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LINCOLN LABORATORY

TRAJECTORY EQUATIONS OF MOTION IN RADAR POLAR COORDINATES

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ABSTRACT

The equations of motion of a body under the influence of the earth's gravity and atmospheric drag are obtained in radar polar coordinates. The choice of this coordinate system may have important advantages in various tracking and filtering algorithms. A rotating ellipsoidal earth with gravitational terms up through the second (easily extendable) harmonic is assumed. The effects of a non-spherical earth along with the corresponding effects of the higher gravitational harmonics can be important in the estimation of ballistic coefficients at very low deceleration levels.

Accepted for the Air Force
Franklin C. Hudson
Chief, Lincoln Laboratory Office

GLOSSARY

$\vec{r}$	vector from earth center to target
$\vec{R}$	earth radius vector to "foot" of radar site
$\vec{S}$	vector from radar to target
S, ε, α	radar range, elevation, and azimuth, respectively
$\hat{s}, \hat{\varepsilon}, \hat{\alpha}$	unit orthogonal vectors in the directions of increasing S, ε, and α, respectively
X, Y, Z	rectangular coordinates with origin at radar and X East, Y North, and Z vertically up
$\hat{i}, \hat{j}, \hat{k}$	unit vectors along X, Y, Z, respectively
$\hat{\xi}$	unit vector along earth axis ξ, pointing toward North Pole
$\hat{r}$	unit vector from earth center to target
ω	earth rotation rate
$\vec{\omega}$	earth rotation vector ($\vec{\omega} = \omega \hat{\xi}$)
μ	geodetic latitude of radar
μ_c	geocentric latitude of radar
φ	geocentric latitude of target
R_{φ}	earth radius at latitude φ
a	equatorial radius of earth (6, 378. 160 km)
H	height of radar above ellipsoidal earth (i.e., above MSL)
I_{ξ}	moment of inertia of earth about its axis ξ
I_e	moment of inertia of earth about an axis in equatorial plane
GM	product of gravitational constant and earth mass (3.986032×10^5 km^3/sec^2)
$\vec{A}_g$	gravitational acceleration vector at the target position

$\vec{V}$	drag velocity vector (i.e., relative to air)
$\vec{A}_o$	inertial acceleration vector
$\vec{A}_d$	drag acceleration vector
h	height of target above ellipsoidal earth (i.e., above MSL)
ρ	atmospheric mass density at target position ($\rho = \rho(h)$)
g	acceleration of gravity at target position ($g = \lvert \vec{A}_g \rvert$)
m	mass of target
β	ballistic coefficient $= m/C_D A$ (i.e., mass-to-drag ratio)
e	eccentricity of reference ellipsoidal earth ($e^2 = .0066945$)
J	dimensionless constant $= 3(I_g - I_e)/(2Ma^2) = 1.624 \times 10^{-3}$

Trajectory Equations of Motion in Radar Polar Coordinates

INTRODUCTION

Trajectory equations of motion in radar rectangular coordinates (the X, Y, Z system below) are readily available in the literature. They are not to the author's knowledge available in radar polar coordinates (Range S, Elevation ϵ, and Azimuth α). Perhaps because of this (and possibly because of the fear that such equations would be too complex anyway), it has been common practice in problems of radar tracking to do the estimation and prediction in radar rectangular coordinates even though the data are gathered and the radar is directed in radar polar coordinates. This procedure involves wasteful coordinate transformations in at least one direction which can be avoided completely if radar polar coordinates are used throughout. These coordinate transformations are particularly complex if range rate ($\dot{S}$) and range acceleration ($\ddot{S}$) are used. On the other hand, the equations of motion in radar polar coordinates, derived in this report, turn out to be only moderately more complex than in radar rectangular coordinates. Quantitative comparison of the computational economy of using one coordinate system or another will depend on the details of the specific application. For example, if the data rate is high and large step sizes can be tolerated in the integration of the equations of motion, it may be more economical to use radar polar coordinates throughout, while in the reverse situation, the use of radar rectangular coordinates may be advantageous. Additionally, the following important advantages of radar polar coordinates in connection with track initiation in Kalman filtering have been brought out in discussions with H. M. Jones and R. P. Wishner of Lincoln Laboratory:

