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PART III

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HYPERSONIC DYNAMIC STABILITY
PART III. UNSTEADY FLOW FIELD PROGRAM

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GENERAL ELECTRIC COMPANY

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FOREWORD

The information released in this report has been generated in support of studies on the Hypersonic Dynamic Stability Characteristics of Lifting and Non-lifting Re-entry Vehicles. These studies were conducted by the General Electric Company, Reentry Systems Department, for the Stability and Control Section of the Air Force Flight Dynamics Laboratory, Research and Technology Division. The program was sponsored under Air Force Contract Number AF 33(657)-11411, Project Number 8219, and Task Number 821902. Mr. J. Jenkins of The Control Criteria Branch, RTD, is the project engineer on the contract. The authors are Mr. H. Rie, engineer — theoretical aerodynamics, Systems and Technology Section, Reentry Systems Department, General Electric Co.; Mr. E. A. Linkiewicz, mathematician-problems analysis, Engineering Section, Spacecraft Department, General Electric Co.; and Mr. F. D. Bosworth, scientific programmer. Finance and Contract Management Operation, General Electric Co. The project supervisor was Mr. L. A. Marshall. The development of the unsteady flow field program covered in this report was started in May 1963 and completed in June 1964.

This technical report has been reviewed and is approved.



C. B. WESTBROOK
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ABSTRACT

A method of computing static and dynamic force and moment coefficients has been prepared for the Research and Technology Division. The method provides for a complete inviscid flow field solution to be computed on the IBM 7094 machine. The solution provides values of all pertinent flow field parameters at a large number of points in the shock layer surrounding a pointed or spherically blunted body of revolution (or the analogous two-dimensional shapes) in supersonic or hypersonic flight. Perturbations (of each flow field parameter) due to angle of attack, rate of change of angle of attack, and pitch velocity are also computed. The force and moment coefficients are obtained by integrating the appropriate perturbations in pressure over the body surface.

The above flow field capabilities have been delivered to the Research and Technology Division in the form of magnetic tapes. A sample solution was demonstrated on the Wright-Patterson AFB computer installation on June 30 - July 1, 1964. The analysis and programming of this solution is described in this report.

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SYMBOLS

a, b	γ^* coefficients
C_m	moment coefficient
C_n	force coefficient
c_o	speed of sound
$\frac{D}{Dt}$	material derivative
D_B	base diameter
dt	differential distance along Mach lines
$d\alpha_o$	distance along zero yaw streamlines
e_o^2	$\left(\frac{\partial p}{\partial s}\right)_\rho$
F	function
F_n	normal force
h	enthalpy
L	reference length
M	Mach number
n	order of derivative of angle of attack or pitch velocity being considered
\vec{n}	unit inward vector normal to the surface indicated by its subscript
P, p	pressure
q	pitch rate
r	coordinate direction (Figure 5). For the two-dimensional problem, this direction is identical with the y-direction shown in Figure 5.
R	gas constant

R, ω, ϕ	coordinate directions (Figure 3)
$\vec{R}, \vec{\omega}, \vec{\phi}$	unit vectors in the coordinate directions
S	entropy/gas constant
t	time
T	temperature
u, v, w	components of \vec{V} in the R, ω, ϕ directions respectively for the pointed cone solution, and in the $x, r,$ and ϕ directions elsewhere.
\vec{r}	unit vector in r direction
V	magnitude of velocity in meridional plane
\vec{V}	fluid velocity relative to the body fixed coordinate system
V_{∞}	free stream speed
x, r, ϕ	coordinate directions (Figure 5)
$\vec{x}, \vec{r}, \vec{\phi}$	unit vectors in the coordinate directions
x, y, z	coordinate directions (Figure 2)
x_0	distance from x, y, z origin to center of spherical nose
$\vec{x}, \vec{y}, \vec{z}$	unit vectors in the coordinate directions
Z	compressibility factor
α	angle of attack
γ	ideal gas ratio of specific heats
γ^*	$\frac{\rho C_o^2}{P} = \left(\frac{\partial P}{\partial \rho} \right)_S \times \frac{\rho}{P}$
∇	gradient operator
δ	an indicator which is defined to be $\delta = 1$ for the axisymmetric case and $\delta = 0$ for the two-dimensional case

$\delta_{\alpha, \beta}$ Kronecker delta = 1 if $\alpha = \beta$
= 0 if $\alpha \neq \beta$

ϵ arbitrary small values ($\epsilon \ll 1$)

θ flow direction

μ Mach angle

ρ density

ψ stream function

$\vec{\Omega}$ Angular velocity of the body

σ shock angle

Subscripts

B body surface

cg center of gravity (center of rotation)

j this subscript indicates whether the perturbation is caused by
angle of attack ($j = 1$) or pitch rate ($j = 2$)

n this subscript indicates the order of derivative of angle of attack
or pitch velocity being considered

N normal to shock

0 zero yaw

S isentropic process

s immediately downstream of shock wave

∞ free stream

1 perturbations due to α and its derivatives

2 perturbations due to q and its derivatives

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1. BACKGROUND AND INTRODUCTION

Usually ballistic re-entry vehicles enter the Earth's atmosphere at some angle of attack. As the vehicle penetrates to regions of increasing density, the magnitude of the angle of attack oscillates and should damp out to very small values by the time peak dynamic pressure is reached in order to prevent excessive heating and loading. The rate at which this damping occurs is dependent on dynamic damping coefficients in the equations of motion. Unfortunately these dynamic damping coefficients, which are of paramount importance in predicting peak heating and loading along a trajectory, could not be adequately predicted at hypersonic speeds by analytic methods available when this study was initiated (e. g. , potential theory, Newtonian theory, small disturbance theory).

The potential theory approach to unsteady supersonic aerodynamics has been extensively developed in the literature. Van Dyke (Reference 7) has clearly defined the first order, and combined first and second order (hybrid) theories for simple shapes. In these methods the boundary conditions are applied at the body and along a Mach line originating at the apex of the body. Consequently the solution becomes invalid when the Mach number becomes high enough for the Mach lines to coincide with the body. Unfortunately, for bodies of practical interest, this condition occurs at relatively low Mach numbers, so that this method can only be applied in the hypersonic regime by extrapolation or by assuming Mach number independence. Further restrictions exist in

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that the geometries which can be considered are limited, inclusion of unsteady boundary layer effects is awkward, and frequency dependence is normally indeterminate.

The Mach number limitation encountered in potential theory can be overcome by using hypersonic small disturbance theory, where the boundary conditions are applied at the body and at the shock. This method, in conjunction with perturbation theory, has been applied to the determination of the hypersonic dynamic stability of oscillating wedges (Reference 8). This study has permitted an evaluation of Mach number and frequency effects on the stability derivatives for the simple two-dimensional shapes.

Newtonian impact theory is quite useful in estimating trends with variation in body geometry. However, when comparisons are made with experimental data, it is found that the predicted levels are considerably in error.* In spite of this, it should be possible to make semi-empirical modifications to this theory and obtain reasonable results, particularly in the case of three-dimensional bodies, since this is the only method which can presently account for the influence of complex geometry.

Because of these limitations in previously available analytical methods for predicting dynamic damping coefficients of bodies in hypersonic flight, an inviscid flow field approach has been taken, and small perturbation techniques have been applied to it. The objective of this work is to provide more reliable analytical means of determining

*Reference 9 provides experimental data typical of this disagreement with Newtonian predictions.

dynamic damping coefficients of pointed and spherically blunted bodies of revolution in hypersonic flight.

Since the forces and moments acting on a body can be found by a simple integration if the pressure at the body surface is known, it is apparent that the stability derivatives will result if the appropriate derivatives of pressure are integrated over the body surface. The flow field approach, with small perturbation techniques, provides these derivatives of pressure.

Because the mathematical character of the equations governing fluid flow is different in supersonic and subsonic flows, different methods must be used to solve them. For this reason, the flow field about a blunt body has been divided into subsonic, transonic, and supersonic regions (see Figure 1). A description of the solution used in each of these regions for flight at zero yaw and in unsteady motion is given in the present report.

Solutions for flow fields about pointed bodies of revolution at zero yaw and in unsteady motion have been obtained by combining the solutions for the supersonic regions (Sections 3.3 and 4.4) with solutions for pointed cones (Sections 3.2 and 4.3). These solutions for blunt and pointed bodies have been used to provide flow field information for a variety of shapes and flight conditions.

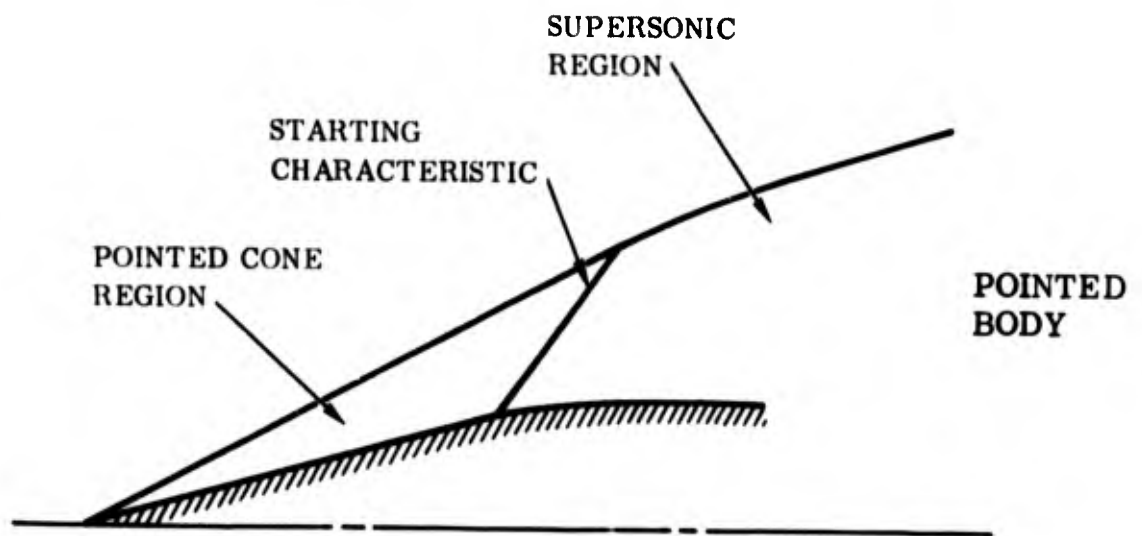
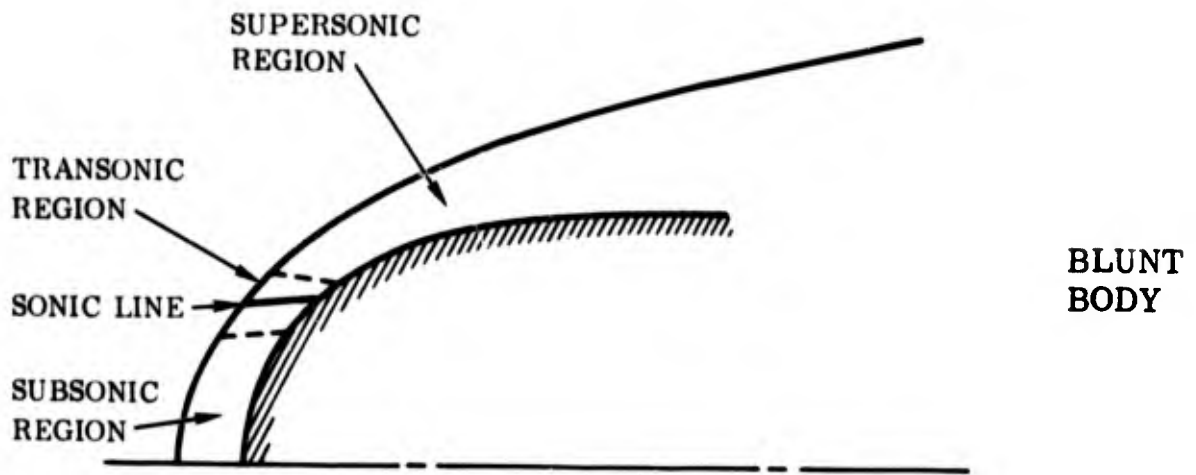


Figure 1. Regions of Flow Field

2. THE FLOW FIELD SOLUTIONS

Two reports have been written explaining the unsteady flow field solutions about pointed cones (Reference 1) and in the supersonic regions of any axisymmetric or 2-D bodies (Reference 2). Many more reports have been written about the zero yaw steady state flow field solutions (e. g. References 3, 4 and 5). The information provided in these reports is compiled here along with additional work to provide a complete description of the solution. Section 3 of this report describes the zero yaw solution. Sections 4 and 5 explain the analysis of the unsteady flow problem. The remaining sections of this report deal with programming of the solution and numerical results.

3. THE ZERO YAW STEADY STATE SOLUTIONS*

The General Electric (GE) flow field solution is capable of both real and ideal gas computations in shock layers surrounding a variety of axisymmetric and two-dimensional bodies. It is a numerical solution of the laws of conservation of mass, momentum, and energy:

$$\nabla \cdot (\rho \vec{V}) = 0 \quad (3.1)$$

$$\frac{D\vec{V}}{Dt} + \frac{1}{\rho} \nabla p = 0 \quad (3.2)$$

$$\frac{DS}{Dt} = 0 \quad (3.3)$$

and the state relations:

$$\frac{p}{\rho} = ZRT \quad (3.4)$$

$$S = S(\rho, T) \quad (3.5)$$

$$Z = Z(\rho, T), \quad Z = Z(\rho, S) \quad (3.6)$$

*Reference 3 provides greater detail on these solutions.

$$p = p(\rho, S) \quad \text{and} \quad (3.7)$$

$$h = h(\rho, S) \quad (3.8)$$

The last five of these relations are in tabular form, for air treated as a real gas in chemical equilibrium.

The GE solution also uses a parameter γ^* , where γ^* is defined as:

$$\gamma^* = \frac{\rho}{p} \left(\frac{\partial p}{\partial \rho} \right)_S \quad (3.9)$$

and is computed as a function of S and p, using the expression:

$$\gamma^* = \frac{a}{p} + b \quad (3.10)$$

The coefficients a and b are tabulated as functions of entropy and pressure. Reference 3 gives a more detailed description of this parameter.

If it is desired to do an ideal gas calculation, Z becomes unity, γ^* becomes the ratio of specific heats,⁺ and the real gas tabulations (Equations 3.5, 3.7 and 3.8) are reduced to several ideal gas relations:

⁺For an ideal gas a = 0 and b = the ratio of specific heats.

$$p \rho^{-\gamma} = \text{constant along streamlines} \quad (3.11)$$

$$\frac{1}{2} V^2 + \frac{\gamma}{\gamma-1} \frac{p}{\rho} = \text{constant} \quad (3.12)$$

The boundary conditions imposed on the problem consist of the free stream conditions upstream of the shock wave,* and the condition that no mass flows through the body surface. Since the mathematical character of the governing equations is different on opposite sides of the sonic line, the solution is carried out in a different way in the transonic and supersonic regions of the shock layer. The solution in both regions is carried out on an IBM 7094 computer.

3.1 THE TRANSONIC REGION

The computation for a blunt body is started in the transonic region, which includes the sonic line and a small part of the shock layer on each side of the sonic line. A coordinate grid of streamlines and the lines normal to them is used. The solution is a direct one. It is started by making an initial estimate of the shock shape and of the pressure distribution at the body surface. The location of a streamline a small distance from this body is then computed (as well as the values of the flow field variables on it), to satisfy the governing equations. This process of stepping to the next streamline is repeated until a new shock wave, which satisfies the conservation of mass law, is reached. The shape of this new shock wave, as well

*Applied through the Rankine-Hugoniot relations.

as the pressures just downstream of it, are compared with the shape and corresponding pressures for the initial estimate. New estimates of shock shape and body pressure distribution are based on this comparison and on a general inspection of the results obtained in the entire transonic region. This iterative cycle is repeated until the estimates and computed values agree closely. * It is usually possible to obtain pressures downstream of the estimated and computed shock waves to agree within 2 percent, and to get the estimated and computed shock waves to coincide within 0.004 of the body radius of curvature at the stagnation point. At each step of the iteration the choice of a new estimate of shock shape and body pressure distribution is made by the operator. All other iterative cycles are automatic.

3.2 THE POINTED CONE SOLUTION

When a flow field solution is needed for a pointed body, the supersonic program provides it. However, the supersonic program requires a solution along a starting line. This starting solution is obtained by approximating a small portion of the nose of the pointed body by a pointed cone. The solution for a pointed cone is then obtained by the method of Taylor and Maccoll (Reference 6), modified to provide a real gas solution, if desired. This solution has been included in a single computer program with the unsteady solution for the pointed cone (Section 4.3). A detailed derivation of this solution appears in Reference 5.

*A guide to converging the zero yaw transonic solution is given in Appendix IV.

3.3 THE SUPERSONIC REGION*

The steady state solution in the supersonic region is carried out by the method of characteristics for both pointed and blunt bodies. Three basic directions are used: the flow direction (constant ψ , constant S) and the directions of the Mach lines. The angles between the Mach lines and the flow direction are:

$$\pm \mu = \sin^{-1} \left(\frac{1}{M} \right) \quad (3.13)$$

The Mach lines are the characteristics (in the mathematical sense) of the continuity and momentum equations. Changes along these lines are defined by Equation (3.14):

$$\frac{d\theta}{d\ell} \pm \frac{\delta}{r} \sin \mu \sin \theta \pm \frac{\cot \mu}{\rho V^2} \frac{dp}{d\ell} = 0 \quad (3.14)$$

where θ = the flow angle measured from the axial.

ℓ = the distance measured along a characteristic.

The + and - signs apply to the left and right Mach lines, respectively.

In addition to Equation (3.14), the equation of state, the condition of constant total energy, and the condition of isentropic flow along streamlines must be satisfied. The

*This Section is taken from Reference 3.

numerical solution is started from a line along which the solution has previously been computed by either the transonic or pointed cone program.

4. THE UNSTEADY FLOW FIELD SOLUTIONS

4.1 DESCRIPTION OF THE PERTURBATION SCHEME*

The flow field about a body in flight is determined by the solution of the non-linear boundary value problem stated in Section 4.2.1. This boundary value problem is stated in a coordinate system fixed in the body, and consequently, the motions of the body, \vec{V}_{cg} and $\vec{\Omega}$, appear as "driving functions" in the problem. For the simple planar trajectory considered here, the motions of the body are given in terms of two functions of time, $\alpha(t)$ and $q(t)$, and the constant speed $V_\infty = |\vec{V}_{cg}|$, by equations (4.18). In order to solve the boundary value problem defined in Section 4.2.1, it is necessary to specify the two functions of time $\alpha(t)$ and $q(t)$. The flow field variables are then functionals of the functions $\alpha(t)$ and $q(t)$ in that they depend on all the values taken on by $\alpha(t)$ and $q(t)$ in the interval from the initial time, $t = 0$, to the current time, $t = t_c$ and are ordinary functions of position as given by the three coordinates, (R, ω, φ) .

In the perturbation scheme utilized in this work, $\alpha(t)$ and $\frac{q(t)L}{V_\infty}$ are taken to be small quantities on the order of ϵ_1 and ϵ_2 , respectively, which then become the perturbation parameters, with the substitutions:

$$\alpha(t) = \epsilon_1 \bar{\alpha}(t) \tag{4.2}$$

$$q(t) = \epsilon_2 \bar{q}(t) \left(\frac{V_\infty}{L} \right) \tag{4.3}$$

*This section appeared originally in Reference 1.

the pressure, for example, will be a functional of the functions $\bar{\alpha}(t)$ and $\bar{q}(t)$ and an ordinary function of the parameters ϵ_1 and ϵ_2 . It is assumed that p can be expanded in Taylor's series in the parameters, ϵ_1 and ϵ_2 , to give a series of the form:

$$p = p_0 + \epsilon_1 p_1 + \epsilon_2 p_2 + \dots \text{ (H. O. T.)}$$

The coefficient, p_0 , is the pressure field produced by the body in steady flight at zero angle of attack and can be found by established methods. The coefficients, p_1 and p_2 , give the first order effects of angle of attack and pitching rate, respectively, and are to be determined by solution of Equations (4.21) through (4.24). They are functionals of the functions $\bar{\alpha}(t)$ and $\bar{q}(t)$ and ordinary functions of the spatial coordinates. (In Section 4.3) it is shown that they can be represented formally by series of the type

$$p_1 = \sum_{n=0}^{\infty} p_{1,n} \left(\frac{R}{V_{\infty}} \right)^n \frac{d^n \bar{\alpha}}{dt^n} \cos \varphi = \left\{ p_{1,0} \bar{\alpha} + p_{1,1} \left(\frac{R}{V_{\infty}} \right) \frac{d\bar{\alpha}}{dt} + \dots \right\} \cos \varphi \quad (4.5)$$

(and a similar series for p_2) where the $p_{1,n}$ are functions only of the ray angle, ω . This solution holds after "starting transients" have died out. The coefficient, $p_{1,0}$ gives the effect of small yaw in the steady state. The coefficients, $p_{1,1}$, $p_{1,2}$, etc., give the effects of time varying angle of attack.

It is the coefficients, $p_{j,n}$, and corresponding quantities for the other flow field variables, which are found as a result of this analysis. The method of finding them is numerical and is described in the following sections. The pressure coefficients yield corresponding force and moment coefficients which are dynamic stability derivatives.

4.2 DERIVATION OF THE PERTURBATION EQUATIONS*

4.2.1 Statement of the Boundary Value Problem in General Form

The inviscid flow field boundary value problem can be stated to an observer in a body-fixed coordinate system, x, y, z , (Figure 2) by a transformation of coordinates from an inertial system. Lamb (Reference 10), gives the appropriate transformed continuity and Euler equations.

The continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{V} = 0; \quad (4.6)$$

The Euler equation:

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} + 2 (\vec{\Omega} \times \vec{V}) + \frac{\nabla p}{\rho} = - \frac{d\vec{V}_{cg}}{dt} - \vec{\Omega} \times \vec{V}_{cg} + \vec{r} \times \frac{d\vec{\Omega}}{dt} + (\vec{\Omega} \times \vec{r}) \times \vec{\Omega} . \quad (4.7)$$

The form of the continuity Equation (4.6) is unaltered by the transformation. The form of the Euler Equation (4.7) differs from Equation (3.2) in that the acceleration of a fixed point in the moving frame of reference appears as a body force involving the vector velocity of the center of gravity, $\vec{V}_{cg}(t)$, and the vector rotation of the body,

*A more detailed derivation appears in Reference 1.

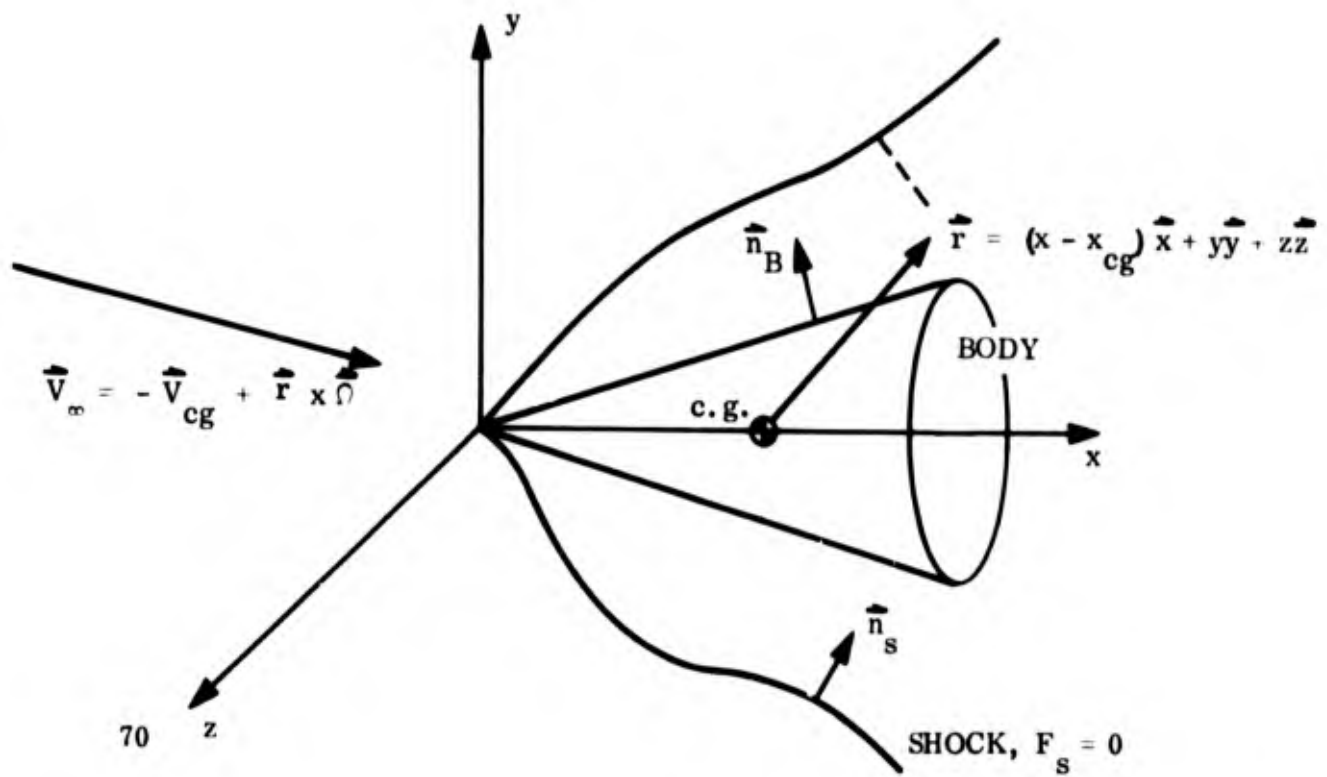


Figure 2. Inviscid Flow Field Boundary Value Problem

$\vec{\Omega}(t)$, on the right-hand side. The vector rotation causes a given fluid particle to experience a Coriolis acceleration, $2(\vec{\Omega} \times \vec{V})$, which is present on the left-hand side.

The continuity and Euler equations must be complemented by the energy equation for the adiabatic flow,

$$\frac{\partial S}{\partial t} + \vec{V} \cdot \nabla S = 0 \quad (4.8)$$

and an equation of state for the gas, here taken to be,

$$p = p(\rho, S) \quad (4.9)$$

The precise form of the relation indicated in Equation (4.9) depends on whether the gas is an ideal gas, a gas at chemical equilibrium, or a gas in some "frozen" composition. The development of the equations can, however, proceed without specification of the precise form of the equation of state.

Equations (4.6) through (4.9) are a complete set of flow equations for the determination of the pressure, p , the density, ρ , the entropy, S , and the three components of the fluid velocity (measured in the moving frame of reference), \vec{V} , for prescribed motions of the body, \vec{V}_{cg} and $\vec{\Omega}$. In order to solve them, initial conditions at some instant of time and boundary conditions at the body and shock, must be given.

As initial conditions, it is assumed that at $t = 0$, the field is the steady-state, axisymmetric field produced by a uniform forward translation at speed V_∞ , of the body along its axis of symmetry.

At the body surface, the flow must be tangent to the body surface and the boundary condition is:

$$\vec{V} \cdot \vec{n}_B = 0. \quad (4.10)$$

On the shock surface,

$$F_s(x, y, z, t) = 0, \quad (4.11)$$

(which must be found as a part of the solution), the shock equations give the flow variables as functions of the relative velocity of the shock and the free stream flow,

and the instantaneous unit inward* normal to the shock,

$$\vec{n}_s = \frac{\pm \nabla F_s}{|\nabla F_s|} \quad (4.12)$$

The shock equations can be reduced to the following set of three algebraic equations:

$$\left. \begin{aligned} \rho_s V_{Ns} &= \rho_\infty V_{N\infty} \\ \rho_s V_{Ns}^2 + p_s &= \rho_\infty V_{N\infty}^2 + p_\infty \\ h(p_s, \rho_s) + \frac{V_{Ns}^2}{2} &= h(p_\infty, \rho_\infty) + \frac{V_{N\infty}^2}{2} \end{aligned} \right\} \quad (4.13)$$

These equations express, respectively, conservation of mass, momentum normal to the shock, and energy, in a form which utilizes the result that the velocity components tangent to the shock are unchanged in crossing the shock. In these equations, $V_{N\infty}$ is the component normal to the shock, of the relative velocity between the shock and the flow on the upstream side of the shock, and V_{Ns} is the corresponding component on the downstream side of the shock.

The flow velocity on the upstream side of the shock is given by:

$$\vec{V}_\infty = -\vec{V}_{cg} + (\vec{r} \times \vec{\Omega}) \quad (4.14)$$

*Either the + or - will be used in equation (4.12), whichever provides the inward normal.

and the component of shock velocity along its normal is given by:

$$-\frac{\frac{\partial F_s}{\partial t}}{|\nabla F_s|}$$

Therefore, $V_{N\infty}$, is given by:

$$V_{N\infty} = \vec{V}_\infty \cdot \vec{n}_s + \frac{\frac{\partial F_s}{\partial t}}{|\nabla F_s|} \quad (4.15)$$

$$= -\vec{V}_{cg} \cdot \vec{n}_s + (\vec{r} \times \vec{\Omega}) \cdot \vec{n}_s + \frac{\frac{\partial F_s}{\partial t}}{|\nabla F_s|}$$

If Equation (4.13) is presumed to be solved in the form*

$$\left. \begin{aligned} p_s &= p_s(V_{N\infty}, p_\infty, \rho_\infty) \\ \rho_s &= \rho_s(V_{N\infty}, p_\infty, \rho_\infty) \\ V_{Ns} - V_{N\infty} &= \Delta V_N = \Delta V_N(V_{N\infty}, p_\infty, \rho_\infty) \end{aligned} \right\} \quad (4.16)$$

The first two equations of Equation (4.16) give the pressure and density downstream of the shock explicitly in terms of the function giving the shock, F_s , and the

*There are numerous procedures in the literature for obtaining such a solution numerically. Reference 11 presents the procedure used in the subsequent numerical work.

motions of the body, \vec{V}_{cg} and $\vec{\Omega}$ by use of Equation (4. 15). The third equation of Equation (4. 16) gives the three velocity components of the flow on the downstream side of the shock by further use of Equation (4. 17):

$$\vec{V}_s = -\vec{V}_{cg} + (\vec{r} \times \vec{\Omega}) + \Delta V_N \vec{n}_s. \quad (4. 17)$$

4. 2. 2 Reduction of the Boundary Value Problem to Perturbation Form

The equations and conditions of Section 4. 2. 1 define the boundary value problem for determination of the inviscid flow field about a body whose motions, \vec{V}_{cg} and $\vec{\Omega}$, are known. In this Section, the boundary value problem is reduced to a perturbation form applicable to bodies of revolution which move at constant speed, $|\vec{V}_{cg}| = \text{const.} = V_\infty$, in such a manner that the z-axis is always parallel to a fixed reference direction. *

For the type of motion just described, the vector velocity and rotation can be written in terms of two time variable functions, $\alpha(t)$ (the angle of attack) and $q(t)$:

$$\left. \begin{aligned} \vec{V}_{cg} &= V_\infty (-\vec{x} \cos \alpha + \vec{y} \sin \alpha) \\ \vec{\Omega} &= q \vec{z} \end{aligned} \right\} \quad (4. 18)$$

*Although this analysis does not employ stability axes, the force and moment derivatives are conventional.

To put the problem in perturbation form it is assumed that $\alpha \ll 1$ and $\frac{qL}{V_\infty} \ll 1$, and parameters ϵ_1 and ϵ_2 which measure the magnitudes of α and $\frac{qL}{V_\infty}$ are introduced by the Equations:

$$\left. \begin{aligned} \alpha &= \epsilon_1 \bar{\alpha} \\ \text{and } \frac{qL}{V_\infty} &= \epsilon_2 \bar{q} \end{aligned} \right\} \quad (4.19)$$

In Equation (4.19), $\bar{\alpha}(t)$ and $\bar{q}(t)$ are assumed to be of the order of one in the time interval of interest, and ϵ_1 and ϵ_2 (which are constants) are assumed to be much less than one. The flow field variables can be regarded as functionals of the functions $\bar{\alpha}(t)$ and $\bar{q}(t)$ (in the sense defined in Reference 12) and ordinary functions of the perturbation parameters ϵ_1 and ϵ_2 .

In the perturbation scheme, it is assumed that each flow field variable can be expanded in Taylor's series in the variables, ϵ_1 and ϵ_2 . For example, it is assumed that the pressure can be written in the form:

$$p = p_0(x, y, z) + \epsilon_1 p_1(x, y, z, t) + \epsilon_2 p_2(x, y, z, t) + \dots \text{H.O.T.} \quad (4.20)$$

These expanded forms are then substituted into the equations defining the boundary value problem and the resulting set of perturbed equations is separated into a number of sets of perturbation equations by the usual process of equating terms in like powers of ϵ_1 and ϵ_2 . (Only the three lowest order terms are considered).

The boundary value problem so defined for the lowest order coefficients, p_0 , ρ_0 , S_0 and \vec{V}_0 , is, of course, the problem of determining the steady-state, axisymmetric field produced by the body as it translates with constant speed, V_∞ , parallel to its axis of symmetry. *

Substitution of Equation (4.20) and like expressions for the density, entropy, and fluid velocity into Equations (4.6) through (4.9) to obtain the two sets of equations governing the coefficients of ϵ_1 and ϵ_2 yields,

$$\frac{\partial \rho_j}{\partial t} + \nabla \cdot (\rho_0 \vec{V}_j + \rho_j \vec{V}_0) = 0; \quad (4.21)$$

$$\frac{\partial \vec{V}_j}{\partial t} + (\vec{V}_0 \cdot \nabla) \vec{V}_j + (\vec{V}_j \cdot \nabla) \vec{V}_0 + \frac{\nabla p_j}{\rho_0} - \frac{\nabla p_0}{\rho_0^2} \rho_j = \vec{F}_j; \quad (4.22)$$

$$\frac{\partial S_j}{\partial t} + \vec{V}_0 \cdot \nabla S_j + \vec{V}_j \cdot \nabla S_0 = 0 \quad (4.23)$$

$$p_j = C_0^2 \rho_j + e_0^2 S_j; \quad (4.24)$$

where:

$$\vec{F}_j = \begin{cases} -V_\infty \frac{d\bar{\alpha}}{dt} \vec{y}, j = 1 \\ 2(\vec{V}_0 \times \vec{z}) \bar{q} \frac{V_\infty}{L} + \frac{V_\infty^2}{L} \bar{q} \vec{y} + (\vec{r} \times \vec{z}) \frac{V_\infty}{L} \frac{d\bar{q}}{dt}, j = 2 \end{cases}$$

*The solution of the zero yaw field is described in Section 3.

The boundary condition at the body surface, Equation (4. 10), separates into the two sets of conditions,

$$\vec{V}_j \cdot \vec{n}_B = 0, \quad (j = 1, 2) \quad (4. 25)$$

4. 3 THE SOLUTION FOR THE POINTED CONE*

For the conical geometry, the equation giving the shock, Equation (4. 11), is most conveniently expressed in terms of the spherical coordinates, R, ω, φ , (Figure 3), in a form giving the ray angle, ω , as an explicit function of the radial coordinate, R , meridional angle, φ , and time, t ,

$$F_s = \omega_s(R, \varphi, t) - \omega = 0 \quad (4. 26)$$

The perturbation form for the shock is then (to the three lowest order terms),

$$\omega = \omega_s = \omega_{s0} + \epsilon_1 \omega_{s1}(R, \varphi, t) + \epsilon_2 \omega_{s2}(R, \varphi, t), \quad (4. 27)$$

where:

ω_{s0} is the constant shock angle from the steady-state, axisymmetric, conical flow solution. The unit inward normal to the shock is given by:

$$\vec{n}_s = -\vec{\omega} + \sum_{j=1}^2 \epsilon_j \left\{ \vec{R} \frac{R \partial \omega_{sj}}{\partial R} + \frac{\vec{\varphi}}{\sin \omega_{s0}} \frac{\partial \omega_{sj}}{\partial \varphi} \right\} \cdot \quad (4. 28)$$

*This solution is given in greater detail in Reference 1.

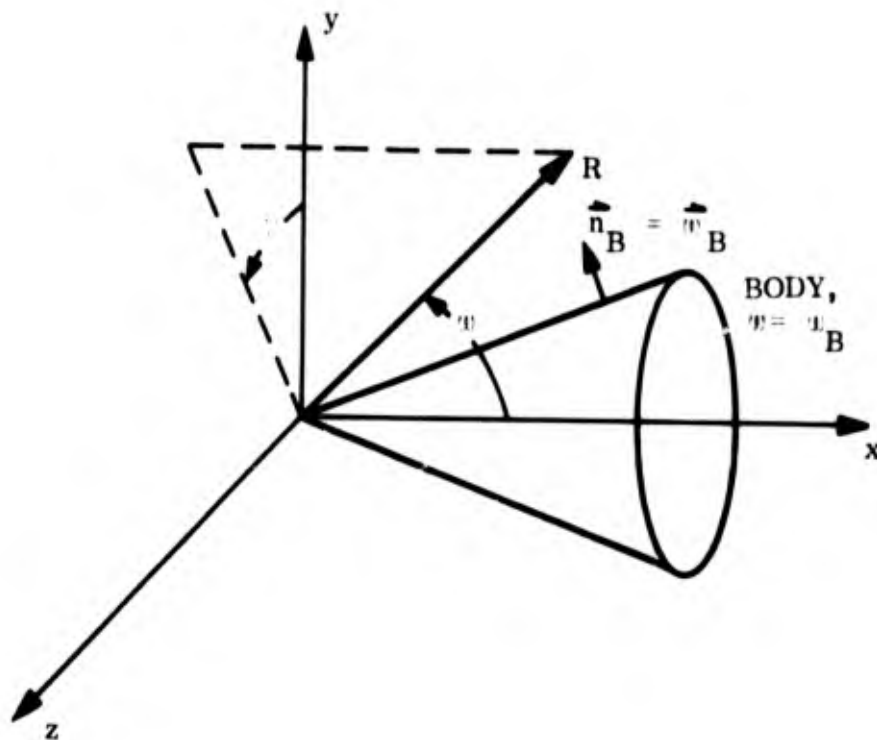


Figure 3. The Spherical Coordinate System

and the flow velocity on the upstream side of the perturbed shock is:

$$\vec{V}_\infty = \vec{R} \{ u_{\infty 0} + \epsilon_1 u_{\infty 1} + \epsilon_2 u_{\infty 2} \} + \vec{w} \{ v_{\infty 0} + \epsilon_1 v_{\infty 1} + \epsilon_2 v_{\infty 2} \} + \vec{\varphi} \{ \epsilon_1 w_{\infty 1} + \epsilon_2 w_{\infty 2} \} \quad (4.29)$$

where:

$$\left. \begin{aligned} u_{\infty 0} &= V_\infty \cos \omega_{s0} \\ u_{\infty 1} &= -V_\infty (\alpha \sin \omega_{s0} \cos \varphi + \omega_{s1} \sin \omega_{s0}) \\ u_{\infty 2} &= \frac{\bar{q} V_\infty}{L} x_{cg} \sin \omega_{s0} \cos \varphi - V_\infty \omega_{s2} \sin \omega_{s0} \end{aligned} \right\} \quad (4.30)$$

$$v_{\infty 0} = -V_{\infty} \sin \omega_{s0}$$

$$v_{\infty 1} = -V_{\infty} (\bar{\alpha} \cos \omega_{s0} \cos \varphi + \omega_{s1} \cos \omega_{s0})$$

$$v_{\infty 2} = \frac{\bar{q} V_{\infty}}{L} (x_{cg} \cos \omega_{s0} - R) \cos \varphi - V_{\infty} \omega_{s2} \cos \omega_{s0} \quad (4.30)$$

Cont'd)

$$w_{\infty 1} = \bar{\alpha} V_{\infty} \sin \varphi$$

$$w_{\infty 2} = \frac{\bar{q} V_{\infty}}{L} (R \cos \omega_{s0} - x_{cg}) \sin \varphi$$

The consequential perturbation form for the normal component of relative velocity between the upstream flow and the shock is obtain from Equation (4. 15) as:

$$V_{N\infty} = -v_{\infty 0} + \sum_{j=1}^2 \epsilon_j \bar{V}_{Nj} \quad (4.31)$$

where:

$$V_{Nj} = u_{\infty 0} R \frac{\partial \omega_{sj}}{\partial R} + R \frac{\partial \omega_{sj}}{\partial t} - v_{\infty j}; j = 1, 2$$

Substitution of Equation (4. 31) into Equations (4. 16) and (4. 17) gives the perturbation forms of the pressure, density, and velocity components on the downstream side of the perturbed shock. These resulting expressions must be equated to expressions for the corresponding variables in the field, evaluated at the perturbed shock, to obtain the proper boundary conditions for the perturbation Equations. The resulting conditions are:

$$\left. \begin{aligned}
p_j(R, \omega_{s0}, \varphi, t) &= - \frac{\partial p_o}{\partial \omega} \Big|_{\omega = \omega_{s0}} \omega_{sj} + \frac{\partial p_s}{\partial V_{N\infty}} \Big|_{V_{N\infty} = -v_{\infty o}} \bar{V}_{Nj} \\
\rho_j(R, \omega_{s0}, \varphi, t) &= - \frac{\partial \rho_o}{\partial \omega} \Big|_{\omega = \omega_{s0}} \omega_{sj} + \frac{\partial \rho_s}{\partial V_{N\infty}} \Big|_{V_{N\infty} = -v_{\infty o}} \bar{V}_{Nj} \\
\vec{V}_j(R, \omega_{s0}, \varphi, t) &= (u_{\infty j} \vec{R} + v_{\infty j} \vec{\omega} + w_{\infty j} \vec{\varphi}) - \left(\frac{\partial u_o}{\partial \omega} \Big|_{\omega = \omega_{s0}} \vec{R} + \frac{\partial v_o}{\partial \omega} \Big|_{\omega = \omega_{s0}} \vec{\omega} \right) \omega_{sj} + \\
\Delta V_N(-v_{\infty o}, p_{\infty}, \rho_{\infty}) &\left(R \frac{\partial \omega_{sj}}{\partial R} \vec{R} + \frac{1}{\sin \omega_{s0}} \frac{\partial \omega_{sj}}{\partial \varphi} \vec{\varphi} \right) - \frac{\partial \Delta V_N}{\partial V_{N\infty}} \Big|_{V_{N\infty} = -v_{\infty o}} \bar{V}_{Nj} \vec{\omega}
\end{aligned} \right\} \begin{array}{l} (4.32) \\ j = 1, 2 \end{array}$$

In Equation (4.32) the partial derivatives with respect to ω arise as a result of evaluating the variables in the field at the perturbed shock. The partial derivatives with respect to $V_{N\infty}$ are to be obtained by differentiation of the expressions indicated in Equation (4.16); expressions for them are given in Section 4.4.4.1.

