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RESEARCH TRANSLATION

Concerning the Origin of a Thin, Elevated, Nocturnal Fog Layer

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Translation of

K voprosu o zarozhdenii tonkogo sloia pripodniatogo nochnogo tumana

by

L. Z. Prokh and L. M. Roev

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The heat balance equations of a turbid isothermal layer of air warmer than the ground provided a criterion for calculating the rate of temperature change within this layer under specified conditions.

Mesoaerological measurements [2,10] show that in certain weather conditions the atmospheric surface boundary layer is essentially stratified. Thin layers with temperatures lower than those of the adjacent layers have been detected against a general background of monotonic change of air temperature with height [5,6,9]. This was observed, for example, during the evolution of certain types of fog, where the stratification is related to a vertical optical inhomogeneity (i.e., when smoke, dust, or droplets of thick haze or fog are present).

Theoretical studies [4] have shown that in some cases the air mass in question must contain strata with considerably different humidity, if radiation processes are to play an important role in the production of a layer containing products of atmospheric condensation.

In view of these facts and the theoretical assumptions, it was felt necessary to make rough calculations of the heat balance conditions for a turbid, isothermal layer of air situated between layers of different temperature. For example, the applicability of radiation nomograms to this problem is examined in [11], and terms of the heat balance equation for a turbid layer of air are estimated in [8] on the basis of a series of hypothetical limitations.

This paper will consider the special case of a thin, isothermal layer of turbid air $z_1 \le z \le z_2$ with temperature T_2 above a colder, ground layer $z_0 \le z \le z_1$ with temperature T_1 and below a warmer layer

$z_2 \leq z \leq z_3$ with temperature T_3. The coordinates are: z_0 = ground, z_1 = base of turbid layer, z_2 = top of turbid layer, and z_3 = layer of air above the turbid layer. Hence, the surface layer is divided into three parts, hereafter identified as α = 1, 2, and 3.

A similar model using a slightly different approach to the problem is examined in [8] and [11].

The general system of equations, by which the nocturnal temperature variation is determined, can be written as follows [3] for the model used

$$\left.\begin{aligned}
c_p\rho_a \frac{\partial T_\alpha}{\partial t} &= \frac{\partial}{\partial z} K \frac{\partial T_\alpha}{\partial z} + \sum_j k_{j,\alpha}\rho_\alpha (U_{j,\alpha} + G_{j,\alpha} - 2p_j B_\alpha) - L\frac{dm}{dt}, \\
\frac{\partial U_{j,\alpha}}{\partial z} &= k_{j,\alpha}\rho_\alpha (p_j B_\alpha - U_{j,\alpha}), \\
\frac{\partial G_{j,\alpha}}{\partial z} &= k_{j,\alpha}\rho_\alpha (G_{j,\alpha} - p_j B_\alpha), \\
\alpha &= 1;\ 2;\ 3.
\end{aligned}\right\} \quad (1)$$

The boundary and initial conditions are:

$$\left.\begin{aligned}
U_{j,1}(z_0) &= p_j B_1, \\
U_{j,2}(z_1) &= (1-\beta)\, U_{j,1}(z_1), \\
G_{j,1}(z_1) &= G_{j,2}(z_1) + \beta U_{j,1}(z_1), \\
G_{j,2}(z_2) &= (1-\beta)\, G_{j,3}(z_2), \\
U_{j,3}(z_2) &= U_{j,2}(z_2) + \beta G_{j,3}(z_2), \\
G_{j,3}(z_3) &= p_j B_3, \\
T_1\big|_{t=0} = T_{1(0)};\ T_2\big|_{t=0} &= T_{2(0)};\ T_3\big|_{t=0} = T_{3(0)}.
\end{aligned}\right\} \quad (2)$$

The following symbols are used in these equations: t is time; c_p and ρ_a are the specific heat and density of the air; K is the coefficient of turbulence, ρ is the density of the absorbant; k_j is the coefficient of absorption of radiation by an absorbant of density ρ in the j-th spectral interval; $U_{j,\alpha}$ and $G_{j,\alpha}$ are the upward and downward fluxes of long-wave radiation in the j-th spectral interval; L is the heat of condensation; dm/dt is the mass of water condensing per unit time; β is the coefficient of reflection from the turbid layer; $B = \sigma T^4$, where σ is the Stefan-Boltzmann constant; and p_j is the fraction of the radiant flux in the j-th interval of the spectrum.