1. The covariance matrix of the estimation parameters in the Kalman filter is more nearly diagonal in radar polar coordinates and thereby less susceptible to the effects of ill-conditioning with short lengths of track
2. The usual linearity assumptions of Kalman filters are more nearly correct in radar polar coordinates. This can result in better track initiation since initial estimates can be less accurate and still yield a nearly optimum correction with later data.

DERIVATION

Assuming that only gravity and drag forces act on the body of mass, m , we have for the equation of motion

$$\vec{F} = \vec{F}_d + \vec{F}_g$$

or

$$m\vec{A}_o = m\vec{A}_d + m\vec{A}_g \quad . \tag{1}$$

The drag force is taken to be

$$\vec{F}_d = m\vec{A}_d = -\frac{m\rho V}{2\beta}\vec{V} \quad , \tag{2}$$

where $\beta = m/C_D A$ is taken as the definition of ballistic coefficient* and ρ is the atmospheric mass density at the body position. Using Eq. (2) in Eq. (1) and dividing by m gives for the equation of motion

$$\vec{A}_o = -\frac{\rho V}{2\beta}\vec{V} + \vec{A}_g \quad . \tag{3}$$

$\vec{V}$ is the target velocity relative to the moving radar site (or equivalently, relative to the surrounding air, assuming the atmosphere to rotate rigidly with the earth and no local winds) and can be immediately expressed in the $\hat{s}$, $\hat{\epsilon}$, $\hat{\alpha}$ system as

$$\vec{V} = V_s\hat{s} + V_\epsilon\hat{\epsilon} + V_\alpha\hat{\alpha} \quad , \tag{4}$$

where

$$V_s = \dot{S}$$
$$V_\epsilon = S\dot{\epsilon}$$
$$V_\alpha = S\dot{\alpha}\cos\epsilon \quad .$$

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* This definition avoids the needless confusion which the definition $\beta = mg/C_D A$ introduces as to whether to evaluate g at sea level or at the target.

$\overline{A}_g$ is the gravitational acceleration vector and, following McCuskey,* can be expressed as

$$\overline{A}_g = g_r \hat{r} + g_\xi \hat{\xi} \quad , \qquad (5)$$

where

$$g_r = -\frac{GM}{r^2}\left[1 + J\left(\frac{a}{r}\right)^2 (1 - 5\sin^2\varphi)\right]$$

$$g_\xi = -\frac{2GM}{r^2} J\left(\frac{a}{r}\right)^2 \sin\varphi \quad .$$

Its magnitude is given by

$$g = |\overline{A}_g| = [\overline{A}_g \cdot \overline{A}_g]^{1/2} = [g_r{}^2 + g_\xi{}^2 + 2g_r\, g_\xi \sin\varphi]^{1/2} \quad .$$

In order to express Eq. (5) in the $\hat{s}$, $\hat{\epsilon}$, $\hat{\eta}$ system, it is necessary to express $\hat{r}$ and $\hat{\xi}$ in that system. This is outlined in the next paragraph.

The geocentric target position vector $\overline{r}$ is given by (see Fig. 1 and glossary)

$$\overline{r} = \overline{R} + H\hat{k} + \overline{S} \quad .$$

Since

$$\overline{S} = X\hat{i} + Y\hat{j} + Z\hat{k}$$

and

$$\overline{R} = -R\sin(\mu - \mu_c)\,\hat{j} + R\cos(\mu - \mu_c)\,\hat{k} \quad ,$$

we have

$$\overline{r} = [X]\,\hat{i} + [Y - RQ_2]\,\hat{j} + [Z + RQ_1]\,\hat{k}$$

where

$$Q_1 = \cos(\mu - \mu_c) + H/R$$

$$Q_2 = \sin(\mu - \mu_c)$$

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* S. W. McCuskey, "Introduction to Celestial Mechanics," pp. 163-166 (1962).

