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ENGINEERING DESIGN HANDBOOK

SERVOMECHANISMS

SECTION 2, MEASUREMENT AND SIGNAL CONVERTERS

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FOREWORD

INTRODUCTION

This is one of a group of handbooks covering the engineering information and quantitative data needed in the design and construction of military equipment, which (as a group) constitutes the Army Materiel Command Engineering Design Handbook Series.

PURPOSE OF HANDBOOK

The handbook on Servomechanisms has been prepared as an aid to designers of automatic control systems for Army equipments, and as a guide to military and civilian personnel who are responsible for setting control-system specifications and ensuring their fulfillment.

SCOPE AND USE OF HANDBOOK

The publications are presented in handbook form rather than in the style of textbooks. Tables, charts, equations, and bibliographical references are used in abundance. Proofs and derivations are often omitted and only final results with interpretations are stated. Certain specific information that is always needed in carrying out design details has, of necessity, been omitted. Manufacturers' names, product serial numbers, technical specifications, and prices are subject to great variation and are more appropriately found in trade catalogs. It is essential that up-to-date catalogs be used by designers as supplements to this handbook.

To make effective use of the handbook during the design of a servo, the following procedure is suggested. The designer should turn first to Chapters 16 and 17 where design philosophy and methods are discussed. Implementation of the design procedure may require a review of certain theoretical concepts and methods which can be achieved through reference to Chapters 1 through 10. As the design proceeds, a stage will be reached at which the power capacity of the output member has been fixed. Reference to Chapters 14, 15, and 16 will then illustrate the salient features of output members having the required power capacity. After the designer has chosen the output member, he will find the information dealing with sensing elements and amplifiers (Chapters 11, 12, and 13) helpful in completing the design.

FEEDBACK CONTROL SYSTEMS AND SERVOMECHANISMS

Servomechanisms are part of a broad class of systems that operate on the principle of feedback. In a feedback control system, the output (response) signal is made to conform with the input (command) signal by feeding back to the input a signal that is a function of the output for the purpose of comparison. Should an error exist, a corrective action is automatically initiated to reduce the error toward zero. Thus, through feedback, output and input signals are made to conform essentially with each other.

In practice, the output signal of a feedback control system may be an electrical quantity such as a voltage or current, or any one of a variety of physical quantities such as a linear or angular displacement, velocity, pressure, or temperature. Similarly, the input signal may take any one of these forms. Moreover, in many applications, input signals belong to one of these types, and the output to another. Suitable transducers or measuring devices must then be used. It is also common to find multiple feedback paths or loops in complicated feedback control systems. In these systems, the over-all system performance as characterized by stability, speed of response, or accuracy can be enhanced by feeding back signals from various points within the system to other points for comparison and initiation of correction signals at the comparison points.

At present, there is no standard definition of a servomechanism. Some engineers prefer to classify any system with a feedback loop as a servomechanism. According to this interpretation, an electronic amplifier with negative feedback is a servo. More frequently, however, the term servomechanism is reserved for a feedback control system containing a mechanical quantity. Thus, the IRE defines a servomechanism as "a feedback control system in which one or more of the system signals represents mechanical motion." Some would restrict the definition further by applying the term only to a special class of feedback control system in which the output is a mechanical position.

APPLICATION OF SERVOMECHANISMS TO ARMY EQUIPMENT

Servomechanisms are an important part of nearly every piece of modern mechanized Army equipment. They are used to automatically position gun mounts, missile launchers, and radar antennas. They aid in the control of the flight paths of jet-propelled rockets and ballistic missiles, and play an important role in the navigational systems of those vehicles. As instrument servos, they permit remote monitoring of physical' and electrical quantities and facilitate mathematical operations in computers.

No single set of electrical and physical requirements can be stated for servomechanisms intended for these diverse military applications. The characteristics of each servomechanism are determined by the function it is to perform, by the characteristics of the other devices and equipments with which it is associated, and by the environment to which it is subjected. It will often be found that two or more servo-system configurations will meet a given set of performance specifications. Final choice of a system may then be determined by such factors as ability of the system to meet environmental specifications, availability of components, simplicity, reliability, ease of maintenance, ease of manufacture, and cost. Finally, the ability to translate any acceptable paper design into a piece of physical equipment that meets electrical and physical specifications and works reliably depends to a great extent upon the skill of the engineering and manufacturing groups responsible for building the system. The exercise of care and good judgment when specifying electrical, mechanical, and thermal tolerances on components and subsystems can contribute greatly to the successful implementation of servo-system design.

The handbook on Xervomechaiisms was prepared under the direction of the Engineering Haiidbook Office, Duke University, under contract to the Army Research Office-Durham. The material for this pamphlet was prepared by Jacksoii & Moreland, Boston, Massachusetts, under subcoiitract to the Engineering Handbook Office. Jacksoii & Moreland was assisted in their work by consultants who are recognized authorities in the field of servomechanisms.

PREFACE

The Engineering Design Handbook Series of the Army Materiel Command is a coordinated series of handbooks containing basic information and fundamental data useful in the design and development of Army materiel and systems. The handbooks are authoritative reference books of practical information and quantitative facts helpful in the design and development of Army materiel so that it will meet the tactical and the technical needs of the Armed Forces. The presenthandbook is one of a series on Servomechanisms.

Section 2 of the handbook contains Chapters 11 and 12, which describe those servomechanism components used as sensing elements and signal converters. Chapter 11 covers various sensing elements such as potentiometers, rotary transformers, linear variable differential transformers, tachometer generators, gyroscopes and analog-to-digital converters. Chapter 12 covers three types of signal converters: modulators, demodulators and digital-to-analog converters.

For information on other servomechanism components and on feedback control theory and system design, see one of the following applicable sections of this handbook:

AMCP 706-136	Section 1	Theory (Chapters 1-10)
AMCP 706-138	Section 3	Amplification (Chapter 13)
AMCP 706-139	Section 4	Power Elements and System Design (Chapters 14-20)

An index for the material in all four sections is placed at the end of Section 4.

Elements of the U. S. Army Materiel Command having need for handbooks may submit requisitions or official requests directly to Publications and Reproduction Agency, Letterkenny Army Depot, Chambersburg, Pennsylvania 17201. Contractors should submit such requisitions or requests to their contracting officers.

Comments and suggestions on this handbook are welcome and should be addressed to Army Research Office-Durham, Box CM, Duke Station, Durham, North Carolina 27706.

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CHAPTER 11

SENSING ELEMENTS

11-1 INTRODUCTION*

This chapter deals with those devices that measure or sense the input or output of a servomechanism and express it in electrical form. The more common types of sensing elements used in ordnance applications convert motion, either translational or rotational, into a corresponding electrical representation. They are part of a class of devices called *transducers*, the function of a transducer being to receive information from some source and to express that information in a different form for use by another operating component.

The variety of sensing elements that have been developed is very large. No attempt is made here, however, to list or discuss all of them. Rather, the selection is restricted to those that find wide use in ordnance applications.

In the remainder of this chapter, emphasis is placed on discussing the characteristics and limitations of the various sensing elements considered. The aim is to provide the servo designer with sufficient information to select a suitable type of sensing element for a given application and to match it properly to the rest of the system utilizing the servo.

11-2 POTENTIOMETERS*

11-2.1 DESCRIPTION AND BASIC THEORY

11-2.2 Definition

A potentiometer is a resistor with a sliding contact. The electrical resistance between the slider and either end point of the total resistance element is a predetermined function of the distance of the slider from the end point so that the potentiometer converts slider position into electrical resistance.

Figure 11-1 shows the electrical representation of a potentiometer. The arrow indicates the movable slider; A and B are the end points of the total resistance element across which the excitation voltage is applied; C and D are the end points of that portion of the total resistance element across which the variable output voltage is taken. In an actual potentiometer, one terminal suffices for end points B and D combined.

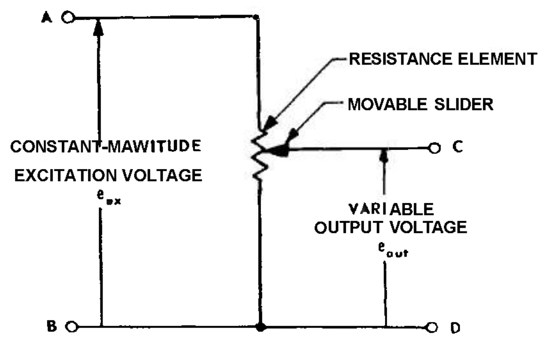

Fig. 11-1 Electrical representation of a potentiometer.

*By A.K. Susskind

11-2.3 Types of Potentiometers

There are two basic types of potentiometers:[1] *rotary* and *translatory*. The majority of potentiometers produced today are intended for applications where the slider motion is rotary. The basic rotary type can be broken down into two subtypes: the ***single-turn*** potentiometer and the *multiturn* potentiometer.

Single-turn potentiometers are designed for slider-travel limits of a full revolution or less (e.g., 300°). Multiturn potentiometers are designed for slider-travel limits of several revolutions (e.g., some multiturn potentiometers are now being made with as many as 60 revolutions between stops).

11-2.4 Principle of Operation

The basic performance equation for a single-turn or multiturn potentiometer can be written[1]

$$R(\theta) = \int_0^{\theta} \rho(\theta)\, d\theta \qquad (11\text{-}1)$$

where

θ = slider angle (i.e., the angular displacement of the slider with respect to the zero end of the potentiometer)

$R(e)$ = resistance between the slider and the zero end of the potentiometer (i.e., the resistance between points C and D in Fig. 11-2)

$\rho(6)$ = change in resistance per unit angular displacement as a function of slider angle 8 (in units consistent with those chosen for θ)

The total winding resistance R_w can be written

$$R_w = \int_0^{\theta_{max}} \rho(\theta)\, d\theta \qquad (11\text{-}2)$$

where ę is the slider angle that represents the angular displacement of the slider from the zero end of the potentiometer to the other end.

If $\rho(e)$ is a constant, k_1, then

$$R(\theta) = k_1\theta \qquad (11\text{-}3)$$

and

$$R_w = k_1\theta_{max} \qquad (11\text{-}4)$$

When $\rho(\theta)$ is constant, the potentiometer is known as a ***linear*** potentiometer. If $\rho(6)$ is not constant, the potentiometer is said to be ***nonlinear***. The form of Eqs. (11-1) and (11-2) indicates that the resistivity function $\rho(\theta)$ of a potentiometer must be the derivative of the desired resistance function; i.e.,

$$\rho(\theta) = \frac{dR(\theta)}{de} \qquad (11\text{-}5)$$

When an excitation voltage e_{ex} of fixed amplitude is applied across the potentiometer, the output voltage e_{out} measured between the zero end of the potentiometer and the slider is a function of slider position; i.e.,

$$e_{out} = e_{ex}\frac{R(\theta)}{R_w} \qquad (11\text{-}6)$$

The relationships for a translatory potentiometer that correspond to Eqs. (11-1) through (11-6) can be written directly as follows:

$$R(x) = \int_0^{x} \rho(x)\, dx \qquad (11\text{-}7)$$

$$R_w = \int_0^{x_{max}} \rho(x)\, dx \qquad (11\text{-}8)$$

$$\left.\begin{array}{l} R(x) = k_2 x \quad (11\text{-}9) \\ R_w = k_2 x_{max} \quad (11\text{-}10) \end{array}\right\} \text{for } \rho(x) = \text{constant} = k_2$$

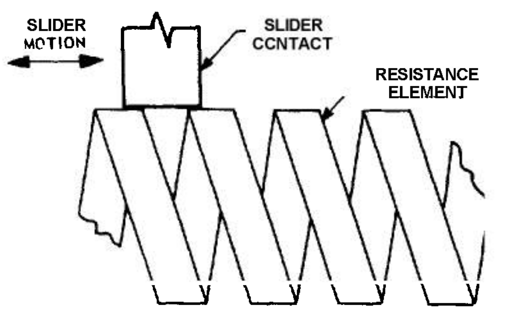

Fig. 1 7-2 Wire-wound element and slider.

$$\rho(x) = \frac{dR(x)}{dx} \qquad (11\text{-}11)$$

$$e_{out} = e_{ex} \frac{R(x)}{R_w} \qquad (11\text{-}12)$$

where

x = slider displacement (i.e., the linear distance of the slider from the zero end of the potentiometer)

$R(x)$ = resistance between the slider and the zero end of the potentiometer

$\rho(x)$ = change in resistance per unit linear displacement as a function of slider displacement x (in units consistent with those chosen for x)

x_{max} = distance represented by slider displacement from the zero end of the potentiometer to the other end

K_2 = constant value of $\rho(x)$

11-2.5 Use

An important application of potentiometers in servos is as an electromechanical transducer for converting a mechanical displacement into a corresponding voltage signal. A potentiometer is one of the simplest means of accomplishing this function. If a direct or alternating excitation voltage of fixed amplitude is connected across the potentiometer, the output voltage (measured between the slider and the zero-end terminal) is a function of slider position and is given by either Eq. **(11-6)** or whichever is applicable. It follows that only when a linear potentiometer is used will the output be a linear function of slider position. Since a linear sensing element is required in most servomechanisms, linear potentiometers are nearly always used and the required tolerance in $R(\theta)$ is so close that only precision potentiometers are employed.

Nonlinear potentiometers are used primarily in computing circuits. For example, by making

$$\rho(\theta) = \cos e \qquad (11\text{-}13)$$

the resistance $R(\theta)$ [see Eq. **(11-1)**] becomes

$$R(e) = \int_0^{\theta} \cos e \, d\theta = \sin \theta \qquad (11\text{-}14)$$

Hence, this particular nonlinear potentiometer can be used to express electrically the sine of the slider position.

11-2.6 Construction Features

A potentiometer consists of three main parts : the resistance element ; the slider ; and the housing. The housing holds the other two parts in proper relationship to each other and also serves as a mount for the complete unit. In rotary-motion units, a shaft for mechanical coupling to the slider is brought out through the housing ; in linear-motion units, a rod is brought out.

Resistance elements of four types are used in precision potentiometers for servomechanism applications : wire wound ; slide wire ; film ; and conductive plastic. Film elements (made of carbon or metal) have not yet been developed as fully as wired-wound and slide-wire elements. At the present time, most commercially available precision potentiometers contain wire-wound resistance elements.

Wire-wound elements are made by wrapping an insulated resistance wire helically around either an insulating card or an insulated metallic rod that, for rotary units, is then bent into a circular shape. The wire insulation is removed along the path that the slider contacts. When moving, the slider successively contacts each turn of the winding as shown in Fig. **11-2.** In the position shown, the slider is in contact with two turns of the resistance element. **As** the slider moves to the right, it leaves the first turn and makes contact with only the second until further motion brings it into contact with both the second and third. The resistance between the slider and one end point of the resistance element therefore varies in discrete steps, as does the output voltage when a fixed voltage is applied across the entire element. The size of the voltage steps is of the order of e_{ex}/n, where n is the number of turns of the resistance wire. The exact size of the steps varies with slider position and structural details.

Because of these voltage steps, the use of a wire-wound potentiometer as an output sensing element in a servomechanism leads tc small oscillations around the particular step that most nearly corresponds to the servomechanism command. The more turns the element has, the smaller the amplitude of the oscillations. Hence, in selecting a potentiometer, care must be taken to assure that the size of the resistance steps is small enough to result in oscillations of negligible amplitude, or compensation can be added to the servomechanism in the form of coulomb friction at the output shaft. Gottling[18] has shown that for a simple positional servomechanism consisting of an amplifier and motor, with voltage feedback from the output shaft obtained by means of a wire-wound potentiometer, the output will settle on the wire step where the voltage is nearest to the input if the coulomb friction, q, is in accordance with the following criterion

$$q \geqq \frac{1}{2} \times \frac{e_{ex}}{n} \times K_a K_m \qquad (11\text{-}15)$$

where

K_a = amplifier gain

K_m = stalled motor torque per unit control-phase voltage

For a given potentiometer diameter, the smaller the wire diameter, the larger the total resistance and the greater the number of turns. The greater the number of turns, the smaller the individual voltage steps and the smaller the travel of the slider arm over which the output voltage is constant. Hence, the number of wire turns also determines the accuracy to which a desired voltage can be achieved.

At the present time, the smallest practical wire diameter used is 0.9 mil. In typical commercial units presently available, the maximum case diameter is a few inches and the maximum number of wire turns per shaft revolution is approximately 5000, so that the upper limit in resolution, obtained in the highest-resistance units, is approximately one part in 10,000 per revolution. Where a circuit design calls for a low impedance level and the inherently poor resolution of a low-resistance potentiometer cannot be tolerated, one can use a high-resistance, good-resolutior, potentiometer and shunt it across the input terminals with a fixed low resistance. This arrangement provides a low impedance level at the, input terminals, without sacrificing the desired resolution.

The stepped nature of the resistance function is the most serious disadvantage of wire-wound elements. It is overcome in the carbon, metal, and conductive-plastic film elements, which represent a nearly smooth, unbroken surface to the slider and exhibit voltage steps smaller than those of a high-resistance, wire-wound element by a factor of several tens. At present, however, only a small variety of Elm units are commercially available and these are not yet as stable under extreme environmental conditions as are wire-wound elements. (Another form of a smooth resistance element is the composition type. However, its resistivity function cannot be sufficiently well controlled in manufacture to permit its use as a sensing element for servomechanisms.)

While very good resolution can be obtained with potentiometers of the slide-wire type, these units have relatively low total winding resistance. This can result in excessive loading of the electrical source. The total resistance of the element is limited by the size of the potentiometer. Typical single-turn units have a total resistance of a few hundred ohms; typical multiturn units have a total resistance of a few thousand ohms.

11-2.7 LINEAR POTENTIOMETERS

11-2.8 Types of linearity

The characteristic of greatest interest in a linear potentiometer is the degree of linearity; i.e., the closeness with which the output voltage or resistance is a linear function of slider position. The inherent stepped nature of the output of a wire-wound potentiometer yields a linearity deviation that is one half the size of the maximum step. Manufacturing tolerances add further deviations from perfect linearity.

Figure **11-3** shows a typical plot of percent output voltage as a function of percent slider position, with percent deviation from linearity $(D)E_{out}/E_{ex}$ exaggerated. The maximum percent deviation of the actual curve from the ideal curve is called ***terminal linearity.*** Terminal linearities as low as **0.01** percent have been achieved, but linearities of 0.1 to 0.5 percent are more common in most commercial units.

A different measure of potentiometer performance commonly used is ***independent linearity,*** which is defined by the Radio, Electronic and Television Manufacturers' Association (RETMA) as "the deviation when the slope and position of a straight reference line are chosen to make the maximum deviations a minimum over the actual effective travel or any specified portion thereof." The straight reference line used does not coincide with the line corresponding to ideal performance. The independent linearity figure is smaller than the terminal linearity figure.

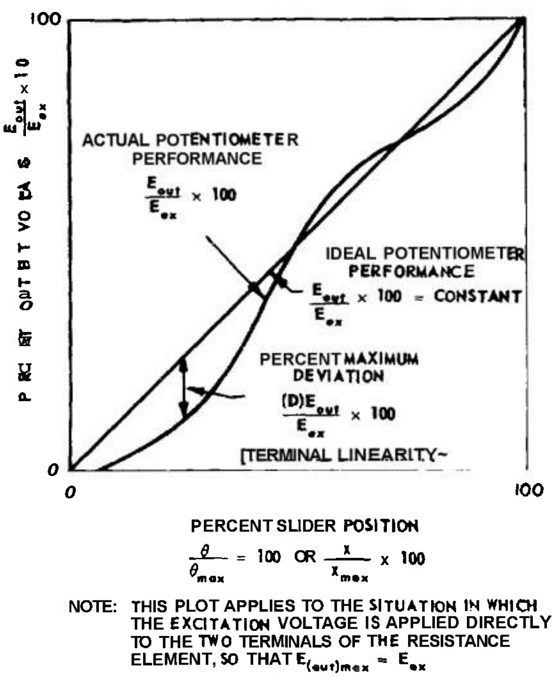

Fig. 11-3 Potentiometer linearity performance compared with ideal performance.

The best straight line for the actual characteristics previously shown is drawn in Fig. **11-4,** which also shows that the maximum deviation now occurs at a different slider position and is smaller than that in Fig. 11-3. Note that the best straight line does not pass through the origin nor does it intersect the actual performance curve at the 100-percent point. The best straight line can be made to coincide with the required straight line by trimming techniques, so that the actual performance deviation from an ideal straight line will differ by only the independent linearity figure. Trimming is accomplished by referring the slider voltage not to the voltage applied across the resistance element but to a value that corresponds to ***DE*** in Fig. **11-4.**

A typical trimming circuit is shown in Fig. 11-5. The upper and lower trimmer potentiometers are center-tapped and the voltage between the two center taps is taken as the reference. The sliders on the two trimmer potentiometers are adjusted so that when the slider of the precision potentiometer is at each end, the percentage output voltage differs from the best straight line by ***m*** and *n*, respectively, of Fig. **11-4.**

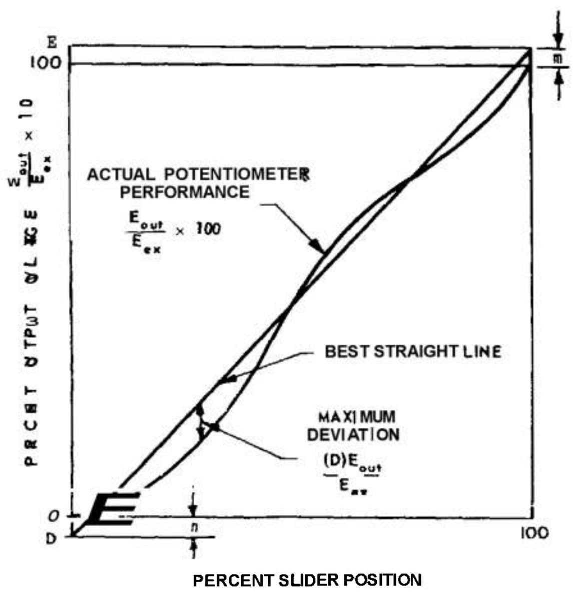

Fig. 11-4 Potentiometer linearity performance compared with the best straight line.

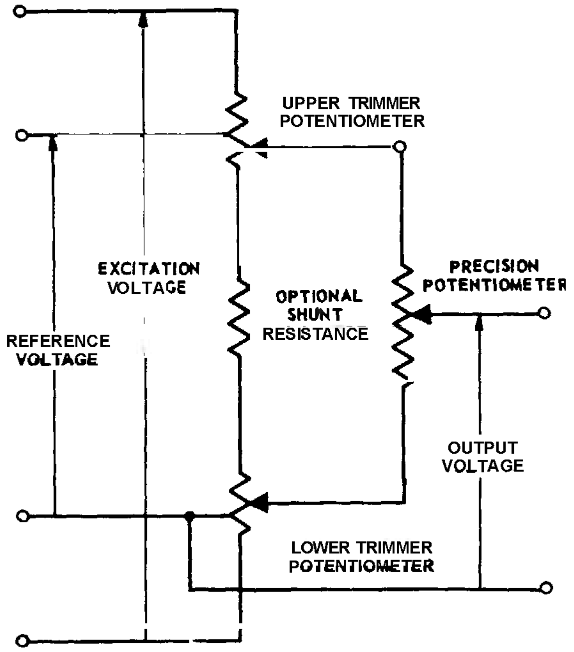

Fig. 11-5 Potentiometer with a typical trimming circuit.

Another definition of linearity that is sometimes used is *zero-based linearity.* This is defined by the RETMA as "the deviation from a straight reference line through zero applied voltage as the starting end point, with its slope chosen to make the maximum deviation minimum over the theoretical effective travel or any portion thereof." The addition of a trimming circuit at the upper end of the potentiometer can cause the actual performance to differ from a straight line by no more than the zero-based linearity.

11-2.9 Effect of Load Impedance

In a circuit application, the linearity of the output voltage is a function not only of the potentiometer linearity itself, but also of the load resistance that is connected to the slider. The larger the load resistance with respect to the total potentiometer resistance, the smaller the deviation of the output voltage from the potentiometer linearity curve. The output voltage in the presence of loading, assuming an ideal linear potentiometer, is[2]

$$E_{out} = E_{ex} \frac{(R_l/R_w)\,[R(\theta)/R_w]}{R_l/R_w + R(\theta)/R_w\,[1 - R(\theta)/R_w]} \qquad \textbf{(11-16)}$$

where

R_l = load resistance

$R(\theta)$ = resistance between the slider and the zero end of the potentiometer

R_w = total winding resistance of the potentiometer

A plot of the ratio of the output voltage to the excitation voltage is given in Fig. **11-6** as a function of the ratio $R(\theta)/R_w$ for various values of the ratio R_l/R_w.

There are several methods of reducing the error due to loading. One means, applicable to servomechanisms deriving the input signal from a potentiometer, is to use identical potentiometers for both the input and the output and to load both by the same amount.

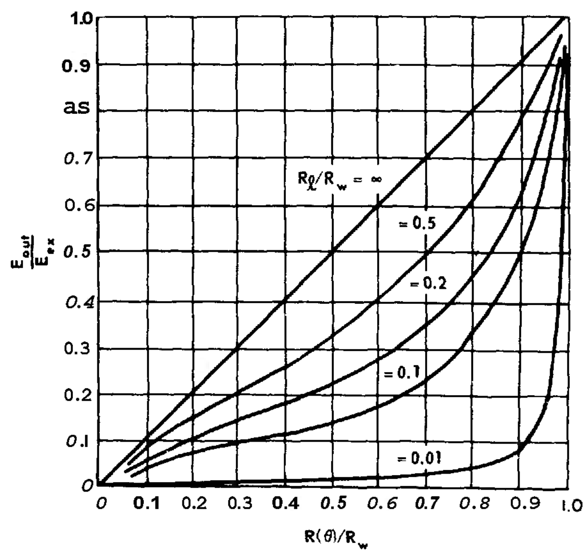

fig. 11-6 Effect of loading on potentiometer output.

Adapted by permission from Electronic *Instruments,* MIT Radiation Laboratory Series, Vol. 21: edited by I. A. Greenwood. Jr., J. V. Holdman, Jr., and D. MacRae, Jr.; Copyright 1948; McGraw-Hill Book Company, Inc.

A second method consists of adding a series resistor R_s as shown in Fig. 11-7. Nettleton and Dole[3] have shown that for $R_l >> R_w$ (by at least a factor of 10) the output voltage cf the uncompensated circuit has the following maximum error, expressed as a percent of the maximum output voltage :

$$\text{percent maximum error} = \frac{100R_w}{11R_l + R_w} \tag{11-17}$$

When R, with a value of $0.28R_w$ is added, however, the maximum error expressed as a percent of the maximum output voltage is

$$\text{percent maximum error} = \frac{17.86R_w}{9R_l + 2R_w} \tag{11-18}$$

A reduction in the loading error in the ratio of about four- or five-to-one can thus be accomplished. However, in order to achieve a desired output voltage for $R(\theta) = R_w$, the supply voltage for the compensated case must be made higher than in the uncompensated case. For $R_s = 0.28R_w$, the supply voltage must be 28 percent higher than the desired output voltage for $R(\theta) = R_w$.

A third method of load compensation consists of using a tapped potentiometer and a parallel resistor R_p as shown in Fig. 11-8. Gilbert[4] has shown that when R_p is $0.31R_l$ and the tap is located so that tap resistance

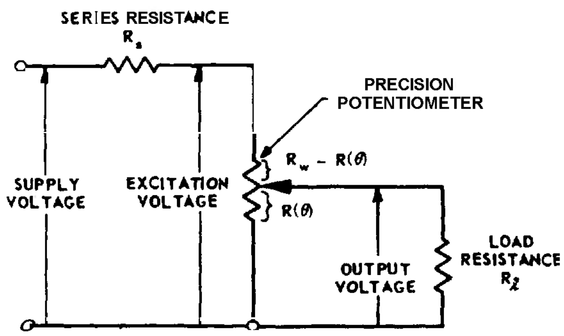

Fig. 17-7 Load compensation by the addition of a series resistor.

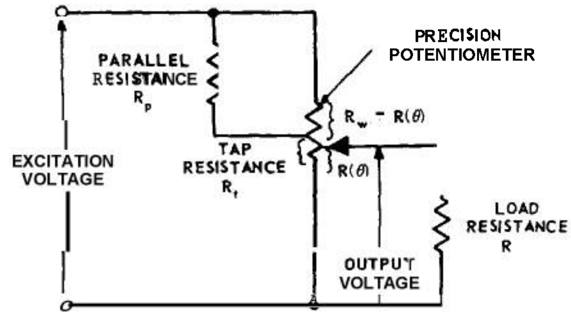

Fig. 11-8 Load compensation by the addition of a potentiometer tap and a parallel resistor.

R_t is $0.74R_w$, the maximum deviation from linearity between the end points of the potentiometer is approximately

$$(DE_{out} = 0.02 \frac{R_w}{R_l} E_{ex} \tag{11-19}$$

for $R_l >> R_w$ (by at least a factor of 10). The maximum deviation without tap compensation is over seven times greater.

Table 11-1[4] shows the results of combining series and tap compensation. The deviation of the output voltage from linearity, $(D)E_{out}/E_{(out)\,max}$, is the deviation voltage divided by the voltage across the precision potentiometer when the slider is at its maximum displacement. This table is applicable for R_l greater than R_w by a factor of at least 10.

Table 11-2 gives this deviation from linearity between end points when an optimum tap point and a parallel resistor are used.

11-2.10 NONLINEAR POTENTIOMETERS

Nonlinear potentiometers of two types are manufactured. In the first type, the resistivity $\rho(\theta)$ varies almost smoothly, so that $R(\theta)$ is a nearly smooth function. This type is restricted to functions where $\rho(\theta)$ need not vary by a factor of more than 5 over the length of the element. The other type of nonlinear potentiometer approximates the required function $R(\theta)$ by a series of straight lines, $\rho(\theta)$ therefore varying in steps. Larger variations in $\rho(\theta)$ can be achieved with this type than with the smooth type of unit.

TABLE 11-1 LOAD COMPENSATION BY THE ADDITION OF BOTH SERIES AND PARALLEL RESISTORS†

$\frac{R_s}{R_w}$	$\frac{R_p}{R_l}$	$\frac{R_t}{R_w}$	$\frac{(D)E_{out}}{E_{(out)max}}$
0.10	0.338	0.77	$\frac{0.017}{(R_l/R_w)}$
0.25	0.255	0.86	$\frac{0.022}{(R_l/R_w)}$

† $\frac{R_l}{R_w} > 10$

Adapted by permission from *Control Engineering,* Volume 2, No. 2, February, 1955, from article entitled 'Here's a Shortcut in Compensating Pot Loading Errors', by J. Gilbert.

TABLE 11-2 LOAD COMPENSATION BY THE ADDITION OF A PARALLEL RESISTOR

$\frac{R_l}{R_w}$	$\frac{R_t}{R_w}$	$\frac{R_p}{R_l}$	$\frac{(D)E_{out}}{E_{(out)max}}$
1	0.73	0.281	0.025
2	0.73	0.304	0.011
3	0.73	0.315	0.0067
5	0.735	0.312	0.0038
10	0.74	0.305	0.002
over 10	0.74	0.311	$\frac{0.019}{(R_l/R_w)}$

Adapted by permission from *Control Engineering.* Volume 2. No. 2, February, 1955, from article entitled 'Here's a Shortcut in Compensating Pot Loading Errors', by J. Gilbert.

The term conformity is used to describe the degree to which the nonlinear potentiometer approximates the desired nonlinear function. *Independent* conformity is the maximum percentage deviation, with respect to the excitation voltage, of the actual electrical output at any point from the best specified function cprve drawn through the output versus rotation data. Terminal *conformity* is the maximum percentage deviation, with respect to the excitation voltage, of the actual electrical output at any point from the specified function curve drawn through the end points. By use of trimmer potentiometers similar to those discussed under linear potentiometers, the performance of a nonlinear potentiometer can be made to coincide with its independent conformity.

Nonlinear as well as linear potentiometers are subject to loading errors, and compensation for nonlinear units is accomplished by the same circuits as those discussed under linear potentiometers. For example, Gilbert[5] shows that where parallel compensation only is used, the tap should be located at 74 percent of total winding resistance R_w and parallel resistor R_p (connected between the tap and the upper end of the total winding resistance) should be 31 percent of load resistance R_l. However, since nonlinear potentiometers very often must be made up specially for a specific application, the custom design can take into account the effect of a specified load and compensate for it in the element construction.

A tapped linear potentiometer can be used to approximate nonlinear functions by the addition of shunting resistors. This technique is described as follows:

"The first step in the design of a shunting circuit is to draw the required nonlinear function as shown in Fig. 11-9. A series of connected straight lines are next drawn to best represent the function. If the tapped potentiometer has already been constructed, the straight lines must join at angular points corresponding to the tap points. If the tapped potentiometer is to be custom-designed for the particular nonlinear application, the best straight-line approximation can be drawn and the tap points placed at the resulting points of intersection. It is usually necessary to space the taps closely in the region of maximum function curvature.

"An important characteristic of the function of Fig. 11-9 is that the slope does not change its algebraic sign throughout the entire function. The curve commences with a positive slope and continues positive throughout the remainder of the function.

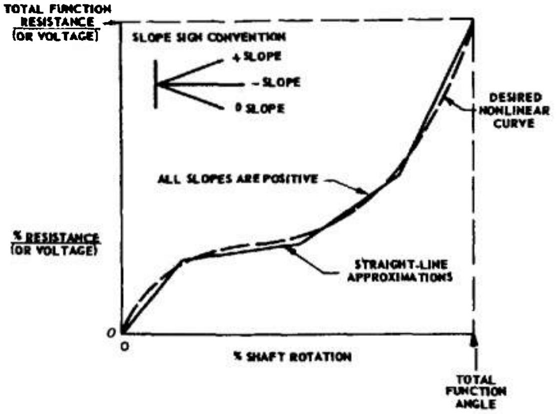

Fig. 11-9 Straight-line approximation of smooth nonlinear function — no change in slope sign.

Reprinted by permission from the *Potentiometer Handbook*, 1956, Technology Instrument Corp.

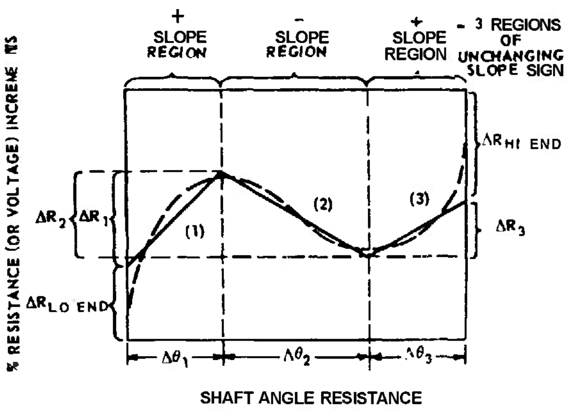

Fig. 71-70 Resistance and shaft angle increments for straight-line approximations to nonlinear function — 3 regions of unchanging slope design.

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"In contrast, the nonlinear function of Fig. **11-10** changes its slope sign twice and thus has three separate regions of unchanging slope sign. Also indicated on Fig. **11-10** are the resistance increments ΔR and shaft angle increments Δθ defined by the straight-line approximation to the curve. These resistance and angle increments are used in the calculation of shunt resistance values.

"The next step is to draw a resistance diagram corresponding to the function diagram of Fig. **11-10.** In the graphical representation of Fig. **11-11A,** it can be seen that the potentiometer resistance element follows exactly the straight-line approximation of Fig. **11-10.** A shunt resistance R_s is connected across each of the three segments of the potentiometer resistance element. End resistors R_e are required at each end of the winding to provide the necessary low-end and high-end voltages. At the points where there is reversal in the algebraic sign of the function, 'pull-in' resistors R_p are used. The resistance diagram can now be redrawn into the more conventional circuit of Fig. **11-11B.**

"In the calculation of resistors R_s, R_e, and R_p, it is helpful to break the complex network problem into problems of a simpler form. This is done by drawing a separate resistance branch for each region of unchanging slope sign. This results in the three resistance branches of Fig. **11-12.** The network division is performed simply by replacing each pull-in resistor R_p with a pair of shunt resistors of value $2R_p$. This form of network division results in equal branch currents i_b flowing in each of the separate resistance branches. Since the same excitation voltage appears across all three branches, the total resistance of each branch, R_b, must be the same.

"This latter condition facilitates the design of a potentiometer shunting network which will have a final resistance, measured between the excitation terminals, of a predetermined value. If the desired resistance of the final shunted potentiometer is called R_d, it can be seen that the branch resistance R_b is simply $R_b = B \times R_d$, where B is the number of resistance branches corresponding to the number of regions of unchanging slope sign. This relation means simply that each resistance branch of Fig. **11-12** must have a total resistance 3 times greater than the desired resistance of the final shunted potentiometer.

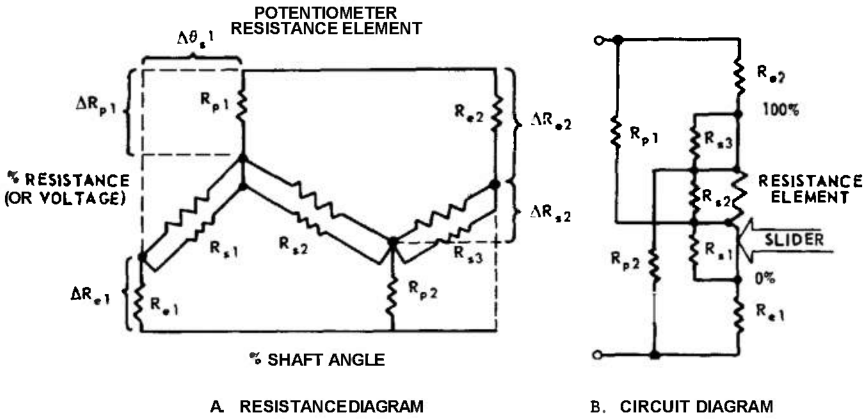

Fig. 11-71 Resistance diagram and resultant circuit diagram for the tapped nonlinear function of Fig. 11-10.

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"To achieve a desired value resistance R_d, it is essential that the winding resistance of the potentiometer to be shunted be greater than a certain minimum value. This results from the fact that shunt resistors can only reduce the net resistance, and the potentiometer winding resistance must be chosen high enough to allow for this reduction effect.

"The minimum allowable winding resistance $R_{w(min)}$ can be determined from the straight-line segment having the greatest slope $(\Delta R/\Delta\theta)_{max}$. The equation relating these variables is

$$R_{w(min)} = \left(\frac{\Delta R}{\Delta\theta}\right)_{max} \times B \times R_d \qquad (11\text{-}20)$$

It is necessary only to choose the potentiometer having a winding resistance R_w that is greater than this defined minimum value.

"The steps required in the design of a potentiometer shunting network can be summarized as follows:

(a) Plot the desired function $\%R$ (or $\%V$) versus $\%\theta$. These percentages are commonly expressed in decimal form with values ranging from 0 to 1.0.

(b) Approximate the desired function with connected straight lines, joined at selected tap points or at tap points predetermined by the potentiometer construction.

(c) Determine the number of regions of the function having unchanging slope sign. This defines the number of resistance branches B (Fig. 11-12). For functions such as that of Fig. 11-9, B simply equals unity.

(d) Draw a resistance diagram (Fig. 11-11) and define the *AR* and $\Delta\theta$ increments. Label the required shunt resistors *R,,* the end resistors *R,,* and the pull-in resistors *R,.*

(e) Select the desired resistance R_d for the final shunted potentiometer.

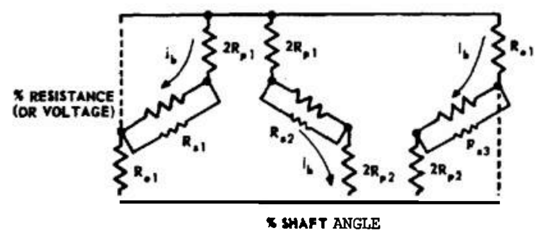

Fig. 11-72 Equivalent resistance diagrom showing a resistance branch for each region of unchanging slope sign.

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(f) Determine the value of slope of the straight-line segment having the maximum slope.

(g) Calculate the minimum allowable value of potentiometer winding resistance $R_{w(min)}$ that is capable of producing the desired function with the specified final resistance R_d [see **Eq. (11-19)**].

(h) Select a tapped potentiometer having a winding resistance greater than this minimum value, $R_w > R_{w(min)}$.

(i) Calculate the required end, pull-in, and shunt resistors from the relations

$$R_e = \Delta R_e \times B \times R_d \qquad (11\text{-}21)$$

$$R_p = \frac{\Delta R_p \times B \times R_d}{2} \qquad (11\text{-}22)$$

$$\frac{1}{R_s} = \frac{1}{\Delta R_s \times B \times R_d} - \frac{1}{\Delta\theta_s R_w}\,^{*} \qquad (11\text{-}23)$$

*Quoted by permission from the *Potentiometer Handbook,* 1956, Technology Instrument Corp.

Nonlinear functions can also be approximated by loading the slider and adding series resistors to a linear potentiometer. Some typical configurations and the corresponding transfer functions are given in Table **11-3.**

Another method of generating nonlinear functions through the use of tapped linear potentiometers is based on clamping the taps at the voltage levels that the desired function has at these points. This is illustrated in Fig. **11-13,** which shows the resultant straight-line approximation of the desired function. It is best to derive the clamping voltages from a very low impedance bleeder as shown, for not only does this eliminate the need for many separate bias voltages, but it permits adjustment of the tap voltages when the slider load resistance is connected. If the load resistance is low, the resultant voltage segments between tap points will be concave and can be made to coincide with the desired function more closely by increasing the tap voltages. If the function slope between taps is steep, care must be exercised that the resultant large voltage drop does not result in currents greater than the potentiometer rating. Thus, the output-voltage scale is determined primarily by the greatest function slope required and the current rating of the potentiometer.

