# MASSACHUSETTS IN STITUTE OF TECHNOLOGY LINCO N LABORATORY

# DESIGN OF SEISMIC ARRAYS FOR EFFICIENT ON-LINE BEAMFC //ING

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#### ABSTRACT

The signal-to-noise ratio improvement obtained with delay-and-sum (DS) processing is discussed for short-period seismic data. It is shown that at least 3 km spacing should be maintained between seismometers at the Large Aperture Seismic Array (LASA) or at another site with a similar noise environment. It is shown that noise coherency measurements are of use in determining the sensor spacing at a new array location. The parameters required for a site survey coherency measurement, such as length of data and resolution, are also presented. The design of subarrays of approximately 20 km aperture is described and array patterns are given.

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### I. INTRODUCTION

This report is concerned with the relationship between the configuration of a short-period seismic subarray and the signal-to-noise (S/N) improvement which can be obtained with delay and sum (DS) processing, also known as beamforming. This simple form of processing is desirable because of its ease of implementation, and its relative immunity to slight signal anomalies among the seismometers.

It was pointed out by Capon, et al<sup>(1)</sup> that in the 0.6 to 2.0 Hz body-wave signal band, DS processing of data obtained from a large subarray can give nearly the S/N improvement of more elaborate processing such as maximum-likelihood processing. In the present work the results of a more exhaustive study of DS processing are given, and it is shown that measurements of noise coherency give a useful means of determining suitable short-period seismometer spacing for a future array. The basic result at LASA is that a minimum spacing between seismometers of 3 km gives a noise reduction close to the 10 log K (K = number of seismometers) expected for independent noise. The coherency measurements at LASA support this result. Suitable parameters for site survey coherency measurements will be discussed.

#### **II.** COHERENCY MEASUREMENT

It is shown in Appendix A that the gain of DS processing is related to the coherency between the seismometers. In order to obtain a gain of 10 log K, the minimum seismometer spacing should be large enough so that the noise coherency between seismometers is small, say less than 0.2.

Measurements were carried out to ascertain the manner in which the noise coherency varies as a function of seismometer separation. The direct segment method of spectral estimation<sup>(2)</sup> allowed the averaging of many seismometer pairs with similar separations to obtain an average value for coherency over small ranges of separations. Coherencies were computed at five frequencies over the 0.6 to 2.0 Hz signal band. Estimates were made for three different time periods and several different LASA subarrays. In addition, coherencies between seismometers covering a 40 km aperture were measured so as to relate these results with delay-and-sum results described later in this report.

More precisely, the procedure used to measure coherency is as follows. Let  $r_{ij}$  be the separation between the i<sup>th</sup> and j<sup>th</sup> seismometers, and let  $Z_{ij}(f)$  be an estimate of the true noise coherency,  $C_{ij}(f)$ , between them. Here  $C_{ij}(f)$  is  $\frac{|P_{ij}(f)|}{\sqrt{P_{ii}(f) P_{jj}(f)}}$ , where  $P_{ij}(f)$  is the cross power at the frequency, f. The estimate  $Z_{ij}(f)$  is associated with the k<sup>th</sup> range of seismometer separations if  $R_k \leq r_{ij} \leq R_{k+1}$ , where  $R_k$  and  $R_{k+1}$  define the k<sup>th</sup> range. All the coherency estimates which correspond to a given separation range, k, were averaged to give  $\overline{Z}_k(f)$ .

The estimates for the  $C_{ij}(f)$  were obtained using the direct segment method to estimate  $P_{ij}(f)$ . In all cases, the data blocks were 10 seconds long giving a spectral resolution of 0.1 Hz. A whitening filter was applied to the data before the spectral estimates were made. The number of degrees of freedom, n, used in the estimation is easily shown to be equal to the number of data blocks used. Since a finite length of data was used, the  $Z_{ij}(f)$  are biased estimates of the  $C_{ij}(f)$ . It has been shown by Goodman<sup>(3)</sup> that if the true value of the coherency is  $C_{ij}$  and n blocks are used that the estimate  $Z_{ij}(f)$  will have the probability density function

$$\Lambda[Z_{ij}(f), C_{ij}(f), n] = \frac{2(n-1)(1-C_{ij})^n Z_{ij}(1-Z_{ij})^{n-2}}{\Gamma^2(n)} \sum_{k=0}^{\infty} \frac{\Gamma^2(n+k)}{\Gamma^2(k+1)} (C_{ij} Z_{ij})^{2k}$$

where  $\Gamma(n)$  is the gamma function. The expected value of  $Z_{ii}(f)$  is then

$$E[Z_{ij}(f)] = F[C_{ij}(f), n] = \int_{0}^{1} \Lambda[Z_{ij}(f), C_{ij}(f), n] Z_{ij}(f) dZ_{ij}(f)$$

The bias exists since  $F[C_{ij}(f), n] \neq C_{ij}(f)$ .

