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CORNELL AERONAUTICAL LABORATORY, INC.
BUFFALO, NEW YORK

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WRIGHT-PATTERSON AIR FORCE BASE, OHIO

FOREWORD

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ABSTRACT

An analysis is made of thermal and diffusion effects on the droplet growth phenomena in a supersaturated vapor and inert carrier gas. Two cases are considered: 1) constant fluid conditions, and 2) changing fluid conditions due to condensation effects. The analysis is so formulated as to describe the continuous growth process as the droplet size increases from microscopic (free-molecular) to "rarefied", even to macroscopic (continuum).

Equations for the conservation of mass and energy are derived by application of the "Langmuir model" in the rarefied ("slip") regime and two correlation parameters for the mass transfer and the energy transfer are introduced for analyzing this regime. Analytic solutions are obtained for the droplet growth with time by expressing the saturation vapor pressure as a linear function of temperature.

The results indicate that the choice of these slip-regime parameters influence the droplet growth only when the ratio of the droplet size to the mean free path is of order one. However, when the droplet size is very small or very large compared to the mean free path, the values chosen for these parameters have a negligible effect on the droplet growth.

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LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

A, B	Constants defined in Eq (2-10)
c	Vapor concentration (molecules/cm^3)
c_p	Specific heat at constant pressure (erg/gr-°K)
D	Droplet diameter (cm)
e	$= 2\Delta_e$ (cm)
H_c	Heat of condensation, (erg/gr)
j	Number of degrees of freedom of a molecule
k	Boltzmann constant = 1.3803 x 10^{-16} erg/°K
K_{th}	Thermal conductivity of inert gas (erg/cm-sec-°K)
m	Mass of molecule (gr/molecule)
p	Partial pressure (dyne/cm^2)
q	Heat flux from a droplet (erg/cm^2-sec)
r	Radial distance from the center of a droplet (cm)
T	Temperature (°K)
t	Time (sec)
U	Internal energy (erg/gr)
V	Flux to a droplet (molecules/cm^2-sec)
W	Droplet number density (droplets/cm^3)
z	$= 2\Delta_d$ (cm)
α	Sticking coefficient
γ	Ratio of specific heats
Δ	Region I around a droplet (Sec. 2) (cm)
δ_{12}	Diffusion coefficient of vapor in inert gas (cm^2/sec)

λ	Mean free path of the gas mixture (cm)
ξ	Slip-regime parameter for energy transfer (Eq(2-28))
ρ	Density (gr/cm^3)
σ	Surface tension (dyne/cm)
Φ	Thermal accommodation coefficient (Eq (2-14))
Ψ	Slip-regime parameter for mass transfer (Eq(2-28))

SUBSCRIPTS

c	Condensing vapor
ℓ	Condensing vapor in liquid phase
D	Droplet
g	Inert gas
r	Reflected condition
sat	Saturated equilibrium condition
d,e	Designations of regions of diffusion and of energy transfer, respectively (Fig. 1)
o	Initial condition
∞	Condition at large distance from droplet

1. INTRODUCTION

The purpose of the present analysis is to investigate the growth of condensation droplets in rarefied gas when influenced by both thermal and mass-diffusion effects in the supersaturated surrounding fluid. By "rarefied gas" it is meant that the droplet size is of the same order of magnitude as the mean free path of the surrounding gas. As such, the droplet is in the so-called "slip regime". The present study extends an earlier analysis performed by Kang and Weatherston,[1] in which a first-order approximation was employed for the slip regime to investigate the droplet growth from a microscopic size (free-molecular regime) to a macroscopic size (continuum). In that analysis, only a small portion of the total droplet growth process occurred in the slip regime and, consequently, it was not necessary to be too exacting in analyzing that regime. However, if the droplet is in the slip regime, then a more accurate analysis is required. The present analysis deals mainly with the droplet growth in the free-molecular and slip regimes. The analysis, however, is also valid for large final droplet sizes in the continuum regime.

Langmuir[2] pointed out that there is a jump in the vapor concentration at the surface of an evaporating droplet in the transition regime, analogous to the well-known temperature jump at the droplet surface. His observation has led many investigators[3-6] to use the Langmuir model in analyzing either a heat-conduction problem or a mass-transfer process in this regime.

In the present report, the Langmuir model is extended to simultaneously consider both the energy-transfer and mass-transfer processes in the condensation droplet growth under the influence of thermal and diffusion effects. Moreover, in the present analysis, the droplet temperature is left as a variable and is obtained as part of the solution.

In line with the objectives of the present analysis, it is necessary to develop a formulation for droplet growth processes that will apply throughout the free-molecular, slip, and continuum regimes. In both the free-molecular regime, where the mean free path is large compared with the droplet diameter, and the continuum regime, where the droplet is large compared with the mean free path, there are convenient mathematical formulations for analyzing both the mass and energy fluxes. In the slip regime, however, the formulation is not so well established. To bridge the gap between the free-molecular and the continuum regimes, the mass flux at a distance Δ_d from the droplet surface is assumed to be the same for the free-molecular or continuum considerations. Similarly, the energy fluxes are matched at a distance Δ_e. Thus, a uniform free-molecular field is assumed within the distance Δ from the droplet surface, and continuum conditions are assumed to prevail at greater distances (Fig. 1).

The results obtained represent a mathematical fairing between the free-molecular and continuum regimes. In Ref. 1, the free-molecular and continuum effects were equated at the droplet surface proper. This procedure was known to be somewhat in error, but led to satisfactory results for the purpose of that analysis. Actually, the marked temperature jump within one mean free path of droplet undergoing heat transfer to a fluid would suggest the validity of the Langmuir model, but the mathematical model used here is somewhat of an artifice. However, there is good agreement between experiment and theory if the free-molecular and continuum mechanisms are equated at one mean free path from the droplet surface.

It will be shown that the values assigned to the slip-regime parameters influence the droplet growth only when the ratio of the droplet diameter to the mean free path is near one. When the droplet is very small or very large compared with the mean free path, the values chosen for these parameters have a negligible effect on the droplet growth.

Two situations are considered in the present analysis. In Sec. 2, the growth of a single droplet is studied in a constant fluid medium which consists of an inert carrier gas and the condensing vapor. In Sec. 3, the droplet growth is analyzed for the condition that the surrounding fluid is changing due to condensation, depletion of the mass of vapor in the fluid medium, and release of the heat of condensation to the surroundings.

Closed-form solutions are obtained for the droplet size as a function of time by expressing the saturation vapor pressure as a linear function of temperature. Sample calculations are performed for a nitrogen-mercury gas mixture under various conditions of pressure and temperature. The initial droplet size is taken to be much smaller than the mean free path of the surrounding gas, so that initially the free-molecular mechanisms will prevail for the droplet growth process. Results are shown to be in general agreement with those obtained in the earlier analysis.[1]

2. ANALYSIS FOR CONSTANT FIELD CONDITIONS

The formulation of the equations for the solution of the droplet growth and temperature history is similar to that used previously,[1] although the mass and energy transfer in the slip regime is treated somewhat differently in the present analysis.

2.1 MASS BALANCE

It is desired to derive an equation for the mass balance which is applicable not only in the free-molecule and the continuum regimes, but also to the intermediate slip regime. Following Fuchs,[3] the field around a droplet is divided into two regions, as illustrated in Fig. 1. In the immediate vicinity of the droplet (Region $\mathrm{I_d}$), between $r = D/2$ and $r_d = \frac{D}{2} + \Delta_d$ (where D is the diameter of the droplet), the free-molecular mechanism is assumed to prevail with the vapor concentration being constant. Outside this region, i.e., in Region $\mathrm{II_d}$, continuum mechanisms hold. Then, the mass balance yields the following equations.

Region I_d

It follows, from kinetic theory,[7] that the net number of vapor molecules condensing on the droplet per unit time is:

$$V_I = \alpha \pi D^2 \left(\frac{p_d}{\sqrt{2\pi m_c k T_e}} - \frac{p_D}{\sqrt{2\pi m_c k T_D}} \right) \qquad (2\text{-}1)$$

where α is the sticking coefficient of the vapor and p_D is the equilibrium pressure on the surface of a droplet with diameter D at temperature T_D. From the physics of small droplets, it is shown that[3]

$$p_D = p_{sat}(T_D) \exp\left(\frac{4\sigma_c m_c}{\rho_\ell k T_D D} \right) \qquad (2\text{-}2)$$

For a droplet larger than 5×10^{-6} cm in a mercury-nitrogen gas mixture at moderate temperature (of the order of 500°K), the exponential term in Eq. (2-2) is very close to unity, and thus p_D can be considered equal to p_{sat}.* In addition, the term $(T_e/T_D)^{1/2}$ is only slightly different from unity, although $T_e - T_D$ is not necessarily negligible, as will be shown later in the numerical examples. The error thus introduced into the analysis by setting $(T_e/T_D)^{1/2}$ equal to unity is negligible. Hence, Eq. (2-1) may be written

$$V_I = \frac{\alpha \pi D^2}{\sqrt{2\pi m_c k T_D}} \left(p_d - p_{sat} \right) \qquad (2\text{-}3)$$

Region II_d

In the continuum, the net number of the vapor molecules passing through $r = r_d$ per unit time is,[8] using $(T_d/T_\infty)^{1/2} \approx 1$,

$$V_{II} = \frac{2\pi (D + 2\Delta_d)\, \delta_{12}}{k T_\infty} \left(p_\infty - p_d \right) \qquad (2\text{-}4)$$

Since the net number of the vapor molecules leaving Region II_d per unit time should all condense on the droplet, we may put $V_I = V_{II}$. Therefore, combining Eqs. (2-3) and (2-4) yields:

$$p_d = \frac{2\delta_{12}(D + 2\Delta_d)\, p_\infty + \alpha D^2 \sqrt{\frac{k T_\infty}{2\pi m_c}}\; p_{sat}}{2\delta_{12}(D + 2\Delta_d) + \alpha D^2 \sqrt{\frac{k T_\infty}{2\pi m_c}}} \qquad (2\text{-}5)$$

---

*The time required for a droplet to grow from an arbitrarily small size to 5×10^{-6} cm is infinitesimally small compared with the growth times of interest here, which are of the order of 0.2 to 10 milliseconds.

and introducing $z = 2\Delta_d$

$$V = \frac{\pi D^2 \left(\frac{2\delta_{12}}{k T_\infty}\right)\left(p_\infty - p_{sat}\right)}{D - \frac{zD}{D+z} + \frac{\delta_{12}}{\alpha}\sqrt{\frac{8\pi m_c}{k T_\infty}}} \tag{2-6}$$

Now, mass balance on a droplet yields

$$\frac{\pi D^2}{m_c}\,\frac{\rho_l}{2}\,\frac{dD}{dt} = V \tag{2-7}$$

Substituting Eq. (2-6) and Eq. (2-7) gives

$$\frac{dD}{dt} = \frac{M_1}{D + M_2 - z + \frac{z^2}{D+z}}\left(p_\infty - p_{sat}\right) \tag{2-8}$$

where

$$M_1 = \frac{4 m_c \delta_{12}}{\rho_l k T_\infty}$$

$$M_2 = \frac{\delta_{12}}{\alpha}\sqrt{\frac{8\pi m_c}{k T_\infty}} \tag{2-9}$$

For moderate temperature changes, the saturation vapor pressure of the condensing gas may be assumed to vary linearly with temperature. We may set

$$p_{sat}(T_D) = A + B\left(T_D - T_\infty\right) \tag{2-10}$$

where A is the saturated vapor pressure at T_∞ and B is the proportionality constant which is determined for a specific vapor being considered. Combining Eqs. (2-8) and (2-10) gives:

$$\frac{dD}{dt} = \frac{M_3 - M_4\left(T_D - T_\infty\right)}{D + M_2 - z + \frac{z^2}{D+z}} \tag{2-11}$$

where

$$M_3 = M_1\left(p_\infty - A\right)$$

$$M_4 = M_1 B \tag{2-12}$$

2.2 ENERGY BALANCE

As in the mass diffusion case, we wish to obtain an equation for the energy balance between a droplet and its environment which is valid in the rarefied case, as well as in the continuum and free-molecular regimes. In the Langmuir model used here, the environment surrounding a droplet is divided into two regions, as illustrated in Fig. 1B. In Region I_e, free-molecular mechanism is assumed to prevail; in Region II_e, exterior to Region I_e, the continuum mechanism holds.

