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ENGINEERING DESIGN HANDBOOK

SERVOMECHANISMS

SECTION 3, AMPLIFICATION

HEADQUARTERS, U. S. ARMY MATERIEL COMMAND APRIL 1965

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FOREWORD

INTRODUCTION

This is one of a group of handbooks covering the engineering information and quantitative data needed in the design and construction of military equipment, which (as a group) constitutes the Army Materiel Command Engineering Design Handbook Series.

PURPOSE OF HANDBOOK

The handbook on Servomechanisms has been prepared as an aid to designers of automatic control systems for Army equipments, and as a guide to military and civilian personnel who are responsible for setting control-system specifications and ensuring their fulfillment.

SCOPE AND USE OF HANDBOOK

The publications are presented in handbook form rather than in the style of textbooks. Tables, charts, equations, and bibliographical references are used in abundance. Proofs and derivations are often omitted and only final results with interpretations are stated. Certain specific information that is always needed in carrying out design details has, of necessity, been omitted. Manufacturers' names, product serial numbers, technical specifications, and prices are subject to great variation and are more appropriately found in trade catalogs. It is essential that up-to-date catalogs be used by designers as supplements to this handbook.

To make effective use of the handbook during the design of a servo, the following procedure is suggested. The designer should turn first to Chapters 16 and 17 where design philosophy and methods are discussed. Implementation of the design procedure may require a review of certain theoretical concepts and methods which can be achieved through reference to Chapters 1 through 10. As the design proceeds, a stage will be reached at which the power capacity of the output member has been fixed. Reference to Chapters 14, 15, and 16 will then illustrate the salient features of output members having the required power capacity. After the designer has chosen the output member, he will find the information dealing with sensing elements and amplifiers (Chapters 11, 12, and 13) helpful in completing the design.

FEEDBACK CONTROL SYSTEMS AND SERVOMECHANISMS

Servomechanisms are part of a broad class of systems that operate on the principle of feedback. In a feedback control system, the output (response) signal is made to conform with the input (command) signal by feeding back to the input a signal that is a function of the output for the purpose of comparison. Should an error exist, a corrective action is automatically initiated to reduce the error toward zero. Thus, through feedback, output and input signals are made to conform essentially with each other.

In practice, the output signal of a feedback control system may be an electrical quantity such as a voltage or current, or any one of a variety of physical quantities such as a linear or angular displacement, velocity, pressure, or temperature. Similarly, the input signal may take any one of these forms. Moreover, in many applications, input signals belong to one of these types, and the output to another. Suitable transducers or measuring devices must then be used. It is also common to find multiple feedback paths or loops in complicated feedback control systems. In these systems, the over-all system performance as characterized by stability, speed of response, or accuracy can be enhanced by feeding back signals from various points within the system to other points for comparison and initiation of correction signals at the comparison points.

At present, there is no standard definition of a servomechanism. Some engineers prefer to classify any system with a feedback loop as a servomechanism. According to this interpretation, an electronic amplifier with negative feedback is a servo. More frequently, however, the term servomechanism is reserved for a feedback control system containing a mechanical quantity. Thus, the IRE defines a servomechanism as "a feedback control system in which one or more of the system signals represents mechanical motion." Some would restrict the definition further by applying the term only to a special class of feedback control system in which the output is a mechanical position.

APPLICATION OF SERVOMECHANISMS TO ARMY EQUIPMENT

Servomechanisms are an important part of nearly every piece of modern mechanized Army equipment. They are used to automatically position gun mounts, missile launchers, and radar antennas. They aid in the control of the flight paths of jet-propelled rockets and ballistic missiles, and play an important role in the navigational systems of those vehicles. As instrument servos, they permit remote monitoring of physical and electrical quantities and facilitate mathematical operations in computers.

No single set of electrical and physical requirements can be stated for servomechanisms intended for these diverse military applications. The characteristics of each servomechanism are determined by the function it is to perform, by the characteristics of the other devices and equipments with which it is associated, and by the environment to which it is subjected. It will often be found that two or more servo-system configurations will meet a given set of performance specifications. Final choice of a system may then be determined by such factors as ability of the system to meet environmental specifications, availability of components, simplicity, reliability, ease of maintenance, ease of manufacture, and cost. Finally, the ability to translate any acceptable paper design into a piece of physical equipment that meets electrical and physical specifications and works reliably depends to a great extent upon the skill of the engineering and manufacturing groups responsible for building the system. The exercise of care and good judgment when specifying electrical, mechanical, and thermal tolerances on components and subsystems can contribute greatly to the successful implementation of servo-system design.

The handbook on Servomechanisms was prepared under the direction of the Engineering Handbook Office, Duke University, under contract to the Army Research Office-Durham. The material for this pamphlet was prepared by Jackson & Moreland, Boston, Massachusetts, under subcontract to the Engineering Handbook Office. Jackson & Moreland was assisted in their work by consultants who are recognized authorities in the field of servomechanisms.

PREFACE

The Engineering Design Handbook Series of the Army Materiel Command is a coordinated series of handbooks containing basic information and fundamental data useful in the design and development of Army materiel and systems. The handbooks are authoritative reference books of practical information and quantitative facts helpful in the design and development of Army materiel so that it will meet the tactical and the technical needs of the Armed Forces. The present handbookis one of a series on Servomechanisms.

Section 3 of the handbook contains Chapter 13, which covers the different amplifiers used in servo controllers. This chapter describes the following types of amplifiers: electronic, transistor, magnetic, rotary electric, relay, hydraulic, pneumatic and mechanical. The advantages and disadvantages of the various types are discussed to help the circuit designer in making a choice for his particular application.

For information on other servomechanism components and on feedback control theory and system design, see one of the following applicable sections of this handbook:

AMCP 706-136	Section 1	Theory (Chapters 1-10)
AMCP 706-137	Section 2	Measurement and Signal Converters (Chapters 11-12)
AMCP 706-139	Section 4	Power Elements and System Design (Chapters 14-20)

An index for the material in all four sections is placed at the end of Section 4.

Elements of the U. S. Army Materiel Command having need for handbooks may submit requisitions or official requests directly to Publications and Reproduction Agency, Letterkenny Army Depot, Chambersburg, Pennsylvania 17201. Contractors should submit such requisitions or requests to their contracting officers.

Comments and suggestions on this handbook are welcome and should be addressed to Army Research Office-Durham, Box CM, Duke Station, Durham, North Carolina 27706.

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CHAPTER 13

AMPLIFIERS USED IN CONTROLLERS

13-1 ELECTRONIC AMPLIFIERS*

13-1.1 VACUUM TUBES

Vacuum tubes are thermionic devices in which a number of electrodes are contained inside a sealed envelope of glass, metal, or ceramic, which is evacuated to the lowest practical gas pressure. Types of tubes generally used in amplifiers include diodes, triodes, tetrodes, pentodes, and beam power tubes. Table 13-1 lists the symbols normally used with these types of tubes and their associated circuits.

13-1.2 Diodes

The simplest form of vacuum tube is a diode, which contains two electrodes (a plate and a cathode) and a heater. In the symbolic representation of Fig. 13-1, the top element is the plate, the middle element is the cathode, and the heater is at the bottom. The

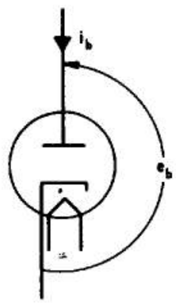


Fig. 13-1 Symbolic representation of a diode.

*By A. K. Susskind

TABLE 13-1 SYMBOLS

Control-grid supply voltage	E_{cc}
Plate supply voltage	E_{bb}
Instantaneous total grid voltage	e_c
Instantaneous total plate voltage	e_b
Instantaneous total plate current	i_b
Quiescent (zero excitation) value of plate voltage	E_{bo}
Quiescent (zero excitation) value of plate current	I_{bo}
Instantaneous value of alternating component of grid voltage	e_g
Instantaneous value of alternating component of plate voltage	e_p
Instantaneous value of alternating component of plate current	i_p

heater, usually enclosed within the cathode, is used to raise the temperature of the cathode to the point where the cathode acts as a source of free electrons. The electrons leave the cathode and a great many of them travel to the plate, forming a unidirectional path for controllable current flow from the plate to the cathode (called ***plate current).***

13-1.3 Control of electron flow. Useful control of electron flow from the cathode cannot be achieved by varying the heater potential. Therefore, heaters are always connected to sources of fixed potential, and heater symbols are frequently omitted from electronic-circuit diagrams.

In a diode, the only useful way to contrcl electron flow is by control of the plate voltage e_b (the potential difference between the plate and cathode). The relationship between plate voltage e_b and plate current i_b for constant cathode temperature is shown in Fig. **13-2,** where the curve is drawn for positive values of e_b. For all negative values of e_b, plate current i_b is zero. Reference directions (indicated by arrows in Fig. **13-1,** for example) are chosen so that e, is a positive number whenever the plate is actually positive with respect to the cathode, and i_b is a positive number when current flows from plate to cathode.

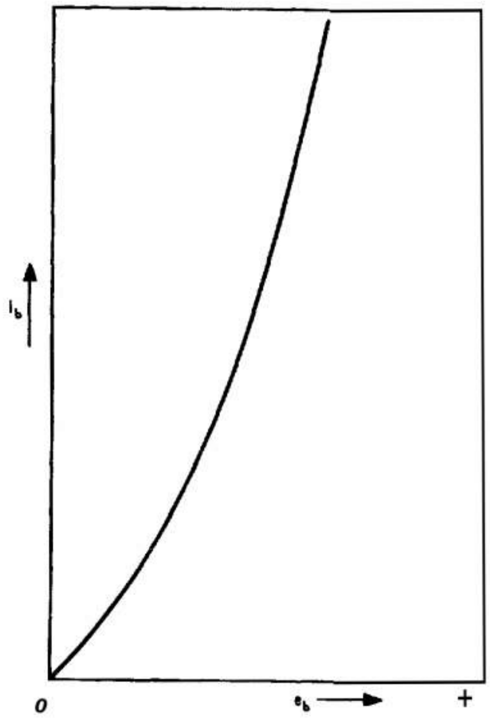


Fig. 73-2 Volt-ampere curve of a diode cathode temperature constant.

13-1.4 Diodes as rectifiers. The diode is a nonlinear device. One useful application of this nonlinearity is in a rectifying circuit such as Fig. **13-3.** If the simplifying assumption is made that the e_b-i_b curve has a constant slope R_b in the conducting region, and infinite impedance in the nonconducting region, output current is given by

$$i_b = \frac{E_m}{R_b + R}\left(\frac{1}{\pi} + \frac{1}{2}\sin \omega t - \frac{2}{3x}\cos 2\omega t - \frac{2}{15\pi}\cos 4wt - \ldots\right) \tag{13-1}$$

The time-invariant (or d-c) component of this waveform can be isolated by adding a low-pass filter to the output. See **Gray**[1] for a more detailed discussion of rectifier circuits.

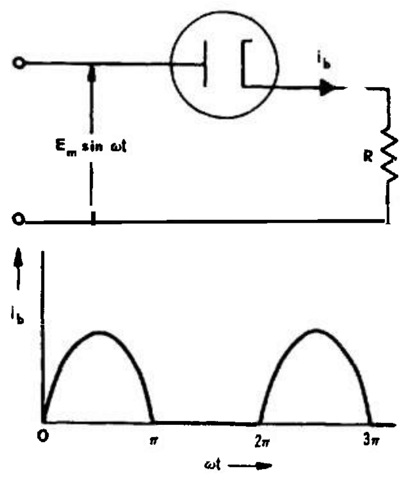


Fig. 13-3 Diode used as rectifier.

13-1.5 Triodes

If a third electrode, called the ***control grid,*** is inserted between the plate and cathode of a diode, the tube becomes a triode. The control grid provides an additional means of controlling electron flow. Plate current is now a function of both the control-grid and plate voltages and, since a change in grid voltage can result in a larger change in plate voltage, amplification can occur. The triode is shown symbolically in Fig. **13-4.**

13-1.6 Plate characteristics. The volt-ampere (e_b-i_b) characteristics of a triode comprise a family of curves with grid voltage e_c (grid-to-cathode voltage) a parameter. These curves are called ***plate characteristics*** ; a typical family is shown in Fig. **13-5.** These curves demonstrate that :

(a) Plate current can flow only when the plate is positive with respect to the cathode.

(b) For a given value of plate voltage e_b, increasingly negative values of grid voltage e_c decrease plate current i_b and, if e_c is made sufficiently negative, conduction ceases. This is called *cutoff,* and the value of e_c that causes cutoff is called the ***cutoff voltage.***

(c) The curves are not straight lines.

(d) Curves for successive values of e_c are displaced by unequal amounts.

It follows that, even when the plate voltage is positive and the grid voltage is above cutoff, the triode behaves as a nonlinear device. For precise results, therefore, nonlinear analysis must be used.

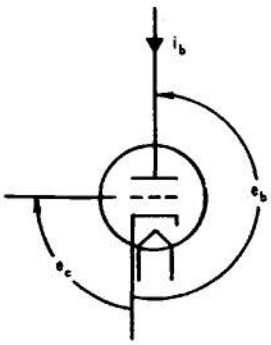


Fig. 13-4 Symbolic representation of a triode.

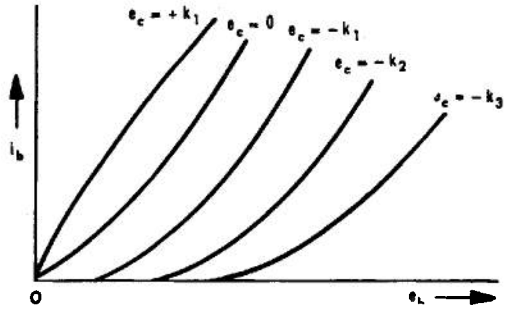


Fig. 13-5 Plate characteristics of a triode.

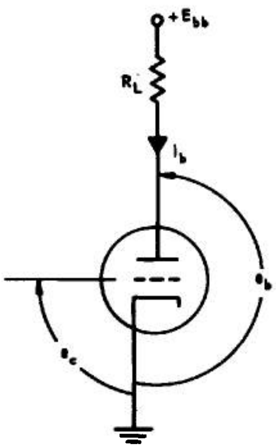


Fig. 13-6 Triode amplifier.

13-1.7 Graphical analysis. The most effective technique is graphical analysis. Using the typical triode amplifier in Fig. 13-6, which includes a load resistance R_L in its plate circuit, graphical analysis can be accomplished as follows :

(a) On the plate-voltage axis of the plate characteristics, mark the point corresponding to the plate supply voltage E_{bb}.

(b) From this point, draw a straight line with a slope $-1/R_L$ to the i_b axis. This sloping line is called the ***load line.***

(c) For each value of e_c, determine the corresponding values of e_b and i_b using the abscissa and ordinate, respectively, of the intersection of the load line with the e_c line of interest.

An example of this construction is given in Fig. 13-7. For a more complete discussion, see Preisman's book.[2]

13-1.8 Linear approximations. **As** an analysis tool, graphical construction is reasonably straightforward, particularly when the circuit associated with the tube is relatively simple. In many cases, however, complex configurations are involved and graphical analysis becomes tedious. It is especially so when nonresistive elements are encountered. It is often possible to make a good linear approximation of the tube characteristics, if the region of operation on the plate characteristics is small enough, so that

(a) The slope of the plate characteristics is approximately constant over that region.

(b) The plate characteristics are almost uniformly spaced over that region.

13-1.9 Region of operation. The region of operation is determined by the choice of E_{bb}, R_L, and the range over which e_c varies due to the input signal. In amplifiers for servomechanisms, it is usually possible (in all but the last stage) to select E_{bb}, R_L, and the tube type so that the grid is never operated positively or in the nonlinear region near cutoff. Under these conditions, the assumption of linear behavior usually leads to satisfactory results.

13-1.10 Linear equivalent circuits. The dynamic voltage-source linear equivalent circuit for a triode is given in Fig. 13-8. Here, e_p, e_g, and i_p are time-varying quantities and the grid-to-cathode impedance is shown as infinite. This is primarily justified by the assumption that the grid is never allowed to become positive with respect to the cathode and, as a result, no significant grid current flows. In the equivalent circuit, the amplification factor μ is defined as

$$\mu = -\left.\frac{\partial e_b}{\partial e_c}\right|_{i_b = \text{constant}} \qquad (13\text{-}2)$$

The factor μ can be computed from the plate characteristics of Fig. 13-5 by finding (at the approximate center of the operating region, and along a horizontal line) the change in plate voltage due to a change in grid voltage and then forming the quotient of these two variables. The plate resistance r_p, in **ohms**, is defined as

$$r_p = \left.\frac{\partial e_b}{\partial i_b}\right|_{e_c = \text{constant}} \qquad (13\text{-}3)$$

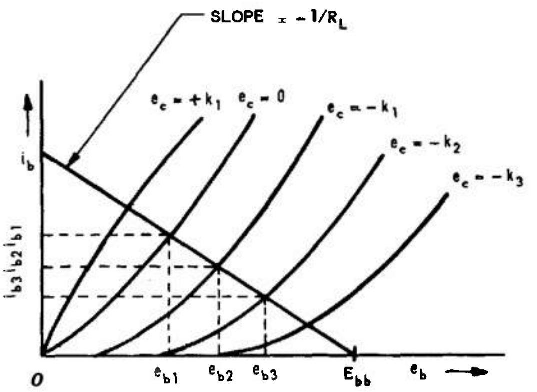


Fig. 73-7 Graphical analysis of a triode amplifier.

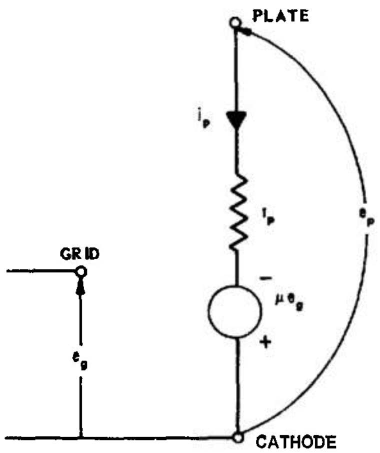


Fig. 13-8 Linear equivalent circuit of a triode.

The factor r_p can be computed by measuring the slope of an $e_b - i_b$ curve at the approximate center of the operating region. A third parameter, the transconductance, is also convenient to use at times. Transconductance, in mhos, is defined as the change in plate current per unit change in control-grid voltage and is given by

$$g_m = \left.\frac{\partial i_b}{\partial e_c}\right|_{e_b = \text{constant}} = \frac{\mu}{r_p} \quad (13\text{-}4)$$

For different types of triodes, typical values of μ range from **2** to **100,** r_p from **1000** to 20,000 ohms, and g_m from **1000** to **5000** micromhos (1×10^{-6} mho = 1 micromho).

13-1.11 Alternate linear equivalent circuit. An alternate linear equivalent circuit is shown in Fig. **13-9.** In this circuit, a current source $g_m e_g$ and a parallel resistance r_p are used instead of the voltage source μe_g in series with r_p as shown in Fig. **13-8.** The choice of which equivalent circuit to use in an analysis is entirely a matter of convenience because the two circuits shown give identical results.

13-1.12 Quiescent operating point. As indicated above, the linear model usually applies only to variations in e_b and i_b from the quiescent operating point, which is the point $e_b = E_{bo}$, $i_b = I_{bo}$, corresponding to zero input signal. The input signal e, is usually superimposed on a negative bias E,,, so that positive values of e, do not result in positive values of e,. It follows that the quiescent operating point is found at the point where the zero-signal load line intersects the e, $= E$,, characteristic. This point should be determined from the plate characteristics.

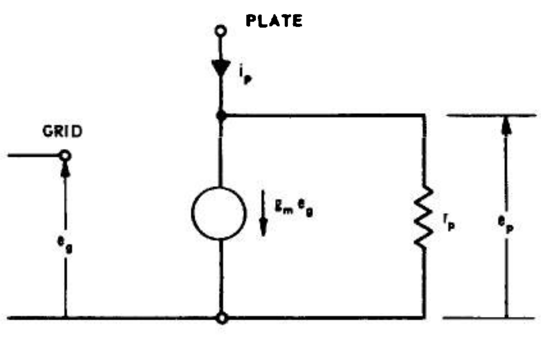


Fig. 73-9 Alternate equivalent circuit of a triode.

13-1.13 Phase shift. The zero-signal load is usually simple to determine because it takes into consideration only the resistive components of the load (capacitors being considered as open circuits and inductances as short circuits). Variations in e_b and i_b due to input signals are denoted as e, and i_p. Using these definitions, the following may be written:

$$e_b = E_{bo} + e_p \quad (13\text{-}5)$$

$$i_b = I_{bo} + i_p \quad (13\text{-}6)$$

$$e_c = E_{cc} + e_g \quad (13\text{-}7)$$

The signs of e, and i_p may be negative, but the signs of e_b and i_b are not. In fact, Figs. **13-8** and **13-9** show that e, is negative when e, is positive, and e, is positive when e, is negative. Thus, a triode causes a phase shift of **180°** between grid and plate.

13-1.14 Pentodes and Beam-Power Tubes

Among vacuum tubes containing more than three electrodes, beam-power tubes and pentodes are of primary importance in servomechanism applications. A beam-power tube contains a cathode, a control grid, a screen grid, beam-forming electrodes, and a plate. A pentode contains a cathode, a control grid, a screen grid, a suppressor grid, and a plate. These two tube types are shown symbolically in Fig. **13-10.** For servomechanism applications, the significant change brought about by the addition of the screen grid is the ability to achieve higher gain in beam-power and pentode circuits than is possible in triode circuits. However, since the screen grid is usually operated at a fixed d-c voltage (relative to the cathode), which is intermediate between plate supply and cathode potentials, an additional power supply is often required. This frequently outweighs the advantage of higher gain when over-all amplifier complexity is considered. In this respect, the triode is

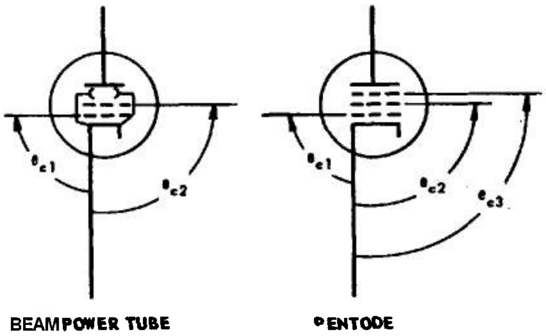


Fig. 13-10 Symbolic representation of a beam-power tube and a pentode.

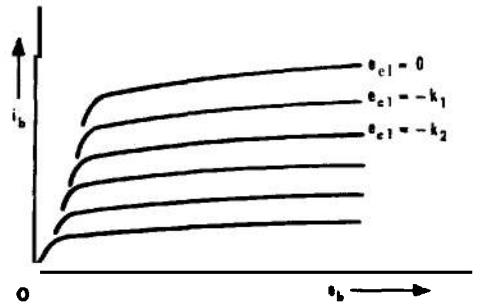


Fig. 13-11 Plate characteristics of a beam-power tube with constant screen voltage.

superior to multigrid tubes because it requires no screen supply. It is often simpler to add more stages of amplification than to provide additional power supplies. Another feature that triodes possess is greater linearity of operation with large signal-voltage inputs.

13-1.15 Plate characteristics. Figure 13-11 shows typical plate characteristics of a beam-power tube, while those for a pentode are shown in Fig. 13-12. Both figures show that the plate current changes very little with changes in plate voltage when operation is above the knee of the curves and when all grid voltages are constant. It follows, therefore, that the plate resistance r_p of these tubes is very high, usually ranging from about 100,000 ohms to 10 megohms, as compared with a triode r_p of about 1000 to 20,000 ohms. Since triode transconductance is approximately equal to pentode transconductance, when similar construction is used, it follows that the amplification factor μ, given by

$$\mu = g_m r_p \tag{13-8}$$

is much higher for multigrid tubes than for triodes.

13-1.16 Linear equivalent circuits. When screen-grid and suppressor-grid voltages are constant, and operation is above the knee of the curves, the linear equivalent circuits of a multigrid tube have the same form as the

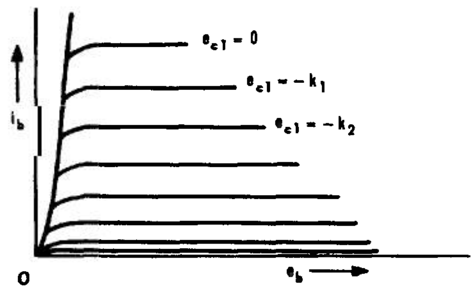


Fig. 13-12 Plate characteristics of a pentode with constant suppressor and screen voltages.

triode linear equivalent circuits of Figs. 13-8 and 13-9. For analysis of pentode operation, Fig. 13-9 is usually more convenient. When r_p is high compared with any load impedance connected to the plate, it is possible to consider the plate resistance as infinite. In Fig. 13-9, the tube can then be considered simply as a current source $g_m e_g$

13-1.17 Graphical analysis. Graphical analysis of multigrid tubes is similar to that of triodes.

13-1.18 Interelectrode Capacitance

In the linear equivalent circuits heretofore discussed, no interelectrode capacitances were shown. These capacitances actually exist, but their values are so small (a few micromicrofarads) that they are of no consequence at the signal frequencies encountered in servomechanisms. Therefore, interelectrode capacitance may be ignored, except when amplifier stability is considered.

13-1.19 Tube Specifications

In addition to the plate characteristics and the parameter values for the equivalent circuit, there are other important tube specifications, listed in tube-manufacturers' catalogs, which the circuit designer must take into consideration. While these additional specifications do not affect the theoretical operation, they do represent application limits. If these limits are exceeded, tube performance will differ from the given characteristics, or the period of operation over which the tube parameters remain at approximately the published values will be materially narrowed. These additional specifications must therefore be ascertained and adhered to. Of particular interest is the average plate dissipation P_p, which is defined as the time average (over a cycle) of the product of total plate voltage and total plate current. For sinusoidal operation

$$P_p = \frac{1}{2\pi}\int_0^{2\pi} e_b i_b d(\omega t) \qquad (13\text{-}9)$$

For linear operation, the plate dissipation is greatest under quiescent conditions and is given by

$$P'_p = E_{bo} I_{bo} \qquad (13\text{-}10)$$

where E_{bo} and I_{bo} are the plate voltage and plate current, with zero input signal. Since servo amplifiers usually receive no error signals for extended periods of time, quiescent-condition plate dissipation frequently determines whether the dissipation rating is being exceeded. The power dissipation of the screen is determined in a similar manner; i.e., by computing the time average of the product of screen voltage and total screen current.

13-1.20 LINEAR ANALYSIS OF SIGNAL-STAGE VACUUM-TUBE VOLTAGE AMPLIFIERS

In servomechanisms, voltage amplifiers are used primarily to increase the level of the control signal.

13-1.21 Characteristics of Tubes Used

The tubes used in low-level voltage amplifiers have the following characteristics :

(a) Triodes have high values of μ (e.g., 100) and pentodes have high values of g_m (e.g., 4000 micromhos).

(b) Maximum current ratings are moderate (e.g., 10 ma).

(c) Plate-dissipation ratings are moderate (e.g., 1 watt).

13-1.22 Simple Amplifier

The simplest voltage-amplifier circuit is that of Fig. 13-6, the linear equivalent of which is shown in Fig. 13-13. The gain of such an amplifier is defined as the ratio of the variational component of the output voltage to the variational component of the input voltage. The mathematical expression for this gain is

$$K = \frac{e_o}{e_{in}} = \frac{-\mu R_L}{R_L + r_p} \qquad (13\text{-}11)$$

(The minus sign results from the 180° phase relationship between e_o and e_{in}.) In designing amplifiers, the usual practice is to attempt to achieve the required gain with the least number of stages of amplification. This naturally leads to an attempt to realize the maximum

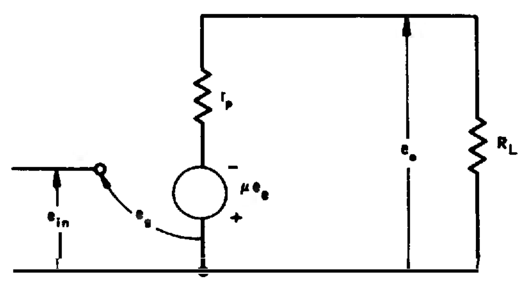


Fig. 13-13 Equivalent circuit of a simple plate-loaded amplifier

gain from each stage. Equation (13-11) indicates that the maximum possible gain equals μ and can be attained by making R_L much larger than r_p. To prevent this large value of R_L from producing plate-current values so low as to make r_p large (r_p is inversely related to &), a high plate supply voltage is required, reaching a value that is usually impractical. Practical values of E_{bb} do not exceed a few hundred volts and usually restrict the gain to a level that is only 60 to 80 percent of the amplification factor. Similar restrictions apply to a pentode, since g_m is approximately proportional to i_b.

13-1.23 Series Tube Triode Amplifier

In one method of attempting to realize the maximum possible gain, R_L is replaced by a second vacuum-tube circuit, as in the series tube triode amplifier of Fig. 13-14. Tube V2 and its cathode resistor R_k are approximately the equivalent of a battery of $\mu_2 E_{cc}$ volts plus a series resistor of value $r_{p2} + R_k(1 + \mu_2)$. As an example, consider a series tube circuit using both sections of a Type 6SL7 dual-triode, where

$$\mu_1 = \mu_2 = 70$$

$$r_{p1} = r_{p2} = 44{,}000 \text{ ohms}$$

$$R_k = 22{,}000 \text{ ohms}$$

$$E_{bb} = 350 \text{ volts}$$

$$E_{cc} = 45 \text{ volts}$$

The equivalent plate supply voltage is

$$E'_{bb} = E_{bb} + \mu_2 E_{cc} = 350 + (70 \times 45) = 3500 \text{ volts}$$

the equivalent plate load on V2 is

$$R'_L = r_{p2} + (1 + \mu_2)\, R_k = 44{,}000 + (71 \times 22{,}000) \approx 1.6 \text{ megohms}$$ and the gain is

$$K = \frac{e_o}{e_{in}} \approx \mu = 70$$

The series tube amplifier has one drawback, however, that makes it impractical except in rare instances. This drawback is the necessity for a floating supply E_{cc}.

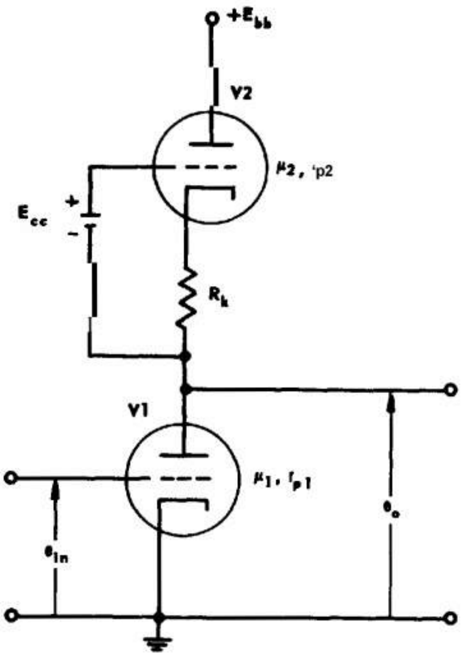


Fig. 73-14 Series tube amplifier.

13-1.24 Cascode Amplifier

A more practical circuit for realizing maximum possible gain is the cascode triode amplifier of Fig. 13-15. With tubes V1 and V2 identical, the gain of this circuit is

$$K = \frac{e_o}{e_{in}} = \frac{-\mu\,(\mu + 1)\, R_L}{(\mu + 2)\, r_p + R_L} \qquad (13\text{-}12)$$

which reduces to

$$K = -g_m R_L \qquad (13\text{-}13)$$

when

$$(\mu + 2) r_p >> R_L \text{ and } \mu >> 1$$

This demonstrates that the gain of a cascode triode amplifier equals that of a single pentode amplifier with infinite r_p. However, the pentode amplifier requires a screen-bias supply and the triode cascode amplifier does not. On the other hand, the disadvantage of the cascode triode amplifier is the necessity for a second tube. If two tubes are to be used, the second tube could be more advantageously

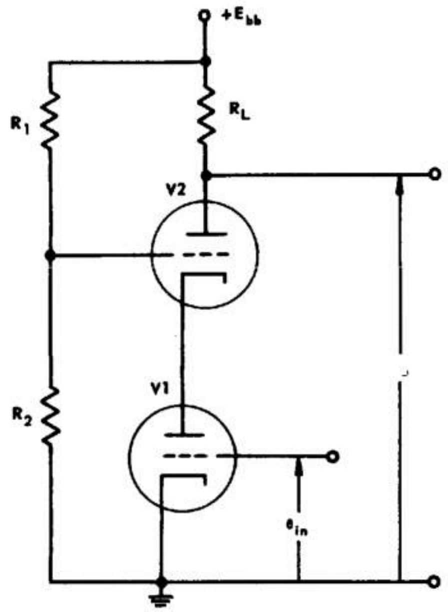


Fig. 13-15 Cascode amplifier circuit.

employed as a second voltage amplifier in cascade with the first, in which case an even larger over-all gain than either the cascode or pentode amplifier would produce might be realized. Cascode circuits are used primarily in direct-coupled amplifiers where the problem of cascading amplifier stages makes it desirable to maximize the gain per stage, even at the expense of extra tubes.

13-1.25 Cathode Followers

In equivalent circuits, the symbol e, represents the variational component of grid-to-cathode voltage. When the tube circuit has internal feedback, e, does not equal the externally applied input voltage. For example, in Fig. 13-16, which is the circuit schematic of a cathode follower, e, is expressed as

$$e, = e_{in} - e_k$$

where e_{in} is the time-varying component of the input voltage and e_k is the time-varying component of the cathode-to-ground voltage.

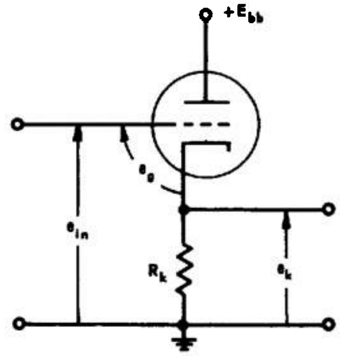


Fig. 13-16 Cathode follower.

Successive manipulations of the linear equivalent circuit (Fig. 18-17) demonstrate the nature of e,. When the generator voltage μe_k (which has the same polarity as e_k) is considered as due to a fictitious resistor μR_k and is then lumped with R_k in the final step of Fig. 13-17, the identity of the cathode terminal vanishes. Therefore, additional cathode loading cannot be applied to the final equivalent circuit of Fig. 13-17. The equations for e_k and gain K are

$$e_k = i_p R_k = \frac{\mu e_{in} R_k}{R_k (1 + \mu) + r_p} \qquad (13\text{-}14)$$

$$K = \frac{e_k}{e_{in}} = \frac{\mu R_k}{R_k (1 + \mu) + r_p}$$

$$= \frac{\mu R_k}{(1 + \mu)\left(R_k + \dfrac{r_p}{1 + \mu}\right)} \qquad (13\text{-}15)$$

The voltage gain of a cathode follower is always less than unity, no matter how large R_k is made. To a load connected from cathode to ground, the cathode follower appears as a qenerator with an open-circuit voltage as given by **Eq. (13-14),** and the variational output impedance R_o is given by

$$R_o = \frac{R_k\left(\frac{r_p}{1+\mu}\right)}{R_k + \left(\frac{r_p}{1+\mu}\right)} \approx \frac{r_p}{1+\mu} \approx \frac{1}{g_m} \quad \textbf{(13-16)}$$

if

$$R_k >> \frac{r_p}{1+\mu} \text{ and } \mu >> 1$$

Practical values of R_o range from **100** to **1000** ohms.

The cathode follower is nearly always used to couple a low-impedance load to a high-impedance signal source, thereby reducing the severe loading effect (on the source) that is imposed by other types of coupling circuits. Where even lower output impedances are desired, the White cathode follower, comprising two tubes interconnected as shown in Fig. **13-18,** can be used. The equivalent circuit (when **V1** and **V2** are identical) is shown in Fig. **13-19.** This configuration can result in practical circuits with an output impedance of **10** to **100** ohms and with a gain approximately the same as that for a single-tube cathode follower.

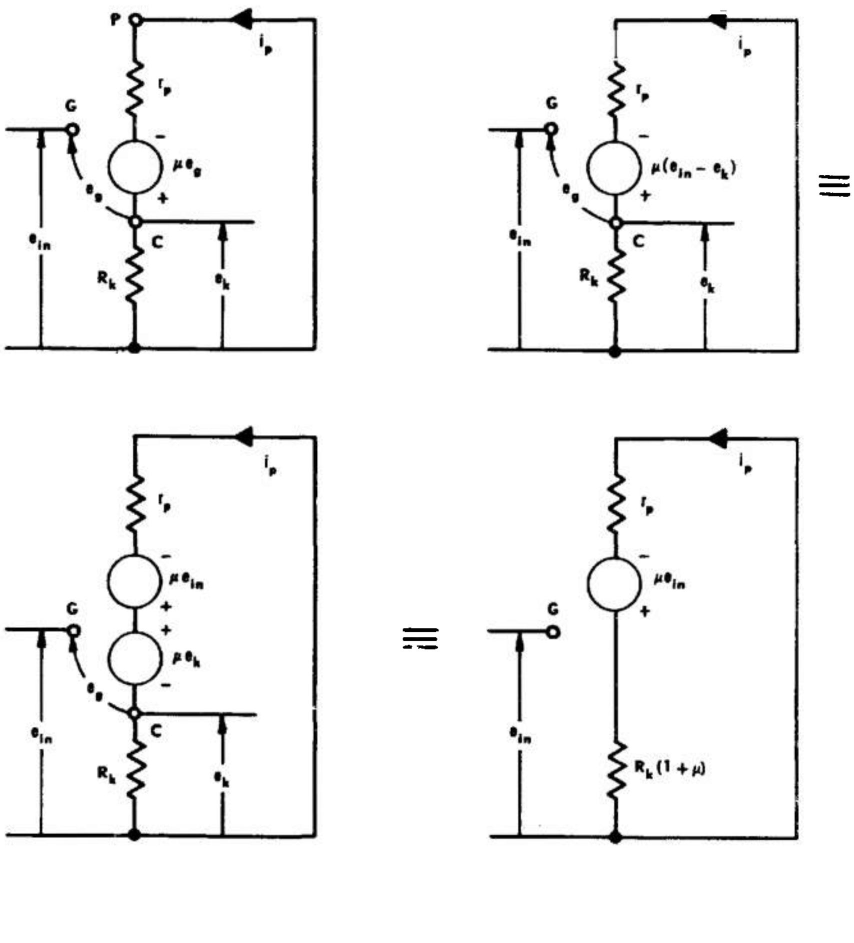


Fig. 13-77 Equivalent circuit of a cathode follower and its manipulation.

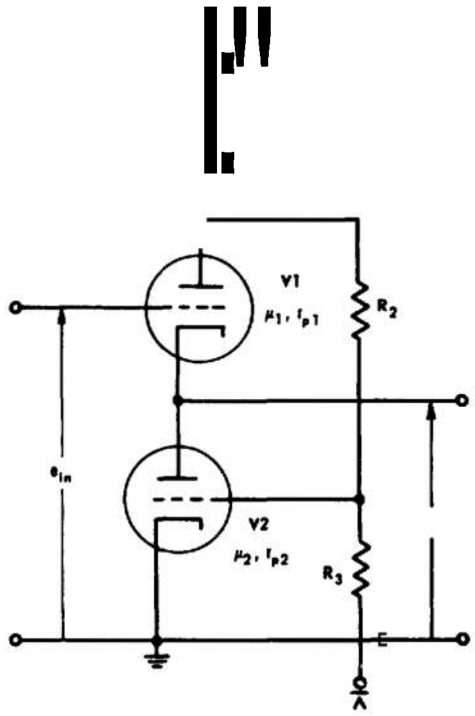


Fig. 13-18 White cathode follower.

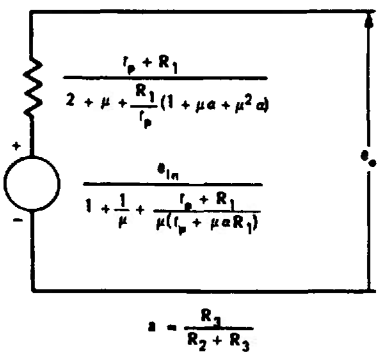


Fig. 13-19 Thevenin equivalent circuit of White cathode follower.

Where the voltage gain of a cathode follower must approach unity as closely as possible, the resistor R_k in Fig. **13-16** can be replaced by a second tube as in Fig. **13-20.** The gain expression then becomes

$$K = \frac{e_o}{e_{in}} = \frac{\mu_1[r_{p2} + R_k(1 + \mu_2)]}{(1 + \mu_1)\left[r_{p2} + R_k(1 + \mu_2) + \frac{r_{p1}}{1 + \mu_1}\right]} \approx 1 \qquad (13\text{-}17)$$

when

$$r_{p2} + R_k(1 + \mu_2) >> \frac{r_{p1}}{1 + \mu_1} \text{ and } \mu_1 >> 1$$

These inequalities can exist in a practical circuit without reducing the *V1* plate current to such a small value that r_{p1} is large.

13-1.26 Simple Feedback Amplifier

Another common form of voltage amplifier is the circuit of Fig. **13-21,** which is evolved by adding a feedback resistor R_k to the circuit of Fig. **13-6.** The grid-to-plate gain expression for this circuit is

$$K = \frac{-\mu R_L}{R_L + r_p + R_k(1 + \mu)} \qquad (13\text{-}18)$$

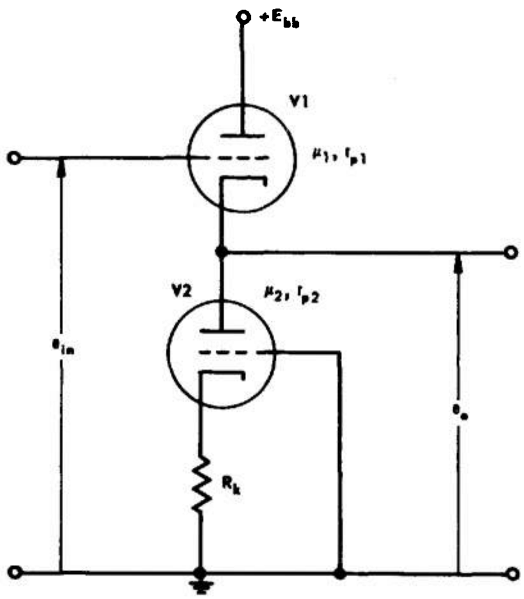


Fig. 13-20 Two-tube cathode follower.

and the variational output impedance that the load sees is

$$R_o = r_p + R_k(1 + \mu) \qquad (13\text{-}19)$$

Because of the appearance of the term $R_k(1 + \mu)$ and also because of the approximate constancy of μ over a range of operating conditions (also during aging of the tube), the addition of R_k increases the gain stability in the presence of variations in r_p. It also lowers the gain and increases the effective output resistance. Output voltage e_o is expressed as

$$e_o = \frac{-\mu R e_{in}}{2R + \mu R + r_p} = -e_k \qquad (13\text{-}20)$$

when

$$R_L = R_k = R$$

The circuit can be used as a phase splitter; i.e., a circuit with two outputs equal in magnitude but *180"* out of phase. It can also be used as a coupling circuit between a single-ended input and a push-pull stage. The two output voltages must always be less than the input voltage.

13-1.27 Differential Amplifiers

Figure *13-22* shows the general form of a commonly used class of amplifiers called differential amplifiers. The time-varying components of the output voltages are given by the expressions

$$e_{o1} = -R_{L1}\left\{\frac{-e_{in1}\mu_1[R_k(1+\mu_2) + R_{L2} + r_{p2}] + e_{in2}\mu_2 R_k(1+\mu_1)}{D}\right\} \qquad (13\text{-}21)$$

$$e_{o2} = -R_{L2}\left\{\frac{e_{in1}\mu_1 R_k(1+\mu_2) - e_{in2}\mu_2[R_k(1+\mu_1) + R_{L1} + r_{p1}]}{D}\right\} \qquad (13\text{-}22)$$

where

$$D = (1+\mu_1)(1+\mu_2)R_k^2 - [(1+\mu_1)R_k + R_{L1} + r_{p1}][(1+\mu_2)R_k + R_{L2} + r_{p2}]$$

If both tubes are identical, $R_{L1} = R_{L2} = R_L$, and the output is taken between the plate of $V1$ and the plate of $V2$, then

$$e_o = e_{o1} - e_{o2} = \frac{\mu R_L}{R_L + r_p}(e_{in2} - e_{in1}) \qquad (13\text{-}23)$$

This equation shows that the plate-to-plate output voltage of a differential amplifier is proportional to the *difference* between the input signals. If the output is taken between the plate of one tube and a point of fixed potential (such as ground), the output can still be made approximately proportional to the difference between the input signals by making

$$R_k(1 + \mu) >> R_L + r_p$$

Then

$$e'_{o1} = \frac{-\mu R_L}{2(R_L + r_p)}(e_{in1} - e_{in2}) \qquad (13\text{-}24)$$

$$e'_{o2} = \frac{\mu R_L}{2(R_L + r_p)}(e_{in1} - e_{in2}) \qquad (13\text{-}25)$$

Because it produces an output signal that is either positive or negative, and proportional to the difference between the two input signals, this circuit is useful as a summing device in a servomechanism. If one of the grids is grounded, one output of the differential amplifier is in phase with the input, and the other output is *180"* out of phase with the input. For this reason, the differential amplifier may be used as a phase splitter. It is then necessary that the two output voltages be of equal magnitude. For this case it is sufficient, but not necessary, that

$$R_k(1 + \mu) >> R_L + r_p$$

It is only necessary that

$$R_{L1} = \frac{R_k R_{L2}}{R_k + \dfrac{r_p + R_{L2}}{1 + \mu}}$$

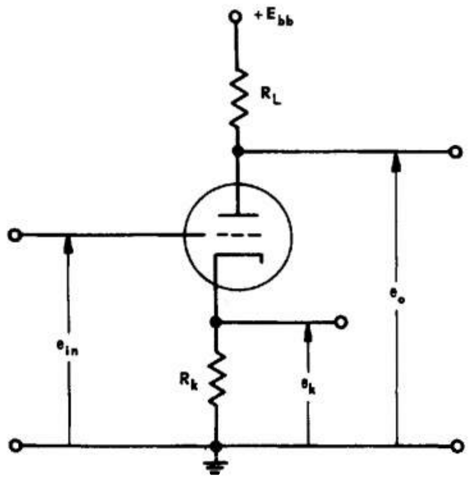


fig. 13-21 Plate-and-cathode-loaded amplifier.

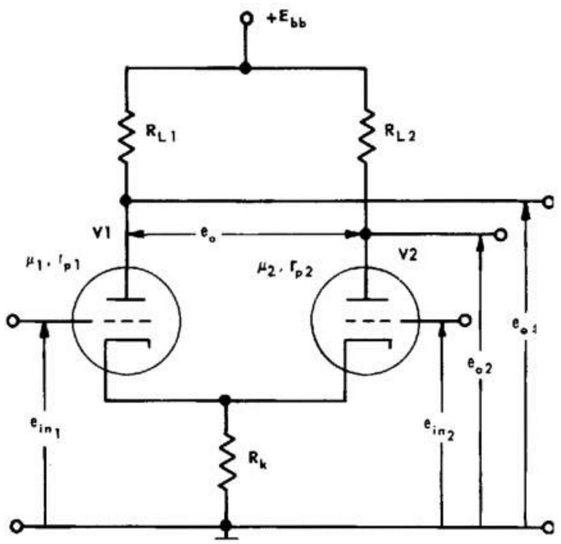


fig. 13-22 Differential amplifier.

Because the output of a differential amplifier depends almost entirely upon the difference between the two input signals, the circuit is insensitive to equal changes in both signals. Thus, equal variations in the input signals, such as drift in the reference potentials to which they are returned, result in small output-voltage variations. This property makes differential amplifiers very useful for d-c amplifier applications.

13-1.28 Use of Pentodes

In all circuits previously discussed where a resistor R_k is connected in series with the cathode, the use of pentode tubes is not convenient. This is because maintenance of proper screen voltage (which must be constant with respect to the cathode) would require a separate power supply for the screen of each pentode stage, with the supply connected between screen and cathode. However, when signal frequencies are,high enough (at least **10** cps) , pentodes can be used without a separate or floating screen supply for each stage. This is accomplished by connecting all pentode screens to a single source of positive voltage through individual resistors R_s and by connecting a large capacitor (typically, 0.1 to **1** microfarad) between the screen and cathode of each pentode.

13-1.29 POWER AMPLIFIERS

In the previous discussion of voltage-amplifier circuits, no attention was paid to the current levels in the amplifier load because the power supplied to that load is of little consequence. However, in the final stage of a servo amplifier, the amplifier load is the output member of the servomechanism. Since the load is usually a motor or a similar power-consuming device, the final amplifier stage must be a power amplifier with adequate output.

13-1.30 Tubes Used in Power Amplifiers

The tubes used for power amplification differ from those used for voltage amplification. These differences include :

(a) Greater maximum plate-dissipation ratings (e.g., 10 watts)

(b) Greater plate-current ratings (e.g., 200 ma)

(c) Greater range of grid voltage over which operation is linear (e.g., 15 volts)

Pentodes and beam-power tubes (e.g., Type 6L6) are commonly used as power-amplifier tubes because they produce a larger a-c load voltage than triodes do, for a given plate supply voltage and given maximum permissible harmonic distortion. In addition, pentodes are less likely than triodes to produce unwanted oscillations, when feeding inductive loads such as motors, because the grid-to-plate capacitance of a pentode is lower than that of a triode.

13-1.31 Bush-Pull Power Amplifiers

In servomechanisms, the push-pull power amplifier is more generally used than the single-ended, or any other type of power amplifier. The push-pull circuit comprises two tubes of the same type connected as shown in Fig. **13-23.** The two grid-signal voltages are of equal magnitude and 180° out of phase. The input transformer T1 can be replaced by a phase-splitting circuit as discussed in connection with Figs. **13-21** and **13-22.** The major advantages of the push-pull circuit over the single-ended circuit (load connected to one tube or two tubes in parallel) can be summarized as follows:

(a) Higher plate-circuit efficiency (ratio of output power to power dissipated in tube). Efficiency can be as high as **65** percent in a push-pull amplifier, as against up to **30** percent for a single-ended amplifier. Hence, for a given pair of tubes dissipating a fixed amount of power, the push-pull circuit delivers a greater output.

(b) An output, or load, signal that is completely free of even harmonics. Tube nonlinearites introduce these harmonics, but push-pull circuit action cancels them. This permits operation of each tube with a bias that is high enough to cut off the tube during part of the negative grid-voltage half-cycle when there is an input sinusoid of maximum amplitude. Under these conditions, the higher values of plate-circuit efficiency (up to *65* percent) are achieved. Therefore, in servo amplifiers, E_{cc} is often adjusted for plate-current cutoff during some part of the negative portion of the maximum input sinusoid, but for less than 180" if self-bias is used.

(c) There is no net d-c component of plate current in the output-transformer primary. Therefore, the transformer may be lighter and smaller than one that is used to match a load to a single-ended power amplifier which must carry d-c current.

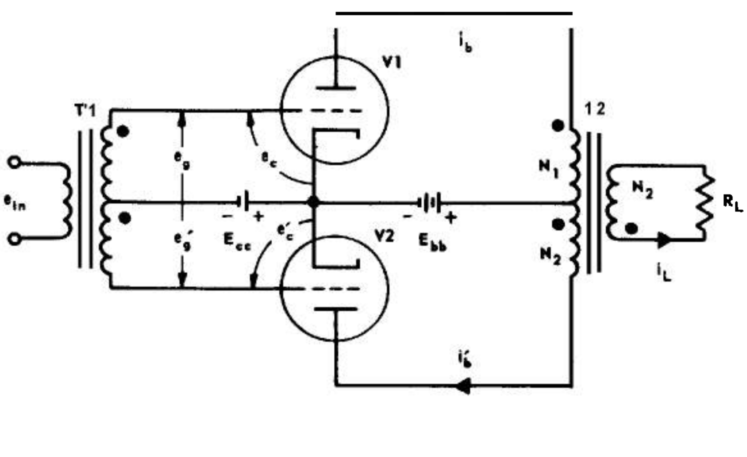


Fig. 13-23 Push-pull amplifier.

(d) Ripple in the plate supply voltage does not appear in the load, thus reducing the filtering requirements on the power supply.

13-1.32 Analysis of push-pull power amplifiers. Practical push-pull amplifiers must be analyzed by graphical techniques because they nearly always involve nonlinear tube operation. The analysis procedure is as follows:

(a) First, form the composite plate characteristics of the two tubes. This is done by arranging two sets of plate characteristics, as shown in Fig. 13-24A, so that the origin of the bottom set coincides with the point $e_b = 2E_{bb}$ of the top set.

(b) Add algebraically the two curves e„ E,,.In the example of Fig. 13-24B, E,,is assumed to be equal to $-k_3$, and the dashed curve is the result of the addition. Relabel the resultant curve e, $= 0$.

(c) Add algebraically the upper curve for e, $= -k_2$ and the lower curve for e‘, $= -k_4$. Relabel the resultant curve e, $= -k_2 + k_3 = k_1$.

(d) Add algebraically the upper curve for e, $= -k_4$ and the lower curve for e', $= -k_2$. Relabel the resultant curve e, $= -k_1$.

(e) Continue this procedure by combining upper e, and lower e', until the complete set of composite characteristics is obtained (Fig. 13-24C).

(f) Through the point $e_b = E_{bb}$, draw a straight line with slope $-1/(N_1/N_2)^2R_L$. Relabel the current axis $(N_2/N_1)i_L$ and the voltage axis $(N_1/N_2)e_L$. This is shown in Fig. 13-24D. Load voltage e_L and load current i_L may then be read off as a function of e,

13-1.33 Push-pull amplifiers with a-c supply. When the amplifier signal is an amplitude-modulated carrier, a power amplifier similar to the one shown in Fig. 13-25 is sometimes used. Its advantage over the conventional push-pull configuration is that it does not require a d-c power supply. The resulting signal waveforms are indicated in Fig. 13-26. The pulse-like nature of the plate-current waveform makes analysis exceedingly difficult, particularly when pentodes are used. Therefore, circuit values for this configuration are more easily determined by experimental procedures.

13-1.34 Efficiency

In practice, vacuum-tube power amplifiers are used only where the load requirements are moderate (up to approximately 100watts) because such amplifiers are relatively inefficient, with a maximum of about 65 percent.

13-1.35 CASCADING AMPLIFIER STAGES

When a single amplifier stage cannot provide the required gain, additional cascaded stages must be supplied until the requirement is met. In a cascaded amplifier, each successive stage amplifies the output of the preceding stage. There are two methods of cascading, each named according to the type of coupling circuit used between stages:

(a) Direct coupling, in which the total instantaneous value of the output of one stage affects the next stage.

(b) A-c coupling, in which only the time-varying component of the output of one stage affects the next stage.

13-1.36 Direct-Coupled Amplifiers

An example of a two-stage direct-coupled amplifier is given in Fig. 13-27. Assuming that $R_1 + R_2 >> R_{L1}$, the gain of this amplifier is expressed as

$$K = \frac{\mu_1\mu_2 R_{L1} R_{L2}}{(R_{L1} + r_{p1})(R_{L2} + r_{p2})} \times \frac{R_2}{R_1 + R_2} \qquad (13\text{-}26)$$

The voltage divider R_1, R_2 reduces the gain but is needed to make the V2 grid negative with respect to its cathode. The voltage divider can be eliminated by coupling the plate of V1 to the grid of V2 by means of a battery E (Fig. 13-28). However, the separate battery makes this coupling method impractical from many viewpoints. Another way of eliminating the voltage divider is to couple the plate of V1 directly to the grid of V2. The cathode of V2 must then be returned to a positive-potential point, and R_{L2} to a still more positive point.

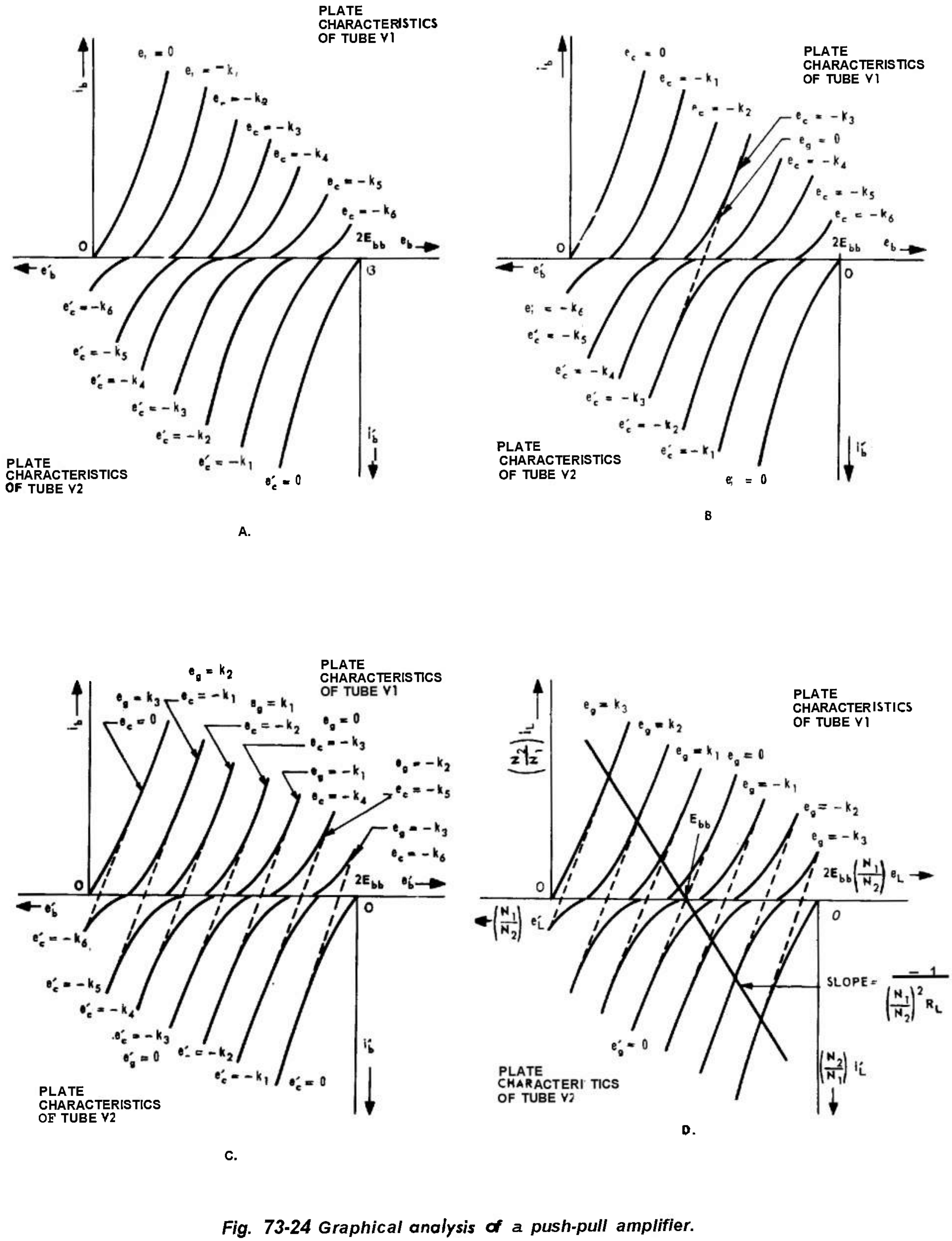


Fig. 73-24 Graphical analysis of a push-pull amplifier.

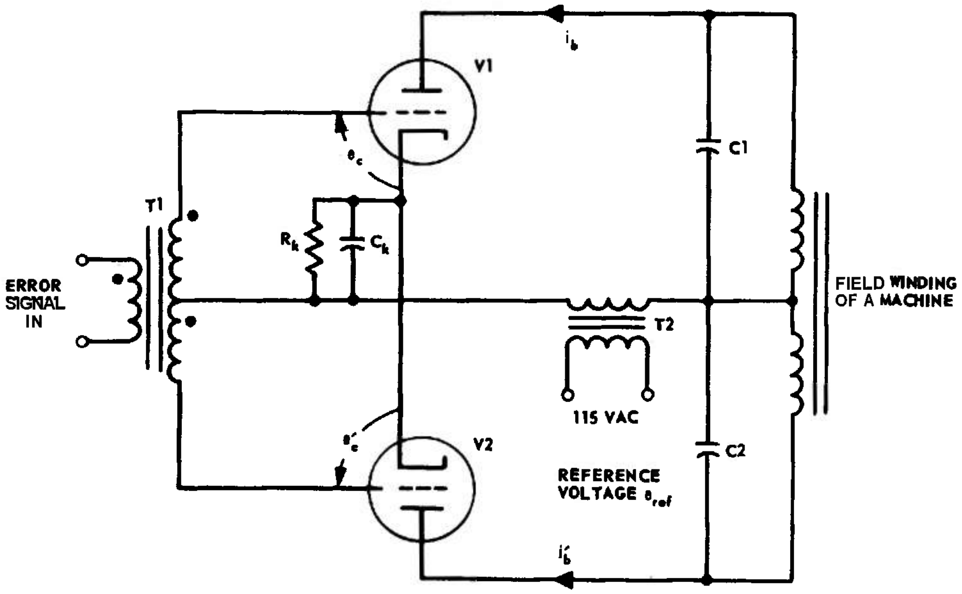


Fig. 73-25 Power amplifier with a-c supply.

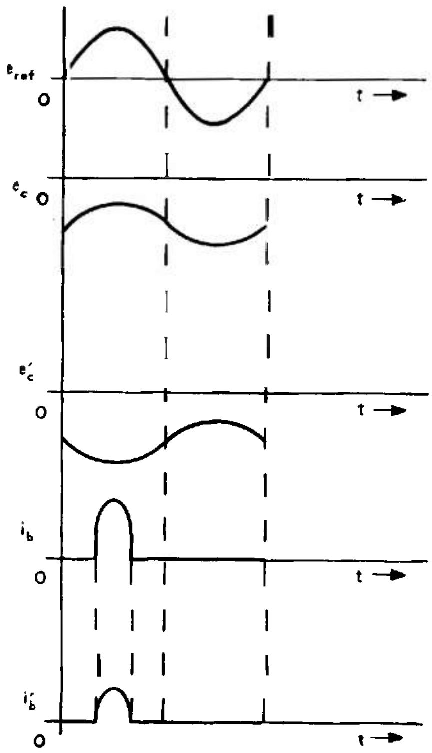


Fig. 73-26 Wuveforms of power amplifier in Fig. 73-25.

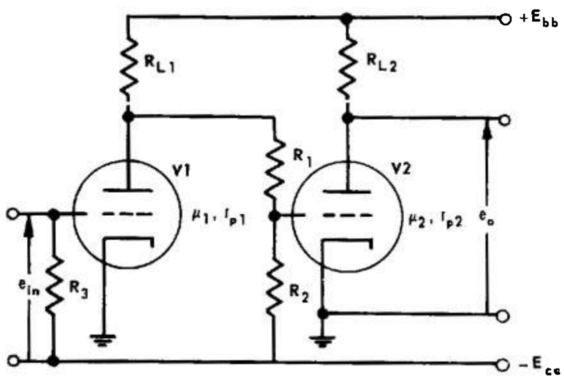


Fig. 73-27 Voltage-divider-coupled d-c amplifier.

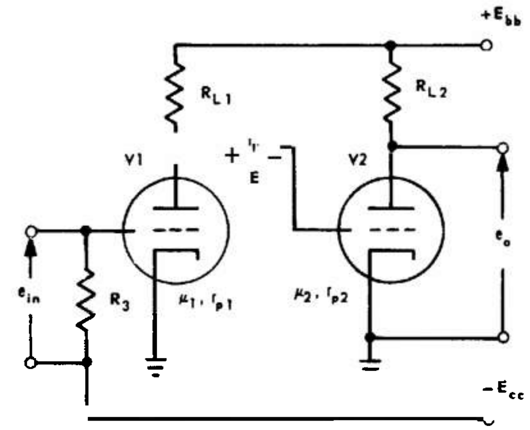


Fig. 13-28 Battery-coupled, d-c amplifier.

Four separate power supplies are therefore required (bias of V1, plate of V1, cathode of V2, and plate of V2). Again, an impractical situation arises. However, the need for four separate supplies can be obviated by using a single high-voltage supply and tapping off the required d-c voltages at appropriate points on a bleeder as shown in Fig. 13-29. Typical voltage levels corresponding to zero input signal are indicated on this figure. If the bleeder current is large compared with the tube currents, then the gain of the amplifier is

$$K = \frac{\mu_1 \mu_2 R_{L1} R_{L2}}{(R_{L1} + r_{p1})(R_{L2} + r_{p2})} \qquad (13\text{-}27)$$

The heavy drain on the power supply, due to the large bleeder current, makes this method undesirable in many applications. Figure 13-30 shows still another method of eliminating the effect of the voltage divider. Resistance R_1 in Fig. 13-27 is replaced by a gas-discharge tube, such as a Type VR105 voltage regulator. A gas-discharge tube is characterized by a very nearly constant terminal voltage over a limited current range (e.g., 10 to 30 ma) so that it acts as an equivalent battery. Figure 13-30 is the simplest coupling circuit where signal levels are high enough so that the gas-discharge-tube noise (somewhat reduced by R_N and C_N) does not reduce the amplifier signal-to-noise ratio below acceptable limits. Iannone[3] discusses in detail the use of gas-discharge tubes as coupling devices in d-c amplifiers.

13-1.37 Problems encountered in **direct-coupled amplifiers.** Not only are there practical difficulties in the design of coupling circuits in direct-coupled amplifiers, but the very nature of direct coupling makes the output level of such an amplifier dependent upon all amplifier components, tube characteristics, and power supply levels. Any variation

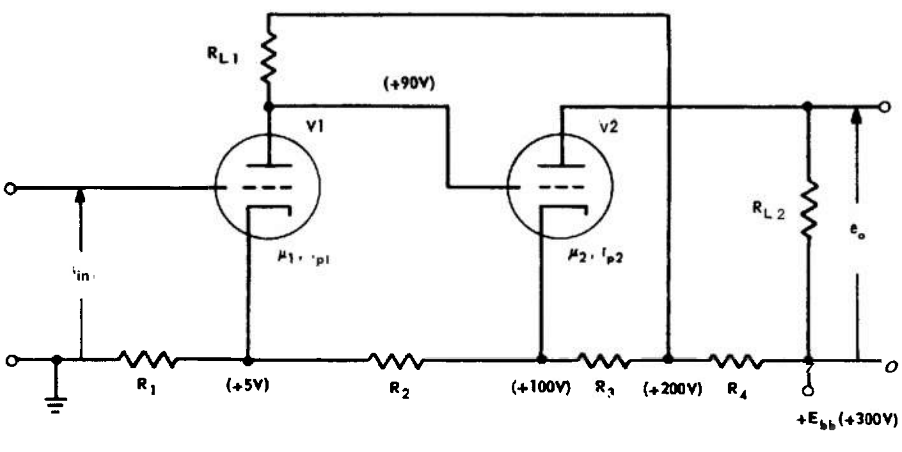


Fig. 73-29 D-c umplifier with single supply voltage.

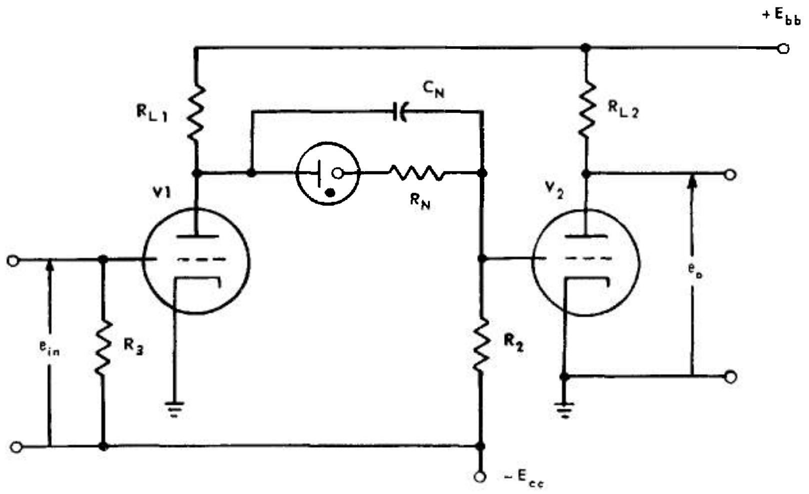


Fig. 73-30 D-c amplifier with gas-discharge tube coupling.

in these parameters (caused by aging of components, changes in environmental conditions, power supply ripple, etc.) changes the output level even though the input remains constant. The following measures are generally used to reduce variation (drift) in the output level:

(a) Use only components that are stabilized for age and changes in environmental conditions; e. g., tubes that have been "broken in" (operated for 100 to 200 hours before use) and temperature-stable metalized-type resistors.

(b) Incorporate good regulation of all power supply voltages, including filament voltage. (A change of 20 percent in filament voltage is approximately equivalent to a grid-voltage change of 0.2 volt. Over the nonlinear range of operation, changes in filament voltage also produce changes in r_p.)

In practice, these measures cannot be carried out to perfection. Tubes show the effects of aging, practical power supplies rarely have voltage regulation better than 0.1 percent, and good regulation of filament supplies often results in excessive weight and space requirements. The reduction of drift, therefore, requires the use of special circuit designs (particularly for the first stage of a direct-coupled amplifier) that inevitably lead to greater over-all amplifier complexity. One example of added complexity is the use of feedback, which can *reduce* the effect of drift but cannot *eliminate* it.

13-1.38 Drift-compensated direct-coupled amplifier. Figure **13-31** shows a circuit in which the output is independent of filament-voltage and supply-voltage variations, provided that the two tubes are identical. Assuming linear operation, the ratio of total output voltage to total input voltage is

$$\frac{e_o}{e_{in}} = \frac{-\mu R_3}{2r_p + R_3} \tag{13-28}$$

13-1.39 Bridge circuits. Bridge circuits, such as those in Figs. **13-32** and **13-33,** eliminate drift due to changes in E_{bb}, filament voltage, and r_p. Assuming linear operation in Fig. 13-**32,** the ratio of output voltage to input voltage is

$$\frac{e_o}{e_{in}} = \frac{-\mu R_L}{r_p + 2R_k + 2R_L + R} \tag{13-29}$$

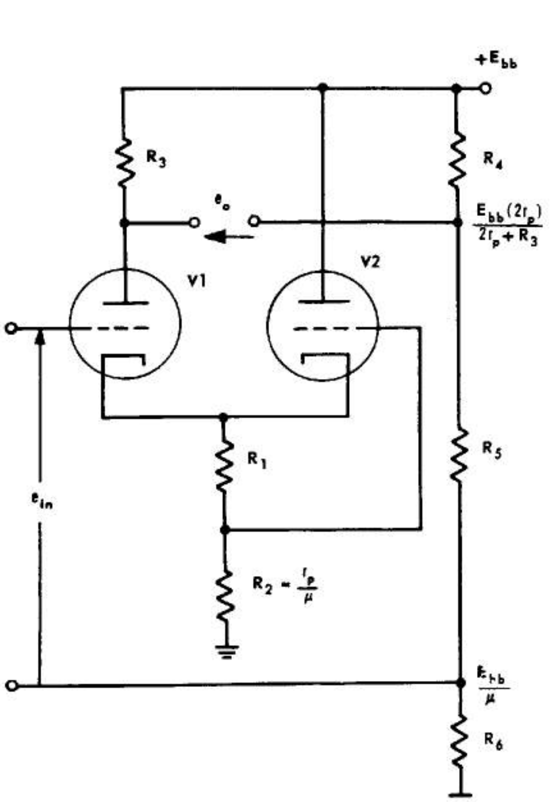


Fig. 73-31 Drift-compensated d-c amplifier.

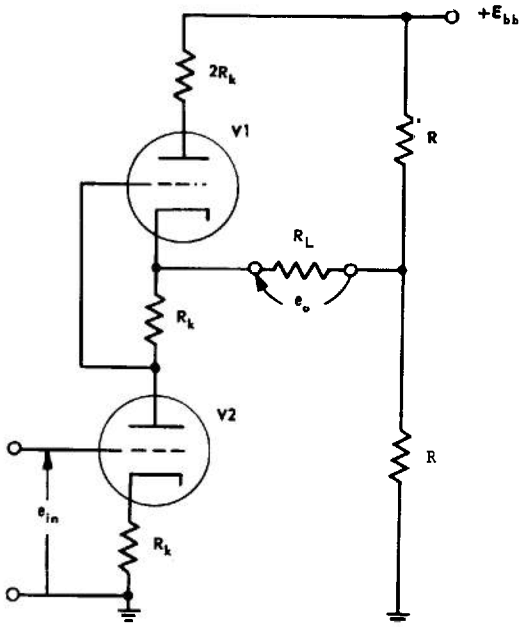


Fig. 13-32 Drift-compensated d-c amplifier.

and the output impedance R, seen by the load is

$$R_o = R_k + \frac{r_p + R}{2} \tag{13-30}$$

Assuming linear operation in **Fig. 13-33, a** low output impedance circuit, it follows that

$$\frac{e_o}{e_{in}} = \frac{-\mu R_L}{r_p + R_k(1+\mu) + R_L(2+\mu) + R} \tag{13-31}$$

and

$$R_o = \frac{R_k(1+\mu)}{2+\mu} + \frac{r_p + R}{2+\mu} \tag{13-32}$$

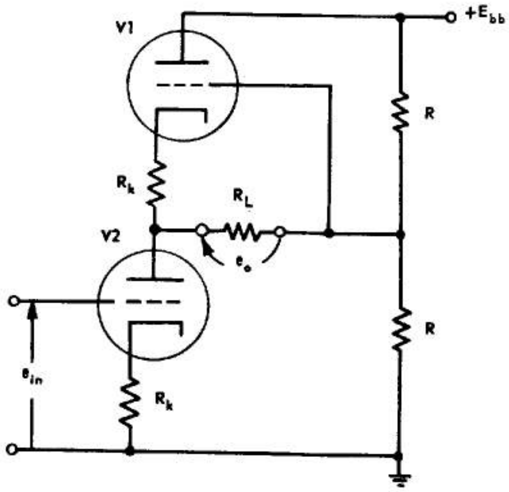


Fig. 13-33 Drift-compensated d-c amplifier.

13-1.40 A-C Coupled Amplifiers

Because the reduction of drift to a negligible amount often leads to considerable circuit complexity, direct-coupled amplifiers are generally avoided in servomechanism design. Instead, a-c coupled amplifiers are used. However, in an a-c coupled amplifier, low-frequency signal components are attenuated and, since typical signal frequencies in a servomechanism occur in the 0 to 20 cps band, the signal must first be modulated; i.e., superimposed on an a-c carrier. Typical carrier frequencies are **60** cps and **400** cps. The block diagram of a servomechanism incorporating an a-c coupled amplifier is shown in Fig. **13-34.** If the amplifier load responds to modulated a-c signals, the demodulator is not used. An example of such a load is a 2-phase induction motor. If the input signal is already in the form of a modulated a-c signal, such as the output of a synchro control transformer, the modulator is not used. Servo compensation is shown ahead of the modulator in Fig. **13-34** because d-c compensating networks are simpler to design than a-c compensating networks.

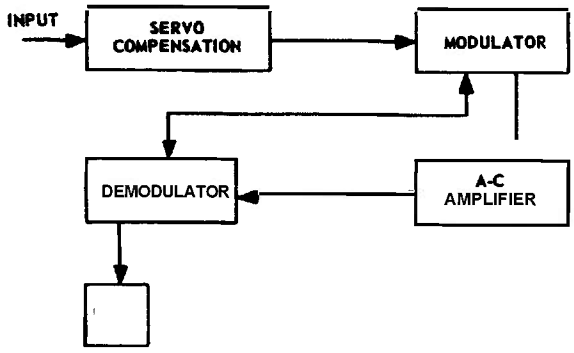


Fig. 73-34 Use of a-c amplifier to replace d-c umplifier.

13-1.41 Two-stage a-c coupled amplifier. Figure **13-35** is the circuit schematic of a simple two-stage a-c coupled amplifier. Resistors R_k and capacitors C_k furnish self-bias by raising the average cathode potential above ground. The grid circuit may then be returned to ground potential, as shown, and the need for a grid-bias supply is eliminated. C_k is large

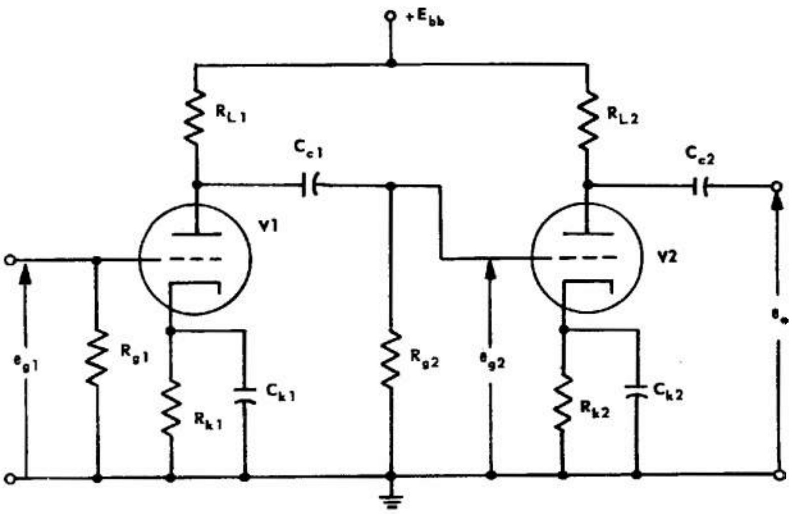


Fig. 73-35 Two-stage a-c amplifier (resistance-capacitance coupled).

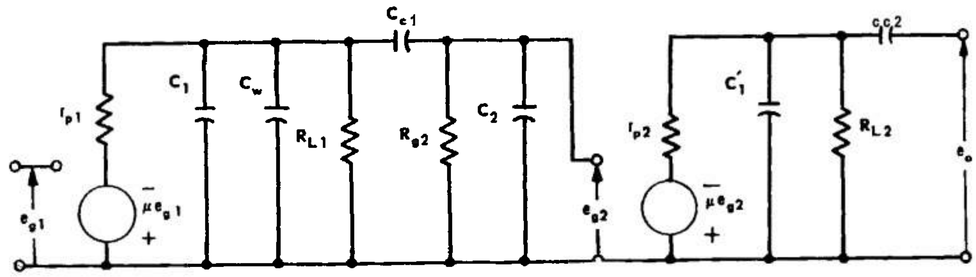


Fig. 13-36 Equivalent circuit of two-stage a-c amplifier in Fig. 73-35.

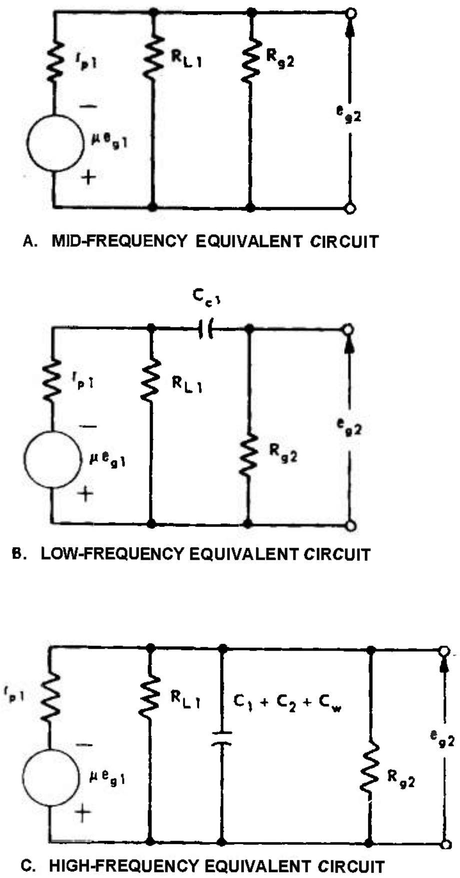


Fig. 73-37 Simplified equivalenf circuits of first stage of Fig. 73-35.

enough (1 to 40 microfarads) so that its reactance is negligible at the frequencies of interest, thereby eliminating the effect of R_k on dynamic response. For linear operation, the value of R_k is chosen so that

$$|E_{cc}| = I_{bo} R_k \tag{13-33}$$

where

$|E_{cc}|$ = absolute value of desired bias

I_{bo} = quiescent plate current

The large capacity of C_k is provided in compact form by low-voltage electrolytic capacitors.