To complete specification of the boundary value problem for the perturbation variables, it is necessary to give the initial conditions at $t=0$. Corresponding to the assumption stated in Section 4.2.1, this condition is that the perturbation variables vanish at $t=0$.

Equations (4.21) through (4.24) together with the boundary condition at the body surface, Equation (4.25), the boundary conditions at the shock, Equation (4.32), and the initial conditions, define two separate linear boundary value problems for the

perturbation variables. In these problems, the "motions" of the body, $\bar{\alpha}(t)$ and $\bar{q}(t)$, appear in inhomogeneous terms and play the role of "driving functions." Because of the linearity of the problems, the solutions for arbitrarily prescribed functions, $\bar{\alpha}$ and \bar{q} , can be divided into two parts which can be called the particular solution and the complementary solution. The particular solutions are defined as solutions of the inhomogeneous problems which reduce to the trivial solution (all perturbation variables equal zero) when $\bar{\alpha}$ and \bar{q} are identically zero; the complementary solutions are defined as solutions of the homogeneous problems (obtained by setting $\bar{\alpha}$ and \bar{q} equal to zero) which cause the complete solution to satisfy the initial conditions.

On physical grounds, it is known that the complementary solutions must "die out" in time, and their decay time is some nominal multiple of the time it takes the body to move through its own length, $\frac{L}{V_\infty}$. Subsequent to this decay time the flow field about the body is given by the particular solutions alone. It is, therefore, only the particular solutions which are of interest here.

The problem of determining the particular solutions of the boundary value problems for arbitrarily prescribed functions, $\bar{\alpha}(t)$ and $\bar{q}(t)$, can be approached in several ways, e.g., by use of the Laplace transform. The approach taken here is one which reduces each problem to a problem of solving an infinite sequence of sets of ordinary differential equations having the ray angle, ω , as independent variable. The substitutions which accomplish this reduction are all of the form:

$$p_1(R, \omega, \varphi, t) = \sum_{n=0}^{\infty} p_{1,n}(\omega) \left(\frac{R}{v_{\infty}}\right)^n \frac{d^n \bar{\alpha}}{dt^n} \cos \varphi. \quad (4.33)^*$$

In writing these series, it is assumed that $\bar{\alpha}(t)$ and $\bar{q}(t)$ are analytic functions. Also, it is recognized that there is no a-priori guarantee that these series are convergent for any given functions $\bar{\alpha}(t)$ and $\bar{q}(t)$.

The two infinite sequences of sets of ordinary differential Equations for the variables, $p_{j,n}(\omega)$, etc., which arise from the substitutions of the expressions represented by Equation (4.33) can be written in the common form, ($j=1, 2$), ($n=0, 1, 2, 3, \dots$).

$$\frac{d}{d\omega} \left\{ (\rho_o v_{j,n} + v_o \rho_{j,n}) \sin \omega \right\} + (n+2+\delta_{j,2}) (\rho_o u_{j,n} + u_o \rho_{j,n}) \sin \omega + \rho_o w_{j,n} = \mathcal{A}_{j,n} \quad (4.34a)$$

$$v_o \frac{du_{j,n}}{d\omega} - v_o v_{j,n} + (n+\delta_{j,2}) \left(u_o u_{j,n} + \frac{p_{j,n}}{\rho_o} \right) = \gamma_{j,n} \quad (4.34b)$$

$$v_o \frac{dv_{j,n}}{d\omega} + u_o v_{j,n} + v_o u_{j,n} + v_{j,n} \frac{dv_o}{d\omega} + \frac{1}{\rho_o} \frac{dp_{j,n}}{d\omega} - \frac{\rho_{j,n}}{\rho_o^2} \frac{dp_o}{d\omega} + (n+\delta_{j,2}) u_o v_{j,n} = G_{j,n} \quad (4.34c)$$

$$v_o \frac{dw_{j,n}}{d\omega} + u_o w_{j,n} + v_o w_{j,n} \cot \omega - \frac{p_{j,n}}{\rho_o \sin \omega} + (n+\delta_{j,2}) u_o w_{j,n} = \mathcal{H}_{j,n} \quad (4.34d)$$

$$v_o \frac{ds_{j,n}}{d\omega} + (n+\delta_{j,2}) u_o s_{j,n} = \mathcal{L}_{j,n} \quad (4.34e)$$

$$p_{j,n} = C_o^2 \rho_{j,n} + e_o^2 s_{j,n} \quad (4.34f)$$

*The "zeroth" derivative, $\frac{d^0 \bar{\alpha}}{dt^0}$, is equal to $\bar{\alpha}$, by convention.

where:

$$Q_{j,n} = - (1 - \delta_{o,n}) V_{\infty} \rho_{j,n-1} \sin \omega$$

$$Y_{j,n} = - (1 - \delta_{o,n}) V_{\infty} u_{j,n-1} - \delta_{1,n} \delta_{j,1} V_{\infty}^2 \sin \omega + \delta_{o,n} \delta_{j,2} V_{\infty} (2v_o + V_{\infty} \sin \omega)$$

$$G_{j,n} = - (1 - \delta_{o,n}) V_{\infty} v_{j,n-1} - \delta_{1,n} V_{\infty}^2 (\delta_{j,1} \cos \omega + \delta_{j,2}) + \delta_{o,n} \delta_{j,2} V_{\infty} (-2u_o + V_{\infty} \cos \omega)$$

$$Z_{j,n} = - (1 - \delta_{o,n}) V_{\infty} w_{j,n-1} + \delta_{1,n} V_{\infty}^2 (\delta_{j,1} + \delta_{j,2} \cos \omega) +$$

$$\delta_{o,n} \delta_{j,2} \left\{ 2 V_{\infty} (u_o \cos \omega - v_o \sin \omega) - V_{\infty}^2 \right\}$$

$$S_{j,n} = - (1 - \delta_{o,n}) V_{\infty} S_{j,n-1}$$

The boundary conditions for these Equations can also be written in a common form.

At the body, $\omega = \omega_B$, the condition of tangency of the flow becomes:

$$v_{j,n} = 0 \text{ at } \omega = \omega_B \quad (4.35)$$

At the shock, $\omega = \omega_{SO}$, the perturbation variables, $p_{j,n}$, etc., are related to the quantities, $\omega_{j,n}$, which describe the perturbation in the shock shape by:

$$p_{j,n} = \left\{ - \frac{\partial p_o}{\partial \omega} \Big|_{\omega = \omega_{SO}} + (n+1 + \delta_{j,2}) V_{\infty} \cos \omega_{SO} \frac{\partial p_s}{\partial V_{N_{\infty}}} \Big|_{V_{N_{\infty}} = -v_{\infty o}} \right\} \omega_{j,n} +$$

$$\lambda_{j,n} \frac{\partial p_s}{\partial V_{N_{\infty}}} \Big|_{V_{N_{\infty}} = -v_{\infty o}} \quad (4.36a)$$

$$\rho_{j,n} = \left\{ -\frac{\partial \rho_0}{\partial \omega} \Big|_{\omega=\omega_{so}} + (n+1+\delta_{j,2}) V_{\infty} \cos \omega_{so} \frac{\partial \rho_2}{\partial V_{N\infty}} \Big|_{V_{N\infty}=-v_{\infty 0}} \right\} \omega_{j,n} + \quad (4.36b)$$

$$\lambda_{j,n} \frac{\partial \rho_2}{\partial V_{N\infty}} \Big|_{V_{N\infty}=-v_{\infty 0}}$$

$$S_{j,n} = \frac{P_{j,n}}{e_0^2} - \frac{C_0^2}{e_0^2} \rho_{j,n} \quad (4.36c)$$

$$u_{j,n} = (n+1+\delta_{j,2}) \Delta V_n(-v_{\infty 0}, p_{\infty}, \rho_{\infty}) \omega_{j,n} - \delta_{o,n} \delta_{j,1} V_{\infty} \sin \omega_{so} \quad (4.36d)$$

$$v_{j,n} = \left\{ -\left(u_0 + \frac{\partial v_0}{\partial \omega} \Big|_{\omega=\omega_{so}} - (n+1+\delta_{j,2}) V_{\infty} \cos \omega_{so} \frac{\partial \Delta V_N}{\partial V_{N\infty}} \Big|_{V_{N\infty}=-v_{\infty 0}} \right) \omega_{j,n} \right. \quad (4.36e)$$

$$\left. - \left(\delta_{o,n} V_{\infty} \left[\delta_{j,1} \cos \omega_{so} + \delta_{j,1} \right] + \lambda_{j,n} \frac{\partial \Delta V_N}{\partial V_{N\infty}} \Big|_{V_{N\infty}=-v_{\infty 0}} \right) \right\}$$

$$w_{j,n} = \left\{ -\frac{\Delta V_N(-v_{\infty 0}, p_{\infty}, \rho_{\infty})}{\sin \omega_{so}} \right\} \omega_{j,n} + \delta_{o,n} V_{\infty} (\delta_{j,1} + \delta_{j,2} \cos \omega_{so}) \quad (4.36f)$$

where:

$$\lambda_{j,n} = \delta_{o,n} V_{\infty} (\delta_{j,1} \cos \omega_{so} + \delta_{j,2}) + (1-\delta_{o,n}) V_{\infty} \omega_{j,n-1}$$

Equations (4.34) are ordinary differential equations with variable coefficients, which are determined by the solution for the axially symmetric conical field, u_0, v_0, p_0 and ρ_0 .

For $j=1$ and $n=0$ the equation is equivalent to those derived by Stone (Reference 13) for the problem of cones at small yaw. For either value of j and any value of n (except $n=0$), the Equations and boundary conditions are complete, provided that the solution for the same value of j and a value of n which is one less, has been previously solved. For $n=0$, the equations and boundary conditions are complete without a knowledge of the solution for any other value of j and n . For any values of j and n , the problem is a two-point boundary value problem in which the boundary conditions contain an unknown constant, $\omega_{j,n}$. The equations have a singular behavior at the body surface where $v_o = 0$.

4.3.1 The Numerical Solution of The Pointed Cone Problem.

The numerical method of solution of the problem is based on manipulated forms of Equation (4.34a through f). The objective of these manipulations is to reduce as much as possible of the numerical work to performing quadratures, and also to provide a convenient method of handling the singular point at the body surface.

The manipulated forms of the equations are:

from Equation (4.34e),

$$S_{j,n} = \left(\frac{S_{j,n}}{\psi_o^m} \right) \Big|_{\omega = \omega_{so}}^{\omega} \psi_o^m + \psi_o^m \int_{\omega_{so}}^{\omega} \frac{d_{j,n}}{v_o \psi_o^m} d\omega, \quad (4.37)$$

where:

$$\psi_o = \sqrt{-\rho_o v_o \sin \omega} \quad ;$$

and

$$m = n + \delta_{2,j}$$

from Equations (4.34b, c and e):

$$H_{j,n} = \left(\frac{H_{j,n}}{\psi_o^m} \right) \Bigg|_{\omega = \omega_{so}} \psi_o^m + \psi_o^m \int_{\omega_{so}}^{\omega} \frac{u_o \gamma_{j,n} + v_o G_{j,n} + T_o J_{j,n}}{v_o \psi_o^m} d\omega \quad (4.38)$$

and

$$J_{j,n} = \frac{du_{j,n}}{d\omega} = (m+1)v_{j,n} = \frac{\gamma_{j,n} - mH_{j,n} + mT_o S_{j,n}}{v_o} \quad (4.39)$$

where:

$H_{j,n}$ is a quantity related to perturbations in the stagnation enthalpy,

$$H_{j,n} = \frac{p_{j,n}}{\rho_o} + u_o u_{j,n} + v_o v_{j,n} + T_o S_{j,n}; \quad (4.40)$$

from Equations (4.34d) and (4.39), and (4.40),

$$W_{j,n} = w_{j,n} + \frac{u_{j,n}}{(m+1) \sin \omega} = \left(\frac{w_{j,n} \sin \omega + \frac{u_{j,n}}{m+1}}{\psi_o^{m+1}} \right) \Bigg|_{\omega = \omega_{so}} \frac{\psi_o^{m+1}}{\sin \omega} + \frac{\psi_o^{m+1}}{\sin \omega} \int_{\omega_{so}}^{\omega} \frac{J_{j,n} \sin \omega + \left(H_{j,n} - T_o S_{j,n} + \frac{v_o J_{j,n}}{(m+1)} \right)}{v_o \psi_o^{m+1}} d\omega \quad (4.41)$$

and, from the preceding Equations and Equations (4.34a and f),

$$\frac{d^2 u_{j,n}}{d\omega^2} + \frac{1}{A} \frac{dA}{d\omega} \frac{du_{j,n}}{d\omega} + B u_{j,n} = \frac{(m+1) \Gamma}{\rho_o \left(1 - \frac{v_o^2}{C_o^2}\right) \sin \omega} \quad (4.42)$$

where:

$$A = \rho_o \left(1 - \frac{v_o^2}{C_o^2}\right) (\sin \omega) \text{ EXP} \left[- (2m+3) \int_{\omega_{so}}^{\omega} \frac{u_o v_o}{C_o^2 - v_o^2} d\omega \right] \quad (4.43)$$

$$B = \frac{-1}{\left(1 - \frac{v_o^2}{C_o^2}\right) \sin^2 \omega} + (m+1)(m+2) \frac{C_o^2 - u_o^2}{C_o^2 - v_o^2} - \rho_o \left(1 - \frac{v_o^2}{C_o^2}\right) \sin \omega \frac{d}{d\omega} \left(\frac{\rho_o u_o v_o \sin \omega}{C_o^2} \right) \quad (4.44)$$

$$\Gamma = \frac{d\Gamma_1}{d\omega} + \Gamma_2 \quad (4.45)$$

with:

$$\Gamma_1 = \frac{\rho_o \sin \omega}{(m+1)} \left(1 - \frac{v_o^2}{C_o^2}\right) J_{j,n} - \frac{\rho_o v_o \sin \omega}{C_o^2} H_{j,n} + v_o \sin \omega \left(\frac{\rho_o T_o + e_o^2}{C_o^2} \right) S_{j,n} \quad (4.46)$$

$$\Gamma_2 = \mathcal{R}_{j,n} - \rho_o W_{j,n} - (m+2) \sin \omega \left\{ \frac{\rho_o u_o}{C_o^2} H_{j,n} + \frac{\rho_o u_o v_o}{C_o^2} \frac{J_{j,n}}{(m+1)} - u_o \left(\frac{\rho_o T_o + e_o^2}{C_o^2} \right) S_{j,n} \right\} \quad (4.47)$$

Equations (4.37, 4.38, 4.39, and 4.41) give $S_{j,n}$, $H_{j,n}$, $J_{j,n}$ and $W_{j,n}$ as linear functions of the shock perturbation parameter, $\omega_{j,n}$ by substitution from the boundary conditions, Equation (4.36). That is, they give the variables in the forms:

$$\left. \begin{aligned} S_{j,n} &= S_{j,n}^{(1)} + S_{j,n}^{(2)} \omega_{j,n} \\ H_{j,n} &= H_{j,n}^{(1)} + H_{j,n}^{(2)} \omega_{j,n} \\ J_{j,n} &= J_{j,n}^{(1)} + J_{j,n}^{(2)} \omega_{j,n} \quad \text{and} \\ W_{j,n} &= W_{j,n}^{(1)} + W_{j,n}^{(2)} \omega_{j,n} \end{aligned} \right\} \quad (4.48)$$

where the superscripted variables are known functions of ω (or are known constants in some instances) which can be determined numerically by quadratures. With the superscripted variables in Equations (4.48) known, the quantities Γ_1 and Γ_2 which determine the right-hand side of Equation (4.42), are also known in the sense that the superscripted quantities in the expressions:

$$\left. \begin{aligned} \Gamma_1 &= \Gamma_1^{(1)} + \Gamma_1^{(2)} \omega_{j,n} \\ \Gamma_2 &= \Gamma_2^{(1)} + \Gamma_2^{(2)} \omega_{j,n} \end{aligned} \right\} \quad (4.49)$$

are known functions of ω . Thus, Equation (4.42) is a second-order ordinary differential Equation of the form:

$$\frac{d^2 u_{j,n}}{d\omega^2} + \frac{1}{A} \frac{dA}{d\omega} \frac{du_{j,n}}{d\omega} + B u_{j,n} = \tau_1 + \tau_2 \omega_{j,n} \quad (4.50)$$

where:

τ_1 and τ_2 are known functions of ω , and $\omega_{j,n}$ is an unknown constant. The solution of this Equation involves two additional constants of integration, K_1 and K_2 , and can be written in the form.*

$$u_{j,n} = u_{j,n}^{(1)} + u_{j,n}^{(2)} + g_{j,n} K_1 + \zeta_{j,n} K_2 \quad (4.51)$$

where:

$$\zeta_{j,n} \text{ is a solution of the equation } \frac{d^2 \zeta_{j,n}}{d\omega^2} + \frac{1}{A} \frac{dA}{d\omega} \frac{d\zeta_{j,n}}{d\omega} + B \zeta_{j,n} = 0$$

$$\text{satisfying the conditions } \zeta_{j,n}(\omega_B) = 0, \quad \left. \frac{d\zeta_{j,n}}{d\omega} \right|_{\omega = \omega_B} = -2u_o(\omega_B)$$

(For $m = 0$, the solution is $\zeta_{1,0} = v_o$)

$$g_{j,n} = \zeta_{j,n} \int_{\omega_{so}}^{\omega} \frac{d\omega}{A \zeta_{j,n}^2} \quad (4.52)$$

*This form is obtained from the standard form for the general solution of a second order ordinary differential Equation - e.g. as given in Reference 14 by performing an integration by parts.

$$u_{j,n}^{(k)} = - (m+1) g_{j,n} \left(\zeta_{j,n} \Gamma_1^{(k)} \right) \Big|_{\omega = \omega_{so}} + (m+1) \left\{ \zeta_{j,n} I_g^{(k)} - g_{j,n} I_\zeta^{(k)} \right\} \quad (4.53)$$

where:

$$I_g^{(k)} = \int_{\omega_{so}}^{\omega} \left\{ \Gamma_1^{(k)} \left(\frac{dg_{j,n}}{d\omega} + \xi g_{j,n} \right) - g_{j,n} \Gamma_2^{(k)} \right\} e^{\int_{\omega_{so}}^{\omega} \xi d\omega} d\omega \quad (k = 1, 2)$$

$$I_\zeta^{(k)} = \int_{\omega_{so}}^{\omega} \left\{ \Gamma_1^{(k)} \left(\frac{d\zeta_{j,n}}{d\omega} + \xi \zeta_{j,n} \right) - \zeta_{j,n} \Gamma_2^{(k)} \right\} e^{\int_{\omega_{so}}^{\omega} \xi d\omega} d\omega \quad (k = 1, 2)$$

$$\xi = (2m+3) \frac{u_o v_o}{v_o^2 - C_o^2}$$

The perturbation variables, $v_{j,n}$, can also be expressed in terms of the three constants, $\omega_{j,n}$, K_1 and K_2 , by use of Equations (4.39), (4.48), (4.51) and (4.53),

$$v_{j,n} = v_{j,n}^{(1)} + v_{j,n}^{(2)} \omega_{j,n} + \frac{K_1}{(m+1)} \frac{dg_{j,n}}{d\omega} + \frac{K_2}{(m+1)} \frac{d\zeta_{j,n}}{d\omega} \quad (4.54)$$

where:

$$v_{j,n}^{(k)} = - \left(\zeta_{j,n} \Gamma_1^{(k)} \right) \Big|_{\omega = \omega_{so}} \frac{dg_{j,n}}{d\omega} + \frac{\Gamma_1^{(k)} e^{\int_{\omega_{so}}^{\omega} \xi d\omega}}{A} - \frac{J_{j,n}^{(k)}}{(m+1)}$$

$$I_g^{(k)} \frac{d\zeta_{j,n}}{d\omega} - I_\zeta^{(k)} \frac{dg_{j,n}}{d\omega}$$

The Equations, as written, contain several limiting forms of the type $0x^\infty$ and $\infty-\infty$ as the body surface is approached. With one exception, $J_{j,n}^{(k)}$, these limiting forms have

finite values and the final results for the perturbation variables are all finite at the body surface. The following limiting forms are encountered.

1. Equations (4.37), (4.38), and (4.41) contain terms of the type $I = \psi_o^m \int_{\omega_{so}}^{\omega} \frac{F(\omega)}{v_o \psi_o^m} d\omega$

where $m \neq 0$ (for $M=0$, $F \equiv 0$ in all cases) and $F(\omega)$ is non-singular at the body surface. As the body surface is approached $\psi_o^m \rightarrow 0$ and $\left| \int_{\omega_{so}}^{\omega} \frac{F(\omega)}{v_o \psi_o^m} d\omega \right| \rightarrow \infty$. How-

ever, use of L'Hopital's rule and the equations given in Appendix II gives $\lim_{\omega \rightarrow \omega_B} I =$

$$\frac{F(\omega_B)}{m u_o(\omega_B)}. \text{ These terms are handled numerically by use of a quadrature}$$

formula having the same singularity as the integral.

2. Equation (4.39) requires division by zero at the body surface. For $m=0$, the numerator of the fraction is identically zero so the limit is zero; for other values of m the limit is not finite. However, this infinity does not appear in the subsequent numerical work as explained in items 4 and 5.
3. Equation (4.52) for $g_{j,n}$ approaches the form $0 \times \infty$ at the body surface due to the boundary conditions imposed on $\zeta_{j,n}$. The limit is again finite and is:

$$g_{j,n}(\omega_B) = \frac{1}{2 u_o(\omega_B) A(\omega_B)} \tag{4.55}$$

The first derivative of $g_{j,n}$ which is needed in Equations (4.53) and (4.54), approaches the form $\infty - \infty$ at the body surface and the finite value of this limit is:

$$\left. \frac{dg_{j,n}}{d\omega} \right|_{\omega=\omega_B} = - \left(\frac{1}{A \zeta_{j,n} \frac{d\zeta_{j,n}}{d\omega}} \right) \Big|_{\omega=\omega_{so}}^{(2u_o)(\omega_B) - 2u_o(\omega_B)} \int_{\omega_{so}}^{\omega_B} \frac{B d\omega}{A \left(\frac{d\zeta_{i,j}}{d\omega} \right)^2}$$

4. In the computation of $u_{j,n}^{(k)}$, (Equation 4.53) integrals of the form:

$$I = \int_{\omega_{so}}^{\omega_B} \varphi J_{j,n} d\omega, \quad (4.57)$$

where:

φ is free of singularities, are required. Although $J_{j,n}$ is infinite at the body surface, these integrals are finite. By use of the following expression for $J_{j,n}$ (equivalent to Equation (4.39)).

$$J_{j,n} = \frac{\frac{dH_{j,n}}{d\omega} - T_o \frac{dS_{j,n}}{d\omega} - G_{j,n}}{u_o}, \quad (4.58)$$

and an integration by parts, the finite values of the integrals can be obtained numerically from Equation (4.59):

$$I = \left\{ \frac{\varphi}{u_o} (T_o S_{j,n} - H_{j,n}) \right\} \Big|_{\omega=\omega_{so}} + \frac{\varphi}{u_o} (H_{j,n} - T_o S_{j,n}) \quad (4.59)$$

$$+ \int_{\omega_{so}}^{\omega} \left\{ S_{j,n} \frac{d}{d\omega} \left(\frac{T_o \varphi}{u_o} \right) - H_{j,n} \frac{d}{d\omega} \left(\frac{\varphi}{u_o} \right) - \frac{G_{j,n} \varphi}{u_o} \right\} d\omega$$

5. At the body surface, the term $\frac{\Gamma_1^{(k)} e^{\int_{\omega_{so}}^{\omega} \xi d\omega}}{A} - \frac{J_{j,n}^{(k)}}{(m+1)}$ appearing in Equation (4.54),

assumes the limiting form $\infty - \infty$ (except when $m = 0$). However, when the expression for $I_1^{(k)}$ arising from Equation (4.46) is substituted algebraically into this expression, the infinities cancel without the application of L'Hopital's rule to give a finite result.

The numerical solution of the problem proceeds in the following order (the axisymmetric conical flow solution is assumed to have been pre-computed).

1. Compute $S_{1,0}^{(1)}$ and $S_{1,0}^{(2)}$ from the formulas which arise from substituting the boundary conditions at the shock, Equations (4.36a, b, and c) into Equation (4.37).
2. Compute $H_{1,0}^{(1)}$ and $H_{1,0}^{(2)}$ from the formulas which arise from substituting the boundary conditions at the shock Equations (4.36a through 4.36e) into Equation (4.38).
3. Compute $W_{1,0}^{(1)}$ and $W_{1,0}^{(2)}$ from the formulas which arise from substituting the boundary conditions at the shock, Equations (4.36d and 4.36f), into Equation (4.41).
4. Compute $\zeta_{1,0}$ and $\frac{d\zeta_{1,0}}{d\omega}$ as described following Equation (4.51), using finite difference methods when $m \neq 0$.
5. Compute $g_{1,0}$ and $\frac{dg_{1,0}}{d\omega}$ from Equation (4.52) and its derivative.
6. Compute $I_g^{(1)}$, $I_g^{(2)}$, $I_\zeta^{(1)}$, and $I_\zeta^{(2)}$ and the remaining factors present in Equation (4.53).
7. Compute $v_{1,0}^{(1)}$ and $v_{1,0}^{(2)}$ from Equations (4.54).
8. Compute the constants, $\omega_{1,0}$, K_1 and K_2 from the boundary conditions, Equations (4.35, 4.36d, and 4.36e) and Equations (4.51 and 4.54).
9. Compute $u_{1,0}$ from Equation (4.51), $v_{1,0}$ from Equation (4.54), $S_{1,0}$, $H_{1,0}$ and $W_{1,0}$ from Equation (4.48), $w_{1,0}$ from Equation (4.41), $p_{1,0}$ from Equation (4.40) and $\rho_{1,0}$ from Equation (4.34f)

10. Repeat the preceding sequence for $n = 1, 2, 3, \dots, n_{\max}$, and for $j = 2, n = 0, 1, 2, \dots, n_{\max}$ to generate as many perturbation coefficients as desired.

4.4 THE SOLUTION IN THE SUPERSONIC REGION*

4.4.1 The Coordinate Systems.

The coordinate systems used in the unsteady supersonic solution, are shown in Figures 4, 5, and 6.

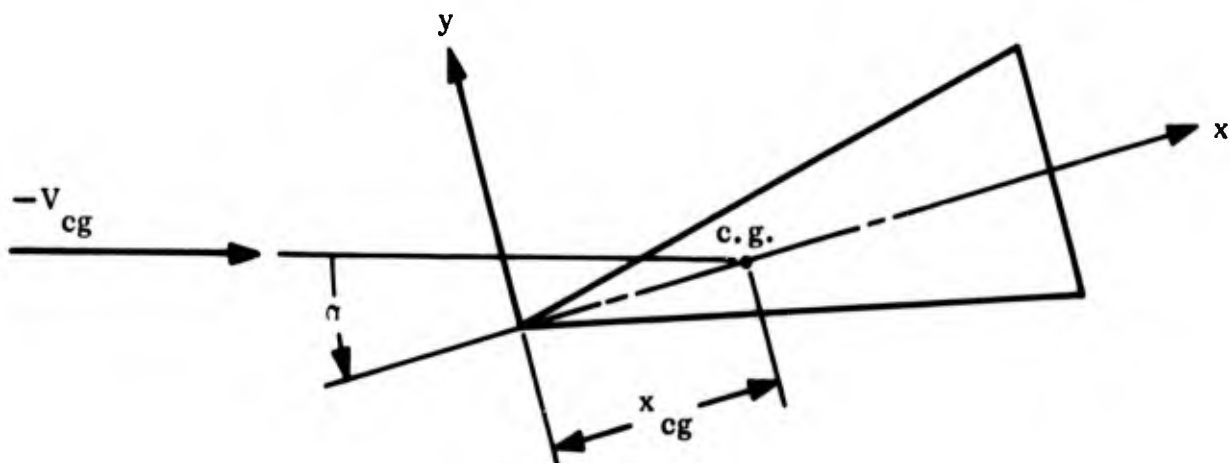


Figure 4. The Coordinate System and Angle of Attack

*This section appeared originally in Reference 2.

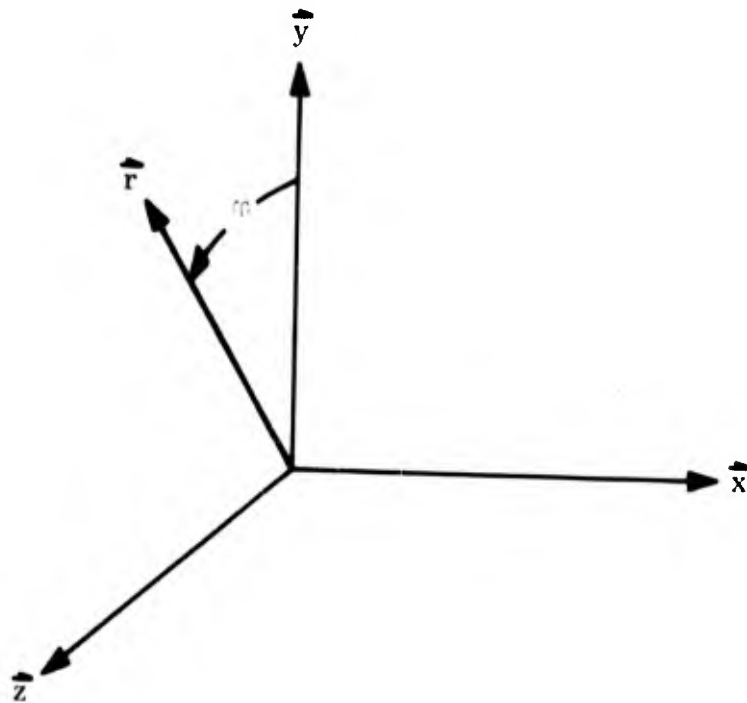


Figure 5. The Cylindrical Coordinate System

The x direction coincides with the body axis. The y axis lies in the plane defined by the x axis and the vector direction (\vec{v}_{cg}) of the velocity of the center of gravity. The z axis is in the direction which makes the x, y, z system orthogonal and right-handed. The r and ϕ directions lie in the y-z plane. The x, r, ϕ system is an orthogonal cylindrical coordinate system.

The flow at any point in the field can be thought of as being composed of two components: V which lies in the meridional planes (x - r planes); and w which is normal to the meridional planes. The flow direction in the meridional planes will be called θ , and the two components of V in the meridional planes will be labelled u and v. These quantities are illustrated in Figure 6.

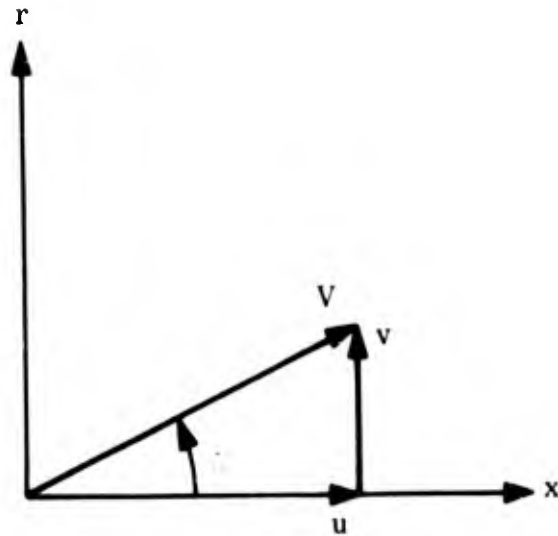


Figure 6. The Velocities in the Meridional Plane

4.4.2 Governing Equations

The Equations which govern the unsteady flow of an inviscid, non-conducting gas Equations (4.21) through (4.24), are written in the triorthogonal coordinate system described by the following unit vectors:

a unit vector, \vec{s} , in the direction of the projection of the velocity vector (\vec{V}) in the meridional planes

$$\vec{s} = \vec{x} \frac{u}{V} + \vec{r} \frac{v}{V} = \vec{x} \cos \theta + \vec{r} \sin \theta \quad (4.60)$$

a unit vector, \vec{n} , normal to \vec{s} and also in the meridional plane.

$$\vec{n} = -\vec{x} \frac{v}{V} + \vec{r} \frac{u}{V} = -\vec{x} \sin \theta + \vec{r} \cos \theta \quad (4.61)$$

and a unit vector, $\vec{\phi}$, perpendicular to the meridional plane (axisymmetric case only).

In this coordinate system, with the velocity vector expressed as

$$\vec{V} = v \vec{e}_1 + w \vec{\phi} \quad (4.62)$$

the governing Equations become:

Continuity Equation *

$$\begin{aligned} & \frac{V_\infty}{L} (1 - \delta_{o,n}) \rho_{j,n-1} + \frac{d}{d\mathcal{L}_o} (\rho_o V_{j,n} + \rho_{j,n} V_o) + \theta_{j,n} \frac{d(\rho_o V_o)}{dn_o} \\ & + \frac{\delta}{r} \frac{d}{d\phi} (\rho_o w_{j,n}) + \rho_o V_o \frac{d\theta_{j,n}}{dn_o} - \rho_o V_o \theta_{j,n} \frac{d\theta_o}{d\mathcal{L}_o} + \frac{\delta \rho_o V_o \theta_{j,n} \cos \theta_o}{r} \quad (4.63) \\ & + (\rho_o V_{j,n} + \rho_{j,n} V_o) \frac{d\theta_o}{dn_o} + \frac{\delta (\rho_o V_{j,n} + \rho_{j,n} V_o)}{r} \sin \theta_o = 0 \end{aligned}$$

The three components of the momentum Equation: **

$$\frac{V_\infty}{L} (1 - \delta_{o,n}) \rho_o V_{j,n-1} + \frac{dp_{j,n}}{d\mathcal{L}_o} + \theta_{j,n} \frac{dp_o}{dn_o} + \rho_o q_o \frac{dV_{j,n}}{d\mathcal{L}_o} + \rho_o V_o \theta_{j,n} \frac{dV_o}{dn_o} \quad (4.64)$$

$$+ (\rho_o V_{j,n} + V_o \rho_{j,n}) \frac{dV_o}{d\mathcal{L}_o} = \rho_o F_{j,n}$$

$$\frac{V_\infty}{L} (1 - \delta_{o,n}) V_o \rho_o \theta_{j,n-1} + \frac{dp_{j,n}}{dn_o} - \theta_{j,n} \frac{dp_o}{d\mathcal{L}_o} + \rho_o V_o^2 \left(\frac{d\theta_{j,n}}{d\mathcal{L}_o} + \theta_{j,n} \frac{d\theta_o}{dn_o} \right) \quad (4.65)$$

$$+ \left(2 \rho_o V_o V_{j,n} + V_o^2 \rho_{j,n} \right) \frac{d\theta_o}{d\mathcal{L}_o} = \rho_o G_{j,n}$$

*The symbol δ is defined to be

$\delta = 1$ for the axisymmetric case

$= 0$ for the two dimensional case.

**Equation (4.66) does not apply in the two-dimensional case.

$$\frac{V_\infty}{L} (1-\delta_{o,n}) \rho_o w_{j,n-1} + \frac{1}{r} \frac{dp_{j,n}}{d\varphi} + \rho_o V_o \frac{dw_{j,n}}{d\mathcal{L}_o} + \rho_o V_o w_{j,n} \frac{\sin \theta_o}{r} = \rho_o H_{j,n} \quad (4.66)$$

The energy Equation:

$$\frac{V_\infty}{L} (1-\delta_{o,n}) S_{j,n-1} + V_o \left(\frac{dS_{j,n}}{d\mathcal{L}_o} + \theta_{j,n} \frac{dS_o}{dn_o} \right) = 0 \quad (4.67)$$

and the state relations:

$$\rho_{j,n} = \frac{\partial \rho}{\partial p} p_{j,n} + \frac{\partial \rho}{\partial S} S_{j,n} \quad (4.68)$$

$$Z_{j,n} = \frac{\partial Z}{\partial \rho} \rho_{j,n} + \frac{\partial Z}{\partial S} S_{j,n} \quad (4.69)$$

$$H_{j,n} = \frac{\partial h}{\partial p} p_{j,n} + \frac{\partial h}{\partial S} S_{j,n} \quad (4.70)$$

$$T_{j,n} = \frac{1}{Z_o R \rho_o} \left[p_{j,n} - \rho_{j,n} \left(Z_o RT_o + RT_o \rho_o \frac{\partial Z}{\partial \rho} \right) - S_{j,n} \left(RT_o \rho_o \frac{\partial Z}{\partial S} \right) \right] \quad (4.71)*$$

The terms $F_{j,n}$, $G_{j,n}$ and $H_{j,n}$ which appear in Equations (4.64), (4.65), (4.66) are acceleration terms resulting from the fact that the coordinate system is moving with the body. They are the components of \vec{F}_j , Equation (4.22) and are:

$$\begin{aligned} F_{j,n} &= -\delta_{1,n} V_\infty \sin \theta_o \cos \varphi \text{ for } j = 1 \\ &= \delta_{o,n} V_\infty \sin \theta_o \cos \varphi + \delta_{1,n} \left[r \cos \theta_o - (x-x_{cg}) \sin \theta_o \right] \cos \varphi \text{ for } j = 2 \end{aligned}$$

* Equation (4.71) is derived from the equation of state by differentiating with respect to the appropriate motion.

$$G_{j,n} = -\delta_{1,n} V_{\infty} \cos \theta_0 \cos \varphi \text{ for } j = 1$$

$$= -\delta_{0,n} [2V_0 - V_{\infty} \cos \theta_0] \cos \varphi - \delta_{1,n} [(x-x_{cg}) \cos \theta_0 + r \sin \theta_0] \cos \varphi \text{ for } j=2$$

$$H_{j,n} = \delta_{1,n} V_{\infty} \sin \varphi \text{ for } j = 1$$

$$= \delta_{0,n} [2V_0 \cos \theta_0 - V_{\infty}] \sin \varphi + \delta_{1,n} (x - x_{cg}) \sin \varphi \text{ for } j = 2$$

The following Sections will combine these equations with the boundary conditions and prepare them for numerical solution.