Region I_e

Kinetic theory yields, for the energy balance on a droplet (Fig. 2),

$$\frac{\rho_\ell U_\ell}{2}\frac{dD}{dT} = \frac{\phi_g p_g}{\sqrt{2\pi m_g k T_e}}\left(\frac{j_g+1}{2}\right)k\left(T_e - T_D\right)$$
$$+\frac{\left[\alpha+(1-\alpha)\phi_c\right] p_d}{\sqrt{2\pi m_c k T_e}}\left(\frac{j_c+1}{2}\right)k\left(T_e - T_D\right)+\alpha\left[\frac{p_d}{\sqrt{2\pi m_c k T_e}}-\frac{p_D}{\sqrt{2\pi m_c k T_D}}\right]\left(\frac{j_c+1}{2}\right)kT_D \qquad (2\text{-}13)$$

where the thermal accommodation coefficient is defined as:

$$\phi = \frac{T_r - T_e}{T_D - T_e} \qquad (2\text{-}14)$$

and T_r is the temperature corresponding to the energy of a reflected molecule. In Eq. (2-13), the surface-energy term and the term accounting for the droplet-temperature adjustment to the droplet size are not included, due to their negligible effects on the growth process.[1]

By use of Eqs. (2-1) and (2-7), the last term on the right-hand side of Eq. (2-13) may be replaced with

$$\frac{\rho_\ell}{2}\left(U_c + \frac{kT_D}{2m_c}\right)\frac{dD}{dt}$$

since the internal energy of the vapor per unit mass in gaseous phase is

$$U_c = \left(j_c k T\right)/\left(2m_c\right)$$

Rearranging Eq. (2-13) yields:

$$f_I = M_5 \frac{dD}{dt} = \left(M_6 + M_7\right)\left(T_D - T_e\right) \qquad (2\text{-}15)$$

where

$$\begin{aligned} M_5 &= \rho_\ell H_c/2 \\ M_6 &= \phi_g N_1 \\ M_7 &= \left[\alpha + (1-\alpha)\phi_c\right] N_2 \\ N_1 &= \frac{k}{2}\left(\frac{\gamma_g+1}{\gamma_g-1}\right)\frac{p_g}{\sqrt{2\pi m_g k T_\infty}} \end{aligned} \qquad (2\text{-}16)$$

and

$$N_2 = \frac{k}{2}\left(\frac{\gamma_c+1}{\gamma_c-1}\right)\frac{p_\infty}{\sqrt{2\pi m_c k T_\infty}}$$

Region II_e

At $r_e = \frac{D}{2} + \Delta_e$, Fourier's equation for continuum heat conduction per unit area of droplet surface gives

$$q_{II} = \frac{2K_{th}(T_e - T_\infty)}{\left(\frac{D^2}{D+2\Delta_e}\right)} \qquad (2\text{-}17)$$

Now the boundary condition at the juncture of regions I_e and II_e requires that $q_I = q_{II}$.

Therefore, combining Eqs. (2-14) and (2-17), we obtain

$$T_e = \frac{(M_6+M_7)T_D + \frac{2K_{th}}{D^2}(D+2\Delta_e)T_\infty}{M_6+M_7+\frac{2K_{th}}{D^2}(D+2\Delta_e)} \qquad (2\text{-}18)$$

and

$$M_5 \frac{dD}{dt} = \frac{(M_6+M_7)\,2K_{th}(D+2\Delta_e)}{(M_6+M_7)D^2 + 2K_{th}(D+2\Delta_e)}(T_D - T_\infty) \qquad (2\text{-}19)$$

Introducing $e = 2\Delta_e$ and some manipulation of Eq. (2-19) yields

$$\frac{dD}{dt} = \frac{M_8}{D + M_9 - e + \frac{e^2}{D+e}}(T_D - T_\infty) \qquad (2\text{-}20)$$

where

$$M_8 = \frac{4 K_{th}}{\rho_l H_c}$$
$$M_9 = \frac{2 K_{th}}{M_6 + M_7} \qquad (2\text{-}21)$$

and M_6 , M_7 are defined in Eq. (2-16).

Thus, by combining the mass-balance equation (2-11), and the energy-balance equation (2-20), we obtain an expression for the rate of droplet growth.

$$\frac{dD}{dt} = \frac{M_3 M_8 (D+z)(D+e)}{(N_6 D + N_7)(D+z)(D+e) - M_4 e D(D+z) - M_8 z D(D+e)} \qquad (2\text{-}22)$$

where

$$N_6 = M_4 + M_8$$
$$N_7 = M_2 M_8 + M_4 M_9 \qquad (2\text{-}23)$$

Integration of Eq. (2-22) with the boundary condition $D = D_0$ at $t = 0$, the solution for the droplet size as a function of time is obtained as

$$t = N_3 (D^2 - D_0^2) + N_4 (D - D_0) + \frac{z^2}{M_3} \ln\left(\frac{D+z}{D_0+z}\right) + N_5 e^2 \ln\left(\frac{D+e}{D_0+e}\right) \qquad (2\text{-}24)$$

where

$$N_3 = \frac{1}{2M_3}\left(1 + \frac{M_4}{M_8}\right)$$
$$N_4 = \frac{1}{M_3}\left[M_2 - z + \frac{M_4}{M_8}(M_9 - e)\right] \qquad (2\text{-}25)$$
$$N_5 = \frac{M_4}{M_3 M_8}$$

The droplet temperature T_D as a function of the droplet size is, from Eqs.(2-11) and (2-20)

$$T_D - T_\infty = \frac{M_3 (D + M_9)(D+z)(D+e) - M_3 e D(D+z)}{(N_6 D + N_7)(D+z)(D+e) - M_4 e D(D+z) - M_8 z D(D+e)} \qquad (2\text{-}26)$$

For convenience, we introduce the correction parameters ψ and ξ in terms of the mean free path λ :

$$z = 2\lambda\psi \qquad (2\text{-}27)$$

$$e = 2\lambda\xi$$

or

$$\psi = \frac{\Delta_d}{\lambda} \qquad (2\text{-}28)$$

$$\xi = \frac{\Delta_e}{\lambda}$$

where ψ and ξ are multiplication factors for mass diffusion and energy transfer, respectively.

Fuchs[3] contends that the values of ψ and ξ may be between unity and 10. On the other hand, Springer and Tsai[6] take the value of unity and results obtained appear to be in good agreement with experimental data,[10, 11] at least for heat conduction in rarefied gases (see Ref. 6 for comparison). Moreover, Gyarmathy[12] suggests taking from 0.5 to unity for these values as deduced from Millikan's oil drop drag experiments.

Numerical examples were calculated for $\psi = 1, 3, 5$ and $\xi = 1, 3, 5$ at $T_\infty = 455°K$, $p_T = 5$ atm, $p_\infty = 0.08$ atm, $\alpha = 1$, $\phi_T = 1$, and $D_0 = 0.001$ micron. The results are shown in Figs. 3 to 5. It is seen that the droplet growth at a given time t depends greatly upon the values of ψ and ξ .

Two observations may be made on the droplet growth from the results. The first is that once the diffusion effect on the droplet growth is taken into account, the heat transfer effect is rather small. This may be seen from Fig. 4, where comparison is made between the growth for $\psi = \xi = 1$ and that for $\psi = 1$, $\xi = 5$. The difference in the growths is very small and, for all practical purposes, may be neglected. On the other hand, we observe in Fig. 5 that increasing the value of ψ from unity to 5 at a fixed value of ξ (e.g. $\xi = 1$) causes an appreciable difference in the droplet-size growths. Thus, for the numerical examples calculated, the effects of diffusion are more pronounced than those of energy transfer. This is the same conclusion that was obtained in the previous analysis (Ref. 1) which corresponds to $\psi = \xi = 0$.

The second observation is that this limiting case of $\psi = \xi = 0$ is usually adequate. The comparison of the curves for $\psi = \xi = 1$ and $\psi = \xi = 0$ in Fig. 3A reveals that the curves are very close to each other. In other words, if the final droplet is sufficiently large to be in the continuum regime, it makes little difference whether the diffusion and energy-transfer conditions are taken at one mean free path from the droplet ($\psi = \xi = 1$), or at the droplet surface itself ($\psi = \xi = 0$). However, if the droplet size is in the slip regime, the relative influence of ψ and ξ on the droplet growth is appreciable, as shown in Fig. 3B, which shows the data of Fig. 3A enlarged and replotted on semi-logarithmic coordinates.

The same conclusion may also be obtained from the consideration of the rate of droplet growth for given ψ and ξ (Eq. (2-22)). For $\psi = \xi = 0$, the growth rate becomes, from Eq. (2-22),

$$\left(\frac{dD}{dt}\right)_{0=\psi=\xi} = \frac{M_3 M_8}{N_6 D + N_7} \qquad (2\text{-}29)$$

We now compare these two rates by defining

$$\theta = \frac{dD/dt}{(dD/dt)_{0=\psi=\xi}} - 1 \qquad (2\text{-}30)$$

Substituting Eqs. (2-22) and (2-29) in Eq. (2-30) gives

$$\theta = \frac{M_4 e D(D+z) + M_8 z D(D+e)}{(N_6 D+N_7)(D+z)(D+e) - M_4 e D(D+z) - M_8 z D(D+e)} \qquad (2\text{-}31)$$

Calculations were performed for the nitrogen-mercury gas mixture under the same conditions as before for various droplet sizes. The mean free path is about 0.02 micron under these circumstances, and the numerical example is calculated for the droplet sizes varying between 0.001 micron and 1.0 micron, which corresponds to the Knudsen number λ/D between 20 and 0.02.

The results are shown in Fig. 6 for various values of ψ and ξ. It is seen that there are considerable differences in the rates of the growth, but the resulting droplet sizes, as shown in Fig. 3, are not greatly affected by the particular choice of the parameters ψ and ξ as long as the final droplet size is sufficiently large to be in the continuum.

The droplet-temperature history was also obtained for various values of ψ and ξ, and the results are plotted in Fig. 7. It shows that the droplet temperature jumps about 11 °K from its initial value of T_∞ at the onset of condensation process, decreases slightly and reaches a constant value in a matter of several milliseconds, signifying the balance between the thermal-gradient limitation and the diffusion limitation. This conclusion was also reached in the previous analysis for the case where $\psi = \xi = 0$.

3. CHANGING FLUID CONDITIONS

In Sec. 2, the droplet-size growth and the droplet temperature as functions of time were analyzed under thermal and diffusion limitations while the surrounding fluid conditions were assumed to be constant. In this section, the assumption of constant conditions is removed, so that the environmental conditions such as the fluid temperature and the vapor pressure are allowed to change with time due to the release of the heat of condensation to the surrounding medium as condensation progresses. The changes and the droplet growth now depend upon the droplet number density W, in addition to the other conditions considered in Sec. 2.