The system of equations (1) with boundary conditions (2) cannot be solved in final form, therefore, it is of interest to simplify the problem. The simplification amounts to estimation of the sign of the term $\partial T_2 / \partial t$ (the rate of nocturnal temperature fluctuation in the turbid layer z ... z_2) and to calculation of the parameters that determine this sign. The water vapor in the turbid layer is assumed to be saturated, hence fog will begin to form in this turbid layer when $\partial T_2 / \partial t < 0$. Similar assumptions are made in [8] and [11].

Let us consider calm conditions, in which 1) there is no eddy flux, 2) the term containing $\frac{\partial}{\partial z} K \frac{\partial T}{\partial z}$ can be dropped from eq. (1), and 3) molecular heat exchange between the layers can be disregarded.

It is evident that

$$\rho_s(T+\Delta T)-\rho_s(T)=dm,$$

ρ_s is the saturated vapor density and ΔT is the cooling required to condense a mass of water dm.

Assuming that the temperature change ΔT is small compared with the temperature itself, we get

$$\frac{dm}{dt}=\frac{\partial \rho_s}{\partial T}\Big|_{T_{2(0)}}\frac{\partial T}{\partial t}=\rho_s'(T_{2(0)})\frac{\partial T}{\partial t}. \quad (3)$$

Then, the system of equations (1) is written

$$\left.\begin{aligned} (c_p\rho_a+L\rho_s'(T_0))\frac{\partial T_a}{\partial t} &= \sum_j k_{j,a}\rho_a(U_{j,a}+G_{j,a}-2p_jB_a),\\ \frac{\partial U_{j,a}}{\partial z} &= k_{j,a}\rho_a(p_jB_a-U_{j,a}),\\ \frac{\partial G_{j,a}}{\partial z} &= k_{j,a}\rho_a(G_{j,a}-p_jB_a),\\ a &= 1;\ 2;\ 3. \end{aligned}\right\} \quad (4)$$

The boundary conditions (2) remain unchanged.

The solutions to the second and third equations of (4) are of the form

$$\left.\begin{aligned} G_{j,\alpha}(z) &= e^{k_{j,\alpha}\rho_\alpha(z-z_\alpha)}\left\{A_{j,\alpha} + p_j B_\alpha\left\{e^{-k_{j,\alpha}\rho_\alpha(z-z_\alpha)} - 1\right\}\right\}, \\ U_{j,\alpha}(z) &= e^{-k_{j,\alpha}\rho_\alpha(z-z_{\alpha-1})}\left\{C_{j,\alpha} + p_j B_\alpha\left\{e^{k_{j,\alpha}\rho_\alpha(z-z_{\alpha-1})} - 1\right\}\right\} \end{aligned}\right\} \quad (5)$$

where $A_{j,\alpha}$ and $C_{j,\alpha}$ are integration constants which must be found from the boundary conditions (2). $A_{j,2}$ and $C_{j,2}$ must be determined in order to find $\partial T_2/\partial t$.

Substituting (5) into (2) and solving the resulting system of equations with respect to $A_{j,\alpha}$ and $C_{j,\alpha}$, we find the following expressions for $A_{j,2}$ and $C_{j,2}$

$$\left.\begin{aligned} A_{j,2} &= p_j B_3 (1-\beta), \\ C_{j,2} &= p_j B_1 (1-\beta). \end{aligned}\right\} \quad (6)$$