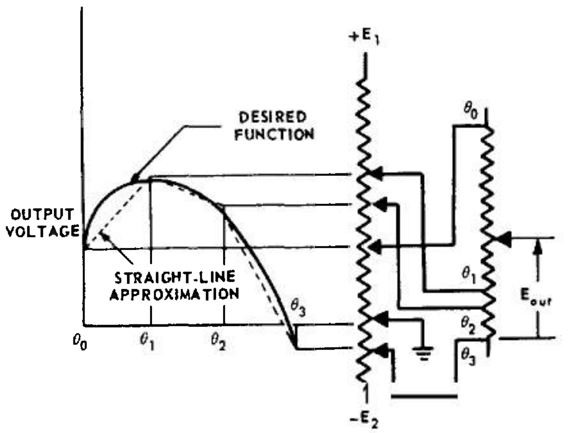


Fig. 11-13 Tapped potentiometer for generating a nonlinear function.

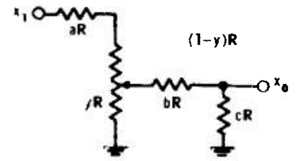
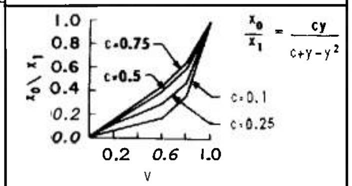
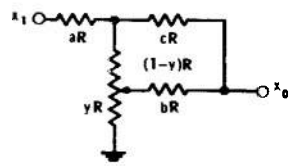
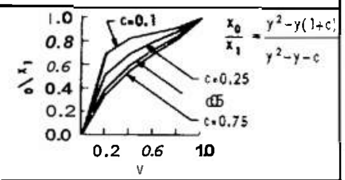
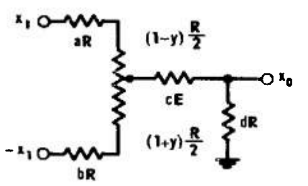
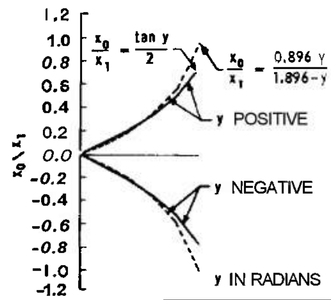
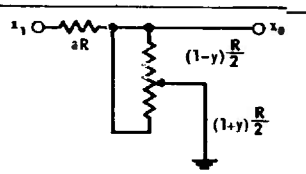
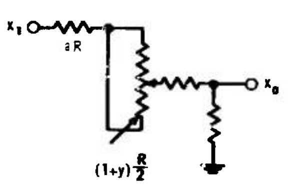
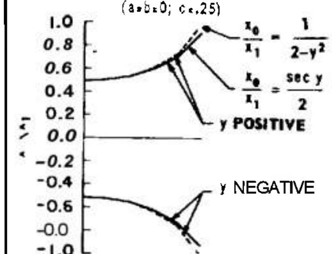
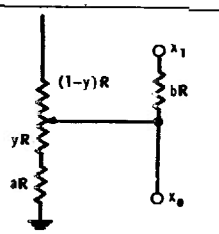
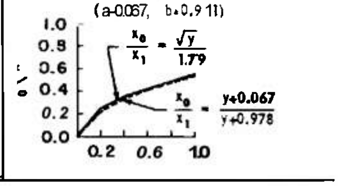
11-2.11 APPLICATION FACTORS

11-2.12 Noise

There are two types of noise voltages, usually not exceeding a few hundred microvolts, that appear at the output of a potentiometer: ***active*** and ***passive.*** The former is due to the motion of the slider over the resistance element. The latter is due primarily to the fluctuating contact resistance between the slider and the resistance element and may be reduced by increasing the pressure of the slider against the element. However, this will shorten the life of the potentiometer and increase the driving torque.

Total noise is expressed as an equivalent noise resistance *ENR,* which is measured **as** shown in Fig. **11-14.** This figure, together

TABLE 11-3 APPROXIMATION OF NONLINEAR FUNCTIONS THROUGH SLIDER LOADING AND SERIES RESISTORS

CIRCUIT	TRANSFER FUNCTION	TYPICAL FUNCTIONS
	$\frac{cy}{(b+c)(a+1)+(a+1)y-y^2}$	
	$\frac{y^2-y(1+c)-b}{y^2-y(1-a)-a-(a+1)(b+c)}$	
	$\frac{4(b+y-a)d}{[2(a+b)+4(c+d+ab+bd+ad+ac+bc)+1]+2(a-b)y-y^2}$	APPROXIMATES Tan y ($a=b=c=0$; $d=.224$) 
	$\frac{1-y^2}{4a+1-y^2}$	APPROXIMATES $\frac{1-y^2}{4a}$ a LARGE
	$\frac{4c}{4(a+b+c)+1-y^2}$	APPROXIMATES Sec y ($a=b=0$; $c=.25$) 
	$\frac{y+a}{y+a+b}$	APPROXIMATES $\frac{\sqrt{y}}{1.79}$ ($a=0.067$, $b=0.911$) 

with the following explanation, is taken from Altieri :[6]

"A source of constant current is placed across one arm of the potentiometer and the slider. The slider and the other arm of the potentiometer are joined to a voltage-measuring circuit having a high input impedance. Owing to the high impedance of the voltage output circuit, the entire source current i_s flows through the left-hand portion of the potentiometer and through the slider.

"If the source current is zero, the noise voltage measured at the circuit output can be attributed entirely to the active noise potential at the slider contact. When the source current is applied, a voltage drop appears across any passive resistance which may exist at the slider point of contact. This passive noise voltage adds directly to the active noise voltage.

"The total voltage output of the circuit is the sum of active and passive residual noise components. Although the two types of noise are of quite different nature (one is a generated voltage and the other is a passive resistance) it is helpful to express both active and passive noise components as ohmic resistances. The total of these resistances is called the ***equivalent noise resistance*** and is defined as the total residual noise voltage divided by the source current (Fig. **11-14**).

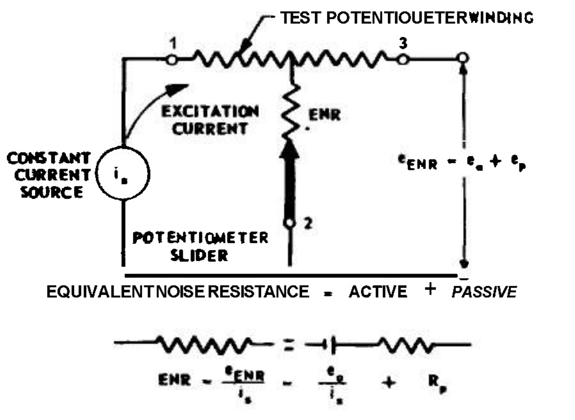


Fig. 11-14 *Circuit* **for** *measuring equivalent noise resistance, ENR.*

By permission from *Instruments*, Volume 26, No. 11, November, 1953, from article entitled 'Residual Potentiometer Noise', by J. R. Altieri.

"The equivalent passive noise resistance appears directly as resistance R_p. The equivalent active noise resistance, however, is the ratio of the active noise voltage *(e_a)* to the source current **(i_s)**. In standardization of equivalent-noise-resistance measurements, the source current has been arbitrarily set at **1** milliampere flowing from the contact to the winding, and the standard rate of slider rotation has been chosen to be **4** rpm."*

***Quoted by permission from *Instruments* Vol. 26, No. 11, November, 1953, from article entitled "Causes and Measurement of Residual Potentiometer Noise" by J. R. Altieri.**

11-2.13 Power Rating

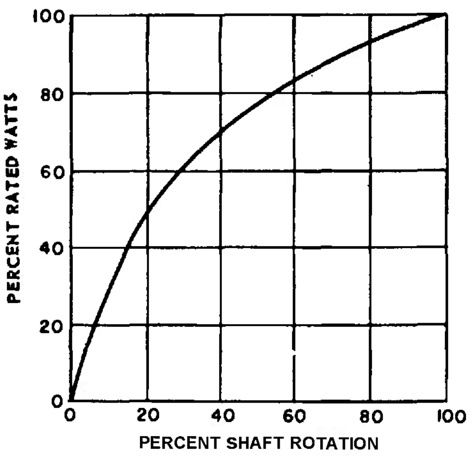
The power rating of a potentiometer states the maximum recommended power that it can dissipate continuously and still satisfy all performance specifications. This power rating is usually specified at a given ambient temperature. Above this temperature, the power rating must be derated. For example, in the instance of potentiometers for which the power rating is given at **40°C,** it is customary to apply a linear derating curve with zero dissipation at approximately ***85°C.***

If a potentiometer is used as a rheostat, the maximum permissible power dissipation must also be derated. Typical derating curves are given in Figs. **11-15** and **11-16.** Because of the better heat-conducting characteristics of metal, the derating curve for a metal-base potentiometer is not as severe as for a bakelite-base potentiometer.

The power ratings of potentiometers suitable as sensing elements range from **0.1** to 10 watts. Small units (a fraction of an inch in diameter) have power ratings in the lower end of the range and large units (several inches in diameter) have power ratings in the upper end of the range.

11-2.14 Environmental Effects

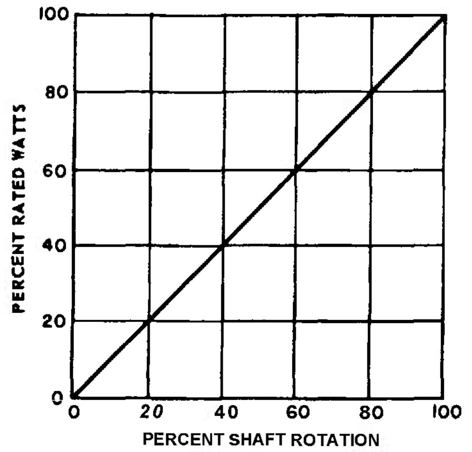
While most potentiometers are intended for use at temperatures no greater than 40°C, units are now available that can withstand temperatures as high as 200°C. In selecting a



OPERATING TEMPERATURE 80°C OR LOWER
AMBIENT TEMPERATURE 25°C
% WATTS = $(-0.8\,\theta^2 + 114\,\theta - 49)^{1/2} + (0.46\,\theta - 5.6)$
WHERE θ IS % ROTATION

Fig. 11-15 Wattage derating curve for rheostat-connected metal-base potentiometers.

Reprinted by permission from the *Potentiometer Handbook,* 1956, Technology Instrument Corp.



OPERATING TEMPERATURE 80°C OR LOWER
AMBIENT TEMPERATURE 25°C

fig. 11-16 Wattage derating curve for rheostat-connected bakelite-base potentiometers.

Reprinted by permission from the *Potentiometer Handbook,* 1966, Technology Instrument Corp.

potentiometer, care must be taken to select one that can withstand the highest expected temperature, which is a function of the ambient temperature as well as the presence of any nearby heat sources. It must be remembered that temperature affects the total element resistance. From a knowledge of the temperature coefficient of the element and the expected operating temperature (which is a function of the ambient temperature, the presence of heat sources, and the power dissipation in the potentiometer), the resultant element resistance can be computed.

Humidity, fungus, salt spray, and altitude are also factors that must be taken into consideration when selecting a potentiometer. Potentiometer components can be protected against the effects of these factors to various degrees, with hermetically sealed units offering the greatest protection.

Acceleration is another environmental factor that affects the choice of a potentiometer. Units have been designed to withstand acceleration as high as 10 g's for frequencies up to 500 cps.

11-2.15 Life

The life of a potentiometer depends upon the condition of its use. It does not follow that the less the slider rotates the longer the life, for slider motion helps keep the surface between slider and resistance element free of oxide and dirt particles. Well-made potentiometers have been cycled continuously several million times, and many manufacturers now rate the life of their units at one million cycles.

11-2.16 Mechanical Loading

The torque load of the slider on the driving shaft varies with individual designs and ranges Erom several thousandths of an ounce-inch to several ounce-inches. Most units require about one ounce-inch of starting torque and half that amount for running torque. Moment of inertia figures also vary greatly for different designs and range from less than a thousandth of a gram-square centimeter to several hundred gram-square centimeters.

Most designs result in a slider-assembly inertia of several tens of gram-square centimeters.

11-2.17 Lubrication

In some designs, it has been found desirable to immerse the wire-wound resistance element in a light mineral oil. The flushing action of the oil reduces electrical noise, disperses wear products which otherwise would tend to lodge between the wire turns, and increases life because of its lubricating action. An example of lubricated potentiometers can be found in the NIKE AJAX Instruction Manual, Vol. VI, Chapter 11.

11-3 ROTARY TRANSFORMERS*

11-3.1 GENERAL DESCRIPTION

A rotary transformer is a device in which the coupling between a set of stator coils and a set of rotor poles (not necessarily wound) can be varied by a rotation of either the rotor shaft, the stator assembly, or both. Such a device may be used for the following purpose~(:~~)

(a) To transmit angular information to remote points.

(b) To modulate an electrical signal with mechanical information.

(c) To demodulate an electrical signal and furnish the output information in electrical or mechanical form.

11-3.2 GENERAL CLASSIFICATIONS

Rotary transformers are classified by many different methods according to system, application, construction, basic principles of operation, or manufacturers' trade names.

11-3.3 Use in Positional Systems

In system work, a common use for rotary transformers is the transfer of positional information from one point to another ; e.g., in a positional servo where an output shaft position is required to be compared with an input shaft position. For this purpose, two or more rotary transformers are used' in a subsystem or in conjunction with the servo system. In such cases, one or more of the devices may be used to receive, modulate, or transmit data while another may be used to produce a torque that is a function of the input received from the data-handling units. For the present discussion, the torques produced by rotary transformers will be considered to be in the order of a few inch-ounces because, in a strict sense, rotary transformers are small-torque devices.

11-3.4 Miscellaneous System Uses

Rotary transformers are also used in systems that require modulation of electrical waveforms, resolution of vectors into components, and vector combining processes. In general, such uses are concerned only with data-handling operations.

11-3.5 General Functional Classification

Considering the above, rotary transformers may be classified according to their use in systems, such as : data-handling units ; and torque-producing units. The category into which a unit falls depends upon the load requirements. If the required torque output is furnished by the device handling the data, the device is classified as a data-handling unit. On the other hand, if the required torque output exceeds the capacity of the data-handling device, a torque-producing unit must be used.

**By A. Kusko*

11-3.6 General Unit Classification

Other general classifications of rotary transformers involve their construction, basic principles of operation, or a combination of these categories. This method is used to designate the general type of rotary transformers discussed in the following sections; i.e., synchros, induction resolvers, induction potentiometers, toroid-wound rotary transformers, and microsyns. Further classifications of these general types are pointed out as they occur.

11-3.7 SYNCHROS

Synchros are rotary transformers that, with minor changes in basic design, may be used for either data-handling or torque-producing functions. Most synchros have a configuration similar to that of a conventional 3-phase alternator of fractional horsepower size. The form of the rotor and the arrangement of the rotor winding identify the type of synchro and its function.

11-3.8 Stator Construction

In general, the synchro stator is a cylindrical slotted and laminated structure with three windings arranged in the slots at 120° spatial displacement from each other. In most units, the slots are skewed one slot pitch to avoid "slot lock" and resulting angular errors. Units that do not have skewed stator laminations are constructed with skewed rotor laminations.

The stator windings are not 3-phase in the usual sense because all induced voltages are in time phase. They can be either Y-connected or delta-connected (Fig. 11-17) and serve as the secondary winding of the synchro. Stator connections are usually brought out as three leads and labelled S_1, S_2, and S_3.

11-3.9 Rotor Construction

Rotors of standard synchros are of two-pole construction, the most common type being the salient-pole rotor used in transmitter or repeater units. The rotor is a slotted and laminated structure of the "dumbbell" or "H" type. The structure is mounted on a shaft that turns on ball bearings. It carries a machine-wound single-phase spool winding that serves as the primary winding of the synchro. Connections are made to the primary winding through two slip rings and brushes that have full excitation voltage impressed upon them at all times. Rotor lead connections are usually labelled R_1 and R_2.

11-3.10 Synchro Supply

All standard synchros are designed to operate from one of the following supplies : (a) 26 volts at 400 cps, (b) 115 volts at 430 cps, or (c) 115 volts at 60 cps.

11-3.11 Nomenclature

Military Specification MIL-S-20708 describes the method of classification to be used for standard synchros of new design. The type designation of a standard synchro identifies its voltage, size, function, supply frequency, and modification as follows :

(a) Both 26-volt and 115-volt synchros are classified in the same manner, except that the type designation of 26-volt synchros is prefixed by "26V".

(b) The first two digits indicate the ***maximum diameter*** in tenths of an inch. If the diameter is not a whole number of tenths, the next higher tenth is used.

(c) The succeeding group of letters indicates the function in accordance with the following :

First Letter	*Function.*
C	Control
T	Torque

Succeeding Letters	*Function*
D	Differential
R	Receiver
T	Transformer
X	Transmitter
B	Rotatable Stator Winding

When two synchros are enclosed within the same housing, the type designation indicates both units, e.g., 37 TR-TR6a.

(d) The succeeding digit indicates the *frequency* of the power supply in accordance with the following:

Number	*Supply Frequency (cps)*
6	60
4	400

(e) The lower-case letter "a" following the frequency digit indicates the original issue of a standard synchro type. The first modification that affects the external mechanical dimensions or the electrical characteristics of the basic type is indicated by the letter "b", succeeding modifications being indicated by "c", "d", etc.

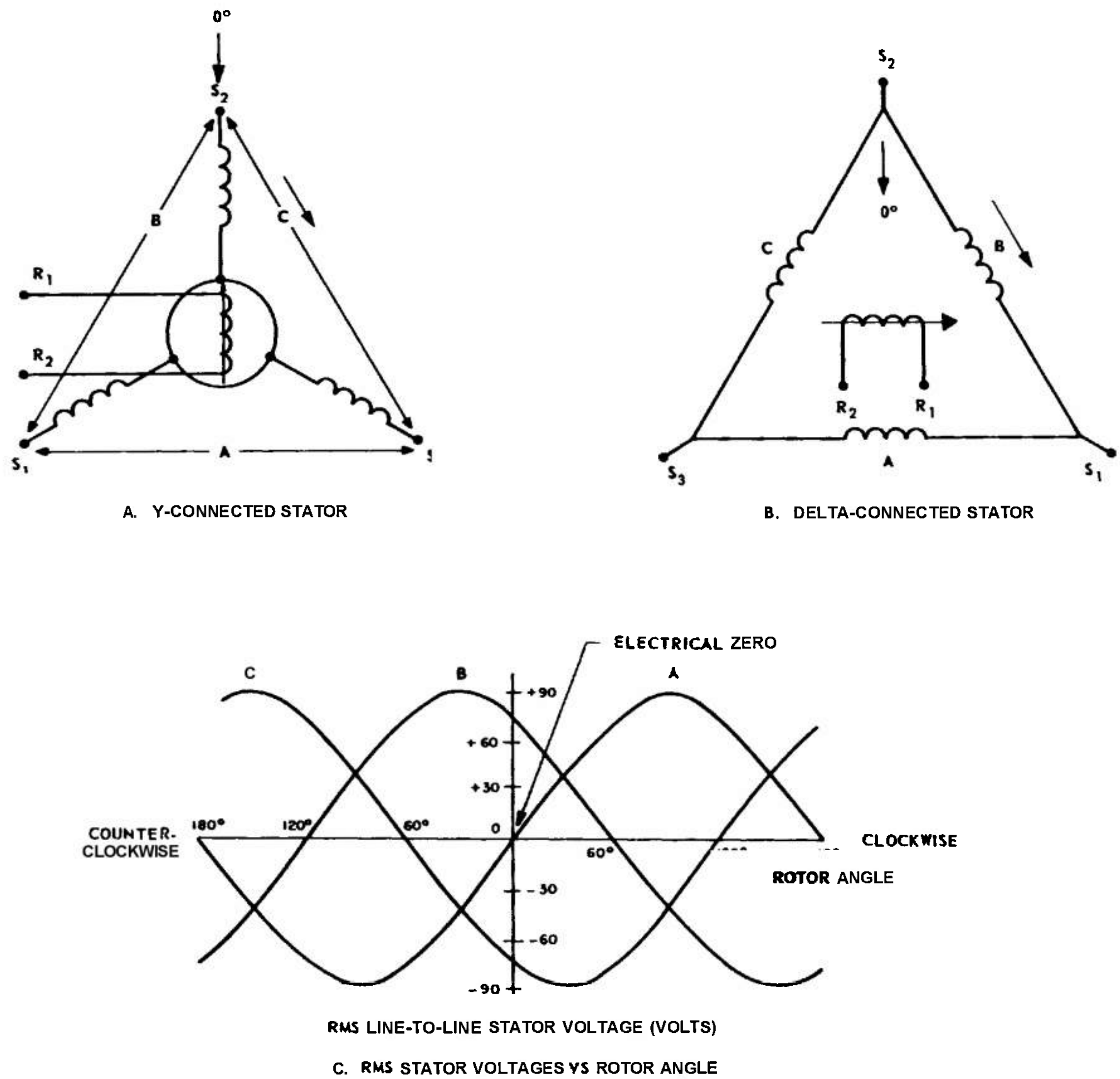


Fig. 11-17 Electrical characteristics of a typical synchro.

As an illustration of the interpretation of the type designation, an 18CDX4b synchro is the first modification of a 115-volt, 400-cycle, control differential transmitter whose largest diameter is between 1.71 and 1.80 inches. If it were a 26-volt synchro, its designation would be 26V-18CDX4b.

11-3.12 Other Methods of Nomenclature

The various services formerly used individual methods of nomenclature which will still be encountered in old designs. The Army method of nomenclature is much like the system described in the previous paragraph, while the Navy method for 115-volt 60-cps synchros differs greatly. The latter synchros have a code designation such as 5 HCT Mark 2 Mod 3B, which identifies the approximate size, special design features, function, type designation, and manufacturer as follows:

(a) The first digit indicates a size group in the following table:

Size	Approx Wt (lb)	Approx Length (in.)	Approx Dia (in.)
1	2	3.9 - 4.2	2.25
3	3	5.2 - 5.51	3.1
5	5	6.0 - 6.8	3.39 - 3.625
6	8	6.4 - 7.5	4.5
7	18	8.9 - 9.2	5.75

Other synchros such as Army units or types with trade names are often referred to as "Size 1" etc., if they approximate the corresponding Navy type.

(b) One or more modifying letters following the first digit indicate that the unit includes *special fittings* as follows:

B —bearing-mounted

F —flange-mounted ; this letter is omitted if letters other than "H" and "S" occur in a receiver type designation

H —special bearings and brushes for high-speed operation (1200 rpm for 1500 hours)

N —nozzle-mounted

S — special unit; i.e., synchro does not conform to standard specifications and therefore is not interchangeable with other units having the same designation but different "Mod" number

(c) One or more letters following the special-fitting letters indicate the function of the unit from Table 11-4.

(d) The *Mark number* signifies the design of the particular unit being specified.

(e) The *Mod number* designates the manufacturer as assigned by the Bureau of Ordnance.

11-3.13 Transmitter Characteristics

The transmitter generates and transmits signal voltages with a magnitude corresponding to the angular position of the rotor. Consider the rotor (or primary) winding to be supplied with a single-phase a-c excitation current. This excitation sets up a flux that varies sinusoidally with the excitation frequency and links each of the three stator (or secondary) windings. The extent of the linkage depends upon the angular position of the rotor, the distribution (120° spatial displacement) of the stator windings, and the configuration of the rotor and stator pole faces. For a given unit, therefore, the magnitude of the single-phase voltage induced in each of the stator windings is a function of the angular position of the rotor.

A plot of the induced stator voltages for different rotor positions is shown in Fig. 11-17. The curves in this plot apply to either Y-connected or delta-connected transmitters. This result makes it difficult to tell whether the stator of a given machine is Y-connected or delta-connected because either type of connection yields exactly the same stator voltages for each position of the rotor.

It follows from the above that, for a given distribution and polarity of the stator voltages, there is but one corresponding rotor position. Conversely, for a given position of the

TABLE 11-4 FUNCTIONAL CLASSIFICATION OF SYNCHROS

Functional Designation	Symbol	Function
Synchro transmitter	X	Generates and transmits electrical signal corresponding to the angular position of the mechanically driven rotor of the unit.
Synchro receiver	R	Produces a torque through the turning of the rotor shaft as a function of an electrical input signal received from the transmitter.
Synchro control transformer	CT	Produces an electrical output signal having a magnitude proportional to the angle of rotation of the unit rotor with respect to the magnetic field set up by the unit stator. Indirectly, the output signal is a sine function of the angle between the rotor shaft position and the shaft position of the transmitter that generates the transformer input.
Synchro differential transmitter	DT	Modifies a received signal and transmits an electrical signal corresponding to the sum or difference of the impressed and modifying signals. The modifying signal is proportional to the angle of rotation of the mechanically driven rotor shaft.
Synchro differential receiver	DR	Produces a mechanical output signal when the rotor shaft of the unit turns in accordance with the sum or difference of electrical signals received from two sources.

rotor, there is but one stator voltage condition. The electrical zero position for Y-connected and delta-connected stators is shown in Fig. 11-17.

11-3.14 Receiver Characteristics

A receiver has the same electrical configuration as a transmitter but the unit output is an angular position of the rotor corresponding to an electrical signal input. The rotor winding of the receiver is excited from the same single-phase a-c source as the transmitter rotor, and the receiver stator terminals are connected to corresponding terminals of the transmitter (Fig. 11-18). Assume for the moment that the rotor winding of the receiver is open and that voltages are induced in the stator windings of the transmitter. These voltages are applied to the stator windings of the receiver and the resulting current flow sets up a magnetic flux in the receiver air gap. Moreover, because the transmitter and the receiver windings have the same configuration and voltages, the orientation of the magnetic field in the receiver is identical with that in the transmitter. Now assume that the receiver rotor winding is switched to the same a-c excitation source that supplies the transmitter rotor. This operation sets up a second polarized field in the receiver air gap. As the second field tends to align itself with the magnetic field induced by currents in the stator windings, the rotor turns until the two fields are aligned. The receiver rotor, therefore, is forced to take up an angular position corresponding to the position of the transmitter rotor.

A distinguishing feature of a receiver is an oscillation damper in the form of a flywheel that is free to rotate relative to the rotor shaft within limit stops set about 45" apart. The flywheel has approximately the same moment of inertia as the rotor. A friction coupling between the rotor shaft and the flywheel serves to dissipate energy when the rotor oscillates as it does when coming into a position of alignment. The added inertia furished by the oscillation damper also prevents the rotor from "running away" when it is forced to follow the transmitter rotor in a continual process. Without the oscillation damper, "running away" would occur at high angular velocities when the torque becomes large enough to overcome the "aligning" torque and thereby prevents the transmitter from maintaining control.

Receivers excited from a 60-cps source can be used as transmitters but the converse is not true. In general, receivers designed for 400-cps application have little tendency to run away and therefore are not provided with dampers.

11-3.15 Transformer Characteristics

A transformer is used with a transmitter to indicate the difference between the angular positions of the shafts of each unit. The electrical connections for this purpose are shown in Fig. **11-19.** The synchro capacitors included in this figure are discussed in Par. **11-3.18.**

The stator windings of a transformer are similar to those of a transmitter except that each winding of the transformer has more turns of wire and therefore a higher impedance. This characteristic reduces the excitation current drawn from the system by the addition of the transformer. A transformer rotor differs from a transmitter rotor in that

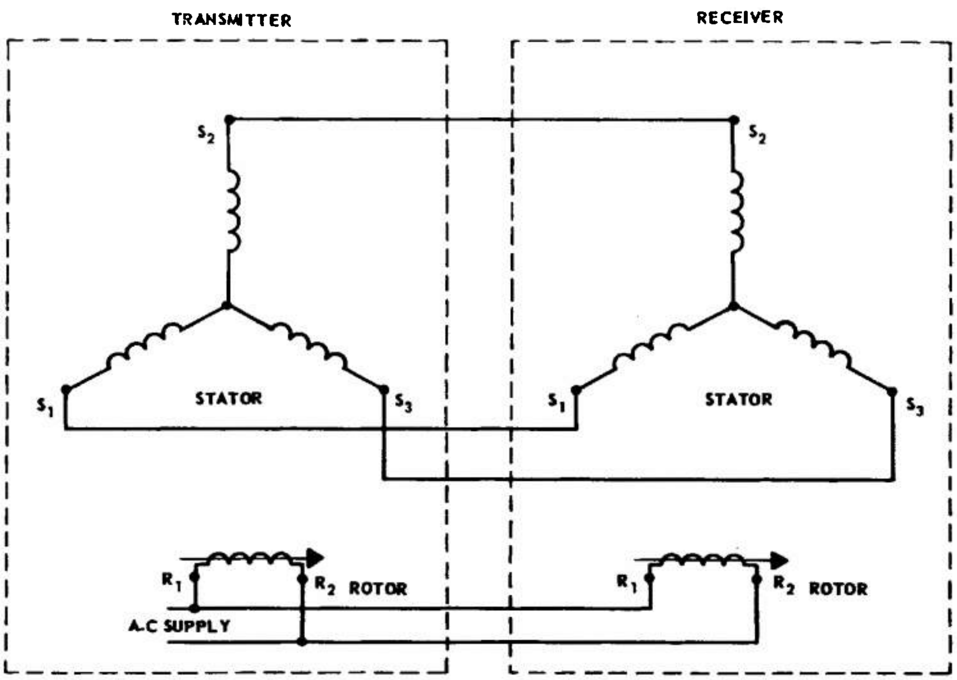


Fig. 11-18 ***Electrical** connections for transmitter-receiver system.*

it has a cylindrical instead of a salient-pole shape and a single high-impedance winding with a large number of turns of fine wire. The cylindrical rotor provides for a constant input impedance regardless of the rotor angular position. The high impedance of the rotor winding provides for a higher and more useful output-voltage gradient.

In a transmitter-transformer system, the transmitter stator voltages cause excitation currents to flow in the transformer stator windings. These currents produce a **flux** corresponding to the angular position of the transmitter rotor. The voltage induced by the flux in the transformer rotor has a magnitude that depends upon the position of the transformer rotor with respect to the flux and therefore with respect to the position of the transformer rotor. It follows that the induced voltage is at a maximum when the transformer rotor is in a position to link the maximum flux. Hence, a 90° displacement of the rotor from its maximum position places the rotor coil across the flux and the induced voltage is zero. The latter position (often called the "null" position) is the normal condition of transformer operation.

From the above, the transformer output voltage is a function of the error angle between the relative rotor positions of the transmitter and the transformer. Moreover, by transformer design, the magnitude of the output voltage is proportional to the sine of this error angle.

The impedances of the stator and rotor windings of a transformer are considerably higher than those of an equivalent-sized transmitter or receiver. Hence, a transformer should never be used to feed a low-impedance load. Because the output of a transformer is normally connected to an amplifier with high input impedance, the rotor current and therefore the developed torque are negligible.

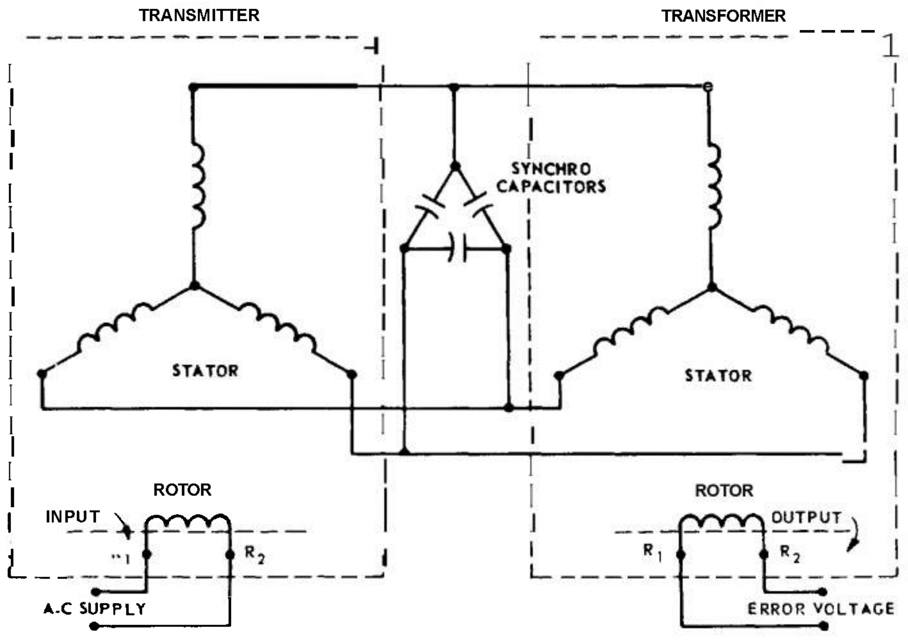


Fig. 11-79 **Electrical connections for transmitter-transformer system.**

11-3.16 Differential Transmitter Characteristics

A differential transmitter is commonly used as a component of a differential synchro system. Such a system is used when it is desired to have the angular position of an output shaft equal the sum or difference of the angular positions of two input shafts.

A differential transmitter has a stator identical with that of a transmitter or receiver but the rotor is cylindrical instead of salient-pole and has three windings spaced 120° apart like the stator. The turns ratio between the rotor and stator windings is approximately 1:1. Three slip rings and brushes are used for connections to the rotor windings (Fig. 11-20). Both Y-connections and delta-connections may be used. Normally, the differential transmitter is an intermediate unit connected between the output of a transmitter and the input of a receiver or a transformer. By this arrangement, the differential transmitter receives an electrical signal corresponding to the angular position of the transmitter rotor., modifies this electrical signal by an amount corresponding to its own rotor position, and transmits the modified electrical signal to the stator of the receiver or the transformer. This electrical input to the receiver creates a magnetic field having an orientation that is either the sum or difference of the rotor angles (or inputs) of the transmitter and the differential transmitter.

The output signal of the differential transmitter system equals the sum or difference of the two input signals depending upon the following:

(a) The relative directions of rotation of the transmitter rotor and the differential transmitter rotor.

(b) The electrical connections between the transmitter and the differential transmitter.

(c) The electrical connections between the differential transmitter and the receiver or transformer.

Different relative directions of rotation can be obtained by interchanging pairs of the input and the output leads of the differential transmitter. An arrangement of leads as shown in Fig. 11-20 is called a *symmetrical* connection. The receiver angle for the symmetrical connection, and changes thereto, equals the sum or difference of the two input angles as follows:

(a) Symmetrical connection (Fig. 11-20): receiver angle = transmitter angle − differential transmitter angle

(b) Two input and output leads interchanged: (e.g., transmitter S_1 and S_3 to differential transmitter S_3 and S_1, respectively, and differential transmitter R_1 and R_3 to receiver S_3 and S_1, respectively)

receiver angle = transmitter angle + differential transmitter angle

(c) Two input leads interchanged: (e.g., transmitter S_1 and S_3 to differential transmitter S_3 and S_1, respectively)

receiver angle = − (transmitter angle + differential transmitter angle)

(d) Two output leads interchanged: (e.g., differential transmitter R_1 and R_3 to receiver S_3 and S_1, respectively)

receiver angle = differential transmitter angle − transmitter angle

When the rotor windings of the differential transmitter in Fig. 11-20 are connected to a transformer instead of a receiver, the transformer output voltage is a measure of the error angle between the transformer rotor angle and the combined rotor angle of the transmitter and differential transmitter that positions the transformer flux.

11-3.17 Differential Receiver Characteristics

Differential synchro systems also employ a differential receiver as an intermediate unit between two transmitters (see Fig. 11-21). The construction of a differential receiver is similar to that of a differential transmitter except that the differential receiver, as an ordinary receiver, has an oscillation damper. In a differential receiver system, the stator of the differential receiver is connected to the stator of one of the associated transmitters and its rotor is connected to the stator of the

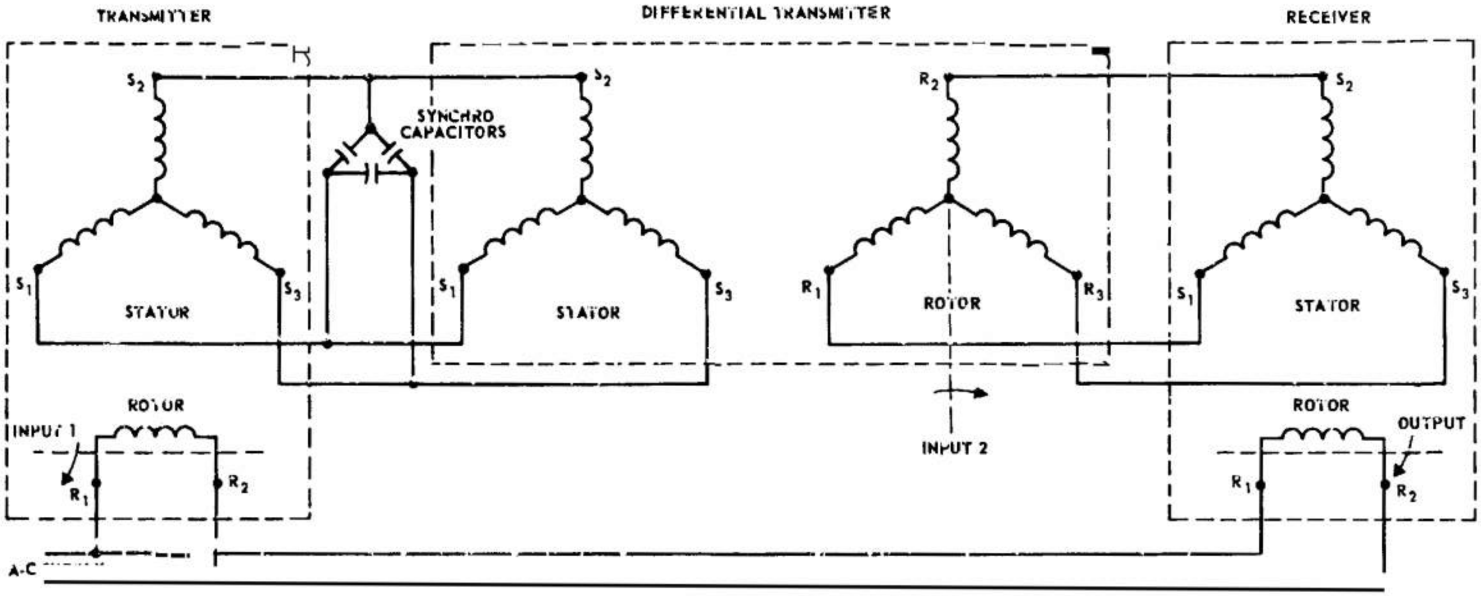


Fig. 11-6 Electrical connections for a differential transmitter system.

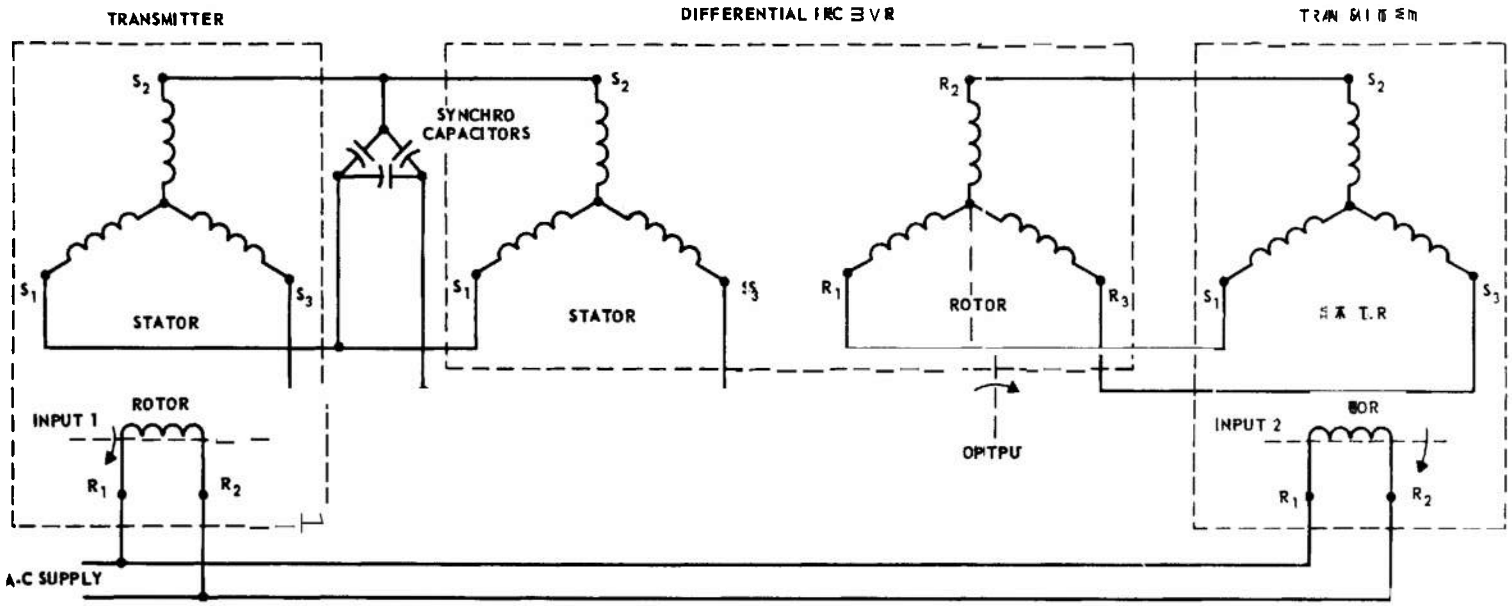


Fig. 11-21 Electrical connections for a differential receiver system.

other transmitter. The differential receiver therefore has two electrical inputs and its shaft output is the mechanical output of the system.

The voltages applied to the stator and to the rotor of the differential receiver depend upon the rotor positions of the supplying transmitters. It follows that two magnetic fields are created in the gap of the differential receiver, one depending upon the rotor position of one transmitter and the other depending upon the rotor position of the other transmitter. Hence, if the rotor of the differential receiver is free to turn, it assumes a position in which the flux vectors of both magnetic fields are aligned. In this position, the rotor angle of a differential receiver connected as in Fig. **11-21** equals the rotor angle of the left transmitter minus the rotor angle of the right transmitter. Reversing pairs of connections at the differential receiver changes the relative directions of motion on the two transmitter shafts as previously described for the differential transmitter.

11-3.18 Synchro Capacitors

Synchro capacitors are used with transformers, differential transmitters, and differential receivers to neutralize the lagging component of the magnetizing current drawn from the transmitters supplying the stators of the units concerned. For this purpose, three matched capacitors are delta-connected as shown in Figs. **11-19** to **11-21.** The leading current drawn by the capacitors at almost zero power factor neutralizes the lagging component of the magnetizing current and thus reduces the total current drawn from the transmitter. To ensure proper operation, the capacitance of the three legs of the capacitor must be balanced to within **1** percent but the absolute value of the legs may vary over $\pm$**10** percent. For maximum effectiveness, the leads connecting the capacitor bank to its associated transformer **or** differential unit should be as **short** as possible.