In formulating the equations, it is necessary to obtain expressions for the changes in the vapor concentration in the fluid medium (p_∞), the saturation pressure of the condensing vapor (p_{sat}), and the fluid temperature T_∞ in terms of W, D_0, and other variables. Of course, the usual mass and energy balances are required. Since these expressions have already been obtained in the previous analysis,[1] their derivations will be omitted here. From Eq.(3-1) of Ref. 1, the fluid temperature T_∞ is

$$T_\infty - T_0 = G_5 W \left(D^3 - D_0^3\right) \tag{3-1}$$

where

$$G_5 = \frac{\pi k T_0 \rho_\ell H_c}{6 m_g c_{p_g} (p_T - p_0)} \tag{3-2}$$

The partial vapor pressure of the condensing gas is[1]

$$p_\infty = G_6 - G_7 W D^3 - G_8 W^2 D^6 \tag{3-3}$$

where

$$\begin{aligned}
G_6 &= p_0 - \frac{p_0}{T_0} G_5 W D_0^3 + \frac{\pi \rho_\ell k T_0}{6 m_c} W D_0^3 - \frac{\pi \rho_\ell k G_5}{6 m_c} W^2 D_0^6 \\
G_7 &= \frac{\pi \rho_\ell k T_0}{6 m_c} - \frac{p_0 G_5}{T_0} - \frac{\pi \rho_\ell k G_5}{3 m_c} W D_0^3 \\
G_8 &= \frac{\pi \rho_\ell k G_5}{6 m_c}
\end{aligned} \tag{3-4}$$

The saturation pressure of the condensing gas is[1]

$$p_{sat} = \left(A - B G_5 W D_0^3\right) + B G_5 W D^3 + B\left(T_0 - T_\alpha\right) \tag{3-5}$$

Combining Eqs.(3-1),(3-3), (3-5), along with the previously developed mass and energy balance Eqs. (2-10) and (2-21), we obtain

$$\left(K_2 D + K_1 - K_6 + \frac{K_7}{D+z} + \frac{K_8}{D+e}\right)\frac{dD}{dt} = K_3 - K_4 W D^3 - G_8 W^2 D^6 \tag{3-6}$$

where

$$K_1 = \frac{M_2}{M_1} + BM_8 M_4 \,, \qquad K_2 = \frac{1}{M_1} + BM_8 \,,$$

$$K_3 = G_6 - A + BG_8 W D_0^3 \,, \qquad K_4 = G_7 + BG_5 \,,$$

$$K_6 = \frac{z}{M_1} + BM_8 e \,, \qquad K_7 = \frac{z^2}{M_1} \,, \qquad K_8 = BM_8 e^2 \tag{3-7}$$

Integrating Eq. (3-6) by use of integral tables with the boundary condition $D = D_0$ at $t = 0$, we obtain

$$\begin{aligned}
t = {} & \frac{1}{6NH^2\Omega}\left[Q_1\Omega - Q_3 H - \frac{H^3}{W}(Q_2\Omega - Q_4 H)\right]\left\{\ln\frac{(H+\Omega D)^2}{(H-\Omega D)^2 + H\Omega D}\right. \\
& \left. - \ln\frac{(H+\Omega D_0)^2}{(H-\Omega D_0)^2 + H\Omega D_0}\right\} + \frac{1}{\sqrt{3}\,NH^2\Omega^2}\left[Q_1\Omega + Q_3 H - \frac{H^3}{W}(Q_2\Omega + Q_4 H)\right] \times \\
& \left\{\tan^{-1}\frac{\Omega D\sqrt{3}}{2H - \Omega D} - \tan^{-1}\frac{\Omega D_0\sqrt{3}}{2H - \Omega D_0}\right\} + \frac{1}{6NF^2\Omega^2}\left[Q_1\Omega + Q_3 F + \frac{F^3}{W}(Q_2\Omega + Q_4 F)\right] \times \\
& \left\{\ln\frac{(F+\Omega D)^2 - F\Omega D}{(F-\Omega D)^2} - \ln\frac{(F+\Omega D_0)^2 - F\Omega D_0}{(F-\Omega D_0)^2}\right\} + \frac{1}{\sqrt{3}\,NF^2\Omega^2}\left[Q_1\Omega - Q_3 F + \right. \\
& \left. + \frac{F^3}{W}(Q_2\Omega - Q_4 F)\right]\left\{\tan^{-1}\frac{F\sqrt{3}}{F + 2\Omega D_0} - \tan^{-1}\frac{F\sqrt{3}}{F + 2\Omega D}\right\} \\
& + K_7 r_6 \ln\left(\frac{D+z}{D_0+z}\right) + K_8 u_6 \ln\left(\frac{D+e}{D_0+e}\right) \\
& - \frac{Q_6}{3WK_4}\ln\left(\frac{K_3 - K_4 W D^3 - G_8 W^2 D^6}{K_3 - K_4 W D_0^3 - G_8 W^2 D_0^6}\right) \\
& + \frac{(Q_5 - Q_6)}{3NW}\left\{\ln\left(\frac{WD^3 + H^3}{WD^3 - F^3}\right) - \ln\left(\frac{WD_0^3 + H^3}{WD_0^3 - F^3}\right)\right\}
\end{aligned} \tag{3-8}$$

where

$$\Omega = W^{1/3} \qquad N = \sqrt{K_4^2 + 4K_3 G_8}$$

$$H = \left(\frac{N + K_4}{2G_8}\right)^{1/3} \qquad F = \left(\frac{N - K_4}{2G_8}\right)^{1/3} \tag{3-9}$$

and

$$Q_1 = K_1 - K_6 + K_7 r_0 + K_8 u_0$$

$$Q_2 = K_7 r_3 + K_8 u_3$$

$$Q_3 = K_2 + K_7 r_1 + K_8 u_1$$

$$Q_4 = K_7 r_4 + K_8 u_4$$

$$Q_5 = K_7 r_2 + K_8 u_2$$

$$Q_6 = \frac{K_4}{2G_8 W}\left(K_7 r_5 + K_8 u_5\right) \tag{3-10}$$

The terms r_0 , r_1 , r_2 , ... and u_0 , u_1 , ... are defined as

$$r_0 = r_6 z^2 W \left(K_4 - G_8 W z^3\right)$$

$$r_1 = -r_0/z$$

$$r_2 = r_0/(z)^2$$

$$r_3 = r_6 W^2 z^2 G_8$$

$$r_4 = -r_3/z$$

$$r_5 = r_3/(z)^2$$

$$r_6 = \frac{1}{K_3 + W z^3 \left(K_4 - G_8 W z^3\right)} \tag{3-11}$$

and

$$
\begin{aligned}
u_0 &= u_6 e^2 W\left(K_4 - G_8 W e^3\right) \\
u_1 &= -u_0/e \\
u_2 &= u_0/(e)^2 \\
u_3 &= u_6 G_8 W^2 e^5 \\
u_4 &= -u_3/e \\
u_5 &= u_3/(e)^2 \\
u_6 &= \frac{1}{K_3 + We^2\left(K_4 - G_8 We^3\right)}
\end{aligned}
\qquad (3\text{-}12)
$$

Thus, an implicit solution is obtained for the droplet size as a function of time in terms of ψ, ξ, D_0, etc.

Numerical examples are calculated under the same initial conditions as in Sec. 2 and the results are plotted in Fig. 8 for $W = 10^6$ and, in Fig. 9, for $W = 10^8$ with $\psi = \xi = 0, 1, 3,$ and 5. Fig. 10 gives the droplet growth as a function of time when $\psi = \xi = 1$ for various values of the droplet number density W. The fluid temperature T_∞ is plotted in Fig. 11 as a function of time for various values of W and $\psi = \xi = 1$. Also, the influence of ψ and ξ on the fluid temperature is shown in Fig. 12 where $W = 10^7$ and ψ, ξ are varied from 0 to 5.

The results indicate that the effect of ψ, ξ on the droplet growth is less pronounced in the changing fluid case (Figs. 8, 9) than in the constant fluid case. These figures show that, at a given time t, the droplet-size differences between, say, $\psi = \xi = 1$ and $\psi = \xi = 5$ become smaller as the magnitude of the droplet number density W is increased. The same trend is also observed for the rate of increase in the fluid temperature as condensation proceeds (Figs. 10, 11). However, in the slip regime itself, the effect of ψ and ξ on the droplet growth is not necessarily small (Fig. 9) and determination of physically reasonable values of ψ and ξ is important in predicting droplet growth in the slip regime. As yet, this question is not settled to everyone's satisfaction. However, in the case of heat conduction in rarefied gases at least, taking unity[6] for the energy-transfer correlation parameter seems to agree well with the experiment results.[10, 11] It appears reasonable to take unit value for the mass transfer parameter as well, since both these processes display similar "jump" characteristics in the slip regime.

4. CONCLUSIONS

The growth characteristics of condensation droplets and the histories of droplet and fluid temperatures in rarefied gases have been analyzed when thermal and diffusion effects are present. Both energy and mass transfer processes are simultaneously considered for the analysis, in which the Langmuir model is employed for the slip regime. Two cases were considered: 1) constant environmental fluid conditions, and 2) the more practical case of the changing fluid conditions due to condensation effects, such as depletion of the mass of the vapor in the fluid medium and the release of the heat of condensation to the surroundings.

Analytic solutions were obtained for these cases for the droplet size as a function of time by expressing the saturation vapor pressure as a linear function of temperature.

The results indicate that when the droplet is very small or very large compared with the mean free path of the surrounding fluid, the values chosen for the slip-regime parameters ψ and ξ have a negligible effect on the droplet growth. On the other hand, in the slip regime, the droplet growth depends greatly on the choice of these parameters. Experimental studies are in progress to establish the correct values of those parameters. However, on the basis of limited available heat-transfer data, it appears that taking unit values for these parameters is reasonable.

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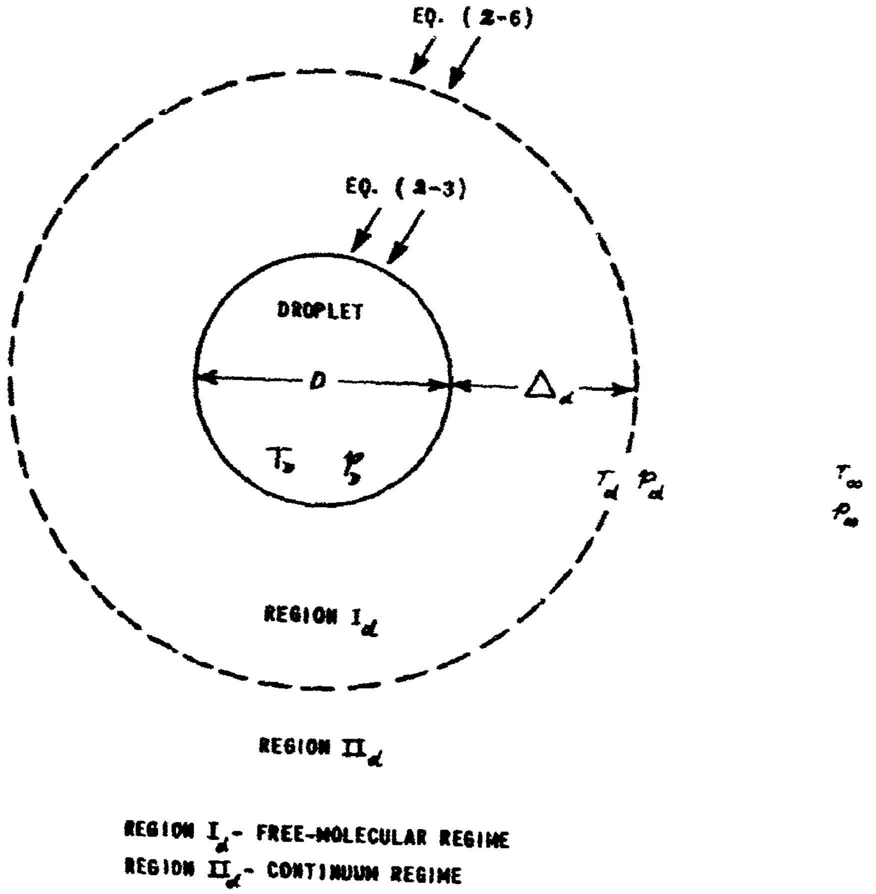

