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FOREWORD

This report was prepared by the Plastics and Composites Branch, Nonmetallic Materials Division and was initiated under Project No. 7340, "Nonmetallic and Composite Materials," Task No. 734003 "Structural Plastics and Composites." The work was administered under the direction of the Air Force Materials Laboratory, Research and Technology Division, with S. W. Tsai as the project engineer.

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This technical report has been reviewed and is approved.

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ABSTRACT

This report presents a method for determining shear modulus and shear strength of unidirectional fiber-reinforced composites. The method incorporates data from uniaxial tensile and/or compressive tests at fiber orientations of 0, 45, and 90 degrees. The approach presented requires fewer specimens for shear strength determinations than current existing procedures.

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INTRODUCTION

This report covers the experimental determination of shear modulus G and shear strength S of unidirectional composites by using specimens with fiber orientations of 0, 45, and 90 degrees. The elastic and strength properties of unidirectional composites as a function of fiber orientation are known to follow certain mathematical relations. The values of G and S which represent two of the intrinsic properties can be extracted from the uniaxial tensile and/or compressive properties of the 0-, 45-, and 90-degree specimens if the data are interpreted in accordance with the theories of anisotropic elasticity and strength.

Shear properties of unidirectional composites are independent material properties. Their numerical values are closely related to the shear properties of the matrix and the strength of the interfacial bond. Based on available information derived from micromechanics analysis, the measurement of the shear and transverse properties may provide a better assessment of the contribution of the matrix and the interfacial bond to the composite than that of the axial properties.

There are a number of existing methods for the determination of shear properties. The twisting of a circular tube is probably the best method because a state of pure shear is imposed. This test method, however, requires a large amount of material and a knowledge of the techniques of filament winding. In addition, a torsion testing machine is required. This is not available in most testing laboratories.

The plate twisting method, as reported recently by Hennessey et al. (Reference 1), and Tsai (Reference 2), is relatively easy to perform if plate-form composites can be made. A special loading fixture is required. The test can be performed on standard tensile testing machines. This test method is effective for the determination of shear modulus. Shear strength, on the other hand, is difficult to obtain from this method because the ultimate load required may cause excessive indentation or local failure near the loading points.

Another test method for the shear properties is the panel shear or picture-frame test (Reference 3). This method requires a complicated fixture. The actual boundary conditions at the clamped edges and the friction in the fixture are difficult to assess. Both will have strong influence on the interpretation of data.

A test method similar to that proposed here has been in use for some time by the Forest Products Laboratory (Reference 3). A new feature is the comparison of various existing strength theories and a resulting procedure that requires fewer specimens (no 0-degree specimen) for the shear strength determination. The test specimens are made from composite materials in the form of plates. It is recommended that, wherever possible, the twisting test of square plates be performed for the determination of the shear modulus before individual tensile or compressive specimens are cut from the plate specimen.

SHEAR MODULUS

The transformation equation for the uniaxial stiffness of an orthotropic material is as follows (Reference 4):

$$\frac{1}{E_\theta} = \frac{m^4}{E_{11}} + \left(\frac{1}{G} - \frac{2\nu_{12}}{E_{11}} \right) m^2 n^2 + \frac{n^4}{E_{22}} \tag{1}$$

where E_θ = transformed uniaxial stiffness, E_{11} = axial stiffness, E_{22} = transverse stiffness, G = shear modulus, ν_{12} = major Poisson's ratio, m = cos θ, n = sin θ, θ = fiber orientation. This equation can be derived from the well-known transformation equation of a fourth-rank tensor. The principal elastic moduli for an orthotropic plate-form material are E_{11}, E_{22}, ν_{12}, and G.

From an 0-degree specimen, E_{11} and ν_{12} can be readily determined by means of two-element strain rosettes. From a 90-degree specimen, E_{22} and ν_{21} (minor Poisson's ratio) can also be determined from two-element strain rosettes. The major and minor Poisson's ratios are known to satisfy a reciprocal relationship, that their ratio is equal to E_{11}/E_{22}.

From a 45-degree specimen, the transformed stiffness E_{45} can be measured either by means of a strain gage or extensometer. For this fiber orientation one can rearrange Equation 1 as follows:

$$\frac{1}{G} = \frac{4}{E_{45}} - \frac{1-\nu_{12}}{E_{11}} - \frac{1-\nu_{21}}{E_{22}} \tag{2}$$

The shear modulus G can be computed from this equation if the E_{45} (measured with θ = 45°) and E_{11}, E_{22}, and ν_{12} are known.