11-3.19 Dual-Speed Synchro Systems

To provide synchro systems with a greater over-all accuracy, it is possible to gear up the transmitter input shafts from their driving devices and to gear down the shaft output of the associated receiver or transformer by the same ratio. Such gearing reduces the effect of the synchro no-load errors by a factor equal to the gear ratio. Load errors are reduced even more because the torque at the load shaft causes less torque on the receiver and a smaller error at the receiver shaft. However, the friction and backlash of the gears introduce errors; and a geared system is not completely sef-synchronous because it is possible for the receiver to "slip a pole" and find a synchronous position other than the correct one.

Consider a 36-speed system where one revolution of the driving shaft turns the transmitter rotor through 36 revolutions, and 36 revolutions of the transformer or receiver shaft correspond to **1** revolution of the system output shaft. In such a system, there are **35** incorrect positions of the input-output relationship for one correct position. To obtain accuracy without such false indications, it is common practice to add a duplicate system of synchros operating at a low ratio (usually **1:1**). The combination of the 36-speed and the l-speed systems is referred to as a *dual-speed* system. Incorrect alignment of the 36-speed system is prevented because, when the error is large, the l-speed system has control and reduces the error to a small value. The 36-speed system then takes control and maintains its inherent high accuracy.

The use of speed in connection with the system as a whole should not be confused with the gear ratio; e.g., an n-speed synchro is one that turns n times for one turn of the actuating device. Common speeds are **1, 2,** 10, **18,** and 36. Note also that, in a dual-speed system, all elements in the system must be duplicated. For example, in Fig. 11-21, two pairs of transmitters and also two differential receivers would be required.

11-3.20 Zeroing

For correspondence between the angular indications at the transmitting and receiving ends of a synchro system, it is necessary to orient the units at each end with respect to each other. For this purpose, a definite relative position of the rotor with respect to the stator has been designated as the "electrical zero" position for *all standard synchro units.* This position is the relative angular position of the rotor and the stator that results in a minimum, or null, output voltage. The process of rotating the stator or rotor with respect to one another to obtain the "electrical zero" point is called *zeroing.*

To facilitate the zeroing of synchros, the units are constructed with standardized flanges concentric with the shaft. In some synchros, the flanges are located on the head end of the frame; in others, near the center. In either case, the synchros are mounted and held in place by a ring retainer or by dogs acting on the flange. Thus, by loosening the ring or the dogs, the stator may be rotated until the synchro output corresponds to the zero shaft position. When this procedure is followed for all synchros in a system, the system will operate properly.

In Fig. **11-17,** it is shown that the voltage between stator terminals S_1 and S_3 is zero at zero rotor angle. However, this one condition is not sufficient to specify "electrical zero" because the $S_1 - S_3$ voltage is also zero at a 180° rotor angle. For zeroing purposes, therefore, use is made of the fact that the effective voltages of the stator and rotor windings are in the same direction at "electrical zero."

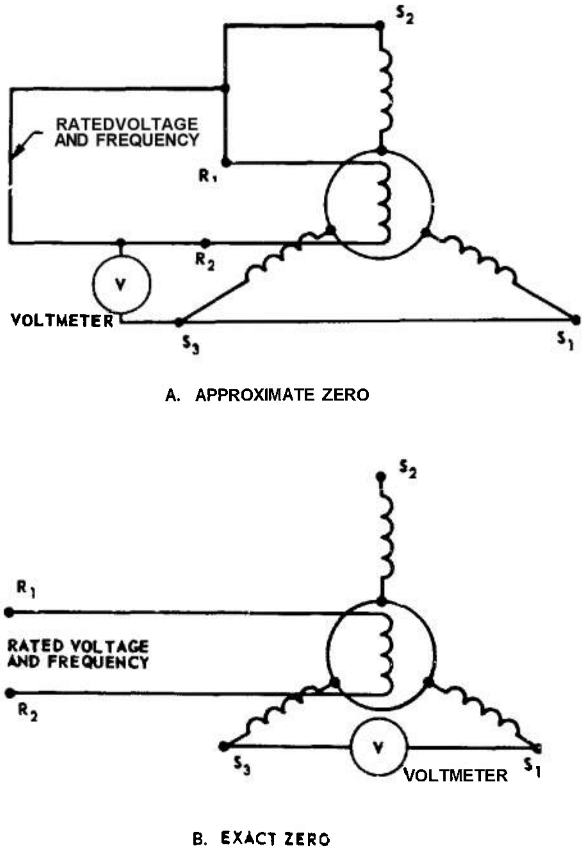


Fig. 11-22 Connections for zeroing a transmitter or receiver.

11-3.21 Zeroing a Transmitter (or Receiver)

The zeroing of a transmitter or receiver may be accomplished with the connections shown in Fig. 11-22. A supply of rated voltage and frequency is used. In determining the approximate zero, the shaft must be free to rotate when power is applied to prevent damage to the synchro.

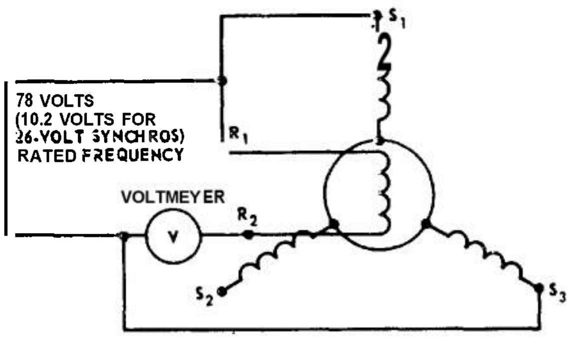
Using the connections shown in Fig. **11-22A,** the rotor will turn by itself to an approximate zero position. An accurate zero is then obtained with the stator terminals open and the voltmeter connected across S_1 and S_3 (Fig. 11-22B). With this arrangement, the rotor or stator is turned slowly until the voltage $S_1 - S_3$ is a minimum.

11-3.22 Zeroing a Control Transformer

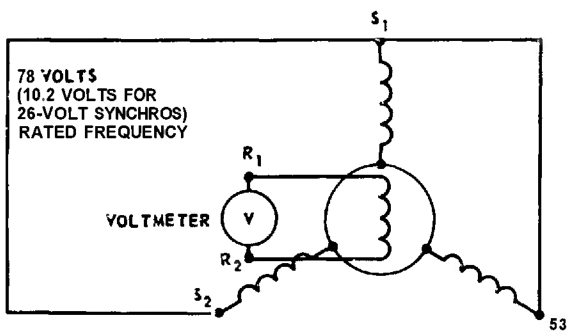
An approximate zero position **of** a control transformer is obtained with the connections shown in Fig. **11-23A.** Approximate zero is the position obtained by rotating the rotor or stator until the voltage reading is a minimum. Then, using the connections of Fig. **11-23B,** a fine adjustment to a zero voltage reading will provide an accurate zero setting.

11-3.23 Zeroing a Differential Synchro Unit

Using the connections shown in Fig. **11-24A,** the rotor of a differential synchro unit will turn by itself to an approximate zero position. For an accurate zero determination, connect the unit as shown in **Fig.** 11-24B and adjust until the voltage across R_1 and R_3 is zero.



A. APPROXIMATE ZERO



B. EXACT ZERO

Fig. 11-23 Connections for zeroing a control transformer.

11-3.24 Torque Relationships

Many synchro applications are based on a continuous and accurate alignment between the transmitter and receiver shafts. For this purpose, the receiver must generate a torque to provide shaft motion and acceleration because the receiver plus the load connected to it always have some friction and inertia. This fact makes perfect alignment impossible because no torque is developed when the two rotor positions coincide.

The torque or rotor current as a function of the displacement angle is shown in Fig. 11-25. Note that the rotor current varies with angular displacement and has its minimum or null point at zero displacement. Note also that the torque is approximately a sinusoidal function of the angle between the transmitter and receiver shafts. The maximum torque occurs at 90° and this torque determines the maximum load that can be handled by the receiver. For accuracy, the receiver should be operated at light loads and over small continuous angular displacements (e.g., not exceeding 20°) so that momentary overloads will not pull the motor out of step. Operation at light loads also prevents overheating of the units.

11-3.25 Torque gradient. The torque gradient of a synchro is defined as the slope of the torque curve at the point of zero displacement (Fig. 11-25). This gradient is expressed in inch-ounces per degree of rotor displacement. Because an increase in the torque gradient gives a proportional decrease in the static error, it is desirable that the receiver have a large torque gradient.

11-3.26 Performance prediction. At times, it is helpful to predict the performance of a synchro system in which the receiver is connected to a transmitter of different size and construction. The method used for such a prediction is based on the following known operating characteristics:

(a) The unbalanced stator voltages of a standard transmitter or receiver are the same regardless of the size of the units.

(b) The only electrical characteristic that varies with the size of a synchro is the internal impedance of the stator windings.

(c) When two standard synchros of the same size are connected together, the internal impedance determines the amount of current flowing and thereby the torque produced.

(d) The torque produced by the synchros in (c) can also be determined by measuring the torque gradient of the receiver. A torque gradient obtained in this manner is called the ***unit torque gradient*** and is inversely proportional to the internal impedance of the stator windings.

In view of the above, consider a receiver driven by a transmitter having a unit torque gradient equal to *R* times that of the receiver.

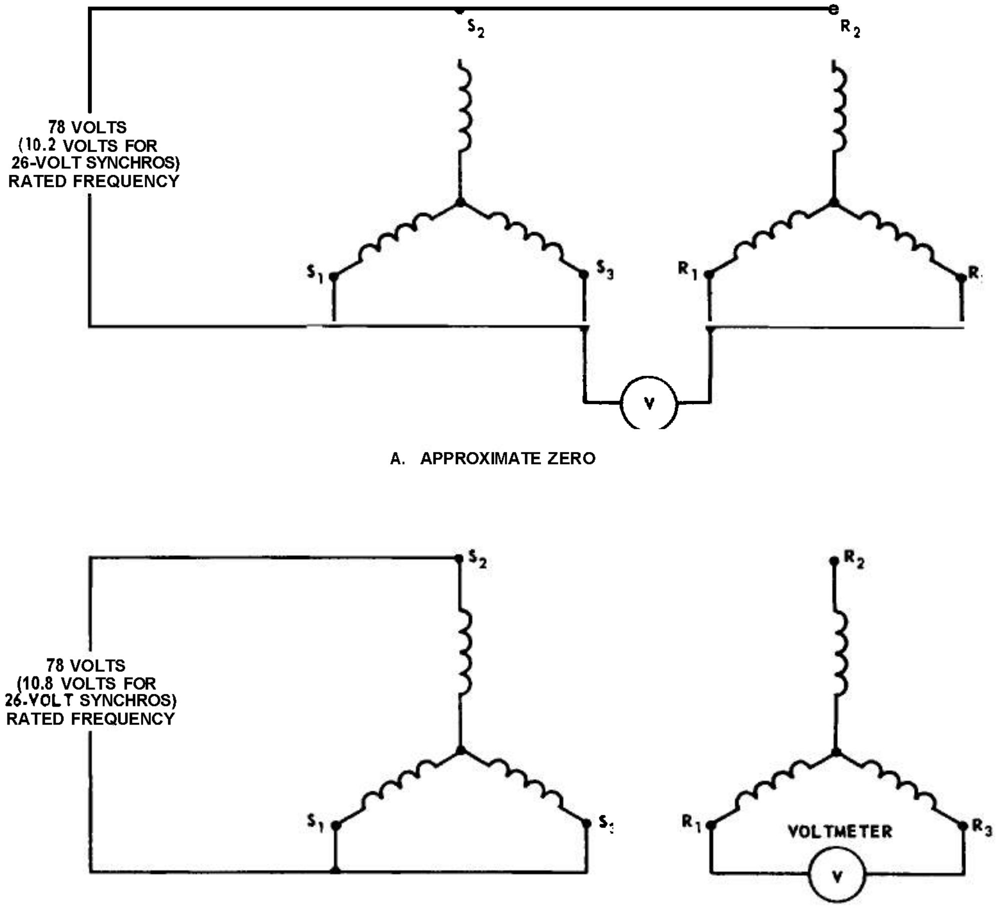


Fig. 1 1-24 Connections for zeroing a differential synchro unit.

Then the actual receiver torque gradient will be [redacted] $(1 + R)$ times the receiver unit torque gradient. Consider now that a large transmitter is used to drive a number (N) of small receivers connected in parallel, each having an equal load. Under these conditions, each receiver will develop an actual torque gradient of $2R/(N + R)$ times its unit torque gradient. This relationship does not apply when the receivers are loaded unequally **or** in systems using differential units or control transformers.

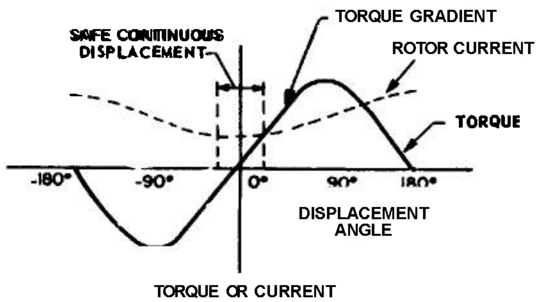


fig. 1 7-25 Torque and rotor current as a function of displacement angle.

11-3.27 Synchro Accuracy

Under static conditions, all synchros and synchro systems are subject to errors created by unavoidable imperfections in design, manufacture, or operation. Torque-producing synchros are also subject to dynamic errors introduced while overcoming friction and inertia. For specified operating conditions, the maximum total error of typical data-handling synchros (transmitters, differential transmitter, and control transformers) corresponds to an angle in the order of 10 minutes of arc. For torque-producing synchros (receivers and differential receivers) the maximum total error is in the order of 1.5". These maximum error values may be increased or decreased slightly depending upon the design of the unit, its manufacture, and its arrangement in a system.

As mentioned previously, perfect alignment between the transmitter and receiver shafts at the null point is an impossibility. This case is an example of an unavoidable design imperfection. An example showing how synchro accuracy can be improved by proper design is explained in Par. 11-3.18 on synchro capacitors. Manufacturing imperfections that produce static errors are as follows:

(a) Irregularities of the stator bore

(b) Rotor eccentricity

(c) A nonuniform air gap resulting from slot openings around the stator periphery.

The above imperfections introduce electrical errors as shown in Fig. 11-26. Another source of static error is electrical noise.

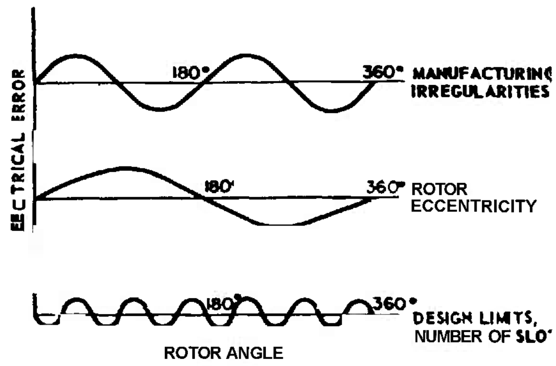


Fig. 17-26 Sources of error in a synchro unit.

11-3.28 Static errors. Static errors tend to increase the null voltage. The amount of null voltage that can be tolerated depends upon the amplifier gain and the ability of the amplifier to distinguish between the quadrature and the in-phase signal. The maximum fundamental component of null voltage in typical synchros is 30 millivolts. Depending upon the type of synchro, service specifications usually restrict the value of null voltage to a range of **40** to 60 rms millivolts.

Because an increase in torque gradient gives an approximately proportional decrease in static error, the error of a transmitter-receiver system can be predicted by the method used previously for determining the actual torque gradient of a receiver. The static error of a transmitter-control transformer system may be predicted approximately from the no-load electrical errors of the individual units. If the units are connected so that their shafts turn in the same direction, the no-load electrical error of the system equals the electrical error of the control transformer minus the electrical error of the transmitter. Hence, system accuracy improves if the no-load electrical error curves of both units follow approximately similar variations of the shaft position.

The above methods cannot be used to determine the over-all error of a system employing a differential synchro because the electrical error of such a unit is a function of the rotor positions of other units connected to it In general, the over-all error of a differential synchro system is the sum of the errors of the individual units.(22)

11-3.29 Dynamic errors. The dynamic error of a receiver and differential receiver is the angle by which the receiver shaft lags behind the transmitter shaft during a slow rotation of the transmitter shaft. The chief sources of this type of error are the friction of the receiver bearings and brushes and the inertia of the receiver rotor and load. Another type of dynamic is called a *speed* error because it is a function of the synchro operating speed. The speed error is explained by examining the voltage induced in the stator windings of a transmitter. This voltage is the result of a time rate of change of the flux set up by the currents in the rotor winding. The stator flux ϕ_s linked by a stator coil, the axis of which is inclined to the rotor axis at an angle θ, may may be expressed as

$$\phi_s = KE \cos \omega t \sin \theta \qquad (11\text{-}24)$$

where

E = applied rotor voltage

K = winding and frequency constant for the synchro

$\omega = 2\pi f$ = radian frequency of the voltage applied to the rotor

Thus, the voltage induced in a stator coil of N turns is

$$e = N\frac{d\theta_s}{dt} = NKE \cos \omega t \cos \theta \frac{d\theta}{dt} - NKE\omega \sin \omega t \sin \theta \qquad (11\text{-}25)$$

The desired induced voltage is in the form

$$e = K \sin \theta \sin \omega t \qquad (11\text{-}26)$$

Identification of the constant K' in **Eq. (11-26)** with the factor $-NKE\omega$ in **Eq. (11-25)** shows that the term $NKE \cos \omega t \cos \theta \frac{d\theta}{dt}$ in **Eq. (11-25)** is an additional or error voltage. This error voltage is present only while the rotor **of** the transmitter is rotating, since it is proportional to $\frac{d\theta}{dt}$. If the transmitter is driven at constant speed, $\frac{d\theta}{dt}$ is a constant,

TABLE 11-5 LIMITS OF TOLERANCES FOR MILITARY SYNCHROS (115 VOLTS, 60 CPS)

Type of Synchro	Transmitter					Receiver		
Size	1 G or 1 HG	3 G or 3 HG	5 G or 5 HG	6 G or 6 HG	7 HG or 7 HG	1 HF 1 F	3 F 3 HF 3 B	5 F 5 HF 5 B
Static accuracy max error (minutes)	18	18	18	18	18	90 (1 F) 150 (1 HF)	36 (3F, 3B) 60 (3 HF)	36 (5F, 5B) 45 (5 HF)
Torque gradient min (oz-in./deg)	0.06	0.25	0.40	1.2	3.4	0.06	0.25	0.40
Secondary peak-voltage limits min (volts) / max (volts)	88.2 91.8	88.2 91.8	88.2 91.8	88.2 91.8	88.2 91.8	88.2 91.8	88.2 91.8	88.2 91.8
Primary current max (amp)	0.30	0.40	0.60	1.3	3.0	0.30	0.40	0.60
Primary power max (watts)	4.8	5.5	7.0	15.0	22.0	4.8	5.5	7.0
Secondary load current max (amp)	0.04	0.20	0.35	0.70	1.50	0.04	0.20	0.35
Temperature rise max (°C)	50	50	50	50	50	50	50	50

and the actual induced voltage consists of a suppressed-carrier wave rather than the carrier-frequency wave desired. A typical value of the speed error for a control transformer operating at 1/4 synchronous speed is 4°, and it is due to the difference in amplitude between the actual suppressed-carrier wave and the desired carrier-frequency wave.

11-3.30 Military Specifications

The general specification for 60- and 400-cps synchros is military specification MIL-S-20708. Specification MIL-S-2335 applies to 115-volt, 60-cps synchros and lists the allowable tolerances for such units. These tolerances are given in Table 11-5, in which the synchros are classified in accordance with the Navy nomenclature of Par. 11-3.12. For 400-cps synchros, the military specification is MIL-S-16892; there is also a tentative specification FXS-1066, covering 31, 23, and 15 size 400-cps synchros. The limiting values in Tables 11-6 and 11-7 were taken from MIL-S-16892.

TABLE 11-5 LIMITS OF TOLERANCES FOR MILITARY SYNCHROS (cont)
(115 VOLTS, 60 CPS)

Type of Synchro	Differential Transmitter					Differential Receiver		
Size	1 DG 1 HDG	3 HDG	5 DG or 5 HDG	6 DG or 6 HDG	7 DG or 7 HDG	1 D 1 HD	3 D	5 D
Static accuracy max error (minutes)	18	18	18	18	18	180	81	54
Torque gradient min (oz-in./deg)	0,035	0.15	0.3	1.4	4.0	0.035		
Secondary peak-voltage limits min (volts)	88.2	88.2	88.2	88.2	88.2	88.2	88.2	88.2
max (volts)	91.8	91.8	91.8	91.8	91.8	91.8	91.8	91.8
Primary current (max) (amp) without capacitor	0.3	0.7	1.0	2.0	3.5	0.15	0.20	0.25
with capacitor	0.15	0.20	0.25	0.55	0.55	0.30	0.70	1.0
Primary power max (watts)	8.0	13.0	15.0	23.0	30.0	8.0	13.0	15.0
Secondary load current max (amp)	0.020	0.18	0.25	0.80	2.00	0.020	0.18	0.25
Temperature rise max (°C)	50	50	50	50	50	50	50	50

TABLE 11-5 LIMITS OF TOLERANCES FOR MILITARY SYNCHROS (cont)
(115 VOLTS, 60 CPS)

Type of Synchro	Control Transformer		
Size	1 CT 1 HCT	3 HCT	5 CT, 5 CTB or 5 HCT
Static accuracy max error (minutes)	18	18	18
Voltage gradient (volts-per-degree)	1.0 ± 5%	1.0 ± 5%	1.0 ± 5%
Null voltage max above null (volts rms)	0.06	0.04	0.04
Primary current (max) (amp) without capacitor	0.035	0.035	0.035
with capacitor	0.020	0.020	0.020
Secondary peak-voltage limits min (volts)	54	54	54
max (volts)	60	60	60
Primary power max (watts)	0.7	0.7	0.7
Rotor impedance max (nominal) (ohms)	950	400	200

TABLE 11-6 LIMITING VALUES FOR 115-VOLT, 400-CPS SYNCHROS

Type of Synchro		Torque Differential Transmitter (TDX)				Torque Differentia Receiver (TDR)	
Size		15 & 16	18 & 19	23	31	23	31
Characteristic	Units						
Primary voltage (nominal)	volts	90	90	90	90	90	90
Transformation ratio ± 1%		1.154	1.154	1.154	1.154	1.154	1.154
No-load temperature rise (max)	°C	60	60	60	45	60	45
Receiver error (max)	minutes					60	48
Electrical error, rotor (max)	minutes	10	10	8	8	8	8
Electrical error, stator (max)	minutes	10	10	8	8	8	8
Torque gradient (min)	oz-in./deg	0.011	0.06	0.09	0.60	0.09	0.60
Friction torque at −55°C	oz-in.	0.1	0.2	0.5	0.5		

TABLE 11-6 LIMITING VALUES FOR 115-VOLT, 400-CPS SYNCHROS (cont)

Type of Synchro		Torque Transmitter (TX)		Torque Receiver (TR)			
Size		23	31	15 & 16	18 & 19	23	31
Characteristic	Units						
Primary voltage (nominal)	volts	115	115	115	115	115	115
Transformation ratio ± 1%		0.783	0.783	0.783	0.783	0.783	0.783
No-load temperature rise (max)	°C	60	60	60	60	60	60
Electrical error (max)	minutes	8	8	12	8	8	8
Receiver error (max)	minutes			60	60	60	48
Total null voltage (max)	millivolts	100	100				
Fund. comp. of null voltage (max)	millivolts	75	75				
Torque gradient (min)	oz-in./deg	0.13	0.67	0.013	0.036	0.13	0.67
Friction torque at −55°C (max)	oz-in.	0.5	0.5				

Type of Synchro		Control Transmitter (CX)			Control Differential Transmitter (CDX)		
Size		15 & 16	18 & 19	23	15 & 16	18 & 19	23
Characteristic	Units						
Primary voltage (nominal)	volts	115	115	115	90	90	90
Transformation ratio ± 1%		0.783	0.783	0.783	1.154	1.154	1.154
No-load temperature rise (max)	°C	30	25	25	30	25	25
Electrical error (max)	minutes	12	8	8			
Electrical error, rotor (max)	minutes				10	8	8
Electrical error, stator (max)	minutes				10	8	8
Null voltage, total (max)	millivolts	125	115	100	125	115	100
Fund. comp. of null voltage (max)	millivolts	75	65	50	75	65	50
Friction torque at $-55°C$ (max)	oz-in.	0.1	0.2	0.5	0.1	0.2	0.5

TABLE 11-6 LIMITING VALUES FOR 115-VOLT, 400-CPS SYNCHROS (cont)

Type of Synchro		Control Transformer (CT)		
Size		15 & 16	18 & 19	23
Characteristic	Units			
Primary voltage (nominal)	volts	90	90	90
Transformation ratio ± 1%		0.735	0.735	0.735
Null voltage, total (max)	millivolts	75	65	60
Fund. comp. of null voltage (max)	millivolts	40	35	30
Electrical error (max)	minutes	10	8	6
Friction torque at −55°C	oz-in.	0.1	0.2	0.5

TABLE 11-7 LIMITING VALUES FOR 26-VOLT, 400-CPS SYNCHROS

Type of Synchro		Control Transformer (CT)
Size		11
Characteristic	Units	
Primary voltage (nominal)	volts	11.8
Transformation ratio ± 1%		2.203
Null voltage, total (max)	millivolts	18
Fund. comp. of null voltage (max)	millivolts	15
Electrical error (max)	minutes	7
Friction torque at room temperature	oz-in.	0.05

11-3.31 INDUCTION RESOLVERS

An induction resolver is a rotary transformer similar to a synchro in that it has rotor and stator windings and behaves like a varible transformer. Unlike a synchro, however, a resolver usually has two stator windings acting as the primary and two rotor windings acting as the secondary. Some resolvers have separate connections for each winding ; others use one common lead for the primaries and another for the secondaries. Some models have extra windings for feedback and other models omit either one primary or one secondary (Fig. 11-27). Resolvers are used primarily in electrical computations involving: (1) the interchange of rectangular and polar coordinates; and/or (2) the rotation of coordinate axes. A resolver may also be used as a variable phase-shifting element.

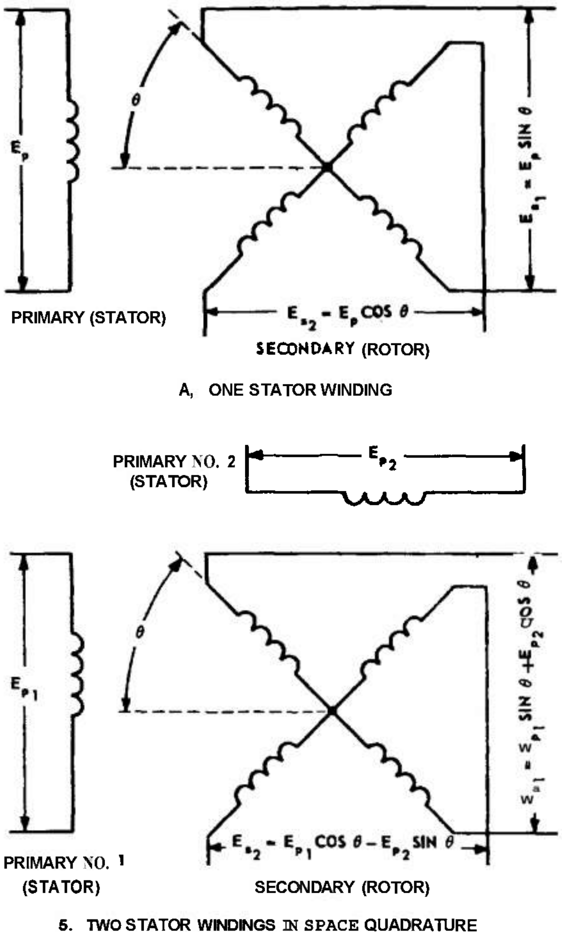


Fig. 11-27 Induction resolver windings.

11-3.32 Basic Operation

The output of a resolver depends upon the primary voltages and upon the position of the rotor with respect to the stator. Currents flowing in the two stator windings set up magnetic fields that are at right angles to one another. The total magnetic field produced by the combined primary currents is the vector sum of the separate fields. The resultant flux may be represented, therefore, by a vector having a magnitude proportional to the instantaneous magnitudes of the primary currents and a direction determined from the arc tangent of the ratio of the two primary currents. The voltage induced in each secondary winding is proportional to the component of the resultant flux vector that lies in a plane perpendicular to the plane of the secondary winding. It is customary to adjust the ratio of primary and secondary turns so that the output equals the input when the rotor is turned to maximize the coupling. For example, in Fig. 11-27A, E_{s_1} equals E_p and E_{s_2} is zero when the rotor angle is 90°. It follows that the output of the windings for other rotor angles are $E_p \sin\theta$ and $E_p \cos\theta$, respectively.

In Fig. 11-27B, the signal applied to one of the primary windings is resolved along the two axes of the secondary rotor windings. A similar resolution is made on the signal applied to the other primary winding. The outputs of each of these resolutions are induced voltages along the secondary windings and produce the outputs shown in Fig. 11-27B. In computing applications, this process is similar to a rotation of coordinates.

11-3.33 Design Principles

To meet the requirements of most resolver applications, the units are designed for operation on a sinusoidal input at either 60 cps or 400 cps. The use of a single operating frequency makes the frequency response relatively unimportant and permits fairly accurate temperature and load compensation. To maintain the desired accuracy, the units should be loaded as lightly as possible. It is possible, however, to use booster amplifiers to drive low-impedance loads from the resolver outputs. Resolvers are normally used with approximately sinusoidal voltages at frequencies that vary slightly from the designed frequency of the unit. However, if the exciting voltages are properly chosen, the units may be operated at widely different frequencies but with a somewhat decreased accuracy.

11-3.34 Applications

Common resolver uses for operation on either 60 cps or 400 cps are as follows:

(a) Vector resolution (Fig. 11-27).

(b) Vector addition. Addition may be performed by a resolver system arranged (Fig. 11-28) to provide either electrical or mechanical outputs, or both. In this arrangement, one of the rotor windings acts as the rotor of a control transformer and is connected to a servo amplifier and drive unit. By virtue of the servo action, this winding is maintained at right angles to the vector magnetic field at all times. The second rotor winding is at right angles to the first and therefore is always aligned with the stator field. The induced voltage in this winding is proportional to the amplitude of the magnetic field. Thus, a vector output is obtained, the magnitude of which is the magnitude of the signal on the second rotor winding and the phase of which is given by the angle between the rotor and the stator. If the stator windings are two in number and are at right angles to each other, a resolver may be considered as a coordinate transformer that transforms rectangular coordinates into polar coordinates.

(c) Computer applications. For computer applications, a resolver is generally used where the voltages serve to represent positive and negative quantities. Positive voltages are those in phase with some convenient reference voltage, while negative voltages are those 180° out of phase with the same reference voltage.

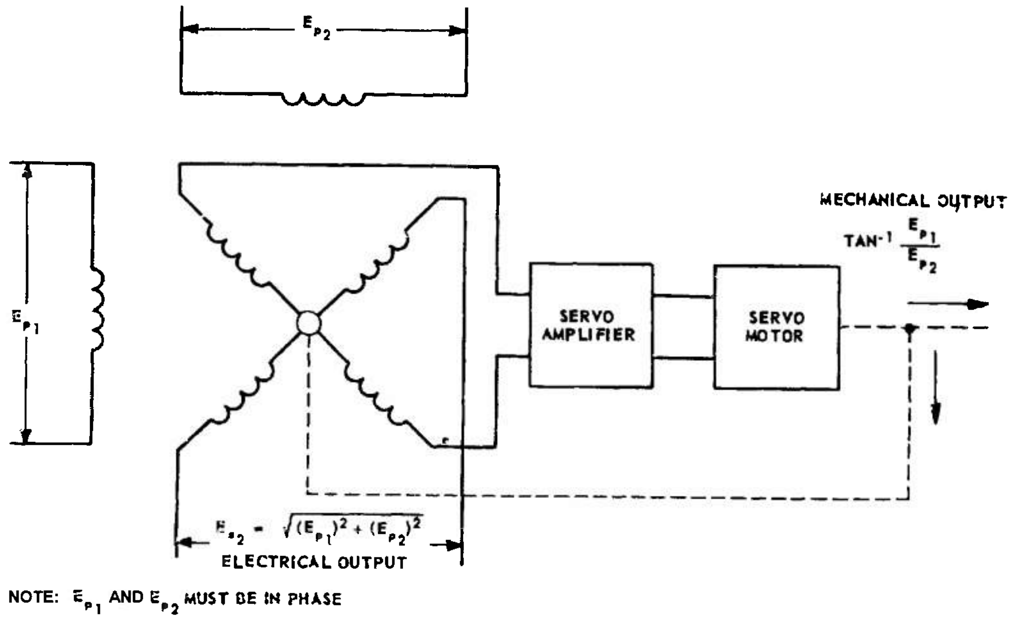


Fig. 11-28 Resolver arrangement for vector addition.

In addition to the common uses mentioned above, resolvers are used for multifrequency (or irregular waveshape) operation; e.g., sweep voltages. Special. units are manufactured for this purpose.

11-3.35 Booster Amplifiers

Because resolvers are not adaptable to the dual-speed operation used with synchro systems, a booster amplifier is used to minimize the inherent resolver errors. A booster amplifier (sometimes called a *resolver amplifier)* eliminates resolver errors resulting from the winding resistance and the leakage reactance in the primary and secondary.

11-3.36 Application of booster amplifiers. Resolvers are frequently cascaded in chains of three or fcur. Hence, the final resolver may be supplied with one input that has passed through all preceding resolvers, whereas the other input has only passed through one. To preserve the proper phases and levels in such cases, it is imperative that there is no change in levels except the one called for by the rotor position. This condition can be fulfilled if neither secondary draws more than a negligible amount of current and if the proper magnetic field is produced by each primary. Both of these requirements are fulfilled for the general case by the use of two separate booster amplifiers with each resolver, one for each primary. Figure 11-29 shows a booster amplifier connected to one primary. The output of one of the secondary windings of the preceding resolver is applied to the booster amplifier, which has a high input impedance and, therefore, draws almost no current. During this process, the field strength produced by the main primary winding is measured by the voltage induced in an auxiliary winding on the stator structure. This induced voltage is applied to the booster input in opposition to the external signal normally received from the secondary of the preceding resolver. The actual booster input is, therefore, the difference between the externa! input and a voltage proportional to the error in the strength of the field. As a result, the booster delivers the current required to produce the proper field. In some cases, all computing voltages may originate at the two secondaries of a resolver having an input on only one primary. In such cases, the booster driving the unused primary may be omitted because any phase shift and change in level will be the same in both secondaries. In almost all other cases, one auxiliary winding and one booster amplifier are used with each primary.

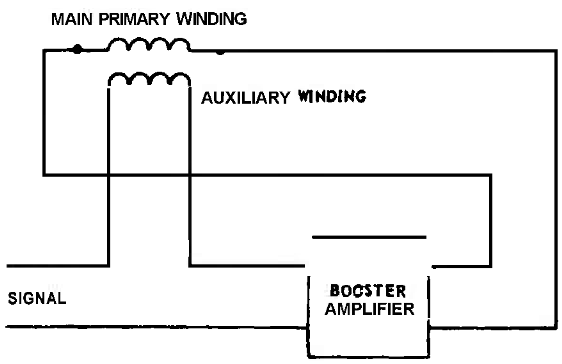


Fig. 17-29 Connections for booster amplifier to primary winding of resolver.

11-3.37 Nomenclature

Military specification **MIL-R-14346** (ORD) describes the classification system used for resolvers. The type designation of a resolver identifies its size, function, impedance, compensation, excitation frequency, and modification as follows:

(a) The first two digits indicate the *maximum diameter* in tenths of an inch. If the diameter is not a whole number of tenths, the next higher tenth is used.

(b) The *function* is identified by the letter *R* to distinguish a resolver from a synchro which might have a similar designation.

(c) The next group of digits is the *nominal input impedance* in hundreds of ohms. If the impedance is not a whole number of hundreds, the next higher hundred is used.

(d) *Compensation* is indicated by a one-letter symbol as follows:

Symbol	*Type of Compensation*
R	Resistor Compensation
W	Winding Compensation
B	Resistor and Winding Compensation
N	No Compensation

(e) The *excitation frequency* is identified by one digit as follows:

Digit	*Supply Frequency (CPS)*
6	60
4	400

(f) A *modification* that affects the mechanical dimensions or electrical characteristics is identified by a lower-case letter, the first modification being assigned "a", etc.

A typical type designation is 23R32N4d, which indicates the fourth modification of an uncompensated 400-cps resolver having a maximum diameter between 2.21 and 2.3 inches and a nominal input impedance between 3110 and 3200 ohms. Typical characteristics of commercial resolvers are listed in Table 11-8.

TABLE 11-8 TYPICAL CHARACTERISTICS OF COMMERCIAL RESOLVERS

Size	11	15	23
Excitation winding	rotor	rotor or stator	stator
Windings: stator	2	1	1
Windings: rotor	2	2	2
Frequency (cps)	400	7 × 103 (nominal)	400
Voltage rating (volts ac)	26/22		26/26
Transformation ratio at 90"	0.864 ± .034		$1.00^{+.00}_{-.02}$
Time phase shift	13.5° ± 1°		1.0" ± 10′
Angular distance between null voltages	90" ± 30'		90" ± 5
Total null voltage (mv)	60	20	20
Fundamental component of null voltage	40		15
Nominal test voltage	26	2	26
Angular accuracy (max) (minutes)	10	60	
Input impedance (ohms)	1510 /71°		550 /86°

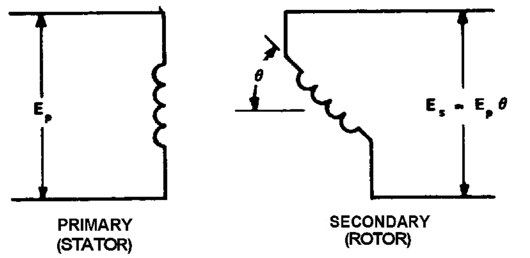


Fig. 11-30 Induction potentiometer.

11-3.38 INDUCTION POTENTIOMETERS

Induction potentiometers are rotary transformers that generate an output voltage which is a linear function of the rotor angular displacement (Fig. 11-30). These devices are commonly used in place of conventional potentiometers to control positioning servos. In positioning servos, the position order is set on a potentiometer shaft and transmitted to the servo as an electrical voltage. A follow-up potentiometer 'at the servo develops an output voltage proportional to the actual servo position, the difference between the two potentiometer outputs being the error voltage. 'When conventional potentiometers are used for this purpose, small errors cause the servo to hunt back and forth between positions corresponding to adjacent turns of the resistance element. This fault is minimized by the use of induction potentiometers which have a higher resolution than most wire-wound resistance potentiometers.

11-3.39 Construction

An induction potentiometer has a configuration similar to that of a synchro or a resolver except that it has only one stator (primary) winding and one rotor (secondary) winding. Both the stator and the rotor are cylindrical and are constructed from magnetic material. Like other rotary transformers, the rotors are mounted on ball bearings. By shaping the pole pieces, the output voltage is generated as a linear function of the shaft angle over a limited range (±90° from zero position). Induction potentiometers are designed for a-c operation only, the rotor winding being supplied through slip rings and brushes.

11-3.40 Characteristics

In comparison with a wire-wound resistance potentiometer, induction potentiometers have lower restraining torques and lower angular errors. Induction potentiometers also have isolated input and output. circuits, low noise levels, and low null-voltage values. Their merit, however, lies in the fact that they have a higher resolution than most wire-wound resistance potentiometers.

11-3.41 TOROID-WOUND ROTARY TRANSFORMERS

Toroid-wound rotary transformers are used primarily for remote position indicators. They are not power transmitting units and therefore should be used only to drive pointers or similar devices when connected back-to-back. A single unit, however, may be connected to a synchro through an amplifier or other matching device and the combination then used as a data-input system to a servomechanism.

11-3.42 Construction

Toroid-wound rotary transformers consist of a strong permanent-magnet cylindrical rotor surrounded by a toroid-wound stator which, in turn, is surrounded by a cylindrical stack of core laminations. The rotor is bearing-mounted and located at the center of the toroid. Its cylindrical shape produces a uniform field around the peripheral toroid. The stator winding is wound on a laminated core of soft, easily saturable material such as permalloy. A cylindrical stack of core laminations surroundingthe stator winding serves to complete the magnetic path when the permalloy has been saturated by a-c excitation. Two taps are taken off at 120" and 240" from the ends of the stator coil dividing it into three equal segments.

11-3.43 Principles of Operation

As shown in Fig. **11-31A,** the permanent magnet causes a steady magnetomotive force H_{st} to be set up in each half of an annular core that is bisected by the axes of the poles of the magnet. Moreover, the core is made of homogeneous material, symmetrical in shape and concentric with the magnet; hence, the reluctances of the two magnetic paths are the same and H_{st} is equal in both halves of the ring. An alternating magnetomotive force H_{a-c} is also set up in the core by the a-c excitation in the stator winding.

