), Dept. of the Army, FCDD-291, Sperry Publication No. 14-8094, November 1, 1953.

16 *Notes on Development Type Materiel — Primary Fire Control System, Tank, 90 MM Gun, M48* (prepared by Chrysler Corporation), Dept. of the Army, FCDD-305, Ordnance Project No. TT2-**760,** July 10, 1955.

CHAPTER 18

AUXILIARIES ASSOCIATED WITH SERVOMECHANISMS*

18-1 AUXILIARY PUMPS

18-1.1 PURPOSE

In a servo system incorporating a hydraulic transmission, hydraulic fluid (oil) lost by leakage from the main hydraulic pump and motor must be replaced, otherwise the system eventually runs dry and ceases to function. Automatic, continuous replacement of lost oil is accomplished by means of auxiliary pumps operating in conjunction with regulating devices. When it is used for this purpose, an auxiliary pump is called a replenishing pump, a supercharging pump, or a make-up pump. The replenishing pressure is usually below 100 psi.

Many hydraulic-transmission servo systems also include hydraulic amplifiers, which control the stroke of the main hydraulic pump. Hydraulic amplifiers require oil at a pressure that may be as high as **1000** psi. Because of high pressure in large servo systems of this type, two auxiliary pumps may be used : one to feed the amplifier and one to feed the replenishing system. In smaller servo systems of this type, one auxiliary pump may perform both functions. In a servo system utilizing a valve-controlled ram or hydraulic motor, the pump used to feed the valve is often considered a subsidiary device because the pump is not in the closed-loop control system.

By J.O. Silvey

18-1.2 TYPES OF AUXILIARY PUMPS

Any type of pump that supplies the necessary pressure and rate of flow is usually adequate for auxiliary service in **a** servo system. The type generally used in practice is the fixed-displacement pump driven at a constant speed. The fluid pressure is controlled by means of automatic pressure regulators or unloading valves and accumulators. Occasionally, variable-delivery piston-type pumps are used to maintain a constant pressure at all flow rates between zero and their rated maximum. When used for this purpose, however, the pump is considered as merely one component of a pressure-control system. Variable-delivery pumps are discussed in Par. **13-6.** The fixed-displacement pump types discussed in the following paragraphs are :

(a) Gear pumps
(b) Vane pumps
(c) Piston pumps

18-1.3 Gear Pumps

A gear pump comprises two meshed gears enclosed by a close-fitting housing equipped with an inlet (suction) port and a discharge port (Fig. 18-1). One gear is shaft-driven **by** an external motor. Oil is carried from the inlet port to the discharge port in the spaces between the gear teeth, and is forced from these spaces where the gears mesh.

Usually, spur or helical gears are used in gear pumps. Gear materials range from cast

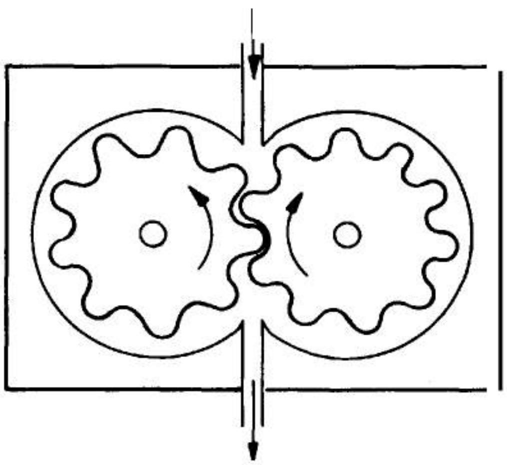


Fig. 18-1 Gear pump.

iron through hardened alloy steel. The material used depends upon the service expected from the pump.

Capacities of gear pumps normally used as replenishing pumps range from 1 to 60 gpm (gallons per minute). Rated output pressures range from 15 psi for replenishing service to 2000 psi for intermittent heavy duty.

Oil leakage from the high-pressure (discharge) side of the gears to the low-pressure (inlet) side is held to a minimum by the close fit of the housing along the sides of the gears as well as at the tips of the teeth. Because clearances cannot be held lower in a small pump than in a large one, internal leakage is larger fraction of the output flow in a small pump. For this reason, small-size gear pumps are less efficient than large pumps and are rated at lower pressures. The maximum efficiency obtained from a large gear pump is about 80 percent. Small-pump efficiencies are below this figure. Leakage is also largely dependent upon operating pressure and the viscosity of the hydraulic fluid. The pump manufacturer's recommendations should be followed when choosing a hydraulic fluid. If a particular hydraulic fluid has been specified, a pump designed to operate efficiently with that fluid should be used.

In some gear pumps, the pressure on opposite sides of the gears is equalized by means of passages cut into the shaft of the driven gear. This prevents the gears from being forced against one side of the housing by unbalanced pressure, thereby decreasing wear and increasing efficiency. Some gear pumps are equipped with "wear plates" which are forced against the sides of the gears by discharge pressure. Wear plates decrease oil leakage, but they also increase mechanical friction within the pump, necessitating more driving power and increasing the wear rate. In some gear-pump designs, discharge pressure is used to lubricate the gear-shaft bearings through small holes in the housing between the discharge port and the bearings.

18-1.4 Vane Pumps

The simplest form of vane pump (Fig. 18-2) consists of a stationary housing (stator) within which is a rotating, slotted cylinder (rotor) fitted with movable, radial vanes. The rotor is mounted eccentrically in the cylindrical interior (bore) of the stator. As the rotor turns, the vanes are forced against the hardened-steel ring (cam) on the outer periphery of the bore by centrifugal force, assisted by discharge pressure. In this way, the vanes move the oil from the inlet port to the discharge port.

Most vane-pump designs specify hardened steel for the slotted rotor and the cam. High-speed steel is specified for the vanes in order to prevent rapid wear caused by heating of the vane tips as they rub against the cam.

During each revolution of the rotor, the eccentricity of the device forces the vanes in and out of their slots. Thus, the volume of oil between any two vanes decreases and increases accordingly. The ports, at the top and bottom of the stator bore, extend along arcs concentric with the circumference of

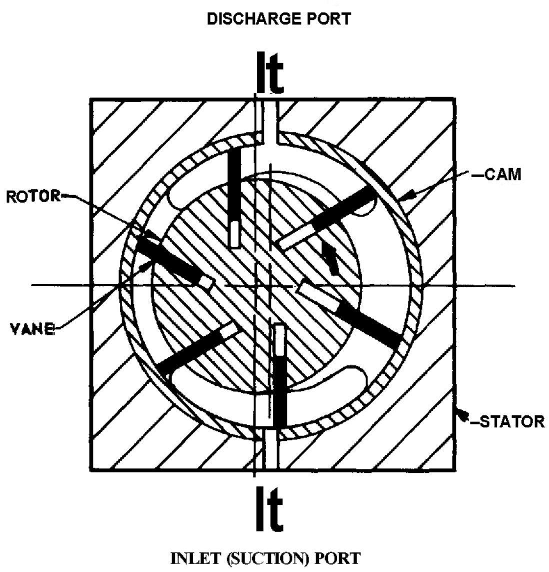


Fig. 18-2 Vane pump.

the bore. The angular distance between the ports is made slightly greater than the angle between two adjacent vanes to prevent intermittent direct connection between the ports. In some vane-pump designs, the stroke can be varied by shifting the stator in relation to the rotor, thereby varying the eccentricity.

High radial loads act on the shaft bearings of the vane pump shown in Fig. 18-2 because the discharge pressure prevails over one half of the rotor. Some vane-pump designs avoid these radial loads by using a double cam in the housing, as well as dual and diametrically opposite inlet and discharge ports, In such a design, each vane passes through two suction and two discharge cycles during each shaft revolution. Since the two rotor areas subjected to high pressure are equal and since the forces acting on the areas oppose each other, no net force acts on the shaft bearings.

The rated discharge of vane pumps ranges from 1 to 60 gpm. Continuous-pressure ratings are usually about 1000 psi ; intermittent ratings run to 1500 psi. Some vane pumps have been built that could deliver 3000 psi. The maximum over-all efficiency of the larger sizes of vane pumps is about 85 percent; smaller vane pumps are somewhat less efficient.

18-1.5 Piston Pumps

Piston pumps are almost always selected for auxiliary service when the required delivery pressure is more than 1500 psi. They may be used either as adjustable-delivery or fixed-delivery pumps. Their maximum deliveries range from 1 to 60 gpm. Rated pressures are as high as 5000 psi. The maximum mechanical efficiency obtained is about 85 percent for pressures below 2000 psi and usually less than that above 2000 psi. There is no significant difference between the characteristics of axial and radial piston pumps. The operation of piston pumps is described in Par. 13-6.

18-1.6 MAINTENANCE

Most hydraulic pumps used on ordnance equipment evolved from industrial designs. Since industrial pumps are subjected to a use more severe than that found in most military systems, they are usually rugged enough for most types of military service, provided they receive an adequate supply of clean oil. The clean-oil requirement is the price that must be paid for continuous, trouble-free operation. Foreign particles in the oil abrade the close-fitting operating surfaces of a pump and ruin them. Excessive heating can also produce this result. Routine maintenance consists almost entirely of periodic inspection of the oil supply and filters. The oil supply should be maintained at an adequate level; the filters should be cleaned regularly and replaced when they no longer perform their function.

18-1.7 LEAKAGE AND DRAINAGE

A properly assembled pump should not leak at any point, except possibly around the drive shaft where it is almost impossible to make an absolutely oil-tight seal. Most pumps are fitted with shaft seals which control oil leakage. These seals are usually not intended to sustain case pressures of more than 10 psi but, in fact, depend upon the hydraulic fluid in the pump for lubrication of the shaft (see Par. 18-3). Most pumps are also fitted with drain ports to prevent build-up of excessive pressure within the pump case. Drain ports should be vented to the system sump from the top of the pump case. The drain ports should not be vented to the suction line because the vacuum of the suction line may entrain pump-case air in the oil.

18-1.8 COST

Of the three types of pumps described above, vane pumps are the least expensive and piston pumps are the most expensive. The cost of gear pumps is between the costs of the other two.

18-2 HYDRAULIC AUXILIARIES

18-2.1 HYDRAULIC SYSTEMS INCORPORATING AUXILIARIES[1]

Figure 18-3 shows the hydraulic circuit and auxiliaries that are commonly used in a hydraulic transmission. Relief valves are connected across the output of the variable-delivery main pump to limit pressure if the hydraulic motor is overloaded. A fixed-delivery auxiliary pump supplies fluid to the hydraulic stroke control and replenishes any leakage from the main pump and hydraulic motor. Pressure to the hydraulic stroke control is held at pressure P_1 by a pressure regulator, while the pump-replenishing pressure is held at pressure P_2 by a throttling pressure regulator. Replenishing pressure is admitted to the main pump lines through check valves to make sure that it is admitted to the low-pressure line. In practice, the throttling pressure regulator is sometimes omitted and the hydraulic stroke control pressure is also used for replenishing.

The auxiliaries used in a control-valve and hydraulic-ram system are shown in Fig. 18-4. Fluid is supplied by a constant-delivery pump to an unloading valve. An accumulator performs two functions: (1) it stores fluid under pressure when system demand is low; and (2) it supplies fluid to the system when the unloading valve is returning pump delivery to the sump or when the system demand cannot be met by the pump alone. Pressure-relief valves prevent excessive pressures in the lines to the ram.

18-2.2 CHECK VALVES

Check valves are automatic devices that permit a liquid or gas under pressure to flow through them only in one direction. In the flow direction, the valves open when the applied pressure reaches a small predetermined level and close again when the pressure drops below that level. Pressure in the opposite direction reinforces the force that keeps the valve closed.

18-2.3 Ball Check Valves

Figure 18-5 illustrates the ball check type of valve. When fluid under pressure is applied to port C, ball *A* is forced upward against compression spring *B*, and the fluid flows through the valve and out through port D, as the flow arrows indicate. If the pressurized fluid were applied to port D instead of to port C, ball *A* would be held more firmly in place on its seat and no flow would result. Valve spring *B* is made with a low spring constant and is installed with only a slight preload to minimize pressure drop through the valve. Other types of check valves operate on the same principle as the ball check valve, but use flat buttons instead of balls, and either lapped or resilient-plastic seating surfaces.

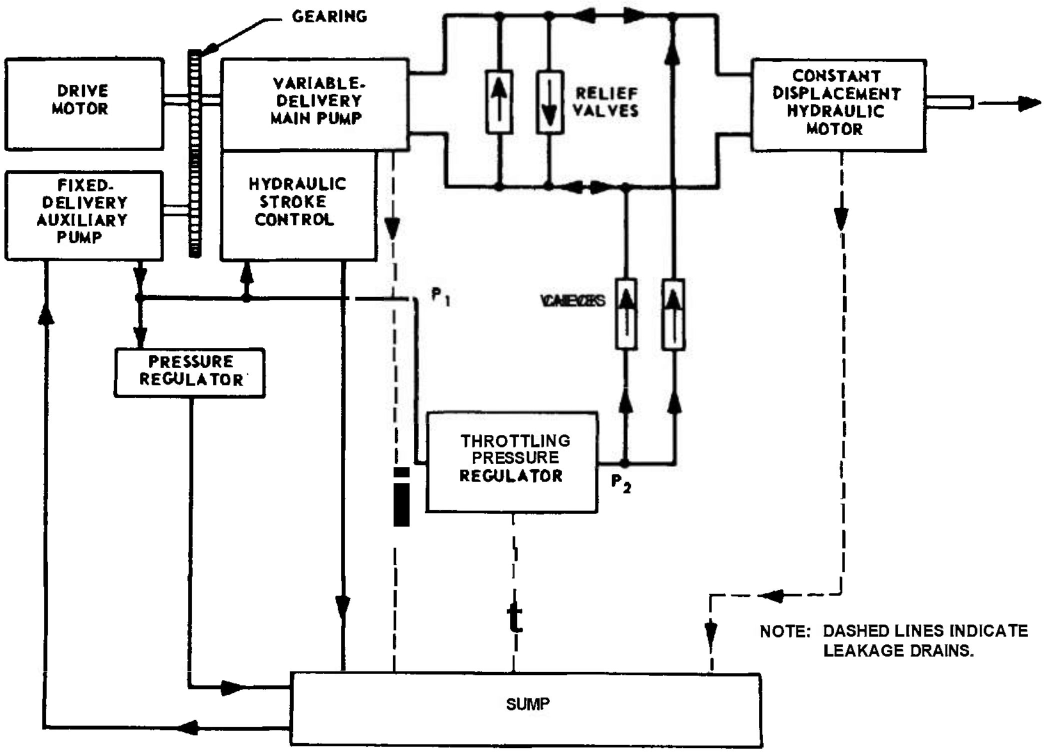
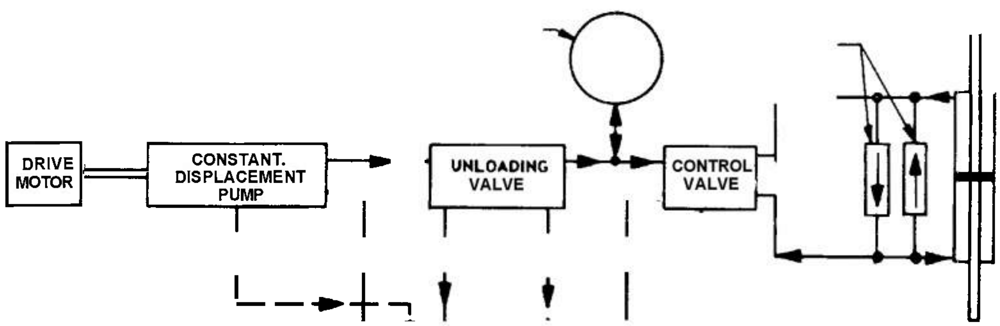


Fig. 18-3 Typical auxiliaries used with hydraulic transmission.



NOTE: DASHED LINES INDICATE LEAKAGE DRAINS

Fig. 18-4 Auxiliaries used in control-valve and hydraulic-ram system.