Since  $\overline{Z}_{k}(f)$  is the average of the biased quantities  $Z_{ij}(f)$ , it is also biased. In the present study the bias in  $\overline{Z}_{k}(f)$  was removed as follows. It was assumed that  $C_{ij}(f)$  had the value  $C_{k}(f)$  for all seismometer pairs in the k<sup>th</sup> separation range, so that the differences in the estimated  $Z_{ij}(f)$  were due to sampling error. Then

$$E[\overline{Z}_{k}(f)] = E[\frac{1}{N_{k}} \sum_{\substack{\text{Pairs in}\\k^{\text{th}} \text{ range}}} Z_{ij}(f)] = E[Z_{ij}(f)]$$

=  $F[C_k(f), n]$ 

Using the computed value of  $\overline{Z}_k(f)$  for its expected value, an unbiased estimate  $\widetilde{C}_k(f)$  for  $C_k(f)$  was obtained by graphically solving

$$\overline{Z}_{k}(f) = F[C_{k}(f), n]$$

The tabulated values of  $F[\tilde{C}_k(f), n]$  given by Amos and Koopmans<sup>(4)</sup> were used.

This procedure was checked by taking 15 seismometers from sites F4 and A0, and computing the average coherency between each of the 15 seismometers in site F4 with each of the 15 seismometers in site A0. As a consequence of the large separation of 100 km between the two sites, the noise should be incoherent between the two sites. In the run made to check the procedure, the number of degrees of freedom n was 41. The tables of Amos and Koopmans give an expected value of  $\overline{Z}_k(f)$  equal to 0. 139 for completely incoherent noise. The measured value of  $\overline{Z}_k(f)$  for the five frequencies 0.7, 1.0, 1.3, 1.6, and 2.0 Hz were respectively, 0. 169, 0. 153, 0. 137, 0. 142, and 0. 151, which is in very good agreement with the expected value for incoherent noise.

The results obtained by the procedures described are given in Figs. la to le. The various separation ranges are delineated by vertical dashed lines and the results for different subarrays and time periods are plotted at convenient abscissas between these lines. These results show that the noise is more coherent at the lower part of the frequency band. For most of the data the coherency is low for separations of over 3 km. This implies that for a separation of 3 km or more between seismometers, the noise reduction in the signal band, obtainable by simple delay-and-sum processing should be close to 10 log K (db). ł,

In Figs. 2a and 2b coherencies obtained by the same procedure are given for 11 November 1965 and 13 November 1965 data. These data were obtained using 40 of the 58 seismometers that were used to form beams. In the next section these coherencies will be referred to in discussing the beamforming results.

#### III. DELAY-AND-SUM PROCESSING RESULTS

The results of many examples of delay-and-sum processing will be discussed in this section. The n.ajor interest will be centered on data which have been prefiltered to reduce the noise outside of the 0.6 to 2.0 Hz signal band. It has been shown by Capon, et al<sup>(5)</sup> that by use of a phaseless convolutional filter, this prefiltering can be achieved without significant distortion of the signal. In the present processing, a 0.6 to 2.0 Hz three-pole Butterworth prefilter, with amplitude characteristics similar to the convolutional filter was used, the response of this filter is also given in Capon, et al<sup>(5)</sup>.

The array parameters which are significant are the aperture covered by the seismometers used to form the beam and the minimum spacing between seismometers. The noise reduction given by the processing and degradation of the signal are affected by these parameters. The results are tabulated in Table I. The data for 6 km minimum spacing show noise reductions of almost 10 log K, and this indicates the noise was almost independent between sensors. For 3 km minimum spacings, the gain was approximately 3 db less than the theoretical gain for completely incoherent noise. This latter result was obtained using both a cluster of seismometers near the center of LASA, and the enlarged E3 subarray shown in Fig. 10a. Using the 25 closely spaced seismometers in the LASA subarrays, the noise reduction is only 6 db as compared with the theoretical 14 db for random noise.