Consider H_{a-c} to cause a clockwise flux in the core at a specific instant (Fig. 11-31A). Then the total magnetomotive forces in each half of the core may be expressed as

$$H_1 = H_{a-c} + H_{st} \tag{11-27}$$

$$H_2 = H_{a-c} - H_{st} \tag{11-28}$$

These magnetomotive forces give rise to flux densities B_1 and B_2, each of which is made up of two components; namely, B_{a-c}, which is common to both halves of the ring; and an additional flux, B_x, which exists because of the presence of H_{st} combined with H_{a-c}. Hence

$$B_1 = B_{a-c} + B_x \tag{11-29}$$

$$B_2 = B_{a-c} - B_x \tag{11-30}$$

From Eqs. (11-29) and (11-30), it follows that

$$B_{a-c} = \frac{B_1 + B_2}{2} \tag{11-31}$$

$$B_x = \frac{B_1 - B_2}{2} \tag{11-32}$$

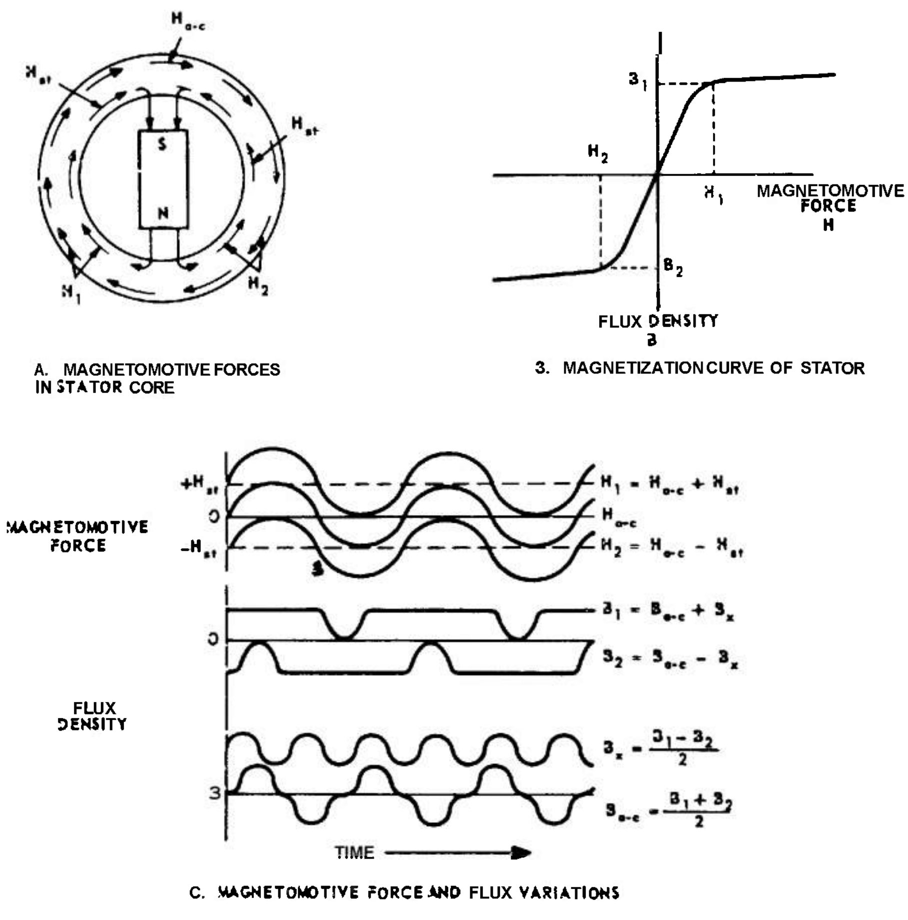


Fig. 11-31 Magnetomotive force and flux relationships of a toroid-wound rotary transformer.

Consider now that the permanent magnet is in a position that creates a magnetizing field corresponding to the point H_1 in Fig. 11-31B. Then the magnetomotive force and flux variations with time will be as shown in Fig. 11-31C. Note that B_x varies at twice the frequency of the fundamental flux B_{a-c} and therefore is a second-harmonic flux. By the above, and the voltage distribution shown in Fig. **11-32A,** the following voltage and flux relationships may be written :

$$E = e_1 + e_2 = e_{a-c} \qquad (11\text{-}33)$$

and

$$e_1 = -k\frac{dB_1}{dt} = -k\left(\frac{dB_{a-c}}{dt} + \frac{dB_x}{dt}\right)$$
$$= \frac{e_{a-c}}{2} - e_x \qquad (11\text{-}34)$$

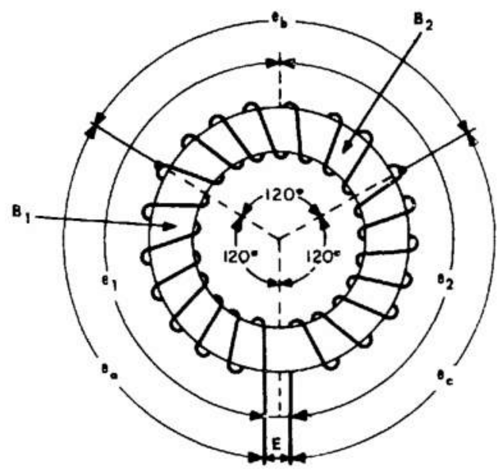
$$e_2 = -k\frac{dB_2}{dt} = -k\left(\frac{dB_{a-c}}{dt} - \frac{dB_x}{dt}\right)$$
$$= \frac{e_{a-c}}{2} + e_x \qquad (11\text{-}35)$$

where

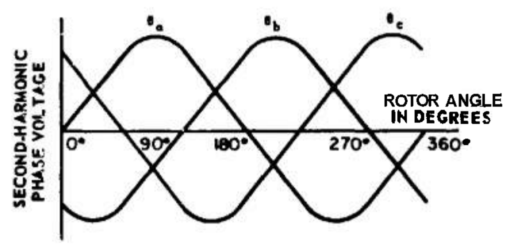
$$e_x = k\,\frac{dB_x}{dt}$$

The frequency of the voltage e_x in the above equations is double that of the voltage E and therefore is a second harmonic of the impressed voltage. This voltage is a maximum at the points of the coil adjacent to the poles of the magnet. With taps located 120° apart (Fig. 11-32A), the phase-voltage distribution is as shown in Fig. 11-32B.

Fundamental-frequency voltages are also present during the above operation but their amplitudes are independent of the rotor position. Hence, when two units are connected back-to-back (Fig. 11-33), the same fundamental frequency voltages appear at corresponding taps of the two units regardless of the position of the rotor. However, the second-harmonic voltages change with rotor position and any difference in the position of the rotors of the two units will produce a voltage unbalance. This unbalance causes currents to flow, thereby producing restoring torques that bring the rotors into alignment.



A. VOLTAGE DISTRIBUTION IN TOROIDAL COIL



B. SECOND-HARMONIC PHASE-VOLTAGE DISTRIBUTION

Fig. 17-32 Voltage distribution in a toroid-wound rotary transformer.

11-3.44 Electrical Characteristics

Toroid-wound rotary transformers are designed for use with 400-cycle 28-volt power and draw approximately 50 ma for the smaller units and 80 to 100 ma for the larger units.[19] The units are built to give accuracies of 1/4° per unit, thus being capable of 1/2° when used in a back-to-back system. The units are also constructed in linear form but this type has a high inherent friction and should be used only as a generator. When linear units are used to drive a rotary unit, errors up to 2° may be expected.

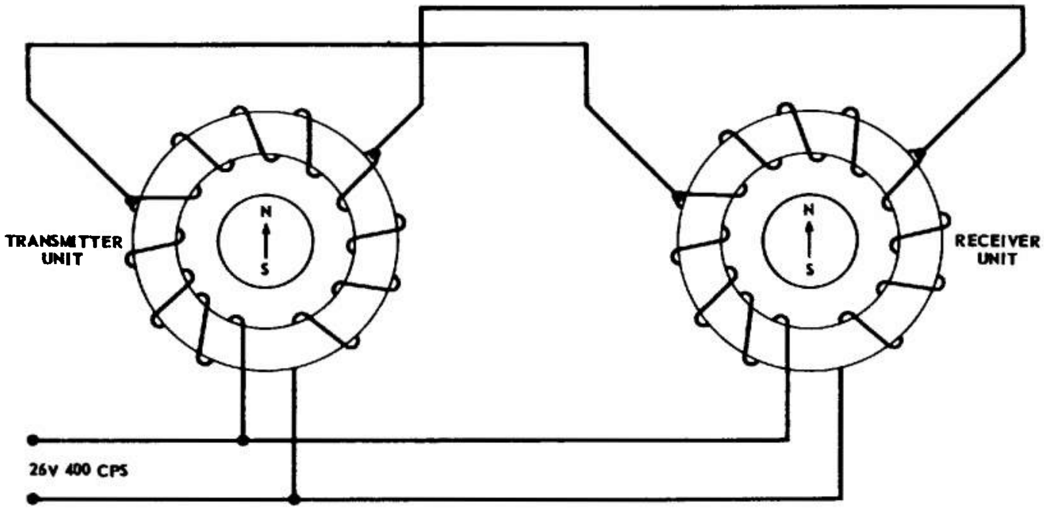


Fig. 11-33 Toroid-wound rotary transformers in back-to-back circuit.

11-3.45 MICROSYNS

Microsyns are rotary transformers of the variable-reluctance type. They consist of a four-pole stator and a special-shaped two-pole rotor, both constructed of silicon-steel laminations (Fig. 11-34). Depending upon the application of the unit, the stator is wound with either one or two continuous windings. The rotor is mounted on ball bearings and, being unwound, requires no slip rings.

11-3.46 Principles of Operation

Microsyn operation depends upon the principle that a magnetic body (the rotor) tends to orient itself in a magnetic field so that its position accommodates the maximum flow of flux. This tendency of the rotor to seek a position of stable equilibrium is accompanied by a force or torque in the direction of the stable position. Once the rotor reaches the position of equilibrium, the torque disappears. Conversely, the displacement of the rotor influences the configuration of an alternating electromagnetic field and produces a detectable change in the stator current. Based on the above, microsyns are designed to provide a linear relationship between the electrical signals and rotor angular displacement when the angular displacement is within ±12° from its null position. This condition is easily met in servo applications.

11-3.47 General Classification

Microsyns are classified by two general types, depending upon their functions. One type, called a ***microsyn torque generator,*** is used to provide a torque that is proportional to a command signal current. Another type called a ***microsyn signal generator,*** is used to provide a voltage output signal that, for a given excitation voltage, is proportional to the angular displacement of the rotor from a null position.

11-3.48 Microsyn Torque Generators

Microsyn torque generators are also classified into types according to the operation specified for each unit. One classification concerns a-c or d-c operation. For a-c operation, the units are designed with low-impedance windings to avoid excessively high terminal voltages at high torque levels. For d-c operation, high-impedance windings are desirable, because the high direct currents required for high torque levels are difficult to control by conventional techniques.

11-3.49 Double-winding type. Another classification of torque generators concerns the specific use of the units. In some cases, it is desired to produce an output torque that is independent of the rotor position within the normal limits of operation. A torque generator designed for this purpose is employed as a component of a single-degree-of-freedom gyro unit (see Par. **11-6**). The stator of this unit has a double winding that leads to the following operating characteristics (Fig. **11-34A**):

(a) With the rotor in the position shown, poles **1** and 3 produce a counterclockwise torque on the rotor; poles 2 and **4** produce a clockwise torque.

(b) When both I_1 and I_2 are zero, the magnetic flux in the poles is balanced and the net output torque is zero.

(c) Poles **1** and 3 produce a torque proportional to $(I_1 + I_2)^2$; poles **2** and **4** produce a torque proportional to $(I_1 - I_2)^2$. Hence, because poles **1** and 3 oppose poles 2 and **4,** the net output torque is proportional to the difference between the two squared quantities, or $I_1 I_2$.

(d) If the currents in the coils are reversed simultaneously, the direction of the torque output is independent of the direction of current flow between the terminals. For this reason, either a-c or d-c excitation current may be used.

11-3.50 Single-winding type. Another specific use of a torque generator is to convert an angular displacement of the rotor shaft into a proportionate torque that restrains the motion of the rotor shaft. For this purpose, the stator is wound with a single winding as shown in Fig. **11-34B.** When the rotor of this unit is displaced from its position of stable

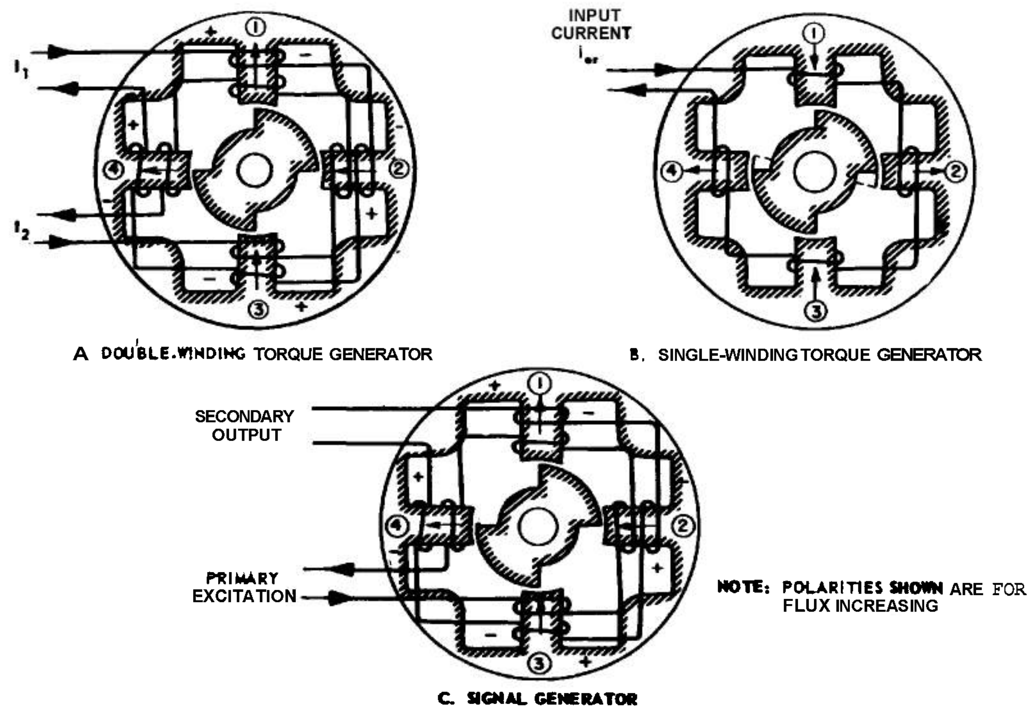


Fig. 11-34 Winding schematics of typical microsyns.

equilibrium (shown by solid lines), the unit produces an output torque that is directly proportional to the square of the current and to the displacement of the rotor from its position of equilibrium. To explain this operation, consider first the condition when the rotor is in its position of stable equilibrium. Then the air gaps and overlap areas of all four poles are equal and the rotor accommodates a maximum flux for a given excitation current. The direction of this magnetic flux is determined by the right-hand rule for flux flow and is indicated by the arrows in Fig. 11-34B.

Now consider the rotor to be displaced clockwise by a small amount as shown by the dotted lines in Fig. 11-34B. Then the overlap of the rotor decreases at poles **1** and **3** and increases at poles 2 and **4.** The rotor nc longer accommodates maximum flux from poles **1** and **3,** and the magnetizing forces lend to return the rotor to its previous position of equilibrium. Thus, a restoring torque is applied to the rotor. in a direction opposite to the angular displacement of the rotor. Because the square of the current is involved in this procedure, the direction of current flow (or the winding sense) of all coils can be reversed without altering the direction of the output torque.

11-3.51 Microsyn Signal Generator

Microsyn signal generators receive an angular displacement of the rotor shaft and convert this angular displacement into a proportionate electrical voltage for indication or control. In brief, a signal generator operates as **a** transformer with a magnetic coupling that varies with shaft rotation. **As** shown in Fig. **11-34C,** one set of stator coils serves as **a** primary and another set, isolated from the first, serves as **a,** secondary.

The flux directions indicated in Fig. **11-34C** are obtained by applying the right-hand rule to the specified instantaneous directions of excitation current flow. Figure **11-34C** alsc indicates the relative and instantaneous polarities of the four secondary coils for increasing the flux. This condition occurs when a change in magnitude of the magnetic flux during an excitation a-c cycle induces voltages in the secondary coils. With the coils connected as shown, the output voltages in the secondary winding cancel if the rotor overlap is symmetrical, the air gaps are uniform, and all coils are identical.

If the rotor is displaced from its neutral position, there is a proportionate increase of flux through two opposite poles and a reduction in flux through two alternate poles. It follows that the sum of the voltages in the series-connected secondary coils is no longer zero. Moreover, the net output voltage has a magnitude proportional to the difference in flux in alternate poles and consequently to the angular displacement of the rotor from its null position. If the flux is caused by **a,** sinusoidal a-c excitation, the output voltage will be an a-c sinusoid. **A** reversal of the direction of rotor displacement from null causes a reversal in the polarity of the output voltage. From the foregoing, it follows that the output voltage of **a** signal generator is proportional to the magnitude of the excitation current, the frequency of the excitation current, and the angular displacement of the rotor from null. Excitation currents for signal generators may be selected with magnitudes up to **400** ma and with frequencies up to 1000 cps.

11-3.52 Typical Performance Characteristics

The performance characteristics of a microsyn unit depend upon several factors, the most important of which are the unit size, the excitation current, the excitation frequency, and appropriate filtering or quadrature bucking techniques. Microsyns have been manufactured in sizes from 1.0 to **2.5** inchee in diameter. Excitation currents may be selected with magnitudes up to **400** ma and with frequencies up to 1000 **cps,** but the most commonly used values are **100 ma** and 400 cps. Typical performance characteristics **for** microsyn units having **a** nominal diameter **of** 2.0 inches are listed in Table 11-9.

TABLE 11-9 TYPICAL PERFORMANCE CHARACTERISTICS OF MICROSYN UNITS

Characteristic	Signal Generator	Torque Generator (a-c)	Torque Generator (d-c) †
Nominal diameter (in.)	2.0	2.0	2.0
Excitation current (nominal) magnitude (ma) frequency (cps)	100 ± 1% 400 ± 0.5%	† † 400 ± 0.5%	†† dc
Continuous operating range	‡	††; ‡	††
Total null voltage (max) (mv rms)	9		
Output voltage-input angle sensitivity (mv/millirad)	25		
Output torque-current squared input sensitivity (dyne-cm/ma^2)		1	10
Angle rotation limit away from signal null 7.5 ± 2.5 in either direction (deg)	7.5 ± 2.5		

†The torque generator (d-c) is designed to be adaptable to conventional hysteresis erasing circuits.

††The excitation current rms magnitude of both the a-c and d-c torque generators depends upon the maximum rate required by each particular application. This maximum rate is usually achieved with equal signal and excitation currents. Maximum continuous rates up to 2.5 rad/sec (corresponding to 160 ma in both circuits) are possible.

‡The signal generator and the a-c torque generator are required to provide a usable signal over a frequency range from 60 to 2000 cps. The signal generator is required to operate continuously at 400 cps over an excitation current range of 0 to 400 ma.

Based on data from "Suggested Specifications for the BuOrd Standard Integrating Gyro 20IG", by D. G. Hoag, Instrumentation Laboratory, Report No. R-91, December, 1955, Massachusetts Institute of Technology.

11 4 LINEAR VARIABLE DIFFERENTIAL TRANSFORMERS*

114.1 GENERAL DESCRIPTION

A linear variable differential transformer modulates the amplitude of an a-c carrier voltage in accordance with the linear displacement of an armature free to move axially inside a cylindrical coil structure. The linear variable differential transformer is used as a transducer for linear displacement in the positive and negative direction with respect to a reference or null, or as a transducer for other physical quantities which are first converted into a linear displacement.

The coil structure consists of a primary winding to which a source of alternating voltage is connected, and a secondary winding made up of two coils connected in series opposition (Fig. 11-35). When the armature is located in the center of the structure as shown in the figure, equal voltages are induced in both secondaries and the net output

*By A. K. Susskind

voltage is almost zero. If, however, the armature is displaced axially, the voltage induced in that secondary coil toward which the armature moves is increased, while the voltage induced in the other secondary coil is decreased. A net voltage then results at the secondary output and has a fixed phase angle ϕ with respect to the primary excitation when the armature is displaced in one direction from the reference or null position, and a fixed phase angle $\phi - 180°$ when the armature is displaced in the opposite direction. The magnitude of the net secondary voltage is a measure of the displacement distance and the unit is designed to have a nearly linear relationship between magnitude of net secondary voltage and magnitude of displacement over its operating range.

Accuracy of linearity for various units ranges from less than ± 0.1 percent to ± 1.0 percent, with the better accuracy figures holding for units designed for small maximum travel (e.g., 1/8 inch) and the poorer accuracy figures holding for units designed for several inches of maximum travel.

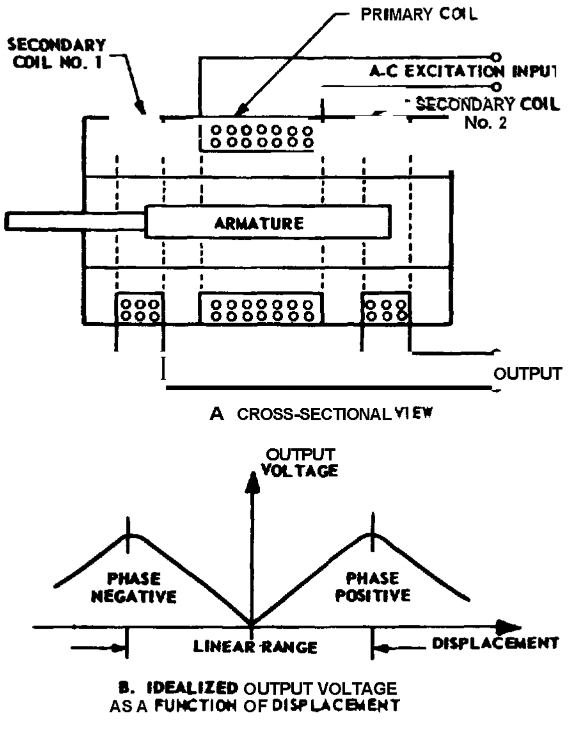


Fig. 11-35 Linear variable differential transformer.

11-4.2 DESIGN CHARACTERISTICS

Linear variable differential transformers are designed for maximum armature motions, in either direction from null, ranging from a few thousandths of an inch to several inches. Transducer dimensions depend upon the maximum travel specifications and range in length and diameter from a fraction of an inch to several inches.

11-4.3 Sensitivity Rating

The sensitivity rating of a linear variable differential transformer relates the net change in secondary voltage to the change in armature position. It is expressed as output volts per unit armature motion per unit primary excitation voltage. Typical sensitivity ratings range from a fraction of a millivolt to several millivolts net output voltage per 0.001-inch armature motion per volt excitation. The sensitivity as well as the total output voltage for maximum travel are functions of the primary-voltage amplitude and frequency. In general, increasing primary voltage and frequency increases sensitivity and output voltage over a considerable part of the audio spectrum, but some units exhibit a decrease in sensitivity as frequency and input voltage are raised above a certain point. Because of the light coupling between the primary and the secondaries, increasing the primary voltage and frequency does not result in excessive waveform distortion. Moreover, an increase in frequency will, broadly speaking, reduce the phase angle and in some cases the desired phase angle may be obtained by suitable selection of frequency. The peak value of the actual output voltage for a given displacement may be found by multiplying the given sensitivity figure for the unit (at the excitation frequency used) by the displacement in thousandths of an inch and then multiplying again by the peak value of the primary excitation.

11-4.4 Input and Output Characteristics

Typical input voltage levels are a few volts, with 6.3 volts (generally available in any system) commonly used. The maximum input voltage is limited by the heat dissipating capabilities of the primary windings. Since the output voltage is proportional to the input voltage, the supply must be well regulated unless a bridge configuration, to be discussed in Par. 11-4.9, is used.

Excitation frequencies commonly used are 60 cps and 400 cps, but units can be designed for almost any frequency in the audio spectrum. The choice of an excitation frequency depends not only upon convenience (i.e., selection of a frequency already available in the remainder of the system), but also upon the highest frequency component of the armature motion to be measured. The excitation frequency should be at least ten times greater than the highest frequency component of the armature motion. Since sensitivity is a function of input frequency, stability of the input frequency is required. Because of manufacturing tolerances, the output at the center position of the armature is not truly zero, but has a small value that is usually held to a fraction of a percent of the maximum output. Often the residual voltage can be appreciably reduced by the addition of a capacitor in parallel with one of the secondary coils. The proper value of capacitance is best found by experiment.

114.5 Phase Angle

The phase angle of the secondary voltage is very nearly constant for all displacements in a given direction from null, varying less than 1" over the linear range. The secondary phase angle is related to the primary-voltage phase by the coil and load parameters. A simplified equivalent circuit of the linear differential transformer is shown in Fig. 11-36, where

R_p = primary resistance

L_p = primary inductance

I_p = primary current

I_s = secondary current

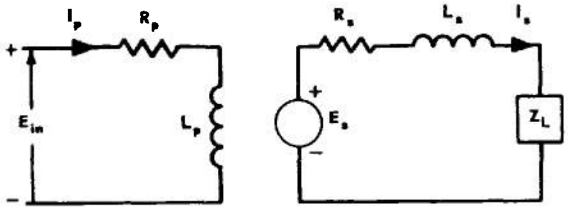


Fig. 77-36 Approximate equivalent circuit of linear differential transformer.

R_s = total secondary resistance

L_s = total secondary inductance

$Z_L = R_L + jX_L$ = load impedance

E_{in} = applied voltage

E_s = induced secondary voltage with Z_L infinite

f = frequency of applied voltage

The phase angle θ of E_s with respect to E_{in} is approximately

$$\theta = 90° - \tan^{-1}\left(\frac{2\pi f L_p}{R_p}\right) \qquad (11\text{-}36)$$

The phase angle ϕ of the secondary current with respect to E_{in} is approximately

$$\phi = \theta - \tan^{-1}\left(\frac{2\pi f L_s + X_L}{R_s + R_L}\right) \qquad (11\text{-}37)$$

114.6 Output Impedance

Typical values of secondary impedance range from hundreds to thousands of ohms. For largest output voltage, the value of Z_L should be as high as possible (e.g., grid circuit of a vacuum tube). For maximum power transfer, required for example when the output is connected to a magnetic amplifier, the load impedance should match the secondary impedance.

114.7 Loading

The armature loads the driving member due to its mass, its friction, and the magnetic pull. The effects of this loading are usually negligible. The axial and radial pull on the armature are proportional to the square of the primary current, and for the same applied voltage amplitude, these forces generally decrease as the frequency is increased, because the current decreases with the increased impedance. The radial pull is zero when the armature is radially centered and can be eliminated by guiding the armature along its axis. This also eliminates the scraping friction of the armature against the inner coil wall. Typical maximum values of axial magnetic pull range from a few thousandths to a few hundredths of a gram for units with no iron in the magnetic return path. The axial pull is zero at null.

11-4.8 Construction

Linear variable differential transformers can be built to withstand temperature ranges of several hundred degrees Fahrenheit. They are mechanically rugged and, like other electromechanical devices, are limited in life only by the wear of the mechanical components when operated within their electric: , ratings.

Unshielded units are sensitive to the proximity of electrically conducting or magnetic materials, particularly when these materials are located near the ends of the units. If unshielded, a unit should be separated from such materials by an inch or two. Where this cannot be done, shielded units should be used. The rod that connects the armature to the moving member to be measured should never be made of a magnetic material and, preferably, should consist of an electrically nonconducting material. Where a conducting material is used, identical armature rods should be used at both ends of the armature so that magnetic symmetry is maintained inside the unit.

11-4.9 APPLICATION

In applying a linear variable differential transformer to a servomechanism, a null-type circuit is often used. Here, the armature is connected to the input member and the coil structure moves with the output of the servomechanism. The secondary voltage is then a measure of the system error and, since this is usually held small, a unit with only small maximum rated displacement between coil and armature can be used even where a large range in input (and thus output) variation is involved. Static positioning accuracy is then independent of transformer linearity, source voltage, and frequency. It depends only upon the null voltage.

For remote control, a null-type circuit with two units U_1 and U_2, as shown in Fig. 11-37, can be used. The secondaries of the two units are connected in phase opposition so that the

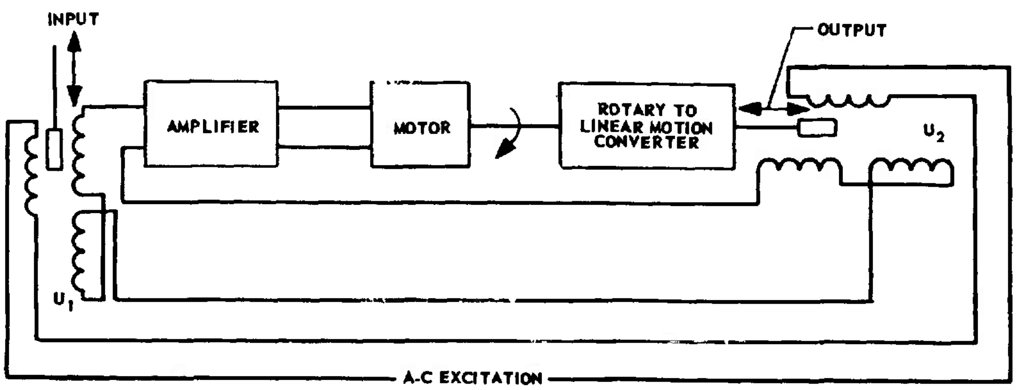


fig. 11-37 Linear variable differential transformers used in remote control.

amplifier input voltage expresses the difference, or error, in the two armature positions. The resultant motor signal tends to reduce the error to zero. This circuit exhibits two desirable characteristics: its static accuracy is independent of the excitation voltage and frequency; and it is also independent of variations in coil resistance with temperature. Several units can be connected in series with the single input unit shown in Fig. **11-37** so that the output is the algebraic sum of several inputs.

11-5 TACHOMETER GENERATORS*

11-5.1 GENERAL DESCRIPTION

A tachometer generator is a device that generates an output voltage essentially proportional to the angular velocity of its shaft. The voltage signal may be alternating or direct; however, in general, the direct-voltage type is not practical for high-accuracy applications because of commutator noise, which is excessive at low speeds. When **a** tachometer generator is used to provide a feedback signal in servomechanisms, the output-voltage frequency must be fixed and must correspond to the reference frequency of the control system. The alternating-current tachometer generator commonly used for this purpose operates on the drag-cup principle.

11-5.2 DRAG-CUP A-C TACHOMETER GENERATOR

This type of tachometer generator has two fixed windings located in close proximity to a rotating drag-cup (Fig. **11-38).** Both windings may be located either inside or outside the cup; or one winding may be located inside and the other outside. The latter arrangement makes possible adjustment of the winding positions relative to each other. When both windings are inside, the cup is larger and therefore has a relatively larger moment of inertia. Such an arrangement, however, simplifies the winding process and facilitates the accurate machining of the cup. The main reason for locating the windings outside the cup is to achieve a lower moment of inertia for the cup. This arrangement is also necessary for miniature applications, as in missiles. In very small units, the cup is usually affixed to the inner core and rotates with it. This arrangement, however, is not satisfactory for high-precision applications such as analog computers.

11-5.3 Theory of Operation

The theory of operation of a drag-cup a-c tachometer generator is explained by reference to the two-pole device shown in Fig. **11-39.** The main, or reference, winding (r) is excited from a source of fixed voltage and frequency. If the tachometer construction is symmetrical and of homogeneous materials, the currents induced in a stationary cup are symmetrically distributed with respect to the axis of the reference winding. For these conditions, the signal winding (s) experiences no flux linkage because its axis is at right angles to the axis of the reference winding. When the drag cup rotates, the resultant current distribution in the cup is skewed slightly from the zero-speed distribution, thereby creating a **flux** linkage with the signal winding. This linkage induces a voltage in the signal winding having the same frequency as the exciting source and an amplitude that is essentially proportional to the angular velocity of the cup. Moreover, **a** reversal of the cup rotation is accompanied by a phase reversal of the output signal.

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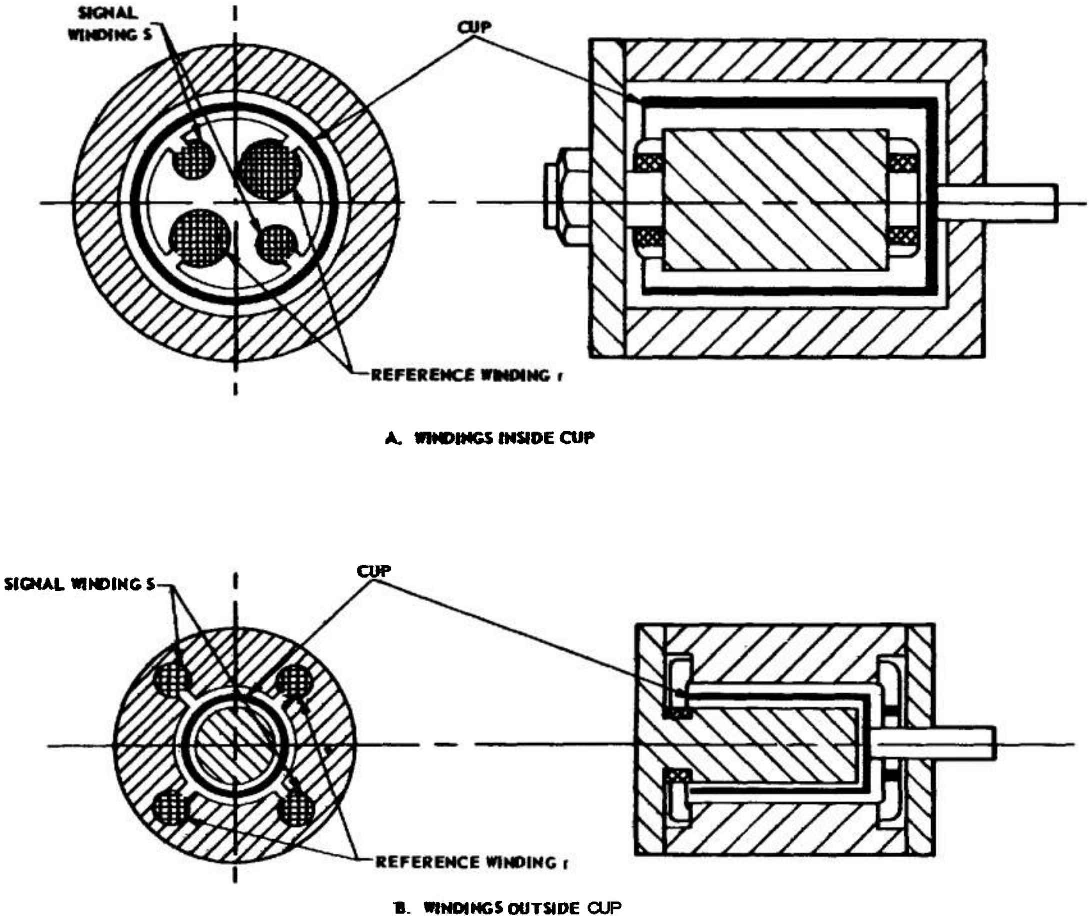


Fig. 11-38 Construction of drag-cup a-c tachometer generator.

11-5.4 Equivalent Circuits and General Equations[32,33,34]

Equivalent circuits of a drag-cup a-c tachometer generator operating from a voltage source and from a current source are shown in Figs. 11-40A and 11-40B, respectively. These circuits represent ideal operating conditions and therefore do not account for the errors that exist in an actual device.

11-5.5 General equation for circuit having a voltage source.[32] Figure **11-40A is** an equivalent circuit of a drag-cup a-c tachometer generator operating from a voltage source and having an open-circuit signal winding as shown in Fig. 11-39. The conditions represented by this circuit are closely approximated in practice if the signal winding circuit has such a high impedance that it draws negligible current. The general equation for the signal voltage of an equivalent circuit of this type is as follows:

$$V_s = \frac{aV_r n}{A_1 + jB_1 + (A_3 - jB_3)n^2} \qquad (11\text{-}38)$$

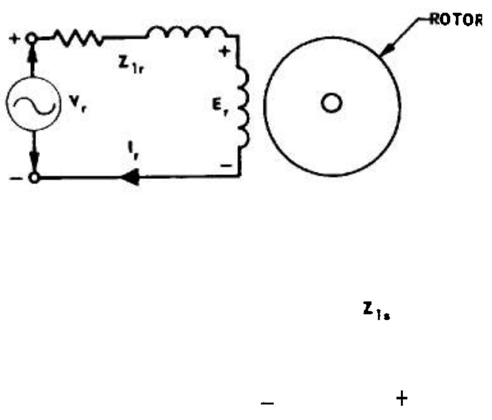


fig 11-39 Schematic diagram of drag-cup a-c tachometer generator.

where

V_s = output voltage of signal winding

V_r = voltage impressed on reference winding

a = turns ratio, signal-to-reference winding

n = per unit speed, based on synchronous speed

$A_1 = (b - a^2 b) x_{11} + 2ab r_{1r} + a$

$B_1 = 2ab x_{11} - (b - a^2 b) r_{1r} - 1$

$A_3 = a^2 b x_{11} - a$

$B_3 = a^2 b r_{1r}$

$j = \sqrt{-1}$

$x_{11} = x_\phi + x_{1r}$ = self-reactance of reference winding, including external series reactance or equivalent

r_{1r} = self-resistance of reference winding, including external series resistance or equivalent

x_{1r} = leakage reactance of reference winding with respect to cup, plus external series reactance or equivalent

x_ϕ = magnetizing reactance

$a = x_{22}/r_2$

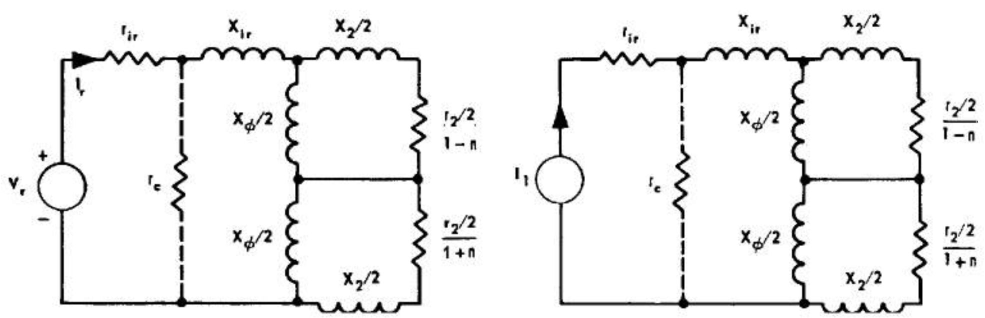
$b = r_2/x_\phi^2$

$x_{22} = x_2 + x_\phi$ = self-reactance of cup, referred to reference winding

x_2 = leakage reactance of cup with respect to reference winding, referred to reference winding

r_2 = resistance of cup, referred to reference winding

Quantities r_{1r} and x_{11} and ratios a and b are measurable; the quantities involved in a and b are not measurable. Core loss can be taken into account approximately by means of r_c (shown dotted in Fig. **11-40**) which can be



A. WITH VOLTAGE SOURCE

B. WITH CURRENT SOURCE

fig. 11-40 Equivalent circuits of drag-cup a-c tachometer generator.

measured or estimated. To use Eq. (11-38), a Thevenin transformation should be made to the left of x_{1r}, so that V_r and r_{1r} can be replaced by the equivalent series voltage and resistance combination. Although voltage V, does not appear in Fig. 11-40A, this voltage is ja times the difference between the voltages across the respective paralleled branches. When the signal winding operates into a significant load, a more complex equivalent circuit than that shown in Fig. 11-40A is required. For this more complex circuit, the general signal-voltage equation has the same form as Eq. (11-38) but the parameters of the signal winding and the load are involved in addition to the parameters of the reference winding.[32]

11-5.6 General equation for a circuit **having a** current **source.**[34] Theoretically, if the tachometer generator could be excited by a so-called current source (Fig. 11-40B), the fixed exciting flux should give extreme linearity of operation, as can be determined by a study of the following equation :

$$V_s = \frac{aI_r n}{b[2a - j(1 - a^2) - ja^2n^2]} \qquad (11\text{-}39)$$

where I_r is the current existing in the reference winding and the remaining symbols are defined following Eq. (11-38). Furthermore, the use of a current source should eliminate troubles caused by a change of winding resistance with temperature because then the winding resistance and the external impedance do not influence the operation of the tachometer generator. However, the realization of these theoretical advantages is doubtful in a practical instrument due to the tachometer-generator design difficulties, temperature effects other than changes in winding resistance, core-loss effects, and difficulties in the design of a highly stable current source.[34,35]

11-5.7 Speed Error

In the denominator of Eqs. (11-38) and (11-39), the term with n^2 indicates that the tachometer generator would have an inherent speed error even if it were: (1) constructed perfectly with regard to geometric symmetry and homogeneous materials ; (2) operated from a perfectly steady sinusoidal source; and (3) free from undesirable environmental effects. The magnitude of this error is primarily a matter of design since it depends upon the relative magnitudes of the tachometer-generator parameters. Basically, the drag-cup tachometer generator is a simple device, but the severe reduction of small errors requires very careful workmanship, selection of materials, and substantial tailoring of individual units. The high cost of these units may not be justified unless the operating conditions are such that they minimize additional errors that might otherwise predominate. Tachometer generators can be purchased which have been calibrated for use at specified voltages and frequencies and also have the linearity of output guaranteed within specified tolerances over a specified speed range. Some of these units have ratio errors as low as 0.1 percent and phase errors as low as 0.1° at speeds up to about 30 percent of synchronous speed. However, the error magnitude and phase departure may be decreased by the use of the terminal networks associated with either the reference winding or the signal winding, or both. These networks can be used for calibrating purposes.

11-5.8 Calibration for speed errors. Ordinarily, the signal voltage should be in phase (or in opposite phase) with the reference voltage.[32] Essentially, this in-phase condition implies that

$$B_1 = 0 \qquad (11\text{-}40)$$

$$\rho = \frac{A_3n^2}{A_1} \text{ (a ratio error per unit)} \qquad (11\text{-}41)$$

$$\theta = \frac{B_3n^2}{A_1} \text{ (a phase error, in radians)} \qquad (11\text{-}42)$$

Theoretically, if calibration is made at speed

$$n_c = \frac{n_f}{\sqrt{2}} \qquad (11\text{-}43)$$

where n_f is the maximum per unit speed, the errors indicated by Eqs. (11-41) and (11-42) are halved.[32,34] In general, the most satisfactory methods of tachometer-generator calibration are those that use an a-c potentiometer or a summing network. The calibration is conducted by checking a number of points within the desired speed range until all of them fall within specified tolerances of ratio and phase error. The error is often expressed in terms of in-phase and quadrature components. In many applications, a small quadrature error can be tolerated because the error-sensing device may be responsive only to the in-phase error. However, because the quadrature component is not cancelled at the summing point, it may cause overloading of amplifiers.