4.4.3 Removal of Dependence on Meridional Angle*

Each of the perturbation parameters, $p_{j,n}$, $\rho_{j,n}$, etc., is a function of the three variables, x , r , and φ , the time variable having been removed by means of the perturbation expressions Equation (4.33). Each of the perturbation parameters can be expressed as a Fourier series in φ . Before doing this, however, certain symmetry properties for an axisymmetric body in unsteady motion should be noted. For motion about the z axis; i. e., the angle of attack and pitch motions, the following symmetry relations can be deduced at any given x and r :

for $j = 1, 2$ and $n = 0, 1, 2, 3 \dots \infty$

$$p_{j,n} \left(\frac{d^n \alpha_j}{dt^n}, \varphi \right) = p_{n,j} \left(\frac{d^n \alpha_j}{dt^n}, -\varphi \right) = p_{j,n} \left(-\frac{d^n \alpha_j}{dt^n}, \pi - \varphi \right) \quad (4.72)$$

$$v_{j,n} \left(\frac{d^n \alpha_j}{dt^n}, \varphi \right) = v_{j,n} \left(\frac{d^n \alpha_j}{dt^n}, -\varphi \right) = v_{j,n} \left(-\frac{d^n \alpha_j}{dt^n}, \pi - \varphi \right) \quad (4.73)$$

*This section does not apply to the two-dimensional problem.

Identical relations exist for all perturbation parameters except $w_{j,n}$, for which the following is true

$$w_{j,n} \left(\frac{d^n \alpha_j}{dt^n}, \varphi \right) = - w_{j,n} \left(\frac{d^n \alpha_j}{dt^n}, -\varphi \right) = - w_{j,n} \left(-\frac{d^n \alpha_j}{dt^n}, \pi - \varphi \right) \quad (4.74)$$

Consideration of Equations (4.72), and (4.73), and (4.74) leads to the conclusion that if the perturbation parameters are expanded in Fourier series, it is possible to express each parameter with either a sine or a cosine series. None of the parameters requires a complete Fourier series, i. e., they are either even or odd functions.

For the pitch and angle of attack motions ($j = 1, 2$), the Fourier series expansions are:

$$\left. \begin{aligned} p_{j,n} &= \sum_{m=1}^{\infty} \bar{p}_{j,n,m} \cos m \varphi & j = 1, 2 \\ v_{j,n} &= \sum_{m=1}^{\infty} \bar{v}_{j,n,m} \cos m \varphi & n = 0, 1, 2, \dots, \infty \end{aligned} \right\} \quad (4.75)$$

etc., except for the meridional velocity which requires a sine series:

$$\left. \begin{aligned} w_{j,n} &= \sum_{m=1}^{\infty} \bar{w}_{j,n,m} \sin m \varphi & j = 1, 2 \\ & & n = 0, 1, 2, \dots, \infty \end{aligned} \right\}$$

The purpose of expanding the perturbation parameters in Fourier series is to eliminate the φ - dependence from the system of perturbation Equations. This is done by substituting Equation (4.75) into Equations (4.63) through (4.71). Before doing this, it is useful to note that no terms with m greater than 1 are present either

in the governing equations, or in the boundary conditions which are applied to them. The absence of terms with m greater than 1 in the governing equations and boundary conditions means that all the Fourier coefficients $\bar{p}_{j,n,m}$, $\bar{V}_{j,n,m}$, $\bar{w}_{j,n,m}$ etc., except those for $m = 1$, can be set equal to zero. Because this leaves only one non-zero term in each of the Fourier series expansions (Equation (4.75)) the subscript, m , will be dropped from our notation. Substitution of the Fourier series expansions into the governing Equations, Equations (4.63) through (4.71) results in the following system of Equations. The results of this substitution are the same for both values of j . These equations apply to the axisymmetric case only, hence δ has been set equal to unity.

$$\begin{aligned} \frac{V_\infty}{L} (1 - \delta_{o,n}) \bar{\rho}_{j,n-1} + \frac{d}{d\mathcal{L}_o} [\rho_o \bar{V}_{j,n} + \bar{\rho}_{j,n} V_o] + \bar{\theta}_{j,n} \frac{d(\rho_o V_o)}{dn_o} + \frac{\rho_o \bar{w}_{j,n}}{r} \\ + \rho_o V_o \frac{d\bar{\theta}_{j,n}}{dn_o} - \rho_o V_o \frac{d\theta_o}{d\mathcal{L}_o} \bar{\theta}_{j,n} + \rho_o V_o \frac{\cos \theta_o}{r} \bar{\theta}_{j,n} \end{aligned} \quad (4.77)$$

$$+ \left(\frac{d\theta_o}{dn_o} \right) (\rho_o \bar{V}_{j,n} + \bar{\rho}_{j,n} V_o) + \frac{\sin \theta_o}{r} (\rho_o \bar{V}_{j,n} + \bar{\rho}_{j,n} V_o) = 0$$

$$\frac{V_\infty}{L} (1 - \delta_{o,n}) \rho_o \bar{V}_{j,n-1} + \frac{d\bar{p}_{j,n}}{d\mathcal{L}_o} + \frac{dp_o}{dn_o} \bar{\theta}_{j,n} + \rho_o V_o \frac{d\bar{V}_{j,n}}{d\mathcal{L}_o} + \rho_o V_o \frac{dV_o}{dn_o} \bar{\theta}_{j,n}$$

$$+ \rho_o \frac{dV_o}{d\mathcal{L}_o} \bar{q}_{j,n} + V_o \frac{dV_o}{d\mathcal{L}_o} \bar{\rho}_{j,n} = \rho_o \bar{F}_{j,n} \quad (4.78)^*$$

* $\bar{F}_{j,n}$, $\bar{G}_{j,n}$ and $\bar{H}_{j,n}$ are $\left(\frac{1}{\cos \varphi} \right)$ times $F_{j,n}$, $G_{j,n}$ and $H_{j,n}$ respectively.

$$\frac{V_\infty}{L} (1 - \delta_{o,n}) \rho_o V_o \bar{\theta}_{j,n-1} + \frac{d\bar{p}_{j,n}}{dn_o} - \bar{\theta}_{j,n} \frac{dp_o}{d\mathcal{L}_o} + \rho_o V_o^2 \left(\frac{d\bar{\theta}_{j,n}}{d\mathcal{L}_o} + \bar{\theta}_{j,n} \frac{d\theta_o}{dn_o} \right) + \left(2 \rho_o q_o \bar{v}_{j,n} + V_o^2 \bar{\rho}_{j,n} \right) \frac{d\theta_o}{d\mathcal{L}_o} = \rho_o \bar{G}_{j,n} \quad (4.79)$$

$$\frac{V_\infty}{L} (1 - \delta_{o,n}) \rho_o \bar{w}_{j,n-1} - \frac{\bar{p}_{j,n}}{r} + \rho_o V_o \frac{d\bar{w}_{j,n}}{d\mathcal{L}_o} + \frac{\rho_o V_o}{r} \sin \theta_o \bar{w}_{j,n} = \rho_o \bar{H}_{j,n} \quad (4.80)$$

$$\frac{V_\infty}{L} (1 - \delta_{o,n}) \bar{s}_{j,n-1} + V_o \left(\frac{d\bar{s}_{j,n}}{d\mathcal{L}_o} + \bar{\theta}_{j,n} \frac{ds_o}{dn_o} \right) = 0 \quad (4.81)$$

$$\bar{\rho}_{j,n} = \bar{p}_{j,n} \frac{\partial \rho}{\partial p} + \bar{s}_{j,n} \frac{\partial \rho}{\partial S} = \frac{1}{C_o^2} \bar{p}_{j,n} - \frac{e_o^2}{C_o^2} \bar{s}_{j,n} \quad (4.82)$$

$$\bar{h}_{j,n} = \bar{p}_{j,n} \frac{\partial h}{\partial p} + \bar{s}_{j,n} \frac{\partial h}{\partial S} \quad (4.83)$$

$$\bar{T}_{j,n} = \frac{1}{Z_o R \rho_o} \left[\bar{p}_{j,n} - \bar{\rho}_{j,n} \left(Z_o RT_o + RT_o \rho_o \frac{\partial Z}{\partial \rho} \right) - \bar{s}_{j,n} \left(RT_o \rho_o \frac{\partial Z}{\partial S} \right) \right] \quad (4.84)$$

Equations (4.77) through (4.82) are projected onto the zero-yaw characteristic grid and form the basis of the numerical solution. * Equation (4.83) is helpful in applying the boundary conditions at the shock wave. Equation (4.84) is used to compute the temperature perturbations after the pressure, density, and entropy perturbations have been calculated. Since the zero-yaw flow field parameters appear in these

*The reasons for projecting these Equations on the zero-yaw characteristic grid will become apparent in Section 4.4.5 of this report.

equations, it is apparent that the zero-yaw problem must be solved before the perturbation flow field is calculated.

4.4.4 Boundary Conditions at the Shock Wave

The pressure and density just downstream of an oblique shock, as well as the change in magnitude of the component of velocity normal to the shock, can in general be expressed as functions of upstream pressure, density, and normal component of velocity:

$$\left. \begin{aligned} p_s &= F_1(p_\infty, \rho_\infty, V_{N_\infty}) \\ \rho_s &= F_2(p_\infty, \rho_\infty, V_{N_\infty}) \\ V_{N_s} - V_{N_\infty} &= F_3(p_\infty, \rho_\infty, V_{N_\infty}) \end{aligned} \right\} \quad (4.85)$$

In each case the only argument which is dependent on the perturbations in the motion of the body is V_{N_∞} . * The Taylor series expansions of Equation (4.85) about their values in the zero yaw field then are:

$$\begin{aligned} p_s &= F_1(p_\infty, \rho_\infty, V_{N_\infty}) + \sum_j \sum_n \frac{\partial F_1}{\partial V_{N_\infty}} \frac{dV_{N_\infty}}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} \frac{d^n \alpha_j}{dt^n} \\ \rho_s &= F_2(p_\infty, \rho_\infty, V_{N_\infty}) + \sum_j \sum_n \frac{\partial F_2}{\partial V_{N_\infty}} \frac{dV_{N_\infty}}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} \frac{d^n \alpha_j}{dt^n} \end{aligned} \quad (4.86)$$

*The velocity V_{N_∞} must be given relative to the shock wave.

$$V_{N_s} - V_{N_\infty} = F_3(p_\infty, \rho_\infty, V_{N_\infty}) + \sum_j \sum_n \frac{\partial F_3}{\partial V_{N_\infty}} \frac{dV_{N_\infty}}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} \frac{d^n \alpha_j}{dt^n} \quad (4.86) \quad (\text{cont.})$$

p_s , ρ_s , and V_{N_s} are the conditions which exist just downstream of the perturbed shock (point s in Figure 7).

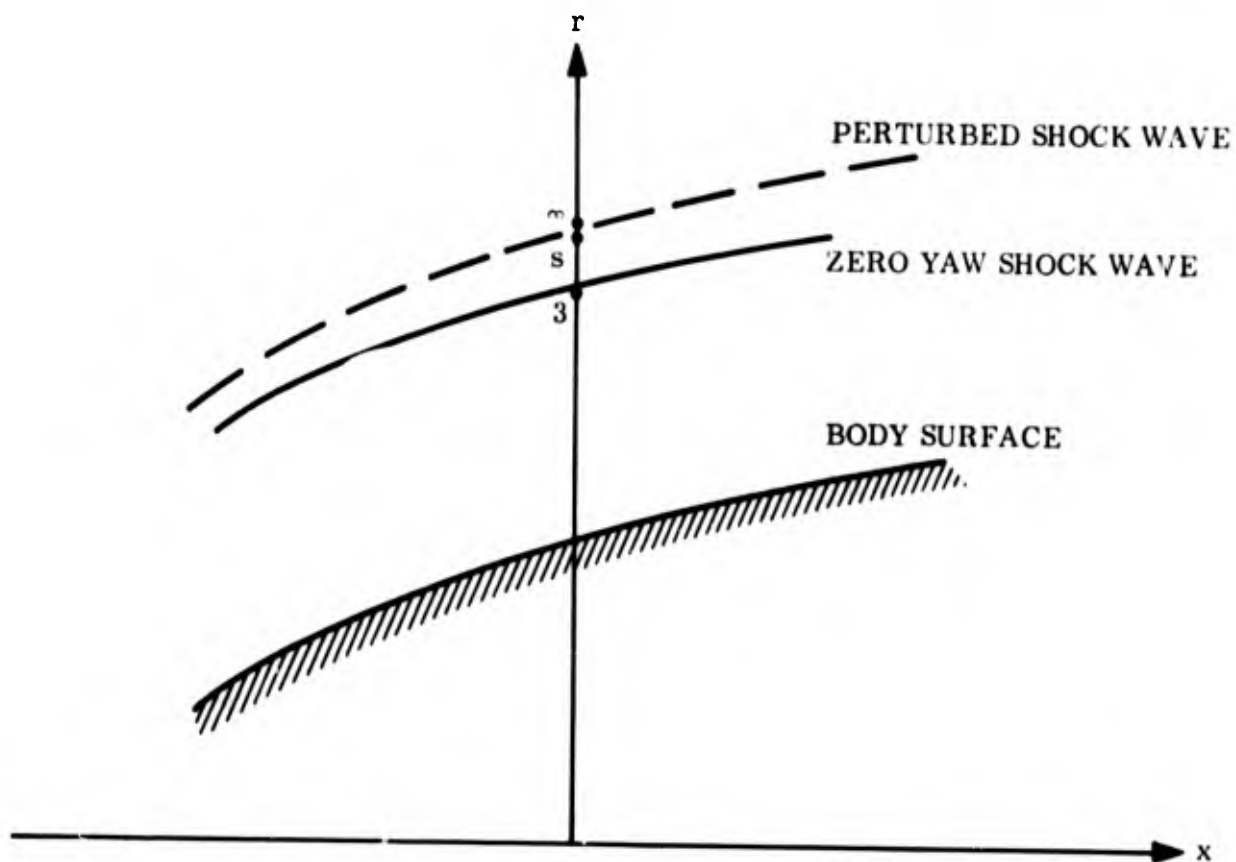


Figure 7. The Shock Wave Perturbation

Since the present calculations are being carried out on the zero-yaw grid, the boundary conditions are needed at point 3, rather than at point s. The pressure and density at point 3 are given by:

$$p_3 = p_s - \frac{\partial p_o}{\partial r} \sum_j \sum_n R_{s,j,n} \frac{d^n \alpha_j}{dt^n} \quad (4.87)$$

where R_s is the value of r at the shock wave.

and

$$\rho_3 = \rho_s - \frac{\partial \rho_o}{\partial r} \sum_j \sum_n R_{s_{j,n}} \frac{d^n \alpha_j}{dt^n} \quad (4.88)$$

From Equations (4.86), (4.87), and (4.88)* it can be concluded that:

$$\bar{p}_{j,n} = \frac{\partial F_1}{\partial V_{N_\infty}} \frac{d\bar{V}_{N_\infty}}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} - \frac{\partial \rho_o}{\partial r} \bar{R}_{s_{j,n}} \quad (4.89)$$

and

$$\bar{p}_{j,n} = \frac{\partial F_2}{\partial V_{N_\infty}} \frac{d\bar{V}_{N_\infty}}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} - \frac{\partial \rho_o}{\partial r} \bar{R}_{s_{j,n}} \quad (4.90)$$

The zero-yaw values of $\frac{\delta p}{\delta r}$, $\frac{\delta \rho}{\delta r}$, $\frac{\delta F_1}{\delta V_{N_\infty}}$ and $\frac{\delta F_2}{\delta V_{N_\infty}}$ are to be used. Equations (4.89) and (4.90) will provide two of the boundary conditions at each shock point, once the other derivatives on the right hand side are determined. Expressions for these derivatives are obtained in Sections 4.4.4.1 and 4.4.4.2.

Three other boundary conditions will be needed at the shock points. If expressions similar to Equations (4.87) and (4.88) are written for the velocities u, v, and w, then the following statements result:

*See also Equations (4.33), (4.75), and (4.76).

$$\begin{aligned}
u_3 + \frac{\partial u_0}{\partial r} \sum_j \sum_n R_{s_{j,n}} \frac{d^n \alpha_j}{dt^n} &= u_\infty + (V_{N_s} - V_{N_\infty}) (\vec{n}_s \cdot \vec{x}) \\
&= u_\infty + F_3 (\vec{n}_s \cdot \vec{x})
\end{aligned}
\tag{4.91}$$

$$v_3 + \frac{\partial v_0}{\partial r} \sum_j \sum_n R_{s_{j,n}} \frac{d^n \alpha_j}{dt^n} = v_\infty + F_3 (\vec{n}_s \cdot \vec{r})
\tag{4.92}$$

$$w_3 + \frac{\partial w_0}{\partial r} \sum_j \sum_n R_{s_{j,n}} \frac{d^n \alpha_j}{dt^n} = w_\infty + F_3 (\vec{n}_s \cdot \vec{\varphi})
\tag{4.93}$$

Equations 4.91 through 4.93 can be put in "perturbation" form through the following definitions:

$$\begin{aligned}
u_\infty &= v_\infty + \sum_j \sum_n u_{\infty,j,n} \frac{d^n \alpha_j}{dt^n} \\
v_\infty &= \sum_j \sum_n v_{\infty,j,n} \frac{d^n \alpha_j}{dt^n}
\end{aligned}
\tag{4.94}$$

$$w_\infty = \sum_j \sum_n w_{\infty,j,n} \frac{d^n \alpha_j}{dt^n}$$

$$\vec{n}_s \cdot \vec{x} = \tau_o + \sum_j \sum_n \tau_{j,n} \frac{d^n \alpha_j}{dt^n}$$

$$\vec{n}_s \cdot \vec{r} = \eta_o + \sum_j \sum_n \eta_{j,n} \frac{d^n \alpha_j}{dt^n}$$

(4.95)

$$\vec{n}_s \cdot \vec{\phi} = \xi_o + \sum_j \sum_n \xi_{j,n} \frac{d^n \alpha_j}{dt^n}$$

Expressions for $u_{\infty j,n}$, $v_{\infty j,n}$, $w_{\infty j,n}$ and the other terms on the right hand side of Equations (4.94) and (4.95) will be derived in Section 4.4.4.3. If Equations (4.86), (4.94), and (4.95) are used in Equations (4.91) through (4.93), the perturbations in u , v , and w can be expressed as:

$$\bar{u}_{j,n} = \bar{u}_{\infty j,n} + F_{3_o} \bar{\tau}_{j,n} + \tau_o \frac{\partial F_3}{\partial V_{N_\infty}} \frac{dV_{N_\infty}}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} - \frac{\partial u_o}{\partial r} \bar{R}_{s_{j,n}} \quad (4.96)$$

$$\bar{v}_{j,n} = \bar{v}_{\infty j,n} + F_{3_o} \bar{\eta}_{j,n} + \eta_o \frac{\partial F_3}{\partial V_{N_\infty}} \frac{dV_{N_\infty}}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} - \frac{\partial v_o}{\partial r} \bar{R}_{s_{j,n}} \quad (4.97)$$

$$\bar{w}_{j,n} = \bar{w}_{\infty j,n} + F_{3_o} \bar{\xi}_{j,n} + \xi_o \frac{\partial F_3}{\partial V_{N_\infty}} \frac{dV_{N_\infty}}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} \quad (4.98)$$

The symbol F_{3_0} is used above to denote $F_3(p_\infty, \rho_\infty, V_{N_\infty})$ evaluated at $V_{N_\infty} = V_{N_\infty_0}$.

Finally, the perturbation scheme can be applied to the trigonometric relations:

$$u = q \cos \theta \quad (4.99)$$

$$v = q \sin \theta \quad (4.100)$$

To get:

$$\bar{V}_{j,n} = \bar{u}_{j,n} \cos \theta_0 + \bar{v}_{j,n} \sin \theta_0 \quad (4.101)$$

$$\bar{\theta}_{j,n} = -\frac{\bar{u}_{j,n}}{V_0} \sin \theta_0 + \frac{\bar{v}_{j,n}}{V_0} \cos \theta_0 \quad (4.102)$$

where $V_{j,n}$ is the change in the magnitude of \bar{V} . It is not $(u_{j,n}^2 + v_{j,n}^2)^{1/2}$.

Note that the θ_0 which appears in Equations (4.101) and (4.102) is the zero yaw flow direction downstream of the shock. The θ_∞ which appeared in the preceding Equations is the flow direction upstream of the shock.

Equations (4.101) and (4.102) together with Equations (4.96) and (4.97) give:

$$\begin{aligned} \bar{V}_{j,n} = & \left[\bar{u}_{\infty,j,n} + F_{3_0} \bar{\tau}_{j,n} + \tau_0 \frac{\partial F_3}{\partial V_{N_\infty}} \frac{d\bar{V}_{N_\infty}}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} - \frac{\partial u_0}{\partial r} \bar{R}_{s,j,n} \right] \cos \theta_0 \\ & + \left[\bar{v}_{\infty,j,n} + F_{3_0} \bar{\eta}_{j,n} + \eta_0 \frac{\partial F_3}{\partial V_{N_\infty}} \frac{d\bar{V}_{N_\infty}}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} - \frac{\partial v_0}{\partial r} \bar{R}_{s,j,n} \right] \sin \theta_0 \end{aligned} \quad (4.103)$$

$$\bar{\theta}_{j,n} = -\frac{1}{V_0} \left[\bar{u}_{\infty,j,n} + F_3 \bar{\tau}_{j,n} + \tau_0 \frac{\partial F_3}{\partial V_{N_\infty}} \frac{d\bar{V}_{N_\infty}}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} - \frac{\partial u_0}{\partial r} \bar{R}_{s,j,n} \right] \sin \theta_0 \quad (4.104)$$

$$+\frac{1}{V_0} \left[\bar{v}_{\infty,j,n} + F_3 \bar{\eta}_{j,n} + \eta_0 \frac{\partial F_3}{\partial V_{N_\infty}} \frac{d\bar{V}_{N_\infty}}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} - \frac{\partial v_0}{\partial r} \bar{R}_{s,j,n} \right] \cos \theta_0$$

Equations (4.89), (4.90), (4.98), (4.103), and (4.104) are the boundary conditions needed in the shock point calculation. Expressions for τ_0 , η_0 , ξ_0 , $\tau_{j,n}$, $\eta_{j,n}$,

$\xi_{j,n}$ and $dV_{N_\infty}/d\frac{d^n \alpha_j}{dt^n}$ are given in Section 4.4.4.2. $\partial F_1/\partial V_{N_\infty}$, $\partial F_2/\partial V_{N_\infty}$, and $\partial F_3/\partial V_{N_\infty}$ in Section 4.4.4.1.

4.4.4.1 Determination of $\partial F_1/\partial V_{N_\infty}$, $\partial F_2/\partial V_{N_\infty}$ and $\partial F_3/\partial V_{N_\infty}$ from the Governing Relations for a Normal Shock

The governing Equations for a normal shock are:

continuity:

$$\rho_s V_{N_s} = \rho_\infty V_{N_\infty} \quad (4.105)$$

momentum:

$$\rho_s V_{N_s}^2 + p_2 = \rho_\infty V_{N_\infty}^2 + p_\infty \quad (4.106)$$

energy:

$$h_s + \frac{1}{2} V_{N_s}^2 = h_\infty + \frac{1}{2} V_{N_\infty}^2 \quad (4.107)$$

and state:

$$h = h(p, \rho) \quad (4.108)$$

where subscript ∞ indicates conditions just upstream of the shock, and the subscript s indicates conditions just downstream of the shock. Equations (4.105) through (4.107) can be differentiated with respect to V_{N_∞} . If this is done, and the definition of F_1 , F_2 , F_3 in Equation (4.85) used, the following equations result:

$$\rho_s \frac{\partial F_3}{\partial V_{N_\infty}} + \rho_s + V_{N_s} \frac{\partial F_2}{\partial V_{N_\infty}} = \rho_\infty \quad (4.109)$$

$$2 \rho_s V_{N_s} \frac{\partial F_3}{\partial V_{N_\infty}} + V_{N_s}^2 \frac{\partial F_2}{\partial V_{N_\infty}} + \frac{\partial F_1}{\partial V_{N_\infty}} = 0 \quad (4.110)$$

$$V_{N_s} \frac{\partial F_3}{\partial V_{N_\infty}} + V_{N_s} + \frac{\partial h_s}{\partial p} \frac{\partial F_2}{\partial V_{N_\infty}} + \frac{\partial h_s}{\partial p} \frac{\partial F_1}{\partial V_{N_\infty}} = V_{N_\infty} \quad (4.111)$$

Equations (4.109) through (4.111) can be solved algebraically for $\partial F_1 / \partial V_{N_\infty}$,

$\partial F_2 / \partial V_{N_\infty}$, and $\partial F_3 / \partial V_{N_\infty}$:

$$\begin{aligned}
\frac{\partial F_1}{\partial V_{N_\infty}} &= \frac{\rho_\infty (V_{N_s} - V_{N_\infty})}{\Delta} \left[V_{N_s} (V_{N_\infty} - V_{N_s}) + 2\rho_s \frac{\partial h_s}{\partial p} \right] \\
\frac{\partial F_2}{\partial V_{N_\infty}} &= \frac{\rho_\infty (V_{N_s} - V_{N_\infty})}{\Delta} \left[\frac{V_{N_s} + V_{N_\infty}}{V_{N_s}} - 2\rho_s \frac{\partial h_s}{\partial p} \right] \\
\frac{\partial F_3}{\partial V_{N_\infty}} &= \frac{1}{\Delta} \left[V_{N_s} (V_{N_\infty} - V_{N_s}) + \rho_\infty V_{N_s} (V_{N_s} - V_{N_\infty}) \frac{\partial h_s}{\partial p} - \frac{\rho_\infty}{V_{N_s}} (V_{N_s} - V_{N_\infty}) \frac{\partial h_s}{\partial \rho} \right]
\end{aligned}
\tag{4.112}$$

where

$$\Delta = V_{N_s}^2 \left(1 - \rho_s \frac{\partial h_s}{\partial p} \right) - \rho_s \frac{\partial h_s}{\partial p}
\tag{4.113}$$

Since the right hand sides of Equations (4.112) and (4.113) require only values that are known in the zero-yaw flow field, the three derivatives $\partial F_1/\partial V_{N_\infty}$, $\partial F_2/\partial V_{N_\infty}$, and $\partial F_3/\partial V_{N_\infty}$ are completely determined.

Equations (4.112) and (4.113) will be used in the shock boundary conditions, Equations (4.89), (4.90), (4.98), (4.103), and (4.104).

4.4.4.2 Determination of Quantities Dependent on Perturbations in the Shock Wave Shape and on the Perturbations In the Velocity Upstream of the Shock.

- a. Derivation of $\tau_o, \eta_o, \xi_o, \tau_{j,n}, \eta_{j,n}, \xi_{j,n}$.

If the Equation of the shock surface is expressed as:

$$R_s \left(x, \varphi, \frac{d^n \alpha_j}{dt^n} \right) - r = 0 \quad (4.114)$$

The unit inward normal to the shock surface is given by:

$$\vec{n}_s = \frac{\nabla R_s}{|\nabla R_s|} = \frac{\vec{x} \frac{\partial R_s}{\partial x} - \vec{r} + \vec{\varphi} \frac{\delta}{R_s} \frac{\partial R_s}{\partial \varphi}}{\sqrt{\frac{\partial R_s}{\partial x}^2 + 1 + \left(\frac{\delta}{R_s} \frac{\partial R_s}{\partial \varphi} \right)^2}} \quad (4.115)$$

Equation (4.114) can be expressed in the perturbation scheme:

$$R_s = R_{s0}(x) + \sum_j \sum_n R_{s,j,n}(x, \varphi) \frac{d^n \alpha_j}{dt^n} \quad (4.116)$$

then

$$\frac{\partial R_s}{\partial x} = \frac{\partial R_{s0}}{\partial x} + \sum_j \sum_n \frac{\partial R_{s,j,n}}{\partial x} \frac{d^n \alpha_j}{dt^n} \quad (4.117)$$

and

$$\frac{\partial R_s}{\partial \varphi} = \sum_j \sum_n \frac{\partial R_{s,j,n}}{\partial \varphi} \frac{d^n \alpha_j}{dt^n} \quad (4.118)^*$$

*Equations (4.118), (4.121), and (4.124) do not apply in the two-dimensional case.

and

$$\frac{\partial R_s}{\partial t} = \sum_j \sum_n R_{s,j,n} \frac{d^{n+1} \alpha_j}{dt^{n+1}} \quad (4.119)$$

$$\left(\frac{\partial R_s}{\partial x} \right)^2 = \left(\frac{\partial R_{s0}}{\partial x} \right)^2 + 2 \frac{\partial R_{s0}}{\partial x} \sum_j \sum_n \frac{\partial R_{s,j,n}}{\partial x} \frac{d^n \alpha_j}{dt^n} \quad (4.120)$$

and

$$\left(\frac{1}{R_s} \frac{\partial R_s}{\partial \varphi} \right)^2 = 0, \text{ if higher order terms are neglected, then } \vec{n}_s \text{ becomes:} \quad (4.121)$$

$$\vec{n}_s = \frac{\left[\frac{\partial R_{s0}}{\partial x} + \sum_j \sum_n \frac{\partial R_{s,j,n}}{\partial x} \frac{d^n \alpha_j}{dt^n} \right] \vec{r} + \varphi \left[\frac{\sum_j \sum_n \frac{\partial R_{s,j,n}}{\partial \varphi} \frac{d^n \alpha_j}{dt^n}}{R_{s0} + \sum_j \sum_n R_{s,j,n} \frac{d^n \alpha_j}{dt^n}} \right] \delta}{\sqrt{\left(\frac{\partial R_{s0}}{\partial x} \right)^2 + 2 \frac{\partial R_{s0}}{\partial x} \sum_j \sum_n \frac{\partial R_{s,j,n}}{\partial x} \frac{d^n \alpha_j}{dt^n} + 1}} \quad (4.122)$$

It can be noted that:

$$\frac{\partial R_{s0}}{\partial x} \equiv \tan \sigma_0 \quad (4.123)$$

(where σ is the local shock angle measured from the x-axis) and that:

$$\frac{\sum_j \sum_n \frac{\partial R_{s_{j,n}}}{\partial \varphi} \frac{d^n \alpha_j}{dt^n}}{R_{s_0} + \sum_j \sum_n R_{s_{j,n}} \frac{d^n \alpha_j}{dt^n}} = \frac{1}{R_{s_0}} \sum_j \sum_n \frac{\partial R_{s_{j,n}}}{\partial \varphi} \frac{d^n \alpha_j}{dt^n} \quad (4.124)$$

to first order accuracy.

Substituting Equations (4.123) and (4.124), into Equation (4.122) and multiplying both numerator and denominator by $\cos \sigma_0$:

$$\vec{n}_s = \frac{\vec{x} \left[\sin \sigma_0 + \sum_j \sum_n \cos \sigma_0 \frac{\partial R_{s_{j,n}}}{\partial x} \frac{d^n \alpha_j}{dt^n} \right] - \vec{r} \cos \sigma_0 + \vec{\varphi} \left[\frac{\cos \sigma_0}{R_{s_0}} \sum_j \sum_n \frac{\partial R_{s_{j,n}}}{\partial \varphi} \frac{d^n \alpha_j}{dt^n} \right]}{\sqrt{\sin^2 \sigma_0 + 2 \sin \sigma_0 \cos \sigma_0 \sum_j \sum_n \frac{\partial R_{s_{j,n}}}{\partial x} \frac{d^n \alpha_j}{dt^n} + \cos^2 \sigma_0}} \delta \quad (4.125)$$

The denominator of Equation (4.125) can be rationalized if the numerator and denominator are multiplied by

$$\left[1 - \sin \sigma_0 \cos \sigma_0 \sum_j \sum_n \frac{\partial R_{s_{j,n}}}{\partial x} \frac{d^n \alpha_j}{dt^n} \right]:$$

$$\vec{n}_s = \vec{x} \left[\sin \sigma_0 - \sin^2 \sigma_0 \cos \sigma_0 \sum_j \sum_n \frac{\partial R_{s_{j,n}}}{\partial x} \frac{d^n \alpha_j}{dt^n} + \cos \sigma_0 \sum_j \sum_n \frac{\partial R_{s_{j,n}}}{\partial x} \frac{d^n \alpha_j}{dt^n} \right] - \vec{r} \left[\cos \sigma_0 - \sin \sigma_0 \cos^2 \sigma_0 \sum_j \sum_n \frac{\partial R_{s_{j,n}}}{\partial x} \frac{d^n \alpha_j}{dt^n} \right] \quad (4.126)$$

$$+\varphi \left[\frac{\cos \sigma_0 \sum_j \sum_n \frac{\partial R_{s_{j,n}}}{\partial \varphi} \frac{d^n \alpha_j}{dt^n}}{R_{s_0}} \right] \delta \quad (4.126)$$

(cont.)

Comparisons of Equation (4.95) with Equation (4.216), permits the following conclusions:

$$\left. \begin{aligned} \tau_0 &= \sin \sigma_0 \\ \eta_0 &= -\cos \sigma_0 \\ \xi_0 &= 0 \end{aligned} \right\} \quad (4.127)$$

$$\left. \begin{aligned} \tau_{j,n} &= \cos^3 \sigma_0 \frac{\partial R_{s_{j,n}}}{\partial x} = \cos^3 \sigma_0 \frac{\partial \bar{R}_{s_{j,n}}}{\partial x} \cos \varphi \\ \eta_{j,n} &= \sin \sigma_0 \cos^2 \sigma_0 \frac{\partial R_{s_{j,n}}}{\partial x} = \sin \sigma_0 \cos^2 \sigma_0 \frac{\partial \bar{R}_{s_{j,n}}}{\partial x} \cos \varphi \\ \xi_{j,n} &= \frac{\cos \sigma_0}{R_{s_0}} \frac{\partial R_{s_{j,n}}}{\partial \varphi} = -\frac{\cos \sigma_0}{R_{s_0}} \bar{R}_{s_{j,n}} \sin \varphi \end{aligned} \right\} \quad (4.129)$$

or after removal of dependence on φ :

$$\left. \begin{aligned} \bar{\tau}_{j,n} &= \cos^3 \sigma_0 \frac{\partial \bar{R}_{s_{j,n}}}{\partial x} & \eta_{j,n} &= \sin \sigma_0 \cos^2 \sigma_0 \frac{\partial \bar{R}_{s_{j,n}}}{\partial x} \\ \bar{\xi}_{j,n} &= -\cos \sigma_0 \frac{\bar{R}_{s_{j,n}}}{R_{s_0}} \end{aligned} \right\} \quad (4.130)$$

b. Derivation of $\partial V_{N_\infty} / \alpha \left(\frac{d^n \alpha_j}{dt^n} \right)$

The component of free stream velocity normal to the shock wave consists of the normal component of free stream (absolute) velocity, and the normal component of the shock wave velocity:

$$\vec{V}_{N_\infty} = \vec{V}_\infty \cdot \vec{n}_s - \vec{n}_s \cdot \vec{r} \frac{\partial R_s}{\partial t} \quad (4.131)$$

where:

\vec{V}_∞ is the free stream velocity

\vec{n}_s is the unit vector normal (positive inward) to the shock wave

R_s is the shock wave radius. The free stream velocity vector can be expressed in the terms of its components:

$$\vec{V}_\infty = \vec{x} (V_\infty + \sum_j \alpha_j u_{\infty j}) + \vec{r} \sum_j v_{\infty j} \alpha_j + \vec{\phi} \sum_j w_{\infty j} \alpha_j \delta \quad (4.132)$$

where:

V_∞ is the free stream velocity in the zero yaw steady state case, and

$u_{\infty j}$, $v_{\infty j}$, and $w_{\infty j}$ are perturbations in the free stream velocity (see

Section 4.4.4.3).

The components of the vector normal to the shock have been expressed in the perturbation scheme: (Equation (4.133) is the same as Equation 4.95)

$$\begin{aligned}
\vec{n}_s \cdot \vec{x} &= \tau_o + \sum_j \sum_n \tau_{j,n} \frac{d^n \alpha_j}{dt^n} \\
\vec{n}_s \cdot \vec{r} &= \eta_o + \sum_j \sum_n \eta_{j,n} \frac{d^n \alpha_j}{dt^n} \\
\vec{n}_s \cdot \vec{\varphi} &= \xi_o + \sum_j \sum_n \xi_{j,n} \frac{d^n \alpha_j}{dt^n}
\end{aligned}
\tag{4.133}$$

Equations (4.119), (4.126), (4.132), (4.94) and (4.133) can be combined in Equation (4.131):

$$\begin{aligned}
V_{N_\infty} &= V_\infty \tau_o + \sum_j \sum_n V_\infty \tau_{j,n} \frac{d^n \alpha_j}{dt^n} + \sum_j \left(\tau_o u_{\infty j} + \eta_o v_{\infty j} + \xi_o w_{\infty j} \right) \delta \alpha_j \tag{4.134}* \\
&- \eta_o \sum_j \sum_n R_{s_{j,n}} \frac{d^{(n+1)} \alpha_j}{dt^{(n+1)}}
\end{aligned}$$

* $m = n$ when $j = 1$; $m = n + 1$ when $j = 2$.

Differentiating Equation (4.134):

$$\begin{aligned} \partial V_{N_\infty} / \partial \left(\frac{d^n \alpha_j}{dt^n} \right) &= V_{\infty} \tau_{j,n} + \delta_{o,n} (\tau_o u_{\infty j} + \eta_o v_{\infty j} + \xi_o w_{\infty j} \delta) \\ &- (1 - \delta_{o,n}) \eta_o R_{s_{j,n-1}} \left(\frac{V_{\infty}}{L} \right) \end{aligned} \quad (4.135)*$$

Using Equations (4.127) and (4.130), Equation (4.135) becomes:

$$\begin{aligned} \partial \bar{V}_{N_\infty} / \partial \left(\frac{d^n \alpha_j}{dt^n} \right) &= V_{\infty} \cos^3 \sigma_o \frac{\partial \bar{R}_{s_{j,n}}}{\partial x} + \delta_{o,n} (\bar{u}_{\infty j} \sin \sigma_o - v_{\infty j} \cos \sigma_o) \\ &+ (1 - \delta_{o,n}) \left(\bar{R}_{s_{j,n-1}} \right) \frac{V_{\infty}}{L} \end{aligned} \quad (4.136)$$

*In the equations leading up to Equation (4.134) the term $\frac{d^n \alpha_j}{dt^n}$ which appears within

the double summation should, in every instance, be multiplied by $\left(\frac{L}{V_{\infty}} \right)^m$ in order

to be consistent with our notation (i. e. with Equation (4.33)). It has been omitted in

order to simplify notation. However, in going from Equation (4.134) to (4.135) we

divide by $\left(\frac{L}{V_{\infty}} \right)^m$ (See Appendix VI for a similar step). It is this division which

causes the quantity $\frac{V_{\infty}}{L}$ to appear in the last term of Equation (4.135).