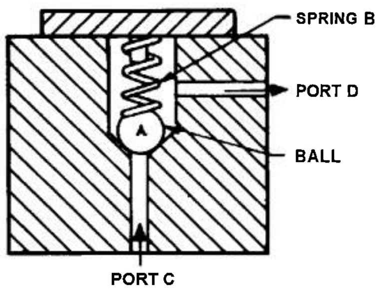


Fig. 78-5 Ball check valve or relief valve.

18-2.4 PRESSURE-RELIEF VALVES

For equipment protection, the pressure in hydraulic lines is relieved, or prevented from exceeding a predetermined value, by pressure-relief valves. Ball relief valves (Fig. **18-5**) are frequently used for this purpose. The ball spring is preloaded to the point where the valve will open when the pressure reaches the predetermined value. The valve opens when

$$P = \frac{F}{A} \qquad \textbf{(18-1)}$$

where

P = pressure at port C above the pressure at port D, in psi

A = cross-sectional area of the ball at its seating surface, in in.^{2}

F = spring preload, in lb

As the flow increases, the pressure at port C (Fig. **18-5**) rises somewhat, because the spring must be compressed to accommodate the higher flow and because the high velocity **of** the oil between the ball and its seat reduces the pressure applied to a small area of the ball surface.

For prevention of damage, it is desirable to limit the pressure in the two main lines of **a** hydraulic system and to permit excess oil to flow from the high-pressure line to the low-pressure line (see Fig. 18-4). Two ball relief valves are often used for this purpose and are connected between the main lines so that they permit flow in opposite directions. The main line carrying the higher pressure (beyond the predetermined level) discharges the excess oil to the other main line through the ball check valve connected for that flow direction.

A double-acting relief valve (Fig. **18-6**) can be used in place of the two ball check valves discussed above. In Fig. **18-6,** ports A and B are connected to the main lines to be relieved and the drain i nnected to the system sump. Pressure P_1 pplied over a piston area of $\pi(a/2)^2$, and ressure P_2 over a piston area of $(\pi/4)(b^2 - a^2)$. If pressure P_1 is higher than pressure P_2 and also high enough to force the piston downward against the compression spring, oil will flow from port A to port B when the passage between the ports is opened. The flow is in the opposite direction (from port B to port A) if pressure P_2 is higher than pressure P_1 and also high enough to force the piston down far enough to open the passage between the ports.

18-2.5 PRESSURE-REGULATING VALVES

In some small hydraulic systems, it is economically advantageous to use a single fixed-delivery pump that has a delivery rate slightly higher than the maximum system demand. The pressure in the system is controlled by a pressure-regulating valve similar to that in Fig. **18-7.** When the hydraulic system uses the entire pump delivery, keeping pump pressure low, the valve is held in the closed position by the compression spring. When the system uses only part of the delivery, the pump pressure rises and the piston is forced downward against spring tension until the port connected to the sump is uncovered. In this way, the unused portion **of** the delivery is returned to the system sump. If none of the delivery is used by the system, the entire delivery returns to the sump through the pressure-regulating valve.

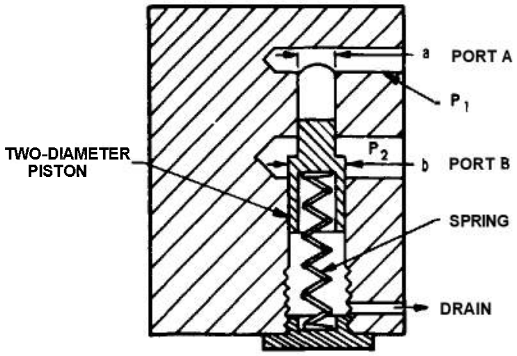


Fig. **18-6** *Double-acting relief valve.*

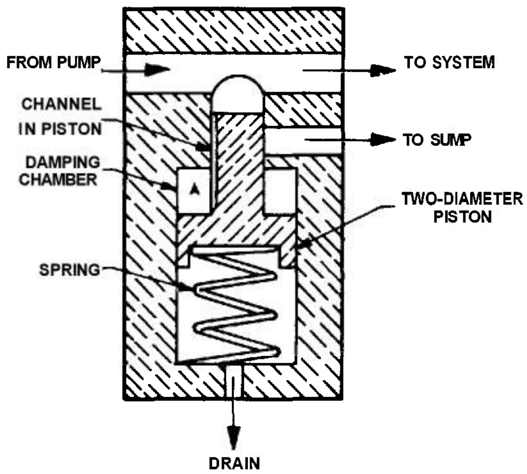


Fig. 18-7 Pressure-regulating valve.

One problem presented to the designer of the pressure-regulating valve shown in Fig. **18-7** is the noise produced by the mechanical oscillation **of** the piston about its regulating position. Oscillations of the piston are reduced by means of damping chamber **A** formed by using a valve piston with two diameters. The damping chamber is connected to the pressure side of the piston by a small channel cut along the length **of** the smaller part **of** the piston. When the piston is forced downward by oil pressure, a small amount of this pressure is used to force oil through the small channel into the damping chamber, thereby slightly reducing the pressure on the small diameter of the piston. At the same time, the damping chamber is filled with oil. After the excess delivery has been discharged to the sump, the normal tendency of the compression spring is to rapidly force the piston upward. However, the oil trapped in the damping chamber opposes this action by compelling the spring-driven piston to force the trapped oil through the small channel in the piston. In this way, rapid motion of the piston is damped out. The channel is small enough to stop oscillations of the piston, yet large enough to permit adequate regulator response speed. Actual channel dimensions are usually determined by experiment.

Ball check valves (Fig. 18-5) are sometimes used as pressure regulators, but are almost invariably noisy. However, some commercial regulators use ball check valves as pilot valves to actuate the main regulator. This combination of valves is usually quiet, but the speed of response may not be rapid enough for some applications.

One important disadvantage of the pressure-regulating valve of Fig. 18-7, as well as the ball check valve used as a pressure regulator, is the amount of heat generated in the pumping of oil not used by the system. The work lost in generating this heat reduces the efficiency of a system using one of these pressure-regulating valves. In addition, the heat must be dissipated in some manner, either through water-cooled heat exchangers which occupy relatively little space or through air-cooling of the sump. The air-cooling method must be used in most ordnance equipment because water is unavailable.

A throttling pressure regulator (Fig. 18-8) drops pressure from a regulated high pressure to a regulated lower pressure. When the lower pressure is too **low,** the compression

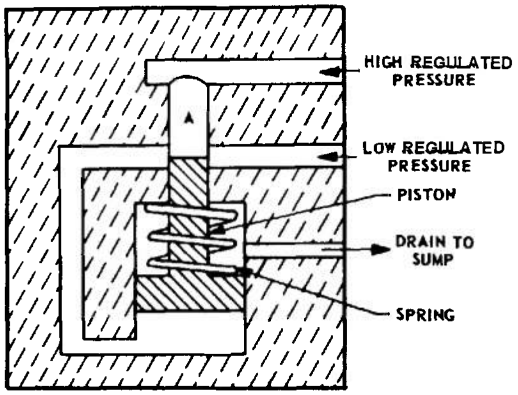


Fig. 18-8 Throttling pressure regulator.

spring opens the valve by pushing the piston downward until passage A between the high-pressure and low-pressure ports of the valve is open. This permits the high-pressure oil to flow to the low-pressure side of the system. When the pressure in the low-pressure side reaches its normal value, the piston is forced upward, closing the passage. This type of regulator permits no more oil to flow from the high-pressure side than that required by the low-pressure side (the load). For this reason, it is not suitable for use as the sole regulator of the output of a fixed-delivery pump. When supplied with a regulated pressure of approximately 500 psi, throttling pressure regulators of this type can be used to regulate the replenishing pressure (approximately 50 psi) of a hydraulic transmission.

18-2.6 ACCUMULATORS[2]

Accumulators are used to store hydraulic fluid under pressure. They therefore contain stored energy that is available on demand. Accumulators are used in some guided missiles as the sole source of energy used by the control system of the missile during flight. In other applications, accumulators store energy when the system demand is low and make it available for use when the system demand suddenly increases, thereby decreasing the average power demand of the system.

18-2.7 Gravity Accumulators

The simplest form of accumulator (Fig. 18-9) consists of: (1) a vertical cylinder with a port at the bottom ; (2) a single-ended piston inside the cylinder, with the piston rod pointed upward; and (3) a weight secured to the piston rod. Oil under pressure is forced into the cylinder port, pushing the weighted piston upward to its limit of travel in the cylinder. Assuming there are no friction losses, the stored energy thus made available is the product of the weight (in lb, oz, etc.) and the downward distance the weighted piston travels to its lower limit when discharging the oil to the system. The gravity accumulator is suitable only for fixed installations and normally only for low pressures. However, It does have the advantage that its discharge pressure is constant for the entire distance of piston travel. Another

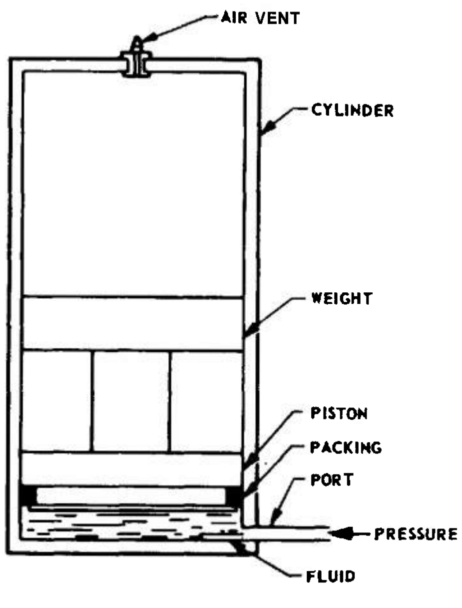


Fig. 18-9 Gravity accumulator.

version of the gravity accumulator uses compression springs instead of a weight, but this type is usually large and high pressures are difficult to obtain.

18-2.8 Hydropneumatic Accumulators

Gas-filled, or hydropneumatic, accumulators are often used in ordnance equipment because of their small size and high pressure and because of the large quantity of available energy that they provide.

A piston-type hydropneumatic accumulator (Fig. 18-10A) usually consists of a cylinder within which the gas and fluid are confined, and uses either a single-piece piston or a pair of opposed pistons operating in cylinder bores of different diameters. A drain is usually provided, as shown in Fig. 18-10A, to avoid possible seepage of hydraulic fluid into the pneumatic system, or vice versa. The gas is first brought up to specified pressure and the gas shutoff valve is closed. Then the fluid is brought up to pressure, thereby forcing the piston to compress the gas still further. The relation between the two pressures, assuming no friction at the piston seals, is

$$P_g A_g = P_h A_h \tag{18-2}$$

where

P_h = hydraulic pressure, in psi

P_g = gas pressure, in psi

A_h = area of hydraulic piston, in in.^{2}

A_g = area of gas piston, in in.^{2}

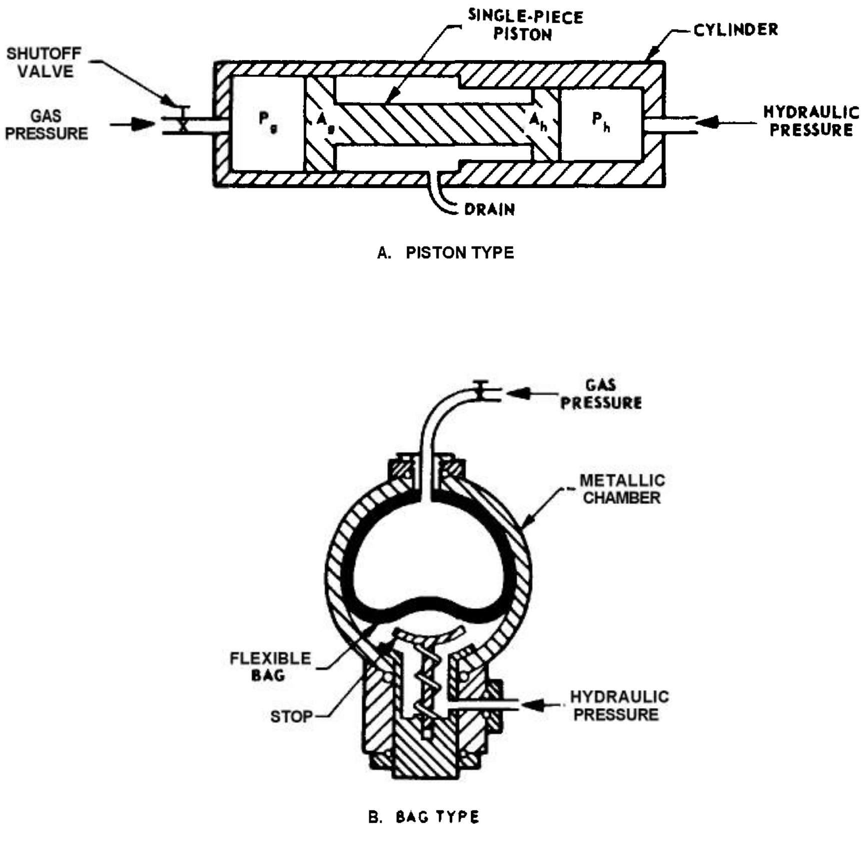


Fig. 18-10 Hydropneumatic accumulators.

Assuming that the gas expansion is isothermal, the energy available when the accumulator is discharged can be expressed by

$$W = P_o V_o \ln\left(\frac{P_1}{P_2}\right) \tag{18-3}$$

where

W = energy released, in lb-in.

P_o = pressure in gas chamber when fluid chamber is empty, in psi

V_o = maximum volume of gas chamber, in in.^{3}

P_1 = initial gas pressure, in psi

P_2 = final gas pressure, in psi

$\ln$ = natural logarithm (base e)

Piston-type hydropneumatic accumulators have been made with capacities up to approximately 230 in.^{3}, for a nominal system pressure of 3000 psi, but there is no reason why accumulators with larger capacities could not be made. For system pressures much above 3000 psi, however, providing adequate piston seals in this type of accumulator is a difficult problem.

The bag-type hydropneumatic accumulator (Fig. 18-10B) consists of an outer metallic chamber and an inner flexible synthetic-rubber bag which separates the gas and the fluid. Customarily, the chamber is spherical or cylindrical with hemispherical ends. When an initial charge of gas is placed in the bag, a spring-actuated piston-type stop covers the hydraulic port to prevent extrusion of the bag into the port. Then hydraulic fluid is forced into the space between the bag and the interior wall of the chamber, thus producing higher compression of the gas within the bag, until the gas and hydraulic pressures are about equal. The stored energy thus obtained is the same as in the piston-type accumulator. Commercial bag-type accumulators are available with total capacities ranging from 2 to 2300 in.^{3}; maximum pressure is 6000 psi.

Low-pressure hydropneumatic accumulators have been made in which the gas and fluid are in direct contact (no bag or piston is used). This type of accumulator is not generally recommended because the gas dissolves in the fluid at high pressure and comes out of solution when the pressure is removed, producing gas bubbles that may cause serious difficulties in the hydraulic system.

18-2.9 UNLOADING VALVES

Unloading valves (Fig. 18-11) are devices that: (1) permit delivery of pumped oil to the load when the load pressure is below a particular value; (2) permit return of pumped oil to the sump at a low pressure when the load pressure is above a preset value; and (3) prevent return of the system fluid to the sump when the pump is unloaded. Unloading valves are generally used with an accumulator connected to the load to provide a source of available power while the pumped oil is being returned to the sump. Unloading valves actually comprise a number of components within a common housing. In the type shown in Fig. 18-11, port P is connected to the pump, port S to the load, and port R to the sump. Port D is also connected to the sump, but through a separate line. When the system pressure is low, pump pressure opens check valve C which permits oil to flow to the system. Pilot valve V is held against its smaller upper seat by compression spring A, thereby connecting piston chamber L to port D. Bypass valve M is held against its seat by compression spring B and, consequently, full pump delivery flows to the system.

When the system pressure rises above a predetermined value, it forces pilot valve V downward against the force of compression spring A (Fig. 18-11). Since the lower sphere and seat diameters of pilot valve V are larger than those of the upper sphere and seat, entrance of pressure into the upper valve chamber forces V to close on its larger seat. Oil from the system pressure line then flows into piston chamber L, forcing bypass valve M downward against the force of spring B.