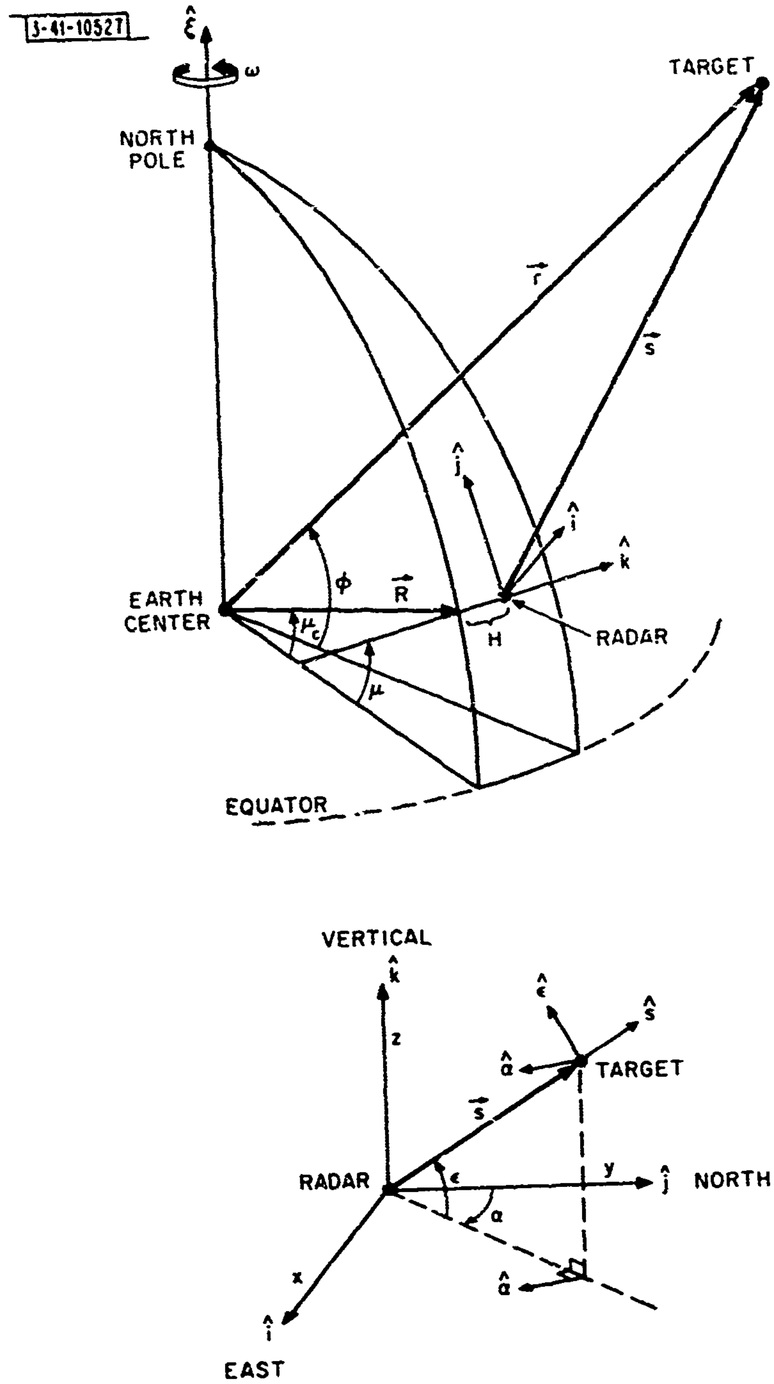
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Fig. 1