Figure IA MODEL FOR DIFFUSION ANALYSIS - MASS TRANSFER

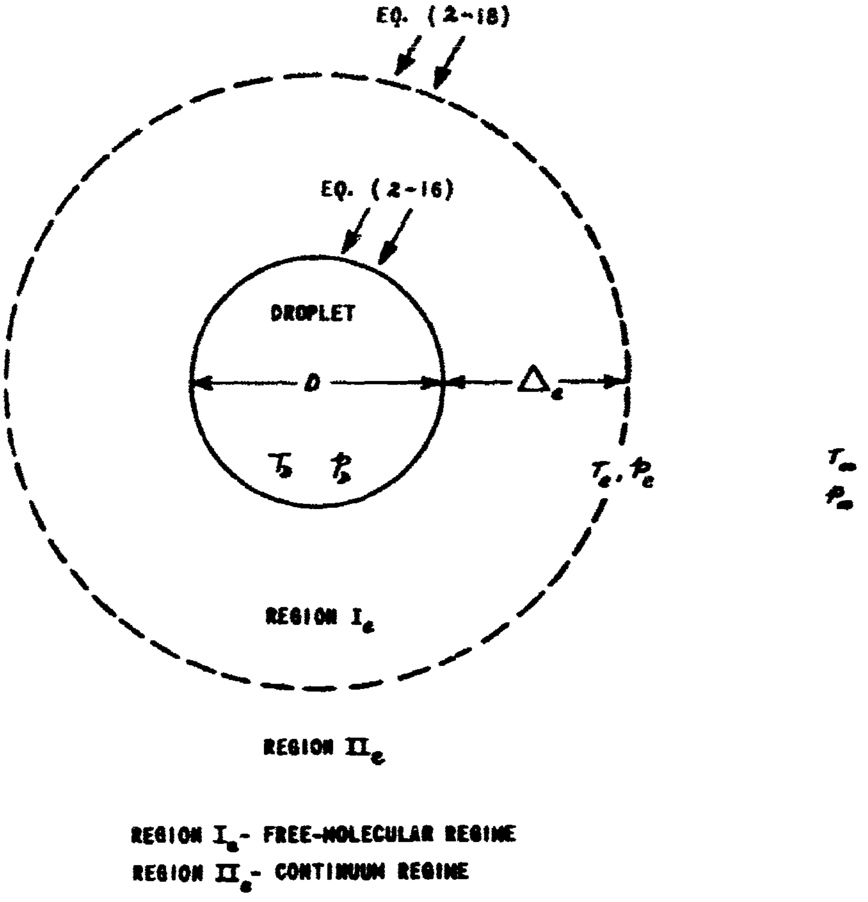

Figure 1B MODEL FOR ANALYSIS - ENERGY TRANSFER

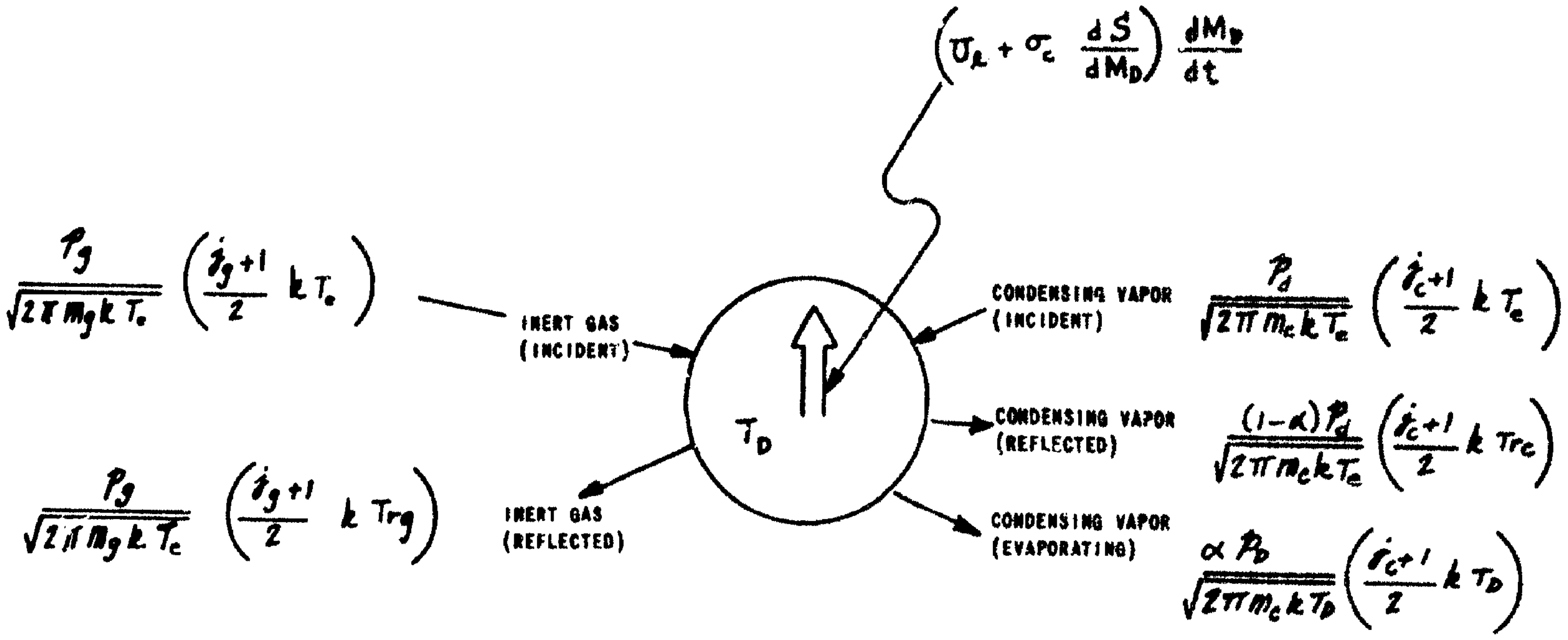

Figure 2 ENERGY CONSIDERATIONS IN THE FREE-MOLECULAR REGIME

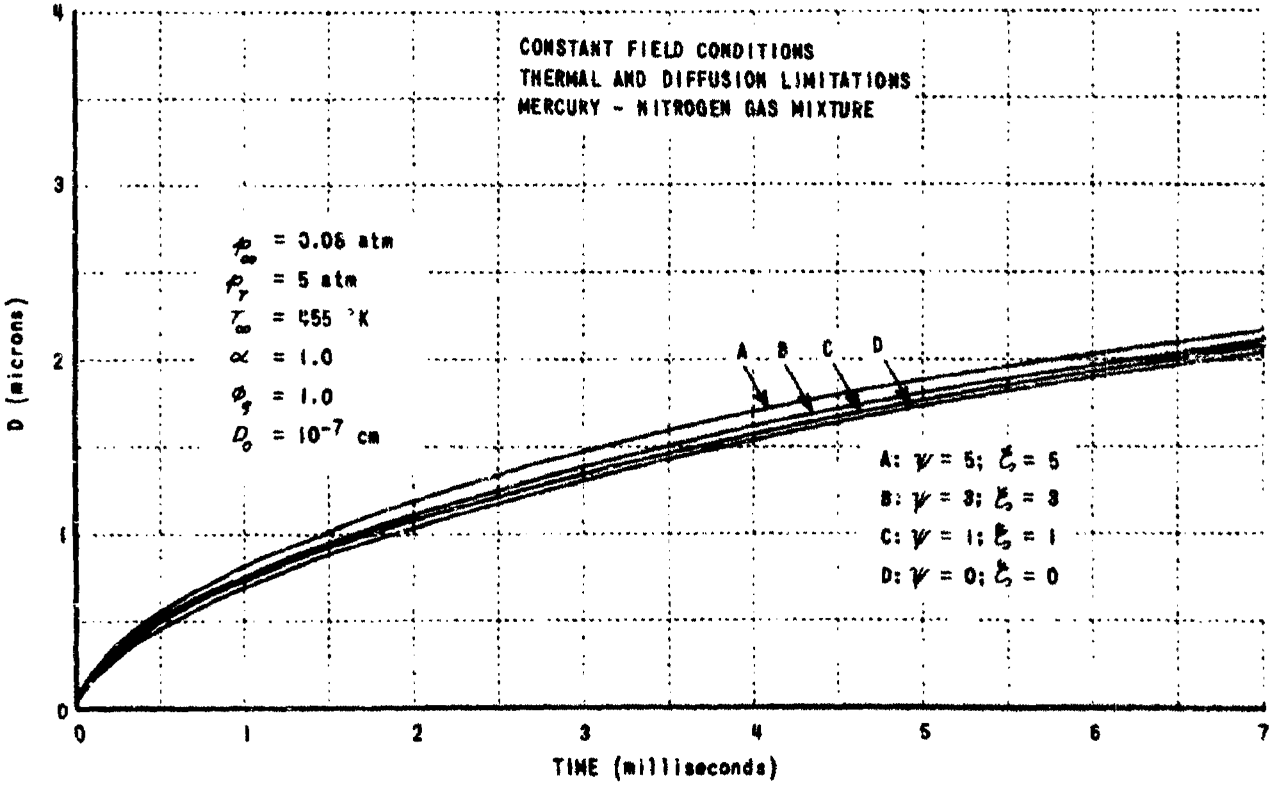

Figure 3A EFFECT OF THE VARIATIONS OF SLIP-REGIME PARAMETERS ON DROPLET GROWTH

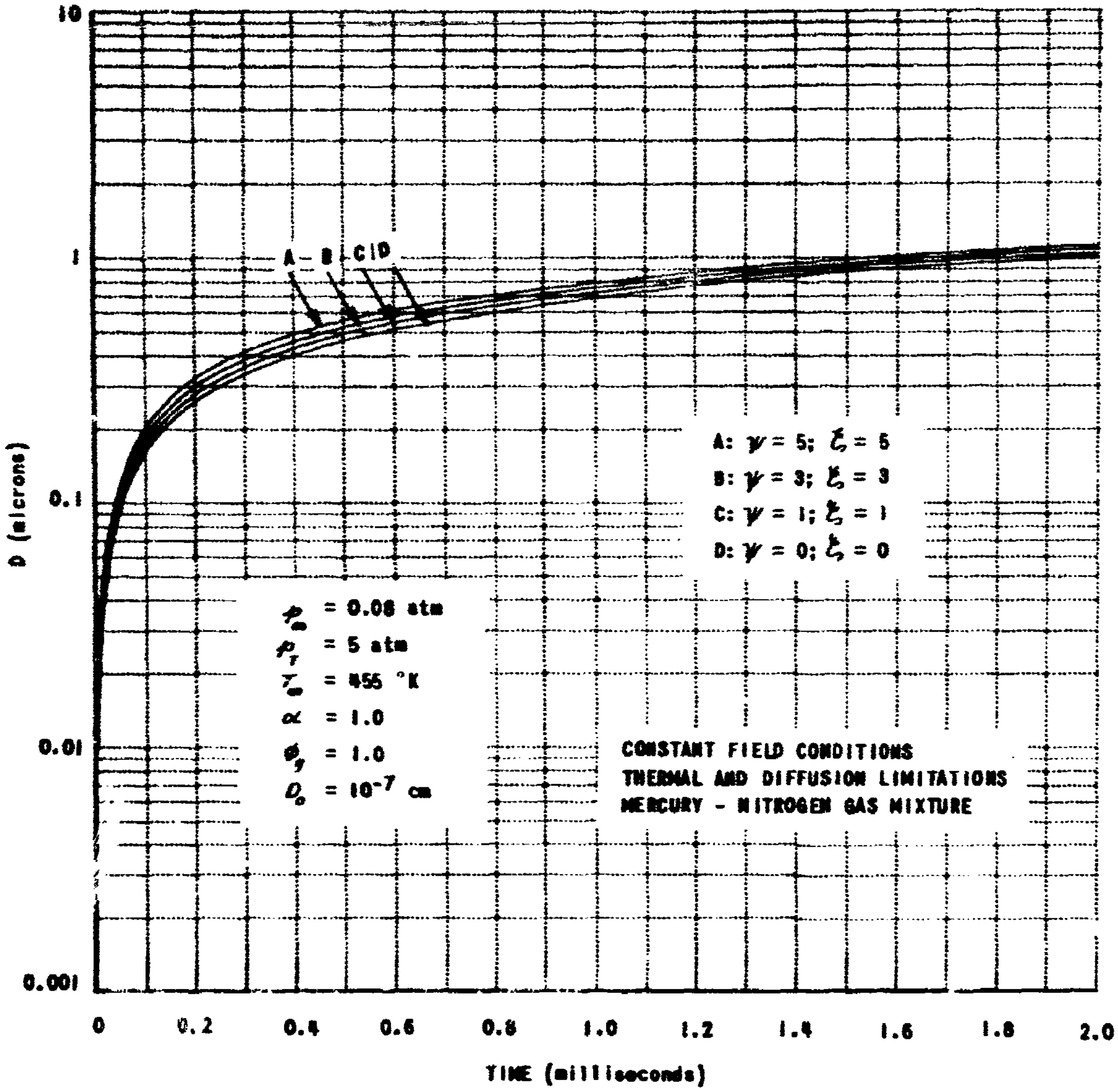