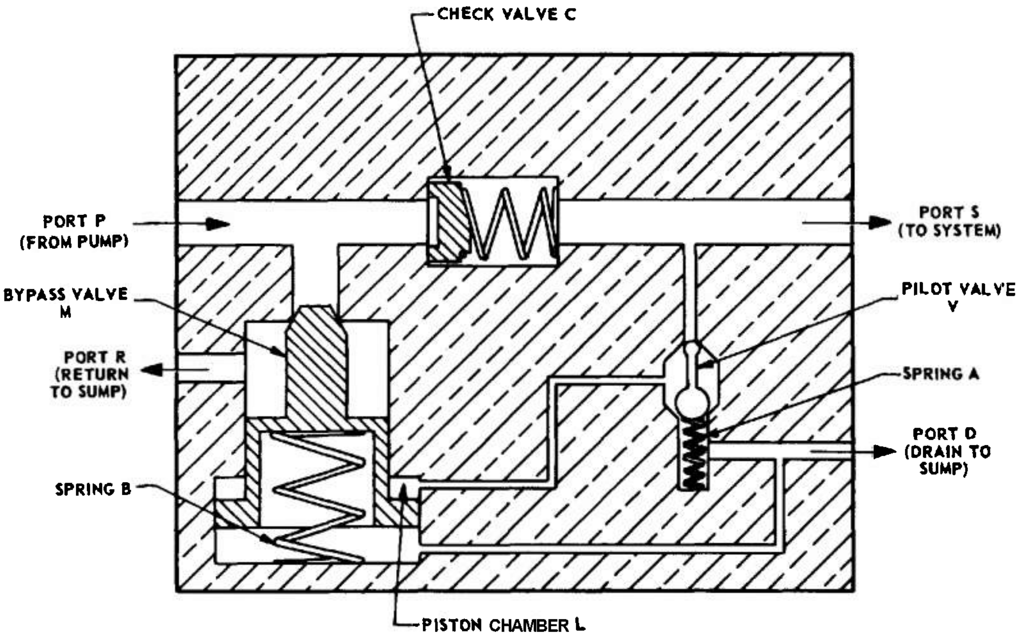


fig. 18-11 Unloading valve.

Pumped oil is now free to flow back to the sump from port P, past bypass valve M, and out through port R. Check valve *C* closes (as shown in Fig. 18-11) because the pumped oil is bypassed to the sump, making pump pressure lower than the system pressure.

When the system pressure drops low enough to permit spring A to lift pilot valve V from its larger seat, V closes again on its smaller seat because of the valve proportions; piston chamber *L* is connected to port D through V; spring *B* forces bypass valve ***M*** to close the passage between *P* and R; and pump pressure opens check valve C, permitting delivery to the system.

In some unloading valves of the type shown in Fig. 18-11, the passage to chamber *L* is restricted to prevent flutter of bypass valve M. Most unloading valves require extremely clean oil because even the slightest amount of foreign matter in the oil could prevent proper operation of pilot valve V, which moves only a few thousandths of an inch between its upper and lower seats.

18-3 ROTARY JOINTS

18-3.1 DYNAMIC SEALS

Dynamic seals must be used when a rotating or translating shaft must pass through the wall of a chamber in which fluid must be confined. To obtain a reasonable life expectancy from the seal, it must be lubricated by the confined fluid. For this reason, successful seals always leak enough fluid to keep the shaft moist at the sealing surface, but this leakage should be extremely small.

18-3.2 Glands

One of the oldest and simplest forms of dynamic seal is the stuffing box or packing gland shown in Fig. **18-12.** A soft, compressible material (for hydraulic fluid this is usually lead wool or asbestos) is packed into the hollow recess of the gland and compressed by the screw. These seals usually have relatively high friction and leakage, and their use is confined to applications involving low-speed shafts with rotary or translatory motion. When properly installed, glands are effective for low-pressure applications. In most cases, glands are considerably larger than some other types of seals.

18-3.3 O-Rings[3, 6, 7]

O-rings are employed primarily to seal translating shafts. They are not recommended for high-speed continuously rotating shafts, although they have been used for sealing oscillating shafts that rotate a maximum of one revolution or less. As some compression of the O-ring is required to produce a seal, some friction is always present, even with zero pressure differential across the ring. For this reason, O-rings should not be used when friction forces must be minimized. Compared with O-rings, U-cups and V-ring seals produce somewhat lower friction forces at low pressure differentials. Two applications of O-rings are shown in Fig. **18-13.** The dimensions of standard O-rings are given in Air Force-Navy Aeronautical Standard AN6227. O-rings are usually fitted into rectangular grooves. For applications similar to the piston seal of Fig. **18-13A,** O-rings can be used with leather backing rings to increase the permissible pressure differential across the piston from **1500** psi without the backing ring to **3000** psi with the backing ring. When O-rings are being installed, they must be protected from scratches or cuts because even small surface defects produce high oil leakage. The surface against which the O-ring slides, as well as the bottom of the groove in which it is installed, must be well-polished.

18-3.4 U-Cup and V-Ring Packings

These types of packings are commonly used to seal translating surfaces. Applications and characteristics of U-cup and V-ring packings are given in the Mechanical Engineer's Handbook.[4]

18-3.5 Shaft Seals[5]

Shaft seals are used to seal continuously rotating shafts when the pressure is below 35 psi. Three types of construction are in general use (Fig. **18-14).** In the construction

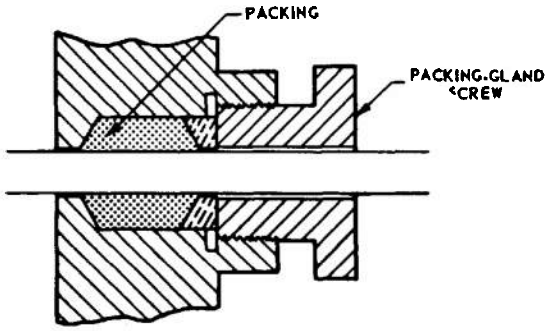


Fig. 78-12 Packing gland.

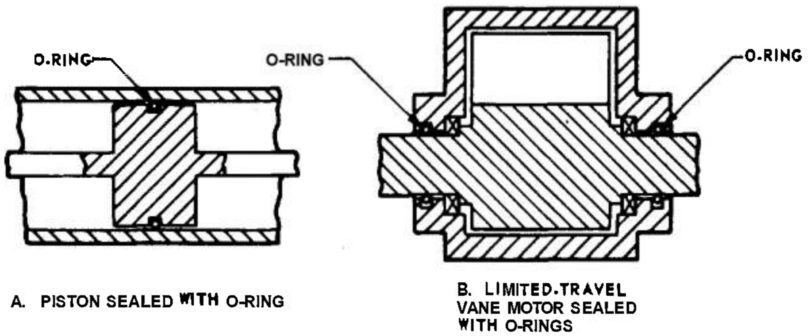


Fig. 18-13 Applications of O-rings.

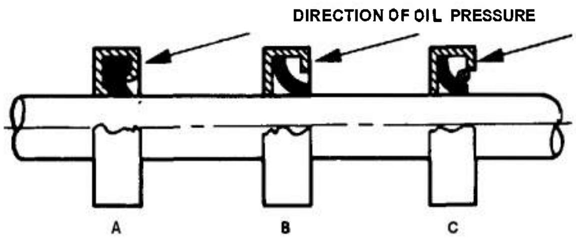


Fig. 18-14 Shaft seals.

illustrated in Fig. 18-14A, no back-up spring is used. The elasticity of the sealing material is utilized to keep the sealing material in contact with the shaft. In Fig. 18-14B, leaf-spring fingers are used to reinforce the sealing material elasticity. In Fig. 18-14C, a helical spring formed into a circle serves to keep the sealing lip in contact with the shaft. There is little difference in sealing efficiency between these three types. The sealing-lip material is usually leather or synthetic rubber.

Installation of shaft seals requires great care in order to avoid even the slightest damage to the sealing lips. An arbor or collar, with one end rounded or tapered, is sometimes used to open the seal to the shaft diameter and to start it over a smooth portion of the shaft. Before shaft seals with synthetic-rubber sealing lips are installed, both the shaft and the installing tool must be oiled. Shaft seals are installed by pressing the external metal shell of the seal into a recess in the housing, with the seal oriented so that pressure of the fluid being sealed forces the sealing lip against the shaft (unless the pressure is very low).

18-3.6 Face Seals

High-speed shafts are sometimes sealed with face seals (Fig. 18-15) when the oil pressure ranges as high as 2000 psi. In this type of seal, two materials that have a low coefficient of friction are used to establish the dynamic seal and the dynamic sealing surfaces are lapped to obtain a very smooth finish. The static seal between the shaft and the rotating face of the dynamic seal must be flexible enough to accommodate a small amount of runout between the surfaces of the dynamic seal. Many arrangements are used to drive the rotating face and to provide the static seal. The dynamic sealing surfaces must be lubricated at all times; otherwise, the face seal will be damaged quickly by the heat resulting from dry friction. Descriptions of seals used to prevent low-pressure

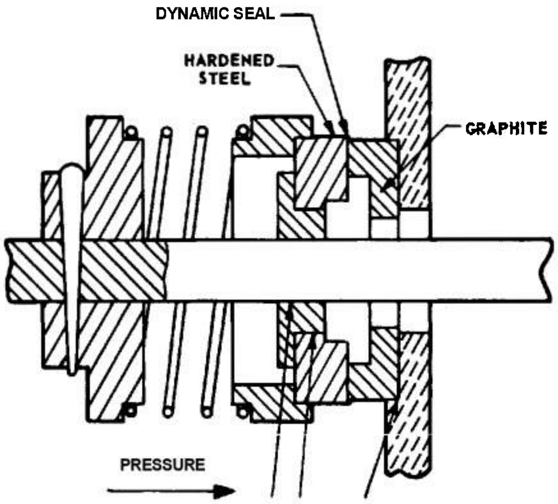


Fig. 78-75 Face seal.

gas from leaking past a small shaft are given in references 8 and 9.

18-3.7 High-pressure Seals

For high-pressure seals with low friction, an arrangement similar to that in Fig. 18-16 is sometimes used. This seal is not recommended for high-speed shafts or in applications requiring extremely low leakage from the pressure vessel. The seal will reduce leakage from the end of the shaft to a very small amount. Basically, the seal comprises a series of grooves around the shaft approximately 1/16-inch wide and 1/32-inch deep. The effect of the grooves is to equalize the pressure around the entire shaft circumference and to keep the shaft centered in the housing. Oil leaking past the grooves is carried away by a drain line to the sump. Oil flow from the end of the shaft is prevented by a commercial shaft seal. The shaft and housing should be made of materials having low coefficients of friction, preferably hardened

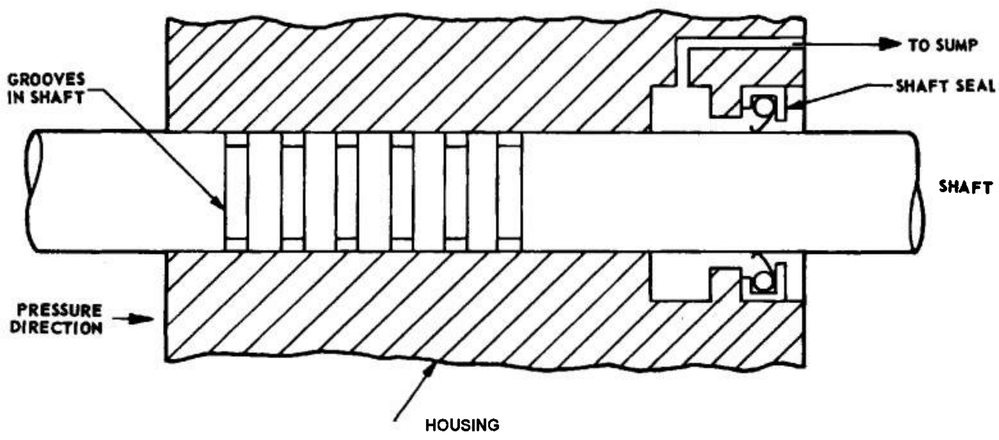


Fig. 18-16 High-pressure seal.

steel which also prevents foreign particles from being embedded in the material. Clearance between the shaft and its passage in the housing should be low, and the edges of the grooves at the shaft surface should be sharp and clean.

18-3.8 Friction

All dynamic seals produce some friction and should not be used on shafts where friction must be kept to an absolute minimum. For example, the shafts of torque motors or free synchro receivers should not be sealed. These devices, provided they have no slip rings, can usually be submerged in oil with no ill effects; in fact, submerging them in oil appreciably decreases their temperature rise.

Little quantitative information about seal friction is available, probably because of the wide variations that occur between seals that should produce the same frictional torque or force. Nevertheless, the following general comments can be made:

(a) Synthetic-rubber packings have a tendency to break through the lubricating-oil film while a shaft is at rest and, consequently, may exhibit an initial starting friction that is considerably higher than the subsequent running friction. This tendency is much less pronounced with leather packings, possibly because leather can absorb oil and thus become partially self-lubricating.

(b) A smooth surface on the moving member of the seal produces less friction than a rough surface.

(c) Synthetic-rubber seals cannot be used with fluids that produce excessive swelling of the rubber because high friction and rapid wear will result.

(d) For O-rings:[7]

(1) The use of leather backing rings increases seal friction on the shaft.

(2) The squeeze of the O-ring should not be excessive.

(3) The clearance between the shaft and the bore in which the O-ring moves should be held as low as practicable on the low-pressure side to prevent extrusion of the ring into the clearance.

(4) O-rings with a durometer hardness reading of approximately 65 have somewhat lower friction than O-rings with a durometer hardness reading of approximately 90, provided there is no extrusion.

18-4 LIMIT STOPS AND POSITIVE STOPS

18-4.1 PURPOSE

Stops are often desirable or even necessary in servomechanisms to limit output motion or travel. For example, azimuth drives of antiaircraft guns might well be limited to avoid the possibility of firing in a direction that would cause shells to fall on friendly troops. The elevation motions of tank and artillery guns are always limited, and stops are necessary to prevent equipment damage that might occur if it were possible to drive the guns into the travel limits at full speed and torque.

18-4.2 CHARACTERISTICS

Limiting stops may be either positive stops, which halt the output motion by overpowering the servomotor, or limit stops. Limit stops disconnect the normal servomechanism error signal and replace it with a signal originated by the output motion, which causes the output to stop. Because positive stops are easy to make, and operate directly on the output, some devices are equipped with both positive and limit stops. Positive stops are often necessary to avoid damage during manual operation of a device that is operable either manually or by means of a servomechanism.

When used alone (without limit stops), positive stops possess the disadvantage that it is possible to drive into them at full servomotor speed. Under such conditions, the positive stops and the output drive system must be capable of withstanding the full output torque plus whatever acceleration torques may be originated in the stopping process. Usually, positive stops are not easily changed, However, because they are simple mechanical devices, they ordinarily require little or no maintenance.

184.3 LIMIT STOPS

Limit stops restrict the travel of a servomechanism by replacing the normal error signal with a signal that is a function of the system output. The replacement signal can reverse the direction of output motion, cut off output power, or cause the output to stop until a signal to move in the opposite direction is received. The simplest form of limit stop merely disconnects power from the control circuit. Limit stops of this type are seldom used because of the difficulty of starting the drive in the opposite direction, away from the limit.

A limit-stop system utilizing switches in a simple reversing circuit is shown in Fig. **18-17.** In this system, two limit switches are actuated by a cam that is driven by the servo output. When either switch is actuated by the cam, the associated limit relay is energized, thereby disconnecting the error signal from the servomotor and connecting one of the fixed reversing voltages (V_1 or V_2).

The reversing voltage drives the servomotor cam away from the limit but, because of the switching arrangement involved, the servo output oscillates at the limit until the error signal is reversed. However, if the error signal is reversed, the servomotor moves the cam away from the limit switch and normal operation is resumed. Two variations of this circuit are: (1) omission of the relays by using switches capable of carrying the motor current; and (2) insertion of the reversing voltage ahead of the amplifier where current and voltage requirements are low. The switching circuit of Fig **18-17** is not recommended for large servos, but operates very well with servos of moderate or low power that only go into their limits occasionally.