4.4.4.3 Derivation of the Expression for Free Stream Velocity

Figure 8 shows a two-dimensional sketch of a right handed rectangular coordinate system (x, y, z) with origin a distance lp forward of the center of gravity of the body. The x -axis coincides with the body axis. The y -axis is perpendicular to the x -axis, and lies in the plane defined by the x -axis and the vector direction of the velocity of the center of gravity.

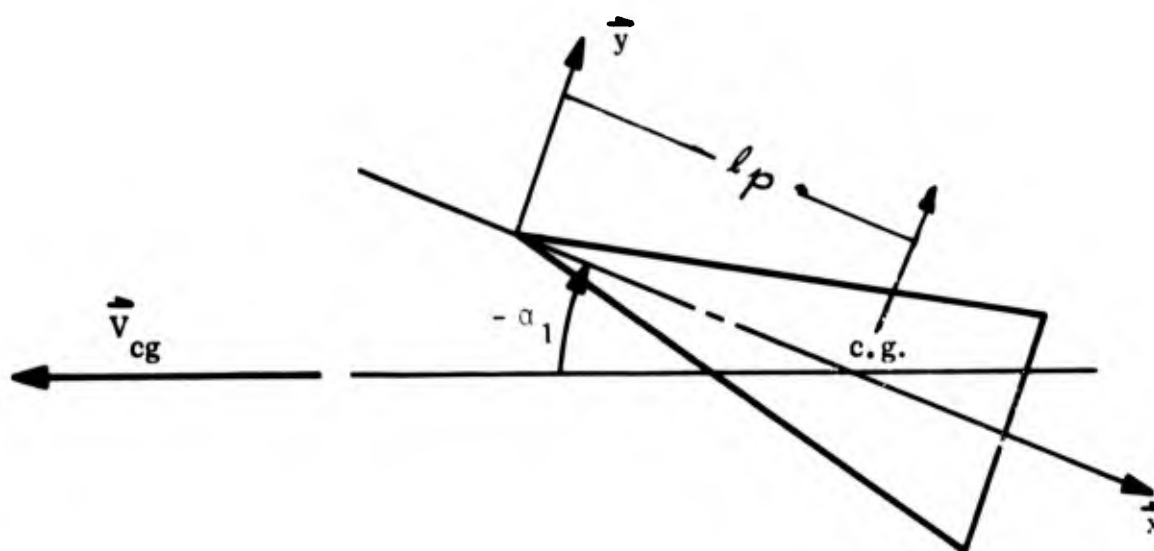


Figure 8. * Two-Dimensional Sketch of a Right-Handed Rectangular Coordinate System (x, y, z)

A second coordinate system, (x, r, φ) is shown in Figure 9. The unit vectors, \vec{r} and $\vec{\varphi}$, lie in the $y - z$ plane. The fluid velocity, relative to the body-fixed coordinate systems, at any point in the free stream is:

$$\vec{V}_{\infty} = -\vec{V}_{cg} - \vec{\Omega} \times (\vec{x} [x-lp] + \vec{r}r) \quad (4.137)$$

*The body in the Figure 8 is intended to represent either a pointed or blunt body of revolution, or a two-dimensional body.

where:

$$\vec{\Omega} = \vec{z} \alpha_2 \quad (4.138)$$

$$\vec{V}_{cg} = -\vec{x} V_\infty \cos \alpha_1 + \vec{y} V_\infty \sin \alpha_1 \quad (4.139)$$

Note that α_2 is an angular velocity, while α_1 is angle of attack. This inconsistency in nomenclature is permitted because it results in a considerably simplified notation throughout this work.

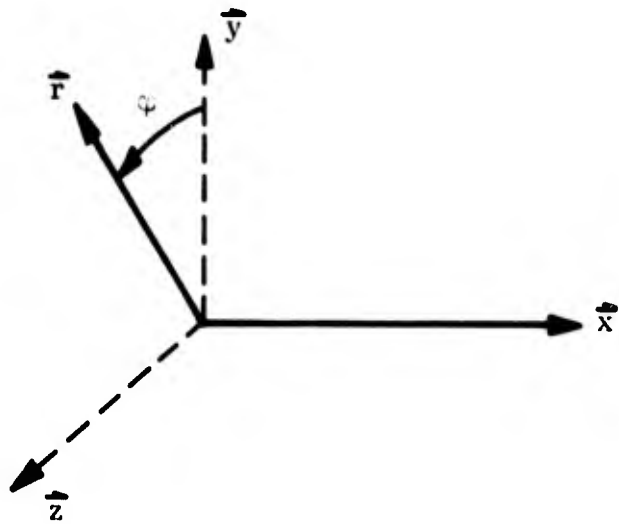


Figure 9. Cylindrical Coordinate System (x, r, φ)

The \vec{y} and \vec{z} unit vectors can be given in the (x, r, φ) system:

$$\vec{y} = \vec{r} \cos(\varphi \delta) - \vec{\phi} \sin(\varphi \delta) \quad (4.140)$$

$$\vec{z} = \vec{r} \sin(\varphi \delta) + \vec{\phi} \cos(\varphi \delta) \quad (4.141)$$

Substituting Equations (4.138) through (4.141) in Equation (4.137):

$$\begin{aligned} \vec{V}_\infty = & \vec{x} [V_\infty + \alpha_2 r \cos(\varphi \delta)] \\ & + \vec{r} [-V_\infty \alpha_1 \cos(\varphi \delta) - \alpha_2 (x - lp) \cos(\varphi \delta)] \\ & + \vec{\phi} [V_\infty \alpha_1 \sin(\varphi \delta) + \alpha_2 (x - lp) \sin(\varphi \delta)] \end{aligned} \quad (4.142)$$

The approximations:

$$\cos \alpha_1 = 1$$

$$\sin \alpha_1 = \alpha_1$$

have been used in Equation (4.142).

In Section 4.4.4.2, the free stream velocity was expressed as:

$$\vec{V}_\infty = \vec{x}(V_\infty + \sum_j \alpha_j u_{\infty j}) + \vec{r} \sum_j v_{\infty j} \alpha_j + \vec{\varphi} \sum_j w_{\infty j} \alpha_j \delta \quad (4.143)$$

Comparison of Equation (4.142) with Equation (4.143) leads to the conclusion that:

$$\begin{aligned} u_{\infty j} &= 0 && \text{for } j = 1 \\ &= + r \cos(\varphi \delta) && \text{for } j = 2 \end{aligned} \quad (4.144)$$

$$\begin{aligned} v_{\infty j} &= -V_\infty \cos(\varphi \delta) && \text{for } j = 1 \\ &= -(x - lp) \cos(\varphi \delta) && \text{for } j = 2 \end{aligned} \quad (4.145)$$

$$\begin{aligned} w_{\infty j} &= V_\infty \sin(\varphi \delta) && \text{for } j = 1 \\ &= (x - lp) \sin(\varphi \delta) && \text{for } j = 2 \end{aligned} \quad (4.146)$$

4.4.5 The Characteristics Solution

Solution by the method of characteristics entails finding those lines in the flow field which may contain discontinuities in the spatial derivatives of the flow field parameters, but not in the parameters themselves. This can be done at any point in the field by arbitrarily placing a line at an angle β to the zero-yaw streamline direction. If a differential length along this line is called dl , then the relation,

$$\frac{dF}{d\ell} = \cos \beta \frac{dF}{d\ell_0} + \sin \beta \frac{dF}{dn_0} \quad (4.147)$$

is seen to hold, where F represents each of the flow field parameters which appear in Equations (4.77) through (4.82). Equation (4.147) then represents a matrix of 6 Equations. In order to determine what values of β (if any) will determine the desired lines along which the various $\frac{dF}{d\ell_0}$ and $\frac{dF}{dn_0}$ may be discontinuous, Equations (4.77) through (4.82) and Equation (4.147) must be treated as twelve algebraic Equations in the twelve derivatives represented by $\frac{dF}{d\ell_0}$ and $\frac{dF}{dn_0}$. The desired values of β are found by setting the determinant of the resulting coefficient matrix equal to zero.

The values of β for which this determinant is zero are:

$$\beta = 0 \quad (4.148)$$

and

$$\beta = \pm \sin^{-1} \left(\frac{1}{M_0} \right) = \pm \mu_0 \quad (4.149)$$

Equation (4.148) represents the zero-yaw streamlines, Equation (4.149) the zero-yaw characteristic lines. If the twelve algebraic Equations are further investigated to determine under what conditions the derivatives $\frac{dF}{d\ell_0}$ and $\frac{dF}{dn_0}$ are indeterminate (rather than infinite), no further conditions (beyond Equation (4.148)) are found necessary on the streamlines. On the characteristic lines, however, the following conditions must hold:

on the left running characteristic $\left(\beta = + \sin^{-1} \left(\frac{1}{M_0} \right) \right)$.

$$\frac{d\bar{\theta}_{j,n}}{d\ell_+} + \frac{1}{\rho_o V_o^2} \sqrt{M_o^2 - 1} \frac{d\bar{p}_{j,n}}{d\ell_+} - \frac{\lambda_1 C_o}{\rho_o V_o^2} - \frac{\lambda_2 \sqrt{1 - M_o^2}}{\rho_o V_o^2} = 0 \quad (4.150)$$

on the right running characteristic $\left(\beta = -\sin^{-1}\left(\frac{1}{M_o}\right)\right)$,

$$\frac{d\bar{\theta}_{j,n}}{d\ell_-} - \frac{1}{\rho_o V_o^2} \sqrt{M_o^2 - 1} \frac{d\bar{p}_{j,n}}{d\ell_-} + \frac{\lambda_1 C_o}{\rho_o V_o^2} - \frac{\lambda_2 \sqrt{1 - M_o^2}}{\rho_o V_o^2} = 0 \quad (4.151)$$

In Equations (4.150) and (4.151) the differential lengths along the left and right running characteristic lines are denoted by $d\ell_+$ and $d\ell_-$ respectively. λ_1 and λ_2 are known functions of the unknown flow field parameters (but not of their derivatives), which result when the Equations being investigated are solved for the derivatives of the flow field parameters. They are given in Equations (4.155) and (4.156).

The solution can be carried on to solve for eleven of the derivatives $\frac{dF}{d\mathcal{A}_o}$ and $\frac{dF}{dn_o}$ in terms of the six perturbation flow field parameters F , and the twelfth derivative $\frac{d\bar{p}_{j,n}}{d\mathcal{A}_o}$. Some of these solutions will be in indeterminate form, and therefore trivial. The non-trivial solutions are:

$$\frac{d\bar{w}_{j,n}}{d\mathcal{A}_o} = \frac{\lambda_3}{\rho_o V_o} \quad (4.152)$$

$$\frac{d\bar{s}_{j,n}}{d\mathcal{A}_o} = \frac{\lambda_4}{V_o} \quad (4.153)$$

$$\frac{d\bar{q}_{j,n}}{d\mathcal{L}_o} = -\frac{1}{\rho_o V_o} \frac{d\bar{p}_{j,n}}{d\mathcal{L}_o} + \frac{\lambda_5}{\rho_o V_o} \quad (4.154)$$

λ_3 , λ_4 , and λ_5 , like λ_1 and λ_2 , are known functions of the flow field parameters.

The expressions for the five λ are:

$$\lambda_1 = -\bar{V}_{j,n} \left[\frac{d\rho_o}{d\mathcal{L}_o} + \rho_o \frac{d\theta_o}{dn_o} + \delta\rho_o \frac{\sin\theta_o}{r} - \frac{\rho_o}{V_o} \frac{dV_o}{d\mathcal{L}_o} \right] \quad (4.155)$$

$$- \bar{w}_{j,n} \left[\frac{\rho_o}{r} \right] - \bar{\theta}_{j,n} \left[\frac{V_o e_o^2}{C_o^2} \frac{dS_o}{dn_o} - \rho_o V_o \frac{d\theta_o}{d\mathcal{L}_o} + \frac{\delta\rho_o V_o \cos\theta_o}{r} \right]$$

$$+ V_o \frac{d\rho_o}{dn_o} - \frac{1}{V_o} \frac{dp_o}{dn_o} + \bar{p}_{j,n} \left[\frac{V_o e_o^2}{C_o^2} \frac{d\left(\frac{C_o^2}{e_o^2}\right)}{d\mathcal{L}_o} - V_o \frac{d\theta_o}{dn_o} \right]$$

$$- \delta V_o \frac{\sin\theta_o}{r} \left] - \bar{p}_{j,n} \left[V_o \frac{e_o^2}{C_o^2} \frac{d\left(\frac{1}{e_o^2}\right)}{d\mathcal{L}_o} \right]$$

$$+ (1-\delta_{o,n}) \left[\frac{\rho_o}{V_o} \bar{V}_{j,n-1} - \frac{e_o^2}{C_o^2} \bar{S}_{j,n-1} - \bar{p}_{j,n-1} \right] + \frac{\delta_{1,n} \rho_o V_o \sin\theta_o}{V_o}$$

$$\lambda_2 = -\bar{V}_{j,n} \left[2\rho_o V_o \frac{d\theta_o}{d\mathcal{L}_o} \right] - \bar{p}_{j,n} \left[V_o^2 \frac{d\theta_o}{dn_o} \right] - \bar{\theta}_{j,n} \left[\rho_o V_o^2 \frac{d\theta_o}{dn_o} - \frac{dp_o}{d\mathcal{L}_o} \right] \quad (4.156)$$

$$- (1-\delta_{o,n}) \left(\rho_o V_o \bar{\theta}_{j,n-1} \right) - \delta_{1,n} \rho_o V_o \cos\theta_o$$

$$\lambda_3 = -\bar{w}_{j,n} \left[\frac{\rho_o V_o \sin \theta_o}{r} \right] + \bar{p}_{j,n} \left[\frac{1}{r} \right] - (1-\delta_{o,n}) (\rho_o \bar{w}_{j,n-1}) + \delta_{1,n} \rho_o V_\infty \quad (4.157)^*$$

$$\lambda_4 = -\bar{\theta}_{j,n} \left[V_o \frac{dS_o}{dn_o} \right] - (1-\delta_{o,n}) \bar{S}_{j,n-1} \quad (4.158)$$

$$\lambda_5 = -\bar{V}_{j,n} \left[\rho_o \frac{dq_o}{dn_o} \right] - \bar{p}_{j,n} \left[V_o \frac{dq_o}{dn_o} \right] - \bar{\theta}_{j,n} \left[\frac{dp_o}{dn_o} + \rho_o V_o \frac{dV_o}{dn_o} \right] \quad (4.159)$$

$$- (1-\delta_{o,n}) (\rho_o \bar{V}_{j,n-1}) - \delta_{1,n} \rho_o V_\infty \sin \theta_o$$

Equations (4.150) through (4.154), along with Equations (4.82) and (4.84) permit us to begin a numerical solution of the flow field.

4.4.6 The Numerical Solution in the Supersonic Region

The numerical solution in the supersonic shock layer consists of the repeated application of three "unit processes." Each unit process requires that all the perturbation flow field parameters be known at two points in the flow field, and permits the calculation of the same parameters at a third point downstream of the other two. ** If in Figure 10, the perturbation flow field parameters are known at points A and B, and are to be computed at point C, the three unit processes are:

*Equation (4.157) is not applicable in the two-dimensional case.

**Values of the flow field parameters which would exist if the body were in steady flight at zero yaw, must be known at all three points.

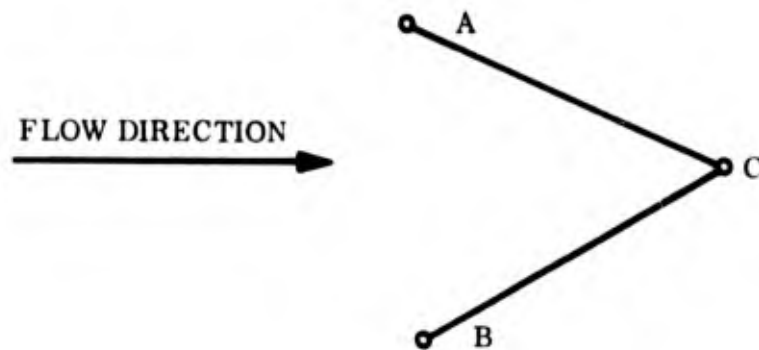


Figure 10. Location Of Points A, B and C On The Characteristics Grid

1. The body point process which is used when points B and C lie on the surface of the body. In this case, C lies on the same zero-yaw right running characteristic as point A;
2. The shock point process which is used when points A and C lie on the shock wave. In this case, point C lies on the same zero-yaw left running characteristics as point B;
3. The field point process which is used when none of the points lies on either the body surface or the shock wave. In this case, point C lies on the same zero-yaw right running characteristic as point A, and the same zero-yaw left running characteristic as point B.

A description of these three processes is given in following sections. The application of these three unit processes to the solution of the supersonic portion of the flow field requires:

1. A complete description of the zero-yaw flow field in the region where calculations are to be performed, and
2. The values of the perturbation flow field parameters along a "starting line" in the flow field.

The former is obtained from the zero-yaw solution (Section 3). The latter is obtained from a solution for a pointed cone in unsteady motion (Section 4.3), if a pointed body is being investigated, or from a transonic solution for a spherically blunted body in unsteady motion (Section 4.5), if a spherically blunted body is being investigated.

The starting line in the case of a pointed body will always be a left running zero-yaw characteristic, which intersects the body surface on the conic section. This starting line was chosen because it fits in effectively with the sequence in which the computations in the rest of the field are carried out. Providing the information along this starting line presents no difficulties in the pointed cone solution for flow field perturbations. In figure 11, points 1 through 5 are on the starting line. The solution begins with computation of the perturbation flow field parameters at point 6 using the body point unit process and the known perturbation parameters at points 1 and 2. It continues to point 7, using the field point process and the known values at points 3 and 6. It then continues to points 8 and 9 using the same unit process. Finally the solution at point 10 is obtained from the known values at points 5 and 9, using the shock point

process. The computation then moves to point 11 and continues to points 12, 13, etc., until the aft end of the body is reached.

The computations for a spherically blunted body follow a similar scheme. They are only slightly complicated by the fact that the starting line may not be a characteristic line. The complications appear only in programming the solution, they do not affect the theory.

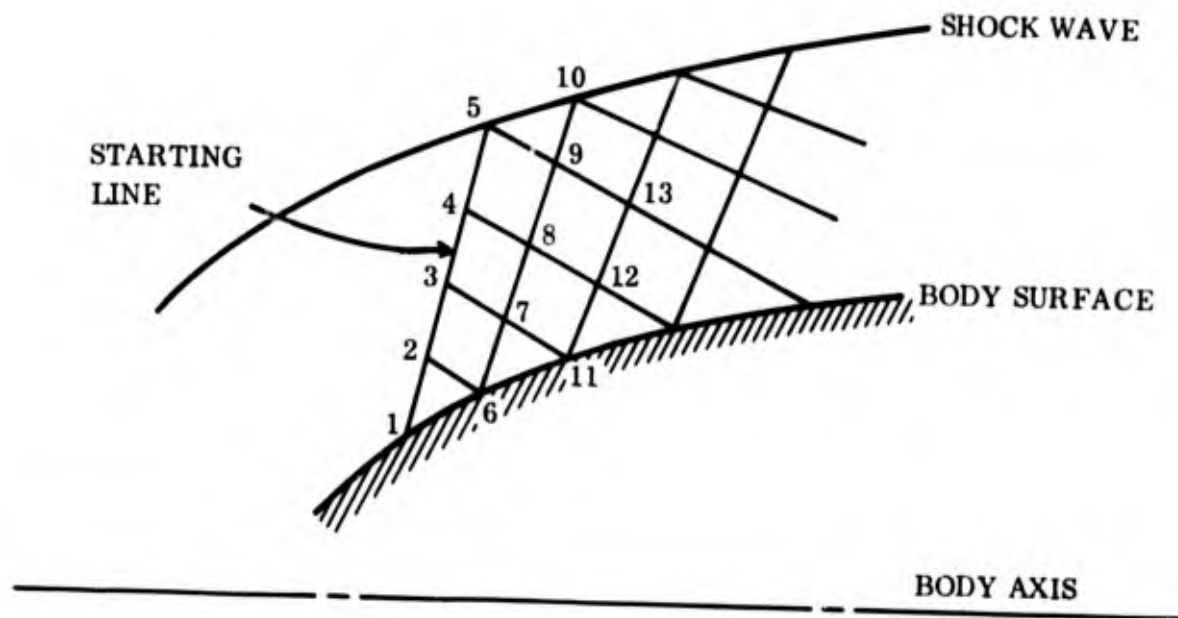


Figure 11. Sequence Of Computations In The Unsteady Supersonic Solution

4.4.6.1 The Field Point Process

In the field point process, the values of the perturbation parameters are known at points A and B (figure 12). The zero-yaw flow field is known at points A, B, and C. It is desired to compute the values of the perturbation parameters at point C. Point D is defined to lie on the same zero-yaw streamline as point C and on a straight line

with points A and B. If it is assumed that all the zero-yaw and perturbation parameters vary linearly between points A and B, then the values of the perturbation parameters as well as the zero-yaw parameters at point D are given by:

$$F_D = F_B + \begin{bmatrix} \Psi_{oB} & -\Psi_{oD} \\ \Psi_{oB} & -\Psi_{oA} \end{bmatrix} (F_A - F_B) \quad (4.160)$$

F represents each of the parameters, and the subscripts A, B, C, D refer to the points in Figure 12. Equation (4.160) is useful since $\Psi_{oD} = \Psi_{oC}$.

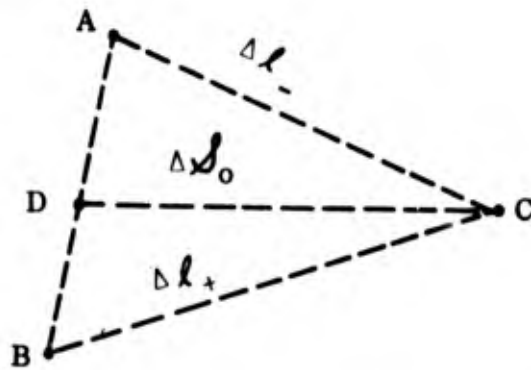


Figure 12. Grid Points For The Field Point Process

Having obtained values of the perturbation parameters at point D, Equations (4.150) through (4.154) can be put in finite difference form, and the final step in the computation can be indicated. In the following Equations the subscripts A, B, C, and D will again indicate the points (Figure 12) at which the subscripted values are to be taken. Combinations of two of these subscripts will indicate an average, e.g. V_{OAC} indicates that the average of the values of V_o at points A and C is to be used.

$$\bar{\theta}_{j,n_C} - \bar{\theta}_{j,n_B} + \left[\frac{1}{\rho_o V_o^2} \sqrt{M_o^2 - 1} \right]_{BC} (\bar{p}_{j,n_C} - \bar{p}_{j,n_B}) = \Delta \ell_+ \left[\frac{\lambda_1 C_o}{\rho_o V_o^2} + \frac{\lambda_2 \sqrt{1 - \frac{1}{M_o^2}}}{\rho_o V_o^2} \right]_{BC} \quad (4.161)$$

from Equation (4.151):

$$\bar{\theta}_{j,n_C} - \bar{\theta}_{j,n_A} - \left[\frac{1}{\rho_o V_o^2} \sqrt{M_o^2 - 1} \right]_{AC} (\bar{p}_{j,n_C} - \bar{p}_{j,n_A}) = -\Delta \ell_- \left[\frac{\lambda_1 C_o}{\rho_o V_o^2} - \frac{\lambda_2 \sqrt{1 - \frac{1}{M_o^2}}}{\rho_o V_o^2} \right]_{AC} \quad (4.162)$$

from Equation (4. 152):

$$\bar{w}_{j,n_C} - \bar{w}_{j,n_D} = \Delta \left[\frac{\lambda}{\rho_o V_o} \right]_{CD} \quad (4. 163)^*$$

from Equation (4. 153):

$$\bar{s}_{j,n_C} - \bar{s}_{j,n_D} = \Delta \left[\frac{\lambda}{V_o} \right]_{CD} \quad (4. 164)$$

from Equation (4. 154):

$$\bar{v}_{j,n_C} - \bar{v}_{j,n_D} + \left[\frac{1}{\rho_o V_o} \right]_{CD} (\bar{p}_{j,n_C} - \bar{p}_{j,n_D}) = \left[\frac{\lambda}{\rho_o V_o} \right]_{CD} \Delta \quad (4. 165)$$

Equations (4. 161) through (4. 165), together with Equations (4. 166) and 4. 167) which are identical to Equations (4. 82) and (4. 84):

$$\bar{p}_{j,n_C} = \left[\frac{1}{C_o^2} \right]_C \bar{p}_{j,n_C} - \left[\frac{e_o^2}{C_o^2} \right]_C \bar{s}_{j,n_C} \quad (4. 166)$$

$$\bar{T}_{j,n_C} = \left[\frac{1}{Z_o R \rho_o} \right]_C \left\{ \bar{p}_{j,n_C} - \bar{p}_{j,n_C} \left[Z_o R T_o + R T_o \rho_o \frac{\partial Z}{\partial \rho} \right]_C - \bar{s}_{j,n_C} \left[R T_o \rho_o \frac{\partial Z}{\partial S} \right]_C \right\} \quad (4. 167)$$

*Equation (4. 163) is not applicable in the two-dimensional case

constitute a set of seven algebraic equations in the seven unknowns \bar{p}_{j,n_C} , $\bar{\rho}_{j,n_C}$, \bar{v}_{j,n_C} , $\bar{\theta}_{j,n_C}$, \bar{w}_{j,n_C} , \bar{s}_{j,n_C} , and \bar{T}_{j,n_C} . They need only be solved for the seven unknowns.

Note that the solution of this set is somewhat more complicated than it may at first seem, because the unknowns (but not their derivatives) are present in λ_1 through λ_5 .

4.4.6.2 The Body Point Process

Again the values of the perturbation parameters are known at points A and B (Figure 13). The zero-yaw field is known at points A, B, and C. It is desired to compute the perturbation parameters at point C.

Since flow cannot pass through the body surface, at point C the flow direction must be the same as it was in the zero-yaw field. Therefore,

$$\bar{\theta}_{j,n_C} = 0 \quad (4.168)$$

This fact reduces Equation (4.162)

$$-\bar{\theta}_{j,n_A} - \left[\frac{1}{\rho_o V_o^2} \sqrt{M_o^2 - 1} \right]_{AC} (\bar{p}_{j,n_C} - \bar{p}_{j,n_A}) = -\Delta \ell - \left[\frac{\lambda_{1C}}{\rho_o V_o^2} - \frac{\lambda_2 \sqrt{1 - \frac{1}{M_o^2}}}{\rho_o V_o^2} \right]_{AC} \quad (4.169)$$

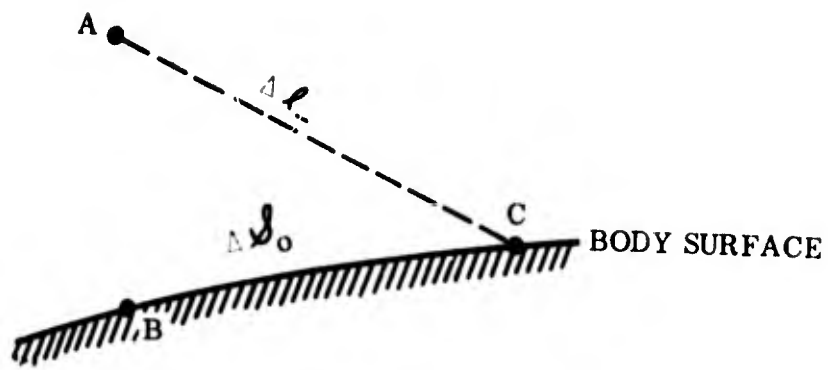


Figure 13. Grid Points For The Body Point Process

Equations (4. 152) through (4. 154) in finite difference form are (note that line BC is a zero-yaw streamline):

$$\bar{w}_{j,n_C} - \bar{w}_{j,n_B} = \left[\frac{\lambda_3}{\rho_0 V_0} \right]_{BC} \Delta s_0 \quad (4. 170)^*$$

$$\bar{s}_{j,n_C} - \bar{s}_{j,n_B} = \left[\frac{\lambda_4}{V_0} \right]_{BC} \Delta s_0 \quad (4. 171)$$

$$\bar{v}_{j,n_C} - \bar{v}_{j,n_B} + \left[\frac{1}{\rho_0 V_0} \right]_{BC} (\bar{p}_{j,n_C} - \bar{p}_{j,n_B}) = \left[\frac{\lambda_5}{\rho_0 V_0} \right]_{BC} \Delta s_0 \quad (4. 172)$$

*Equation (4. 170) is not applicable in the two-dimensional case.

Equations (4.169) through (4.172) together with Equations (4.173) and (4.174) (which are identical to Equations (4.82) and (4.84)) and (4.166) and (4.167),

$$\rho_{j,n_C} = \left[\frac{1}{C_o^2} \right]_C \bar{p}_{j,n_C} - \left[\frac{e_o^2}{C_o^2} \right]_C \bar{s}_{j,n_C} \quad (4.173)$$

$$\bar{T}_{j,n_C} = \left[\frac{1}{Z_o R \rho_o} \right]_C \left\{ \bar{p}_{j,n_C} - \bar{\rho}_{j,n_C} \left[Z_o RT_o + RT_o \rho_o \frac{\partial Z}{\partial \rho} \right]_C \right. \quad (4.174)$$

$$\left. - \bar{s}_{j,n_C} \left[RT_o \rho_o \frac{\partial Z}{\partial S} \right]_C \right\}$$

constitute a set of six algebraic equations in the six unknowns \bar{p}_{j,n_C} , $\bar{\rho}_{j,n_C}$, \bar{v}_{j,n_C} , \bar{w}_{j,n_C} , \bar{s}_{j,n_C} , and \bar{T}_{j,n_C} . They must again be solved for the unknowns. Again the unknowns are present in λ_1 through λ_5 .

4.4.6.3 The Shock Point Process

The values of the perturbation parameters are again known at points A* and B (Figure 14). The zero-yaw field is known at points A, B, and C. It is desired to compute the perturbation parameters at point C.

*At the shock wave, an additional parameter, R_s , enters, and the perturbations in R_s must be known at point A and calculated at point C.

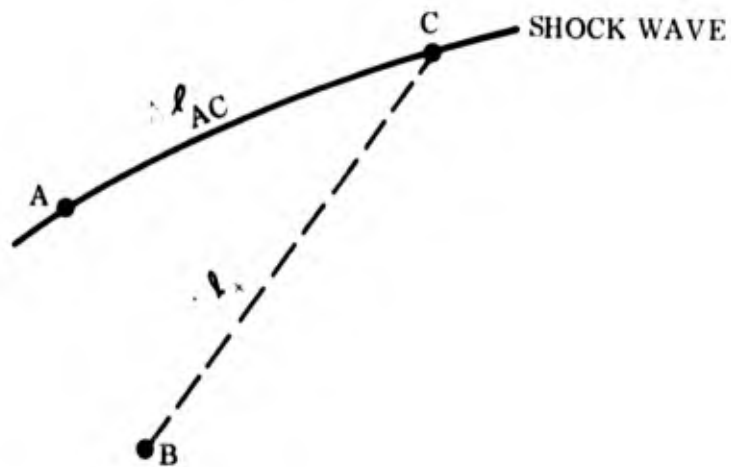


Figure 14. Grid Points For The Shock Point Process

Equation (4.150), in finite difference form is valid between points B and C.

$$\bar{\theta}_{j,n_C} - \bar{\theta}_{j,n_B} + \left[\frac{1}{\rho_o V_o^2 \sqrt{M_o^2 - 1}} \right]_{BC} (\bar{p}_{j,n_C} - \bar{p}_{j,n_B}) = \Delta \ell + \left[\frac{1}{\rho_o V_o^2} \right]_{C} \quad (4.175)$$

$$\left[\frac{\lambda}{2 \sqrt{1 - \frac{1}{M_o^2}}} \right]_{BC} \rho_o V_o^2$$

The shock boundary conditions (See Section 4.4.4, Equations (4.89), (4.90), (4.98), (4.103), and (4.104) are:

$$\bar{p}_{j,n_C} = \left[\frac{\partial F_1}{\partial V_{N_\infty}} \frac{d(\bar{v}_{N_\infty})}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} \right]_C - \left[\frac{\partial p_o}{\partial r} \bar{R}_{S_{j,n}} \right]_C \quad (4.176)$$

$$\bar{\rho}_{j,n_C} = \left[\frac{\partial F_2}{\partial V_{N_\infty}} \frac{d(\bar{v}_{N_\infty})}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} \right]_C - \left[\frac{\partial \rho_o}{\partial r} \bar{R}_{S_{j,n}} \right]_C \quad (4.177)$$

$$\bar{w}_{j,n_C} = \left[\bar{w}_{\infty,j,n} + F_{3_o} \bar{\xi}_{j,n} + \xi_o \frac{\partial F_3}{\partial V_{N_\infty}} \frac{d(\bar{v}_{N_\infty})}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} \right]_C \quad (4.178)^*$$

*Equation (4.178) is not applicable in the two-dimensional case.

$$\bar{v}_{j,nC} = \left[\bar{u}_{\infty j,n} + F_{3o} \bar{\tau}_{j,n} + \tau_o \frac{\partial F_3}{\partial V_{N_{\infty}}} \frac{d\left(\frac{\bar{v}_{N_{\infty}}}{dt^n}\right)}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} - \frac{\partial u_o}{\partial r} \bar{R}_{s,j,n} \right]_C \cos \theta_{oC} \quad (4.179)$$

$$+ \left[\bar{v}_{\infty j,n} + F_{3o} \bar{\eta}_{j,n} + \eta_o \frac{\partial F_3}{\partial V_{N_{\infty}}} \frac{d\left(\frac{\bar{v}_{N_{\infty}}}{dt^n}\right)}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} - \frac{\partial v_o}{\partial r} \bar{R}_{s,j,n} \right]_C \sin \theta_{oC}$$

$$\bar{\theta}_{j,nC} = -\frac{1}{V_{oC}} \left[\bar{u}_{\infty j,n} + F_{3o} \bar{\tau}_{j,n} + \tau_o \frac{\partial F_3}{\partial V_{N_{\infty}}} \frac{d\left(\frac{\bar{v}_{N_{\infty}}}{dt^n}\right)}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} - \frac{\partial u_o}{\partial r} \bar{R}_{s,j,n} \right]_C \sin \theta_{oC} \quad (4.180)$$

$$+ \frac{1}{V_{oC}} \left[\bar{v}_{\infty j,n} + F_{3o} \bar{\eta}_{j,n} + \eta_o \frac{\partial F_3}{\partial V_{N_{\infty}}} \frac{d\left(\frac{\bar{v}_{N_{\infty}}}{dt^n}\right)}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} \right]$$

$$- \frac{\partial v_o}{\partial r} \bar{R}_{s,j,n} \right]_C \cos \theta_{oC}$$

Equations (4.181) and (4.182) which are identical to Equations (4.84) and (4.82), respectively, apply:

$$\bar{T}_{j,n_C} = \left[\frac{1}{Z_o R \rho_o} \right]_C \left\{ \bar{p}_{j,n_C} - \bar{\rho}_{j,n_C} \left[Z_o RT_o + RT_o \rho_o \frac{\partial Z}{\partial \rho} \right]_C - \bar{s}_{j,n_C} \left[RT_o \rho_o \frac{\partial Z}{\partial s} \right]_C \right\} \quad (4.181)$$

$$\bar{p}_{j,n_C} = \left[\frac{1}{C_o} \right]_C \bar{p}_{j,n_C} - \left[\frac{e_o}{C_o} \right]_C \bar{s}_{j,n_C} \quad (4.182)$$

Equations (4.175) through (4.182) constitute a set of eight algebraic equations in the nine

unknowns \bar{p}_{j,n_C} , $\bar{\rho}_{j,n_C}$, \bar{v}_{j,n_C} , \bar{w}_{j,n_C} , $\bar{\theta}_{j,n_C}$, \bar{s}_{j,n_C} , \bar{T}_{j,n_C} , \bar{R}_{j,n_C} and $\left[\frac{\partial \bar{R}_{s_{j,n}}}{\partial x} \right]_C^*$.

The ninth equation is simply the backward difference expression for $\bar{R}_{s_{j,n}}$.

$$\bar{R}_{s_{j,n_C}} = \bar{R}_{s_{j,n_A}} + \frac{1}{2} \left(\left[\frac{\partial \bar{R}_{s_{j,n}}}{\partial x} \right]_C + \left[\frac{\partial \bar{R}_{s_{j,n}}}{\partial x} \right]_A \right) (x_C - x_A) \quad (4.183)$$

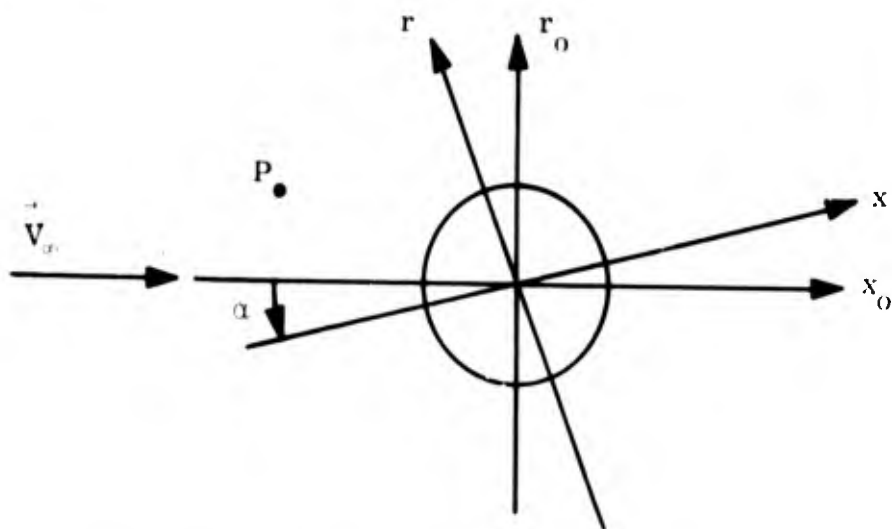
*Although $\frac{\partial \bar{R}_{s_{j,n}}}{\partial x}$ does not appear explicitly in Equations (4.175) through (4.182),

it is included in the terms $\bar{\tau}_{j,n}$ and $\bar{\eta}_{j,n}$.