$$R = \left[\frac{a^2(1 - e^2)}{1 - e^2\cos^2\mu}\right]^{1/2}$$

$$\mu_c = \tan^{-1}[(1 - e^2)\tan\mu] \quad .$$

Using the transformation (12), we obtain $\vec{r}$ in the $\hat{s}$, $\hat{\epsilon}$, $\hat{a}$ system as

$$\vec{r} = r_s\hat{s} + r_\epsilon\hat{\epsilon} + r_a\hat{a} \tag{6}$$

where

$$r_s = S + Q_1 R \sin\epsilon - Q_2 R \cos\epsilon \cos a$$

$$r_\epsilon = Q_1 R \cos\epsilon + Q_2 R \sin\epsilon \cos a$$

$$r_a = Q_2 R \sin a \quad .$$

Its magnitude is given by

$$r = |\vec{r}| = R\{(S/R)^2 + 2(S/R)(Q_1 \sin\epsilon - Q_2 \cos\epsilon \cos a) + Q_1^2 + Q_2^2\}^{1/2} \quad . \tag{7}$$

The unit earth rotation vector $\hat{\xi}$ can be expressed as

$$\hat{\xi} = \cos\mu\hat{j} + \sin\mu\hat{k} \quad .$$

Again, utilizing the transformation (12), this can be expressed in the $\hat{s}$, $\hat{\epsilon}$, $\hat{a}$ system as

$$\hat{\xi} = \xi_s\hat{s} + \xi_\epsilon\hat{\epsilon} + \xi_a\hat{a} \quad , \tag{8}$$

where

$$\xi_s = \cos\mu \cos\epsilon \cos a + \sin\mu \sin\epsilon$$

$$\xi_\epsilon = -\cos\mu \sin\epsilon \cos a + \sin\mu \cos\epsilon$$

$$\xi_a = -\cos\mu \sin a \quad .$$

The required quantity $\sin\varphi$ is then simply obtained as

$$\sin\varphi = \frac{\vec{r}\cdot\hat{\xi}}{r} = \frac{1}{r}\{S\cos\epsilon\cos\alpha\cos\mu + (H + S\sin\epsilon)\sin\mu + R\sin\mu_c\} \quad . \tag{9}$$

Using Eqs. (6) and (8) in (5) yields $\vec{A}_g$ in the $\hat{s}$, $\hat{\epsilon}$, $\hat{\alpha}$ system as

$$\vec{A}_g = \left(\frac{g_r r_s}{r} + g_\xi \xi_s\right)\hat{s} + \left(\frac{g_r r_\epsilon}{r} + g_\xi \xi_\epsilon\right)\hat{\epsilon} + \left(\frac{g_r r_\alpha}{r} + g_\xi \xi_\alpha\right)\hat{\alpha} \quad . \tag{10}$$

Equations (7) and (9) are used to evaluate g_r and g_ξ. All that remains is to express $\vec{A}_o$ in the $\hat{s}$, $\hat{\epsilon}$, $\hat{\alpha}$ system which follows.

$\vec{A}_o$, the earth-centered inertial acceleration of an object observed from a site on a uniformly rotating earth, can be expressed as*

$$\vec{A}_o = (\dot{V}_s\hat{s} + \dot{V}_\epsilon\hat{\epsilon} + \dot{V}_\alpha\hat{\alpha} + V_s\dot{\hat{s}} + V_\epsilon\dot{\hat{\epsilon}} + V_\alpha\dot{\hat{\alpha}}) + 2\vec{\omega}\times\vec{V} + \vec{\omega}\times(\vec{\omega}\times\vec{r}) \quad , \tag{11}$$

where the term in parentheses $(\dot{\vec{V}})$ is the apparent (i.e., relative to the moving radar site) acceleration of the body.

