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RESEARCH ON ANALYSIS OF VARIANCE AND DATA INTERPRETATION

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FOREWORD

This interim technical report was prepared on Contract AF 33(615)-1737 between Iowa State University of Science and Technology and Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force. It summarizes the research accomplished under the direction of Professors Oscar Kempthorne and George Zyskind, principal investigators, during the eighteen-month period July 1964 through December 1965. The contract has been extended through December 1967 at which time a final report is scheduled.

The work performed under contract was initiated and coordinated by Mary D. Lum, Research Mathematical Statistician, Applied Mathematics Research Laboratory, Aerospace Research Laboratories, and supported by funds for Project 7071, Research in Applied Mathematics, Work Unit 7071-00-10, Analysis of Variance and Probability.

ABSTRACT

Research on Analysis of Variance and Data Interpretation is described. Section I discusses estimation problems in variance component and mixed model problems. Section II considers the combination of information on estimable functions from distinct uncorrelated sources and justifies some of the common applications in experimental design problems. Section III discusses size and power under experiment randomization of several competitive tests for the paired design and presents conclusions about the high relative merits of the variance ratio randomization and the Wilcoxon tests. Section IV discusses the development of high speed computational methods for the calculation of fourth degree generalized polykays of variances and covariances of estimated variance components for balanced samples from balanced populations. Section V summarizes briefly papers on the design of experiments and multivariate responses in experiments and the 1965 Fisher Memorial lecture on experimental inference.

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INTRODUCTION

The research described in this report deals with aspects of linear model methodology and with a search for greater understanding of consequences of sampling and randomization in experiments. The present report summarizes briefly work performed on the contract and dealt with in detail in separate reports and papers now in final stages of preparation. The separate but related accounts deal with the following general topics:

I. Unbiased Estimation in Variance Component Models

II. Simple Linear Combinability of Information from Independent Sources

III. Size and Power of Certain Tests under Experiment Randomization

IV. Computation of Variances of Estimated Variance Components in Finite Balanced Population Structures

V. General Related and Broader Matters

The ensuing sections delineate briefly the main results and general viewpoints arrived at in investigating the above problems.

AUTHORSHIP OF THE REPORT

The introduction was written by O. Kempthorne and G. Zyskind. Section I is based on work of R. P. Basson with advice of G. Zyskind. Section II is by F. Martin and G. Zyskind. Section III is by T. E. Doerfler and O. Kempthorne. Section IV is by E. J. Carney with advice from O. Kempthorne. Section V is by O. Kempthorne.

I. UNBIASED ESTIMATION IN VARIANCE COMPONENT MODELS

With regard to variance component models we have considered the problem of minimum variance (M. V.) unbiased estimation of regression parameters and variance components in the mixed model

$$y = \sum_{i=0}^{r} X_i \gamma_i + \sum_{i=r+1}^{k+1} X_i \beta_i$$

where γ_i's are fixed effects, β_i's are random effects with distributional properties to be further specified, and X_i's are known fixed matrices whose elements are not necessarily restricted to be 0's or 1's. We assume throughout that $X_{k+1} = I$, $E(\beta_i \beta_j') = 0 \quad (i \neq j)$, and $E(\beta_{k+1} \beta_{k+1}') = I\sigma^2_{k+1}$. A model representation is defined to be balanced$_2$ if $X_i X_i' X_j X_j' = X_j X_j' X_i X_i' \quad (i \neq j, \ i, j = 0, \ldots, k+1)$. A representation that is not balanced$_2$ is unbalanced.

Completeness of the sufficient set of statistics is established by a restriction on the number of roots of $V = E(yy') - E(y)E(y')$. Several theorems, on the minimum variance properties under normality of Model I type A. o. V. estimators for variance components, and simple least squares estimators of estimable functions of regression parameters for balanced$_2$ mixed models, are proved. Certain optimality properties for the same estimators, when the normality assumption is replaced by a less stringent condition, are obtained.

Two results for the model

$$y = j_n\mu + \sum_{i=1}^{k+1} X_i\beta_i = \sum_{i=0}^{k+1} X_i\beta_i$$

where $E(\beta_i\beta_i') = I\sigma_i^2$ which are due to Graybill and Hultquist (1961), and which we have refined are:

(1) If (a) all σ_i^2 are estimable (b) $X_iX_i'X_jX_j' = X_jX_j'X_iX_i'$ $(i,j = 0,\ldots,k+1)$ and (c) the random β_i vectors are normally distributed then there is a complete sufficient statistic for the parameters $(\mu, \sigma_1^2,\ldots,\sigma_{k+1}^2)$ if, and only if, $W = \sum_{i=1}^{k+1} X_iX_i'\sigma_i^2 + X_0X_0'\mu^2$ has $k+2$ distinct latent roots. The set of complete sufficient statistics consists of $\bar{y}$ and $y'P_i'P_iy$ $(i = 1,\ldots,k+1)$ where P_i's are collections of vectors of P, an orthogonal matrix such that $PWP' = \Delta$ (diagonal), and where all vectors of P_i (say) correspond to the same latent root of W.

We define the class of situations of type for which commutativity of $X_iX_i'X_jX_j'$ $(i,j = 0,\ldots,k+1)$ holds and W has $k+2$ distinct roots to be the class P.

(2) If β_i and β_j are independent for all i and j $(i \neq j)$ and finite fourth moments exist for all random variables, and within every given vector β_i, all fourth moments are equal, and all third moments are equal, then the same estimators, i.e., the usual Model I. A.o.V. mean square estimators for the $\delta_i = E(y'P_i'P_iy)$, that are M.V. unbiased under normality, are best quadratic unbiased (b.q.u.) estimators under present assumptions.

We have obtained results analogous to 1, and under slightly more extended restrictions results analogous to 2 above, for the completely random model under the assumption that $E(\beta_i \beta_i') = (a_i \setminus b_i)$ $(i = 1, \ldots, k)$ where $(a_i \setminus b_i)$ is a matrix with a_i on the diagonal and b_i off it. The same estimators as before are complete sufficient for $(\mu, a_1 - b_1, \ldots, a_k - b_k, \sigma^2_{k+1})$.

For the mixed model, under the assumptions (a) normality of β_i vectors (b) $E(\beta_i \beta_i') = I\sigma_i^2$ $(i = r+1, \ldots, k+1)$ (c) $X_i X_i' X_j X_j' = X_j X_j' X_i X_i'$ $(i, j = 0, \ldots, k+1)$ (d) the matrix $\overline{W} = X_0 X_0' \mu^2 + \sum_{i=1}^{k+1} X_i X_i' \sigma_i^2 = J\mu^2 + V$, where V is the variance matrix of y in the corresponding completely random case, has $k+2$ distinct roots and (e) $P_i X_j$ $(i \neq 0) \neq (j \neq k+1) = 0$, where the P_i's are as defined previously, we have shown that the sufficient statistic $(X\hat{\gamma}_{LS}, s^2_{r+1}, \ldots, s^2_{k+1})$ for the parameters $(X\gamma, \sigma^2_{r+1}, \ldots, \sigma^2_{k+1})$ is complete. We have also given the counterpart of 2 above for the mixed model, namely best linear unbiased (b.l.u.) estimators for estimable functions of regression parameters and b.q.u. estimators for variance components. We have also presented analogous results under slightly more extended restrictions for a mixed model with $E(\beta_i \beta_i') = (a_i \setminus b_i)$ $(i = r+1, \ldots, k)$.