These noise reduction results are consistent with the separation distances at which the measured noise coherences become small. For the bandpass filtered data · 'n

the noise power is highest at the lower frequency end of the signal band. It is therefore the 0.7 Hz and 1.0 Hz coherencies that are most relevant to the DS noise reduction. At these frequencies, although the data of Figs. 1a to 1e are scattered, the coherencies are leveling off at 3 or 4 km separation. Therefore, at LASA a separation of at least 3 km is desirable for DS processing.

As further evidence of the qualitative relation between large DS noise reduction and low values of the noise coherency consider again Figs. 2a and 2b. In these figures, the noise coherencies at 0.7 and 1.0 Hz for data from 11 November 1965 and 13 November 1965 are plotted as a function of distance. The peak power in the filtered data occurs at about 0.7 Hz. It is seen that the 11 November 1965 noise is the more coherent. Measured DS noise reductions were greater for the 13 November 1965 data (11.6 db) than for the 11 November 1965 data (16.1 db).

For the larger aperture seismometer groups, some signal degradation occurred. For a 65 km aperture, the average degradation was 3.1 db. The beams were formed with plane wave delays and no station corrections, and it is probable that signal degradation could be reduced by the introduction of station corrections.

A pitfall to be avoided in designing a short-period subarray whose primary use is for DS processing is that of greatly varying the spacings between seismometers in different parts of the subarray. To show how the addition of closely spaced seismometers can actually lower the noise reduction, we present the results of a series of DS experiments.

Runs were made for a 4 February 1966 data sample at LASA Site A0 both filtered (0.6 to 2.0 Hz bandpass) and unfiltered, and also for unfiltered 11 November 1965 data from Site A0. The geometry of the subarray is shown in Fig. 3. For each sample of data two series of beams were formed, different seismometers being used in each beam. In the first series, a small number of seismometers near the subarray center were used in the first sum. On each successive sum seismometers were added to fill the subarray outward. In the second series, the first sum used seismometers on the outmost rings of the subarray, then, successively, seismometers from the inside rings were used.

The results from these runs are shown in Figs. 4, 5, and 6. The figures give noise reduction in db <u>vs</u> number of seismometers. The curve of 10 log K, which is the gain for uncorrelated noise, has also been plotted. The noise reduction does not come close to what is expected for independent noise. The plots show, in all three cases, that the addition of seismometers near the center of the array decreases the amount of noise reduction from that given by the outer seismometers alone.

We believe that this result may be explained by a model for correlated noise similar to the simple one discussed in Appendix B. The seismometers in the outer rings are separated from each other (all separations are greater than 3.0 km for the 7 and 8 rings) by much more than the separation between the inner seismometers(e.g. 0.5 km betw.on 1 ring seismometers). The experimentally determined coherencies discussed in Section II show that the noise is a rrelated between close seismometers.

In this instance, the addition of seismometers which are close together has decreased rather than increased the noise reduction.

## IV. SITE SURVEY MEASUREMENTS OF COHERENCY

In this section parameters will be discussed for making coherency measurements to be used in designing a short-period seismic array. The purpose of a site survey wil. be to determine how close seismometers may be and still have noise which has low correlation between sensors. The 0.1 Hz resolution used in obtaining the results of the previous section should be suitable.

It is assumed that in the survey only a few sensors will be operating simultanecusly, so no averaging of coherency estimates over sensor pairs will be done to improve the stability of the estimates. It is then necessary to use long enough data lengths to give an accurate measure of the coherency. A 95% confidence limit of 0.20 about the true value should be accurate enough for a site survey. Using the sampling distributions tabulated by Amos and Koopmens<sup>(4)</sup> we find a sample length with n = 100 degrees of freedom is long enough. In Fig. 6 the mean or expected value of the sample coherency, Z, is plotted <u>vs</u> the true coherency, C. This shows the bias in the estimate. The 95% confidence limits are also drawn. When a measurement is made, the bias may be removed using the curve of the mean, by finding the C which corresponds to the measured sample coherency, Z. As an example, if we measure Z = 0.1, then this implies C = 0.05. For n = 100, the worst error which occurs within the 95% limits occurs when C = 0. Then within the 95% confidence limits Z can be as much as 0.19 corresponding to an implied value of C = 0.18.