The cartesian and polar unit vectors are related according to the transformation

$$\begin{bmatrix}\hat{s}\\ \hat{\epsilon}\\ \hat{\alpha}\end{bmatrix} = M \begin{bmatrix}\hat{i}\\ \hat{j}\\ \hat{k}\end{bmatrix} \quad \text{or} \quad \begin{bmatrix}\hat{i}\\ \hat{j}\\ \hat{k}\end{bmatrix} = M^{-1} \begin{bmatrix}\hat{s}\\ \hat{\epsilon}\\ \hat{\alpha}\end{bmatrix} \tag{12}$$

where

$$M = \begin{bmatrix}\cos\epsilon\sin\alpha & \cos\epsilon\cos\alpha & \sin\epsilon\\ -\sin\epsilon\sin\alpha & -\sin\epsilon\cos\alpha & \cos\epsilon\\ \cos\alpha & -\sin\alpha & 0\end{bmatrix} \tag{13}$$

and $M^{-1} = M^T$ (i.e., transpose of M). The unit vectors $\hat{s}$, $\hat{\epsilon}$, $\hat{\alpha}$ are related to their time rates as

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* L. Page, "Introduction to Theoretical Physics," 3rd Ed., 4th Printing, p. 121 (1959).

$$\begin{bmatrix} \dot{\hat{s}} \\ \dot{\hat{\epsilon}} \\ \dot{\hat{\alpha}} \end{bmatrix} = \begin{bmatrix} 0 & \dot{\epsilon} & +\dot{\alpha}\cos\epsilon \\ -\dot{\epsilon} & 0 & -\dot{\alpha}\sin\epsilon \\ -\dot{\alpha}\cos\epsilon & \dot{\alpha}\sin\epsilon & 0 \end{bmatrix} \begin{bmatrix} \hat{s} \\ \hat{\epsilon} \\ \hat{\alpha} \end{bmatrix} \tag{14}$$

The relation (14) is easily verified by differentiating Eq. (12) with $\dot{\hat{i}} = \dot{\hat{j}} = \dot{\hat{k}} = 0$ and solving for $\dot{\hat{s}}$, $\dot{\hat{\epsilon}}$ $\dot{\hat{\alpha}}$. It can also be verified directly by referring to Fig. 1 and using relations of the type

$$\dot{\hat{s}} = \frac{d\hat{s}}{dt} = \frac{\partial\hat{s}}{\partial\epsilon}\frac{\partial\epsilon}{\partial t} + \frac{\partial\hat{s}}{\partial\alpha}\frac{\partial\alpha}{\partial t} \quad , \quad \frac{\partial\hat{s}}{\partial\epsilon} = \hat{\epsilon} \quad , \text{ and } \frac{\partial\hat{s}}{\partial\alpha} = \cos\epsilon\,\hat{\alpha} \quad .$$

Forming the dot product of Eq. (11) with $\hat{s}$, $\hat{\epsilon}$, $\hat{\alpha}$, successively, yields

$$\begin{aligned} \vec{A}_o \cdot \hat{s} &= [\dot{V}_s + (V_\epsilon \dot{\hat{\epsilon}} + V_\alpha \dot{\hat{\alpha}}) \cdot \hat{S}] + 2(\vec{\omega} \times \vec{V}) \cdot \hat{s} + [\vec{\omega} \times (\vec{\omega} \times \vec{r})] \cdot \hat{s} \\ \vec{A}_o \cdot \hat{\epsilon} &= [\dot{V}_\epsilon + (V_s \dot{\hat{s}} + V_\alpha \dot{\hat{\alpha}}) \cdot \hat{\epsilon}] + 2(\vec{\omega} \times \vec{V}) \cdot \hat{\epsilon} + [\vec{\omega} \times (\vec{\omega} \times \vec{r})] \cdot \hat{\epsilon} \\ \vec{A}_o \cdot \hat{\alpha} &= [\dot{V}_\alpha + (V_s \dot{\hat{s}} + V_\epsilon \dot{\hat{\epsilon}}) \cdot \hat{\alpha}] + 2(\vec{\omega} \times \vec{V}) \cdot \hat{\alpha} + [\vec{\omega} \times (\vec{\omega} \times \vec{r})] \cdot \hat{\alpha} \quad . \end{aligned} \tag{15}$$