This set of nine equations must be solved for the unknowns. Expressions for the terms on the right hand side of these equations are given in Sections 4.4.4.1 through 4.4.4.3. **

4.5 THE STARTING (TRANSONIC) SOLUTION FOR SPHERICALLY BLUNTED BODIES

4.5.1 The Small Yaw ($j = 1, n = 0$) case



If the solution for flow about a spherical body is known in the x_0, r_0 coordinate system in the above sketch, it can be expressed in the x, r system as follows (as long as the angle α is small):

**Section 4.4.4.1 provides expressions for $\frac{\partial F_1}{\partial V_{N_\infty}}, \frac{\partial F_2}{\partial V_{N_\infty}},$ and $\frac{\partial F_3}{\partial V_{N_\infty}}$. Section 4.4.4.2 provides $\tau_0, \eta_0, \xi_0, \bar{\tau}_{j,n}, \bar{\eta}_{j,n}, \bar{\xi}_{j,n},$ and $\frac{d \bar{V}_{N_\infty}}{d \left(\frac{d^n \alpha_j}{dt^n} \right)}$. Section 4.4.4.3 provides expressions for $\bar{u}_{\infty j,n}, \bar{v}_{\infty j,n},$ and $\bar{w}_{\infty j,n}$. All other terms come from the zero-yaw solution or from thermodynamic tables.

$$p = p_o + \frac{\partial p_o}{\partial x} \frac{dx}{d\alpha} \alpha + \frac{\partial p_o}{\partial r} \frac{dr}{d\alpha} \alpha \quad (4.184)$$

where $\frac{dx}{d\alpha}$ and $\frac{dr}{d\alpha}$ represent the rate of change of the coordinates of a general point P, with α , when going from the x_o, r_o system to the x, r system. Since the flow field surrounding a spherical body does not change when the body is rotated about its center, the above representation will be useful if the x_o axis is taken to coincide with the axis of a spherically blunted* body at zero angle of attack; and the x axis is taken to coincide with the axis of the same body at small yaw. The above equation then fits our perturbation scheme:

$$p = p_o + p_{1,0} \alpha \cos(\varphi \delta),$$

if we note that $\frac{dx}{d\alpha}$ and $\frac{dr}{d\alpha}$ have a sinusoidal dependence on φ , i. e. that

$$\frac{dx}{d\alpha} = -r \cos(\varphi \delta)$$

and

$$\frac{dr}{d\alpha} = x \cos(\varphi \delta)$$

*For the two dimensional case this discussion is applicable to a cylindrically blunted body.

so that

$$p_{1,0} = \left\{ -r \frac{\partial p_o}{\partial x} + x \frac{\partial p_o}{\partial r} \right\} \cos(\varphi \delta) \quad (4.185)$$

all other scalar quantities have the same form as Equation (4.185). Because the derivatives of the unit vectors \vec{x} , \vec{r} and $\vec{\varphi}$ with respect to α are non-zero, the velocity components take on a slightly different appearance:

$$\theta_{1,0} = \left\{ -1 - r \frac{\partial \theta_o}{\partial x} + x \frac{\partial \theta_o}{\partial r} \right\} \cos(\varphi \delta) \quad (4.186)$$

and

$$w_{1,0} = V_o \left\{ \cos \theta_o - \frac{x}{r} \sin \theta_o \right\} \sin(\varphi \delta) \quad (4.187)$$

The perturbation in shock radius is obtained from

$$\begin{aligned} R_s &= R_{s_o} + \frac{\partial R_{s_o}}{\partial x} \frac{dx}{d\alpha} \alpha + \frac{\partial R_{s_o}}{\partial r} \frac{dr}{dx} \alpha \\ &= R_{s_o} - \alpha \left\{ R_{s_o} \tan \sigma_o + x \right\} \cos(\varphi \delta) \end{aligned} \quad (4.188)$$

In Equations (4.184) through (4.188), x is assumed to be measured from the center of the blunting sphere, and in equation (4.188) $\tan \sigma_o$ is:

$$\tan \sigma_o = \frac{dR_{s_o}}{dx} \quad (4.189)$$

4.5.2 The $j = 1, n = 1$ and $j = 2, n = 0$ Cases

Since the angular velocity (due to the $\dot{\alpha} + q$ motions) of a sphere* about its own center will not affect the inviscid flow field surrounding the sphere, the sum of the $\dot{\alpha}$ and q derivatives of scalar values must be zero. The sum of the derivatives of the velocity components must however reflect the fact that the $\vec{x}, \vec{r}, \vec{\phi}$ unit vectors are dependent on $\dot{\alpha}$ and q . Within these requirements and the need to make the solution compatible with the solution for $j = 1, n = 0$ the $\dot{\alpha}$ and q derivatives may be assigned arbitrary values. For example, $\bar{p}_{1,1}$ can be given any value as long as $\bar{p}_{2,0}$ is set equal to minus $\bar{p}_{1,1}$. In the present discussion, the following values will be assigned:

$$\left. \begin{aligned}
 \bar{R}_{s_{1,1}} &= \bar{R}_{s_{2,0}} = 0 \\
 \bar{p}_{1,1} &= \bar{p}_{2,0} = 0 \\
 \bar{\rho}_{1,1} &= \bar{\rho}_{2,0} = 0 \\
 \frac{\bar{s}_{1,1}}{R} &= \frac{\bar{s}_{2,0}}{R} = 0
 \end{aligned} \right\} \quad (4.190)$$

*For the two dimensional case, this discussion is applicable to a cylindrically blunted body.

Substituting Equations (4. 190) into Equations (4. 176) and (4. 177) however, indicates that by assigning these values we have implied that

$$\frac{d(\bar{V}_{N_\infty})}{d\left(\frac{d^n \alpha_j}{dt^n}\right)} = 0 \quad (4. 191)$$

Using this fact in Equation (4. 136) leads us to conclude that (note the definition of

$R_{s_{1,0}}$ in Equation (4. 188):

$$\frac{\partial \bar{R}_{s_{1,1}}}{\partial x} = - \frac{\partial \bar{R}_{s_{2,0}}}{\partial x} = \frac{R_{s_0} \sin \sigma_0 + x_s \cos \sigma_0}{L \cos^3 \sigma_0} \quad (4. 192)$$

Substitution of this equation in Equations (4. 178) through (4. 180) yields:

$$\bar{V}_{1,1} = - \left(\frac{V_\infty \sin \theta_s}{L \cos \sigma_0} \right) \left(R_{s_0} \sin \sigma_0 + x_s \cos \sigma_0 \right) \quad (4. 193)$$

$$\bar{\theta}_{1,1} = - \frac{\sin \theta_s \sin (\sigma_0 - \theta_s) (R_{s_0} \sin \sigma_0 + x_s \cos \sigma_0) (x \cos \theta_0 + r \sin \theta_0)}{L \cos^2 \sigma_0 (x_s \cos \theta_s + R_{s_0} \sin \theta_s)} \quad (4. 194)$$

$$\bar{w}_{1,1} = 0 \quad (4. 195)$$

$$\bar{V}_{2,0} = -\bar{V}_{1,1} + \frac{V_\infty}{L} (r \cos \theta_0 - x \sin \theta_0) \quad (4. 196)$$

$$\bar{\theta}_{2,0} = -\bar{\theta}_{1,1} - \frac{V_{\infty}}{V_0 L} (r \sin \theta_0 + x \cos \theta_0) \quad (4.197)$$

$$\bar{w}_{2,0} = V_{\infty} \frac{x}{L} \quad (4.198)$$

In the preceding equations, the subscript s used with x and θ implies that values of x and θ at the shock point on the starting line are to be used. Unsubscripted values of x and r imply local values of x and r at each point on the starting line.

In the definition of $\theta_{1,1}$ the ratio:

$$\frac{(x \cos \theta_0 + r \sin \theta_0)}{(x_s \cos \theta_s + R_{s0} \sin \theta_s)}$$

is used in order to satisfy the boundary requirement at the body surface, that:

$$\bar{\theta}_{1,1} = \bar{\theta}_{2,0} = 0$$

at the body surface. Inspection of Equations (4.193) through (4.198) indicates that $(\bar{V}_{1,1} + \bar{V}_{2,0})$, $(\bar{\theta}_{1,1} + \bar{\theta}_{2,0})$ and $(\bar{w}_{1,1} + \bar{w}_{2,0})$ do in fact express the apparent velocities due to the angular velocity of the coordinate axes.

5. FORCE AND MOMENT COEFFICIENTS*

Once the pressure perturbations at the body surface have been calculated, it is a simple matter to integrate them over the body surface in order to obtain the force and moment coefficients of interest. Figure 15 shows a two-dimensional view of a right-handed cartesian x, y, z coordinate system, in which the coefficients will be given. The body sketched in Figure 15 is intended to represent either a pointed or spherically blunted body of revolution. The normal force F_n will be taken to be positive in the negative y -direction.

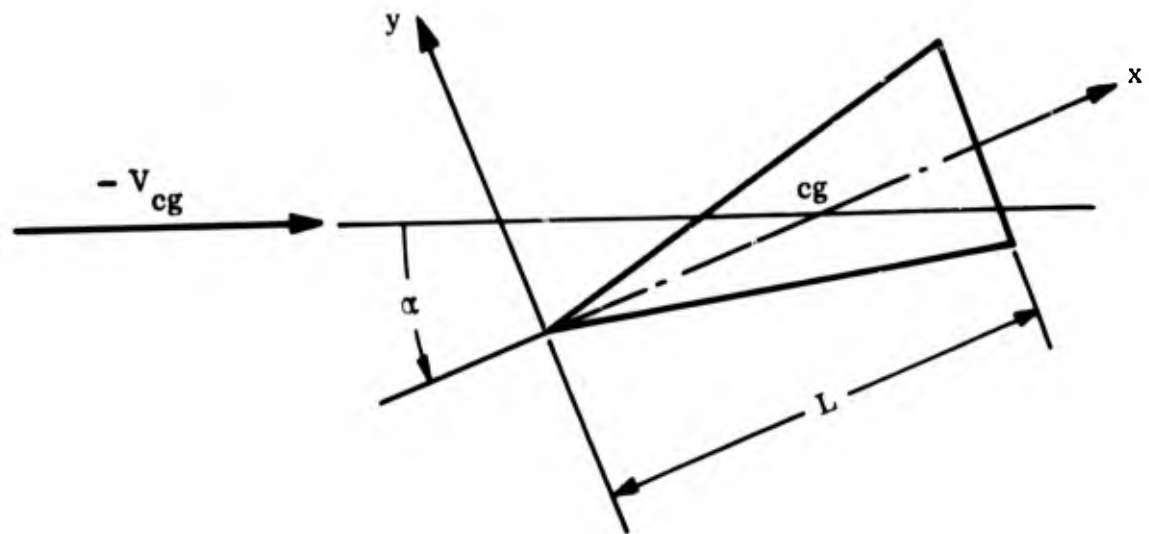


Figure 15. Two-Dimensional View of a Right-Handed Cartesian x, y, z , Coordinate System

*This Section appeared originally in Reference 2.

The pressure is given by Equations (4.20) and (4.33):

$$p = p_0 + \sum_{n=0}^{\infty} \bar{p}_{1,n} \left(\frac{L}{V_{\infty}} \right)^n \frac{d^n \alpha}{dt^n} \cos(\varphi \delta) + \sum_{n=0}^{\infty} \bar{p}_{2,n} \left(\frac{L}{V_{\infty}} \right)^{n+1} \frac{d^n q}{dt^n} \cos(\varphi \delta) \quad (5.1)$$

So that the normal force is:

$$F_N = \iint_{\text{Body Surface}} \left[p_0 + \sum_{n=0}^{\infty} \bar{p}_{1,n} \left(\frac{L}{V_{\infty}} \right)^n \frac{d^n \alpha}{dt^n} \cos(\varphi \delta) + \sum_{n=0}^{\infty} \bar{p}_{2,n} \left(\frac{L}{V_{\infty}} \right)^{n+1} \frac{d^n q}{dt^n} \cos(\varphi \delta) \right] (-\vec{y}) \cdot d\vec{A} \quad (5.2)$$

where the vector $d\vec{A}$ is a differential area of the body surface. Similarly, the moment about an axis which is parallel to the \vec{Z} direction and passes through the center of gravity is:

$$M = \iint_{\text{Body Surface}} (x - l_p) \left[p_0 + \sum_{n=0}^{\infty} \bar{p}_{1,n} \left(\frac{L}{V_{\infty}} \right)^n \frac{d^n \alpha}{dt^n} \cos(\varphi \delta) + \sum_{n=0}^{\infty} \bar{p}_{2,n} \left(\frac{L}{V_{\infty}} \right)^{n+1} \frac{d^n q}{dt^n} \cos(\varphi \delta) \right] (-\vec{y}) \cdot d\vec{A} \quad (5.3)$$

where l_p is the distance from the coordinate origin to the moment center.

The usual force and moment derivatives

$$\frac{\partial C_N}{\partial \left(\frac{d^n \alpha_j}{dt^n} \right)} = \frac{1}{\frac{1}{2} \rho^\infty V^\infty{}^2 A_{\text{base}}} \frac{\partial F_n}{\partial \left(\frac{d^n \alpha_j}{dt^n} \right)} \quad (5.4)$$

and

$$\frac{\partial C_M}{\partial \left(\frac{d^n \alpha_j}{dt^n} \right)} = \frac{1}{\frac{1}{2} \rho^\infty V^\infty{}^2 A_{\text{base}} D_{\text{base}}} \frac{\partial M}{\partial \left(\frac{d^n \alpha_j}{dt^n} \right)} \quad (5.5)$$

can be obtained by differentiating Equations (5.2) and (5.3) with respect to the appropriate motions:

$$\frac{\partial C_N}{\partial \left(\frac{d^n \alpha_j}{dt^n} \right)} = \frac{1}{\frac{1}{2} \rho^\infty V^\infty{}^2 A_{\text{base}}} \int_0^L (\pi r_b)^\delta \bar{p}_{j,n} dx \quad (5.6)$$

$$\frac{\partial C_M}{\partial \left(\frac{d^n \alpha_j}{dt^n} \right)} = \frac{1}{\frac{1}{2} \rho^\infty V^\infty{}^2 A_{\text{base}} D_{\text{base}}} \int_0^L (\pi r_b)^\delta (x - \ell_p) \bar{p}_{j,n} dx \quad (5.7)$$

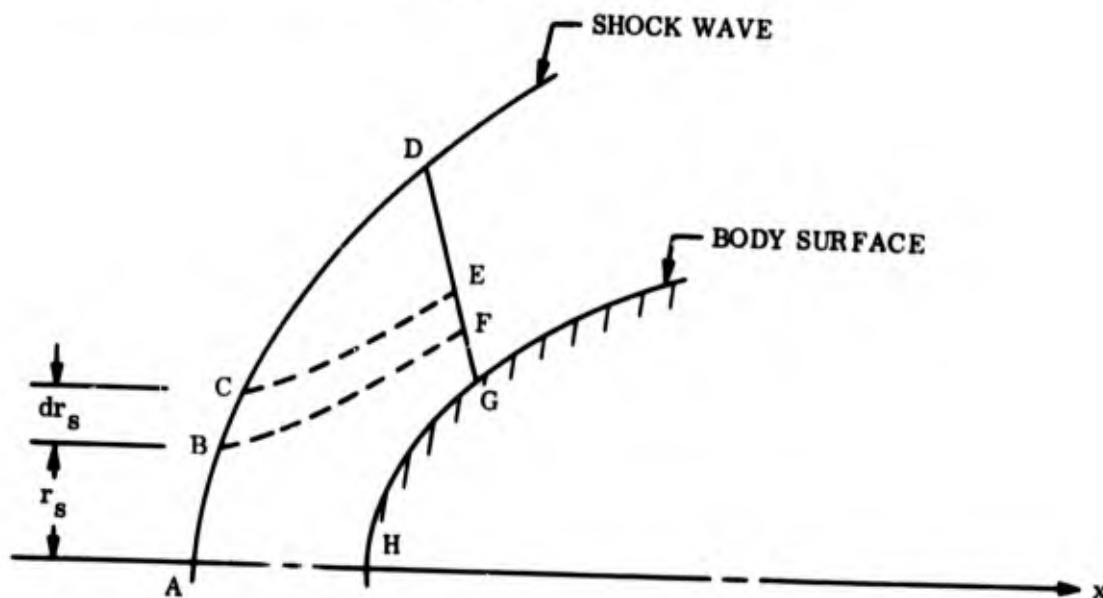
In obtaining Equations (5.6) and (5.7) from Equations (5.2) and (5.3) the integration in the meridional (φ) direction has been carried out, since $\bar{p}_{j,n}$ is independent of φ , and the dependence of $(-\vec{y}) \cdot d\vec{A}$ on φ is easily expressed:

$$(-\vec{y}) \cdot d\vec{A} = (r_B \cos \varphi d\varphi)^\delta dx \quad (5.8)$$

In the two dimensional case ($\delta = 0$), substitution of Equation (5.8) in Equations (5.2) and (5.3) will result in a force and moment per unit depth in the \vec{z} direction. The base area, A_{base} , which appears in Equations (5.6) and (5.7) must, therefore, be base area per unit depth in the two dimensional case.

5.1 FORCE AND MOMENT COEFFICIENTS FOR FRONT END OF BLUNT BODIES

The foregoing discussion of force and moment coefficients is applicable on any portion of a body where the various perturbations in pressure are known, and, therefore, can be used with both the pointed cone and supersonic solutions. In the case of a blunt body, however, the pressure perturbations are not calculated upstream of the starting line. In this case, use is made of the knowledge that $\frac{\partial p}{\partial q} + \frac{\partial p}{\partial \alpha} = 0$ on the spherical portion of the body, to conclude that the spherical portion does not contribute to $(C_{M_q} + C_{M_{\alpha}})$ or to $(C_{N_q} + C_{N_{\alpha}})^*$. As a result, it is only necessary to compute the $C_{N_{\alpha}}$ and $C_{M_{\alpha}}$ contributions of the spherical nose. This is done by a momentum balance as indicated in the following sketch.



*In this part of the discussion, rotation is assumed to be about the center of the spherical nose.

Line DEFG is the starting line. The dotted lines CE and BF are the bounding surfaces of a differential stream tube. If we define dF_x to be:

$$dF_x = \text{flux of axial momentum crossing BC} + \text{axial force on the stream tube due to pressure on BC} - \text{flux of axial momentum crossing EF} - \text{axial force on the stream tube due to pressure on EF},$$

then total axial force on the body (due to the portion of the flow field upstream of the starting line DEFG) results from integrating dF_x over all the stream tubes between the stagnation streamline AHG and the point D.

If r_s is defined to be the radius to point B, r the radius to point F, dr_s the difference in radius at points B and C and dr the difference in radius at points E and F, the above equation for dF_x becomes:

$$dF_x = \left\{ V_\infty d\dot{m}_s + p_\infty (2\pi r_s)^\delta dr_s \right\} - \left\{ V \cos \theta d\dot{m} + p (2\pi r)^\delta dr \right\}$$

where $d\dot{m}_s$ and $d\dot{m}$ are the mass flow rates across BC and EF respectively. They are

$$d\dot{m}_s = \rho_\infty V_\infty (2\pi r_s)^\delta dr_s$$

and

$$d\dot{m} = \rho V_n (2\pi r)^\delta dl$$

where V_n is the component of velocity normal to EF, and dl is the distance between E and F.

The conservation of mass law requires that $d\dot{m} = d\dot{m}_s$, so that

$$(2\pi r_s)^\delta dr_s = \frac{\rho V_n}{\rho^\infty V^\infty} (2\pi r)^\delta dl \quad (5.9)$$

We also note that:

$$dr = (\sin \lambda) dl$$

$$V_N = V \sin (\lambda - \theta)$$

where λ is the angle between the line EF and the axial. Substitution into the expression for dF_x yields:

$$dF_x = \left\{ \left(V^\infty + \frac{p^\infty}{\rho^\infty V^\infty} \right) - \left(V \cos \theta + \frac{p}{\rho V} \frac{\sin \lambda}{\sin (\lambda - \theta)} \right) \right\} (2\pi r)^\delta \rho V \sin (\lambda - \theta) dl \quad (5.10)$$

Integrating from point G to point D provides axial force:

$$F_x = \int_G^D \left\{ V^\infty + \frac{p^\infty}{\rho^\infty V^\infty} - \left(V \cos \theta + \frac{p}{\rho V} \frac{\sin \lambda}{\sin (\lambda - \theta)} \right) \right\} (2\pi r)^\delta \rho V \sin (\lambda - \theta) dl \quad (5.11)$$

The normal force coefficient (for the $j = 1, n = 0$ case) is obtained by integration of the pressure perturbation over the body surface:

$$\frac{\partial F_n}{\partial \alpha} = \iint_{\text{Body surface}} p_{1,0} (-\hat{y}) \cdot d\hat{A}$$

where $-\hat{y} \cdot d\hat{A}$ is given in Equation (5.8). The pressure perturbation $p_{1,0}$ on the sur-

face of the spherical* forebody is given in Equation (4.185) so that:

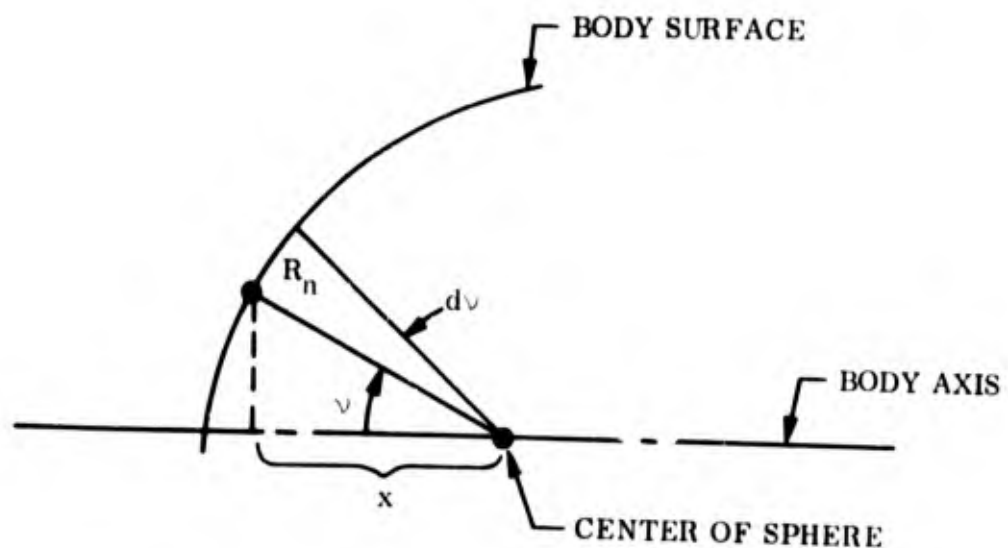
$$\frac{\partial F_n}{\partial \alpha} = \int_{x_{\text{axis}}}^{x_{\text{sl}}} \int_0^{2\pi} \left[-r \frac{\partial p_o}{\partial x} + x \frac{\partial p_o}{\partial r} \right] (r \cos^2 \varphi d\varphi) \delta dx$$

The coordinate origin is placed at the center of the sphere in this discussion, and x_{sl} is the x value at the body surface on the starting line.

Integration with respect to φ yields:

$$\frac{\partial F_n}{\partial \alpha} = \int_{x_{\text{axis}}}^{x_{\text{sl}}} \left[-r \frac{\partial p_o}{\partial x} + x \frac{\partial p_o}{\partial r} \right] (\pi r) \delta dx$$

At this point it is convenient to change to the variable ν which is defined in the following sketch



*For the two dimensional case the present discussion applies to a cylindrical forebody.

Then

$$x = -R_n \cos \nu$$

$$r = R_n \sin \nu$$

and

$$\frac{\partial p_o}{\partial \nu} = \frac{\partial p_o}{\partial x} \frac{\partial x}{\partial \nu} + \frac{\partial p_o}{\partial r} \frac{\partial r}{\partial \nu} = r \frac{\partial p_o}{\partial x} - x \frac{\partial p_o}{\partial x}$$

where R_n is
the radius of
the sphere

so that the force coefficient becomes:

$$\frac{\partial F_n}{\partial \alpha} = \int_0^{\nu_{sl}} \left(-\frac{\partial p_o}{\partial \nu} \right) \left(\pi R_n \sin \nu \right)^\delta R_n \sin \nu d\nu$$

Integration by parts yields:

$$\frac{\partial F_n}{\partial \alpha} = -p_o (\pi r)^\delta r + \int_0^{\nu_{sl}} p_o R_n (\pi R_n)^\delta (\delta + 1) (\sin \nu)^\delta \cos \nu d\nu$$

The integral in the above equation, however, is seen to be the axial force F_x ,

since

$$F_x = \iint_{\text{Body surface}} p_o \vec{x} \cdot d\vec{A} = \int_0^{\nu_{sl}} p_o (2\pi R_n \sin \nu)^\delta R_n \cos \nu d\nu$$

so that the normal force coefficient is:

$$F_{n_\alpha} = \frac{\partial F_n}{\partial \alpha} = -p_o (\pi r)^\delta r + F_x$$

where F_x is given by Equation (5.11).

Finally, it is apparent that in the inviscid case surface pressure on a sphere cannot cause a moment about the center of the sphere, so that $M_{Z\alpha} = 0$ if the moment center is taken at the center of the sphere.

6. PROGRAMMING OF THE ZERO YAW STEADY STATE FLOW FIELD SOLUTIONS

This Section provides a brief outline of the programming of the zero yaw steady state flow field solutions. Its purpose is to provide a link between the analysis of the problem and the listing of the computer program. *

CHAIN 1

- A. Main program.
- B. Calling sequence: None.
- C. This program calls in link 1 of the chain tape.
- D. Error indications: None.

FFSYS

- A. Main program.
- B. Calling sequence: None.
- C. This program is always link 1 on the chain tape. It reads in the first two input cards, (FFCASE) and (IDPR \bar{O} G & KENSE) and then calls for the link containing the program to be executed.
- D. Error indications: None.

ATBNTP

- A. Main program.
- B. Calling sequence: None

*The listing is to be published as a separate report by RTD.

- C. This program uses as input the BCD thermodynamic tables tape and writes the binary tape which is used in the program.
- D. Error indications: None.

MNPRNT

- A. Main program.
- B. Calling sequence: None.
- C. This program will print:
 - 1. Thermodynamic tables.
 - 2A. Oblique shock tables $0^\circ \leq \sigma \leq 90^\circ$ in increments of 0.5°
 - 2B. Normal shock $\sigma = 90^\circ$
- D. Error indications: None.

MNEXP

- A. Main program.
- B. Calling sequence: None.
- C. This program will compute the properties at an expansion corner.
- D. Error indications: None.

BINTP1

- A. Subroutine.
- B. Calling sequence: (IND)
- C. In this routine, the "standard flow field binary tape" is written or read.

The various options are determined by the value of IND

1. IND = 0. The tape numbers (logical) are read in, NBTPW (writing) and NBTPR (reading).
2. IND = -1. Preliminary information is written.
3. IND = 1. Preliminary information is read.
4. IND = -2. Streamlines or characteristics are written.
5. IND = 2. Streamlines or characteristics are read. Also, the reference table SREFS is created, when appropriate.
6. IND = 3. Final record is written (NVAL = 0), and tape is rewound.
7. IND = 3. Tape is rewound.

D. Error indications: None.

CLCINT

A. Integration.

B. Type

FORTRAN 2 Function.

C. Purpose.

To compute the definite integral of $F(X) \cdot DX$ from $X = A$ to $X = B$ by

1. Trapezoidal rule.
2. Simpson's rule.

D. Usage.

1. The calling sequence is: $F = \text{CLCINT} (\text{IND}, \text{DX}, \text{FX}, \text{TEMP})$

A. F is the value of the integral.

B. IND = 0 Trapezoidal rule.

 IND = 1 Simpson's rule.

C. $DX = 0$ when $X. = A.$

$DX = X(N) - X(N-1)$ when $X. \neq A.$

D. FX is the integrand.

E. $TEMP$ is an array containing 5 cells which must not be used for any other purpose while the integration is being performed.

E. Method.

1. The first interval is always computed by Trapezoidal rule

$$G(1) = DX/2.0 * (FX(1) + FX(2))$$

$$F(1) = G(1)$$

2. Subsequent intervals are computed by Trapezoidal rule when that option is specified and when the current and previous values of DX are unequal.

$$G(N) = DX/2.0 * (FX(N+1) + FX(N))$$

$$F(N) = F(N-1) + G(N)$$

3. Otherwise, subsequent intervals are computed by Simpson's rule as follows:

$$G(N) = DX/3.0 * (FX(N+1) + 4.0 * FX(N) + FX(N-1)) - G(N-1)$$

$$F(N) = F(N-1) + G(N)$$

Assuming a constant DX , the net effect of this procedure is: if N is odd, the integral consists of Trapezoidal rule integration over the first interval, and Simpson's rule integration over the remaining $N-1$ intervals. If N is even, the integral consists of Simpson's rule integration over the N intervals. This is in keeping with the fact that Simpson's rule must be performed over an even number of intervals.

- F. Restrictions. None.
- G. Additional Subprograms Required. None.
- H. Sample Case

Suppose we wish to find by Simpson's rule the definite integral of $X^{**2} * EXP(-X)$ from $X = 1.0$ to $X = 5.0$ with a DX of 0.5 .

Dimension SAVINT (5)

```

1   X = 1.0

      DO 91 J1 = 1, 9

      If (J1. = 1) go to 41

21  DX = 0.5

      Go to 61

41  DX = 0.0

61  X = X + DX

      A = CLCINT (1, DX, X**2*EXP(-X), SAVINT)

91  Continue

```

CLPOLY

A. POLYNOMIAL Evaluation.

B. Type

FORTRAN 2 Function.

C. Purpose

To evaluate the Polynomial (SUM (A(I)*X**(I-1), I = 1, N+1)).

D. Usage

1. The calling sequence is:

$$Y = \text{CLPOLY}(X, A, N)$$

- A. Y is the value of the polynomial.
- B. X is the independent variable.
- C. A is the name of the array containing the coefficients.
- D. N is the degree of the polynomial.

E. Method

1. The standard nesting process is used.

F. Restrictions. None.

G. Additional Subprograms Required. None.

CONENT

A. Subroutine.

B. Calling sequence: (IND)

C. In this routine, the properties ρ , V, M, S/R, T are computed assuming values for P and ψ . The method of solution is explained in the write-up (Reference 3) of

$$\gamma^* = \frac{A}{P} + B$$

The use of IND is as follows:

1. $|\text{IND}| = 1$. The reference properties are taken from the reference tables (SREFP or SREFS).

2. $|\text{IND}| = 2$. The reference properties are taken from a previously computed point (stored in the SB array). This option is used in the pointed cone and expansion corner programs.
3. $\text{IND} < 0$. This insures that in the expansion corner program, no more than one pressure break is crossed at a time.

Error indications:

NOVAL = 121: ψ out of range of reference tables.

NOVAL = 1501: Pressure out of range in A, B tables.

NOVAL = 1511: Reference pressure out of range in A, B tables.

NOVAL = 2291: Failed to converge in temperature iteration.

CRVINT

- A. Subroutine.
- B. Calling sequence: (IND).
- C. If $\text{IND} = 1$, this routine reads in the body curve coefficients and the x-coordinates of the intersections, if any. It computes for these points of intersections the value of r, the downstream flow angle, and the upstream flow angle. It also prints the information on tape.
If $\text{IND} = 0$ (i. e. a restart), only the printing is done.
- D. Error indications: None.

CRVOUT

- A. Subroutine.
- B. Calling sequence: (A, J, N)

C. Given an array of I sets of J curve coefficients of the form A(J, I), this routine prints out the first N of these sets.

D. Error indications: None.

EXPAND (The unsteady solution as developed here, does not include the expansion corner capability.)

A. Subroutine.

B. Calling sequence: (IND).

C. This is the routine in which the properties at an expansion corner are computed. There are the following 3 entries to the subroutine.

1. IND = 1. The information required at each expansion corner (up to 5) is read in and the first expansion corner is initialized.

2. IND = 2. The properties on the first fan of the expansion corner are computed.

3. IND = 3. The properties on the succeeding fans are computed. If it is the last fan, the next expansion corner (if any) is initialized.

D. Error indications: None

FFCASE

A. Subroutine.

B. Calling sequence: None.

C. This is an initialization routine. It will read input cards until it finds one with the letters FFCASE in cols. 1-6. It will print out cols. 7-72 of that card and then read the next card which will identify which link of the chain tape the program is on (IDPROG) and the external parameters (KENSE).

D. Error indications: None.

FLOWRD

A. Subroutine

B. Calling sequence: (IND)

C. This routine reads into core storage:

A. From the input tape:

1. RIDEAL, GIDEAL, ZIDEAL, RHOO, RHOOO, GREAL.

B. From the atmospheric tables tape (real gas only).

1. $S/R = f(\rho, T)$

2. $a, b = f(\rho, S/R)$

3. $Z = f(\rho, T)$ (curve fits)

4. $\gamma = f(\rho, T)$ (curve fits)

5. $Z = f(\rho, S/R)$

If $IND < 0$, this information is printed out.

D. Error indications: None.

FLOWTL

A. Subroutine.

B. Calling sequence: (IND).

C. This routine computes for real gas:

1. $Z = f(\rho, S/R)$ IND = 1 or -1

2. $\frac{dZ}{d\rho} = f(\rho, S/R)$ IND = -1

3. $Z = f(\rho, T)$ IND = 2

- | | | |
|----|--|---------------|
| 4. | $S/R = f(\rho, T)$ | IND = 3 |
| 5. | $\gamma = f(\rho, T)$ | IND = 4 |
| 6. | $b \text{ \& } a + b \cdot \rho = f(\rho, S/R)$ | IND = 5 or -5 |
| 7. | $\frac{da}{dS/R} \text{ \& } \frac{db}{dS/R} = f(\rho, S/R)$ | IND = -5 |

For ideal gas:

$$Z = Z_{IDEAL}$$

$$\frac{dZ}{d\rho} = 0$$

$$S/R = f(\rho, T)$$

$$\gamma = G_{IDEAL}$$

$$b = G_{IDEAL}$$

$$a + b \cdot \rho = G_{IDEAL} \cdot \rho$$

$$\frac{da}{dS/R} = \frac{db}{dS/R} = 0$$

- D. Error indications: Various for arguments being out of range.

FSSTAB

- A. Subroutine
- B. Calling sequence: (IND).
- C. This routine calculates:
 - 1. Certain free stream conditions.

$$A. \quad T_{\infty} = \frac{P_{\infty}}{P_{\infty} Z_{\infty} R_{GAS}}$$

Since Z is a function of T, an iteration process is required.

B. Z_{∞} , γ_{∞} , S/R_{∞} (Table look-ups)

It also prints alt., P_{∞} , ρ_{∞} , V_{∞} , T_{∞} , M_{∞} .

2. Shock and SIGMA Tables.

The shock tables are merely calculated and printed and consist of the properties:

σ , P , ρ , T , S/R , Z , γ , γ^* , V , M , θ .

computed at increments of $.5^{\circ}$ between $\sigma = 0^{\circ}$ and $\sigma = 90^{\circ}$.

The SIGMA tables consist of σ , T , γ , and are computed and stored for use in the SHOCK routine as a time saving feature.

The options are as follows:

1. $IND = 0$. Only the free stream conditions are printed. Used only in a restart.
2. $IND < 0$. The normal shock values ($\sigma = 90^{\circ}$) only are calculated and printed.
3. $IND = 1$. The shock and SIGMA tables are not computed.
4. $IND = 2$. The shock and SIGMA tables are computed.

D. Error indications: $NOVAL = 291$. Failed to converge on T_{∞} .

QUDINT

A. Function

B. Calling sequence: (X, Y)

C. This routine computes the value of definite integral

$$A = \int_{x_1}^{x_2} y dx$$

by the four-point quadrature formula:

$$A = y_1 h_1 + \frac{\delta_{11} h_1^2}{2} - \frac{\delta_{21} h_1^3}{6} - \frac{\delta_{31} h_1^3}{12} \quad (h_1 - 2h_2)$$

where

$$h_1 = x_2 - x_1$$

$$h_2 = x_3 - x_1$$

$$\delta_{11} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\delta_{21} = \frac{\delta_{12} - \delta_{11}}{x_3 - x_1}$$

$$\delta_{12} = \frac{y_3 - y_2}{x_3 - x_2}$$

$$\delta_{22} = \frac{\delta_{13} - \delta_{12}}{x_4 - x_2}$$

$$\delta_{13} = \frac{y_4 - y_3}{x_4 - x_3}$$

$$\delta_{31} = \frac{\delta_{22} - \delta_{21}}{x_4 - x_1}$$

The array X contains x_1, x_2, x_3, x_4

The array Y contains y_1, y_2, y_3, y_4

D. Error indications: None.

RF

A. Function

B. Calling sequence: (IND, A, X)

C. Given the general body curve

$$r = A_1 + A_2 \bar{x} + A_3 \bar{x}^{-2} + A_4 \bar{x}^{-3} + A_5 \bar{x}^{-4} + A_6 \bar{x}^{-5} + A_7 \bar{x}^{-8} + A_9 \sqrt{A_{10}^2 - (\bar{x} - A_{11})^2}$$

where: $\bar{x} = x - A_{12}$

This routine computes:

1. r IND = 1
2. r' IND = 2
3. r'' IND = 3

If we let $u = x - A_{11}$

$$v = \sqrt{A_{10}^2 - u^2}$$

Then,

$$v' = -\frac{u}{v}$$

$$u' = 1$$

$$v'' = -\frac{v^2 + u^2}{v^3}$$

Therefore,

$$r = A_1 + A_2 \bar{x} + A_3 \bar{x}^2 + A_4 \bar{x}^3 + A_5 \bar{x}^4 + A_6 \bar{x}^5 + A_7 \bar{x}^8 + A_9 v$$

$$r' = A_2 + 2A_3 \bar{x} + 3A_4 \bar{x}^2 + 4A_5 \bar{x}^3 + 5A_6 \bar{x}^4 + A_7 A_8 \bar{x}^{A_8-1} + A_9 v'$$

$$r'' = 2A_3 + 6A_4 \bar{x} + 12A_5 \bar{x}^2 + 20A_6 \bar{x}^3 + A_7 A_8 (A_8-1) \bar{x}^{A_8-2} + A_9 v''$$

A is an array containing $A_1 - A_{12}$.

D. Error indications: None.

SHOCK

- A. Subroutine.
- B. Calling sequence: (IND, SU)

C. In this routine, the properties behind a shock are computed. The process is explained in PROPERTIES BEHIND A SHOCK, *SU is the name of the array containing the upstream properties. The use of IND is to determine whether the iteration process on T and γ is to be performed. If IND = 1, the process is performed. If IND = 2, we interpolate for T and γ in the SIGMA tables calculated in FSSTAB and skip the iteration.

D. Error indications:

NOVAL = 311. σ not found in range of SIGMA tables.

NOVAL = 411. $\sigma < 0^\circ$.

NOVAL = 421. $\sigma > 90^\circ$.

NOVAL = 1491. Failed to converge on temperature.

NOVAL = 1691. Failed to converge on γ .

STREFP

A. Subroutine.

B. Calling sequence: (IND, N)

C. In this routine, the reference properties are stored in one of the two reference tables.

1. IND. = 1. The properties are stored in SREFP

1. ψ

2. P

3. ρ

*Reference 4.

- 4. S/R
- 5A. V
- 5B. Z
- 2. IND = 2. The properties are stored in SREFS.
 - 1. ψ
 - 2A. K
 - 2B. S/R
- D. Error indications:
 - NOVAL = 311. More than 500 entries in SREFS, exceeding dimension.

TLU1

- A. Table Search
- B. Type
 - FORTRAN 2 Subroutine.
- C. Purpose
 - To locate the position in a table of a specified number.
- D. Usage
 - 1. The calling sequence is: CALL TLU1(ARG, NTAB, TAB, J, IERR)
 - A. ARG is the specified number.
 - B. NTAB is the number of elements in the table.
 - C. TAB is the name of the table.
 - D. J and IERR are outputs as follow

	J	IERR
1. ARG. < TAB(1)	1	-1
2. ARG. = TAB(K)	K	0

	J	IERR
3. TAB(K). < ARG. < TAB(K+1)	K	0
4. ARG. > TAB(NTAB)	NTAB	1

E. Restrictions

1. The elements of the table must be in monotonic ascending order.

G. Additional subprograms required: None.

TNT1

A. Single interpolation (Lagrangian).

B. Type

FORTRAN 2 Function.

C. Purpose

To do a single table look-up and Lagrangian interpolation of specified order.

D. Usage

1. The calling sequence is: $Y = \text{TNT1}(X, \text{NTAB}, \text{XTAB}, \text{YTAB}, \text{NPT}, \text{IERR})$
 - A. Y is the interpolated value.
 - B. X is the independent argument.
 - C. NTAB is the number of elements in the table.
 - D. XTAB is the name of the independent variable table.
 - E. YTAB is the name of the dependent variable table.
 - F. NPT is the number of points over which the interpolation is performed.
 - G. IERR is an error return as follows:

	IERR
1. $X. < XTAB(1)$	-1
2. $XTAB(1). \leq X. \leq XTAB(NTAB)$	0
3. $X. > XTAB(NTAB)$	1

E. Method

1. The order of interpolation is $N = \text{MIN}(NPT-1, NTAB-1)$. The best $N+1$ points are selected for the interpolation.
2. Special cases are:
 - A. $N = 0$ No interpolation.
 - B. $N = 1$ Linear interpolation.
 - C. $N = 2$ Parabolic interpolation.
3. When the argument is outside the range of the independent variable table, TNT1 is set to 0

F. Restrictions

1. The elements of the independent variable table must be in monotonic ascending order.

G. Additional subprograms required: TLU1.

MNSUP1

- A. Main program.
- B. Calling sequence: None.
- C. This program performs initialization for the supersonic plus the calculation of all left hand characteristics to and including the first starting from the body.
- D. Error indications: None.