11-5.9 Voltage Errors

Theoretically, except for the influence of magnetic material, the fluctuations of reference-voltage amplitude should cause no change in voltage ratio or phase and the signal voltage should vary in amplitude in direct proportion to the variation in the amplitude of the reference voltage. Good practice dictates that the magnetic structure of a drag-cup tachometer be of high-permeability low-loss magnetic materials worked at relatively low flux densities. Moreover, to permit mechanical clearances on both sides of the drag cup, the tachometer must be designed with a relatively long air gap. The air-gap reluctance predominates and hence any errors due to a change of reference voltage are secondary effects and, except for extreme changes, are generally negligible in comparison with the other errors.

11-5.10 Frequency Errors

Errors caused by change of frequency are more serious than voltage errors because a change of frequency has a direct influence on the reactances involved in A, and B_1. The frequency error may be of the order of a hundredth of a percent in ratio and of the order of a tenth of a degree in phase for a frequency change of one cps. If the reference source is a rotating machine, changes in voltage and frequency occur together and in proportion ; hence, the errors are superposed.

11-5.11 Harmonics

Even if the reference source generates a pure sinusoid, the signal voltage may contain small harmonics due to nonlinear magnetic-core materials. If small harmonics are not recognized by the error-sensing device, the only harm that might be created would be an overloading of an amplifier. Harmonics in the reference voltage are carried through to the signal voltage in various proportions. Except for a small modifying effect from the magnetic material, the signal-voltage harmonics carried over from the reference voltage can be determined by applying Eq. (11-38) to each harmonic.

11-5.12 Temperature Error and Its Compensation

A resistance change in the reference winding or the cup may result in prohibitive errors unless ambient temperature control, compensation, or both are utilized. The extent of the temperature error can be determined by examining the influence of r_{1r} and r_2 on A, and B_1. Control of ambient temperature is necessary if the cup is made of copper or aluminum, or if the reference-circuit resistance is composed, for the most part, of the resistance of the copper winding. Ambient temperature control requires the use of a heater unit and thermostat as an integral part of the tachometer generator. Small temperature errors may be created by the thermal lags even though the input-current (and thus temperature,) variations due to speed changes are small. This situation can be improved considerably by constructing the cup from a low-temperature-coefficient alloy and by compensating for the change in reference-winding resistance through the use of an external series zero-temperature-coefficient resistance. Substantial compensation may also be achieved by shunting a thermistor, located in the tachometer-generator housing, across part of the external resistance. By virtue of the forementioned selection of materials and compensation, the resulting tachometer performance over a considerable temperature range without ambient control is practically as good as the performance of a tachometer using copper and aluminum electrical parts and having close temperature control. Of course, the addition of temperature control to the other remedies results in further error reduction. Temperature control may also be required to stabilize capacitors used in calibrating or compensating networks. Satisfactory tachometer-generator performance requires a 30- to 60-minute warm-up period so that the temperature of the unit can reach a steady state.

11-5.13 Residual (or Zero-Speed) Error

An inference is made in the theory of operation (Par. **11-5.3)** that a zero signal voltage exists at zero speed only if the tachometer generator is constructed with perfect geometric symmetry and homogeneous materials. Because these conditions are never met in actual tachometer-generator construction, the signal winding has a flux linkage at zero speed. This linkage causes a zero-speed voltage to be induced in the signal winding, and the level of this voltage determines the degree of resolution obtainable from a normal tachometer-generator signal. Either one or a combination of the following methods may be used to minimize the undesirable zero-speed voltage :

(a) Mechanical correction. The core inside the cup may be given some magnetic dissymmetry by means of a small flat or a few nicks; or, some eddy-current paths may be specially provided in the core or in an auxiliary end member. Then, by a rotation or an axial motion of the core, or a combination of both these motions, a position may be found in which the effects of natural dissymmetries and inhomogeneities are essentially neutralized. For any one cup position, these methods usually lead to the elimination of the in-phase component and a substantial reduction in the quadrature component.

(b) Use of auxiliary coils. Small auxiliary coils may be wound in the same slots with the signal winding. One coil is connected in series with a high resistance across the reference winding. The other coil is connected in parallel with a high resistance and the combination is then connected in series with the reference winding. By adjusting the magnitudes and phases of the currents in the auxiliary windings, the residual voltage can be practically eliminated.

(c) Positioning of reference and signal windings. If the reference and signal windings are located on separate parts of the magnetic structure (e.g., one winding inside and the other outside the cup), the windings may be adjusted relative to each other until the residual voltage is at a minimum.

(d) Neutralization with compensating voltage. An external compensating voltage may be introduced to neutralize the residual voltage.

11-5.14 Wobble

If the cup is not fabricated from homogeneous materials or is not constructed and arranged in perfect symmetry with respect to the stator and the bearings, an adjustment that minimizes the zero-speed voltage for one cup position does not do so for another. As a result, the residual voltage varies, or wobbles, through a complete cycle as the rotor turns through **180** electrical degrees. The in-phase and quadrature components **of** residual voltage can be observed separately as a function of rotor position. Because these components are created by the lack of coincidence between the magnetic axis and the geometric axis, the average values **of** the components are called **axis *errors.*** Under operating conditions, these errors appear in the output signal as a component that is amplitude modulated at a frequency corresponding to the rotor speed. Wobble can be minimized by corrective efforts **such as:**[32]

(a) Improved homogeneity **of** cup material

(b) Closer machining and bearing tolerances (*c*) Scraping the cup or depositing metal on it. Compensation may also be employed to center the in-phase and quadrature components about zero. When the cup is attached to the core, the nonhomogeneity of the magnetic material greatly aggravates the wobble. In mounting or in calibrating the tachometer generator, care should be exercised to ensure that no substantial magnetic material is placed near the device in such a position as to cause appreciable field distortion. Otherwise, the slight distortion of magnetic-field distribution may nullify the residual-error adjustments.

11-5.15 Acceleration Error

The equations given for signal voltage as a function of cup speed, Eqs. (11-38) and (11-39), apply to steady-state conditions. They indicate that the signal voltage has a corresponding amplitude (rms value) for a constant speed. As the speed changes, and for a time thereafter, the electrical signal is in a transient state. Hence, a knowledge of how closely the electrical signal follows the mechanical speed becomes an important consideration. A complete solution of this problem cannot be achieved by analytical methods; however, the limiting case of a signal response to a step in speed can be readily solved.[32,34] Although the equivalent circuits of Fig 11-40 do not apply to the transient case, the characteristic equation for this case for a circuit having a voltage source may be written as

$$(L_{11}L_{22}^2 - L_\phi^2 L_{22})s + (r_{1r}L_{22}^2 + 2r_2L_{11}L_{22} - r_2L_\phi^2)s^2 + (r_2^2L_{11} + 2r_{1r}r_2L_{22} + L_{11}L_{22}^2\omega^2n^2 - L_\phi^2L_{22}\omega^2n^2)s + (r_{1r}r_2^2 + r_{1r}L_{22}^2\omega^2n^2) = 0 \qquad (11\text{-}44)$$

and the characteristic equation for a circuit having a current source may be written as

$$(L_{11}L_{22}^2 - L_\phi^2 L_{22})s^3 + (r_cL_{22}^2 + 2r_2L_{11}L_{22} - r_2L_\phi^2)s^2 + (r_2^2L_{11} - 2r_cr_2L_{22} + L_{11}L_{22}^2\omega^2n^2 - L_\phi^2L_{22}\omega^2n^2)s + (r_cr_2^2 + r_cL_{22}^2\omega^2n^2) = 0 \qquad (11\text{-}45)$$

where

L_{11}, L_{22}, L_ϕ = inductances associated with corresponding reactances listed after Eq. (11-38)

ω = angular speed of cup, in electrical radians per second

s = roots of equation in accordance with Laplace transform notation and the definitions of the remaining symbols follow Eq. (11-38).

11-5.16 Influence **of** parameters on **acceleration** error. The use of r_c in Eq. (11-45) is an approximation. Moreover, to be consistent with Eq. (11-44) the equivalent parallel resistance $r_{1r}r_c/(r_{1r} + r_c)$ should be used in Eq. (11-45) instead of r_{1r}. This refinement is not used in Eq. (11-45) because r_c is very large compared with r_{1r} and thus the equivalent parallel resistance differs very little from r_{1r}. Moreover, because the speed step (ωn) over the entire speed range has very little effect on the roots (s), the transient is determined for the most part from the electrical parameters. In practical cases, no root has a time constant longer than approximately 0.5×10^{-3} second. Hence, because the speed cannot change in a step (which is infinite acceleration), the signal voltage lags behind the speed by a value less than 10^{-3} second. These figures are based on representative values; some existing tachometer generators may have longer electrical time constants than the value given.

11-5.17 Commercial Units

Drag-cup a-c tachometer generators may be purchased in various sizes and configurations. They are available as separate units or are mounted on the same shaft and in the same housing with their associated 2-phase servomotor. The common shaft arrangement is advantageous from the point of view of bearing spacing. Otherwise, the cup may have considerable overhang and the bearings, placed close together at one end, may permit appreciable play in the cup position. Tachometer generators with speed errors of one or two percent are not regarded as precision devices. They are relatively inexpensive and are generally used as damping generators in servomechanisms or as actuating devices for speed indicators. Units with speed errors of one percent or less are regarded as precision devices and are relatively expensive. They are used primarily in computers and speed-control systems. The limit for the minimum speed error of commercial instruments is about 0.1 percent.[35]

11-5.18 Selection

The preceding discussion indicates that errors other than speed errors are of considerable interest in the selection of precision tachometer generators. Information along this line, however, is not always complete in manufacturers' data sheets. To illustrate the use of manufacturers' data in the selection of tachometer generators, reference is made to the sample data in Table 11-10 and to the following considerations:

(a) *Voltage and frequency ratings.* These ratings are selected to match the available supply. A transformer or other input network may be used to match the tachometer generator to a voltage source; however, this procedure introduces additional errors and may require recaiibration of the instrument. In general, the voltage ratio of the unit need not be an exact match with the designed ratio of a feedback system. This is due to the fact that the gain of the amplifier into which the signal voltage is applied can usually be adjusted over a substantial range. Tachometer generators are available for voltages and frequencies other than those listed in Table 11-10, but 24 volts and 115 volts at 400 cps are the most common.

(b) *Speed range and associated errors.* Speed errors are sometimes expressed in millivolts or in rpm rather than in percent of ratio or degree of phase.

(c) *Impedance levels and power rating.* These two factors are important only if matching or power economy is important. At full speed, the mechanical input is generally less than 1 watt because the electromagnetic drag torque is in the order of a few hundredths of an inch-ounce per 1000 rpm and the friction and windage are in the order of a few tenths of an inch-ounce.

(d) *Moment of inertia.* The moment of inertia is an important factor if fast response is a requirement.

(e) *Weight and size.* Weight and size are especially important for airborne installations. Tachometer generators for this purpose are designed to meet military specifications for environmental conditions. They are usually very small especially for missile application. Tachometer generators for missiles cannot provide the same accuracy as units designed for stationary installations under controlled conditions because of their small size and the adverse conditions that must be withstood.

11-5.19 D-C TACHOMETER GENERATORS

As previously stated, d-c tachometer generators are not practical for high-accuracy applications. The primary difficulties involved are commutator and tooth ripple in wound machines, insufficient voltage in homopolar machines, and contact difficulties in both types.

11-5.20 Deficiencies in a Homopolar Unit

A homopolar machine of practical size and speed with either a rotating cup or a rotating disc generates a very small voltage. As a result, the uncertainties due to slip-ring contacts are emphasized. For example, a 1-inch diameter cup, 2 inches long, rotating at 3000 rpm in a field of 10,000 gausses, generates approximately 0.2 volt. Amplification of a voltage of this magnitude introduces the common difficuities of d-c amplification. Voltages of such machines can be increased by utilizing a multiple-bar type of construction instead of a cup and by connecting the sets of bars in series externally. This arrangement, however, multiplies the slip-ring problem in direct proportion to the number of external series connections and results in a considerable increase in size. The use of bars instead of a continuous cup also introduces voltage ripple due to the nonuniform distribution of flux density in the air gap.

TABLE 11-10 SAMPLE DATA FOR DRAG-CUP A-C TACHOMETER GENERATORS

Tachometer Generator	A	B	C	D	E	F
Input voltage (volts)	115†	115	24t	115	115 or Æ	115
Input current (amp)*	0.10	0.073	0.13	0.42		0.082
Input power (watts)*	8	5.4	1.9	4.5		8.2
Frequency (cps)	400	400	400	60	400	400
Output voltage (volts/1000 rpm)	2.0	3.2	10	5.0	0.5	0.36
Input impedance .(ohms)*	$190 + j750$	$1010 + j1210$	$110 + j150$†			
Output impedance (ohms)*	$360 + j1390$	2200	$850 + j1700$	2200		
Speed error						
ratio (percent)	0.15	0.2	0.1	0.1		
phase (deg)	0.3	0.5	0.1			
max range (rpm)	4000	5000	4000	1800		
Synchronous speed (rpm)	12,000	12,000				8000
Output load (ohms)			50,000			500
Output load (uuf)			1000			
Temperature error						
ratio (percent)	0.15	12	0.3	1		
phase (deg)	0.3	5	0.8			
Temperature range (°C)	15 to 70‡	20 to 70	15 to 70‡	−10 to 55		
Residual error						
total (mv)		13		100	20	10
in-phase axis (mv)	3	1	1			
quadrature axis (mv)	10	8	1			
harmonics (mv)	10		10			
Wobble						
in-phase (mv)			2			
quadrature (mv)			10	30		
Moment of inertia (oz-in.^{2})	0.57	0.011	0.1		0.0021	0.063
Static friction (oz-in.)	0.2		0.1			
Weight (oz)**	32	6.7	8		3.5	8
Diameter × length (in. × in.)***	2.25 × 4.0					..1 × 1.71

* At zero speed; these quantities do not change greatly with speed.
** Not counting auxiliary network, if any.
*** Not including shaft extension.
t Includes input network.
‡ Thermistor compensation.

11-5.21 Deficiencies in a Conventional-Wound Unit

If a conventional-wound d-c machine is used, having either a permanent-magnet or an electromagnet field structure, the voltage can be as high as necessary and the sliding contact problem is relatively less severe. The voltage ripple, however, is usually intolerable because of the relatively few teeth and commutator segments possible in a practical small tachometer generator. The calibration is also somewhat unstable because of the residual-magnetism changes in devices using electromagnets and because of the possibility of demagnetization in devices using permanent magnets. Another possible disadvantage is that a d-c tachometer generator with a saturated electromagnetic field or a permanent magnet does not reproduce a signal-voltage change in direct proportion to a reference-voltage change. All of these disadvantages preclude the wide use of d-c tachometer generators for high-accuracy feedback applications. The use of alternating current, rather than direct current, is also considered to be more advantageous from the viewpoint of control system design in general.[36]

11-6 GYROSCOPES*

11-6.1 INTRODUCTION

The terms "gyroscope", "gyro unit", and "gyro" all identify a family of sensing elements that respond primarily to change in orientation with respect to inertial space (see Par. 11-6.4 for the definition of inertial space). In this publication, the term "gyro unit" will generally be used throughout. Physically, a gyro unit is a small, compact, and precise operating component that contains a specially mounted spinning rotor together with various arrangements of mechanical and electromagnetic elements necessary for the gyro unit to carry out its function.

As a sensing element, a gyro unit may be used as a measuring device to produce an output that is either a measure or a function of some input quantity. For example, the gyroscopic rate-of-turn indicator commonly used in aircraft employs a gyro unit that responds to the angular velocity of the aircraft about its yaw axis.[37] Under steady-state conditions, the position of the index is an indication or measure of this angular velocity, which is the rate of turn of the aircraft.

In present and future applications, however, gyro units can be expected to exhibit their greatest value when operated as control monitoring devices, rather than as measuring instruments. As such, they will usually function as one of the vital elements in a particular type of operating system. In one type of operation, for example, the gyro unit controls, in accordance with an input command signal, the orientation (relative to inertial space) of a reference direction associated with the unit. This "space-integrator" mode of operation is described by Draper and others.[38,39] For such applications, the gyro operating characteristics — sometimes called gyro performance characteristics — play a very important part.

Although the gyro units used in modern fire-control systems, flight-control systems, navigation systems, etc., are comparatively recent developments (late World War II and subsequently), gyroscopes in their rudimentary form are well-established in scientific history and literature. For example, the theory of the gyroscope as a rigid body and its subsequent development has been discussed by many writers.[38 to 45]

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11-6.2 DESCRIPTION AND BASIC THEORY

Before proceeding with the details of particular types of gyro units, a brief description applicable to all gyro units will be given. Also included will be a brief discussion on the basic theory of operation.

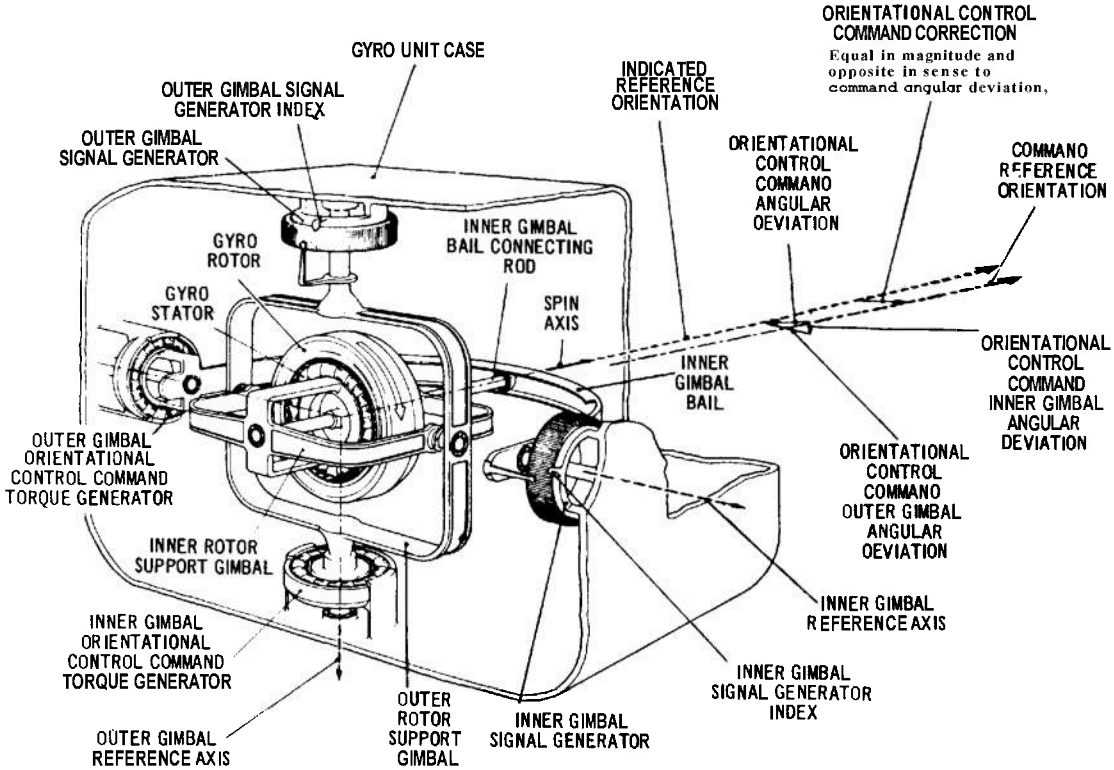
11-6.3 Description

The gyro units described in this publication are operating components whose basic element, known as the gyroscopic element, is a symmetrical rotor spinning at constant speed about the axis of symmetry and having its entire angular momentum concentrated about the spin axis. In addition to this basic element of a gyro unit, means are also included for providing (not always concurrently) damping, elastic restraint, one or more torque generators for receiving desired input signals, and one or more signal generators (or pick-offs) for producing output signals. Through various physical arrangements of these component elements, gyro units can be designed to respond as desired to orientation changes to which they may be subjected. Figure **11-41** shows the various elements in pictorial form for the three basic types of gyro units : the two-degree-of-freedom gyro unit, the single-axis rate gyro unit, and the single-axis integrating gyro unit. In the two-degree-of-freedom gyro unit (see Fig. **11-41A**), the rotor spins about one of the three axes of the unit and has freedom to turn about both of the other two axes. In the two single-degree-of-freedom units (see Figs. 11-41B and **11-41C**), the rotor spins about one of the three axes, but has freedom to turn about only one of the other two axes. It is important to note that the over-all gyro units, as well as the component elements shown on these figures, are highly pictorialized and should not be expected to be found in practice as shown.

11-6.4 Basic Principle of Operation

The behavior of the gyroscopic element of a gyro unit is explained by tbe application of Newton's Second Law of Motion to the rotation of the gyroscopic element. By this law, it may be stated that ***the time rate*** of ***change*** of ***angular momentum*** of ***the spinning rotor with respect to inertial space equals the torque applied to the element.*** This statement is the starting point for describing the principle of operation of any gyro unit. A brief treatment of the vector relationships of gyroscopic-element performance is given in Derivation Summary **11-1**. This summary shows that a gyroscopic element responds to an applied torque in such a manner that it tends to align its angular momentum vector with the applied torque vector ; this response is known as precession. The rate of precession is proportional to the applied torque. When the applied torque is zero, there is no precession and the gyroscopic element maintains its spin axis fixed with respect to inertial space. A concise and up-to-date treatment of gyros, complete with derivations of applicable equations, has been prepared by well-known authorities on gyro units.[38] A condensed version of this treatment is also available.[39]

Before proceeding further, it is necessary to have a clear understanding of the phrase "with respect to inertial space." This phrase implies that the change in orientation to which the gyro unit responds is measured with respect to inertial space rather than with respect to the earth, which is the reference frame in general everyday use. Inertial space is the space in which Newton's Law of Inertia is valid; i.e., it is the reference frame in which a force-free body is unaccelerated. As such, it is difficult to identify inertial space with any known part of the universe. In practice, however, it has been found that celestial space (as determined by "fixed stars") may be considered as being identical with inertial space, even when dealing with extremely accurate gyro units. In applications where the accuracy requirements are less stringent, on the other hand, the earth frequently serves satisfactorily as an effective inertial space ; i.e., the error incurred by neglecting earth rate lies within the error tolerance of the system concerned.



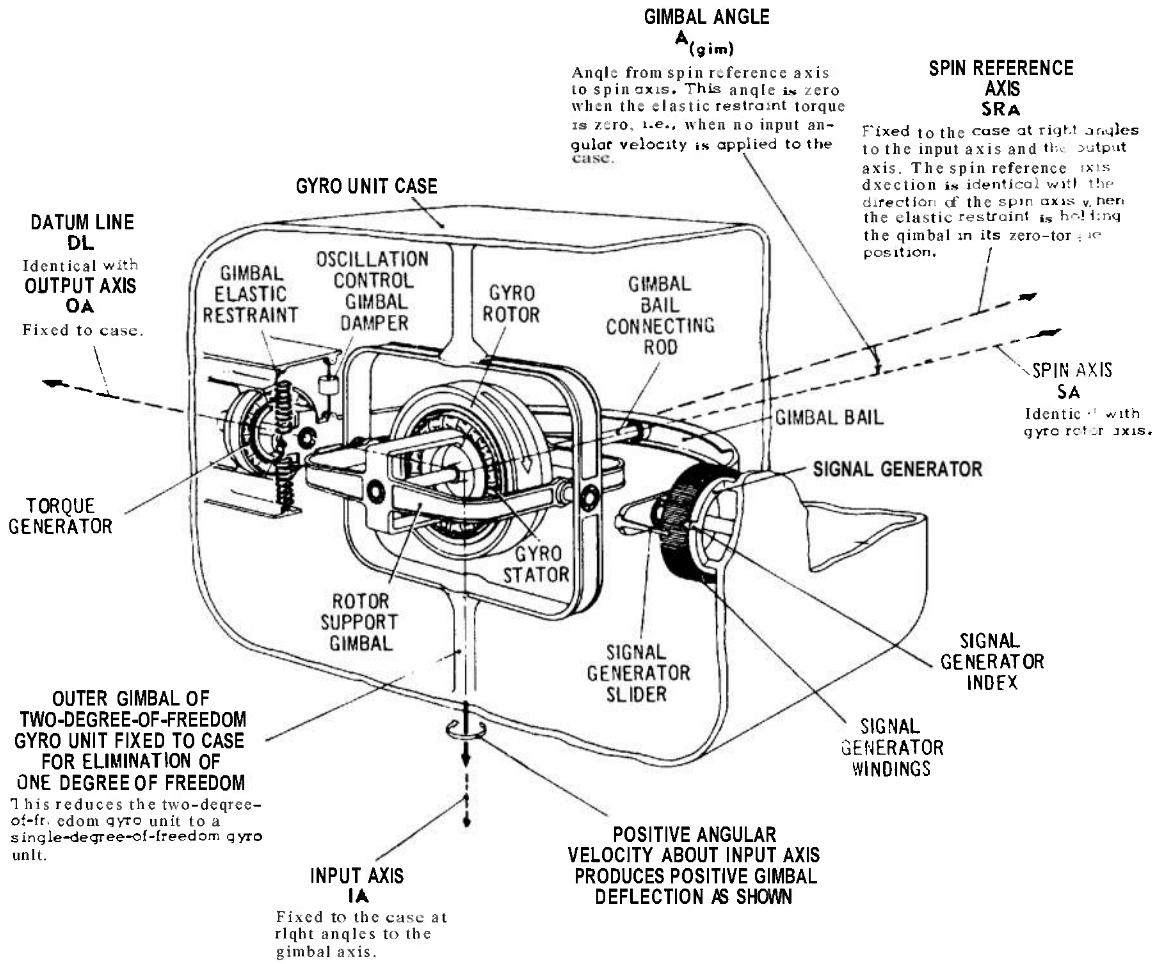
Notes: 1. Gyro units with the general features shown in the illustrative diagram of this figure are commonly used in automatic pilots for aircraft.

2. The range of satisfactory operation for a unit of this kind is restricted to indicated angular deviation magnitudes of small size, i.e., to angles of less than five degrees.

A. Essential Elements of a Two-Degree-of-Freedom Gyro Unit

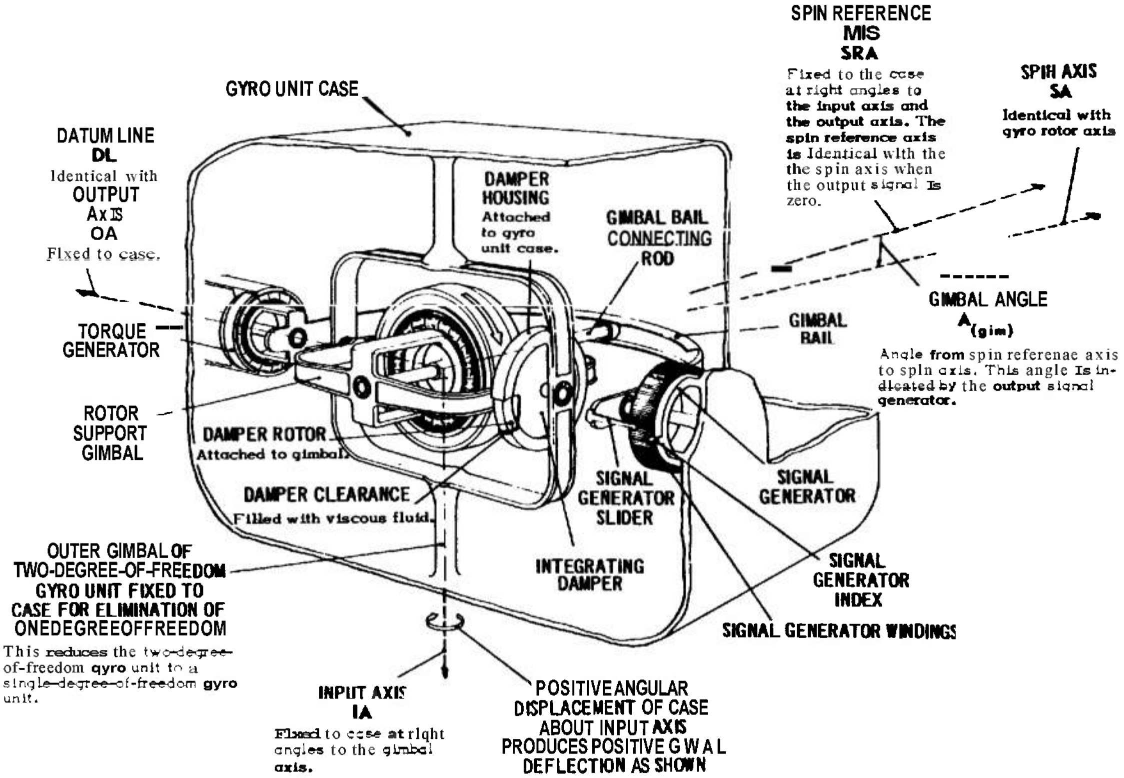
Fig. 11-41 Pictorial diagrams of the three basic types of gyro units. (Sheet 1 of 3)

.Adapted by permission from 'The Floating Integrating Gyro and Its Application to Geometrical Stabilization Problems on Moving Bases', by C. S. Draper, W. Wrigley, and L. R. Grohe. Sherman M. Fairchild Fund Paper No. FF-13. Institute of the Aeronautical Sciences.



B. Essential Elements of a Single-Axis Rate Gyro Unit

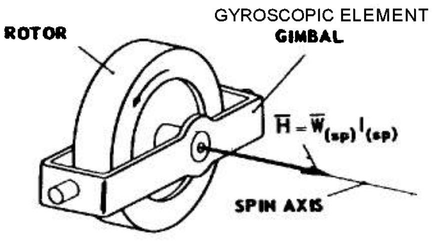
Fig. 11-41 Pictorial diagrams of the three basic types of gyro units. (Sheet 2 of 3)



Note: The action of the viscous liquid in the damper clearance space causes a torque that opposes the angular velocity of the gimbal. This torque is proportional to the magnitude of the gimbal angular velocity with respect to the unit case.

C. Essential Elements of a Single-Axis Integrating Gyro Unit

Fig. 11-41 Pictorial diagrams of the three basic types of gyro units. (Sheet 3 of 3)



$\bar{H}$ = VECTOR REPRESENTING ANGULAR MOMENTUM OF GYROSCOPIC ELEMENT ABOUT SPIN AXIS

$\bar{W}_{(sp)}$ = VECTOR REPRESENTING ANGULAR VELOCITY OF ROTOR ABOUT SPIN AXIS

$I_{(sp)}$ = MOMENT OF INERTIA OF ROTOR ABOUT SPIN AXIS

BY DEFINITION, A GYROSCOPIC ELEMENT IS A MECHANICAL SYSTEM WITH THE FOLLOWING PROPERTIES:

1) SYSTEM CONTAINS A ROTOR SPINNING ABOUT AN AXIS OF SYMMETRY.
2) ROTOR IS SPINNING AT CONSTANT SPEED; I.E., $W_{(sp)}$ IS CONSTANT SO THAT $dH/dt = 0$.
3) ALL ANGULAR MOMENTUM OF SYSTEM IS CONCENTRATED ABOUT SPIN AXIS.

THE ANGULAR MOMENTUM VECTOR $\bar{H}$ COMPLETELY SPECIFIES A GYROSCOPIC ELEMENT FOR PURPOSES OF ANALYSIS AND DESIGN.

a) Vector representation of gyroscopic element

BY NEWTON'S LAW FOR ROTATION:

$$\bar{M}_{(ge)(opp)} = \left[\frac{d\bar{H}}{dt}\right]_I \quad (1)$$

WHERE

$\bar{M}_{(ge)(opp)}$ = TORQUE APPLIED TO GYROSCOPK ELEMENT

$\left[\frac{d\bar{H}}{dt}\right]_I$ = TIME RATE OF CHANGE OF ANGULAR MOMENTUM WITH RESPECT TO INERTIAL SPACE

THE EQUATION OF CORIOLIS IS:

$$\left[\frac{d\bar{R}}{dt}\right]_{(ref)} = \left[\frac{d\bar{R}}{dt}\right]_{(mov)} + \bar{W}_{[(ref)-(mov)]} \times \bar{R} \quad (2)$$

(SEE CHAPTER 7 OF "VECTOR ANALYSIS" BY J. G. COFFIN, JOHN WILEY & SONS)

WHERE

$\left[\frac{d\bar{R}}{dt}\right]_{(rot)}$ = TIME RATE OF CHANGE OF R WITH RESPECT TO REFERENCE SPACE ; $\left[\frac{d\bar{R}}{dt}\right]_{(mov)}$ = TIME RATE OF CHANGE OF R WITH RESPECT TO MOVING SPACE

$\bar{R}$ = ANY VECTOR ; $\bar{W}_{[(ref)-(mov)]}$ = ANGULAR VELOCITY OF MOVING SPACE WITH RESPECT TO REFERENCE SPACE

WHEN THE GYROSCOPIC ELEMENT GIMBAL IS TAKEN AS THE MOVING SPACE AND INERTIAL SPACE AS THE REFERENCE SPACE:

$$\left[\frac{d\bar{H}}{dt}\right]_I = \left[\frac{d\bar{H}}{dt}\right]_{(gim)} + \bar{W}_{[I-(gim)]} \times \bar{H} \quad (3)$$

WHERE

$\left[\frac{d\bar{H}}{dt}\right]_{(gim)}$ = TIME RATE OF CHANGE OF ANGULAR **MOMENTUM WITH RESPECT TO GIMBAL** ; $\bar{W}_{[I-(gim)]}$ = ANGULAR VELOCITY OF GIMBAL WITH RESPECT TO INERTIAL SPACE

SINCE (1) THE MAGNITUDE OF THE ROTOR ANGULAR VELOCITY IS CONSTANT AND (2) THE SPIN AXIS OF THE ROTOR IS RIGIDLY FIXED TO THE GIMBAL,

$$\left[\frac{d\bar{W}_{(sp)}}{dt}\right]_{(gim)} = 0 \quad \textbf{AND} \quad \left[\frac{d\bar{H}}{dt}\right]_{(gim)} = 0$$

THEREFORE

$$\left[\frac{d\bar{H}}{dt}\right]_I = \bar{W}_{[I-(gim)]} \times \bar{H} \quad (4)$$

FROM NEWTON'S LAW STATED ABOVE:

$$\bar{M}_{(ge)(opp)} = \left[\frac{d\bar{H}}{dt}\right]_I = \bar{W}_{[I-(gim)]} \times \bar{H} \quad (5)$$

FROM NEWTON'S LAW OF ACTION AND REACTION:

$\bar{M}_{(ge)(out)} = -\bar{M}_{(ge)(opp)}$ = OUTPUT TORQUE OF GYROSCOPIC ELEMENT (REACTION IS SUPPLIED BY RESTRAINTS ACTING ON THE GYROSCOPIC ELEMENT)

SO THAT

$$\bar{M}_{(ge)(out)} = \bar{H} \times \bar{W}_{[I-(gim)]} \quad (6)$$

b) Performance equation for gyroscopic element

Deviation Summary II-1. Vector representation and basic performance equation for the gyroscopic element.

The relative angular motion of the earth with respect to inertial space arises from several causes, the major elements of which are as follows:

(a) Because the earth rotates about its axis once a day, an angular velocity of the earth with respect to inertial space of 15°/hr, or approximately 0.0727 milliradian/sec, results. This angular velocity must be considered in the use of gyro units for navigational purpose, but is unimportant in flight-control problems and most fire-control problems.

(b) The earth travels in its orbit around the sun at an average speed of 18.5 mps. This orbital travel at a radius of 93 million miles contributes an additional 0.0002-milliradian/sec angular velocity with respect to inertial space. Consideration of this phenomenon is necessary only in the more precise uses of gyro units in navigation because sidereal (star) time rather than solar (conventional) time must be used.

(c) The solar system, as a whole, travels through the space of the universe at about 12.4 mps due to the general rotation of the stellar galaxy (about once in 200 million years) at a radius of 30,000 light years. This effect is completely negligible. As far as can be determined, the unknown rotation of the stellar galaxy about the center of mass of the universe does not affect mechanical or electrical measurements. Hence, the "fixed stars" of the stellar galaxy may be considered as inertial space for extremely accurate gyro applications.

The point to keep in mind from the preceding discussion is that some gyro applications require that the relative motion of the earth with respect to inertial space be taken into account. For other applications, however, this relative motion is unimportant and does not require consideration in the instrumentation of the system. In determining whether the earth's motion is significant compared with other environmental motions, the following facts should be considered:

(a) At any point on the earth, the earth's daily rotation causes a gyro unit fixed with respect to the earth to rotate with respect to inertial space at earth rate about an intersecting axis that is parallel to the north-south axis of the earth.

(b) About an axis in the east-west direction, the angular velocity of this gyro unit with respect to inertial space is zero.

(c) About the local vertical, this gyro unit has an angular velocity that varies with the sine of the latitude, being zero at the equator and equal to full earth rate at either pole. The value of the earth-rate component about the local vertical at any latitude may be determined from Fig. 11-42. A corresponding rotation component, proportional to the cosine of the latitude, exists about the direction of true north.

11-6.5 GYRO UNIT PERFORMANCE

The performance of a gyro unit depends upon a number of factors, which may be grouped in two general categories; component performance and drift rate. Both of these categories must be used to determine the ultimate gyro unit performance. For example, if the requirements for component performance are satisfied, the drift rate becomes the significant yardstick,

11-6.6 Component Performance

In general, the over-all performance of a gyro unit depends upon:

(a) The linearity of the various torque-generating components (including the gyroscopic element) over their operating ranges.

(b) The accuracy of the signal generator in converting its input angles into proportional electrical signals.

11-6.7 Drift Rate

This designation is used to specify the net error in the gyro unit output due to unwanted and uncontrollable torques acting on various components of the unit; i.e., drift rate is the fictitious angular velocity the gyro unit responds to because of these torques. Some sources of unwanted torques are mechanical unbalance, distortion of parts, flexure of gyro-rotor power leads, and electromagnetic reactions of the torque generator and the signal generator. A certain amount of unwanted torque in a gyro unit is consistent and repeatable. The remainder, however, is variable and inconsistent. Thus, drift rate consists of a systematic component and an unpredictable component. The systematic component is caused by the following:

(a) Action of gravity and linear accelerations on the mechanical unbalance of the gimbal.

(b) Effects associated with the signal generator, the torque generator, the current lead-ins to the gyro wheel, and elastic deformations of the bearings and the gimbal structure.

To a large degree, the mechanical unbalance noted in item (a) can be compensated for by means of small balance nuts. Similarly, adjustments can be applied to compensate for the effects noted in item (b).

Methods of measuring and minimizing drift rate are described by **Denhard,**[46] **Grohe,**[47] and **Hoag.**[48] It is important to note that an unpredictable component of drift rate remains, even when the systematic component is completely eliminated. This residual drift rate, which is termed drift rate uncertainty, determines the ultimate performance of the gyro unit.

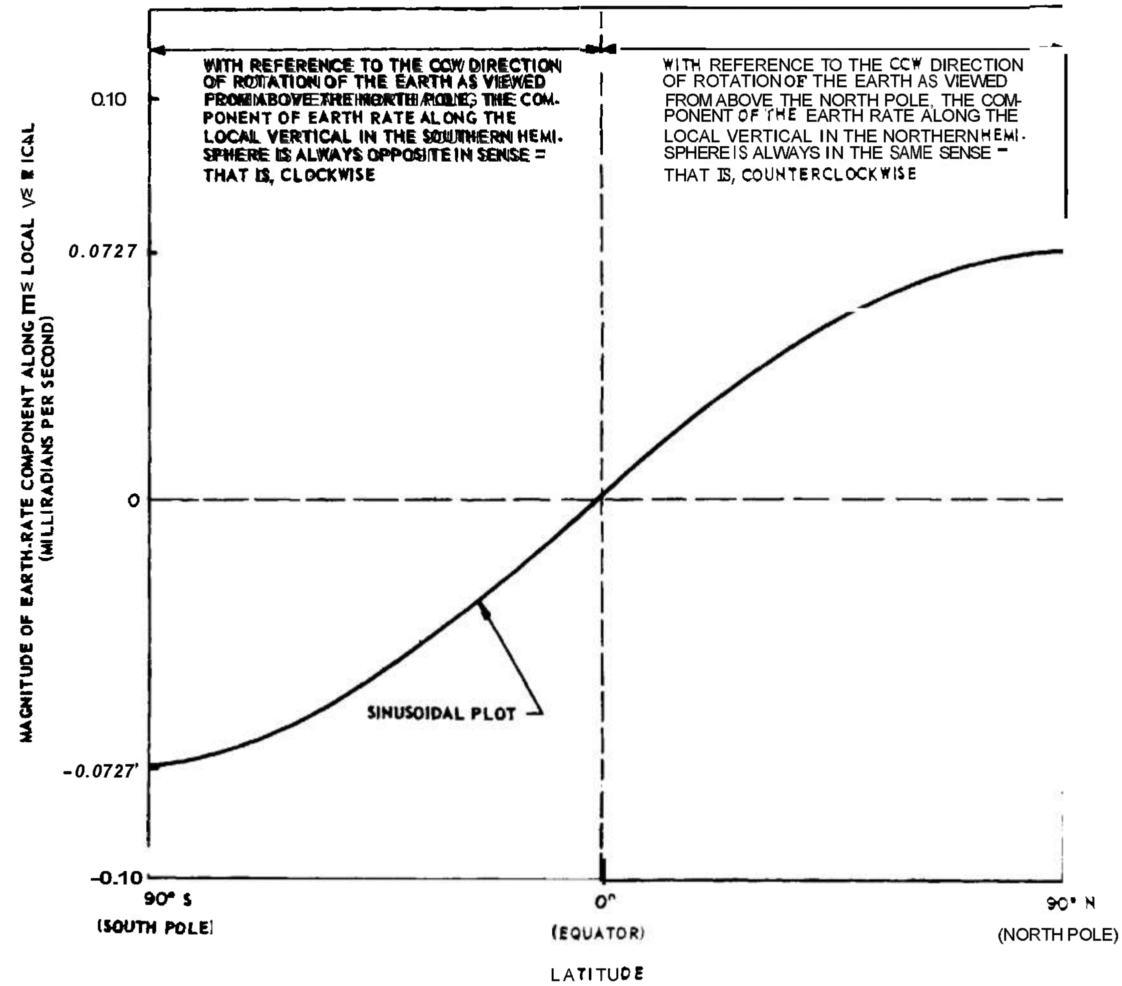


fig. 11-42 Plot for determining the value of the earth-rate component along the local vertical at any latitude.