A data length of 1000 sec or 15.7 minutes is necessary to obtain 100 degrees of freedom with 0.1 Hz resolution. This assumes that 100 blocks of 10-second duration are used to obtain spectra by the direct segment method. Judging from LASA experience, seismometer separations varying by 0.5 km or 1.0 km up to about a 6.0 km maximum should be used.

### V. BEAM PATTERNS

Array beam patterns in wave number space determine the azimut'.al and velocity selectivity of the array<sup>(6)</sup>. Such patterns have been computed for a number of 20 km subarrays. Figures 7, 8, and 9 show the arrays and wave number patterns. The patterns are contoured in db down from the center of the main lobe. Note that the size and shape of the main beam is essentially determined by the subarray size, not by the specific distribution of sensors. Only the details of more distant side lobe structure changes significantly. Since it is the extent of the main lobe which is of prime interest for short-period data, (noise reduction tends to be established by seismometer separation and the number of sensors) it is clear that apertures can be picked to establish a main lobe. As long as sensors within the aperture are not widely nonuniform and have sufficient minimum spacing, the resulting subarray will be satisfictory.

#### VI. CONCLUSIONS

It has been shown that for LASA short-period seismic data DS processing can get close to 10 log K noise reduction in the body-wave signal frequency band of 0.6 to 2.0 Hz if a minimum seismometer spacing in excess of 3 km is maintained. The 3 km distance is that at which coherency measurements show the noise in the relevant frequency bands to become almost uncorrelated. Therefore, coherency measurements are a convenient tool in site survey work for determining minimum spacing between seismometers. The parameters for such field coherency measurements have been given.

The larger the aperture of the subarray the greater the number of sensors which fit in with a fixed minimum spacing. Therefore, more noise reduction can be obtained by an increase in aperture, but at apertures larger than about 20 km, the signal coherency decreases and some signal degradation occurs. Furthermore, since the beam width is inversely proportional to aperture, the larger the subarray is made, the greater is the number of subarray beams which are required for on-line surveillance. A subarray of no more than about 20 km seems desirable. Large subarrays of approximately 20 km aperture have been described and their beam patterns given.

It is important not to include any group of closely clustered seismometers, as their coherent noise will give a high noise level on the output beam.

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### APPENDIX A

## RELATION BETWEEN NOISE COHERENCE AND DELAY-AND-SUM GAIN

The output y(t) of delay-and-sum processing is

$$y(t) = \frac{1}{K} \sum_{k=1}^{K} X_k(t + \tau_k)$$
 (1)

#### where

 $X_k(t+\tau_k)$  is the noise on the k<sup>th</sup> seismometer, and is assumed to be stationary, K = number of seismometers,

$$\tau_{\mathbf{k}} \equiv$$
 steering delay of the k<sup>th</sup> seismometer,

$$\operatorname{var} \{ \mathbf{y}(\mathbf{t}) \} = \frac{1}{K^2} \sum_{k=1}^{K} \sum_{k'=1}^{K} R_{kk'} (\tau_k - \tau_{k'})$$
(2)

where

$$R_{kk}, (\tau_k - \tau_k) = E \{X_k(t + \tau_k) X_k, (t + \tau_k)\}$$

Equation (2) may be written

$$\operatorname{var} \{ y(t) \} = \int_{-\infty}^{\infty} df \{ \frac{1}{K^2} \sum_{k=1}^{K} \sum_{k'=1}^{K} P_{kk'}(f) \exp \left[ -i2\pi f(\tau_k - \tau_{k'}) \right]$$
(3)

where  $P_{kk'}(f)$ , k, k' = 1,..., K are the cross spectra. The term in brackets is the output at the frequency, f. The processing noise reduction at the frequency, f, is given by

$$G(f) = -10 \log \frac{\frac{1}{K^2} \sum_{k=1}^{K} \sum_{k'=1}^{K} P_{kk'}(f) \exp \left[-2\pi f(\tau_k - \tau_{k'})\right]}{\frac{1}{K} \sum_{k=1}^{K} P_{kk'}(f)} db \qquad (4)$$

We will consider a lower bound,  $\hat{G}(f)$  for G(f)

$$\hat{G}(f) = -10 \log - \frac{\frac{1}{K} \sum_{k=1}^{K} \sum_{k'=1}^{K} |P_{kk'}(f)|}{\sum_{k=1}^{K} P_{kk'}(f)}$$
(5)