The equivalent circuit of the two-stage a-c coupled amplifier is shown in Fig. 13-36. For analysis, this equivalent circuit can be broken down into three separate simplified circuits as shown in Fig. 13-37. In these circuits, C_1 is the output capacitance of tube V1, C_2 is the input capacitance of tube V2, and C_w is the wiring capacitance. The gain expressions for the three frequency regions are

$$K_{mid} = \frac{e_{g2}}{e_{g1}} = \frac{-\mu R_{L1} R_{g2}}{r_{p1} R_{L1} + r_{p1} R_{g2} + R_{L1} R_{g2}} \tag{13-34}$$

$$K_{low} = \frac{K_{mid}}{1 - j\dfrac{f_1}{f}} \tag{13-35}$$

$$K_{high} = \frac{K_{mid}}{1 + j\dfrac{f}{f_2}} \tag{13-36}$$

where

f = frequency of input signal

f_1 = lower half-power frequency (response down to 3 db)

$$= \frac{r_{p1} + R_{L1}}{2\pi C_{c1}(r_{p1}R_{L1} + r_{p1}R_{g2} + R_{g2}R_L)}$$

f_2 = upper half-power frequency (response down 3 db)

$$= \frac{r_{p1}R_{L1} + r_{p1}R_{g2} + R_{L1}R_{g2}}{2\pi(C_1 + C_2 + C_w)r_{p1}R_{L1}R_{g2}}$$

Typical values of f_1 range from 1 to 20 cps, typical values of f_2 from 50 to 500 kc. Thus, in a servomechanism application, the high-frequency equivalent circuit needs consideration only when stability questions are involved. In a properly designed amplifier, it is usually permissible to assume that the gain is constant and given by the mid-frequency value K_{mid}. Plots of output voltage, gain, and phase shift as a function of frequency are shown in Fig. 13-38. When pentodes are used, and the effect of R_k and C_k is to be taken into account, then

$$\frac{\text{actual output voltage}}{\text{output voltage with fixed-bias operation}} = \frac{1}{1 + \dfrac{g_m R_k}{1 + j\dfrac{f}{f_3}}} \quad (13\text{-}37)$$

where

f = frequency of input signal

$$f_3 = \frac{1}{2\pi C_k R_k}$$

13-1.42 FEEDBACK AMPLIFIERS

13-1.43 Advantages

Negative feedback applied to an amplifier produces the following advantages:

(a) Improved *stability*,

$$\frac{dK'}{K'} = \frac{1}{1 - \beta K}\,\frac{dK}{K} \quad (13\text{-}38)$$

where

$\frac{dK'}{K'}$ = per unit change in gain with feedback

K = open-loop gain

β = fraction of output signal fed back

$\frac{dK}{K}$ = per unit change in gain without feedback

Since $|1 - \beta K| > 1$ for negative feedback, then

$$\frac{dK'}{K'} < \frac{dK}{K}$$

(b) *Modification* of *frequency* ***response.*** For example, addition of negative feedback to a single-stage a-c coupled amplifier (Fig. 13-39) decreases the lower half-power frequency and increases the upper half-power frequency. Thus,

$$f_{1f} = f_1/(1 - \beta K_{mid}) \quad (13\text{-}39)$$

$$f_{2f} = f_2(1 - \beta K_{mid}) \quad (13\text{-}40)$$

where

f_{1f} = lower half-power frequency with feedback

f_1 = lower half-power frequency without feedback

f_{2f} = upper half-power frequency with feedback

f_2 = upper half-power frequency without feedback

K_{mid} = mid-frequency gain of amplifier without feedback

(c) *Modification of input and output impedances.*

(1) For voltage feedback (signal proportional to output voltage fed back), the output impedance is given by

$$Z'_o = \frac{Z_o}{1 - \beta K_o} \quad (13\text{-}41)$$

where

Z'_o = output impedance with voltage feedback

Z_o = output impedance with input voltage to feed-forward section short-circuited

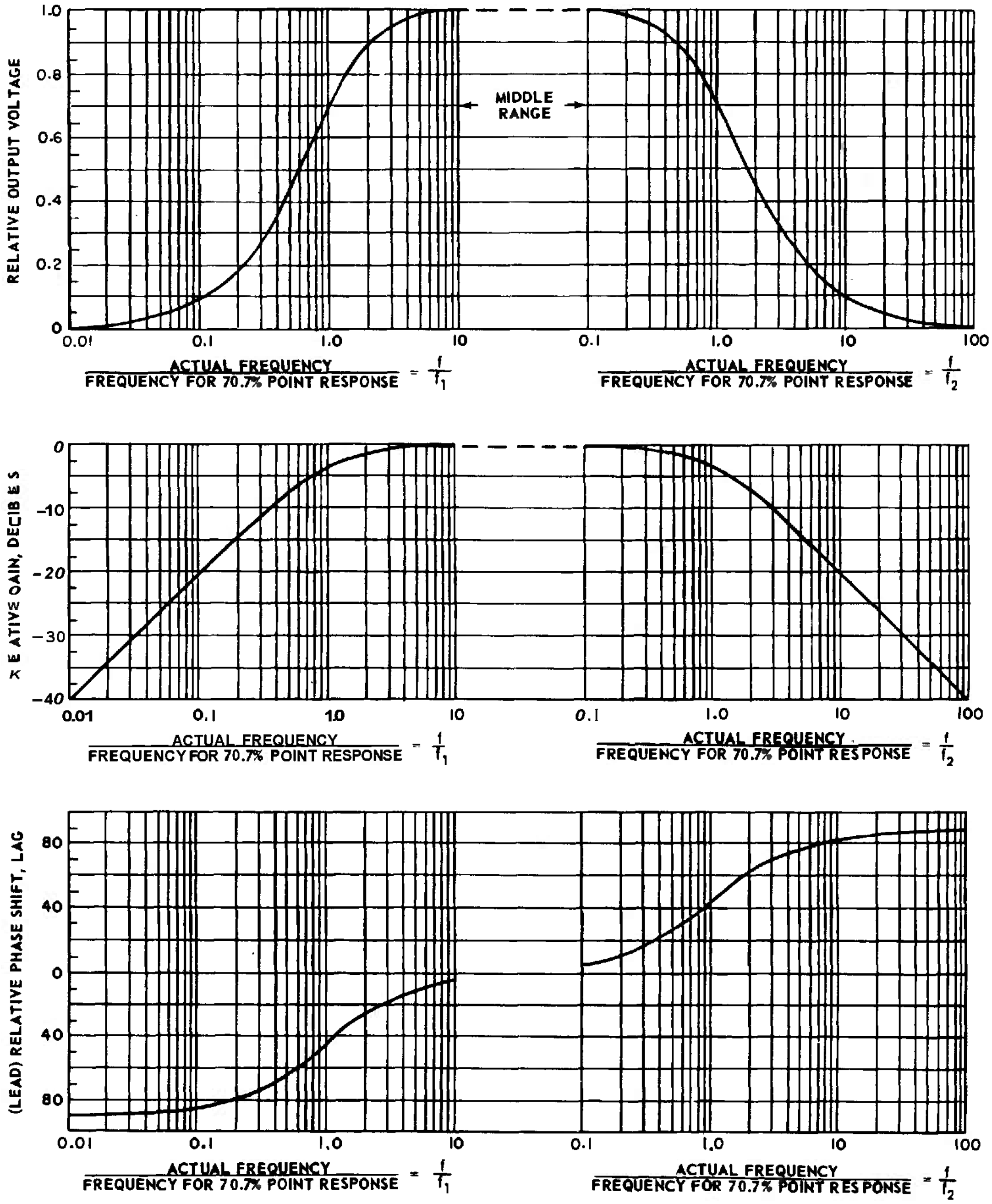


Fig. 13-38 Frequency response of single-stage u-c coupled amplifier using resistance-capacitance coupling.

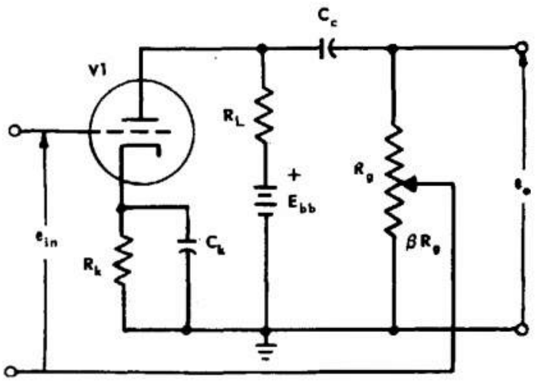


Fig. 13-39 Single-stage a-c coupled amplifier with voltage feedback.

β = fraction of output voltage fed back

K_o = open-loop gain of amplifier with load open-circuited

Since $|1 - \beta K_o| > 1$ for negative feedback, then

$Z'_o < Z_o$.

For example, consider the output impedance seen by R_k in the cathode follower of Fig. 13-16. Here,

$Z_o = r_p$

$\beta = -1$

$K_o = \mu$

$$Z'_o = \frac{r_p}{1+\mu}$$

(2) For current feedback (signal proportional to output current fed back), the output impedance is given by

$$Z''_o = Z_o + Z_f(1 - K_o) \tag{13-42}$$

where

Z''_o = output-impedance with current feedback

Z_f = impedance across which feedback is developed

Since $|1 - K_o| > 1$ for negative feedback, then

$Z''_o > Z_o$

For example, consider the output impedance seen by the load resistor R_L in Fig. 13-21. Here,

$Z_o = r_p + R_k$

$Z_f = R_k$

$K_o = -\mu$

$Z''_o = r_p + R_k(1 + \mu)$

(3) When both voltage and current feedback are used, then

$Z''_o{}'$ = effective output impedance with both types of feedback

$$= \frac{Z_o + Z_f(1 - K_o)}{1 - \beta K_o} \tag{13-43}$$

Techniques for finding impedance levels at any terminal pair are given in the book by Bode.[4]

(d) ***Improvement in signal-to-noise ratio when noise is generated internally (not at input to amplifier).*** The addition of feedback itself does not increase the signal-to-noise ratio. It does, however, permit an increase in the signal-to-noise ratio because it permits an increase in the input-signal level or an increase in the gain of the stages preceding the noise source without overloading the output stage. This means, for example, that the power supply for the output stage of a feedback amplifier requires less filtering as would normally be required.

(e) ***Reduction of nonlinear distortion.*** A feedback amplifier reduces the percentage of harmonic distortion in the output by a factor of $1/1 - \beta K$.

13-1.44 Disadvantages

The disadvantages of negative feedback are:

(a) Stability problems arise. These problems are analyzed and solved in the manner outlined in Section I of this publication.

(b) Over-all gain is less than in an open-loop amplifier with the same number of stages. That is,

$$K' = \frac{K}{1 - \beta K} \tag{13-44}$$

Since $|1 - \beta K| > 1$ for negative feedback, $K' < K$. To achieve a specified gain, a negative-feedback amplifier requires more stages of amplification than an open-loop amplifier.

13-1.45 PROBLEMS ENCOUNTERED IN USE OF ELECTRONIC AMPLIFIERS AS SERVO COMPONENTS

13-1.46 Reliability

The greatest single cause of failure in electronic servo amplifiers is vacuum-tube breakdown. In many modern designs, vacuum tubes have been replaced by transistors and magnetic amplifiers, but there are still cases where dynamic requirements or environmental conditions dictate the use of vacuum tubes. In these cases, the designer can do much toward achieving amplifier reliability. It is possible to design amplifiers that will operate for thousands of hours without failure by taking the following measures :

(a) ***Care in selection of tube types.*** There are now available higher-quality tube types than those originally developed for consumer use. The highest-quality tubes that are well-adapted to military applications are listed in MIL-STD-200. Tube types should be selected from this list whenever possible.

(b) ***Derating of tubes.*** Particular care must be taken to ensure that maximum tube ratings are not exceeded. These ratings are published in tube manufacturers' catalogs and in Military Specification MIL-E-1. Circuits should be designed for operation well below maximum tube ratings because tube life is appreciably increased when the tubes are thus derated. While there is no available analytical relationship giving tube life as a function of derating, ample experimental evidence shows that derating of tube characteristics leads to increased tube life. Maximum plate dissipation, maximum plate voltage, maximum cathode current, and maximum bulb temperature are the major characteristics that should be derated.

(c) ***Cooling of tubes.*** Vacuum tubes should not be located near other heat-radiating parts and proper ventilation should always be provided. Where convection cooling cannot be made adequate, either by natural circulation or forced air flow (fans), conduction cooling can be provided by means of a metallic bond between the tube envelope and the chassis (e.g., by using a properly designed tube shield).

(d) ***Interchangeability of tubes.*** Circuits should be designed so that malfunction cannot be caused by variations in characteristics between tubes of the same type (as listed in MIL-E-1) or by reasonable variations in tube characteristics caused by aging. Negative feedback, for example, can do much to stabilize circuit performapce in the presence of tube-characteristic variations. A properly designed circuit does not require a specially selected tube. When circuit design necessitates the selection of tubes with particular characteristics, unmanageable maintenance problems inevitably follow.

(e) ***Protection from shock.*** In mobile applications, tubes must be protected from mechanical shock. Shock mounts on the amplifier chassis are very helpful and, in addition, the amplifier should be located as far as possible from shock-producing members. Where mechanical shock is unavoidable, select tube types specifically designed to tolerate shock and use their acceleration ratings as a guide in this selection.

(f) ***Filament-voltage regulation.*** Filament voltages in excess of nominal ratings are particularly detrimental to tube reliability. If the source of heater voltage is known to be variable, it may be possible to select a center value that is lower than the tube rating, providing the resulting decrease in tube performance does not produce circuit malfunction. Otherwise, some means of heater-voltage control must be devised, or a more stable source provided.

(g) *Derating of passive circuit components.* Circuit components such as resistors and capacitors (called *passive* components) must be carefully chosen for highest quality. The effects of variations in environmental conditions on component electrical characteristics must be considered in the over-all. amplifier design. Derating of manufacturers' specifications is helpful in extending the useful life of passive components as well as tubes. Many designers now derate the power-dissipation ratings or resistors and the maximum voltage ratings of capacitors (at normal ambient temperatures) by as much as 50 percent. Additional derating of these specifications is made for operation under unusual or extreme conditions. To include aging effects in resistors, some design groups also derate the resistance-tolerance specification and use a 1-percent resistor where a 5-percent tolerance is initially required, and a 5-percent resistor where 10- to 20-percent tolerance is required. While these measures lead to higher initial equipment cost, they often pay for themselves in reduced maintenance costs.

13-1.47 Construction

In the physical construction of an amplifier, the dominant considerations are: adherence to weight and space requirements; freedom from unwanted electrical coupling between components; proper ventilation to prevent excessive temperatures; freedom from mechanical vibration; and accessibility of compcnents for maintenance.

13-1.48 Maintenance

Equipment down-time can be materially reduced by using plug-in subassemblies that can be quickly replaced by spares in case of failure. To reduce the required inventory of spare units, as many as possible should be identical. For example, it is frequently possible to use several identical *complete amplifiers* at different places in a system. The single design, in this case, must be capable of satisfying different requirements and may therefore require greater design effort. However, the resultant operational and maintenance convenience often makes this approach very desirable.

13-1.49 Quadrature Signals

In some applications, a servo amplifier is required to amplify a signal that contains two components, one component in time-phase with respect to a signal to which the output member of the servomechanism can respond and representing the loop error, and the other component 90° out of phase with the first. The second component, called the *quadrature* signal, serves no useful purpose. Its presence, however, causes the amplifier to saturate earlier than it otherwise would, thereby reducing the effective gain of the amplifier. For this reason, it is desirable to remove the quadrature-signal component. One method employs a keyed demodulator that is triggered at the time the instantaneous value of the quadrature voltage is zero. Since the output of a keyed demodulator will then be a function of the in-phase signal only, remaining constant between triggering points, it follows that the quadrature component is eliminated from the demodulator output.

13-1.50 Complete Amplifier

The design of a complete amplifier is not a rigidly determined procedure. The designer must take into consideration not only the amplifier specifications (load characteristics, input signal voltage and impedance, amplifier dynamics, space and weight restrictions), but also such characteristics of the over-all system as environmental conditions and available power supplies. However, the designer may very well be able to design several different amplifiers that satisfy the requirements. For example, after a push-pull power-amplifier stage has been designed, a freedom of choice often exists for selecting the means of obtaining the two grid signals. This choice could be a transformer with a center-tapped secondary, a plate- and cathode-loaded amplifier, or a differential amplifier with one grid grounded. In general, the objective is to achieve the required gain in the voltage-amplifier section with the least number of stages so that the tube complement is held to a minimum. On the other hand, it might be desirable to incorporate negative feedback for gain stability. This decreases the open-loop gain and, as a result, the design might use more than the minimum possible number of tubes. Here, as in so much of systems design, trial-and-error procedures are used and even after a design is completed, building and testing a breadboard model nearly always results in a considerable number of design changes.

13-1.51 Details of a Typical Servo Amplifier

Figure 13-40 shows the circuit schematic of the main elements of a complete amplifier designed to drive a 2-phase motor. Tubes V4 and V5 form a push-pull amplifier. The control-grid signal for V4 is derived from voltage amplifier V3A, one of two triode sections in the same tube envelope. The control-grid signal for V5 is derived from V3B, the output of which is of the same magnitude as, but 180° out of phase with, the output of V3A. The input signal for V3B is part of the output of V3A, and hence V3B inverts that signal. The ratio $R_{22}/R_{21} + R_{22}$ and the gain of V3B are adjusted so that the output of V3B is exactly equal in magnitude to the output of V3A. V2B is another simple voltage amplifier and V2A is an electronic phase shifter comprising a phase splitter similar to the one discussed in connection with Fig. 13-21. By connecting $R9$ and C4 between the plate and cathode of V2A, the output voltage at the junction of $R9$ and C4 can be varied in phase from 7° to 170° without appreciable change in amplitude simply by varying $R9$. V1 is a summing amplifier for the two input signals. Each stage of the servo amplifier uses a cathode resistor to provide negative feedback within the stage and also to provide bias voltage, thereby eliminating the need for a negative bias-voltage supply. Series grid resistors R25 and R26 are used to suppress parasitic oscillations that frequently occur when the resistors are omitted. The capacitor across the output transformer compensates for the inductance of the motor; the capacitor value depends upon the particular motor used. Amfier gain is controlled by varying R28 in the feedback loop. The gain factors for the individual stages are indicated on the figure. Other specifications are:

Power Requirements:

For V4 and V5, Type 6V6 —plate current 80 ma at +300v, filament current 2.7 amp at 6.3 vac

For V4 and V5, Type 6L6 —plate current 110 ma at +300v, filament current 3.6 amp at 6.3 vac

Gain: 1000 min, 5000 max

Power ouput: 15 watts max

Load impedance: adjustable from 200 to 20,000 ohms through transformer taps

Noise level: less than 1 volt output with both inputs grounded

Internal phase shift: adjustable from 7" to 170°

Linearity: linear for input signals up to ±50 mv

Response: flat within ± 1 db from 50 to 10,000 cps for resistive load of 1200 ohms; flat to ±50 cps of the modulating frequency (3" phase shift at 50 cps), for motor loads, with carrier frequency constant; carrier frequencies of 60, 400, and 1000 cps can be used (for 60 cps carriers, the coupling capacitors should be increased to 0.05 microfarad).

13-1.52 THYRATRON AMPLIFIERS

Amplifiers employing thyratron tubes can be used in certain servomechanism applications to supply resistive, capacitive, or inductive loads. The thyratron is an efficient amplifying device that provides high power gain.

13-1.53 Description of Thyratron

A thyratron is a thermionic tube containing a plate, a cathode, and one or more grids. The thyratron envelope is filled with hydrogen, mercury vapor, or a noble gas such as xenon. Hydrogen-filled thyratrons are used only for high-voltage pulse work; they will not be considered in the following discussion. Mercury-vapor-filled thyratrons must be kept between fairly narrow operating-temperature limits, usually between 40°C and 80°C, thus limiting the ambient temperatures in which these tubes can be used. However, nearly all

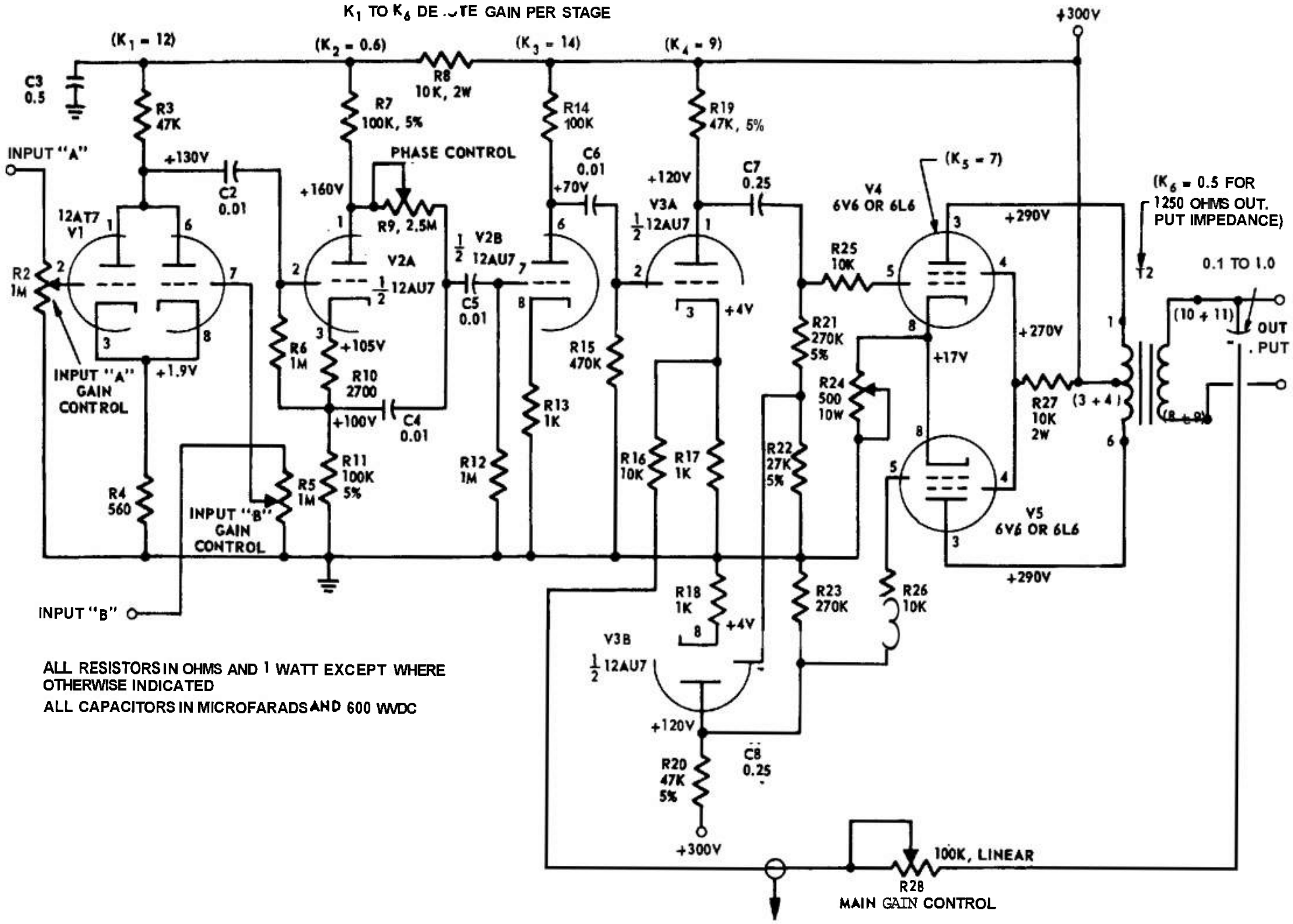


Fig. 73-40 A-c servo amplifier.

thyratrons filled with noble gas can be used in ambient temperatures ranging from —55°C to +75°C, with some tubes having an even wider ambient-temperature operating range. The control power required by thyratrons is quite small, being generally about 100 milliwatts. Load-current ratings range from about 40 milliamperes to 18 amperes, and more. Peak-forward and peak-inverse voltage ratings range from about 350 to 2000 volts, with ratings from 750 to 1500 volts being the most common for the larger tubes. Plate-to-cathode drop is generally 8 to 15 volts.

13-1.54 Thyratron Characteristics

The thyratron acts essentially as a switch capable of conducting current in one direction only. As long as the voltage between control grid and cathode is more negative than the critical grid voltage, the thyratron does not conduct plate current. When the grid-to-cathode voltage is more positive than the critical grid voltage, the thyratron conducts plate current, provided that the plate is positive with respect to the cathode. However, the control grid loses all control over the plate current as soon as the thyratron begins to conduct, and current flows as long as the plate-to-cathode voltage is positive, even if the control-grid voltage is again made more negative than the critical grid voltage. Figure 13-41 shows typical control characteristics of a thyratron. The grid voltage at which conduction starts is called the *critical grid voltage* and is a function of plate voltage. The shaded area in the figure indicates a region of uncertainty where conduction may, or may not, start. This uncertainty region is the result of manufacturing variations, variation in tube temperature, and tube aging. Conduction in a thyratron is carried on by ionized gas. At the end of the conduction period, a certain time is required for the gas to be deionized. Usually, this deionization time is about 1 millisecond, thus placing a limit on the frequency of the alternating voltage that can be controlled by a thyratron. Nearly all thyratrons require a warm-up period, ranging from about 10 seconds to one minute, every time the equipment is started. This warm-up period prevents damage to the cathode by positive-ion bombardment.

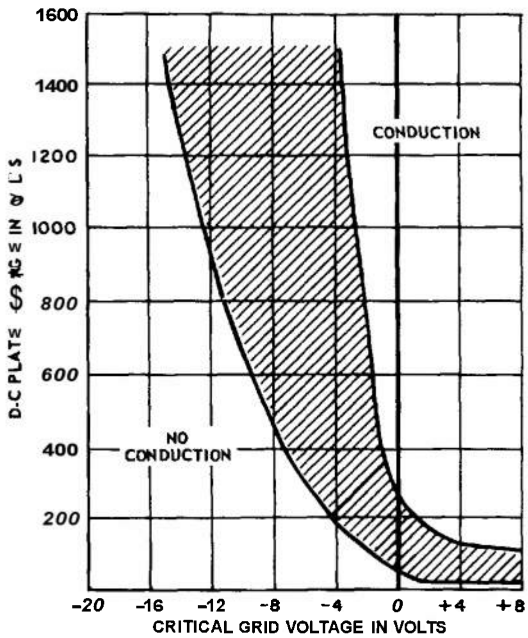


Fig. 13-41 Typical thyratron control characteristics.

Once a thyratron begins to conduct, the control grid has no further influence on the magnitude of the plate current, and grid control can only be re-established by removing the plate voltage or by reversing it. Removal of the plate voltage is nearly always a cumbersome procedure, but reversal can be accomplished by the simple expedient of using ac instead of dc to supply the plate voltage. For this reason, thyratrons are hardly ever supplied from a d-c source. Reversal of the alternating plate voltage interrupts the plate current during each cycle, thus re-establishing grid control fur the next cycle.

13-1.55 Thyratron Amplifier with Resistive Load

Figure 13-42 shows a simple half-wave thyratron amplifier working into a purely resistive load. The output current of this amplifier is a pulsating direct current, the magnitude of which depends upon the grid voltage. A common method of applying grid control is shown in this figure, where a constant sinusoidal voltage is phase-shifted by an R-C network so that it lags the plate voltage of the thyratron by 90°. This phase-shifted voltage, frequently called the ***rider,*** is added to a variable d-c control voltage E_c; the sum of the two voltages is then applied between grid and cathode of the thyratron. A very small capacitor *C2* is connected directly between grid and cathode to prevent transient disturbances in the plate voltage from producing excessive voltages on the control grid through the plate-to-grid capacitance.

Figure **13-43** shows the waveforms of plate voltage and critical grid voltage of the thyratron as a function of time. The actual grid voltage produced by the circuit of Fig. **13-42** is shown for two values of the control voltage E_c, together with the resulting load voltage. If E_c is positive (Fig. **13-43A**), the grid voltage exceeds the critical grid voltage early in the positive half-cycle of plate voltage. The thyratron thus "fires" early, permitting the plate voltage to be applied to the load over the major portion of the positive half-cycle. The rectifying properties of the thyratron prevent a reversal of the load current; therefore, no negative voltage can be applied to the resistive load. If E_c is negative (Fig. **13-43B**), the thyratron fires late in the positive half-cycle, and the plate voltage is applied to the load for only a small portion of the positive half-cycle.

13-1.56 Control of load Voltage

With the grid circuit shown in Fig. **13-42,** the load voltage can be controlled by varying the control voltage E_c. With resistive loads, the change in the d-c value of the load voltage is proportional to the change in the d-c control voltage. Thyratrons are often used in full-wave amplifiers, in which case a transformer or other means must be used to supply each tube with the appropriate rider. Occasionally, the a-c rider is purposely distorted by means of diodes, etc., to obtain more favorable control characteristics for inductive or capacitive loads.

The angle α at which the thyratron begins to conduct is often called the ***firing angle,*** or ***angle of retard.*** This firing angle can also be controlled by providing an a-c grid voltage,

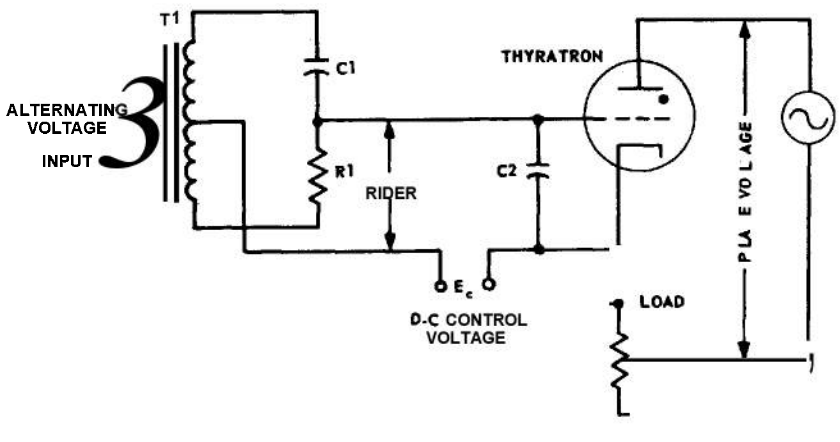


Fig. 13-42 Half-wave thyratron amplifier.

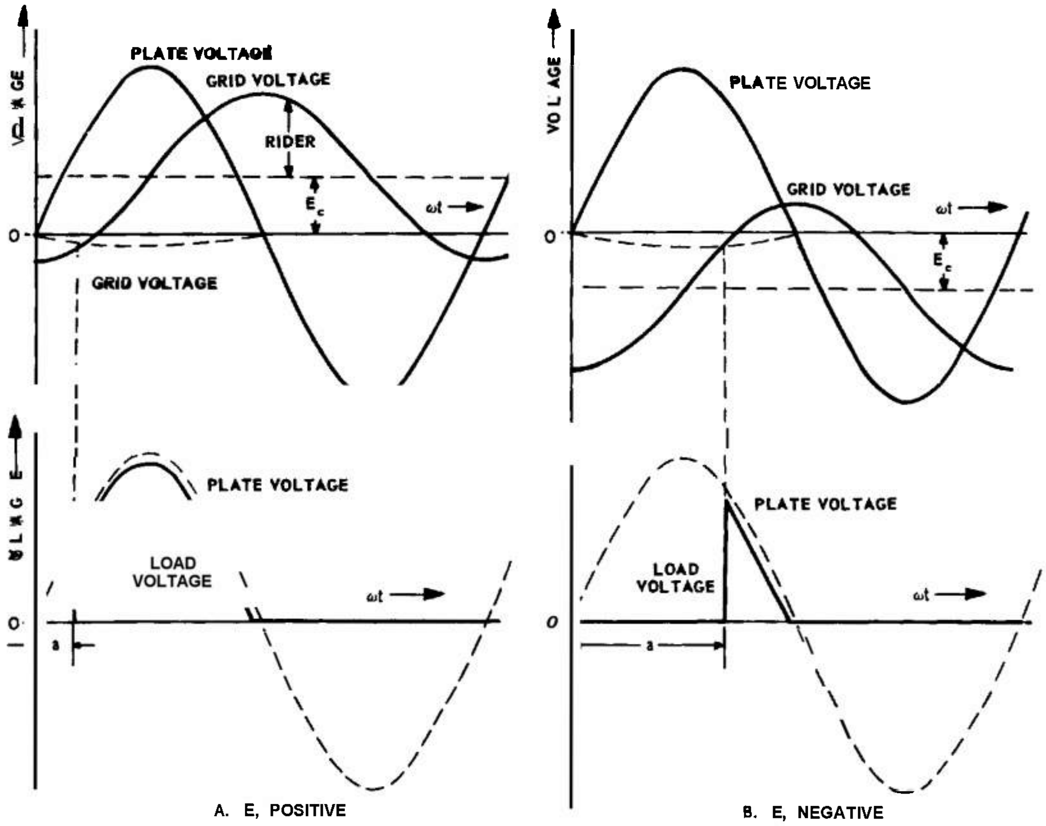


fig. 73-43 **Waveforms of** *circuit in fig. 73-42.*

the phase angle of which (with respect to the plate voltage) can be adjusted by external means. **A** simplified circuit of this type is shown in Fig. **13-44.** The phase angle of the alternating grid voltage may be varied by means of various phase-shift circuits and devices, depending upon the application.

Control schemes furnishing a steep voltage pulse to fire the thyratron offer certain advantages in some applications. These pulses may be produced by electronic circuits or by magnetic devices such as pulse transformers. Another control scheme in this general category employs a magnetic amplifier to provide pulse signals, the phase angle of which (with respect to the plate voltage) varies in response to the control current of the magnetic amplifier.

13-1.57 Thyratron-Amplifier loads

Thyratrons can be used to supply loads consisting of resistive, inductive, or capacitive elements, either singly or in combination. If a d-c output is required, the thyratrons can be connected as single-phase **or** polyphase rectifiers of either the half-wave, full-wave, or bridge type. The back-to-back connection is used if an a-c load, such as the control winding of a servomotor, is to be supplied. Some of these circuits are shown in Fig. **13-45.**

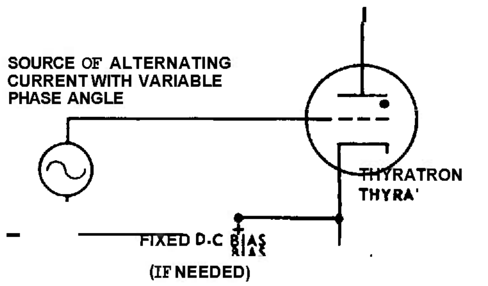


Fig. 13-44 Control of a thyratron by means of a phase-variable a-c signal.

13-1.58 Resistive loads. The load-voltage waveforms found in multitube circuits with resistive loads can be obtained by methods similar to those used to obtain the waveforms in Fig. **13-43.**

13-1.59 Inductive loads. If the load is inductive, the picture is somewhat more complicated. When the firing angle is very large, the load current is very small because, during any conduction period, the load inductance prevents build-up of the load current as shown in Fig. **13-46. As** the firing angle is reduced, the load current increases slowly until the point is reached at which the load current supplied by one tube persists until the next tube

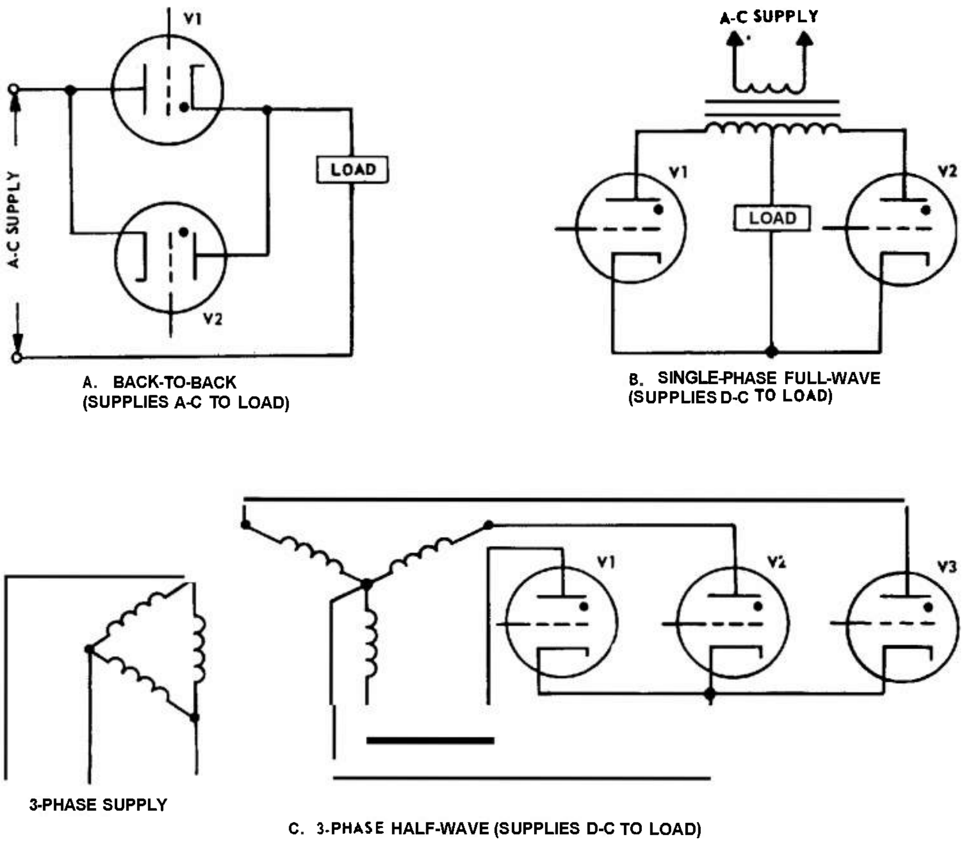
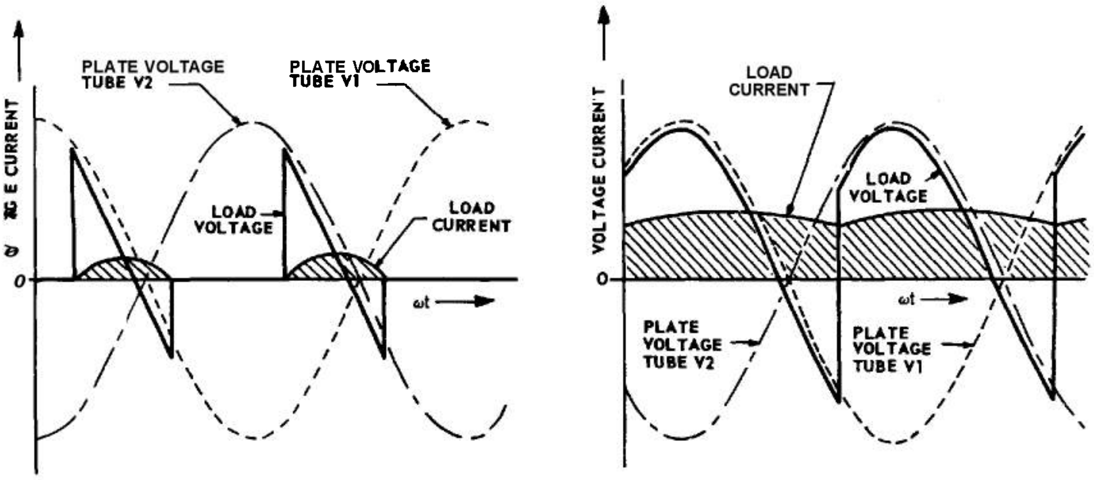


Fig. 73-45 Plate and load connections of typical thyratron amplifiers.



A. FIRING ANGLE GREATER THAN 90”

B. FIRING ANGLE LESS THAN 90°

Fig. 73-46 Load voltage and current supplied by a single-phase full-wave thyratron amplifier to a highly inductive load.

begins to conduct. When this point is reached, conduction is nearly continuous and the average value of the load current, being limited only by the load resistance, increases rapidly as the firing angle is further reduced. Continuous conduction is possible in 3-phase half-wave circuits when the firing angle is somewhat less than 120°. In single-phase full-wave circuits, the firing angle for continuous conduction must be less than 90°. Continuous conduction is impossible in single-phase half-wave circuits.

If gas-filled thyratrons are used to supply inductive loads by continuous conduction, provision must be made for the prevention of gas “cleanup” due to bombardment of the plate by positive ions.[12] This bombardment takes place at the end of each conduction period, at which time a large negative voltage is applied to the plate while there are still positive ions present within the tube envelope. Manufacturers’ instructions should be followed when applying gas-filled thyratrons to inductive loads.

13-1.60 Battery, capacitive, and separately excited d-c motor loads. The back voltage caused by a battery, by a charge on a capacitor, or by motor counter-emf prevents current flow until the plate voltage exceeds the back voltage. Once conduction has started, current is limited by the series impedance present between the source of alternating power and the back voltage. Conduction continues until the plate voltage drops to a value approximately equal to that of the back voltage. Conduction ceases at this point, unless an inductive element is present, in which case conduction continues until the energy stored in the inductive element is dissipated.

When an inductive element is present in the impedance between the source of alternating power and the back voltage, conduction may or may not be continuous, depending upon the magnitude of the inductive element, the magnitude of the back voltage, and the configuration of the thyratron amplifier.[13,16] Continuous conduction is particularly desirable in motor-control circuits since the speed regulation and transient response of the motor are much better with continuous conduction than with discontinuous conduction.

13-1.61 Dynamic Performance

The response of a thyratron-controlled device is generally determined by the time constants of the device. The response of most devices is not affected by the substitution of a thyratron amplifier for other sources of electrical power, except that a small random delay is introduced. The magnitude of this delay is roughly equal to the period of the supply frequency divided by the number of thyratrons used in the amplifier. Thus, a single-phase full-wave amplifier introduces a random time delay of about one-half cycle, a 3-phase half-wave amplifier introduces a delay of one-third cycle, etc. This delay is caused by the time difference between firing of successive thyratrons; i.e., once a thyratron is fired, a change in its grid voltage has no effect until the next thyratron is ready to conduct.

13-1.62 Exception **to** time-constant rule. An important exception to the time-constant rule outlined above is the separately excited d-c motor. As long as thyratron conduction is continuous, the response of the motor is determined by the mechanical and electrical time constants of the motor. However, the response becomes much more sluggish as soon as conduction becomes discontinuous. This is because the discontinuous nature of the conduction makes it impossible to build up the armature current to the value required for rapid acceleration. Regenerative braking for rapid deceleration is impossible because the rectifying action of the thyratron amplifier prevents a reversal of the armature current. Discontinuous conduction, therefore, leads to a much slower response than would be expected from the mechanical and electrical time constants of the motor. This effect is discussed in detail in the article by Chin and Walter.[14]

13-1.63 D-C POWER SUPPLIES FOR ELECTRONIC AMPLIFIERS

A d-c power supply converts an a-c input voltage into a unidirectional voltage which, after filtering and smoothing, becomes the desired d-c voltage. D-c power supplies are used primarily to furnish tube plate and screen voltages as well as control-grid bias when self-bias is not used.

13-1.64 Types of Rectifiers

Diodes or thyratrons are commonly used as rectifiers in d-c power supplies. Many low-current power supplies, however, use semiconductor devices such as junction diodes or selenium rectifiers because they are smaller, require no filament voltage, and last longer. Thyratrons and gas diodes are used where load currents are heavy; i.e., in the order of amperes.

13-1.65 Power-Supply Circuits

Circuit schematics of the two most common diode-rectifier configurations are given in Figs. 13-47 and 13-48. The advantages of the

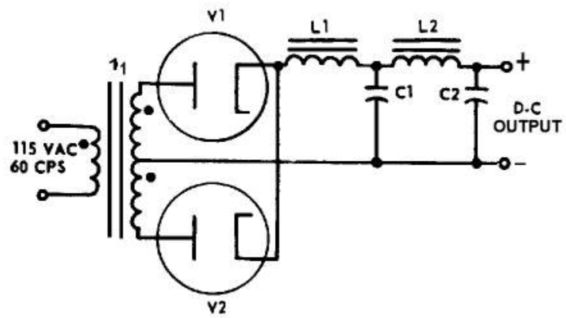


fig. 13-47 Full-wave rectifier with typical L-C filter.

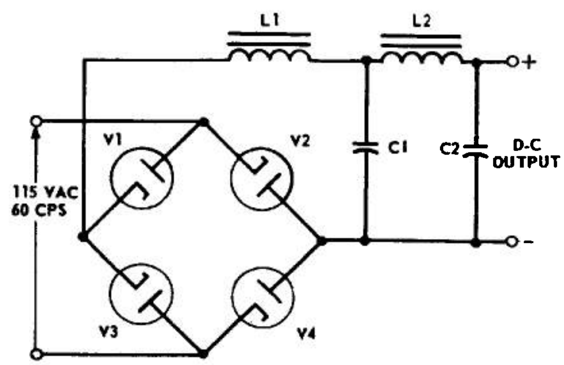


Fig. 13-48 Bridge rectifier with typical L-C filter.

bridge rectifier over the standard full-wave rectifier are as follows:

(a) A transformer is not required to provide d-c isolation between the a-c input and the output. (A transformer is necessary only if a voltage step-up is required.)

(b) The peak inverse (negative) voltage across each rectifier is one-half that in a full-wave circuit. The one disadvantage of the bridge circuit is that twice as many rectifiers are required.

13-1.66 Design of D-C Power Supplies

The design of rectifier-filter combinations is given detailed treatment in many publications.[5,6] In specifying a power supply, the following characteristics are of interest:

(a) Load voltage and current.

(b) Regulation. *Load regulation* is a measure of the steady-state change in output voltage due to a change in load current from zero to full load, with the a-c line voltage constant at the design center. *Line regulation* is a measure of the steady-state change in output voltage due to a change in a-c line voltage from minimum to maximum specified value, with load current held constant at 50 percent of the specified rating. Regulation is usually expressed as a percentage R, where

$$R = \frac{\Delta E_o}{E_o} \times 100 \qquad (13\text{-}45)$$

(c) Output-voltage ripple. This term describes the time-variant component of the output voltage. Ripple usually has a complex waveform and is specified either as the peak-to-peak value or as the rms ripple, which defines the "heating value" of the ripple component.

(d) Internal or output impedance. Output impedance $Z_o(s)$ relates dynamically the change in output voltage caused by a change in output current and is given by

$$Z_o(s) = -\frac{\Delta E_o(s)}{\Delta I_o(s)} \qquad (13\text{-}46)$$

where

$\Delta E_o(s)$ = Laplace transform of the change in output voltage

$\Delta I_o(s)$ = Laplace transform of the change in output current

The magnitude of the output impedance is a function of the frequency of the load variations and, because power supplies are nonlinear devices, is also a function of the magnitude of these variations and the center value around which they are taken.

Regulation, ripple, and internal impedance are largely determined by the type of filter used. These characteristics can be improved (i.e., lower percentage regulation, ripple voltage, and output impedance) by inserting a regulator between the rectifier-filter combination and the load, as shown in Fig. **13-49.** In most practical regulated power supplies, the reference voltage is derived from the plate-to-cathode voltage drop of a gas-discharge tube called a *voltage-regulator* tube. This voltage drop is nearly constant and is continually compared with the regulator output voltage. The drop is usually a fraction of the output voltage because practical voltage-regulator tubes have voltage drops lower than the required regulator output voltage. The resultant difference voltage, or error, is used to modify the voltage drop in the regulator in such a way that the output voltage remains nearly constant in the presence of input and load variations. For high-precision applications, standard cells are used as reference elements.

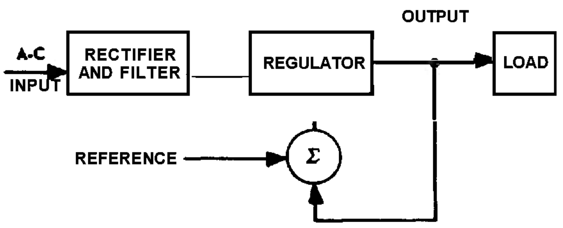


fig. 73-49 Block diagram of regulated power supply.

13-1.67 Typical electronic regulator. The circuit schematic of a typical electronic regulator is given in Fig. **13-50,** where V1 is the error amplifier, V2 is the series regulator tube, and V3 is the voltage-regulator tube. Because the full load current flows through V2, and because V2 has a plate-to-cathode voltage of at least a few tens of volts, the plate dissipation of V2 is quite high. Where the full load current is appreciable, several tubes may be connected in parallel, each tube dissipating (wasting) considerable power. If the circuit of Fig. **13-50** is assumed to operate in its linear range, the change in output voltage caused by a change in input voltage is

$$\frac{dE_o}{dE_{in}} = \frac{\mu_2 A + 1}{1 + \mu_2(1 + \beta K) + \dfrac{Ir_{p2}}{E_o}} \qquad \textbf{(13-47)}$$

where

$$A = \frac{r_{p1}}{r_{p1} + R_3}$$

$$\beta = \frac{R_2}{R_1 + R_2}$$

$$K = \frac{\mu_1 R_3}{r_{p1} + R_3}$$

E_o = load voltage

I = current through series regulator tube

The smaller dE_o/dE_{in} is made, the better the regulation and the smaller the output-voltage ripple. Values of dE_o/dE_{in} as low as 0.0005 can be achieved, and ripple voltages of only a few millivolts are common. For a regulator connected to a filter with output impedance $Z_F(s)$, the output impedance of the combination becomes approximately

$$Z_o(s) = \frac{dE_o}{dE_{in}} Z_F(s) + \frac{r_{p2}}{1 + \mu_2(1 + \beta K)} \qquad \textbf{(13-48)}$$

In practical circuits, the magnitude of the over-all output impedance is made considerably less than the magnitude of the filter output impedance. The latter may be as high as a few hundred ohms, while the former may be made as low as a few tenths of an ohm, so that the addition of the regulator results in a substantial improvement in load regulation. Regulated power supplies with **0.1** percent regulation are not unusual.

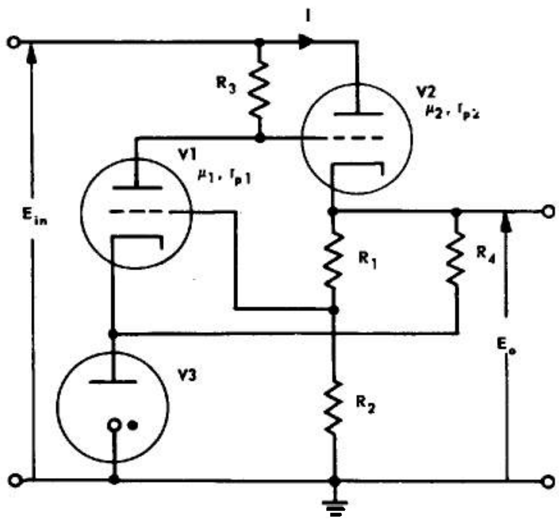


Fig. 13-50 Circuit schematic of series regulator.

13-2 TRANSISTOR AMPLIFIERS*

13-2.1 BASIC PRINCIPLES(17)

The principles of semiconductor electronics are based on quantum theory and the structure of solids —subjects that are beyond the scope of this publication. However, the electrical properties of a transistor are quite similar to those of a temperature-limited vacuum-tube diode, and this simple analogy gives a good qualitative picture of the nature of a transistor.

13-2.2 Operating Characteristics of Temperature—Limited Vacuum Diode

If a temperature-limited vacuum-tube diode is connected to a load as shown in Fig. 13-51, its operating characteristics are as follows:

(a) For small input signals, the plate current depends primarily upon the filament power and is practically independent of plate voltage.

**By R.D. Thornton (Par. 13-2.42 by R.E. Marshall)*

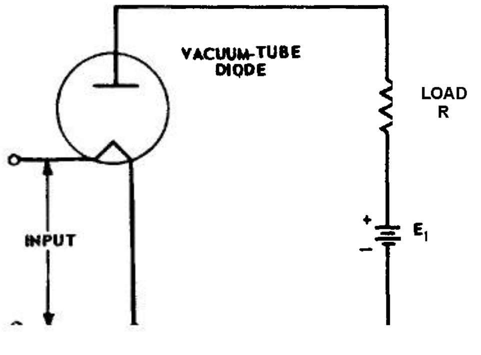
(b) If enough power is supplied to the filament, the diode saturates, and its current-voltage characteristic is equivalent to a low impedance that is practically independent of the input signal.

(c) The input impedance is relatively low and also highly nonlinear. An appreciable amount of power is required to bias the device into the amplification region, and the plate current depends exponentially upon filament temperature.

(d) The diode is likely to have poor frequency response for three reasons:

(1) The thermal capacity of the filament makes it difficult to change the temperature suddenly. The physical dimensions of the filament must thus be small for fast response.

(2) If a large input signal is supplied, the filament will emit more electrons than can be collected by the plate, thus driving the diode into saturation. As a result, there will be an accumulation of excess electrons (space



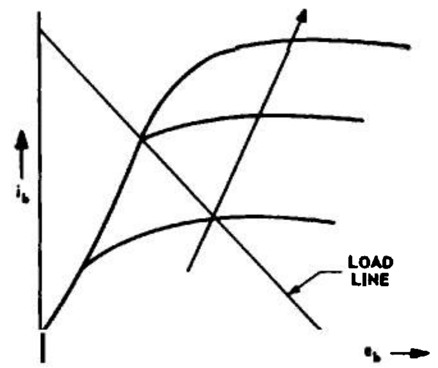


Fig. 13-51 Temperature-limited diode amplifier.

charge) near the filament. To turn the tube off, these excess electrons must be removed before the load current will be reduced. This space-charge effect is an important problem in large-signal applications, even if the thermal time constant is small.

(3) Interelectrode capacitance between the plate and the filament is similar to the capacity in multielement vacuum tubes. If the diode is to be operated at high frequencies, this capacitance is a factor that must be considered since it limits the use of the tube above certain frequencies.

(e) The voltage, current, and power that can be handled by the diode are limited by the emission capabilities of the filament, the breakdown voltage between the filament and the plate, and the thermal properties and maximum allowable temperature of the plate.

13-2.3 Transistor Operation

In a transistor, electrons are in a normally bound state within the body of the solid until an input signal is supplied that has sufficient energy to free the electrons and make them available for conduction. Temperature, light, electric fields, or even X-rays can be used to supply this energy. In a semiconductor, it is possible to conduct current with either positive or negative charge carriers (holes or electrons). Thus, the equivalent of a positron tube, as well as an electron tube, can be built. It is also possible to avoid the space-charge limitations common to most vacuum-tube devices.

13-2.4 Advantages and disadvantages of transistors. The obvious advantages of a transistor are:

(a) Small size
(b) Low power consumption
(c) No filament or heater power required
(d) No warm-up time required
(e) Good ruggedness and reliability

There are also a number of more subtle features of transistors that merit consideration when designing servo-system amplifiers and control devices. For low-power applications, the transistor produces very low vibrational noise and is free from many of the problems of electromagnetic pickup and hum that are troublesome in vacuum-tube circuits. In properly designed circuits using low-noise transistors, the noise above about 500 cps may be less than that of an equivalent vacuum tube. In fact, noise figures as low as 4.5 db at 1 kc have been reported. Below 500 cps, however, the transistor becomes more noisy until, at normal servo frequencies, the noise is large enough to interfere with proper operation. For this reason, there is a marked advantage in using a carrier-amplifier system, the carrier frequency being higher than normal power-line frequencies for best results. Transistor circuits are so simple and small that the additional difficulty of designing and building a local oscillator to generate the carrier is small by comparison with the benefits gained in the form of reduced noise, small physical size, and low power requirements.

13-2.5 High-frequency operation. Operation of transistors at radio frequencies presents a number of problems with which the servo-system designer is not concerned; hence, these problems will not be discussed here. The interested designer may refer to several good texts on the subject.[18,19,22]

13-2.6 Medium-frequency operation. For medium audio frequencies (300 to 3000 cps) and for medium power levels from 10^{-9} watts to 10 watts, the transistor is, at present, unexcelled in almost all amplifier applications. For example, it is possible to construct a capacitor-coupled transistor amplifier with more than 40 db power gain per stage; in fact, a two-stage transistor amplifier will often prove superior to three or more vacuum-tube stages. The emitter bias power of a transistor is analogous to the filament power of a vacuum tube, but it may be a thousand times smaller and therefore does not represent an appreciable power drain. Thus, the fact that transistors can operate with only a few microwatts of bias power make it practicable to consider battery-operated equipment for continuous duty. For instance, a properly designed two-stage transistor amplifier will operate on less than 100 microwatts of bias power, providing about five years of continuous operation from a single mercury cell. The very small physical size of transistor amplifiers makes it possible to mount them directly on control equipment. For low-power signals, amplification can be provided at the control-signal source, thus avoiding the necessity of transmitting the low-power signals over long lengths of shielded cable. Some work has also been done with transistors in unusual applications, such as providing commutating action in the armature circuit of a d-c motor, thus eliminating mechanical commutation.

13-2.7 High-power applications. Originally, transistors were limited in power applications because they were not capable of dissipating more than about 50 milliwatts of power. However, rapid progress has been made in the power-transistor field in recent years. There is every reason to expect that, while present-day power transistors are capable of up to about one kilowatt of output, future power transistors with an output of tens of kilowatts will be available. It will then be possible to replace magnetic amplifiers with transistor amplifiers, with an attendant substantial reduction in over-all size and weight but without a loss in ruggedness and reliability.

13-2.8 Switching applications. The fine switching properties of transistors make transistors ideally suited for modulators, demodulators, and on-off control systems. In addition, the application of transistors to digital computers is now widespread. If a digital computer is to be included as part of a servo system, transistors should definitely be considered in the design of both computer and servo system.

13-2.9 Summary. The similarities and differences between transistors and vacuum tubes must be borne in mind at all times when designing a transistorized servo system. It is not enough to replace conventional vacuum-tube circuits with the nearest equivalent transistor circuits. The characteristics of the transistors must also be considered. Servo systems are particularly suited to the application of transistors, regardless of system size or power range.

13-2.10 BASIC THEORY OF JUNCTION DIODES AND TRANSISTORS

13-2.11 Electron Current

The ability of a metal to conduct current depends upon the existence of charge carriers (electrons) and the ability of these carriers to move when acted upon by an external field. In a normal metal, there is one free electron for each atom of the metal. When an electric field is applied, these free electrons move with an average velocity that is proportional to the strength of the applied field. The resistance of the metal is linear for small currents and is fairly low because of the large number of free electrons. In a semiconductor, there are relatively few free electrons and the resistivity is comparatively high—up to 47 ohm-centimeters for germanium and 63,000 ohm-centimeters for silicon at 25°C. Moreover, the number of current carriers in a semiconductor depends exponentially upon temperature. Thus, the resistivity becomes very high for temperatures near 0°K.

13-2.12 Hole Current

In addition to the electron current in a semiconductor, there is also a so-called *hole* current. The hole is a fictitious particle that has no counterpart in the simple vacuum tube; it is only a concept and does not actually exist. The hole concept was devised as a means of simplifying the analysis of a semiconductor and has a very simple analog; namely, if a large number of objects in a periodic array are to be counted, it is often easier to count the number of empty sites than to count the objects actually present. The hole, therefore, represents the lack of an electron and behaves much as a positively charged electron might behave. In the pure semiconductor, there are an equal number of holes and electrons, and the product of the hole and electron concentrations depends exponentially upon the temperature of the solid. However, by "doping" (adding impurities or alloys), the total number of charge carriers can be controlled, even though the product of hole and electron densities is essentially independent of impurity concentration. By controlling the total number of charge carriers, the resistivity can be controlled and amplification is possible.

13-2.13 Material Types

The semiconductor crystal, as a whole, displays charge neutrality; i.e., the total number of electrons is almost exactly balanced by the nuclear charge of the crystal atoms. If there are more free electrons than holes, the material is called *negative* or *N-type*, while if the reverse is true, the material is *positive* or *P-type*. In N-type material, the electrons are the majority carriers and the holes are the minority carriers. In P-type material, the electrons are the minority carriers and the holes are the majority carriers.

13-2.14 Junctions

If a sharp junction is formed between a mass of P-type material and a mass of N-type material, rectification is possible. This is demonstrated as follows: If a battery is connected to the junction so that holes are driven from the P region to the N region, the junction resistance in this direction is low, and current can flow because there are many carriers able to move across the junction; if the polarity of the battery is reversed, relatively few carriers can move across the junction, the resistance in this direction is high, and little current can flow.

13-2.15 Junction Diodes

Ideally, a junction diode would be described approximately by the relationship

$$i = I_s(e^{v}/v_o - 1) \tag{13-49}$$

where

i = diode current, in amperes

I_s = characteristic of diode

e = base of natural logarithms = 2.718

v = diode voltage, in volts

$V_o = kT/q$ = 0.025 volt at 20°C

k = Boltzmann's constant = 1.37×10^{-23} joule/°K

T = absolute temperature, in °K

q = charge on electron = 1.6×10^{-19} coulomb

Various other effects contribute series resistance, back-leakage current, and voltage breakdown, so that the v-i characteristics of a typical junction diode are as shown in Fig. 13-52. Three important characteristics of the junction diode should be noted because of their importance in transistor operation:

(a) The back current of a junction diode varies exponentially with temperature and may double for every 10 or 11 degrees increase in junction temperature. Figure 13-52 shows the typical temperature dependence of a silicon junction diode.

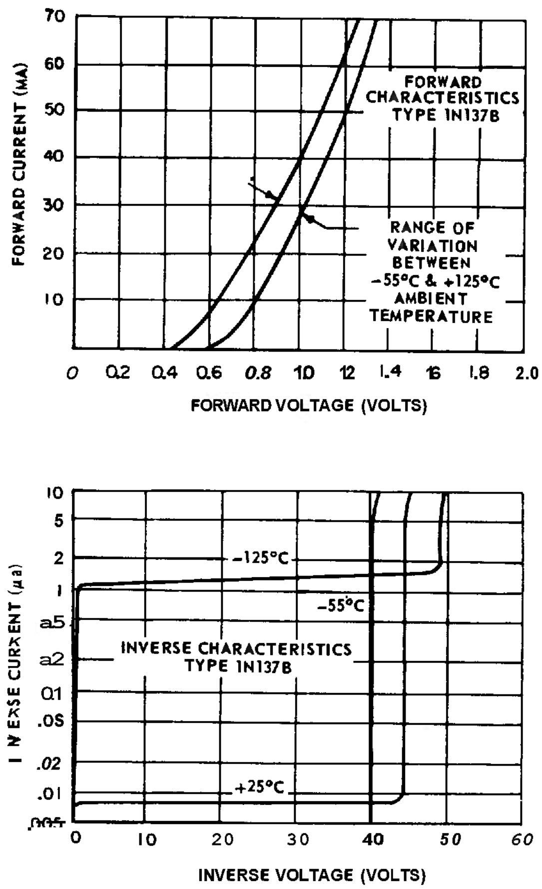


Fig. 13-52 Typical characteristics of Type 7N1370 silicon junction diode.

Courtesy Transitron Electronic Corporation.

(b) The forward current depends exponentially upon the forward voltage and increases by a factor $e = 2.718$ for every kT/q increase in applied voltage. For very large currents, the junction diode behaves approximately like a small resistance in series with a battery. For some silicon junction diodes, the forward current increases as $e^{v}/2v_o$ and for high-voltage units the forward characteristics may become quite complicated. Some diodes also exhibit a forward characteristic similar to the space-charge-limited characteristic of a vacuum-tube diode. In general, however, junction diodes conduct a much larger current for a given voltage drop than is practicable with vacuum-tube diodes.

(c) A junction diode will break down when back voltages become large enough. This breakdown may be temperature-induced or it may be the result of a Townsend discharge (sometimes called Zener breakdown), and it imposes limitations on the permissible levels of back voltage. The Townsend discharge may be used as a voltage reference similar to a gas-tube regulator; however, for transistors, it is normally undesirable.

13-2.16 Junction Transistors

A junction-type transistor is little more than a junction diode with provisions for increasing the number of minority carriers in the vicinity of the junction. Since the back current of a junction diode depends primarily upon the minority-carrier concentration, it is possible to control the back current of the junction diode much as the forward current is controlled in the temperature-limited vacuum diode. Figure 13-53A shows schematically the construction of a PNP transistor in which the minority carriers in the N region are controlled by injecting majority carriers from the adjacent P region. Figure 13-53B shows an NPN transistor in which the minority carriers in the P region are controlled by injecting electrons from the adjacent N region. The characteristics of the PNP and NPN transistors are similar, but all voltage and current polarities are opposite. The actual construction of practical transistors differs from this schematic picture, but the principle is the same; namely, the back current of a junction diode is controlled by controlling the number of minority carriers in the vicinity of the junction.

13-2.17 Special transistor types. Special types of transistors, such as tetrodes, double-base drift and field effect transistors have been proposed and constructed. Since most of them were specifically designed for high-frequency operation, they will not be discussed here. Some of the newer power transistors have two base connections (on opposite sides of the base region) by which the minority-carrier concentration is controlled. By passing a bias current through the base region, it is possible to improve the temperature stability, increase the gain, and/or make the amplification more linear. Some transistors also use an intrinsic or I-region between the P and N regions to increase the breakdown voltage. In I-material, the number of holes approximately equals the number of electrons. Although other types probably will be designed and manufactured, it is likely that many will use the junction diode as a basic building block. For this reason, the properties of the junction diode should be fully understood by the transistor-circuit designer.

13-2.18 Characteristics of transistor materials. Although the most common present-day transistor materials are germanium and silicon, newer alloys should be available in the near future. Germanium transistors are more frequently used than other types because a simplified refining process has made the germanium type less expensive and much easier to produce. However, germanium has a relatively low resistivity (compared with other semiconductor materials), thus permitting diode back currents to become significant at temperatures of 50°C or less. In addition, the indium-alloy construction commonly used with germanium limits the permissible temperature rise in the junction because of the low melting point of indium. At present, silicon is more expensive than germanium, but this differential will probably be reduced by improved processing methods. Silicon has a relatively high resistivity; it appears that a desirable value is probably somewhere between that of germanium and silicon. However, silicon diodes can operate at junction temperatures as high as 200°C with the same back current that would be present in a similar germanium diode at 50°C. For this reason, silicon diodes are almost always required when the operating temperatures are expected to be in excess of 75°C. The forward voltage drop of a silicon diode is larger than that of a similar germanium diode, thereby making the silicon diode inferior to the germanium diode in switching characteristics at room temperatures. The low reverse-leakage current of a silicon diode, however, may be a big advantage if very high impedance levels are required in the amplifier circuit.

13-2.19 ANALYSIS OF TRANSISTOR CHARACTERISTICS

The low-frequency characteristics of a transistor are best described by a plot of collector current versus collector-to-emitter voltage, with the base current as a parameter. Typical plots for a low-power PNP germanium unit and a medium-power NPN silicon unit are shown in Fig. 13-54. The PNP curves are similar to the NPN curves, except that all voltage and current polarities are opposite.

13-2.20 Transistor Model

For analytical purposes, it is convenient to represent the transistor by an approximate model; a typical large-signal model is shown in Fig. 13-55. A simplified theory of transistor

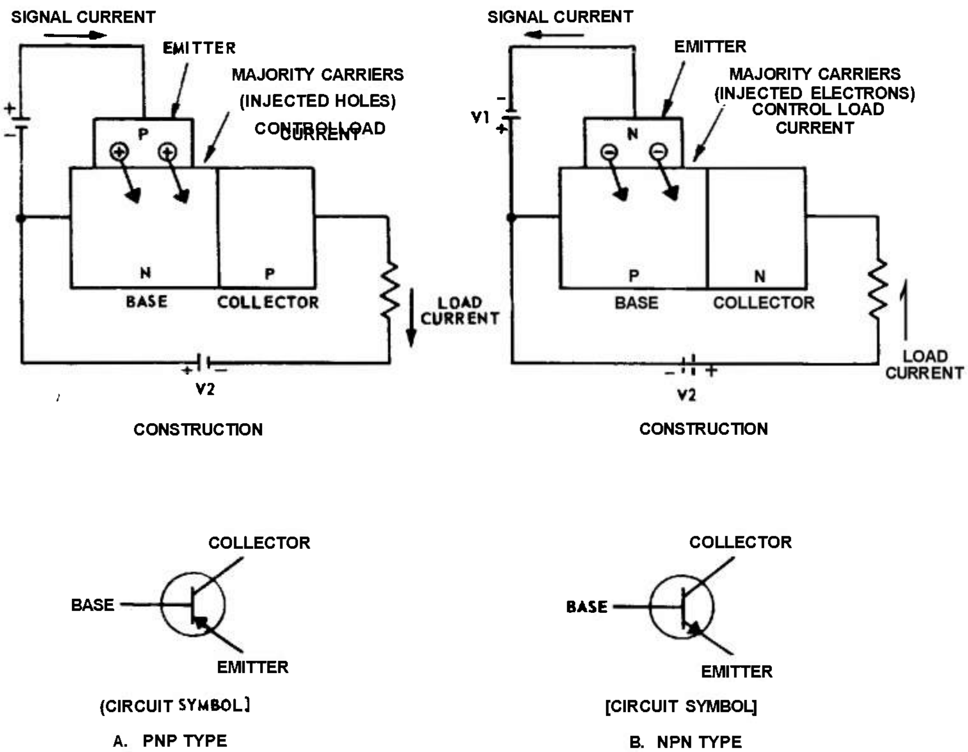
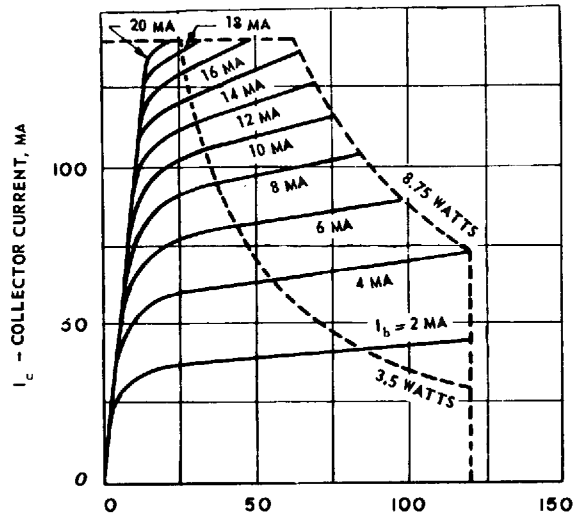
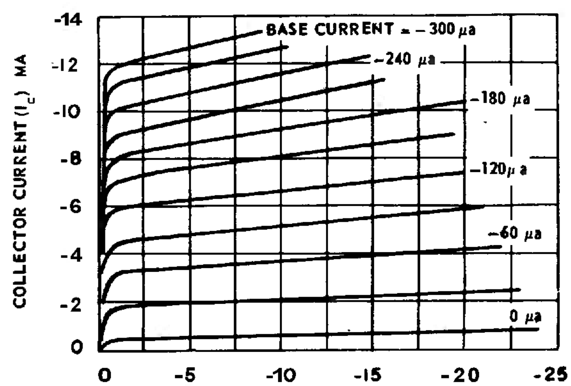


Fig. 13-53 Types of junction transistors.



V_c –COLLECTORVOLTAGE–VOLTS
COMMON EMITTER OUTPUT CHARACTERISTICS
A. TYPE 970 SILICON – NPN

GROUNDED EMITTER
TYPICAL COLLECTOR CHARACTERISTICS



COLLECTOR TO EMITTER VOLTAGE (V_{ce})
THIS FAMILY IS A FUNCTION OF 1-a AND THUS CHANGES APPRECIABLY WITH SMALL CHANGES IN a
B. TYPE 2N64 GERMANIUM – PNP

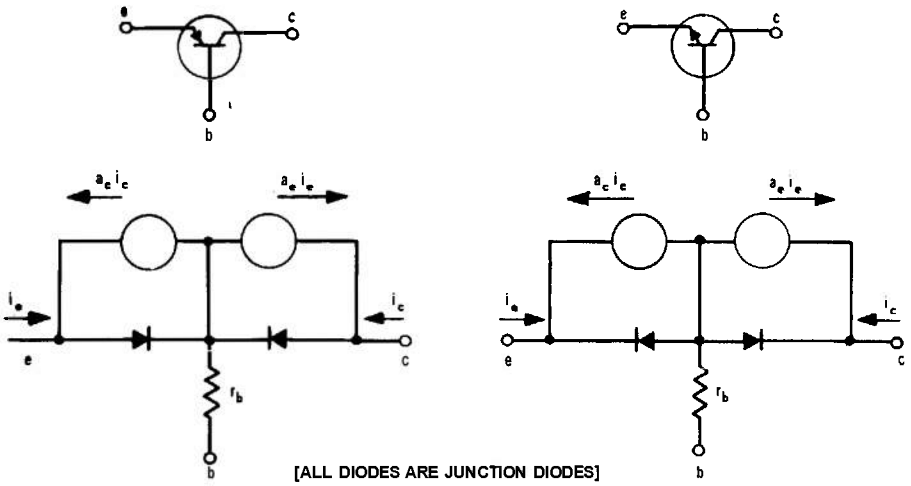
Fig. 13-54 Typical collector characteristics.

(Type 970 characteristics courtesy *Texas Instruments Incorporated;* Type 2N64 characteristics courtesy *Raytheon Manufacturing Company.*)

operation predicts that this model is a good approximation for small-to-medium signals. The two diodes in this model are junction diodes, with characteristics as discussed previously, and the transistor action is represented by the two dependent current generators $a_c i_c$ and $a_e i_e$. For normal transistor operation, the emitter diode is closed and short-circuits the $a_c i_c$ generator, thus making the $a_e i_e$ generator the important gain-producing element in the transistor. Some circuits, however, use the reverse-current gain (due to the $a_c i_c$ generator) and it often becomes important in large-signal applications. In the output characteristics of Fig. 13-54, the a, generator produces the amplification in the regions shown. Typical numerical values of a, (often called just a and very nearly equal to the short-circuit current gain a) are 0.95 to 0.995, while a_c is often less than 0.5. Special symmetrical transistors have been made with both a_e and a_c greater than 0.95.

13-2.21 Piecewise linear model. For analysis of large-signal circuits, the model of Fig. 13-55 is often used, but with the diodes replaced by an appropriate piecewise linear model. For high-power applications, the voltage drop of the forward-biased diode is of major importance while, for low-power circuits, the current in the back-biased collector diode is important and produces biasing problems that will be discussed later.

13-2.22 Incremental model. For a small-signal incremental analysis, the emitter diode is usually closed and may be replaced by a small resistance r_e, while the collector diode is back-biased and may be replaced by a large resitance r_c. Based on these approximations, the incremental model of Fig. 13-56A is appropriate and has a direct physical interpretation. From a circuit point of view, however, the model of Fig. 13-56B is more useful because the transistor is most often used in a grounded-emitter circuit. Numerous other types of models have been **proposed,**[2] each with an advantage for some particular application.

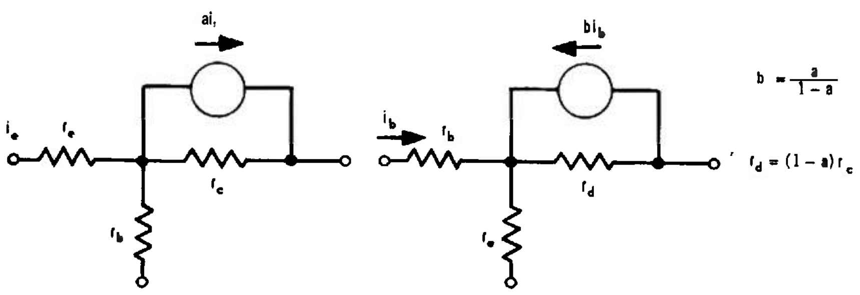


A. PNP TWO DIODE MODEL

B. NPN TWO DIODE MODEL

e = EMITTER
c = COLLECTOR
b = BASE
i_e = EMITTER CURRENT
i_c = COLLECTOR CURRENT
r_b = BASE RESISTANCE
$a_c\ i_c$ = DEPENDENT CURRENT GENERATOR
$a_e\ i_e$ = DEPENDENT CURRENT GENERATOR

Fig. 13-55 Approximate junction transistor model,



A. COMMON-BASE MODEL

B. COMMON-EMITTER MODEL

Fig. 73-56 Incremental transistor models.

13-2.23 Hybrid Parameters

The problem of determining the incremental parameters from measurements has led to a system of hybrid parameters. These parameters, plus associated circuit models, are shown in Fig. **13-57.** (There is a marked degree of nonuniformity in notation in transistor literature, various authors using different symbols for the same parameters.) These h (hybrid) parameters are probably the easiest to measure experimentally. They can also be determined directly from the slopes and intercepts of the input and output v-i characteristics, as in the case of a vacuum tube. Table **13-2** relates the h parameters to r_b, r_c, r_e, and a.

13-2.24 Frequency Dependence

For most servo applications, the frequency dependence of a transistor's parameters is not too important, but it does deserve mention because, for high-power transistors, it may be significant at frequencies as low as **1 kc.** Three effects are especially important:

(a) ***Alpha cutoff.*** At high frequencies, the current gain a decreases because of the time required for current carriers to diffuse across the base region. Current gain can usually be approximated analytically by the relation

$$a = \frac{a_0}{1 + j\omega T} \tag{13-50}$$

$$\text{alpha cutoff frequency} = f_{co} = \frac{1}{2\pi T} \tag{13-51}$$

For grounded-emitter applications, the current gain is approximately

$$\beta \approx b \equiv \frac{a}{1-a} = \frac{a_0}{(1-a_0) + j\omega T} \tag{13-52}$$

and, since $(1 - a_0) << 1$, the effect of alpha cutoff will become important for frequencies well below the cutoff frequency f_{co}. The grounded emitter cutoff frequency is approximately $(1 - a)f_{co}$.

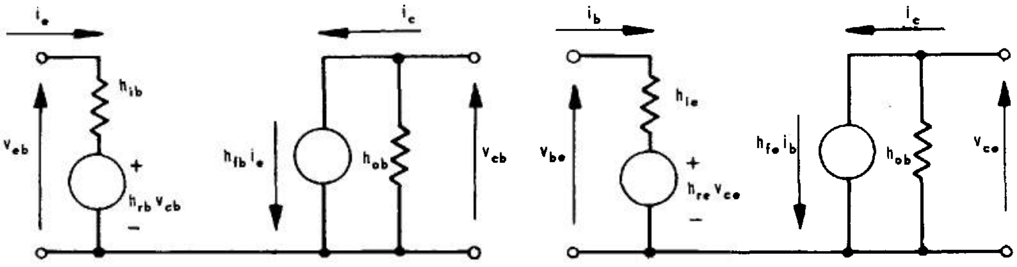
(b) *Collector capacitance.* The large collector resistance r_c is bypassed by a capacitor C_c, usually called the ***collector capacitance***; this capacitance is similar to the plate capacitance of a vacuum tube. For audio frequencies, however, C_c is usually less important than f_{co}. There is also an effective diffusion capacitance in parallel with r_e, but the $C_e r_e$ time constant is normally quite small.

(c) ***Hole and electron storage.*** The storage of current carriers in a transistor is a nonlinear effect and cannot be expressed simply in terms of a circuit model. The effect is similar to the transit-time electron storage in a vacuum tube, and its importance will be discussed in connection with transistor circuits.

TABLE 13-2 RELATIONS BETWEEN INCREMENTAL PARAMETERS

Common base	$h_{ib} = r_e + (1-a)\dfrac{r_b r_c}{r_b + r_c}$, $h_{ob} = \dfrac{1}{r_c + r_b}$, $h_{rb} = \dfrac{r_b}{r_b + r_c}$, $h_{fb} = -\dfrac{r_b + a r_c}{r_b + r_c}$ $r_e = h_{ib} - \dfrac{(h_{fb}+1)h_{rb}}{h_{ob}}$, $r_b = \dfrac{h_{rb}}{h_{ob}}$, $r_c = \dfrac{1 - h_{rb}}{h_{ob}}$, $a = -\dfrac{h_{fb} + h_{rb}}{1 - h_{rb}}$
Common emitter	$h_{ie} = r_b + (b+1)\dfrac{r_e r_d}{r_e + r_d}$, $h_{oe} = \dfrac{1}{r_d + r_e}$, $h_{re} = \dfrac{r_e}{r_e + r_d}$, $h_{fe} = \dfrac{b r_d - r_e}{r_d + r_e}$ $r_b = h_{ie} - \dfrac{(h_{fe}+1)h_{re}}{h_{oe}}$, $r_e = \dfrac{h_{re}}{h_{oe}}$, $r_d = \dfrac{1 - h_{re}}{h_{oe}}$, $b = \dfrac{h_{fe} + h_{re}}{1 - h_{re}}$

$b \equiv \dfrac{a}{1-a}$, $r_d \equiv (1-a)r_c$; usually $h_{re} << 1$, h_{fe}; $h_{rb} << 1$, $-h_{fb}$



$$v_{eb} = h_{ib}i_e + h_{rb}v_{ce}$$
$$i_c = h_{fb}i_e + h_{ob}v_{ce}$$

A. COMMON BASE h PARAMETERS

$$v_{be} = h_{ie}i_b + h_{re}v_{ce}$$
$$i_c = h_{fe}i_b + h_{oe}v_{ce}$$

B. COMMON EMITTER h PARAMETERS

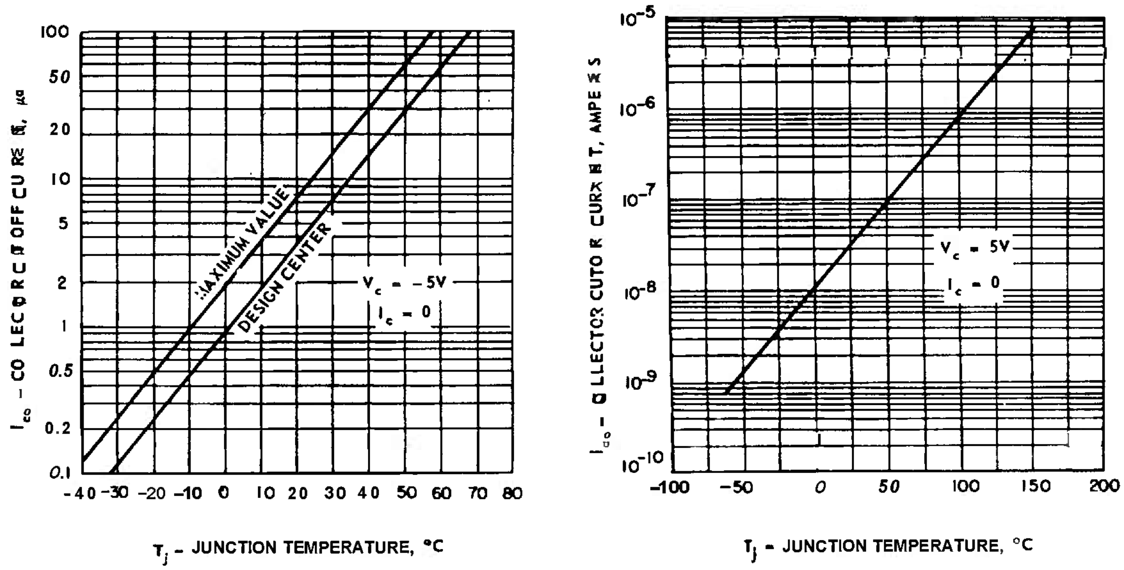
Fig. *73-57 Models of h-parameter transistor.*

13-2.25 Temperature Sensitivity

Most transistor parameters are significantly sensitive to temperature and, in servo application, temperature extremes are likely to be the most important limitations to the use of transistors. The parameter that is probably the most sensitive to temperature is the back current of the junction diode. This back current is relatively independent of the back voltage as long as the back voltage is in excess of about **0.5** volts. However, back current depends exponentially upon temperature and may double in value for every **10°C** increase in junction temperature. Since a part of the collector current is the back current of a junction diode, this current may become quite important for low-level amplifiers. The back current of the collector junction (for zero emitter current) is commonly called I_{co}, with a temperature dependance typified by Fig. 13-58. The incremental transistor parameters are sensitive to bias and temperature, with typical variations as shown in Fig. 13-59. **If** a circuit is to operate over a wide range of temperatures, it is almost imperative that stabilization of both bias and gain be provided.

13-2.26 Nonuniformity

Transistors of the same type vary from unit to unit because the manufacturers are unable to exercise precise control over production. This nonuniformity is likely to be much greater than in vacuum tubes unless the transistors are carefully selected units that conform closely to mean parameter values. Such selected units command premium prices. Therefore, most manufacturers specify permissible parameter ranges as well as *mean value* thereof. **It** is common procedure for a manufacturer to produce transistors according to a single process and then sort them for sale according to the value of β. Variations in $\beta \approx \alpha/(1-\alpha)$ are perhaps the most serious variations between transistors **of** the same type. In general, units having higher values of β are better transistors and command higher prices, especially in the power-transistor category. Table **13-3** shows the typical spread of characteristics for a low-power PNP transistor.

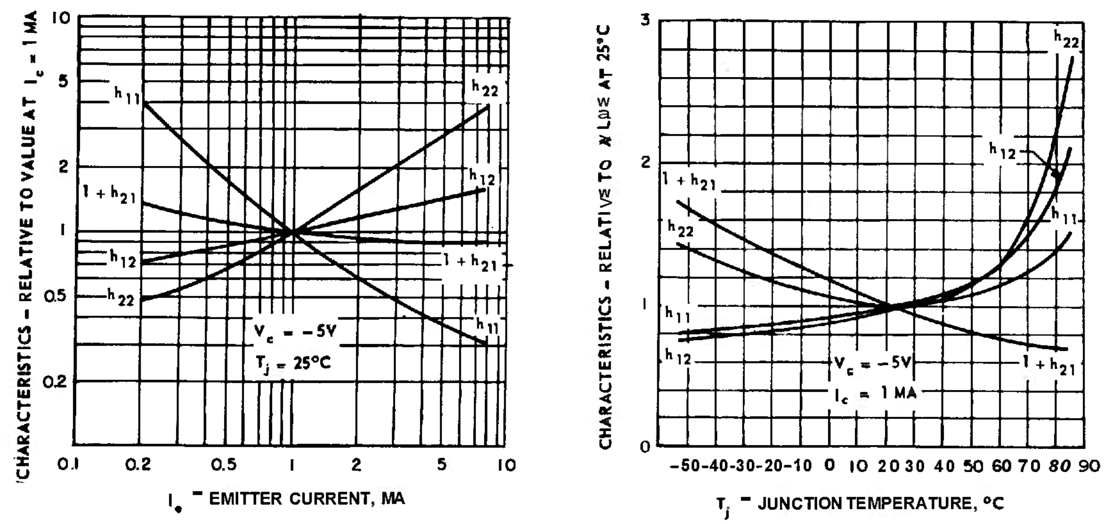


A. TYPE 301 - PNP - GERMANIUM

B. TYPE 904 - NPN SILICON

Fig. 73-58 I,, temperature dependence.