Because some unwanted torque effects change with the size of the unit while others do not, there tends to be a balancing that creates an optimum-size gyro unit for best performance. The particular size giving the best performance varies with the design, materials used, and manufacturing techniques. For the conventional type of design, materials, etc. used in the past, the performance appeared to improve with increasing size, leading to the belief that the largest-size gyro unit consistent with space-and-weightconsiderations should be used in a given application. With the improved designs, materials, etc. developed recently, however, performance appears to improve with increasing size only until a certain optimum size is reached and to then decrease as the size is increased beyond this optimum.

11-6.8 BASIC TYPES OF GYRO UNITS

The pictorial diagrams in Fig. 11-41 illustrate the three basic types of gyro units in use at the present time. The primary purpose of these diagrams is to indicate the essential elements of each basic type and to reveal the functional differences between the three types. As a result, the diagrams bear only slight physical resemblance to actual gyro units. A discussion of the three basic types of gyro units is given in subsequent paragraphs. Although many variations of these types are found in practice, the operation of a particular unit can generally be determined from an understanding of its basic type.

114.9 TYPICAL APPLICATIONS

The gyro units described in this publication are used as integral operating components in the followingkinds of systems :

(a) Fire-control systems for tanks, aircraft, and ships[49 to 52]

(b) Missile control systems[53,54]

(c) Inertial navigation systems[54 to 59]

(d) Stable verticals[60,61]

(e) Automatic pilots and flight-control systems[62,63]

(f) Stabilization equipment, such as for a fire-control-radar antenna or a reconnaissance camera

11-6.10 TWO-DEGREE-OF-FREEDOM GYRO UNITS

The basic physical characteristics of a two-degree-of-freedom gyro unit are shown in Fig. 11-41A. This unit contains a rotor spinning at constant speed and mounted in gimbals so that its spin axis has two rotational degrees of freedom. Torque generators are provided to permit the application of command inputs for orientation control of the spin axis. Signal generators are also provided to generate electrical signals proportional to the orientation of the spin axis relative to the case of the gyro unit.

The diagram in Fig. 11-41A is sufficiently generalized to show the functional details of all elements that might be required in a two-degree-of-freedom gyro unit. In some units, the functions are carried out by elements of an altogether different type than those shown. In other units, some of the elements shown in Fig. 11-41A are omitted.

An actual two-degree-of-freedom gyro unit whose general functional operation resembles that of Fig. 11-41A is described by Klass.[57]

A two-degree-of-freedom gyro unit that has no orientational control of the gyro rotor is commonly called a *free* gyro. This designation implies that the gyro spin axis assumes a fixed arbitrary orientation with respect to inertial space and, except for any drift effects that may exist, is completely free from the effect of any motion of the gyro unit case. Hence, the angular displacement of the gyro unit case with respect to the rotor is a true measure of the angular displacement of the gyro unit case with respect to inertial space. It is important to note that gyro units commonly used in aircraft (i.e., for artificial horizons, gyro-compasses, automatic pilot systems, etc.) are designed to receive orientational command, or control, signals. Gyros of this type are therefore not free-gyros.

11-6.11 Typical Applications

A typical application of two-degree-of-freedom gyro units is one in which the gyro output signals are used to control the position of a stabilized platform.[38,57] Control is provided by feeding the output signals to servomotors that drive the platform with respect to a base that is undergoing arbitrary motion with respect to inertial space. This operation is performed in one or two different modes, depending upon the manner in which the platform is to be positioned.

11-6.12 Geometrical stabilization. In one case, the platform is required to be maintained in a fixed position with respect to inertial space. This mode of operation, known as "geometrical stabilization", does not require that orientational command signals be applied to the gyro unit. Assuming that the servo components and the other gyro unit components are operating ideally, ideal system performance requires that the gyro-rotor spin axis remain fixed with respect to inertial space.

11-6.13 Space integration. In this case, the platform is required to be rotated with respect to inertial space in a specified manner ; i.e., at an angular velocity that, for any one axis, is directly proportional to the orientational command signal applied to the torque generator associated with that axis. This mode of operation is known as "space integration". The over-all system performance is ideal when the platform is rotated about each of its axes at an angular velocity directly proportional to the respective orientational command signal. Note that geometrical stabilization can be considered as space-integrator action for zero command signal.

11-6.14 Angular-displacement measuring device. The aforementioned servo system and orientational command signal are not necessary when the two-degree-of-freedom gyro unit is used to measure angular displacement. In this application, the gyro unit is operating ideally when the output signal from each signal generator is directly proportional to the angular displacement of the case about the gimbal axis associated with the respective signal generator.

11-6.15 Departures from Ideal Performance

The performance of a two-degree-of-freedom gyro unit is governed by the component performance and the drift rate uncertainty, mentioned in Pars. 11-6.6 and 11-6.7, respectively. In a geometrical stabilization system, drift rate uncertainty prevents the gyro-rotor spin axis from remaining fixed with respect to inertial space. This effect, in turn, prevents the system from maintaining the platform in its fixed position with respect to inertial space. Drift rate uncertainty also introduces errors into the output of the gyro unit used as an angular-displacement measuring device. The use of flotation and thin wires in place of bearings is one technique that has been employed to reduce the drift rate uncertainty in a two-degree-of-freedom gyro unit. Mechanical unbalance torques in this type of gyro unit are difficult to eliminate, however, because this requires that the center of gravity of the floated part be located at a particular point ; namely, the gimbal-axes intersection.

In a space-integration system, the departure from ideal performance may be due to a combination of drift rate uncertainty, nonideal servo performance, and nonideal performance of the torque generator and the signal generator. As a result of this departure, the angular velocity of the platform about each axis is not directly proportional to the respective orientational command signal.

The actual deviation of a two-degree-of-freedom gyro unit from ideal performance can be determined by applying standard test procedures similar to those used for single-degree of freedom units.[46,47,48]

11-6.16 Nutation. An undesirable characteristic peculiar to two-degree-of-freedom gyro units is an oscillatory nodding of the spin axis. This nodding, called nutation, may occur in practice because angular momentum components of appreciable magnitude build up about axes other than the spin axis. Nutation is manifested as a wobbling of the spin axis (relative to inertial space) in an essentially conical path about the total angular momentum vector. It can be considered as a form of rotational resonance in which the nutation frequency and the spin frequency are of the same order of magnitude. Nutation effects can be damped out only by techniques that operate relative to inertial space. Conventional damping methods are unsatisfactory for high-performance instruments because such methods introduce an undesirable coupling between the gyro rotor and the gyro unit case. Nutation effects can usually be satisfactorily eliminated in practice, but they should always be considered for the particular application at hand, especially if high performance is required.

11-6.17 SINGLE-DEGREE-OF-FREEDOM GYRO UNITS

A single-degree-of-freedom gyro unit operates on the same basic principle as the two-degree-of-freedom unit but, as its name implies, has only one degree of freedom. This suppression of full freedom of rotation makes the unit respond, for the most part, to angular velocity components about one particular axis called the *input* axis (see Figs. 11-41B and 11-41C). Although there is some response to angular velocity components about axes other than the input axis, the magnitude of this response is very small in units that are properly designed and applied. For simplicity, single-degree-of-freedom gyro units are commonly referred to as *single-axis* gyro units. Single-axis gyro units are classified as either *rate* gyro units or *integrating* gyro units, depending upon the nature of the output signal. A single-axis rate gyro unit generates an output signal that is proportional to the input angular velocity of the gyro unit case with respect to inertial space. The output of a single-axis integrating gyro unit, on the other hand, is proportional to the time integral of the same type of input angular velocity. (A third type of single-axis gyro unit—the *doubly-integrating* gyro unit, or unrestrained gyro—is sufficiently similar, basically, to the integrating gyro unit so that it is not treated here. However, there are marked differences in the characteristics of the servo loops used with the two gyro units.)

11-6.18 SINGLE-AXIS RATE GYRO UNIT

The basic physical characteristics of a single-axis rate gyro unit are compared with those of a two-degree-of-freedom gyro unit in A and B of Fig. 11-41. The differences between these characteristics are as follows:

(a) One degree of freedom of the two-degree-of-freedom unit is eliminated by essentially freezing the outer gimbal to the gyro unit case.

(b) Means are added for the application of an elastic restraint torque about the free gimbal axis.

(c) A damper is added for the dynamic control of oscillations about the free gimbal axis.

Several publications[38,39,47,48] describe a single-axis rate gyro unit that has been designed, built, and tested by the Instrumentation Laboratory of the Massachusetts Institute of Technology. A number of other similar designs are in production and are described in publications[64,65] prepared by the manufacturers.

From a design standpoint, it is important to note that equally effective damping may be produced by either one or a combination of the following: eddy-current drag effects; rate-controlled feedback effects; or viscous shear in a fluid. It is also pointed out that the required functions of the associated torque generators, signal generators, and elastic-restraint devices may be produced by different types of mechanisms.

11-6.19 Departures from Ideal Performance

Under quasi-static conditions, ideal performance of a single-axis rate gyro unit is attained when the voltage output signal is directly proportional to the angular velocity component about the gyro unit input axis over its specified range of operation. Deviations from ideal performance usually occur at the low end of the input-angular-velocity range because drift effects predominate in this region.[38] Operation at the upper end of the input-angular-velocity range is restricted by gyro gimbal stops. These stops are set to prevent the angular displacement of the gimbal from exceeding a value that will cause excessive interaxis coupling and thereby lead to erroneous outputs.

Drift-rate measurements for a rate gyro unit are made by observing the output voltage for zero input angular velocity with respect to inertial space, and then dividing this voltage by the angular velocity input – voltage output sensitivity of the unit. A major cause of this drift rate, in addition to those noted in Par. 11-6.7, is a misalignment that causes the minimum output (or signal-generator null) to occur at a gimbal angle that differs from the mechanical zero set through the elastic-restraint spring. This misalignment includes adjustment tolerances and mechanical hysteresis effects.

11-6.20 Design Considerations

When deciding on the magnitude of the gimbal deflection angle for a given input velocity, the following conflicting factors must be kept in mind:[38]

(a) If the magnitude is to be small (this requires a strong elastic restraint), the resolution with which small angular velocities can be determined is limited.

(b) If the magnitude is to be large, the necessary decrease in the elastic restraint will cause a corresponding undesirable reduction in the speed of response. In addition, large gimbal angles lead to problems of interaxis coupling. For example, angular rates about the spin reference axis are liable to create undesirable gimbal torques and thereby cause a false rate indication.

A compromise gimbal angle that meets all the requirements of a given application is very difficult to achieve. For example, the use of rate gyros in geometrical stabilization systems will not forestall platform position errors caused by servo-amplifier drift, unless additional instrumentation is provided to furnish extra integration. Rate gyros may also fail to detect small angular velocities and thereby introduce a drift in the platform.

11-6.21 SINGLE-AXIS INTEGRATING GYRO UNIT

An illustrative comparison of the basic physical characteristics of a single-axis integrating gyro unit with those of a single-axis rate gyro unit is shown in B and C of Fig. 11-41. The differences between these characteristics are as follows:

(a) The elastic restraint shown in Fig. 11-41B is eliminated.

(b) The damping is made relatively heavy to serve as the primary restraining torque instead of merely controlling oscillations.

Except for the above, the components of a single-axis integrating gyro unit are arranged to have the same configuration as the components of a single-axis rate gyro unit., Several publications[38,39,47,48] describe a single-axis integrating gyro unit that is one of a series designed, built, and tested by the Instrumentation Laboratory of the Massachusetts Institute of Technology. A number of other similar designs are in production and are described in publications[64,65] prepared by the manufacturers.

11-6.22 Qperating Arrangement

Unlike the two-degree-of-freedom and the single-axis rate gyro units, the single-axis integrating gyro unit must be operated in a closed-loop servo system. By this arrangement, the gyro gimbal is forced to assume a zero angular displacement at all times, thereby maintaining a position of null output voltage. This method of operation results in extremely high performance, especially because the interaxis coupling problem is eliminated.

11-6.23 Modes of operation. Single-axis integrating gyro units are operated in either of two modes, depending upon the method of applying the gyro output to the other components of the feedback system.[38,39] These modes of operation are classified as follows:

(a) Gyro output signal used to control the servo components that drive the gyro supporting structure. In this mode of operation (Fig. 11-43), an angular velocity command signal is applied to the gyro unit. The other components of the feedback system, together

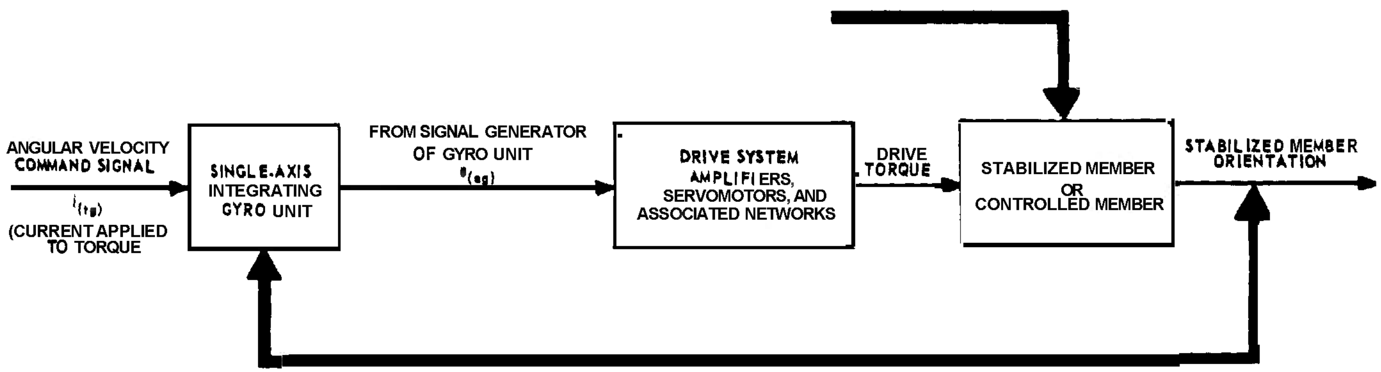


Fig. 11-43 Single-axis feedback system using an integrating gyro unit to drive the gyro supporting member according to an angular velocity command signal.

with the gyro unit, are arranged to operate as a space **integrator.**[38] When the command signal is applied to the torque generator of the gyro unit, the gyro output signal controls the servo drive system so that the gyro supporting structure undergoes a specific angular displacement about the gyro input axis. This angular displacement is directly proportional to the time integral of the input command signal. During the foregoing procedure, the feedback system is simultaneously carrying out an additional and separate function (geometrical stabilization) in which the supporting structure is geometrically stabilized with respect to inertial space from any uncommanded rotation about the gyro input axis. In other words, the space integrating and the geometrical stabilizing functions are performed concurrently.

(b) Gyro output signal used as the torque-generator input signal. In this mode of operation (Fig. **11-44**), the torque-generator input signal is directly proportional to the angular velocity of the gyro supporting structure about the input axis of the gyro unit. Thus, the feedback system serves as an angular velocity measuring device and is designated as a feedback gyro rate indicating system.

11-6.24 Ideal performance. The single-axis integrating gyro units in the systems mentioned above are considered to have ideal performance when the following conditions are fulfilled:

(a) Space integration systems. The ratio of the angular velocity of the stabilized member about the gyro unit input axis (with respect to inertial space) to the command current remains constant over the entire operating range of the system.

(b) Geometrical stabilization systems. The angular velocity of the stabilized member about the gyro unit input axis (with respect to its base) remains equal and opposite to the angular velocity of the base (with respect to inertial space) over the entire operating range of the system.

(c) Angular velocity measuring system. The ratio of the feedback current **to** the angular velocity of the gyro unit about its input axis (with respect to inertial space) remains constant over the entire operating range of the system.

11-6.25 Departures from ideal performance. Static performance characteristics in each of the three cases mentioned above show that the performance falls off rapidly from the ideal values at the low angular-velocity end of the operating **range.**[38] This deteriorating effect occurs because drift effects predominate in the low angular-velocity region. Except in the case of geometrical stabilization, the performance also falls off rapidly from the ideal values at the upper end of the angular-velocity region. This deteriorating effect is due to the saturation that takes place in the magnetic circuits of the torque generator when a large input current is applied. No saturation occurs when geometrical stabilization is the only function to be performed because, in such a case, no input current is applied to the torque generator. The performance of single-axis integrating gyro units may be determined by the procedures outlined by Denhard,[46] **Grohe,**[47] and Hoag.[48]

11-6.26 APPLICATION FACTORS

Intrinsic with any design problem involving gyro units are various application factors, such as electrical power requirements, special operational problems, impedance levels, and availability of design information on **gyro-**unit components.

11-6.27 Electrical Power Requirements

Because of the variety of gyro-unit configurations, even within each of the major types covered in the preceding paragraphs, a brief summary of power requirements for gyro units is not feasible. For reference, see the report by Rawlings and Rankin[66] on the gyro units of various manufacturers. Each application of gyro units must be considered individually. With regard to the power supply for gyro-wheel-drive hysteresis motors, however, it should be noted that the power must be closely regulated to avoid variations in the gyro-motor excitation current and frequency. Variations in current can shift the center of gravity of the torque-summing gimbal by a combination of thermal and magnetic effects and thereby create gravity-sensitive drift rates. Variations in frequency affect the angular velocity input —angular displacement output sensitivity of the gyro unit, which should ideally remain constant.

11-6.28 Special Operational Problems

(a) Warm-up time, Some applications require that the gyro unit have an extremely rapid warm-up time. Therefore, it may be necessary to either include a special integral heater for supplying heat quickly in the vicinity of the damper float or to provide for an extra supply of heat from an external non-integral heater during the warm-up time. For normal operation, external heaters have an advantage because they can be chosen to fit particular temperature environments. In addition, external heaters can be made more rugged than internal ones which, upon failure, may ruin the entire gyro unit.

(b) Exposure to extreme temperatures. It is important that a gyro unit be stored and used only within the temperature extremes for which it is designed. Use at temperatures beyond the rated limits will result in decreased accuracy and may possibly cause internal damage to the unit.

(c) Mounting problems. In contrast with many operating components, gyro units are very rugged and therefore do not usually need shock mounting for protection. This is especially true of gyro units in which the internal elements are supported principally by liquid flotation. For many applications, in fact, gyro units should be connected directly to the components they are stabilizing or controlling, since shock mounting may interfere with the operation of the units.

(d) Inherent limitations of two-degree-of-freedom gyro units. Gimbal lock, which may result when the vehicle carrying a two-degree-of-freedom gyro unit changes its attitude during radical maneuvers, and coupling effects associated with gyro-erection systems are factors that must be considered in applications of two-degree-of-freedom gyro units. These problems can be eliminated by the use of high-performance single-degree-of-freedom gyro units.

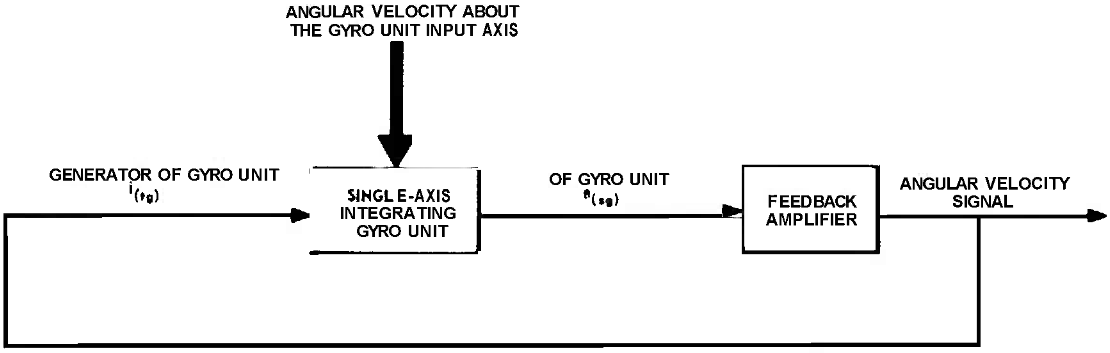


Fig. 11-44 Feedback system using an integrating gyro unit to measure the angular velocity about the gyro unit input axis.

11-6.29 Impedance Levels

Because of the variety of gyro-unit configurations, it is not feasible to summarize briefly the various impedance levels to be found in the input and output circuits of gyro units. For reference, see the report by Rawlings and Rankin.[66] As is the case when determining power requirements, each gyro-unit application must be considered individually.

11-6.30 Availability of Design Information

Although a tremendous amount of effort has been expended in recent years on the design of gyro units and their components, this information is often either never published or never adequately noted in bibliographies. To remedy this situation, reference[67] was prepared. This bibliography, prepared late in 1956, is as complete a list as possible of available information relating to lubrication, friction, manufacturing and assembly techniques, stress analysis, etc., of the various components used in gyro units. Much valuable design information is also available in publications relating to specific gyro unit designs; for example, Grohe.[68,69]

11-6.31 INERTIAL-GUIDANCE APPLICATIONS OF GYRO UNITS

One group of authorities on inertial guidance[70] has defined *guidance* as the "art and science of causing objects to follow desired paths with respect to designated reference points." ***Inertial guidance*** is the particular type of guidance in which the control equipment involved generates all orientation and position data by means of internal systems that are operated by gravitation forces and inertia reaction effects. These two types of inputs are not subject to interference from any currently known source—either natural or artificial. As a result, inertial guidance offers the following outstanding advantages over any other type of guidance.[54]

(a) The serious treat usually posed by enemy jamming activities in military applications of guidance is eliminated.

(b) The lack of any dependence on radiation contacts with the environment external to the inertial guidance system means that the guidance capability is unaffected by such problems as weather disturbances and altitude. Inertial guidance systems have applications for either terrestrial operations[59] or interplanetary operations,[58] and operations that are either manned or unmanned.

Because of its inherent nature, inertial guidance is a process that is describable by the laws of mechanics. As such, it involves the measurement of time, gravitation, acceleration, and angular velocity. Accordingly, gyro units and specific force receivers are the components most uniquely associated with inertial guidance systems. The purpose of the following paragraphs is to show how these components fulfill their functions in inertial guidance systems. Because of space limitations, consideration of inertial guidance here is limited to this particular facet. Complete treatments of inertial guidance systems are available in the references, however, for those who may wish to investigate this subject further.

In practice, the over-all function of an inertial guidance system is to cause the vehicle in which it is carried to arrive at some preselected destination by following a more or less closely defined path. The particular functions that such equipment must provide in order to accomplish a typical terrestrial guidance mission, for example, would be as follows:

(a) Establish an initial orientation that is nonrotating with respect to inertial space. For this purpose, a three-axis geometrical stabilization system made up of a combination of three single-axis servo loops such as shown in Fig. 11-43 will suffice. The operation of such a system is described in detail by Draper and Woodbury.[71] It is of primary importance that the gyro units be characterized by low drift rate, for the lower the drift rate the more closely is the reference orientation held with respect to inertial space.

(b) Establish an initial orientation that is nonrotating with respect to the earth. As discussed in reference,[70] such a nonrotating reference can be obtained by the addition of a sidereal time drive or computation based on accurate knowledge of sidereal time. This will give a reference orientation that rotates at earth rate with respect to inertial space and therefore remains fixed with respect to the earth. High-quality sidereal time signal generators can readily be achieved.

(c) Establish an orientation related to the position of the guided vehicle with respect to the earth. Fortunately, each position on the earth's surface is uniquely associated with one, and only one, direction of the true vertical (that is, the local direction of gravity). Indications of this true vertical are obtained by means of a *vertical indicating system.* Specific force receivers are the crucial components here, for it is the accuracy with which specific force is received that determines the ultimate accuracy with which the **true** vertical is indicated. The operation of a vertical indicating system is described in following paragraphs.

(d) Determine the movement of the guided vehicle over the surface of the earth by appropriate coupling of the vertical indicating system, which provides local-position information, and the geometrical stabilization system, which in connection with a sidereal time drive provides reference-position information. The distance between the local position and the reference position on the surface of the earth in nautical miles is given by the angle between the local vertical and the reference vertical measured in minutes of arc (ref. Fig. 11-45).

(e) Compare the actual **course** of the guided vehicle with the programmed course and provide the necessary correction signals to the vehicle's control system-

As shown in Wrigley, **Woodbury,** and **Hovorka,**[72] there are three basic methods of coupling between the vertical indicating system and the geometrical stabilization system to obtain the required position data. Listed in order of decreasing accuracy and a corresponding decrease in complexity, these three methods can be **described** as follows :

(a) The first method uses direct measurement of the angle between the reference orientation that is fixed with respect to the earth and the true vertical that passes through the instantaneous position of the guided vehicle.

(b) The second method uses integration of the angular velocity of the indicated vertical as the local position of the vehicle changes over the earth's surface.

(c) The third method uses double integration of the angular acceleration of the indicated vertical. This method involves direct use of the measured acceleration outputs of the specific force receivers.

Detailed discussion here of these coupling methods is prohibited by space limitations. For such details, the reader is referred to the reference noted and also to Draper, Wrigley, and **Woodbury.**[70]

11-6.32 INDICATION OF THE TRUE VERTICAL

The local direction of gravity, that is, the direction of the true vertical, can be most readily indicated by means of a simple plumb bob. Unfortunately, this method will produce satisfactory results only when measurement is made from a base that remains fixed with respect to the earth. In practical guidance applications, the base from which measurements must be made is subject to many kinds of motion, arising from roll, pitch and yaw of the guided vehicle, maneuvers, and atmospheric turbulence. Under these conditions, the plumb bob is completely incapable of indicating the true vertical.

A possible remedy for this situation that involves the use of a pendulum was proposed by Max Schuler many years **ago.**[73] This remedy was based on Schuler's recognition of the fact that as the base from which measurements were to be made moves over the surface of the earth, the angular acceleration of the pendulum arm about its pivot must equal the angular acceleration of the vertical about the center of the earth. To accomplish this condition, known as ***Schuler tuning,*** the separation of the pivot and the center of mass of the pendulum must be equal to the radius of gyration of the pendulum divided

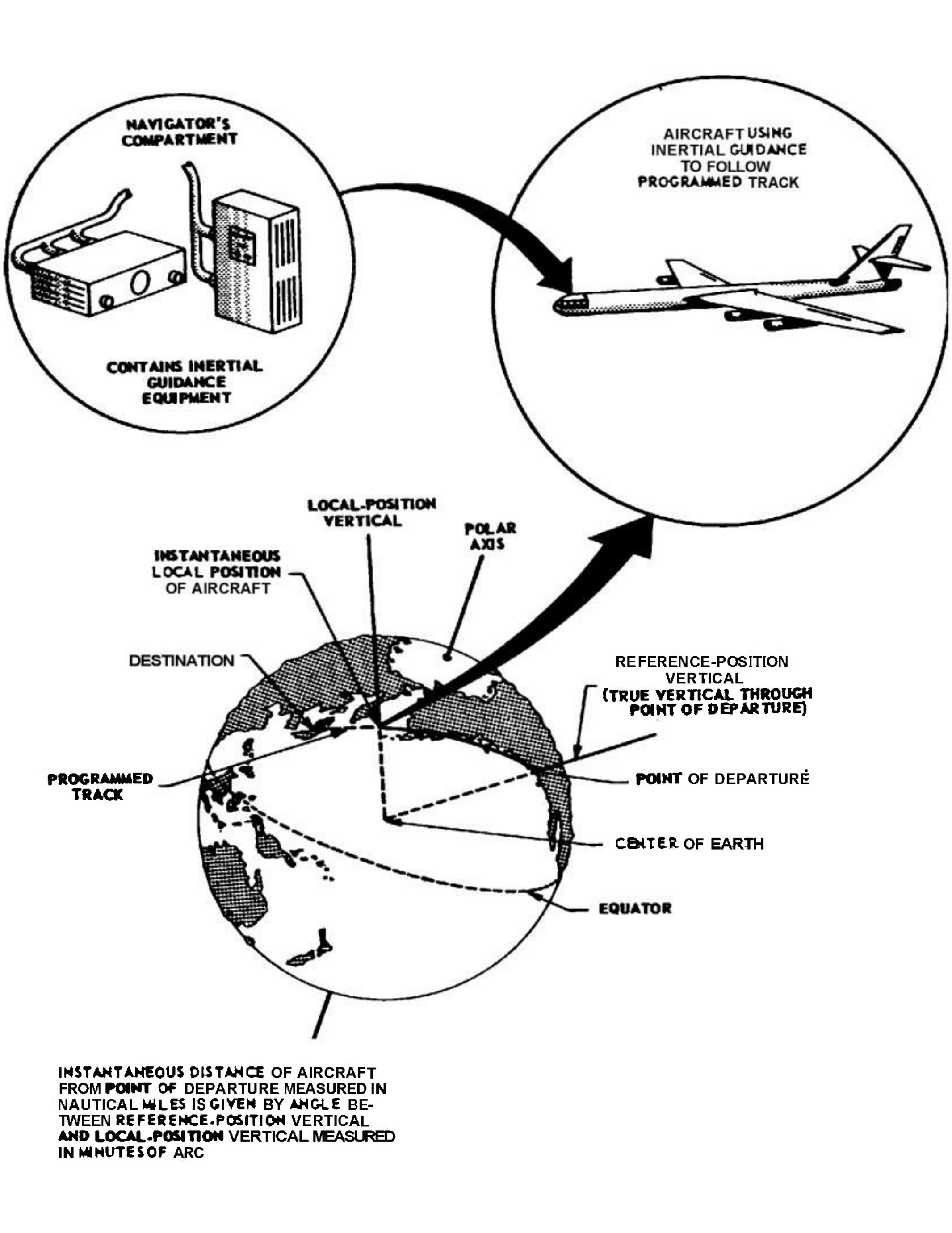


Fig. 11-45 Determination of present position of an aircraft by means of inertial guidance.

by the radius of the earth. Were such a pendulum to operate undamped in the earth's gravity field, it would have a natural period of about 84 minutes. Hence it is often referred to as the 84-minute pendulum.

The requirements of pivot to center-of-mass separation make it highly improbable that an 84-minute pendulum can ever be constructed. Such a device will not be needed, however, since the desired Schuler-tuning condition can effectively be obtained by an equivalent electromechanical closed-loop servo system. This system utilizes a space integrator whose controlled-member orientation is monitored by acceleration signals furnished by specific force receivers that must have natural periods much shorter than 84 minutes, so that they measure – rather than filter – the pertinent force components. By using appropriate coupling between the specific force receivers and the space integrator, the dynamics of the system can be controlled to give the equivalent Schuler tuning required of high-performance vertical indicating systems. Because Schuler's contributions to the field of inertial guidance have been so great, this essential dynamic characteristic of vertical indicating systems has been given the designation *Schuler tuning,* in his honor.

Figure 11-46 shows the physical arrangement employed in a typical vertical indicating system. The controlled member, the inertial reference package, the gimbal system, and the associated resolvers and drive motors constitute a three-axis space integrator. The operation of this space integrator for a single-axis is represented functionally by Fig. 11-43 and is discussed earlier. The specific force receiving package shown in Fig. 11-46 contains two specific force receivers. These receivers can be one of many designs; for example, see Part II of Draper, Wrigley, and Lees.(54) Two such units, with their input axes perpendicular and in the horizontal plane, are required for the complete indication of the vertical. The components that couple the specific force receivers and the space integrator are omitted from the pictorial representation of Fig. 11-46 inasmuch as they have no particular geometrical relationship to the over-all system.

Figure 11-47 represents, for a single axis only, the functional operation of the vertical indicating system shown in Fig. 11-46. As indicated by these two figures, the two specific force receivers are rigidly mounted on the controlled member of the space integrator. This means that they are subject to the same specific forces that act on the vehicle being guided by the inertial guidance system of which the vertical indicating system is a subsystem. The specific force receivers are so oriented that they generate signals proportional to the specific forces acting along two directions at right angles to the inner gimbal axis. These specific force signals are then integrated and modified by one of several forms of damping as required to produce high-quality angular velocity command signals for the space integrator. These signals are applied to the two gyro units that control the outer and middle gimbal drive motors. This action will cause a direction fixed to the controlled member (in this case, along the inner gimbal axis) to be maintained in continuous alignment with the true vertical. The accuracy with which this is done is determined by the particular signal-integrator-plus-damping configuration employed to couple the specific force receivers to the space integrator. For a mathematical analysis of several typical configurations, together with performance plots as a function of the frequency of the specific-force inputs, see Wrigley, Woodbury, and Hovorka.(72) The particular Schuler-tuning conditions are specified for each case.

For a description of some actual inertial guidance systems, see Part III of Draper, Wrigley, and Lees.(54)

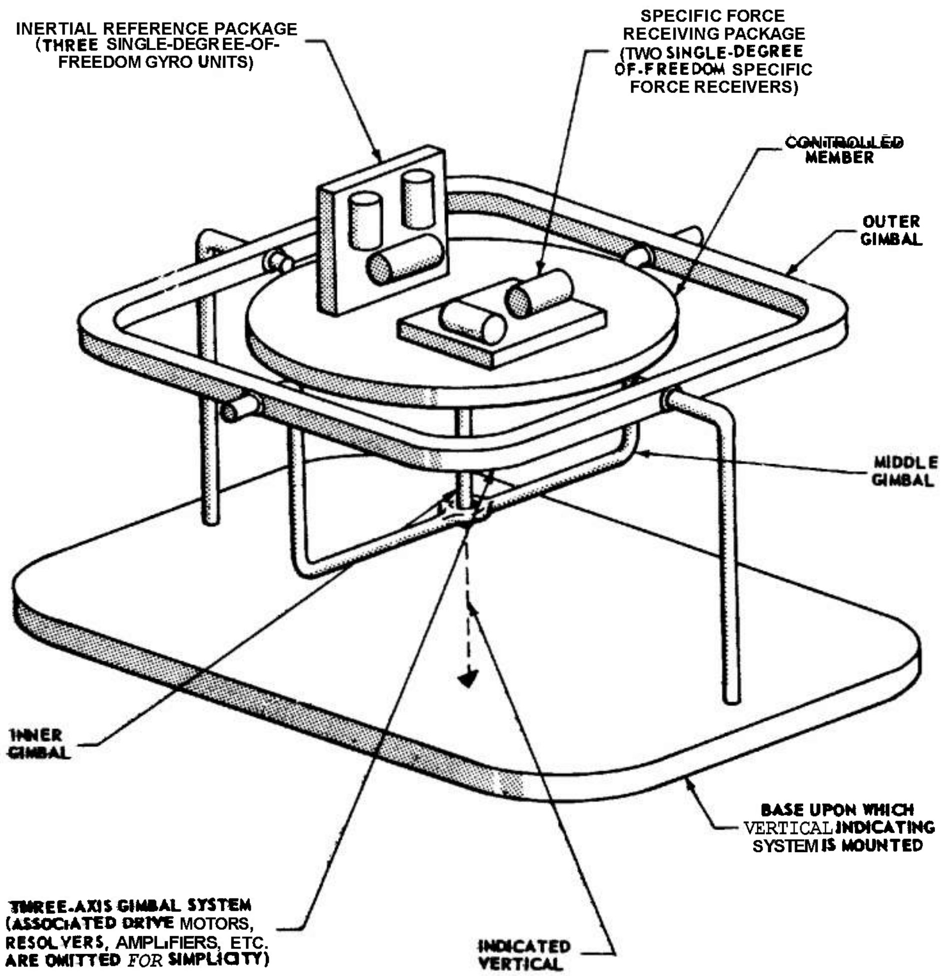


Fig. 11-46 Physical arrangement employed in a typical vertical indicating syst--

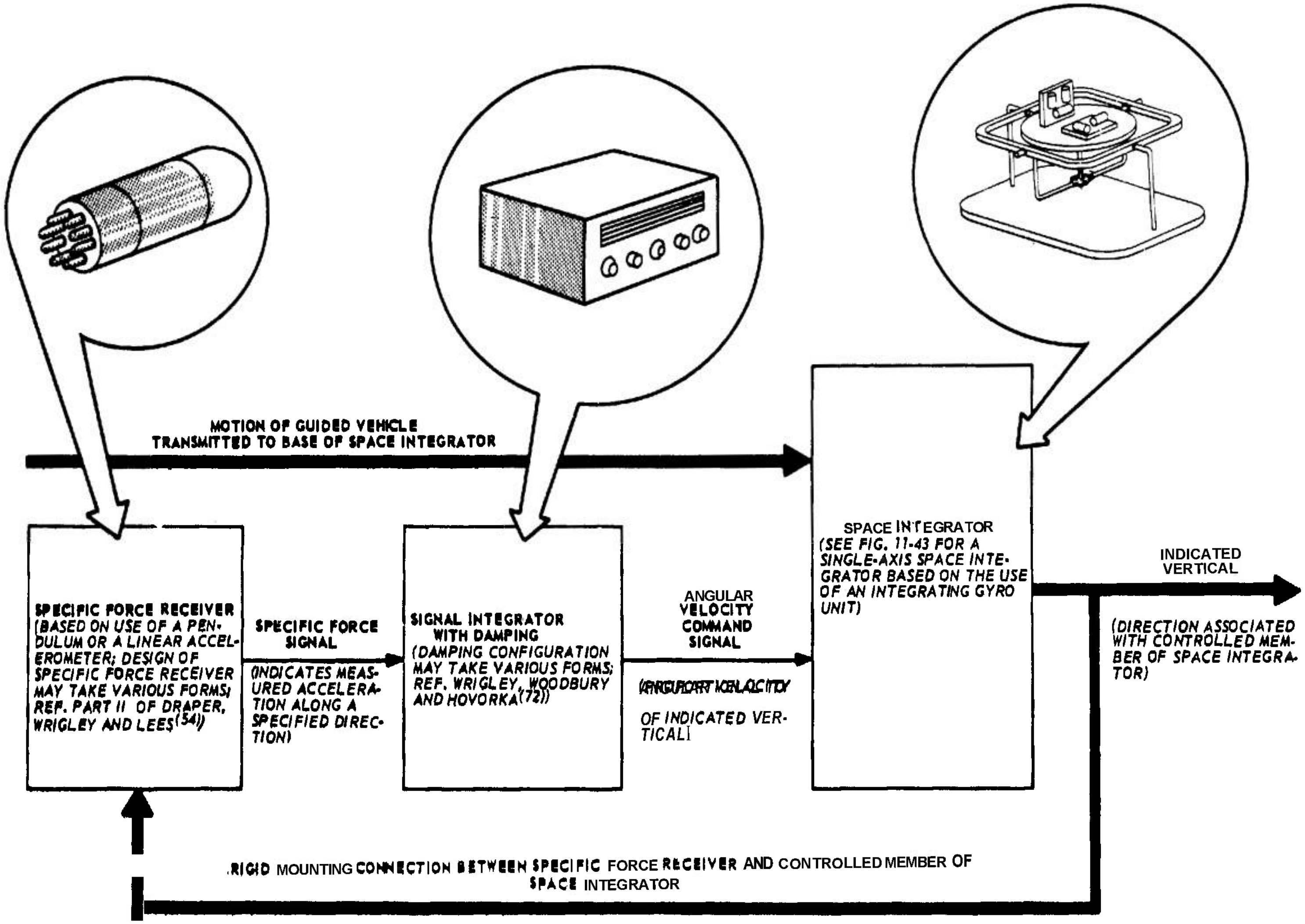


Fig. 11-47 Functional diagram, showing single-axis operation only, of a vertical indicating system.

11-7 ANALOG-TO-DIGITAL CONVERTERS*

11-7.1 INTRODUCTION

When the command for a servomechanism originates from digital data-processing equipment and is therefore expressed as a number, it is often necessary that the response of the servomechanisms also be expressed in numerical form. Sensing elements are then required to convert analog quantities, such as a voltage or a shaft rotation denoting the response of the servomechanism, into a corresponding numerical representation. These sensing elements are called **analog-to-digital converters** or **coders.** In this section, only the most common types of coders are discussed. More complete surveys may be found in references 78 through 81.

11-7.2 NUMERICAL REPRESENTATION

Decimal numbers, which are used in everyday life, are numbers having ten possible digits (0 to 9), the values of successive digit columns differing from each other by a power of ten. Thus, the decimal number 3108 means

$$3 \times 10^3 + 1 \times 10^2 + 0 \times 10^1 + 8 \times 10^0$$

In general, a decimal number q with $(n+1)$ digits d may be expressed as

$$q = d_n\, d_{n-1}\, d_{n-2} \ldots d_0 \qquad (11\text{-}46)$$

and means

$$d_n 10^n + d_{n-1} 10^{n-1} + d_{n-2} 10^{n-2} + \ldots + d_0 10^0 \qquad (11\text{-}47)$$

In machines, it is more convenient to express numbers in terms of only two possible digits, ZERO and ONE. For example, the digit ZERO may be instrumented as an open contact, a de-energized relay, or a tube that is cut off. Then, the digit ONE could be instrumented by these same devices as a closed contact, an energized relay, or a tube that conducts.

11-7.3 Binary Numbers

Numbers in which each digit has only two possible values are called **binary** numbers. In these numbers, the values of successive digit columns differ by a power of two. The rightmost digit column has the value 2^0, the second column from the right has the value 2^1, the third column from the right has the value 2^2, etc. Table 11-11 shows the decimal numbers from 0 to 31 and their binary equivalents. To write numbers larger than decimal 31 in binary form, additional digit columns are required. A sixth binary digit column covers numbers up to and including 63, a seventh covers numbers up to 127, etc. Given a binary number p of $(n+1)$ digits b, it follows that

$$p = b_n b_{n-1} b_{n-2} \ldots b_0 \qquad (11\text{-}48)$$

and its numerical value is

$$d = b_n 2^n + b_{n-1} 2^{n-1} + b_{n-2} 2^{n-2} + \ldots + b_0 2^0 \qquad (11\text{-}49)$$

For example, the decimal equivalent of the binary number 11001101 is

$$d = 1 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 205$$

11-7.4 Binary Coded Decimal Numbers

Binary coded decimal numbers are also used in machines as a numerical representation of information. In this system, each individual decimal digit is represented by a group of four binary digits (bits). Since four binary digits can express 16 different codes, only ten of the 16 are needed to express the ten decimal digits. Therefore, six of the binary codes are not required and it is entirely a matter of system convenience as to which six are eliminated.