The class of model situations with $E(\beta_i \beta_i') = I\sigma_i^2$ and for which for at least some i, j $(i \neq j)$, $X_i X_i' X_j X_j' \neq X_j X_j' X_i X_i'$ or the number of roots of

W (or $\overline{W}$) exceeds k+2 we designate as the class S-P. In the class S-P, a class containing many design situations, some common and others less so, the condition of balance 2 is often not satisfied and in all of the examples that we have thus far examined, even if normality of β_i's is assumed, the minimal sufficient set of statistics is not complete. It is not known whether U.M.V. estimators exist in these cases, and if they do, how to proceed to obtain them. Since here the assumption of normality cannot apparently be profitably used, and later removed, we favor obtaining alternative estimators directly, and comparing them at different points of the parameter space by means of the variances of each variance component estimator.

We have given consideration to the simple "least squares" method of estimation in unbalanced cases. We present a transformation procedure, which is actually a single degree of freedom breakdown of sums of squares, and which in random models provides one means of finding variances of variance component estimators. The procedure suggests theoretically, at least, an alternative way of weighting single degree of freedom sums of squares to find estimators with smaller variance than those given by simple least squares.

We have attacked the problem of the variance of a quadratic form, and the covariance between two forms that arise in mixed and random models. We have found considerable simplifications in the case of a usual least squares method of estimation, also known as Method 3 of Henderson (1953), and we have found a further simplification under the assumption

of normality of random effects. We have applied the general results derived to obtain variance formulae for various sums of squares which have been suggested for finding estimators of variance components in random models with added concomitants.

II. SIMPLE LINEAR COMBINABILITY OF INFORMATION FROM INDEPENDENT SOURCES

The issue of combining information from independent sources has long been of general interest to experimenters and statisticians alike. A common procedure has been to combine estimates of scalar parameters by weighting inversely as the variances. This procedure is not generally best for vector parameters. We have therefore examined combinability with data arising from linear models of the type

$$y = X\beta + e$$

where X is an $n \times p$ matrix, and β is a $p \times 1$ vector of unknown parameters. The vector of errors e has non-singular covariance matrix V.

If one has several independent sets of data $y_i = X_i\beta + e_i$ with the same parameter vector β and respective non-singular variance matrices V_i, there is immediate interest in the simplest possible method of combining information from the several sources to get the best linear unbiased estimator (b.l.u.e.) of a parametric function $\lambda'\beta$ estimable from the full set of data. A commonly used assumption will be that V_1 and V_2 are essentially known. The report is concerned with the specific conditions under which the b.l.u.e. of a $\lambda'\beta$, estimable in each independent source, can be obtained by simple weighting of the information available in each independent source. Special attention is given to the situation of exactly two sources of information as, for example, in the case of inter and intra block information in incomplete block designs.

The particular question examined may be stated as follows: if $\lambda'\beta$ is estimable from the data $y_1 = X_1\beta + e_1$ and also estimable from the

independent data $y_2 = X_2\beta + e_2$, when is the b.l.u.e. $\lambda'\beta^*$ given by

$$\lambda'\beta^* = w\lambda'\hat{\beta} + (1-w)\lambda'\tilde{\beta} ,$$

where $\lambda'\hat{\beta}$ and $\lambda'\tilde{\beta}$ are the b.l.u.e.'s from the first and the second sources respectively?

Definition: An estimable parametric function $\lambda'\beta$ is said to be best combinable by simple weighting (b.c.s.w.) if

$\lambda'\beta^* = w\lambda'\hat{\beta} + (1-w)\lambda'\tilde{\beta}$, $\quad 0 < w < 1$, or

$\lambda'\beta^* = \lambda'\hat{\beta}$ or $\lambda'\beta^* = \lambda'\tilde{\beta}$.

The extension of this definition to $k > 2$ uncorrelated sources of information is obvious.

The following main theorems have been proved.

Theorem 1: A necessary and sufficient condition for $\lambda'\beta$ to be b.c.s.w. is that the set of solutions of the conjugate normal equation

$$(X_1'X_2') \begin{vmatrix} V_1^{-1} & 0 \\ 0 & V_2^{-1} \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} \rho = \lambda$$

is identical with the set of solutions to either the pair of conjugate equations

$$X_1'V_1^{-1}X_1\rho = w\lambda$$

$$X_2'V_2^{-1}X_2\rho = (1-w)\lambda$$

or

$$X_1'V_1^{-1}X_1\rho = \lambda \qquad X_1'V_1^{-1}X_1\rho = 0$$

or

$$X_2'V_2^{-1}X_2\rho = 0 \qquad X_2'V_2^{-1}X_2\rho = \lambda$$

Corollary 1.1: A necessary and sufficient condition for a $\lambda'\beta$, estimable in both sources, to be b.c.s.w. is that λ be in the image of a subspace S such that the mapping $X_1'V_1^{-1}X_1$ restricted to S is a scalar multiple of the mapping $X_2'V_2^{-1}X_2$ restricted to S, i.e.,

$$X_1'V_1^{-1}X_1\Big|_S = kX_2'V_2^{-1}X_2\Big|_S .$$

Corollary 1.2: A necessary and sufficient condition that $\lambda'\beta$ be best estimated from source one alone, i.e., $\lambda'\beta^* = \lambda'\hat{\beta}$, is that λ be in the image under the mapping $X_1'V_1^{-1}X_1$ of a subspace S such that S is contained in the null space of $X_2'V_2^{-1}X_2$.

Denoting the row space of X_i by χ_i, we may state another corollary of theorem 1.

Corollary 1.3: A necessary and sufficient condition for $\lambda'\beta^* = \lambda'\hat{\beta}$ for every λ in χ_1 is that $\chi_1 \cap \chi_2 = 0$.

To simplify notation and facilitate the discussion we shall hereafter, with no real loss of generality, restrict V_i to be of the form $\sigma_i^2 I$. If we restrict attention to vectors λ' in $\chi_1 \cap \chi_2 \neq 0$, we may further

characterize the set of corresponding b.c.s.w. $\lambda'\beta$'s by observing that $\lambda'\beta$ is b.c.s.w. if and only if λ is the image under $X_1'X_1$ or $X_2'X_2$ of a vector ρ such that ρ is a generalized eigenvector of $X_1'X_1 - kX_2'X_2$ for some generalized eigenvalue $k \neq 0$, i.e.,

$$(X_1'X_1 - kX_2'X_2)\,\rho = 0\,. \tag{1}$$

Lemma: If A and B are real $p \times p$ positive semi-definite matrices, with rank (A) = a $\leq$ rank (B) = b, then there exists a real non-singular matrix T and real diagonal matrices A* and B* such that A* = T'AT and B* = T'BT where

$$A^* = \begin{vmatrix} I_a & 0 \\ 0 & 0 \end{vmatrix}\,, \quad B^* = \begin{vmatrix} \mu_1 & & & & & \\ & \ddots & & & & \\ & & \mu_r & & & \\ & & & 0_{a-r} & & \\ & & & & I_{b-r} & \\ & & & & & 0_{p-a-b+r} \end{vmatrix} \tag{2}$$

and the μ_i, $i = 1, \ldots, r$, are positive.