Courtesy Texas Instruments Incorporated.



COMMON BASE CHARACTERISTICS VS EMITTER CURRENT

COMMON BASE CHARACTERISTICS VS JUNCTION TEMPERATURE

TYPE 301 - PNP - GERMANIUM TRANSISTOR

Fig. 13-59 Variation of h-parameter with bias and temperature.

Court— Texas Instruments Incorporated.

TABLE 13-3 TYPICAL PARAMETER VARIATION FOR A LOW-POWER TRANSISTOR

Common-Base Design Characteristics at Tj = 25°C Type 301 Germanium PNP Transistor

Symbol	Description	Collector Voltage	Emitter Current	Min	Design Center	Max	Units
I_{co}	collector cutoff current	$V_c = -30V$	$I_e = 0$	—	8	30	μa
I_{co}	collector cutoff current	V, = — 5 v	$I_e = 0$	—	5	10	μa
h_{ib}	input impedance	$V_c = -$ 5v	$I_e = 1$ ma	25	33	50	ohms
h_{ob}	output admittance	$V_c = -$ 5V	$I_e = 1$ ma	0.30	0.71	1.5	μmhos
h_{rb}	feedback voltage ratio	$V_c = -$ 5v	$I_e = 1$ ma	200	425	1000	$\times 10^{-6}$
h_{fb}	current transfer ratio	$V_c = -$ 5v	$I_e = 1$ ma	0.950	0.973	0.980	—
β	beta, common emitter	$V_c = -$ 5V	$I_e = 1$ ma	19	36	49	—
NF	noise figure*, common emitter	$V_c = -$ 2.5V	$I_e = 0.5$ ma	—	22	35	db
f_{co}	frequency cutoff	V, = — 5 v	$I_e = 1$ ma	0.40	1.0	—	mc
C_{ob}	output capacitance	f = 10 kc		—	33	50	μμf

*Conventional noise – compared with 1000-ohm resistor, 1000 cps, and 1-cycle bandwidth.

This table is adapted from Bulletin No. DL-Sd11 (August, 1955), Texas Instruments Incorporated.

13-2.27 Noise Factor

Noise is likely to be greater in a transistor amplifier than in a vacuum-tube amplifier for frequencies below about 500 cps. Above about 1 kc, the two are comparable but, at frequencies approaching the alpha cutoff frequency of the transistors, the transistor noise increases again. If a choice is permitted, a transistor carrier amplifier should be operated at frequencies in the range from 1 to 10 kc, though 400 cps is acceptable. Special low-noise transistors are available with noise figures in the order of 3 db, but the'average transistor has a noise figure of more nearly 20 db. To achieve minimum noise in a transistor amplifier, the unit should be operated with extremely small biases, typical values being v, = 0.5 volt, $i_c = 0.1$ ma. The available gain is lower at these low biases but, if noise is important, this is a necessary compromise.

13-2.28 Microphonics and Vibration Effects

In many applications, thermal and shot noise are not as important as freedom from microphonic and vibrational noises. For this type of application, transistors offer decided advantages over vacuum tubes. Transistors are also less easily affected by stray magnetic and electrostatic fields and, since they have no filaments, power-frequency hum is virtually nonexistent.

13-2.29 Maximum Collector Voltage

For high-power applications, transistors are limited by a maximum allowable collector voltage. The maximum collector-voltage limitation may be due to several causes, and the maximum allowable peak voltage is likely to be considerably greater than the maximum allowable junction temperature, the thermal are usually described fully on manufacturers' specification sheets.

13-2.30 Maximum Power Dissipation

The maximum allowable power dissipation in a transistor is determined by the maximum allowable junction temperature, the thermal resistance from the collector junction to the chassis, and the chassis ambient temperature. If the maximum allowable junction temperature is T_j, the thermal resistance is R°C/watt, and the chassis temperature is T_c, then the maximum allowable power dissipation P is given by

$$P = \frac{T_j - T_c}{R} \tag{13-53}$$

If there is electrical insulation between the transistor and the chassis, the thermal resistance of the insulation must be included in R. It should be noted that the chassis temperature is not the same as the room temperature. For high-power transistors, it may be necessary to use forced-air cooling to maintain a reasonable T_c. For lower-power transistors that are not attached to a heat sink, the maximum power is often specified at a normal ambient-air temperature. For large-signal applications, it is important to know the thermal time constant of the collector junction. For high-frequency operation, the average power dissipation is the important item; for very low frequencies, it is the peak power that limits operation. The thermal time constant of the collector junction is used to establish the maximum peak junction temperature which, in turn, determines the maximum power dissipation.

13-2.31 TRANSISTOR AMPLIFIER CIRCUITS

13-2.32 Grounded-Emitter Amplifier

The grounded-emitter amplifier, shown schematically in Fig. 13-60A, is the most useful for servo applications because it provides both voltage and current gain and can therefore be cascaded without using coupling transformers. At low frequencies, h_{re} is usually negligible. Therefore, the input resistance is very nearly h_{ie}, the short-circuit gain is h_{fe}, and the output conductance is h_{oe}. If several stages are cascaded with capacitive coupling, the voltage and current gain per stage will be very nearly h_{fe}. The high-frequency gain is limited primarily by the β cutoff and the collector capacitance, but these effects should not be important below about 5 kc, except possibly for high-power transistors.

13-2.33 Grounded-Base Amplifier

The grounded-base amplifier, shown schematically in **Fig.** 13-60B, is used primarily where an exceptionally high output impedance is required. This configuration produces no current gain but, if a large load impedance is used, a voltage gain of 10,000 or more may be realized.

13-2.34 Grounded-Collector Amplifier

The grounded-collector amplifier, shown schematically in Fig. 13-60C, is useful for providing current gain without voltage gain. For normal load impedance, the voltage gain is slightly less than unity, while the current gain is very nearly $1 + h_{fe}$ and may be 50 or more for a high-gain transistor. This circuit has very low power gain, but is capable of providing a low output impedance if the source impedance is not too high. The input and output impedance depend significantly upon the load and source impedances, making this circuit quite different from the vacuum-tube cathode follower, which provides almost perfect isolation between input and load.

13-2.35 Phase-Inverter and Difference-Amplifier Circuits

The split-load phase inverter and the difference amplifier shown in Fig. 13-61 are similar to their vacuum-tube counterparts, but it should be borne in mind that the base current of a transistor is much larger than the grid current of a vacuum tube and, therefore, must be considered when analyzing these circuits. The base current causes the input impedance to be much smaller than in the corresponding vacuum-tube circuit.

13-2.36 High-Power Amplifiers

For high-power operation, it is desirable to use Class B amplification, thereby avoiding excessive dissipation in the transistors. The circuit shown in Fig. 13-62A is a possible configuration using a small voltage bias. The bias must be large enough to place the transistor in the linear region of operation without causing excessive collector dissipation, and the impedance of the bias supply must be reasonably low to avoid the loss of power gain. If two batteries are available, the output transformer may be eliminated by using a circuit similar to Fig. 13-62B. However, if both NPN and PNP transistors are available, the greatly simplified circuit of Fig. 13-62C is possible. Unfortunately, it is difficult to obtain matched NPN-PNP power transistors and, for this reason, the "complementary symmetry" circuit of Fig. 13-62C is not widely used.

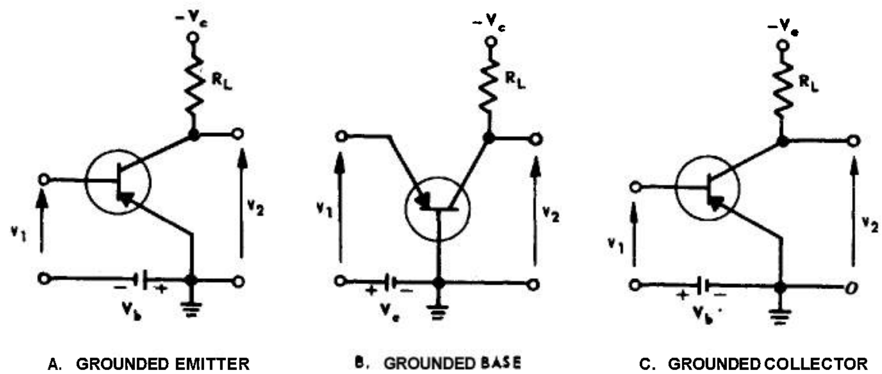
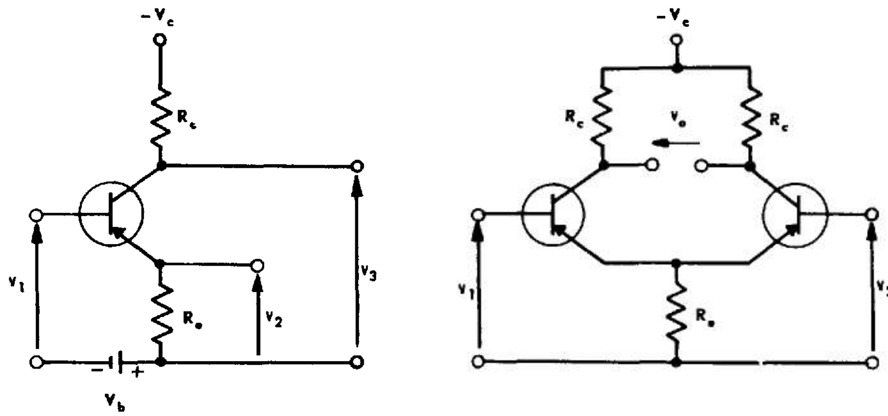


Fig. 13-60 Basic transistor amplifier configurations.



A. SPLIT-LOAD PHASE INVERTER

B. DIFFERENCE AMPLIFIER

Fig. 73-67 Typical amplifier circuits.

13-2.37 Maximum power output. For maximum power output, the load resistance of a high-power transistor stage should be selected according to the voltage, current, and power limitations of the transistor and ***not*** according to the incremental parameters. If maximum available output power is not required, however, the load impedance can be increased, with a corresponding increase in power gain. For normal applications, the load conductance is so large compared with h_{oe} that the gain can be approximated by

$$K_p = G_p R_L \tag{13-54}$$

where

K_p = power gain

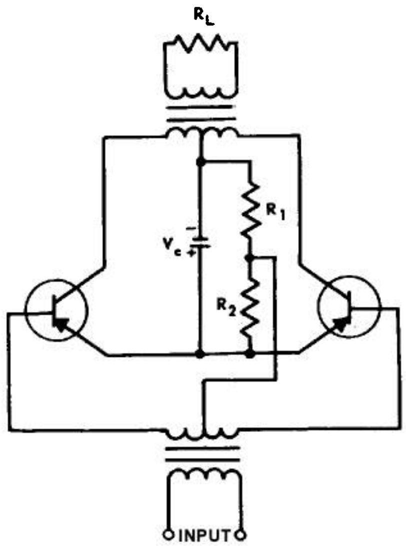
G_p = power conductance = h^2_{fe}/h_{ie}

R_L = load resistance

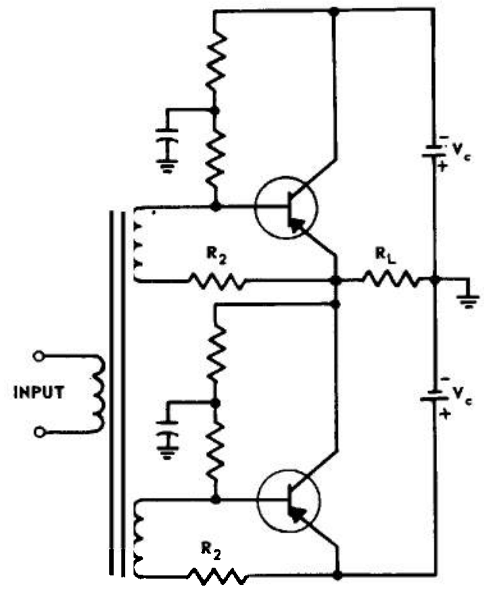
If low-frequency operation is anticipated (i.e., ω^{-1} greater than the thermal time constant of the collector junction), the load should not permit the peak power dissipation to exceed the maximum rated dissipation of the transistor. For carrier frequencies in the order of 400 cps, the peak power can be considerably greater, provided the average dissipation is within the limits of the rating. The limiting factor on power dissipation is the maximum allowable junction temperature. Almost any instantaneous dissipation is allowable if this temperature limit is not exceeded. There is a good evidence that the life of a transistor depends markedly upon the junction temperature. Above a critical value, the junction tends to destroy itself by thermal diffusion of impurity centers. Thus, derating may be necessary if long life is desired.

13-2.38 Biasing

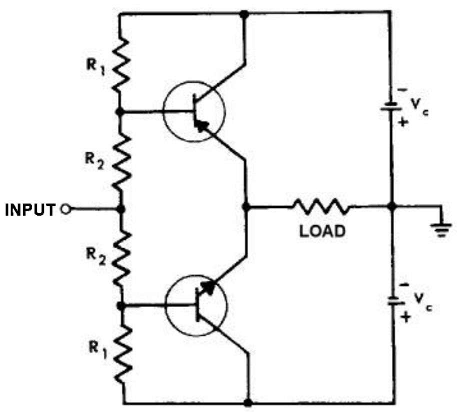
The choice of a suitable biasing scheme for a transistor is very important because of the wide manufacturing and temperature variations of transistor parameters. The guiding principle is to supply the bias in such a way that the emitter current and the collector voltage will be more or less independent of reasonable variations in transistor parameters, particularly I_{co} and h_{fe}. The circuit of Fig. 13-63A uses current feedback and is very effective if the load impedance is small. For a circuit with a large load resistance, the voltage-feedback scheme of Fig. 13-63B may be better. In both cases, the bypass capacitors short-circuit the feedback for signal frequencies, so that the biasing circuit has very little effect on the gain. If transformer coupling is



A. TRANSFORMER INPUT, TRANSFORMER OUTPUT



B. TRANSFORMER INPUT, DIRECT-COUPLED OUTPUT



NOTE: R_2 IS SOMETIMES REPLACED BY A DIODE CONNECTED SO AS TO BE FORWARD BIASED.

C. COMPLEMENTARY SYMMETRY (GROUNDED COLLECTOR)

Fig. 13-62 Push-pull Class 0 power-amplifier circuits.

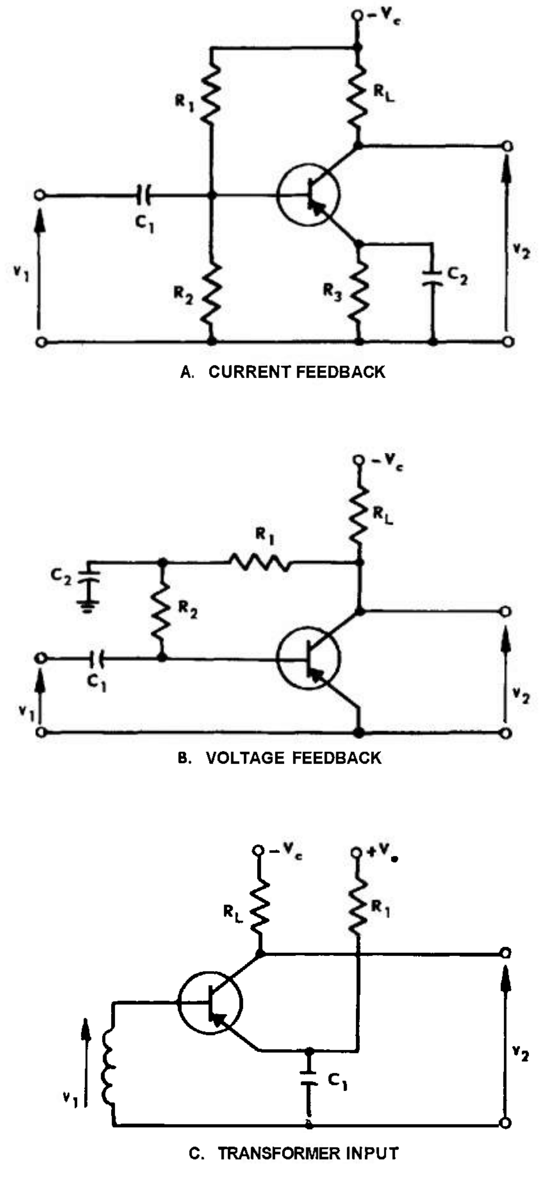


Fig. 13-63 Typical biasing circuits.

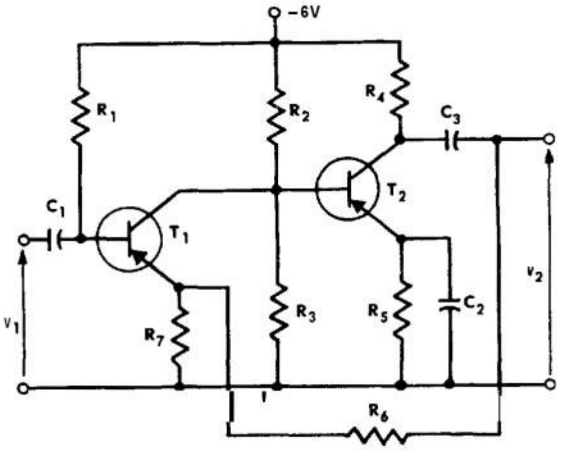
permissible, a better impedance match between stages can be achieved, and a simple biasing arrangement such as the one shown in Fig. 13-63C may be used. Usually, it is more convenient to increase the gain by using more amplifier stages than by resorting to transformer coupling. However, if tuned amplifiers are desired, the transformer action is readily incorporated into the tuned circuit, and the impedance transformation makes it easier to achieve high-Q operation.

13-2.39 Typical Two-Stage Transistor Amplifier

The two-stage transistor amplifier of Fig. 13-64 illustrates some of the important principles of good design. The input stage uses a low-noise transistor, biased with a collector voltage of about -1 volt and an emitter current of about 100 microamperes. This low-power bias makes it possible to achieve a noise figure of 6 db or less, and the emitter current is so small that bias stabilization can be omitted if feedback is used to stabilize the gain. The collector voltage of the first stage and the emitter current of the second stage are determined by R_2 and R_3 and are thus reasonably independent of transistor parameters. The lack of a coupling capacitor does not produce any instability problems, because the d-c gain of the second stage is very low. The bypass capacitor effectively grounds the emitter of the second-stage transistor, which can then provide a voltage gain of 100 or more. The feedback increases the input impedance and makes the voltage gain very nearly equal to $(R_6 + R_7)/R_f$, and independent of the transistor parameters.

13-2.40 Direct-Coupled Amplifiers

High-gain direct-coupled amplifiers have many undesirable features and should be avoided unless no other type of amplifier will accomplish the desired results. In general, silicon transistors offer some advantages over germanium transistors in low-level direct-coupled amplifiers. However, for signal levels below about 10^{-6} watt, the drift becomes excessive unless elaborate temperature-compensation schemes are used. In addition, below about 500 cps, the noise increases as $1/f$ and must be considered for microwatt power levels. On the other hand, if the signal levels are in the milliwatt range, direct-coupled amplifiers are quite feasible and, in fact, may be preferable to capacitance-coupled or transformer-coupled amplifiers. For example, a



T_1 = LOW-NOISE TRANSISTOR
T_2 = HIGH-GAIN TRANSISTOR

TYPICAL VALUES

R_1 = 2.2M R_2 = 4.7K
R_3 = 1K R_4 = 4.7K
R_5 = 2.2K R_6 = 4.7K
R_7 = 4712
INPUT RESISTANCE R_{IN} > 10K
VOLTAGE GAIN K_V = 100

Fig. 73-64 Two-stage transistor **feedback** *amplifier.*

direct-coupled driver for a Class **B** amplifier stage eliminates the problems of capacitive blocking or inductive overshoot.

13-2.41 State of the Art

Both the design and manufacture of transistors are in a continual state of change and improvement, more so than for most other electronic devices. While the capabilities and limitations of vacuum tubes are fairly well established and stabilized, quite the opposite is true of transistors. Therefore, it is recommended that the very latest manufacturers' data and literature be obtained and studied before evaluating the use of transistorized circuits in servomechanisms.

13-2.42 A-C POWER AMPLIFIER DESIGN

Refer to Table 13-4 for a comparison of typical design information on silicon and germanium power transistors. Inspection of this table indicates that for class B power amplifiers which operate at audio frequencies, have outputs of five watts or more, and are powered from the commonly used 28-vdc source, germanium transistors are preferable to silicon transistors in cases where ambient air temperatures remain below about 60°C. Thus, in designing a 10- to 15-watt a-c power amplifier for Servo applications or as a power source, germanium power transistors are of greater interest. If ambient temperatures approach the 60°C limit, it generally is preferable to use silicon transistors in low-level circuits where incremental current gains are equivalent; the problem of stabilizing low-level circuits using germanium transistors at these temperatures becomes increasingly difficult due to the large temperature-dependent leakage currents. For most purposes, the design can be divided into the two areas, signal-level (<25 mw) circuits and power-level circuits. Signal-level circuits with silicon transistors can employ capacitive or direct coupling effectively.

For all but the most stringent design problems, the class B common-emitter circuit is preferable, since both voltage gain and power gain are obtained. The relatively high output impedance of the common-emitter configuration affects output transformer design adversely in much the same manner as beam tetrode vacuum tubes do, requiring larger transformers with correspondingly narrower bandwidth capabilities. In precision power sources, or in servo applications where an advantage is taken of internal motor damping by having a low-impedance source obtained through considerable feedback, it is advantageous in terms of crossover distortion, transformer size, and greater gain bandwidth, to use the emitter-follower (common-collector) circuit. The gain and temperature stability and low output impedance of the emitter-follower circuit makes it feasible to reduce over-all system feedback and to minimize the frequency stability problem involved in a loop containing two transformers.

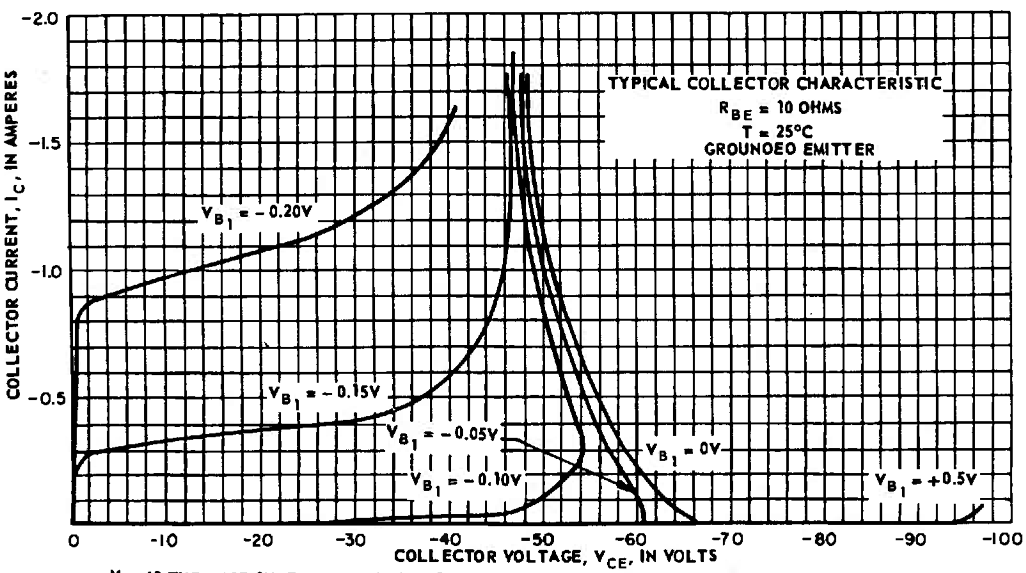
In the common-emitter design, a 12- to 15-watt amplifier requires approximately one ampere peak current from a 28-vdc supply. Servo motors with a center-tapped (20-0-20 volt rms) control winding are available for transistor applications. A matching transformer should present an equivalent primary impedance of approximately 110 ohms. Power dissipation within each transistor consists of (1) the ideal class B dissipation which can approach 20 per cent of maximum design power out under some conditions, (2) the "cut-off" current-collector voltage product for the ideally non-conducting half cycle, and (3) the temperature-dependent leakage. The third component is generally treated adequately by published curves and data, but the second is neglected somewhat because published data are often not complete. See Fig. 13-65, which shows adequate power transistor characteristics for the circuit designer.

Typical power stages that provide 24 to 30 db of power gain would be similar to that shown in Fig. 13-62A. Cascaded emitter followers can be added between the input transformer and the output transistor bases in applications where 25 to 100 watts of power are required. This straightforward method

TABLE 13-4 COMPARISON OF TYPICAL DESIGN INFORMATION ON SILICON AND GERMANIUM POWER TRANSISTORS

	Germanium	Silicon
Maximum junction temperature	85-100°C	150-200°C
Principal types	PNP	NPN
Typical large signal gains at		
0.5 amp max	100	85
1 amp max	55	30
5 amp max	35	10
10 amp max	25	
Saturation resistance $\frac{\text{\_\_\_\_ volts}}{\text{at \_\_\_\_ amps}}$	0.3/3 0.5/10	1.5/0.2 3/0.5 2/1 5/1
Internal thermal resistance	0.5-2° c/w	1.5-19° c/w
Base-collector diode rating	60-100v	60-120 v
Common emitter cutoff frequency (3 db)	5-20 kc	0.4-0.5 mc
Cost per unit	$5-20	$25-50

reduces the input power requirement by possibly a factor of 50, a level that can be attained with a class A silicon transistor stage (less than 25 milliwatts at 60°C ambient, convection cooled). Commercial transformers are available for both interstage and output applications. Many solutions and power gains are feasible with signal-level circuits similar to that shown in Fig. 13-64. As an indication of gains and accuracies attainable, power amplifiers with six to eight transistors (15-watt output at 400 and 1000 cps) with a 40-db gain and accurate to 0.1 per cent amplitude over a temperature range of —20° to 65°C have been designed.



V_{B_1} IS THE BASE SUPPLY VOLTAGE AND IS CONNECTED TO THE BASE THROUGH A 10 OHM RESISTOR (R_{BE})

Fig. 73-65 Collector characteristics of typical power transistor.

13-3 MAGNETIC AMPLIFIERS*

13-3.1 BASIC CONSIDERATIONS

13-3.2 Functions of Magnetic Amplifiers in Servo Systems

The chief function of a magnetic amplifier in a servo system is to act as the power amplifier for driving the system output element, which is usually an a-c or d-c motor, but may be any other form of end electrical device. Magnetic amplifiers may also be employed as intermediate-stage or signal amplifiers, handling either a-c carrier or d-c signals. Therefore, where electronic devices cannot be used, magnetic amplifiers *can* be employed to perform all of the amplification, sensing, and demodulating functions of a servo system. Magnetic amplifiers are also used in voltage regulators for power supplies and generators in servo systems.

13-3.3 Features of Magnetic Amplifiers

The principal, outstanding feature of magnetic amplifiers, and the main reason for their widespread use, is their all-static construction which uses only magnetic components and rectifiers. Theoretically, the magnetic amplifier should have infinite reliability compared with electronic equipment and zero maintenance compared with rotating equipment. Magnetic amplifiers can be built for output powers ranging from microwatts to kilowatts; they can be cascaded; they can be fed from multiple input circuits; they can operate from single-phase or multi-phase power ; and they require no electronic-amplifier-type power supplies.

13-3.4 Application Problems of Magnetic Amplifiers

The output of a magnetic amplifier is an a-c or rectified carrier that has been modulated in some manner in accordance with the control signal. The maximum signal frequency is limited to some fraction of the carrier or supply frequency; hence, the theoretical pass band can be raised only by raising the carrier frequency. High carrier frequencies are used, therefore, to increase the bandwidth and to reduce the physical size of the amplifier. Furthermore, the magnetic amplifier always introduces a time delay between application of the control signal at the amplifier input and realization of the output signal. The lower limit of this time delay is one half-cycle of the carrier frequency; the upper limit is a function of amplifier construction.

13-3.5 Temperature Limitations

Magnetic amplifiers are temperature-limited in several ways. They are environment-temperature limited by the rectifiers. For example, this limit is in the 125°C to 150°C region' when silicon rectifiers are used. In addition, the characteristic of the magnetic amplifier may shift as a result of the effect of temperature on the back leakage of the rectifier and also on the magnetic characteristics of the core material. This temperature-drift effect is a serious difficulty in cascaded magnetic amplifiers used for amplifying low-level signals, just as it is in electronic d-c amplifiers.

13-3.6 Design Difficulties

The design and development of magnetic-amplifier systems may be considerably more difficult than that of their electron-tube counterparts, principally because magnetic-amplifier cores are not standardized and catalogued. When a magnetic amplifier is in the "breadboard" stage of development, circuit revisions often require rewinding of cores and, sometimes, substitution of different cores as well. Furthermore, the analysis of magnetic-amplifier circuits has not yet been developed to the point where all combinations of cores and loads can be analyzed beforehand and the operation of the amplifier accurately predicted. For greater detail, consult the books by Storm[26] and Geyger.[27] These books are fairly comprehensive references on magnetic amplifiers.

**By A. Kusko*

13-3.7 PRINCIPLES OF OPERATION

13-3.8 Single-Core, Single-Rectifier Circuit

The basic principles of operation of magnetic amplifiers can be studied conveniently by means of the basic single-core circuit shown in Fig. **13-66.** The function of this simple circuit is to control the flow of energy from the source voltage e_l to the load resistance R_L. Control is effected by means of the voltage E_c. To obtain a simple analysis, several idealizations are made in the circuit, and the core material is assumed to have a B-H characteristic as shown in Fig. **13-67.** In this figure, the core material is shown as the square-loop type, which features a sharp saturation point and zero width. This means that the core requires neither ampere turns for obtaining a change in flux density nor energy-loss for operation. The rectifier is assumed to be ideal; i.e., it has zero resistance in the forward direction and infinite resistance in the back direction. The source voltage e_l is assumed to be sinusoidal and the control voltage E_c is assumed to be a constant direct voltage. The core is assumed to have an area A, with two windings thereon —one winding with N_g turns, called a ***gate*** winding; and the second winding with N_c turns, called a ***control*** winding.

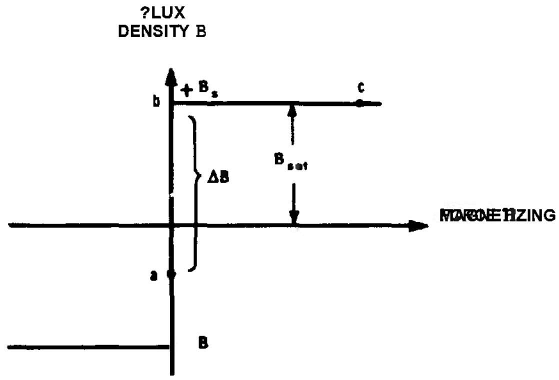


Fig. 13-67 Simplified B-H characteristic of core material.

13-3.9 Operating cycle of a single-core circuit. The single-core circuit will be studied over one cycle of source voltage to note the manner in which the control voltage E_c controls the energy transfer from the source to the load. The waveforms of flux density and load current over one cycle are shown in Fig. **13-68.** The cycle of operation can be divided into three parts. The first part is called the "exciting period" of the cycle. This is the portion in which the magnetic-amplifier core blocks the source from the load and prevents the flow of current. The second portion of the cycle is called the "conducting period" and is that part of the cycle during which the magnetic core is saturated, permitting the source voltage to be applied directly to the load resistance. The third portion of the cycle is called the "reset period". During this part of the cycle, the control voltage operates on the core to determine the load current in the next period.

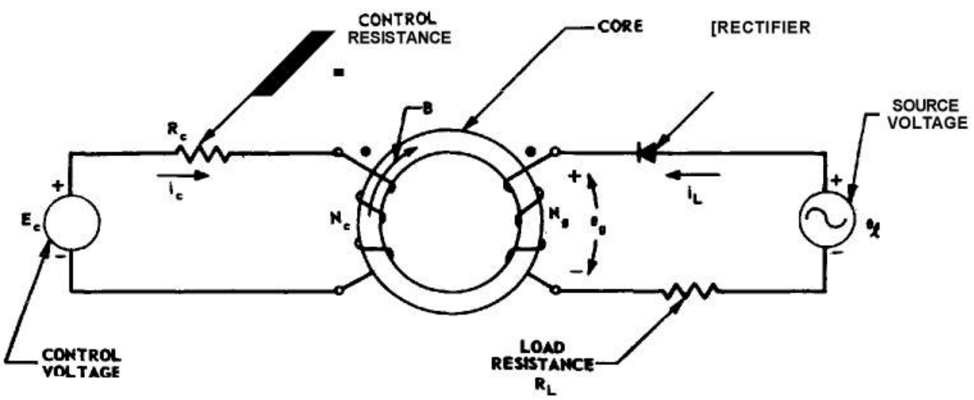


Fig. 13-66 Simple single-core magnetic amplifier.

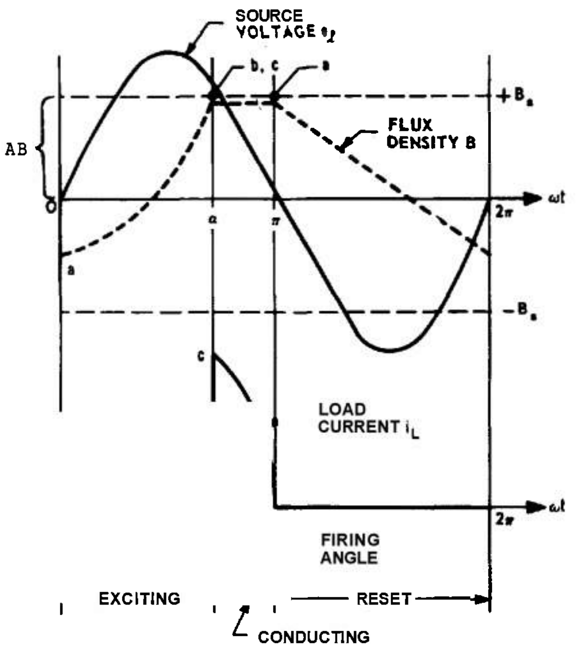


Fig. 73-68 Waveforms of source volfage, flux density, and load current over one cycle.

13-3.10 Exciting and conducting periods. Assume that the cycle of operation commences with the core in the magnetic state shown as point a on the *B-H* characteristic of the core material (Fig. **13-67).** This corresponds to $\omega t = 0$ on the waveform diagram (Fig. **13-68).** As the cycle commences, the source voltage e_t is positive, causing the rectifier to conduct. If the control resistance R, is relatively large compared with the load resistance R_L (referred to the same side of the magnetic core), almost all of the source voltage will appear directly across the gate winding N_g. The load current i_L will be zero at this time because the core represents, in effect, an infinite impedance. The effect of the voltage e, appearing at the gate-winding terminals is to cause the flux density in the core to rise from point a toward point b, the saturation value. At the angle $\omega t = a$, called the ***firing angle*** in the waveform diagram (Fig. **13-68),** the core flux has reached point b and the core saturates. When this occurs, the core no longer introduces impedance into the gate circuit, and the source voltage e_t is applied directly to the load resistance R_L. The load current i_L now jumps to the value shown at point *c,* this value being merely the source voltage (at that instant) divided by the load resistance. For the remainder of the half-cycle, the load current i_L is determined by the source voltage and the load resistance only, as shown in Fig. **13-68.** At the angle $\omega t = n$, the source voltage reverses polarity, the rectifier blocks, and the load current i_L remains at zero until the angle finally reaches $\omega t = 2\pi$.

13-3.11 Reset period. During the interval from $\omega t = \pi$ to $\omega t = 2\pi$, called the reset period, the control voltage E, takes charge of the operation, since the rectifier has blocked the source voltage e_t from the core. If the control voltage is negative, its effect will be to drive the flux density in the core downward from point b toward the value a. Since the idealized core material requires no exciting current, the full value of control voltage is applied to control winding N,, producing a straight-line decrease in flux density *B,* as shown in Fig. **13-68.** At the angle $\omega t = 2\pi$, the control voltage has reset the core back to point a, and the cycle of operation is ready to start again. The magnitude of the control voltage determines how far down on the *B-H* characteristic the core is returned during the reset period. This, in turn, determines how long the core will block during the next half-cycle and also determines the value of the firing angle a. Consequently, the firing angle determines I_L, the average value of the load current, in that half-cycle. To summarize the chain of operation: the control voltage *E,* determines the amount of core reset; this, in turn, determines the firing angle *a* in the next half-cycle and thus determines the average value of load current in that half-cycle.

13-3.12 Control limits. For a sinusoidal source voltage $e_t = \sqrt{2E_t} \sin \omega t$, the combination of frequency ω, saturation flux density B_s, gate-winding turns N_g, and core area **A,** should be chosen in accordance with the' following equation:

$$\sqrt{2}\,E_l = \omega B_s N_g A_c \qquad (13\text{-}55)$$

Hence, a half-cycle of source voltage will produce a flux change in the core that is twice the saturation value. The lower limit of control voltage E_c is zero control voltage, at which level the control circuit does no resetting of the core and the firing angle a is zero. When this occurs, the core is continuously saturated and the load current consists of alternate half-cycle sinusoidal pulses. This represents the maximum output of the magnetic amplifier. The upper limit of control voltage E_c is the value that is sufficient to reset the core by twice the saturation flux density. The upper limit is determined by

$$(E_c)_{lim} = -4fB_sN_cA_c \qquad (13\text{-}56)$$

When this occurs, all of the next half-cycle of time is required by the source voltage to bring the core back up to saturation, and the firing angle a is π. The output of the magnetic amplifier is zero. The relationship between the firing angle a and the control voltage E_c is

$$\cos\alpha = \left[1 - \frac{\Delta B}{B_s}\right] = \left[1 - \frac{2E_c}{(E_c)_{lim}}\right] \qquad (13\text{-}57)$$

The relationship between the average load current I_L and the control voltage E_c is

$$I_L = \frac{2\sqrt{2}}{\pi}\frac{E_l}{R_L}\left[1 - \frac{\Delta B}{2B_s}\right] = (I_L)_{max}\left[1 - \frac{E_c}{(E_c)_{lim}}\right] \qquad (13\text{-}58)$$

13-3.13 Control characteristics. A plot of the control characteristics of the simplified magnetic amplifier is shown in Fig. **13-69.** The theoretical control characteristic is a straight line, as shown by the $I_L - E_c$ curve. The load current is zero at the limiting value of control voltage $(E_c)_{lim}$ and maximum when the control voltage is zero. For reference, the firing angle a is plotted on the same curve, with the value π for zero output and the value zero for maximum output. Practically all magnetic amplifiers exhibit this straight-line-control characteristic over which linear input-output relationships are obtained.

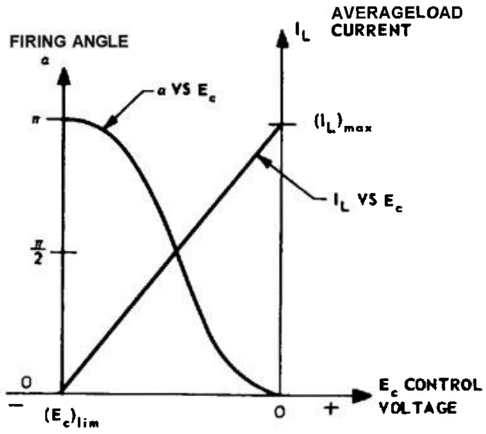


Fig. 13-69 Curves showing relationship between average load current I_L, firing angle a, and control voltage E_c.

13-3.14 Analysis limitations. There are several limitations to this idealized analysis when it is applied to an actual single-core magnetic amplifier. First, the core material always has a finite-width hysteresis loop, as compared with the zero width in the theoretical analysis. This means that, during the reset portion of the operation cycle, a finite control current I_c exists in the control circuit. If the control resistance is made very large, the voltage drop in the control resistance reduces the effect of the control voltage E_c on the core. The power gain of the idealized circuit is infinite, because it is assumed that no control current exists. However, with a finite-width hysteresis loop, the control-voltage source has to supply a finite amount of power, both to the control resistance and to the core itself, limiting the power gain. The second limitation of the idealized analysis is that, in practice, a rectifier is never ideal. The rectifier may have some leakage in the back direction, allowing the source voltage to influence the reset operation during the negative half-cycle. Actually, the simple single-core circuit is rarely used as shown, but is modified into either the Ramey circuit or multicore circuits (both described later).

13-3.15 Analysis extension. The analysis given here for the highly simplified circuit can be extended to more complicated single-core and multicore circuits. In general, the extended analysis is carried out by dividing the cycle of circuit operation into its various parts, as was done for thesimple circuit. The circuit equations are then written for each part of the cycle, the correct boundary conditions are inserted, and the equations are solved for the operation of the amplifier. In the usual application and design of magnetic amplifiers, it is often unnecessary to make a complete analysis as shown in this section. Instead, the operation of a magnetic amplifier can be briefly summarized in terms of gains and time constants, which are sufficient for describing the operation of the amplifier in most servo applications.

13-3.16 TYPICAL CIRCUITS

There is a wide variety of magnetic-amplifier circuits designed for various functions and power levels and employing from 1 to 12 cores in circuits of varying degrees of complexity. A good survey of these circuits is given by Geyger.[27] The three classes of circuits discussed below represent the majority of those used in magnetic-amplifier work.

13-3.17 Half-Cycle (Ramey) Circuit

The basic Ramey circuit, shown in Fig. 13-70, is somewhat similar to the idealized circuit of Fig. 13-66. The Ramey circuit, however, utilizes a second rectifier and an a-c source in the control circuit. This minimizes the coupling between the control and load circuits during the exciting interval when the source voltage is operating on the core. A salient feature of the Ramey circuit is the high response speed. Theoretically, the circuit responds to the command of a control signal in a half-cycle of carrier-frequency time. This occurs because the circuit has no memory from one operating cycle to the next. In the resetting half-cycle, the control voltage *e*, resets the core to some value. In the following half-cycle, the average value of the load-current pulse is dependent upon the degree to which the core was previously reset. In effect, then, the amplifier operates over any single operating cycle independently of previous cycles. Although a single-core circuit is shown in Fig. 13-70, the same principle may be applied to multicore circuits to obtain the advantage of half-cycle response speed.

13-3.18 Single-Ended, Two-Core Circuits

Single-ended circuits fulfill most of the requirements for magnetic-amplifier applications. They are known by various names, such as *doubler* circuit, *bridge* circuit, and *full-wave* circuit. The doubler circuit in Fig. 13-71 utilizes two cores and two self-saturating rectifiers, and features a-c output. Single-ended circuits may be designed for either a-c

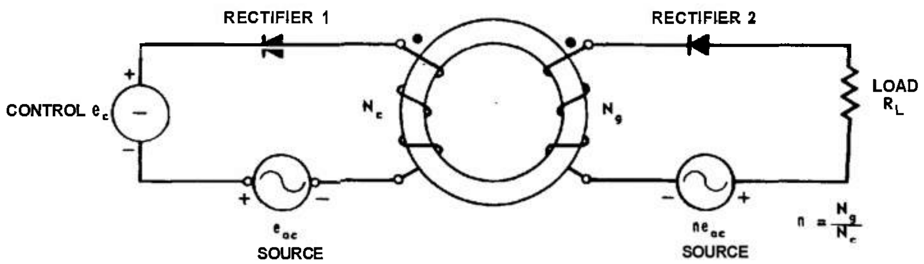


Fig. 13-70 *Ramey circuit.*

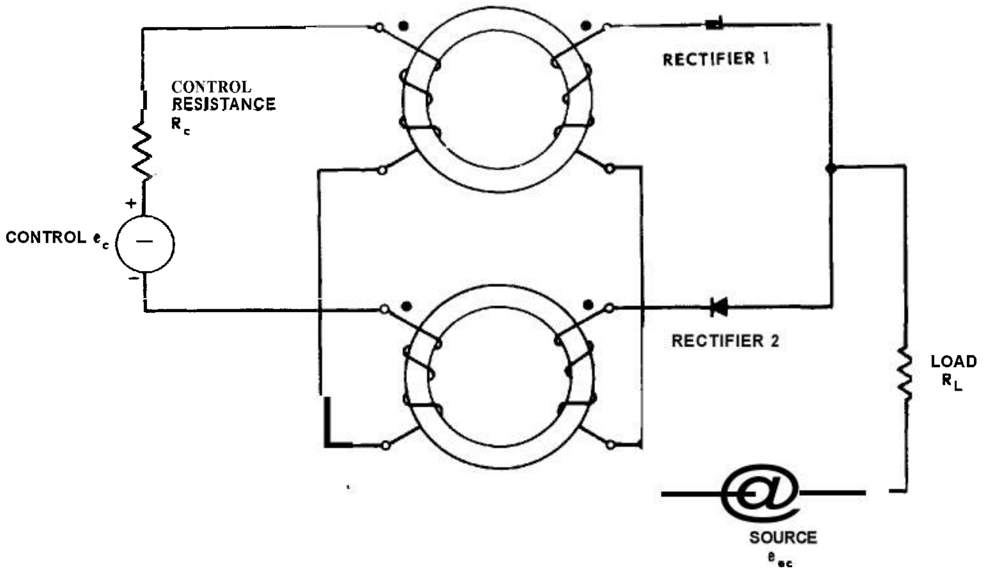


Fig. 13-77 **Doubler** *circuit.*

or d-c output. They are characterized by relatively high power gain. The time constants can be made to have a minimum of approximately three cycles of carrier-frequency time. The control circuit may consist of many windings for signal mixing, instead of the single winding shown in Fig. 13-71.

13-3.19 Operation of two-core circuits. Basically, two-core circuits operate by utilizing the saturation characteristics of the core on alternate half-cycles. During any half-cycle, one core is resetting while the other core is passing through its exciting and conducting periods. Each core operates much as the single-core amplifier of Fig. 13-66 does. However, since the two cores are coupled through the control circuit, the induced voltage in one core during its exciting period simultaneously acts as a resetting voltage on the other core. For this reason, all of the energy for resetting the individual cores does not come from the control circuit during each half-cycle. This accounts for the relatively high power gain of two-core circuits as compared with single-core circuits.

13-3.20 Reversible-Polarity and Reversible-Phase Circuits

To obtain reversible-polarity or reversible-phase output from a magnetic amplifier, the single-core and two-core circuits heretofore described (which are not reversible by themselves) must be combined in some manner. For example, the outputs of two single-ended circuits can be mixed in resistive networks or in multiwinding inductive devices to obtain reversible outputs.

13-3.21 Example of reversible-phase amplifier. An example of a reversible-phase magnetic-amplifier circuit, known as the half-wave bridge circuit, is shown in Fig. 13-72. The circuit load is a 2-phase servomotor that reverses its torque and direction when the phase of the main-winding current is reversed with respect to the excitation winding. This represents quite an important kind of behavior in a large class of servomechanisms. In the circuit shown, all cores excite and conduct during the same half-cycle. However, the control signal causes two cores

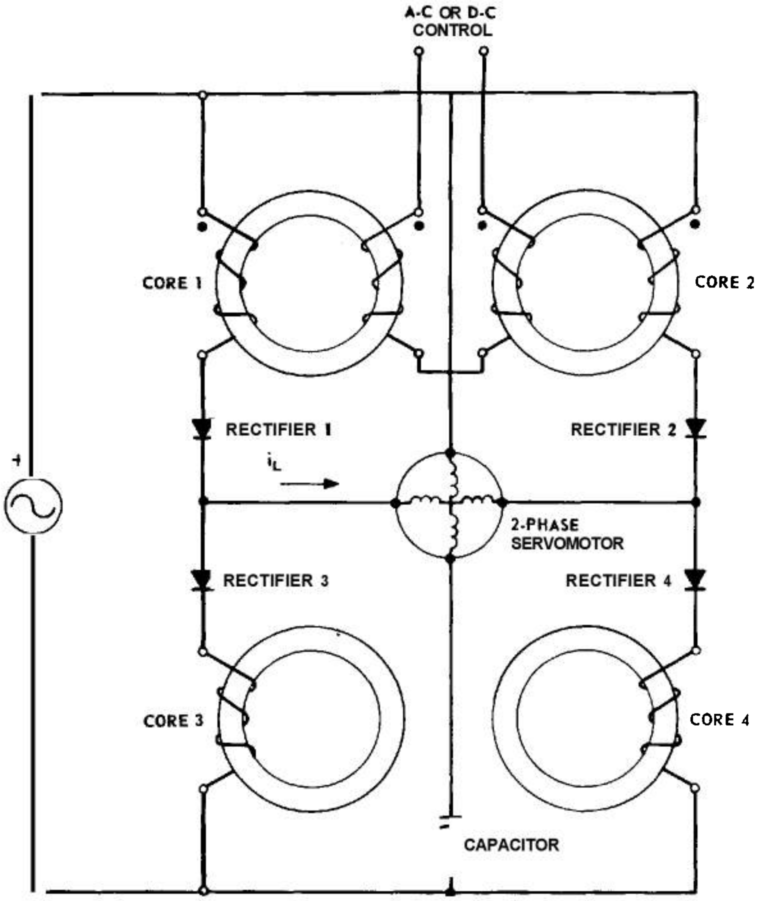


Fig. 73-72 Half-wave magnetic servo amplifier bridge circuit.

(either 1 and 4, or 2 and 3) to saturate early, in a symmetrical manner about the center of the half-wave. For this reason, the load current consists of a pulse that is centered at the 90° point of the half-cycle, its polarity being dependent upon the pair of cores that saturates first in the half-cycle. The circuit features high-speed operation because phase reversal of load current can be made to take place from one half-cycle of operation to the next half-cycle of operation.

13-3.22 ANALYTICAL REPRESENTATION OF MAGNETIC AMPLIFIERS

13-3.23 Dynamic Performance

The output of a magnetic amplifier is a series of unipolar or bipolar pulses derived from the pulse-width-modulation action on the a-c carrier waveform. The envelope of the pulse is sinusoidal for a sinusoidal carrier and rectangular for a square-wave carrier. The pulses start during each half-cycle at the firing angle a and terminate at integral multiples of π.

13-3.24 Dynamic response. The dynamic response of the output to a step-change in control voltage is a transient change in the firing angle, from the initial value to the final value, over a period of time ranging from one half-cycle to many cycles. This is shown in Fig. 13-73. Since the load current, or output, is usually measured in terms of the average rectified current, the change in firing angle causes a change in pulse area over each half-cycle of the transient period.

13-3.25 Accuracy of prediction. For most analytical problems, the assumption that the output is smooth and continuous at the average value leads to an adequate accuracy of prediction. However, it is apparent that the discreteness of the output pulses must be retained in the analysis of other special problems where the load or a following stage does not serve as a low-pass filter.

13-3.26 Analytical Representation

For servo-system analysis, the magnetic amplifier can be treated in one of several ways, depending upon the required accuracy of prediction. The simplest form of representation stems from the assumption that the gain of the amplifier can be expressed by

$$K = \frac{\text{change in average output voltage}}{\text{change in control ampere turns}}$$

This gain can be read directly from the slope of the steady-state control characteristic. The average load voltage of the amplifier is assumed to change exponentially with time, with a time constant

$$T = \frac{L_c}{R_c} \tag{13-59}$$

where

L_c = equivalent control-circuit inductance proportional to N_c^2

R, = total control-circuit resistance

A transfer function can thus be written for the amplifier as

$$\frac{E_L}{E_c}(j\omega) = \frac{K\dfrac{N_c}{R_c}}{1 + j\omega T} \tag{13-60}$$

The use of this transfer function is described further by Decker[28] and Johannessen.[29]

13-3.27 Effective control-circuit resistance. In magnetic amplifiers having more than one control circuit (i.e., bias circuits, reference circuits, etc.), the control-circuit resistance R, must reflect the resistance of the other circuits as they are coupled to the control

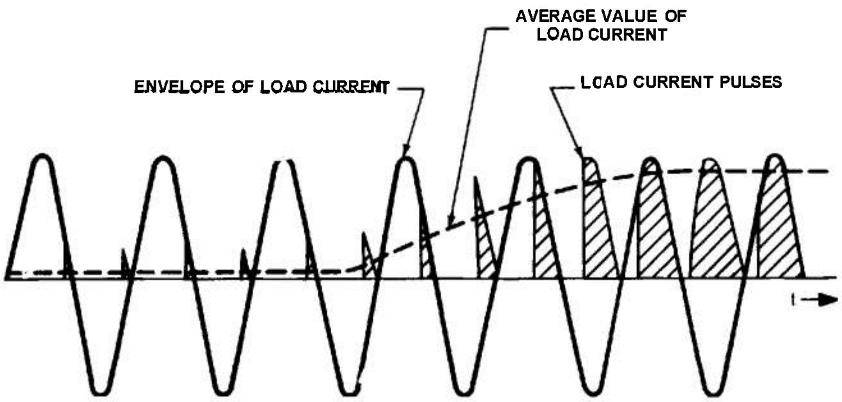


Fig. 73-73 Wuveforms of loud current during a transient change in output from minimum to maximum output (resistive loud).

circuit. The effective control-circuit resistance for dynamic operation is

$$\frac{1}{R'_c} = \frac{1}{R_c} + \left(\frac{N_1}{N_c}\right)^2 \frac{1}{R_1} + \cdots + \left(\frac{N_n}{N_c}\right)^2 \frac{1}{R_n} \qquad (13\text{-}61)$$

where R_n is the resistance and N_n is the number of turns in the nth control circuit. The time constant becomes $T' = L_c/R'_c$. Where a more accurate representation of the amplifier is desired, several alternative approaches are possible. For example, the transfer functions proposed by Johannessen,[29] in which the time delay appears explicitly, may be used. The servo system can also be represented on an analog computer, with the magnetic amplifier represented by the methods developed by Woodson.[30] Alternately, the discrete nature of the load-current pulses can be included, and the amplifier represented as a cyclical pulse-width modulator by sampled-data techniques.

13-3.28 Limitations in analytical representation. Limitations in the analytical representation of magnetic amplifiers for servo-system analysis and design arise in two ways. First. there are limitations mentioned previously, when an attempt is made to represent a given and measurable physical device by a mathematical expression. These are the limitations imposed by approximating the dynamic behavior by a single time constant, characterizing the amplifier by a fixed value of gain, assuming continuous output, and so forth. Second, there are the limitations in predicting the behavior of a magnetic amplifier in a servo system, where the amplifier itself is in a design state. Here is where difficulties arise in predicting the behavior of core material, the effect of rectifiers, and the effect of temperature.

13-3.29 Conclusions. Because of the present state of the art, it is still necessary to build, test, and adjust a prototype magnetic amplifier to determine its transfer function to an adequate degree of reliability. However, typical control-characteristic curves, and charts of typical Figures of Merit (discussed later), are extremely useful to the servo-system designer. From these, one can determine approximately what to expect from particular magnetic amplifiers.

13-3.30 PERFORMANCE OF MAGNETIC AMPLIFIERS

13-3.31 Ranges of Input and Output Power

Magnetic amplifiers can be designed to cover an extremely wide range of input and output power levels. The lowest control power that can be utilized (by using instrument-like techniques) is in the order of 10^{-12} watt. Practical input-power levels range from 10^{-4} to 10^{-6} watt and are obtainable without extreme precautions. Power outputs in the kilowatt region are possible. Table 13-5 shows complete data on a line of 60-cps industrial magnetic amplifiers with output powers ranging from 30 milliwatts for the smallest unit to 150 watts for the largest single-phase unit. Figure 13-74 shows several sets of curves giving the relationship between magnetic-amplifier power output and reactor weight in pounds, with the type of core material and the carrier frequency as parameters.

13-3.32 Control Characteristics

The steady-state characteristics of magnetic amplifiers are frequently given in the form known as a *control characteristic*. This is a plot of load current versus control current, or ampere turns, with the load resistance as a parameter. Typical control characteristics for a 30-milliwatt, 60-cycle magnetic amplifier are shown in Fig. 13-75. Because the magnetic amplifier appears basically to its load circuit as a controllable voltage source, each control characteristic is different for each value of ohmic load resistance. The usual region of operation is the linear rising portion; i.e., the portion ranging from —0.7 to —0.9 control ampere turns for the curves shown. The slope of this portion of the control characteristic is a measure of the gain of the magnetic amplifier.

13-3.33 Figure of Merit

A convenient measure of the performance of magnetic amplifiers whose dynamic behavior can be characterized by a time constant

TABLE 13-5 TYPICAL CHARACTERISTICS OF MAGNETIC AMPLIFIERS

Amplifier Type Number	A	B	C	D	E	F	G
Type of circuit	doubler	doubler	doubler	doubler	doubler	doubler	3-phase doubler
Nominal power output (watts)**	0.03	0.3	8	40	75	150	325
Nominal ampere turns for control***	0.3	0.45	1	2.2	2.2	2.5	1.5
A-c supply							
volts	6.3	12.6	30	115	115	230	230
frequency (cps)	60	60	60	60	60	60	60
phase	1	1	1	1	1	1	3
Max usable incremental Figure of Merit$	200	330	840	1300	2000	2300	3300
Max incremental power amplification (3-cycle response) ‡‡	225	440	1100	1200	2500	2200	8000 (6-cycle response)
Output current (amp)#							
rated			0.6	0.6	1	0.93	1.7
max power output	0.015	0.068	0.71	1.25	2.1	2.3	5
Load resistance (ohms)							
rated (55°C, temp rise)			28	148	85	175	130
max power output	150	100	20	45	28	50	25
max incremental power amplification	150	100	20	100	50	100	90
Weight of reactor assembly (lb)	0.27	0.3	1.1	2.9	5	6.5	19.5
Size of assembly (in.)							
diameter	1-5/8	1-5/8	2-1/2	3-1/2	4-1/2	4-1/2	4-1/2
height	1-3/4	1-3/4	2-5/8	3-1/4	3-3/4	4-3/4	4-3/4*

*Three units required.

**Maximum power output obtainable within the linear range of the control characteristic curve for rated load.

***Control ampere turns necessary to change the output from cutoff to the nominal power output point.

‡The ratio of the maximum power amplification to the response time.

$$Power amplification corresponding to 3-cycle response time taken over the linear range of the control characteristic curve for the optimum load resistance.

#Rms value if output is ac; average value if output is dc.

This table is adapted from technical data 52-600 (October, 1954), Westinghouse Electric Corporation, table a, p. 10.

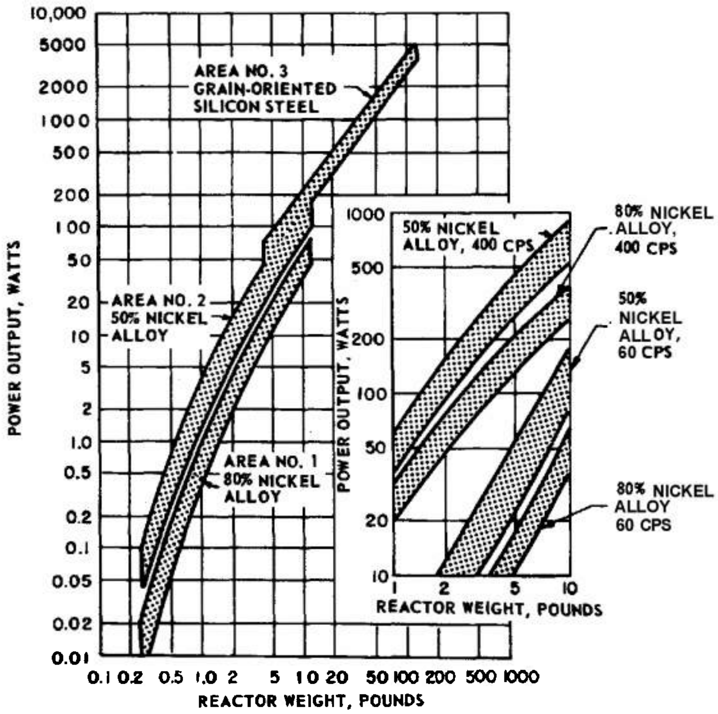


Fig. 73-74 (Left) Power output vs total weight of 60-cycle standard self-safurating magnetic amplifiers.

(Right) Comparison of power output vs reactor weight at 60 cps and 400 cps.

By permission from *Electrical Manufacturing*, Volume 48, No. 3, September 1951, from article entitled 'Applying magnetic amplifiers —IV,' by W. J. Domhoefer and V. H. Krummenacher.

is a term called the *Figure of* Merit. This term is defined as the ratio of the incremental power gain to the time constant of the amplifier. The time constant is usually measured in cycles of the carrier frequency. The Figure of Merit is a function of the material and of the geometry of the core. It is independent of the control-circuit resistance. Changes in control-circuit resistance change the relative amounts of gain and time constant, but preserve their ratio.

13-3.34 Typical values of Figure of Merit. Typical values of Figure of Merit are listed in Table 13-5 and shown in Fig. 13-76. In Table 13-5, the Figure of Merit increases from a value of 200 for the 30-milliwatt magnetic amplifier to 2300 for the 150-watt single-phase amplifier. In Fig. 13-76, the Figure of Merit, or power gain per cycle, is shown as a function of the rated output power of the amplifier. It increases with the power rating and with the quality of the core material. The maximum power gain and the maximum time constant are obtained for zero resistance in the external control circuit. To reduce the time constant, resistance is added until the practical limit of two to three cycles is reached. Since the Figure of Merit is measured in terms of a time constant in cycles, the performance of the amplifier for a time constant measured in seconds can be improved by raising the Carrier frequency. To offset this, there is some deterioration in amplifier performance at

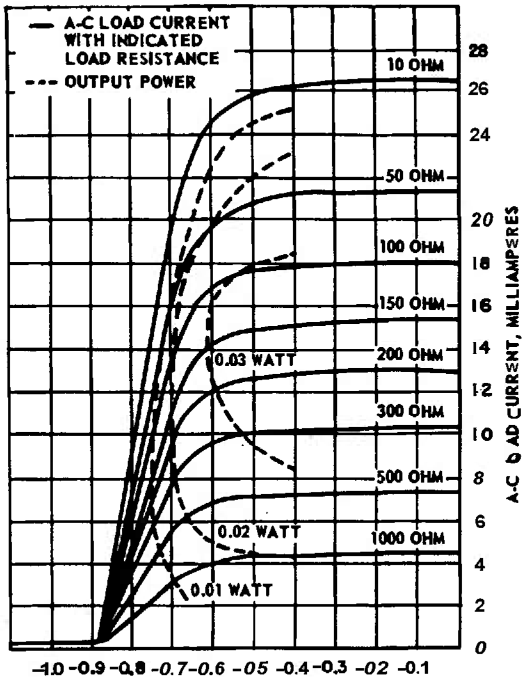


Fig. 73-75 Control characteristics for $N^2/R = 7.0$ zero bias current.

Courtesy Westinghouse Electric Corporation

higher carrier frequencies, and a resultant reduction in the Figure of Merit, because of eddy current and other magnetic phenomena as well as increased rectifier leakage.

13-3.35 Expression for Figure of Merit. From a design standpoint, the Figure of Merit can be expressed as[31]

$$K_f \sim \frac{JA_w}{l_i \Delta H_i} \text{ per cycle} \qquad (13\text{-}62)$$

where

$A\omega$ = effective total gate-winding copper area

J = gate-winding current density

ΔH_i = change in H in core material necessary to obtain gross change in load current

l_i = mean length of magnetic core

The core material enters into the expression for the Figure of Merit through the term ΔH_i; the geometry enters through the terms for the winding area A, and the core length l_i. On a dimensional basis, the Figure of Merit varies in proportion to the dimensions of magnetic cores when the cores are made of the

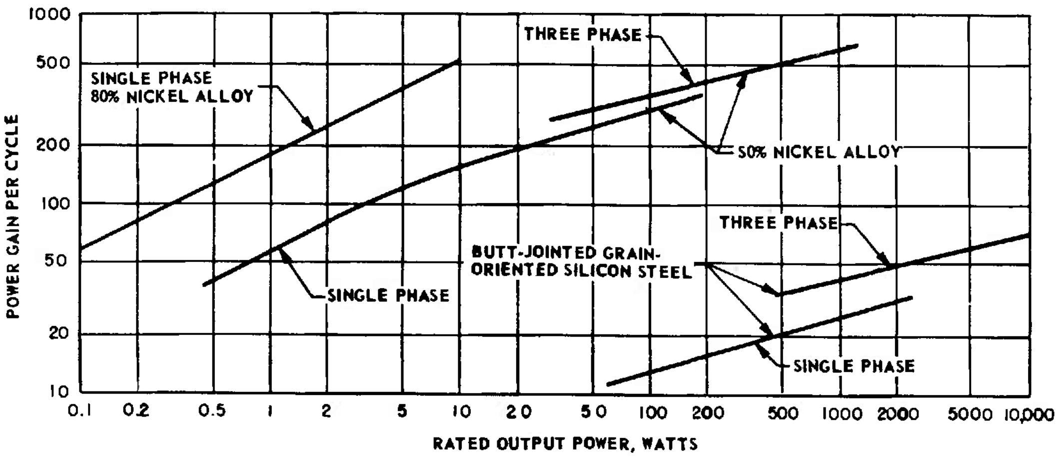


Fig. 73-76 Power gain per cycle for self-saturating magnetic amplifiers with several core materials in single and three-phase bridge circuits, with d-c output and 60-cycle supply.

By permission from *Electrical Manufacturing*. Volume 48, No. 3. September 1961. from article entitled 'Applying magnetic amplifiers—IV,' by W. J. Domhoefer and V. H. Krummenacher.

same material. Hence, an increase is expected in the Figure of Merit with an increase in the size of the magnetic amplifier. This is confirmed by the curves of Figure of Merit versus power rating, where power rating is an indication of amplifier size.

13-3.36 Largest factor. The largest factor controlling the Figure of Merit is the value of ΔH_t, which is determined by the core material. ΔH_t can be made to vary over a range of probably **50** to 1 by the selection of either square-loop nickel-iron alloys, which have a low ΔH_t, or grain-oriented silicon steel, which has a higher ΔH_t.

13-3.37 Effects of dimensions. The increase in the Figure of Merit through dimension makes it possible to use larger values of ΔH_t (i.e., less expensive core materials) for large-power amplifiers and still obtain Figures of Merit that are suitable for most applications. The thickness of the magnetic tape or the core laminations also influences ΔH_t. Eddy currents in the core materials introduce an increase in the value of ΔH_t. An increase in ΔH_t also results from an increase in frequency with a fixed tape thickness. Hence, as the carrier frequency is raised, the core laminations should be made thinner to maintain ΔH_t within acceptable limits.

13-3.38 Leakage effects. Although the effect of rectifier leakage does not appear explicitly in the expression for the Figure of Merit, the effect of this leakage is to increase the apparent value of ΔH_t.

13-3.39 Temperature effects. Temperature has one of the most pronounced effects on the performance of magnetic amplifiers. Temperature affects both the leakage of the rectifiers and the effective ΔH_t of the core material. Hence, the Figure of Merit of a magnetic amplifier, or the power gain for a fixed value of time constant, may shift with variation in temperature unless the amplifier is carefully designed. In magnetic amplifiers used in servo systems, this change in gain introduces a change in the loop gain, which may lead to an unfavorable change in steady-state and dynamic performance. In addition, the effect of temperature on the Figure of Merit makes it difficult to design magnetic amplifiers for open-loop systems with calibrated or predetermined gain.

13-3.40 CONSTRUCTION OF MAGNETIC AMPLIFIERS

13-3.41 Methods of Core Construction

The choice of a method of constructing magnetic-amplifier cores is a compromise between geometries that favor superior electrical behavior and those that favor ease and economy of construction. For best electrical performance, the magnetic potential developed in the core by the control signal should produce the largest possible change in core-flux density in a prescribed time. To achieve such results, the core should have the lowest possible reluctance over the reset range; this means that there can be no air gaps, high-permeability material should be used, and low levels of eddy-current shielding should exist (thin materials of high resistivity must be used). On the other hand, for economy of construction, the use of conventional transformer-type magnetic materials and construction is favored. The amplifier design is a compromise between these factors. The following types of core construction are commonly used :

(a) Spiral. The core material is made in tape form, in thicknesses ranging generally from 0.001 to **0.004** inch, and is wound into a gapless ring or spiral core. Toroidal windings are used, placed on the core by a special winding machine.

(b) D-U laminations. The core material is made in sheet form as thin laminations, cut in a U shape, with the base of the U twice as wide as the sides. The core is formed by stacking the laminations with long, overlapping joints. Conventional coil-winding techniques are used.

(c) C-cores. The core material is in tape form and is wound to form a closed ring of rectangular shape, which is then cut into two parts. The parts are inserted into a winding and then strapped together again. By etching and lapping the cut faces of the cores, the air gaps at the cut faces are made extremely small when the cores are reassembled. Conventional coil-winding techniques are used.

(d) Laminations. The core material is stamped from sheets into laminations, or strips, and assembled into cores by stacking in accordance with the usual transformer-construction techniques. Windings are conventional.

13-3.42 Relative merits. The above methods of core construction are listed in the order in which they are employed for producing increasing values of magnetic-amplifier power capability. Where the control signal is low-level and must be conserved, the spiral core construction produces amplifiers with maximum gain and sensitivity. Such amplifiers are used for low-power applications (below the range of 10 to 100 watts of power output). The D-U core construction is favored by some manufacturers for ranges of power output between 1 and 1000 watts, whereas other manufacturers may not employ it at all. Magnetic amplifiers employing C-cores range in output power from 0.5 to 10 kw. Finally, laminated core construction might be used in amplifiers for output powers of 5 kw and higher. However, amplifiers for the usual range of servomechanism applications employ primarily the spiral and D-U types of core construction.

13-3.43 Core Materials

Core materials are identified with various sections of the power spectrum just as types of core construction are, and for the same reasons. The core materials are distinguished from one another by the alloys used, the manner of processing and magnetization orientation. Their electrical behavior is distinguished primarily by the saturation flux density, which is a measure of the power output per core, and also by the magnetic ΔH necessary to obtain a change of flux density ΔB, which is a measure of the sensitivity and gain per core. There are many other factors, such as susceptibility to shock, dependence upon temperature, and so forth, which are functions of particular applications. Complete information is available from the manufacturers of special core materials.[26] The following are the principal types of core materials, arranged in the order of power capability :

(a) Supermalloy. This is a 79%-Ni, 16%-Fe, 5%-Mo alloy that is used in the lowest-power and most-sensitive magnetic amplifiers. The saturation flux density is 7900 gauss and the coercive force is 0.004 oersted.

(b) Mo-Permalloy. This alloy and its heat treatment differ slightly from Supermalloy. The material is also known by the trade names Hy-Mu-80 and Squaremu. Amplifiers using Mo-Permalloy are nearly as sensitive as those using Supermalloy cores, and the material costs about the same. The saturation flux density is 8800 gauss and the coercive force is 0.035 oersted.

(c) 50% Ni-Fe Oriented. This alloy is known by the trade names of Deltamax, Orthonol, and Hypernik V, and is the core material most used for the 0.1-watt to 100-watt range of spiral (toroidal) core magnetic amplifiers. It has a saturation flux density of 15,500 gauss and a coercive force of 0.08 oersted, and its cost per pound is about 25 percent of the cost of Supermalloy and Mo-Permalloy.

(d) 3%-Si Oriented. This material is used for grain-oriented transformer cores, either as C-cores or as laminations of some kind. The saturation flux density is 19,700 gauss and the coercive force is 0.4 oersted for the 0.002-inch thickness.

(e) Silicon steel. Conventional, nonoriented, 5% silicon steel has a saturation flux density of 19,700 gauss and a coercive force of 1.0 oersted, but requires 1400 oersteds to reach saturation flux density. The cost per pound is the lowest of the core materials described here.

13-3.44 Applications. In the design of magnetic amplifiers, the core material that is just adequate to handle the sensitivity and power requirements of the application will produce the most economical, stable, and easily manufactured amplifier. For this reason, spiral cores using materials such as Supermalloy or Mo-Permalloy are used only for low-power, sensitive stages, whereas 3%-Si Oriented steel in C-cores suffices for high-power stages. The **50%** Ni-Fe Oriented material, in spiral or D-U core form, is used for intermediate-power applications.

13-3.45 Rectifiers

There are three types of rectifiers currently in use for magnetic amplifiers—selenium plate, germanium junction, and silicon junction. In the selenium rectifier, rectification takes place over a large surface, at current densities in the order of 0.3 ampere per square inch. In the semiconductor rectifier (germanium or silicon), rectification takes place at a PN junction within the material, at current densities in the order of 30 amperes per square inch. The three types of rectifiers have definite spheres of use in magnetic amplifiers.

13-3.46 Characteristics of selenium rectifiers. Existing types of selenium rectifiers must be derated for operation above 35°C ambient temperature and cannot be operated above **75°C.** However, there is promise of a new high-temperature selenium rectifier that can be operated at ambient temperatures up to 85°C without derating and at an ultimate ambient of 125°C. The back leakage current of selenium rectifiers, which so strongly determines magnetic-amplifier performance, ranges from 0.1 to 1.0 percent of the forward current at cell inverse voltages in the order of 10 to 15 volts.

13-3.47 Characteristics of germanium rectifiers. The germanium rectifier is rapidly growing in acceptance and can be used both as a rectifier in very sensitive stages and as a power-stage rectifier. The nominal ambient temperature for germanium rectifiers is 55°C, with an ultimate of 85°C, while the junctions themselves may be operated up to 100°C. In diodes of the 5-ma type, such as the 1N54, the back leakage is approximately 0.2 percent of the forward current at 55°C and 10 volts peak inverse voltage. In the intermediate-current diode, such as the 1N315, the leakage is 0.2 percent of the forward current for a 300-volt peak inverse voltage and a 75-ma forward current.

13-3.48 Characteristics of silicon rectifiers. In silicon rectifiers, there is considerably lower leakage current than in germanium rectifiers operated at the same temperature. Silicon units can also be operated at higher temperatures than selenium or germanium units. The rated case temperature is in the range from 100°C to 135°C, while the ultimate is in the range from 150°C to 200°C. In typical performance, a I-ampere silicon power rectifier (Type 1N253) at 135°C case temperature has a maximum leakage current of 0.01 percent of the forward current (0.1-ma leakage) for **95** volts peak inverse voltage. A corresponding Type 1N256 rectifier has a forward current rating of 0.2 ampere and a leakage in the order of 0.1 percent at a peak inverse voltage of 570 volts.

13-3.49 Factors governing choice of rectifier type. The rectifier field is progressing so rapidly at present that few rules can be given for the selection of rectifiers for magnetic-amplifier work. In sensitive amplifiers, the rectifier with the lowest leakage over the operating temperature range should be used. In power-stage application, considerations of temperature, space, and cost govern the choice of rectifier types. Many of these types require forced-air or water cooling unless sharp derating is applied. These factors may be more important in the selection of the power-stage rectifier than the electrical characteristics of the rectifier.

13-3.50 SPECIFICATIONS AND DESIGN

13-3.51 Specifications

Since magnetic amplifiers are frequently supplied as a package to fit between input, output, and source terminals, the specifications of the amplifier must include as much of the following information as can be supplied to the, designer:

(a) Load. The electrical nature of the load; the required maximum power output; the minimum tolerable power output; the requirement for ac or dc; the polarity, phase, and ripple requirement ;the class of magnetic-amplifier circuit.

(b) Control. The range and maximum value of control power ; the internal impedance and voltage of the control-power source ; the range of acceptable response speeds, or time constant; the required linearity **of** response ; the drift level.

(c) Source. The frequency and voltage **of** the source; the ranges of fluctuation; the power and capacity of the source; the availability of bias power.

(d) Environment. The range of operating temperatures ;storage temperature ;the availability of coolants ; environmental conditions of humidity, corrosive fumes, and radiation.

(e) Physical. The size, weight, and shape requirements ; shock ; requirements for impregnation, packing, and so forth.

13-3.52 Approach to Design

Where a packaged magnetic amplifier that meets the specifications is not already available, it may be necessary to originate an amplifier design. **A** specific design procedure will not be given here, because the number **of** factors that must be treated is too great. However, the steps of design will be outlined so that detailed procedures can be inserted when necessary.[32]

(a) Circuit. Select a circuit to accomplish the amplifier function.

(b) Figure of Merit. From the required gain and time constant, compute the Figure **of** Merit and check with curves, such as Fig. **13-76,** to determine the realizability and expected value.

(c) Cores. Select several possible cores and materials from manufacturers' catalogs and curves, such as Fig. **13-73,** that may be satisfactory.

(d) Gate winding. Design the gate winding so that the core is able to absorb the necessary voltage to reduce the load current to zero (or to the prescribed value). Take source-voltage variation into account. Select wire size by current and heating or by winding resistance. Check space availability in cores and check temperature rise.

(e) Rectifiers. Select rectifiers for amplifier function, peak inverse voltage, forward current, and temperature conditions. Evaluate leakage current and its effect on the core.

(f) Control circuits. Design the control windings for required reset ampere turns, leakage effect, and desired range of control. Correct design may have to be determined experimentally. Design control-circuit resistance for the required time constant. Design bias windings and bias circuits.

With this design information, a prototype amplifier can be built and tested for compliance with the specifications. Since it is almost impossible to design an amplifier with a predictable performance over a wide temperature range, an experimental evaluation is required.

13-4 ROTARY ELECTRIC AMPLIFIERS*

134.1 TYPES OF ROTARY ELECTRIC AMPLIFIERS

13-4.2 Basic Principles

Functionally, a rotary electric amplifier is a rotating machine in which the power flow from a mechanical drive to an electrical system is controlled **or** modulated **by** an electrical command or signal injected into the control field of the machine. The mechanical drive acts as a power source and the electrical system serves **as** the load. Amplification is achieved because a relatively small amount **of** input power to the control circuit can **con**trol a much larger amount **of** power to the load.

13-4.3 Power-Handling and Time-Constant Characteristics

Compared with magnetic and electronic amplifiers, rotary electric amplifiers as a class

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are characterized by their much larger power-handling capabilities (up to **5000 kw**) and by their large time constants (from a few milliseconds to a few seconds). In the low-power class, however, there is some overlap between rotary and magnetic amplifiers.

13-4.4 Basic Features

Figure 13-77 is a pictorial representation of the basic features of the rotary electric amplifier as viewed from its terminals. **As** a rule, the field system comprises several windings, where input control is effected. The shaft constitutes the mechanical source of power and the electrical load appears at the output circuit, also called the armature. In Fig. 13-77, the electrical terminals are defined in terms of voltages v and currents i, while the mechanical terminals are defined in terms of an applied torque T and an angular speed ω.

13-4.5 Types of Rotary Electric Amplifiers

Many different rotating machines operating as electric generators can be considered as power-amplifying devices, in accordance with the above characterization of a rotary amplifier. **For** example, a synchronous generator (alternator) may qualify as a rotary

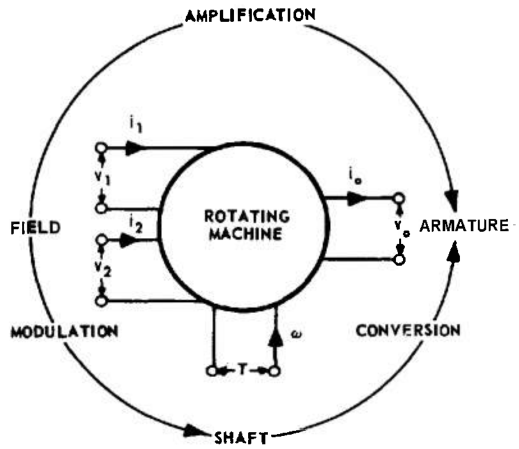


Fig. 13-77 A rotary electric amplifier.

power amplifier in which **d-c control is effected** in the field and a-c modulated signals appear in the armature. However, in servo systems, it is customary to restrict the term rotary amplifier to those commutator-type machines that have d-c inputs and outputs. Therefore, this section will deal exclusively with d-c generators especially suited to control applications. The most widely used d-c rotary amplifiers are:

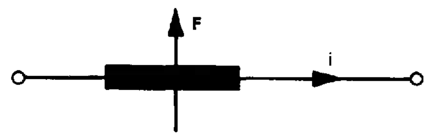
(a) Single-stage generators
(1) Conventional types
- a. Separately excited
- b. Self-excited or shunt-excited
- c. Compound-excited

(2) Special types
- a. Split-field
- b. Multified
- c. Tuned-field

(b) Multistage generators
(1) Cross-field machines
- a. Amplidyne
- b. Metadyne

(2) Superposed heterogeneous-pole systems

13-4.6 **Excitation.** The various types **or** rotary amplifiers are the result of combining several basic types of excitation inside the machine, as illustrated in Fig. 13-78. For simplicity, only two-pole generators are shown. **A** field coil is represented by the symbol



where the open-head arrow F indicates the direction of magnetomotive force that corresponds to a current flow in the direction of arrow i. This symbol may be interpreted as a profile view of the winding. Single-stage arrangements employ excitation **of** the type

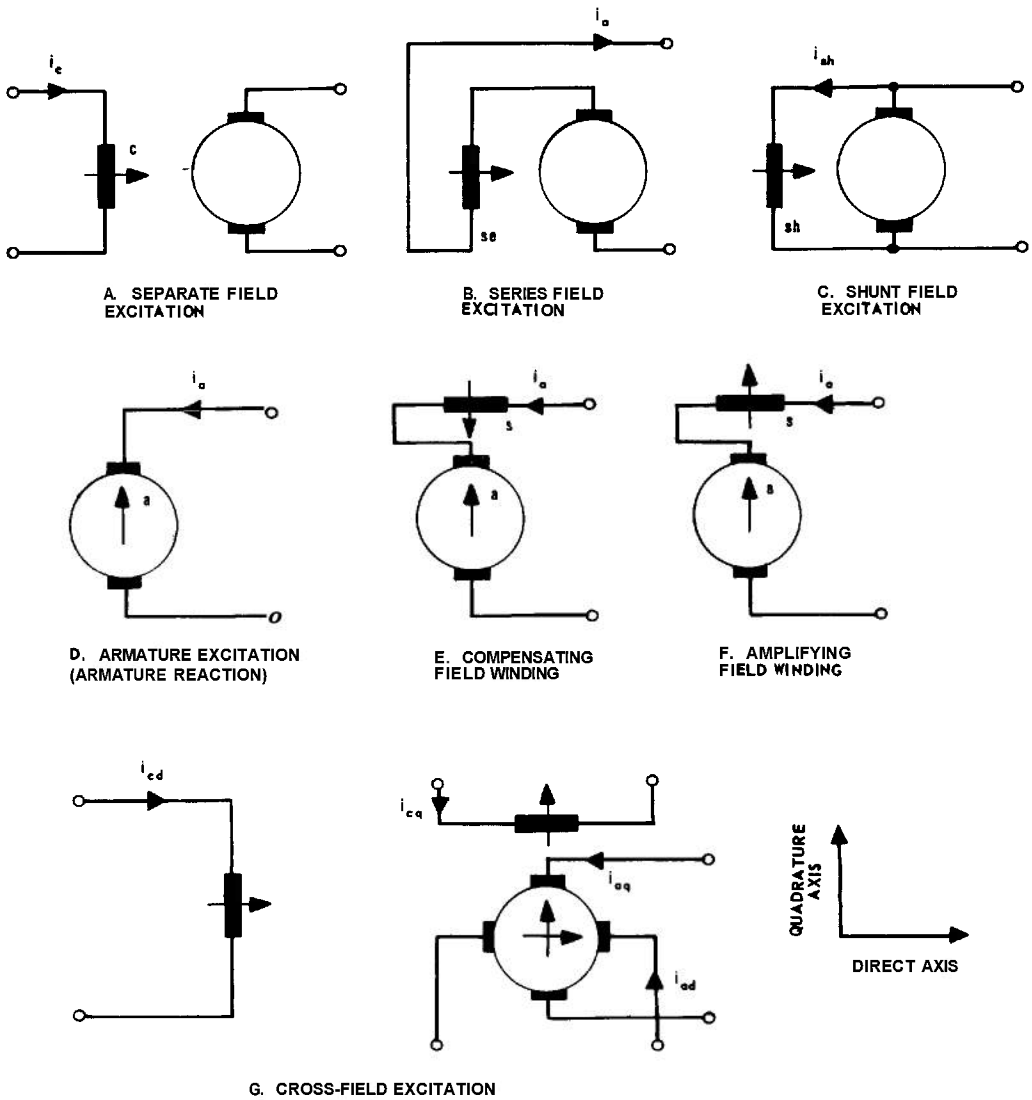


Fig. 73-78 Types of excitation in d-c rotary amplifiers.

shown in A, B, or C of Fig. 13-78. In these arrangements, one or more field windings on the stator create the necessary excitation in the direct axis to develop an output-speed voltage in the quadrature axis. In control applications, a separately excited control winding, denoted here by the letter **c**, is usually energized by a control-voltage signal. In addition to this control winding, other windings provide various forms of series (current) or shunt (voltage) excitation. Each of these may oppose or aid the original control excitation, thereby producing negative (suppressive) or positive (cumulative) feedback. A three-field rotary amplifier may be equipped with a control winding **c**, a positive-feedback series winding **se**, and a negative-feedback shunt winding *sh*.