Since major Poisson's ratio has an approximate numerical value of 0.3, one can set up convenient tables and diagrams showing the dimensionless shear modulus as a function of the dimensionless stiffness at θ = 45 degrees, i.e., E_{45}/E_{22} for various ratios of E_{11}/E_{22}. The numerical results are plotted in Figure 1 which gives an approximate numerical value for the shear modulus by knowing the uniaxial stiffness derived from a 45-degree specimen.

SHEAR STRENGTH

The determination of shear strength S of a unidirectional composite depends on the applicable strength theory. Unlike the elastic moduli which are governed by the transformation of a fourth-rank tensor, there is no unique strength theory. For orthotropic materials, which include unidirectional composites, various strength theories have been proposed in recent years by Norris, Hill, Werren and Norris, Marin, Jackson and Cratchley, and Cooper (References 5, 6, 3, 7, 8, and 9, respectively). A comparison of the first four references was made by Tsai (Reference 10) and the last two references may be described as maximum stress theories.

The theory presented by Norris (Reference 5) covers an interesting historical account of works dating back to the 1920's, including those produced by the predecessor of the Air Force

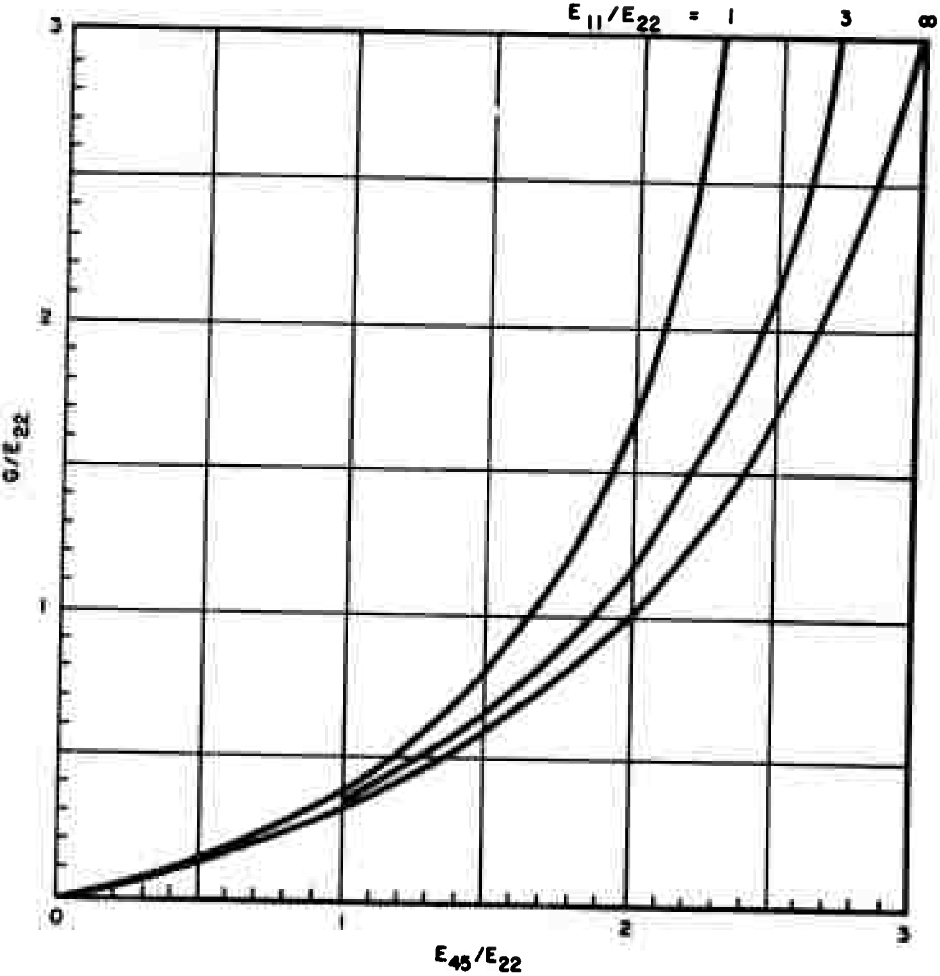

Figure 1. Dimensionless E_{45} Versus Shear Modulus G

Materials Laboratory--the Materials Section of the Air Service. Norris' strength theory, in case of plane stress, can be expressed by

$$\frac{1}{\sigma_\theta^2} = \frac{m^4}{X^2} + \left(\frac{1}{S^2} - \frac{1}{XY}\right) m^2 n^2 + \frac{n^4}{Y^2} \tag{3}$$

where σ_θ = uniaxial strength of a specimen with θ fiber orientation, X = axial or longitudinal strength, Y = transverse strength, and S = longitudinal shear strength. This equation has been referred to as the "interaction formula" because of the 1/(XY) term. The theory of Reference 3, which does not contain the interaction term is:

$$\frac{1}{\sigma_\theta^2} = \frac{m^4}{X^2} + \frac{m^2 n^2}{S^2} + \frac{n^4}{Y^2} \tag{4}$$