Figure **18-18** shows a limit-stop system that is applicable to an electric servomechanism. In this system, a cam driven by the

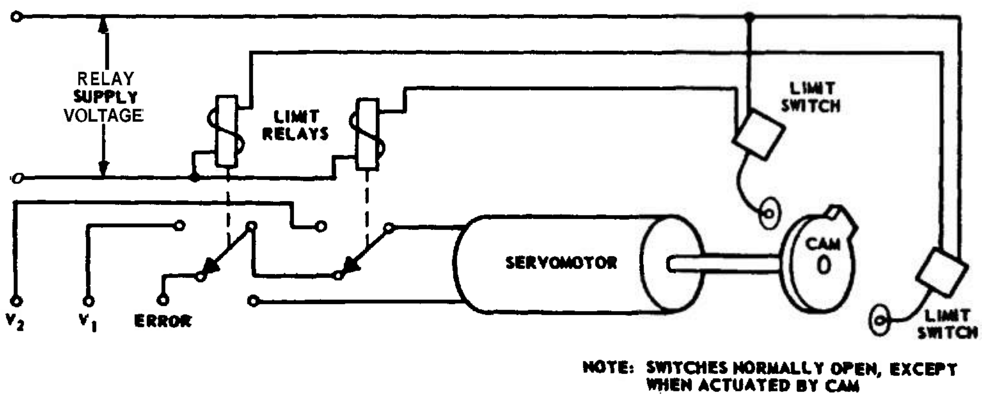


Fig. 18-17 Switching limit-stop system.

servomotor actuates the arm of a spring-centered potentiometer, near either end of the prescribed output travel. The output voltage of the potentiometer network is zero when the potentiometer arm is centered and increases when the arm is moved in either direction from center. The phase or polarity of the output voltage is governed by the direction of arm movement.

The maximum output voltage of the potentiometer network (Fig. 18-18) is higher than the maximum output voltage of the saturating error amplifier. The potentiometer and the saturating error amplifier are both connected to the output-amplifier input through a summing network, the entire arrangement being set up in such a way that the potentiometer-network voltage always subtracts from the error voltage. When servomotor rotation nears either limit, the cam moves the potentiometer arm, thereby increasing the potentiometer-network output voltage. Thus, the voltage input to the output amplifier is reduced to zero and the servomotor is stopped. This limit-stop system is a position servomechanism that uses the output of the saturating error amplifier as its command signal. Therefore, the exact position at which the servo output stops depends upon the magnitude of the saturating-amplifier output voltage. This system operates with little difficulty if d-c voltages are being summed. However, some difficulty can be expected if a-c voltages are used, because of the phasing of the two signals and because the waveforms of the mixed signals differ (the output of the saturating error amplifier is clipped, while the output of the potentiometer network is essentially sinusoidal).

A mechanical limit-stop system, a form of position servo, is shown in Fig. 18-19. Error motion at a reasonably high force level is transmitted through both the spring-loaded force limiter and the box around the spring-centered limit-stop arm to the control valve. The valve controls the direction of rotation as well as the starting and stopping of the hydraulic motor, thereby controlling the position of the cam that actuates the limit-stop arm. When the cam is well clear of the limits, error motion is transmitted without interference to the control valve, which opens one valve port and causes rotation of the motor. If the cam eventually turns the limit-stop arm to one side, the boss on the arm shifts its enclosing box, forcing the control-valve piston back to neutral. This stops the hydraulic motor. The control valve

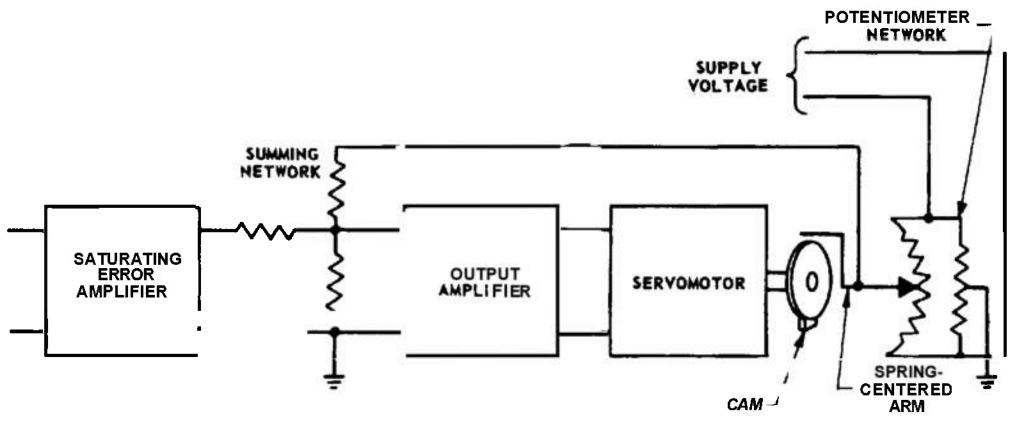


Fig. 18-18 Electrical limit-stop system.

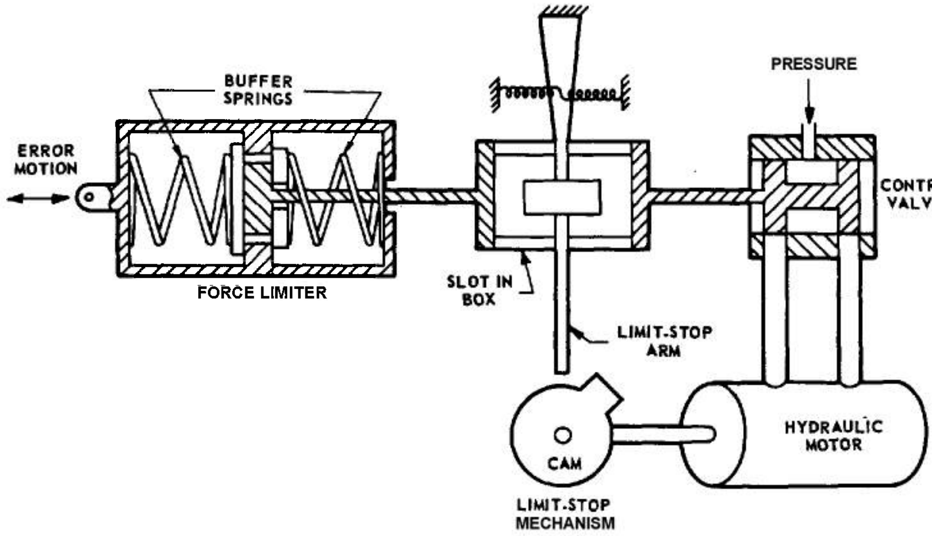


Fig. 78-79 Mechanical limit-stop system.

can be moved far enough to stop the motor without damaging the error-input mechanism because of the spring buffers provided in the force limiter. If error motion is obtained from a source that is not damaged when it is forcibly reversed against its full torque (as would be the case if a synchro or torque motor were used), the force limiter can be omitted.

When the limit-stop arm is actuating the control valve, the motor, cam, and valve comprise a closed loop which can be considered as a position servomechanism. As a result, the mechanical limit-stop system must be designed to suit the requirements of the servomechanism with which it is to operate in order to obtain stability (nonoscillating operation) of the device at the limits.

In general, limit stops should be designed to rely on a minimum number of other devices for their operation. Electronic amplifiers or other devices that can be expected to fail occasionally should not be placed in the limit-stop closed loop.

18-4.4 POSITIVE STOPS

The simplest form of positive stop consists of a lug, attached to the moving member, that strikes a fixed member at the end of the desired travel (Fig. 18-20). If the output motion exceeds one revolution, lugged washers can be stacked on the output shaft so that their lugs overlap, as shown in Fig. 18-21. The lug of the washer at one end of the stack is engaged by a lug on the shaft. As the shaft rotates, the lug pick-up action is cumulative until the lug on the washer at the other end of the stack is engaged by a fixed block or pin at the completion of the required number of revolutions. Lugged washers are often used on instrument servomechanisms.

18-4.5 Buffers

Springs are often incorporated in positive stops to produce buffer action that reduces the acceleration produced when the stops are engaged. Often, the addition of a piece of thick synthetic rubber to each side of the

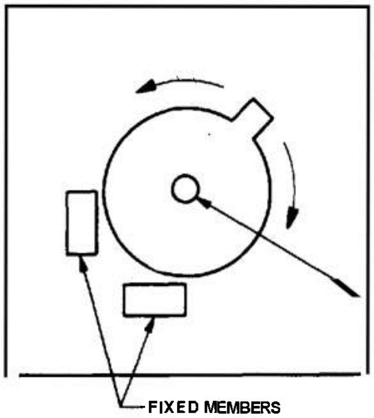


Fig. 78-20 Positive stop.

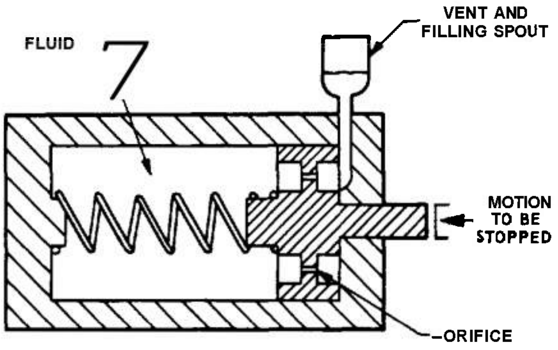


Fig. 78-22 Spring and buffer stop.

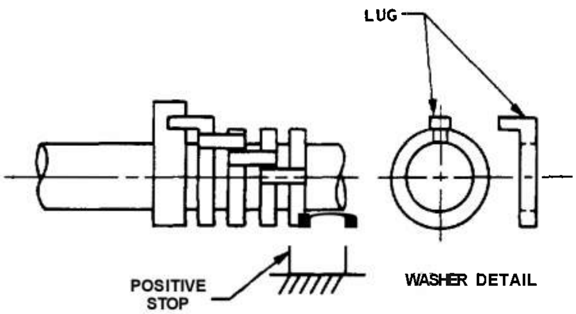


Fig. 78-27 Stop with lugged washers.

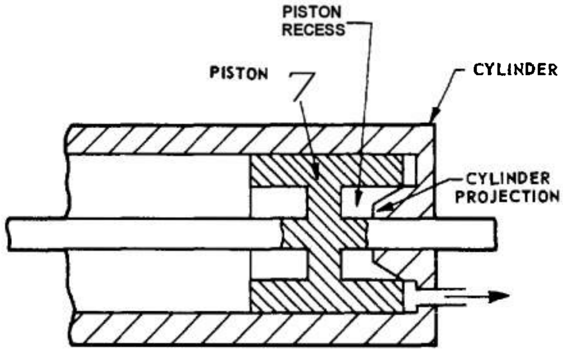


Fig. 78-23 Buffer stop in hydraulic cylinder.

lug arrangement of Fig. 18-20 produces sufficient shock-absorbing action to prevent damage. In other applications, it is necessary to resort to a combination of spring and dash-pot type damping, similar to the system of Fig. **18-22,** to obtain the desired deceleration without damage.

Hydraulic cylinders can be equipped with buffer stops at each end of piston travel, as shown in Fig. **18-23.** In this system, oil is trapped in the recessed end of the piston by the cylinder projection as the piston nears the end of the cylinder. The trapped oil can escape from the piston recess only through the restricted space between the wall of the piston recess and the surface of the cylinder projection, thereby producing viscous damping of piston movement. Either the cylinder projection or the piston recess, or both, can be shaped to produce any desired retardation rate. Check valves are sometimes used to permit unrestricted flow of oil from the supply line into the damping chamber but to force oil flowing from the chamber to the line to pass through the restricted passage. This arrangement reduces the time required to move the piston away from the end of the cylinder.

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CHAPTER 19

CONSTRUCTIONAL TECHNIQUES*

19-1 BASIC CONSIDERATIONS

In the design of any servomechanism or control system, a great many factors besides the response characteristics must be considered. Many systems that have adequate response characteristics are completely unsatisfactory because too many adjustments are required, the adjustments must be made too often, maintenance is difficult, or the equipment cannot withstand environmental conditions. In ordnance equipments, it is particularly important that much attention be paid to these details, not only because the equipment must be made reliable and simple to operate under conditions normally imposed upon equipment and operator, but also to minimize the time required to train operators and maintenance organizations.

19-2 COMPONENT LAYOUT

19-2.1 PHYSICAL ARRANGEMENT

Several physical aspects of component layout should be considered carefully during the design of a servomechanism.[1,2] Controls that must be used by the operator for successful operation of the device should be accessible at all times. For example, an amplifier balance control that must be adjusted each time the amplifier is turned on, or readjusted as the amplifier warms up, should be easily available to the operator. However, the control should not be placed where it can be moved accidentally. Switches should be placed for convenient use and the switch position (on, off, or other) should be marked. If several switches are included on a panel, the function of the switch should also be marked.

Controls requiring readjustment during maintenance should be inaccessible to operating personnel. However, the controls should be easily accessible to maintenance personnel, perhaps requiring only the removal of a simple cover or cap. It should not be necessary to remove an entire unit to readjust the gain control of the main loop of a servomechanism incorporating an electronic amplifier. Also, this control should not be accessible to the operator so that he can misadjust the system gain to an extremely oscillatory or badly overdamped condition. Access for routine maintenance should require only the removal of a few covers, not the removal of an entire assembly. Also, parts with finite useful lives should be easily replaced. The replacement of electron tubes, for example, should be a simple task. Filters for hydraulic fluid should be arranged so that they can be removed for cleaning or

*By J. O Silvey

replacement without disturbing anything but the filter element ; they should be connected so that only a negligible quantity of fluid is lost when a filter is removed. Controls and adjustments necessary for the initial alignment of the device, but not requiring subsequent readjustment, should be arranged to prevent unintentional readjustment. Furthermore, these controls should be placed where only authorized ordnance maintenance personnel are permitted to make adjustments.

Dials and instruments that must be read quickly and accurately should be placed where they can be seen easily and should be marked distinctively. Considerable psychological testing to determine the most readable instrumentation layouts has been done by the military[2]. Although the problems encountered in ordnance equipment are not identical with those for which the experiments were made, the general results of the tests are applicable.

Components should be arranged within an assembly so that it can be disassembled and reassembled with no special knowledge or special tools. It is also very desirable that the number of parts be kept as low as practical and that there be only one way of putting the parts together.

19-2.2 THERMAL CONSIDERATIONS AND HEAT GENERATION[3, 4, 5]

Many of the devices used in servomechanisms develop undesired heat which must be dissipated. Where space is not a limitation and air is free to circulate around the device, heat dissipation to the atmosphere may be accomplished with a temperature rise of only a few degrees above the ambient temperature. However, if the volume and hence the area of the devices is small, the resulting temperature rise may be sufficiently high to damage the equipment. The high ambient temperatures that cap be encountered by ordnance devices should be considered when designing control systems.

The sources of heat in various control systems differ so greatly from one another that it is necessary to evaluate each source separately. Some of the most common heat sources, as well as the representative rates at which they liberate heat, are listed in Table **19-1.** If the temperature at which a control system of a particular design is to operate appears to be excessive, the heating rates of the actual components should be obtained under service conditions.

The heat generated in a control system must be adequately dissipated to prevent excessive temperature rises. In general, it is preferable that auxiliary cooling devices such as fans and heat exchangers should not be used. However, in some instances, auxiliary cooling devices may be required (for example, to avoid excessive size of the control system).

The mounting of electronic components so that air can flow freely past the heat sources, particularly in an upward direction, is a well-established method of cooling amplifiers. Attaching a heat source (for example, a control motor) to a large plate or structural member of high heat conductivity helps to decrease the temperature rise of the heat source. In hydraulic systems, the temperature of electrical equipment can often be lowered by submerging the electrical device in the hydraulic fluid. Because the fluid is in motion and has much greater thermal capacity than air, the temperature rise of the submerged electrical device will be much less than it would be in air. Normally, hydraulic fluid can be kept adequately cool by using a sufficiently large sump which, in turn, is air cooled.