The application of lemma to $A = X_1'X_1$ and $B = X_2'X_2$ is evident. Equation (1) becomes

$$0 = T'(X_1'X_1 - kX_2'X_2)\,T\,T^{-1}\rho$$

$$= \begin{vmatrix} 1-k\mu_1 & & & & \\ & \ddots & & & \\ & & 1-k\mu_r & & \\ & & & I_{a-r} & \\ & & & & -kI_{b-r} \\ & & & & 0 \end{vmatrix} \tau\,, \qquad (3)$$

where $\rho = T\tau$. Because of the diagonal forms of $T'X_1'X_1T$ and $T'X_2'X_2T$ in (2), bases for the row spaces of $T'X_1'X_1T$ and $T'X_2'X_2T$ are respectively the transposed columns of $E_1 = (\epsilon_1, \ldots, \epsilon_a)$ and $E_2 = (\epsilon_1, \ldots, \epsilon_r, \epsilon_{a+1}, \ldots, \epsilon_{a+b-r})$, where ϵ_i is the column of zeros with 1 in the i-th position. Since T is non-singular, any vector $\delta = T\tau$ for some unique τ. Thus for any vector δ the image

$T'X_1'X_1\delta = T'X_1'X_1T\tau = \sum_{i=1}^{a} a_i \epsilon_i$ for the proper coefficients a_i, and

thus for any vector δ the image $X_1'X_1\delta = \sum_{i=1}^{a} a_i (T')^{-1}\epsilon_i$. Hence the linearly independent columns of $(T')^{-1}E_1 = (t_1, t_2, \ldots, t_a)$ form a basis for χ_1. Similarly the linearly independent columns of $(T')^{-1}E_2 = (t_1, \ldots, t_r, t_{a+1}, \ldots, t_{a+b-r})$ form a basis for χ_2. Since the full set $\{t_1, \ldots, t_{a+b-r}\}$ is also linearly independent, the set $\{t_1, \ldots, t_r\}$ is a basis for $\chi_1 \cap \chi_2$.

For a corresponding $k = \mu_i^{-1}$, $i = 1, \ldots, r$, ϵ_i is a solution to (3) and $\rho_i = T \epsilon_i$ is a generalized eigenvector of $X_1'X_1 - \mu_i^{-1} X_2'X_2$. Thus, the image of ρ_i under either mapping $X_1'X_1$ or $X_2'X_2$ is a vector λ_i such that $\lambda_i'\beta$ is b.c.s.w. But $X_1'X_1 \rho_i = X_1'X_1 T \epsilon_i = (T')^{-1} \epsilon_i = t_i$ for $i = 1, \ldots, r$. Thus the basis, $\{t_1, t_2, \ldots, t_r\}$ (the first r rows of T^{-1}), of $\chi_1 \cap \chi_2$ constitutes a set of r independent coefficient vectors of b.c.s.w. linear parametric functions. We have therefore proved the following theorem.

Theorem 2: If the rank of the intersection space of the row spaces of X_1 and X_2 is r then there exist r linearly independent vectors λ' in $\chi_1 \cap \chi_2$ such that $\lambda'\beta$ is b.c.s.w.

Theorem 3: If a subset of s generalized eigenvalues $k_i = \mu_i^{-1}$, $i \leq r$, of (1) are equal then there exists a corresponding s dimensional subspace of $\chi_1 \cap \chi_2$ in which every vector λ' is the coefficient of a b.c.s.w. parametric function.

Theorem 4: A sufficient condition that $\lambda'\beta$ be b.c.s.w. is that λ be a common eigenvector of $X_1'X_1$ and $X_2'X_2$.

Theorem 5: If $\lambda_1, \ldots, \lambda_r$ is a set of common eigenvectors of $X_1'X_1$ and $X_2'X_2$ then $(\sum_{i=1}^{r} a_i \lambda_i)'\beta$ is b.c.s.w. if and only if

$k = c_{11} c_{12}^{-1} = \cdots = c_{r1} c_{r2}^{-1}$, where c_{i1} and c_{i2} are the eigenvalues of λ_i and $X_1'X_1$ and $X_2'X_2$ respectively.

The above two theorems apply to the case of $k > 2$ uncorrelated sources of information.

Making use of the sufficiency of eigenvectors and the fact that the interblock and intrablock information matrices in any incomplete block design have a common orthogonal diagonalization we deduced the following theorems.

Theorem 6: For incomplete designs, a linear function of the treatments, $\lambda'\tau$, is b. c. s. w. from the interblock and intrablock sources of information if and only if λ is an eigenvector of NN' where N is the treatment by block incidence matrix.

Theorem 7: In an incomplete block design (t, r, b, k, s_{ij}) a necessary and sufficient condition for the interblock and intrablock estimates of the set of treatment effects denoted by $\{t_1, t_2, \ldots, t_a\}$ to be b. c. s. w. is that all treatments occur the same number of times with treatments $T_1, T_2, \ldots, T_a$.

Corollary 7. 1: In an incomplete block design (t, r, b, k, s_{ij}), if the treatment effects $\{t_1, \ldots, t_a\}$ are b. c. s. w. then so are any linear combinations of the set.

Corollary 7. 2: In an incomplete block design (t, r, b, k, s_{ij}), all treatment effects t_i are b. c. s. w. if and only if the design has a b. i. b. structure.

The theorem to follow and the necessity conditions of Theorem 7 and Corollary 7.2 were established by Sprott (1956) using manipulations of solutions to the normal equations under the restrictive assumption of estimability in both sources.

Theorem 8: In an incomplete block design (t, r, b, k, s_{ij}) a necessary and sufficient condition that there exist a subset of treatments $T_1, \ldots, T_a$, such that $t_i - t_j$ is b.c.s.w. for all possible pairs in the subset, is that all pairs T_i and T_j, $i \neq j$ and $i, j \leq a$, occur together in a block a constant number of times and that any other treatment T_u, $u > a$, occur in a block a constant number of times s_u with $T_1, \ldots, T_a$.

In factorial designs in incomplete blocks, resolvable into uncorrelated replications, each replicate consists of uncorrelated interblock and intrablock sources of information on the treatment parameter vector τ. Thus there are $2r$ uncorrelated sources of information on τ. If we denote the interblock information matrix of the single i-th replicate by $N_i N_i'$ the following theorem was easily established.

Theorem 9: In a symmetric factorial design, with complete confounding of full sets of effect or interaction degrees of freedom within replicates, any effect or interaction degree of freedom contrast $\lambda'\tau$ is such that λ is an eigenvector of $N_i N_i'$, $i = 1, \ldots, r$.

Corollary 9.1: In a symmetric factorial design, with complete confounding of full sets of effect or interaction degrees of freedom within replicates, any effect or interaction degree of freedom contrast is b.c.s.w. for the whole set of $2r$ interblock and intrablock sources of information.