Hill (Reference 6) postulated an orthotropic strength criterion which in the case of plane stress becomes:

$$\frac{1}{\sigma_\theta^2} = \frac{m^4}{X^2} + \left(\frac{1}{S^2} - \frac{1}{X^2}\right) m^2 n^2 + \frac{n^4}{Y^2} \tag{5}$$

For an isotropic material, obeying the von Mises yield condition

$$X = Y = \sqrt{3}\, S \tag{6}$$

Substitute this condition into Equations 3, 4, and 5, and σ_θ becomes invariant (constant strength) from Equations 3 and 5, but not from Equation 4 unless $X = Y = S/\sqrt{2}$. For this reason, Equation 4 may not be valid because it does not reduce to the isotropic case under normal conditions, i.e., Equation 6. The derivation of Equations 3, and 5 (References 5 and 6) contain the von Mises criterion as a special case.

The axial and transverse strength can be measured from 0- and 90-degree specimens. This is done by substituting θ = 0 and 90 degrees, respectively, in Equations 3, 4, and 5. Both tensile and compressive properties can be obtained by imposing a tensile or a compressive load, respectively. For θ = 45 degrees, Equations 3, 4, and 5 become, after some rearrangement, respectively:

$$\frac{1}{S^2} = \frac{4}{\sigma_{45}^2} - \frac{1}{Y^2} + \frac{1}{X}\left(\frac{1}{Y} - \frac{1}{X}\right) \tag{7}$$

$$\frac{1}{S^2} = \frac{4}{\sigma_{45}^2} - \frac{1}{Y^2} - \frac{1}{X^2} \tag{8}$$

$$\frac{1}{S^2} = \frac{4}{\sigma_{45}^2} - \frac{1}{Y^2} \tag{9}$$

Shear strength S can be deduced from measuring X, Y, and σ_{45}, except in the last equation where X is not needed. When X is very large in comparison with Y, all of these equations reduce to essentially the same relation. This is the case with glass and boron composites, where X/Y is approximately 40. When X = Y, Equations 7 and 9 become the same, but Equation 8 is different unless $\sigma_{45} << Y$. A comparison of the three equations is shown in Figure 2.

Equation 7 is shown with X/Y = 2. This ratio represents the maximum deviation from Equation 9. If the ratio is either greater or smaller than 2, Equation 7 approaches 9 very rapidly.

Equation 8 is shown in Figure 2 with X/Y = 1. This ratio again represents the maximum deviation from Equation 9. As X/Y becomes larger than 1 (it is assumed that X/Y is always equal or greater than 1), Equation 8 will approach 9.

Also shown in Figure 2 is the maximum stress theory, which is used in References 8 and 9. This theory can be derived by assuming that the maximum uniaxial strength is governed by one of the following conditions, whichever yields the lowest value:

$$\begin{aligned} \sigma_\theta &= X/m^2 \\ \sigma_\theta &= S/mn \\ \sigma_\theta &= Y/n^2 \end{aligned} \tag{10}$$

Assuming that the shear strength S is lower than X and Y, a shear failure will be the governing mechanism when $\theta = 45°$. We can then measure the shear strength by imposing a tensile or compressive load on a 45-degree specimen. From Equation 10,

$$S = \sigma_{45}/2 \tag{11}$$

According to this strength theory (the maximum stress theory), the uniaxial tensile and compressive strengths of a 45-degree specimen, σ_{45}^{+} and σ_{45}^{-}, are identical. This strength theory is also plotted in Figure 2.

When σ_{45}/Y is less than 1, all four strength theories (Equations 7, 8, 9, and 11) approximately agree with one another. According to available data (Reference 10), the distortional work criterion is apparently better than that of maximum stress. For the materials tested (glass and boron composites) all three "work" theories (Equations 7, 8, and 9) predict essentially the same results because X/Y is large. Available data do not seem to substantiate or repudiate any one of the work theories, except they all are better than the maximum stress theory. One may, however, disregard Equation 8 on the basis that it does not satisfy isotropy in the limit, and is not associated with distortional work. Equation 7 is also on a less than rigorous basis, according to not only the author himself (Reference 5) but also the comparison made in Reference 10.