In most ordnance control equipment, the generated heat is transmitted to the surroundings by radiation or by convection of the air. Unfortunately, the laws of convection and radiation are so complicated that it is not usually practical to establish a single simple relation that will quickly give the heat dissipated by a hot body.

TABLE 19-1 HEAT LIBERATED BY CONTROL DEVICES

Device		Rate of Heat Liberation	Remarks
Vacuum-tube voltage-amplifier triode		4 watts	
Vacuum-tube voltage-amplifier pentode		3 watts	
Vacuum-tube power pentode (Type 6L6)		25 watts	
Constant-speed 60-cps induction motor	no load	9% output rating	
	full load	18% output rating	
Armature-controlled d-c motor	field only	6% power rating	
	no load	15% power rating	
	full load	25% power rating	
2-phase control motor	no load	300% of max output power	Heating with only reference field excited is approximately equal to maximum power output.
	stalled rated voltage	400% of max output power	
Auxiliary pump		20% of input power	Minimum heat equals 15% of input power. Actual loss depends upon duty cycle and pressure.
Hydraulic motor		15% of input, minimum	
Hydraulic amplifier		50% of input power	Minimum heat usually about 30% of input.
Variable-delivery pump		15% of input, minimum	

19-2.3 Radiation

Heat radiated by a hot body to a large, cool, black body that completely surrounds the hot body can be determined by

$$Q_1 = 36.9 \times 10^{-12} e (T_1^4 - T_2^4) \qquad (19\text{-}1)$$

where

Q_1 = radiated heat, in watts/in.^{2}

e = emissivity of hot body (see Table 19-2)

T_1 = absolute temperature of hot body, in °K
(Temperature in degrees Kelvin = 273 + temperature in degrees Centigrade.)

T_2 = absolute temperature of cool surrounding body, in °K

The heat radiated by a hot body can also be determined by

$$Q_2 = 17.2 \times 10^{-10} e (T_a^4 - T_b^4) \qquad (19\text{-}2)$$

TABLE 19-2 EMISSIVITY OF VARIOUS SURFACES

Surface (100°F)	Emissivity
Polished aluminum	0.04
Polished copper	0.03 (approx.)
Copper covered with oxide	0.78
Polished iron	0.24
Oil paint	0.94
White paint	0.90 to 0.95
Black paint	0.96
Aluminum paint	0.3 to 0.6*

*Dependingupon amount of aluminum and kind of lacquer.

where

Q_2 = radiated heat, in Btu/(hr) (ft^2)

e = emissivity of hot body (see Table 19-2)

T_a = absolute temperature of hot body, in °R
(Temperature in degrees Rankine = 460 + temperature in degrees Fahrenheit.)

T_b = absolute temperature of cool surrounding body, in °R

When the radiation loss is to be computed for a device that is enclosed in a chamber, the temperature to be used for T_2 or T_b in Eqs. (19-1) or (19-2) is the temperature of the chamber walls. If the device is outdoors, the temperature to be used for the surroundings is not so 'easily obtained. The safe approach is to use the temperature of the surrounding outdoor air as the value of T_2 or T_b, although this will, in nearly all cases, yield values for heat dissipation which are lower than those actually encountered. A clear, blue sky reflects little radiant energy. In calculations involving heat dissipation of devices exposed to direct sunlight, it is necessary to dissipate the heat absorbed from solar radiation as well as the heat generated within the device. Neglecting atmospheric absorption and assuming that the solar radiation falls on a black body, the heat resulting from the sunlight is approximately 420 Btu/ft^2(hr) if the surface is normal to the solar rays.[5] Dust, water vapor, ozone, and other substances in the atmosphere reduce the solar heat to a value of approximately 290 Btu/ft^2(hr), or lower, at the surface of the earth.

19-2.4 Free Convection by Air

Free convection by air occurs when the air is initially at rest; subsequent air motion results from the expansion produced by the heat extracted from the warm body. In ordnance equipment that is operated outdoors, this type of convection occurs only when there is no wind. Heat loss due to convection is dependent upon the attitude of the surface relative to the vertical, the dimensions of the surface, and the temperature difference between the surface and the surrounding air.

For a high vertical surface, the process of free convection by air is somewhat as follows: Cool air is drawn in toward the surface at the bottom. However, because of the viscosity of the air, the layer of air in immediate contact with the surface does not move, forming a boundary layer through which the heat must be conducted. When the air close to the surface becomes heated, it rises slowly, and the flow is laminar or viscous. As the heated air moves upward along the surface, the air becomes increasingly warmer, its cooling ability decreases, and its velocity increases. Eventually, the air flow becomes turbulent, and as cooler air is brought in by the eddies of the turbulence, the cooling ability of the air increases.

Experiments[4] indicate that heat loss by air convection from a surface may be approximated by the expression

$$H_c = A_c \theta_c^{5/4} \qquad (19\text{-}3)$$

where

H_c = heat loss, in microcalories/cm^2(sec)

θ_c = temperature difference between surface and air, in °C

$$\log (A_c - 50) = 2.22 - \frac{h}{20}$$

h = height of surface, in cm

A more convenient approximate relation for heat loss by air convection is

$$Q_c = A_F H_F \theta_F \qquad (19\text{-}4)$$

where

Q_c = heat loss, in Btu/hr

A_F = area of vertical surface, in ft^2

θ_F = temperature difference between surface and air, in °F

H_F = variable whose value is obtained from Table 19-3

The values of H_F in Table 19-3 are based on values in Table 8B of reference 4 that were obtained from tests made with heated cylinders which were either suspended in air

TABLE 19-3 VALUES OF H_F [H_F in Btu/(hr)(ft^2)(°F)]

Surface Height (in.)	θF							
	20°F	40°F	60°F	80°F	100°F	120°F	140°F	180°F
100	0.60	0.75	0.87	0.93	1.01	1.06	1.12	1.23
69	0.65	0.75	0.88	0.96	1.03	1.10	1.15	1.24
35	0.65	0.8	0.87	0.92	0.98	1.02	1.07	1.13
23	0.7	0.8	0.88	0.95	1.0	1.05	1.10	2.17
11.3	0.75	0.9	1.00	1.06	1.12	1.17	1.23	1.32
5.9	1.10	1.3	1.38	1.44	1.51	1.55	1.60	1.66
3.2	1.65	1.8	1.95	2.03	2.12	2.2	2.27	2.38
1.8	2.0	2.4	2.55	2.70	2.86	2.98	3.10	3.28

by thin supports or supported by columns no larger in diameter than the heated cylinder. Other tests made with a vertical, flat plate (34 inches square) indicate that a decrease of approximately four percent in convection cooling results when a floor is placed immediately below the vertical plate. The reduction in natural convection cooling that would result from a floor placed immediately below the cylindrical surfaces used for Table 19-3 is not known. A second plate placed parallel to, and at a distance of more than two inches from, a vertical heated surface has no effect on the air convection cooling. Large horizontal heated surfaces facing upward release approximately 30 percent more heat than the vertical cylinders tested for Table 19-3. Also, large horizontal heated surfaces facing downward release approximately 30 percent less heat by convection than the cylinders tested for Table 19-3.

19-2.5 Conduction

Heat transmitted through a material by conduction follows the law

$$Q_T = \frac{AK\theta}{L} \tag{19-5}$$

where

Q_T = heat transmitted, in Btu/hr

A = area of path of heat flow (perpendicular to direction of heat flow), in ft^2

K = thermal conductivity of material, in Btu/hr (ft) (F")
(F° = temperature difference, in °F)

θ = difference in temperature of two sides of material, in °F

L = thickness of material, in ft

The thermal conductivities of some materials are listed in Table 19-4. The conversion factors for the coefficients of heat transfer are given in Table 19-5.

TABLE 19-4 THERMAL CONDUCTIVITY OF METALS AND ALLOYS AT 212°F

Material	Thermal Conductivity [Btu/(hr) (ft) (F°)]
Aluminum	119
Brass (70-30)	60
Copper	218
Lead	20
Silver	238
Steel AISI-1020	26
Stainless steel AISI-304	9.4
Pine wood — across grain	0.09

19-2.6 EQUIPMENT AND PERSONNEL SAFETY MEASURES

Ideally, any failure of a servomechanism should cause an alarm indication and result in complete cessation of the output. Also, the device should be constructed so that personnel injury or equipment damage is impossible. It is difficult, however, to eliminate a servomechanism output for all system failures and to make an alarm system that will indicate failures reliably. The practical requirements that arise in the use of many service equipments make the exercise of common sense and the fulfillment of an adequate training program the only real insurance against injury to personnel. Nevertheless, much can be done to reduce the probability of equipment damage and injury to personnel. Each device should be studied with regard to its specific requirements in order to properly evaluate the safety devices that should be included.

Limit stops and positive stops (Par. 18-4) are often used to prevent damage to equipment. Positive stops are usually accompanied by slip clutches, over-current relays, or hydraulic relief valves so as to avoid damage to the servomotor or associated devices when the servomechanism output strikes the positive stop.

Interlocking relays are often used to prevent operation of a servomechanism when a component has failed; for example, loss of plate voltage to a vacuum-tube amplifier can be used via an interlocking relay to de-energize the motor of an associated amplidyne. Solenoid-operated bypass valves and disconnect-linkages are often used to prevent motion of hydraulic or mechanical devices when the servo electric power fails. When it is necessary that the output of the servo go to one limit in case of failure, it is sometimes possible to use .relays or spring-loaded solenoids to apply a false error signal that will cause the output to slew to the **prop** limit.

Most power servomechanisms used to drive guns and tank turrets have more than enough torque to crush any portion of the human anatomy that happens to be in the path of the output. It is important that the output be equipped with guards to prevent personnel injuries where moving members are close to fixed members. These guards must not interfere with normal operation of the weapon. Adequate protection against accidental contact with high-voltage electric circuits should also be provided.

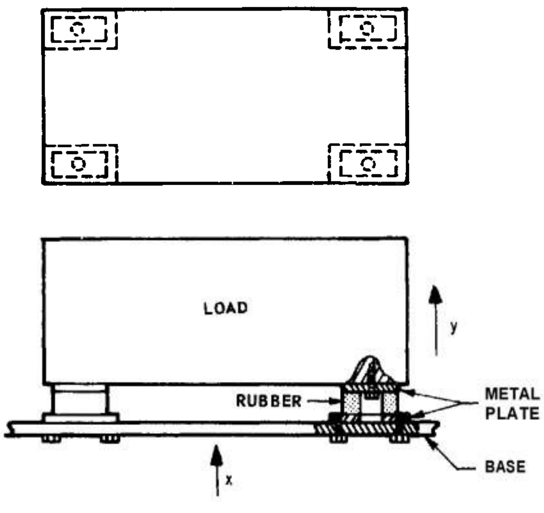
TABLE 19-5 CONVERSION FACTORS FOR COEFFICIENTS OF HEAT TRANSFER

To obtain ↓ / By / Multiply number of →	Btu/(hr) (ft²) (F°)	Btu/(hr) (in.²) (F°)	Watts/(in.²) (F°)	Watts/(cm²) (C°)	Kg-calories/(hr) (cm²) (C°)	Gram-calories/(sec) (cm²) (C°)
Btu/(hr) (ft²) (F°)	1	144	490	1760	2050	7380
Btu/(hr)(in.²) (F")	6.94 $\times 10^{-3}$	1	3.424	12.2	14.2	51.3
Watts/ (in.²) (F°)	2.04 $\times 10^{-3}$	0.292	1	3.58	4.16	15.0
Watts/ (cm²) (C°)	5.68 $\times 10^{-4}$	8.19 $\times 10^{-2}$	0.279	1	1.16	4.23
Kg-calories/ (hr) (cm²) (C°)	4.88 $\times 10^{-4}$	7.04 $\times 10^{-2}$	0.240	0.859	1	3.60
Gram-calories/ (sec) (cm²) (C°)	1.355 $\times 10^{-4}$	1.95 $\times 10^{-2}$	6.67 $\times 10^{-2}$	0.236	0.277	1

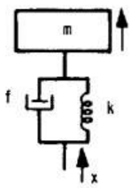
19-3 VIBRATION ISOLATION[6,8,9,10,11,12]

The necessity of vibration isolation for the proper functioning of any piece of ordnance equipment is undesirable. A thorough search for an alternate device should be made before using a device that requires vibration isolation. Some devices are sufficiently delicate to require isolation from the vibration of the base upon which they will be mounted; other devices produce vibration which must not be transmitted to the base. For such devices, vibration isolation is necessary.

Figure 19-1A shows a typical vibration isolation mounting. The vibration mounts consist of a piece of rubber bonded to two pieces of metal in such a way that any motion transmitted from one piece of metal to the other must pass through the rubber. One piece of metal is attached to the base, and the other is attached to the device that is to be isolated. The elastic and internal damping properties of rubber make it well-suited for use in vibration isolation devices, although metallic springs, cork, and other materials are also used. Vibration isolation is obtained by making the resonant frequency of the device and its vibration mounts appreciably lower than the forcing frequency which is to be attenuated.



A.



B.

Fig. 19-1 Vibration isolation mounting.

By analysis of the equivalent circuit (Fig. 19-1B) of the vibration isolation mounting (Fig. 19-1A), an expression for the amplitude of vibration of the isolated load is given as

$$y = x \frac{2\zeta \frac{s}{\omega_n} + 1}{\frac{s^2}{\omega_n^2} + 2\zeta \frac{s}{\omega_n} + 1} \qquad (19\text{-}6)$$

where

y = amplitude of vibration of isolated load in ft

x = amplitude of vibration of base, in ft

$\zeta = \frac{f}{2}\sqrt{\frac{g}{mk}}$ = damping ratio

f = damping of vibration mounts, in lb-sec/ft

g = 32.2 ft/sec^2 = acceleration of gravity

m = mass of isolated load, in lb

k = combined spring constant of vibration mounts, in lb/ft

s = Laplace operator

$\omega_n = \sqrt{\frac{kg}{m}}$ = undamped resonant frequency in radians/sec

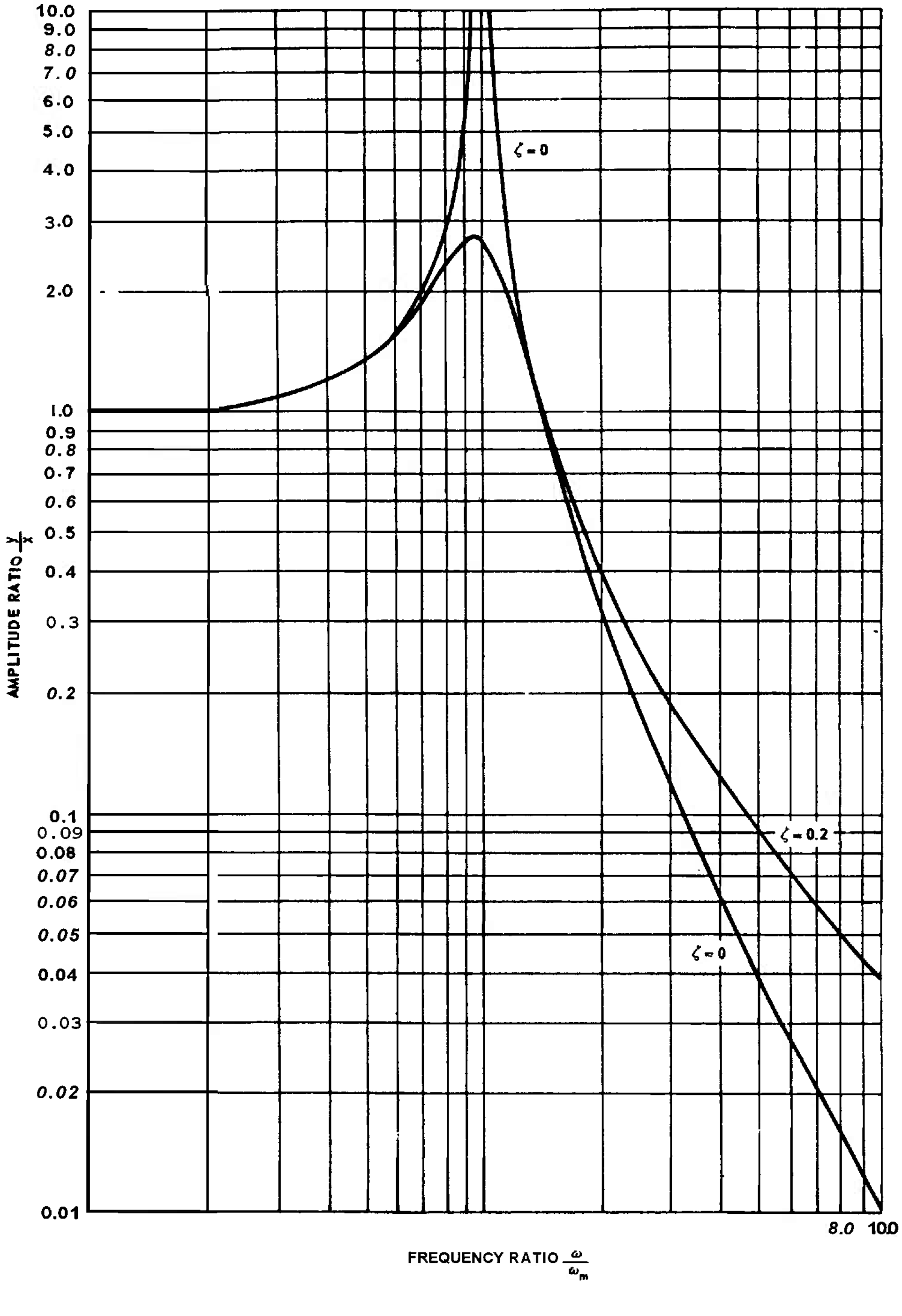


Fig. 19-2 Frequency response of vibration isolation mounting.