13-4.7 Single-shunt-winding machine. A d-c generator with only a single shunt winding (Fig. 13-78C) relies on the nonlinear relationship between excitation and flux (or voltage output at a given speed) for its operation. The output voltage is controlled by adjusting the resistance in the shunt-field circuit. This type of control, which may be termed parameter-type control in contradistinction to the more common source-type control, is achieved by some on-off mechanism in the field circuit, such as vibrating-finger contacts, or by an externally actuated variable resistance, such as a carbon pile or rheostat. Parameter-type control of a d-c generator is frequently used in regulating systems.

13-4.8 Armature reaction. Currents in the armature of a d-c machine can also provide excitation. This so-called armature reaction as illustrated in Fig. 13-78D is in line with the brushes. In a single-stage amplifier, the brushes are normally located in the neutral zones, so that armature and field excitations are in space quadrature. If the machine is not saturated, armature reaction (the armature excitation due to load current) does not affect the total working field flux, although it distorts the field distribution and affects the commutation of the machine at various loads. When saturation is present, the armature-flux and field-flux systems interact, so that armature currents create a coupling effect with the main field. This deleterious effect is often referred to as the demagnetizing action of armature reaction. To combat the ill effects of armature reaction, a single-stage rotary amplifier is usually equipped with a series winding, called a compensating winding, which is excited by armature current. This produces a field opposing that of armature reaction, thereby nullifying the reaction (Fig. 13-78E). To improve commutation, commutating windings are also used on commutating poles located above the neutral zones.

13-4.9 Cross-field machine. In some specially designed d-c rotary amplifiers, armature reaction is actually put to use to provide the main working excitation. An armature-excited machine is also called a cross-field machine because two systems of flux in space quadrature are made to interact in the same machine. This cross-field arrangement is the result of adding extra pairs of brushes between those normally used in single-stage construction. To assist commutation, openings are provided in the main poles above the commutating zones, and commutating poles are sometimes fitted in these spaces. The cross-field machine thus appears to have $2p$ polar projections, although its stator and rotor flux systems are still arranged for p poles. The basic cross-field type of excitation is illustrated in Fig. 13-78G. Because of its double utilization of the armature, a cross-field machine essentially combines two-stage amplification with various forms of internal feedback, thereby providing inherent characteristics that are specially suited for control purposes. These characteristics originate from the various ways in which feedback may be used (by interconnecting stator and armature windings in different configurations).

13-4.10 Other multistage machines. In addition to the two-stage cross-field amplifier, there are other types of multistage single-unit machines that involve the superposition of flux systems corresponding to different numbers of poles. However, because of the resulting complexity in design, these heterogeneous-pole multistage amplifiers are seldom used in servo applications, and are therefore not discussed in this section.

13-4.11 CHARACTERISTICS OF ROTARY ELECTRIC AMPLIFIERS

13-4.12 Steady-State and Transient Characteristics

The steady-state and transient characteristics of the various types of rotary electric amplifiers previously described may be obtained in two steps. In the first step, consideration is given to the general case of a multifield single-stage generator with independent field excitation ; the second step considers the particular configuration of an amplifier with specific interconnections of the windings to provide desired feedback actions. In the derivation of the characteristics of rotary electric amplifiers, the following assumptions are usually introduced :

(a) The brushes are assumed to be located in the neutral zones.

(b) Any shift in brush position can be considered to produce an effect equivalent to that of a series field winding.

(c) The effects of the short-circuited armature coils undergoing commutation are neglected.

(d) The effects of saturation, hysteresis, and eddy currents are ignored.

(e) The generator is assumed to be driven at a constant speed n_o.

13-4.13 Deriving input-output characteristics. Each field circuit, assuming a total of q fields, is governed by a differential equation of the form

$$V_k = R_k i_k + \sum_{j=1}^{q} M_{kj} \frac{di_j}{dt} \quad (k = 1,2,3, \ldots, q) \tag{13-63}$$

where M_{kj} represents the mutual inductance between windings k and j, and M_{kk} (also denoted &) is the self-inductance of winding k. The q equations may be solved for the field currents i_k. The generated speed voltage resulting from the combined action of all q windings, with due regard to excitation polarities, is then obtained from

$$V_o = \sum_{k=1}^{q} K_k i_k \tag{13-64}$$

where K_k is the linear speed coefficient (or sensitivity) associated with winding k, in volts per ampere at base speed n_o. The terminal or output voltage differs from this generated voltage V, by the amount of armature-impedance drop due to load current i_o. That is,

$$V_t = V_o - \left[R_a i_o + L_a \frac{di_o}{dt} \right] \tag{13-65}$$

where R_a and L_a represent the armature resistance and inductance, respectively. In addition to Eqs. (13-63), (13-64), and (13-65), an equation describing the load is needed to completely specify the problem of determining the input-output characteristics of the amplifier.

13-4.14 Simplifying characteristics derivation. The derivation of amplifier characteristics is greatly simplified if two further assumptions are made. The first assumption is to neglect the armature-impedance drop compared with the generated voltage [the second term in Eq. (13-65)] or to consider this impedance as lumped with the load supplied by the generator. The second assumption is to regard all q field windings as being perfectly coupled because they are placed on the same poles and link the same total field flux. A fixed portion of this field flux links the armature and is accounted for by a constant coefficient of dispersion ξ_f, defined as the ratio of field flux to air-gap flux. Both assumptions are reasonably warranted in rotary electric amplifiers used in control applications. With these assumptions, the machine equations can be written as

$$V_k = R_k i_k + p \xi_f N_k \frac{d\phi}{dt} \quad (k = 1,2,3, \ldots, q) \tag{13-66}$$

for the q field circuits and, for the armature,

$$V_t = V, = \frac{p}{a} z n_o \phi \tag{13-67}$$

where

p = number of poles

a = number of parallel paths in armature

z = total number of armature conductors

N_k = number of turns per pole of winding k

n_o = rated speed

4 = air-gap flux per pole

The air-gap flux ϕ per hole in the linear region of operation is related to the resultant ampere turns by the expression

$$\phi = \mathcal{P}_g \sum_{k=1}^{q} N_k i_k \tag{13-68}$$

in which $\mathcal{P}_g$ denotes the permeance of a single air gap. Equation **(13-67)** may be rewritten as

$$V_t = V_o = \frac{p}{a} z n_o \mathcal{P}_g \sum_{k=1}^{q} N_k i_k = \sum_{k=1}^{q} K_k i_k \tag{13-69}$$

Thus, the sensitivity of a field winding is, in terms of dimensions,

$$K_k = \frac{p}{a} z n_o \mathcal{P}_g N_k \tag{13-70}$$

The unsaturated inductance L_k of field winding k is given by

$$L_k = p \xi_f \mathcal{P}_g N_k^2 \tag{13-71}$$

After solving Eq. **(13-66)** for the field currents, using the Laplace operational method, and substituting these values into Eq. **(13-69),** the over-all input-output function may be written as

$$V_o = \frac{\sum_{k=1}^{q} (K_k/R_k) V_k}{1 + s \sum_{k=1}^{q} L_k/R_k} = \frac{\sum_{k=1}^{q} \alpha_k V_k}{1 + s \sum_{k=1}^{q} \tau_k} \tag{13-72}$$

where

$\alpha_k = K_k/R_k$ = voltage gain at zero frequency or d-c condition

$\tau_k = L_k/R_k$ = time constant

V_k = signal voltage of the kth field circuit

Equation **(13-72)** shows that the multifield d-c rotary amplifier behaves dynamically as a single time-lag system in which the equivalent time constant equals the sum of the individual time constants for all field windings.

13-4.15 Effects of saturation and variable speed. When the effects of saturation or variable-speed operation must be included in the analysis, a transfer function such as Eq. **(13-72)** cannot be derived. However, the machine dynamics can be expressed in the form of a block diagram such as Fig. **13-79.** The saturation effects are accounted for by load saturation curves, which are nonlinear relationships of the form

$$V_o = S(\sum K_k i_k ; i_o) \tag{13-73}$$

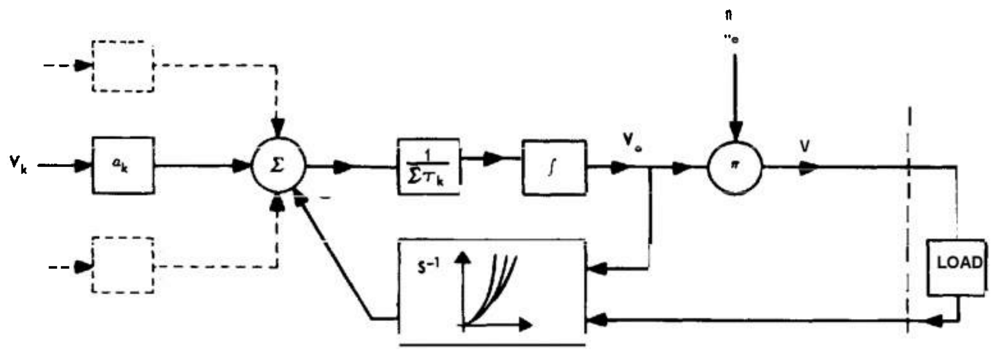


Fig. 13-79 Block diagram representation of multifield d-c rotary amplifier.

where S denotes a saturation function that has a unity initial slope corresponding to the linear portion of the magnetization characteristic of the machine. (S^{-1} represents the inverse saturation function.)

13-4.16 Results of saturation. As a result of saturation, the armature reaction caused by load current i_o creates a demagnetizing action on the main field so that the field and armature fluxes are effectively coupled together, even though they act in space quadrature. In many servo-application designs of single-stage rotary amplifiers in which compensating windings are used, the effect of armature reaction is very small. A single no-load saturation function can then be used to represent saturation non-linearity in the amplifier.

13-4.17 Results of speed variations. Speed variations depend upon the dynamics of the rotary-amplifier drive and introduce a product-type nonlinearity, as indicated by the multiplier in the block diagram of Fig. **13-79.** If the speed is constant, this multiplier is simply replaced by a gain factor that represents a per-unit speed.

13-4.18 Linear operation. If the amplifier is linearized around an operating point, the relationships between incremental changes can be obtained from the block diagram of Fig. **13-79** by replacing the feedback nonlinear block S^{-1} with an incremental gain factor equal to the slope of the inverse saturation function at the operating point. Saturation then produces an equal diminution of gain and time constant which is a function of the operating point. If operation is confined to the linear portion of the magnetization curve, the feedback becomes unity.

13-4.19 Derivation of dynamic characteristics. The dynamics of most types of generally used rotary electric amplifiers can be derived from the general representation of the multifield d-c generator by suitable inter-connections and feedback combinations. Table **13-6** summarizes the characteristics obtainable from some basic configurations of rotary electric amplifiers. The second column in the table shows block-diagram representations particularly suited to analog computer studies because integrators are used. These representations include the effects of saturation nonlinearity but, for simplicity, assume constant base speed operation of the amplifier. The third column in the table presents the transfer functions for machines operating in the linear region and at constant speed; the corresponding root loci are shown in the last column of the table. The distinguishing features of the various configurations in the table are briefly discussed below:

(a) Configuration A. The separately-excited generator is one of the most common types of rotary electric amplifiers. Dynamically, it is characterized by a gain $\alpha_f = K_f/R_f$ and a time constant $\tau_f = L_f/R_f$.

(b) Configuration B. Essentially, this arrangement provides external positive-voltage feedback with a resulting increase in both gain and time constant.

(c) Configuration C. This amplifier operates in its nonlinear saturating region in response to an input-signal change, which is the field-circuit resistance. The system then involves a parameter-type control for which only a block-diagram representation is given in Table **13-6.**

(d) Configuration D. This configuration is similar to B, but the feedback is provided inside the machine instead of externally. The gain a_c, due to the control winding acting alone, is increased by the positive-feedback gain α_{sh} of the shunt winding. However, the equivalent amplifier time constant $\tau_c + \tau_{sh}$ is also increased.

(e) Configuration E. When the shunt feedback is adjusted to unity (a process often known as *tuning*), the amplifier behaves as a perfect integrator, implying an infinite steady-state gain. This is theoretically true; however, in practice, the gain (which may be quite large) is very sensitive to the tuning condition $K_{sh} = R_{sh}$ as it is affected by slight changes in speed, temperature, and nonlinearities in the magnetization characteristics.

(f) Configuration F. To increase the gain of this amplifier, series positive feedback is used. The system dynamics are then dependent upon the type of load supplied by the amplifier.

(g) Configuration G. A particular form of amplifier is achieved by using a tuned series-field generator to excite the field winding of another main generator. The tuned condition is realized when the total armature-circuit resistance R_o (which includes the resistance of the main generator field) equals the sensitivity K_{se} associated with the series winding. A tuned series-field machine operating linearly with an inductive load provides a form of integration as expressed in the transfer function given in Table 13-6. A series-tuned rotary electric amplifier is normally not suited for use with a capacitive-type load, such as a motor in a Ward-Leonard control system (see Par. 14-2).

All seven of the above configurations refer to single-stage d-c generators in which positive feedback is used in various ways to increase the gain. Other arrangements can be devised by using different combinations of series and shunt feedback, which act positively as well as negatively. The effects of these feedback combinations may readily be studied by examining the corresponding block diagram derived from the general case given in Fig. 13-79. The last two configurations in Table 13-6 depict rotary electric amplifiers in which two stages of amplification are combined in a single machine. Essentially, these cross-field generators use armature reaction as a source of excitation by providing an extra set of brushes in the direct axis (across which a load is connected) and by short-circuiting the quadrature brushes. The direct-axis excitation is the result of a control-field excitation and an opposing direct-axis armature reaction due to load current, thereby creating a generated voltage in the quadrature axis. The resulting current in the short-circuited axis, in turn, creates the generated output voltage in the direct axis. Therefore, a cross-field armature-excited machine can be considered as two machines in cascade, with inherent negative output current feedback.

(h) Configuration H. The characteristics of a cross-field machine are basically those of a constant-current source because of the output current feedback. Dynamically, the feedback improves the response speed of the amplifier, but at the expense of a decrease in over-all amplification. Therefore, when such a machine is used as an amplifier, it is usually equipped with a compensating winding in the direct axis to partially overcome the output armature reaction. With 60 to 90 percent compensation, the characteristics of the machine are essentially those of a constant-current source with inherent large power amplification.

(i) Configuration I. If output compensation is perfect (100 percent), the characteristics of the amplifier approach those of a constant-voltage source. With the output feedback thus cancelled, the amplification becomes very large.

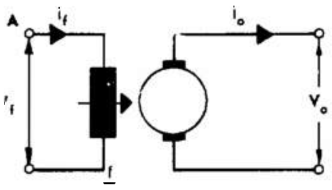
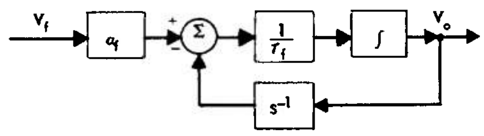
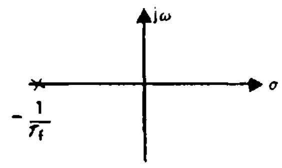
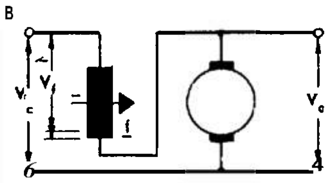
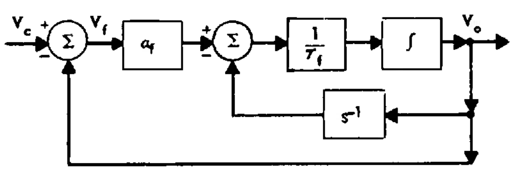
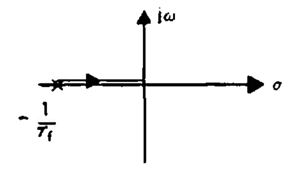
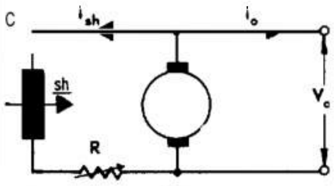
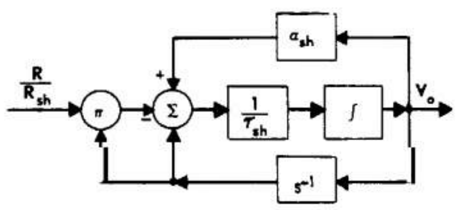
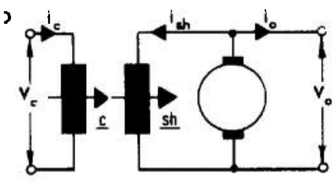
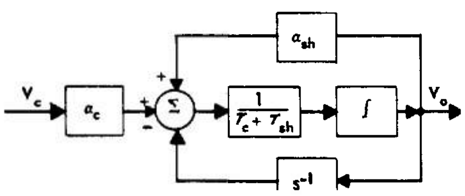
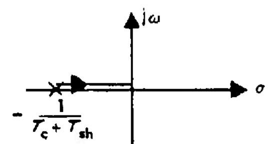
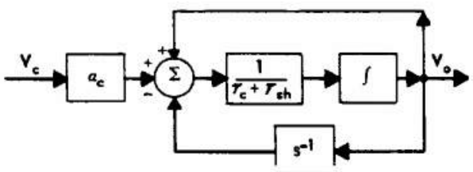
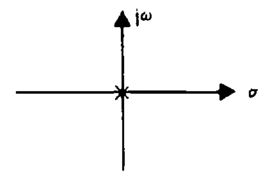
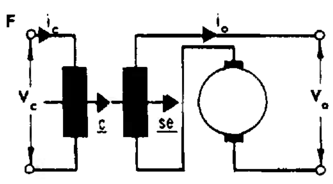
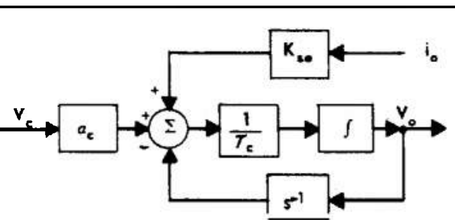
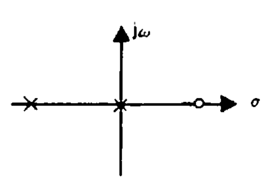
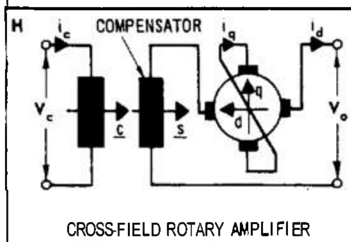
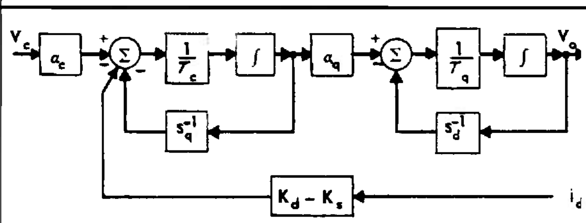
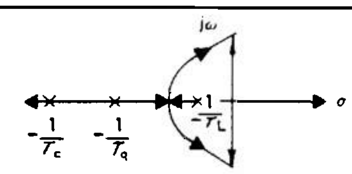
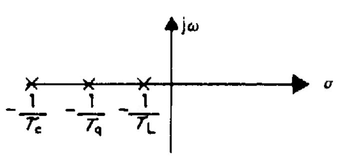
The dynamic characteristics of the two basic forms of cross-field amplifiers (configurations H and I) are expressed in the form of block diagrams and transfer functions in Table 13-6. As far as saturation in these machines is concerned, the given representations are based on the assumption that the cross-flux systems do not interfere with each other. This assumption is warranted in most control applications in which the machine amplifiers do not normally operate under pronounced saturating conditions. Many other configurations, in addition to those depicted in Table 13-6, can be achieved by adding other stator windings (shunt and series) that provide various forms of positive or negative feedback along the two axes of the cross-field machine. The action of these windings may be studied in the manner used for single-stage machines.

13-4.20 PARAMETERS OF D-C ROTARY AMPLIFIERS

13-4.21 Fundamental Requirements

A fundamental requirement for a rotary electric amplifier is that the inductive or stored energies must be kept small in relation to the transmitted power. Therefore, it is

TABLE 13–6 DYNAMIC CHARACTERISTICS OF SOME BASIC CONFIGURATIONS OF ROTARY ELECTRIC AMPLIFIERS

CONFIGURATION OF ROTARY ELECTRIC AMPLIFIER	ANALOG COMPUTER BLOCK DIAGRAM	TRANSFER FUNCTION (FOR LINEAR REGION OF OPERATION)	ROOT LOCUS (LINEAR OPERATION)
A  SEPARATELY EXCITED GENERATOR		$\frac{V_o}{V_f} = \frac{\alpha_f}{1+\tau_f s}$ $\alpha_f = K_f/R_f$ $\tau_f = L_f/R_f$	
B  BUCK-BOOST SYSTEM		$\frac{V_o}{V_f} = \frac{\alpha_f}{1-\alpha_f}\,\frac{1}{1+\frac{\tau_f}{1-\alpha_f}s}$ $\alpha_f < 1$	 LOOP GAIN = α_f
C  SHUNT GENERATOR WITH RHEOSTATIC FIELD CONTROL		$\alpha_{sh} = K_{sh}/R_{sh}$ $\tau_{sh} = L_{sh}/R_{sh}$ $\alpha_{sh} > 1$	—
D  AMPLIFIER WITH SHUNT POSITIVE FEEDBACK		$\frac{V_o}{V_c} = \frac{\alpha_c}{1-\alpha_{sh}}\,\frac{1}{1+\frac{\tau_c+\tau_{sh}}{1-\alpha_{sh}}}$ $\alpha_c = K_c/R_c$ $\alpha_{sh} = K_{sh}/R_{sh} < 1$	 LOOP GAIN = α_{sh}
E SAME CONFIGURATION AS D WITH TUNED SHUNT FIELD		$\frac{V_o}{V_c} = \frac{\alpha_c}{\tau_c+\tau_{sh}}\,\frac{1}{s}$ $\alpha_{sh} = 1$	
F  AMPLIFIER WITH SERIES POSITIVE FEEDBACK		—	—
G SAME CONFIGURATION AS F WITH TUNED SERIE FIELD AND INDUCTIVE-TYPE LOAD	— $K_{se} = R_o$	$\frac{i_o}{V_c} = \frac{\alpha_c}{K_{se}(\tau_c+\tau_L)}\,\frac{1-\frac{L_{se}}{K_{se}}s}{s\left(\frac{\tau_c\tau_L}{\tau_c+\tau_L}s+1\right)}$ $\tau_c = L_c/R_c$; $\alpha_c = K_c/R_c$ τ_L = LOAD TIME CONSTANT; K_{se} = RESISTANCE)F ARMATURE LOAD CIRCUIT R_o	
H  CROSS-FIELD ROTARY AMPLIFIER		$\alpha_c = K_c/R_c$; $\tau_c = L_c/R_c$ $\alpha_q = K_q/R_q$; $\tau_q = L_q/R_q$ $\frac{K_s}{K_d}$ = PER UNIT COMPENSATION	 .OCUS OF POLES OF I_c/V_c WITH INDUCTIVE .OADING [LOOP GAIN = $\alpha_q/R_L \times (K_d-K_s)$] .
I SAME CONFIGURATION AS H WITH 100% COMPENSATION	— $K_d = K_s$	$\frac{V_o}{V_c} = \frac{\alpha_c\alpha_q}{(1+\tau_c s)(1+\tau_q s)}$	 LOCUS OF POLES OF I_c/V_c WITH INDUCTIVE LOADING (LOOP GAIN = ZERO)

important to be able to evaluate the various inductances, gains, and time constants that govern the dynamic characteristics in terms of the design and dimensions of the machine. Furthermore, it is important to be able to relate these parameters to the power-transfer capabilities of the amplifier.

13-4.22 Stored energy. When a separately excited d-c generator is operated in its unsaturated region, the function of the field system is to produce the necessary excitation magnetomotive force (mmf) for a specified flux ϕ in the air gap of the machine. The establishment of such a flux is paralleled by storage of energy in the field, resulting in the time lags exhibited by the machine. This stored energy in the air gap can be written in the following equivalent forms:

$$W_f = \frac{1}{2} L_f I_f^2 = \frac{1}{2} p\, (N_f I_f)\, \phi = \frac{1}{2} p \frac{\phi^2}{\mathcal{P}_g} \tag{13-74}$$

For rated conditions, the power dissipated in the field winding (omitting external resistance) is

$$Pf = R_f I_f^2 \tag{13-75}$$

Therefore,

$$Pf \times \tau_f = 2W_f \tag{13-76}$$

This relationship, which states that the product of field-power loss and time constant is twice the stored magnetic energy, is of fundamental significance in the selection of d-c generators as power amplifiers in control systems. The relationship shows the importance of keeping the energy storage small. This is accomplished by an increase in permeance or by a decrease in flux. A decrease in flux means a reduction in power output. With a required flux, permeance may be maximized within the limitations of the design, but the time constant and power loss are then in inverse proportion.

13-4.23 Resistance of field winding. The resistance of the field winding may be expressed as

$$R_f = p\, \rho l_m \frac{N_f}{a_f} = p\, \rho l_m \frac{N_f^2}{A_f} \tag{13-77}$$

where

a_f = cross-sectional area of the wire

A_f = area occupied by the field winding

l_m = mean length of a field-winding turn

ρ = resistivity per unit length

A given copper cross-sectional area A_f is assumed for each pole of the field winding, subject to a constant winding-space factor so that $A_f = N_f a_f$.

13-4.24 Dissipated power. Since, from Eq. (13-71), the unsaturated field inductance is

$$L_f = p\, \xi_f \mathcal{P}_g N_f^2$$

the field time constant is

$$\tau_f = \frac{L_f}{R_f} = \frac{\xi_f}{\rho l_m} A_f \mathcal{P}g \tag{13-78}$$

This relationship shows that the time constant of the field is proportional to the available winding space and to the permeance of the air gap, but is independent of the number of turns in the field winding. Similarly, the power dissipated in the field may be written as

$$P_f = R_f I_f^2 = \frac{R_f}{N_f^2} (N_f I_f)^2 = p\, \rho l_m \frac{(N_f I_f)^2}{A_f} \tag{13-79}$$

which shows that the dissipated power is directly proportional to the square of the field ampere turns and inversely proportional to the winding area.

13-4.25 Basic information for field-system design. When designing the field system of a d-c rotary amplifier, the following information is generally required:

(a) The ampere turns developed by each coil per pole

(b) The mean length l_m of a field-winding turn

(c) The permissible loss P_f in the total winding

Item (c) defines the power level of the input. If the voltage level at the field input is further restricted to a fixed value V_f, then the cross-sectional area of the conductor is fixed and is given by

$$a_f = p \, \rho l_m \frac{(N_f I_f)}{V_f} \tag{13-80}$$

The selection of the number of turns is then determined by the permissible field loss. From Eq. (13-70), the linear speed coefficient K_f associated with the field winding is

$$K_f = \frac{p}{a} z n_o \mathcal{P}_g N_f$$

or

$$K_f = 2p^2 N_a n_o \mathcal{P}_g N_f \tag{13-81}$$

in terms of the equivalent number of armature turns per pole, $N_a = \frac{z}{2ap}$. Equations (13-77) and (13-81) show that the voltage gain $\alpha_f = K_f/R_f$ for field winding f is a function of the ratio of armature turns to field turns and is a variable parameter that depends upon the choice of wire sizes. Table 13-7 gives the relationships required to approximately evaluate the inductances and sensitivities associated with the various windings of single-stage and cross-field rotary electric amplifiers.

13-4.26 Power amplification. Whereas voltage gain may assume widely differing values for the same size machine, power amplification is a direct function of the design (i.e., the extent to which the electrical, magnetic, and mechanical properties have been stressed) and also of the time constant. An expression for power amplification may be written as

$$\frac{P_a}{P_f} = \frac{\text{armature power}}{\text{field power}} = \left[\mu_o \left(\frac{Q}{B_g}\right) \frac{v}{g}\right] \times \tau_f \tag{13-82}$$

where

Q = electric loading = $\frac{\quad}{a\pi D}$

B_g = magnetic loading = maximum air-gap flux density

v = peripheral armature speed

g = length of air gap

D = diameter of armature

μ_o = permeability of air

This relationship shows that power amplification can only be achieved as dictated by a certain time constant. This is dependent upon the quantity in the brackets, which is the Figure of Merit of the machine. The Figure of Merit, defined as the ratio of amplification to time constant, is an expression of the dynamic qualities of the amplifier. This is because it provides a measure of the extent to which the inductive or stored energies have been diminished in relation to the transmitted power. A large Figure of Merit may be achieved by designing the following characteristics into the machine:

(a) Large rotational speed

(b) Large diameter and small air gap

(c) Large electric loading, but small magnetic loading

13-4.27 PROBLEMS ENCOUNTERED IN THE USE OF ROTARY ELECTRIC AMPLIFIERS IN SERVO APPLICATIONS

13-4.28 Design Factors

Several design factors, not previously discussed in detail, are described herein. These factors may affect the performance of a rotary electric amplifier used in a control application by causing an appreciable discrepancy between calculated and measured characteristics. In some cases, these factors may influence the choice of a particular type of amplifier for the application.

(a) Hysteresis. In rotary amplifiers with short air gaps, hysteresis is likely to be quite noticeable, especially if there are two stages of amplification. To minimize this effect, cross-field machines often include "killer" windings, in which alternating currents are injected.

TABLE 13-7 INDUCTANCES AND SENSITIVITIES ASSOCIATED WITH THE WINDINGS OF ROTARY ELECTRIC AMPLIFIERS

Winding	Single-Stage Generator	Cross-Field Generator
Field	$L_f = p\,\xi_f N_f^2 \mathcal{P}_g$ $M_{12} = p\,\xi_f N_1 N_2 \mathcal{P}_g$ $K_f = 2p^2 N_f N_a \mathcal{P}_g n_o$	(Stator has $2p$ polar projections) $L_f = p\,\xi_f N_f^2 \mathcal{P}_g$ $M_{12} = p\,\xi_f N_1 N_2 \mathcal{P}_g$ $K_f = 2p^2 N_f N_a \mathcal{P}_g n_o$
Armature	$L_a = \frac{1}{3} p\,\xi_a (\propto N_a)^2 \mathcal{P}_g$	(Armature has a full pitch lap winding) $L_a = \frac{1}{4} p\,\xi_a \left(1 + \frac{\propto^2}{3}\right) N_a^2 \mathcal{P}_g$ $K_a = p^2 N_a^2 \mathcal{P}_g n_o$

where

$\mathcal{P}_g =$ permeance of a single air gap for one pole or two polar projections

$$= \mu_o \times \frac{\text{area of pole face (including fringe)}}{\text{effective air-gap length}}, \text{ in webers/ampere turn}$$

$\mu_o = 4\pi \times 10^{-7}$

a = number of parallel paths in armature

$= p$ (for a lap winding)

$= 2$ (for a wave winding)

ξ_a = coefficient to account for leakage fluxes in slots, end windings, and interpoles $\cong$ *1.2*

ξ_f = field leakage coefficient = 1.1 to 1.25 *(13-126)*

$\propto$ = ratio of pole arc to pole pitch $\cong$ *0.67*

(b) Eddy currents. Eddy currents in the iron may be reduced by using laminated magnetic circuits. However, eddy currents may also appear in other places. For example, the short-circuited armature coils under the brushes (undergoing commutation) constitute a damping circuit with respect to the main pole flux. In the previous analysis, the effect of eddy currents may be conveniently included if they are represented by an equivalent short-circuited field winding that has appropriate resistance and inductance values.

(c) Commutation. Even though a d-c machine may commutate properly through the use of commutating and compensating windings, the mere existence of the commutator may create noise and interference problems. Commutator action is essentially substitution or switching of circuits under the brushes, producing discontinuities that are responsible for the appearance of high-frequency noise in the output signal. The lowest frequency of this noise, in cps, is the product of the number of commutator bars and the speed of the armature, or

$$f_{ripple} = bn_o \tag{13-83}$$

where b is the number of commutator bars and n_o is the speed in revolutions per second. This high-frequency ripple can be filtered out of the signal, but the filter may affect the dynamic response of the amplifier. High rotational speed is desirable from a noise standpoint. (Operation at high altitudes or at high temperatures imposes severe problems in commutation. Such environmental conditions may require the use of specially treated brushes, as well as special construction of the machine itself. The brush contact voltage contributes to the effective resistance of the armature circuits and this contribution, though often small, is highly nonlinear.)

13-4.29 SELECTION OF ROTARY ELECTRIC AMPLIFIERS FOR CONTROL PURPOSES

13-4.30 Controlling Factors

The selection of a rotary electric amplifier is governed by three main considerations:

(a) Output or load requirements. These reqcirements include the type of load (e.g., the armature of a motor, or the field of another rotating machine), the power level, and the specifications imposed by over-all system behavior.

(b) Input or control requirements. These requirements include the function to be performed by the rotary amplifier and, to a major degree, govern the design and choice of control-field excitation. In many applications, a d-c machine serves as an amplifying element as well as a regulator in the control system and, for this reason, often includes stabilizing and limiting features. The type of control source (or preamplifier) employed in the system imposes an impedance-matching problem with respect to the input control field of the amplifier because, in many systems, the preamplifier may be either an electronic or magnetic amplifier operating at a different impedance or voltage level. In push-pull type amplifiers, it is convenient to use a split-field d-c generator as a power amplifier. When the rotary amplifier also functions as a regulator, the error-measuring means may be achieved by a differential arrangement of dual-field windings. In many practical circuits, stabilization is included in the rotary electric amplifier because of the inherent large amplification. Stabilization may be in the form of a negative output voltage feedback, so proportioned as to reduce the time lag of the amplifier to secure stability without decreasing the amplification by a large amount. In other instances, feedback through frequency-sensitive elements such as transformers and capacitor-resistor arrangements is used to provide the required stabilizing action.

(c) Drive. The availability of a suitable drive is an important factor in the selection of a rotary electric amplifier. Where a separate drive motor is not available, an amplifier with an integral drive motor'is often indicated as the logical choice. The speed regulation of any drive motor must be held very close under steady-state and transient conditions **for** proper amplifier operation. Especially in the case of a tuned-field amplifier, the drive-motor speed must be constant;otherwise, the amplifier gain and stability will be noticeably affected. Figure 13-80 shows some practical control-circuit arrangements commonly encountered in the use of rotary electric amplifiers for control systems.

13-4.31 TYPICAL CHARACTERISTICS AND DESIGN DATA OF SOME ROTARY ELECTRIC AMPLIFIERS

13-4.32 Information Available from Manufacturers

In addition to the rating and nameplate data, the following is typical of the information available from manufacturers of rotary electric amplifiers especially designed for control applications :

(a) Machine field data. This usually gives a complete description of all control windings, including terminal markings, resistance, turns per pole, percent of total winding area for each winding, and maximum allowable current.

(b) Magnetic characteristics. Typical saturation curves for a rotary electric amplifier are shown in Fig. 13-81. The control ampere turns vary in accordance with the type and size of the amplifier. In a single-stage machine with no positive feedback, the base ampere-turn figure corresponding to one per unit may be in the order **of** 1000 ampere turns per pole; in a two-stage, 100-percent compensated cross-field amplifier, the figure may be only 30 ampere turns. In addition, the shape of a characteristic varies from one design to the next. In one case, the characteristic may exhibit a large residual voltage or indicate a pronounced armature reaction at high saturation levels. In another case, the characteristic may have a sharp bend in its nonlinear portion, as is common for **a** two-stage machine.

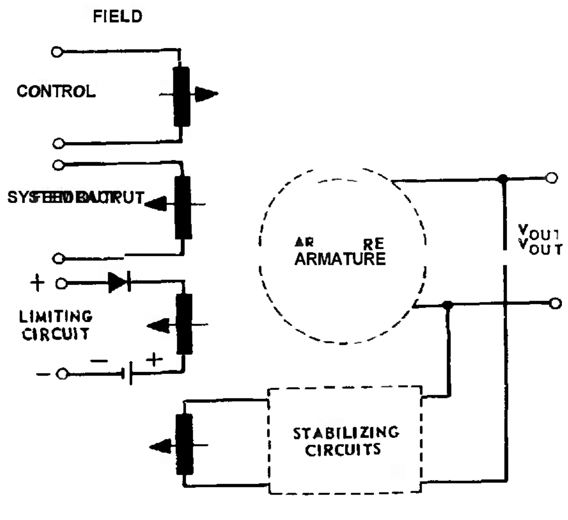


fig. 73-80 Types of control circuits used in rotary electric amplifiers.

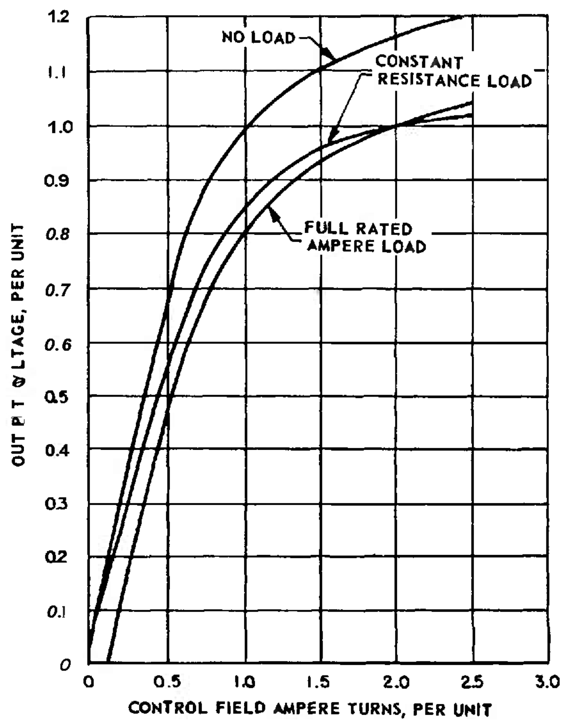


Fig. 73-87 Typical saturation curves for a rotary electric amplifier.

From the above data, it is possible to calculate the sensitivity associated with the different windings, thereby enabling the designer to choose the amplifier configuration that promises to produce the desired steady-state characteristics.

13-4.33 Dynamic Performance

Usually, little information is available concerning the inductances and time constants of the various windings. In this respect, Table **13-7** may prove helpful in estimating these constants from a knowledge of machine dimensions. Table **13-8** is suggested as a preliminary guide to probable values **of** parameters that may be encountered in rotary electric amplifiers. These values may vary widely, even for the same design, depending upon the type of added feedback or the type of preamplifier. However, these values indicate the range likely to be met.

TABLE 13-8 SOME TYPICAL VALUES OF PARAMETERS FOR ROTARY ELECTRIC AMPLIFIERS

Configuration	Power Amplification	Time Constant (seconds)	
Single-stage	**50** to 100	0.1 to 0.5	
Tuned-field	70 to **120**	**0.3** to 0.8	
Cross-field		field	quadrature
(100 percent compensation)	7,000 to 10,000	0.01 to *0.08*	0.05 to 0.2

13-5 RELAY AMPLIFIERS*

13-5.1 DEFINITION

A relay amplifier is essentially a compact, fast-acting switching device used to control the operation of an electric motor. Such an amplifier comprises one or more relays, depending upon the type of control to be effected and the magnitude **of** the power to be switched, together with associated sensing devices and controls. The input to a relay amplifier is a low-level control signal; the output is a control order to the motor at the power level **of** the motor input. In a servo system, relay amplifiers are used for positional and speed control of servomotors. **(A** detailed discussion of the theory **of** relay servomechanisms and other nonlinear devices is given in Ch. 10.)

13-5.2 ADVANTAGES AND DISADVANTAGES

Relay amplifiers may be used where space is at a premium and weight must be carefully controlled. However, electric power must **be** available for operation. One advantage provided by relay amplifiers in a servo system is that full power is instantly available to start a motor load against its starting or breakaway friction. When the input signal fluctuates over a narrow range, the relay amplifier can be designed to ignore these small fluctuations, thus avoiding needless output motion of the controlled motor. Low standby power is possible with the use of relay amplifiers because it is unnecessary to energize a motor winding when the motor is not running. For

**By P. E. Smith, Jr.*

a given motor and a given available power, the motor responds most rapidly to any change in the relay-amplifier input signal that calls for maximum motor speed. For a given motor, given input signal, and given available power, the relay amplifier is the smallest amplifier that produces the desired results, with the possible exception of a device using switching transistors. The relay amplifier, however, is more flexible than switching transistors in that a multiplicity of relay contacts can be used. Major disadvantages of the relay amplifier are life limitations and operating-time lag.

13-5.3 RELAY CHARACTERISTICS

13-5.4 Description of Operation

A relay is an electromechanical device capable of operating one **or** more switches. The relay can be arranged so that the application of relay-coil current causes an open single-throw switch to close or a closed single-throw switch to open. The same type of relay can be used to operate a double-throw switch from one position to the other upon application of coil current. A slightly different kind of relay (polarized) operates a double-throw switch but requires one polarity of current to close the switch in one direction, the opposite polarity closing the switch in the other direction. Zero current corresponds to the switch being open. The static and dynamic characteristics of relays are discussed in Pars. **13-5.20** and **13-5.24,** respectively.

13-5.5 Usage of Relays

Relays and the closely related contactors are used in a very wide range of applications, including motor and other electrical-equipment power controllers, digital computers, process controllers, and servomechanisms. The remainder of this section applies to the latter two fields.

13-5.6 SINGLE-SIDED RELAY AMPLIFIERS FOR SPEED CONTROL

13-5.7 Typical Amplifiers

Systems in which the controlled variable never becomes negative are frequently used to control temperature, pressure, liquid level, flow, and nonreversible motors. In such systems, single-sided control **or** two-position on-off control may be used. **A** single-stage relay circuit for a speed-control system is shown in Fig. **13-82.** Larger motors that draw a starting current of more than five amperes may require a second amplifier stage with a slave relay (Fig. **13-83).**

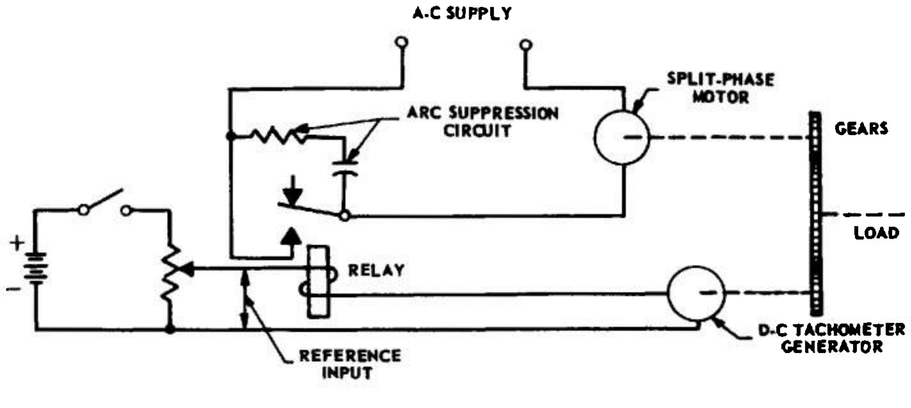


Fig. 13-82 Speed control using single-sided single-stage relay umplifier.

13-5.8 Operation. When the system of Fig. 13-82 is at rest, the battery switch is open. Operation is initiated by closing the switch, thereby energizing the relay which, in turn, connects the a-c supply to the split-phase motor. As the motor speed increases, so does the d-c tachometer-generator output, the polarity of which is opposite to that of the reference input. The resulting voltage across the relay coil is the difference between the reference input voltage and the tachometer-generator output voltage. As the tachometer-generator output approaches the magnitude of the reference input, the difference voltage becomes increasingly smaller, reaching zero when the opposing voltages are approximately equal. However, before the zero-voltage point is reached, the current through the relay coil decreases (in direct relationship to the decreasing difference voltage) until it is insufficient to keep the relay energized, and the supply to the motor is disconnected. This relay opening point is adjusted by means of the reference-input potentiometer, which thus becomes a speed control for the motor. The de-energized motor will slow down under the influence of inertia or the load. When it has slowed down enough so that sufficient current is again flowing through the relay coil, the relay will be re-energized and the supply will again be connected. The motor's speed will therefore rise once more to the relay opening point. With a sensitive relay and tachometer-generator, the time lag between relay opening and closing is quite small, and, as a result, the motor speed remains nearly constant. To increase the sensitivity of the control, thereby maintaining a more constant motor speed, a preamplifier may be required between the error-sensing device and the relay. Either an electron-tube or transistor amplifier is recommended for this purpose, in preference to a magnetic amplifier (see Par. 13-5.33).

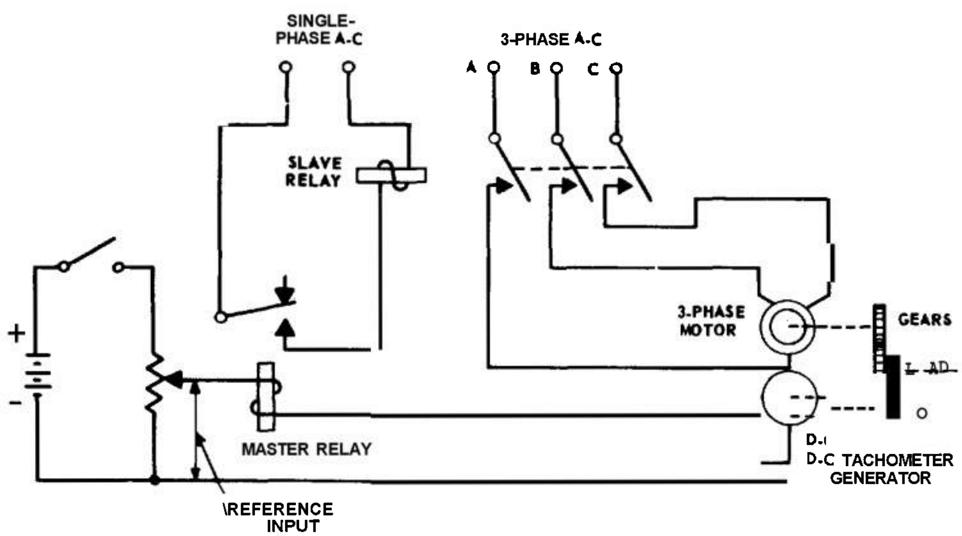


Fig. 13-83 Speed control using single-sided ccscade relay amplifier.

13-5.9 REVERSIBLE MOTOR-SHAFT ROTATION

13-5.10 Servomechanism Applications

In almost all servomechanisms, reversibility of motor-shaft rotation is required to satisfy the requirements of the system for positional control. This feature is provided by the relay amplifiers described below.

13-5.11 Relay-Amplifier Operation for Positional Control

Relay amplifiers for positional control are usually phase-sensitive devices, comprising one or more stages, that respond to d-c control (error) signals of either polarity. The direction of shaft rotation of the controlled motor, being directly dependent upon this polarity, is therefore completely reversible.

13-5.12 Basic operation. Ordinarily, three-position control is used in relay servomechanisms. When the error is less than the magnitude of the dead space $|\delta|$*, the relay remains inoperative. When the error is greater than the dead space, the relay operates, thereby applying power to the controlled motor and causing the motor shaft to rotate in the direction that decreases the error. When the error becomes less than $|\delta - \Delta_h|$, the relay drops out and the motor, in turn, is de-energized. If the motor shaft has sufficient momentum to coast in the same direction until the error is of the opposite sense and greater than $|\delta|$, the relay will be re-energized and will reverse the direction of motor-shaft rotation.

*Refer to Par. 13-6.20 for definitions of δ and Δ_h.

13-5.13 Phase-Sensitive Relay Amplifiers

13-5.14 Typical types. A phase-sensitive relay amplifier can be any one of several different types: single-stage with externally polarized (series-rectifier) relays; single-stage with polarized relay; two-stage with externally polarized master relays; or two-stage with polarized master relays. The operation of the first type, which is illustrated in Fig. 13-84, is described below.

13-5.15 Operation **of** single-stage phase-sensitive relay amplifier. Figure 13-84 shows a position-control circuit illustrating the use

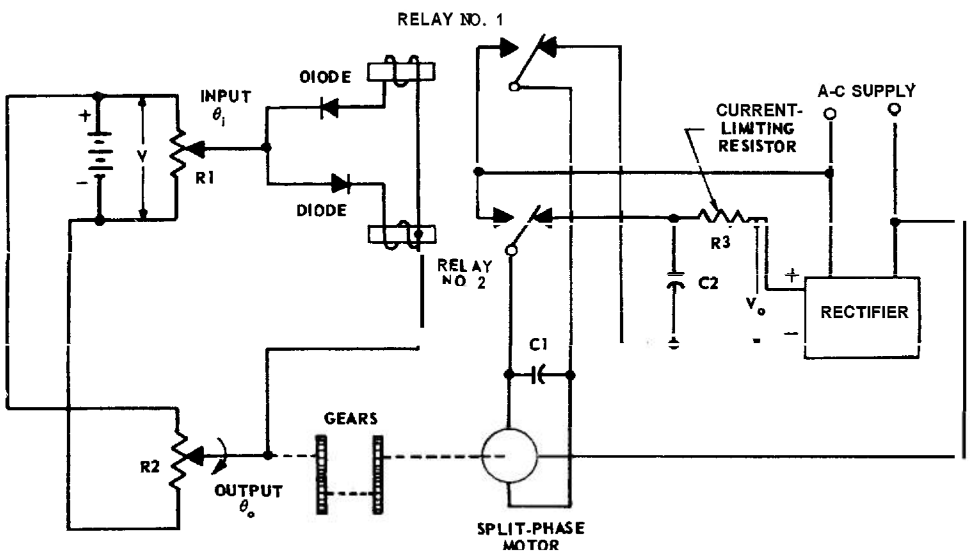


Fig. 13-84 Position control illustrating use of double-sided single-stage phase-sensitive relay amplifier.

of a single-stage, phase-sensitive relay amplifier. In its steady-state, the motor shaft is held motionless by dynamic breaking action (dc on the motor windings). When an error signal greater than $|\delta|$ is applied, current flows through the coil of relay No. **1 or** No. **2,** depending upon the polarity of the error. The contacts of the energized relay close, disconnecting one side of the d-c supply and applying ac to the split-phase motor. This causes the shaft of the motor to turn in the direction that decreases the error. When the error becomes less than $|\delta - \Delta_h|$, the relay is de-energized, disconnecting the motor from the a-c supply and reconnecting it to the d-c supply. Application of dc to the motor windings causes dynamic braking by eddy-current action, and the motor shaft will come to a stop without overshoot and reversal, **or** perhaps with only one reversal. The magnitude of the dc is held to a level that is just adequate for the desired braking action. **A** separate eddy-current brake may be used in place of the motor windings. Alternatively, if **it** is possible to obtain a specified Figure of Merit without the braking circuit and without an excessive number of reversals, the braking circuit may be omitted. Capacitor C2 charges when the motor is running and discharges during the braking period. An electrolytic-type capacitor may be used for this purpose because the d-c polarity is never reversed. To achieve higher Figure of Merit values, "anticipation" may be required to offset the effect of relay-closure time. One system using anticipation is shown in Fig. 13-85. In this circuit, the tachometer-generator signal subtracts from the error so that a reduced error is indicated at a time prior to that at which the value $|\delta - \Delta_h|$ is reached.

13-5.16 Choice of circuit components. The procedure for selecting components **for** the circuits in Figs. 13-84 and 13-85 is as follows:

(a) The motor should be chosen from a knowledge of the required stall torque and transient response. The motor should be no larger than required and should have minimum inertia for the indicated size.

(b) From the motor specifications, the relay manufacturer can indicate the size of

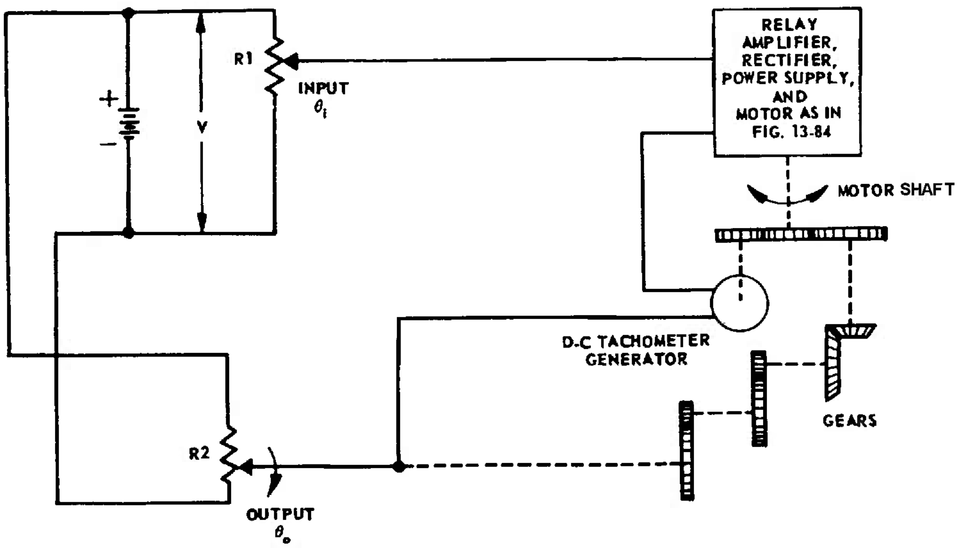


Fig. 73-85 Position control with single-stage relay amplifier showing rate compensation or anticipation.

relay contacts required for a given life expectancy (number of contact operations before failure). The type of relay coil is determined by the contacts used. If a choice is offered, select the fastest-operating most sensitive relay that is compatible with cost, ruggedness, and availability. The effect of relay operating time is discussed in Par. *13-5.25.*

(c) The voltage V, the resistance R_1, and the coil resistance R, are chosen next. The choice of the relay fixes the relay-coil power P,. At this point, the voltage V can be set arbitrarily at a convenient value; e.g., *24, 75,* 90, or *105* volts. Potentiometers $R1$ and $R2$ provide two voltage sources of magnitudes $V\theta_i/\theta_{max}$ and $V\theta_o/\theta_{max}$, respectively, where θ_{max} is the total angle (measured in the same units as θ_i and e,) through which the potentiometer slider turns as the voltage varies from zero to V volts. The maximum internal resistance of each voltage source occurs at the potentiometer mid-point, and is $R_1/4$ for source No. *1* and $R_2/4$ for source No. *2.* Assuming that $R_1 = R_2$ and combining the sources produces a source voltage (V/θ_{max}) $(\theta_i - \theta_o)$ in series with resistance $R_1/2$. The relay-coil current is then $(V/\theta_{max}) \dfrac{\theta_E}{R_c + R_1/2}$, where $\theta_E = \theta_i -$ 8,. The value of error voltage at which the coil is to be energized is chosen from the specified servo accuracy so that θ_E/θ_{max} is fixed and denoted as θ'_E. The coil power is then given by

$$\mathrm{P}_c = \frac{R_c(V\theta'_E)^2}{\left(R_c + \dfrac{R_1}{2}\right)^2} \qquad (13\text{-}84)$$

Two other factors must be considered. One is the maximum desirable standby (no-signal) power to be dissipated by potentiometer resistances R_1 and R_2. The other is the resolution of the potentiometers. For a fixed V, the standby power is $P_R = 2V^2/R_1$. If P_R is fixed, R_1 is immediately specified. The resolution of the potentiometer with this value of resistance should be checked. The size of the indicated angle discontinuity (minimum resolved angle) should be substantially less than the designed dead space. If this is not the case for the value of R_1, calculated from the allowable standby power, revise the choice. Assuming that R, is chosen in this manner, R_o can be found from the expression for P,. Use the nearest-size commercially available coil. The smaller the coil resistance for a given P,, the easier it is to manufacture the coil. It may be worthwhile to connect an external resistor in series with the relay coil to reduce the relay operating time. If this is done, the additional external resistance should be at least five times the value of R, to obtain a worthwhile improvement. The revised P, equation for this case is

$$P_c = \frac{R_c(V\theta'_E)^2}{\left(6R_c + \dfrac{R_1}{2}\right)^2} \qquad (13\text{-}85)$$

(d) The remaining values to be chosen are R_3, C_1, C_2, and V,. The value of C_1 is fixed by motor performance. (As a rule, one value yields maximum starting torque for a given motor.) Since this value is larger than that needed for running conditions, an intermediate value may be **used.**[42] The d-c voltage V, should be of such magnitude that the current surge from C_2, when C_2 is connected across the motor windings, is commensurate with the magnitude of the a-c current drawn by the motor (when running). That is, V, approximately equals the product of twice the resistance of one motor winding and the motor current per winding. Capacitor C_2 should be chosen by the trial-and-error method. As a start, select a value of capacitance C_2 such that the time constant of C_2 and the motor resistance is approximately equal to one half the time for one motor-shaft revolution at no load. If substantial load inertia is reflected to the motor, a larger value of C_2 may be required. An electrolytic-type capacitor may be used, but its capacitance ***must not*** be larger than that required. Current-limiting resistor R_3 should be chosen in accordance with the rectifier manufacturer's specifications for the minimum allowable series resistance, keeping in mind that if a transformer is used in the d-c supply, its windings also have resistance. A transformer so used should not have a resistance much greater than that of the motor because such a high resistance results in an oversized charging time for C_2, thus preventing damping for small transient changes in input. The choice of tachometer voltage depends upon: (1) allowable dead space $|\delta|$; (2) motor runaway velocity; and (3) motor inertia and friction. If the motor is running at its top velocity R, it will take a certain distance δ_c to stop after being de-energized. Define the tachometer voltage at full speed as V_{tR}. A preliminary choice of V_{tR} is to make it equal to $V(\delta_c - |\delta|)$. If this is done, the motor will coast to a stop at the center of the dead space, following a transient change large enough to bring about motor runaway speed. Some readjustment of tachometer voltage may be needed for best all-around operation.

13-5.17 POLARIZED RELAYS

13-5.18 Purpose

To reduce the number of relay-amplifier components, a single polarized relay may be substituted for the two separate relays and diode rectifiers used in the circuit of Fig. 13-84. A circuit using such a polarized relay is shown in Fig. 13-86. Coils No. 1 and No. 2 may be used to increase the detent of the relay; i.e., to increase Δ_h. Two system requirements may necessitate use of these coils:

(a) A desire to lower the frequency of motor actuation during slowly changing (ramp) inputs, under which condition the relay acts like a speed-control device. In such a device, the repetition frequency depends upon the size of Δ_h and may be excessive for small values of Δ_h.

(b) Faulty operation due to vibration, hum, or magnetic or conductive coupling may occur without the added detent. The diodes and resistance-capacitance filters are used to convert the ac applied to the motor into dc for detent control of the relay (this is normally the case for polarized relays).

If the polarized relay is used in a two-stage circuit, as in Fig. 13-87, series detent coils may be used. The number of turns in the detent coils is chosen to give the desired Δ_h for a given load current. A reasonable Δ_h/Δ ratio is about 0.25, if no reason for a different choice exists.

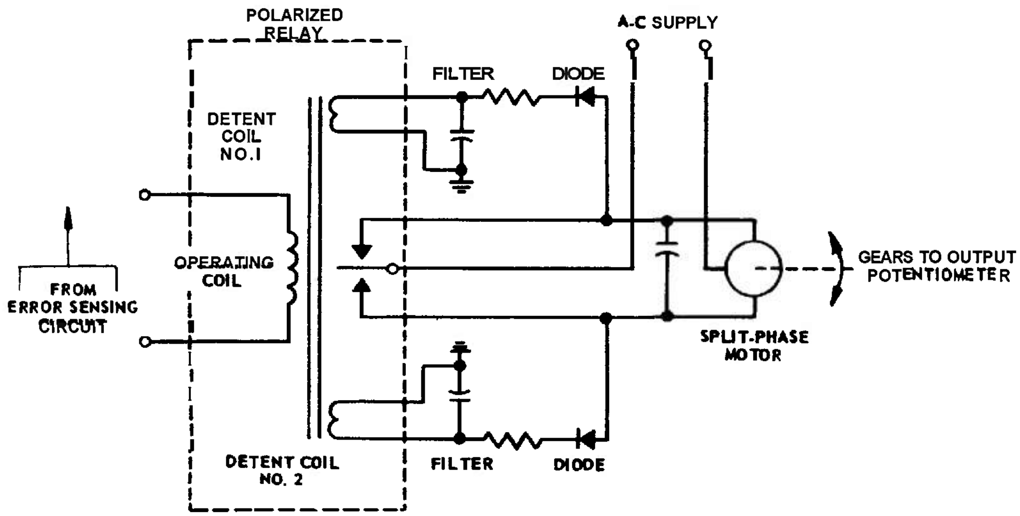


Fig. 13-86 Position control illustrating use of single-stage phase-sensitive relay amplifier.

TABLE 13-9 COMPARISON OF COMMERCIAL RELAYS

General-Purpose D-C Relays

Relay No.	Nominal Coil Voltage (volts)	Contact Arrangement	Contact Current Rating (amperes)	Weight (grams)	Coil Resistance at 25°C (ohms)	Temp Rise Above 65°C Ambient (°C)	Operating Power (watts)			Release Factor[a] (percent)	Operate Time (millisec)			Release Time (millisec)	
							normal	min	percent		max	average	min	max	min
1	24	DPDT	5	90	250	60	2.30	1.10	50	36-54	24	12.6	8.6	8.6	3
2	24	DPDT	5	100	276	65	2.10	1.06	60	59-69	37	14	8.6	9	3
3	24	DPDT	15	127	230	60	2.60	1.47	60	36-48	34	27	17	5	5
4	24	4PDT	16	158	235	65	2.45	1.37	60	74-78	36	22	16	10	10
5	24	DPDT	10	141	635	35 (175)	0.91	0.40	40	22	36	24	12	13	5
6	87	DPDT	10	140	0,250	40 (185)	0.74	0.29	40	37-43	38	26	16	16	8
7	24	6PDT	10	297	175	60	3.30	2.76	80	37-68	60	42	28	6	6
8	24	DPDT	5	108	2665	0	0.22	0.15	70	42	55	33	26	–	–
9	24	DPDT	1	50	300	50	1.92	0.32-0.66	20-30	44-62	18	10	6	5	4
10	24	DPDT	1	54	296	60	1.96	1.40	70	48-66	30	19	12	5	4
11	24	SPDT	1	33	600	30	0.96	0.28	30	44-66	20	9	6	4	2
12	24	DPDT	1	37	380	46	1.61	0.77	60	44	30	12	7	5	4
13	24	SPDT	1	39	3000	5	0.19	0.09	60	76	76	30	13	5	3
14	24	4PDT	1	52	610	35	1.13	0.46	40	53-68	32	16	9	3	3
15[b]	50	DPDT	10	339	1,000	10 (260)	0.23	0.16	70	26-29	–	180	36	5	6
16	24	4PDT	3	53	180	70	3.20	1.39	40	61-63	20	13	8	6	3
17	50	BPDT	3	331	2500	45 (260)	1.00	0.69	70	59-73	–	80	30	6	6
18	24	SPDT	3	35	300	60	1.92	0.40	20	61	10	6	4.6	3	3
19	26	SPDT	3	38	300	55 (115)	2.26	0.35	20	44-48	16	6	4.6	4	4
20[c]	24	–	3	42	300	45	1.92	0.85	40	56	16	9.6	7	4	4
21[d]	24	DPDT	3	70	300	65	1.92	0.56	30	53-75	26	9	6	5	3
22	14	SPDT	20	107	72	40 (107)	2.72	0.62	20	48	26	13	11	18	18
23	24	DPDT	8	175	108	80	6.34	2.26	40	27-32	32	24	17	26	23

24	70	DPDT	2	157	10,000	20 (215)	0.49	0.34	70	53	90	60	18	5	5
25	3.5	SPST	15	161	17	15 (230)	0.72	0.50	70	82	–	48	12	–	–
26	24	SPDT	4	170	133	50	4.33	1.33	30	23	35	22	17	5	5
27	24	SPDT	15	144	283	30	2.04	0.63	30	21-27	35	27	19	7	7
28	2.5	DPDT	15	160	16	55 (320)	0.39	0.23	60	27	40	24	12	6	6
29	70	DPDT	15	160	5700	30 (185)	0.86	0.60	50	33	–	55	18	8	7
30	50	4PDT	4	165	3600	25 (200)	0.70	0.60	90	38-46	–	–	12	6	6
31	70	3PDT	4	160	10,500	20	0.47	0.43	90	31-40	–	–	28	10	10
32	24	SPDT	1	273	2400	5	0.24	0.06	30	50-54	120	60	46	7	7
33	24	8PDT	1	340	240	25	2.40	0.51	20	66-71	70	49	37	7	7
34	26	SPDT	12	197	170	45 (115)	3.98	1.70	40	32-35	45	34	28	9	9
35	24	DPDT	12.5	137	150	70	3.84	2.06	50	40-48	40	33	24	13	10
36	10	4PDT	4	157	9950	20 (118)	1.22	1.03	80	33-42	65	47	20	25	20
37	24	DPDT	6	156	165	55	3.50	1.02	30	21-23	26	17	12	10	10
38	24	SPDTDB	6	215	168	55	3.42	1.62	50	59	40	29	22	42	42
39	24	DPDT	6	230	160	75	3.60	0.72	20	22-28	22	16	13	5	2.5
40	24	SPSTNBDB	25	167	255	40	2.26	0.24-0.55	10-20	44-55	27	19	16	3	3
41	24	SPDT	3	96	300	40	1.92	1.11	60	73-78	9	6	5	–	–
42	24	4PDT	3	128	295	40	1.95	0.31	40	42-57	40	24	15	10	8
43	24	SPDT	3	52	515	35	1.12	0.31	30	57	17	9	7	16	11
44	24	SPDT	5	121	235	50	2.45	0.64	30	37	24	13	10	5	5
45	45	DPSTNODB	5	138	175	15	0.82	0.35	40	37	20	18	14	–	–
46	10-14	DPDT	0.25	71	77	40	1.87	1.02	60	33	33	23	15	5	4
47	8-12	3PSTNO	0.25	55	77	40 (150)	1.30	0.65	50	61	28	20	9	–	–
48	26	4PDT	1	66	250	55 (115)	2.70	0.47	20	29	18	10	7	5	5
49	26	SPDT	1	41	290	60 (115)	2.33	0.46	20	49-54	19	7	5	5	4
50(e)	90	SPDTDB	1	111	11,000	30	0.74	0.64	90	39-42	–	19	6	10	10
51(f)	2.5	SPDT	1	156	47	40 (360)	0.13	0.11	80	86	–	–	25	0.5	0.5

TABLE 13-9 COMPARISON OF COMMERCIAL RELAYS (cont)

General-Purpose D-C Relays

Relay No.	Nominal Coil Voltage (volts)	Contact Arrangement	Contact Current Rating (amperes)	Weight (grams)	Coil Resistance at 25°C (ohms)	Temp Rise Above 65°C Ambient (°C)	Operating Power (watts)			Release Factor[a] (percent)	Operate Time (millisec)			Release Time (millisec)	
							normal	min	percent		max	average	min	max	min
						Small D-C Motor Contactors									
71	24	SPDTDB	50	310	100	65	5.76	2.03	35	22-41	30	21	15	14	11
72[g]	24	SPSTNODB	200	408	8.5	45	169.0	–	–	41	30	21	16	–	–

Slug-type Lag Relays

Relay No.	Nominal Coil Voltage (volts)	Contact Arrangement	Contact Current Rating (amperes)	Weight (grams)	Coil Resistance at 25°C (ohms)	Temp Rise Above 65°C Ambient (°C)	Operating Power (watts)			Release Factor[a] (percent)
							normal	min	percent	
81	24	DPDT	10	223	230	60	2.50	0.37	20	47-54
82	–	SPDT	4	211	83	75 (20 V)	–	0.25	–	59-67
83	24	SPSTNO	3	347	2000	30 (365)	0.29	0.19	20	19-95
84	–	SPDT	4	211	80	55 (20 V)	–	0.20	–	25-38
85	–	SPDT	4	328	250	40 (30 V)	–	0.17	–	26-46
86	24	DPDT	15	198	190	50	2.96	0.17	20	26-46

General-Purpose A-C Relays

Relay No.	Nominal Coil Voltage (volts)	Contact Arrangement	Contact Current Rating (amperes)	Weight (grams)	Coil Resistance at 25°C (ohms)	Temp Rise Above 65°C Ambient (°C)	Release Factor[a] (percent)	Operate Time (millisec)		Release Time (millisec)	
								max	min	max	min
101	110	DPDT	5	92	645	70	93	11	4	8	5

102	110	DPDT	5	101	640	65	87-94	4	3	10	5
103	110	DPDT	15	140	450	30	89-95	14	6	12	11
104	110	3PDT	15	161	300	50	95	12	5	7	7
105	110	4PDT	15	169	300	50	94-98	10	4	10	10
106	110	6PDT	10	298	175	35	95	11	5	13	5
107	115	4PDT	3	183	150	80(120)	94	14	7	11	11
108	115	DPDT	4	167	215	40(115)	92	7	5	7	5
109(h)	120	DPDT	3	73	215	35(120)	85-90	4	2	4	4
110	110	3PDT	12.5	198	385	55	94	13	LO	25	22
111	iio	DPDT	4	137	540	50	88-93	8	8	12	12
112	110	DPDT	6	149	520	50	93-95	10	7	19	19
113	110	3PDT	6	150	510	55	82-95	10	6	18	15
114	110	1NODB+1NC	25	191	360	65	88-91	13	10	25	23
115	110	DPDT	6	226	525	55	92-94	20	15	27	27
116	110	SPNODB	25	226	520	50	8[illegible]-94	11	6	—	—
117	115	SPDT	3	100	540	40	86-92	10	5	13	13
118	110	4PDT	3	112	540	35	88-94	12	7	17	17
119	115	DPDT	3	50	3130	25	86-90	10	7	20	20
120	110	SPDTDB	5	118	1235	35	87-90	11	7	9	7

Notes Applicable to Table 13-9

(a) The release factor is 100 times the maximum release current (or voltage) divided by the minimum operate current (or voltage). [6]

(b) Uses two microswitches.

(c) Contacts SPDT + 1NO make before 1 NC break.

(d) Hermetically sealed on octal base.

(e) Sealed dust cover on 6-pin plug-in base.

(f) Hermetically sealed.

(g) Has double winding; low-resistance winding cut out by series NC contacts on operation. Normal input is 69 watts to low-resistance coil; holds on less than 1 watt.

(h) Relay No. 109 intended for 400-cps operation. All other a-c relays intended for 60-cps operation.

(i) Actual percent voltage used (or applied voltage in case of relays for which no nominal voltage is specified) is given in parentheses. Most relays tested at 126 percent of nominal coil voltage.

13-5.19 Advantages and Disadvantages

The advantages of a polarized relay are:

(a) Less total coil power is required.

(b) The balanced armature is inherently less sensitive to vibration and shock.

(c) Less additional equipment is required than in comparable rectifier-relay systems.

The disadvantages of a polarized relay are:

(a) Special detent coils may be required.

(b) No normally closed contacts are available for a damping circuit. (In the circuit of Fig. **13-86,** for example, the lack of normally closed contacts on the polarized relay precludes the use of a damping circuit like that shown in Fig. **13-84.)**

(c) Contact ratings on sensitive polarized relays are usually low. Because of these disadvantages, use of secondary (slave) relays is desirable, as in Fig. **13-87.** Slave relays may also be used with externally polarized master relays.

13-5.20 STATIC CHARACTERISTICS OF RELAYS

13-5.21 idealized Relay

The idealized relay has static characteristics as shown in Fig. 13-88, where 6 is the pull-in value and $6 - \Delta_h$ is the drop-out value. The total dead space is equal to 26. The pull-in to drop-out ratio $\delta/(\delta - \Delta_h)$ is a measure of relay quality. The highest possible value of this ratio should be used when it is consistent with reasonable detent action, as discussed in connection with Fig. **13-86.** (Typical ratio values are listed in Table **13-9** under the heading of "Release Factor".) It is possible to produce a servomechanism with zero *average* error, even in the presence of the dead space, by using what is called an *oscillating center*. This is produced by the introduction of a "dither" signal having a peak value of δ, thereby causing the servo to oscillate about the center of the dead space. Oscillation frequency should be high, in relation to the frequency at which the servo tends to oscillate without the dither, when the servo gain (or R) is increased. On the other hand, dither produces additional wear on relays and motors and should be used with due consideration of such undesirable results.

13-5.22 Sensitivity[38, 39]

The sensitivity of a relay is commonly expressed in terms of the power required to

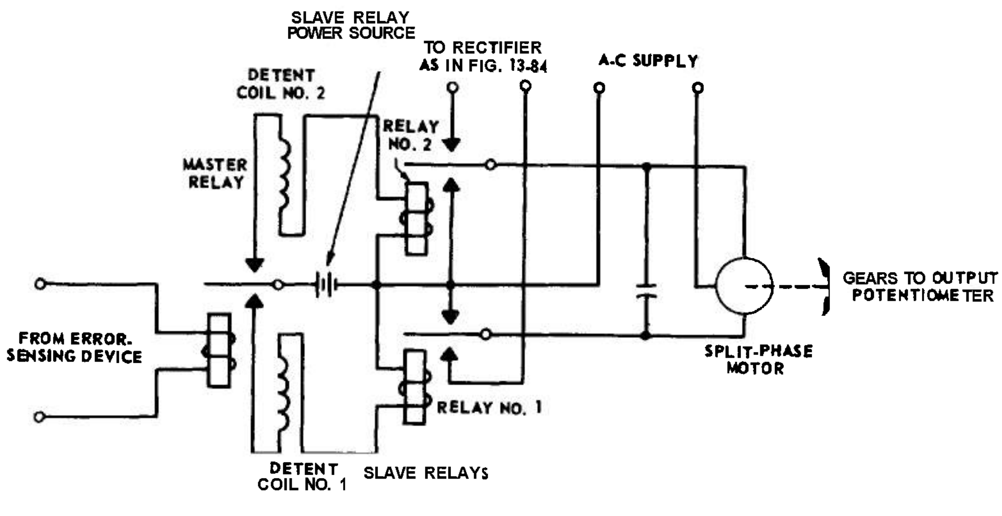


Fig. 13-87 Position control illustrating use of two-stage phase-sensitive relay amplifier.

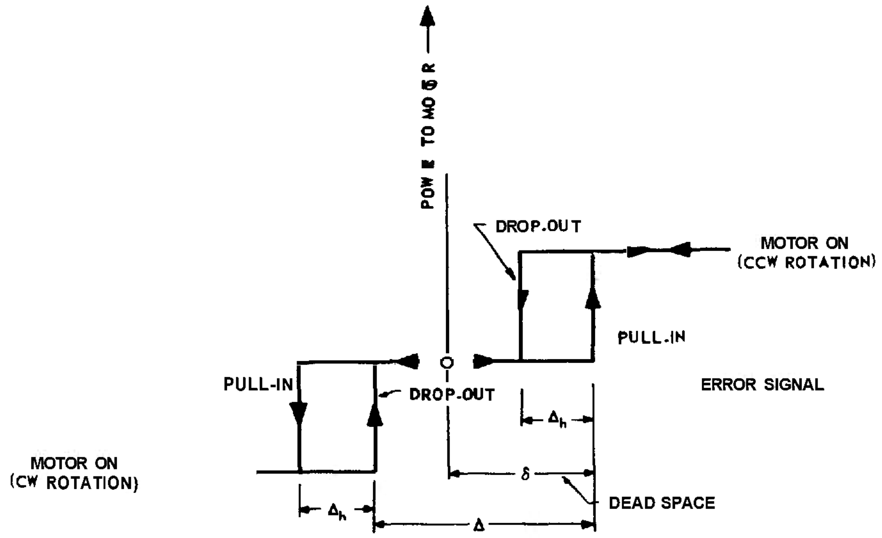


Fig. 13-88 Static characteristics of idealized relay.

bring about contact closure. Changing the number of turns in a given winding space varies the impedance of the winding, without varying the power requirements significantly or affecting the time constant of the relay coil. Insertion of external resistance is required if "forcing action" is desired. For the same coil, more contacts on a relay reduce sensitivity and/or speed of response. Therefore, do not select a relay with more contacts than are actually required. Methods of increasing relay sensitivity are listed in Table 13-10.

TABLE 13-10 METHODS OF INCREASING RELAY SENSITIVITY

Method	Limitation
Reduce air gap.	Gap must remain wide enough for arc extinction.
Reduce armature spring-rate.	Increased sensibility to vibration and shock.
Increase armature-iron area.	Increased inertia slows response.
Increase winding space.	Saturation of iron.

13-5.23 Contact Rating[38, 40]

Contact rating is a significant static characteristic of relays. Manufacturers' ratings should be followed, subject to variation for extra-long or extra-short life specifications. Contact rating may be increased by the methods listed in Table 13-11. A special note on the

TABLE 13-11 METHODS OF INCREASING CONTACT RATING

(a)	Increase contact size.
(b)	Lower operating temperature and use heavier construction for removal of heat from contacts.
(c)	Use a-c instead of d-c loads.
(d)	Use mercury-wetted contacts.
(e)	Increase contact pressure.
(f)	Eliminate corrosive atmosphere.
(g)	Use arc suppression to reduce effective load inductance (see Fig. 13-92).
(h)	Use special alloys for contact materials.
(i)	Operate contacts in a vacuum.

use of hermetically sealed relays : It has been **found**[44] that sealing must be accompanied by the most extreme care in manufacture to prevent the formation of a nonconducting carbonaceous film on the contacts due to "gassing" of organic compounds remaining on the relay from manufacturing processes.

13-5.24 DYNAMIC CHARACTERISTICS OF RELAYS

13-5.25 Response Time

An undesirable feature of the relay is its relatively slow response time. The effect of relay response time on servo stability may be analyzed by a describing-function or phase-plane approach, as described in Ch. 10. For a simpler explanation, refer to Fig. 13-89 where the error following a large step-change in input is plotted. The relay should turn off the motor at an error value of 6 — A,. If this occurs, and the damping circuit stops the motor instantly, curve A applies. Curve B applies if

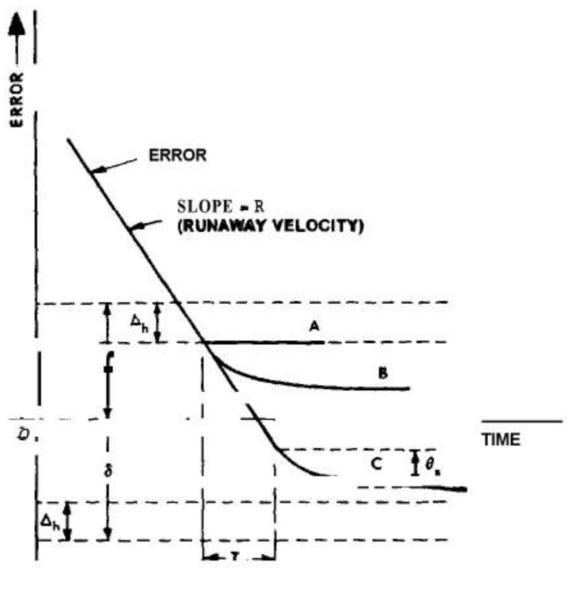


Fig. 13-89 Error response of contactor servo with large input change.

the motor does not stop instantly. If the relay contacts do not open for T_d seconds, curve C applies. If θ_s represents the distance that the motor coasts, and if the runaway velocity is R, the maximum allowable relay delay time (if overshoot and reversal are to be avoided) is $(26 - A_h - \theta_s)/R$ seconds, where 6, Δ_h, and R are expressed in terms of the same angular quantity $(26 - \Delta_h = A)$. If a tachometer-generator is used, as in Fig. 13-85, it must introduce a signal equivalent to RT_d into the error channel to counteract the relay delay time in addition to the voltage $V(\theta_s - |\delta|)$ suggested in Par. 13-5.16(d). Random variations in relay delay time, as well as variations with small steps, make it impossible in practice to achieve coasting that results in the final error being zero, although it appears possible theoretically.

13-5.26 Nature of time delay. The time required to close or open relay contacts, following a change in voltage across the coil, depends upon the nature of the change. Fast operation occurs following a sudden (step) change in voltage which is far greater than that required to vary current through the pull-in or pull-out value. A gradually changing signal may cause "bounce" or "chatter", especially if accompanied by hum or ripple. A-c relays have a tendency to chatter under such circumstances and are therefore usually equipped with split pole pieces and a shading coil. A-c relays also have release times that are slower (by a factor of 1.5) than obtained with d-c relays of the same rating.[41] The principal cause for delay is the inertia of moving parts, although the electrical delay due to coil inductance also affects the action. The speed of response may be increased by the methods listed in Table 13-12.