*By A.K. Susskind

TABLE 11-11 BINARY NUMBERS

Decimal Number	Binary Number	Decimal Number	Binary Number
	16 8 4 2 1		16 8 4 2 1
0	00000	16	10000
1	00001	17	10001
2	00010	18	10010
3	00011	19	10011
4	00100	20	10100
5	00101	21	10101
6	00110	22	10110
7	00111	23	10111
8	01000	24	11000
9	01001	25	11001
10	01010	26	11010
11	01011	27	11011
12	01100	28	11100
13	01101	29	11101
14	01110	30	11110
15	01111	31	11111

Example: $\underbrace{15}_{\text{Decimal}} = \overbrace{8 + 4 + 2 + 1}^{\text{Binary}}$

11-7.5 CODING DISCS

The most common type of digital sensing element is the d i n g disc. An example of a primitive coding disc that divides one shaft revolution into 16 parts is shown in Fig. 11-48. The center of the disc is connected to the shaft to be coded. The black areas are made of electrically conducting material and the white areas are made of nonconducting material. A common voltage is connected to all conducting areas. Four small carbon brushes (B_0 to B_3) are arranged along a fixed radius and pressed against the disc by springs. As the shaft and hence the disc rotate counterclockwise, the following brushes make contact with conducting areas:

Disc Position, Sector Number	Brushes Making Contact	Binary Number
0	—	0000
1	B_0	0001
2	B_1	0010
3	B_1, B_0	0011
4	B_2	0100
5	B_2, B_0	0101
6	B_2, B_1	0110
7	B_2, B_1, B_0	0111
8	B_3	1000
9	B_3, B_0	1001
10	B_3, B_1	1010
11	B_3, B_1, B_0	1011
12	B_3, B_2	1100
13	B_3, B_2, B_0	1101
14	B_3, B_2, B_1	1110
15	B_3, B_2, B_1, B_0	1111

Note that the appearance of the voltage on brush B_0 represents a ONE in the 2^0 digit column, on brush B_1 a ONE in the 2^1 digit column, etc.

11-7.6 Techniques for Avoiding Misalignment Errors

Suppose that brush B_3 were imperfectly lined up and slightly displaced in a counterclockwise direction (see Fig. 11-48). Then, as the shaft rotates in a clockwise direction, brush B_3 makes contact with the conducting area in the 2^3 digit column before any of the others do, and the number 1000 is read out, corresponding to sector 8. This is entirely false, for either 0000 (sector 0) or 1111 (sector 15) should have been read out. Similar mistakes result if the individual rings (zones) of the code pattem do not line up perfectly along a radius. To avoid these errors, two techniques have been devised.

11-7.7 Use of two brushes. In one technique, two brushes are used to read each binary digit.[74,75] The two brushes are arranged so that when one brush is located over a transition between conducting and nonconducting areas and therefore could result in a reading error, the mating brush is located completely within a conducting or nonconducting area. Additional circuits are provided to always select the brush that is not over a transition.

11-7.8 Cyclic code. In the other technique, a cyclic code (also called reflected or Gray code) is used in place of the binary code. A feature of the cyclic code is that successive numbers differ from each other in only one digit column. This code eliminates reading mistakes if the brushes are slightly misaligned or the d e pattern is not perfectly accurate. Table 11-12 lists decimal and binary numbers together with the corresponding cyclic code for numbers up to 31. In cyclic code, successive digit columns have the absolute values 1, 3, 7, 15, 31, 63, 127, etc. In general, the kth digit column (counting the rightmost column as one) has the absolute value $\sum_{0}^{k-1} 2^j$. Moreover, the sign of successive ONES alternates, with the most significant ONE having a positive value. For example, the decimal equivalent of the cyclic code number 110111001 is

$$+1\times\sum_{0}^{8}2^j-1\times\sum_{0}^{7}2^j+1\times\sum_{0}^{5}2^j-1\times\sum_{0}^{4}2^j+1\times\sum_{0}^{3}2^j-1\times\sum_{0}^{0}2^j$$

$$=+1\times(2^8+2^7+2^6+2^5+2^4+2^3+2^2+2^1+2^0)-1$$

$$\times(2^7+2^6+2^5+2^4+2^3+2^2+2^1+2^0)+1\times(2^5+2^4+2^3+2^2+2^1+2^0)-1$$

$$+(2^4+2^3+2^2+2^1+2^0)+1\times(2^3+2^2+2^1+2^0)-1\times 2^0$$

$$=2^8+2^5+2^3+2^2+2^1=256+32+8+4+2=302$$

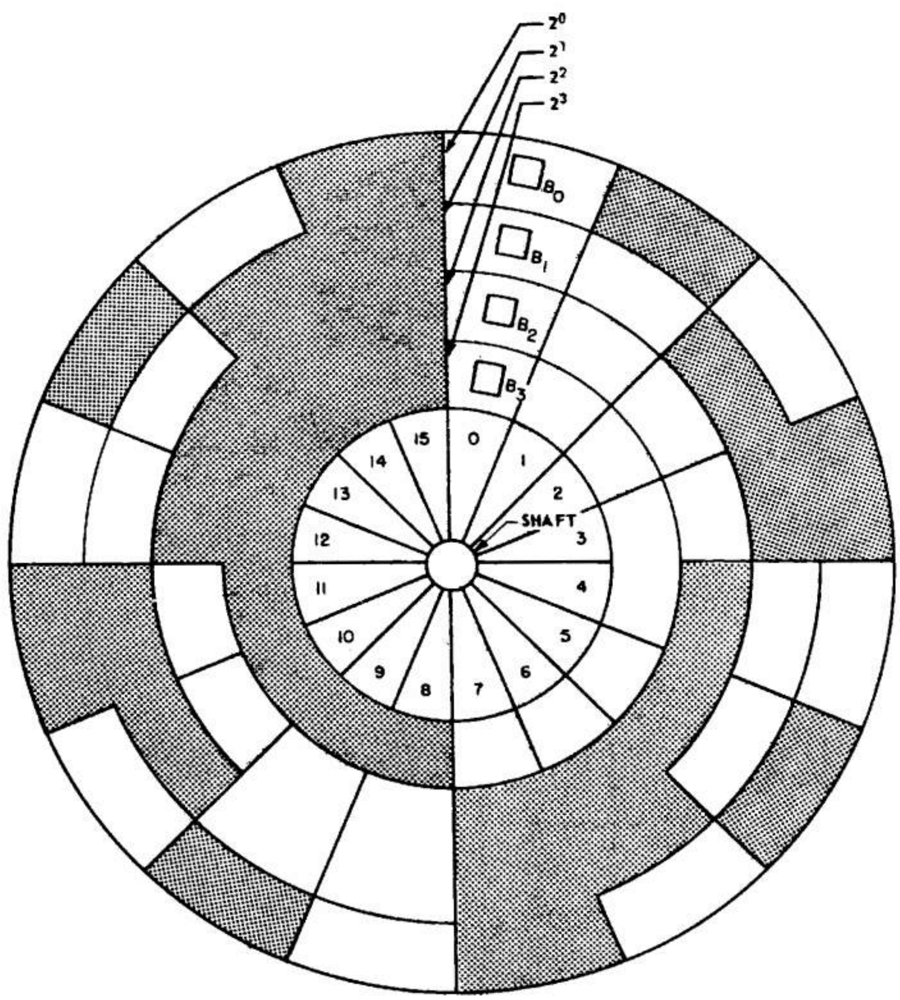


Fig. 11-48 Binary coding disc.

Decimal Number	Binary Number	Cyclic Code	Decimal Number	Binary Number	Cyclic Cude
	16 8 4 2 1	31 15 7 3 1		16 8 4 2 1	31 15 7 3 1
0	00000	00000	16	10000	11000
1	00001	00001	17	10001	11001
2	00010	00011	18	10010	11011
3	00011	00010	19	10011	11010
4	00100	00110	20	10100	11110
5	00101	00111	21	10101	11111
6	00110	00101	22	10110	11101
7	00111	00100	23	10111	11100
8	01000	01100	24	11000	10100
9	01001	01101	25	11001	10101
10	01010	01111	26	11010	10111
11	01011	01110	27	11011	10110
12	01100	01010	28	11100	10010
13	01101	01011	29	11101	10011
14	01110	01001	30	11110	10001
15	01111	01000	31	11111	10000

Example: $\underbrace{9}_{\text{Decimal}} = \underbrace{8 + 1}_{\text{Binary}} = \underbrace{+15 - 7 + 1}_{\text{Cyclic}}$

11-7.9 Cyclic coding disc. Figure 11-49 shows a coding disc that converts shaft rotation to cyclic (or Gray) code numbers. It is readily verified that small misalignments of the brushes or code rings do not result in an error larger than one sector. Usually, however, it is required that the output of the converter be in binary form. Hence, the cyclic code output must be converted into binary code. This can be done with conventional, digital data-processing circuits. Follingstad et al[76] describe an example of equipment developed to perform the code conversion. The complexity of the equipment is approximately the same regardless of whether a cyclic code disc plus a code converter or a binary disc with two reading brushes per digit plus selection circuits are used.

Coding discs have been widely used. They are commercially available with as many as 1024 sectors (10 binary digits). A typical disc diameter is four inches. Where finer resolution is required, discs that are read optically can be used. In these discs, the code pattern is represented by opaque and transparent areas. On one side of the disc is mounted a light source, the illumination of which is confined to a narrow radial line. Photoelectric devices are mounted on the other side of the disc along the same radial line. Those photoelectric devices that receive light through the disc conduct current; the others do not. Photoelectric discs of reasonable diameter (a few inches) have been made with an accuracy of one part in 65,536 (16 binary digits).

11-7.10 VOLTAGE-TO-DIGITAL CODERS

When the quantity sensed is already available in the form of a voltage and it is necessary to express this quantity in digital form, a voltage coder can be used. Voltage coders are not simple transducers and frequently consist of several pieces of equipment.

11-7.11 Voltage Coder Operating on Time-Interval Principle

One example of a voltage coder is shown in block diagram form in Fig 11-50. The principle of operation is based on converting the voltage into a proportional time interval and measuring that time interval by

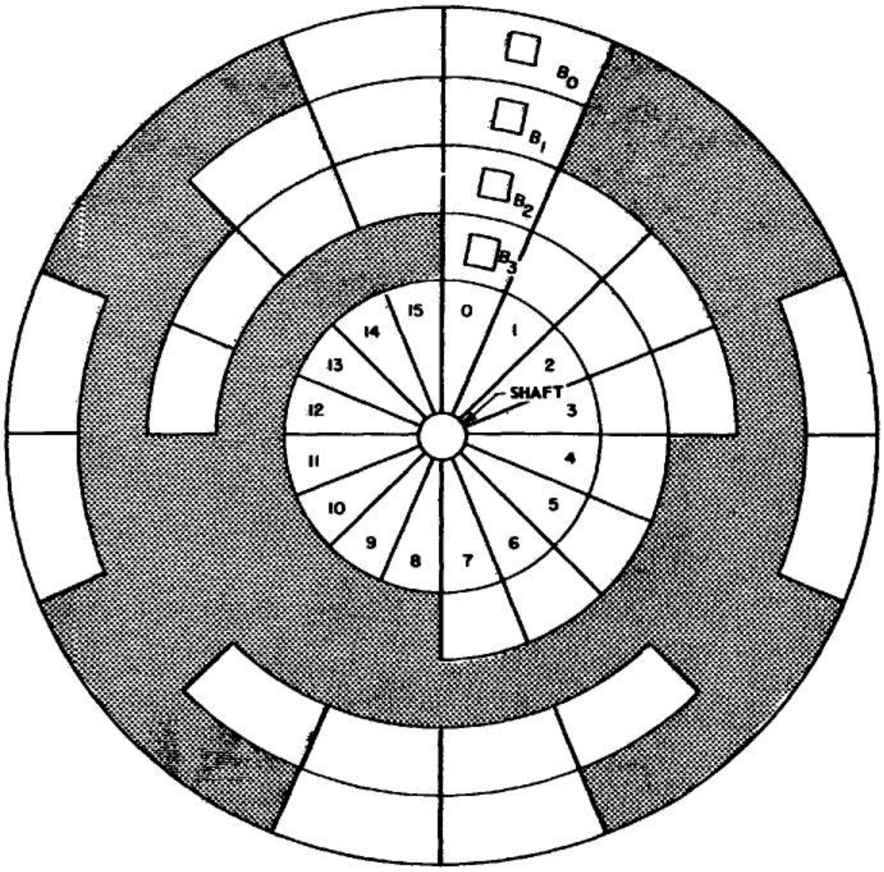


Fig. 11-49 Cyclic coding disc.

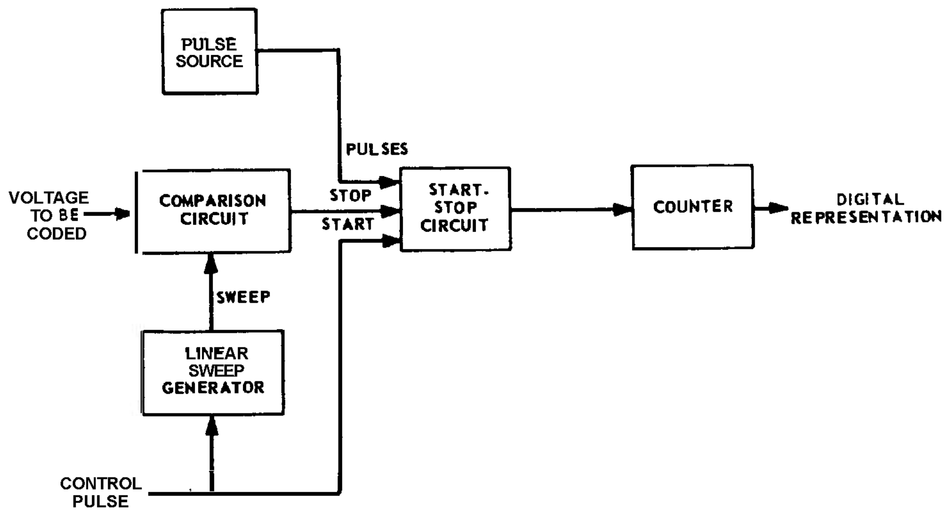


Fig. 11-50 Voltage coder operating on time-interval principle.

means of a counter and a fixed-frequency pulse source. In Fig. 11-50, the conversion into a time interval is accomplished by a sweep generator and a comparison circuit. A linear sweep is started from a reference voltage by the occurrence of a control pulse. At the same time, fixed-frequency pulses from the pulse source are passed through the start-stop circuit and the counting operation begins. When the sweep voltage equals the voltage to be coded, the comparison circuit stops the flow of pulses into the counter. For a linear sweep, the time interval between the occurrence of a control pulse and the instant at which counting stops is linearly related to the voltage level to be coded. Since the counter receives pulses only between the start of the sweep and the point at which its level agrees with the input voltage, it follows that the number of pulses accumulated by the counter is proportional to the voltage to be coded and forms the desired digital representation.

The accuracy of a time-interval coder is governed by the linearity and slope of the sweep and by the accuracy of the comparison circuit. A typical accuracy figure for a coder of this type is 0.1 percent. The time required to perform a conversion is determined by the maximum counting rate of the counter, which has a practical upper limit of several million counts per second. An approximate expression for the conversion time is

$$\tau = \frac{s}{f_{max}} \qquad (11\text{-}50)$$

where

s = number developed by counter

f_{max} = maximum counting rate

For example, when the number 1000 is developed and the counter operates at 5×10^6 counts per second (i.e., the frequency of the pulse source), then

$$\tau = \frac{1000}{5 \times 10^6} = 200 \text{ microseconds}$$

11-7.12 Voltage Coder Operating on Voltage-Comparison Principle

Another example of a voltage coder is shown in Fig. 11-51. In this case, a closed-loop configuration is used. The principle of operation is based on successive comparisons between trial outputs of a decoder and the voltage to be coded. The decoder is a device that converts numbers into corresponding voltages (see Par. 12-4). For decoding an *n*-digit number, there are *n* electrical sources interconnected by a network. Each source corresponds to a particular digit. In Fig. 11-51, the decoder source corresponding to the most significant digit is turned on first by the control circuit. If the resultant decoder output does not exceed the voltage to be coded, the comparison circuit leaves that source on. If the output is greater than the input, the comparison circuit causes the control circuit to turn off the most significant source. Next, the control circuit turns on the second most significant source. Again a comparison is made. If the new output is smaller than the input, the second source is left on. If not, the second source is turned off. In the third step, the third most significant source is turned on and a third comparison occurs. In the following steps, the remaining sources are tried in order of decreasing significance until all sources have been tried. Those sources that have remained on at the end of the comparison period indicate the number corresponding to the voltage to be coded. More details on voltage-comparison coders are given in reference 77.

The above type of coder is much faster than the one described in Par. 11-7.11. The individual comparisons can be made very rapidly and an entire conversion can be completed in but a few microseconds. The accuracy of the conversion is determined by the accuracy of the decoder and the comparison circuit. A typical accuracy figure for a practical system is 0.1 percent.

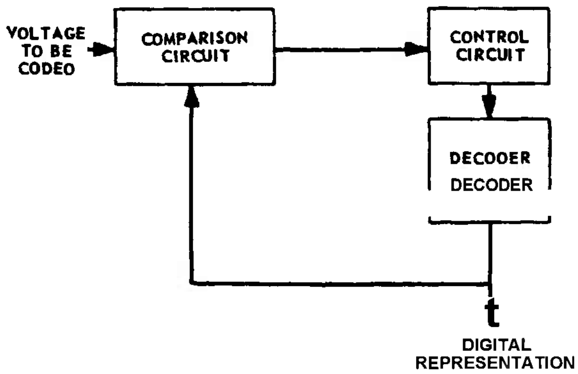


Fig. 11-51 *Voltuge coder operating on voltage-comparison principle.*

11-8 OTHER FORMS OF SENSING ELEMENTS*

11-8.1 INTRODUCTION

The earlier sections of this chapter described various devices used to convert motion or velocity into an electrical signal. Error sensing devices yielding an electrical signal proportional to angular error include potentiometers and synchros. The purpose of Par. 11-8 is to describe some other sensing elements.

*By P. E. Smith, Jr.

11-8.2 DIFFERENTIALS

If the servo input and output are angelar quantities measured at points not too far separated, it is possible to detect the error (input-output) by mechanical means using a geared differential. If input and output are linear motions, it is possible to use a differential-type linkage. Differentials are discussed in Par. 15-2. The equation for error **for** the geared differential discussed in Par. 15-2 is

$$\theta_3 = \frac{-}{2} \qquad (11\text{-}51)$$

where

$\varepsilon = \theta_i - \theta_o$

$\theta_i = \theta_1$ of Eq. (15-18)

$\theta_o = \theta_2$ of Eq. (15-18)

and

θ_3 is from Eq. (15-18). The equation for a single rigid-bar linkage as shown in Fig. 11-52 is

$$Y_o = \frac{1}{2}\varepsilon \qquad (11\text{-}52)$$

where

$\varepsilon = x_i - x_o$

x_i = servo input motion

x_o = servo output motion

and the angles α_1, α_2, and α_3 are nearly 90°.

Various other linkage and lever combinations are discussed in Pars. 15-2 and 15-3.

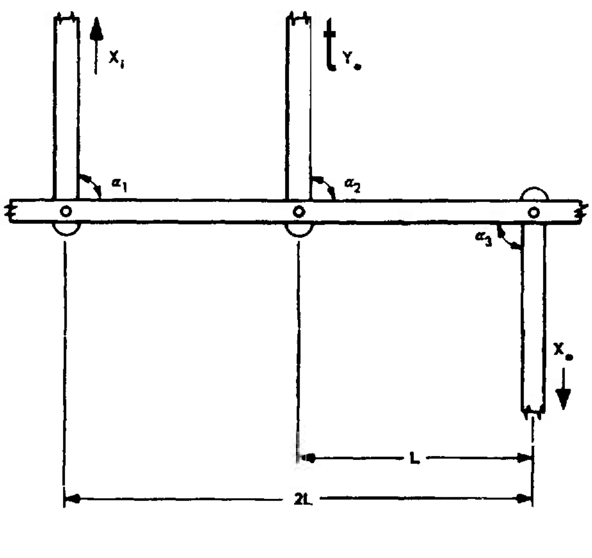


Fig. 11-52 Simple differential linkage.

11-8.3 HYDRAULIC- OR MECHANICAL-AMPLIFIER INPUTS

The input to a hydraulic amplifier (hydraulic servo-valve or control valve) is a motion as is mentioned in Par. 13-6. This input motion can be produced from an electrical signal by the use of a force motor (see, for example, Par. 14-2). If the force required to position the input to the hydraulic amplifier is small compared with the force available from the source supplying the servo input motion, it may be feasible to use a differential to sense $(\theta_i - \theta_o)$. The differential output is used to supply the hydraulic amplifier input. Similarly, the differential might be used to drive the input of a mechanical amplifier directly as discussed in Par. 13-8.

11-8.4 PRESSURE-SENSITIVE DEVICES

It is perfectly possible to transmit information (signals) through media other than the electrical one. A hydraulic or pneumatic transmission is an example. By use of some sort of converter, such as that shown in Fig. 11-53, it is possible to convert motion into a pressure variation proportional to the motion. This variation can be transmitted to some more-or-less remote point where reconversion to motion can be carried out. If such a system were used to transmit servo output signals, then reconversion to motion would be carried out at a point where servo input is available. Servo input — reconverted output would be the error. A suitable differential would accomplish the subtraction. The pressure-to-motion sensing device or receiver used in such a system could be either a bellows, a capsule, or a piston with suitable opposing spring (see Par. 14-4). The steady-state behavior of such a sensing device is given by

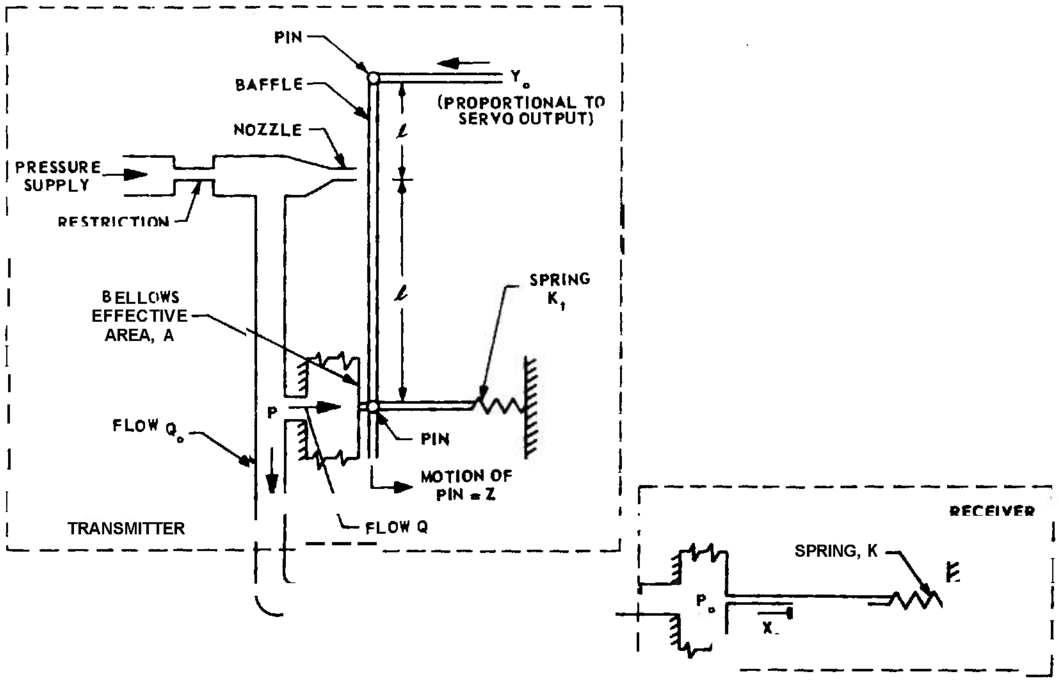
$$x_o = P_o/K \qquad (11\text{-}53)$$

where

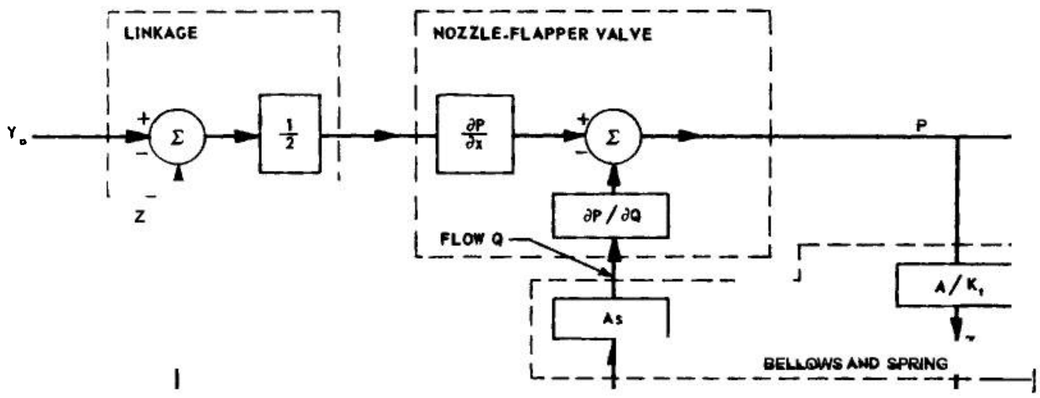
P_o = transmitted pressure

K = spring rate

The dynamic behavior **of** the signal transmission system can be analyzed in the man-



A. SCHEMATIC DIAGRAM OF A PRESSURE TRANSMITTER



B. BLOCK DIAGRAM OF A PRESSURE TRANSMITTER

(1) ALL VARIABLES SHOWN REPRESENT INCREMENTAL CHANGES ABOUT SOME OPERATING POINT.
(2) REFER TO PARAGRAPHS 13.6.5, 13.6.6, AND 13.6.14 FOR DISCUSSION OF:

$\frac{\partial P}{\partial x}$ = NOZZLE-FLAPPER NO-LOAD PRESSURE GAIN

$\frac{\partial P}{\partial Q}$ = NOZZLE-FLAPPER INTERNAL HYDRAULIC SOURCE RESISTANCE

Fig. 11-53 Pressure transmitter.

ner used to analyze hydraulic amplifiers (Par. 13-6) by use of a lumped-element hydraulic equivalent circuit *provided* that the line length is very short compared with the wavelength of signals of significant frequencies. The wavelength is given by

$$\lambda = \frac{v_s}{f} \text{ (inches)} \qquad (11\text{-}54)$$

where

v_s = velocity of sound, in in./sec

f = frequency of transmitted signals, in cps

The velocity is given by

$$v_s = \sqrt{\frac{B_M}{\rho}} \text{ (in./sec)}$$

where

B_M = bulk modulus of medium (lb/in.^{2})

ρ = mass density (lb-sec^2/in.^{4})

Typical values are: $B_M = 250{,}000$; $\rho = 0.031/386$; and $v_s = 54{,}000$ in./sec. For f below 45 cps, the wavelength is greater than 100 feet and a 10-foot transmission line could be treated as a lumped-parameter system. Gibson and Tuteur[82] discuss the effects of distributed-parameter lines.

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CHAPTER 12

SIGNAL CONVERTERS*

12-1 INTRODUCTION

12-1.1 TYPES

Three types of signal converters are discussed in this chapter: modulators, which superimpose a signal on a carrier ; demodulators, which recover the signal that has been superimposed on a carrier ; and digital-to-analog converters, which express a number in electrical form as a proportionate voltage or as a shaft rotation.

12-1.2 Modulators

Modulators are used in servomechanisms in which the error signal is a direct voltage, but the output member, such as a 2-phase motor, will respond only to a-c signals. The modulator converts the d-c signal to ac. Another use for modulators in servomechanisms is the case where it is undesirable to empley d-c amplification of the d-c error signal because of drift problems in such an amplifier. Here, to avoid the drift problems, the signal is first modulated, then amplified by an a-c amplifier, which is drift free ; the amplifier output is then demodulated to recover the d-c signal for application to the output member.

12-1.3 Demodulators

Demodulators are used in servomechanisms in which the error signal is an a-c voltage (such as the output of a synchro control transformer) and the output member responds only to d-c signals (e.g., an amplidyne).

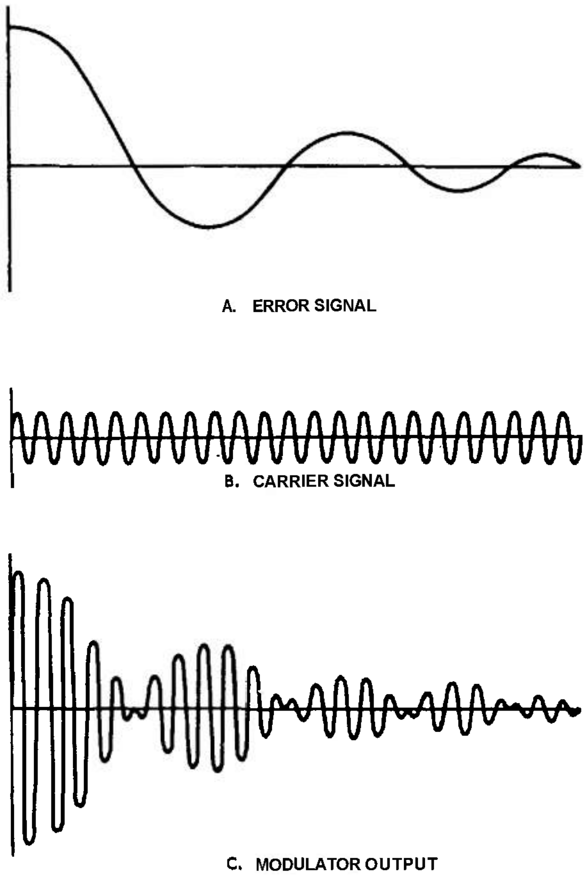

Fig. 12-1 **Modulator waveforms.**

*By A. K. Susskind

12-1.4 Form of Modulation

The form of modulation used in servomechanism is called *suppressed-carrier* modulation. This means that the output of the modulator contains frequency components which are the sum and difference of the carrier and the signal frequencies, but contains no frequency component equal to the carrier frequency. Figure 12-1A shows an error signal and Fig. 12-1C shows the corresponding output of a suppressed-carrier modulator. The modulated signal is zero when the error signal is zero, and the phase of the carrier sinusoids reverses when the sign of the error signal reverses. Therefore, demodulators used in servomechanisms must have zero output when the input is zero, and the polarity of the demodulator d-c output must be determined by the phase of the input sinusoids, with respect to the carrier. Because of this requirement, the demodulators are called *phase-sensitive detectors.* Since the input to the demodulator contains only frequency terms which are the sum and difference of the carrier and modulating frequencies, the demodulator must be separately supplied with a signal at the carrier frequency (carrier reinsertion). Commonly used modulators are described in Par. 12-2. Paragraph 123 discusses demodulators.

12-2 MODULATORS

12-2.1 CHOPPER MODULATORS

Choppers are electromechanical modulators and, because of their simplicity and high zero stability (very nearly zero output when the signal to be modulated is zero), find wide application in servomechanisms.

12-2.2 Description

The essential elements of an electromechanical chopper are shown schematically in Fig. 12-2. A source of periodic voltage, usually sinusoidal, is connected to the drive coil. The resultant magnetic field causes the reed to vibrate between the upper and lower contacts at a frequency equal to that of the driving voltage. When V, the voltage to be modulated, is connected as shown in Fig. 12-3, the square-wave output waveform has the

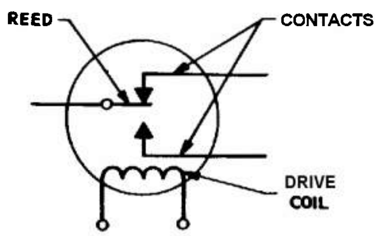

Fig. 72-2 Chopper elements.

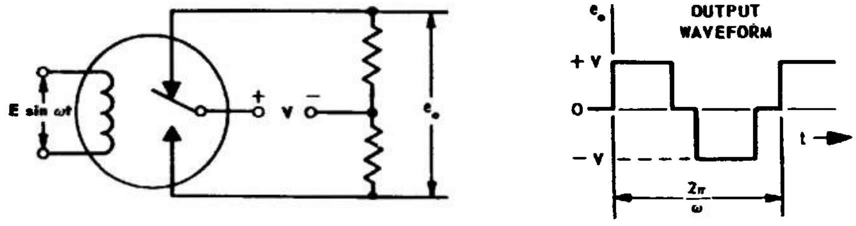

Fig. 12-3 Chopper modulator.

same period as the driving voltage, and the magnitude of each of the positive and negative rectangles of the waveform equals V. The duration of the intervals when there is zero output is a function of the time spent by the reed in moving from one contact to the other. If d denotes the fraction of each half-cycle during which the output is zero, then the output voltage e_o is given by

$$e_o(t) = \frac{4V}{\pi} \sum_{m=1}^{\infty} \frac{1}{2m-1} \cos\left[d \frac{\pi}{2}(2m-1) \right] \sin\left[\omega(2m-1)t\right] \quad (12\text{-}1)$$

where zero time reference is taken midway between the occurrence of $+V$ and $-V$ at the output. This series contains only odd harmonics of the driving-voltage frequency (fundamental, third, fifth, etc.) ; therefore, it is not difficult to filter the chopper output to isolate the desired fundamental-frequency component.

12-2.3 Characteristics

Figure 12-4 shows the chopper output waveform with the driving voltage superimposed. (In Fig. 12-4, the waveform of the output voltage is somewhat idealized. After initial closure, a contact usually bounces once or twice, so that the output waveform contains a few gaps at the leading edge of each rectangle.) The duration of each rectangle is called the **dwell-time,** which varies with different designs, but is typically 150°.

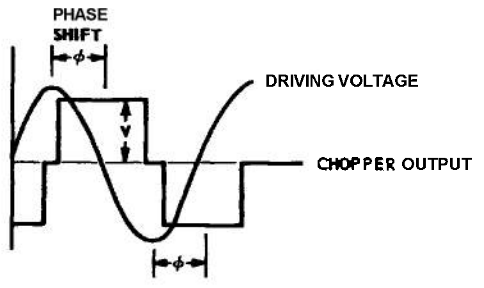

Fig. 12-4 Relationship between driving voltage and chopper output.

12-2.4 Phase shift ϕ. Of particular interest to the user is the phase shift ϕ between the driving voltage and the modulator output. An example is the application in which the chopper is used to modulate a d-c error signal which is then amplified and used to drive a 2-phase motor. The motor responds only to signals that are 90° out of phase with the motor reference-winding voltage, which frequently is also used to drive the chopper. Hence, the over-all phase shift of the chopper and the amplifier must be adjusted to 90°. For this reason, the phase-shift characteristics of the chopper must be known.

Chopper phase shift ϕ is defined as the number of degrees of the driving voltage by which the mid-point between contact make-and-break lags the peak of the driving voltage. The lag is due to the inductive nature of the driving coil and also to the mechanical constants of the reed. The lag angle may be adjusted by inserting a phase-shifting device such as a capacitor, resistor, or a parallel combination of both, in the drive-coil circuit. When these components are used, the input-voltage amplitude must be adjusted so that the rated coil voltage is applied to the coil. The phase angle is a function of temperature, drive-voltage amplitude, and drive-voltage frequency. Where these parameters are expected to vary in an application, the chopper manufacturer should be consulted for quantitative information about the effects of these variations. The phase angle can be substantially stabilized against changes in the drive-voltage frequency by the addition of a resistor in series with the drive coil.

12-2.5 Drive-voltage frequency. Commercially available electromechanical choppers are designed for drive-voltage frequencies ranging up to several thousand cps. However, most units are designed for 60- or 400-cps operation.

12-2.6 Drive voltage. The most commonly specified drive-coil voltage is 6.3 volts, but models with somewhat higher ratings are also made. Typical drive-coil power requirements range from 0.5 to 1 watt.

12-2.7 Contact rating. The useful life .of a chopper is affected most by the contact rating. Most chopper failures are caused by contact wear, pitting, and sticking or the development of contact resistance. These failures can be reduced by strict adherence to the maximum contact ratings which, in typical commercial units, are a few volts and a few milliamperes. Since useful life depends upon how far chopper characteristics can deteriorate before the servomechanism fails, the number of hours of satisfactory operation will vary with the application. It is generally true, however, that where rated-performance limits are not exceeded, several thousand hours of life can be expected.

12-2.8 Chopper noise. Noise generated by an electromechanical chopper varies with the design of the unit, the impedance of the circuit connected to the output, and the bandwidth of the measuring equipment. Since the bandwidth of a servomechanism is small, chopper noise is usually not troublesome, and can be kept low by connecting the output of the chopper to a circuit of moderately low impedance. One design, for example, has a rated noise level of 50 microvolts peak-to-peak when connected to a 2200-ohm load, but this level jumps to 1.5 millivolts when connected to a 1-megohm load. These figures are for a bandwidth of 0.2 to 1000 cps, which is wider than that of a servomechanism.

12-2.9 Temperature effects. Choppers are rugged devices that will operate satisfactorily over a considerable range of temperatures. While temperature does affect phase lag, choppers will operate satisfactorily over a temperature range which, for some designs, is as wide as −55" to +200°C. Hermetically sealed models are available which are rated for operation at altitudes as high as 50,000 feet and in a high-humidity environment. The ruggedness of choppers is illustrated by the vibration rating of one design, which is 30 g's for frequencies between 5 and 500 cps.

12-2.10 Packaging

Most choppers are housed in cylindrical cans, one to two inches in diameter, and one to four inches long.

12-2.11 Practical Circuits

Figures 12-5 through 12-8 show practical connection circuits of chopper contacts, and the corresponding output waveforms. All waveforms have a period $1/f$, where f is the frequency of the drive-coil excitation voltage. The circuit of Fig. 12-5 is more suitable than that of Fig. 12-6 when the source impedance of V is high'. The circuit of Fig. 12-7 produces a push-pull output and is therefore useful when a double-ended signal is desired. Figure 12-8 shows a circuit in which the peak-to-peak output voltage is the difference between the two input signals. This circuit may therefore be used to modulate the error signal in a servomechanism when e_{IN1} represents the command and e_{IN2} represents the response.

12-2.12 MAGNETIC MODULATORS[1, 2]

A magnetic modulator converts the d-c signal to a modulated a-c signal with a carrier frequency twice that of the modulator excitation voltage. The output of the magnetic modulator is phase-sensitive to the polarity of the d-c signal, permitting demodulation of the amplified signal by a phase-sensitive detector and restoration of the original polarity at the output of the amplifier.

12-2.13 Principle of Operation

The basic circuit of a magnetic modulator is shown in Fig. 12-9. The circuit consists of two wound ferromagnetic, high-permeability cores, which are closely matched for magnetic characteristics, each carrying a signal winding, an excitation winding, and an output, or load, winding. The load windings are connected in opposite polarity sense with respect to the excitation windings so that, with no signal current, the load-winding voltages e_1 and e_2 are identical but opposite in phase, thereby cancelling to produce zero load voltage. When a control current is applied from the signal source, the control current produces asymmetry in the operation of the two

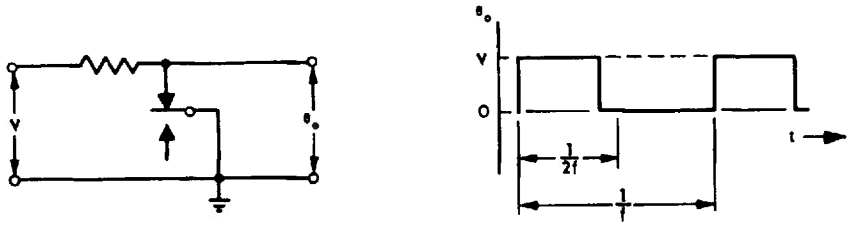

Fig. 12-5 Half-wave connection.

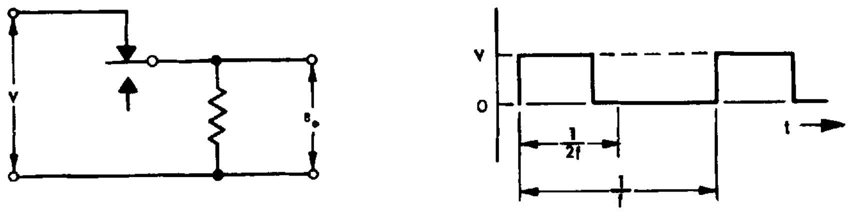

Fig. 12-6 Half-wave connection.

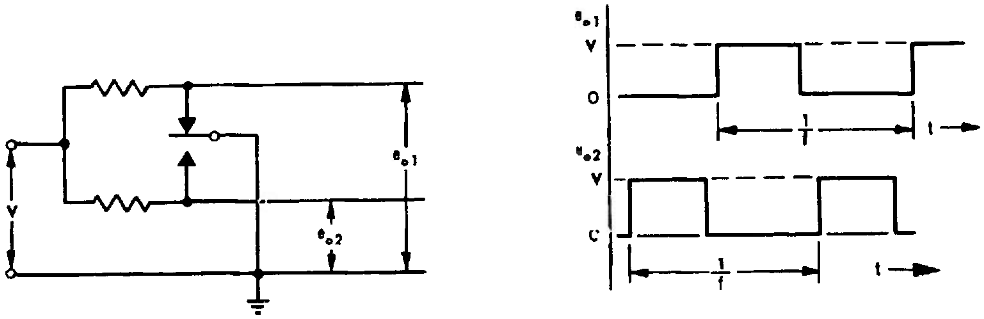

Fig. 12-7 Push-pull half-wave connection.

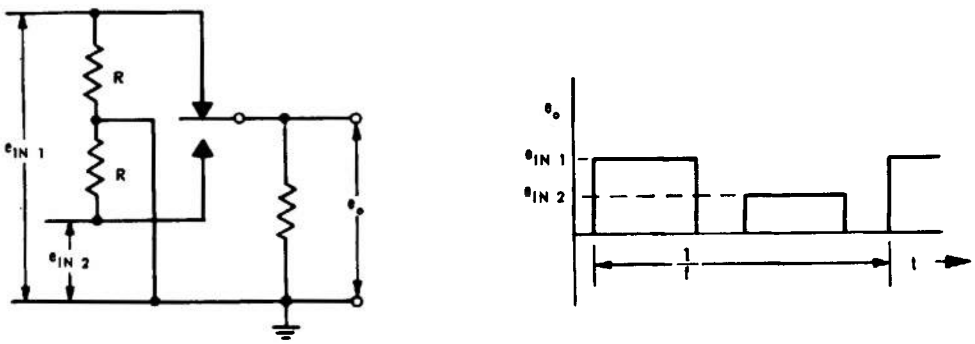

Fig. 12-8 Differential Connection.