The amplitude of the ratio y/x as a function of the ratio of the forcing frequency (ω) to the resonant frequency (ω_n) is plotted in Fig. 19-2 for values of ζ equal to zero and 0.2. It can be seen that, for the damped or undamped condition, extremely large amplitudes occur when the forcing frequency equals the resonant frequency. Most devices equipped with vibration mounts either pass through the mount resonant frequency quickly or are only occasionally excited by a forcing frequency equal to the resonant frequency. Most vibration isolation mountings have a resonant frequency that is less than half the forcing frequency. The resonant frequency must be in the order of one-tenth of the forcing frequency if less than 5 percent of the amplitude of the forcing frequency is to be transmitted through the vibration isolation mounts.

The natural frequency of a mass that is mounted for vibration isolation in the vertical direction is determined by the weight of the load and the static deflection of the mounts when supporting the weight of the load. As the deflection y (ft) produced by a weight w (lb) acting on a spring with a constant k (lb/ft) is

$$y = w/k \tag{19-7}$$

the natural frequency of the mass will be

$$\omega_n = \sqrt{\frac{g}{y}} = \frac{5.67}{\sqrt{y}} \text{ radians/sec} = \frac{0.902}{\sqrt{y}} \text{ cps} \tag{19-8}$$

where y is in feet, or

$$\omega_n = \frac{3.13}{\sqrt{y}} \text{ cps}$$

where y is in inches.

The analysis given here is simplified to include only motion along a line perpendicular to the mounting surface, assuming loads for which the center of mass is equidistant from the isolation supports. In practice, it is necessary to consider rotary vibration of the load, particularly if vibrations can occur parallel to the mounting surface or if rotary forcing vibrations can occur about any axis.

Vibration isolation can be made unnecessary in some cases by eliminating the vibration at its source. Various friction, viscous, and tuned damping devices have been used for this purpose. Static and dynamic balancing are also used to eliminate vibration caused by rotating devices.

19-4 SHOCK ISOLATION[6,7,10,11,12]

The indiscriminate use of shock isolation devices should be avoided because their use often makes conditions worse instead of better. Therefore, servomechanisms should be equipped with shock mounts only when absolutely necessary. A large variety of electrical and mechanical devices are available which, without shock protection, will withstand shocks higher than those normally encountered under service conditions. Even electron tubes, delicate as they may seem, can often operate satisfactorily without shock protection. If precautions are taken in the design of the control system components, there should be no need for devices to reduce shock intensity in the majority of ordnance servomechanisms. Table 19-6 lists typical vibration and shock values encountered in ordnance equipment.

The physical construction of mounts for shock isolation is identical with that of mounts for vibration isolation. In general, the underlying basis for both types of mounts is the same in that they serve to lower the acceleration and hence the force applied to the device to be protected. The resonant frequency of loads supported by resilient shock mounts is often set at approximately 20 cps because a resonant frequency of 20 cps provides adequate protection against most

TABLE 19-6 TYPICAL VIBRATION AND SHOCK VALUES ENCOUNTERED IN ORDNANCE EQUIPMENT

Equipment	Source	Frequency (cps)	Maximum Acceleration in g's†
Trucks and on-carriage guns	springs	2-5	
	tires	8-15	5-7
	structures	60-200	
Tank	———	3 times speed of tank (in mph)	5 (on black top road) 25 (cross country) 2 (on level gravel road)
———	muzzle blast 6 ft from equipment	impulse	1000-1500 (exterior) 300-1250 (interior)
———	moderate non-penetrating ballistic impact	impulse	400
———	heavy non-penetrating ballistic impact	impulse	vehicle: 1800 (fore & aft), 900 (lateral), 275 (vertical)
			equipment: 260 (fore & aft), 375 (lateral)

†Maximum damage due to ballistic impact appears to be caused by accelerations of approximately 3000 g.

Data from letter to Jackson & Moreland, Inc., Boston, Mass., Record No. 5817, from Ordnance Corps, Frankford Arsenal, 7 August 1956.

shocks. Furthermore, most vibrations that might excite the system and cause the spring-mass system to resonate are below 20 cps and hence are not amplified by the shock mounts through sympathetic vibration.

The analytical model of a shock-mounted device is shown in Fig. 19-3. In the figure, k_m is the spring constant and f_m is the damping of the shock mount. The mass of the frame to which the shock mounts are attached to support the load is m_1. The mass of the protected device is m_2, k_d is the spring constant, and f_d is the damping of the support. An example of a shock-mounted device is an instrument mounted in a heavy cabinet which, in turn, is mounted on resilient shock mounts. The amount of protection provided by the shock mounts depends as much upon the dynamic characteristic of the system involving k_d, f_d, and m_2 as it does upon the dynamic characteristics of the shock mounts and the mass m_1. If it is assumed that m_1

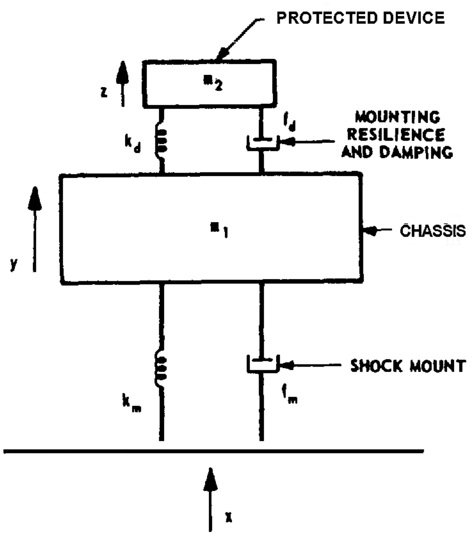


Fig. *19-3 Analytical model* of *shock-mounted device.*

is so large that its motion is unaffected by motion of m_2, the following general characteristics are true of the system:

(a) To be of any significant value, the shock mounts must be such that the resonant frequency of m_1—k_m system is less than half the resonant frequency of the m_2—k_d system. This means that the mounting of the protected device on the chassis or frame must be relatively stiff. If the two frequencies coincide, the acceleration of m_2 becomes extremely large for any reasonable value of damping. If the resonant frequency of the m_1—k_m system is much above the resonant frequency of the m_2—k_d system, the resulting acceleration of m_2 is essentially the same whether or not shock mounts are used.

b) The acceleration of m_2, as well as the acceleration of m_1, rises when the shock-mount damping f_m is increased. However, some damping is necessary to prevent vibration for excessive lengths of time after a shock occurs.

(c) For a shock consisting of an impulse (which might be represented by a sine wave with a time duration of a half-cycle) at position x in Fig. 19-3, the acceleration of m_1 is greatest when the shock duration is half the resonant period of the m_1—k_m system, provided there is no damping (f_m). The peak acceleration occurs after the shock impulse has again returned to zero.

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CHAPTER 20

SUPPLEMENTARY TABLES, FORMULAS, AND CHARTS*

20-1 MASS MOMENT OF INERTIA

20-1.1 DEFINITION

The mass moment of inertia of a body with respect to any axis is defined as

$$J = \int r^2 dm \qquad (20\text{-}1)$$

where J is the moment of inertia of the body, dm is an element of mass of the body, and r is the perpendicular distance of dm from the axis about which the inertia is to be found. The integral must be evaluated over the entire volume of the body. The dimensions of mass moment of inertia are ML^2. The relations between the commonly used units of mass moment of inertia are listed in Table 20-1.

20-1.2 DATA FOR EQUATIONS

Table 20-2 is included to facilitate writing equations involving torque, moment of inertia, and acceleration. When writing equations using torque in units of the first column of the table and moment of inertia in units of the second column, divide the moment of inertia by the number in the third column of the table to obtain acceleration in terms of radians per second per second.

20-1.3 PARALLEL-AXIS THEOREM

If the moment of inertia of a body about an axis through its center of mass is known, and if it is desired to obtain the moment of inertia about an axis parallel to that particular centroidal axis, then

$$I = I_c + Mr^2 \qquad (20\text{-}2)$$

where

I = moment of inertia about the new axis

I_c = moment of inertia about the centroidal axis

M = mass of the body

r = distance between centroidal axis and the new axis

Equation (20-2) is called the ***parallel-axis*** theorem and is proven in most kinematics tests.

20-1.4 PRINCIPAL AXES OF INERTIA

For any particular point within a body, there is one axis about which the moment of inertia is a maximum and another axis about which the moment of inertia is a minimum. These two axes are perpendicular to each other. These axes, together with the axis through their point of intersection and perpendicular to the plane formed by them, are called the ***principal axes*** of ***inertia*** of the body.

20-1.5 PRODUCT OF INERTIA

The product of inertia of a body with respect to two mutually perpendicular planes is defined as

$$G_{ab} = \int ab\, dm \qquad (20\text{-}3)$$

where a and b are the perpendicular distances from mass element dm to the two planes. A body must have three products of inertia

$$G_{xy} = \int xy\, dm,\ G_{yz} = \int yz\, dm,\ G_{zx} = \int xz\, dm \qquad (20\text{-}4)$$

*By J. O. Silvey

TABLE 20-1 RELATIONS BETWEEN UNITS OF MASS MOMENT OF INERTIA

To obtain ↓ / Multiply number of → By	gram-cm^2	oz-in.2	lb-in.2	lb-ft^2	slug-ft^2
gram-cm^2	1	182.9	2926	4.214×10^{-5}	1.356×10^{7}
oz-in.2	5.467×10^{-3}	1	16	2304	7.412×10^{4}
lb-in.^{2}	3.418×10^{-4}	0.0625	1	144	4632
lb-ft^2	2.373×10^{-6}	4.340×10^{-4}	6.944×10^{-3}	1	32.17
slug-ft^2	7.375×10^{-8}	1.349×10^{-5}	2.159×10^{-4}	0.03108	1

TABLE 20-2 RELATIONS BETWEEN TORQUE, MOMENT OF INERTIA, AND ACCELERATION

A torque of	Will accelerate a body having a moment of inertia of	To	
1 dyne-cm	1 gram-cm2	1	rad/sec^2
1 gram-cm	1 gram-cm^2	980.7	rad/sec^2
1 oz-in.	1 oz-in.2	386	rad/sec^2
1 lb-in.	1 lb-in.2	386	rad/sec^2
1 lb-ft	1 lb-ft^2	32.17	rad/sec^2
1 lb-ft	1 slug-ft^2	1	rad/sec^2

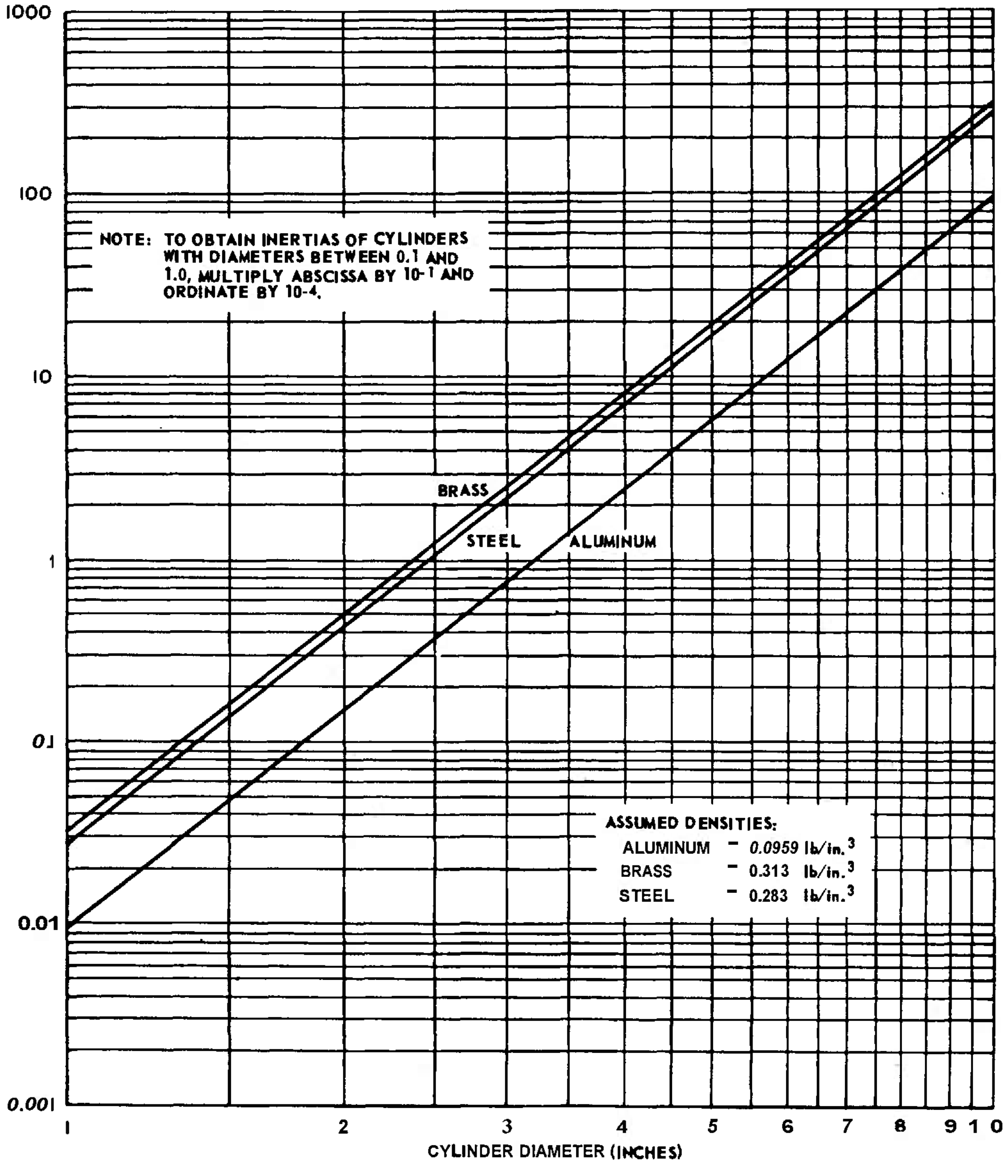

Fig. 20-1 Moments of inertia about the principal axes of cylinders one inch long.

The product of inertia of a body with respect to any two of the planes formed by the principal axes of the body is zero.