For equal noise power on each seismometer,

$$G(f) = (10 \log K - L) db$$

where

$$L = i0 \log \left[1 + \frac{1}{K} \sum_{k \neq k'} C_{kk'}(f)\right] db$$
 (6)

$$C_{kk'}(f) \equiv \frac{|P_{kk'}(f)|}{\sqrt{P_{kk}(f)P_{k''k'}(f)}} \equiv Coherence at frequency, f, between the k and the k'th seismometers (7)$$

The 10 log K term is the familiar gain for uncorrelated noise while L is an upper bound for the loss due to correlation in the noise. The lower bound,  $\hat{G}(f)$  is met only if the coherent part of the noise is in phase at all seismometers after the steering delays. If the seismometers are far enough apart from the noise to be close to incoherent the gain will be almost 10 log K. Therefore, measurements of the coherency as a function of distance are a useful means of determining the spacing needed in order to obtain 10 log K noise reduction.

#### APPENDIX B

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## DISCUSSION OF CLOSELY SPACED SEISMOMETERS

A special case of the result found in Appendix A can be used to point up the deleterious effect a group of closely spaced seismometers can have on the gain obtained with DS processing.

Consider a set of  $N = N_1 + N_2$  seismometers, which for some set of delays have the coherencies

$$C_{ij}(\omega) = 0$$
, if i or j is less than or equal to  $N_1$   
=  $C_0$ , if i and j are greater than  $N_1$  and i  $\frac{1}{2}$  j

Here we have  $N_1$  independent seismometers and  $N_2$  seismometers correlated among themselves.

This type of correlation roughly models a subarray in which the center seismometer and the 0.5 km radius ring of seismometers are close together and are highly correlated, as shown by coherency measurements. For this model the factor L, defined in Appendix A, is

L = 10 log [1+C<sub>0</sub> 
$$\frac{N_2(N_2^{-1})}{N_1 + N_2}$$
]

In Figure 11 the gain G(f) for this case has been plotted as a function of  $N_2$  for  $N_1$  fixed at 20. Curves are given for  $C_0 = 1, 0.5$ , and 0.25. It is apparent that the addition of seismometers with coherencies as small as 0.25 may decrease rather than improve the gain. If the  $N_1$  seismometers are the not completely uncorrelated, the curves of Fig. 11 will be displaced downward.

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			k	4 (qp)	14	14	F1		7 6	19.6					
			A verage SNR	(db)	6.0	6.7	0.11	2 11	0 61	12.2					
			Average Signal Reduction	(qp)	0.1	1.2	0.5	1.5	1.6	3.1					
	ENTS	nal	stiont	Max	0.2	1.7	0.7	2.5	4.2	6.1	1				
Ì	PERIM	Sig	Reduc	Min	0	0.5	0.2	0.4	0.6	1.2					
BLE I	DRMING EX		Average Noise Reduction*	(qp)	6.1	10.9	11.5	13. 2	14.5	15.3	DOVIE:	l amplitude.			
TA	EAMF	ise	tion* b)	Мах	6.5	11.5	12. U	15.4	16.1	16.5	r noise	r signa			
	OF B	Ň	Reduc (d	Min	4.0	10.0	10.0	10.9	11.6	14.3	enso	Senso	Α.		
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			Aperture	(km)	2	19**	22.+	40	40	65 *	*	4	۲۲ ۱	ці * *	

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Fig. 1.  $|\tilde{C}(f)|$  vs seismometer separation.







Fig. 1. Continued.



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Fig. 2.  $|\tilde{C}(f)| \underline{vs}$  seismometer separation.



Fig. 2. Continued.









Fig. 5. Delay-and-sum noise reduction unfiltered data 4 February 1966, Site  $A\emptyset$ .

Fig. 6. Delay-and-sum noise reduction unfiltered data 11 November 1965, Site  $A\emptyset$ .



Fig. 7. 95% confidence limits for sample coherence using 0.1 Hz resolution and 16.7 minutes of data. n = 100.



Fig. 8a. Sels.nometer subarray.



Fig. 8b. Subarray pattern.



Fig. 9a, Seismometer subarray.



Fig. 9b. Subarray pattern.



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Fig. 10a. 25 seismometer E3 LASA subarray.



Fig. 10b. Site E3 subarray pattern.



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Fig. 11. DS gain vs number of correlated seismometers  $N_2$  for a fixed number of uncorrelated seismometers  $N_1 = 20$ .

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