III. SIZE AND POWER OF CERTAIN TESTS UNDER EXPERIMENT RANDOMIZATION

We have conducted an investigation of the size and power of the F test and three non-parametric tests in an attempt to understand more thoroughly the consequences of experiment randomization. In particular we have studied the behavior of tests applicable to a paired design and have further restricted the investigation to include small samples only. The test procedures examined in detail were the Fisher randomization test, the Sign test, the Wilcoxon paired test and the normal theory F test. A specification of these tests is as follows.

(a) The Fisher Randomization Test:

The observed total difference is Σx_i. Let C_{obs} equal the absolute value of this. Consider the absolute values of the possible quantities $\sum_i (\pm) x_i$, where each of the 2^n different patterns of + or - are enumerated. Let the absolute values be $C_1, C_2, \ldots, C_M$, where M equals 2^n. The significance level is the proportion of the C_i which equal or exceed C_{obs}. Actually one need enumerate only 2^{n-1} different patterns, because the criterion is the absolute total difference.

(b) The Sign Test:

Let the maximum of the number of positive x_i's and the number of negative x_i's be S_{obs}. Follow the same procedure

with this criterion. Actually we do not need to perform the details, because the possible values of the criterion are $n, n-1, \ldots, \left[\frac{n}{2}\right]$, where $\left[\frac{n}{2}\right]$ is $\frac{n}{2}$ if n is even and $\frac{n+1}{2}$ if n is odd, and their frequencies are given by combining the tails of the binomial distribution for n trials with probability of success equal to $\frac{1}{2}$.

(c) The Wilcoxon Paired Test:

The x_i are ranked from smallest to largest disregarding signs. Let the maximum of the sum of the ranks of the negative observations, and the sum of the ranks of the positive observations be W_{obs}. Follow the same procedure with this criterion. The critical values for small values of n and the possible significance levels are given in tables, for example by Hodges and Lehmann (1963).

(d) The F Test:

In the case of the paired design, the F test is very simple: calculate the criterion [treatment mean squares / error mean square] and compare this value with the chosen percentage point of the F distribution with the chosen percentage point of the F distribution with 1 and (n-1) degrees of freedom, where n is the number of pairs.

The objective of the study was the determination of the relative and absolute performance of these test procedures with regard to the population

of repetitions induced by physical randomization. If we view the significance level as a summary statistic, a complete characterization of the situation is given by the distribution of the significance level under the null hypothesis, and the distribution of the significance level under the alternative. Since this is an overwhelming task, a common procedure is to examine power of tests which is essentially tail areas of the distribution of the significance level under the alternative hypothesis. Thus, size and power served as reasonable criteria with which to measure test performance.

With N pairs observed in the experiment, there are 2^N possible ways of applying two treatments within each pair, one of which is randomly chosen by the experimenter. The null hypothesis of no treatment difference is then tested against various shift alternatives. In this way it is possible to evaluate critically the influential characteristics inherent in the problem of paired tests. By examining size and power, we obtain the role of the test criterion, significance level, experiment size, true treatment difference and the underlying distribution from which the basal yields are generated.

Since the parametric F test and the Sign test have been dealt with extensively in the literature, emphasis was concentrated on the performance of the Fisher and Wilcoxon techniques as applied to paired data. Completely general integration formulas were developed to enable power computations to be performed for experiments involving three or four pairs of differences. A perfect agreement of the three non-parametric tests at the lowest achievable test size was exhibited, regardless of the

experiment size, with the correspondence extending to the three smallest levels for the Fisher and Wilcoxon criteria.

To extend the investigation to larger experiments it was necessary to perform an empirical study. With a set of differences randomly generated from various representative distributions and an imposed treatment effect Δ, it was possible to generate the totality of conceptual experiments that might have arisen. Each test criterion was then evaluated for every possible randomization, and the appropriate significance levels recorded in each case. In this way exact power probabilities were computed for each test over the population defined by the randomization process. By performing these calculations for a representative number of samples of observed differences, an indication of the small-sample behavior of the four tests of interest was established. Experiments of 3, 4, 5, 6 and 8 pairs were examined in this manner, and where theoretical comparisons exist the results indicate excellent agreement with true power values. Since the power under experiment randomization does not behave with a noticeable regularity for individual experiments, comparisons of tests were based on average power values determined from several random samples of differences. Because of the considerable computing time involved, various sampling techniques were utilized for a limited investigation of the Fisher criterion for experiments involving ten differences.

The general conclusion is that with small samples of differences from any of the distributions considered, the average powers of the Fisher randomization test and the Wilcoxon paired test are essentially identical.

The power curve of the Sign test is somewhat inferior to that of the other tests at comparable sizes greater than $\frac{1}{2^{N-1}}$. It is also shown that knowledge of the power of the Sign test at the lowest achievable test size is complete in the sense that power at all other levels is uniquely related. The relative behavior of the F test and the non-parametric tests is somewhat irregular, but in most cases the power values are quite close. There is evidence that departures from normality do not drastically affect the relative performances of the tests examined, but for extreme non-normal configurations power is low and erratic in its behavior. The average size of the F test was generally quite close to the nominal normal distribution size even when the underlying distribution of differences was decidedly non-normal. The distribution of the size of the F test under experiment randomization was examined in some detail, and it was found that the probability of detecting significance at level α is distributed with considerable spread about the true test size α. The spread is greatly dependent on the underlying distribution of differences.

It appears that except for their inability to achieve any prechosen size, the non-parametric tests are to be preferred because their behavior under the null hypothesis is known a priori regardless of the underlying pattern of basal yields. If one admits Fisher's concept of sensitivity relative to the problem of evaluating significance, the Fisher randomization test is slightly superior to the Wilcoxon test, while both are considerably more sensitive than the Sign test. In this framework we look upon the significance level as a summary statistic giving the weight of evidence

against a null hypothesis with reference to a particular class of alternatives. For the paired design we have seen that the Fisher criterion includes more levels for the declaration of significance than the other non-parametric tests. From this point of view the Sign test should be recommended only when none of the other procedures are applicable.

It is evident that usage of the Fisher randomization test or the Wilcoxon paired test is advantageous to the experimenter when testing two treatments. We have seen that the test criteria can be quickly enumerated over all possible randomizations when the number of observed differences is small. For moderate sample sizes, excellent approximations were observed by sampling a reasonable proportion of randomizations.

IV. COMPUTATION OF ESTIMATES OF VARIANCES AND COVARIANCES OF VARIANCE COMPONENT ESTIMATES FROM FINITE BALANCED POPULATIONS

INTRODUCTION

Dayhoff (1964) has shown that the variances and covariances of variance component estimates for certain simple balanced structures obtained in the usual way, by equating the expected mean squares to the observed mean squares in the analysis of variance and solving the resulting linear equations for the variance components, can be formulated as linear functions of quantities called generalized polykays. The generalized polykays are a natural extension of the bipolykays defined by Hooke (1956a), from which he was able to calculate variances and covariances of estimated variance components in two factor crossed structures, as shown in the papers by Hooke (1954, and 1956b). The bipolykays were an extension of the polykays introduced by Tukey (1950, and 1956).