20-1.6 INERTIA WITH RESPECT TO A LINE THROUGH THE ORIGIN

The moment of inertia of a body with respect to any line passing through the coordinate origin is

$$J = J_x \cos^2 a + J_y \cos^2 \beta + J_z \cos^2 \gamma$$
$$- 2G_{xy} \cos a \cos \beta - 2G_{yz} \cos \beta \cos \gamma$$
$$- 2G_{zx} \cos \gamma \cos a \qquad (20\text{-}5)$$

where

J_x, J_y, J_z = moments of inertia about the x-, y-, and z-axes, respectively

a, β, γ = angles between the x-. y-, and z-axes, respectively

G_{xy}, G_{yz}, G_{zx} = products of inertia with respect to the yz- and xz-planes, the xx- and xy-planes, and the xy- and yz-planes, respectively.

If the coordinate system is so chosen that the x-, y-, and z-axes lie along the principal axes of the body, the products of inertia are zero and Eq. (20-5) reduces to

$$J = J_x \cos^2 a + J_y \cos^2 \beta + J_z \cos^2 \gamma \qquad (20\text{-}6)$$

20-1.7 TABULATED MOMENTS OF INERTIA

Expressions for the moments of inertia of a large number of bodies are tabulated in the Handbook of Engineering Fundamentals[1] and the Handbook of Chemistry and **Physics.**[2] The moments of inertia of cylinders frequently used in control work are plotted in Fig. **20-1.**

20-1.8 COMPLICATED SHAPES

The moment of inertia of a body with a complicated shape can be approximated by dividing the body into a number of parts having simple shapes, computing the inertia of each part, and then adding the inertias of the individual parts. If the shapes of the parts can be approximated by simple geometric shapes, the tabulated expressions for moments of inertia and the parallel-axis theorem can be used to compute the total moment of inertia of the body. If the body is extremely irregular, it sometimes is more convenient to plot its cross section on graph paper, using either Cartesian or polar coordinates (depending upon the problem), and to compute the moment of inertia of the rectangles or arcs covered by the figure. For bodies such as tank turrets, the moment of inertia must be computed at several elevations, and the inertias of the various cross sections must be added to obtain the total inertia.

20-2 DAMPING AND FRICTION

20-2.1 VISCOSITY

20-2.2 Definition

Viscosity is that characteristic of a fluid which produces **a** resistive force when there is relative motion between a solid body and a liquid with which it is in contact. This resistive force, proportional to relative velocity between the solid and the fluid, is known as ***viscous*** *friction* and produces damping. The viscous properties of fluids produce damping whenever relative motion between a fluid and a solid body with which the fluid is in contact occurs. Dampers are made that consist of two parallel discs or concentric cylinders which are separated by a small gap filled with viscous **fluid and which are capable of moving** relative **to one another. Viscous damping also occurs when a viscous fluid flows in pipes or tubing.**

20-2.3 Absolute (Dynamic) Viscosity

If the space between two large parallel plates of area a, separated by a distance l, is filled with a fluid of viscosity μ, and if one plate is moved relative to the other with velocity v, the separation being kept constant, then the force f required to sustain the motion of the moving plate is

$$f = \frac{\mu a v}{l} \tag{20-7}$$

From Eq. (20-7), the viscosity is $\mu = fl/av$ and has the dimensions $ML^{-1}T^{-1}$ or $FL^{-2}T$. Viscosity defined in this way is called *absolute*, or *dynamic, viscosity*. The only unit of absolute viscosity that has a name is the poise, which is one dyne-second per square centimeter, or one gram mass per centimeter-second. Table 20-3 lists the relations between various units of absolute viscosity.

20-2.4 Kinematic Viscosity

Kinematic viscosity is defined as the absolute viscosity of a fluid divided by its density. When a dimension system of force, distance, and time is used, the dimensions of kinematic viscosity are L^2T^{-1}. The centimeter-gram-second (cgs) unit is the stoke, which has the dimensions centimeter squared per second. The English unit, which would be used with the pound force-foot-second system commonly employed in engineering work, is the foot squared per second. The relations between the three commonly used units of kinematic viscosity are:

(a) stokes $\times$ 100 = centistokes

(b) (ft^2/sec) $\times$ 929 = stokes

TABLE 20-3 RELATIONS BETWEEN UNITS OF ABSOLUTE VISCOSITY

To obtain ↓ / By / Multiply number of →	poise* (dyne-seconds per square centimeter)	gram force-seconds per square centimeter	ounce force-seconds per square inch	pound force-seconds per square inch	pound force-seconds per square foot
poise* (dyne-seconds per square centimeter)	1	980.7	4309	6.89 $\times 10^4$	479
gram force-seconds per square centimeter	1.020 $\times 10^{-3}$	1	28.35	454	3.15
ounce force-seconds per square inch	2.33 $\times 10^{-4}$	0.03527	1	16	0.111.
pound force-seconds per square inch	1.451 $\times 10^{-5}$	2.202 $\times 10^{-3}$	0.0625	1	6.94 $\times 10^{-3}$
pound force-seconds per square foot	2.09 $\times 10^{-3}$	0.317	9	144	1

*To obtain centipoises, multiply number of poises by 100.

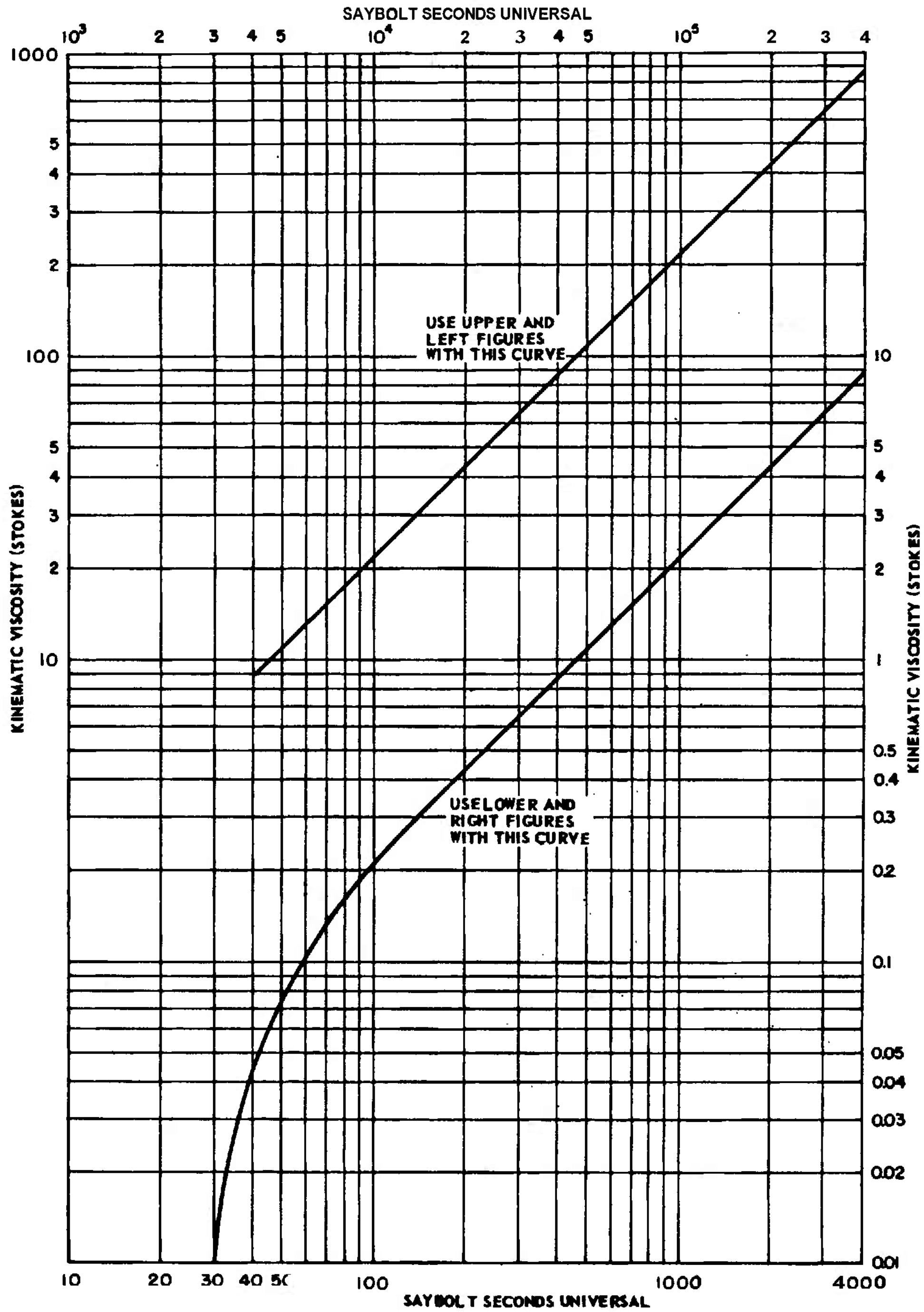

Fig. 20-2 Relation between Saybolt seconds universal and stokes.

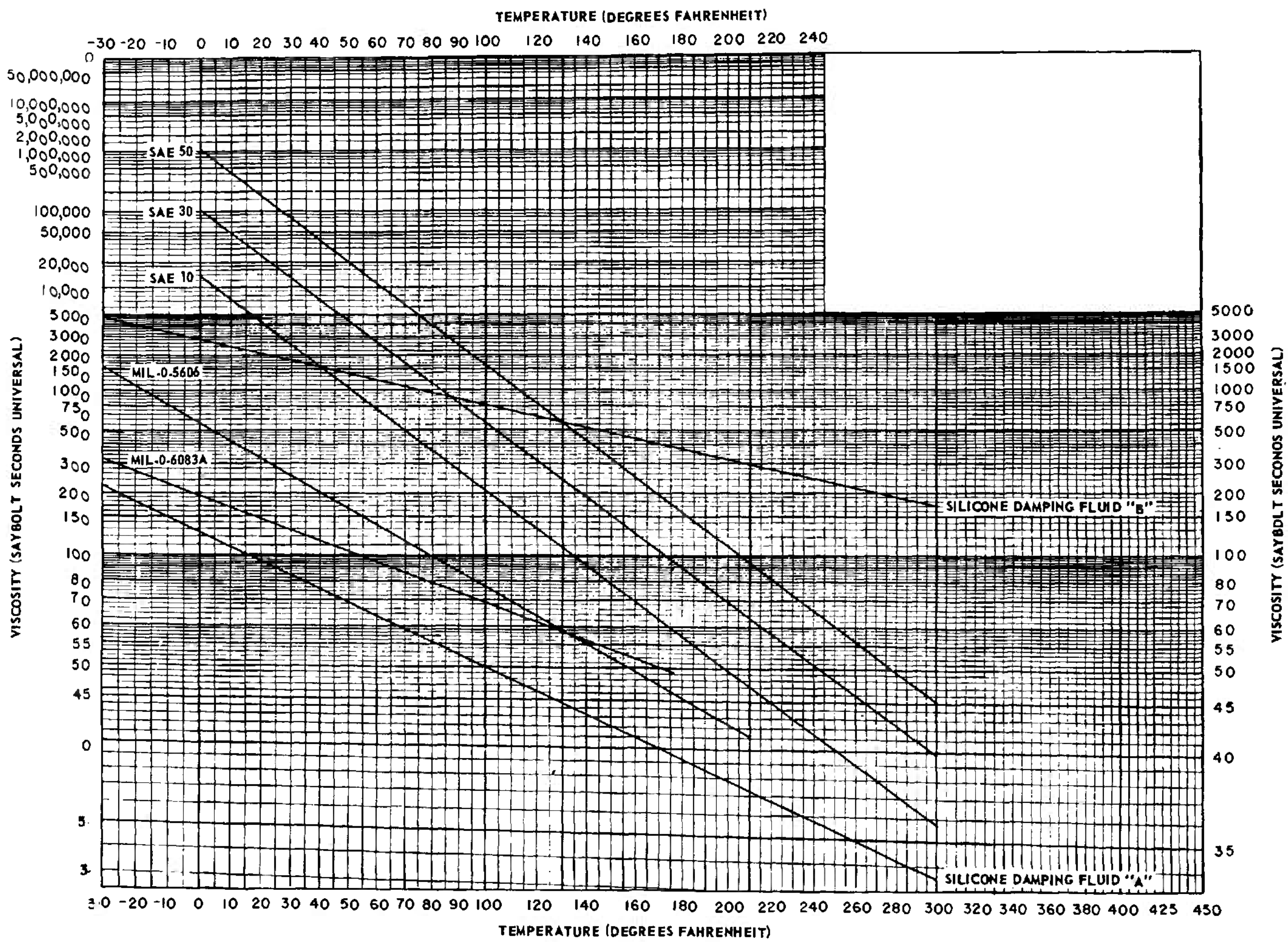

Fig. 20-3 Viscosity versus temperature for some oils and fluids.

Chart used by permission of American Society for Testing Materials.

The time required for a fluid to drain from a viscometer by the action of gravity is a function of the kinematic viscosity of the fluid. In the United States, the kinematic viscosity of lubricating oils and hydraulic fluids is usually expressed in Saybolt seconds universal (ssu). This is the time, in seconds, for a particular quantity of fluid to escape from a specified vessel through a specified capillary tube. The conversion between the time of discharge and the kinematic viscosity of the fluid, in units suitable for calculations, has been determined empirically and is shown in Fig. 20-2. The conversion for various viscometers used in Europe is given in reference 1.

20-2.5 Effect of Temperature

The viscosity of all lubricating oils, hydraulic fluids, and damping fluids increases when their temperature is lowered. However, most hydraulic fluids contain additives that reduce the change in viscosity below that exhibited by pure petroleum-base oils. The relation between temperature and viscosity for some oils and fluids is shown in Fig. 20-3.

20-2.6 FRICTION

Friction that is proportional to velocity is called *viscous friction* and is usually associated with fluids. Friction that is constant regardless of velocity, but which always opposes motion, is called *coulomb friction* and is associated with poorly lubricated sliding surfaces.

20-2.7 Coefficient of Friction

The coefficient of friction between two sliding surfaces is defined as the tangential friction force divided by the normal force. When no motion occurs, the quotient is called the *coefficient of static friction*. The value of the coefficient of static friction is determined by the maximum friction force that will not produce slipping. When one surface slides relative to the other, the quotient is called the *coefficient of kinetic friction*. Both coefficients

TABLE 204 COEFFICIENTS OF FRICTION FOR VARIOUS METALS

Material	Condition	Coefficient of Kinetic Friction	Coefficient of Static Friction
Cast iron on cast iron	dry	0.152	0.162
Cast iron on cast iron	greased	0.08-0.10	0.16
Cast iron on bronze	dry	0.213	
Cast iron on bronze	greased	0.132	
Wrought iron on wrought iron	dry	0.44	
Wrought iron on wrought iron	greased	0.08-0.10	0.11
Wrought iron on bronze	dry	0.18	0.19
Wrought iron on bronze	greased		0.07-0.08
Steel on steel	dry (10 ft/sec)	0.09	0.15
Steel on steel	dry (100 ft/sec)	0.03	
Steel on bronze	dry	0.152	

of friction are independent of pressure on the surfaces, and hence independent of the surface area, when the pressure is between 3/4 psi and 100 psi. For pressures below 3/4 psi, the coefficient of friction increases somewhat. It also increases for extremely high pressures.

20-2.8 Characteristics of Coefficient of Friction

The coefficient of static friction is always higher than the coefficient of kinetic friction. The coefficient of kinetic friction is independent of velocity for average relative velocities. However, the coefficient of kinetic friction decreases at extremely high velocities and increases at extremely low velocities. Since the coefficient of friction, either static or kinetic, is dependent upon the condition of the surfaces and the presence or absence of small amounts of lubricants, any quoted value should be considered as an approximation. The coefficients of friction for various metals are listed in Table 20-4.

20-3 SPRINGS

20-3.1 HELICAL SPRINGS

20-3.2 Stress

Neglecting end effects and curvature correction and assuming round wires, the stress in loose-wound tension helical springs or compression helical springs is

$$S = \frac{8WD}{\pi d^3} \qquad (20\text{-}8)$$

where

S = maximum fibre stress, in psi

W = load, in lb

D = mean coil diameter, in in.

d = wire diameter in in.