13-5.27 General Considerations

Relay design is a compromise. As a rule, high sensitivity and speed of response are incompatible. Response times of 2 to **5** milliseconds are reasonable for small, sensitive relays ; response times of 15 to 20 milliseconds are reasonable for a-c relays with contact ratings of 10 to 15 a-c amperes. The results

TABLE 13-12 METHODS OF INCREASING RELAY RESPONSE SPEED

(a)	Decrease inertia of moving parts.
(b)	Decrease distance that armature must travel to actuate contacts.
(c)	Increase external resistance in series with coil and, simultaneously increase voltage applied to coil and resistance. (A capacitor in parallel with resistance may be helpful.)
(d)	Decrease armature spring-rate.
(e)	For slave relays, use forcing (i.e., supply-coil) current in excess of pull-in value without exceeding power rating of coil.

of tests made at the MIT Radiation Laboratory are summarized in Table 13-9.[41,43] Of the d-c relays tested, 80 percent had operate times between 6 and 30 milliseconds and release times between 3 and 11 milliseconds. Of the a-c relays tested, 80 percent had operate times between 4 and 11 milliseconds and release times between 4 and 18 milliseconds.

When relays are connected in cascade, their operating times are added. Recently, specially designed high-speed relays have been developed with operate times in the order of 1 millisecond. The mercury-wetted-contact relay operates in approximately 3 milliseconds with 2 watts input to the coil; the release time is about 3.5 milliseconds, regardless of coil power prior to release.

13-5.28 PARAMETER MEASUREMENT

13-5.29 Relay Operating Time

Relay operating time may be evaluated by means of a low-frequency function generator capable of producing square waves or by means of any other device for producing stepwise varying-voltage inputs. Evaluation through use of a dual-beam oscilloscope is feasible, with high frequency applied to the contact circuit as shown in Fig. 13-90. If a dual-beam oscilloscope is not available, the low-frequency function-generator signal can be differentiated in a resistance-capacitance network and then coupled to a single-beam oscilloscope input, thereby providing a single

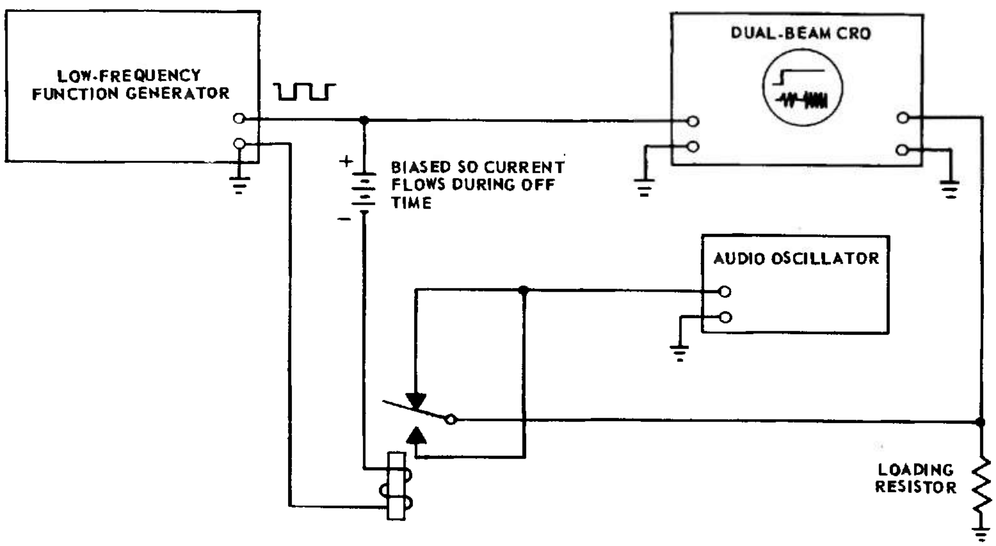
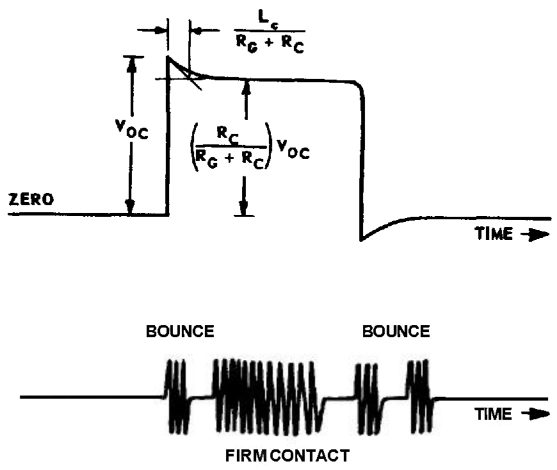


Fig. 73-90 Circuit for measuring relay pull-in and drop-out time.



R_G = FUNCTION-GENERATOR INTERNAL RESISTANCE
R_C = COIL RESISTANCE
L_C = COIL INDUCTANCE

Fig. 13-91 Typical waveforms observed during relay test.

spike at the instant of voltage change. For a timing wave, use an audio frequency of convenient value (about 1000 cps) on the contacts. Typical waveforms are shown in Fig. 13-91.

13-5.30 PROBLEMS ENCOUNTERED WITH RELAY AMPLIFIERS

13-5.31 R/δ, A Figure of Merit

When the relay amplifier is part of a servomechanism, the entire device can be characterized by a convenient Figure of Merit, which is found by dividing the runaway velocity R (the steady-state speed of the motor when the motor is connected to its supply) by the pull-in value δ of the relay. Both quantities must be referred to the same signal point; e.g., the output shaft. The larger the value of R, the quicker the servo response to large step changes. The smaller the value of 6, the more accurate the servo becomes. The ratio R/δ has the units of $\sec^{-1}$ and is similar to the velocity constant of a continuous servo. Servo stability (i.e., lack of steady-state oscillation) becomes more difficult to achieve as the ratio R/δ is increased. For values of R/δ in excess of 10, some compensation techniques will probably be required. Methods of increasing the R/δ ratio are listed in Table 13-13.

13-5.32 Relay Life

Achievement of satisfactory relay life is a serious problem. Limitations on relay life are imposed by contact wear and by wear of moving parts (armature, spring, and pivot). Contact life is increased by the same methods that increase contact rating (see Table 13-11). Arc suppression and the use of sliding or wiping contact action are advisable. Arc suppression may be accomplished by paralleling inductive loads with a series resistance-capacitance network, with parameters as shown in Fig. 13-92A. This results in a load that appears as pure resistance to the'contacts. Another method of arc suppression is shown in Fig. 13-92B. Resistor R prevents contact welding on closure, at which time the previously charged capacitor C is shorted by the contacts. Unless R is used, the capacitor discharge causes an extremely high current, resulting in point or spot welding of the contacts. This suppression method is preferable for loads having variable inductance (such as motors and relay coils). To use this method for high-speed sensitive relays, both the load current and inductance should be taken into account. A nomograph for arc suppression circuits is reproduced in Fig. 13-93. An arc suppressor modification of value, when the primary relay is loaded by a slave-relay coil, uses a resistor in parallel with the load, together with a resistance-capacitance suppressor of the type shown in Fig. 13-92B.[37] A typical value for the shunt resistor is 470 ohms, in parallel with a coil having a resistance of 750 ohms and an inductance of 10 to 13 henrys. A contact-protection circuit for use with mercury-wetted-contact relays may be obtained from the manufacturer. In every case, however, the relay manufacturer should be consulted for advice concerning proper arc suppression for a given load. The importance of cooperation with the

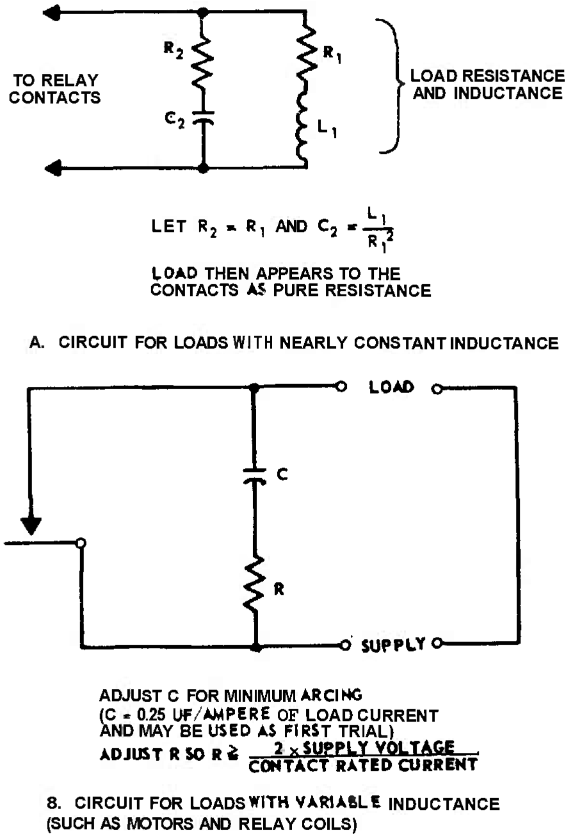


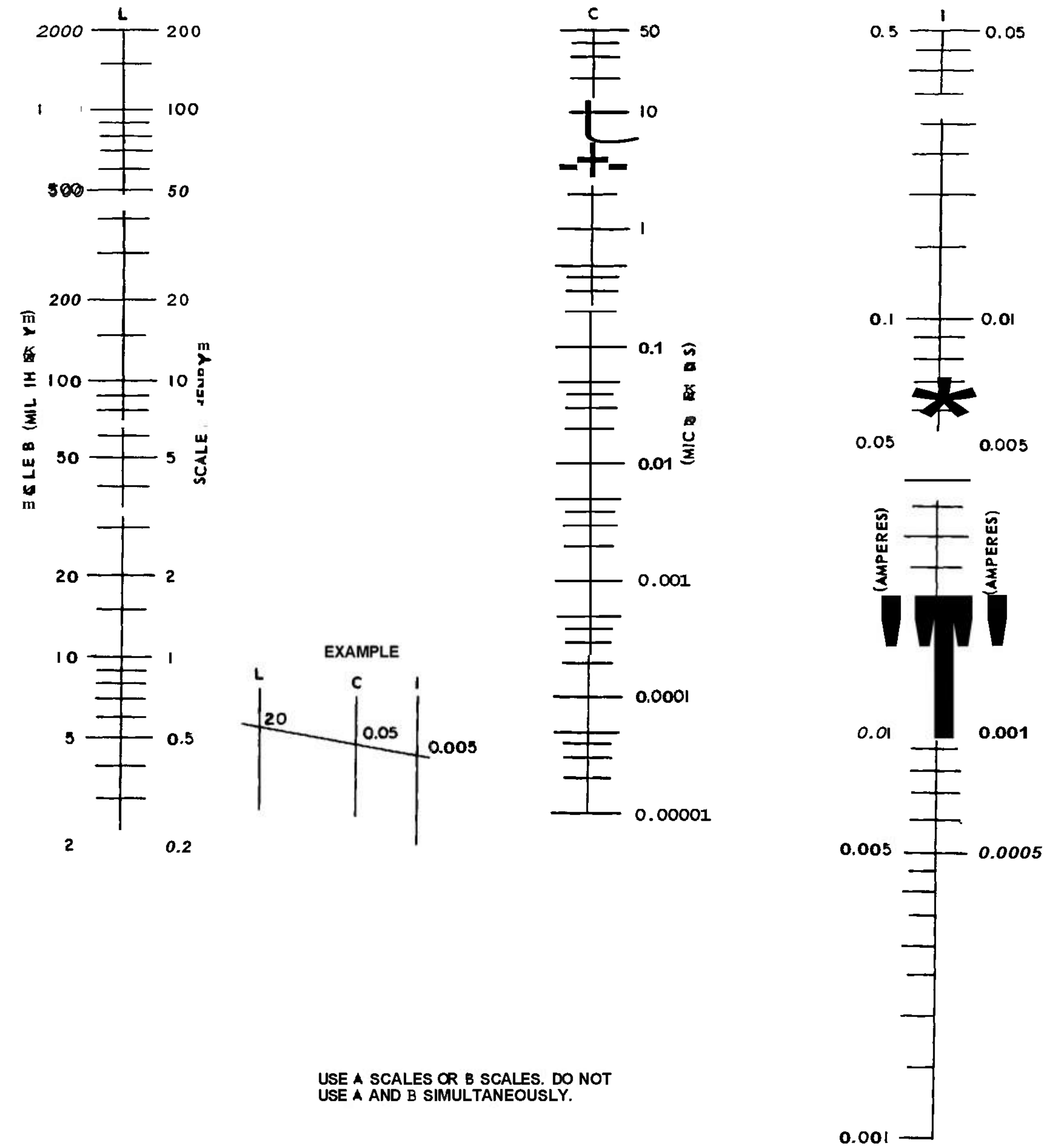
Fig. 13-92 Arc suppression circuit.

manufacturer for the purpose of choosing proper relays cannot be over-emphasized.

Another problem that arises in some cases is the tendency for contact pip-and-crater formation. One contact tends to accumulate material in a conical pip, whereas the other contact loses material and forms a crater. This phenomenon occurs when direct currents that have a low value compared with the nominal contact rating are switched. The use of different contact materials is sometimes very useful.[45] For example, in one test, replacement of coin-silver contacts with tungsten and platinum-ruthenium contacts led to an increase in life from 3 million to 300 million operations. Thedirection of current flow was from the tungsten contact to the platinum-ruthenium contact.

13-5.33 False Operation

False operation can arise from several causes. Acceleration of the structure supporting the relay may cause relative motion of the relay parts. For this reason, relays specified for use in aircraft and missiles are designed to withstand a certain predetermined number of g's without false contact closure. Shock and vibration may also produce acceleration forces sufficient to cause false operation. Chatter is another type of undesirable relay operation that usually occurs if currents nearly equal to the pull-in value are present. Under such circumstances, if a-c hum is present in an amplifier using sensitive, fast-acting d-c relays, the relays will tend to follow the a-c hum. This requires the use of carefully filtered supplies for the reference input as well as for any preamplifier that may be used. If magnetic preamplifiers are used, their output should be filtered or a-c relays should be used. However, filtering or the use of a-c relays causes additional dynamic delays that are undesirable, and prevents achievement of magnetic-amplifier responses with only half-cycle or one-cycle relays, even though such responses might be possible theoretically. Still another type of false operation — relay bounce—is largely dependent upon relay design and must be minimized by the manufacturer. Designing eddy-current damping into the relay, by using a solid armature structure instead of a laminated one, may help. False operation may also occur if a sensitive relay (or preamplifier) is used, due to coupling between motor currents and the relay-coil (or the preamplifier-input) currents. Such coupling may be inductive, capacitive, or resistive. Resistive coupling occurs if a single power supply with a common internal resistance is used to feed all amplifiers as well as the motor. Eliminating hum, vibration, and interaction at the source is preferable to filtering at the relay coil because filtering slows relay response.



INDUCTIVE LOAD

EXAMPLE: LOAD INDUCTANCE (L) = 20 HENRYS
LOAD CURRENT (I) = 0.005 AMPERE

CALCULATIONS: CAPACITOR (C) = 0.05 UF (NOMOGRAPH A SCALES)
RESISTOR (R) = E/0.3 OHM (MIN)
E = D-C SUPPLY VOLTAGE, IN VOLTS

Fig. 73-93 Nomograph and equations for use in calculating the component values for an arc suppression circuit.

TABLE 13-13 METHODS OF INCREASING R/δ

(a)	Use a motor with inherent dynamic braking; e.g., a d-c shunt motor with the armature shorted during the relay dead space.
(b)	Introduce continuously applied coulomb friction. This is simple, but it decreases R at the same time it allows decreased 6.
(c)	Apply coulomb friction by means of a clutch during the dead space only. This is more complex than the continuous method. It does not affect R, but wear of rubbing parts and production of heat may be troublesome.
(d)	Introduce continuously applied eddy-current damping. Use a permanent magnet and a disc of aluminum. This is simple and trouble-free and reduces R.
(e)	Apply eddy-current damping during the dead space only. Use an electromagnet (with aluminum disc) energized during the dead space only. There is no rubbing friction and no loss of R, but the method is more complex.
(f)	Apply eddy-current damping through the motor windings. This is applicable to induction and synchronous motors. A typical circuit is shown in Fig. **13-84.** This method requires no extra coil and disc, but it does require direct current commensurate with normal a-c motor current during the damping period.
(g)	Use "plugging". Motor actuation may be reversed upon entry into the dead space and removed before actual rotation reversal occurs. Difficulty may be encountered in the case of very small steps, during which the motor does not get up to full forward speed. Plugging time is controlled by extra contacts. This method is usually complex.
(h)	Use anticipation. Use a permanent-magnet tachometer-generator, as in Fig. **13-85.** Alternatively, use a resistance-capacitance network in the feedback path; e.g., feedback from the output through a parallel resistance-capacitance circuit. A third method uses a lead network in cascade, after the point where the error is sensed and before the relay coil. A fourth method [37] uses extra contacts on slave relays to energize a resistance-capacitance circuit analog of the motor, the voltage that represents motor speed being fed back in a manner similar to that used to feed back tachometer voltage.
(i)	Use an integral or lag network in cascade between the error-sensing point and the relay coil. Such a network, however, causes difficulty when there are very large inputs in the presence of saturation, which is characteristic of the relay. Mitigation of this difficulty can be achieved by using a charge-limited capacitor in the lag network, as in the case of saturation in otherwise linear conditionally stable systems.

13-6 HYDRAULIC AMPLIFIERS*

13-6.1 INTRODUCTION

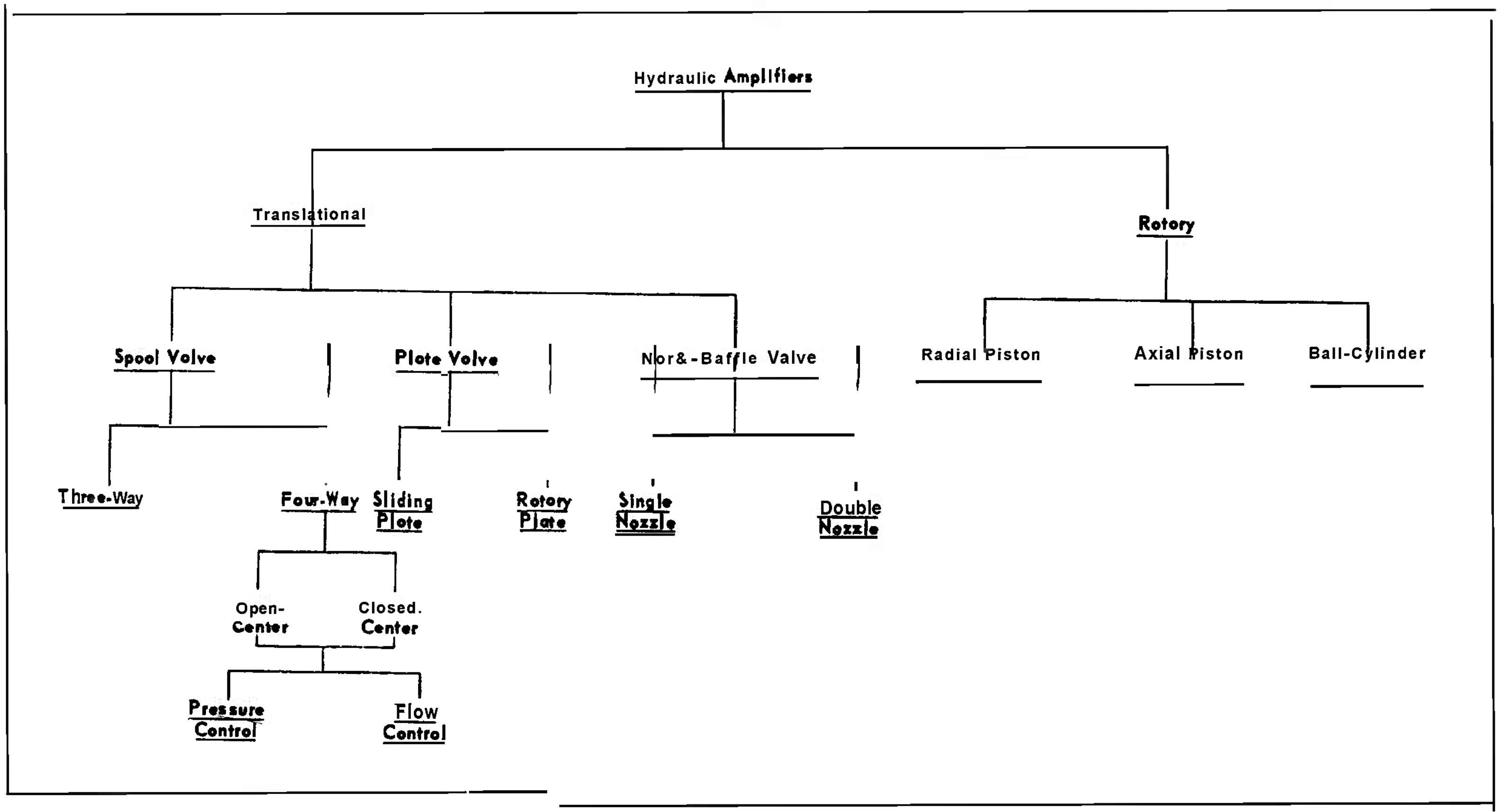
13-6.2 Description and Usage

A hydraulic amplifier is a device for converting ***motion*** at a low force level into ***motion*** at a high force level. It has two principal parts: (1) a valve, which converts a motion into an oil-pressure change; and (2) a load, such as a piston, which converts the pressure into a force, thence into a motion. Hydraulic amplifiers are used where high-gain, fast-response performance is required. They are commonplace in the high-power stages of servomechanisms, while electrical amplification is used in the low-power stages. The transition from electrical to hydraulic amplification occurs at a power level of about 1 to 5 watts. A classification of hydraulic amplifiers is presented in Table 13-14. These amplifiers, and combinations of them, are discussed in detail in this section.

Hydraulic amplifiers may be classed as either rotary or translational. The rotary type (variable-displacement pump or rotary

**By P. E. Smith, Jr.*

TABLE 13-14 CLASSIFICATION OF HYDRAULIC AMPLIFIERS



pump) is normally used at high power levels of 500 watts and up. The translational type may be used over a wide power range, from a few watts to as high as **5000** watts (if a valve-controlled piston can be classed as an amplifier stage).

Translational hydraulic amplifiers may be divided into spool-valve, plate-valve, and nozzle-baffle-valve types. Of these, the nozzle-baffle type is best suited for low power levels, although all three types are used as first-stage hydraulic amplifiers driven by electric transducers (force or torque motors).

Translational hydraulic amplifiers may be used with feedback from output motion to input. The following types of feedback are used :

(a) Position feedback

(b) Force-balance feedback

(c) Position-to-voltage to force-motor feedback

Results somewhat similar to those obtained with position feedback may be achieved by spring-opposing the output. (See Table 13-16 for a detailed mathematical development of amplifiers having either a spring-opposed balanced piston with no feedback, or a balanced piston without feedback, or a balanced piston with position feedback.)

It is possible to use position feedback through mechanical and hydromechanical networks to obtain an over-all lead or lag effect. (This is uncommon in commercial valves presently available.) Schematics of several translational amplifier and feedback configurations are shown in Figs. 13-94 through **13-102.**

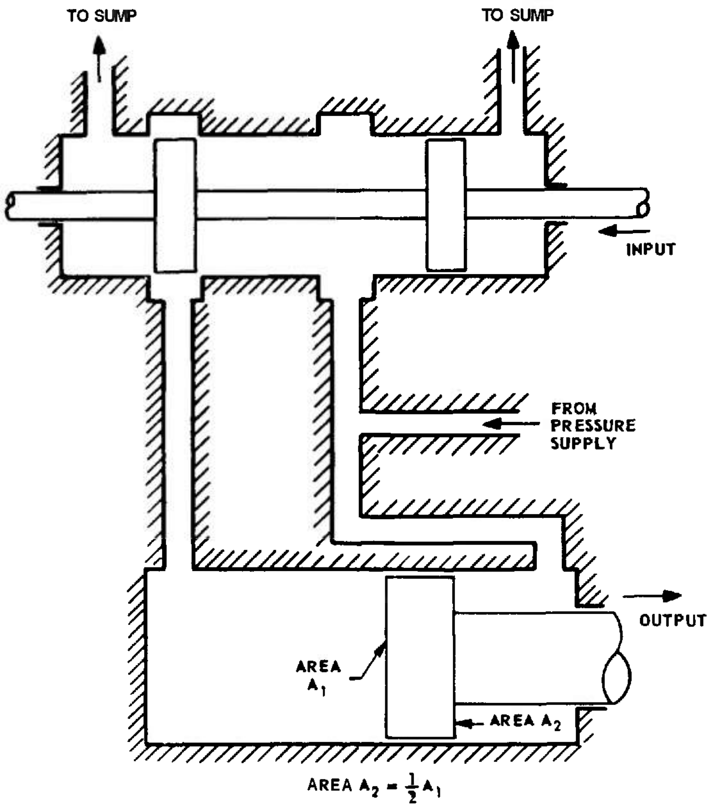


Fig. ***13-94 Three-way spool-valve*** **amplifier.**

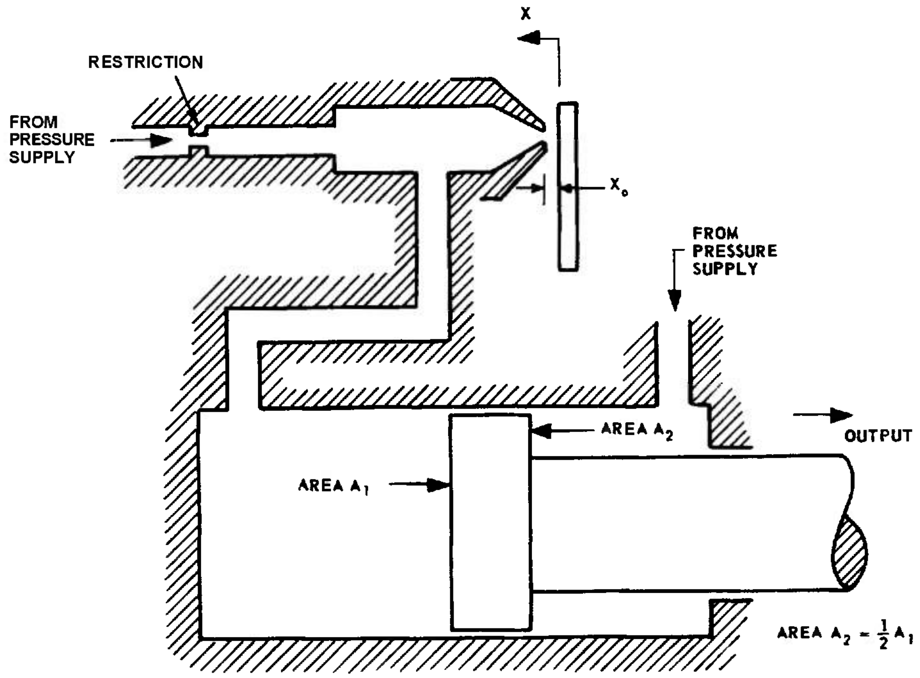


Fig. 13-95 Single nozzle-baffle amplifier.

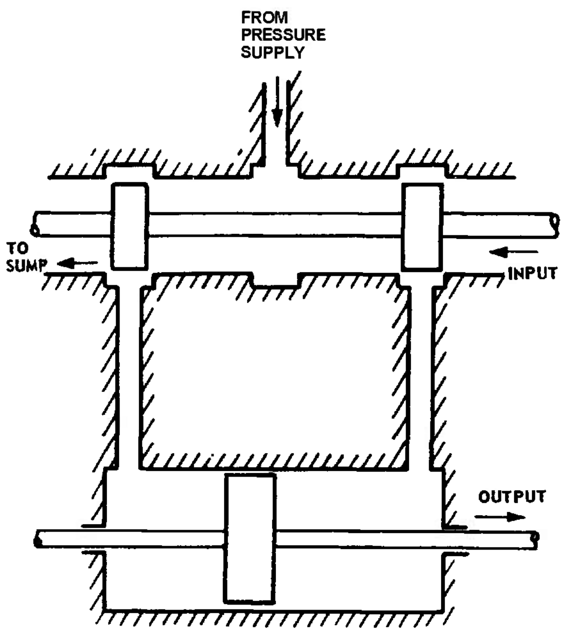


Fig. 13-96 Four-way spool-valve amplifier.

Schematics of three common types of rotary hydraulic amplifiers, or rotary pumps, are shown in Figs. **13-103, 13-104,** and **13-105.** In each case, the moving parts are arranged so that, by varying the location of one part relative to another, the amount of oil pumped per revolution may be varied. The amount of variation in the relative position of the moving parts is termed the stroke of the pump. These pumps are analogous in their mathematical characteristics to rotary electric amplifiers (see Par. **13-4).** Pump speed is usually more or less constant. One exception is the so-called "constant-speed drive" used to supply aircraft alternators with constant input speed. In this case, the prime mover is a variable-speed aircraft engine and the drive acts as a differential transmission to maintain constant generator-shaft speed (see Fig. **13-104).**

A rotary hydraulic amplifier (rotary pump) is generally used with a preamplifier stage that is arranged to vary the pump stroke. The load is usually a rotary hydraulic

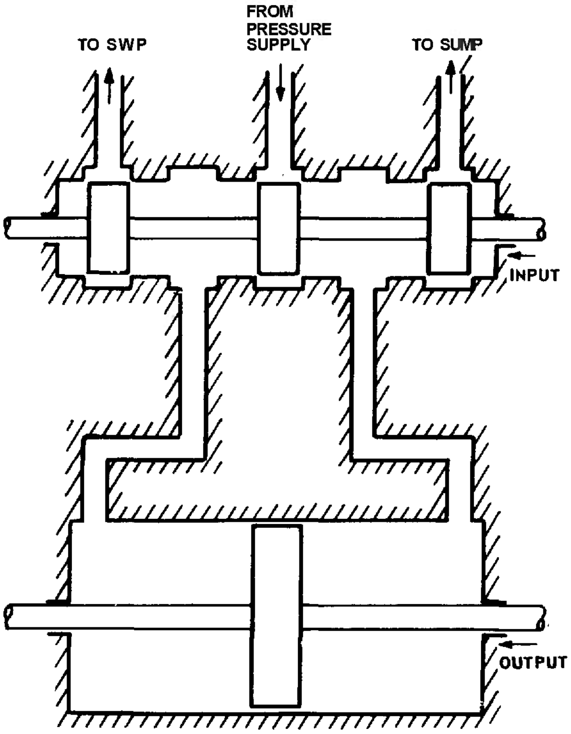


Fig. 73-97 Four-way spool-valve amplifier with open center.

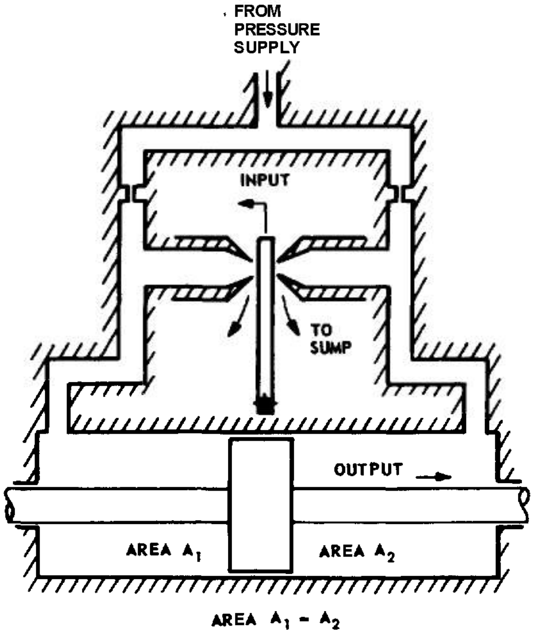


Fig. 73-98 Nozzle-buffle amplifier with balanced load.

motor. This pump-motor combination is termed a *transmission.* A rotary pump is more efficient than a control valve as a means of modulating power, but it is more complex and heavier. These disadvantages are partially offset by the fact that a control valve type of amplifier must be supplied with pressurized oil from a fixed-displacement pump or an accumulator.

13-6.3 Characteristics

Hydraulic amplifiers possess both static and dynamic characteristics. As stated in the introduction, the amplifier consists of valve and load. Since the load is almost universally a piston, that term will be used henceforth. The valve can be considered as a controlled source of energy in the form of a stream of oil. As in the case of an electronic valve (vacuum tube), such an energy source can be characterized as an idealized pressure source in series with a resistance or as an idealized flow

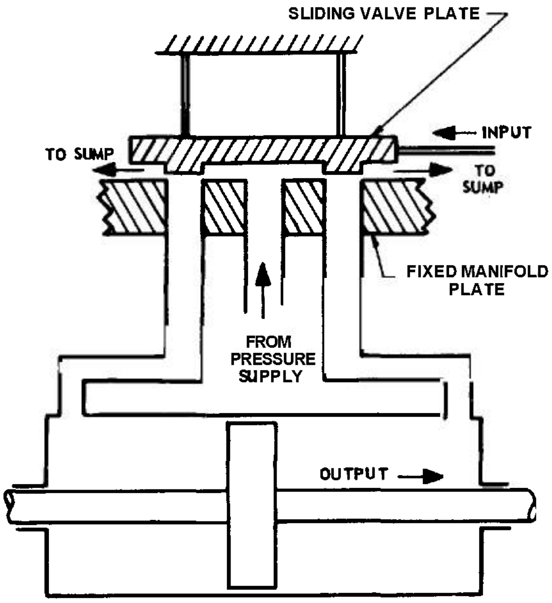


Fig. 13-99 Sliding-plute valve amplifier.

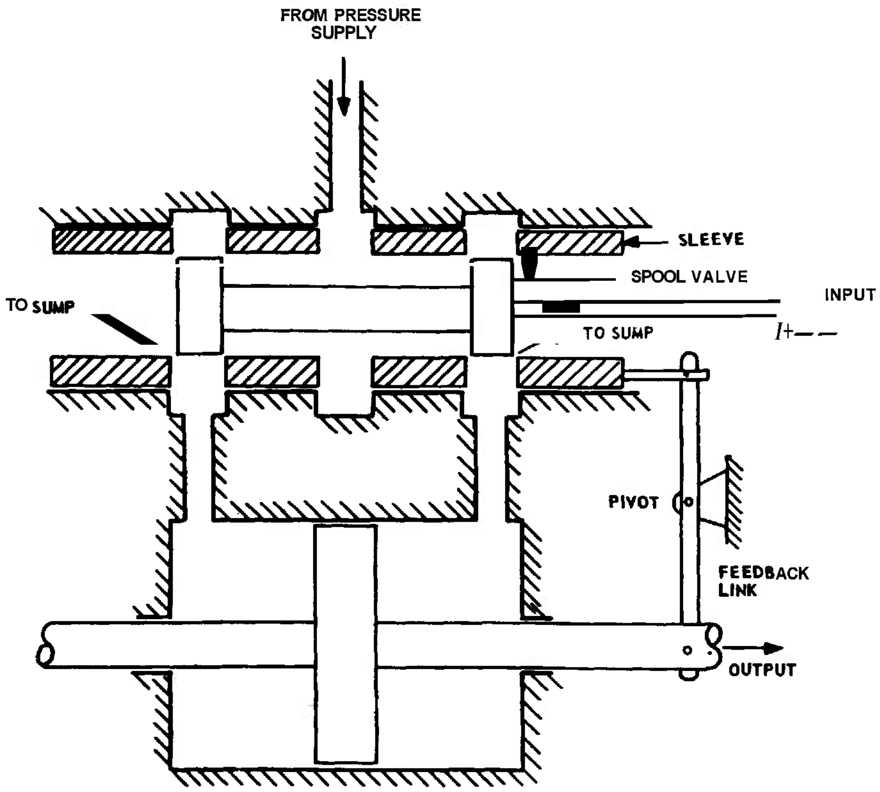


Fig. 13-100 Amplifier with position feedback by means of moving valve sleeve.

source shunted by a conductance (hydraulic conductance is sometimes termed *leakage*). The steady-state pressure-flow properties of the valve can be used to evaluate the pressure-source and resistance properties. The pipe lines connecting valve to cylinder are characterized by hydraulic resistance, capacitance, and inertance. The leakage between cylinder wall and piston constitutes a hydraulic conductance. The volume of oil compressed in the cylinder possesses hydraulic capacitance. The piston has inertia. In addition, there may be a spring (and possibly a dashpot) that opposes piston motion. All of these elements can be represented as lumped hydraulic-circuit parameters. This hydraulic circuit is the load (in a circuit sense) that is supplied by the pressure source (of the valve) through the internal resistance (of the valve). Since the steady-state valve characteristics are used to find the equivalent-source parameters, these will be discussed first. The effect of the remainder of the amplifier, called the load, will then be discussed.

13-6.4 TRANSLATIONAL HYDRAULIC AMPLIFIERS

136.5 Spool-Valve Type

Spool-valve characteristics vary widely, depending upon the amount of underlap and the amount of radial clearance used. The underlap of a spool valve is the axial clearance between the spool and the port when the spool is at the null or center position (axial spool position of minimum supply flow), as shown in Fig. 13-106. Radial clearance is necessary to prevent sticking, whereas underlap improves linearity at small displacements of the

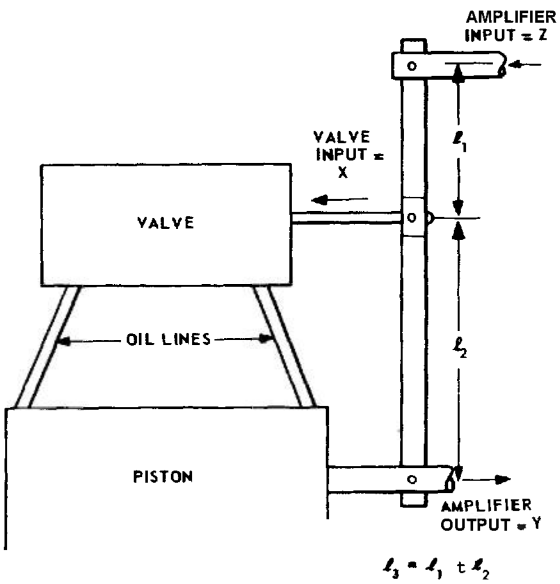


Fig. 73-701 Amplifier with position feedback by means of linkage.

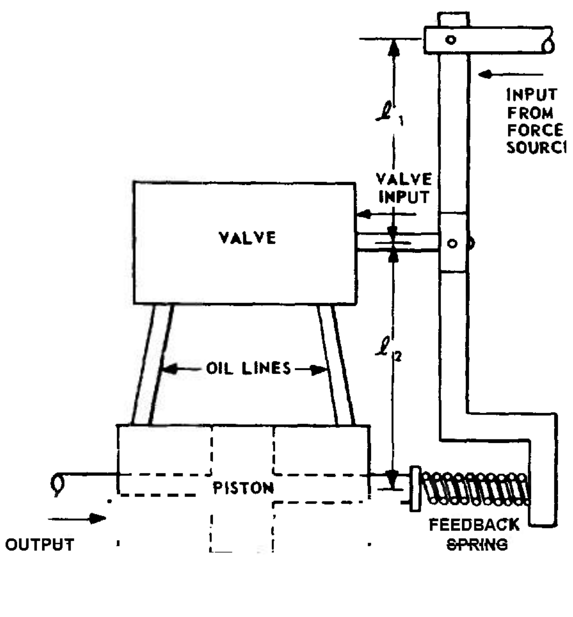


Fig. 73-702 Amplifier with feedback by means of force-balance system.

spool from null (see Fig. 13-108). Since both clearance and underlap result in oil leakage, the amount of oil consumption is increased when the spool is at the null position.

Manufacturers' data is generally in the form of a curve of load pressure versus spool displacement, for zero load flow (P_5 vs $x, Q_5 = Q_2 = 0$). Alternatively, it can be a curve of flow to the load versus valve opening, with a constant pressure maintained at the load (Q_5 vs $x, P_5 =$ constant, $P_2 =$ constant). Figure 13-107 shows typical characteristic curves of a four-way spool valve for various ratios of radial clearance b to underlap X_o. These curves are drawn for relatively small radial clearances; i.e., $0 \leqq b \leqq X_0$. Figure 13-108 shows the effect of negligible underlap. The results of zero underlap in the presence of radial clearances are: serious nonlinearity, and zero gain around the null position.

So-called "pressure-control" valves produce a curve of load pressure versus spool displacement similar to that of the valve with underlap. However, these valves differ from underlapped spool valves in that they have a much lower internal resistance; i.e., their output pressure is less affected by wide variations in load flow.

Curves of steady-state load flow versus spool displacement, for various load pressures, are shown in Figs. 13-109, 13-110, and 13-111 (see Fig. 13-106 for meaning of symbols). The effects of radial clearance and underlap are illustrated by these curves, which are drawn for an uncompensated spool valve. Compensation may be used to make output flow less dependent upon load pressure.

To determine the gain and internal resistance of the four-way spool-valve amplifier, certain curves of static performance must be available. Manufacturers' data is commonly given in dimensionalized form. The curves presented in this section are purposely shown in nondimensionalized form in order to be more generally applicable and also to place in evidence the significant parameters of the amplifier, whenever possible. For spool valves with underlap (i.e., a linear characteristic in the vicinity of null), dimensionalizing the

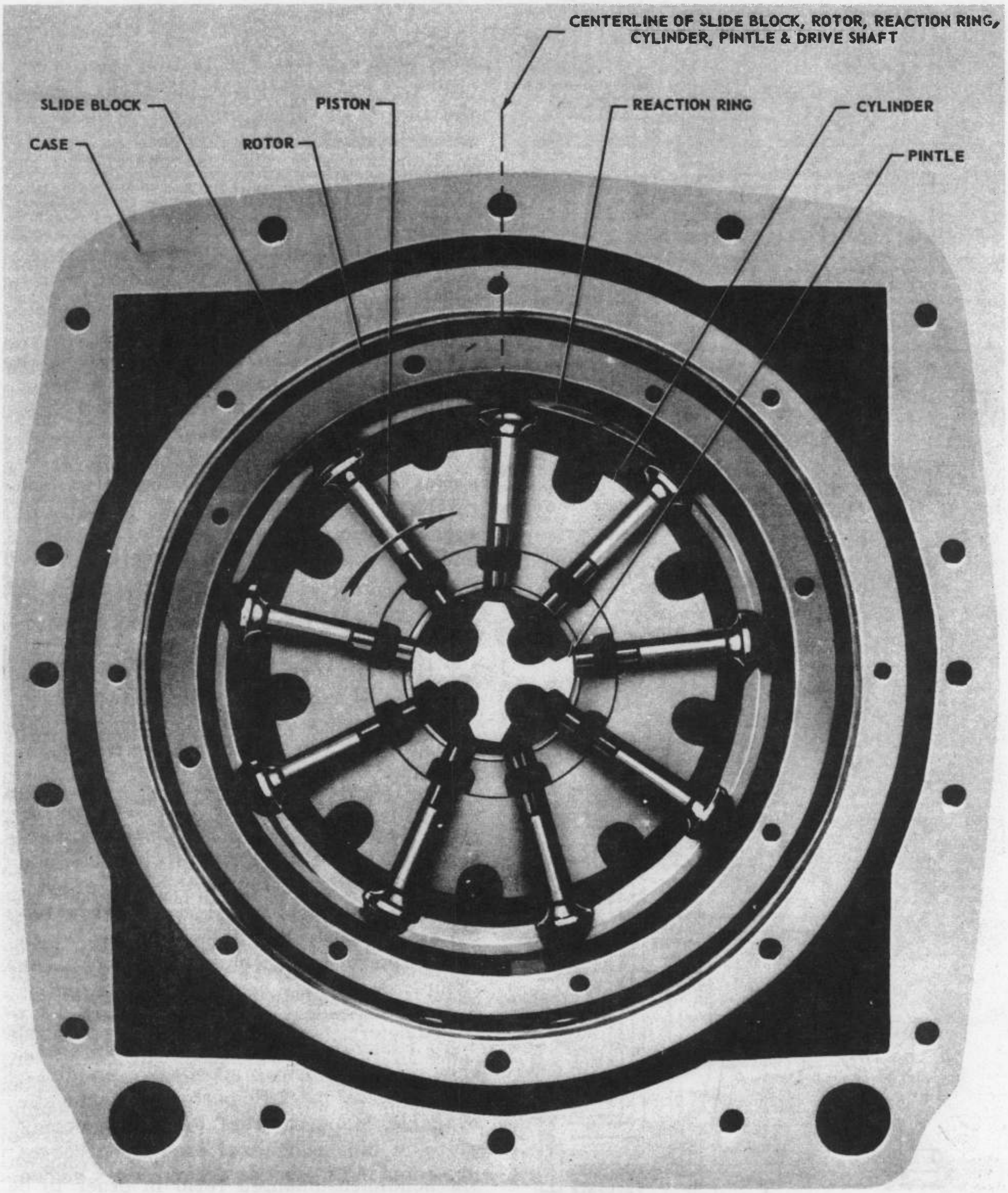


Fig. 13-103 Oil-gear rotary hydraulic amplifier with radial pistons.

Courtesy The Oilgear Company.

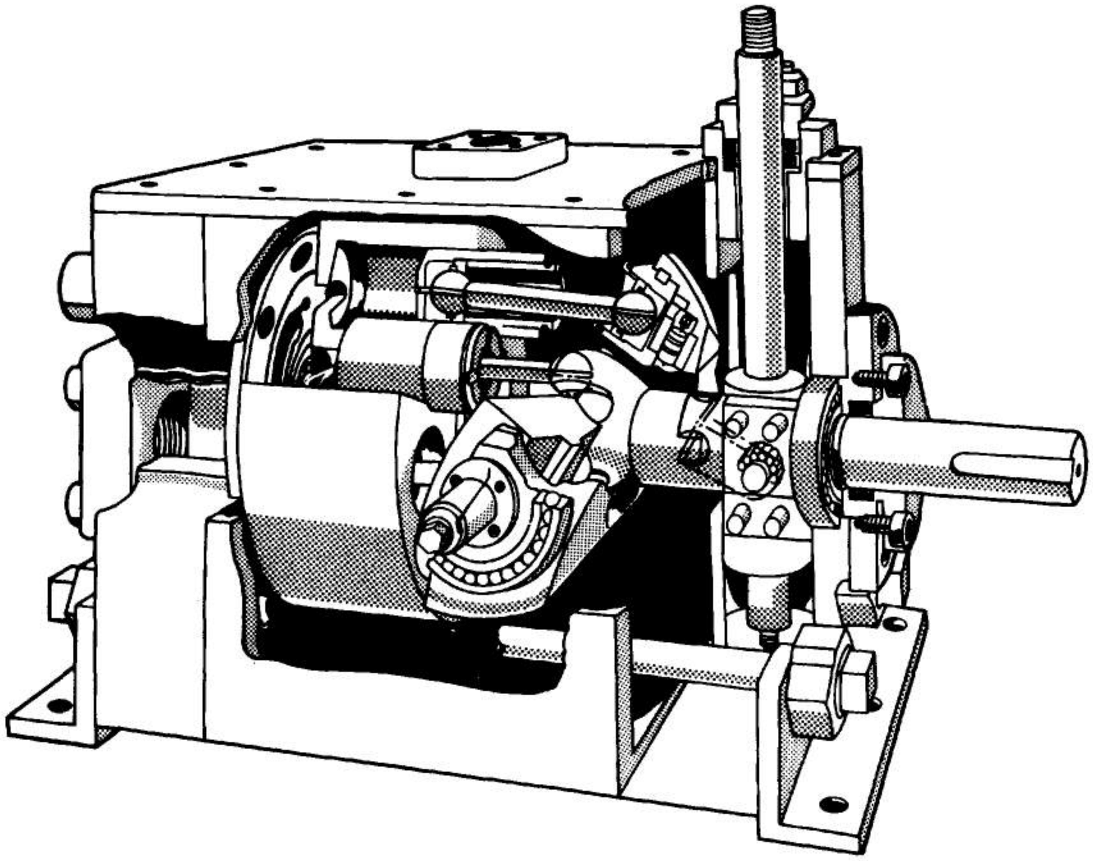


Fig. 13-104 Constant-speed rotary hydraulic amplifier with axial pistons.

Courtesy Northern Ordnance Incorporated.

slope of the curve at null, Fig. 13-107, will suffice to characterize the valve in terms of the gain of a pressure source. The gain **of** a pressure source is the ratio of the change in pressure to the motion required to produce the pressure change. The internal resistance for this case may be found in a manner discussed below. The internal resistance is the coefficient which, when multiplied by the flow from the valve, yields the decrease in load pressure from that value of pressure that would occur in the absence of flow. If the available curves are for flow, as in Figs. **13-109, 13-110,** and **13-111,** an average dimensionalized slope for large spool displacements may be used to characterize the valve in terms of the gain of a flow source. The internal conductance may be found by taking an average value of $\partial Q_5/\partial P_5$ for a given spool displacement.

13-6.6 Equivalent source representation of four-way spool valve. The equivalent hydraulic-source representations of the four-way

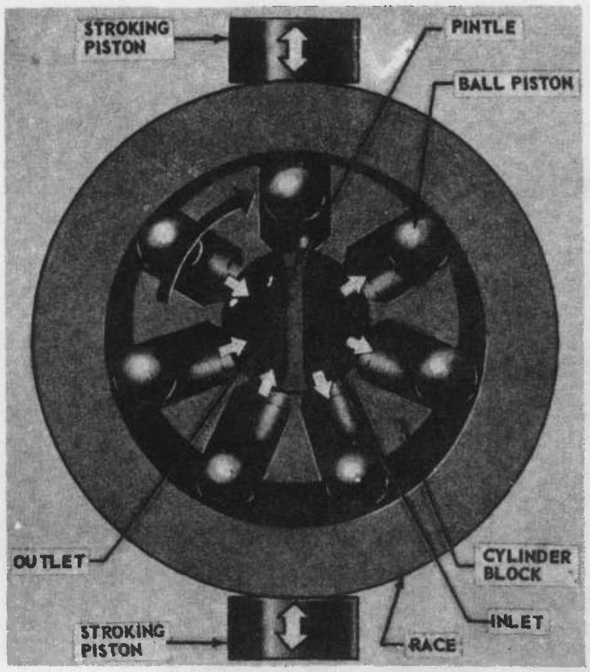


Fig. 13-105 Ball-and-piston type rotary hydraulic amplifier.

Courtesy General Electric Company.

spool-valve amplifier (Fig. 13-112) involve the static parameters discussed above; i.e.,

(a) Pressure-source representation:

$$\text{internal gain} = \frac{\partial P}{\partial x}$$

$$\text{internal resistance} = \frac{\partial P}{\partial Q}$$

(b) Flow-source representation:

$$\text{internal gain} = \frac{\partial Q}{\partial x}$$

$$\text{internal conductance} = \frac{\partial Q}{\partial P}$$

The partial derivatives that express the internal gain and resistance or conductance of the equivalent source appear in the flow equations of the four-way spool-valve amplifier. It is possible to evaluate these partials numerically, by experiment, or from manufacturers' data, and hence numerically evaluate the equivalent hydraulic circuit.

13-6.7 Flow equations of underlapped four-way spool valve. Equations of flow for spool valves with underlap that is large compared with radial clearance ($X_o/b > 1.0$) are presented first. From these equations, the expressions for plotting the parameters of internal gain and resistance are derived. The expressions for incremental load flow are (see Fig. 13-106 for meaning of symbols)

$$dQ_2 = \frac{\partial Q_2}{\partial P_2} dP_2 + \frac{\partial Q_2}{\partial x} dx \tag{13-86}$$

$$dQ_5 = \frac{\partial Q_5}{\partial P_5} dP_5 + \frac{\partial Q_5}{\partial x} dx \tag{13-87}$$

Both Q_2 and Q_5 are determined from the following relationships which hold for

$-X_o \leqq x < X_o$:

$$Q_1 = c\pi d\,(X_o + x)\,\sqrt{P_2} \tag{13-88}$$

$$Q_2 = Q_3 - Q_1 \tag{13-89}$$

$$Q_3 = c\pi d\,(X_o - x)\,\sqrt{P_s - P_2} \tag{13-90}$$

$$Q_4 = c\pi d\,(X_o + x)\,\sqrt{P_s - P_5} \tag{13-91}$$

$$Q_5 = Q_4 - Q_6 \tag{13-92}$$

$$Q_6 = c\pi d\,(X_o - x)\,\sqrt{P_5} \tag{13-93}$$

where

Q = flow, in in.^{3}/sec

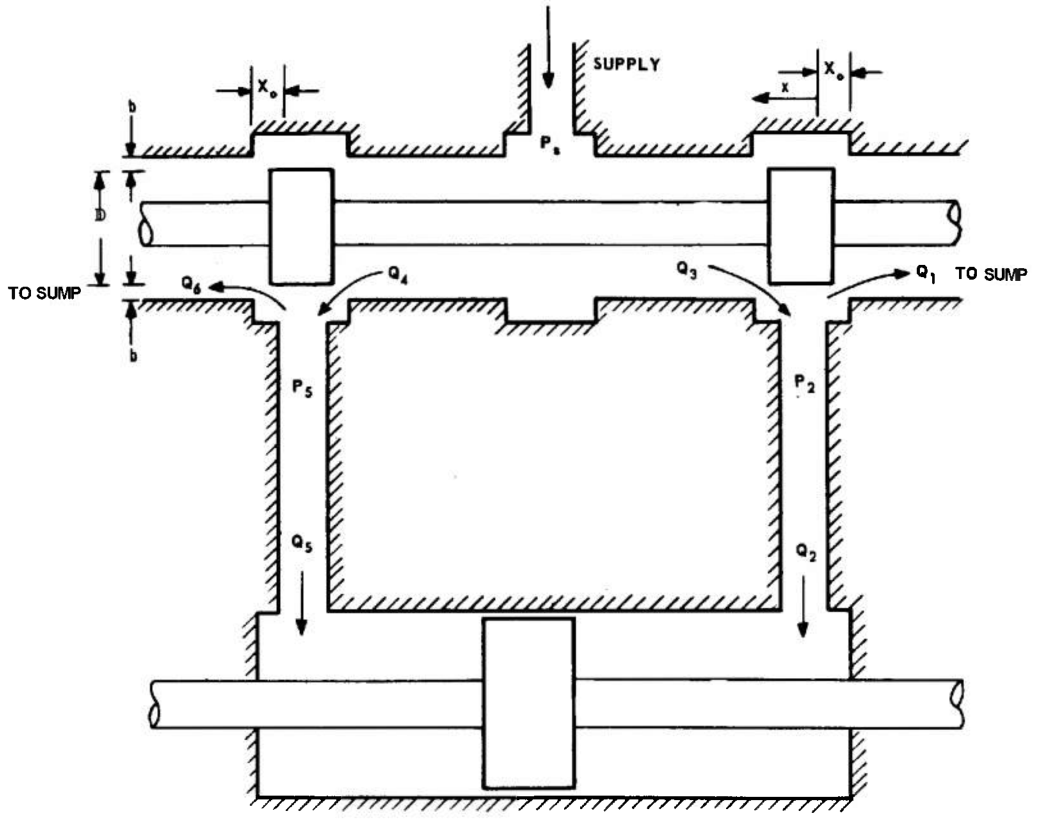
P = pressure measured above sump pressure, in lb/in.^{2}

x = axial spool displacement measured from center or null position (spool position of minimum supply flow), in in.

X_o = underlap, in in.

d = spool diameter plus radial clearance, in in.; i.e., $d = D + b = D$ for case of zero radial clearance

ρ = density of fluid, in lb-sec^2/in.^{4} (see Par. 20-2)



VALVE CENTERED

(a) $\beta = \frac{1}{\gamma} = \frac{b}{X_o}$ = $\frac{\text{RADIAL CLEARANCE}}{\text{UNDERLAP}}$

(b) $P_L = P_5 - P_2$

(c) $Q_L = Q_5 = -Q_2$

(d) $Q_{LO} = Q_3 + Q_4$ WHEN $x = 0$, $P_2 = P_5 = \frac{P_s}{2}$

(e) $p_2 = \frac{P_2}{P_s}$ AND $p_5 = \frac{P_5}{P_s}$

Fig. 73-706 Four-way spool valve, all pressures measured above sump pressure.

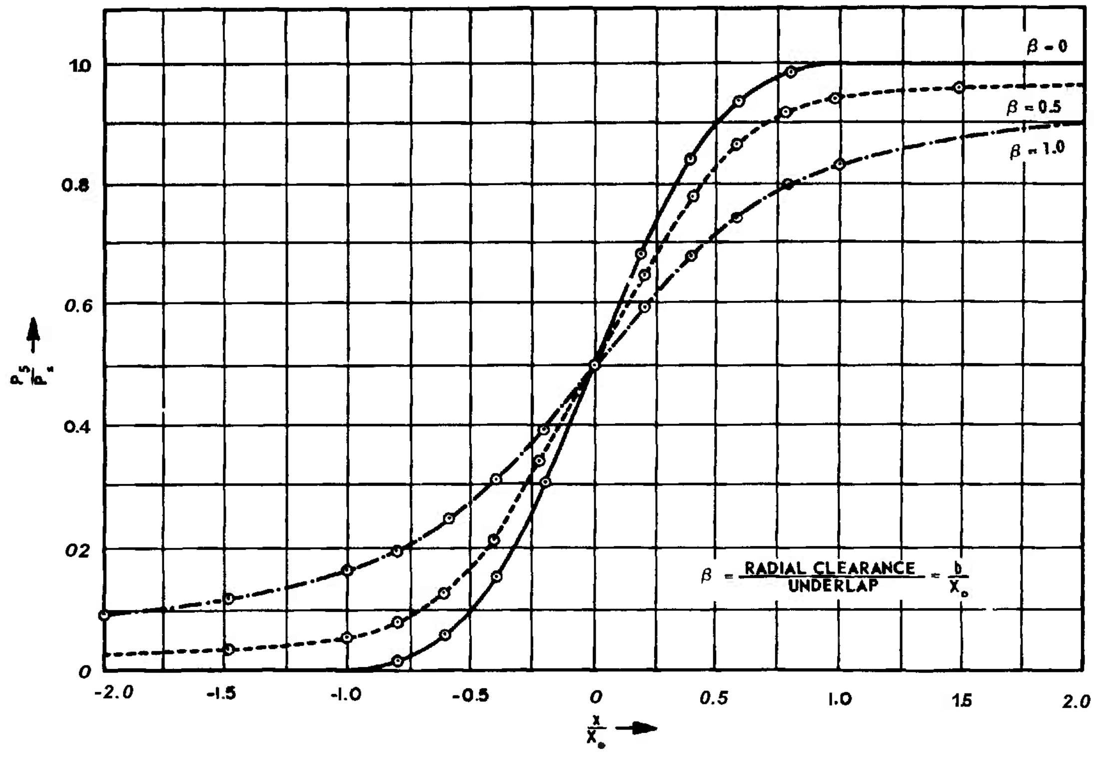


Fig. 73-707 Four-way spool valve, nondimensional plot of load pressure vs spool displacement for zero load flow.

c = a proportionality factor whose value is given by

$$0.61\sqrt{\frac{2}{\rho}} < c < 0.65\sqrt{\frac{2}{\rho}}$$

Expressions for evaluating the partial derivatives in the right-hand side of Eas. **(13-86)** and **(13-87)** may be found by making the appropriate substitutions for Q from Eqs. **(13-88)** through **(13-93).** The results are shown in Eqs. **(13-94)** through **(13-97).** These expressions are true only for axial displacement of the **spool** within the underlap region $(-X_o \leqq x \leqq X_o)$. **For** displacement **of** $x > X_o$ **or** $x < -X_o$, $Q_3 = Q_6 = 0$ and hence Eqs. **(13-88)** through **(13-97)** are somewhat simplified.

$$\frac{\partial Q_2}{\partial x} = -c\pi d\ \sqrt{P_s}\left[\sqrt{1-p_2} + \sqrt{p_2}\right] \tag{13-94}$$

$$\frac{\partial Q_5}{\partial x} = c\pi d\ \sqrt{P_s}\left[\sqrt{1-p_5} + \sqrt{p_5}\right] \tag{13-95}$$

$$\frac{\partial Q_2}{\partial P_2} = \frac{-c\pi d\ X_o}{2\ \sqrt{P_s}}\left[\frac{1-x^*}{\sqrt{1-p_2}} + \frac{1+x^*}{\sqrt{p_2}}\right] \tag{13-96}$$

$$\frac{\partial Q_5}{\partial P_5} = \frac{-c\pi d\ X_o}{2\sqrt{P_s}}\left[\frac{1+x^*}{\sqrt{1-p_5}} + \frac{1-x^*}{\sqrt{p_5}}\right] \tag{13-97}$$

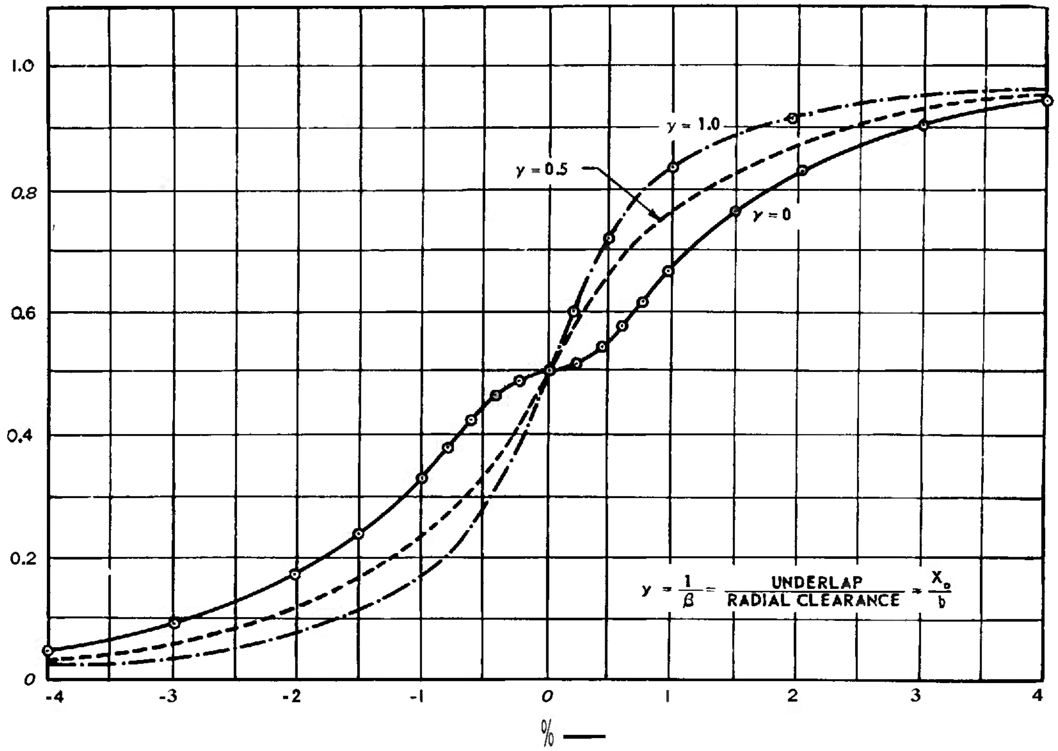


Fig. 13-108 Four-way spool valve, nondimensional plot of load pressure vs spool displacement for zero load flow.

where

$$p_2 = \frac{P_2}{P_s} \quad (13\text{-}98)$$

$$p_5 = \frac{P_5}{P_s} \quad (13\text{-}99)$$

$$x^* = \frac{X}{X_0} \quad (13\text{-}100)$$

P_s = supply pressure, in lb./in.^{2}

and

$$-1 \leqq x^* \leqq 1$$

Values for the pressure gain $\partial P/\partial x$ may be found from the relationship

$$\frac{\partial P}{\partial x} = -\frac{\frac{\partial Q}{\partial x}}{\frac{\partial Q}{\partial P}} \quad (13\text{-}101)$$

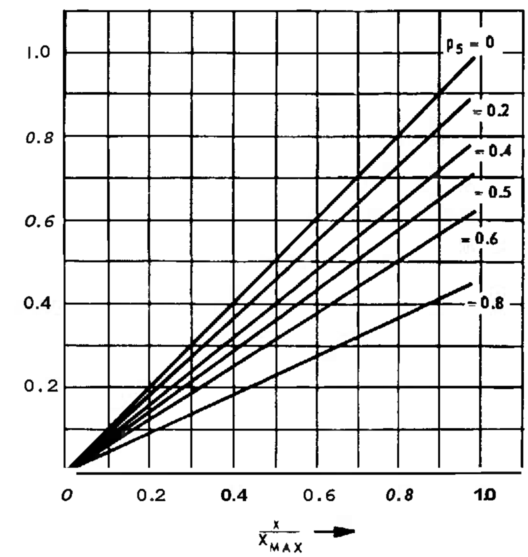


Fig. 13-109 Four-way spool valve, nondimensional plot of load flow vs spool displacement for zero underlap and zero radial clearance ($b = X_0 = 0$).

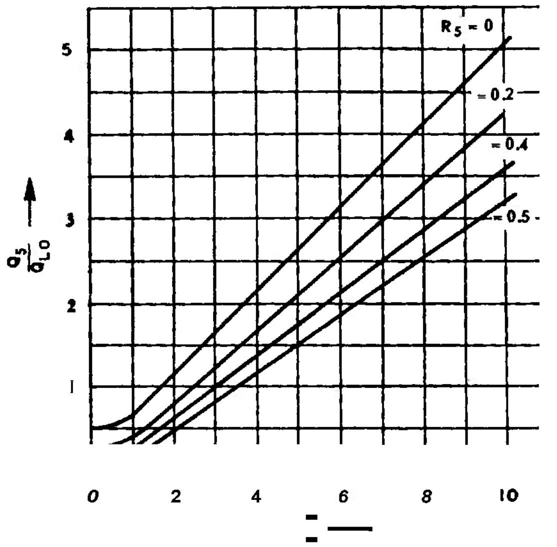


Fig. 73-7 70 Four-way spool valve, nondimensional plot of load flow vs spool displacement for zero underlap ($X_0 = 0$) and finite radial clearance ($6 \neq 0$).

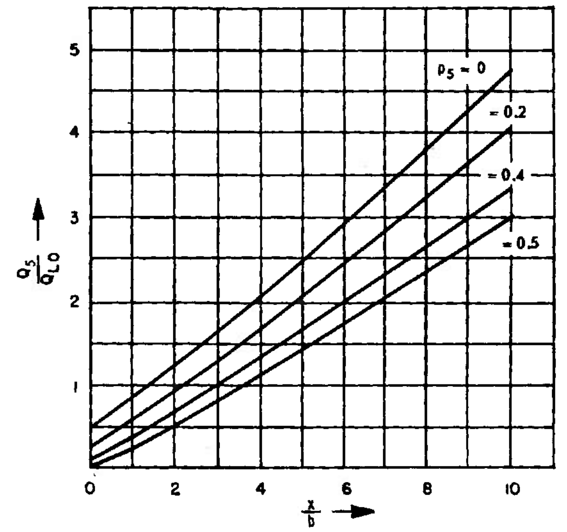


Fig. 73-711 Four-way spool valve, nondimensional plot of load flow vs spool displacement for $y = 0.5$.

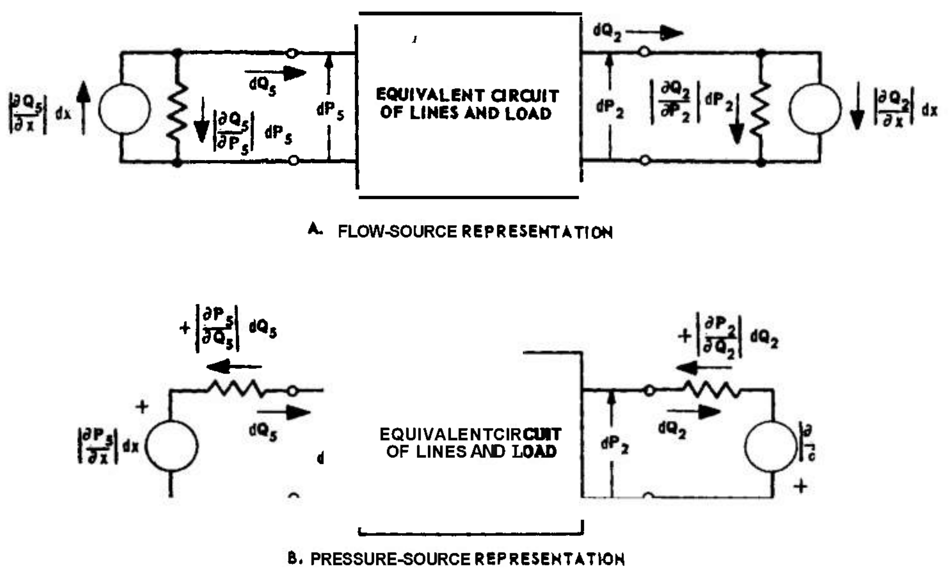


Fig. 13-112 Four-way spool valve, equivalent hydraulic-source representation,

The circuit elements of the equivalent hydraulic-source representation shown in Fig. **13-112** may thus be evaluated by Eqs. **(13-94)** through **(13-101).** Nondimensional plots of the partial derivatives that characterize the source in terms of its flow gain and conductance are shown in Figs. **13-113** and **13-114.** These plots demonstrate how the flow gain and internal conductance of the hydraulic amplifier depend upon the operating point.

13-6.8 Dimensionalizing flow-gain and conductance plots of underlapped four-way spool valve. The dimensionalizing factors that must be applied to the nondimensional plots in Figs. **13-113** and **13-114** are $c\pi d\sqrt{2P_s}$ and $c\pi d X_o \sqrt{2}/\sqrt{P_s}$. These factors can be found from a measured curve of load pressure versus spool displacement for the condition of zero load flow and from a measurement of leakage flow. Let the magnitude of the slope of the measured load pressure versus spool-displacement

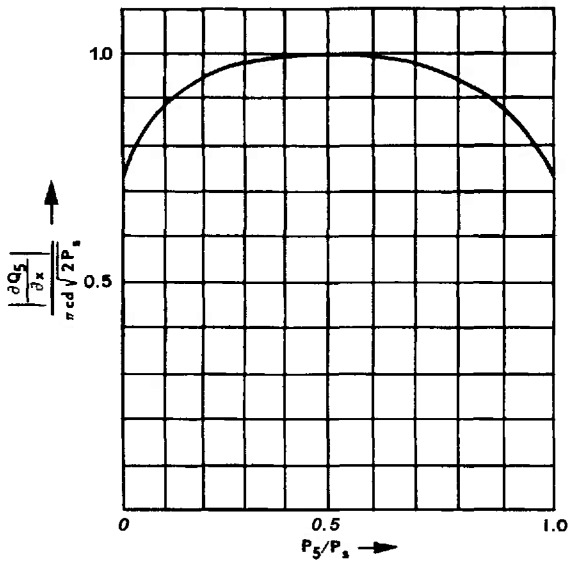


Fig. 13-113 Four-way spool valve nondimensional plot of flow gain vs load pressure for zero radial clearance (b = 0).

curve at zero displacement be $\left.\frac{\partial P_5}{\partial x}\right|_{x=0}$ and let the total flow from the pressure supply to the sump, measured with zero flow to the load and at zero spool displacement, be Q_{L0}. Then, from Eq. **(13-101),** it is found that

$$c\pi d\sqrt{2P_s} = \frac{Q_{L0}}{P_s}\left(\left.\frac{\partial P_5}{\partial x}\right|_{x=0}\right) \tag{13-102}$$

where

$$\begin{aligned} Q_{L0} &= Q_3 + Q_4 \\ &= Q_1 + Q_6 \end{aligned}$$

when

$$x = 0,\ P_2 = P_5 = \frac{1}{2}P_s$$

From Eqs. **(13-90)** and **(13-91),** it is found that

$$\frac{c\pi d X_o\sqrt{2}}{\sqrt{P_s}} = \frac{Q_{L0}}{P_s} \tag{13-103}$$

If Eqs. **(13-102)** and **(13-103)** are combined, then

$$\left.\frac{\partial P_5}{\partial x}\right|_{x=0} = \frac{P_s}{X_o} \tag{13-104}$$

13-6.9 Small-signal gain and conductance parameters of underlapped four-way spool valve. For small-signal variations (i.e., spool displacements) about null, the average load pressure is $P_s/2$ as long as the valve has nonzero underlap or nonzero radial clearance. Therefore, from Eqs. **(13-95)** and **(13-102),** the small-signal flow gain $\partial Q_5/\partial x$ becomes

$$\frac{\partial Q_5}{\partial x} = c\pi d\sqrt{2P_s} = \frac{Q_{L0}}{P_s}\left(\left.\frac{\partial P_5}{\partial x}\right|_{x=0}\right) \tag{13-105}$$

From Eqs. **(13-97)** and **(13-103),** the internal conductance $\partial Q/\partial P$ becomes

$$\frac{\partial Q}{\partial P} = \frac{c\pi d X_o\sqrt{2}}{\sqrt{P_s}} = \frac{Q_{L0}}{P_s} \tag{13-106}$$

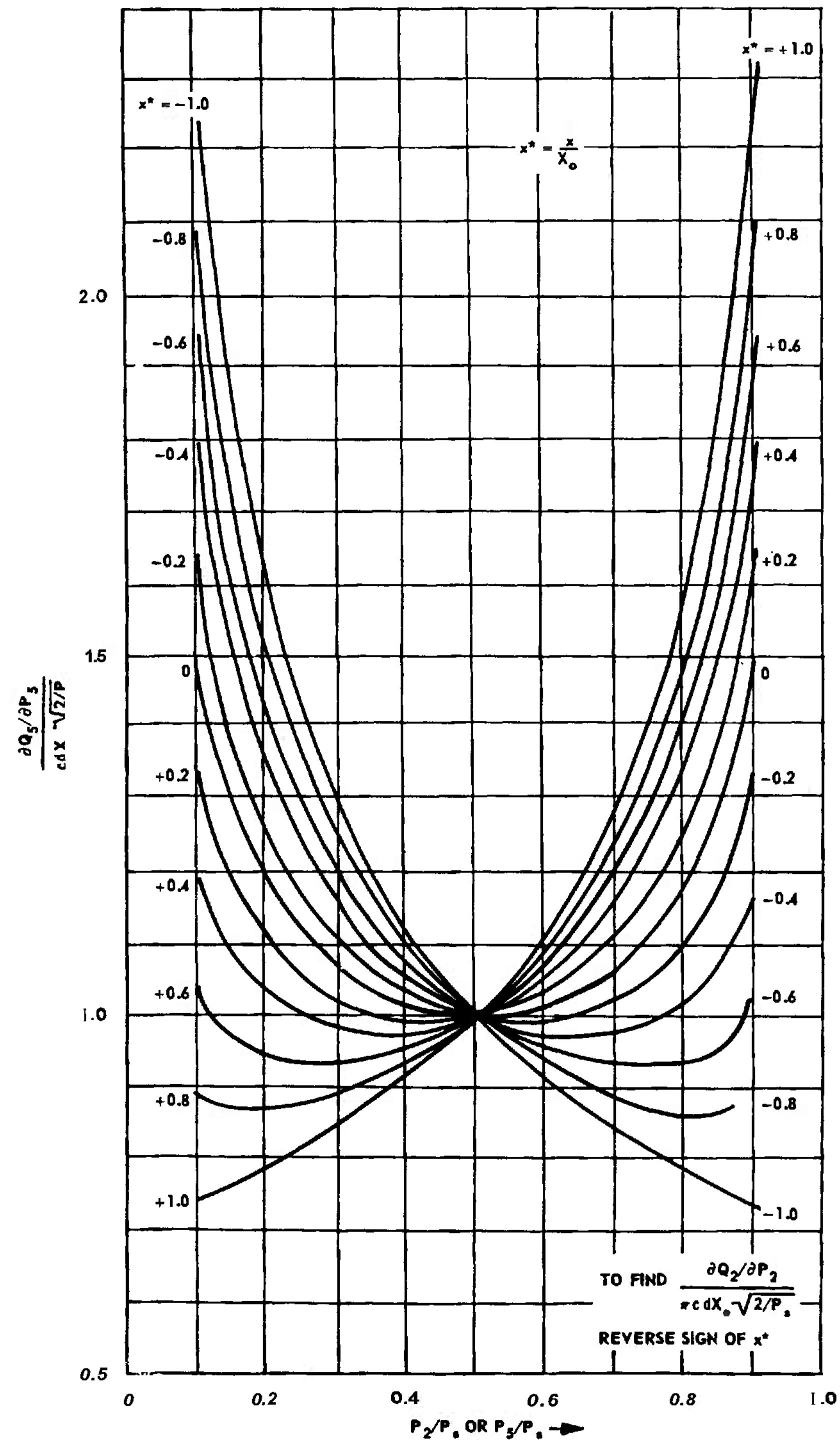
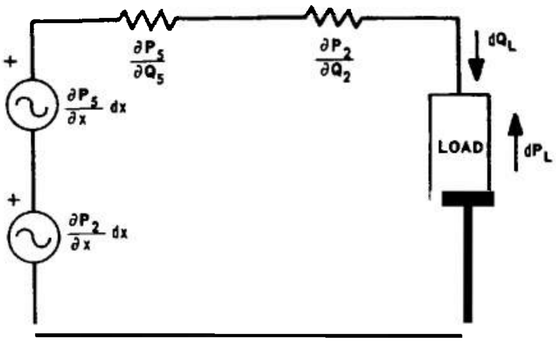


Fig. 13-174 Four-way spool valve with negligible radial clearance, nondimensional plot of internal conductance vs load pressure.

From Eqs. (13-101), (13-105), and (13-106), the pressure gain is

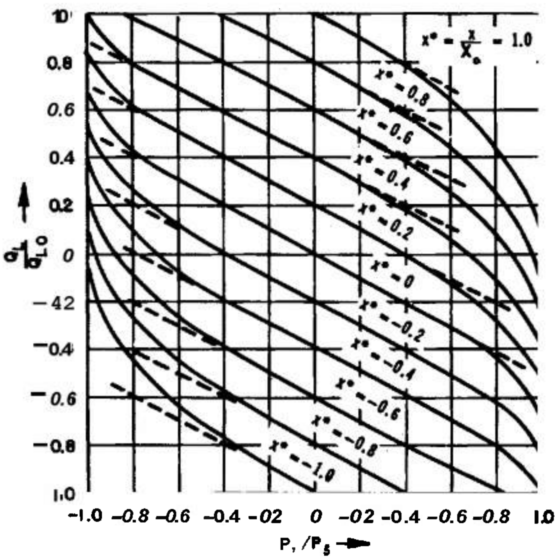
$$\frac{\partial P}{\partial x} = -\frac{\partial Q}{\partial x}\Big/\frac{\partial Q}{\partial P} = -\frac{\partial P_5}{\partial x}\bigg|_{x=0} \tag{13-107}$$

13-6.10 Balanced-load steady-state characteristics of underlapped four-way spool valve. A load is considered balanced when the two sides of the power piston have equal areas. Curves of the differential load pressure-load flow relation for this case (see Fig. 13-115) have been plotted by **Blackburn.**[46] For a balanced load, Fig. 13-112 can be arranged as shown in Fig. 13-116. If $\partial P_2/\partial x = \partial P_5/\partial x$ and $\partial P_2/\partial Q_2 = \partial P_5/\partial Q_5$, as is the case near the null position where $P_2 = P_5 = P_s/2$, Fig. 13-116 simplifies to Fig. 13-117.



P_L = PRESSURE DROP ACROSS LOAD
Q_L = FLOW THROUGH LOAD

Fig. 73-776 Equivalent circuit of four-way spool-valve amplifier with balanced load.



$Q_L = Q_5 = -Q_2$ EQUALS THE FLOW TO THE BALANCED LOAD. P_L IS THE PRESSURE DROP ACROSS THE LOAD
$Q_{LO} = Q_3 + Q_4$ WHEN $x = 0$, $P_2 = P_5 = P_s/2$.
—— COMPUTED FROM FLOW EQUATION
- - - LINEARIZED REPRESENTATION

Fig. 73-715 Four-way spool valve, nondimensional plot of load flow vs differential load pressure for balanced load.

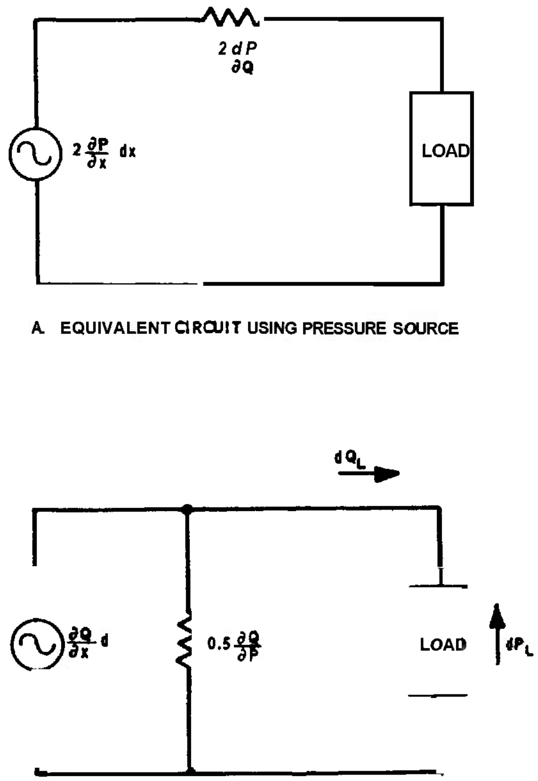


Fig. 13-117 Simplified equivalent circuits of four-way spool-valve amplifier with balanced load.

13-6.11 Flow equations of four-way spool valve with radial clearance. As the amount of underlap X_o is decreased, the radial clearance b becomes more significant until, in the limit, X_o becomes zero. In some applications, overlapped valves may be used. However, their advantage in servo work appears negligible. Equations **(13-88)** through **(13-93)** must be modified to take radial clearance b into account. The orifice area becomes that of the frustrum of a cone (see Fig. **13-118**). Flows and pressures are defined as before (see Fig. **13-106**). For $-X_o \leqq x \leqq X_o$, the flow equations are

$$Q_1 = c\pi d \sqrt{(X_o + x)^2 + b^2} \times \sqrt{P_2} \qquad (13\text{-}108)$$

$$Q_2 = Q_3 - Q_1 \qquad (13\text{-}109)$$

$$Q_3 = c\pi d \sqrt{(X_o - x)^2 + b^2} \times \sqrt{P_s - P_2} \qquad (13\text{-}110)$$

$$Q_4 = c\pi d \sqrt{(X_o + x)^2 + b^2} \times \sqrt{P_s - P_5} \qquad (13\text{-}111)$$

$$Q_5 = Q_4 - Q_6 \qquad (13\text{-}112)$$

$$Q_6 = c\pi d \sqrt{(X_o - x)^2 + b^2} \times \sqrt{P_5} \qquad (13\text{-}113)$$

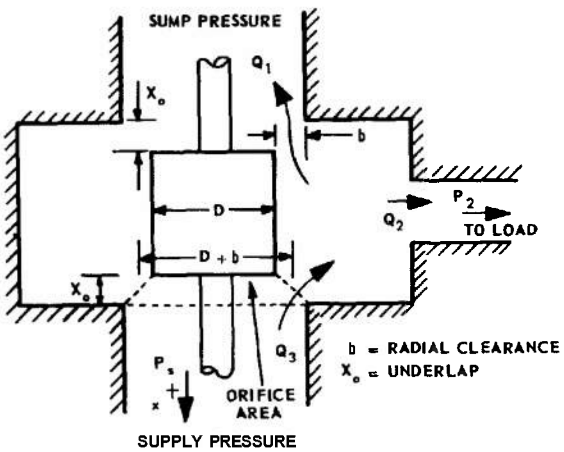


Fig. 13-118 Geometry of spool valve with radial clearance and underlap (spool shown in center or null position).