It is clear from the first equation of (4) that the sign of $\partial T_2/\partial t$ is determined by the term

$$\sum_j k_{j,2}\rho_2 (U_{j,2} + G_{j,2} - 2p_j B_2).$$

Considering that at night the maximum in the radiation spectrum appears in the range of wavelengths 7 to 15 μ, and assuming that the absorption coefficient in the turbid layer in this part of the spectrum is constant, we may replace the sum in this expression by a single term. We can assume that

$$k_{j,2}\rho_2 (z_2 - z_1) \ll 1. \quad (7)$$

Actually, $k_{j,2} \sim 1.5 \times 10^{3}$ cm^2/g [1] and $\rho \sim 10^{-9}$ g/cm^3 for a substance that is becoming turbid; then the following is required to satisfy (7)

$$z_2 - z_1 \ll 1 \text{ км.} \quad (7')$$

Condition (7') is always fulfilled rigorously enough; therefore, we can limit ourselves to the first terms of the series in (5) when expanding in series. In this approximation, we obtain

$$\left.\begin{aligned} G_{j,2} &\approx (1-\beta) p_j B_3, \\ U_{j,2} &\approx (1-\beta) p_j B_1. \end{aligned}\right\} \quad (8)$$

from (5), with consideration of (6). Using (8), we get the following estimation of the sum in the right-hand side of the first equation of (4)

$$I = \sum_j k_{j,2}\rho_2(U_{j,2} + G_{j,2} - 2p_j B_2) \approx$$
$$\approx p_j\,[(1-\beta)(B_1 + B_3) - 2B_2].$$

As has been stated, the sign of $\partial T_2/\partial t$ is the same as that of I. For $I < 0$, β must satisfy the inequality

$$\beta > 1 - \frac{2B_2}{B_1 + B_3} = 1 - \frac{2T_2^4}{T_1^4 + T_3^4}. \qquad (9)$$

If the size of the particles causing turbidity in the layer $\alpha = 2$ is less than 1μ, the Rayleigh theory can be used in calculating β. Then

$$\beta = \frac{8}{3}\pi r^2\left(\frac{2\pi r}{\lambda}\right)^4\left(\frac{n^2-1}{n^2+2}\right)^2 Nh, \qquad (10)$$

where r is the radius of the particles causing turbidity (assuming that this turbidity is monodisperse); N is their concentration; n is the refractive index; h is the height of the turbid layer; and λ is the wavelength.

Substituting (10) into (9), we will find the limitation placed on N, h, and r in order to arrive at $\partial T_2/\partial t < 0$:

$$\frac{8}{3}\pi r^2\left(\frac{2\pi r}{\lambda}\right)^4\left(\frac{n^2-1}{n^2+2}\right)^2 Nh > 1 - \frac{2T_2^4}{T_1^4 + T_3^4}. \qquad (11)$$

From the first equation of (4), we get the expression

$$\frac{\partial T_2}{\partial t} = \frac{\sum_j k_{j,2}\rho_2(U_{j,2} + G_{j,2} - 2p_j B_2)}{c_p\rho_g + L\rho_s'(T_{2(0)})}. \qquad (12)$$

where $U_{j,2}$ and $G_{j,2}$ are defined by eqs. (5) and (6). In the same approximation, we will get the equation

$$w = \rho_s'(T_{2(0)})\frac{\sum_j k_{j,2}\rho_2(U_{j,2} + G_{j,2} - 2p_j B_2)}{c_p\rho_g + L\rho_s'(T_{2(0)})}\,t.$$

for the water content of a nascent fog with a relative humidity of 100% and a temperature around $0^{\circ}C$. The reflection coefficient is not constant for $\partial T_s / \partial t < 0$. In this case, fog forms, leading to an increase of β, thus the process of cooling of the turbid layer and fog formation will reinforce itself. A similar process is described in [7].

Therefore, at night, in the absence of advection and with reduced eddy flux, the condensation in a relatively thin layer of the atmosphere may be attributed to the turbidizing particles in that layer. The particles act as cooling floats and make the top of the layer radiationally active [7] with respect to long-wave radiation.

The occurrence of a turbid layer in a surface inversion, given high relative humidity, plays an important part in creating conditions favorable for the development of an elevated nocturnal fog. It has been demonstrated experimentally [6,9] in a number of cases that even a brief increase of radiation in thin turbid layers can have local and explosive effects, propagating the cooling to adjacent layers of air and leading to fog formation.

Our estimation of the rate of temperature fluctuation gives an idea of the direction and speed of the process taking place under the given conditions in a thin turbid layer within a surface inversion.

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