Equation (20-8) results in an error of more than ten percent if the spring index (D/d) is less than 13. Neglecting end effects, assuming round wire, and taking curvature correction into account

$$S = \frac{8WD}{\pi d^3} K \qquad (20\text{-}9)$$

where S, W, D, and d are defined in Eq. (20-8) and

$$K = \frac{4(D/d) - 1}{4(D/d) - 4} + \frac{0.615}{D/d}$$

K is known as the Wahl correction factor.[9]

20-3.3 Deflection

The deflection of a helical spring, neglecting end effects, is

$$Y = \frac{8WD^3 n}{E_s d^4} \qquad (20\text{-}10)$$

where W, d, and D are defined in Eq. (20-8) and

n = number of active coils

E_s = torsional modulus of rigidity, in psi

Y = spring deflection, in in.

20-3.4 Torsional Elasticity

The torsional elastic properties of some spring materials are listed in Table 20-5. The values of working stress in this table are quite conservative and, in most cases, can be exceeded without over-stressing the spring.

20-3.5 Design Table

Table 20-6 can be used to choose a helical spring size quickly if the desired spring constant is known. The spring constant in pounds per inch of deflection is the spring scale (lower entry) of the table divided by the number of turns in the spring to be used. The upper entry of the table is the spring load that will produce a stress of 100,000 psi, and is independent of the number of turns in the spring. Loads corresponding to higher or lower stresses are equal to the tabulated loads multiplied by the desired stress in psi and divided by 100,000. Linear interpolation to obtain the spring scale and load for any spring coil diameter or any wire size will result in an error of less than 5 percent ; obviously, double interpolation is required if the desired coil diameter and wire diameter both fall between values entered in the table. It should be emphasized that this table is valid only for round wire with a torsional modulus of 11×10^6 psi. An example illustrating the use of this table follows : Find a steel spring with 0.5-inch outside diameter which will deflect 0.5 inch under a load of 7 lb, and which can be deflected 1.0 inch without exceeding 100,000 psi stress. The maximum load will be $7\left(\frac{1.0}{0.5}\right) = 14$ lb. From the table, a wire diameter of 0.059 will produce a spring of 0.5-inch outside diameter which will carry 15.3 lb., which is adequate for this application. From the table, the spring scale for a single coil of this spring will be 204 lb/inch. The desired spring constant is $\frac{7}{0.5} = 14$ lb/inch. The required number of coils is therefore $\frac{204}{14} = 14.6$ coils. Minimum length of a tension spring having the desired characteristics, neglecting length of spring ends, is $(14.6)(.059) = 0.8614$ inch. A compression spring having the desired characteristics would be one inch longer to accommodate the required deflection of one inch before it is compressed solid. Table 20-6 is a condensed version of the table given by Ross.[5] Other spring design tables appear in the literature,[6,7,8] as well as in standard mechanical engineers' handbooks.

TABLE 20-5 TORSIONAL ELASTIC PROPERTIES OF SOME SPRING MATERIALS

Material	Elastic Limit* (psi)	Recommended Working Stress‡ (psi)	Modulus Rigidity (psi)
Music wire	130,000	70,000	11×10^6
Hot rolled steel (SAE 1095) hardened to Rockwell C43	92,000	60,000	11×10^6
Stainless steel (Type 302)	95,000	35,000	11×10^6
Phosphor or silicon bronze	67,000	30,000	6.2×10^6
Beryllium copper	80,000	50,000	6.5×10^6

80%

20-3.6 CANTILEVER SPRINGS

20-3.7 Definition

A cantilever spring (Fig. 20-4) is made by clamping one end of a bar and applying a load at the other end. In effect, the spring is essentially a cantilever beam and ordinary beam theory can be used to obtain stress and deflection.

20-3.8 Stress and Deflection

The stress of a cantilever beam is

$$S = \frac{MC}{I} \qquad (20\text{-}11)$$

where

S = maximum tensile fibre stress, in psi

M = moment on beam, in in.-lb

C = distance from neutral axis to outer fibre, in in.

I = moment of inertia of cross section of the beam about the neutral axis, in in.^{4}

For a cantilever beam with a rectangular cross section along its entire length (Fig. 20-4), the stress is

$$S = \frac{6Wl}{bh^2} \qquad (20\text{-}12)$$

where

S = maximum stress, in psi

W = load, in lb

l = distance between clamp and point of application of load, in in.

b and h = cross-sectional dimensions, in in.

The deflection of a cantilever beam under a load W is

$$Y = \frac{Wl^3}{3EI} \qquad (20\text{-}13)$$

where I, W, and l are defined in Eqs. (20-11) and (20-12), and

Y = beam deflection, in in.

E = Young's modulus for the beam material, in psi

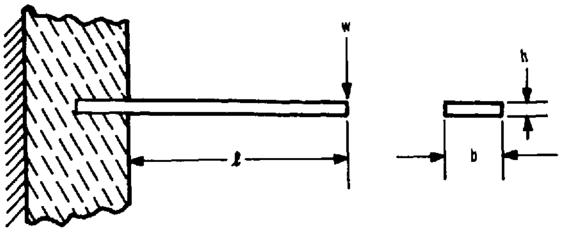

Fig. 20-4 Cantilever spring.

For a rectangular beam (Fig. 20-4), the deflection becomes

$$Y = \frac{4Wl^3}{Ebh^3} = \frac{2Sl^2}{3Eh} \qquad (20\text{-}14)$$

20-3.9 Tensile Elastic Properties

A cantilever beam bends instead of twisting. Therefore, the modulus of elasticity and the allowable stresses are not the same as for a helical spring. Table 20-7 lists the tensile elastic properties of some spring materials.

20-3.10 TORSION BAR SPRINGS

20-3.11 Definition

Torsion bar springs have one end clamped while a torque is applied to the other end.

20-3.12 Stress and Torque

The equation for stress in a torsion bar spring, assuming a round bar of uniform diameter, is

$$S = \frac{E_s r\theta}{l} \qquad (20\text{-}15)$$

where

S = shearing stress, in psi

E_s = torsional modulus of rigidity, in psi

r = bar radius, in in.

θ = angle of twist, in radians

l = length of spring, in in.

TABLE 20-6 PERMISSIBLE LOAD AND SPRING SCALE OF HELICAL COMPRESSION SPRINGS

Wire Diameter (in.)	Outside Diameter of Coil (in.)																			
	1/8	5/32	3/16	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1	-1/4	1-1/2	1-3/4	2	2-1/2	3	3-1/2	4	4-1/2
0.010	0.305 9.47	Load in lb. at 100,000 psi — Wahl's Factor (K) included Spring scale per single turn. in lb/in.																			
0.012	0.522 20.7	0.421 10.0	0.350 5.43																		
0.014	0.823 40.3	0.669 19.4	0.560 10.5	0.422 4.18																	
0.016	1.21 72.9	0.983 34.4	0.824 18.4	0.626 7.37																	
0.018	1.71 124	1.040 57.4	1.17 30.7	0.889 12.1																	
0.020	2.32 200	1.89 91.3	1.60 48.6	1.22 18.9	).972 9.06																
0.024	3.90 464	3.22 208	2.72 108	2.09 41.4	1.68 19.9	1.41 11.1	1.21 6.67														
0.029	6.61 1160	5.55 496	4.71 253	3.64 94.1	2.94 44.6	2.50 24.6	2.13 14.9	1.87 9.75													
0.033		7.97 916	6.85 460	5.29 166	4.31 77.5	3.61 42.4	3.11 25.6	2.74 16.7	2.21 8.26												
0.037		10.9 1580	9.42 785	7.41 280	6.01 128	5.07 69.6	4.41 42.2	3.88 27.3	3.10 13.3	2.61 7.43											
0.043			14.4 1618	11.4 559	9.33 249	7.91 134	6.82 80.0	6.07 51.6	4.89 25.0	4.10 14.0	3.62 8.60										
0.051				18.5 1233	15.3 543	13.0 288	11.3 168	10.0 107	8.01 51.1	6.78 28.6	6.79 17.3	5.14 11.4									
0.059				27.6 2509	23.2 1059	19.9 554	17.3 322	15.3 204	12.4 96.3	10.4 52.7	9.01 32.2	7.89 21.1	6.33 10.3								
0.071					38.7 2663	33.4 1295	29.4 741	26.1 463	21.3 215	18.0 117	15.5 70.6	13.6 45.3	11.1 22.4	9.21 12.6							
0.085						55.5 3083	49.2 1708	44.1 1053	36.1 478	30.5 255	26.6 153	23.3 97.5	18.8 47.4	15.8 26.6							

Wire Diameter (in.)	Outside Diameter of Coil (in.)																			
	1/8	6/32	3/16	1/4	6/16	3/8	7/16	1/2	6/8	3/4	7/8	1	1-1/4	1-1/2	1-3/4	2	2-1/2	3	3-1/2	4	4-1/2
0.092						69.3 4630	61.0 2490	66.1 1616	46.6 683	38.4 363	33.4 216	29.6 136	23.8 66.4	19.9 36.8	17.1 22.6						
0.106							91.0 4973	81.9 2967	68.3 1298	68.1 679	60.8 400	44.8 266	36.2 121	30.1 66.6	26.1 40.9	22.8 26.6					
0.126								130 6710	109 2817	93.8 1436	81.9 832	72.6 626	68.8 246	49.4 136	42.06 82.0	37.6 63.3	30.2 26.2				
0.148										161 3166	133 1802	119 1119	96.3 614	81.2 279	70.6 168	62.0 109	49.7 63.1	41.7 29.7			
0.177										260 7486	222 4160	198 2626	163 1146	137 609	119 361	106 234	84.7 113	71.0 63.2	61.2 38.4		
0.207											341 8867	307 6266	266 2320	217 1218	188 716	166 467	134 218	113 121	97.2 74.1	86.2 48.6	76.0 33.6
0.244												484 11806	406 4988	349 2662	306 1488	269 941	218 443	183 243	168 148	139 96.9	126 66.7
0.283													614 10199	634 6164	467 2919	416 1820	338 849	286 460	247 276	216 180	194 123
0.331														824 10786	733 6048	662 3713	634 1693	463 911	391 643	343 348	308 238
0.376														1166 19966	1038 10903	932 6662	767 2960	662 1676	667 933	497 697	446 406
0.4375															1687 23202	1433 13869	1201 6006	1019 3132	888 1836	783 1166	704 785
0.600																2073 26646	1742 11268	1600 6736	1310 3330	1160 2097	1041 1406
0.6626																	2426 19863	2096 9967	1832 6686	1628 3646	1468 2369
0.626																	3230 33289	2826 16386	2491 9240	2221 6710	1997 3773

Adapted by permission from *Machine Design*, Volume 19, No. 3, March, 1947, from article entitled "Helical Spring Design Tables", by H. F. Roes.

The torque is

$$T = \frac{E_s J \theta}{l} = \frac{\pi E_s r^4 \theta}{2l} \qquad (20\text{-}16)$$

where E_s, θ, and l are defined in Eq. (20-15) and J is the polar moment of inertia of the cross section of the bar about its geometric center, in in.^{4}. The torsional elastic properties of some spring materials are listed in Table 20-5. For discussions of Bellville spring washers, helical springs used as torsion springs, and clock or spiral springs, see Wahl.[4]

20-3.13 VIBRATION IN SPRINGS

All springs can be considered as systems of distributed mass, elastance, and damping. In most springs, the damping is extremely small. Therefore, they will resonate at a large number of frequencies with different modes of vibration. Because the damping is low, the magnitude of the spring motion may become extremely high. In most control applications, however, the resonant frequency of springs is above the system forcing frequency, and resonance in the spring itself is of little interest.

20-3.14 Natural Frequency

The natural frequency of a helical spring with both ends fixed is[9]

$$f = \frac{d}{2\pi r^2 n}\sqrt{\frac{E_s g}{32\rho}} \qquad (20\text{-}17)$$

where

d = spring wire diameter, in in.

ρ = density of spring material, in lb/in.^{3}

g = acceleration of gravity, in in./sec^2

E_s = torsional modulus of rigidity, in psi

r = mean radius of spring coil, in in.

n = number of active turns

f = lowest natural frequency of the spring, in cps

For a steel spring

$E_s = 11.5 \times 10^6$ psi

$\rho = 0.285$ lb/in.^{3}

TABLE 20-7 TENSILE ELASTIC PROPERTIES OF SOME SPRING MATERIALS

Material	Elastic Limit* (psi)	Recommended Working Stress‡ (psi)	Modulus of Elasticity (psi)
Clock spring steel	200,000	100,000	32×10^6
Music wire	200,000	110,000	30×10^6
Hot rolled steel (SAE 1095) hardened to Rockwell C43	120,000	80,000	28×10^6
Stainless steel (Type 302)	150,000	40,000	28×10^6
Phosphor or silicon bronze	80,000	40,000	15×10^6
Beryllium copper	130,000	75,000	17×10^6

*An average value of elastic limit is given here. Actual values vary widely.

‡The recommended working stress given here is approximately 80% of the minimum elastic limit of the material

Thus, the lowest natural frequency of the steel spring is

$$f = \frac{3510d}{r^2n} \text{ cps} \tag{20-18}$$

There will also be natural frequencies at **2, 3, 4,** etc. times the lowest natural frequency. The natural frequency of helical compression springs is also discussed by Maier[8].

20-4 MISCELLANEOUS CONSTANTS OF FLUIDS AND TUBING USED IN HYDRAULIC SYSTEMS

20-4.1 BULK MODULUS

The bulk modulus of a liquid is a measure of the amount the liquid is compressed when the pressure applied to the liquid is increased. Bulk modulus can be defined **as**

$$B = \frac{\Delta P}{\left(\frac{\Delta V}{V}\right)} \tag{20-19}$$

where

B = bulk modulus

ΔP = change in pressure

ΔV = change in volume

V = volume of liquid to which pressure is applied

Bulk modulus has the dimensions of FL^{-2} and is usually expressed in **lb/in.^{2}**, although normal atmospheric pressure is sometimes used as the basic unit. Although bulk modulus changes somewhat with pressure, it is essentially constant for petroleum-base hydraulic fluid at pressures between 15 and **3000** psi. **A** good average value is **2.5** $\times$ **10^5** psi.

20-4.2 COMPRESSIBILITY

Compressibility is the reciprocal of bulk modulus. For most hydraulic fluids, it is approximately **4** $\times$ 10^{-6} **in.^{2}/lb.**

20-4.3 SPECIFIC GRAVITY

The specific gravity of most petroleum products is expressed in terms of degrees Baumé or degrees API. These units are the readings of particular hydrometers. Conversion factors between **API** degrees and specific gravity are given in most handbooks for mechanical engineers. Most hydraulic fluids have a specific gravity of between 0.85 and **0.90** (density between **0.0307** and **0.0325** lb/in.^{3}) at a temperature of 60°F.

20-4.4 ELASTIC PROPERTIES OF TUBING

The elastic properties of the materials most commonly used for tubing in hydraulic systems are listed in Table 20-8. In general, the factor of safety used should depend upon the nature of the driven load. For copper and aluminum, which have no definite yield point, a factor of safety below **2** is never used. It is usually kept above **4.** Heavy pulsating loads require an even higher factor of safety. The tensile modulus of elasticity is included in Table **20-8** for use in computing the increased volume of tubing when the tubing is subjected to internal pressure.

TABLE 20-8 ELASTIC PROPERTIES OF TUBING MATERIAL

Material	Yield Stress (psi)	Ultimate Strength (psi)	Tensile Modulus of Elasticity (psi)
Soft-drawn copper*	7000	33,000	15×10^6
Hard-drawn copper*	49,000	55,000	15×10^6
Soft aluminum*	3000	9000	10×10^6
Alloyed, heat-treated aluminum*	30,000	45,000	10×10^6
Steel (SAE 1020)	40,000	65,000	29×10^6
Stainless steel (AISI 302)	40,000	90,000	29×10^6

***These materials have no well-defined yield point. Values are for 0.2% offset.**

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