Using $\vec{\omega} = \omega\hat{\xi}$ and Eqs. (4), (6), (8), (9), and (14)in (15), and reducing to scalars yields

$$\begin{aligned} \vec{A}_o \cdot \hat{s} &= [\ddot{S} - S(\dot{\epsilon}^2 + \dot{\alpha}^2\cos^2\epsilon)] + 2\omega[\xi_\epsilon V_\alpha - \xi_\alpha V_\epsilon] + \omega^2[\xi_s r \sin\varphi - r_s] \\ \vec{A}_o \cdot \hat{\epsilon} &= [S\ddot{\epsilon} + 2\dot{S}\dot{\epsilon} + S\dot{\alpha}^2 \sin\epsilon\cos\epsilon] + 2\omega[\xi_\alpha V_s - \xi_s V_\alpha] + \omega^2[\xi_\epsilon r \sin\varphi - r_\epsilon] \\ \vec{A}_o \cdot \hat{\alpha} &= [S\ddot{\alpha}\cos\epsilon + 2(\dot{S}\dot{\alpha}\cos\epsilon - S\dot{\alpha}\dot{\epsilon}\sin\epsilon)] + 2\omega[\xi_s V_\epsilon - \xi_\epsilon V_s] + \omega^2[\xi_\alpha r \sin\varphi - r_\alpha] \quad . \end{aligned} \tag{16}$$

Noting that

$$\vec{A}_o = (\vec{A}_o \cdot \hat{s})\,\hat{s} + (\vec{A}_o \cdot \hat{\epsilon})\,\hat{\epsilon} + (\vec{A}_o \cdot \hat{\alpha})\,\hat{\alpha} \quad , \tag{17}$$

we finally have Eq. (3) expressed completely in the $\hat{s}$, $\hat{\epsilon}$, $\hat{\alpha}$ system by using Eqs. (4), (10), and (17). Expressing Eq. (3) in scalar form, we obtain the set of three differential equations of motion in radar polar coordinates. The result is

$$\ddot{S} = S(\dot{\epsilon}^2 + \dot{\alpha}^2\cos^2\epsilon) - 2\omega(\xi_\epsilon V_\alpha - \xi_\alpha V_\epsilon) - \omega^2(\xi_s r \sin\varphi - r_s) - (\frac{\rho V}{2\beta})V_s + \frac{g_r r_s}{r} + g_\xi \xi_s$$

$$\ddot{\epsilon} = \frac{-1}{S}\{2\dot{S}\dot{\epsilon} + S\dot{\alpha}^2 \sin\epsilon\cos\epsilon + 2\omega(\xi_\alpha V_s - \xi_s V_\alpha) + \omega^2(\xi_\epsilon r \sin\varphi - r_\epsilon) + (\frac{\rho V}{2\beta})V_\epsilon - \frac{g_r r_\epsilon}{r} - g_\xi \xi_\epsilon\}$$

$$\ddot{\alpha} = \frac{-1}{S\cos\epsilon}\{2(\dot{S}\dot{\alpha}\cos\epsilon - S\dot{\alpha}\dot{\epsilon}\sin\epsilon) + 2\omega(\xi_s V_\epsilon - \xi_\epsilon V_s) + \omega^2(\xi_\alpha r\sin\varphi - r_\alpha) + (\frac{\rho V}{2\beta})V_\alpha - \frac{g_r r_\alpha}{r} - g_\xi \xi_\alpha\} \quad , \qquad (18)$$