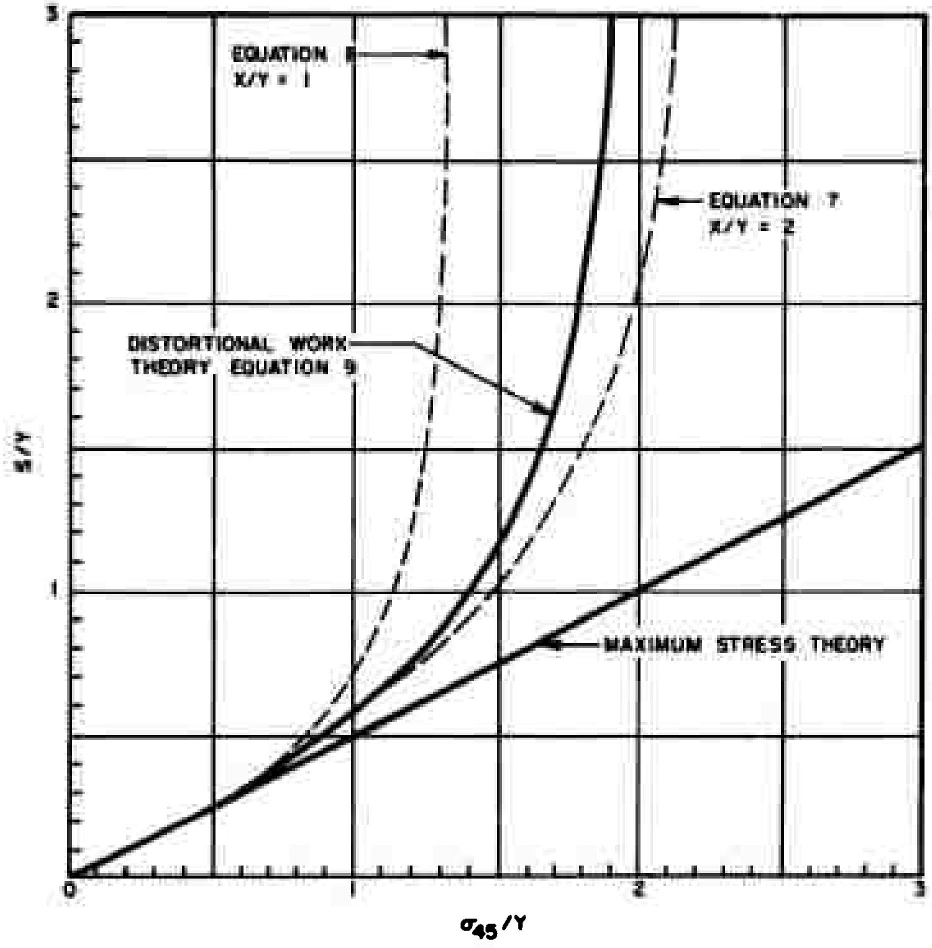

Figure 2. Dimensionless σ_{45} Versus Shear Strength S

CONCLUSIONS

The proposed test method for the determination of shear modulus and shear strength using 0-, 45-, and 90-degree specimens is easy to perform, and requires simple test shapes (dogbone specimens for tension tests and columns for compression tests). All of the intrinsic materials properties can be determined by this method without resorting, for example, to the torsion tube specimens, which may be difficult to fabricate in the case of boron or metallic composites. The proposed test method is also recommended when the availability of constituent materials is limited.

The mathematical equations for the elastic moduli, as shown in Equations 1 and 2, are exact within the realm of linear elasticity theory. The variation of uniaxial strength as a function of fiber orientation depends on the particular strength criteria. Equations 3, 4, and 5 are not as exact, in the mathematical sense, as the case of the elastic moduli. Judging from available experimental evidence, Hill's distortional work criterion (Reference 6) is preferred.

The specimen dimensions should have a gage length at least two or three times larger than the lateral dimensions. Whether the cross-sectional shape of the gage length is circular or rectangular is of no great consequence insofar as the basic theory outlined in this paper is concerned. Although only two-element strain rosettes are needed for the 0- and 90-degree specimens, and one for the 45-degree specimen, it is a recommended practice that three-element rosettes be used for all specimens. Duplicate rosettes should also be used whenever possible.

Because of possible uncertainties of the strength criteria, it is recommended that both tensile and compressive data be obtained from 0-, 45-, and 90-degree specimens. The strength values to be used in Equation 9, for example, should be those obtained from tensile tests (σ_{45}^{+} and Y^{+}), or compressive tests (σ_{45}^{-}, Y^{-}). The shear strength extracted from the tensile tests should, in theory, be the same as that from the compressive tests. Thus, by obtaining S from both tensile and compressive tests, the range of variation of S, if any, may be determined. If the maximum stress theory is applicable, the tensile and compressive strengths of 45-degree specimens would be equal. This will also be true according to Equation 9, if $Y^{+} = Y^{-}$.

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