For $X_o \leqq x \leqq X_{max}$, where X_{max} is the value of x when the load ports are entirely open, the expressions for Q_3 and Q_6 [i.e., Eqs. **(13-110)** and **(13-113)**] become

$$Q_3 = c\pi d\, b \sqrt{P_s - P_2} \qquad (13\text{-}114)$$

and

$$Q_6 = c\pi d\, b \sqrt{P_5} \qquad (13\text{-}115)$$

The expressions **for** the remaining flows —Q_1, Q_2, Q_4, and Q_5 —remain unchanged.

The partial derivatives that characterize the source in terms of its flow gain and conductance are tabulated in Table **13-15.** Nondimensionalized plots of the magnitudes of the partial derivatives in Table **13-15** are shown in Figs. **13-119** through **13-122.** The equiva-

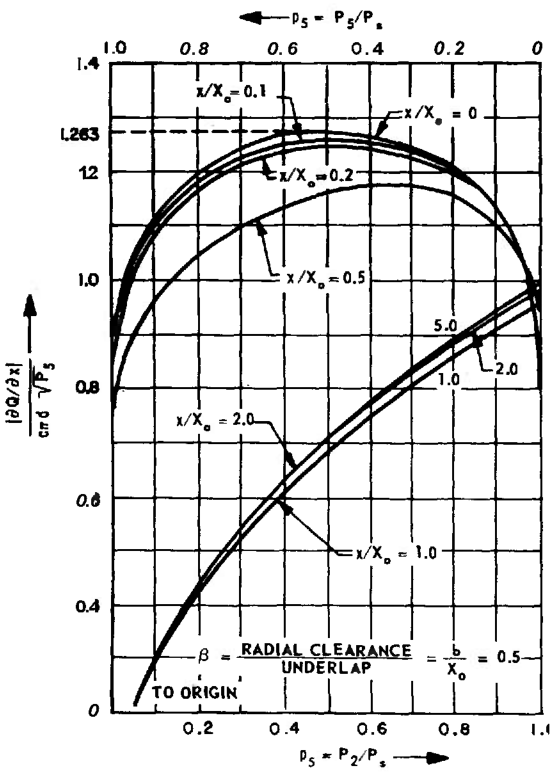


Fig. 13-119 Four-way spool valve, nondimensional plot of flow gain vs load pressure for β = 0.5.

TABLE 13-15 FOUR-WAY SPOOL VALVE WITH APPRECIABLE RADIAL CLEARANCE. EQUIVALENT-SOURCE FLOW GAIN AND CONDUCTANCE PARAMETERS

	Spool Displacement → / ↓ Partial Derivative	$0 \leq x \leq X_o$	$X_o \leq x \leq X_{max}$
Flow Gain	$\dfrac{\partial Q_2/\partial x}{c\pi d\sqrt{P_s}}$	$-\left\{(1-x^*)\left[\dfrac{1-p_2}{(1-x^*)^2+\beta^2}\right]^{1/2} + (1+x^*)\left[\dfrac{p_2}{(1+x^*)^2+\beta^2}\right]^{1/2}\right\}$	$-(1+x^*)\left[\dfrac{p_2}{(1+x^*)^2+\beta^2}\right]^{1/2}$
	$\dfrac{\partial Q_5/\partial x}{c\pi d\sqrt{P_s}}$	$+\left\{(1+x^*)\left[\dfrac{1-p_5}{(1+x^*)^2+\beta^2}\right]^{1/2} + (1-x^*)\left[\dfrac{p_5}{(1-x^*)^2+\beta^2}\right]^{1/2}\right\}$	$+(1+x^*)\left[\dfrac{1-p_5}{(1+x^*)^2+\beta^2}\right]^{1/2}$
Conductance	$\dfrac{\partial Q_2/\partial P_2}{c\pi d X_o/2\sqrt{P_5}}$	$-\left\{\left[\dfrac{(1-x^*)^2+\beta^2}{1-p_2}\right]^{1/2} + \left[\dfrac{(1+x^*)^2+\beta^2}{p_2}\right]^{1/2}\right\}$	$-\left\{\dfrac{\beta}{\sqrt{1-p_2}} + \left[\dfrac{(1+x^*)^2+\beta^2}{p_2}\right]^{1/2}\right\}$
	$\dfrac{\partial Q_5/\partial P_5}{c\pi d X_o/2\sqrt{P_5}}$	$-\left\{\left[\dfrac{(1+x^*)^2+\beta^2}{1-p_5}\right]^{1/2} + \left[\dfrac{(1-x^*)^2+\beta^2}{p_5}\right]^{1/2}\right\}$	$-\left\{\left[\dfrac{(1+x^*)^2+\beta^2}{1-p_5}\right]^{1/2} + \dfrac{\beta}{\sqrt{p_5}}\right\}$
when	$\beta = \dfrac{b}{X_o}$	$p_2 = \dfrac{P_2}{P_s}$ $\quad p_5 = \dfrac{P_5}{P_s}$	$x^* = \dfrac{x}{X_o}$

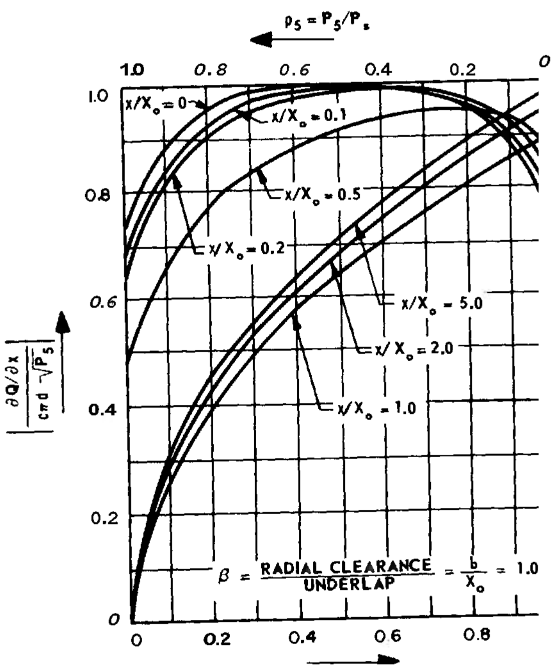


Fig. 73-120 Four-way spool valve, nondimensional plot of flow gain vs load pressure for $\beta = 7.0$.

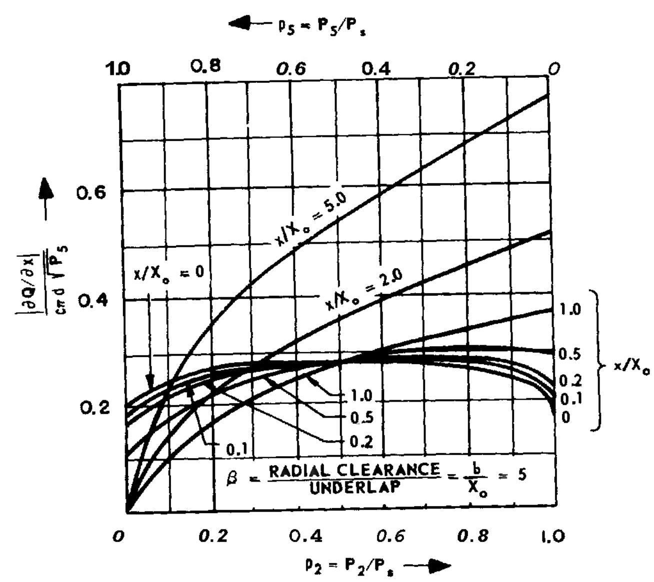


Fig. 73-721 Four-way spool valve, nondimensional plot of flow gain vs load pressure for $\beta = 5$.

lent hydraulic-source representation of the amplifier is shown in Fig. **13-112.** Figures **13-119** and **13-120,** and **13-121** are consistent with Fig. **13-113.** However, note the difference in the scales and also the $\sqrt{2}$ term in the nondimensionalizing factor of Fig. **13-113.**

If compressibility is to be considered, the separate sources set forth in Fig. **13-112** must be used. If a piston with unequal areas is used so that $-Q_5 \neq Q_2$ in the steady state, separate sources must be used together with an ideal transformer. (The circuit diagram for such a case appears in Fig. **13-139.**)

13-6.12 Dimensionalizing flow-gain and conductance plots of four-way spool valve with radial clearance. The nondimensionalized plots of conductance and flow-gain in Figs. **13-119** through **13-122** may be dimensioned if c, d, X_o, b, and P_s are known. For square orifices, which occur in spool valves, plate valves, and at the nozzle of nozzle-baffle valves, it follows that

$$0.61\sqrt{\frac{2}{\rho}} < c < 0.65\sqrt{\frac{2}{\rho}}$$

where ρ is the mass density. A typical value of mass density is **(0.031** lb/in.^{3}) / (**386** in./sec^2). For this value, c can be taken as **100** with reasonable accuracy. To calculate the remaining parameters from measured data of flow and pressure, first obtain a plot of controlled load flow Q_5 (measured at the valve outlet) versus spool displacement x for a given load pressure P_5 and supply pressure P_s. A typical plot is shown in Fig. **13-123.** Then proceed as follows:

(a) Determine d. From Eqs. **(13-112)** and **(13-115)**, it is found that for large values of x (valve opening)

$$\left|\frac{\partial Q_5}{\partial x}\right|_{x \to x_{max}} = c\pi d\sqrt{P_s - P_5}$$

$$\frac{x}{\sqrt{(X_o + x)^2 + b^2}} \approx c\pi d\sqrt{P_s - P_5} \tag{13-116}$$

Thus

$$d \approx \frac{\left|\frac{\partial Q_5}{\partial x}\right|_{x \to x_{max}}}{314\sqrt{P_s - P_5}} \tag{13-117}$$

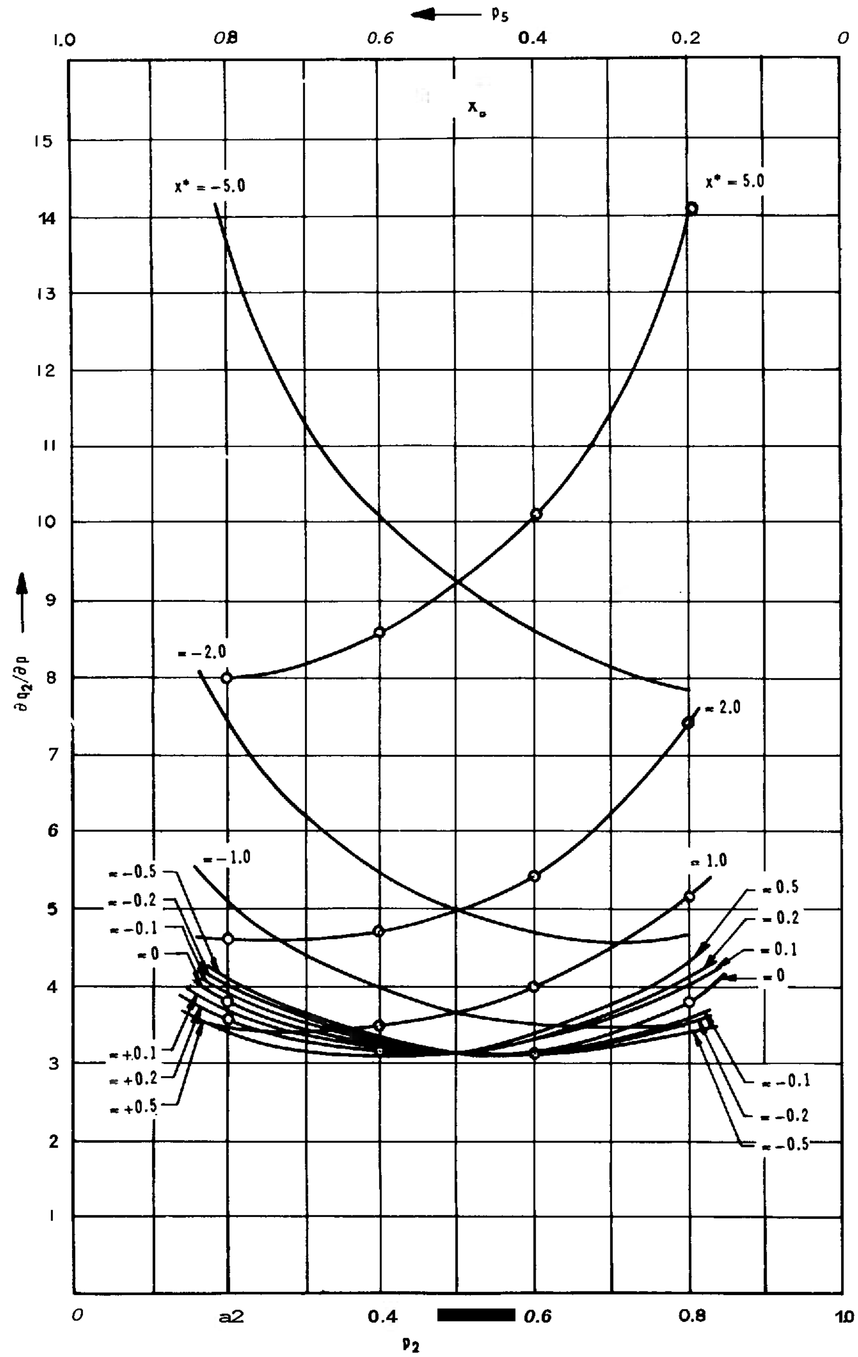


Fig. 73-722 Plot of $\dfrac{\left|\dfrac{\partial Q}{\partial P}\right|}{\left[\dfrac{C\pi d X_o}{2\sqrt{P_s}}\right]}$ *vs* $\dfrac{P}{P_s}$ *with* $\dfrac{x}{X_o}$ *as parameter with* $\beta = 0.5$, *1.0, and 5.0. (Sheet 1 of 3)*

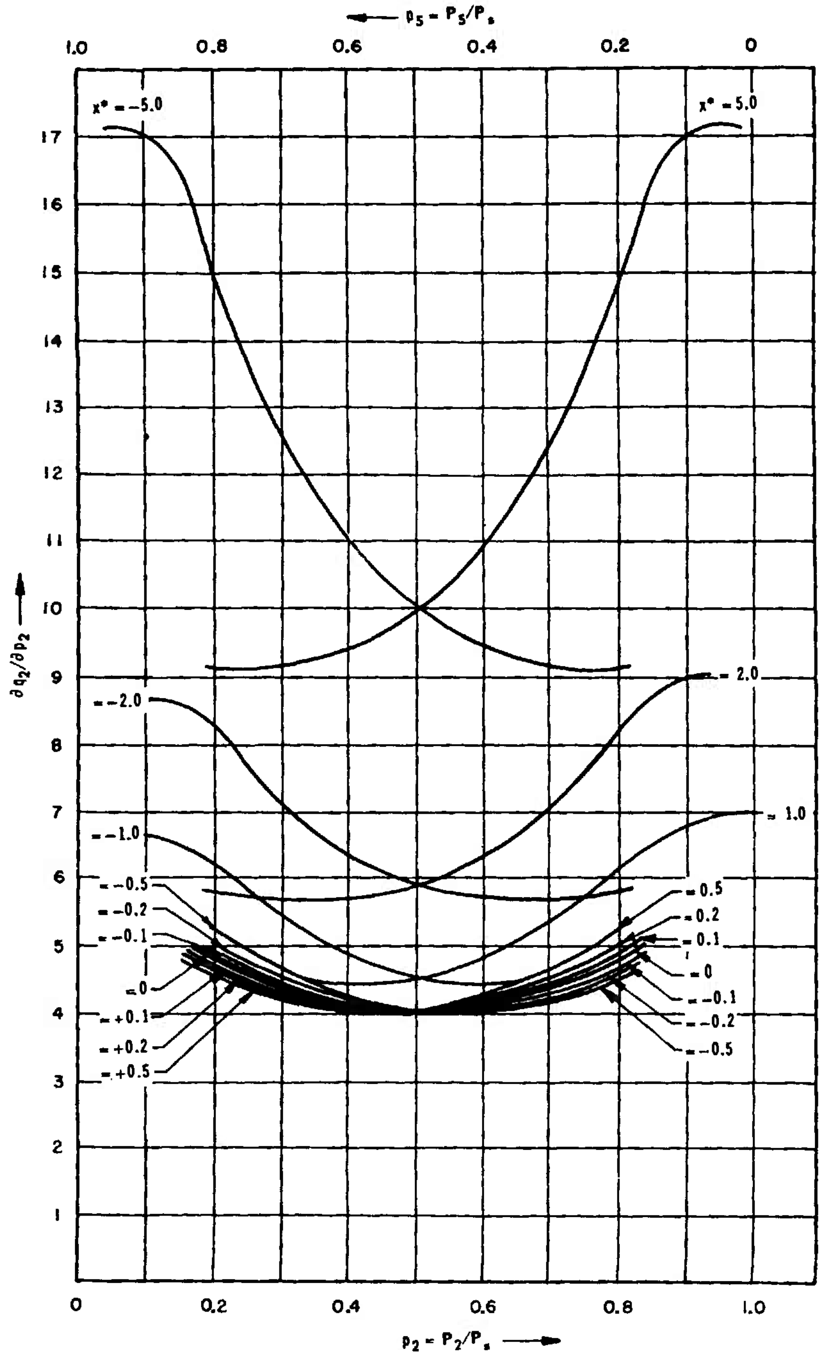


Fig. 13-122 Plot of $\dfrac{\left|\dfrac{\partial Q}{\partial P}\right|}{\left[\dfrac{C\pi d X_o}{2\sqrt{P_s}}\right]}$ *vs* $\dfrac{P}{P_s}$ *with* $\dfrac{x}{X_o}$ *as parameter with* $\beta = 0.5$, *7.0, and 5.0. (Sheet 2 of 3)*

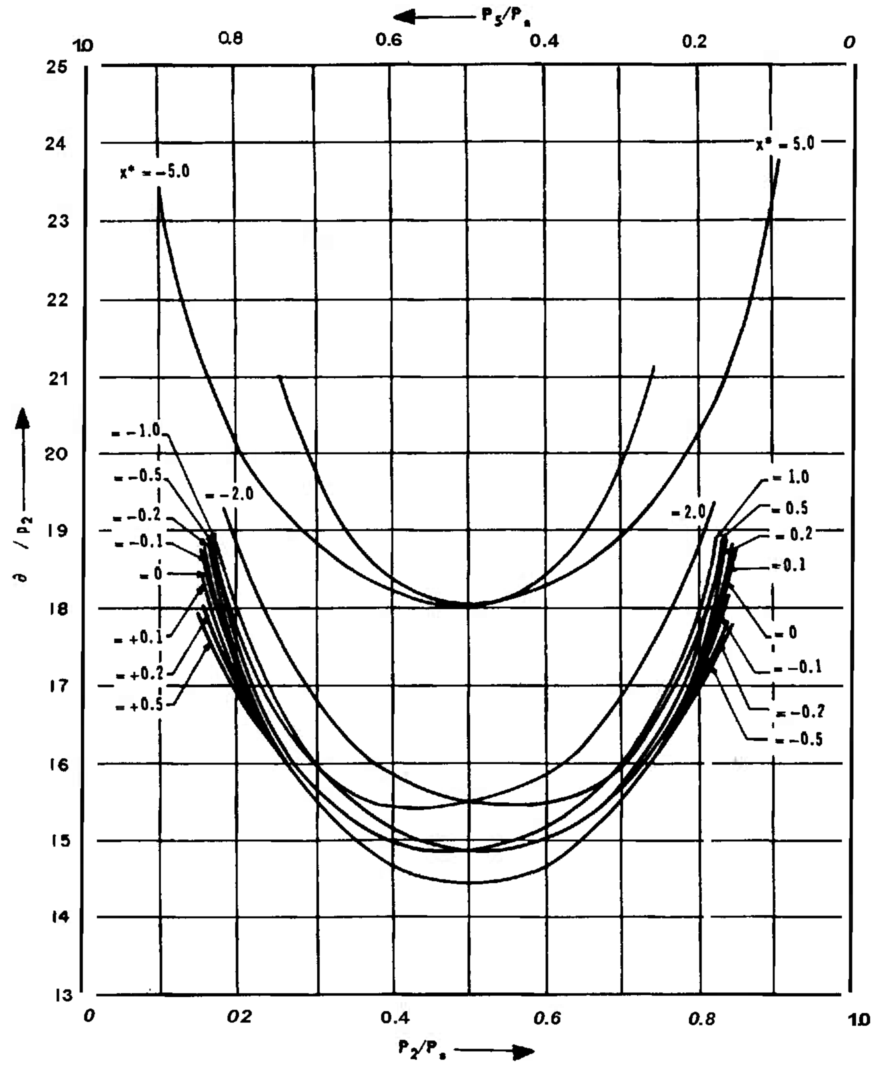


Fig. 73-722 Plot of $\dfrac{\left|\dfrac{\partial Q}{\partial P}\right|}{\left[\dfrac{C\pi d X_o}{2\sqrt{P_s}}\right]}$ vs $\dfrac{P}{P_s}$ with $\dfrac{x^*}{X_o}$ as parameter with $\beta = 0.5$, 7.0, and 5.0. (Sheet 3 of 3)

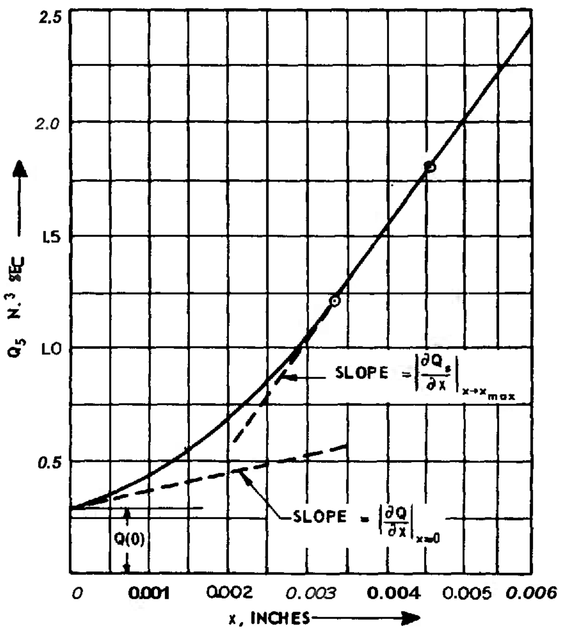


Fig. 73-123 Four-woy spool valve with finite radial clearunce, plot of load flow vs spool displacement for

$$\frac{P_5}{P_s} = \frac{100(lb/in^2)}{350(lb/in.^2)} = constant.$$

where the partial derivative $\left|\frac{\partial Q_5}{Q x}\right|_{x\to x_{max}}$ is measured in the linear portion of the curve of load flow versus spool displacement.

(b) Determine $\beta = b/X_o$. At the origin, the rate of change of load flow with respect to spool displacement is

$$\left|\frac{\partial Q_5}{\partial x}\right|_{x=0} = \frac{c\pi d}{\sqrt{1+\beta^2}}\left(\sqrt{P_s - P_5} + \sqrt{P_5}\right) \tag{13-118}$$

From this expression, it is found that

$$\sqrt{1+\beta^2} = \frac{\left|\frac{\partial Q_5}{\partial x}\right|_{x\to x_{max}}}{\left|\frac{\partial Q_5}{\partial x}\right|_{x=0}}\left(1 + \sqrt{\frac{P_5}{-P_5}}\right) \tag{13-119}$$

where $\left|\frac{\partial Q_5}{\partial x}\right|_{x=0}$ is the slope of the curve df load flow versus spool displacement at the origin, β is found from Eq. **(13-119)** since P_s and P_5 are the supply and load pressures, respectively, at which load flow Q_5 is measured.

(c) Determine X_o. With the spool centered, let the flow to one load line be $Q_5(0)$. Note that this is the controlled flow to one load line, and not the total flow into the valve from the supply. If the inlet or supply flow $Q_s(0)$ is measured with both load lines exhausted to the sump, then $Q_s(0)$ is twice the value given by Eq. **(13-120)**; i.e., $Q_s(0) = 2Q_5(0)$. Since, **from Eq. (13-112),** $Q_5(0) = Q_4(0) - Q_6(0)$, it is found that

$$Q_5\ (0) = c\pi d\, X_o \sqrt{1+\beta^2}\ \sqrt{P_s} \tag{13-120}$$

Substituting Eqs. **(13-118)** and **(13-119)** into Eq. **(13-120)** and solving for X,, it is found that

$$X_o = \frac{Q_5(0)\left|\frac{\partial Q_5}{\partial x}\right|_{x=0}}{\left(\left|\frac{\partial Q_5}{\partial x}\right|_{x\to x_{max}}\right)^2} \tag{13-121}$$

Note that $P_5 = 0$ since all pressures are measured above the sump pressure.

(d) Determine b. This quantity is found from the definition $\beta = b/X_o$ and is

$$b = X_o\beta \tag{13-122}$$

13-6.13 Plate-Valve Type

The plate valve (see Fig. **13-124**) has flow characteristics very similar to those of a four-way spool valve with radial clearance. The orifices of underlapped plate valves are reamed larger than the plugs used in the moving plate, whereas the orifices of overlapped plate valves are reamed smaller than the plugs used in the moving plate.

If the moving valve plate clears the fixed block by an amount b, the resultant orifice area A, is that shown in Fig. **13-124.** The orifice area can be found as follows: The shaded area shown in Fig. **13-124** is the projection of the

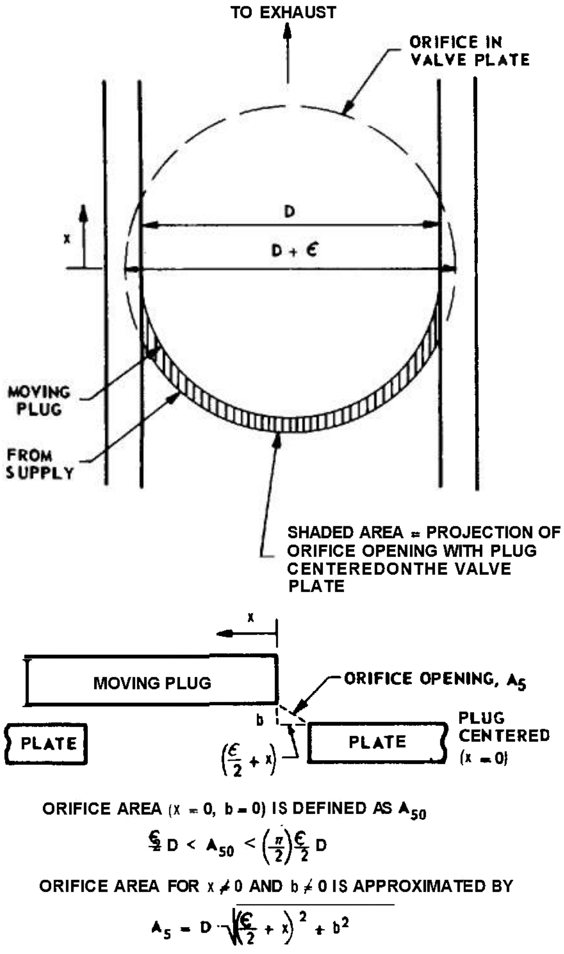


Fig. 73-724 Geometry of orifice plate valve with underlap and clearance.

orifice opening on the plane of the valve plate. The projected area is approximately

$$A_{5p} = \frac{1}{4}\pi\varepsilon D + xD \qquad \textbf{(13-123)}$$

For simplicity, let this area be considered as due to a strip D inches long and $\frac{\varepsilon}{2} + x$ inches wide, an approximation of course. The slant area A_5 may now be found, since the separation of the plug and plate is b inches. The slant height then becomes $\sqrt{b^2 + \left(x + \frac{\varepsilon}{2}\right)^2}$ and the area is given by

$$A_5 = D\sqrt{\left(\frac{\varepsilon}{2} + x\right)^2 + b^2} \qquad \textbf{(13-124)}$$

In a similar fashion, the area of the orifice which decreases with increasing x is

$$A_2 = D\sqrt{\left(\frac{\varepsilon}{2} - x\right)^2 + b^2} \qquad \textbf{(13-125)}$$

By substituting D for π d and $\frac{\varepsilon}{2}$ for X_o in Table **13-15,** the partial-derivative parameters characterizing the plate valve in terms of its flow gain and conductance are obtained. The equivalent source representation of the four-way spool-valve hydraulic amplifier with radial clearance may be used for the plate-valve hydraulic amplifier if the parameter substitutions indicated above are made.

The advantage of the plate valve over the spool valve is mostly one of mechanical construction. The hole-and-plug method of construction results in nearly perfect alignment of all orifices. The suspension type of plate valve illustrated in Fig. **13-124** is only one of several possible **constructions.**[47] *A* disadvantage of the plate valve is that it is somewhat bulkier than a spool-valve amplifier of equal flow-pressure capacity.

13-6.14 Double Nozzle-Baffle-Valve Type

The balanced or double nozzle-baffle-valve hydraulic amplifier (see Fig. **13-125)** is easier to manufacture than the spool-valve amplifier and is about as linear as a spool valve of comparable leakage. *As* is the case with the spool valve, increased leakage results in greater linearity in the flow-pressure versus input-motion characteristics. Because of the high leakage of the double nozzle-baffle valve, the valve is generally used as a first stage, the second stage being a spool valve with little or no underlap. The combination of a double nozzle-baffle valve and a spool valve, in the form of a pressure-control valve, has the properties of both valves. The result is an amplifier with very low (theoretically zero) internal resistance; i.e., an ideal pressure source.

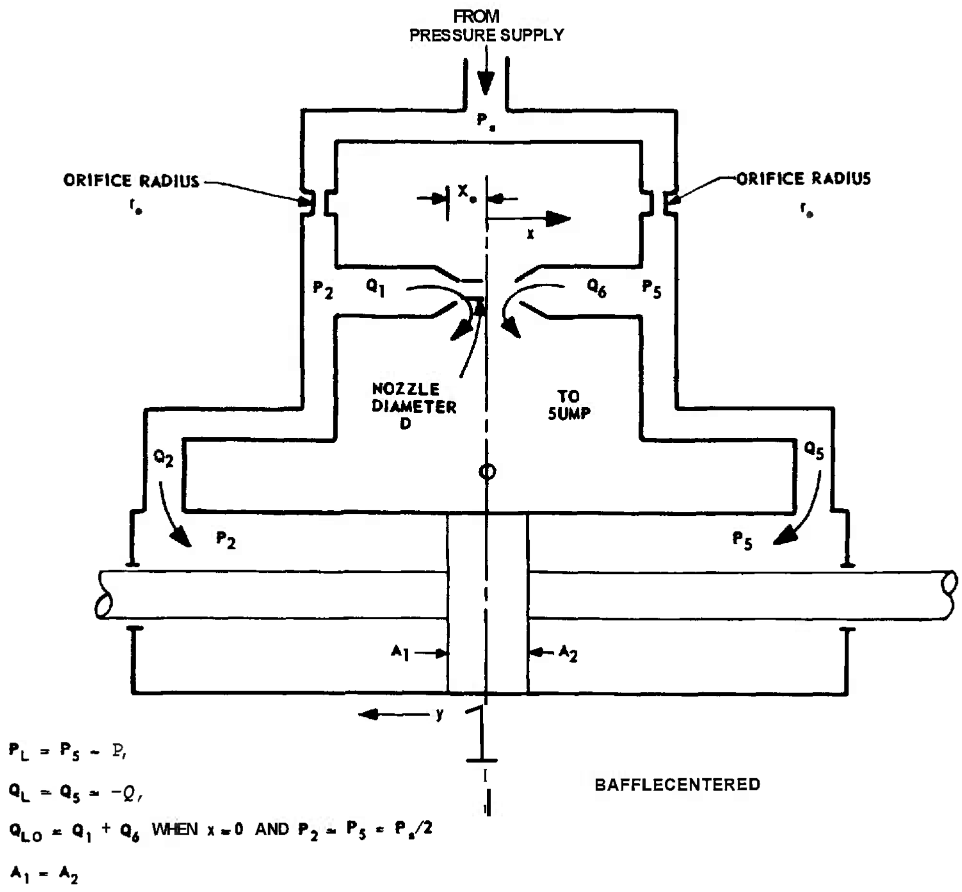


Fig. 13-725 Double nozzle-baffle valve with balanced piston load.

13-6.15 Flow equations of double nozzle-baffle valve. The pertinent flow equations of the double nozzle-baffle valve (see Fig. 13-125 for meaning of symbols not defined below) are

$$Q_1 = c_n \pi D (X_o + x)\sqrt{P_2} \qquad (13\text{-}126)$$

$$Q_2 = Q_3 - Q_1 \qquad (13\text{-}127)$$

$$Q_3 = c_o \pi r_o^2 \sqrt{P_s - P_2} \qquad (13\text{-}128)$$

$$Q_4 = c_o \pi r_o^2 \sqrt{P_s - P_5} \qquad (13\text{-}129)$$

$$Q_5 = Q_4 - Q_6 \qquad (13\text{-}130)$$

$$Q_6 = c_n \pi D (X_o - x)\sqrt{P_5} \qquad (13\text{-}131)$$

where

c_n = nozzle coefficient, where $0.61\sqrt{2/\rho} < c_n < 0.65\sqrt{2/\rho}$

c_o = orifice coefficient, where $0.61\sqrt{2/\rho} < c_o < 0.65\sqrt{2/\rho}$ for sharp-edged orifices

X_o = opening between nozzle and baffle with the baffle in the centered position.

x = motion of baffle from center position, in in.

$p_2 = \dfrac{P_2}{P_s}$

$p_5 = \dfrac{P_5}{P_s}$

D = diameter of nozzle, in in.

r_o = radius of supply orifices, in in.

ρ = density of fluid, in lb-sec^2/in.^{4}

The expressions for the circuit elements of the equivalent flow-source representation of the amplifier are derived from Eqs. (13-127) and (13-130) and are

$$\frac{\partial Q_2}{\partial x} = -c_n \pi D \sqrt{P_s}\,\sqrt{p_2} \qquad (13\text{-}132)$$

$$\frac{\partial Q_5}{\partial x} = c_n \pi D \sqrt{P_s}\,\sqrt{p_5} \qquad (13\text{-}133)$$

$$\frac{\partial Q_2}{\partial P_2} = -\frac{c_n \pi D X_o}{2\sqrt{P_s}}\left(\frac{\alpha}{\sqrt{1-p_2}} + \frac{1+x^*}{\sqrt{p_2}}\right) \qquad (13\text{-}134)$$

$$\frac{\partial Q_5}{\partial P_5} = -\frac{c_n \pi D X_o}{2\sqrt{P_s}}\left(\frac{\alpha}{\sqrt{1-p_5}} + \frac{1-x^*}{\sqrt{p_5}}\right) \qquad (13\text{-}135)$$

where

$$\alpha = \frac{c_o r_o^2}{c_n D X_o} = \frac{P_{5\,(x=0)}}{P_s - P_{5\,(x=0)}}$$

$$x^* = \frac{x}{X_o}$$

The expressions for the circuit elements of the equivalent pressure-source representation of the amplifier are

$$\frac{\partial P_5}{\partial x} = -\frac{\partial Q_5}{\partial x} \Big/ \frac{\partial Q_5}{\partial P_5} = \frac{2P_s}{X_o}\left(\frac{\sqrt{p_5}}{\dfrac{\alpha}{\sqrt{1-p_5}} + \dfrac{1-x^*}{\sqrt{p_5}}}\right) \qquad (13\text{-}136)$$

$$\frac{\partial P_5}{\partial Q_5} = \frac{1}{\dfrac{\partial Q_5}{\partial P_5}} = \frac{-2\sqrt{P_s}}{c_n \pi D X_o}\left(\frac{1}{\dfrac{\alpha}{\sqrt{1-p_5}} + \dfrac{1-x^*}{\sqrt{p_5}}}\right) \qquad (13\text{-}137)$$

A nondimensional plot of the pressure gain $\partial P_5/\partial x$ is shown in Fig. 13-126, while a nondimensional plot of the internal resistance $\partial P_5/\partial Q_5$ is shown in Fig. 13-127. A value of α equal to unity is usually designed for, since this leads to the most linear static behavior. Plots of P_2/P_s and P_5/P_s for zero load flow are shown in Fig. 13-128.

13-6.16 Double nozzle-baffle amplifier with balanced load. For a load consisting of a piston with equal working areas on both sides of the piston, $Q_5 = -Q_2$ (see Fig. 13-125), and the static characteristic relating pressure drop across the load to load flow may be found. This plot is not valid if flow compressibility must be taken into account. For the case of flow compressibility, the representation of the two sources by partial derivatives for both orifices (nozzles) must be used. The plot of $Q_L Q_o$ versus P_L/P_s, with $x^* = x/X_o$ as a parameter, is shown in Fig. 13-129.[46] $Q_L = Q_5 = -Q_2$ equals the flow to the balanced load. $Q_o = Q_1 = Q_6$ when $x = 0$ and $P_5 = P_2 = P_s/2$. P_L is the pressure drop across the load; i.e., $P_L = P_5 - P_2$. The linear equivalent circuit at the point $x^* = 0$ leads to a pressure source of $2(\partial P_5/\partial x)$ in series with a resistance of $2(\partial P_5/\partial Q_5)$ supplying the piston. The partial derivatives are evaluated from Figs. 13-126 and 13-127. The result is

$$2\frac{\partial P}{\partial Q} = 2\frac{\partial P_2}{\partial Q_2}\bigg|_{x^*=0} = -\frac{2\sqrt{P_{s/2}}}{c_n \pi D X_o} = -\frac{P_s}{c_n \pi D X_o \sqrt{P_{s/2}}} = -\frac{P_s}{Q_o} = \frac{\partial P_L}{\partial Q_L} \qquad (13\text{-}138)$$

$$2\frac{\partial P}{\partial x}\bigg|_{x^*=0} = \frac{P_s}{X_o} = \frac{\partial P_L}{\partial x} \qquad (13\text{-}139)$$

$$\frac{\partial (P_L/P_s)}{\partial (Q_L/Q_o)} = -1, \quad \frac{\partial (P_L/P_s)}{\partial (x/X_o)} = 1 \qquad (13\text{-}140)$$

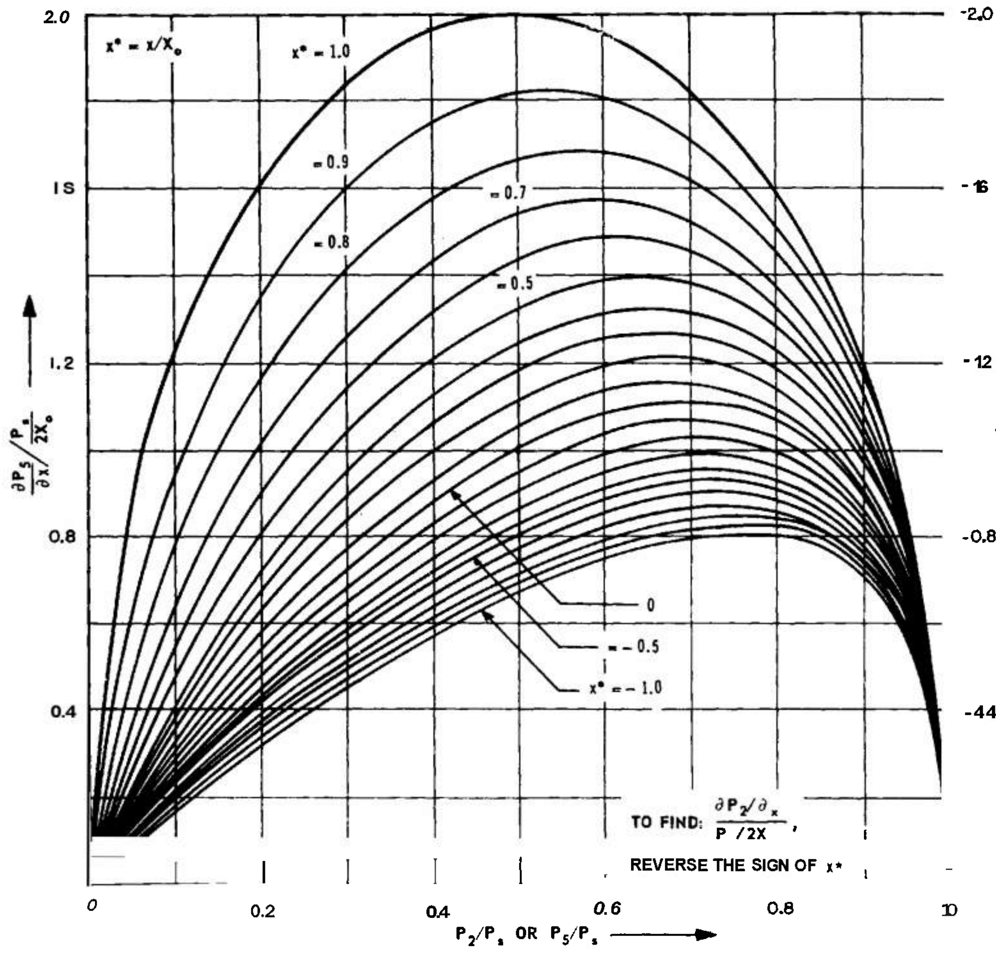


Fig. 73-126 Double nozzle-baffle valve, nondimensional plot of pressure gain vs load pressure.

The dashed lines in Fig. *13-129* represent $\partial(P_L/P_s)/\partial(Q_L/Q_o) = -1$ and $\partial(P_L/P_s)/\partial(x/X_o) = 1$ and demonstrate the linear region of the hydraulic amplifier.

13-6.17 Dimensionalizing pressure-gain and resistance plots of double nozzle-baffle amplifier. The nondimensional plot or Fig. 13-126 may be dimensioned if the abscissa scale is multiplied by P_s and the ordinate scale by $P_s/2X_o$. Similarly, the nondimensional plot of ing the abscissa scale by P, and the ordinate scale by $\sqrt{Ps/2}/c_n\pi DX_o$. For the case of α equal to unity, these coefficients may be found as follows:

(a) Determine $P_s/2X_o$. For the condition of zero load flow, obtain a plot of amplifier output pressure P_5 (at one side of the load) as a function of baffle displacement x. The slope of the nondimensional plot of P_5/P_s Fig. *13-127* may be dimensioned by multiply-

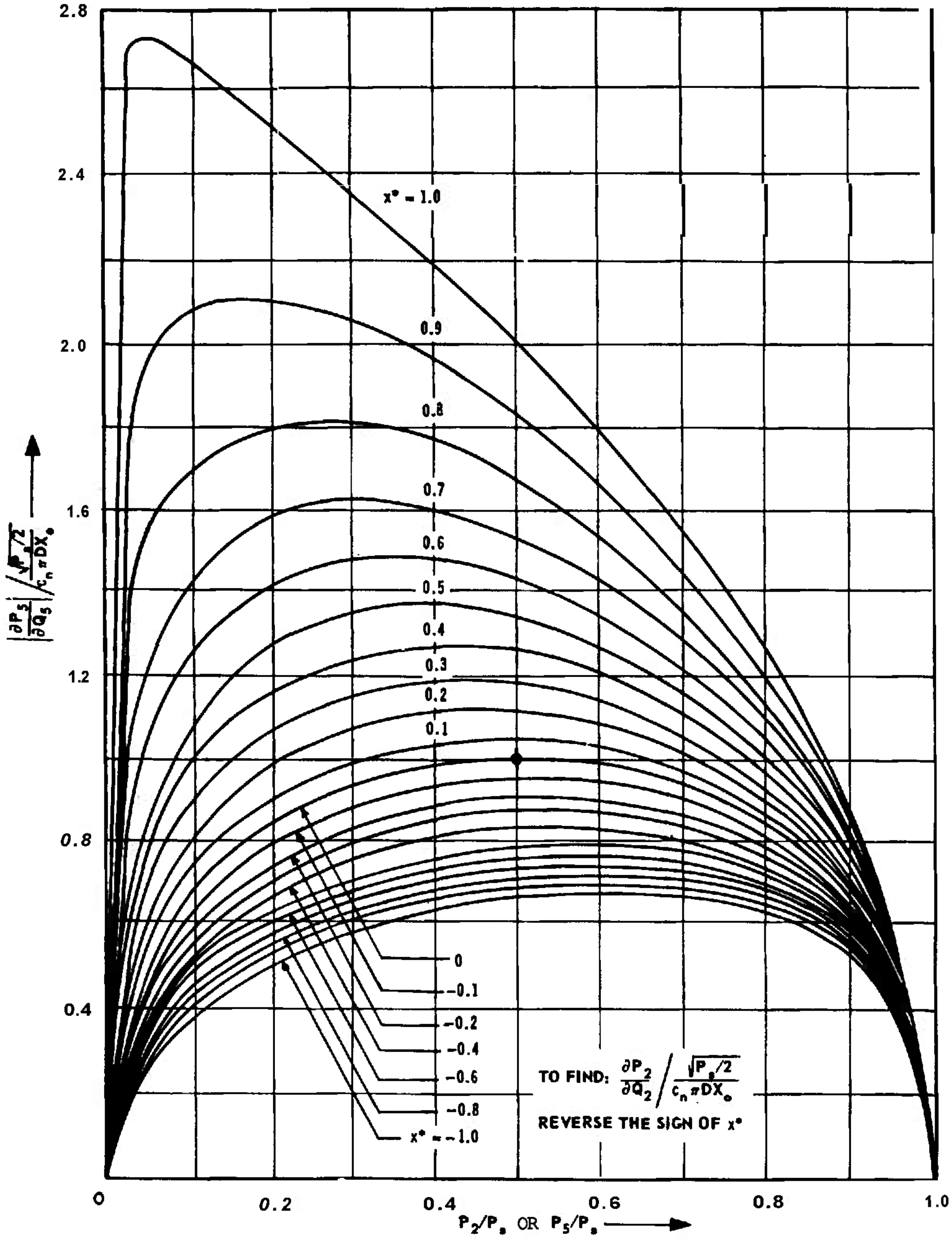


Fig. 73-727 Double nozzle-baffle valve, nondimensional plot of infernal resistance vs load pressure.

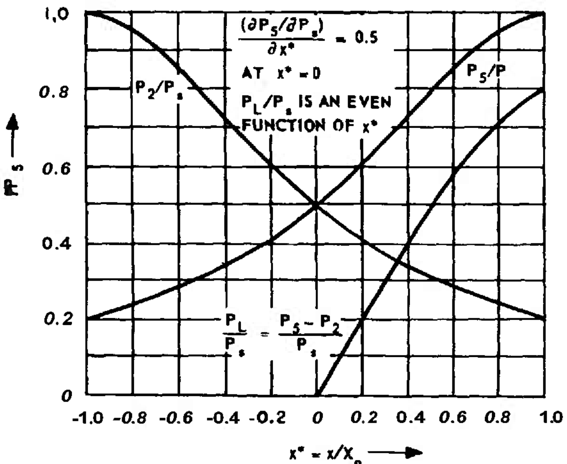


Fig. 13-128 Double nozzle-baffle valve, nondimensional plot of loud pressure vs baffle displacement for zero load flow.

with respect to x^* (see Fig. **13-128**) at the point $x^* = 0$ is found to be

$$\frac{\partial (P_5/P_s)}{\partial (x/X_o)} = \frac{\partial p_5}{\partial x^*}$$

$$= \frac{2(1-x^*)}{[(1-x^*)^2+1]^2}\bigg|_{x^*=0} = \frac{1}{2} \quad \textbf{(13-141)}$$

Equation **(13-141)** may be verified by noting that

$$\frac{\partial p_5}{\partial x^*} = \frac{\partial P_5}{\partial x} - \frac{\partial p_5/\partial P_5}{\partial x^*/\partial x} \quad \textbf{(13-142)}$$

and using Eq. **(13-136)** and the definitions for p_5 and x^*. Therefore

$$\frac{P_s}{2X_o} = \frac{\partial P_5}{\partial x}\bigg|_{x=0} \quad \textbf{(13-143)}$$

(b) Find $\sqrt{P_s/2}/c_n\pi DX_o$. For the condition of zero load flow, obtain a measurement of total flow to the sump with the baffle centered. This is termed the total internal leakage flow with baffle centered, and is defined as Q_{Lo}.

Thus

$$Q_{Lo} = Q_1 + Q_6 \text{ (at } x = 0)$$

$$= 2c_n\pi DX_o\sqrt{\frac{P_s}{2}} = 2Q_o \quad \textbf{(13-144)}$$

From this relationship, it is found that the factor needed for dimensioning the plot of $\partial Q_s/\partial P_5$ is $\sqrt{P_s/2}/c_n\pi DX_o$. Thus

$$\frac{\sqrt{P_s/2}}{c_n\pi DX_o} = \frac{P_s}{Q_{LO}} \quad \textbf{(13-145)}$$

13-6.18 Pressure-Control and Flow-Control Valve Amplifiers

By means of special configurations, the internal resistance of a hydraulic amplifier may be lowered. In this way, the amplifier approaches an ideal pressure source with zero internal resistance. A hydraulic amplifier of this type is known as a pressure-control valve. One method used to achieve low internal resistance is by the use of output-pressure feedback as shown in Fig. **13-130.** A double-sided amplifier is constructed by combining two of the units shown and driving the resultant two baffles out of phase. The amplifier internal resistance can be determined from the transfer function that relates output pressure to input baffle motion and load flow. As the frequency s approaches zero, the transfer function has the following asymptotic form:

$$P_5(s) = \left|\frac{\partial P_{nb}}{\partial x_b}\right| x_b(s) - sA\,\frac{\left|\frac{\partial P_5}{\partial Q_5}\right|}{\left|\frac{\partial P_5}{\partial x}\right|}$$

$$\left(R_o + \left|\frac{\partial P_{nb}}{\partial Q_{nb}}\right|\right) Q_5(s) \quad \textbf{(13-146)}$$

The steady-state relation is found by letting the operator s go to zero in Eq. **(13-146).** When this is done, the result is

$$P_5 = \left|\frac{\partial P_{nb}}{\partial x_b}\right| x_b \quad \textbf{(13-147)}$$

It is apparent that, in the steady state, the output pressure is unaffected by the flow Q_5. Therefore, the zero internal-resistance condition is achieved in the steady state.

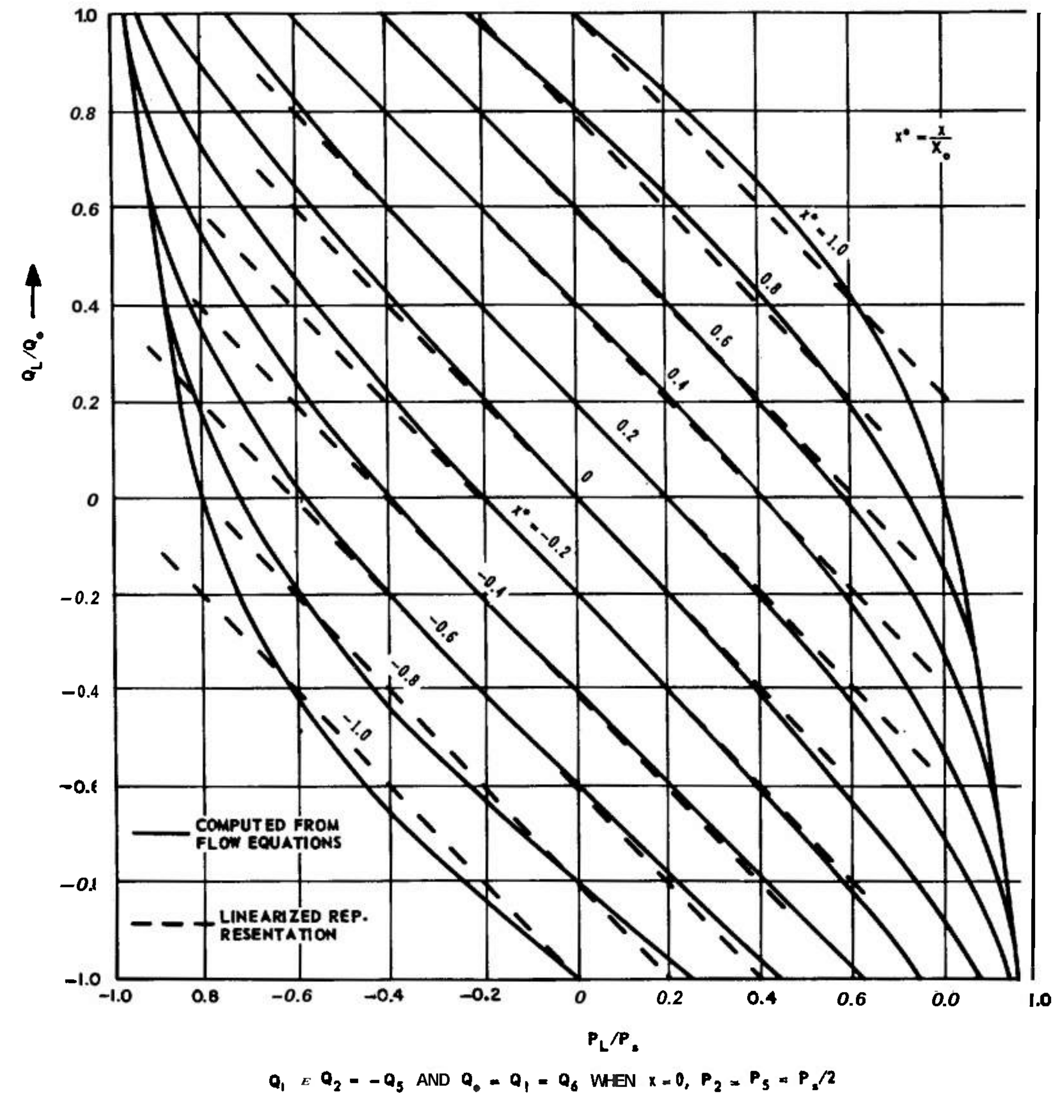


Fig. 13-729 Double nozzle-baffle valve with balanced piston load, nondimensional plot of load flow vs differential load pressure.

STAGE

INT. RES.	$\frac{\partial P_{nb}}{\partial Q_{nb}}$	$\frac{\partial P_5}{\partial Q_5}$

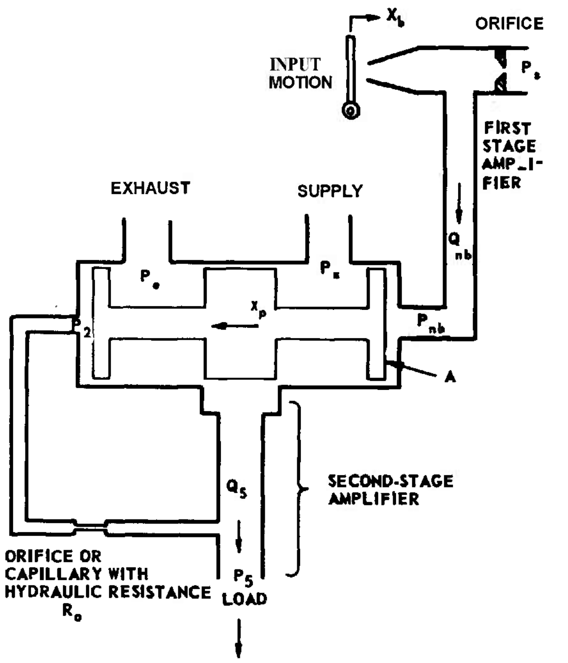


Fig. 13-130 Pressure-control valve amplifier (single-sided amplifier with pressure compensation).

Actual measured steady-state characteristics for a valve of this type with a balanced load are shown in Fig. 13-131. The dotted lines represent the ideal result indicated by Eq. (13-147). Note that the compensation is valid only for static characteristics and that, during changes of flow, the amplifier will exhibit some transient internal resistance.

By using other configurations, it is possible to design a hydraulic amplifier with very low internal conductance. Such a compensated amplifier approaches an ideal flow source. The compensation can be achieved by making use of the static flowforce or by the use of an auxiliary orifice.

13-6.19 ROTARY HYDRAULIC AMPLIFIERS

134.20 Characteristics

The rotary pump is commonly described in terms of a parametric family of curves of flow versus load pressure for various values of stroke (x is the per unit stroke). This family of curves satisfies the following equation of flow:

$$dQ = \frac{\partial Q}{\partial x}dx + \frac{\partial Q}{\partial P}dP \qquad (13\text{-}148)$$

where

Q = flow delivered to line, in in.^{3}/sec

x = per unit stroke

$$= \frac{\text{actual stroke (in in. or deg)}}{\text{full stroke (in in. or deg)}}$$

P = pressure at inlet to load line measured above recharging line pressure in lb/in.^{2}

A plot of typical flow-pressure curves is shown in Fig. 13-132.

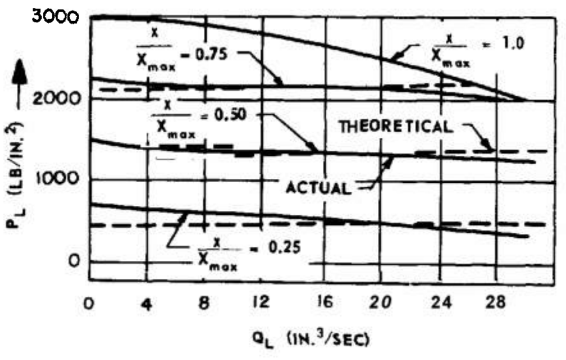


Fig. 13-131 Pressure-control valve amplifier with balanced load, dimensional plot of differential load pressure vs load flow.

The partial derivatives in **Eq. (13-148)** remain relatively constant. In terms of the pump geometry, they are evaluated by the relationships

$$\frac{\partial Q}{\partial x} = D_p \omega_p \qquad (13\text{-}149)$$

$$-\frac{\partial Q}{\partial P} = L_{pu} \qquad (13\text{-}150)$$

where

D_p = displacement of pump at full stroke, in **in.³/radian**

ω_p = pump speed, in **radians/sec**

L_{pu} = pump leakage, in **in.⁵/lb-sec**

Therefore

$$dQ = (D_p\omega_p)\, dx - (L_{pu})dp$$
$$= d_p dx - L_{pu}\, dp \qquad (13\text{-}151)$$

where $d_p = D_p\omega_p$

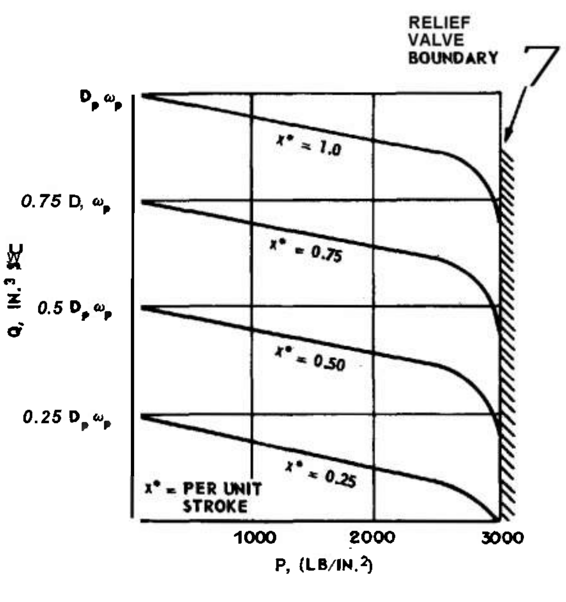


Fig. 13-132 Rotary pump, typical dimensional plot of load flow vs load pressure.

The hydraulic circuit illustrating **Eq. (13-151)** is shown in Fig. **13-133A**; the corresponding block diagram is shown in Fig. **13-133B.** Three schematic diagrams of rotary pumps are shown in Figs. **13-103, 13-104,** and **13-105.**

Pump leakage may be determined by varying the pump load pressure by means of a variable throttling valve and then plotting a curve of flow variation versus load-pressure variation. The slope of this curve is the pump leakage L_{pu}. The pump displacement D_p is generally furnished by the manufacturer. D_p may be determined experimentally by measuring the flow and the pump speed at full stroke ($D_p = Q/\omega_p$ at full stroke).

In the previous discussion, all pressures are measured above that which the replenishing pump maintains in the low-pressure line. The replenishing pump replaces the oil lost because of piston and shaft leakage. It may be noted that the rotary hydraulic amplifier is analogous to the rotary electric amplifier (single-stage separately excited shunt machine).[48]

13-6.21 INTERACTION OF LOAD AND VALVE

In general, the output of a hydraulic amplifier is either a power piston or a rotary hydraulic motor. For the case of the power piston, the mechanical component of the load includes the mass of the power piston and of all the parts attached thereto, any spring that opposes the piston motion, any viscous damping present, the leakage between the power piston and the cylinder wall, and any additional force opposing the power-piston motion. The total mass (referred to the piston) is M, the opposing force is F_L, and the coefficient of piston leakage L, is found in accordance with **Eq. (13-181).** If the **load** consists of a rotary motor of fixed displacement, the load includes the motor inertia, rotational damping, and any opposing load torque. In addition, the capacitance, resistance, and inertance of connecting lines and the output cylinder(s) must be taken into account.

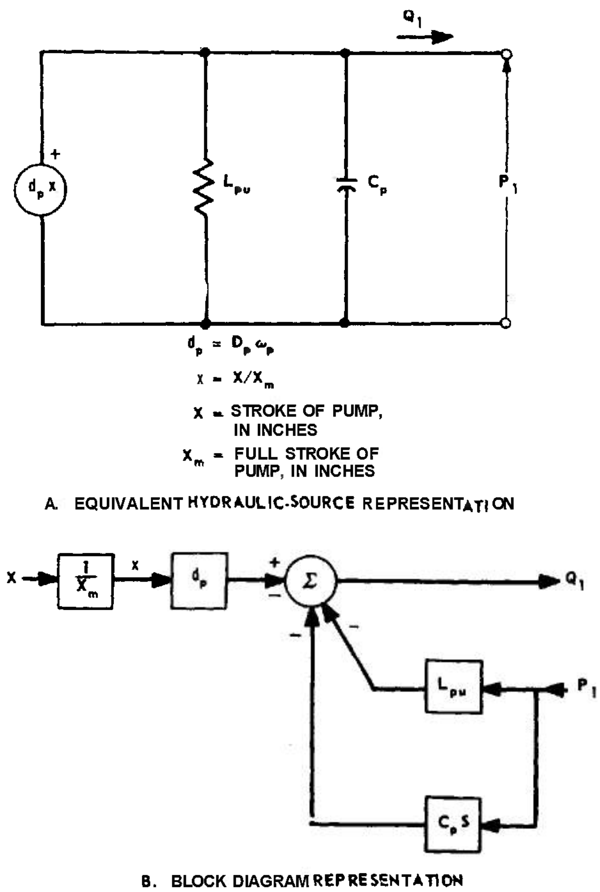


*Fig. 13-133 **Rotary** hydraulic pump.*

The transfer functions of the various configurations are given in Tables 13-16 and **13-17.** Table 13-18 lists the hydraulic-circuit parameters for the rotary amplifier with various loads.

134.22 Equivalent Hydraulic Circuit of Amplifier

The equivalent hydraulic circuit that combines the effect of the lines and the piston load together with that of a valve of given gain and resistance is shown in Fig. **13-134.** If a compensated flow-control type valve is used, the shunting conductances $|\partial Q_5/\partial P_5|$ and $|\partial Q_2/\partial P_2|$ become infinite at low frequencies. If a compensated pressure-control valve is used, the valve is represented by a pressure source with zero internal resistance ($|\partial P_5/\partial Q_5| = |\partial P_2/\partial Q_2| = 0$) at low frequencies. In this case, compressibility C_L of the lines may be neglected. Caution should be used here, however, since the pressure-control valves do not have zero internal resistance for dynamic changes. This is because compensation is obtained by feedback through the resistance R_o (Fig. **13-130)** into the **feed-**back chamber, which has a finite volume and hence hydraulic capacitance. The values of the hydraulic-circuit elements in Fig. **13-134** are discussed in Par. **13-6.27.**

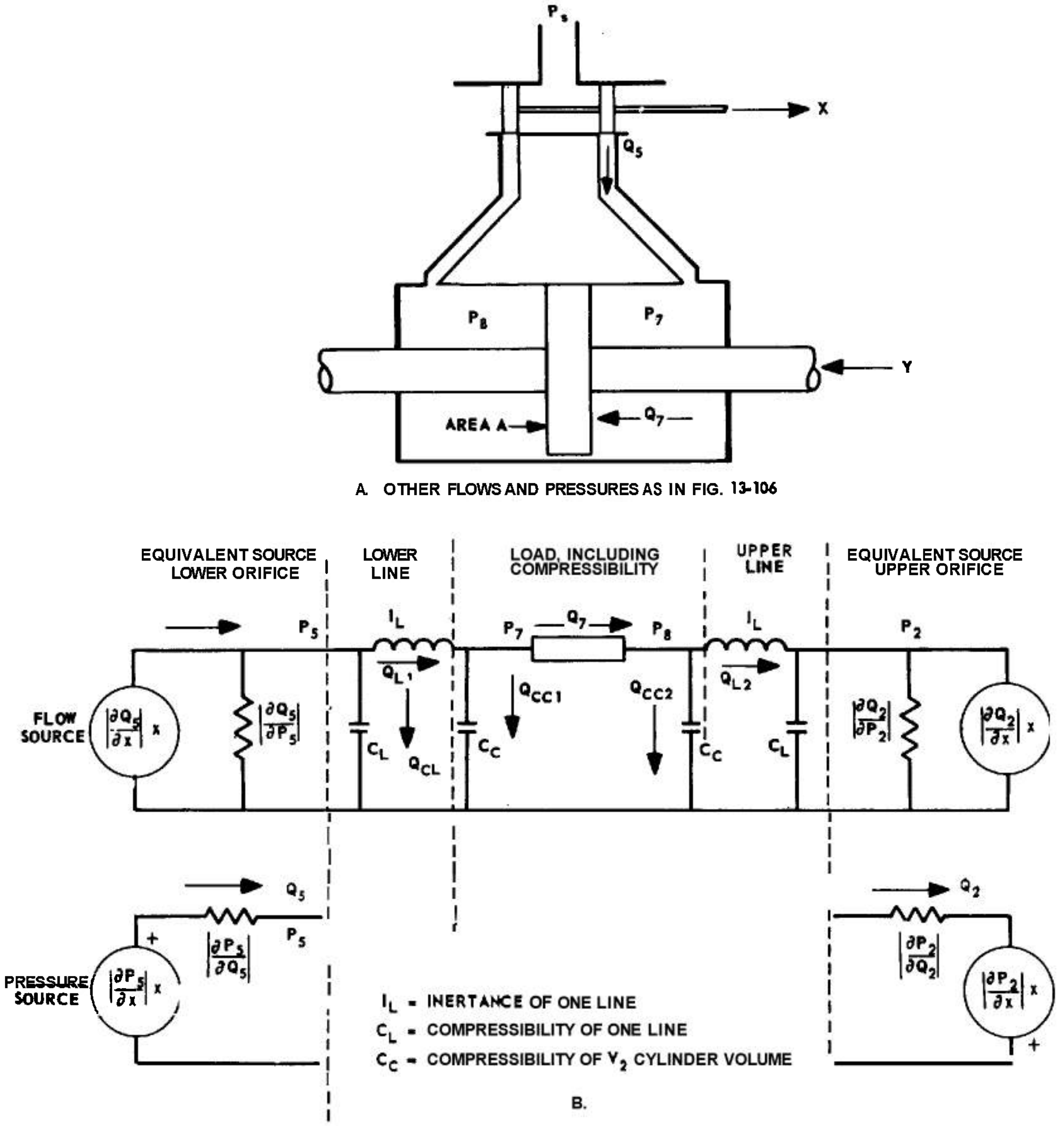


Fig. 13-134 Four-way spool-valve amplifier and load, equivalent hydraulic-circuit representation.

TABLE 13-16 TRANSFER FUNCTIONS OF FOUR-WAY SPOOL-VALVE AMPLIFIER AND LOAD

(a) No Feedback. Spring-Opposed Balanced Power Piston. (See Figs. 13-134 and 13-137.)

The piston and the load moved by the piston have mass M and damping B and are opposed by a spring whose spring constant is K.

$$Y(s) = \frac{\left(\frac{2\left|\frac{\partial P}{\partial x}\right|\frac{A}{K}}{1 + 2L_pR_5}\right) X(s) - \frac{F_L(s)}{K}(1 + b_1s + b_2s^2 + b_3s^3)}{1 + a_1s + a_2s^2 + a_3s^3 + a_4s^4 + a_5s^5}$$

where

$X(s)$ = spool displacement from center position

F_L = load force exerted on power piston. Positive load force is that which oppose motion caused by positive x.

Y(s) = motion of amplifier output (power-piston travel)

$\left|\frac{\partial P}{\partial x}\right|$ = hydraulic-amplifier pressure gain $\left|\frac{\partial P_5}{\partial x}\right|$. Use value of $\left|\frac{\partial P}{\partial x}\right|$ for $x = 0$.

Then, $\left|\frac{\partial P_5}{\partial x}\right| = \left|\frac{\partial P_2}{\partial x}\right| = \left|\frac{\partial P}{\partial x}\right|$.

A = active area of power piston

K = spring constant of power-piston spring

$$b_1 = \frac{C_L + C_C + 2L_pL_5I_L}{L_5 + 2L_p}$$

$$b_2 = C_LI_L$$

$$b_3 = \frac{C_CC_LI_L}{L_5 + 2L_p}$$

$$a_1 = \frac{C_L + C_C + 2L_pI_LL_5 + 2C_K + R_BC_KL_5 + 2R_BC_KL_p}{L_5 + 2L_p}$$

$$a_2 = \frac{(C_KI_M + C_LI_L)(L_5 + 2L_p) + 2C_KI_LL_5 + R_BC_K(C_L + C_C + 2L_pI_LL_5)}{L_5 + 2L_p}$$

$$a_3 = \frac{C_CC_LI_L + 2C_KC_LI_L + R_BC_KC_LI_L(L_5 + 2L_p) + I_MC_K(C_L + C_C + 2L_pI_LI_5)}{L_5 + 2L_p}$$

$$a_4 = \frac{R_BC_KC_CC_LI_L + I_MC_KC_LI_L(L_5 + 2L_p)}{L_5 + 2L_p}$$

$$a_5 = \frac{I_MC_KC_CC_LI_L}{L_5 + 2L_p}$$

$$R_5 = \left|\frac{\partial P}{\partial Q}\right| \qquad L_5 = 1/R_5 = \frac{\partial Q}{\partial P}$$

TABLE 13-16 TRANSFER FUNCTIONS OF FOUR-WAY SPOOL VALVE AMPLIFIER AND LOAD (cont)

$C_L = \dfrac{V_L}{B_M}$

$C_C = V_{C2}/B_M$

L_p = piston-cylinder wall leakage [see **Eq.** (13-181)]

$C_K = A^2/K$

$R_B = B/A^2$

$I_L = \dfrac{\rho L}{A_L}$

$I_M = M/A^2$

V_L = volume of one line from control valve to power piston, in in.^{3}

V_{C2} = volume of oil under compression in one end of power piston, in in.^{3}

B = coefficient of viscous friction associated with load (not to include effect of piston-cylinder leakage), in lb-sec/in.

L = length of one line from control valve to power piston, in in.

A_L = area of line from control valve to power piston, in in.^{2}

M = mass of power piston and parts moved by it, in lb-sec^2/in.

ρ = density of oil (*see* Par. *20-2*), in lb-sec^2/in.^{4}

B_M = bulk modulus of oil (see Par. *20-2*), in lb/in.^{2}

(b) No Feedback. No Spring. Balanced Piston. (See Figs. 13-134 and 13-135.)

The piston and the load moved by the piston have mass M and damping B and are opposed by a force F_L.

$$Y(s) = \frac{\dfrac{2\left|\dfrac{\partial P}{\partial x}\right| X(s)}{A(2R_5 + R_B + 2R_B L_p R_5)} - \dfrac{(1 + 2L_pR_5)(1 + b_1 s + b_2 s^2 + b_3 s^3) F_L(s)}{A^2(2R_5 + R_B + 2R_B L_p R_5)}}{s(1 + d_1 s + d_2 s^2 + d_3 s^3 + d_4 s^4)}$$

where the constants not previously defined are

$$d_1 = \frac{2I_L L_5 + R_B(C_L + C_C + 2L_p I_L L_5) + I_M(L_5 + 2L_p)}{2 + R_B(L_5 + 2L_p)}$$

$$d_2 = \frac{2C_L I_L + R_B C_L I_L (L_5 + 2L_p) + I_M(C_L + C_C + 2L_p I_L L_5)}{2 + R_B(L_5 + 2L_p)}$$

$$d_3 = \frac{C_C R_B C_L I_L + I_M C_L I_L (L_5 + 2L_p)}{2 + R_B(L_5 + 2L_p)}$$

$$d_4 = \frac{I_M C_C C_L I_L}{2 + R_B(L_5 + 2L_p)}$$

TABLE 13-16 TRANSFER FUNCTIONS OF FOUR-WAY SPOOL VALVE AMPLIFIER AND LOAD (cont)

(c) Position Feedback. No Spring. Balanced Piston. (See Fig. **13-101.)**

The feedback lever ratios l_1/l_3 and l_2/l_3 are defined as in Fig. **13-101.** The load has mass M and damping B. The amplifier input is Z, the output is Y, and the valve motion is X.

$$Y(s) = \frac{\dfrac{l_2}{l_1} Z(s) - \dfrac{l_3(1 + 2L_pR_5)}{2l_1A \left|\dfrac{\partial P}{\partial x}\right|} [1 + b_1 s + b_2 s^2 b_3 s^3] F_L(s)}{1 + f_1 s + f_2 s^2 + \text{[redacted]} + f_4 s^4 + f_5 s^5}$$

where the constants not previously defined are

$$f_1 = \frac{l_3 A}{2l_1 \left|\dfrac{\partial P}{\partial x}\right|} (2R_5 + R_B + 2R_BL_pR_5)$$

$$f_2 = \frac{l_3 A}{2l_1 \left|\dfrac{\partial P}{\partial x}\right|} [2I_L + R_B (R_5C_L + R_5C_C + 2L_pI_L) + I_M(1 + 2L_pR_5)]$$

$$f_3 = \frac{l_3 A}{2l_1 \left|\dfrac{\partial P}{\partial x}\right|} [2R_5C_LI_L + R_BC_LI_L(1 + 2L_pR_5) + I_M(R_5C_L + R_5C_C + 2L_pI_L)]$$

$$f_4 = \frac{l_3 A}{2l_1 \left|\dfrac{\partial P}{\partial x}\right|} [R_BC_CR_5C_LI_L + I_MC_LI_L(1 + 2L_pR_5)]$$

$$f_5 = \frac{l_3 A}{2l_1 \left|\dfrac{\partial P}{\partial x}\right|} (I_MC_CR_5C_LI_L)$$

$$l_3 = l_1 + l_2$$

TABLE 13-17 TRANSFER FUNCTIONS OF ROTARY AMPLIFIER AND LOAD (See Fig. 13-142)

(a) Rotary Pump With Load Consisting of Rotary Motor or Piston but without Springs.

$$Q = \frac{\dfrac{d_p x(s)}{1 + R_B(L_M + L_{pu})} - \dfrac{P_L(s)(L_M + L_{pu})}{1 + R_B(L_M + L_{pu})}(1 + n_1 s + n_2 s^2 + n_3 s^3)}{1 + h_1 s + h_2 s^2 + h_3 s^3 + h_4 s^4}$$

TABLE 13-17 TRANSFER FUNCTIONS OF ROTARY AMPLIFIER AND LOAD
(See Fig. 13-142) (cont)

$$h_1 = \frac{R_B\,(C_1 + C_2 + L_M L_{pu} I_L) + I_L L_{pu} + I_M\,(L_{pu} + L_M)}{1 + R_B\,(L_M + L_{pu})}$$

$$h_2 = \frac{R_B\,(C_2 L_{pu} I_L + L_M C_1 I_L) + C_1 I_L + I_M\,(C_1 + C_2 + L_{pu} I_L L_M)}{1 + R_B\,(L_M + L_{pu})}$$

$$h_3 = \frac{R_B C_1 C_2 I_L + I_M\,(C_2 L_{pu} I_L + L_M C_1 I_L)}{1 + R_B\,(L_M + L_{pu})}$$

$$h_4 = \frac{I_M C_1 C_2 I_L}{1 + R_B\,(L_M + L_{pu})}$$

$Q = D_M \omega_M$ (for fixed-displacement motor load)

$$n_1 = \frac{C_1 + C_2 + L_M I_L L_{pu}}{L_M + L_{pu}}$$

$$n_2 = \frac{L_M C_1 I_L + L_{pu} C_2 I_L}{L_M + L_{pu}}$$

$$n_3 = \frac{C_1 C_2 I_L}{L_M + L_{pu}}$$

ω_M = motor-shaft speed, in radians/sec

$Q = A v_P$ (for piston load)

$x(s)$ = Laplace transform of per unit stroke

v_P = piston velocity, in in./sec

For other parameters, see Fig. 13-142.

(b) Rotary Pump With Load Consisting of Spring-Loaded Piston.

$$Z_{piston} = \frac{\dfrac{A}{K}\left[\dfrac{d_p x(s)}{L_M + L_{pu}}\right] - \dfrac{F_L}{K}\,(1 + n_1 s + n_2 s^2 + n_3 s^3)}{1 + m_1 s + m_2 s^2 + m_3 s^3 + m_4 s^4 + m_5 s^5}$$

$$m_1 = \frac{C_1 + C_2 + L_M L_{pu} I_L + R_B C_K\,(L_M + L_{pu}) + C_K}{L_M + L_{pu}}$$

$$m_2 = \frac{C_2 L_{pu} I_L + L_M C_1 I_L + R_B C_K\,(C_1 + C_2 + L_M L_{pu} I_L) + C_K I_M\,(L_M + L_{pu}) + C_K L_{pu} I_L}{L_M + L_{pu}}$$

$$m_3 = \frac{C_1 C_2 I_L + R_B C_K\,(C_2 L_{pu} I_L + C_1 L_M I_L) + C_K I_M\,(C_1 + C_2 + L_M L_{pu} I_L) + C_1 C_K I_L}{L_M + L_{pu}}$$

$$m_4 = \frac{R_B C_K C_1 C_2 I_L + C_K I_M\,(C_2 L_{pu} I_L + C_1 L_M I_L)}{L_M + L_{pu}}$$

$$m_5 = \frac{C_K I_M C_1 C_2 I_L}{L_M + L_{pu}}$$

TABLE 13-18 CIRCUIT PARAMETERS FOR ROTARY HYDRAULIC AMPLIFIER AND LOAD (See Fig. 13-142)

L_{pu} = pump leakage, in in.^{5}/lb-sec

L_M = motor leakage, in in.^{5}/lb-sec

For leakage of rotary motors, see Par. 14-4; for leakage of piston-type motors, use L_f. See Eq. (13-181) for defining equation.

$$C_1 = C_P + C_L/2 = \frac{\frac{1}{2}\left(\frac{D_P}{2\pi}\right) + \frac{1}{2} A_L l_L}{B_M}, \text{ in in.}^5/\text{lb}$$

$$C_2 = C_M + C_L/2 = \frac{\frac{1}{2}\left(\frac{D_M}{2\pi}\right) + \frac{1}{2} A_L l_L}{B_M}, \text{ in in.}^5/\text{lb}$$

I_L = line inertance $= \frac{\rho l_L}{A_L}$

C_M = 1/2 total net cylinder volume divided by B_M for piston-type load. Valid for centered piston only. See Par. 14-4.

l_L = length of one line connecting pump to load, in in.

A_L = Cross-section inner area of line connecting pump to load, in in.^{2}

ρ = mass density of oil (see Par. 20-Z), in lb-sec^2/in.^{4}

TABLE 13-18 CIRCUIT PARAMETERS FOR ROTARY HYDRAULIC AMPLIFIER AND LOAD (See Fig. 13-142) (cont)

	Fixed Displacement Motor Load No Spring	Piston Load No Spring	Piston Load With Spring
P_L	T_L/D_M	F_L/A	F_L/A
I_M	J_L/D_M^2	M_L/A^2	M_L/A^2
$R_B = L_B^{-1}$	B_L/D_M^2	B_P/A^2	B_P/A^2
C_K	∞*	∞	A^2/K

*Infinite capacitance represents a short circuit ; i.e., no circuit element present.

T_L = load torque on motor shaft (not including viscous friction and inertia torque), in in.-lb

F_L = load force on piston (not including viscous friction and inertia forces), in lb

D_M = motor displacement, in in.^{3}/radian

A = net piston area (not including piston-rod area), in in.^{2}

J_L = total load inertia referred to motor shaft (including gear train and motor inertia), in lb-in.-sec^2

M_L = total mass of all parts moved by power piston, in lb-sec^2/in.

B_L = rotary viscous friction constant, in lb-in-sec

B_P = translational viscous friction constant, in lb-sec/in.

K = spring constant of piston spring, in lb/in.

B_M = bulk modulus of oil, in lb/in.^{2}

Note : If entrained air is present, modify capacitance in the manner indicated in Par. 13-6.27.