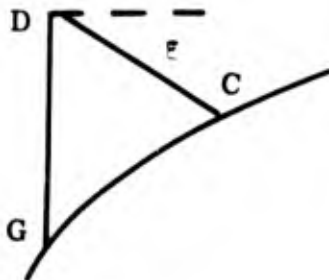
MNSUP2

- A. Main program.
- B. Calling sequence: None.
- C. This program calculates the properties on all characteristics after the first emanating from the body. It includes the calculation of all expansion corners and constant pressure wake as well as regular body points on the surface.
- D. Error indications: None.

BODY1

- A. Subroutine.
- B. Calling sequence: None
- C. This routine computes the properties K , x , r , θ , ψ , p , ρ , V , M , S/R , T at a body point in the supersonic region.

Given the configuration:



and assuming all properties are known at D and G:

$$\psi_C = \psi_G$$

x_C , r_C , and θ_C are obtained in SURFIN.

$$P_C = P_D - Q_{CD} (\theta_D - \theta_C + \xi \cdot G_{CD})$$

where:

$$Q = \frac{V^2 \rho}{\sqrt{M^2 - 1}}$$

$$G = \frac{\delta \sin \theta}{M r}$$

ρ_c , V_c , M_c , S/R_c , and T_c are obtained in CONENT.

$K = -1$. (Used to identify body points).

Convergence must be made on θ and M .

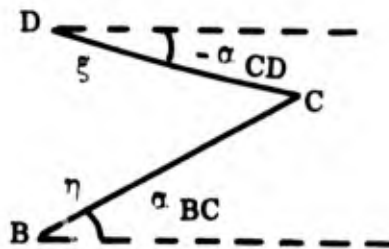
D. Error indications:

$\bar{NOVAL} = 691$: Failed to converge on θ and/or M .

FIELD1

- A. Subroutine.
- B. Calling sequence: None.
- C. This routine computes the properties K , x , r , θ , ψ , p , ρ , V , M , S/R T at a field point in the supersonic region.

Given the configuration:



and assuming all properties are known at B and D:

$$x_C = \frac{r_B - r_D + x_D \cdot A_2 - x_B \cdot A_1}{A_2 - A_1}$$

$$r_C = r_B + (x_C - x_B) \cdot A_1$$

$$\theta_C = \frac{\theta_D \cdot Q_{CD} + \theta_B \cdot Q_{BC} - \eta \cdot G_{BC} \cdot Q_{BC} + \varepsilon \cdot G_{CD} \cdot Q_{CD} + p_B - p_D}{Q_{CD} + Q_{BC}}$$

$$p_C = p_B - Q_{BC} \cdot (\theta_C - \theta_B + \eta \cdot G_{BC})$$

OUTSUP

- A. Subroutine.
- B. Calling sequence: (IND)
- C. This is the main output routine of the supersonic program. The options are:

1. IND = 1. Starting line information is written.

$m, n, x, r, \theta, \psi, P, \rho, V, M, S/R, T.$

Also, P_t and $C_{P_{\max}}$ are computed:

Ideal gas:

$$P_t = P_C \left[1 - \frac{\gamma_{\text{IDEAL}} - 1}{2} \cdot M^2 \right]^{\frac{\gamma_{\text{IDEAL}}}{\gamma_{\text{IDEAL}} - 1}}$$

$$C_{P_{\max}} = \frac{P_t - P_{\infty}}{0.5 \rho_{\infty} V_{\infty}^2}$$

Real gas:

$$C_{P_{\max}} = 2 - \frac{\rho_{\infty}}{\rho}$$

$$P_t = 0.5 \rho_{\infty} V_{\infty}^2 C_{P_{\max}} + P_{\infty}$$

2. IND = 2. Information at each point on left hand characteristic is written.

$m, n, x, r, \theta, \psi, P, \rho, V, M, S/R, T, NITER.$

Shock angle - $K.$

Also, the body and shock data is written on a binary tape to be summarized at the end of the run.

3. IND = 3. The body and shock data is summarized.

Body data:

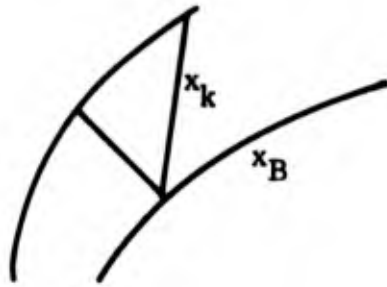
$m, n, x, r, P, P/P_t, P/P_{\infty}, C_P/C_{P_{\max}}, C_x, \theta(\text{rad}), \theta(\text{deg})$

$$\text{where } C_P = \frac{P - P_{\infty}}{.5 \cdot \rho_{\infty} \cdot V_{\infty}^2}$$

and

$$C_x = \frac{F_x}{0.5 \pi r^2 \rho_{\infty} V_{\infty}^2}$$

$$F_x = 2\pi \int_{x_B}^{x_K} P \cdot r \cdot \tan \theta dx + 2\pi \int \left\{ \left[V_{\infty} + \frac{P_{\infty}}{\rho_{\infty} V_{\infty}} \right] - \left[V \cos \theta + \frac{P}{\rho V} \cdot \frac{\sin \gamma}{\sin(\gamma - \theta)} \right] \right\} r \rho V \sin(\gamma - \theta) d\ell$$



Shock data:

m, n, x, r, K (rad.), K (deg).

PSIINT

- A. Subroutine.
- B. Calling sequence: (IND).
- C. In this routine, the values of the stream function ψ are calculated at each point of the starting line. It is computed by the formula

$$\psi = \int r^\delta \cdot \rho \cdot V \cdot \cos \epsilon \, dl$$

where ϵ is computed as follows:

- (a) Arbitrary line normal to streamlines:

$$\epsilon = 0$$

- (b) Arbitrary line not normal to streamlines:

$$\epsilon = \tan^{-1} \left[-\frac{1}{\frac{dy}{dx}} \right] - \theta$$

where:

$$\frac{dy}{dx} = \frac{\left[r_{n+2} - r_{n+1} \right] \left[x_{n+1} - x_n \right]^2 - \left[r_{n+1} - r_n \right] \left[x_{n+2} - x_{n+1} \right]^2}{\left[x_{n+2} - x_{n+1} \right] \left[x_{n+1} - x_n \right]^2 - \left[x_{n+1} - x_n \right] \left[x_{n+2} - x_{n+1} \right]^2}$$

(c) Right or left hand characteristic:

$$\epsilon = \frac{\pi}{2} - \sin^{-1} \frac{1}{M}$$

There are two options currently available:

1. IND = 1. In this option, the integration goes in one direction only. This is to be used for the nozzle program not yet in production.
2. IND = 2. In this option, the integration is done twice, once from body to shock, and second from shock to body, and then a weighted average of the two integrations is taken. The initial values of ψ are:

$$\psi_{\text{BODY}} = 0$$

$$\psi_{\text{SHOCK}} = \frac{r^{\delta+1}}{\delta+1} \cdot \rho_{\infty} \cdot V_{\infty}$$

The weighting formula is:

$$\psi = \frac{\psi_1 \cdot l_2 + \psi_2 \cdot l_1}{l_1 + l_2}$$

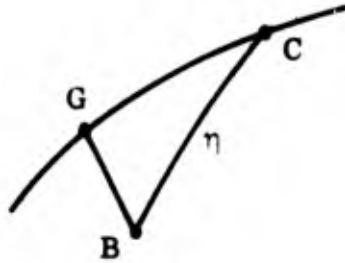
The integration is done by the quadrature formula described in QUDINT.

SHOCK1

- A. Subroutine.
- B. Calling sequence: None

- C. This routine computes the properties K , x , r , θ , ψ , p , ρ , V , M , S/R , T at a shock point in the supersonic region.

Given the configuration:



and assuming all properties are known at B and G:

$$x_C = \frac{r_B - r_G + x_G \cdot A_2 - x_B \cdot A_1}{A_2 - A_1}$$

$$r_C = r_B + (x_C - x_B) \cdot A_1$$

where:

$$A_1 = \tan \alpha_{CG} = \tan \left[\frac{1}{2} (\theta_C + \mu_C + \theta_B + \mu_B) \right]$$

$$A_2 = \tan \alpha_{BC} = \tan \left[\frac{\sigma_C + \sigma_G}{2} \right]$$

$$\sigma_C = K_C$$

$$\mu = \sin^{-1} \left(\frac{1}{M} \right)$$

ρ , V , M , T , S/R , P , θ are obtained in SHOCK.

$$\theta'_C = \theta_B - \frac{P_C - P_B}{Q_{BC}} - G_{BC} \cdot r$$

where:

$$Q = \sqrt{\frac{V^2 \cdot \rho}{M^2 - 1}}$$

$$G = \frac{\delta \cdot \sin \theta}{M \cdot r}$$

$$\psi_C = \frac{\rho_\infty \cdot V_\infty \cdot r_C^{\delta+1}}{\delta + 1}$$

Convergence must be made on M_C and $|\theta_C - \theta'_C|$

The iteration is done on K_C .

D. Error indications:

NOVAL = 691. Failed to converge on θ and/or M .

STPROP

- A. Subroutine.
- B. Calling sequence: (IND).
- C. This routine calculates at each point on the starting line:

$$1. T_C = \frac{P_C}{\rho_C \cdot Z_C \cdot R_{GAS}}$$

Since Z is a function of T , an iteration process is required.

$$2. S/R_C = f(\rho_C, T_C)$$

$$3. V_C = \sqrt{H - \frac{2 \cdot \gamma_C \cdot P_C}{(\gamma_C - 1) \rho_C}}$$

where:

$$H = 2 \left[\frac{V_{\infty}^2}{2} + \frac{\gamma_{\infty} \cdot P_{\infty}}{(\gamma_{\infty} - 1) \rho_{\infty}} \right]$$

4. $M_C = V_C \cdot \sqrt{\frac{\rho_C}{a + b \cdot p}}$: Equilibrium.

$M_C = V_C \cdot \sqrt{\frac{F(Z_C) \cdot P_C}{\rho_C}}$: Frozen.

where: $F(Z) = 1 + \frac{Z}{3(Z-1) + (2-Z) \cdot (2.5 + J_F)}$

There are two options:

1. IND = 1. At the shock point, the free stream conditions and SIGMA tables are also calculated.
2. IND = 2. At the field points and body point, only the starting line information is calculated.

D. Error indications:

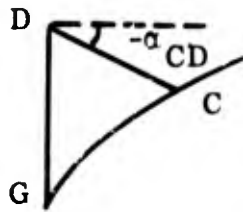
NOVAL = 291. Failed to converge on T_C .

SURFIN

- A. Subroutine.
- B. Calling sequence: (ALPHA).
- C. This routine computes the coordinates and the flow angle at a point determined by the intersection of a left hand characteristic with the body. If there is an expansion corner between this point and the pre-

vious body point, a pseudo body point (i. e., the projection of the upstream body curve) is used, preparatory to entering the expansion corner subroutine.

Given the configuration:



A value is assumed for x_C

$r_C^{(1)}$ can be obtained from the body curve Equation (see RF)

$r_C^{(2)}$ can be obtained from the equation

$$r_C^{(2)} = r_D - (x_C - x_D) \cdot \tan \alpha_{CD}$$

where:

$$\alpha_{CD} = \frac{1}{2} (\theta_C - \mu_C + \theta_D - \mu_D)$$

and

$$\mu = \sin^{-1} \frac{1}{M}$$

Iteration is made on x_C until $r_C^{(1)}$ and $r_C^{(2)}$ agree within specified limit.

θ_C is then obtained in RF.

D. Error indications:

NOVAL = 591: Failed to converge on r.

MNTRN1

- A. Main program.
- B. Calling sequence: None.
- C. This is the main control program for the transonic region.
- D. Error indications: NOVAL = 1686. Failed to converge at continuity point.

DERIV

- A. Subroutine.
- B. Calling sequence: (IND, X₁, X₂, X₃, R₁, R₂, R₃).
- C. This routine computes the first derivative of a function f(x) by means of a parabolic differentiation formula based on the points (x₁, r₁), (x₂, r₂), (x₃, r₃).

$$\Phi = \tan^{-1} \frac{r_3 - r_1}{x_3 - x_1}$$

$$x'_i = (x_i - x_1) \cos \Phi + (r_i - r_1) \sin \Phi \quad 1 \leq i \leq 3.$$

$$r'_i = -(x_i - x_1) \sin \Phi + (r_i - r_1) \cos \Phi \quad 1 \leq i \leq 3$$

$$A = \frac{r'_2 \cdot x'_3 - r'_3 \cdot x'_2}{x'_2 \cdot x'_3 \cdot (x'_3 - x'_2)}$$

$$B = \frac{-r'_2 \cdot x'_3 + r'_3 \cdot x'_2}{x'_2 \cdot x'_3 \cdot (x'_3 - x'_2)}$$

$$\frac{dr'}{dx_i} = A + 2 \cdot B \cdot x'_i$$

$$f'(x)_i = \frac{\frac{dr'}{dx_i} + \tan \phi}{1 - \tan \phi \cdot \frac{dr'}{dx_i}}$$

OUTTRN

- A. Subroutine.
- B. Calling sequence: (IND).
- C. This is the output routine for the transonic program.
 - 1. IND = 1: Continuity point output.
 - 2. IND = 2: Regular output.
 - 3. IND = 3: Summary of continuity point output and first two normals which are supersonic at body.
- D. Error indications: None.

SHKEQ

- A. Function.
- B. Calling sequence: (IND, R)
- C. In this routine, information concerning the shock curves is computed.*

*Provision has been made for up to 7 shock curves.

1. IND = 1. The coefficients of the shock A_i are read in, where

$$x = \sum_{i=0}^{10} A_i \cdot r^i .$$

2. IND = 2. The value of x is computed by the above formula.
3. IND = 3. The shock angle is computed

$$\sigma = \frac{1}{\frac{dx}{dr}}$$

- D. Error indications: None.

TRBDY

- A. Subroutine.
- B. Calling sequence: None.
- C. This routine computes information on the body.

The x and P are read in at each point.

The r , θ , and K may be read in at each point or may be obtained from an analytic body curve (RF).

The ρ , V , M , and T are obtained in CONENT.

- D. Error indications: None.

7. PROGRAMMING OF THE UNSTEADY FLOW FIELD

This Section provides a brief outline of the programming of the unsteady flow field solutions. Its purpose is to provide a link between the analysis of the problem, and the listing of the computer program.

7.1 THE UNSTEADY SUPERSONIC PROGRAM

Using the method of characteristics solution, the program finds the perturbations on the zero yaw flow field properties due to arbitrary values of angle of attack, pitch velocity, yaw velocity and the various time derivatives of angle of attack and pitch and yaw velocity.

Zero yaw properties are found using the Supersonic Zero yaw program. Perturbed properties along an initial line may be obtained from the pointed cone program, input cards provided by the user or internal calculations for a blunt body solution. Using the zero yaw properties and perturbed properties from adjacent left hand characteristics, the perturbations are carried out along left hand characteristics.

The program is written in Fortran II, General Electric MSD version. It utilizes the chain feature, and consists of two links on the flow field system tape.

A brief description of the flow of the program follows.

7.1.1 Link I

1. Read input common to all options.
2. Initialize parameters for options and tape assignments.

3. Read thermodynamic tables if real gas.
4. Decide upon starting line option.
 - a. Arbitrary line - input from cards - go to 5.
 - b. Arbitrary line - input calculated by program - go to 7.
 - c. Left hand characteristic - input from cards - go to 8.
 - d. Left hand characteristic - input from binary tape - go to 8.
5. Locate shock point of left hand characteristic immediately preceding starting line shock point from zero yaw input tape.
6. At each point on starting line, calculate properties not included in input, K_1 , through K_4 , C^2 and e^2 , then continue out the left hand characteristic on which the point lies, until the shock point has been calculated. Each point with its left hand characteristic is calculated for all J's and N's before going on to next point on starting line. After body point on the starting line and its left hand characteristic have been computed, go to 11.
7. In Subroutine STLINE, calculate:
 - a. Perturbed properties for all J's and N's at each point on the starting line from Shock to body, storing data on binary tape (B-5 if set by program).
 - b. Rewind binary tapes containing starting line data and zero yaw solution.
 - c. Read preliminary information from zero yaw tape.
 - d. Return to main program and re-locate starting line in zero yaw solution.
 - e. Proceed as in step 6, except starting line data is read from binary tape instead of cards. When body point is reached, go to 11.
8. Locate starting left hand characteristic from zero yaw solution.

- a. Input from cards - go to 9.
 - b. Input from tape - go to 10.
9. Read in perturbed properties along starting left hand characteristic.
 - a. Calculate K_1 through K_4 , C^2 and e^2 at each point on left hand characteristic.
 - b. Do this for all J's and N's.
 - c. Go to 11.
 10. Read in perturbed properties from Binary tape (A6 if set in program) along starting left hand characteristic.
 - a. Calculate K_1 through K_4 , C^2 and e^2 at each point on left hand characteristic.
 - b. Do this for all J's and N's.
 - c. Go to 11.
 11. Store perturbed properties on one of two temporary tapes (A7 or A8 if set in program).
 12. Go to next link on chain.

7.1.2 Link II

1. Set temporary tape assignments for reading and writing perturbed properties along characteristic for N-1 values. Rewind tapes.
2. Read zero yaw properties along left hand characteristic.
3. Read perturbed properties along previous left hand characteristic for J, N.
4. Find perturbations at:
 - a. Body point

- b. Field points
 - c. Shock point
 - d. Steps 3 and 4 are re-done until all J's and N's have been calculated
 - e. Perturbations are stored on second temporary tape
5. At body point - go to subroutine SUFINT and do integration of forces along body. If blunt body solution, store force coefficients on binary tape (A-6 if set by program).
 6. Print output.
 7. Check to see if this is last characteristic to be calculated.
 - a. Yes - go to 8
 - b. No - go to 1 and repeat process.
 8. Write summation of force integration along body.
 9. Blunt body solution - go to 10.
Pointed cone solution - go to 11.
 10. Call subroutine SUMINT and do summary of force integration for α and $\dot{\alpha}$.
 11. Call control subroutine to either get off machine or do another case.

7.1.3 Tape Assignments.

The standard Fortran peripheral input and output tapes (A2 and A3) are used to read BCD input and write BCD output.

Because of the storage requirements put on the original program, several intermediate tapes are utilized for storing binary information. These tape assignments can be set by the user or set by the program. They are designated, in the

program, as NT2, NT3, NT4 and NT5. If NT2 is read as zero, all tapes will be set by program. If NT2 is non-zero, all tapes must be set by user. The program designates these tapes:

<u>Logical</u>	<u>Physical</u>
NT2 = 17	A7
NT3 = 18	A8
NT4 = 16	A6
NT5 = 25	B5

A7 and A8 are used to store perturbed properties for J, N along a left hand characteristic.

A6, for a pointed cone body, contains perturbations along initial line. This tape is created by the pointed cone program.

A6, for a blunt body solution, is used to store the properties calculated along the starting line.

B5 is used to store the coefficients from the integration of forces along the body for use in final summation tables.

Two additional tapes are used by the program. They contain (1) the zero yaw solution created by the supersonic zero yaw program and (2) the thermodynamic tables.

B7 is used for the thermodynamic table. This tape need not be loaded if atmosphere is an ideal gas. The user has the option of designating tape assignment for the zero-yaw solution.

NBTPR, if input as zero, assigns B6 for the zero yaw solution.

Since the program contains the IO tables for G. E. M. S. D. FORTRAN, the following list must be referred to if the user wishes to change internal tape assignments.

<u>Logical</u>	<u>Physical</u>		<u>Logical</u>	<u>Physical</u>
1	A1*		16	A6
2	A2*		17	A7
3	A4		18	A8
4	A5		19	A9
5	B1		20	A10
6	B2		21	B1
7	B3		22	B2
8	B4		23	B3
9	B5		24	B4
10	A3*		25	B5
11	A1*		26	B6
12	A2*		27	B7
13	A3*		28	B8
14	A4		29	B9*
15	A5		30	B10

*These tapes are not available for intermediate designation as they are used by the system.

7. 1. 4 Restrictions

1. If terminating along right hand characteristic or constant value of X, either must intersect with the shock beyond shock point on left hand characteristic from the first body point.
2. A maximum of 70 points is allowed on a left hand characteristic, including body, field, and shock points. For arbitrary line start, maximum value of NUMPTS is 35.

7. 2 BRIEF DESCRIPTION OF G. E. M. S. D. VARIABLE FIELD INPUT

Floating point fields must begin with F, each variable separated by a comma, and card terminated with an asterisk.

Ex. to input:

X = . 50

R = 1. 78

V = 1700. 06

$\rho = . 5678 \times 10^{-6}$

On one card it will appear as:

F0. 50, 1. 78, 1700. 06, . 5678E-6* beginning in Column 1.

Fixed point fields must begin with X, each variable separated by a comma, and card terminated with an asterisk.

Ex. to input:

N = 1

NT1 = 17

NUMPTS = 20

MSTART = 148.

On one card it will appear as:

X1, 17, 20, 148* beginning in Column 1 (no decimal points may appear on fixed point variables)

To input both floating and fixed point fields on same card, fields must be separated by a comma.

Ex. to input above (information on one card):

(X, R, V, S, N, NT1, NUMPTS, MSTART)

Card will appear as:

F0. 50, 1. 78, 1700. 06, 0. 5678E-6, X1, 17, 20, 148* Beginning in Column 1.

Columns 1-72 may be utilized only. If more than one card is needed to complete list, terminate card with an asterisk after any convenient variable. Continue on next card, beginning card with field designation (F or X) and terminating with an asterisk. Any number of cards may be used to complete a list. A separate card must be used for each input statement in the program.

A list may be terminated, all succeeding variables in that list being set equal to zero, or entire list may be set equal to zero, with use of \$ character.

To terminate a list after a particular variable, place a comma after the variable followed by a \$.

Ex. Array Test (1-10) is to read in where:

Test (1) = 1×10^{-5} , Test (2) = 1×10^{-6} , Test (3-10) = 0

The card will appear as:

F1.0E-5, 1.0E-6, \$ beginning in Column 1.

If entire array were to be set equal to zero, the card would appear as: F\$.

Input Supersonic Unsteady Flow

IDPROG = 7

Variables

	<u>No. of Cards</u>
1. FFCASE	1
2. IDPROG, KENSE (1-10)	1
3. XEND, XTERM, ELP	1
4. MSTART, NMAX, NTERM, NT2, NT3, NT4, NT5	1
5. NBTPW, NBTPR	1
6. RIDEAL, GIDEAL, ZIDEAL, RHOO, RHO00, GREAL	1
I. Pointed body requires Cards 8, 9, 10, 11 as in II below (but Card 7 is omitted for the pointed body).	
II. Arbitrary line input (blunt body) - starting line calculated by program	
7. NUMPTS, XO	1
8. DELX, RN, NRTST, NXINT, NT5	1
9. RTEST (1-NRTST)	1
10. XINT (1-NXINT)	1
11. XMAX	1
III. Arbitrary line input - starting line input from cards*	
7. NUMPTS, XO	
8. (STARTING LINE DATA - SHOCK TO BODY)	

*Neither option III or IV will be needed in the usual unsteady flow field solution.

- A. THCB, VCB, WCB, PCB, RHO�B, SRCB, TCB, DRDX, RX (Shock Pt all J's and N's)
 - B. THCB, VCB, WCB, PCB, RHO�B, SRCB, RB, \$ (Each Field Pt all J's and N's)
 - C. THCB, VCB, WCB, PCB, RHO�B, SRCB, TCB, \$
 - D. COEFN, COEFM (Body Pt. Cards C, D, for each J, N)
- 9. DELX, RN, NRTST, NXINT, NT4
 - 10. RTEST (1-NRTST)
 - 11. XINT (1-NXINT)
 - 12. XMAX (1-NRTST)
- IV. Left hand characteristic input - starting line input from cards
- 7. (STARTING LINE INFORMATION - BODY TO SHOCK)
 - A. COEFN, COEFM
 - B. THCB, VCB, WCB, PCB, RHO�B, SRCB, TCB, DRDX, RX

(ONE CARD FOR EACH POINT BODY TO SHOCK. ENTIRE CHARACTERISTIC is read in for each combination J, N)

7. 2. 1 External Parameters for Supersonic Unsteady Flow

- KENSE (1) = 0 Arbitrary line starting line
- KENSE (1) = +1 Left hand characteristic starting line - input from cards.
- KENSE (1) = -1 Left hand characteristic starting line - input from binary tape.
- KENSE (2) = 0 Arbitrary line starting line properties are calculated by program.
- KENSE (2) = ±1 Arbitrary line starting line properties are read from cards.
- KENSE (3) = 0 Print perturbations of entire flow field
- KENSE (3) = ±1 Print only summation of force integration

7.2.2 Input Variables in the Order in Which They Appear in Input

- ELP Length used in restricting magnitude of perturbed terms.
- XSTRT X-coordinate of body point on initial left hand characteristic.
- XTERM X-coordinate along which left hand characteristics may be terminated.
- NMAX Maximum value of n. No restriction on NMAX.
- NUMJ No. of J's over which solution is to be found. Maximum value of NUMJ=2.
- NT2, NT3 Intermediate tape assignments.
- NT4, NT5 Intermediate tape assignments.
- NBTPW Always set to zero.
- NBTPR Tape designation of zero yaw solution. If input as zero the program sets NBTPR = 26 (B6).
- RIDEAL Ideal gas constant.
- GIDEAL Isentropic relationship $\frac{A}{P} + B$, for ideal gas = 1.4.
- ZIDEAL Compensibility factor in $P = ZR \rho T$. For ideal gas = 1.0.
- RHOO Reference density in density tables.
- RHOOO Reference density in density tables.
- GREAL Actual value of $\frac{A}{P} + B$. For ideal gas = 1.40223.

NOTE: IF RIDEAL, GIDEAL, ZIDEAL, RHOO, RHOOO, GREAL are input as zero, the program sets them equal to the constants for air.

- NUMPTS Number of points on starting line.
- MSTART Value of MC (from zero yaw solution) at shock point on arbitrary line starting line.
- XO Distance from origin of coordinate system to reference center for moments.

COEFN Coefficient of force normal.
COEFM Coefficient of moment.
THCB Perturbation on flow angle θ .
VCB Perturbation on velocity.
WCB Velocity component perpendicular to meridional plane.
PCB Perturbation on pressure.
RHOCB Perturbation on density, ρ .
SRCB Perturbation on entropy divided by Ideal gas constant.
TCB Perturbation on temperature.
DRDX Slope of shock wave, tangent of slope angle.
RX Radial distance to shock wave.
DELX Increment for XCG/LB in TABLE II of force summary.
RN Nose radius. (Coefficient A_{10} of body equation of nose).
NRTST Number of interpolations to be done on summary of body data.
RTST Value of rn/rb at which interpolated values are to be found. There
must be NRTST of these.
XMAX Maximum value of XCG/LB in table II. There must be NRTST of
these.

7.3 BASIC STORAGE ARRAYS

The basic storage arrays used by program are as follows:

SP (J, N, M) Dimensions SP(11, 120, 2) contains zero yaw properties along
left hand characteristic -

J indicates particular PROPERTY.

N indicates point on the characteristic.

M indicates characteristic.

M = 1 characteristic being solved.

M = 2 preceding characteristic.

SPDS(J, N, M) Dimensions SPDS(15, 70, 4) contains perturbed properties along
left hand characteristic -

J indicates particular property

N indicates point on the characteristic

M indicates characteristic

M = 1 contains perturbed properties along characteristic being solved,

N = n.

M = 2 contains perturbed properties along characteristic being solved,

N = n-1.

M = 3 contains perturbed properties along previous characteristic, N = n.

M = 4 contains perturbed properties along previous characteristic, N = n-1.

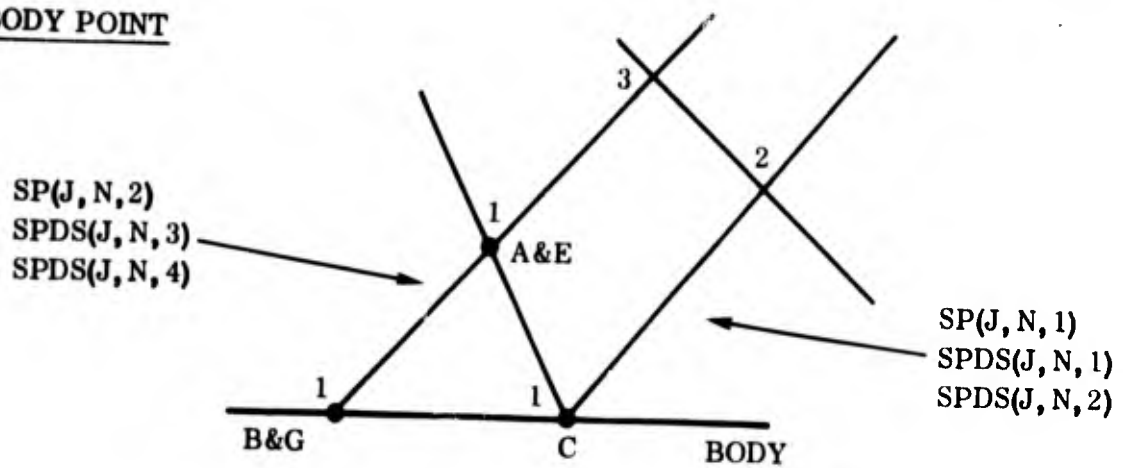
SX(J) Dimension SX(30) "Working" array for using finite difference
equation for method of characteristic solution

X = A, B, C, D, E or G depending upon point on characteristic line

J indicates particular property.

7.3.1 Use of Basic Storage Arrays to Provide Characteristic Method of Solution.

BODY POINT



Point to be calculated is at C

Zero yaw properties: $SA(j) = SE(j) = SP(j, 2, 2)$

$SB(j) = SG(j) = SP(j, 1, 2)$

$SC(j) = SP(j, 1, 1)$

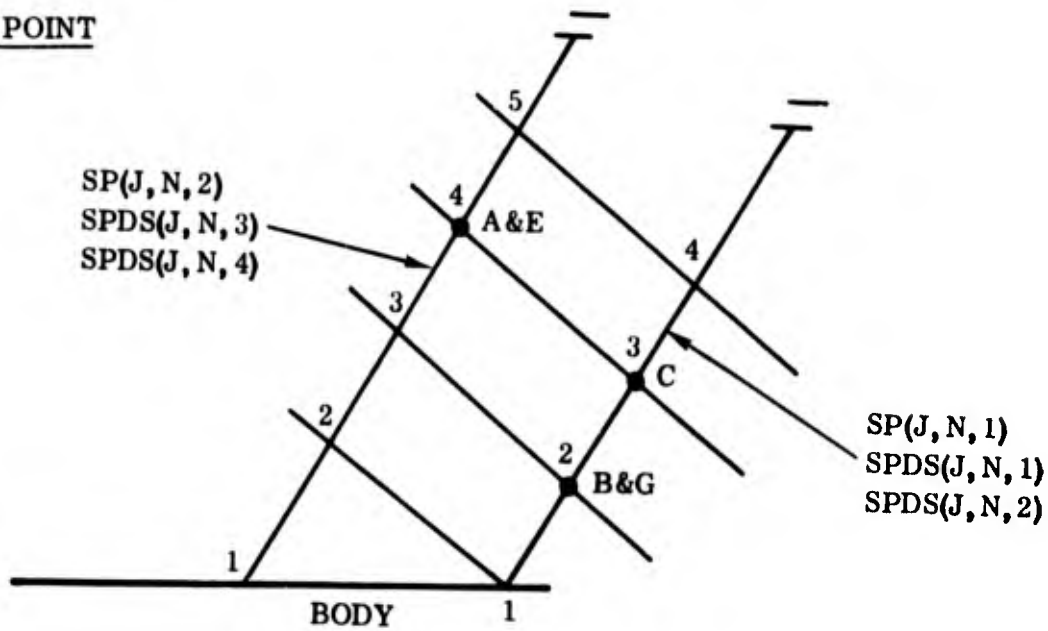
NOTE: Zero yaw properties along any left hand characteristic remain the same for all combinations of j and n along that characteristic.

Perturbed properties:	$SA(j+13) = SPDS(j, 2, 3)$	n terms
	$SE(j+13) = SPDS(j, 2, 4)$	n-1 terms
	$SB(j+13) = SPDS(j, 1, 3)$	n terms
	$SG(j+13) = SPDS(j, 1, 4)$	n-1 terms
	$SC(j+13) = SPDS(j, 1, 2)$	n-1 terms

NOTE: $SC(j)$ array is utilized to store n-1 terms at point being calculated until all calculations have been completed. Then the n terms are placed there for storage in the $SPDS(j, N, M)$ array.

$SPDS(J, 1, 1) = SC(J+13)$ for succeeding characteristics

FIELD POINT



Point to be calculated, point C. 3rd point on left hand characteristic

Zero yaw properties: $SA(J) = SE(J) = SP(J, 4, 2)$

$SB(J) = SG(J) = SP(J, 2, 1)$

$SC(J) = SP(J, 3, 1)$

Perturbed properties: $SA(J+13) = SPDS(J, 2, 4)$ n terms

$SE(J+13) = SPDS(J, 4, 4)$ n-1 terms

$SB(J+13) = SPDS(J, 2, 1)$ n terms

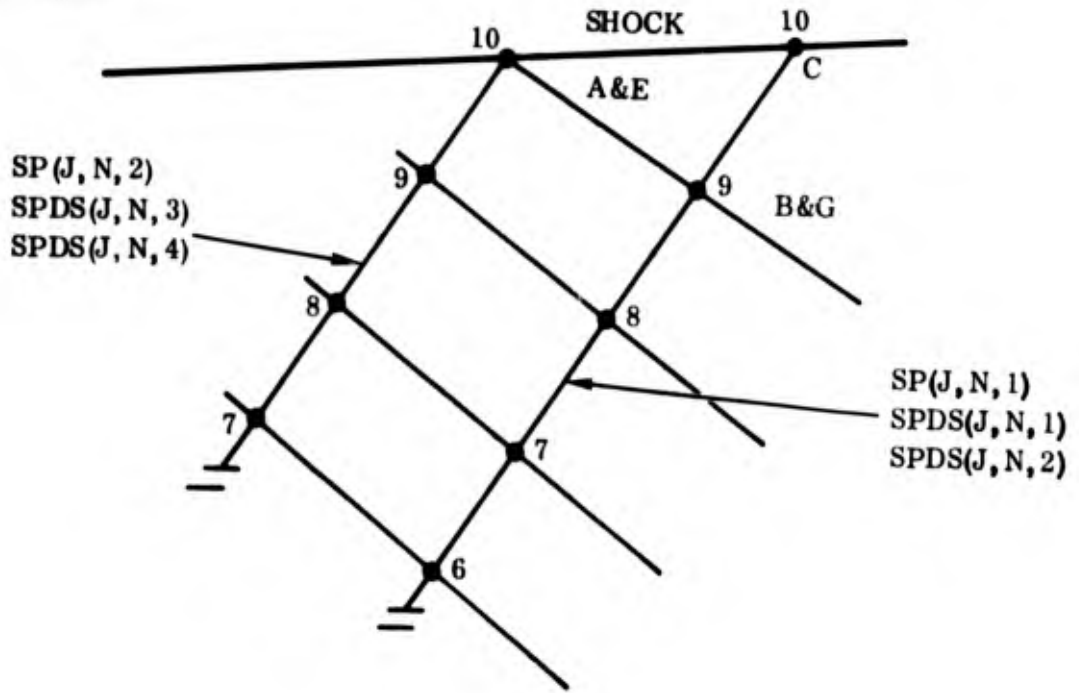
$SG(J+13) = SPDS(J, 2, 2)$ n-1 terms

$SC(J+13) = SPDS(J, 3, 2)$ n-1 terms

Calculations are carried out and

$$SPDS(J, 3, 1) = SC(J+13) \text{ for succeeding calculations.}$$

SHOCK POINT



Point to be calculated, point C-shock point, last point on left hand characteristic.

Zero yaw properties $SA(J) = SE(J) = SP(J, 10, 2)$

$$SB(J) = SG(J) = SP(J, 9, 1)$$

$$SC(J) = SP(J, 10, 1)$$

Perturbed properties $SA(J+13) = SPDS(J, 10, 3)$ n terms

$$SE(J+13) = SPDS(J, 10, 4)$$
 n-1 terms

$$SB(J+13) = SPDS(J, 9, 1)$$
 n terms

$$SG(J+13) = SPDS(J, 9, 2)$$
 n-1 terms

$$SC(J+13) = SPDS(J, 10, 2)$$
 n-1 terms

Calculations are carried out and

$SPDS(J, 10, 1) = SC(J+13)$ for succeeding calculations.

7.3.2 Basic Storage Arrays Showing Assignment of Variables Used in Supersonic

Unsteady Flow Program.

Flow Program.