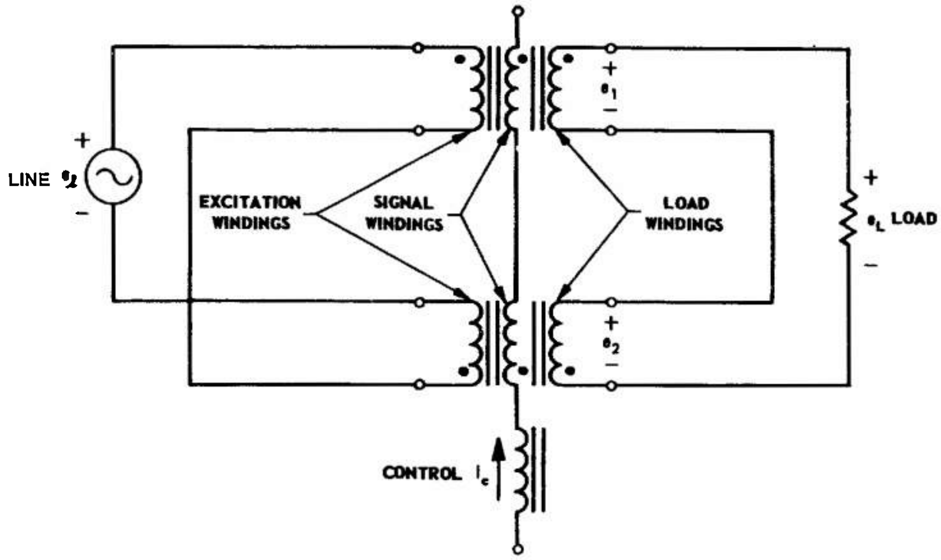

Fig. 12-9 Basic circuit of second-harmonic modulator.

cores, and a voltage is produced in the load winding at twice the frequency of the excitation source and proportional to the magnitude of the control signal.

An understanding of the manner in which this second-harmonic output is produced can be gained by referring to the hysteresis loop of the core material (see Fig. 12-10). The signal current can be assumed to produce a displacement of the initial flux operating point of the core material, from point 1 to the points on the hysteresis loops for the corresponding cores. As a cycle starts, core 1 is operating in a region of low permeability, whereas core 2 is operating in a region of high permeability. Hence, the applied line voltage e_l appears principally across core 2, and the load-winding voltage e_2 (as shown in Fig. 12-11) is larger in magnitude than load-winding voltage e_1 at that time in the cycle. As the cycle progresses, and the operating points of the core material move toward positive saturation, core 2 encounters a region of low permeability near the knee of the saturation curve, while core 1 is now in a region of high permeability. Voltages e_1 and e_2 are now reversed in relative amplitude. When the sum of voltages e_1 and e, is taken as a load voltage e_L (as shown in Fig. 12-11), the net result is a second-harmonic component. If the direction of the signal current is reversed, then the cores interchange positions on the hysteresis loops, and the second-harmonic load voltage reverses in phase. This feature of the magnetic modulator makes is possible to restore the polarity of the original d-c signal by means of a phase-sensitive detector.

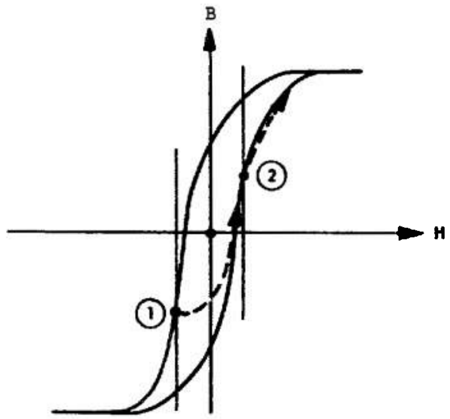

Fig. 12-10 Operation of cores for positive control signal.

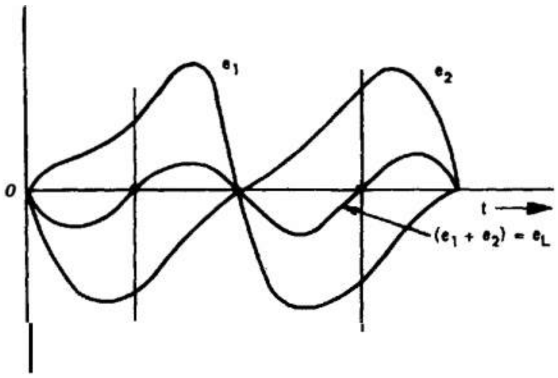

fig. 12-11 Secondary voltage for $i_c = I_c$ condition.

12-2.14 Operating Circuits

There are two categories of operating circuits employing the magnetic modulator : one category is very similar to that shown in Fig. 12-9, in which the signal and load windings are separated; in the second category, the d-c signal and the load are connected to a common winding, each core having only two windings —an excitation winding and a signal-load winding.

12-2.15 Sensitivity

For maximum magnetic-modulator sensitivity, the control-winding impedance is usually matched to the impedance of the signal source, and the load-winding impedance is usually matched to the load impedance, although this may not be true with high-impedance load circuits, such as grid circuits of vacuum-tube amplifiers. It is possible to secure individual load-impedance and signal-impedance matching with the circuit in which the control and load windings are separated. However, the common-winding circuit will result in greater sensitivity and, thus, may be more desirable in certain applications.

12-2.16 Harmonic Attenuation

One feature found in almost all magnetic modulators is a capacitor for tuning the load to the second-harmonic frequency, so that odd harmonics and extraneous noise are attenuated.

12-2.17 Performance and Application of Magnetic Modulators

The control characteristics of a typical, commercially available magnetic modulator are shown in Fig. 12-12. This modulator operates with an excitation voltage of 5 volts in the 300- to 500-cps frequency range. The weight of the unit is approximately 3 ounces. The range of linear output, approximately 5 volts rms, is obtained with about 100 microamperes of signal into a resistance of 1000 ohms, which is equivalent to a signal power of 10^{-5} watt. The limitation on minimum signal power required to operate a magnetic modulator is set by the inherent Barkhausen noise, which is in the order of 10^{-19} watt per cycle of bandwidth. Some workers[3] have reported the design of magnetic modulators in which the zero error can be reduced to 5×10^{-17} watt input, with a random variation of 3×10^{-18} watt over a two-hour period. Two possible hindrances to building extremely sensitive magnetic modulators are: the harmonic level of the oscillator that supplies the excitation power ; and the mismatch. or magnetic asymmetry, in the two cores.

12-2.18 Core Material

Raising the excitation frequency results in higher gain and a reduced time constant for the magnetic modulator, but it may also result in excessive core loss. However, the excitation voltage should saturate the core material to realize optimum performance. Therefore, a suitable core material must have a high maximum permeability and low core loss. A good core material that is not too susceptible to mechanical shock is 4-79 Mo-Permalloy.

12-2.19 ELECTRONIC MODULATORS[4]

Circuit schematics of electronic modulators commonly used in servomechanisms are shown in Figs. 12-13 through 12-16.

12-2.20 Operation

In all of these circuits, except Fig. 12-16, the principle of operation is the same and is similar to chopper operation. Nonlinear elements, either diodes or triodes, are used as switches to connect the d-c signal V to the

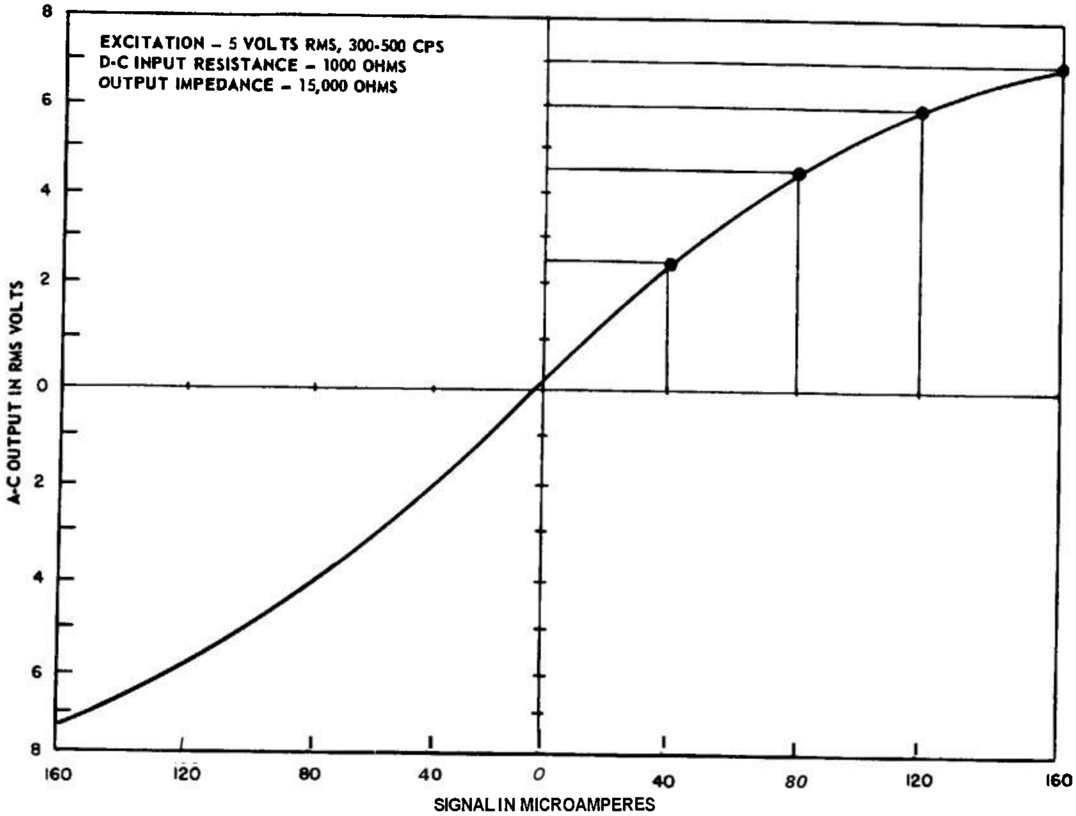

Fig. 12-12 Control characteristics of commercial mugnetic modulutor.

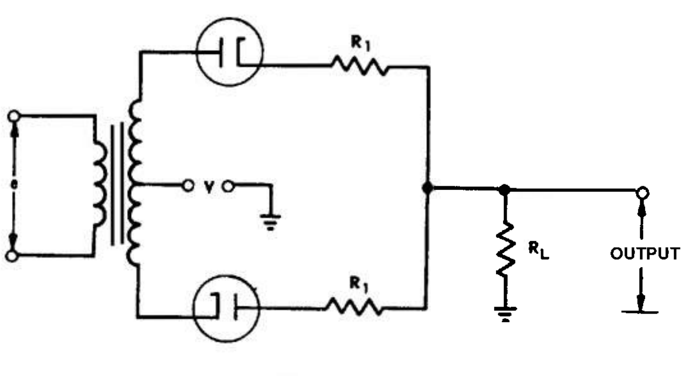

Fig. 12-13 Diode modulator.

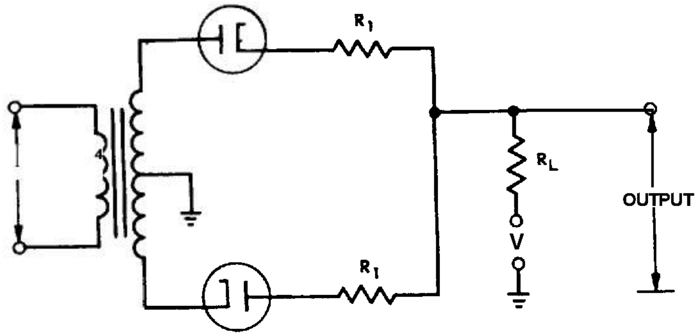

Fig. 12-14 Diode modulator.

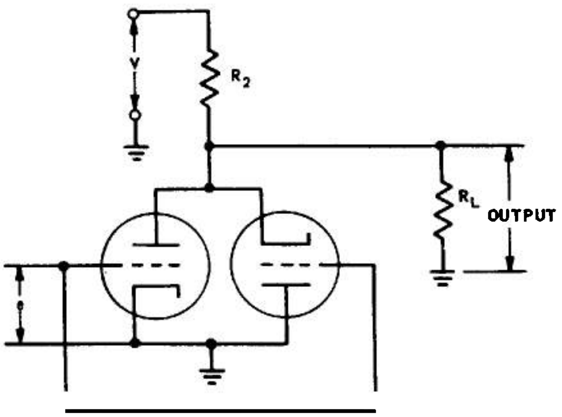

Fig. 12-15 Triode modulator.

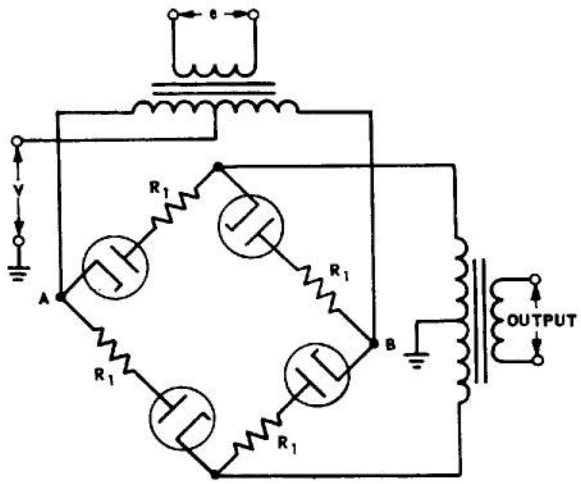

Fig. 12-16 Bridge modulator.

output during one half-cycle of the sinusoidal carrier voltage **e**. During the other half-cycle of *e,* the output is connected to ground. Therefore, the output waveform is a rectangle of amplitude V, with a duration of half **a** period of the carrier and with a repetition rate equal to the modulating frequency. Resistors R_1 are connected in series with all diodes to limit the diode current when the diodes conduct. The value of R_1 should be much less than that of R_L, a typical value being 1000 ohms.

In Fig. **12-13,** the rectangle occurs (i.e., the diodes conduct and connect *V* to the output) when the upper end of the transformer secondary is positive. During the other half-cycle, the output is at ground potential. In Fig. **12-14,** the output is connected to ground when the upper end of the transformer secondary is positive, and the rectangle occurs during the other half-cycle of the carrier. In a variation of this circuit, a second d-c signal is connected to the center tap of the transformer. The output signal then has a peak-to-peak amplitude that is the difference between the two d-c signals, and the modified circuit can therefore be used as a combination error (difference) detector and modulator. In the back-to-back triode circuit of Fig. 12-15, a very large carrier signal is applied to the grids, making one of the two triodes conduct during approximately the entire positive half-cycle of the carrier signal. The left 'tube conducts during the positive half-cycle if V is positive and the right tube conducts if V is negative. The grid signal is so large that the tube drop is very small during conduction and, therefore, the output is nearly at ground potential during conduction. In order for this to occur, R_2 and R_L must be large (e.g., **500,000** ohms). The circuit is used only when large signals, in the order of 100 volts, are to be modulated.

12-2.21 Bridge modulator. Figure 12-16 differs from the other configurations in that two rectangles of opposite polarity are generated during each cycle of the carrier. When the carrier polarity is such that point A is positive with respect to point B, *V* appears across the output transformer with one polarity; if A is negative with respect to B, V appears across the output transformer with opposite polarity.

12-2.22 Typical curve. In the diode circuits, the statement that the rectangles have a duration of half a period of the modulating signal is based on the assumption that the peak amplitude of **e** is large compared with V. Where **e** is small compared with V, saturation occurs. A typical curve is given in Fig. 12-17, showing the rms value of the output voltage as a function of the magnitude of V for a fixed amplitude of **e**. The knee in the curve shows the occurrence of saturation,

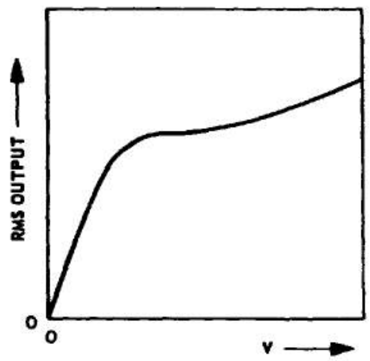

Fig. 72-77 Modulator rms output as a function of amplitude of V for fixed amplitude of e.

which must be avoided if the output is to be proportional to V.

12-2.23 Stability. In all of the diode circuits shown, zero stability (zero input voltage) depends upon the balance between the diodes. For example, in Fig. 12-13, assuming that V is zero, the output will be at zero potential during the half-cycle of the carrier when the upper end of the transformer secondary is positive, but only if the drop is equal in both diodes and their associated resistors R_1. Therefore, zero stability is determined by how nearly identical are the two diodes. For good zero stability, the diodes must be identical, not only when the circuit is first adjusted, but also throughout the aging process of the diodes. The diodes need not be vacuum tubes; they can be semiconductor devices, such as germanium or silicon diodes. When semiconductor diodes are used, it must be remembered that they have measurable reverse current. It is important that the diodes be matched in their forward as well as reverse characteristics. In general, zero stability of vacuum-tube modulators is not as good as that of chopper modulators. Therefore, chopper modulators are preferred for high-precision applications.

12-3 ELECTRONIC DEMODULATORS'''''

12-3.1 DIODE DEMODULATORS

Figure 12-18 shows the schematic of a diode circuit frequently used in servomechanisms for demodulating suppressed-carrier signals; i.e., signals in the form

$$e_{IN} = A \sin \omega_s t \sin \omega_c t \qquad (12\text{-}2)$$

where

ω_s = angular frequency of modulated signal (e.g., error signal)

ω_c = angular frequency of carrier

This signal is connected to the primary of the center-tapped transformer. A signal corresponding to the carrier

$$e_c = B \sin \omega_c t \qquad (12\text{-}3)$$

is connected to the other transformer. The secondary voltages corresponding to e_{IN} and e_c are, respectively

$$e_1 = V_1 \sin \omega_s t \sin \omega_c t \qquad (12\text{-}4)$$

$$e_2 = V_2 \sin \omega_c t \qquad (12\text{-}5)$$

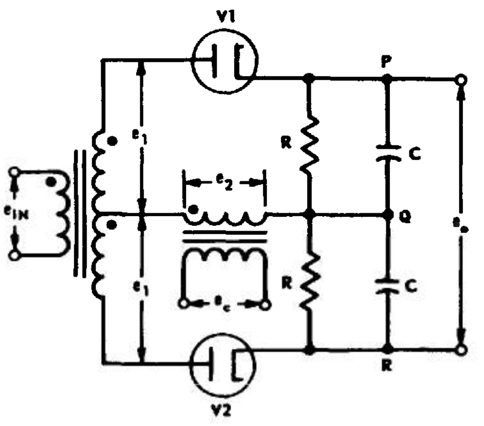

Fig. 12-18 Diode demodulator.

12-3.2 Operation

The operation of the diode demodulator (Fig. 12-18) is as follows: The voltage between Q and the plate of diode V1 is the sum of e_1 and e_2. The voltage between Q and the plate of diode V2 is the difference between e_1 and e_2. Hence, when e_1 is zero, the voltage across PQ equals the voltage across RQ, and the output voltage e_o (which is the difference between the two voltages) is zero. If e_1 is in phase with e_2 (sin $\omega_s t$ is positive), the voltage across PQ is greater than the voltage across RQ, and the output voltage e_o is positive. If e_1 is 180° out of phase with e_2 (sin $\omega_s t$ is negative), the voltage across PQ is smaller than the voltage across RQ, and the output voltage e_o is negative. In general, ignoring ripple in the output due to discharging of capacitors C between conduction periods of the diodes, and ignoring the time lag between input and output, it follows that

$$e_o = 2V_1 \sin \omega_s t \qquad (12\text{-}6)$$

provided V_1 equals or is less than V_2. For V_1 greater than V_2, saturation occurs. If e_1 contains a quadrature component (a component that is phase-shifted 90° with respect to the carrier), the output due to the quadrature component is zero.

12-3.3 Ripple. The output voltage contains a ripple component (the waveform of which approximates a saw tooth) that is amplitude modulated by the variations in the modulated signal. The ripple is caused by the discharge of capacitors C between peaks of the carrier signal. The capacitors are charged during peaks, when the diodes conduct. The output ripple decreases when the RC product increases but, as this product is increased, the output voltage is no longer able to follow variations in the modulated signal. An upper limit for RC is given by

$$RC \leq \frac{1}{m\omega_s}\sqrt{1-m^2} \qquad (12\text{-}7)$$

where

$$m = \frac{V_1}{V_2}$$

The amplitude of the first harmonic of the ripple, which has the same frequency as the carrier, is given by

$$N_1 = \frac{2V_1}{f_c}\sqrt{\frac{1}{(\pi RC)^2} + (2f_s)^2} \qquad (12\text{-}8)$$

where

$$f_c = \frac{\omega_c}{2\pi}$$

$$f_s = \frac{\omega_s}{2x}$$

The ripple factor is defined as the ratio of the amplitude of the first harmonic of the ripple to the .amplitude of the output fundamental and, for values not in excess of 10 percent, is given by

$$r = \frac{1}{f_c}\sqrt{\frac{1}{(\pi RC)^2} + (2f_s)^2} \qquad (12\text{-}9)$$

Since f_s and f_c are fixed in a given application, the specification for r determines the required RC product. The inequality [Eq. **(12-7)**] then determines the maximum value of m and, hence, the upper limit on the ratio of V_1 to V_2 is determined for the design. As r decreases, the RC product increases, and the value of m decreases.

12-3.4 Unbalance. The parallel impedance of R and C at ω_c should be large compared-with the equivalent source impedances e_1 and e_2 and compared with the forward resistance of the diodes so that transformer and diode unbalance is of no practical significance.

12-3.5 Transfer function. An expression for the output voltage that is more accurate than the one stated previously is given by the following transfer function for the demodulator:

$$\frac{E_o(s)}{2E_1(s)} = \frac{1 - \frac{1}{2f_cRC}}{1 + T_d s} \qquad (12\text{-}10)$$

where

$$T_d = \frac{\frac{1}{2f_c}}{1 - \frac{1}{2f_cRC}}$$ = demodulator time constant

s = complex frequency of modulated signal

12-3.6 Output stability. The demodulator output should always be taken across P and R, as shown in Fig. **12-18.** If push-pull outputs are taken across PQ and RQ separately, the ripple at zero input signal results in time-varying outputs and poor stability.

12-3.7 Full-Wave Demodulator

Figure **12-19** shows the schematic diagram of a full-wave diode demodulator. In this circuit, which uses four diodes, conduction occurs during both halves of the carrier cycle. The demodulator time constant and ripple factor are reduced by a factor of 2 over the half-wave circuit of Fig. **12-18.** An equivalent full-wave diode demodulator is shown in Fig. **12-20.**

12-3.8 TRIODE DEMODULATORS

The diodes in Fig. **12-18** may be replaced by triodes. This produces the circuit of Fig. **12-21,** which is characterized by amplification of the modulated signal and by high input impedance for e_{IN}. Quadrature voltages, however, are not rejected; for this reason, the circuit is rarely used in servomechanisms.

12-3.9 Keyed Demodulator

A different demodulator configuration, called a keyed demodulator, is shown in Fig. **12-22.** It is characterized by smaller ripple for a given speed of response.

12-3.10 Operation. The operation of a keyed demodulator is as follows: The transformer supplies large in-phase signals, corresponding to the carrier, to both grids. Self-bias is developed through grid conduction in C_g, and the time constant R_gC_g is chosen to permit very little bias loss between carrier cycles. As a result, the tubes can conduct only during the positive peak of the carrier voltage. At that time, the input is connected to the output, and the demodulator is said to be

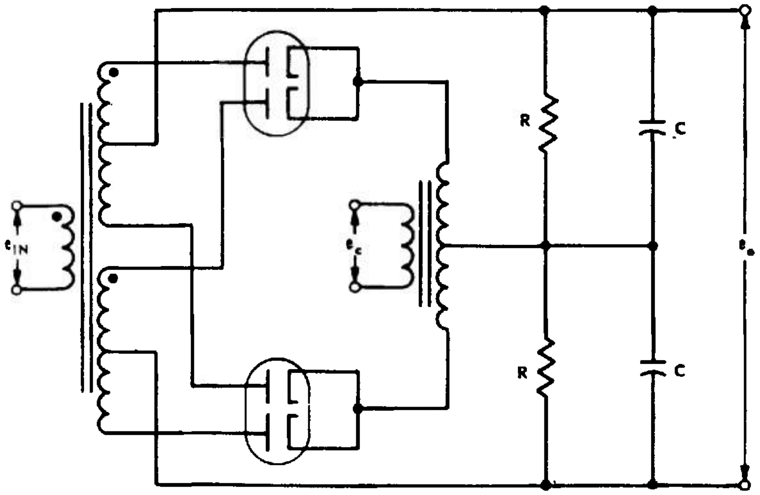

Fig. 12-19 Full-wove diode demodulator.

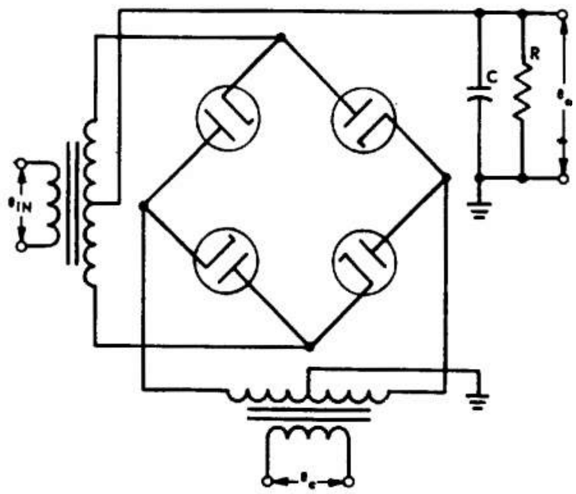

Fig. 12-20 Full-wave (ring) diode demodulator.

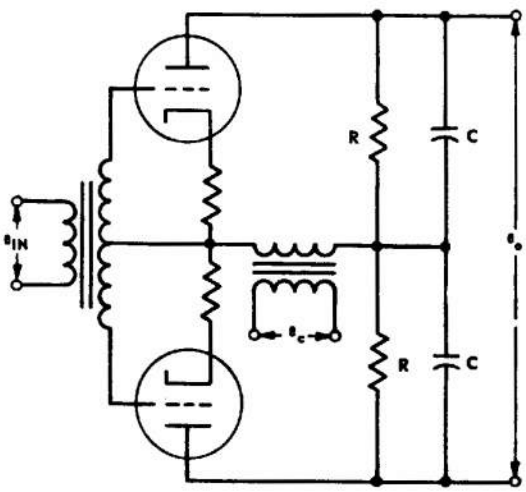

Fig. 72-27 Triode demodulator.

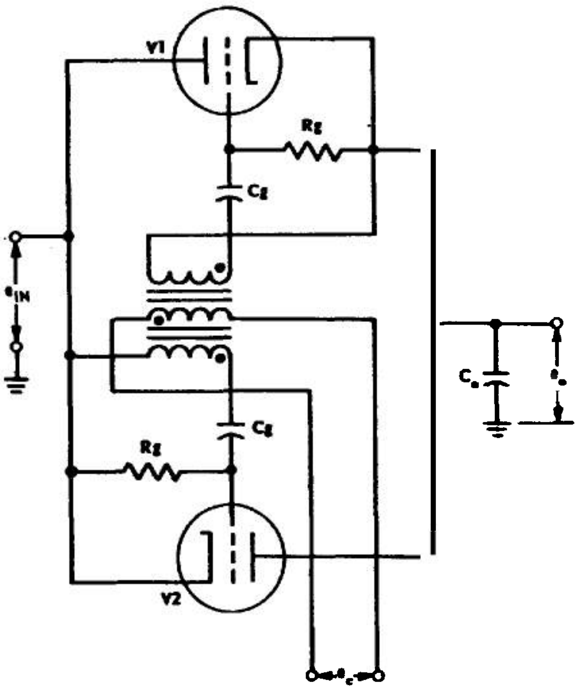

Fig. 12-22 Keyed demodulator.

The ripple voltage is zero when the modulated signal is constant. When the modulated signal is a sinusoid, the ripple voltage is a saw tooth, amplitude modulated by that sinusoid. The ripple factor, which is lower than that of the diode demodulator, is

$$r = \frac{2f_s}{f_c} \qquad (12\text{-}12)$$

Quadrature rejection is equally good, however, for both the keyed demodulator and the diode demodulator. The triodes of the keyed demodulator can be replaced by transistors or magnetic amplifiers.[7]

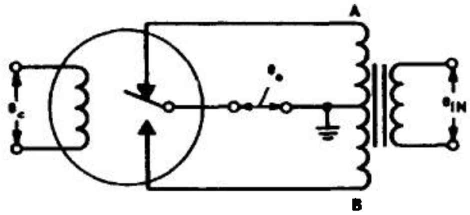

Fig. 12-23 Full-wave chopper demodulator.

keyed by the peak of the carrier signal. Tube **V1 conducts during** keying **if the** input **is** ***larger* than the** output, **and V2 conducts** if the input is *smaller* than the output. The output capacitor C_o charges to the peak input voltage, which is the instantaneous value of the modulated signal and, therefore, the output voltage is a replica of the modulated signal. If the modulated signal is a sinusoid, the output is also a sinusoid, but with a lag of $1/2f_c$ seconds and including a ripple of sawtooth shape.

12-3.11 **Transfer function.** The **transfer function** for the keyed demodulator is

$$\frac{E_o(s)}{E_{IN}(s)} = \frac{1}{1 + T_d s} \qquad (12\text{-}11)$$

where

$$T_d = \frac{1}{2f_c} = \text{demodulator time constant}$$

123.12 CHOPPER DEMODULATORS

Choppers can also be **used as demodulators.** Figure 12-23 shows the schematic diagram of **a full-wave chopper demodulator.**

12-3.13 Operation

The modulated signal e_{IN} is connected to the primary of the center-tapped transformer and the carrier voltage e_c is connected to the drive coil of the chopper. The chopper connects the output to terminal *A* of the transformer during the first half-cycle of the carrier and to terminal *B* during the other half-cycle. Thus, if e_{IN} is phased with respect to **the reed** motion **so that *A* is** positive during the first half-cycle, B will also be positive during the second half-cycle, and the output is a positive voltage. If A is negative during the first half-cycle, B will also be negative during the second half-cycle, and the output is a negative voltage. The polarity of the output voltage e_o is, therefore, a function of the phase relationship between the reed motion and the signal to be demodulated. If these two are 90° out of phase, the output is zero. If they are either in phase or 180° out of phase, the magnitude of the output voltage is determined by the magnitude of the modulated signal.

12-3.14 **Output filtering.** The output voltage contains a ripple voltage that varies at a frequency f_c and at harmonics of f_c. Therefore, a filter is usually required at the output of the demodulator. The design of this filter becomes increasingly less difficult as the highest-frequency component of the modulated signal is made smaller and smaller compared with the carrier frequency. In typical applications, ω_s is less than 10 percent of ω_c, and the filter design is not difficult.

12-3.15 Dual Use

When a chopper is used as a modulator, the same unit *can* also be used as a demodulator. This is shown in Fig. 12-24. The reed and contact *A* of the chopper form a half-wave modulator, like the one shown in Fig. 12-5. The reed and contact *B* of the chopper form a half-wave demodulator, which permits the output of the amplifier to pass to

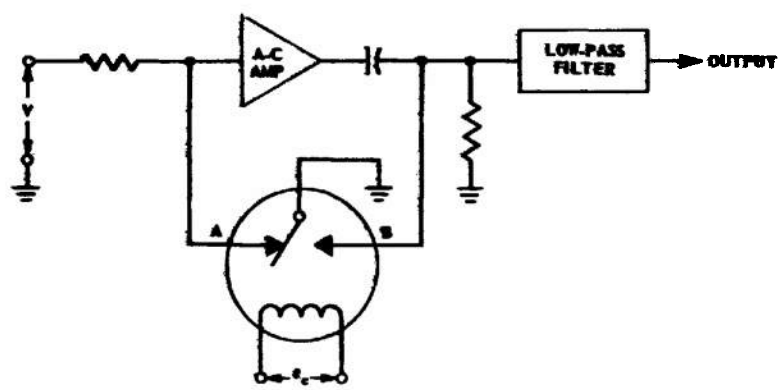

Fig. 12-24 Single chopper used as both modulator and demodulator.

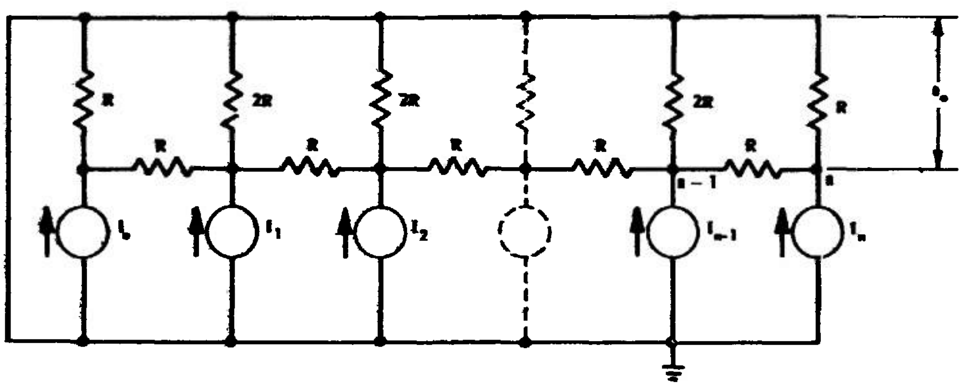

Fig. 12-25 Digital-to-voltage converter.

Adapted by permission from *Notes on Analog-Digital Conversion Techniques*, edited by A. K. Susskind, 1957, Technology Press, Massachusetts Institute of Technology.

the filter during one-half-cycle of reed motion and grounds the filter input on the other half-cycle. This results in a unidirectional filter-input signal, the magnitude of which is determined by the magnitude of the amplifier output, and the polarity of which is determined by the phase of the amplifier output with respect to the reed motion.

12-4 DIGITAL-TO-ANALOG CONVERSION

12-4.1 GENERAL

Control systems are now coming into use in which digital data-processing equipment is employed. In digital equipment, information is expressed as numbers. In the rest of the system, information is represented by analog quantities, such as voltage or shaft rotation. In order to couple the output of the digital equipment to the other parts of the system, digital-to-analog conversion must be performed.

124.2 Networks

Where the required analog representation is to be a voltage, and accuracy requirements are not very high, electrical networks can be used. Examples of several network configurations are shown in Figs. **12-25** through **12-28.**

12-4.3 Operation. In Fig. **12-25,** each current source I_j has an output current I when the associated binary digit (value 2^j) is a ONE, but otherwise has zero output. If the current sources are assumed to have infinite impedance, the output voltage is given by

$$e_o = \frac{IR}{3 \times 2^{n-1}} p \qquad \text{(12-13)}$$

and the output impedance is given by

$$R_o = \frac{2}{3} R \qquad \text{(12-14)}$$

where

p = number to be converted

$n + 1$ = number of stages

In Fig. **12-26,** each voltage source E_j has an output voltage E when the associated binary digit is a ONE, but otherwise has zero output. If the voltage sources are assumed to have zero impedance, the output voltage is given by

$$e_o = \frac{E}{2^{n+1}} p \qquad \text{(12-15)}$$

and the output impedance is given by

$$R' = \frac{R}{2} \qquad \text{(12-16)}$$

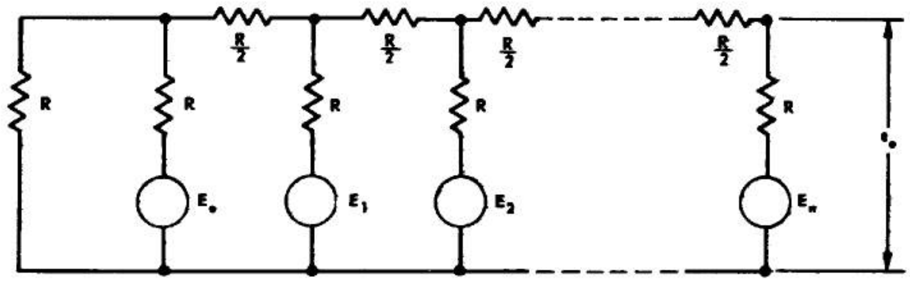

Fig. 12-26 Digital-to-voltage converter.

Adapted by permission from *Notes on Analog-Digital Conversion Techniques,* edited by A. K. Susskind, 1957, Technology Press, Massachusetts Institute of Technology.

Figure 12-27 shows a circuit that is particularly useful when the digital information originates from relays. Here, relay K_j is not energized (contacts in the position shown) when the associated binary digit is a ZERO. The contacts are in the other position, however, when the associated digit is a ONE. The output voltage is given by

$$e_o = \frac{E_1}{\frac{R}{R_c} + 2^{n+1} - 1} p \qquad (12\text{-}17)$$

and the output impedance is given by

$$R_o = \frac{R}{2^{n+1} - 1} \qquad (12\text{-}18)$$

Figure 12-28 shows another circuit that is also useful when the digital information originates from relays. Here again, relay K_j is not energized (contacts in the position shown) when the associated binary digit is a ZERO, and relay K_j is energized (contacts in the other position) when the associated binary digit is a ONE. The output voltage is given by

$$e_o = \frac{E}{2^{n+1} - 1} p \qquad (12\text{-}19)$$

12-4.4 Accuracy. The circuits of Figs. 12-27 and 12-28 have the advantage over those of Figs. 12-25 and 12-26 because they require only one precision resistor per stage. In all of the circuits, accuracy is determined by the accuracy of the precision resistors. In addition, in Figs. 12-25 and 12-26, accuracy is affected by the magnitude of the outputs of the sources and also by their output impedance. The circuits of Figs. 12-27 and 12-28 are therefore somewhat more accurate, but even these can rarely be made more accurate than

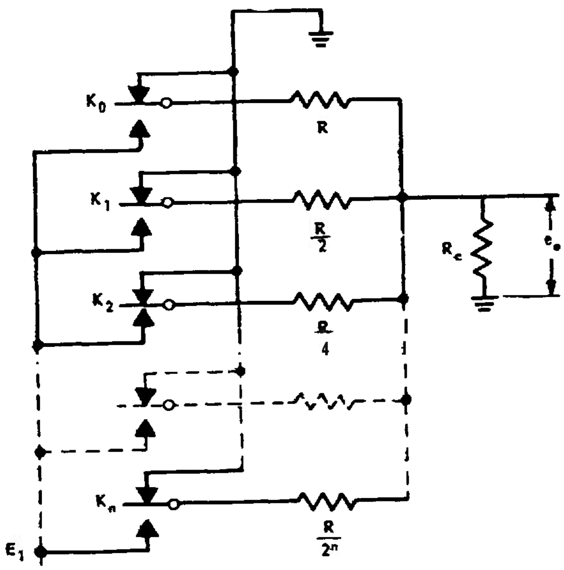

Fig. 12-27 Digital-to-voltage converter.

Adapted by permission from *Notes on Analog-Digital Conversion Techniques*, edited by A. K. Susskind. 1957. Technology Press, Massachusetts Institute of Technology.

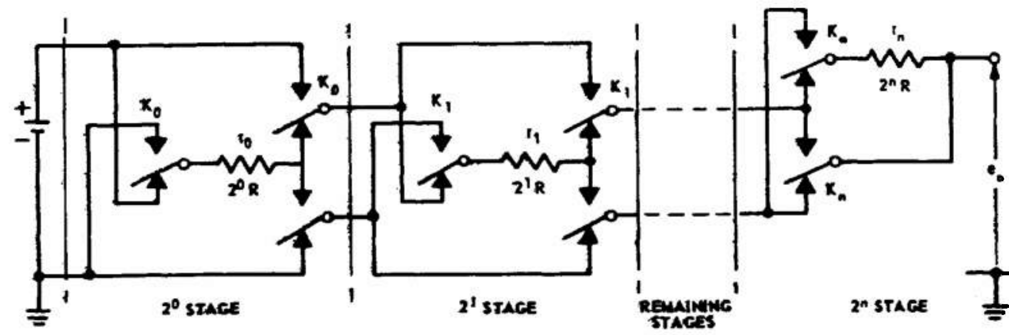

Fig. 12-28 Digital-to-voltage converter.

Adapted by permission from *Notes on Analog-Digital Conversion Techniques*, edited by A. K. Susskind, 1957, Technology Press, Massachusetts Institute of Technology.

one part in 4000 (12 binary digits). For the circuits of Figs. 12-25 and 12-26, typical accuracy figures are one part in 1000 (10 binary digits).

124.5 Servomechanism

Where higher-accuracy requirements must be met, a servomechanism can be used. A block diagram of a digital-to-analog servomechanism is shown in Fig. 12-29. Here, the position of the output shaft is expressed in digital form by a coding disc, such as the one described in Ch. 11. The coded shaft position is then subtracted from the desired shaft position, and the difference number is converted into a voltage which, after dynamic compensation and amplification, is used to drive the output motor. The accuracy of conversion is determined by the coding-disc sensing element. Since coding discs can be made with an accuracy as high as one part in 65,536 (16 binary digits), it follows that the static conversion accuracy in the closed-loop approach can be made very high.

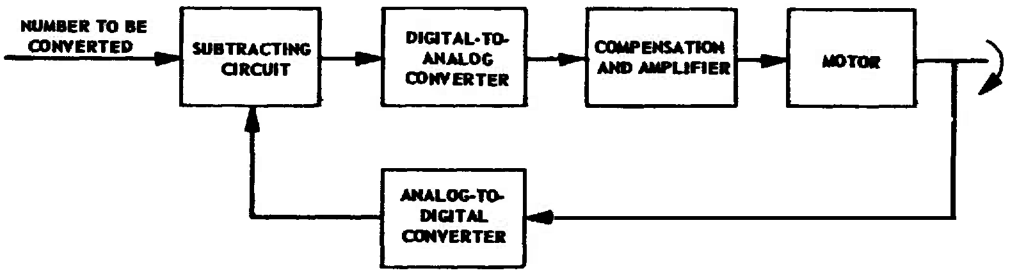

Fig. 12-29 Servomechanism for digital-to-analog conversion.

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