The generalized polykays are, in general, not directly computable, but can be obtained as linear functions of generalized symmetric means, which in the case of polykays of degree four, are fourth moments of the population or sample quantities. Because polykays and symmetric means have the property of inheritance on the average, it is possible to obtain unbiased estimates of the variances and covariances of estimated variance components by taking appropriate linear combinations of generalized sample polykays. The work of Dayhoff is thus complete for the pure random sampling situation in that, by his methods, one may obtain formulas for unbiased estimates of the variances and covariances of the

estimated components of variation.

The implementation of Dayhoff's methods to obtain numerical estimates involves two fairly serious problems. First, the algebra required to obtain the formulas, is, while straightforward in principle, a very tedious and error prone process in execution. As an example, for a three-factor crossed structure a single variance formula involves thirty-seven polykays of degree four with coefficients which are various functions of the numbers of levels of the factors in the sample and in the population. Each of these polykays is, in turn, a linear function of as many as 285 generalized symmetric means of degree four. Because of the heavy burden of algebra required it seems expedient to perform this task on high speed digital computers. Accordingly, algorithms have been developed, which, when presented with an arbitrary balanced complete population structure, obtain the necessary formulas for the variances and covariances of variance components.

The second problem arises in the numerical computation of the generalized symmetric means of degree four, which, in Dayhoff's method, are the basic numerical quantities to be computed. It is a rather simple matter to write a computer program to evaluate a single generalized symmetric mean from its definition and not extremely difficult to write a more general program to compute all the generalized symmetric of degree four in a given structure. This can be extended further, with some difficulty, to compute the generalized symmetric means of degree four for arbitrary balanced complete structures. However, for relatively small numbers of observations the number of multiplications becomes excessive,

and a better approach is necessary. A simple illustration of the approach taken here is given by the familiar identity below

$$\frac{1}{n(n-1)} \sum_{i} \sum_{i' \neq i} y_i y_{i'} = \frac{1}{n(n-1)} \left[\left(\sum_{i} y_i \right)^2 - \sum_{i} y_i^2 \right]$$

The left hand quantity is a simple symmetric mean of degree two and, aside from the divisor requires $n(n-1) / 2$ multiplications and additions, while the expression on the right requires $n + 1$ multiplications and $2n + 1$ additions. Similar identities may be obtained for generalized symmetric means of degree four, and these result in important savings in the amount of computations required. These identities do, of course, increase the amount of algebra required, and care must be taken that one does not exchange the problem of performing an impossibly large number of multiplications for the problem of collecting an impossibly large number of coefficients.

A general method of obtaining all the needed identities in a straightforward way has been developed and implemented in a computer program, so that the generalized symmetric means are formulated in terms of quantities which are computable in a minimum number of operations. These quantities are called D's or "derived terms." The same quantities, for the particular case of two factor crossed structures, are used by Hooke (1954). Further programs have been developed which interpret the D's and compute their numerical values.

DAYHOFF'S PROCEDURE

The theoretical basis for the computations, as developed by Dayhoff (1964) are as follows.

Variance components estimates may be considered as linear combinations of sample cap sigmas. The cap sigmas are in fact the same quantities as generalized polykays of degree two, so that variances and covariances of variance component estimates are linear functions of the variances and covariances of sample generalized polykays of degree two. Variances and covariances of sample generalized polykays of degree two are linear combinations of population generalized polykays of degree four, and unbiased estimates of these are given by the corresponding sample polykays of degree four. The generalized polykays of degree four are linear functions of the generalized symmetric means of degree four, which can be computed.

POLYKAYS AS FUNCTIONS OF SYMMETRIC MEANS

The generalized polykays for a crossed structure are defined as functions of simple polykays by means of symbolic multiplication. Thus let $P = (\alpha/\beta)$ denote a generalized polykay of degree four for two factors. The α and β symbols may be considered as indicating a partition of the subscripts into classes which are equal for each element of the fourth degree product of the "leading" symmetric mean in the definition of (α). We make use of the notation, introduced by Dayhoff, of giving a symbol for each element of the product with the equality of these symbols

indicating equality of the subscripts. Thus to denote a product $y_i^3 y_{i'}$, write 0001, while $y_i y_{i'} y_{i''} y_{i'''}$ is denoted by 0123. If one uses primes to indicate restrictions on the subscripts, then the symbols α, β etc. can be considered simply as a list of the number of primes on the successive y's for the first, second, etc. factors.

These lists can be considered as partitions of the integer four, with a further order restriction; we will call them "ordered partitions." The ordering consideration is not necessary when simple polykays are considered, but when more than one factor is considered the ordering becomes necessary so that the relationships of the restrictions for the various factors will be preserved.

The simple polykays may be expressed as linear combinations of simple symmetric means, so that

$$(\alpha) = \sum_i a_i \langle \alpha_i \rangle$$

The generalized polykays for completely crossed structure are defined by a symbolic multiplication

$$\begin{aligned} P = (\alpha/\beta) &= (\alpha) \otimes (\beta) \\ &= \left(\sum_i a_i \langle \alpha_i \rangle\right) \otimes \left(\sum_j b_j \langle \beta_j \rangle\right) \\ &= \sum_i \sum_j a_i b_j \langle \alpha_i \rangle \otimes \langle \beta_j \rangle \\ &= \sum_i \sum_j a_i b_j \langle \alpha_i / \beta_j \rangle \end{aligned}$$

or, say

$$P = \sum_u c_u g_u \quad .$$

Unfortunately the notation described above is redundant in that we may have $\langle \alpha/\beta \rangle = \langle \alpha'/\beta' \rangle$ with $\alpha \neq \alpha'$, or $\beta \neq \beta'$ or both. No simple notation has been discovered for removing this redundancy, although an algorithm has been developed to give a many-to-one mapping of all the possible symbols for a given set of generalized polykays into a set of distinct ones. Carrying out the symbolic multiplication, combining like terms and collecting coefficients is all that is necessary in obtaining the polykays and their formulas in terms of generalized symmetric means for crossed structures. The handling of arbitrary structures require a few additional operations.