<u>J</u>	<u>SX</u>	<u>SP</u>	<u>SPDS</u>
	X=A, B, C, D, E, or G		
1	M	K, σ	$\bar{\theta}$
2	N	X	\bar{V}
3	K, σ	r	\bar{W}
4	X	θ	\bar{P}
5	r	ψ	$\bar{\rho}$
6	θ	P	\bar{S}/R
7	ψ	ρ	\bar{T}
8	P	V	$\partial\bar{R}/\partial X$
9	ρ	M	$\bar{R}X$
10	V	S/R	K_1
11	M	T	K_2
12	S/R		K_3
13	T		K_4
14	$\bar{\theta}$		C^2
15	\bar{V}		e^2
16	\bar{W}		
17	\bar{P}		

<u>J</u>	<u>SX</u>
18	$\bar{\rho}$
19	\bar{S}/R
20	\bar{T}
21	$\partial\bar{R}/\partial X$
22	\bar{R}_x
23	K_1
24	K_2
25	K_3
26	K_4
27	C^2
28	e^2
29	Z
30	γ

ARYCLC

A. Subroutine

B. Calling sequence: None.

C. This routine computes C_i ($i = 1, 5$); D_i ($i = 1, 19$) and $\bar{F}_{i,j}; \bar{G}_{i,j}; \bar{H}_{i,j}$ ($i = 1, 2; j = 1, 2$) at A, B, C in body, field and shock point calculation along a left hand characteristic. A, B, and C Storage have been set in BODY, FIELD or SHOCK

(depending on type of calculation) before entering ARYCLC.

$$\text{for } n = 0 \quad \delta_{k,1} = 1.0$$

$$\delta_{k,2} = 0.0$$

$$\begin{aligned}
 \text{for } n = 1 \quad \delta_{k,1} &= 0.0 \\
 &\delta_{k,2} = 1.0 \\
 \text{for } n > 1 \quad \delta_{k,1} &= 0.0 \\
 &\delta_{k,2} = 0.0
 \end{aligned}$$

The following equations are solved using zero yaw properties first at point C, then A and B.

$$\bar{F}_{1,1} = 0.0$$

$$\bar{F}_{1,2} = -\rho_c V_\infty \sin \theta_c$$

$$\bar{F}_{2,1} = -\frac{V_\infty}{l'} \bar{F}_{1,2}$$

$$\bar{F}_{2,2} = \frac{V_\infty}{l'} \rho_c (r_c \cos \theta_c - x_c \sin \theta_c)$$

$$\bar{G}_{1,1} = 0.0$$

$$\bar{G}_{1,2} = -\rho_c V_\infty \cos \theta_c$$

$$\bar{G}_{2,1} = \frac{\rho_c V_\infty^2}{l'} \cos \theta_c - 2V_c \frac{V_\infty}{l'} \rho_c$$

$$\bar{G}_{2,2} = -\frac{V_\infty \rho_c}{l'} (r_c \sin \theta_c + x_c \cos \theta_c)$$

$$\bar{H}_{1,1} = 0.0$$

$$\bar{H}_{1,2} = \rho_c V_\infty \delta$$

$$\bar{H}_{2,1} = \frac{V_\infty}{l'} (2 \rho_c V_c \cos \theta_c - \rho_c V_\infty) \delta$$

$$\bar{H}_{2,2} = \frac{V_\infty}{l'} \rho_c x_c \delta$$

a, and b are found in subroutine THERMO from the thermodynamic tables.

$$C_1 = \frac{1}{\rho_c V_c^2} \sqrt{M_c^2 - 1}$$

$$C_2 = \frac{1}{\rho_c V_c M_c}$$

$$C_3 = \frac{C_1}{M_c}$$

$$C_4 = \frac{1}{\rho_c V_c}$$

$$C_5 = \frac{1}{V_c}$$

$\left. \frac{\partial S/R}{\partial \psi} \right|_c$ is found from stream function reference tables (zero yaw)

$$D_{14} = -T_c r_c^\delta \rho_c^2 V_c \left. \frac{\partial S/R}{\partial \psi} \right|_c R_{\text{IDEAL}}$$

$$D_1 = \frac{K_1 + K_2}{V_c^2 \sqrt{1 - \frac{1}{M_c^2}}}$$

$$D_2 = \frac{-\rho_c}{r_c} \delta$$

$$D_3 = \frac{V_c M_c}{2C_c^2} [K_{c_1} - K_{c_2}] + \frac{\rho_c V_c \cos \theta_c}{r_c} \delta$$

$$D_4 = \frac{V_c}{2\rho_c C_c^2 \sqrt{1 - \frac{1}{M_c^2}}} \left[-\frac{K_{c_3}}{e_c} - \frac{P_c K_{c_4}}{e_c} - \frac{1}{M_c^2} \right] [K_{c_1} + K_{c_2}]$$

$$D_5 = -\frac{M_c}{V_c} [K_1 - K_2]$$

$$D_6 = \frac{1}{2} \left(\frac{V_c}{\rho_c} D_5 \right)$$

$$D_7 = -\frac{M_c^2}{2\sqrt{1 - \frac{1}{M_c^2}}} [K_{c_1} + K_{c_2}] - \frac{\rho_c V_c^2 \sin \theta_c}{r_c}$$

$$D_8 = \left[K_{c_1} + K_{c_2} \right] \frac{V_c}{2 \rho_c C_c^4 \sqrt{1 - \frac{1}{M_c^2}}} \left[\frac{K_{c_3}}{e_c} + \frac{P_c K_{c_4}}{e_c} + b \right]$$

$$D_9 = \frac{\rho_c V_c \sin \theta_c}{r_c} \delta$$

$$D_{10} = \frac{\delta}{r_c}$$

$$D_{11} = r_c \rho_c V_c^2 \left. \frac{\partial S/R}{\partial \psi} \right|_c$$

$$D_{12} = \frac{-1}{2V_c \sqrt{1 - \frac{1}{M_c^2}}} \left[K_{c_1} + K_{c_2} \right]$$

$$D_{13} = \frac{V_c}{\rho_c} D_{12}$$

$$D_{15} = \frac{V_\infty}{l'} \left(1 - \delta_{k,1} \right) \left[\frac{\rho_c (\bar{V}_c)_{n-1}}{V_c} - \frac{e_c^2}{C_c^2} \left(\frac{\bar{S}}{R_c} \right)_{n-1} - (\bar{\rho}_c)_{n-1} \right]$$

$$- \frac{V_\infty (\bar{F}_{j,2})_c}{l' V_c} \delta_{k,2} - \frac{(\bar{F}_{j,1})_c}{V_c} \delta_{k,1}$$

$$D_{16} = -\frac{V_{\infty}}{l'} (1 - \delta_{k,1}) \rho_c V_c (\bar{\theta}_c)_{n-1} + (\bar{G}_{j,2})_c \frac{V_{\infty}}{l'} \delta_{k,2} \\ + (\bar{G}_{j,1})_c \delta_{k,1}$$

$$D_{17} = -\frac{V_{\infty}}{l'} (1 - \delta_{k,1}) \rho_c (\bar{W}_c)_{n-1} + (\bar{H}_{j,2})_c \frac{V_{\infty}}{l'} \delta_{k,2} \\ + (\bar{H}_{j,1})_c \delta_{k,1}$$

$$D_{18} = -\frac{V_{\infty}}{l'} (1 - \delta_{k,1}) \left(\bar{S}/R_c \right)_{n-1}$$

$$D_{19} = -\frac{V_{\infty}}{l'} (1 - \delta_{k,1}) \rho_c (\bar{V}_c)_{n-1} + (\bar{F}_{j,2})_c \frac{V_{\infty}}{l'} \delta_{k,2} \\ + (\bar{F}_{j,1})_c \delta_{k,1}$$

D. Error Indications:

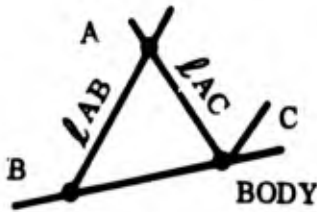
NOVAL = 601: Error in finding a or b in FLOWTL

NOVAL = 621: ψ is outside of reference tables in calculating $\frac{d S/R}{d \psi}$

BODY3

- A. Subroutine.
- B. Calling sequence: (IND)
- C. This routine computes the properties $\bar{\theta}$, \bar{P} , $\bar{\rho}$, \bar{V} , \bar{W} , \bar{T} , \bar{S}/R at a body point on the starting line and at any succeeding body point.

Given the configuration



and assuming zero yaw properties are known at A, B, and C and that the perturbations on these properties are known at A and B.

$$K_2 = \frac{P_C - P_A}{\Delta l_{AC}}$$

$$K_1 = 2\sqrt{1 - \frac{1}{M_C^2}} \left[\frac{P_C - P_B}{\Delta l_{BC}} \right] - K_2 \quad \text{where } \Delta l_{AC} = \sqrt{(XC - XA)^2 + (RC - RA)^2}$$

$$\Delta l_{BC} = \sqrt{(X_C - X_B)^2 + (R_C - R_B)^2}$$

$\frac{dZ}{dS/R}$, $\frac{\partial Z}{\partial \rho/\rho_0}$, K_3 , K_4 , C^2 , e^2 and γ^* are obtained in THERMO.

C_i ($i = 1, 5$) at A, B, C

D_i ($i = 1, 19$) at A, B, C are found in ARYCLC

Average Values of

C_i from A to C and B to C

D_i from A to C and B to C are found.

Body point matrix (a_{ij}) where $i = 1, 4; j = 1, 4$ is initialized as a zero matrix.

Then elements are calculated by:

$$a_{1,1} = 1 - \frac{(D_{AC_8} \ell_{AC} C_{AC_2})}{2 C_{AC_1}}$$

$$a_{1,2} = - \frac{(D_{AC_4} \ell_{AC} C_{AC_2} + D_{AC_6} \ell_{AC} C_{AC_3})}{2 C_{AC_1}}$$

$$a_{1,3} = \frac{(D_{AC_1} \ell_{AC} C_{AC_2} - D_{AC_5} \ell_{AC} C_{AC_3})}{2 C_{AC_1}}$$

$$a_{1,4} = - \frac{D_{AC_2} \ell_{AC} C_{AC_2}}{2 C_{AC_1}}$$

$$a_{2,1} = - \frac{1}{2} D_{BC_{10}} \ell_{BC} C_{BC_4}$$

$$a_{2,4} = 1 + \frac{1}{2} D_{CB_9} \ell_{BC} C_{BC_4}$$

$$a_{3,1} = C_{BC_4}$$

$$a_{3,2} = \frac{1}{2} D_{BC_{13}} l_{BC} C_{BC_4}$$

$$a_{3,3} = 1 + \frac{1}{2} D_{BC_{12}} l_{BC} C_{BC_4}$$

$$a_{4,1} = -\frac{1}{C^2}$$

$$a_{4,2} = 1.0$$

NOTE: All elements not noted are equal to zero.

Compute matrix (ξ_i) , $i = 1, 4$:

$$\xi_1 = \frac{1 + D_{AC_8} l_{AC} C_{AC_2}}{2 C_{AC_1}} P'_A - a_{1,2} \bar{p}_A - a_{1,3} \bar{v}_A - a_{1,4} \bar{w}_A$$

$$- \left(\frac{1}{C_{AC_1}} + \left(D_{AC_3} l_{AC} C_{AC_2} - D_{AC_7} l_{AC} C_{AC_3} \right) \cdot \frac{1}{2 C_{AC_1}} \right) \bar{\theta}_A$$

$$- \frac{l_{AC}}{C_{AC_1}} \left(- D_{AC_{15}} C_{AC_2} + D_{AC_{16}} C_{AC_3} \right)$$

$$\xi_2 = -a_{2,1} \bar{P}_B + (2 - a_{2,4}) \bar{W}_B + D_{BC_{17}} C_{BC_4} \ell_{BC}$$

$$\xi_3 = a_{3,1} \bar{P}_B - a_{3,2} \bar{p}_B + (2.0 - a_{3,3}) \bar{V}_B + D_{BC_{19}} C_{BC_4} \ell_{BC}$$

$$\bar{\theta}_C = 0$$

$$(\bar{S}/R)_C = (\bar{S}/R)_B + C_{BC_5} \ell_{BC} D_{BC_{18}}$$

$$\xi_4 = -\frac{e_C^2}{C_C^2} (\bar{S}/R)_C$$

Invert Body Point matrix:

$$(a_{ij})^{-1} \times (\xi_j) = (b_i) \quad i = 1,4; j = 1,4$$

$$\bar{P}_C = b_1$$

$$\bar{p}_C = \xi_4 - a_{4,1} \bar{P}_C$$

$$\bar{V}_C = b_3$$

$$\bar{w}_C = b_4$$

$$\bar{T}_C = \frac{1}{z_c R \rho_c} \left[\bar{P}'_C - \left\{ \bar{p}_C z_c R T_C + \bar{\rho}_C T_C R \log_{10} e \left(\frac{\partial z}{\partial \log_{10} \rho} \right)_s \right\} \right. \\ \left. + \left(\bar{s}/R \right)_C T_C R \rho_c \left(\frac{\partial z}{\partial s/R} \right)_\rho \right]$$

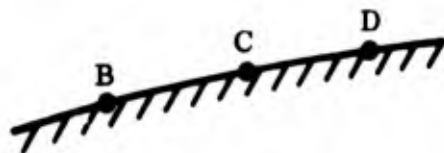
D. Error Indications:

NOVAL = 111: Error in thermodynamic properties (Subroutine THERMO)

NOVAL = 151: Error in finding C_i and D_i (Subroutine ARYCLC)

7.3.3 Initial Line Calculations at Body. (Spherically Blunted Body Only)

Given the configuration for the body point on the starting line.



for $j = 1, n = 0$

$$\bar{P}_{1,0} = \left[\frac{(P_D - P_C)(\theta_C - \theta_B)}{\theta_D - \theta_C} + \frac{(P_C - P_B)(\theta_D - \theta_C)}{\theta_C - \theta_B} \right] \cdot \frac{1}{\theta_D - \theta_B}$$

$$\bar{W}_{1,0} = v_C \left[\cos \theta_C - \frac{(X_C - X_O) \sin \theta_C}{r_c} \right] \times \text{DEL}$$

$$\bar{\theta}_{1,0} = 0$$

$$\overline{s/R}_{1,0} = 0$$

$$\bar{\rho}_{1,0} = \frac{P_{1,0}}{C^2}$$

$$\bar{V}_{1,0} = \frac{-\bar{P}_{1,0}}{\rho_C V_C}$$

$\bar{T}_{1,0}$ is found in the same manner as at succeeding body points.

$$\text{for } j = 1, n = 1 \quad \delta_{k,n} = 1.0$$

$$\text{for } j = 1, n > 1 \quad \delta_{k,n} = 0.0$$

and

$$\bar{V}_{1,n} = -\delta_{k,n} V_{SHK}$$

where V_{SHK} is computed at shock point.

$$\bar{\theta}_{1,n} = -\delta_{k,n} \theta_{SHK} \left[(X_C - X_O) \cos \theta_C + r_C \sin \theta_C \right]$$

where θ_{SHK} is found at shock point.

$$\bar{W}_{1,n} = 0.0$$

$$\bar{P}_{1,n} = 0.0$$

$$\bar{p}_{1,n} = 0.0$$

$$\bar{T}_{1,n} = 0.0$$

$$\text{for } j = 2 \quad n = 0 \quad \delta_{k,n} = 1.0$$

$$\text{for } j = 2 \quad n > 0 \quad \delta_{k,n} = 0.0$$

$$\bar{V}_{2,n} = \delta_{k,n} V_{SHK} + \frac{\delta_{k,n} V_{\infty}}{l'} \left[r_c \cos \theta_c - (x_c - x_o) \sin \theta_c \right]$$

$$\bar{\theta}_{2,n} = \delta_{k,n} \left\{ \left(\theta_{SHK} - \frac{V_{\infty}}{l' V_c} \right) \left[(x_c - x_o) \cos \theta_c + r_c \sin \theta_c \right] \right\}$$

$$\bar{w}_{2,n} = \delta_{k,n} \frac{V_{\infty}}{l'} (x_c - x_o) \delta$$

$$\bar{P}_{2,n} = 0.0$$

$$\bar{p}_{2,n} = 0.0$$

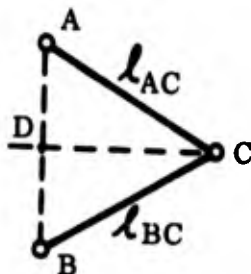
$$\bar{S}/R_{2,n} = 0.0$$

$$\bar{T}_{2,n} = 0.0$$

FIELD3

- A. Subroutine.
- B. Calling Sequence: (IND)
- C. This routine computes the properties $\bar{\theta}$, \bar{P} , \bar{p} , \bar{V} , \bar{w} , \bar{T} , \bar{S}/R at each field point on the starting line, and each field point on succeeding left hand characteristics.

Given the configuration for any field point not on the starting line:



and assuming zero yaw properties are known at A, B and C and that the perturbations on these properties are known at A and B.

$$K_1 = \frac{P_C - P_B}{\Delta l_{BC}}$$

$$K_2 = \frac{P_C - P_A}{\Delta l_{AC}}$$

$$\text{where } \Delta l_{AC} = \sqrt{(X_C - X_A)^2 + (R_C - R_A)^2}$$

$$\Delta l_{BC} = \sqrt{(X_C - X_B)^2 + (R_C - R_B)^2}$$

$\frac{\partial Z}{\partial S/R}$, $\frac{\partial Z}{\partial p/\rho_0}$, K_3 , K_4 , c^2 and e^2 are obtained in THERMO

C_i $i = 1, 5$ at A, B and C

D_i $i = 1, 9$ at A, B, and C are obtained in ARYCLC

Average Values C_i from A to B and A to C

D_i from A to B and A to C are found.

D_i at point D

$$D_{D_i} = D_{B_i} + \left[\frac{\psi_B - \psi_C}{\psi_B - \psi_A} \right] (D_{A_i} - D_{B_i})$$

Average Values D_i from D to C are found.

Field point matrix (a_{ij}) ($i = 1, 5; j = 1, 5$) is initialized as a zero matrix.

$$E_1 = - \frac{(\bar{\theta}_A - \bar{\theta}_B + C_{BC_1} (\bar{P}_A - \bar{P}_B))}{C_{AC_1} + C_{BC_1}}$$

$$E_2 = \frac{C_{BC_2} l_{BC}}{C_{AC_1} + C_{BC_1}}$$

$$E_3 = \frac{C_{BC_3} l_{BC}}{C_{AC_1} + C_{BC_1}}$$

$$E_4 = \frac{C_{AC_2} l_{AC}}{C_{AC_1} + C_{BC_1}}$$

$$E_5 = - \frac{C_{AC_3} l_{AC}}{C_{AC_1} + C_{BC_1}}$$

$$F_1 = \frac{1}{2} \left(E_2 D_{BC_8} + E_4 D_{AC_8} \right)$$

$$F_2 = - C_{AC_1} - \frac{1}{2} C_{AC_2} D_{AC_8} l_{AC}$$

Elements of field point matrix are computed.

$$a_{1,1} = C_{BC_1} - \frac{1}{2} C_{BC_2} D_{BC_8} l_{BC}$$

$$a_{1,2} = - \frac{1}{2} C_{BC_2} D_{BC_4} l_{BC} + \frac{1}{2} C_{BC_3} D_{BC_6} l_{BC}$$

$$a_{1,3} = \frac{1}{2} C_{BC_2} D_{BC_1} l_{BC} + \frac{1}{2} C_{BC_3} D_{BC_5} l_{BC}$$

$$a_{1,4} = - \frac{1}{2} C_{BC_2} D_{BC_2} l_{BC}$$

$$a_{1,5} = 1 + \frac{1}{2} C_{BC_2} D_{BC_3} l_{BC} + \frac{1}{2} C_{BC_3} D_{BC_7} l_{BC}$$

$$a_{2,1} = - C_{AC_1} + \frac{1}{2} C_{AC_2} D_{AC_8} l_{AC}$$

$$a_{2,2} = \frac{1}{2} l_{AC} \left(D_{AC_4} C_{AC_2} + D_{AC_6} C_{AC_3} \right)$$

$$a_{2,3} = -\frac{1}{2} l_{AC} \left(D_{AC_1} C_{AC_2} - D_{AC_5} D_{AC_3} \right)$$

$$a_{2,4} = \frac{1}{2} D_{AC_2} C_{AC_2} l_{AC}$$

$$a_{2,5} = 1 - \frac{1}{2} l_{AC} \left(D_{AC_3} C_{AC_2} - D_{AC_7} C_{AC_3} \right)$$

$$a_{3,1} = -\frac{1}{2} D_{DC_{10}} C_{DC_4} l_{CD}$$

$$a_{3,4} = 1 + \frac{1}{2} D_{DC_9} C_{DC_4} l_{CD}$$

$$a_{4,1} = C_{DC_4}$$

$$a_{4,2} = \frac{1}{2} D_{DC_{13}} C_{DC_4} l_{CD}$$

$$a_{4,3} = 1 + \frac{1}{2} D_{DC_{12}} C_{DC_4} l_{CD}$$

$$a_{4,5} = \frac{1}{2} D_{CD_{14}} C_{DC_4} l_{CD}$$

$$a_{5,1} = \frac{1}{2} e_C$$

$$a_{5,2} = -\frac{C_C^2}{2 e_C}$$

$$a_{5,5} = \frac{1}{2} D_{DC_{11}} C_{DC_5} l_{CD}$$

NOTE: All elements of matrix not in list are equal to zero.

$$\xi_1 = (2 - a_{1,5}) \bar{\theta}_B + (2 C_{BC_1} - a_{1,1}) \bar{P}_B - a_{1,3} \bar{V}_B - a_{1,4} \bar{w}_B - a_{1,2} \bar{\rho}_B$$

$$+ C_{BC_2} D_{BC_{15}} l_{BC} + C_{BC_3} D_{BC_{16}} l_{BC}$$

$$\xi_2 = (2 - a_{2,5}) \bar{\theta}_A + F_2 \bar{P}_A - a_{2,2} \bar{\rho}_A - a_{2,3} \bar{V}_A - a_{2,4} \bar{w}_A$$

$$- l_{AC} \left(D_{AC_{15}} C_{AC_2} - D_{AC_{16}} C_{AC_3} \right)$$

$$\xi_3 = (2 - a_{3,4}) \bar{w}_D - a_{3,1} \bar{P}_D + D_{DC_{17}} C_{DC_4} l_{CD}$$

$$\xi_4 = (2 - a_{4,3}) \bar{V}_D + a_{4,1} \bar{P}_D - a_{4,2} \bar{\rho}_D - a_{4,5} \bar{\theta}_D + D_{DC_{19}} C_{DC_4} l_{CD}$$

$$\xi_5 = (\bar{S}/R)_D - a_{5,5} \bar{\theta}_D + D_{DC_{18}} C_{DC_5} l_{CD}$$

Invert field point matrix:

$$(a_{ij})^{-1} \times (\xi_j) = b_i$$

$$\bar{P}_C = b_1$$

$$\bar{\rho}_C = b_2$$

$$\bar{V}_C = b_3$$

$$\bar{w}_C = b_4$$

$$\bar{\theta}_C = b_5$$

$$(\bar{S}/R)_C = a_{5,1} \bar{P}_C + a_{5,2} \bar{\rho}_C$$

$$\bar{T}_C = \frac{1}{Z_C R_C} \left[\bar{P}_C - \left\{ \bar{\rho}_C Z_C R_C + \bar{\rho}_C T_C R \log_{10} e \left(\frac{\partial t}{\partial \log_{10} \rho} \right)_s \right\} \right. \\ \left. + (\bar{S}/R)_C T_C R \rho_C \left(\frac{\partial t}{\partial S/R} \right) \right]$$

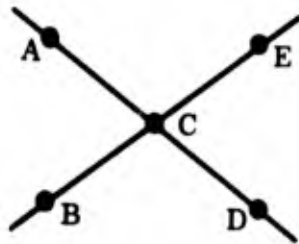
D. Error indicators:

NOVAL = 111: Error in thermodynamic properties (THERMO)

NOVAL = 151: Error in finding C_i or D_i (ARYCLC)

7.3.4 Initial Line Calculations at any Field Point (Spherically Blunted Body Only)

Given the configurations for a field point on the starting line



for $j = 1, n = 0$

$$\frac{dP}{dl^+} = \left[\frac{(P_C - P_B) \Delta l_{CE}}{\Delta l_{BC}} + \frac{(P_E - P_C) \Delta l_{BC}}{\Delta l_{CE}} \right] \cdot \frac{1}{\Delta l_{CE} + \Delta l_{BC}}$$

$$\frac{d\theta}{dl^+} = \left[\frac{(\theta_C - \theta_B) \Delta l_{CE}}{\Delta l_{BC}} + \frac{(\theta_E - \theta_C) \Delta l_{BC}}{\Delta l_{CE}} \right] \cdot \frac{1}{\Delta l_{CE} + \Delta l_{BC}}$$

$$\frac{dS/R}{dl^+} = \left[\frac{(S/R_E - S/R_B) \Delta l_{CE}}{\Delta l_{BC}} + \frac{(S/R_B - S/R_C) \Delta l_{BC}}{\Delta l_{CE}} \right] \cdot \frac{1}{\Delta l_{CE} + \Delta l_{BC}}$$

$$\frac{dP}{dl^-} = \left[\frac{(P_C - P_A) \Delta l_{CD}}{\Delta l_{AC}} + \frac{(P_D - P_C) \Delta l_{AC}}{\Delta l_{CD}} \right] \cdot \frac{1}{\Delta l_{CD} \Delta l_{AC}}$$

$$\frac{d\theta}{dl^-} = \left[\frac{(\theta_C - \theta_A) \Delta l_{CD}}{\Delta l_{AC}} + \frac{(\theta_D - \theta_C) \Delta l_{AC}}{\Delta l_{CD}} \right] \cdot \frac{1}{\Delta l_{CD} \Delta l_{AC}}$$

$$\frac{dS/R}{dl^-} = \left[\frac{(S/R_C - S/R_A) \Delta l_{CD}}{\Delta l_{AC}} + \frac{(S/R_D - S/R_C) \Delta l_{AC}}{\Delta l_{CD}} \right] \cdot \frac{1}{\Delta l_{CD} \Delta l_{AC}}$$

where:

$$\Delta l_{AC} = \sqrt{(X_C - X_A)^2 + (R_C - R_A)^2}$$

$$\Delta l_{BC} = \sqrt{(X_C - X_B)^2 + (R_C - R_B)^2}$$

$$\Delta l_{CD} = \sqrt{(X_C - X_D)^2 + (R_C - R_D)^2}$$

$$\Delta l_{CE} = \sqrt{(X_C - X_E)^2 + (R_C - R_E)^2}$$

$$A = \left[(X_C - X_0) \cos (\theta_C - \mu_C) + R_C \sin (\theta_C - \mu_C) \right] \cdot \frac{1}{\sin 2\mu_C}$$

$$B = \left[(X_C - X_0) \cos (\theta_C + \mu_C) + R_C \sin (\theta_C + \mu_C) \right] \cdot \frac{1}{\sin 2\mu_C}$$

$$\bar{P}_{1,0} = A \frac{dP}{dl^+} + B \frac{dP}{dl^-}$$

$$\bar{\theta}_{1,0} = A \frac{d\theta}{dl^+} + B \frac{d\theta}{dl^-}$$

$$\bar{S/R}_{1,0} = A \frac{d S/R}{dl^+} + B \frac{d S/R}{dl^-}$$

$$\bar{w}_{1,0} = \frac{V_C \left(\cos \theta_C - (X_C - X_O) \sin \theta_C \right)}{R_C} \delta$$

$$\bar{p}_{1,0} = \frac{\bar{P}_{1,0}}{C^2} - \frac{e^2}{C^2} \bar{S/R}_{1,0}$$

$$\bar{V}_{1,0} = \frac{-\bar{P}_{1,0}}{\rho_C V_C} - \frac{RT_C}{V_C} \bar{S/R}_{1,0}$$

$$\text{if } j = 1, n = 1 \quad \delta_{k,n} = 1.0$$

$$\text{if } j = 1, n > 1 \quad \delta_{k,n} = 0.0$$

$$\bar{V}_{1,n} = - \delta_{k,n} V_{SHK} \quad V_{SHK} \text{ from SHOCK3}$$

$$\theta_{1,n} = - \delta_{k,n} \theta_{SHK} \left[(X_C - X_O) \cos \theta_C + R_C \sin \theta_C \right] \quad \theta_{SHK} \text{ from SHOCK3}$$

$$\bar{w}_{1,n} = 0.0$$

$$\bar{P}_{1,n} = 0.0$$

$$\bar{\rho}_{1,n} = 0.0$$

$$\bar{T}_{1,n} = 0.0$$

$$\text{if } j = 2, n = 0 \quad \delta_{k,n} = 1.0$$

$$j = 2, n > 0 \quad \delta_{k,n} = 0.0$$

$$\bar{V}_{2,n} = \delta_{k,n} \left[v_{\text{SHK}} - \frac{v_{\infty}}{l'} \left([x_C - x_O] \sin \theta - (R_C \cos \theta_C) \right) \right]$$

$$\bar{\theta}_{2,n} = \delta_{k,n} \left\{ \left[\theta_{\text{SHK}} - \frac{v_{\infty}}{l' v_C} \right] \left[(x_C - x_O) \cos \theta_C + R_C \sin \theta_C \right] \right\}$$

$$\bar{w}_{2,n} = \delta_{k,n} \frac{v_{\infty}}{l'} (x_C - x_O) \times \delta$$

$$\bar{P}_{2,n} = 0.0$$

$$\bar{\rho}_{2,n} = 0.0$$

$$\bar{S}/R_{2,n} = 0.0$$

$$\bar{T}_{2,n} = 0.0$$

SHOCK3

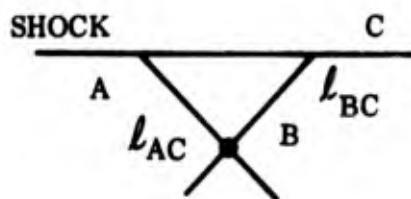
A. Subroutine

B. Calling Sequence: (IND)

C. This routine computes the properties $\bar{\theta}$, \bar{P} , $\bar{\rho}$, \bar{V} , \bar{w} , \bar{T} , \bar{S}/R , \bar{R}_S , $\frac{\partial \bar{R}}{\partial X}$

at a shock point on the starting line and at any succeeding shock point.

Given the configuration for any shock point not on the starting line.



and assuming that the zero yaw properties are known at A, B, and C and that the perturbed properties are known at A and B.

$$\Delta l_{AC} = \sqrt{(X_C - X_A)^2 + (R_C - R_A)^2}$$

$$\Delta l_{BC} = \sqrt{(X_C - X_B)^2 + (X_C - X_A)^2}$$

$$K_1 = \frac{P_C - P_B}{\Delta l_{BC}}$$

$$\mu_C = \sin^{-1} \frac{1}{M_C}$$

$$\beta = \theta_C + \mu_C - \sigma_C$$

$$K_2 = - \frac{\sin(2\mu_C - \beta)}{\sin \beta} K_1 + \frac{\sin 2\mu_C}{\sin \beta} \left[\frac{P_C - P_A}{\Delta l_{AC}} \right]$$

$$\frac{\partial Z}{\partial S/R}, \frac{\partial Z}{\partial \rho/\rho_0}, K_3, K_4, C^2, e^2 \text{ are found in THERMO}$$

$$C_i (i = 1, 5) \text{ at A, B, and C}$$

$$D_i (i = 1, 19) \text{ at A, B, and C are found in ARYCLC}$$

Average Values of

D_i from B to C

C_i from B to C

$$V_{N1} = V_{\infty} \sin \sigma_C$$

$$V_{N2} = V_C \sin(\sigma_C - \theta_C)$$

$$F_3 = V_{N2} - V_{N1}$$

$$\frac{\partial H}{\partial P} = \frac{1}{\rho_C} \left(1 + \frac{P_C}{Z_C e_C^2} \right)$$

$$\frac{\partial H}{\partial \rho} = - \frac{\gamma^* P_C^2}{Z_C \rho_C^2 e_C^2}$$

$$\Delta = V_{N2}^2 - \rho_C V_{N2}^2 \frac{\partial H}{\partial P} - \rho_C \frac{\partial H}{\partial \rho}$$

$$\frac{\partial F_1}{\partial V_{N1}} = \frac{\rho_\infty F_3}{\Delta} \left[- V_{N2} F_3 + 2 \rho_C \frac{\partial H}{\partial \rho} \right]$$

$$\frac{\partial F_2}{\partial V_{N1}} = \frac{\rho_\infty F_3}{\Delta} \left[\frac{(V_{N1} + V_{N2})}{V_{N2}} - 2 \rho_C \frac{\partial H}{\partial P} \right]$$

$$\frac{\partial F_3}{\partial V_{N1}} = -1 + \frac{1}{\Delta} \left[V_{N1} V_{N2} + \rho_\infty V_{N2} (V_{N2} - 2 V_{N1}) \frac{\partial H}{\partial P} - \rho_\infty \frac{\partial H}{\partial \rho} \right]$$

For $n = 0$

$$\bar{C}_1 = - V_\infty \cos \sigma_C$$

$$\bar{C}_2 = - (R_C \sin \sigma_C + X_C \cos \sigma_C) \frac{V_\infty}{l}$$

$$\bar{D}_1 = - V_\infty \sin \theta_C$$

$$\bar{D}_2 = (R_C \cos \theta_C - X_C \sin \theta_C) \frac{V_\infty}{l}$$

$$\bar{E}_1 = V_\infty \cos \theta_C$$

$$\bar{E}_2 = (R_C \sin \theta_C + X_C \cos \theta_C) \frac{V_\infty}{l}$$

$$\bar{Q}_1 = V_\infty$$

$$\bar{Q}_2 = (X_C - X_O) \frac{V_\infty}{l}$$

$$\text{for } n > 0, \bar{C}_i = \bar{D}_i = \bar{E}_i = \bar{Q}_i = 0 \quad (i = 1, 2)$$

$$H_1 = C_{BC_1} - \frac{1}{2} C_{BC_2} l_{BC} D_{BC_8}$$

$$H_2 = -\frac{1}{2} (C_{BC_2} l_{BC} D_{BC_4} - C_{BC_3} l_{BC} D_{BC_6})$$

$$H_3 = \frac{1}{2} (C_{BC_2} l_{BC} D_{BC_1} + C_{BC_3} l_{BC} D_{BC_5})$$

$$H_4 = -\frac{1}{2} C_{BC_2} l_{BC} D_{BC_2}$$

$$H_5 = 1 + \frac{1}{2} (C_{BC_2} l_{BC} D_{BC_3} + C_{BC_3} l_{BC} D_{BC_7})$$

$$H_6 = -\frac{\partial F_1}{\partial V_{N_1}} V_\infty \cos^3 \sigma_C$$

$$H_7 = - \frac{\partial F_2}{\partial V_{N_1}} V_\infty \cos^3 \sigma_C$$

$$u_C = V_C \cos \theta_C$$

$$v_C = V_C \sin \theta_C$$

$$H_8 = - \frac{\cos^2 \sigma_C}{V_C} \left[F_3 (u_C \cos \sigma_C + v_C \sin \sigma_C) + V_\infty \frac{\partial F_3}{\partial V_{N_1}} (u_C \sin \sigma_C \cos \sigma_C - v_C \cos^2 \sigma_C) \right]$$

$$H_9 = \frac{\cos \sigma_C \cos \theta_C}{V_C} \left[\tan \theta_C (F_3 \cos^2 \sigma_C + V_\infty \sin \sigma_C \cos^2 \sigma_C \frac{\partial F_3}{\partial V_{N_1}} \right. \\ \left. - \cos \sigma_C (F_3 \sin \sigma_C - V_\infty \cos^2 \sigma_C \frac{\partial F_3}{\partial V_{N_1}}) \right]$$

$$H_{10} = \frac{F_3 \cos \sigma_C}{R_C}$$

$$H_{11} = - \frac{1}{2} (X_C - X_A)$$

$$H_{12} = \frac{1}{2} C_{C_4} (V_C (1 - M_C^2) D_{C_1} \cos \theta_C + D_{C_5} \sin \theta_C) - \delta \frac{\sin \theta_C \cos \theta_C}{R_C}$$

$$H_{14} = \frac{1}{2} (-V_C \cos \theta_C D_{C_5} + V_C^2 \sin \theta_C D_{C_1})$$

find $\left. \frac{\partial S/R}{\partial \psi} \right|_C$ from reference tables.

$$H_{13} = \frac{H_{14}}{C_C^2} - R_C^{\delta} \rho_C V_C \frac{\partial S/R}{\partial \psi} \frac{e_C^2}{C_C^2} \cos \theta_C$$

$$H_{15} = C_{C_4} \left[(D_{C_{14}} + \frac{1}{2} V_C D_{C_5}) \cos \theta_C + V_C D_{C_{12}} \sin \theta_C \right]$$

$$H_{16} = - (H_1 H_{14} + H_2 H_{13} + H_3 H_{15} + H_4 H_{10} + H_5 H_{12})$$

$$H_{17} = - (H_1 H_6 + H_2 H_7 + H_3 H_8 + H_5 H_9)$$

for $n=0, \delta_k = 1$ for $n > 0 \delta_k = 0$

$$\xi_1 = (2 C_{BC_1} - H_1) \bar{P}_B - H_2 \bar{\rho}_B - H_3 \bar{V}_B - H_4 \bar{w}_B + (2-H_5) \theta_B$$

$$+ l_{BC} (C_{BC_2} D_{BC_{15}} + C_{BC_3} D_{BC_{16}})$$

$$\xi_2 = -\delta_k \bar{C}_j \frac{\partial F_1}{\partial V_{N_1}} + (1-\delta_k) \cos \sigma \frac{V_\infty}{l} \bar{R}_S \Big|_{C_{n-1}} \frac{\partial F_1}{\partial V_{N_1}}$$

$$\xi_3 = -\delta_k \bar{C}_j \frac{\partial F_2}{\partial V_{n1}} + (1-\delta_k) \cos \sigma_C \frac{V_\infty}{l} \frac{\partial F_2}{\partial V_{n1}} \bar{R}_S \Big|_{C_{n-1}}$$

$$\xi_4 = \delta_k D_j - \left[\delta_k \bar{C}_j - (1-\delta_k) \cos \sigma_C \frac{V_\infty}{l} \bar{R}_S \Big|_{C_{n-1}} \right] \frac{\partial F_3}{\partial V_{N1}} \sin (\sigma_C - \theta_C)$$

$$\xi_5 = -\frac{\delta_k \bar{E}_j}{V_C} + \frac{1}{V_C} \left[\delta_k \bar{C}_j - (1-\delta_k) \cos \sigma_C \frac{V_\infty}{l} \bar{R}_S \Big|_{C_{n-1}} \right] \frac{\partial F_3}{\partial V_{N1}} \cos (\sigma_C - \theta_C)$$

$$\xi_6 = \bar{R}_S \Big|_{A_n} - H_{11} \frac{\partial \bar{R}_S}{\partial X} \Big|_A$$

$$\xi_7 = \delta_k \bar{Q}_j$$

$$\xi_8 = \xi_1 - H_1 \xi_2 - H_2 \xi_3 - H_3 \xi_4 - H_4 \xi_7 - H_5 \xi_5$$

$$\frac{\partial \bar{R}_S}{\partial X} \Big|_C = \frac{\xi_8 - H_{16} \xi_6}{H_{17} - H_{16} H_{11}}$$

$$\bar{R}_S \Big|_C = \xi_6 - H_{11} \frac{\partial \bar{R}_S}{\partial X} \Big|_C$$

$$\bar{P}_C = \xi_2 - H_{14} \bar{R}_S \Big|_C - H_6 \frac{\partial \bar{R}_S}{\partial X} \Big|_C$$

$$\bar{\rho}_C = \xi_3 - H_{13} \bar{R}_S \left|_C - H_7 \frac{\partial \bar{R}_S}{\partial X} \right|_C$$

$$\bar{v}_C = \xi_4 - H_{16} \bar{R}_S \left|_C - H_8 \frac{\partial \bar{R}_S}{\partial X} \right|_C$$

$$\bar{w}_C = \xi_7 - H_{10} \bar{R}_S \left|_C$$

$$\bar{\theta}_C = \xi_5 - H_{12} \bar{R}_{S_C} - H_9 \frac{\partial \bar{R}_S}{\partial X_C}$$

$$\left(\frac{\bar{S}}{\bar{R}} \right)_C = \frac{\bar{P}_C - C_C^2 \bar{\rho}_C}{e_C^2}$$

$$\bar{T}_C = \frac{1}{Z_C R \rho_C} \left[\bar{P}_C - \left\{ \bar{\rho}_C Z_C R T_C + \bar{\rho}_C T_C R \log_{10} \rho \left(\frac{\partial Z}{\partial \log_{10} \rho} \right)_S \right\} \right]$$

$$+ \left(\frac{\bar{S}}{\bar{R}} \right)_C T_C R \rho_C \left(\frac{\partial Z}{\partial \frac{\bar{S}}{\bar{R}}} \right)_\rho \right]$$

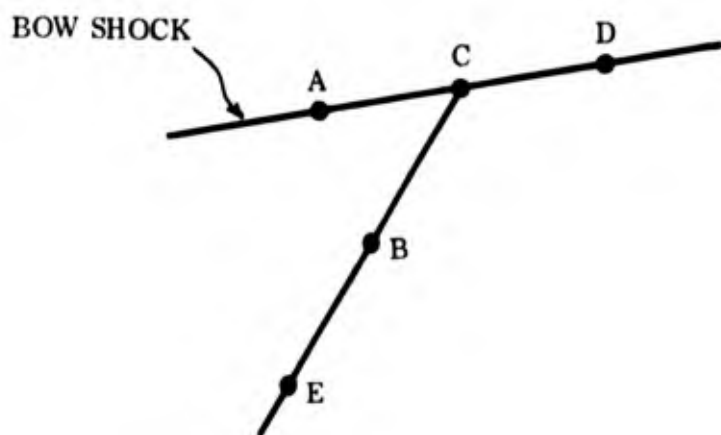
D. Error Indicators:

NOVAL = 111: Error in thermodynamic properties (THERMO)

NOVAL = 151: Error in finding C_i or D_i (ARYCLC)

NOVAL = 321: ψ_C at shock point is not in reference tables.