Figure 3B EFFECT OF THE VARIATIONS OF SLIP-REGIME PARAMETERS ON DROPLET GROWTH

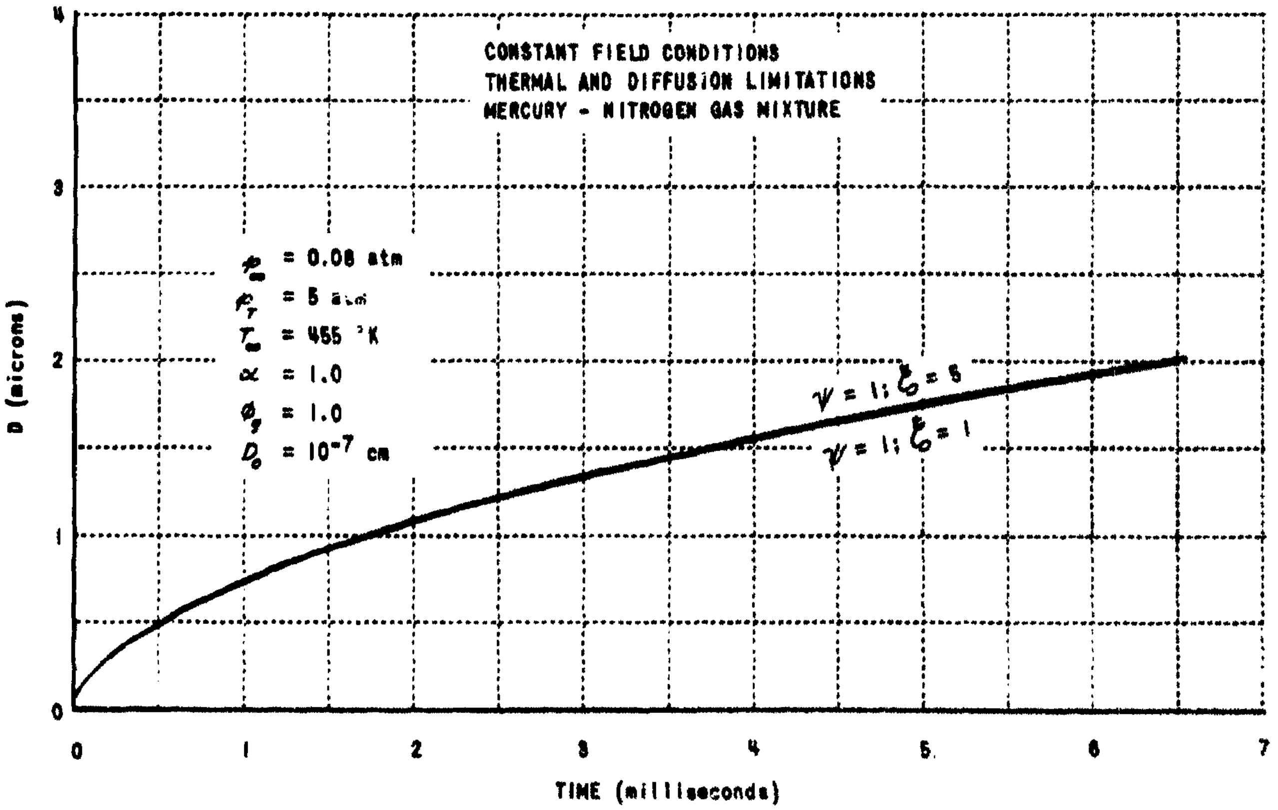

Figure 4 EFFECT OF THE VARIATIONS OF ENERGY-TRANSFER PARAMETER ON DROPLET GROWTH WITH CONSTANT MASS-TRANSFER PARAMETER

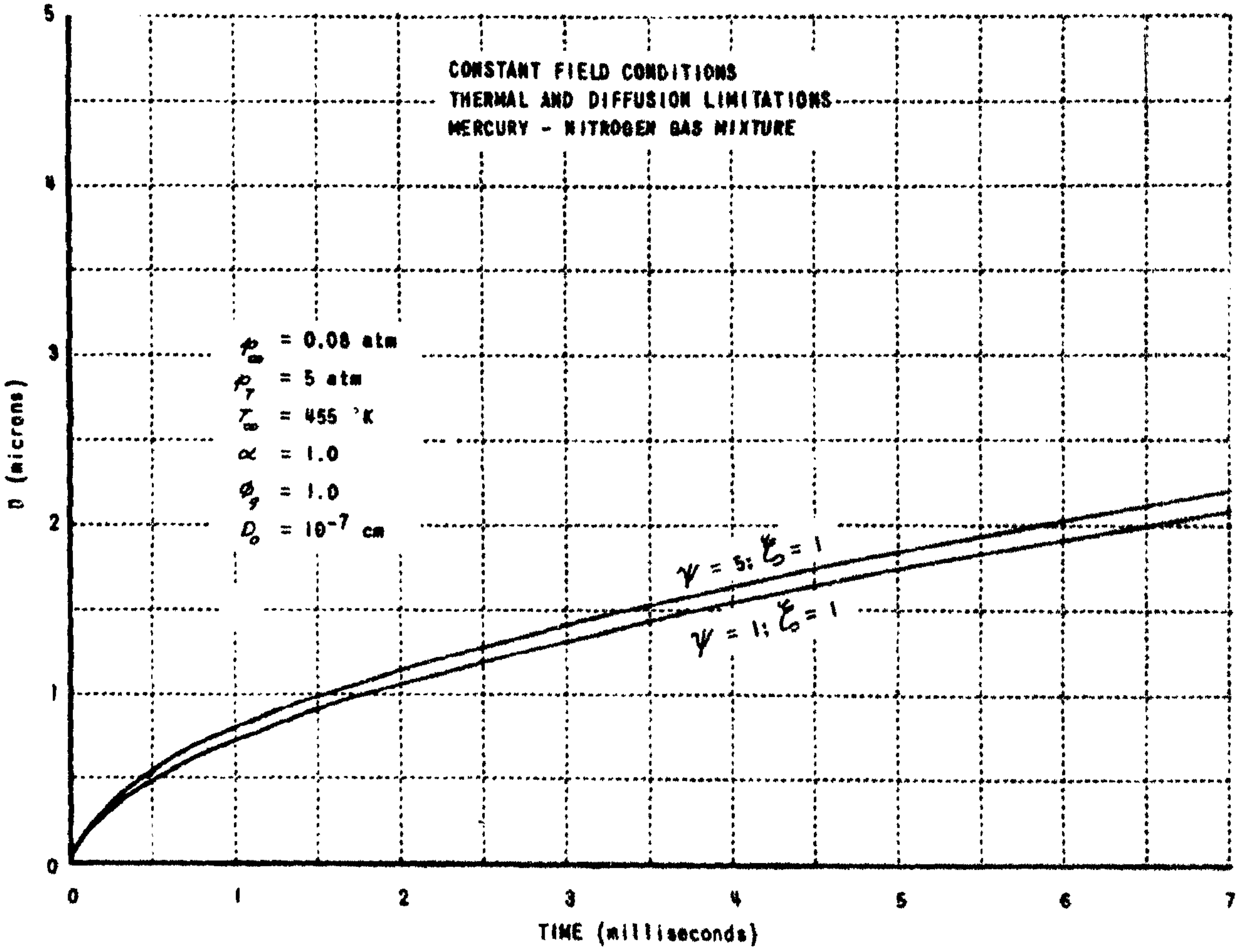

Figure 5 EFFECT OF THE VARIATIONS OF MASS-TRANSFER PARAMETER ON DROPLET GROWTH WITH CONSTANT ENERGY-TRANSFER PARAMETER

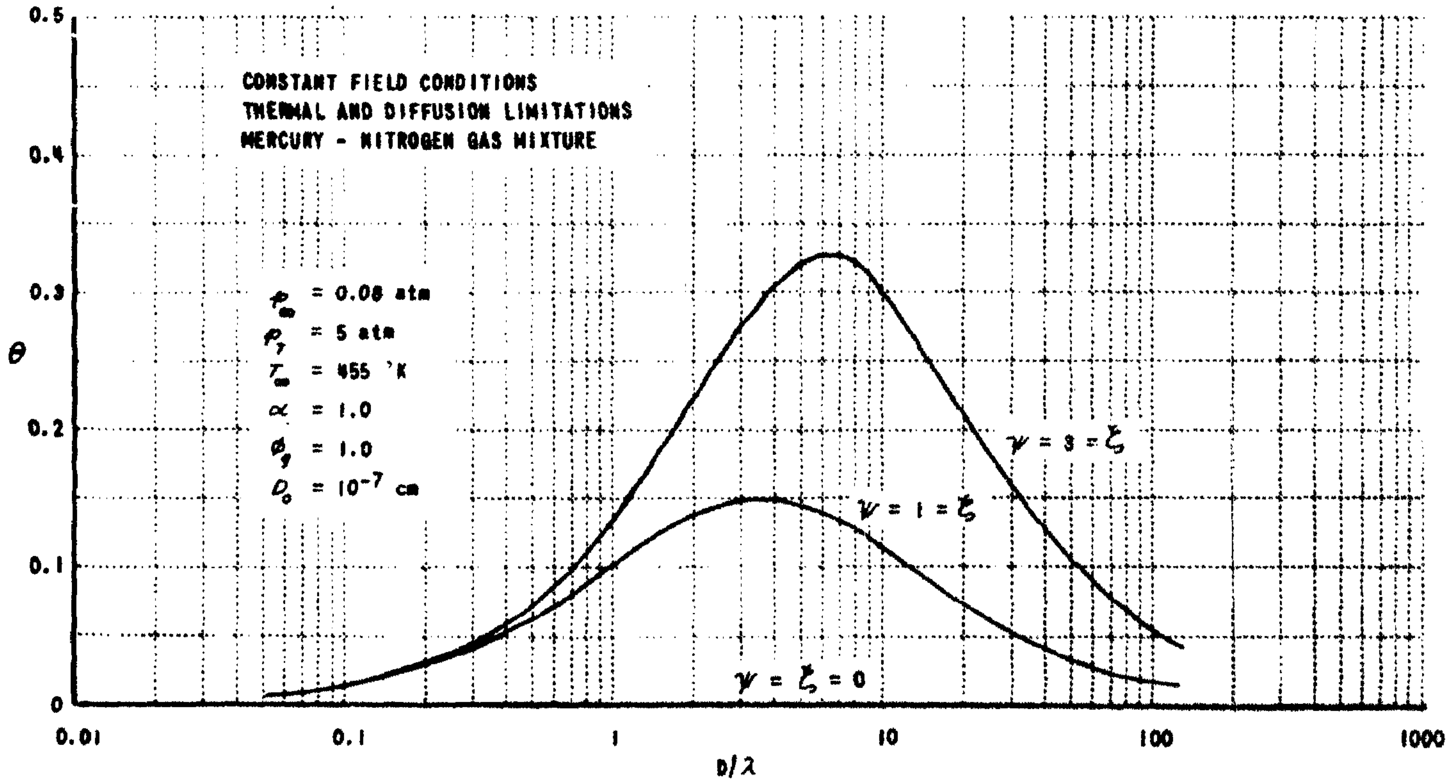