SYMMETRIC MEANS AS FUNCTIONS OF D's

The symbolic multiplication is also applicable in obtaining the expansions of the generalized symmetric means in terms of the D's. Let $N_{\alpha\beta} = n_\alpha n_\beta$ denote the divisor of the generalized symmetric mean $\langle \alpha/\beta \rangle$. (If A is the number of levels of the first factor, then $n_\alpha = A(A-1)\cdots(A - r + 1)$ where r is the number of different symbols in the list α.) Let $\langle a_i \rangle$ denote a simple symmetric mean. Then there exists a formula

$$\langle a_i \rangle = \frac{1}{n_{a_i}} \sum_k d_{ik} \; |a_{ik}|$$

where $|a_{ik}|$ denote the "D" quantities for a single subscript. For

example, consider

$$\langle a_i \rangle = \langle 0012 \rangle = \frac{1}{N(N-1)(N-2)} \sum^{\neq} y_i^2 y_{i'} y_{i''}$$

then

$$\langle 0012 \rangle = \frac{1}{N(N-1)(N-2)} \left[\sum_i y_i^2 y_.^2 - (\Sigma y_i^2)^2 - \Sigma y_i^3 y_. - \Sigma y_i^2 y_. y_i \right.$$

$$\left. + 2 \Sigma y_i^4 \right] = \frac{1}{N(N-1)(N-2)} \left[|0012| - |0011| - |0001| - |0010| \right.$$

$$\left. + 2 |0000| \right] , \text{ where } y_. = \sum_{i=1}^{N} y_i .$$

It can be shown that if $\langle a_i/\beta_j \rangle$ is a generalized symmetric mean for a crossed structure, and $\langle a_i \rangle = \frac{1}{n_{a_i}} \Sigma d_{ik} |a_{ik}|$,

$$\beta_j = \frac{1}{n_{\beta_j}} \sum_k d_{jk} |\beta_{jk}| , \text{ then } \langle a_i/\beta_j \rangle = \langle a_i \rangle \otimes \langle \beta_j \rangle$$

$$= \left(\frac{1}{n_{a_i}} \sum_k d_{ik} |a_{ij}| \right) \times \left(\frac{1}{n_{\beta_j}} \sum_\ell d_{j\ell} |\beta_{j\ell}| \right)$$

$$= \frac{1}{n_{a_i} n_{\beta_j}} \sum_k \sum_\ell d_{ik} e_{j\ell} |a_{ik}| \otimes |\beta_{jk}|$$

$$= \frac{1}{n_{a_i} n_{\beta_j}} \sum_k \sum_\ell d_{ik} d_{j\ell} |a_{ik}/\beta_{j\ell}|$$

$$= \sum_v f_v D_v , \text{ say.}$$

Combining the two symbolic multiplications gives

$$P = (\alpha/\beta) = (\alpha) \otimes (\beta)$$

$$= \sum_i \sum_j a_i b_j \langle \alpha_i/\beta_j \rangle = \sum_i \sum_j a_i b_j \langle \alpha_i \rangle \otimes \langle \beta_j \rangle$$

$$= \sum_i \sum_j \frac{a_i b_j}{n_{\alpha_i} n_{\beta_j}} \sum_k \sum_\ell d_{ik} d_{j\ell} |\alpha_{ik}| \otimes |\beta_{j\ell}|$$

$$= \sum_i \sum_j \sum_k \sum_\ell \frac{a_i b_j d_{ik} d_{j\ell}}{n_{\alpha_i} n_{\beta_j}} |\alpha_{ik}/\beta_{j\ell}| \quad \text{or} \quad P = \sum_s w_s D_s \text{ , say .}$$

Thus the crossed structure polykays can be evaluated as linear functions of the D's, and the proper linear functions are determined by successive symbolic multiplications. When dealing with a structure containing some factors nested in others the above operations are modified in the following ways: (1) Some of the polykays of the crossed structure do not exist in the nested structure. These are eliminated. (2) In performing the first symbolic multiplication those polykays of the crossed structures which do not exist in the nested structure are mapped into other polykays of the nested structure and the terms collected. This procedure gives the proper formulas for the polykays of the nested structure in terms of the generalized symmetric means for the nested structure. (3) Before performing the second symbolic multiplication, some of the terms in the expansion for nested polykays are eliminated in a systematic way depending upon the terms in the expansion for nesting factors. This procedure gives the correct formulas for nested structures.

COMPUTER PROGRAMS

The programming system for obtaining estimated variances and covariance of variance component estimates consists of the following.

(1) PFORM - A program to generate the polykays for a given structure and obtain the formulas for these polykays as linear functions of the generalized symmetric means. This portion of the system is run separately since the formulas depend only upon the structure and not upon the particular data being analyzed. The major subroutines of this program are:

a. SYMPY - A routine for symbolic multiplication

b. UNREP - A routine which maps the various representations of a given polykay into a unique representation.

(2) DCOMP - A routine which computes numerically all the D's for a given structure and sample. This program consists of two major portions; one to interpret the symbolic representation of a D and generate certain tables which determine the base addresses, powers, operations, and sequence of operations required to compute the particular value symbolized, and a second program to follow this sequence of operations and obtain the desired numerical quantity.

(3) DVCMP - A routine to compute the divisors for the generalized symmetric means.

(4) GCOMP - A program which performs the symbolic multiplication to obtain the expansions for generalized symmetric means in terms of the D quantities and uses the D's from DCOMP, and the divisors from

DVCMP to evaluate the generalized symmetric means.

(5) PCOMP - Reads the formulas for generalized polykays in terms of generalized symmetric means (i.e., the output of PFORM) and evaluates these formulas using the values of the generalized symmetric mean computed by GCOMP.

(6) VCVC - This program performs a variety of tasks, largely algebraic in nature in obtaining and evaluating the formulas for the variances and covariances of variance components for the particular structure in terms of the polykays of degree four which have been previously computed by PCOMP. Included are the following operations:

a. The complete model for the present structure and the corresponding completely crossed model are generated. This provides a symbolic list of all the variance components and cap sigmas needed.

b. The formulas for the variances and covariances of the crossed polykays of degree two in terms of crossed polykays of degree four are generated by symbolic multiplication of the formulas for multiplication of simple polykays of degree two. These formulas are then evaluated for the polykays of the present structure to give the numerical values of the estimated variances and covariances of crossed cap sigmas for the present structure.

c. The model terms for the present structure are expanded in the terms of a completely crossed structure to give the formulas for the cap sigmas of the present structure as sums of crossed cap

sigmas. This transformation is then applied to the variance-covariance matrix of crossed cap sigmas to give the variance-covariance matrix of the cap sigmas of the current structure.

d. The tranformation for cap sigmas in terms of variance components are generated, inverted, and applied to the variance-covariance matrix of the cap sigmas to give the estimated variance-covariance matrix for the components of variance.

USE OF THE SYSTEM

Thus far the computations with the system have been made with rather special data for the purpose of checking the computer programs. It is planned to use the system to investigate the variances and covariances of realistic populations and samples and to compare the results with those obtained under infinite model assumptions.

The algorithms are designed to operate for any number of factors. However, the present program is limited to 3 factors so that various arrays need not exceed the storage capacity available with the IBM 7074 FORTRAN Operating System which allows about 8000 ten digit words for program and data. It would not be very difficult to expand the program to 4 or 5 factors with the present 20,000 word IBM 7074 equipment, but this is not contemplated at the present time because this equipment will be replaced in the near future.

Thus far the computations have not proved too costly. For example, with a four by four crossed sample the complete computation required in the neighborhood of 28 seconds. (This included some extra operations required for generating the data). With two or three factors good sized

samples (say 1000 observations) can probably be computed in a matter of a few minutes, say 1 to 5 minutes, depending on the model. By comparison, a program for computing the generalized symmetric means directly would require in the neighborhood of 1000 hours with the same computing equipment.