where

$$\xi_s = \cos\mu\cos\epsilon\cos\alpha + \sin\mu\sin\epsilon$$

$$\xi_\epsilon = -\cos\mu\sin\epsilon\cos\alpha + \sin\mu\cos\epsilon$$

$$\xi_\alpha = -\cos\mu\sin\alpha$$

$$V_s = \dot{S},\ V_\epsilon = S\dot{\epsilon},\ V_\alpha = S\dot{\alpha}\cos\epsilon$$

$$V = \{V_s{}^2 + V_\epsilon{}^2 + V_\alpha{}^2\}^{1/2}$$

$$r_s = S + Q_1 R\sin\epsilon - Q_2 R\cos\epsilon\cos\alpha$$

$$r_\epsilon = Q_1 R\cos\epsilon + Q_2 R\sin\epsilon\cos\alpha$$

$$r_\alpha = Q_2 R\sin\alpha$$

$$Q_1 = \cos(\mu - \mu_c) + H/R$$

$$Q_2 = \sin(\mu - \mu_c)$$

$$r = R\{(S/R)^2 + 2(S/R)(Q_1\sin\epsilon - Q_2\cos\epsilon\cos\alpha) + Q_1{}^2 + Q_2{}^2\}^{1/2}$$

$$g_r = \frac{-GM}{r^2}\{1 + J(\frac{a}{r})^2(1 - 5\sin^2\varphi)\}$$

$$g_\xi = \frac{-2GM}{r^2} J(\frac{a}{r})^2\sin\varphi$$

$$\sin\varphi = \frac{1}{r}\{S\cos\epsilon\cos\alpha\cos\mu + (H + S\sin\epsilon)\sin\mu + R\sin\mu_c\} \quad .$$

The height h required in $\rho = \rho(h)$ is obtained as the distance between the target and the point of intersection of the reference ellipsoid and the line passing through the target and normal to the reference ellipsoid. An excellent approximation to this is

$$h \approx r - R_{\varphi} = r - \left[\frac{a^2(1 - e^2)}{1 - e^2\cos^2\varphi}\right]^{1/2} .$$

This is the distance between the target and the intersection of $\vec{r}$ with the reference ellipsoid.

These equations are only moderately more complex than the equations of motion in radar rectangular coordinates. Only the first two terms in each equation are unique to the polar coordinate system. Coriolis acceleration terms (containing ω), centrifugal acceleration terms (containing ω^2), drag terms (containing $\rho/2\beta$) and gravitational terms (containing g_r and g_φ) appear in both sets of equations.

The relation (3), or equivalently the relations (18), can be used to obtain various expressions which yield estimates of β from radar data. Also, it is easy to specialize to the case of a spherical earth with $1/r^2$ gravity by simply setting $J = e = 0$ and $\mu = \mu_c$. It is then useful to observe what kind of errors can result in β due to earth model choice. Computer runs based on actual radar data have indicated the following. The ballistic coefficient, β, estimated by using the simple model ($J = e = 0$), can differ from that estimated by using the more accurate model (J, e as reported here) by more than a factor of two when the drag acceleration A_{ω} is $g \times 10^{-3}$. The radar data used were at the mid-latitudes where these differences are probably greatest. A good conservative rule would be to require use of an "improved" model when attempting to estimate ballistic coefficients at drag levels in the vicinity of one-thousandth of a g or less. An "improved" model means one at least as good as that reported here (i.e., ellipsoidal shape with gravitational terms through the second harmonics).

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13. ABSTRACT

The equations of motion of a body under the influence of the earth's gravity and atmospheric drag are obtained in radar polar coordinates. The choice of this coordinate system may have important advantages in various tracking and filtering algorithms. A rotating ellipsoidal earth with gravitational terms up through the second (easily extendable) harmonic is assumed. The effects of a non-spherical earth along with the corresponding effects of the higher gravitational harmonics can be important in the estimation of ballistic coefficients at very low deceleration levels.

14. KEY WORDS

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