Figure 6A COMPARISON OF THE RATES OF DROPLET GROWTH FOR VARIOUS VALUES OF SLIP-REGIME PARAMETERS

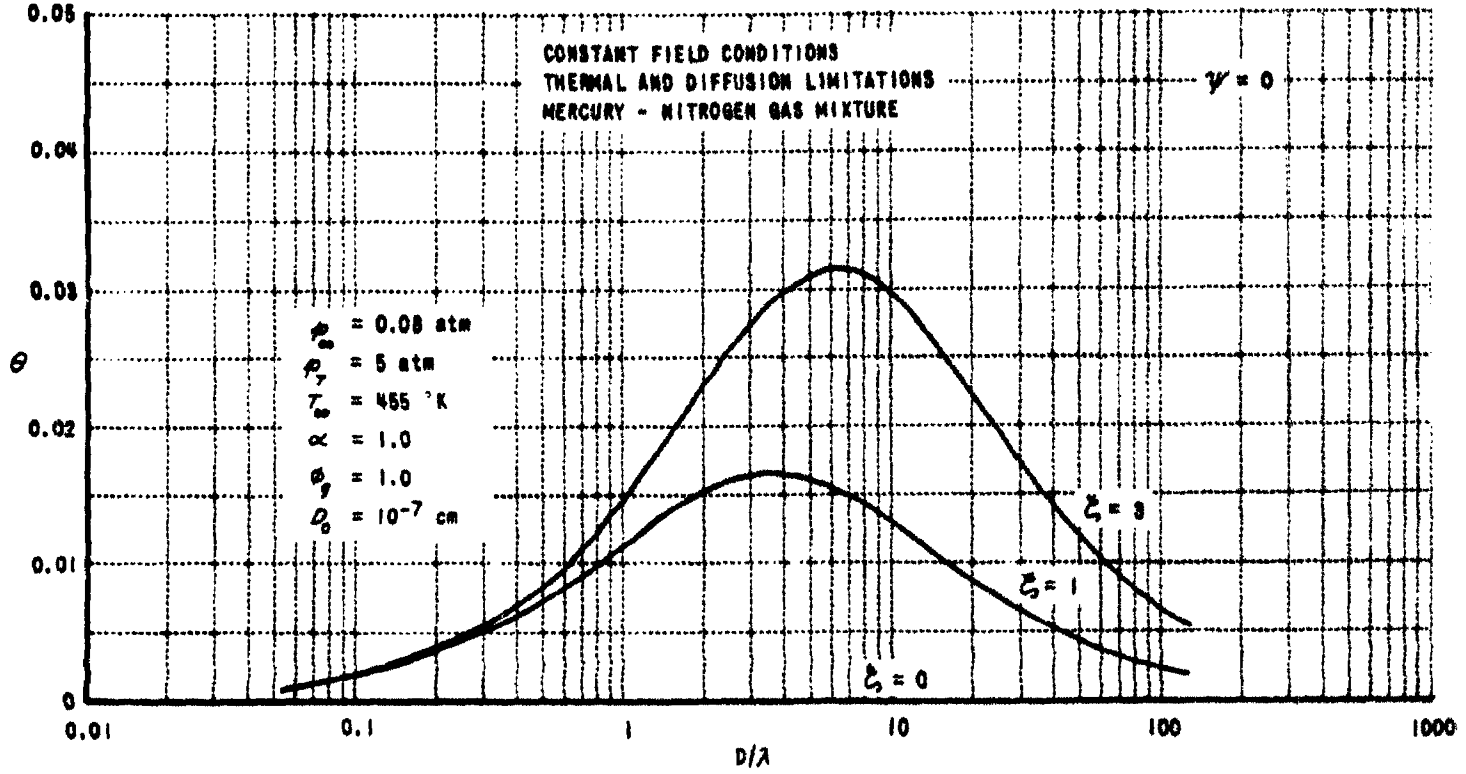

Figure 6B COMPARISON OF THE RATES OF DROPLET GROWTH FOR VARIOUS VALUES OF ENERGY-TRANSFER PARAMETER WITH ZERO MASS-TRANSFER PARAMETER

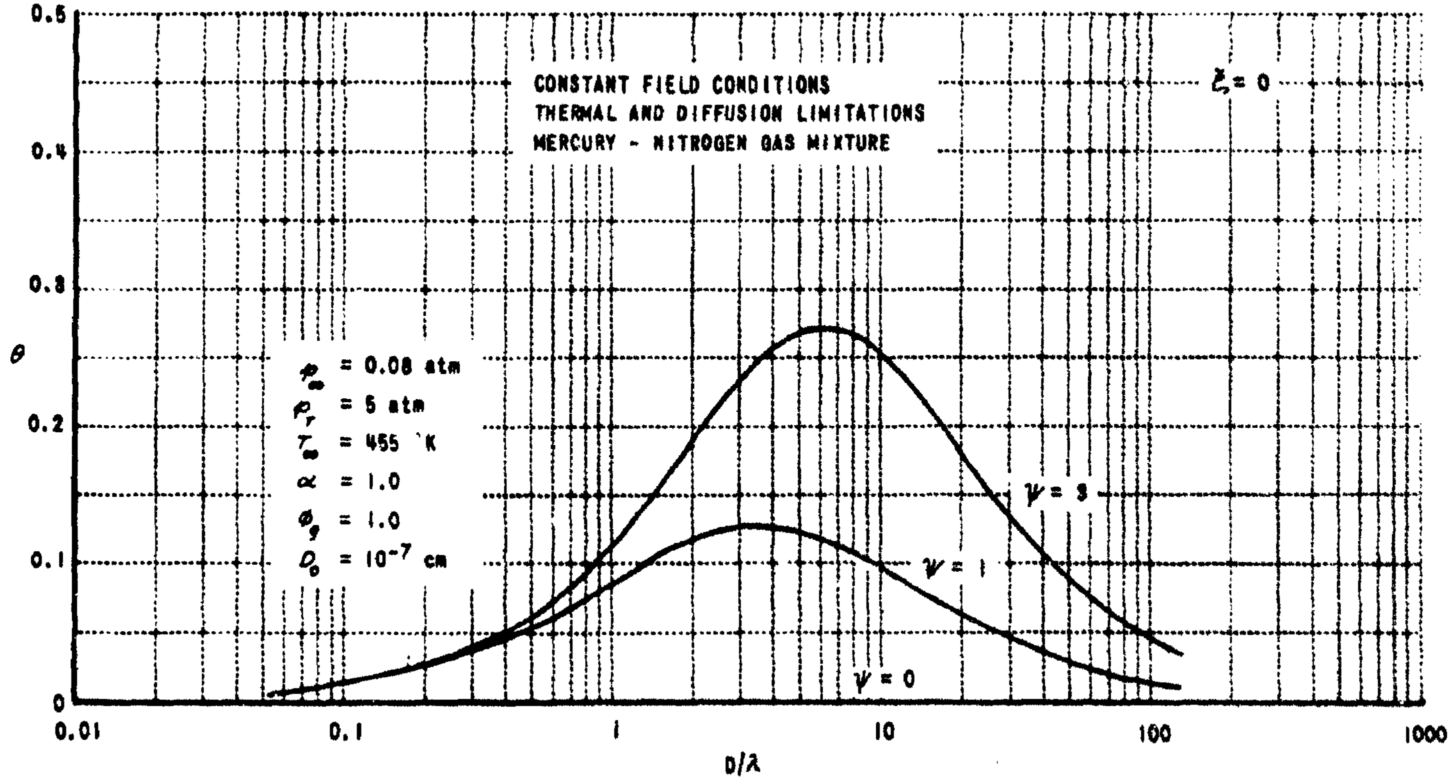

Figure 6C COMPARISON OF THE RATES OF DROPLET GROWTH FOR VARIOUS VALUES OF MASS-TRANSFER PARAMETER WITH ZERO ENERGY-TRANSFER PARAMETER