One familiar with large scale numerical computations will recognize that the type of computations described above may lead to serious truncation errors. This is indeed true, but can be countered to a large extent by the use of double precision arithmetic at selected points in the algorithm and by standardizing the observations. In summary it seems fair to claim that the systems described provides a practical method for obtaining unbiased estimates of variances and covariances of variance components for finite balanced complete structure when few factors are involved, and, while such computation may be of little importance for any particular data set, they are of some importance in the investigation of the properties of variance component estimates in general.

V. OTHER TOPICS

A. THE DESIGN OF EXPERIMENTS

A review of developments in the design of experiments over the past ten years was prepared and presented at the Tenth Conference on the Design of Experiments in Army Research, Development and Testing (Kempthorne, 1965a). The problems of inference from experiments is touched only briefly, and the main area reviewed is the design and analysis of investigations in multifactorial situations. The sequence of developments with regard to qualitative factors is outlined, from the testing of the full factorial set, to the Fisher plans for 2^n-1 factors each at 2 levels in 2^n observations, the Plackett-Burman plans for 4N-1 factors at 2 levels in 4N observations, and then the development of fractional replication by several workers. In the case of continuous or quantitative factors, the developments are reviewed with regard to optimum seeking. The work of Box and Wilson, and the PARTAN method which are essentially strategies based on assumption of ellipsoidality of contours without sizeable error variation are discussed, as is the work of Kiefer and Wolfowitz and others which is concerned with proving convergence with probability one whatever the amount of error present. Work on the general problem of exploring the relationship between control variables, such as temperature and pressure, and yield is discussed. The plans developed by Box and his co-workers are discussed particularly with reference to the problem of scaling of variables. In contrast to this line of work is that of Kiefer and Wolfowitz who make a direct attack on design to achieve optimality with regard to a completely defined aspect of

the investigation. It appears that this approach is informative, but not decisive, because an experimental investigation rarely has a single criterion of value and it is usually the case that a design which is near optimal with respect to one reasonable criterion of value is quite non-optimal with respect to other criteria of value which the experimenter must consider. It would appear then that at best the problem of design can be formulated in programming terms, that is, one would like optimality with respect to one criterion with a reasonable degree of sub-optimality with respect to other criteria. This type of approach to design is being explored currently.

B. MULTIVARIATE RESPONSES IN EXPERIMENTS

A review of the status of procedures for data interpretation and inference for the case of multivariate responses in comparative experiments was presented to the International Symposium on Multivariate Analysis (Kempthorne, 1965b). The view is expressed and substantiated, partially at least, that the theoretical work in multivariate analysis has so far led to quite meager results with regard to the drawing of experimental conclusions. A dichotomy is drawn between experiments the purpose of which is to make terminal decisions, such as the naming of the "best" treatment, and experiments performed to add to knowledge. The obvious names for these are "decision" experiments and "information" experiments. It appears that the great bulk of theoretical work is aimed at "decision" experiments, and that the improvement of data procedures for "information" experiments has been disappointingly small. Some

discussion is given of the rival modern "religions" of statistics, which are associated with the words, "Bayesian", "decision" and "likelihood."

An assessment of what scientists want from the comparative experiment with multivariate response is made, and related to the current availability of techniques. It is concluded that the situation is deplorable. The conclusions of the review are as follows.

(1) The purpose of statistical analysis of experimental informational data is to form opinions about the underlying situation. One can certainly form opinions on the basis of univariate techniques, which are communicable and fairly easily understood. The question of what multivariate analysis can provide over and above separate univariate analyses has an obvious answer at an elementary level, as in the study of the error matrix, but is unanswered beyond this. It is relevant, for instance, to ask why one would get significance at a particular level by correlated univariate tests and not by the corresponding multivariate test. An observation that this happens is in itself, informative of the situation under analysis and requires examination of the data to see "why" it happened. There are, however, situations in which the multivariate analysis tells one something about individual components of the observation vector. Suppose one observed the following in a completely randomized design:

	Mean squares and products		
	x_1^2	x_1x_2	x_2^2
Treatments	500	250	190
Residual	100	75	200

The data indicate differences among treatments with regard to x_1, but not with regard to x_2, if one looks at the univariate analyses. But the product analysis indicates that there are differences between treatments with regard to both x_1 and x_2. Exactly how one can quantify these indications appears to be unknown, but the data illustrate how the multivariate analysis "tells" one more about one component of the observation than a single univariate analysis.

(2) The state of theoretical knowledge about multivariate observations, in spite of very good books on the subject, seems still very primitive. Naturally enough, the theory is dominated by the multivariate normal distribution, but one wonders how robust the procedures for assessing differences of means are. This will probably have to be assessed by Monte Carlo computations. We have some obviously desirable tests of significance for global questions, but have very few informative multivariate data dissection procedures.

(3) The future of data analysis obviously lies in the easy use of high speed computers. The only way "to look at" multivariate data is by means of computers and plotters. Even in the present, after 20 years of modern computation, the problems of communicating with a computer are excessive. Hopefully these problems will be solved soon, and we will

have manuals of data analysis just like manuals of chemical analysis. The presently low amount of truly multivariate analysis is certainly partly due to inadequacy of computing processes.

(4) Many of our univariate procedures arose from looking at real data and trying to make sense of them. The same will hold for multivariate data. The job of thinking of ways of looking at data is different from the job of determining the probability behavior of these ways.

(5) Even though the usual multivariate techniques seem from some points of view to assess the totality of the data, they do so only with regard to linear functions of the observations. Ratios and other indices constructed from the components may well behave in a simple way.

C. EXPERIMENTAL INFERENCE

The Fisher Memorial Lecture sponsored by the American Statistical Association, the Institute of Mathematical Statistics and the Biometric Society was presented on the topic of experimental inference (Kempthorne, 1965c).

The dichotomy presented by Fisher's writings into experimental and non-experimental inference is discussed, and the paper first discusses the more basic of Fisher's ideas on non-experimental inference. These are considered to be (a) tests of significance, (b) the use of the likelihood function, and (c) fiducial probability. Fundamental obscurities with regard to tests of significance are discussed. The use of the likelihood function is examined, and it is concluded that much of the theoretical work on likelihood is at best misleading and at worst utterly

erroneous. The basis for this view is that an intrinsic aspect of data collection is the fact that a continuously distributed random variable is observable only with a definite grouping error, specified by the observer, and that observations of unlimited accuracy are impossible. This fact is surely incontrovertible, but much of the mathematical theory now available and being presented to students assumes the contrary. Some simple consequences of this fact are substantiated in the paper:

(1) the likelihood is <u>not</u> the product of probability densities, but is always a multinomial likelihood, which may or may not be approximated reasonably by the former.

(2) the likelihood properly calculated does not "blow up", that is, become infinitely large as certain values for the parameters are approached, a "fact" which has been stated by many research workers.

(3) the numerous examples in the literature on estimates with variances asymptotically of the form K/n^2 are erroneous.

Some views with regard to fiducial probability are given.

The bulk of the paper consists of a discussion of the concepts "validity of error", "validity of test" in experiments, and it is concluded that Fisher's writings are essentially consistent with regard to these, in that validity has to be judged with reference to the population of repetitions induced by the physical randomization employed.

A short review is made of investigations on the performance in the randomization framework of some tests of significance for the paired design.