A hydraulic amplifier is a device for converting input motion X into output motion Y in the presence of some opposing load force F_L. By combining the flow equations of the valve orifice with those of the load and by taking into account the effects of compressibility, leakage, and inertance, it is possible to find the relationship between the Laplace transform of the input $X(s)$, the output $Y(s)$, and the load $F_L(s)$. This derivation is aided by expressing these equations in the form of an equivalent hydraulic-circuit representation such as Fig. **13-134.** The hydraulic circuit is solved by the same methods used to solve electrical circuits. For example, in Fig. **13-134,** it follows that

$$P_5 - P_7 = I_L s Q_{L1} \tag{13-152}$$

$$Q_{L1} = \left|\frac{\partial Q_5}{\partial X}\right| x - \left|\frac{\partial Q_5}{\partial P_5}\right| P_5 - C_L s P_5 \tag{13-153}$$

$$Q_7 = Q_{L1} - Q_{CC1} = Q_{L1} - C_C s P_7 \tag{13-154}$$

where the differential notation is dropped for convenience. See Table **13-16** for definitions of C_L, I_L, and C_C. In Fig. **13-134,** it also follows that

$$Q_7 = Q_{CC2} + Q_{L2} = C_C s P_8 + \frac{(P_8 - P_2)}{I_L s} \tag{13-155}$$

$$Q_{L2} = C_L s P_2 + \left|\frac{\partial Q_2}{\partial P_2}\right| P_2 + \left|\frac{\partial Q_2}{\partial x}\right| X \tag{13-156}$$

Elimination of the variables not of interest yields the equations for $Y(s)$, in terms of $X(s)$ and $F(s)$, tabulated in Table **13-16.**

Consider the case when a load consisting of a piston and an opposing force is taken into account (see Fig. **13-135).** In terms of the Laplace transform of the variable (assuming zero initial conditions), the load flow becomes

$$Q_7 = AsY + L_p(P_7 - P_8) \tag{13-157}$$

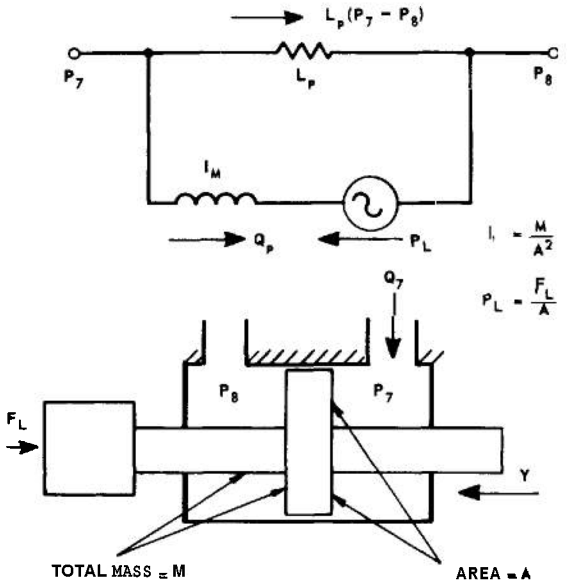


Fig. 73-135 Equivalent circuit representation of load having mass and opposing force.

where L_p is the piston leakage coefficient and A is the cross-sectional area of the piston. If the piston-displacement flow is defined by the relationship

$$Q_p = AsY \tag{13-158}$$

then

$$Q_7 = Q_p + L_p(P_7 - P_8) \tag{13-159}$$

The pressure drop across the piston may be determined from the following relationship:

$$(P_7 - P_8)A = F_L + Ms^2Y$$

and therefore

$$P_7 - P_8 = \frac{F_L}{A} + \frac{MAs^2Y}{A^2}$$

$$= \frac{F_L}{A} + \frac{M}{A^2} sQ_p$$

$$= P_L + I_M s Q_p \tag{13-160}$$

where, by definition

$$P_L = \frac{F_L}{A} \tag{13-161}$$

$$I_M = \frac{M}{A2} \tag{13-162}$$

and F_L is the force opposing the piston motion. Equations **(13-159)** and **(13-160)** are interpreted circuitwise in Fig. **13-135.**

If, in addition to the load force F_L, there is a spring force F_K opposing the piston motion, it may be accounted for in the following manner (see Fig. **13-136)** : By Hooke's Law

$$F_K = KY \tag{13-163}$$

where K is the spring constant. The pressure on the piston required to overcome the spring action is

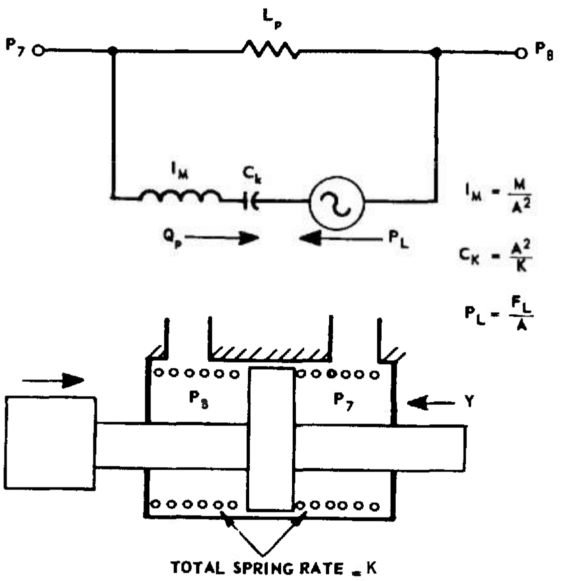


Fig. 13-136 Equivalent circuit representation of load having mass and spring and opposing force.

$$\begin{aligned} P_K &= \frac{F_K}{A} \\ &= \frac{KY}{A} \\ &= \frac{KQ_p}{A^2 s} \\ &= \frac{1}{C_K}\left(\frac{Q_p}{s}\right) \end{aligned} \tag{13-164}$$

where $C_K = \frac{A^2}{K}$ .Therefore

$$\begin{aligned} P_7 - P_8 &= \frac{F_L}{A} + \frac{M}{A^2} sQ_p + \frac{KQ_p}{A^2 s} \\ &= P_L + I_M sQ_p + \frac{1}{C_K}\left(\frac{Q_p}{s}\right) \end{aligned} \tag{13-165}$$

Equation **(13-165)** is interpreted circuitwise in Fig. **13-136.**

If there is a viscous retarding force F_b present, in addition to the spring and inertia, it may be accounted for as follows:

$$F_b = BsY \tag{13-166}$$

and therefore

$$\begin{aligned} &= \frac{F_b}{A} \\ &= \frac{BsY}{A} \\ &= \frac{BQ_p}{A^2} \end{aligned} \tag{13-167}$$

Hence

$$\begin{aligned} P_7 - P_8 &= \frac{F_L}{A} + \frac{M}{A^2} sQ_p + \frac{KQ_p}{A^2 s} + \frac{BQ_p}{A^2} \\ &= P_L + I_M sQ_p + \frac{1}{C_K}\left(\frac{Q_p}{s}\right) + \frac{Q_p}{L_B} \end{aligned} \tag{13-168}$$

where $L_B = \frac{A^2}{B}$.

Equation **(13-168)** is interpreted circuitwise in Fig. **13-137.**

If pistons with different effective areas are used, as shown in Fig. **13-138,** the flow equations become

$$Q_7 = A_7 sY + L_p(P_7 - P_8) \tag{13-169}$$

$$Q_8 = A_8 sY + L_p(P_7 - P_8) \tag{13-170}$$

$$Q_{p7} = A_7 sY \tag{13-171}$$

$$Q_{p8} = A_8 sY \tag{13-172}$$

For **a** load having mass, friction, compliance, and resisting force F_L, the force equation is

$$P_7A_7 - P_8A_8 = F_L + Ms^2Y + KY + BsY \tag{13-173}$$

The circuit representation of Eqs. **(13-169)** through **(13-173)** is shown in Fig. **13-139.**

13-6.23 Block Diagram

The block diagram that follows from the hydraulic circuit **of** Fig. **13-134** is shown in Fig. **13-140.** This diagram may be further simplified to the form shown in Fig. **13-141.** The

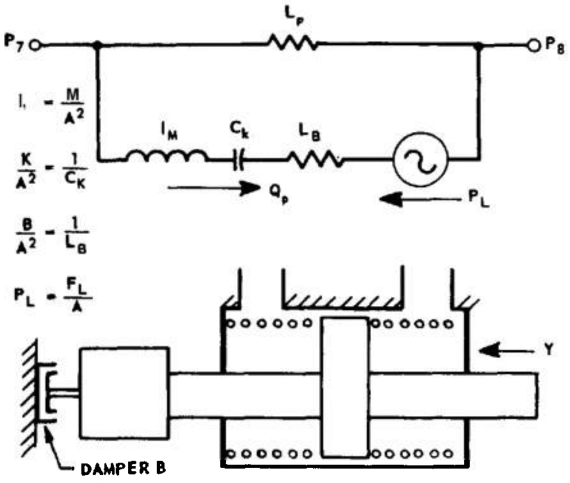


Fig. 13-137 Equivalent circuit representation of load having mass, spring, viscous damping, and opposing force.

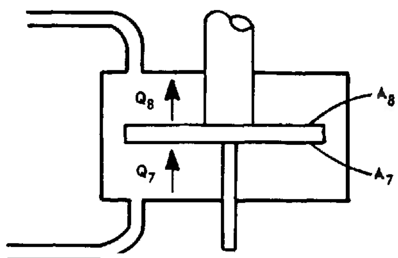


Fig. 13-138 Piston with unequal working areas.

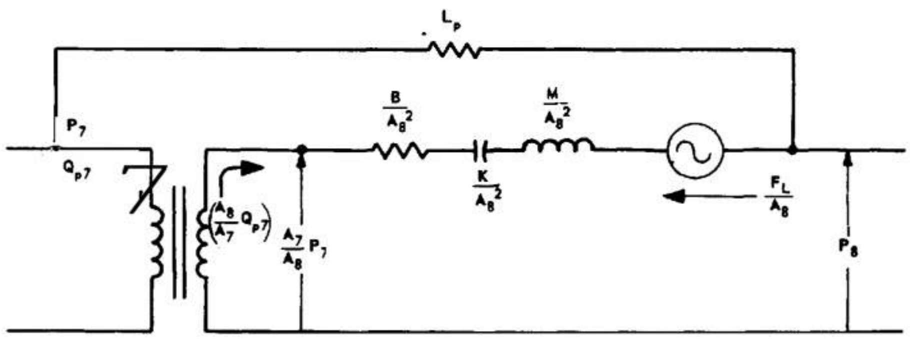


Fig. 13-139 Equivalent circuit representation of load having mass, friction, compliance, and opposing force.

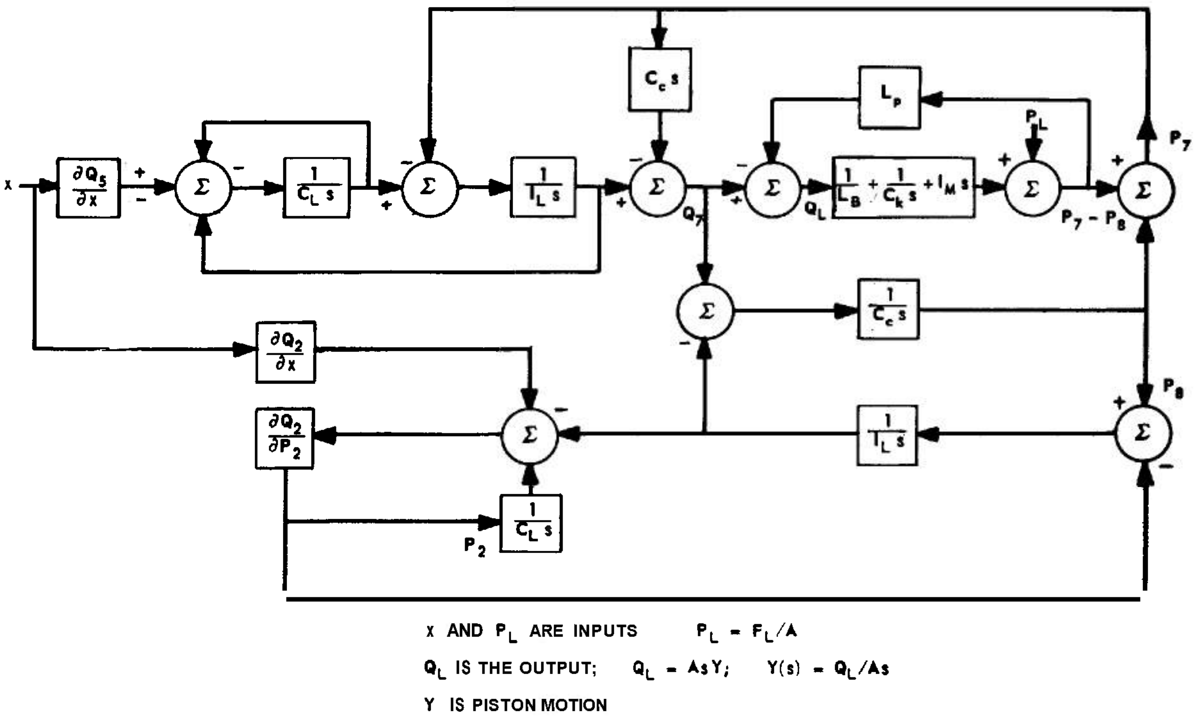


Fig. 73-740 Four-way spool-valve amplifier and load block diagram — hydraulic circuit shown in Fig. 73-734 — load has spring, mass, and dashpot.

form shown in Fig. **13-141** is pertinent to any type of four-way valve used with a balanced piston. If position feedback is used, as shown in Fig. **13-100** or Fig. **13-101,** the appropriate equations are those given in Table **13-16.** Substitution of numerical values for particular cases may be made. Parameters in terms of pipeline and other dimensions and test data may be evaluated by use of the equations given in Par. **13-6.27.** The transfer functions of four-way valves with and without **springs or** feedback are tabulated in Table **13-16.**

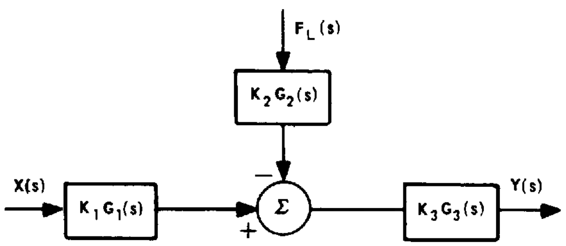


Fig. 73-741 Four-way spool-valve amplifier; simplified block diagram derived from Fig. 73-740. [See Table 73-76 for values of $K_1G_1(s)$, $K_2G_2(s)$, and $K_3G_3(s)$.]

13-6.24 DYNAMIC RESPONSE OF ROTARY PUMP AND LOAD

The dynamic response of a hydraulic amplifier determined in the absence of load is not simply related to the response of a loaded amplifier since the load interacts with the source. Three load cases are of interest: rotary-motor load, translational load (piston), and the spring-opposed piston load. See Fig. **13-142** for the circuit diagram applicable to each of the three cases. The circuit parameters for Fig. **13-142** are given in Table **13-18** and the transfer functions are given in Table **13-17.** The compressibility of a pump is taken as the volume of oil under compression divided by the bulk modulus. This volume is approximately one half the total cylinder volume. The cylinder volume is the displacement D_p divided by 2π.

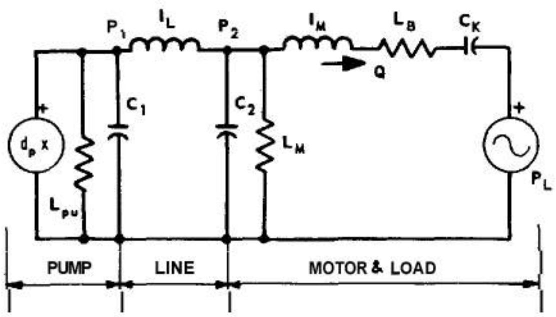


Fig. 13-142 Equivalent hydraulic-circuit representation of rotary amplifier (pump) with load having hydraulic compressibility, leakage, and attached mass, spring, and viscous friction. (See Table 13-78 for values of circuit parameters.)

13-6.25 DYNAMIC RESPONSE OF TRANSLATIONAL AMPLIFIER AND LOAD

Translational amplifiers currently available usually consist of **a** force or torque motor, a first-stage hydraulic amplifier, and a valve. This entire unit is used to drive the output member, which can be either a piston or a rotary motor. Typical commercial configurations are shown in Fig. **13-143.** The dynamic characteristics of the first stage of hydraulic amplification are listed in Table **13-19.** It may be assumed that the tabulated frequency response relates the second-stage spool-valve position to the force-motor coil current.

The block diagram shown in Fig. **13-144** includes the effects of the force motor, $I_c(s)/E(s)$, the first stage of hydraulic amplification, $X(s)/I_c(s)$, and the second-stage hydraulic amplifier (K_1G_1, K_2G_2, and K_3G_3). The force-motor transfer function is given in **Eq. (13-174).** The first-stage dynamics, $G_4(s)$, is obtained from the manufacturer's data (such as shown in Table **13-19)** or else calculated in the manner outlined in the preceding material. The second-stage behavior depends upon the output that is supplied by the valve, and can be obtained from Table **13-16.**

For example, in the case where the second-stage output is a spring-opposed piston, it is apparent from (a) of Table **13-16** that

$$K_1G_1 = \frac{2\left|\frac{\partial P}{\partial x}\right|\frac{A}{K}}{1 + 2L_pR_5}$$

$$K_2G_2 = \frac{1 + b_1s + b_2s^2 + b_3s^3}{K}$$

$$K_3G_3 = \frac{1}{1 + a_1s + a_2s^2 + a_3s^3 + a_4s^4 + a_5s^5}$$

The force-motor coil transfer function that relates coil current to applied voltage can be approximated **by**

$$\frac{I_c(s)}{E(s)} = \frac{1/R}{\frac{L_c}{R}s + 1} \qquad (13\text{-}174)$$

where

$I_c(s)$ = Laplace transform of coil current, in amperes

$E(s)$ = Laplace transform of electronic preamplifier internal voltage, in volts

R = total resistance in coil circuit (including amplifier internal resistance), in ohms

L_c = force-motor coil inductance, in henries.

134.26 PROBLEMS ENCOUNTERED IN USE OF HYDRAULIC AMPLIFIERS

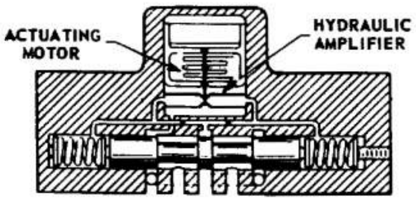
One problem encountered in the design of hydraulic amplifiers is that due to the loading effect **of** the amplifier on the preceding stage. This loading is due to forces caused by the flowing liquid. For a spool valve, the forces are steady-state forces due to flow past the corner of the piston plus a transient force due to the rate of change of flow. Lee and Blackburn[49] express the force as

$$F_x = \rho Q U_2 + \rho l \frac{dQ}{dt} \qquad (13\text{-}175)$$

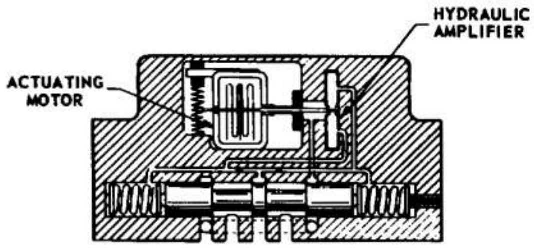
TABLE 13-19 DYNAMIC AND STATIC CHARACTERISTICS OF COMMERCIALLY AVAILABLE ELECTROHYDRAULIC SERVO CONTROL VALVES

Type	Size (in.³)	Wt (lb)	Max Diff Current (ma)	Load Flow (gpm)	Quiescent Flow (gpm)	Typical Frequency Response −3 db ampl. ratio Freq. (cps)	Phase (deg)	Freq. at 90 deg lag (cps)
A†	1.75 × 2.5 × 3.06	0.8	2 to 40 (usually 8)	0.5 to 8.0	0.1 plus 2% rated flow	62	70	92
B†	1.75 × 2.5 × 3.06	0.8	2 to 40 (usually 8)	0.2 to 5.0	0.1 plus 2% rated flow	170	80	193
C†	1.75 × 2.4 × 3.5	0.85	2 to 40 (usually 8)	0.5 to 10.0	0.1 plus 2% rated flow	62	70	92
D	1.75 × 1.9 × 3.06	0.7	2 to 40 (usually 8)	0.5 to 8.0	0.1 plus 2% rated flow	59	70	100
E	1.9 × 1.9 × 3.7	1.1	8 to 10	4.0	0.25	120	71	145
F	2.0 × 2.0 × 3.2	1.3	8.0	7.7	0.25	100††	30††	180††
G	2.32 × 2.65 × 2.14	0.91 to 0.99	6 to 20	up to 9.0	0.12 to 0.15	110 to 190	47 to 68	170 to 270
H	2.0 × 2.0 × 6.0	2.0	30.0	5.0	0.13	185‡	100‡	135‡
I	1.87 × 2.18 × 7.5	3.0	40	10.0	0.21			
J	3.0 × 3.0 × 9.0	15.0	40	20.0	0.26			
K	1.4 × 1.0 × 3.58	0.7	40	4.0	0.13	100‡	105‡	83§
L	2.6 × 3.8 × 7.25	5.75	40	5.5	0.2	100		
M*	3.3 × 3.25 × 4.6	5.0	40	6.8	0.3	60	40	66
N*	2.9 × 2.5 × 4.0	2.8	40	4.3	0.2	50	60	60
O	2.4 × 2.3 × 3.4	2.3	40	2.1	0.1	50	72	52
P	1.4 × 2.6 × 3.3	1.0	10	2.0 to 7.5	0.25	60		90
Q	1.4 × 2.6 × 3.3	1.0	10	0.17 to 2.0	0.18	60		90

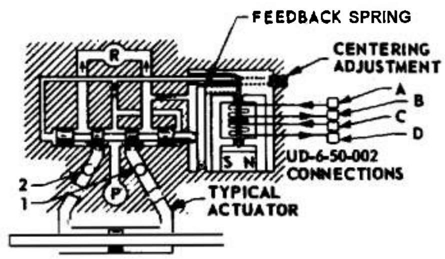
NOTES: †May be gain-compensated
††Differential pressure versus differential current at zero load flow
‡Spool position versus differential current
*Dither generally required



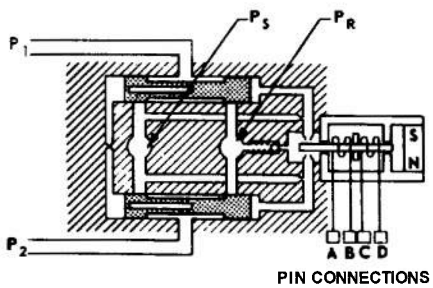
A,B,C



D



E



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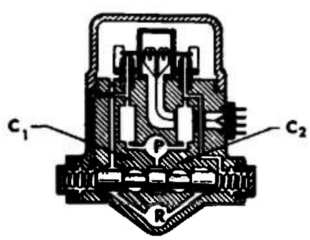

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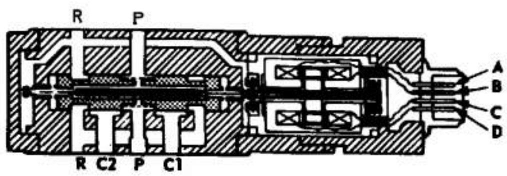

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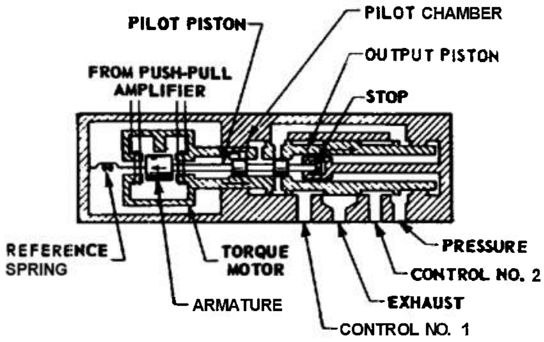
Fig. 13-143 Commercially available electrohydraulic servo valves. (Sheet 1 of 2)

Reprinted from *Control Engineering*, Volume 3, No. 6, June 1956, from article entitled 'What to Look for in Electro-Hydraulic Servo Valves', by B. A. Johnson, L. D. Schmid, and J. Warshawsky.



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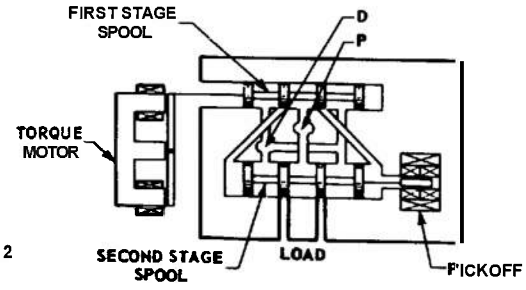

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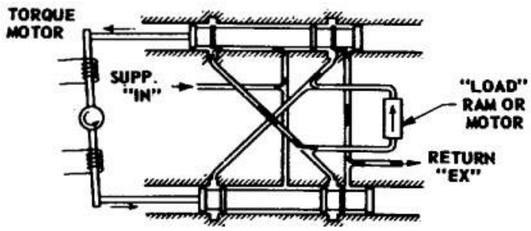
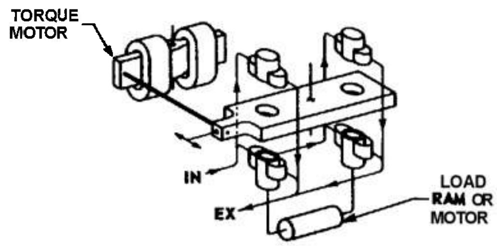


M,N



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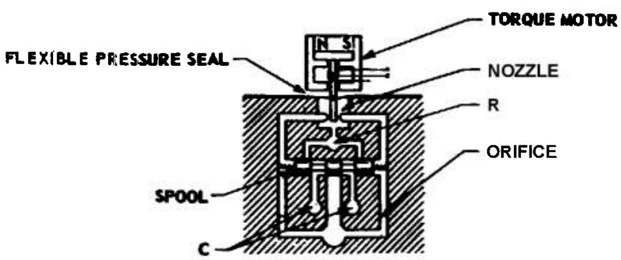


P,Q

Fig. 13-143 Commercially available electrohydraulic servo valves. (Sheet 2 of 2)

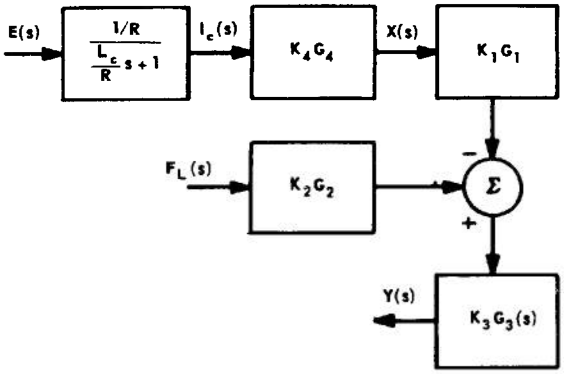


Fig. 13-144 Block diagram of electric amplifier, torque motor, and first- and second-stage hydraulic amplifier.

where

F_x = axial component of force on valve piston due to flow through one chamber, in lb

ρ = density of fluid, in lb-sec²/in.⁴

U_2 = axial component of velocity of fluid through valve orifice, in in./sec

Q = flow through orifice, in in.³/sec

l = axial distance between centers of incoming and outgoing flows, in in.

Additional simplifications lead to

$$F_x = -K_1x - K_2\frac{dx}{dt} \qquad (13\text{-}176)$$

where x is the piston motion, in in.; K_1 has the units lb/in.; and K_2 has the units of viscous damping, i.e., lb-sec/in. The term K_2 may be positive or negative. K_1 is positive and appears as a spring action. By suitable design, K_1 is kept small compared with the constant of the restraining springs. By design, K_2 is made positive, if possible. If K_2 is allowed to be negative, it should be kept less than the coefficient of other viscous-damping terms present. Shaping of valve ports to reduce K_1 is discussed by Lee and Blackburn.[49]

The problem of silting or fouling by particles of foreign matter is always present. Magnetic particles in the oil tend to foul the force motor (especially if a wet coil construction is used, in which case the coil is exposed to the liquid). Filtering is universally used. Manufacturers' specifications for filter capabilities should be obtained and followed strictly. Many systems use a filter rated at 10 microns; i.e., particles larger than 10 microns cannot pass through the filter. A 10-micron particle has a diameter of 0.0004 inch. Hydraulic-amplifier assembly and handling should be done in dust-free rooms. Utmost cleanliness in handling is desirable.

Other difficulties encountered with the use of oil are its flammability (which may preclude its use in some applications), its property variation with temperature, and its tendency for external leakage. The viscosity of oil varies from infinity, as freezing is approached, to nearly zero as decomposition at high temperature occurs. Various oils have been developed in an attempt to circumvent this difficulty.

Entrainment of air in the hydraulic fluid causes serious deterioration of system performance due to the much lower bulk modulus of air (see Par. 14-4). Such entrainment is brought about by the mixing of oil and air in spaces where agitation is present. It may be possible by design to prevent this mixing; e.g., by increasing the replenishing pressure. Sticking or otherwise-faulty check valves in the replenishing line may effectively lower the replenishing pressure.

Sundry auxiliary equipment used in connection with hydraulic amplifiers includes:

(a) Filters

(b) Pressure supplies

(c) Relief valves

(d) Heat exchangers

The physical properties of oil that are of interest to the servo designer are viscosity, density, and bulk modulus. These and other significant properties are tabulated in Par. 20-2.

13-6.27 HYDRAULIC-CIRCUIT ELEMENTS

When studing a hydraulic circuit, one can find the circuit elements in terms of the dimensions of the device and in terms of the properties of the hydraulic fluid used. Methods of hydraulic-circuit analysis are similar to those used in analyzing electrical circuits. The variables are the pressure P, in lb/in.^{2}, and the flow Q, in in.^{3}/sec. The following equations determine the flow components:

Leakage flow $Q_L = LP$ (13-177)

Compressibility flow $Q_c = C\dfrac{dP}{dt}$ (13-178)

Inertance flow $Q_I = \dfrac{1}{I}\int P dt$ (13-179)

The flow coefficients in terms of dimensions are as follows:

(a) Leakage coefficient (L)

(1) Linear flow in pipes (Hagan-Poiseuille Law)

$$L = \frac{\pi D^4}{128\mu l} \quad (13\text{-}180)$$

(2) Linear flow between parallel plates (Stolarik's Law; piston-wall leakage for oil velocity >> piston velocity)

$$L = \frac{bh^3}{12\mu x} \quad (13\text{-}181)$$

(3) Turbulent flow in pipes (Reynolds' number > 2,000)

$$L = \left(\frac{\pi^2}{8}\right)\frac{D^5}{Q_L f l \rho} \quad (13\text{-}182)$$

where

L =leakage coefficient, in in.^{5}/lb-sec

D = inner diameter of pipe, in in.

l = length of pipe, in in.

μ = viscosity, in lb-sec/in.^{2}

b = length perpendicular to flow, in in.

h = separation of plane surfaces, in in.

x =length of surface parallel to flow, in in.

Q_L = flow, in in.^{3}/sec

f = dimensionless friction factor which occurs in the D'Arcy formula: $\Delta P = f\left(\frac{l}{D}\right)\left(\frac{\rho V^2}{2}\right)$

ρ =density, in lb-sec^2/in.^{4} (See Par. 20-2)

AP = pressure drop across pipe of length l, in lb/in.^{2}

and

$$\text{Reynolds' number} = R_e = \frac{\rho VD}{\mu} \quad (13\text{-}183)$$

where

V = velocity, in in./sec

The friction factor has been determined experimentally. It depends upon the Reynolds' number and the roughness ratio e/D. The average height of the roughness projection, in inches, is e. A plot of the friction factor f versus the logarithm of the Reynolds' number, with roughness as a parameter, is called a ***Stanton diagram.*** One such diagram is shown as Table 13-20.*

(b) Compressibility coefficient (C)

(1) Homogeneous liquids

$$C = \frac{V_l}{B_l} \quad (13\text{-}184)$$

(2) Liquid with entrained air

$$C = C_l + C_a = f_a\left[\frac{V}{B_l}\right] \quad (13\text{-}185)$$

(3) Liquid in pipeline with entrained air

$$C = C_l + C_a + C_p \quad (13\text{-}186)$$

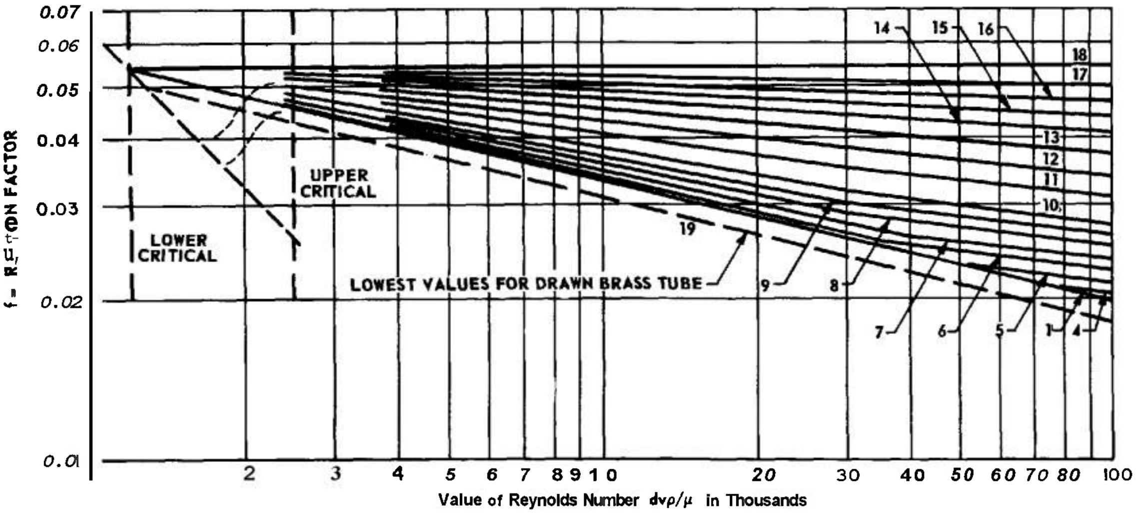
*For additional information, consult more recent handbooks on this subject.

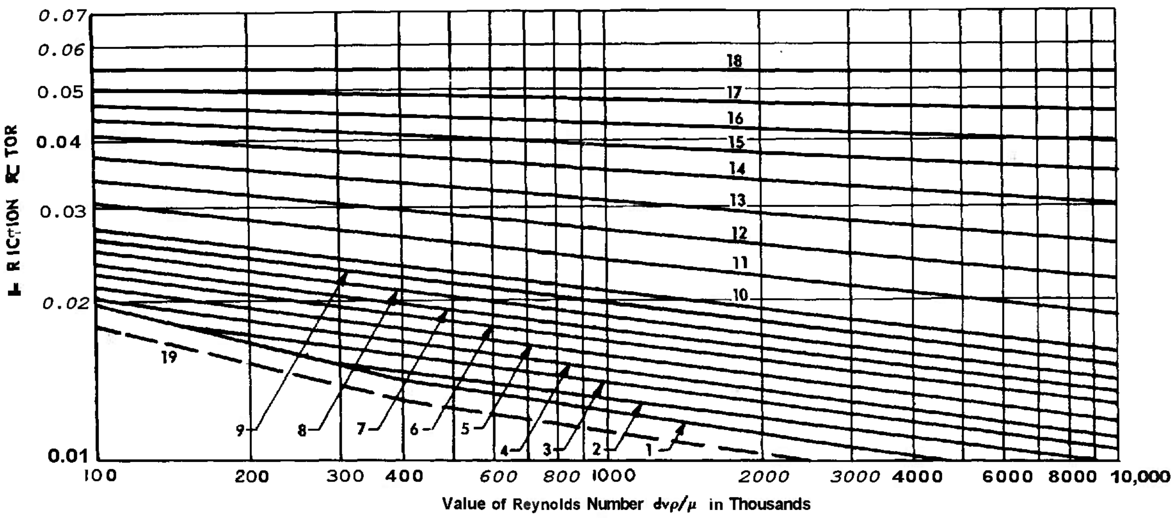
TABLE 13-20 FRICTION FACTORS

SELECTED LOCATION OF f BY ROUGHNESS RELATION

Curve.. ..	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Roughness, per cent .	0.2	0.45	0.81	1.35	2.1	3.0	3.8	4.8	6.0	7.2	10.5	14.5	19.0	24.0	28.0	31.5	34.0	37.5
A	0.35 up	...	...	...	...	...	...	...	...	...	...	...	...	0.125	...	...	...	0.0625
B	72	48 to 66	14 to 42	6 to 12	4 to 5	2 to 3	1%	1 to 1¼	3/4	1/2	3/8	1/4	1/8	...	...	...	...	...
C	...	...	30	10 to 24	6 to 8	3 to 5	2%	1½ to 2	1%	1	3/4	1/2	..	3/8	..	1/4	..	1/8
D	...	...	48 to 96	20 to 48	12 to 16	5 to 10	3 to 4	2 to 2½	1½	1%	1	...	...	...	...	...	...	...
E	...	...	96	42 to 96	24 to 36	10 to 20	6 to 8	4 to 5	3	...	...	...	...	...	...	...	...	...
F	...	...	220	84 to 204	48 to 72	20 to 42	16 to 18	10 to 14	8	5	4	3	...	...	...	...	...	...

A = Drawn tubing, brass, tin, lead, glass, diam., in. B = Clean steel, wrought iron, diam., in. C = Clean, galvanized, diam., in. D = Best cast iron, cement, light riveted sheet ducts, diam., in. E = Average cast iron, rough-formed concrete, diam., in. F = First-class brick, heavy riveted, spiral riveted, diam., in. In drawn tubing, actual inside diameter is given. In pipe, nominal size of standard weight is given.





$$\Delta p = \frac{0.000108\ flpv^2}{d} \text{ for circular pipes} \qquad \Delta p = \frac{0.000027\ flpv^2}{M} \text{ for any shape}$$

Extracted from *Mechanical Engineering*, Volume *66*, August 1933, with permission of the publisher, The American Society of Mechanical Engineers, 29 W. 39th St., New York 18, N. Y.

where

C = compressibility coefficient, in in.^{5}/lb

$C_l = \dfrac{V_l}{B_l}$

$C_a = \dfrac{V_a}{B_a}$

$C_p = \dfrac{\pi D^3 l}{16 E t}\left(1 + \dfrac{t}{2r}\right)$

V_l = volume of liquid under pressure, in in.^{3}

B_l = bulk modulus of liquid, in lb/in.^{2}

V_a = volume of air entrained in liquid and under same pressure as liquid, in in.^{3}

V = geometric volume in which the mixture of oil and air is contained, in in.^{3}

B_a = bulk modulus of air, in lb/in.^{2}

D = inner diameter of pipe, in in.

l = length of pipe, in in.

t = thickness of pipe wall, in in.

r = $D/2$

E = modulus of elasticity for pipe material, in lb/in.^{2}

f_a = relative compressibility constant

and

$P_o < B_a < 1.4\, P_o$

where

P_o = average value of absolute pressure of compressed mixture, in lb/in.^{2}

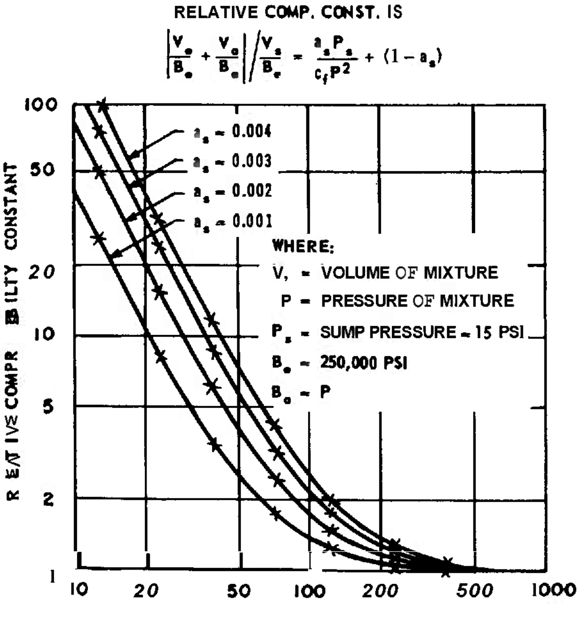
A plot of the relative compressibility coefficient as a function of pressure and amount of entrained air is shown in Fig. 13-145. For convenience, the pipe elastance effect may be found by using the relationship

$$C_p = C_l \frac{B_l}{(E)} f_c \qquad (13\text{-}187)$$

where

f_c = elasticity factor (see Table 13-21)

$$= \frac{1}{2}\left(\frac{r}{t}\right)\left(1 + \frac{t}{2r}\right)$$



a_s = FRACTION OF TOTAL VOLUME THAT IS GAS AT ATMOSPHERIC PRESSURE. NO SOLUTION EFFECTS ARE TAKEN INTO ACCOUNT.

Fig. 13-145 Relative Compressibility coefficient as a function of pressure and amount of entrained gas.

Adapted by permission from the *Journal* of *The Franklin Institute,* Volume 243. No. 6, June 1947. from article entitled 'Hydraulic Variable-Speed Transmissions as Servomotors', by G. C. Newton, Jr.

TABLE 13-21 ELASTICITY FACTORS FOR PIPE [For Use in Eq. (13-187)]

Pipe Size (in.)	I. D. (in.)	Wall Thickness (in.)	Elasticity Factor f_c
1/4	0.186	0.032	1.70
3/8	0.311	0.032	2.67
1/2	0.402	0.049	2.28
3/4	0.527	0.049	3.36
1	0.652	0.049	3.58

For steel tubing and the usual oils, C_p is negligible, since B $\approx$ **250,000** and **E** $\approx$ **30,000,000.**

(c) Inertance coefficient (I)

$$I = \frac{\rho l}{A} \tag{13-188}$$

where

I = inertance coefficient, in lb-sec^2/in.^{5}

ρ = density, in lb-sec^2/in.^{4}

l = length of pipe, in in.

A = cross-sectional area of pipe through which flow occurs, in in.^{2}

13-6.28 ILLUSTRATIVE EXAMPLE

An example will serve to illustrate some of the ideas presented in this section.

Statement of Problem

(a) Find the frequency response of a hydraulic amplifier consisting of a four-way valve and a balanced piston, as shown in Fig. **13-134.**

(b) Find the transfer functions relating output motion to input motion and load force.

(c) Construct the block diagram of the amplifier.

Information

piston net effective area = 1.5 in.^{2}
piston diametric clearance = **0.020** in.
piston thickness = 0.32 in.
piston outside diameter = **1.5** in.
supply pressure = **3000** lb/in.^{2}
volume of oil in one end of cylinder (with piston centered) = **3** in.^{3}
piston opposed by springs having total spring rate = **500** lb/in.
total weight of piston and attached load = **10** lb
each line connecting valve to piston = **4** in. long with internal diameter of **0.3** in.
valve radial clearance (b) = **0.001** in.
valve underlap (X,) = **0.01** in.
valve diameter = **0.1** in.
bulk modulus of oil = **250,000** lb/in.^{2}
viscosity of oil (μ) = 2×10^{-6} lb-sec/in.^{2}
density of oil = 0.031/386 lb-sec^2/in.^{4}

13-6.29 List of Pertinent Equations and Parameters

The circuit diagrams of Figs. **13-134** and **13-136** are pertinent. The transfer function is given by (a) in Table **13-16.** The following parameters must be evaluated :

valve pressure gain	$\left\lvert \frac{\partial P}{\partial x} \right\rvert$
valve resistance	$\left\lvert \frac{\partial P}{\partial Q} \right\rvert$
line capacitance	$C_L = V_L/B_M$
cylinder capacitance	$C_C = V_{C2}/B_M$
piston-cylinder wall leakage coefficient	L_p
line inertance	$I_L = \frac{\rho l}{A_L}$
spring equivalent hydraulic capacitance	$C_K = A^2/K$
equivalent hydraulic resistance of load damping	$R_B = B/A^2$
equivalent hydraulic inertance of load mass	$I_M = M/A^2$

13-6.30 CALCULATION OF VALVE CONSTANTS

The valve constants may be found as follows: By observing Fig. **13-119,** $\lvert \partial Q/\partial x \rvert$ is seen to be a maximum at P_2/P_s = **0.5** and at small values of x/X_0. At the same pressure, the minimum gain is about half the maximum value. The value of β for this problem is **0.1** for which no plots are presented. Refer to Table **13-15** and find the maximum gain. For $x/X_0 = 0$ and for $p_2 = 0.5$, the table gives

$$\left\lvert \frac{\partial Q}{\partial x} \right\rvert = c\pi d \sqrt{P_s} \sqrt{\frac{2}{1+\beta^2}} \tag{13-189}$$

The effect of clearance b is seen to be negligible. In Fig. **13-114,** the flow conductance

$|\partial Q/\partial P|$ is seen to vary parabolically from a minimum value that occurs at $p_2 = 0.5$. At $p_2 = 0.5$ and $x/X_o = 0$, Table **13-15** yields

$$\left|\frac{\partial Q}{\partial P}\right| = L_5 = \frac{c\pi d\, X_o\sqrt{2}\;\;\sqrt{1+\beta^2}}{\sqrt{P_s}} \tag{13-190}$$

Note, from Fig. **13-114,** that the conductance varies from about half to twice this value over a reasonable operating range. By combining **Eqs. (13-189)** and **(13-190)**, the pressure gain at $p_2 = 0.5$ and $x/X_o = 0$ is found to be

$$\left|\frac{\partial P}{\partial x}\right| = \frac{\left|\frac{\partial Q}{\partial x}\right|}{\left|\frac{\partial Q}{\partial P}\right|} = \frac{P_s}{X_o} \tag{13-191}$$

This result was pointed out in **Eq. (13-104)** which could have been used directly. Since no measured value of Q_{L0} is given, it is necessary to use **Eq. (13-190)** to evaluate the valve conductance. According to Par. **13-6.12,** the coefficient c has a value of **100** for the density of oil given above. Substituting the other given values leads to

$$\left|\frac{\partial P}{\partial x}\right| = \frac{3000}{0.01} = 3 \times 10^5 \text{ lb/in.}^3 \tag{13-192}$$

$$\left|\frac{\partial Q}{\partial P}\right| = \frac{(100)(\pi)\,(0.1)(0.01)(\sqrt{2})}{\sqrt{3000}}$$

$$= 9.008 \text{ in.}^5\text{/lb-sec} \tag{13-193}$$

13-6.31 Calculations of Other Constants

The other parameter values are found as follows: For reasonable amounts of entrained air, Fig. **13-145** shows that the effect of entrainment is negligible for pressures above **500** psi. Since the given supply pressure is **3000** psi and the operating point is **1500** psi, the entrainment will be disregarded. Therefore, the line capacitance is

$$C_L = \frac{V_L}{B_M} = \frac{(4)\,(\pi d^2/4)}{250{,}000}$$

$$= 1.14 \times 10^{-6} \text{ in.}^5\text{/lb} \tag{13-194}$$

The cylinder capacitance is

$$C_C = \frac{V_C}{B_M} = \frac{3}{250{,}000} = 1.2 \times 10^{-5} \text{ in.}^5\text{/lb} \tag{13-195}$$

The spring "capacitance" is

$$C_K = \frac{A^2}{K} = \frac{2.25}{500} = 4.5 \times 10^{-3} \text{ in.}^5\text{/lb} \tag{13-196}$$

The line inertance [from Par. 13-6.27] is

$$I_L = \frac{\rho l}{A_L} = \left(\frac{0.031}{386}\right) \frac{4}{(\pi d^2/4)}$$

$$= 0.0045 \text{ lb-sec}^2\text{/in.}^5 \tag{13-197}$$

The mass "inertance" is

$$I_M = \frac{M}{A^2} = \frac{10}{(386)\,(2.25)}$$

$$= 0.0115 \text{ lb-sec}^2\text{/in.}^5 \tag{13-198}$$

The dashpot "resistance' is

$$R_B = 0$$

The cylinder leakage [from **Eq. (13-181)**] is

$$L_p = \frac{bh^3}{12\mu x} = \frac{(\pi d_p)\,h^3}{12\mu x}$$

$$= \frac{(1.5)\,(\pi)\,(0.01)^3}{(12)\,(\mu)\,(0.32)}$$

$$= 0.6 \text{ in.}^5\text{/lb-sec} \tag{13-199}$$

13-6.32 Calculation of Coefficients a and b

The coefficients a_1 through a_5 and b_1 through b_3 of (a) in Table **13-16** are calculated next. When the numerical values are inserted, it becomes apparent that certain approximations are valid. The approximate relations are

$$a_1 = \frac{C_K}{L_p} = 0.0075 \text{ sec}$$

$$a_2 = C_K I_M = 5.18 \times 10^{-5} \text{ sec}^2$$

$$a_3 = (C_K I_M)\left(\frac{C_C}{2L_p}\right) = 5.18 \times 10^{-9} \text{ sec}^3$$

$$a_4 = I_M C_K C_L I_L = 2.7 \times 10^{-13} \text{ sec}^4$$

$$a_5 = (I_M C_K C_L I_L)\left(\frac{C_C}{2L_p}\right) = 2.7 \times 10^{-17} \text{ sec}^5$$

$$b_1 = \frac{C_C}{2L_p} = 10^{-4} \text{ sec}$$

$$b_2 = C_L I_L = 5.13 \times 10^{-9} \text{ sec}^2$$

$$b_3 = (C_L I_L)\left(\frac{C_c}{2L_p}\right) = 5.13 \times 10^{-13} \text{ sec}^3$$

13-6.33 Results, Simplification, and Significance

From (a) in Table **13-16,** the transfer function relating input, load force, and output is

$$Y(s) = \frac{90X(s) - \frac{F_L(s)}{500}[1 + 10^{-4}s + 5.13 \times 10^{-9}s^2 + 5.13 \times 10^{-13}s^3]}{1 + 0.0075s + 5.18 \times 10^{-5}s^2 + 5.18 \times 10^{-9}s^3 + 2.7 \times 10^{-13}s^4 + 2.7 \times 10^{-17}s^5} \quad \textbf{(13-200)}$$

In order to gain some insight into the behavior of the polynomials in s, substitute $u = 10^{-2}s$ in the denominator and $v = 10^{-4}s$ in the numerator. Then

$$Y(s) = \frac{90X(s) - \frac{F_L(s)}{500}[1 + v + 0.513v^2 + 0.513v^3]}{1 + 0.75u + 0.518u^2 + 0.00518u^3 + 0.000027u^4 + 0.00000027u^5} \quad \textbf{(13-201)}$$

Since the coefficients of the last three terms of the denominator are so small compared with the first three, it can be inferred that the first three terms are approximately a factor of the denominator. Dividing them out leaves a small remainder. If added accuracy were required, the method of continued trial division could continue. When the denominator is factored in this approximate manner, the second factor can have $10^{-2}u$ replaced by v. The result is

$$Y(s) = \frac{90X(s) - \frac{F_L(s)}{500}[1 + v + 0.513v^2 + 0.513v^3]}{(1 + 0.75u + 0.518u^2)\,(1 + 0.99v + 0.517v^2 + 0.52v^3)} \quad \textbf{(13-202)}$$

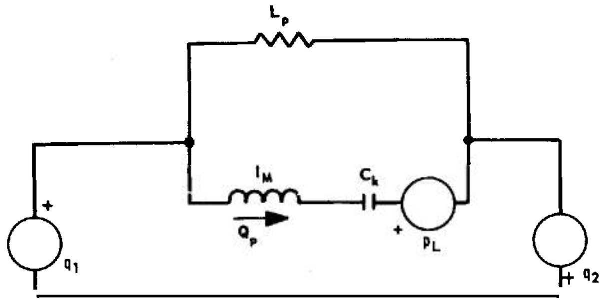
The transfer function relating $F_L(s)$ to $Y(s)$ is seen to consist of only a quadratic, since the two cubics are so nearly identical. The cubic can be neglected in comparison with the quadratic, in any case, since its frequency range is 100 times that of the quadratic; and the fact that all coefficients are near unity indicates that the cubic does not have one low-frequency first-order factor. Therefore, the expression can be simplified with very good accuracy to

$$Y(s) = \frac{90X(s) - \frac{F_L(s)}{500}}{1 + 0.75u + 0.518u^2} \quad \textbf{(13-203)}$$

The natural frequency and damping coefficient of the quadratic are found to be

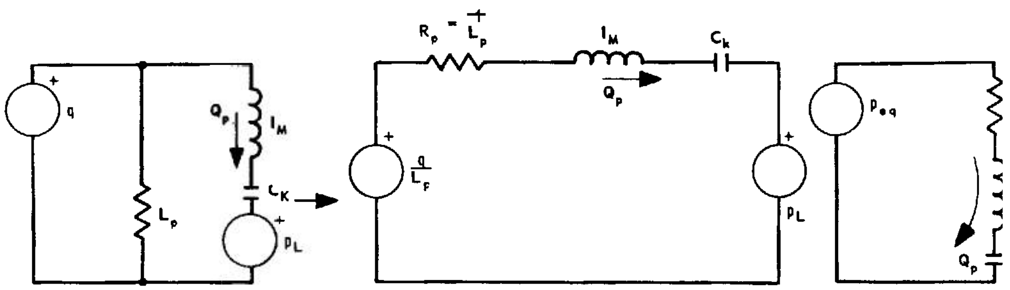
$$u_n = 1.39$$

$$\zeta = 0.53$$



$q_1 = q_2 = q = \left|\frac{\partial Q}{\partial x}\right| X(s)$

FROM FIG. 13-134 DROPPING NEGLIGIBLE ELEMENTS



EQUIVALENT TO ABOVE SINCE EQUAL SERIES FLOW SOURCES ARE EQUIVALENT TO A SINGLE SOURCE

USING SOURCE TRANSFORMATION

$p_{eq} = \frac{q}{L_p} - p_L$

COMBINING PRESSURE SOURCES

Fig. 73-746 Simplified circuit diagram for illustrative example.

In terms of the variable **s**, the natural frequency ω_n is **139** radians/sec. The simplifications that are justified numerically consists of neglecting all parameters except C_k, I_m, and L_p. The natural frequency and damping can be found in terms of these parameters. The zero-frequency gain in absence of load force (numerically **90**) is approximately (from Table **13-16**)

$$\lim_{s\to 0} \frac{Y(s)}{X(s)}\bigg|_{F_L(s)=0} = \frac{2\left|\frac{\partial P}{\partial x}\right|\frac{A}{K}}{2L_pR_5} = \frac{\left|\frac{\partial Q}{\partial x}\right|\frac{A}{K}}{L_p}$$

$$= \frac{(1800)\left(\frac{1.5}{500}\right)}{0.06} = 90 \qquad \textbf{(13-204)}$$

The natural frequency is

$$\omega_n = \sqrt{\frac{1}{C_K I_M}} = \sqrt{\frac{K}{M}} \qquad \textbf{(13-205)}$$

The damping coefficient is

$$\zeta = \frac{1}{2}\sqrt{\frac{1}{KM}}\frac{A^2}{L_p} \qquad \textbf{(13-206)}$$

$$\cong \frac{3\pi}{8}\sqrt{\frac{1}{KM}}\frac{d_p^3}{h^3}\mu X \qquad \textbf{(13-207)}$$

where

X = piston length

d_p = piston diameter

$2h$ = piston diametric clearance

and the piston-rod area has been neglected.

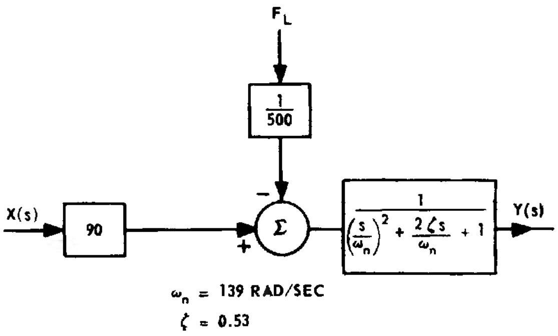


Fig. 13-147 Simplified block diagram for illustrative example.

The effect of the load mass, restraining spring, and piston dimensions on the damping coefficient can be seen in Eq. **(13-207).** The natural frequency is seen to be that of the spring and mass, Eq. **(13-205).** The frequency response will be characterized by a break from a 0 dg/decade slope to a **—20** dg/decade slope **at 139** radians/sec. At **139** radians/sec, the amplitude-response curve has a value which is $0.5/\zeta$ times the low-frequency value. The simplified circuit diagram is shown in Fig. **13-146.** The block diagram is shown in Fig. **13-147.**

The flow Q_p is found from the final simplified circuit by dividing the pressure source by the sum of the impedances; thus

$$Q_p = \frac{P_{eq}}{R_p + I_M s + \dfrac{1}{C_K s}} \qquad \textbf{(13-208)}$$

where

$$Q_p = AsY(s)$$

$$p_1 = F_L/A$$

Equation **(13-208)** can be solved for $Y(s)$ in terms of $X(s)$ and $F_L(s)$ with the same result as given in Eq. **(13-203).** In literal form, there results

$$Y(s) = \frac{\left|\dfrac{\partial Q}{\partial x}\right| \dfrac{A}{KL_p} X(s) - \dfrac{F_L(s)}{K}}{C_K I_M s^2 + \dfrac{C_K}{L_p} s + 1} \qquad (13\text{-}209)$$

13-7 PNEUMATIC AMPLIFIERS*

13-7.1 INTRODUCTION

It is important that the reader keep foremost in mind the following points concerning the material on pneumatic amplifiers:

(a) ***Flow is always expressed herein in terms** of **weight flow (lb/sec).*** Since much of the available literature utilizes volume flow (ft^3/sec), one must take proper account of the type of flow being treated.

(b) ***Approximations for the equivalent source for a pneumatic amplifier in terms of design data hold only for small incremental changes within a restricted region** of **the set of curves describing the amplifiers operating characteristics.*** To determine system behavior over wide limits, one should resort to an analog computer.

A pneumatic amplifier is a device wherein a mechanical input motion modulates the flow of a gas.** The gas is usually derived from a constant pressure source. The modulated gas flow imparts motion to a second movable part, the latter motion occurring at a much higher

**By P. E. Smith, Jr.*

****Although the term "gas" is frequently used, the components discussed herein employ air as the pneumatic fluid.**

force level than that of the input motion. A pneumatic amplifier consists of two parts—a valve and a motor—the dynamic properties of which are inextricably combined. These properties are treated in subsequent paragraphs of this section. The physical characteristics of pneumatic motors are described in Ch. **14**.

13-7.2 PNEUMATIC VALVES

Pneumatic valves include a diversity **of** types, some of which are shown in Figs. **13-148** through **13-150.** The 'choice of a particular type depends upon the pressure level of the pneumatic system employed. Pneumatic systems can be divided into low-pressure and high-pressure systems. Low-pressure systems using a supply pressure of **15** to **20** psi are employed in conventional process control, although some process control systems make use of **60** to **100** psi. Low-pressure systems make extensive use of nozzle-baffle type valves. Three-way valves of various configurations are also used in these systems.

High-pressure systems, such as those used with servos in guided-missile work, employ pressures as high as **1000** to **3000** psi. Such servos might be used to achieve position control of control surfaces, fuel valves, and numerous other objects. The higher-pressure systems frequently use sliding-plate valves or spool valves similar to those used in hydraulic servos.

13-7.3 STATIC CHARACTERISTICS OF PNEUMATIC VALVES

The static characteristics of sliding-plate, double conical-plug (throttling), and nozzle-baffle valves are discussed below. All have three-way valve characteristics. The characteristics of a four-way valve can be obtained from those of the three-way valve. It is essential to consider the four-way valve as a combination of two separate valves, since compressibility usually cannot be neglected. The balanced four-way valve characteristics are useful only when compressibility can be neglected and a balanced load exists. Still further justification **for** the presentation **of** three-way valve characteristics is the widespread use **of** spring-opposed diaphragm or bellows actuators driven by three-way valves or single-sided amplifiers.

13-7.4 Orifice Flow

The equation for flow through an orifice is[52]

$$W = C_d A_o f\left(P_u, T_u, \frac{P_d}{P_u}\right) \qquad (13\text{-}210)$$

where

W = weight rate of flow of air, in lb/sec

C_d = orifice discharge coefficient

A_o = orifice area, in in.[2]

f( ) = function of ( )

P_u = upstream pressure, in psia

T_u = upstream temperature, in "Rankine (°F + 460)

P_d = downstream pressure, in psia

It is convenient to rewrite part of Eq. (13-210) as

$$f\left(P_u, T_u, \frac{P_d}{P_u}\right) = C_k \frac{P_u}{\sqrt{T_u}} f_1\left(\frac{P_d}{P_u}\right) \qquad (13\text{-}211)$$

where

$$C_k = g\sqrt{\frac{\dfrac{Ks}{Ks+1}}{\dfrac{Ks-1}{R_g\left(\dfrac{Ks+1}{2}\right)}}}$$

$$= 0.54, \text{ in } \frac{(\text{"Rankine})^{1/2}}{\text{sec}} \text{ for air}$$

g = acceleration due to gravity = **386** in./sec[2]

K_s = ratio of specific heats $(C_p/C_v) = 1.4$ for air

R_g = gas constant = 2.47×10^5 $\dfrac{\text{in.}^2}{\text{sec}^2 - {}^\circ\text{ Rankine}}$ for air

$f_1(\ )$ = function of ( )

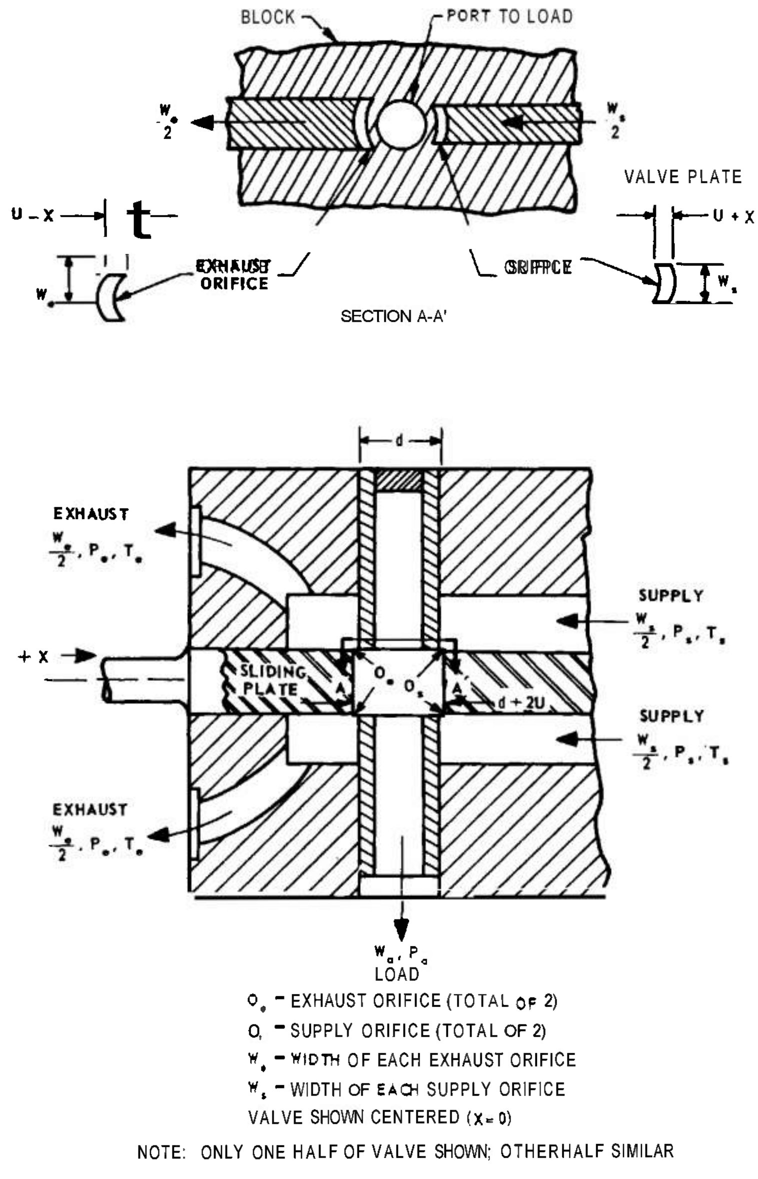


Fig. 73-748 Schematic of sliding-plate valve.

Based on Fig. 4, p. 236, from *Transactions of the ASME*, Volume 78, February 1966, from article entitled 'Study of Pneumatic Processes in the Continuous Control of Motion with Compressed Air—1'. by J. L. Shearer.

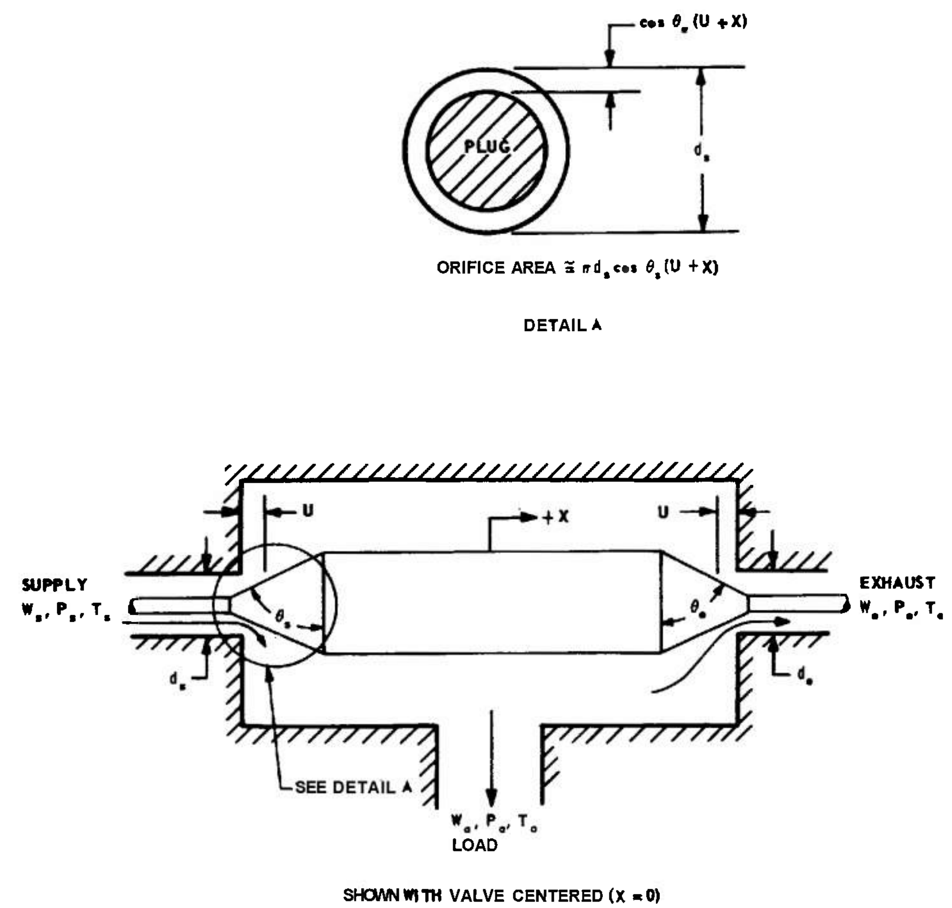


Fig. 73-149 Schematic of conical-plug type three-way valve.

Equation (13-210) can thus be rewritten as

$$W = C_k C_d A_o \frac{P_u}{\sqrt{T_u}} f_1 \left(\frac{P_d}{P_u}\right) \qquad (13\text{-}212)$$

The function $f_1\left(\frac{P_d}{P_u}\right)$ is plotted against $\frac{P_d}{P_u}$ in Fig. 13-151.

Equations for calculations of orifice area A, in terms of valve displacement for sliding-plate, conical-plug, and nozzle-baffle type three-way valves are given in Table 13-22. Therefore, the flow W through an orifice with area A, (corresponding to a given valve position) and with discharge coefficient C_d can be determined for any set of values of the parameters P_d, P_u and T_u.

13-7.5 Nondimensional Flow

Nondimensional flow is used in order to present data that is applicable to most types of valves. It is obtained by dividing the actual flow by the quiescent-leakage flow W_q that occurs through the supply and exhaust orifices

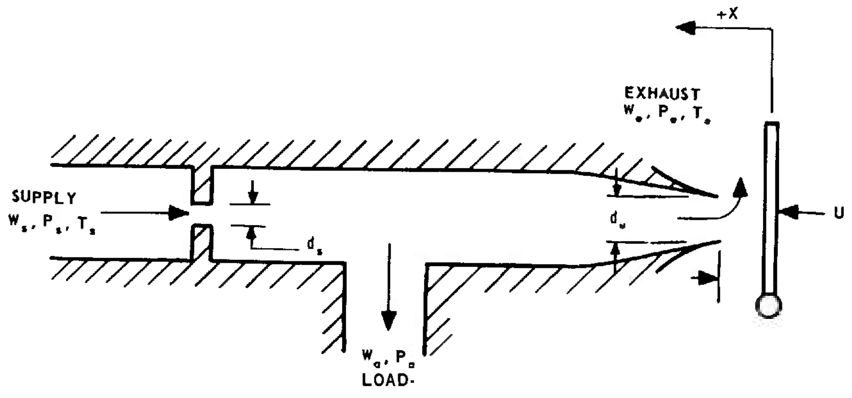


Fig. 73-150 Schematic of nozzle-baffle three-way valve.

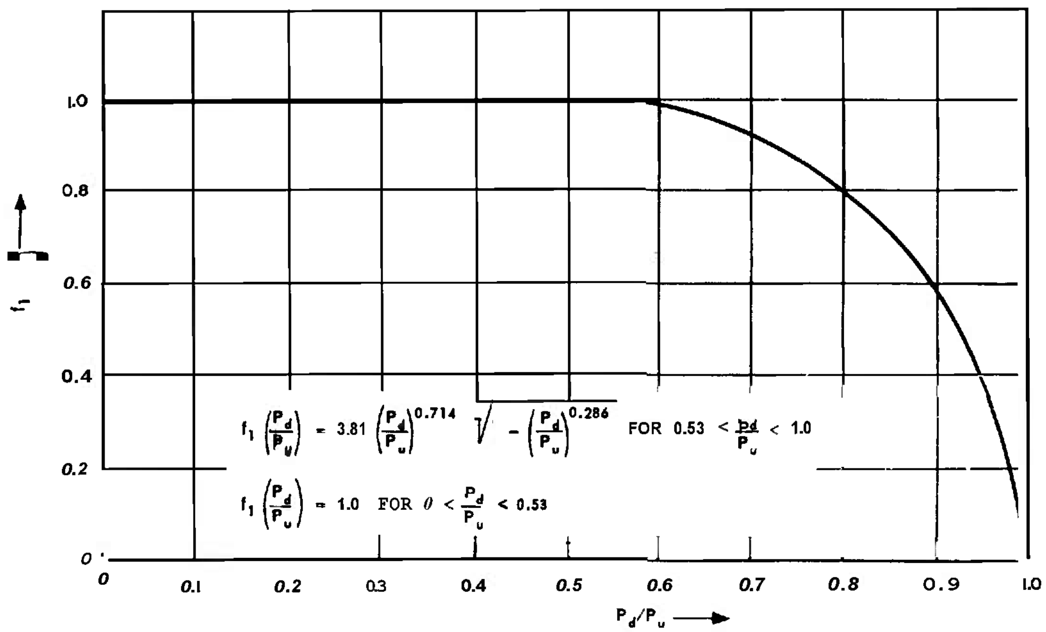


Fig. 13-151 Plot of restriction factor versus pressure ratio for flow of compressible fluid through an orifice.

Extracted from *Transactions of the ASME*, Volume 78, February 1966, with permission of the publisher, The American Society of Mechanical Engineers, 29 W. 39th St., New York 18, N. Y.

when the valve is centered and there exists zero load flow. For air, the nondimensional-flow equation for $\frac{W_a}{W_q}$ is

$$\frac{W_a}{W_q} = \left(1+\frac{X}{U}\right)\frac{f_1\left(\frac{P_a}{P_s}\right)}{f_1\left(\frac{P_{aq}}{P_s}\right)} - \left(1-\frac{X}{U}\right)\frac{C_3C_{de}}{C_{ds}}\frac{P_a}{P_s}\sqrt{\frac{T_s}{T_e}}\frac{f_1\left(\frac{P_e}{P_a}\right)}{f_1\left(\frac{P_{aq}}{P_s}\right)} \tag{13-213}$$

TABLE 13-22 ORIFICE AREA EQUATIONS FOR PNEUMATIC THREE-WAY VALVES

Valve Type	Equation for Orifice Area A_o (in in.^{2})		C_3
	Supply (A_{os})	Exhaust (A_{oe})	$\left(\frac{A_{oe}}{A_{os}}\right)$ at $X = 0$
Sliding-plate	$2w_s(U+X)$ $(X \leqq U)$ (total of 2 orifices)	$2w_e(U-X)$ $(X \leqq U)$ (total of 2 orifices)	$\frac{w_e}{w_s}$
Conical-plug	$\pi d_s \cos\theta_s (U-X)$ $(X \leqq U)$	$\pi d_e \cos\theta_e (U-X)$ $(X \leqq U)$	$\frac{d_e \cos\theta_e}{d_s \cos\theta_s}$
Nozzle-baffle	$\frac{1}{4}\pi d_s^2 = \pi d_e U$ (by design)	$\pi d_e(U-X)$ $(X \leqq U)$	

where

U = valve underlap, in in.

X = valve displacement, measured from center position, in in.

w_s = width of supply port, in in.

w_e = width of exhaust port, in in.

d_s = diameter of supply port or restriction, in in.

d_e = diameter of exhaust port or nozzle, in in.

θ_s = measure of supply-plug taper, in deg

θ_e = measure of exhaust-plug taper, in deg

where

W_a = load **flow,** in lb/sec

W_q = quiescent-leakage flow, in lb/sec

X = valve displacement, measured from center position, in in.

U = valve underlap, in in.

P_a = load pressure, in psia

P_s = supply pressure, in psia

P_{aq} = load pressure for $X = 0$, $W_a = 0$, in psia

C_3 = ratio of exhaust orifice area to supply orifice area at $X = 0$

C_{de} = exhaust orifice discharge coefficient

C_{ds} = supply orifice discharge coefficient

T_s = supply temperature, in °Rankine

T_e = exhaust temperature, in °Rankine

P_e = exhaust pressure, in psia

P_{aq} depends upon the value of C_3. Often, by design, P_{aq} is made equal to approximately one half the difference between P_s and P_e. For a method of finding P_{aq} from design data, refer to Shearer.[52] ***Equation* (13-213) *is not applicable to nozzle-baffle valves with fixed upstream orifices.***

The relationship between W_a/W_q and P_a/P_s for various values of valve displacement is shown by the family of curves in Fig. **13-152.**[52]

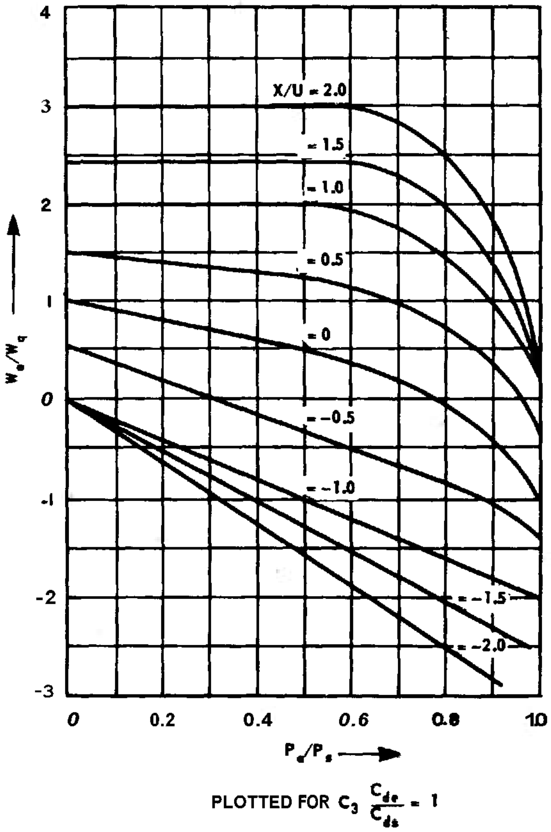


Fig. 13-152 Nondimensional plot of theoretical load flow versus load pressure for open-center (underlapped) three-way valve.

Adapted from *Transactions* of *the ASME,* Volume 78, February 1966, from article entitled 'Study of Pneumatic Processes in the Continuous Control of Motion with Compressed Air— I,' by J. L. Shearer.

For this figure, a value of $C_3C_{de}/C_{ds} = 1$ was used. Since, for this particular case, the supply and exhaust orifices are equivalent to two identical resistances in series, $P_{aq} = 0.5 (P_s - P_e)$. Curves similar to those in Fig. 13-152 can be determined experimentally, rather than theoretically, for a given valve. If the valve is available, the experimental method is preferable, since the necessity for making various simplifying assumptions is eliminated.

13-7.6 Equivalent Source

The equivalent source for a pneumatic valve can be represented by two quantities —pressure gain $\partial P_a/\partial X$ and internal shunt conductance $\partial W_a/\partial P_a$. The equivalent source can be determined from the curves in Fig. 13-152. The change in flow dW_a can be written as

$$dW_a = \frac{\partial W_a}{\partial X} dX + \frac{\partial W_a}{\partial P_a} dP_a$$

and, with $dW_a = 0$, the pressure gain is

$$\frac{\partial P_a}{\partial X} = - \frac{\frac{\partial W_a}{\partial X}}{\frac{\partial W_a}{\partial P_a}} \tag{13-214}$$

From Fig. 13-152, one can determine

$$\frac{\partial(W_a/W_q)}{\partial(X/U)} \text{ and } \frac{\partial(W_a/W_q)}{\partial(P_a/P_s)}.$$

These quantities can be dimensioned by multiplying by W_q/U and W_q/P_s, respectively, to obtain $\partial W_a/\partial X$ and internal shunt conductance $\partial W_a/\partial P_a$. Pressure gain $\partial P_a/\partial X$ can then be obtained by substitution into Eq. (13-214).

13-7.7 Plate-Valve Static Characteristics

By determining $\Delta(W_a/W_q)$ versus $\Delta(X/U)$ along lines of constant (P_a/P_s) in Fig. 13-152, it is observed over a considerable portion of the figure that $\frac{\partial(W_a/W_q)}{\partial(X/U)} \approx 1$. In this region, therefore, an approximate value of $\partial W_a/\partial X$ is

$$\frac{\partial W_a}{\partial X} \approx \frac{W_q}{U} \qquad (13\text{-}215)$$

For the plate valve, the quiescent-leakage flow W_q is

$$W_q = \frac{1.08 C_{ds} w_s U P_s}{\sqrt{T_s}} f_1 \left(\frac{P_{aq}}{P_s}\right) \qquad (13\text{-}216)$$

where w_s is the width of the supply port, in inches, and all other quantities are as previously defined. Since $P_{aq}/P_s \approx 1/2$ (for $P_s >> P_e$) and $f_1(1/2) = 1$ (see Fig. 13-151), the ratio of W_q/U becomes

$$\frac{W_q}{U} = 1.08 C_{ds} w_s \frac{P_s}{\sqrt{T_s}} \qquad (13\text{-}217)$$

A reasonable value for C_{ds} is approximately 0.85.[52] At T_s = 540° Rankine (60°F) and C_{ds} = 0.85, Eq. (13-217) reduces to

$$\frac{W_q}{U} = \frac{(1.08)(0.85)}{\sqrt{520}} w_s P_s = 0.04 w_s P_s \qquad (13\text{-}218)$$

Since $\partial W_a/\partial X \approx W_q/U$, an approximate value for $\partial W_a/\partial X$ (for $P_s >> P_e$, C_{ds} = 0.85, T_s = 540° Rankine) is

$$\frac{\partial W_a}{\partial X} \approx 0.04 w_s P_s \qquad (13\text{-}219)$$

Analysis of Fig. 13-152 shows that the value of $\frac{\partial(W_a/W_q)}{\partial(P_a/P_s)}$ varies more widely than that of $\frac{\partial(W_a/W_q)}{\partial(X/U)}$. However, in a portion of the region bounded by $-0.5 < X/U < 0.5$, $\frac{\partial(W_a/W_q)}{\partial(P_a/P_s)} \approx 2$. In this region, therefore, and approximate value for plate-valve amplifier internal conductance $\partial W_a/\partial P_a$ is

$$\frac{\partial W_a}{\partial P_a} \approx 2\frac{W_q}{P_s} = 2\left(\frac{1.08 C_{ds} w_s U}{\sqrt{T_s}}\right) \qquad (13\text{-}220)$$

For C_{ds} = 0.85 and T_s = 540° Rankine, Eq. (13-220) reduces to

$$\frac{\partial W_a}{\partial P_a} \approx 0.08 w_s U \qquad (13\text{-}221)$$

The equation for plate-valve pressure gain $\partial P_a/\partial X$ is

$$\frac{\partial P_a}{\partial X} = -\frac{\dfrac{\partial W_a}{\partial X}}{\dfrac{\partial W_a}{\partial P_a}} \qquad (13\text{-}222)$$

Substitution of Eqs. (13-219) and (13-221) into Eq. (13-222) yields an approximate value for $\partial P_a/\partial X$ as

$$\frac{\partial P_a}{\partial X} \approx \frac{P_s}{2U} \qquad (13\text{-}223)$$

It is emphasized that Eqs. (13-219), (13-221), and (13-223) are rough approximations. They should be used only as guide markers or sign posts so that a rough appraisal of the valve's internal conductance and gain can be made rapidly from design data; i.e., width of supply slot opening w_s, underlap U, and supply pressure P_s.

Since the compressibility of air varies as $1/P$ and since $\partial W/\partial P$ [see Eq. (13-221)] is not a function of P, the valve's time constant [see Eq. (13-240)] is inversely proportional to P. Bandwidth, therefore, is proportional to P. Gain also is proportional to P [see Eq. (13-223)]. Consequently, *gain-bandwidth* product, a Figure of Merit, is proportional to ***pressure** squared*. This clearly indicates the desirability of using high pressures.

13-7.8 Conical-Plug Valve Static Characteristics

The nondimensional parameters that describe the equivalent source for the conical-plug valve, calculated from the valve equations, are presented in Figs. **13-153** and **13-154**. The calculations are for the case of $W_a = 0$ and $C_3C_{ds}/C_{de} = 1$. With $2w_s = \pi d_s \cos \theta_s$, where d_s is the diameter of the supply port (in in.) and θ_s is the measure of supply-plug taper (in deg) (see Fig. **13-149**), curves such as those in Fig. **13-152** can be adapted to the conical-plug valve. The approximate equations for this valve (for $P_s >> P_a$) are

$$W_q = \left(\frac{C_kC_{ds}}{\sqrt{T_s}}\right) \pi d_s \cos \theta_s P_s U \tag{13-224}$$

$$\frac{\partial W_a}{\partial X} = \frac{W_q}{U} = 0.02\pi d_s \cos \theta_s P_s \tag{13-225}$$

$$\frac{\partial W_a}{\partial P_a} = \frac{2W_q}{P_s} = 0.04\pi d_s \cos \theta_s U \tag{13-226}$$

$$\frac{\partial P_a}{\partial X} = \frac{P_s}{2U} \tag{13-227}$$

13-7.9 Nozzle-Baffle Valve Static Characteristics

The nondimensional parameters that describe the nozzle-baffle valve equivalent source are presented in Figs. **13-155** and **13-156**. The internal conductance and the

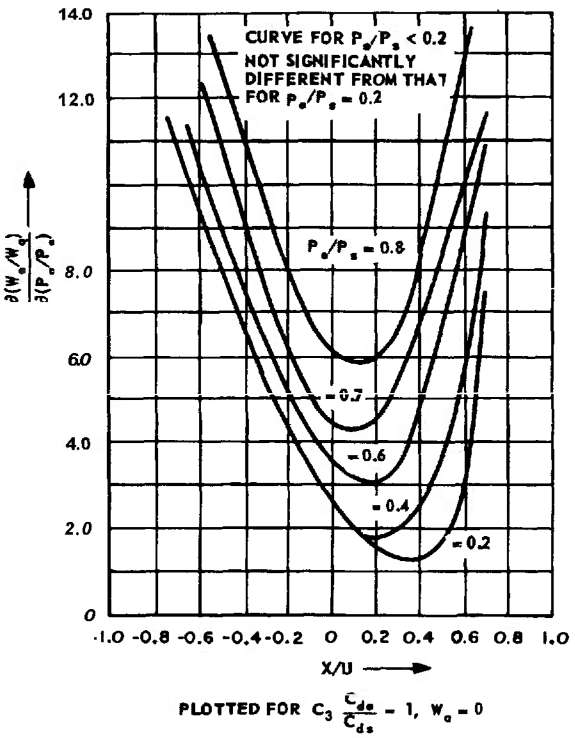


Fig. 13-153 Nondimensional plot of internal shunt conductance versus valve displacement for conical-plug valve.

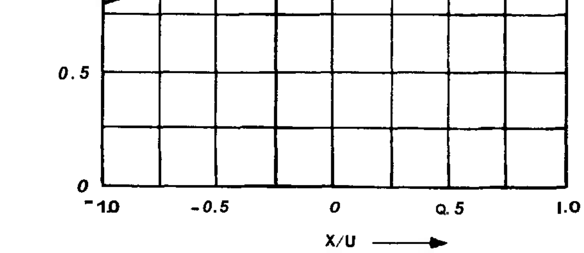


Fig. 13-154 Nondimensional plot of flow gain versus valve displacement for conical-plug valve.