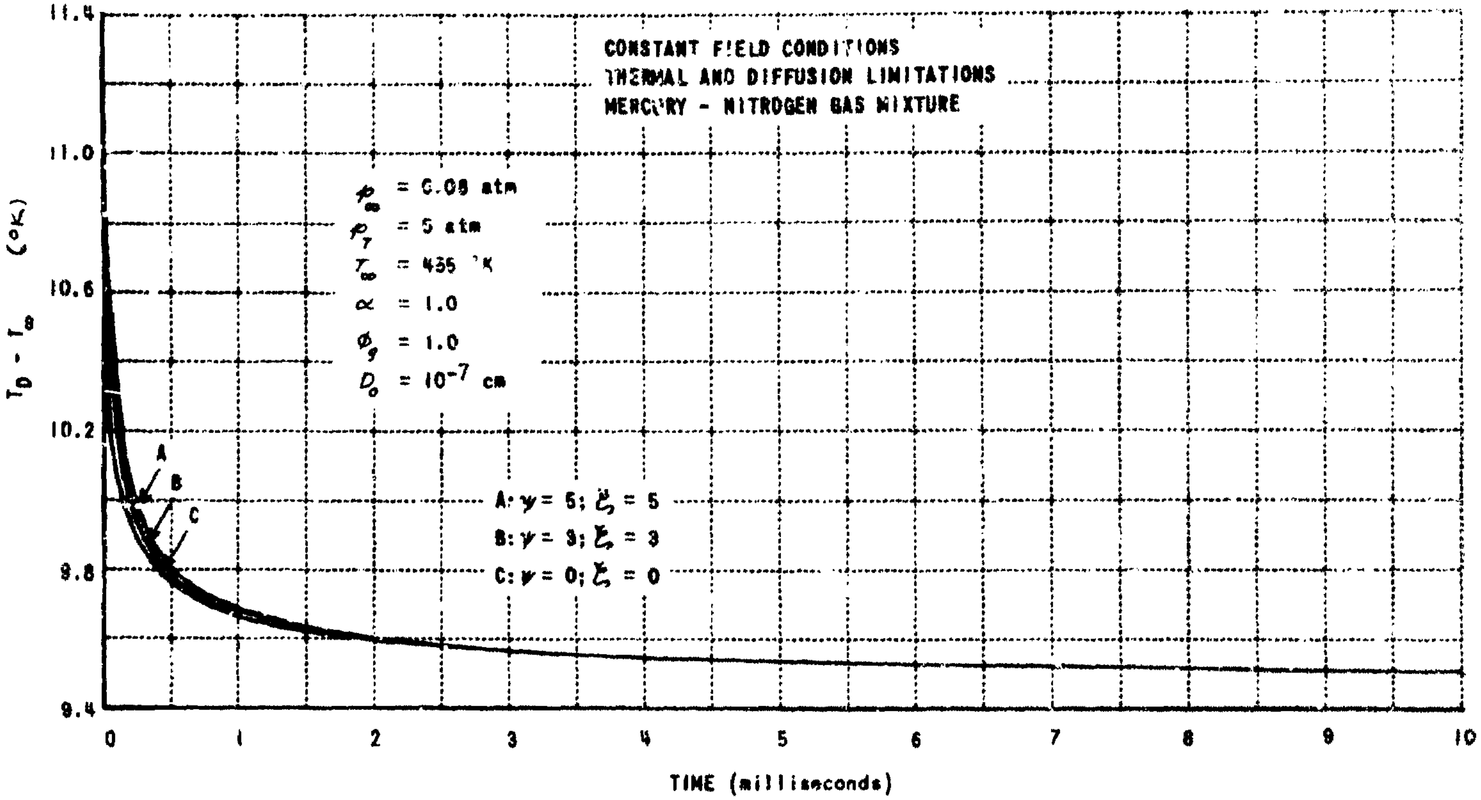

Figure 7A EFFECT OF THE VARIATIONS OF SLIP-REGIME PARAMETERS ON DROPLET TEMPERATURE

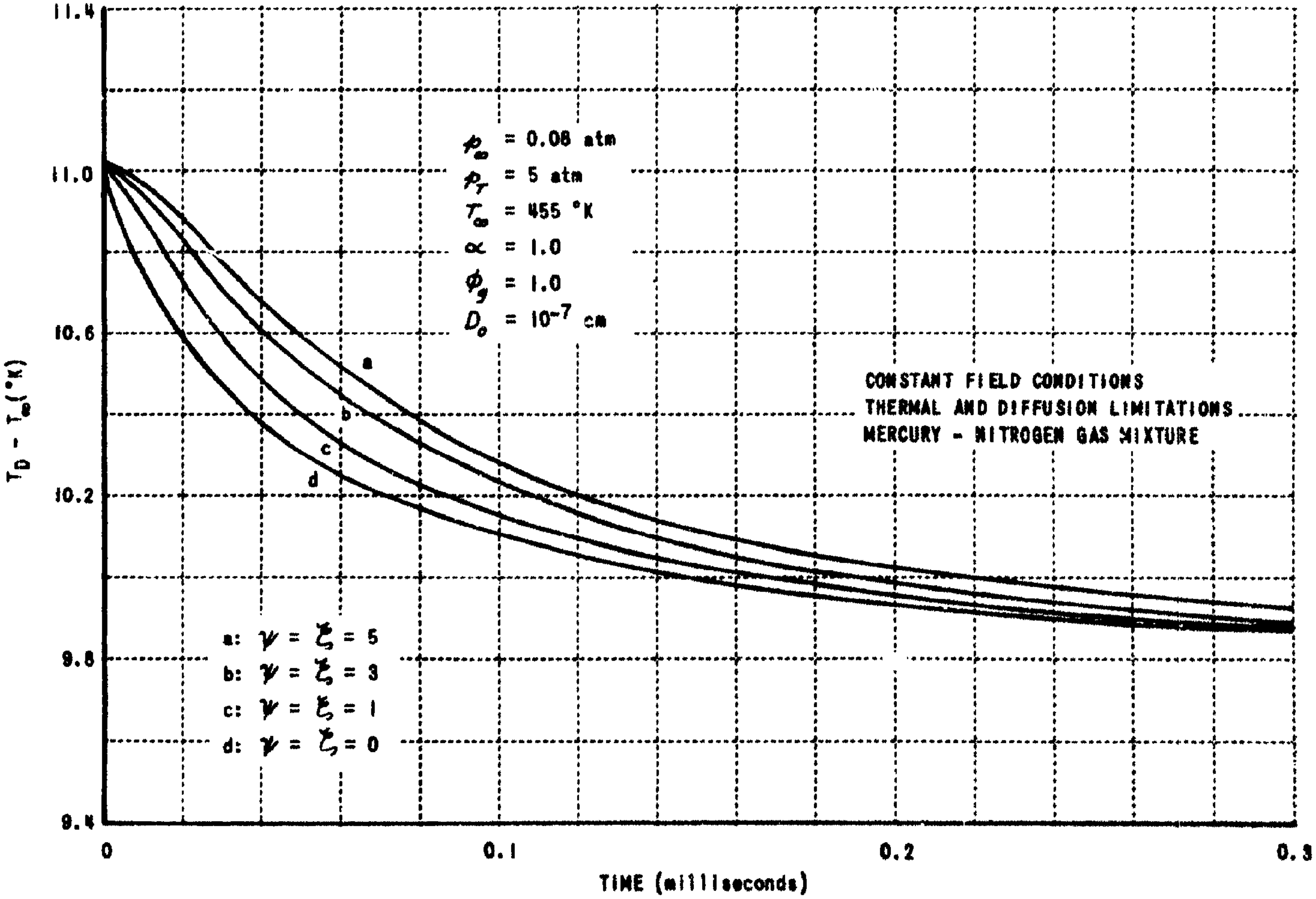

Figure 7B EFFECT OF THE VARIATIONS OF SLIP-REGIME PARAMETERS ON DROPLET TEMPERATURE

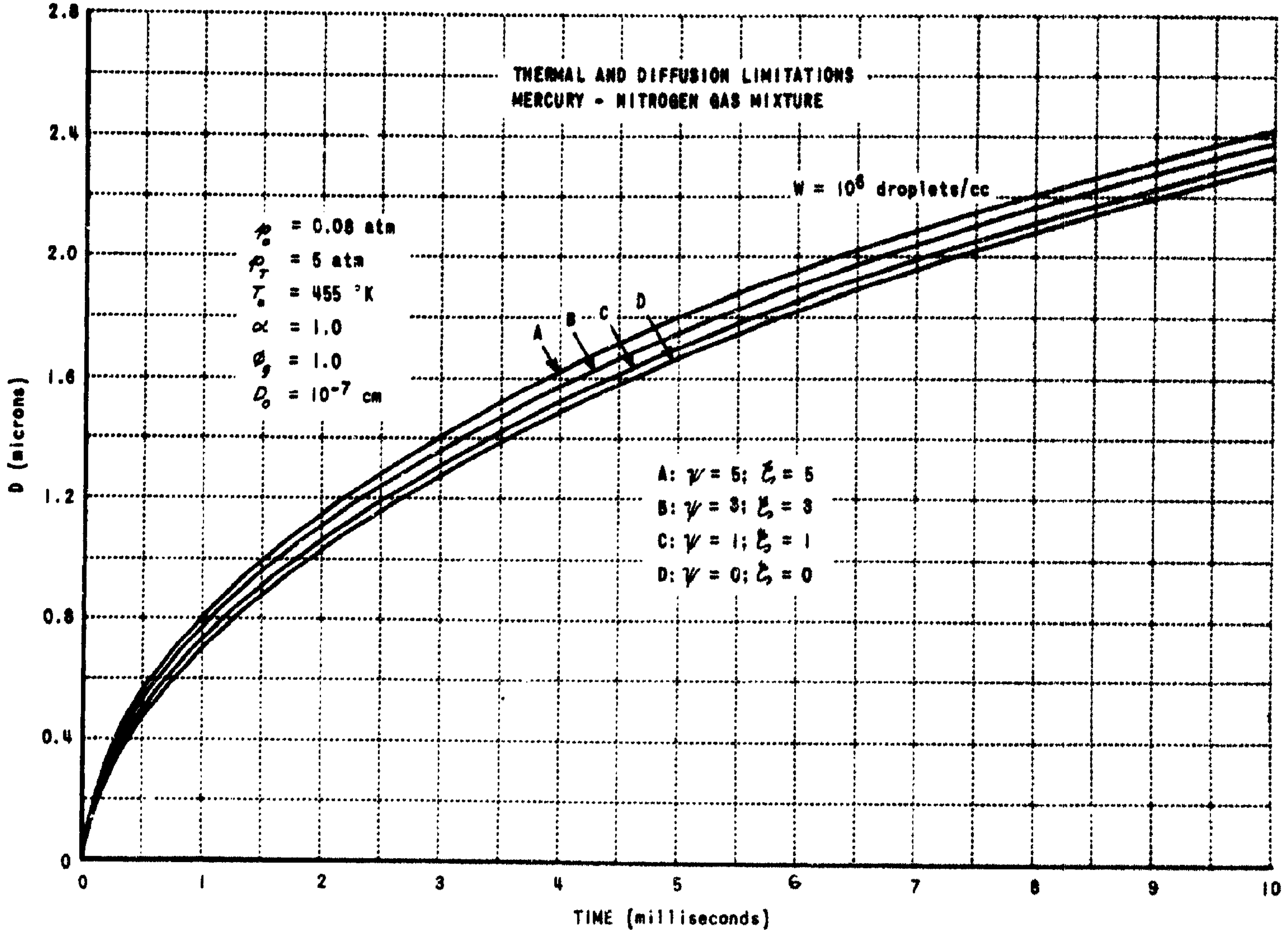

Figure 8 DROPLET GROWTH UNDER CHANGING FLUID CONDITIONS FOR VARIOUS VALUES OF SLIP-REGIME PARAMETERS

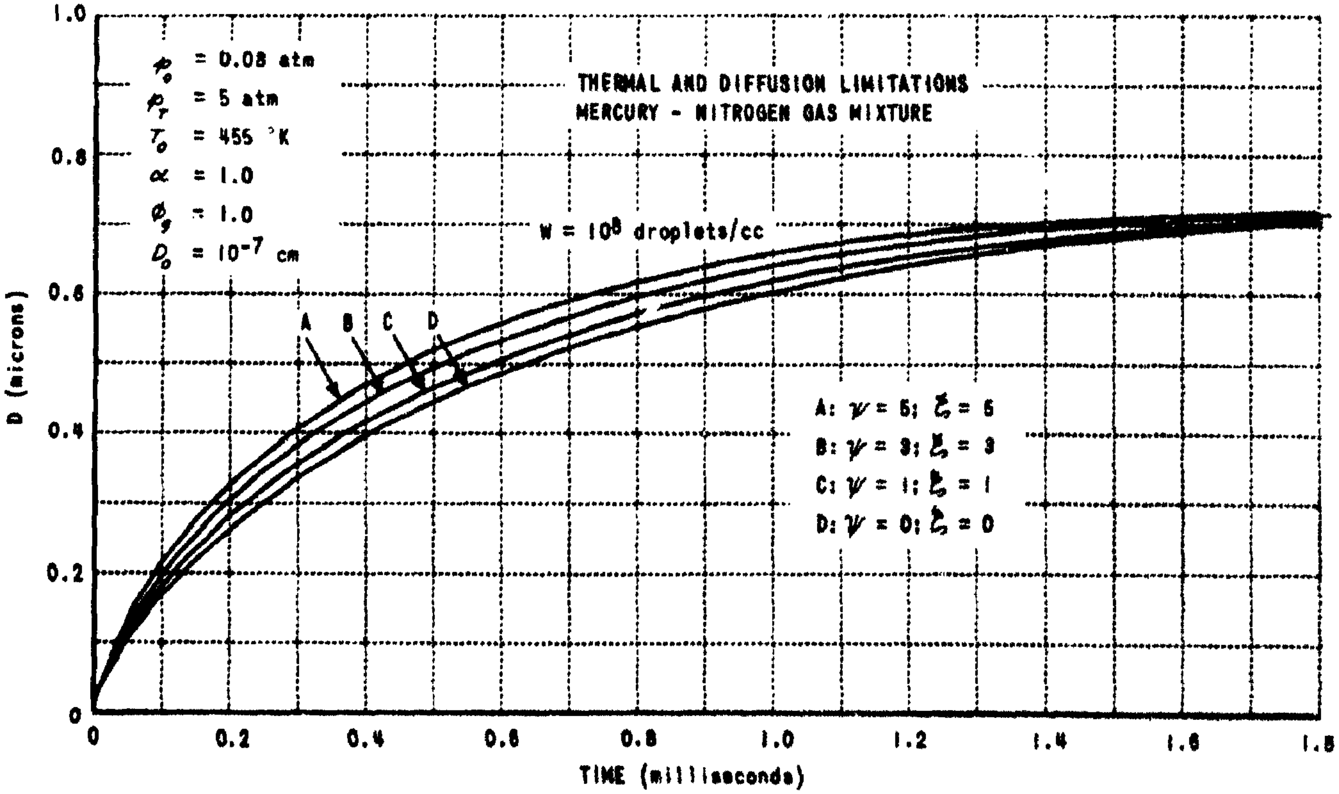

Figure 9 DROPLET GROWTH UNDER CHANGING FLUID CONDITIONS FOR VARIOUS VALUES OF SLIP-REGIME PARAMETERS