REFERENCES

Basson, R. P. (1965). On unbiased estimation in variance component models. Unpublished Ph. D. Thesis. Library, Iowa State University of Science and Technology, Ames, Iowa.

Dayhoff, E. E. (1964). Generalized polykays and application to obtaining variances and covariances of components of variation. Unpublished Ph. D. Thesis. Library, Iowa State University of Science and Technology, Ames, Iowa.

Doerfler, T. E. (1965). Size and power of some tests under experimental randomization. Unpublished Ph. D. Thesis. Library, Iowa State University of Science and Technology, Ames, Iowa.

Graybill, F. A. and R. A. Hultquist. (1961). Theorems concerning Eisenhart's Model II. Ann. Math. Stat. 32: 261-269.

Henderson, C. R. (1953). Estimation of variance and covariance components. Biometrics 9: 226-252.

Hooke, R. W. (1954). The estimation of polykays in the analysis of variance. Stat. Res. Group, Princeton University. Memorandum Report 56.

Hooke, R. W. (1956a). Symmetric functions of a two-way array. Ann. Math. Stat. 27: 55-79.

Hooke, R. W. (1956b). Some applications of bipolykays to the estimation of variance components and their moments. Ann. Math. Stat. 27: 80-98.

Kempthorne, O. (1965a). Development of the design of experiments over the past ten years. Presented at the Tenth Conference on the Design of Experiments in Army Research, Developments and Testing, Washington, D. C.

Kempthorne, O. (1965b). Multivariate responses in comparative experiments. Presented at the International Symposium on Multivariate Analysis, Dayton, Ohio.

Kempthorne, O. (1965c). Some aspects of experimental inference. The Fisher Memorial lecture presented at the joint meetings of the American Statistical Association, the Institute of the Mathematical Society, and the Biometric Society, Philadelphia, Pennsylvania.

Sprott, D.W. (1956). A note on combined interblock and intrablock estimation in incomplete block designs. Ann. Math. Stat. 27: 633-641.

Tukey, John W. (1950). Some sampling simplified. J. Am. Stat. Assoc. 45: 501-519.

Tukey, John W. (1956). Keeping moment - like computations simple. Ann. Math. Stat. 27: 37-54.

Zyskind, G., O. Kempthorne, R.F. White, E.E. Dayhoff, and T.E. Doerfler. (1964). Research on analysis of variance and related topics. ARL report 193, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio.

LIST OF ACTIVITIES ASSOCIATED WITH CONTRACT AF 33(615)-1737
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Speeches

O. Kempthorne

"The Current Status of the Design of Experiments." (Annual meetings of the Institute of Mathematical Statistics - August, 1964 - Amherst, Massachusetts).

"Development of the Design of Experiments over the Past Ten Years." (Tenth Conference on the Design of Experiments in Army Research, Development and Testing - November, 1964 - Washington, D.C.).

"Multivariate Responses in Comparative Experiments." (International Symposium on Multivariate Analysis - June, 1965 - Dayton, Ohio).

"Some Aspects of Experimental Inference." (Joint meetings of American Statistical Association, Biometric Society and Institute of Mathematical Statistics - September, 1965 - Philadelphia, Pennsylvania).

O. Kempthorne - Discussion leader - "Statistical Methods for Data Analysis." (Joint meeting of American Statistical Association, Biometric Society, and Institute of Mathematical Statistics - September, 1965 - Philadelphia, Pennsylvania).

George Zyskind and Frank Martin

"On Simple Combination of Information from Uncorrelated Linear Sources." (Institute of Mathematical Statistics - Purdue University - March, 1966).

George Zyskind - Discussion leader - "Problems in the Analysis of the General Comparative Experiments." (Joint meeting of American Statistical Association, Biometric Society, and Institute of Mathematical Statistics - September, 1965 - Philadelphia, Pennsylvania).

E.J. Carney

"The Lattice of Ordered Partitions and its Relation to Generalized Symmetric Means and Generalized Polykays." (Joint meeting of Biometric Society, Institute of Mathematical Statistics, and American Statistical Association - Brookhaven National Laboratory, Upton, L.I., New York - April, 1966).

Publications

Basson, Rodney - On unbiased estimation in variance component models. Aerospace Research Laboratories, Wright Patterson Air Force Base, Ohio. ARL report submitted.

Carney, E. J. - June, 1966 - The lattice of ordered partitions and its relation to generalized symmetric means and generalized polykays. Ann. Math. Stat. 37: 761-762. (Abstract).

Dayhoff, E. - December, 1964 - On the equivalence of polykays of the second degree and Σ's. Ann. Math. Stat. 35: 1663-1672.

Dayhoff, E. - February, 1966 - Generalized polykays, an extension of simple polykays and bipolykays. Ann. Math. Stat. 37: 226-241.

Kempthorne, O. - March, 1966 - Some aspects of experimental inference. J. Am. Stat. Assoc. 61: 11-34.

Kempthorne, O. - October, 1965 - Development of the design of experiments over the past ten years. Proceedings of the Tenth Conference on the Design of Experiments in Army Research Development and Testing - AROD Report 65-3: 19-46.

Kempthorne, O. - Multivariate responses in comparative experiments. Proceedings of the Multivariate Symposium, Dayton, Ohio. To be published.

Martin, F. and G. Zyskind - October, 1966 - On combinability of information from uncorrelated linear models by simple weighting. Ann. Math. Stat. 37: 1338-1347.

Zyskind, G., O. Kempthorne, R. F. White, E. E. Dayhoff, and T. E. Doerfler. - November, 1964 - Research on analysis of variance and related topics. Aerospace Research Laboratories, Wright Patterson Air Force Base, Ohio. ARL64-193.

Theses

Basson, Rodney - (Under G. Zyskind)

"On unbiased estimation in variance component models." Ph. D.

Dayhoff, E. E. - (Under O. Kempthorne)

"Generalized polykays and application to obtaining variances and covariances of components of variation." Ph. D.

Doerfler, T. E. - (Under O. Kempthorne)

"Size and power of some tests under experimental randomization." Ph. D.

Martin, Frank B. - (Under G. Zyskind)

"On simple linear combinability of information from independent sources." M. S.

Seminars - Iowa State University

O. Kempthorne

November, 1964 - Some developments in the design of experiments over the past ten years.

September, 1965 - Comparison of tests for the paired design.

G. Zyskind

March, 1965 - On the invariance of a class of canonical forms arising in the analysis of experimental structures.

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13 ABSTRACT

Research on Analysis of Variance and Data Interpretation is described. Section I discusses estimation problems in variance component and mixed model problems. Section II considers the combination of information on estimable functions from distinct uncorrelated sources and justifies some of the common applications in experimental design problems. Section III discusses size and power under experiment randomization of several competitive tests for the paired design and presents conclusions about the high relative merits of the variance ratio randomization and the Wilcoxon tests. Section IV discusses the development of high speed computational methods for the calculation of fourth degree generalized polykays of variances and covariances of estimated variance components for balanced samples from balanced populations. Section V summarizes briefly papers on the design of experiments and multivariate responses in experiments and the 1965 Fisher Memorial lecture on experimental inference.