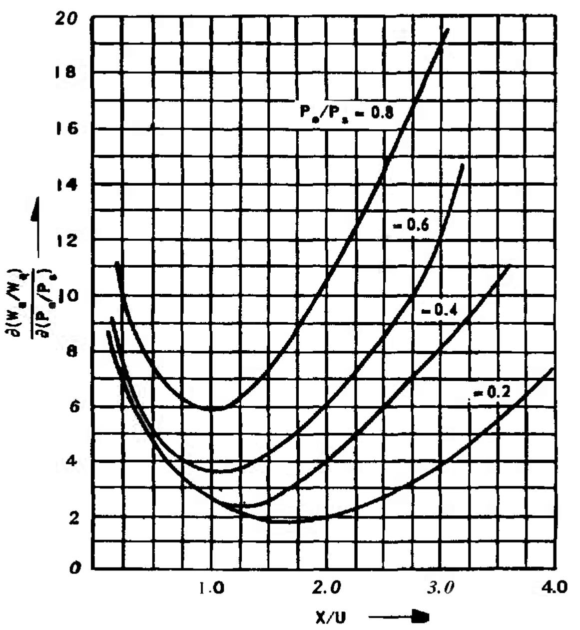


Fig. 73-755 Nondimensional plot of internal shunt conductance versus baffle opening for nozzle-baffle valve.

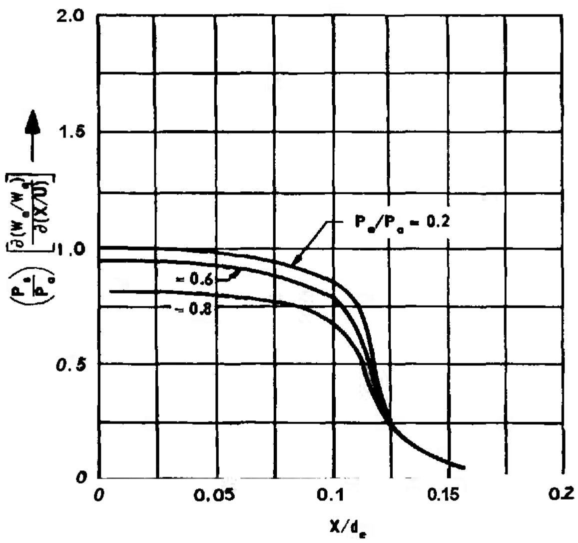


Fig. 73-756 Nondimensional plot of gain versus baffle opening for nozzle-buffle valve.

pressure gain can be determined as follows:

(a) Internal conductance $\partial W_a/\partial P_a$. Dimension $\frac{\partial (W_a/W_q)}{\partial (P_a/P_s)}$ from the curves in Fig. 13-155 by multiplying by W_q/P_s to obtain $\partial W_a/\partial P_a$. W_q/P_s is expressed in terms of design parameters as

$$\frac{W_q}{P_s} = \frac{C_k C_{ds} A_s}{\sqrt{T_s}} \tag{13-228}$$

where A, is the cross-sectional area of the supply orifice, in inches^2, and all other quantities are as previously defined.

(b) Pressure gain $\partial P_a/\partial X$. Dimension $(P_s/P_a)\left[\frac{\partial (W_a/W_q)}{\partial (X/U)}\right]$ from the curves in Fig. 13-156 by multiplying by (W_q/U) (P_a/P_s) to obtain $\partial W_a/\partial X$; then divide $\partial W_a/\partial X$ by $\partial W_a/\partial P_a$ to obtain $\partial P_a/\partial X$. Use the following dimensioning quantities :

(1) $W_,$, as determined from Eq. **(13-228).**

(2) U, corresponding in this case to the orifice opening at which $P_{aq} = 0.5(P_s - P_e)$, $W_a = 0$, given by

$$U = \frac{C_{ds} A_s}{\pi C_{de} d_e} \tag{13-229}$$

where d_e is the diameter of the exhaust orifice, in inches, and all other quantities are as previously defined.

(3) P_a/P_s, taken from a no-load pressure versus opening curve such as that shown in Fig. **13-157.** In general, such a curve is available from the valve manufacturer or can be determined readily by measurement. Alternatively, it can be computed from the basic equations of orifice flow.

The ratio P_a/P_s is actually a function of load flow. The results found using the preceding method, therefore, hold only for relatively small values of load w around an average value of zero. The only practical alternative procedures are to calculate the pressure-flow product versus opening curves from the orifice equations or to acquire

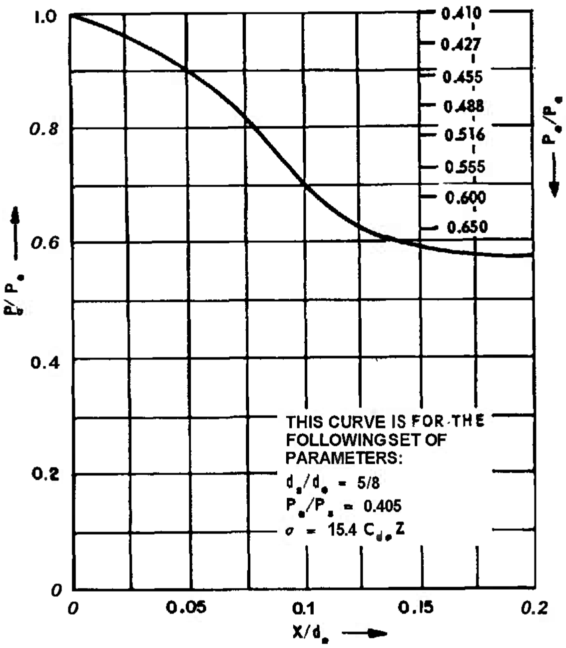


Fig. 13-157 Nondimensional plot of load pressure versus baffle opening for nozzle-baffle valve.

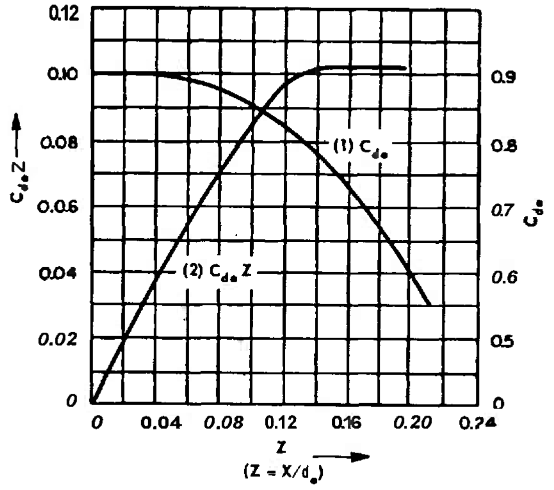


Fig. 13-158 Plots of (1) discharge coefficient versus baffle opening; and (2) discharge coefficient-buffle opening product versus baffle opening for typical exhaust nozzle in nozzle-buffle valve.

similar data from measurements. Since the internal shunt conductance varies over a wide range (see Fig. 13-155), use of the preceding method of approximating the dimensionalized pressure gain appears justified.

The designer of pneumatic servos may be required to design the component pneumatic amplifiers. It is useful to be able to compute curves, such as those given in Fig. 13-157, that establish the pressure gain $\partial P_a/\partial X$. The following method can be used:

(a) Obtain a plot of the measured discharge coefficient C_{de} of the exhaust nozzle to be used. If unavailable, use the plot given in Fig. 13-158.

(b) Multiply C_{de} by $X/d_e = Z$ to obtain $C_{de}Z$, also plotted in Fig. 13-158. It is sufficient to take $0 < Z < 1.0$.

(c) Choose values of d_e and d_s. In order to choose d_e, it is helpful to have an approximate expression for pressure gain $\partial P_a/\partial X$. This approximation is given by

$$\frac{\partial P_a}{\partial X} \approx \frac{P_s - P_e}{2U} = \left(\frac{P_s - P_e}{2}\right)\left(\frac{\pi C_{de}\, d_e}{C_{ds} A_s}\right)$$

$$\approx 2\,\frac{d_e}{d_s^2}\,(P_s - P_c) \qquad (13\text{-}230)$$

If there is no other criterion, let $d_e = d_s$. Then, $\frac{\partial P_a}{\partial X} \approx \frac{2(P_s - P_e)}{d_s}$ and, for a given set of supply and exhaust pressures, choose d_s and thence d_e to achieve the approximate pressure gain desired.

(d) Select a value of X and determine $Z = X/d_e$. The useful range of Z is approximately $0 < Z < 0.1$.

(e) Find $C_{de}Z$ (from plot) for this value of Z.

(f) Compute $X/U = \sigma$ from the relation

$$\sigma = \left(\frac{4d_e^2}{C_{ds}d_s^2}\right) C_{de} Z \qquad (13\text{-}231)$$

To compute σ, use the values of d_e and d_s chosen in step (c) and use the value of $C_{de} Z$ determined in step (e). Use $C_{ds} = 0.85$ if no contrary data is available.

(g) Compute $\lambda = P_e/P_s$ (use absolute values of pressure).

(h) Using the computed values of λ and σ, find P_a/P_s frcm Fig. **13-159** for the value of X assumed in step (d).

(i) Repeat for other values of X.

(j) Plot the curve of P_a/P_s versus X/d_e.

13-7.10 DYNAMIC BEHAVIOR OF PNEUMATIC AMPLIFIERS

The dynamic properties of pneumatic amplifiers (valve and motor combinations) are analyzed most readily by use of an equivalent-circuit representation (see Fig. **13-160**). The methods of determining an equivalent-source representation for a three-way valve have been presented in the preceding material of this section. The remaining elements required to complete the equivalent circuit are defined and equations for determining their values are presented herein. Dynamic behavior of four-way and three-way pneumatic amplifiers is also discussed.

13-7.11 Equivalent-Circuit Elements for Pneumatic Systems

The equivalent-circuit elements for a pneumatic system consist of resistance R, capacitance C, and inertance Z. These elements represent flow coefficients in the pneumatic system and are defined such that

$$\left.\begin{aligned} &\text{leakage flow } W_L = \mathrm{LP} = \frac{P}{R_L} \\ &\text{compressibility flow } W_C = C\,\frac{dP}{dt} \\ &\text{inertance flow } W_I = \frac{\int P\,dt}{Z} \end{aligned}\right\} \tag{13-232}$$

13-7.12 Resistance. Resistance elements fall into two classes—linear and nonlinear.

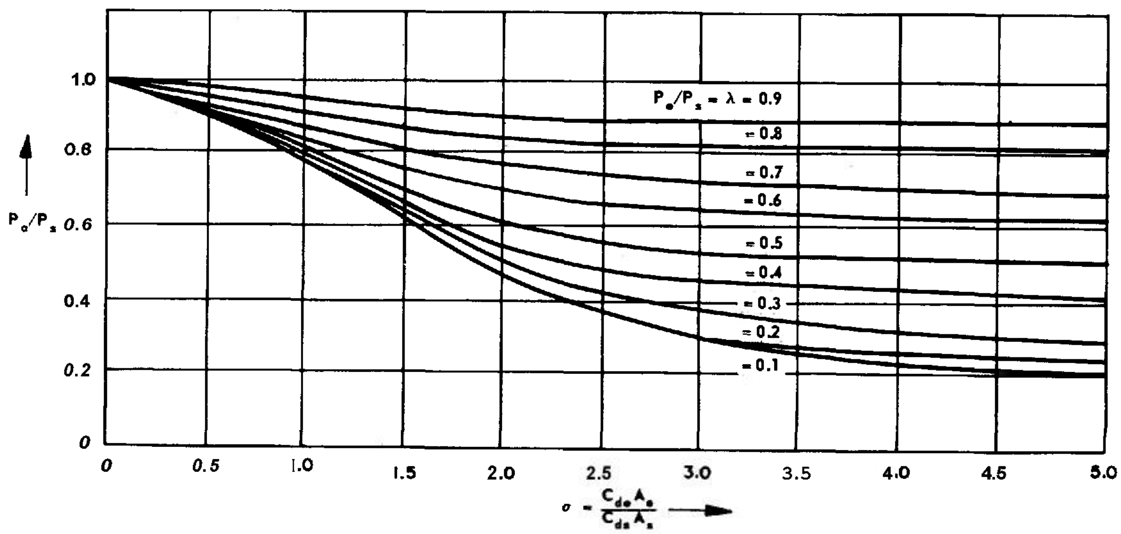


Fig. 13-159 Plot of pressure ratio versus exhaust-to-supply orifice area ratio for nozzle-baffle valve.

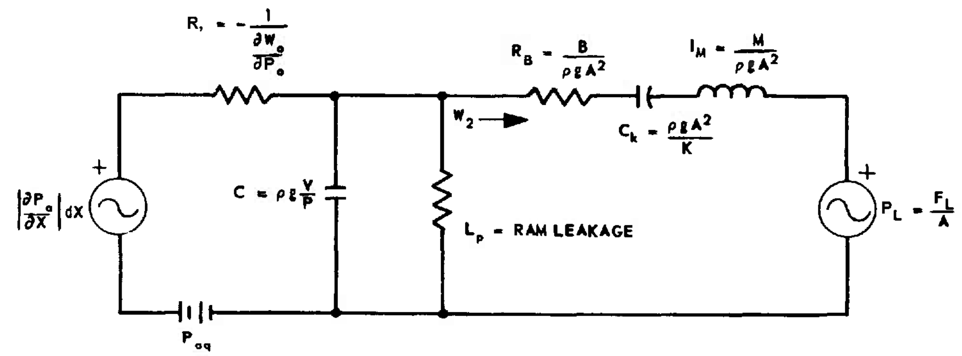


Fig. 13-160 Equivalent-circuit representation for pneumatic amplifier (three-way valve) with spring-of.posed ram load.

Linear resistance is typified by capillaries and by parallel plates spaced close together. The value of resistance R_{cap} for capillary flow[53] is given by the Hagan-Poiseiulle formula, applicable for Reynolds' numbers less than **2000,** as

$$R_{cap} = \frac{128\nu l}{\pi g d^4} = \frac{128\mu l}{\pi g \rho d^4} \tag{13-233}$$

where

R, = resistance **of** capillary, in sec/in.^{2}

ν = kinematic viscosity = 2.28 $\times$ 10^{-2} in.^{2}/sec, for air at 60°F and 14.7 psia

l = length of capillary, in in.

g = acceleration due to gravity = 386 in./sec^2

d = diameter of capillary, in in.

μ = air viscosity = 2.5 $\times$ 10^{-9} $\frac{(C^\circ + 273)^{3/4}}{273}$ sec/in.^{2}

ρ = mass density of air, in lb-sec^2/in.^{4}

The minimum length of tubing l_{min} for linear flow[53] (Reynolds' numbers < 2000) can be found from

$$\frac{\Delta P}{l_{min}} = 6.4 \times 10^4 \frac{\mu^2 R_o T_R g}{P_{av} d^3} \tag{13-234}$$

where

l_{min} = minimum length of tubing for linear flow, in in.

AP = pressure drop across tube, in psi

R_o = gas constant = **663** in./°Rankine for air

P_{av} = average pressure, in psia

T_R = temperature, in °Rankine

Calculated values of l_{min} for tubing of various diameters are listed in Table **13-23.** These values were determined for the case of ΔP = **15** psi and P,, = 25 psia.

Resistance for parallel-plate flow R_{par} is given by

$$R_{par} = \frac{12\mu l}{g\rho b h^3} \tag{13-235}$$

where

R_{par} = resistance of parallel plates, in sec/in.^{2}

l = length of plate parallel to flow, in in.

TABLE 13-23 MINIMUM TUBING LENGTHS FOR LINEAR FLOW

Tubing Diameter (in in. $\times 10^{-3}$)	Minimum Length (in in.)
1	0.006
2	0.05
3	0.17
4	0.4
5	0.8
8	3.1
10	6.3
16	26

b = width of plate perpendicular to flow, in in.

h = distance between plates, in in.

The equivalent resistance of an orifice R_{orf} is a nonlinear quantity, given by

$$R_{orf} = \frac{1}{\dfrac{\partial W}{\partial P}} \tag{13-236}$$

where

R_{orf} = equivalent orifice resistance, in sec/in.^{2}

$P = P_u - P_d$

and W, P_u, and P_d are as defined in Eq. **(13-210).** W can be determined from Eq. **(13-212)** for various values of P, using values of $f_1(P_d/P_u)$ obtained from Fig. **13-213** or from the relationship

$$f_1\left(\frac{P_d}{P_u}\right) = 3.81\left(\frac{P_d}{P_u}\right)^{0.714}\sqrt{1-\left(\frac{P_d}{P_u}\right)^{0.286}}$$

13-7.13 **Capacitance.** The capacitance of a volume is a measure of the storage property of the volume. The value of capacitance of a given volume is dependent upon whether the thermodynamic change within the volume is isothermal or adiabatic. The capacitance of a volume for isothermal changes is

$$C_I = \frac{V_{av}}{R_o T_{R_{av}}} = \frac{\rho g\, V_{av}}{P_{av}} \tag{13-237}$$

where

C_I = capacitance for isothermal changes, in in.^{5}/lb

V_{av} = average volume of air under pressure, in in.^{3}

R_o = 663 in./°Rankine for air

$T_{R_{av}}$ = average temperature of volume, in "Rankine

ρg = weight density = $3 \times 10^{-6} P_{av}$ lb/in.3 for air at 60°F

P_{av} = average pressure of volume, in psia

The capacitance C_A of a volume for adiabatic changes is

$$C_A = \frac{V_{av}}{K_s R_o T_{R_{av}}} = \frac{\rho g\, V_{av}}{K_s P_{av}} \tag{13-238}$$

where

C_A = capacitance for adiabatic changes, in in.^{5}/lb

K, = ratio of specific heats $(C_p/C_v) = 1.4$ for air

13-7.14 **Inertance.** Inertance is a measure of the pressure required to accelerate the mass of a fluid. The inertance Z of a length of air-filled tubing is

$$I = \frac{l}{Ag} \tag{13-239}$$

where

I = inertance of length of air-filled tubing, in sec^2/in.^{2}

I = length of tubing, in in.

A = cross-sectional area of tubing, in in.^{2}

g = 386 in./sec^2

13-7.15 Time constant. The time constant τ_{RC} of a resistance R and a capacitance C in series is the product RC. For a capillary and a volume, τ_{RC} is the product of the resistance R_{cap} of the capillary and the capacitance C of the volume. For a volume experiencing an isothermal change, τ_{RC} is given by

$$\tau_{RC} = R_{cap}\, C_I = \left(\frac{128\mu l}{\pi g \rho d^4}\right)\left(\frac{\rho g\, V_{av}}{P_{av}}\right)$$

$$= \frac{128\mu l\, V_{av}}{\pi d^4\, P_{av}} \qquad (13\text{-}240)$$

13-7.16 Pneumatic Equivalents for Dashpots, Springs, and Masses

The equivalent-circuit representation of a pneumatic amplifier must take account of dashpots, springs, and masses (mechanical elements) of the load. The equations used to account for these mechanical elements are:

$$\text{(a)}\quad R_B = \frac{B}{\rho g\, A^2} \qquad (13\text{-}241)$$

where

R_B = pneumatic resistance to account for a dashpot, in sec/in.^{2}

B = damping constant of dashpot, in lb-sec/in.^{2}

A = $\partial F/\partial P$ = effective area of load element, in in.^{2}

For a piston, $A = \dfrac{F}{P}$

$$= \frac{\text{force exerted on piston}}{\text{pressure exerted on piston}},$$

$$\text{(b)}\quad C_K = \frac{\rho g\, A^2}{K} \qquad (13\text{-}242)$$

where

C_K = pneumatic capacitance to account for a spring, in in.^{5}/lb

K = spring constant, in lb/in.

$$\text{(c)}\quad I_M = \frac{M}{\rho g\, A^2} \qquad (13\text{-}243)$$

where

I_M = pneumatic inertance to account for a mass, in sec^2/in.^{2}

M = mass, in lb-sec^2/in.

13-7.17 Dynamic Behavior of Four-Way Valves

Methods of determining the equivalent source for a three-way valve were presented previously. Two such sources account for the behavior of the four-way valve, as in the case discussed for hydraulic valves in Par. **13-6.** Hydraulic four-way valve equations are found in Table **13-16.** By introducing the definitions of Eqs. (13-232), (13-241), (13-242), and (13-243), the hydraulic equations can be used for predicting the dynamic behavior of pneumatic four-way valves. It should be noted that the definitions of R_B, C_K, and I_M for hydraulic circuits (see Table **13-16**) differ

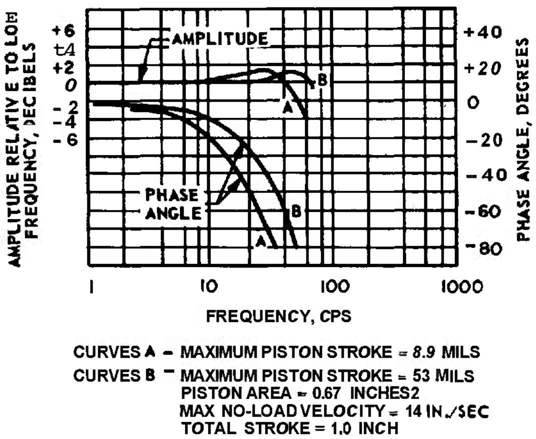


Fig. 13-161 Frequency-response curve for a four-way pneumatic servomechanism with 1000 psi supply pressure (plate-valve and piston combination with position feedback by electrical means).

Adapted by permission from *Proceedings of the National Conference on Industrial Hydraulics,* Volume VIII. 1954, from article entitled "Development of Valves for the Control of Pneumatic Power", by S. Y. Lee and J. L. Shearer.

from those for pneumatic circuits by the factor ρg. For air at 60°F, $\rho g = 3 \times 10^{-6}$ P lb/in.^{3}, where P is taken as the average pressure P_{av} about which the linear model is developed.

An example of the dynamic behavior of a servo using a four-way valve is cited by Lee and Shearer.(54) The servo consists of an electronic amplifier, a force motor, a four-way valve, and a piston. It employs position feedback from the piston to the input of the electronic amplifier by means of an electrical pickoff. The dynamic behavior of this servo is summarized in terms of the servo frequency response shown in Fig. 13-161.

At present, the supply of standard pneumatic equipment capable of high performance in servos operating at high-pressure levels is limited. The example cited, therefore, is illustrative of possible performance attainable in equipment carefully designed and built to order. It is not representative of performance to be expected from standard industrial designs.

13-7.18 Dynamic Behavior of Three-way Valves

The source and load arrangement for a three-way valve with spring-opposed ram is shown in Fig. 13-160. The line and ram capacitance have been combined. The inertance of air, usually negligible, is excluded.

Ram velocity can be found from the mechanical displacement flow W_2 by dividing W_2 by $\rho g A$, where A is the effective area of the ram. The transfer function relating ram displacement to valve (input) displacement can be readily obtained from Fig. 13-160 by writing the circuit equations and solving for the transfer function.

The origin or reference point for valve input displacement is taken as the valve-centered position of the input member for the plate and conical-plug valves and as the orifice opening U for the nozzle-baffle valve. With the valve input member at the origin and with no flow to the load, the cylinder pressure is the quiescent pressure P_{aq}. The input (baffle) motion of the nozzle-baffle valve is denoted by the symbol ξ. The valve opening X for the nozzle-baffle valve is related to displacement from the quiescent point by

$$X = U - \xi \tag{13-244}$$

By choosing ξ in this sense, a positive ξ causes a positive load-pressure increment similar to the case for the other-way valves.

The transfer function relating ram displacement to valve displacement is

$$Y_{out}(s) = \frac{A}{K}\left[\frac{\left(\frac{\partial P_a}{\partial X}\right) Y_{in}(s) - (R'_s Cs + 1) P_{Lf}(s)}{1 + a_1 s + a_2 s^2 + a_3 s^3}\right] \tag{13-245}$$

where

$Y_{out}(s)$ = transform of fluctuation of ram displacement around the average displacement Y_{ss} [see Eqs. **(13-246)** and **(13-247)**]

A = effective area of ram, in in.^{2}

$\frac{\partial P_a}{\partial X}$ = valve pressure gain, in lb/in.^{3}

$Y_{in}(s)$ = transform of valve input displacement from quiescent (transform of X for plate and conical-plug valves, transform of ξ for nozzle-baffle valve)

R'_s $= \frac{1}{G'_s} = \frac{1}{L_P - \frac{1}{\frac{\partial W_a}{\partial P_a}}}$

L_P = ram leakage, in in.^{2}/sec

$\frac{\partial W_a}{\partial P_a}$ = valve internal conductance, in in.^{2}/sec

C = capacitance of system average volume, in in.^{5}/lb

s = transform operator

$P_{Lf}(s)$ = transform of $\frac{F_{Lf}}{A}$ = transform of $\frac{(F_L - F_{L_{av}})}{A}$

F_{Lf} = fluctuating component of load force, in lb

F_L = total load force, in lb

$F_{L_{av}}$ = average value of load force, in lb

K = spring constant of opposing spring, in lb/in.

a_1 $= R'_s C_K + R'_s C + R_B C_K$

a_2 $= R_B R'_s C\, C_K + I_M C_K$

a_3 $= I_M C_K R'_s C$

C_K $= \frac{\rho g A^2}{K}$, in in.^{5}/lb

R_B $= \frac{B}{\rho g A^2}$, in sec/in.^{2}

B = load damping constant, in lb-sec/in.

I_M $= \frac{M}{\rho g A^2}$, in sec^2/in.^{2}

M = load mass, in lb-sec^2/in.

The average displacement Y_{ss}, in inches, is determined from either of the following relations:

(a) For the case of an unstressed spring at $Y = 0$

$$Y_{ss} = \frac{P_{aq} - \frac{F_{L_{av}}}{A}}{K} \tag{13-246}$$

(b) For the case of initial spring compression of F_{KO} pounds at $Y = 0$

$$Y_{ss} = \frac{P_{aq} - \frac{F_{L_{av}} + F_{KO}}{A}}{K} \tag{13-247}$$

If $Y(t) = \mathcal{L}^{-1}[Y_{out}(s)]$, then the total ram displacement Y, in inches, is

$$Y = Y_{ss} + Y(t) \tag{13-248}$$

13-7.19 Typical Performance of Low-Pressure Three-way Valves

The performance of a low-pressure pneumatic amplifier consisting of a conical-plug valve and a diaphragm valve (motor) was investigated. The diaphragm valve had a volume $V$of **46 in.,^{3}** an effective area A of **63.6 in.,^{2}** and an average pressure P_{av} of **29.5** psia. The opposing spring had a constant K of **1000** lb/in. Ram leakage L_P and the fluctuating component of load force F, were zero and the mass M can be neglected

For the above amplifier, **Eq. (13-245)** becomes

$$Y_{out}(s) = \frac{\frac{A}{K}\frac{\partial P_a}{\partial X} Y_{in}(s)}{\tau s + 1}$$

where

$$\tau = R_s \left(\frac{V}{P_{av}} + \frac{A^2}{K}\right) \rho g$$

$$= R_s \left(\frac{46}{29.5} + \frac{63.6^2}{1000}\right) (3 \times 10^{-6} \times 29.5)$$

$$= R_s\ (5.63)(8.85 \times 10^{-5})$$

$$R_s = \frac{1}{G_s} = \frac{1}{\dfrac{\partial(W_a/W_q)}{\partial(P_a/P_s)} \times \dfrac{W_q}{P_s}}$$

By substituting the value of W_q/P_s from Eq. (13-224), it follows that

$$R_s = \frac{1}{\dfrac{\partial(W_a/W_q)}{\partial(P_a/P_s)} \times \dfrac{C_k C_{ds}\, \pi\, d_s \cos\theta_s\, U}{\sqrt{T_s}}}$$

R_s is determined from the following values for constants, coefficients, and parameters :

$$\left[\frac{\partial(W_a/W_q)}{\partial(P_a/P_s)}\right]_{\left(\frac{X}{U} = 0, \frac{P_e}{P_s} = 0.4\right)} = 2.6$$

(from Fig. 13-153)

$C_k = 0.54$

$C_{ds} = 0.67$

$d_s = 0.135$ in.

$\theta_s = 60"$

$U = 0.006$ in.

$T_s = 520°$ Rankine

$\dfrac{P_e}{P_s} = 0.4$

Substitution of the above values into the expression for R, gives

$$R_s = \frac{\sqrt{520}}{(2.6)\ (0.54)\ (0.67)(\pi)\ (0.135)\ (0.5)\ (0.006)} = 1.9 \times 10^4$$

Whence

$$\tau = (1.9 \times 10^4)\ (5.63)\ (8.85 \times 10^{-5}) = 9.5 \text{ sec}$$

The measured value of τ was 5.6 seconds. Calculated values of τ for other values of X/U are given in Table 13-24.

The preceding amplifier was used with position feedback and a loop gain of 80. The resultant closed-loop response was predominantly first order with a break frequency of about 2 cps. Comparing this servo performance using a supply pressure of approximately 37 psia with that reported by Lee and Shearer (Fig. 13-161) of 60 cps at a supply pressure of 1000 psi, one obtains the following :

$$\text{bandwidth ratio} = \frac{60}{2} = 30$$

$$\text{pressure ratio} = \frac{1000}{37} \approx 27$$

The contention that bandwidth is proportional to pressure (see end of Par. 13-7.7) appears to be borne out in practice.

TABLE 13-24 CALCULATED TIME CONSTANTS FOR A THREE-WAY CONICAL-PLUG VALVE

$\dfrac{X}{U}$	Time Constant (in sec)
−0.6	2.4
−0.2	5.4
0	9.5
+0.2	11.0
+0.6	2.11

13-7.20 ADVANTAGES AND DISADVANTAGES OF PNEUMATIC SYSTEMS

The principal advantages and disadvantages of pneumatic systems in relation to systems employing other media are given below.

13-7.21 Advantages

(a) The control fluid can be released to the atmosphere at any convenient point.

(b) Spilled fluid causes no fire hazard and no housekeeping problem.

(c) The variation of viscosity with temperature is less for air than for liquids, allowing air to be used over wider temperature ranges.

(d) A large energy storage capacity is feasible in the form of accumulators or tanks. In rocket work, the same gas that is used to fuel the missile might also be used to control it.

(e) Air is available almost universally (high altitude and underwater environments are notable exceptions).

13-7.22 Disadvantages

(a) Filtering to remove water, oil, dirt, and metal particles is an absolute necessity.

(1) Moisture in air at low temperatures cannot be tolerated because freezing would prevent servo operation. Even at ambient temperatures above freezing, air must be water-free.

(2) Oil, required for lubrication of compressors, might form an explosive mixture if entrained in air.

(3) Dirt and oil can clog small passages.

(b) Air is not a lubricant of much worth; friction forces are troublesome at valve stems, at piston walls, and between moving parts of control valves. If clearance is large enough to avoid friction, excessive leakage occurs.[54]

(c) The speed of response of a pneumatic servo is inherently less than that of a hydraulic servo. The slower response of the pneumatic servo results from the high compressibility constant of air, since a servo's undamped natural frequency ω_n varies inversely with the square root of the compressibility constant. For a cylinder of given size, stroke, and force capability, the compressibility constant of air is between V/P and $V/1.4P$ and that of oil is V/B_m, where P is the air pressure in cylinder volume V and B_m is the bulk modulus of the oil. A reasonable value of P might be 2500 psi, whereas the value of B_m might be 250,000 psi. Air entrainment in oil might, in an extreme case, reduce B_m to 25,000 psi. For the above values of P and B_m, the ratio of the compressibility constant of air to that of oil can be expected to lie within the limits of *100* and *10.* Therefore, whereas a 400-cps hydraulic system is possible today, a *40-* to 80-cps pneumatic system is about the best that can be expected.

(d) Pneumatic systems, as compared with electrical systems, require additional auxiliaries such as filters, compressors, and relief valves.

(e) Transmission of pneumatic signals over an appreciable distance results in a slow speed of response as compared with electrical transmission.

13-8 MECHANICAL AMPLIFIERS*

13-8.1 BASIC TYPES

There are two basic types of mechanical amplifiers. One type produces a variable-speed output **from a constant-speed** source. The second type produces a variable-torque output from a constant-power source.

By P. E. Smith, Jr.

13-8.2 Variable-Speed-Output Mechanical Amplifier

Variable-speed gears can be classified as variable-speed-output mechanical amplifiers since, by displacement of a control member, they convert a constant-speed input into an output at a higher **or** lower speed. Two examples are shown in Figs. **13-162** and **13-163.** Displacement of the control member requires a driver **or** preamplifier such as a solenoid,

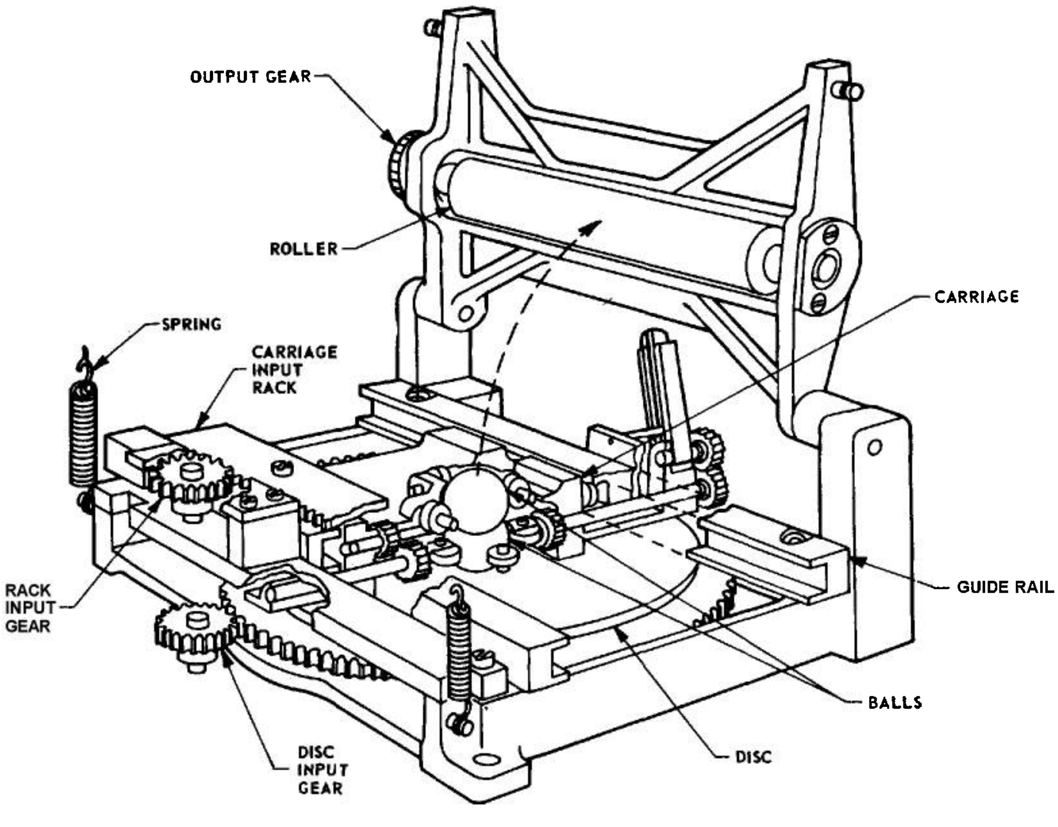


Fig. 13-162 Ball-disc integrator.

Courtesy Ford Instrument Company.

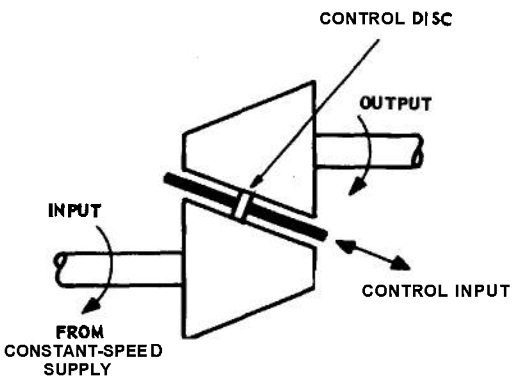


Fig. 13-163 Cone-and-disc amplifier.

By permission from *The Theory of Control in Mechanical Engineering*, by R. H. Macmillan, Cambridge University Press, New York, N. Y.

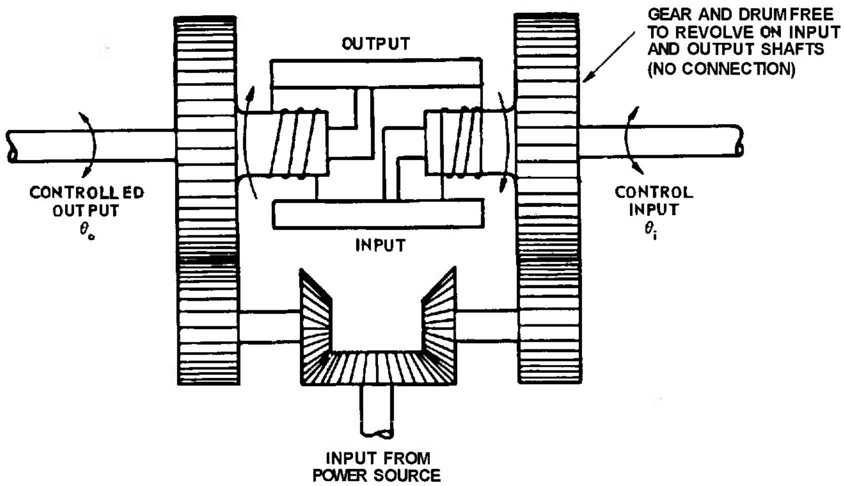
a low-power servomotor, or a force motor. Since almost all servomechanisms utilize electrical error signals, a preamplifier must be designed to accept such signals. In rare cases where a mechanical error signal is used instead of an electrical signal, a series of mechanical amplifiers may prove advantageous from the standpoint of weight and space limitations.

13-8.3 Ball-disc integrator. Mechanical amplifiers with accurate relationship between input motion and output speed are often used as integrating mechanisms in analog computers. One type of widely used integrator is the ball-disc type shown in Fig. **13-162.** The disc input gear turns another much larger gear upon which is mounted a disc. The disc rotates two steel balls, one of which is mounted on top of the other. The top ball, in turn, rotates a roller at the end of which the output gear is fastened. The balls are held in position between the roller and the disc by a movable carriage, which permits positioning of the balls anywhere along the roller axis from one edge of the disc, through its center, to the opposite edge. With constant input-gear speed, the output gear rotates fastest when the steel balls are positioned at the outer edge of the disc and slowest when the balls are near the disc center. The direction of rotation of the output gear is reversed when the balls are moved from one side of the disc center to the other side. Thus, the carriage position controls the speed and direction of the output gear. The springs on the frame carrying the roller provide the necessary pressure to hold the roller, balls, and disc in continuous contact with each other.

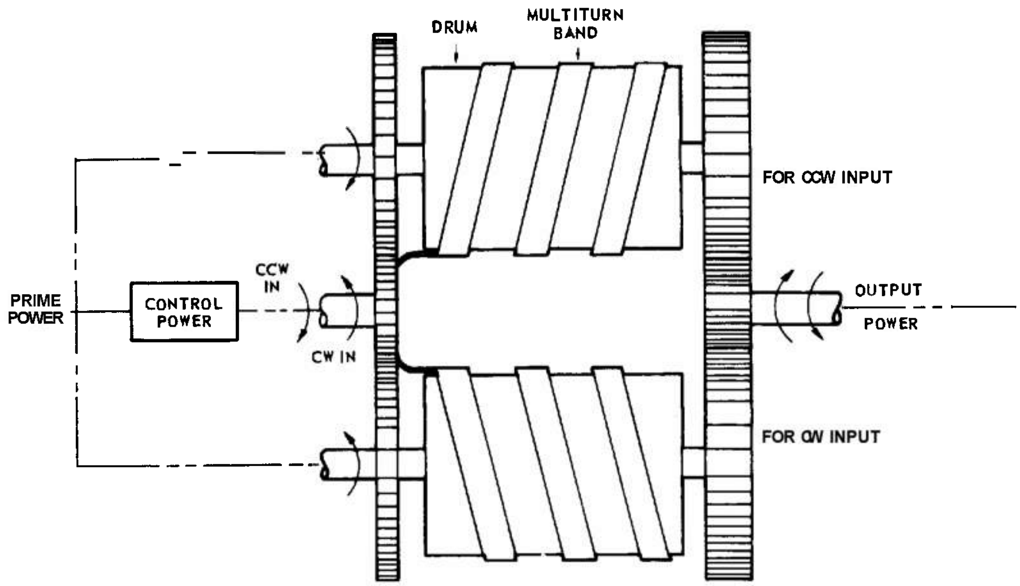
13-8.4 Cone-and-disc amplifier. A somewhat different mechanical amplifier is shown in Fig. **13-163.** Here, two cones are mounted on parallel-axis shafts, with the smaller end of each cone adjacent to the larger end of the other. The cones are separated by a movable control disc that is in pressure contact with both cones. Either shaft can be the constant-speed input, since the device is symmetrical. Output speed is varied by changing the position of the control disc. However, direction of output-shaft rotation cannot be reversed unless the input-shaft rotation is reversed or the device is modified. By contrast, the ball-disc amplifier output can be reversed without reversing the input or modifying the device.

13-8.5 Variable-Torque-Output Mechanical Amplifier

The most widely used form of variable-torque-output mechanical amplifier is based on the principle of the capstan, Two forms of the double capstan amplifier are shown in Fig. **13-164.** Operation is as follows (see Fig. **13-164A)** : The power source causes the bevel gears to rotate. These gears transmit the rotation to the spur gears which rotate freely on the input (θ_i) and output (θ_o) shafts. Attached to the output spur gears are hubs on which the capstan cords are wound. If the input shaft is rotated clockwise (viewed from the right), the right-hand cord wrap-around angle is decreased slightly and the left-hand angle is increased. The result is an increase in the tension in the cord from the left capstan to the output shaft and a decrease in the tension in the right-hand cord. Thus, net torque



A. REVERSIBLE VARIABLE-TORWE-OUTPUT AMPLIFIER



5. REVERSIBLE MECHANICAL POWER AMPLIFIER

Fig. 73-764 Two forms of *double* capstan amplifier.

is produced, tending to rotate the output shaft in the same direction as the left-hand capstan is turning (counterclockwise, viewed from the left side of the amplifier). The torque tends to align θ_o with θ_i; as soon as θ_o is lined up with θ_i, the wrap-around angles of the capstans become equal, and no torque results. For small errors, the torque is proportional to the error angle, $\theta_i - \theta_o$. This particular device is similar to one developed by Bush and described by Crank[57] in which a belt device was used instead of gears.

13-8.6 STATIC CHARACTERISTICS OF MECHANICAL AMPLIFIERS

13-8.7 Ideal and Actual Characteristics

The ideal properties of the speed and torque types of mechanical amplifiers, together with the actual characteristics, are shown in Figs. **13-165** and **13-166**. The characteristic equations are

$$d\Omega = \frac{\partial \Omega}{\partial X} dX + \frac{\partial \Omega}{\partial T} dT \quad \text{(for speed type)} \qquad (13\text{-}249)$$

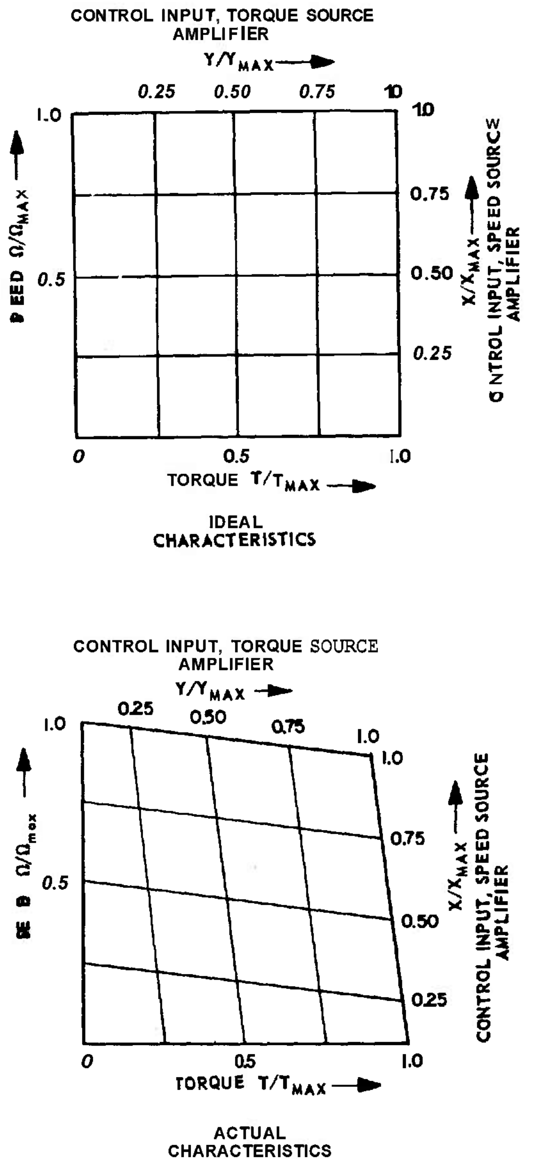


Fig. 13-165 Ideal and actual characteristics of mechanical amplifiers.

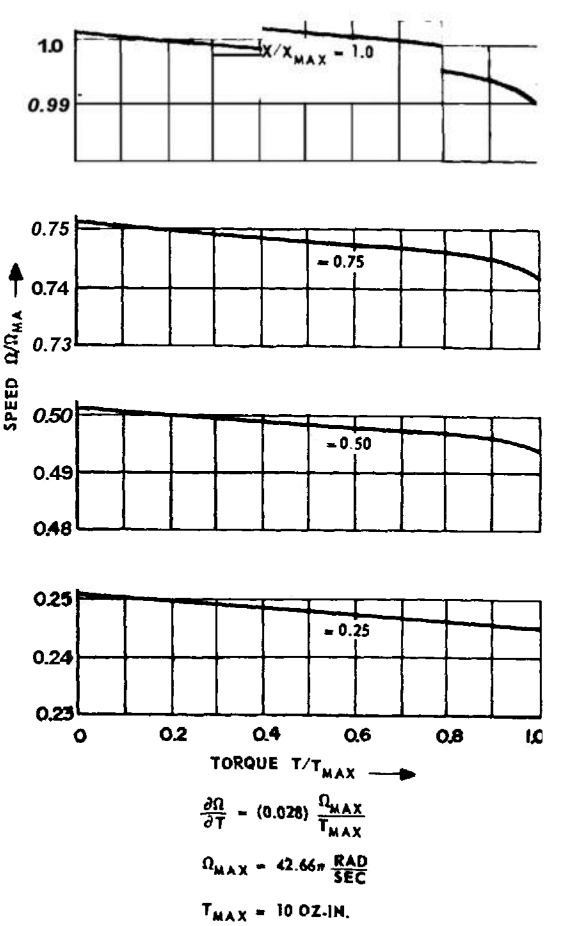


Fig. 13-166 Typical speed-torque characteristics of variable-speed-output mechanical amplifier.

$$dT = \frac{\partial T}{\partial Y} dY + \frac{\partial T}{\partial \Omega} d\Omega \quad \text{(for torque type)} \tag{13-250}$$

where

Ω = speed of output shaft, in radians/sec

T = torque at output shaft, in oz-in.

X, Y = input motion, in in.

The static characteristics of the integrator (ball-disc) and capstan types of amplifiers are discussed in the following paragraphs.

13-8.8 Static Characteristics of Integrator Type Mechanical Amplifiers

Common names for this type of amplifier are ball-disc integrator or VSD (variable-speed drive). A typical set of static characteristics is shown in Fig. **13-166.** The measured values of the constants Ω_{max} and T_{max} are 42.66π radians/sec and **10** oz-inch, respectively. The value of the .amplifier unloaded gain, $\partial\Omega/\partial X$, is **112** radians/inch-sec. The amplifier internal impedance, $\partial\Omega/\partial T$, is **0.375** radian/oz.-in-sec.

The manner in which the amplifier characteristics are related to its dimensions will now be discussed. Figure **13-167** shows a cross section through the cylinder (roller). The force,

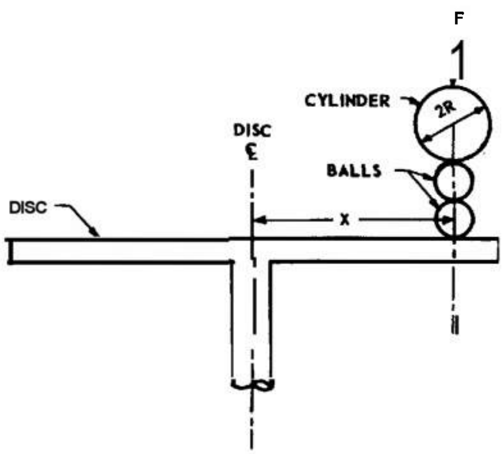


Fig. 73-767 Schematic of working parts of integrator type mechanical amplifier.

F, is the ball (loading) force which holds the cylinder, balls, and disc in mutual contact.

The output speed depends upon the disc speed and the carriage displacement, X, as follows:[60,61]

$$\Omega_o = \frac{X}{R}\omega_D \tag{13-251}$$

where ω_D is the disc speed in radians/sec. From Eq. **(13-251)** it follows that:

$$\frac{\partial \Omega}{\partial X} = \frac{\partial \Omega_o}{\partial X} = \frac{\omega_D}{R} \tag{13-252}$$

The relation betwen speed and load torque is found as follows. Assume that the drive motor speea is unaffected by the load torque required from the VSD. The effect of this loading will be examined later. The percentage slip is defined[61] as follows:

$$S = \frac{\left(\frac{\partial \Omega}{\partial T}\right) dT}{\Omega_{nl}} \times 100 \tag{13-253}$$

where Ω_{nl} = speed at no torque.

The percentage slip[61] has been found, for one type of amplifier, to be:

$$s = 85\,\frac{T_o}{F} \tag{13-254}$$

where

T_o = torque at which slip is measured

F = ball force, in ounces

Substitute Ω_{nl} from Eqs. **(13-251)** and **(13-254)** into Eq. **(13-253)** and let $dT = T_o$. There results

$$\frac{\partial \Omega}{\partial T} = -\frac{s}{100} \times \frac{\Omega_{nl}}{T_o} = -\frac{0.85\omega_D X}{RF} \tag{13-255}$$

The factor of proportionality (0.85) in Eq. (13-255) may not hold for all types and models of integrators; however, the form of the equation is general.

The optimum design of ball-disc integrators has been investigated and **reported.**[61] The following assumptions were made:

(a) The dynamic output torque which will produce one percent slip is equal to 3/8 of the static stall torque at the output. Furthermore, the apparent coefficient of friction of the VSD bearing surfaces does not vary with ball size, lubrication, surface finish, and bearing materials.

(b) The limiting ball force is proportional to the square of the ball size.

(c) The limiting ball force is inversely proportional to the cube root of the expected life of the balls.

(d) The limiting ball force is inversely proportional to the fourth root of the average ball speed.

(e) The weight of the unit is proportional to the cube of the distance between disc face and the center line of the output shaft and cylinder. Assumption (a) is based upon measurements and, in terms of a coefficient of friction, μ, leads to the empirical relation

$$T_r = 3T_s/8 = 3\mu FR/8 \tag{13-256}$$

where

T_r = output torque at 1% slip, in oz-in.

T_s = stall torque, in oz-in.

μ = coefficient of friction

F = ball force, in ounces

R = cylinder radius

Based upon the above assumptions, there results that the cylinder diameter should be twice the ball diameter. Other relationships are:[61]

$$T_r = \frac{3}{8}\mu k d_b^3 \tag{13-257}$$

$$k = k_o f_L f_s \tag{13-258}$$

$$f_L = (3000/L)^{1/3} \tag{13-259}$$

$$f_s = (100/s_B)^{1/4} \tag{13-260}$$

$$s_B = 2s_R \tag{13-261}$$

where

d_b = ball diameter, in in.

k_o = 250 lb/in.^{2}

L = life expectance, in hours

s_B = average speed of balls, in rpm

s_R = average speed of roller (cylinder), in rpm

The characteristics of some typical variable-speed drives are tabulated in Table 13-25.

The effect of a non-ideal disc drive-motor is as follows. If the disc drive-motor has a drooping speed-torque characteristic so that its speed is given by

$$\omega_D = \omega_{Do} - k_M T_D \tag{13-262}$$

where

ω_{Do} = no-load disc drive-motor speed

K_M = a constant

T_D = VSD output torque referred to the disc motor shaft plus torque losses

the disc motor torque is

$$T_D = (X/R)T_o + T_{loss} \tag{13-263}$$

where

T_o = torque at VSD output

T_{loss} = torque losses referred to disc motor

The losses in torque are those due to the losses in the disc shaft bearing(s), the disc thrust bearing, the balls, and the cylinder motor.[60,61] Substitution of Eq. (13-263) into Eq. (13-262) and the result into Eq. (13-251) results in

$$\Omega_o = \left(\frac{X}{R}\right)\left\{\omega_{Do} - k_M\left(\frac{XT_o}{R}\right) - k_M T_{loss}\right\} \tag{13-264}$$

Partial differentiation of Eq. (13-264) leads to the term

$$\frac{\partial\Omega_o}{\partial T_o} = -\left(\frac{X}{R}\right)^2 k_M \tag{13-265}$$

The complete expression for $\partial\Omega/\partial T$ is the sum of the values given by Eqs. (13-265) and (13-255), namely:

$$\frac{\partial\Omega}{\partial T} = -\frac{0.85\omega_{Do}X}{RF} - \left(\frac{X}{R}\right)^2 k_M \tag{13-266}$$

13-8.9 Static Characteristics of Capstan Type Amplifier

The capstan type of torque-output amplifier may be analyzed in the following manner : The law of a single capstan relating the output torque T_o to the tension in the input cord T_i is

$$T_o = T_i r e^{\mu\theta} \tag{13-267}$$

where

r = capstan drum radius

μ = coefficient of friction

e = angle of cord wrap-around

e = base of natural logarithms = 2.718

The double capstan in Fig. 13-164A, with input tension T_m in both cords, has zero output torque as long as the input and output shafts are aligned. If the input shaft rotates slightly so that $\theta_i > \theta_o$ by an amount ξ, the torque exerted on the output shaft by the two cords becomes

$$T_1 = T_m r e^{\mu(\phi+\xi)} \text{ and } T_2 = T_m r e^{\mu(\phi-\xi)} \tag{13-268}$$

where

ϕ is the angle of wrap-around with $\theta_i = \theta_o$. Total output torque is $T_1 - T_2$, or

$$T_o = r e^{\mu\phi} T_m (e^{\mu\xi} - e^{-\mu\xi})$$

$$= 2\mu r e^{\mu\phi} T_m \xi \tag{13-269}$$

provided that $\mu\xi << 1.0$. A typical set of values is(56) $\mu = 0.25$, $\phi = 6\pi$, and $e^{\mu\phi} = 113$. For values r = 1 inch and T_m = 2 ounces, T_o = 113ξ. Thus, $\dfrac{\partial T}{\partial Y} = 113$, where Y is the error ξ, in radians,-and torque is in inch-ounces. The relation between θ_i and θ_o depends upon the load torque. Although the value of $\dfrac{\partial T}{\partial \Omega}$ may not be zero, it is apparently quite small because Crank(57) does mention the necessity for stabilization of the amplifier. The block diagram relating 8, to θ_i, assuming $\partial T/\partial\Omega = 0$, is given in Fig. 13-168. Inspection discloses that, with no viscous damping of the output shaft, the device is characterized by

$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{1}{\left(\dfrac{s}{\omega_n}\right)^2 + 1} \tag{13-270}$$

where

$\omega_n = \sqrt{\dfrac{2\mu r e^{\mu\phi} T_m}{J}}$, in radians/sec

J = load inertia

s = complex Laplace transform

13-8.10 DYNAMIC BEHAVIOR OF MECHANICAL AMPLIFIERS

13-8.11 Capstan Amplifier

The dynamic behavior of the capstan amplifier is exemplified by the amplifier equation [Eq. (13-270)] and block diagram (Fig. 13-168). In the actual system, a dashpot or an aluminum disc and alnico magnets can be used to provide a viscous-damping factor of ζ in the region of $0.3 < \zeta < 0.8$. The natural frequency ω_n is unaffected by this addition. Practically, two such amplifiers can be cascaded. In such a case, the output of the first may be principally the inertia of the input cord carrier of the second plus the viscous friction and whatever additional frictional load is present. Inertias of 0.01 oz-in.-sec² or less appear possible, so that a value of 100 radians/sec for ω_n can be achieved. Since the second stage faces a heavier load inertia, a somewhat lower natural frequency results.

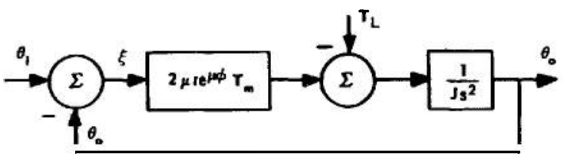


Fig. 73-768 Block diagram of capstan amplifier, with load inertia J and load torque T_L.

TABLE 13-25 CHARACTERISTICS OF VSD UNITS

Unit No.	Approximate Size (inch)	Weight (lb)	Ball Dia. (inch)	Cylinder Dia. (inch)	Usable Disc Dia. (inch)
1	4 x 4 x 3	1.9	.375	.500	2.0
2	4 x 4 x 3	1.1	.374	.500	2.0
3	4 x 2.5 x 4	1.9	.406	.812	1.62
4	3 x 2 x 3	1.1	.250	.625	1.5
5	4 x 2 x 3	1.3	.250	.625	1.5
6	6 x 6 x 4	6.3	.750	.750	2.5
Unit No.	**Cylinder Dia. Ball Dia.**	**Disc Dia. Cyl. Dia.**	**Type Cylinder Bearings**	**Type Disc Bearings**	**Thrust Bearing Dia. (inch)**
1	1.33	4.0	S1R	S5R	2.25
2	1.33	4.0	R4SS-MG	S5R	2.25
3	2.0	2.0	R4SS-MG	R6 MG	0.6
4	2.5	2.4	R4SS-MG	R6 MG	0.6
5	2.5	2.4	R4SS-MG	R6 MG	0.6
6	1.0	3.33	37K	S3K	3.38

TABLE 13-25 CHARACTERISTICS OF VSD UNITS (cont)

Unit No.	Description of Ball Carriage	Ball Force (lb)	Lubrication	Rated Output Torque at 70°F* (in.-oz)	Rated Torque at −65°F* (in.-oz)	Torque Wt (in.-oz/lb)
1	Aluminum with roller bearings	16	MIL-G-3278	3	0	1.6
2	Aluminum with roller bearings	32	MIL-G-7421	6	6	3.2
3	Steel with ball bearings	38	MIL-G-7421	12	12	6.3
4	Aluminum, no bearings	13	MIL-G-7421	2.7	2.7	2.5
5	Aluminum, no bearings	13	MIL-G-7421	12	10	9.2
6	Aluminum with ball bearings	37	MIL-G-7421	9	9	1.4

*Rated for 1% slip

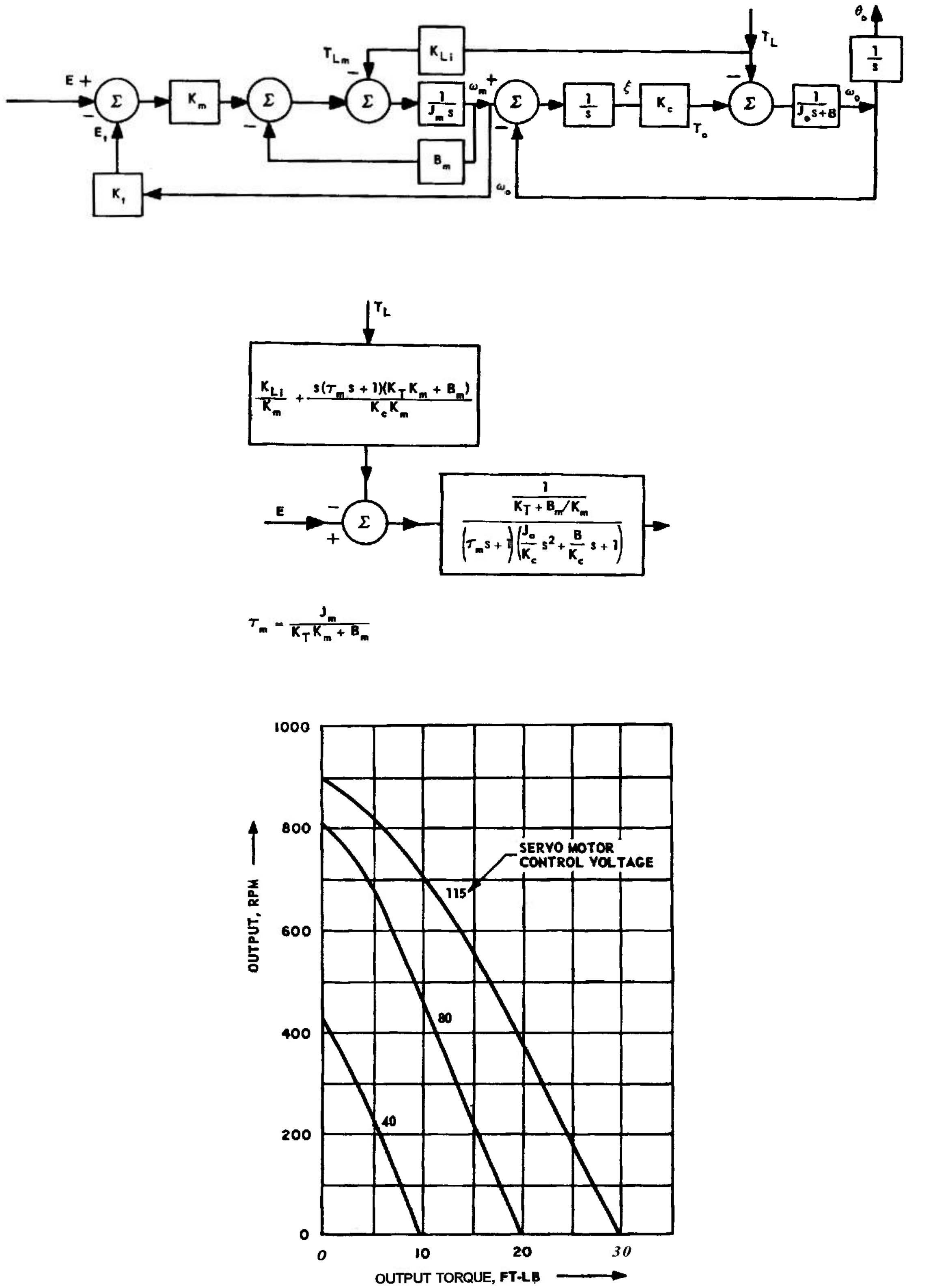


Fig. 13-169 Capstan amplifier with electrical input.

In certain cases, the input to the mechanical amplifier may be furnished by a transducer whose input is electrical, such as a two-phase servomotor[59] or a solenoid. In such cases, the transducer may be loaded by the mechanical amplifier. A somewhat altered block diagram results rather than Fig. **13-168.** The combined diagram of the amplifier and a two-phase servomotor transducer appears as Fig. **13-169A** wherein servomotor control voltage is E, servomotor output speed is ω_m, torque constant is K_m, and damping B_m (see Par. **14-3).** The effect of loading on the servomotor is the torque which is assumed to be related to mechanical amplifier load torque by K_{Li}, a constant. The amplifier is assumed to have some damping, B, such that $\frac{\partial \Omega}{\partial T} = 1/B$. A tachometer can be used with the **servomotor.**[59] The voltage appearing at the motor control winding due to the tachometer is $E_t = K_t\omega_m$. The inertia of the amplifier output shaft is J_o; the inertia of the servomotor and its load is J_m; the constant $K_c = 2\mu r e^{\mu\phi} T_m$; and the amplifier output velocity is ω_o radians/second.

The steady-state relationship for the amplifier driven by servomotor can be found from the block diagram by first simplifying it as shown in Fig. **13-169B** and then letting $s = 0$ therein. The result in terms of output speed, torque, and servomotor control voltage is:

$$\omega_o = \frac{E}{\frac{B_m}{K_m} + K_t} - \frac{\frac{K_{Li}}{K_m} T_L}{\frac{B_m}{K_m} + K_t} \qquad (13\text{-}271)$$

Compare Eq. **(13-271)** to the servomotor speed-torque characteristic [see **Eq. (14-75)** with $s = 0$] which is

$$\omega_m = \frac{E}{\frac{B_m}{K_m}} - \frac{\frac{1}{K_m} T_L}{\frac{B_m}{K_m}} \qquad (13\text{-}272)$$

The effect of the amplifier is to increase the servomotor torque constant by a factor of $1/K_{Li}$ and to multiply the motor damping by the factor $\left\{1 + \left(\frac{K_m}{B_m}\right) K_t\right\}$, this last effect being due to the inclusion of the tachometer, not any property of the mechanical amplifier. The resultant torque-speed characteristic of a typical amplifier [59] is shown in Fig. **13-1696.** Some typical values for an amplifier capable of delivering about one-horsepower output are tabulated in Table **13-26.**

13-8.12 Integrator Amplifier

The dynamic behavior of the integrator amplifier is characterized by results similar to those produced by the capstan amplifier with added damping. These results are illustrated in Fig. **13-170,** where a position feedback is shown dotted.

TABLE 13-26 SOME TYPICAL VALUES OF MECHANICAL AMPLIFIER PARAMETERS

Symbol	Value	Units
K_t	0.5	volt-sec/radian
τ_m	0.02	second
K_{Li}	0.001	dimensionless ratio
K_m	0.05	oz-in./volt
B_m	0.007	oz-in.-sec/radian
K_c	113	oz-in./radian
J_o	0.01	oz-in.-sec^2

13-8.13 OTHER MECHANICAL AMPLIFIERS

13-8.14 Clutch-Type Amplifiers

Clutches of the on-off type may be considered as mechanical relays. Clutch operation can be by electrical means (solenoids) or by application of air or hydraulic (oil) pressure. An example of a solenoid-operated clutch, as used in servomechanisms, is shown in Fig. **13-171.** This device was part of one of the first

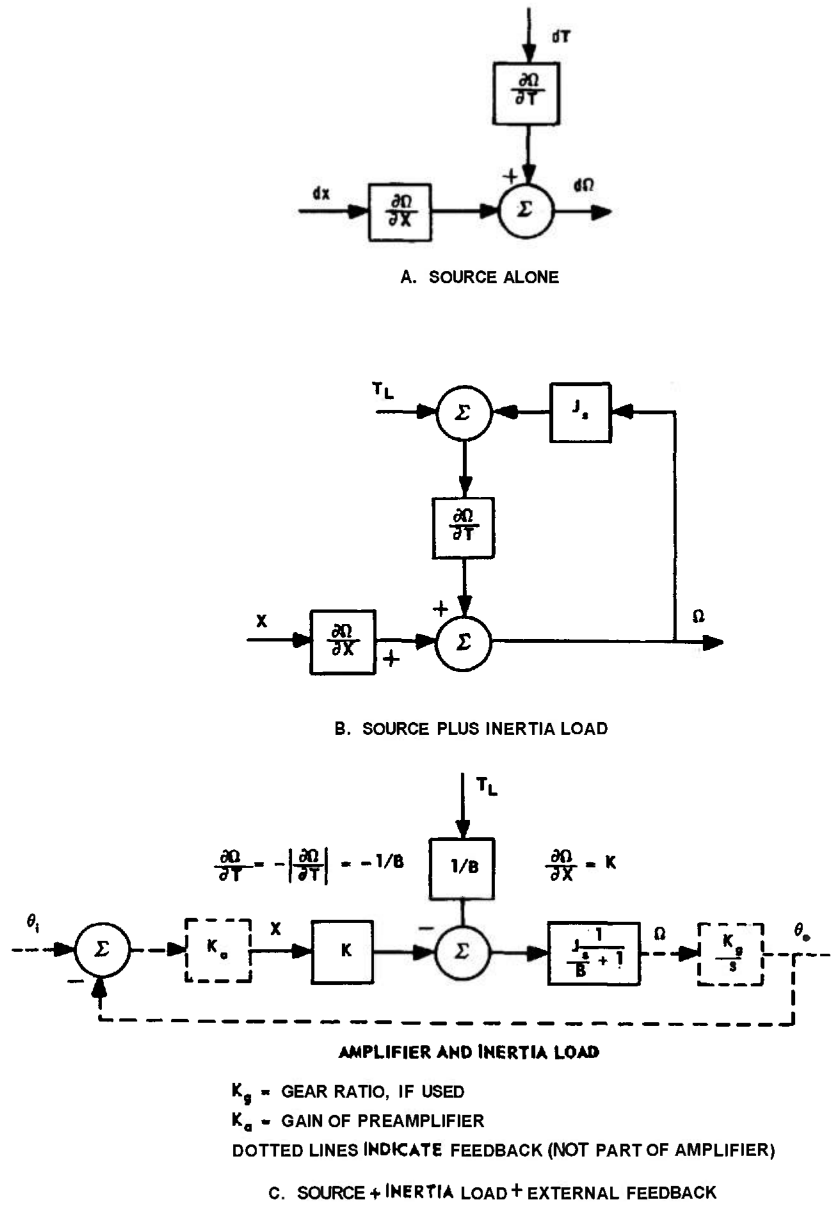


Fig. 13-170 Block diagram of integrator-type amplifier with inertia load and external feedback.

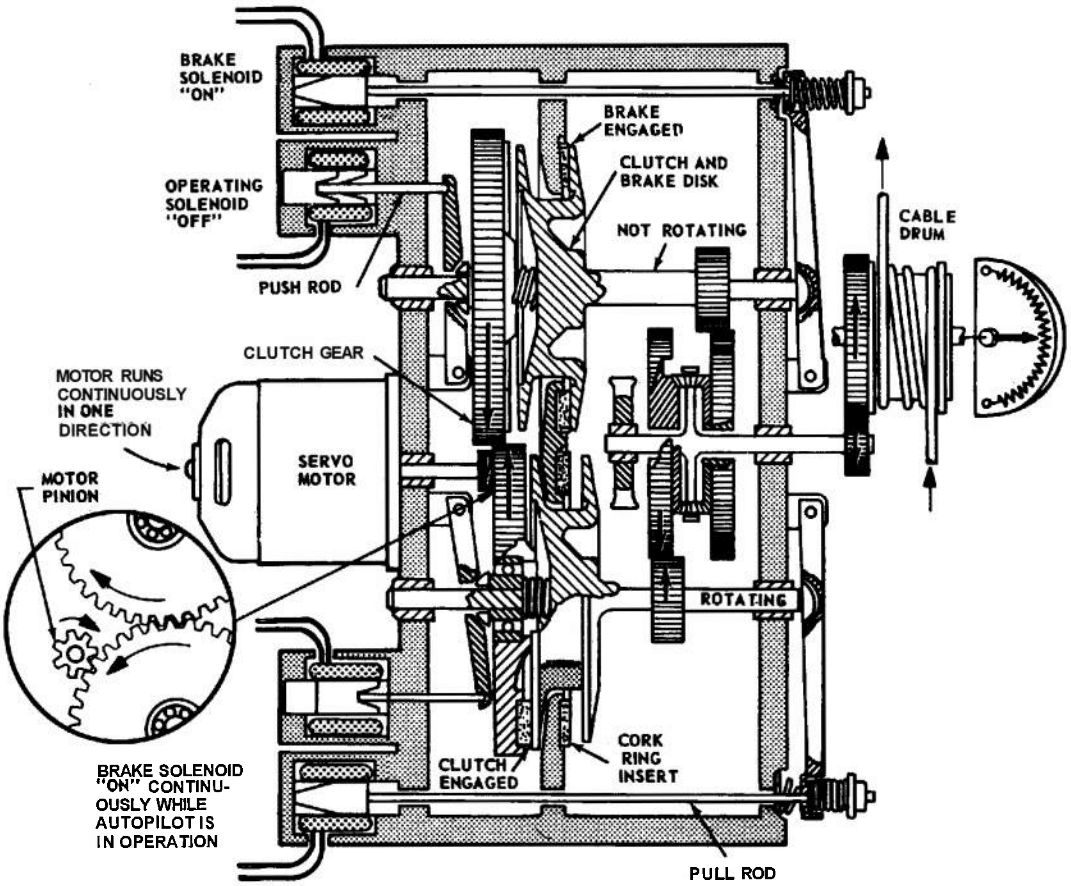


Fig. 13-171 Solenoid-operated clutch.

electrically operated airplane autopilots. Another type of solenoid-operated clutch (operating from direct current) is shown in Fig. 13-172. Two procedures for determining a torque T required for this clutch are as follows:

$$T = \frac{HP \times 5250}{N} \times S \qquad (13\text{-}273)$$

where

T = torque, in lb ft

HP = horsepower of motor or other power source

N = speed of clutch, in rpm

S = a service factor (consult manufacturer's catalog)

A tabulation of typical characteristics for the clutch in Fig. 13-172 is given in Table 13-27. Torque for brakes may be calculated from Eq. (13-273), the resulting torque being the retarding effort. The other equation that is useful is based on the torque necessary to produce constant acceleration and is

$$T = \frac{WK^2 \times N}{308 \times t} \qquad (13\text{-}274)$$

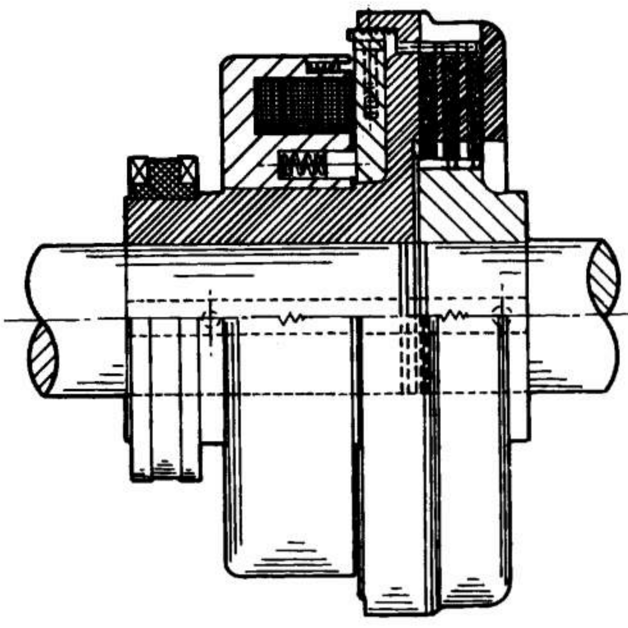
where

t = time required to accelerate rotating parts from 0 to maximum speed, in sec

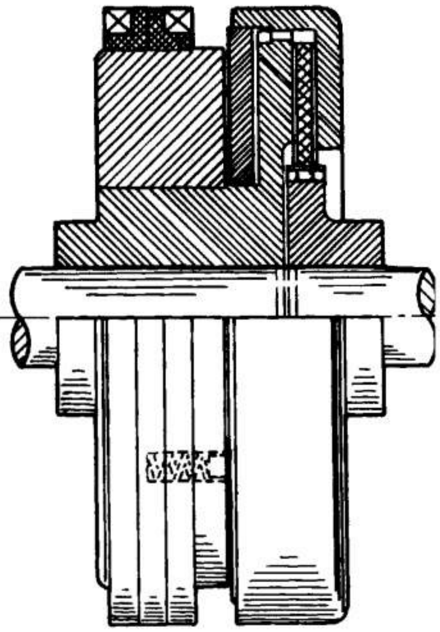
WK^2 = moment of inertia, in lb ft^2

W = weight, in lb

K = radius of gyration, in ft



A.
CLUTCH WITH COLLECTOR RING ON HUB



B.
CLUTCH WITH COLLECTOR RING ONMAGNETBODY

Fig. 13-172 Solenoid-operated clutches.

TABLE 13-27 CHARACTERISTICS OF SOLENOID-OPERATED CLUTCH

Solenoid Size No.	Maximum Torque (lb ft)	Trans-mitted HP (at 100 rpm)	Required D-C Coil Power (watts)
1	7	0.13	24
2	14	0.26	24
3	21	0.39	24
4	13.5	0.25	29
5	27	0.50	29
6	40.5	0.75	29
7	35	0.66	45
8	70	1.3	45
9	105	1.9	45
10	140	2.6	75
11	280	6.2	75
12	420	7.8	75
13	400	7.6	110
14	800	15	110
15	1200	22	110
16	600	11	125
17	1200	22	125
18	1800	33	125
19	900	17	165
20	1800	34	165
21	2700	51	165
22	1500	28	185
23	3000	57	185
24	4500	85	1-85
25	3000	57	200
26	6000	114	220
27	9000	171	220

Courtesy Stearns Electric Corporation.

Equation **(13-274)** is applicable to drives with flywheels or rotating parts to be accelerated in a given time t. Types of driven loads are:

(a) ***Steady.*** Starting torque is low; running load is smooth and uniform, without shocks.

(b) Light *shock.* Starting torque is somewhat greater than running torque, but peak loads are comparatively low and infrequent.

(c) Severe *shock.* Starting torque and peak loads are **200** to **250** percent of the running load. Fluctuating running load, heavy shocks, or reversals occur frequently. Magnetic-particle clutches, also very commonly used, are discussed in Par. **14-6.**

13-8.15 PROBLEMS ENCOUNTERED WITH MECHANICAL AMPLIFIERS

13-8.16 Capstan Amplifiers

The most important problems are those imposed by friction forces, either necessary or parasitic. For example, the capstan operates by means of sliding or slipping friction, which wears out the cord and necessitates frequent replacement thereof. **Crank**[57] cites various types of cords suitable for the purpose, using graphite for a lubricant. Other problems encountered are: uneven, jerky operation due to oil spots on the drums; and loosening of the cords. In addition, the amplifier is unstable unless damping is used, and heat is generated when slippage occurs. Heat must be dissipated artificially.

13-8.17 Integrator Amplifiers

Theoretically, this type of amplifier operates without slippage and friction between the disc, balls, and roller. Practically, some slippage does occur, producing friction and consequent wearing of the parts. This is especially true of the disc, in which an indentation may occur near the center, resulting in a "dead" spot.

Operation of the integrator with an offset carriage causes more wear than in the case where the carriage passes across the center of the **disc.**[61] Dirt particles in the lubricant used cause very severe wear and must be guarded against. Dust housings are recommended. Use of a gear reducer on the output of the integrator with a higher disc motor speed results in an increase in the torque-to-total-weight ratio.[61] Operation at low temperature can cause difficulty due in part to:[61]

(a) insufficient clearance in the ball carriage

(b) radial squeezing of the cylinder bearings by aluminum housing

(c) insufficient thrust clearance on cylinder

(d) high viscosity of lubricant.

Use of all steel housings or steel bushings can reduce some of these difficulties.

13-8.18 Clutch Amplifiers

Clutch linings wear rapidly, limiting the number of engagements before the linings must be replaced.

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Listed below are the Handbooks which have been publirhed or submitted for publication. Handbooks with publication dater prior to 1 August 1962 were published as 20-series Ordnance Corps pamphlets. AMC Circular 310-38, 19 July 1963, redesignated those publications as 706-series AMC pamphlets (i.e., ORDP 20-138 war roderignated AMCP 706-138). All new, reprinted, or revised Handbooks are being publirhed ar 706-series AMC pamphlets.

General and Miscellaneous Subjects

Number	Title
106	Elements of Armament Engineering, Part One, Sources of Energy
107	Elements of Armament Engineering, Part Two, Ballistics
108	Elements of Armament Engineering, Part Three, Weapon Syrtemr and Components
110	Experimental Statistics, Section 1, Basic Concepts and Analysis of Measurement Data
111	Experimental Statirticr, Section 2, Analyrir of Enumerative and Classificatory Data
112	Experimental Statirticr, Section 3, Planning and Analysis of Comparative Experiments
113	Experimental Statistics, Section 4, Special Topicr
114	Experimental Statirticr, Section 5, Tables
121	Packaging and Pack Engineering
134	Maintenance Engineering Guide for Ordnance Design
135	Inventions, Patents, and Related Matters (Revised)
136	Servomechanisms, Section 1, Theory
137	Servomechanisms, Section 2, Measurement and Signal Converters
138	Servomechanisms, Section 3, Amplification
139	Servomechanisms, Section 4, Power Elements and System Design
170(C)	Armor and Its Application to Vehicles (U)
250	Guns--General (Guns Series)
252	Gun Tubes (Guns Series)
270	Propellant Actuated Devices
290(C)	Warheads--General (U)
331	Compensating Elements (Fire Control Series)
355	The Automotive Assembly (Automotive Series)

Ammunition and Explosives Series

Number	Title
175	Solid Propellants, Part One
176(C)	Solid Propellants, Part Two (U)
177	Properties of Explosives of Military Interest, Section 1
* 178(C)	Properties of Explosives of Military Interest, Section 2 (U)
210	Fuzes, General and Mechanical
211(C)	Fuses, Proximity, Electrical, Part One (U)
212(S)	Fuzes, Proximity, Electrical, Part Two (U)
213(S)	Fuzes, Proximity, Electrical, Part Three (U)
214(S)	Fuzes, Proximity, Electrical, Part Four (U)
215(C)	Fuzer, Proximity, Electrical, Part Five (U)
244	Section, 1, Artillery Ammunition--General, , with Table of Contents, Glossary and Index for Series
245(C)	Section 2, Design for Terminal Effects (U)
246	Section 3, Design for Control of Flight Characteristics
247	Section 4, Design for Projection
248	Section 5, Inspection Aspects of Artillery Ammunition Derign
249	Section 6, Manufacture of Metallic Components of Artillery Ammunition
*179	Explosive Trains

Ballistic Missile Serier

Number	Title
281(S-RD)	Weapon Syrtem Effectivenerr (U)
*282	Propulsion and Propellants
284(C)	Trajectoriee (U)
286	Structurer
*283	Aerodynamics

Ballirticr Seriee

Number	Title
140	Trajectories, Differential Effectr, and Data for Projectiles
150	Interior Ballistics of Guns
160(S)	Elements of Terminal Ballistics, Part One, Introduction, Kill Mechanisms, and Vulnerability (U)
161(S)	Elements of Terminal Ballistics, Part Two, Collection and Analysis of Data Concerning Targets (U)
162(S-RD)	Elements of Terminal Ballistics, Part Three, Application to Missile and Space Targets (U)

Carriages and Mounts Series

Number	Title
340	Carriages and Mounts--General
341	Cradles
342	Recoil Systems
343	Top Carriages
344	Bottom Carriages
345	Equilibrators
346	Elevating Mechanisms
347	Traversing Mechanisms

Materials Handbooks

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301	Aluminum and Aluminum Alloys
302	Copper and Copper Alloys
303	Magnesium and Magnesium Alloys
305	Titanium and Titanium Alloys
308	Glass
309	Plastics
310	Rubber and Rubber-Like Materials

Military Pyrotechnics Series

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186	Part Two, Safety, Procedures and Glosrary
187	Part Three, Properties of Materials Used in Pyrotechnic Comporitions

Surface-to-Air Missile Series

Number	Title
291	Part One, System Integration
292	Part Two, Weapon Control
293	Part Three, Computers
294(S)	Part Four, Missile Armament (U)
295(S)	Part Five, Countermeasures (U)
296	Part Six, Structures and Power Sources
297(S)	Part Seven, Sample Problem (U)