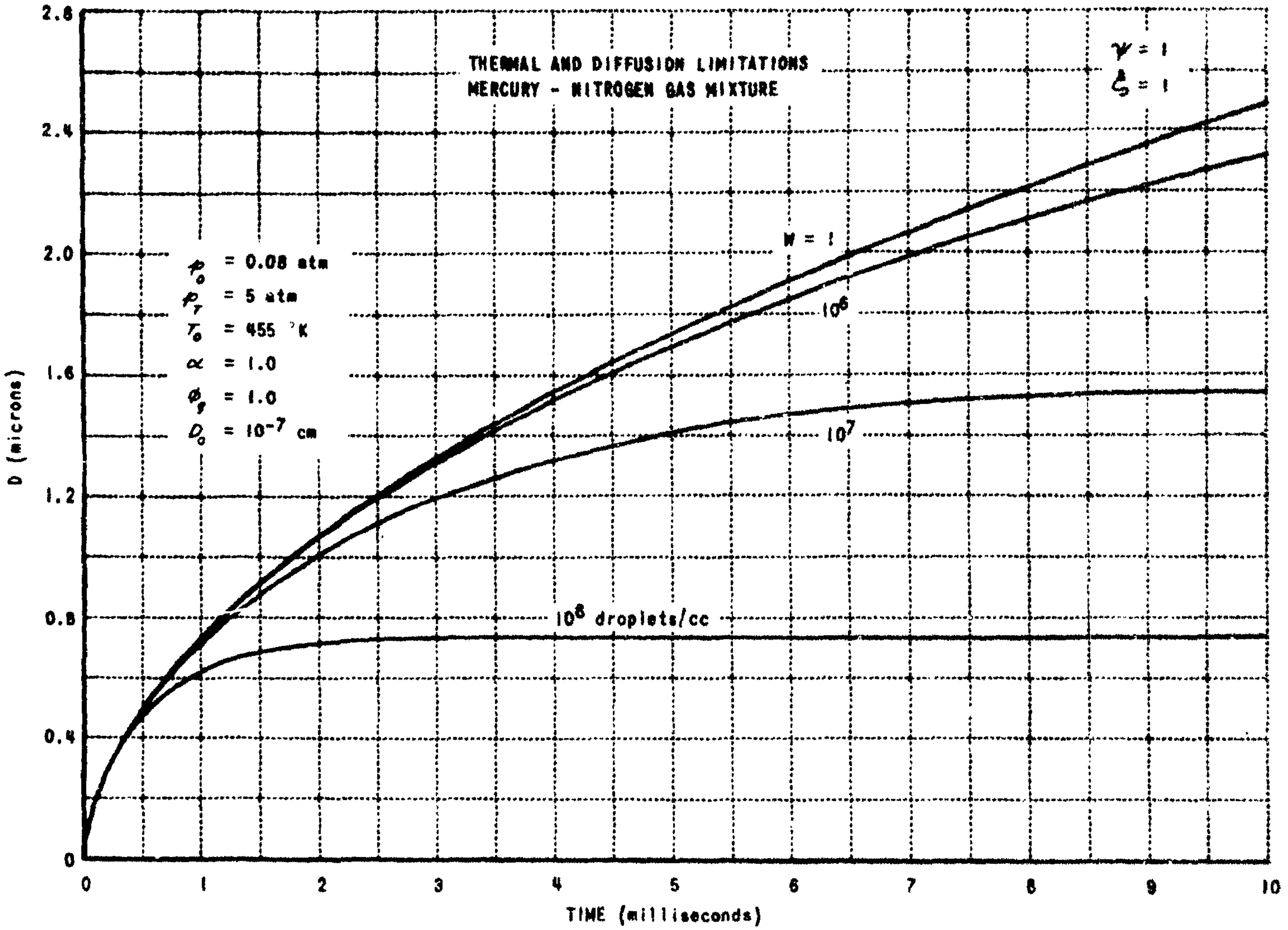

Figure 10 DROPLET GROWTH UNDER CHANGING FLUID CONDITIONS FOR VARIOUS DROPLET CONCENTRATIONS

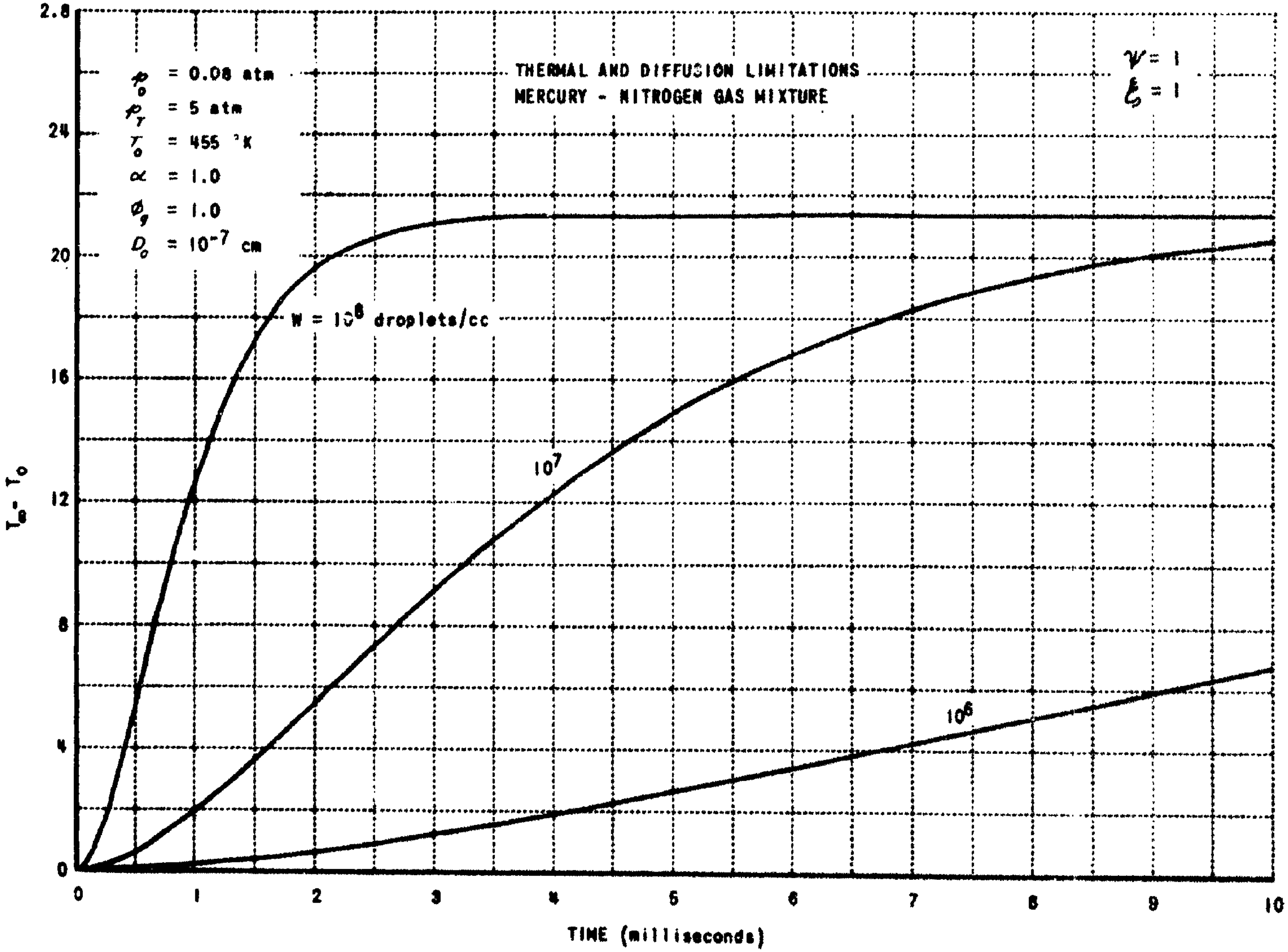

Figure 11 FLUID-TEMPERATURE INCREASE UNDER CHANGING FLUID CONDITIONS FOR VARIOUS DROPLET CONCENTRATIONS

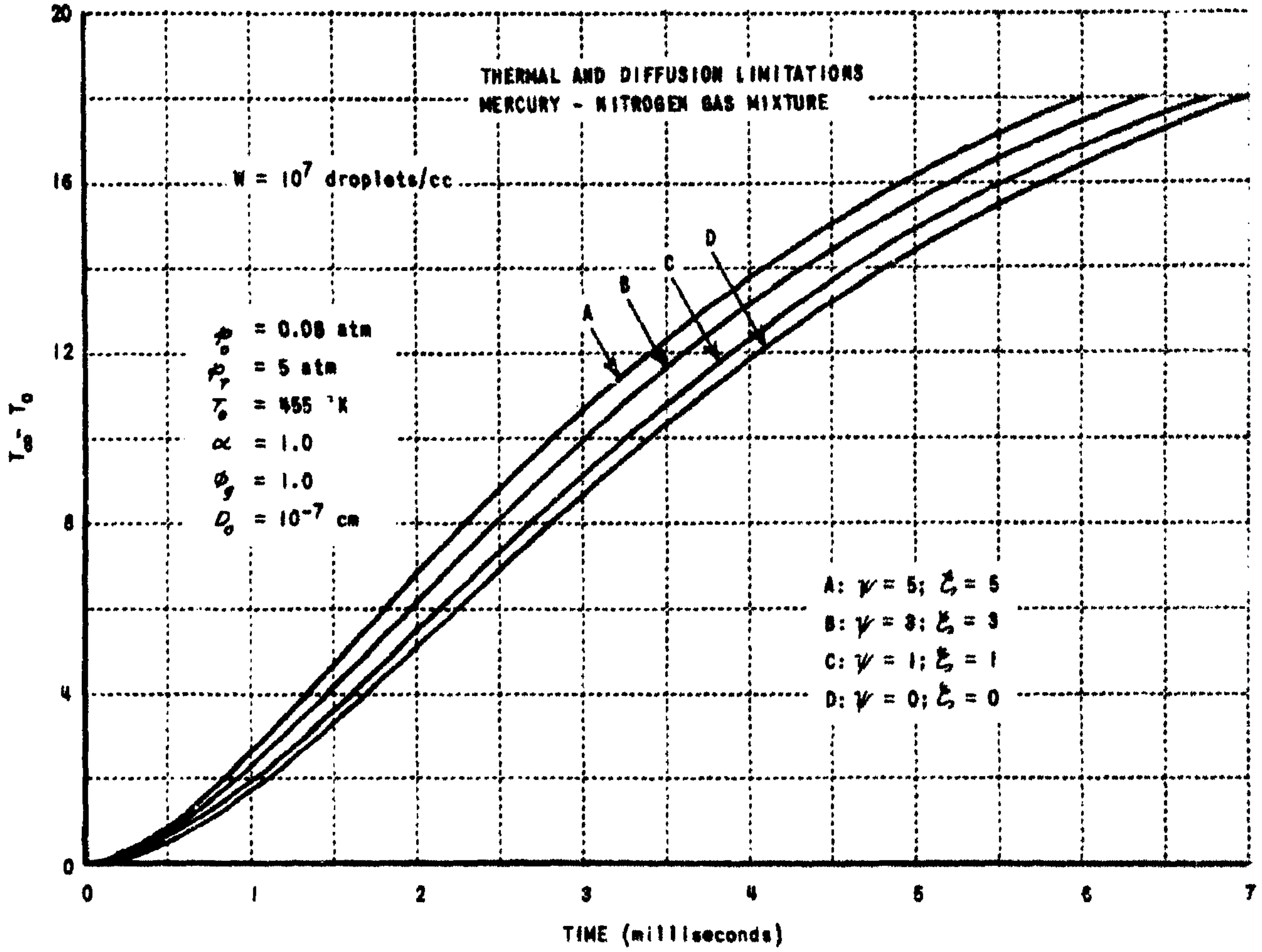

Figure 12 FLUID-TEMPERATURE INCREASE FOR VARIOUS VALUES OF SLIP-REGIME PARAMETERS UNDER CHANGING FLUID CONDITIONS

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