Given the Configuration for the shock point on the starting line (spherically blunted body only).



$$\Delta l_{AC} = \sqrt{(X_C - X_A)^2 + (R_C - R_A)^2}$$

$$\Delta l_{BC} = \sqrt{(X_C - X_B)^2 + (R_C - R_B)^2}$$

$$\mu_C = \sin^{-1} \left(\frac{1}{M_C} \right)$$

$$\bar{R}_X = - \left[R_C \tan \sigma_C + (X_C - X_O) \right]$$

$$\frac{\partial \bar{R}_X}{\partial X} = - \left[1.0 + R_C \frac{d\sigma}{dx} \right] \cdot \frac{1}{\cos^2 \sigma}$$

where:

$$\frac{d\sigma}{dx} = \left[\frac{(\sigma_D - \sigma_C)(X_C - X_A)}{X_D - X_C} + \frac{(\sigma_C - \sigma_A)(X_D - X_C)}{X_C - X_A} \right] \cdot \frac{1}{X_D - X_A}$$

$$\frac{dP}{dl^+} = \left[\frac{(P_C - P_B) \Delta l_{CE}}{\Delta l_{BC}} - \frac{(P_C - P_E) \Delta l_{BC}}{\Delta l_{CE}} \right] \cdot \frac{1}{\Delta l_{CE} - \Delta l_{BC}}$$

$$\frac{d\theta}{dl^+} = \left[\frac{(\theta_C - \theta_B) \Delta l_{CE}}{\Delta l_{BC}} - \frac{(\theta_C - \theta_E) \Delta l_{BC}}{\Delta l_{CE}} \right] \cdot \frac{1}{\Delta l_{CE} - \Delta l_{BC}}$$

$$\frac{dS/R}{dl^+} = \left[\frac{(S/R_C - S/R_B) \Delta l_{CE}}{\Delta l_{BC}} - \frac{(S/R_C - S/R_E) \Delta l_{BC}}{\Delta l_{CE}} \right] \cdot \frac{1}{\Delta l_{CE} - \Delta l_{BC}}$$

$$\frac{dP}{d\mathcal{L}} = \left[\frac{(P_C - P_A) \Delta l_{CD}}{\Delta l_{AC}} + \frac{(P_D - P_C) \Delta l_{AC}}{\Delta l_{CD}} \right] \cdot \frac{1}{\Delta l_{AC} + \Delta l_{CD}}$$

$$\frac{d\theta}{d\mathcal{L}} = \left[\frac{(\theta_C - \theta_A) \Delta l_{CD}}{\Delta l_{AC}} + \frac{(\theta_D - \theta_C) \Delta l_{AC}}{\Delta l_{CD}} \right] \cdot \frac{1}{\Delta l_{AC} + \Delta l_{CD}}$$

$$\frac{dS/R}{d\mathcal{L}} = \left[\frac{(S/R_C - S/R_A) \Delta l_{CD}}{\Delta l_{AC}} + \frac{(S/R_D - S/R_C) \Delta l_{AC}}{\Delta l_{CD}} \right] \cdot \frac{1}{\Delta l_{AC} + \Delta l_{CD}}$$

$$A = \frac{R_C \sin \sigma_C + (X_C - X_O) \cos \sigma_C}{\sin (\sigma_C - \theta_C - \mu_C)}$$

$$B = \frac{R_C \sin(\theta_C + \mu_C) + (X_C - X_O) \cos(\theta_C + \mu_C)}{\sin(\sigma_C - \theta_C - \mu_C)}$$

$$\bar{P}_{1,0} = -A \frac{dP}{d\ell^+} + B \frac{dP}{d\mathcal{L}}$$

$$\bar{\theta}_{1,0} = \left[-A \frac{d\theta}{d\ell^+} + B \frac{d\theta}{d\mathcal{L}} \right]^{-1}$$

$$\bar{S}/R_{1,0} = -A \frac{dS/R}{d\ell^+} + B \frac{dS/R}{d\mathcal{L}}$$

$$\bar{w}_{1,0} = V_C \left[\cos \theta_C - \frac{(X_C - X_O) \sin \theta_C}{R_C} \right] \delta$$

$$\bar{\rho}_{1,0} = \frac{\bar{P}_{1,0}}{C^2} - \frac{e^2}{C^2} \bar{S}/R_{1,0}$$

$$\bar{V}_{1,0} = \frac{\bar{P}_{1,0}}{\rho_C V_C} - \frac{RT_C}{V_C} \bar{S}/R_{1,0}$$

$\bar{T}_{1,0}$ is found in the same manner as succeeding shock points.

$$\text{if } j=1, n=1 \quad \delta_{k,n} = 1.0$$

$$j=1, n > 1 \quad \delta_{k,n} = 0.0$$

$$\frac{d\bar{R}}{dX_{1,1}} = \delta_{k,n} \left[R_C \sin \sigma_C + (X_C - X_O) \cos \sigma_C \right] \cdot \frac{1}{l' \cos^3 \sigma_C}$$

$$\bar{R}_{1,1} = 0.0$$

$$V_{SHK} = \frac{V_\infty}{l'} \left(\frac{\sin \theta_C}{\cos \sigma_C} \right) \left[(X_C - X_O) \cos \sigma_C + R_C \sin \sigma_C \right]$$

$$\theta_{SHK} = \left[\frac{\sin \theta_C \sin (\sigma_C - \theta_C)}{l' \cos^2 \sigma_C} \right] \left[\frac{(X_C - X_O) \cos \sigma_C + R_C \sin \sigma_C}{(X_C - X_O) \cos \theta_C + R_C \sin \theta_C} \right]$$

$$\bar{V}_{1,1} = -\delta_{k,n} V_{SHK}$$

$$\bar{\theta}_{1,1} = -\delta_{k,n} \theta_{SHK} \left[(X_C - X_O) \cos \theta_C + R_C \sin \theta_C \right]$$

$$\bar{w}_{1,1} = 0.0$$

$$\bar{p}_{1,1} = 0.0$$

$$\bar{\rho}_{1,1} = 0.0$$

$$\bar{S}/R_{1,1} = 0.0$$

$$\text{if } j = 2, n = 0 : \delta_{k,n} = 1.0$$

$$j = 2, n > 0 : \delta_{k,n} = 0.0$$

$$\frac{\overline{dR}}{dX} = - \delta_{k,n} \left[(X_C - X_O) \cos \sigma_C + R_C \sin \sigma_C \right] \cdot \frac{1}{l' \cos^3 \sigma_C}$$

$$\overline{R}_x = 0.0$$

$$V_{SHK} = \frac{V_\infty}{l'} \left(\frac{\sin \theta_C}{\cos \sigma_C} \right) \left[(X_C - X_O) \cos \sigma_C + R_C \sin \sigma_C \right]$$

$$\theta_{SHK} = \left[\frac{\sin \theta_C \sin (\sigma_C - \theta_C)}{l' \cos^2 \sigma_C} \right] \left[\frac{(X_C - X_O) \cos \sigma_C + R_C \sin \sigma_C}{(X_C - X_O) \cos \theta_C + R_C \sin \theta_C} \right]$$

$$\overline{V}_{2,n} = \delta_{k,n} \left[V_{SHK} - \frac{V_\infty}{l'} \left([X_C - X_O] \sin \theta_C - R_C \cos \theta_C \right) \right]$$

$$\overline{\theta}_{2,n} = \delta_{k,n} \left[\theta_{SHK} - \frac{V_\infty}{l' V_C} \right] \left[(X_C - X_O) \cos \theta_C + R_C \sin \theta_C \right]$$

$$\overline{w}_{2,n} = \delta_{k,n} \frac{V_\infty}{l'} (X_C - X_O) \delta$$

$$\overline{P}_{2,n} = 0.0$$

$$\overline{S}/R_{2,n} = 0.0$$

$$\overline{T}_{2,n} = 0.0$$

STLINE

- A. Subroutine
- B. Calling Sequence: JARG, NARG, MARG, NTARG
- C. This subroutine calculates perturbations along an arbitrary line to be used as a starting line for a blunt body supersonic unsteady flow run.

Finds shock point which will be first point on arbitrary line, from zero-yaw solution. Assigns storage properties for shock point. Goes to subroutine SHOCK 3 and calculates perturbations for all J's and N's. Stores perturbations on binary tape NT5. Repeats for all field points using subroutine FIELD 3 and body point using subroutine BODY 3. After calculating body point, rewinds NT5 and zero-yaw tape. Reads preliminary information from zero-yaw tape, and returns to main program.

SUFINT

- A. Subroutine
- B. Calling Sequence
- C. This routine calculates the integration of normal force and moment along the body. The integration is done by Simpson's rule.

$$\text{Perturbation normal force: } FN_{j,n} = FN_{o,j,n} + \int_{X_s}^L (\pi R_C)^\delta \bar{P}_{j,n} dx$$

Perturbation Pitching moment:
$$Mz_{j,n} = Mz_{o,j,n} - \int_{X_S}^L (\pi R_C)^{\delta} \bar{P}_{j,n} (X_C +$$

$$R_C \tan \theta_o) dx$$

force and moment coefficients:
$$C_{N_{j,n}} = \frac{F_{N_{j,n}}}{\frac{1}{2} (\pi R_C)^{\delta} R_C \rho_{\infty} V_{\infty}^2}$$

$$C_{M_{j,n}} = \frac{M_{Z_{j,n}}}{(\pi R_C)^{\delta} R_C^2 \rho_{\infty} V_{\infty}^2}$$

Axial Location of center of pressure

$$\frac{X_{Cp}}{L} = - \left[\frac{Mz_{j,n}}{FN_{j,n}} + \frac{X_o}{L} \right] \frac{1}{X_C}$$

D. Along starting line, integration is done from shock to body to get initial data at body point.

at shock point $F_{X_K} = 0.0$.

at each field point

$$\gamma = \tan^{-1} \frac{R_C - R_B}{X_C - X_B}$$

$$\Delta l_{BC} = \sqrt{(X_C - X_B)^2 + (R_C - R_B)^2}$$

$$F_{X_K} = \int_A^B \left\{ V_\infty + \frac{P_\infty}{\rho_\infty V_\infty} - (V \cos \theta + \frac{P}{\rho V} \frac{\sin \gamma}{\sin(\gamma - \theta)}) \right\} r^\delta \rho V \sin(\gamma - \theta) dl$$

at body point

$$F_{X_K} = -2 \pi^\delta F_{X_K}$$

$$F_{N_\alpha} = F_{X_K} - PR (\pi R)^\delta$$

$$Mz_\alpha = \frac{-R}{\cos \theta} F_{N_\alpha}$$

C_{N_α} and C_{M_α} are calculated as in C above.

D. Error Indicators: None

SUMINT

A. Subroutine

B. Calling Sequence: None

C. This routine provides a summary of body data from integration of forces along body.

Reads Input

1) DELX, RN, NRTST, NT4

2) RTST (J1), J1=1, NRTST

3) XMAX (J1), J1=1, NRTST

Reads binary tape from Supersonic unsteady flow run on blunt body NUMJ,
NMAX, XEND, RHOUN, VUN, XO, ELP

$$\text{Computes } TMP = \rho_{\infty} V_{\infty}^2$$

$$\pi = 3.1415926$$

for each body point reads from binary tape for each J, N at each body point

J, N, X_C , R_C , $F_{N\alpha}$, Mz_{α}

$$\text{Computes } C_{N\alpha}_{j,n} = \frac{F_{N\alpha}}{\frac{\pi}{2} TMP R_C^2}$$

$$C_{Mz\alpha}_{j,n} = \frac{Mz_{\alpha}}{\pi TMP R_C^3}$$

Computes Table I

$$CN_{\alpha} = CN_{\alpha 1,0}$$

$$C_{M\alpha} = \frac{2 |R_C|}{X_C} C_{Mz\alpha 1,0}$$

$$(C_{N\dot{\alpha}} + C_{Nq}) = \frac{2 l'}{X_C} \left(CN_{\alpha_{2,0}} + CN_{\alpha_{1,1}} \right)$$

$$(C_{M\dot{\alpha}} + C_{Mq}) = \frac{4 |R_C| l'}{X_C^2} \left(CMz_{\alpha_{2,0}} + CMz_{\alpha_{1,1}} \right)$$

$$X_{CP} = \left[-\frac{M_{z\alpha}}{F_{N\alpha}} + X_0 \right] \frac{1}{X_C}$$

$$\frac{R_N}{R_B} = \frac{R_N}{|R_C|}$$

Stores values in table of two elements. Tests $\frac{R_N}{R_B}$ against RTST. When RTST

is found to lie between two elements of this table, interpolation is done to find above quantities for RTST:

$$X_{xo}, C_{N\alpha_{xo}}, C_{Mz\alpha_{xo}}, \left(C_{N\dot{\alpha}_{xo}} + C_{Nq_{xo}} \right), \text{ and } \left(C_{M\dot{\alpha}_{xo}} + C_{Mq_{xo}} \right)$$

TABLE II is calculated

$\frac{\psi_{CG}}{L_B}$ is incremented from 0.0 by increment DELX until Xmax is reached. For

each value of $\frac{X_{CG}}{L_B}$

$$C_{N\alpha} = C_{N\alpha_{x_0}}$$

$$C_{M\alpha} = C_{M\alpha_{x_0}} + \left(\frac{X_{CG}}{l_B} - \frac{X_o}{l_B} \right) C_{N\alpha_{x_0}}$$

$$\left(C_{N\dot{\alpha}} + C_{Nq} \right) = \left(C_{N\dot{\alpha}_{x_0}} + C_{Nq_{x_0}} \right) - 2 \left(\frac{X_{CG}}{l_B} - \frac{X_o}{l_B} \right) C_{N\alpha_{x_0}}$$

$$\left(C_{M\dot{\alpha}} + C_{Mq} \right) = \left(C_{M\dot{\alpha}_{x_0}} + C_{Mq_{x_0}} \right) + \left(\frac{X_{CG}}{l_B} - \frac{X_o}{l_B} \right) \left(C_{N\dot{\alpha}_{x_0}} + C_{Nq_{x_0}} \right)$$

$$- 2 \left(\frac{X_{CG}}{l_B} - \frac{X_o}{l_B} \right) C_{M\alpha_{x_0}} - 2 \left(\frac{X_{CG}}{l_B} - \frac{X_o}{l_B} \right)^2 C_{N\alpha_{x_0}}$$

Table II is printed and calculations return to Table I. Process is repeated until final body point is computed.

D. Interpolation above can also be done if a value for XINT (X interpolate) is read in as input. The program will use XINT to interpolate as RTST is used above.

E. Error Indicators: None.

THERMO

A. Subroutine

B. Calling Sequence: (IND)

C. This routine computes the thermodynamic properties used in calculating the unsteady flow perturbations and the small yaw angle perturbations.

IND = 1 Unsteady flow perturbations.

IND = 2 Small yaw angle perturbations.

$a, b, \frac{\partial a}{\partial \frac{S}{R}}, \frac{\partial b}{\partial \frac{S}{R}}, Z_C, \frac{\partial z}{\partial \frac{S}{R}}, \frac{\partial z}{\partial \left(\frac{\rho}{\rho_0}\right)}$ are found in FLOWTL

$$\gamma^* = \frac{a + b P_C}{P_C}$$

$$e^2 = \left[\begin{array}{cc} \gamma^* - 1 & \frac{\partial z}{\partial \rho / \rho_0} \\ -\frac{1}{Z_C} & \frac{\partial z}{\partial \rho / \rho_0} \end{array} \right] \frac{P_C}{Z_C}$$

$$C^2 = \frac{\gamma^* P_C}{\rho_C}$$

NOTE: Following equations are not used in supersonic unsteady flow program.

$$K_3 = M_C^2 e^2 + \frac{2 P_C}{Z_C} + M_C^2 \left(\frac{\partial a}{\partial \frac{S}{R}} + P_C \frac{\partial b}{\partial \frac{S}{R}} \right) \cdot \frac{1}{2 \gamma^* P_C M_C^2 \sqrt{M_C^2 - 1}}$$

$$K_4 = 2 + (b-1) M_C^2 \cdot \frac{1}{2 M_C^2 \sqrt{M_C^2 - 1} \rho_C C^2}$$

$$K_5 = -2 \frac{P_C}{Z_C} - e^2 M_C^2$$

D. Error Indicators:

NOVAL = 21: Error in FLOWTL in finding a, b, $\frac{\partial a}{\partial \frac{S}{R}}$ or $\frac{\partial b}{\partial \frac{S}{R}}$

NOVAL = 121: Error in FLOWTL in finding Z, $\frac{\partial Z}{\partial \left(\frac{\rho}{\rho_0}\right)}$ or $\frac{\partial Z}{\partial \left(\frac{S}{R}\right)}$

8. NUMERICAL RESULTS AND COMPARISON WITH TEST DATA.

Some representative results of the unsteady flow field solution are presented in this section. Calculations were carried out at four flight conditions along the trajectory shown in Figure 16. Figures 17 through 23 show the dynamic and static force and moment coefficients plotted against Mach number and against center of gravity location. Test data from Reference 16 is shown on Figures 17 and 19. While agreement is fair, two reasons for discrepancies between computed and measured results can be given. One is the fact that free stream conditions were not the same. The computations were done for entry along a representative trajectory, while the free stream conditions in the test cell were restricted to those available in the facility. The other reason for discrepancy between computed and measured results is the difference in Reynold's numbers. Calculations were carried out for an inviscid fluid while the tests were carried out at finite Reynold's numbers.

The plots demonstrate the wide range of information available from the unsteady, flow field solution. It should be noted that for each Mach number, a single solution provided all the coefficients over the center of gravity location range, for $R_N/R_B = 0.15, 0.3, \text{ and } 0.6$.

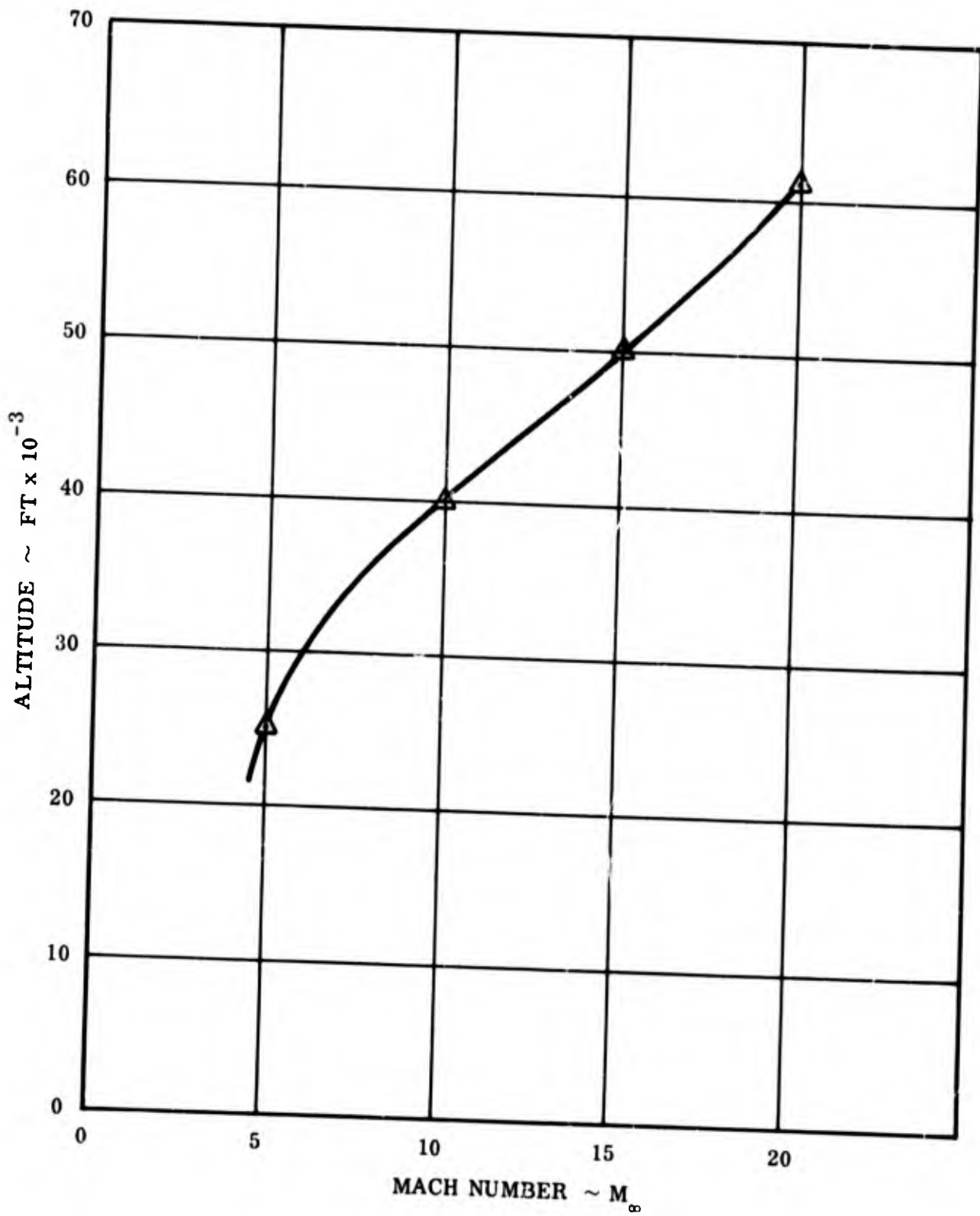


Figure 16. Trajectory Along Which Data is Presented

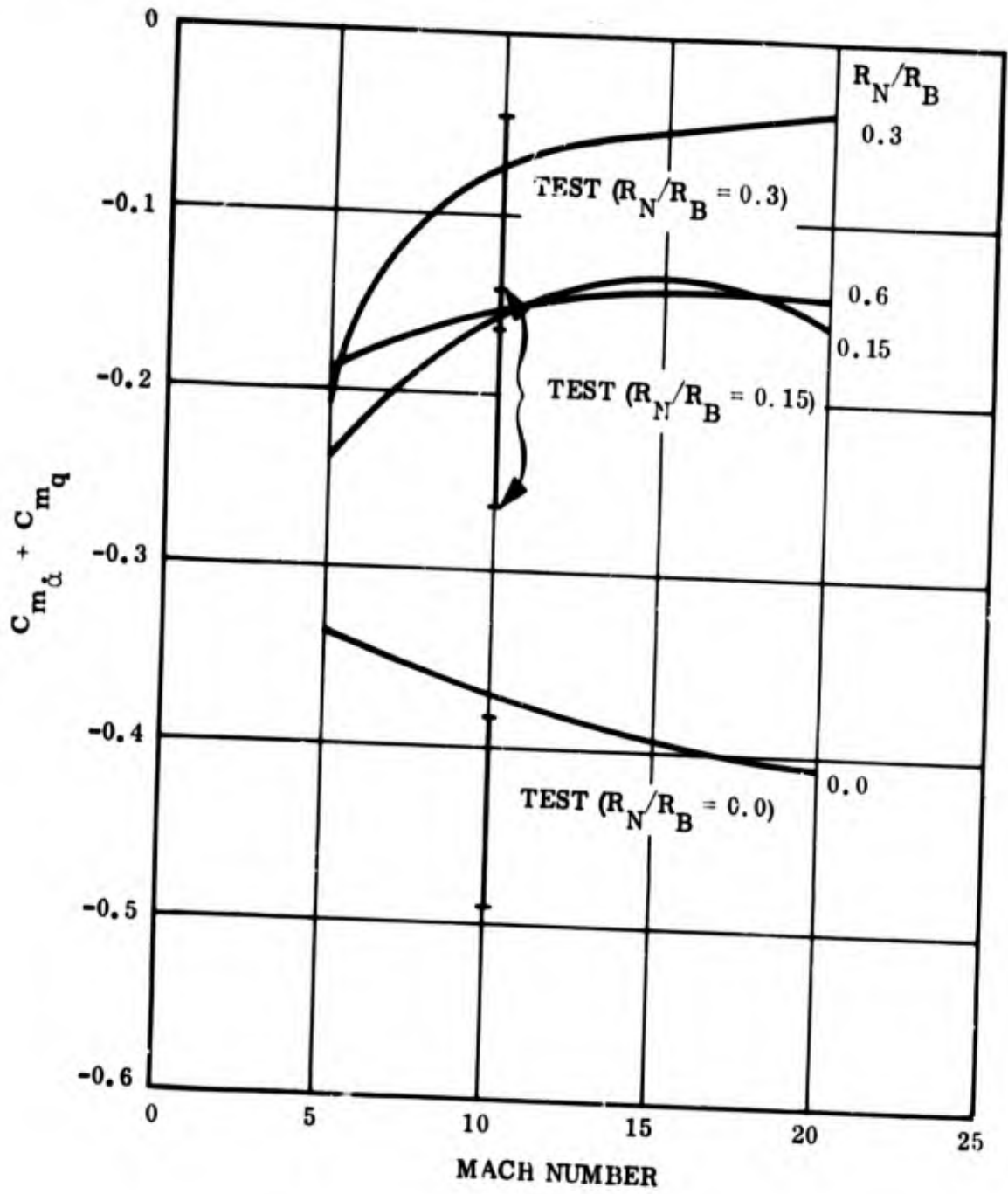


Figure 17. Dynamic Moment Coefficient vs Mach Number for 10-Degree Sphere Cone at $x_{cg}/L = 0.6$

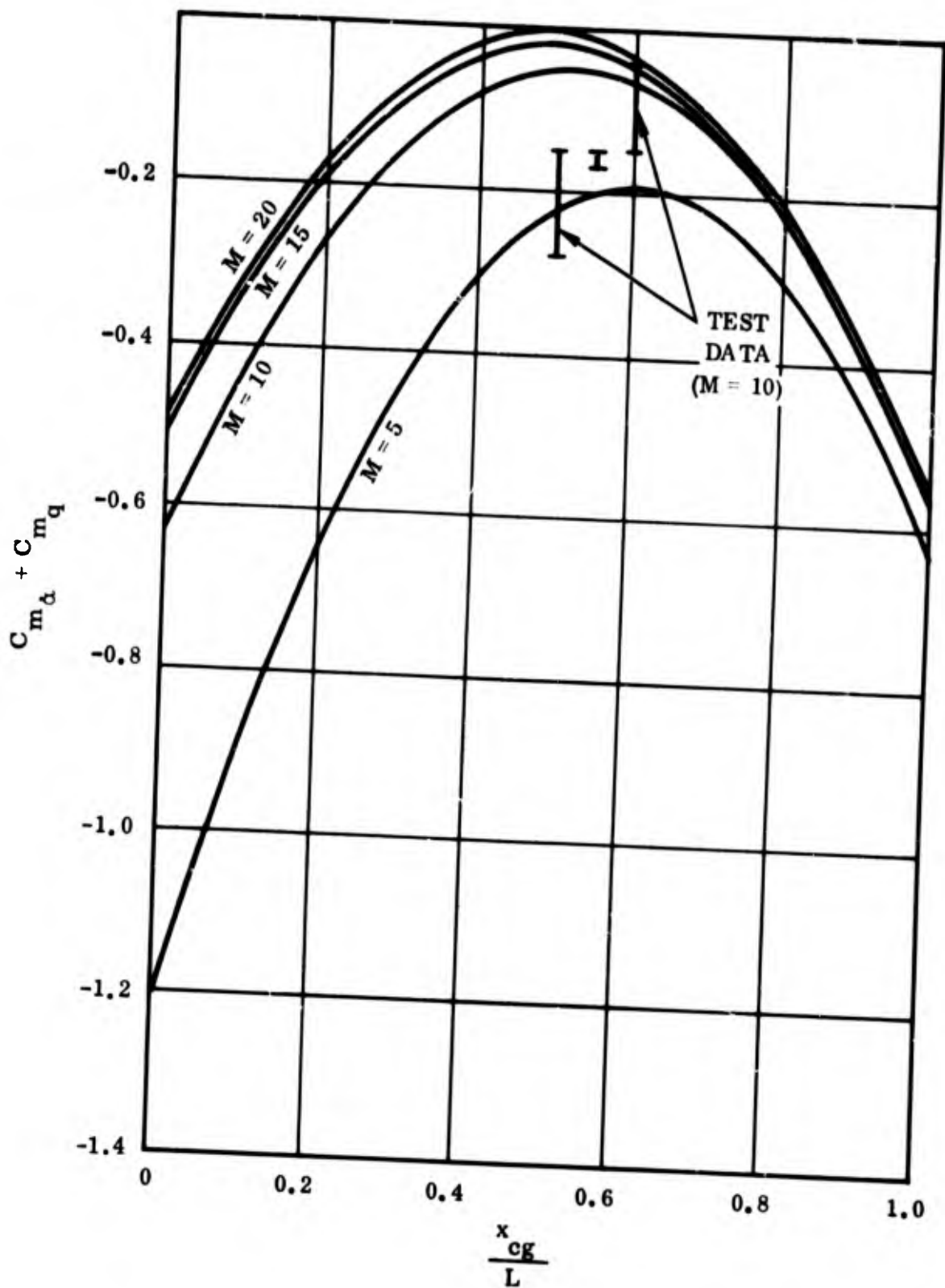


Figure 18. Dynamic Moment Coefficient vs Center of Gravity Location for 10-Degree Sphere Cone ($R_N/R_B = 0.3$)

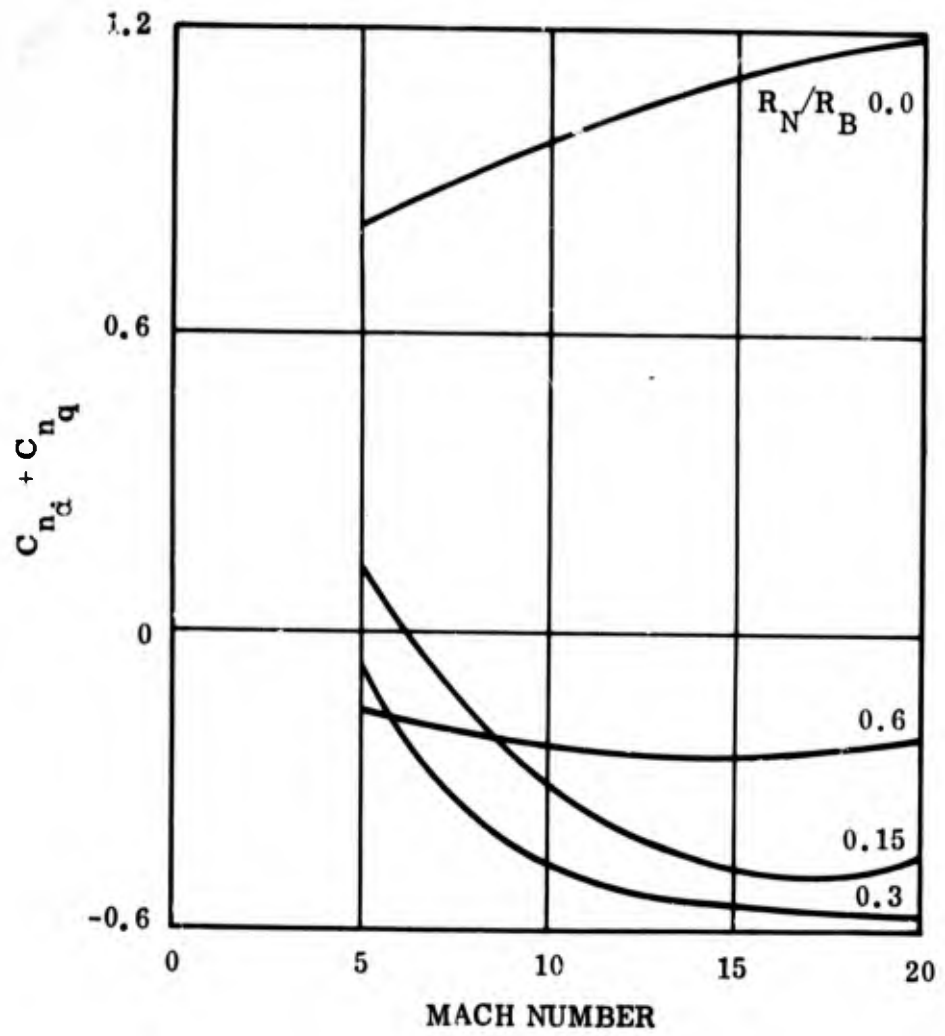


Figure 19. Dynamic Force Coefficient vs Mach Number for 10-Degree Sphere Cone at $x_{cg}/L = 0.6$

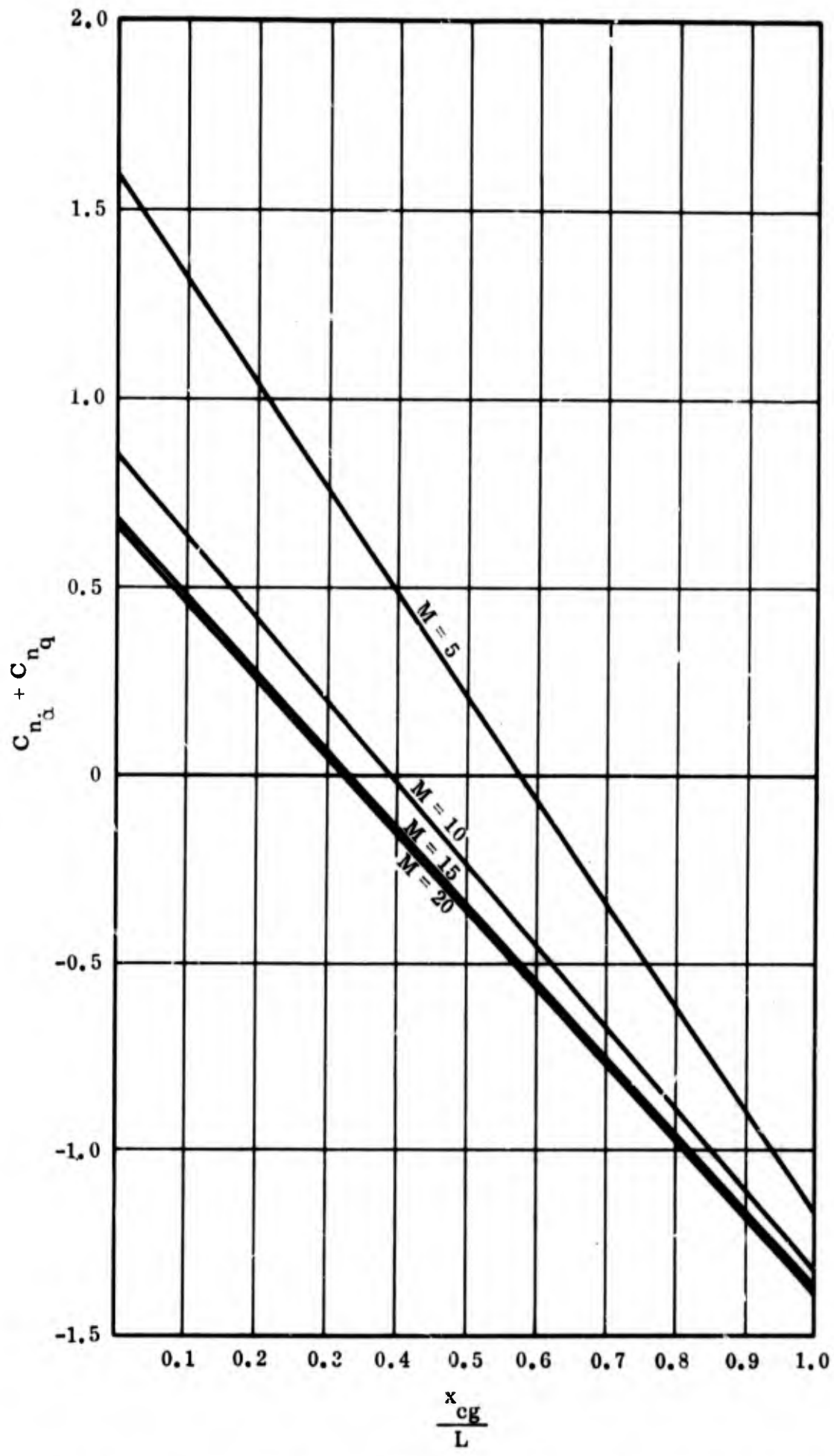


Figure 20. Dynamic Force Coefficient vs Center of Gravity Location for 10-Degree Sphere Cone $R_N/R_B = 0.3$

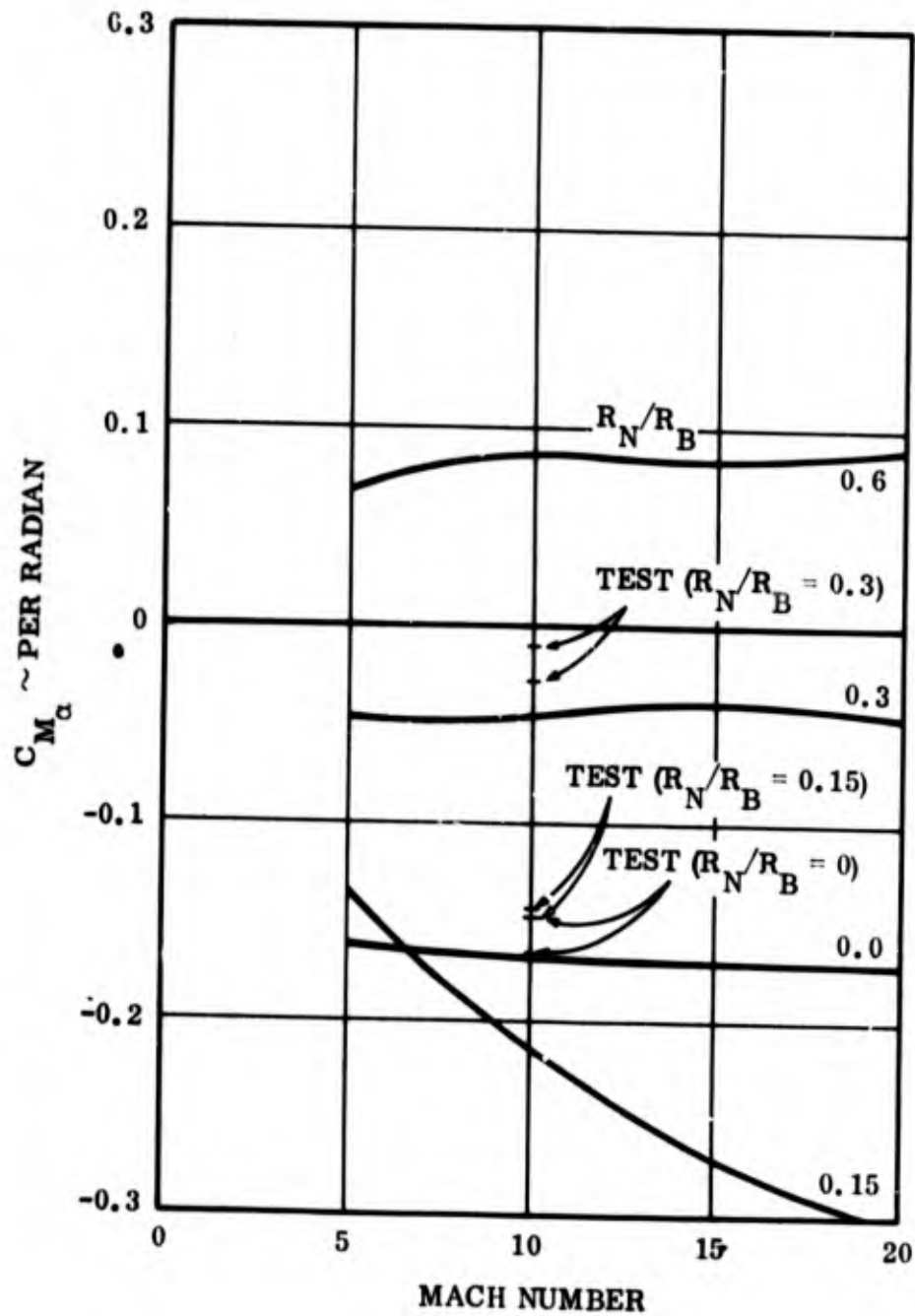


Figure 21. Static Moment Coefficient vs Mach Number for 10-Degree Sphere Cone at $x_{cg}/L = 0.6$

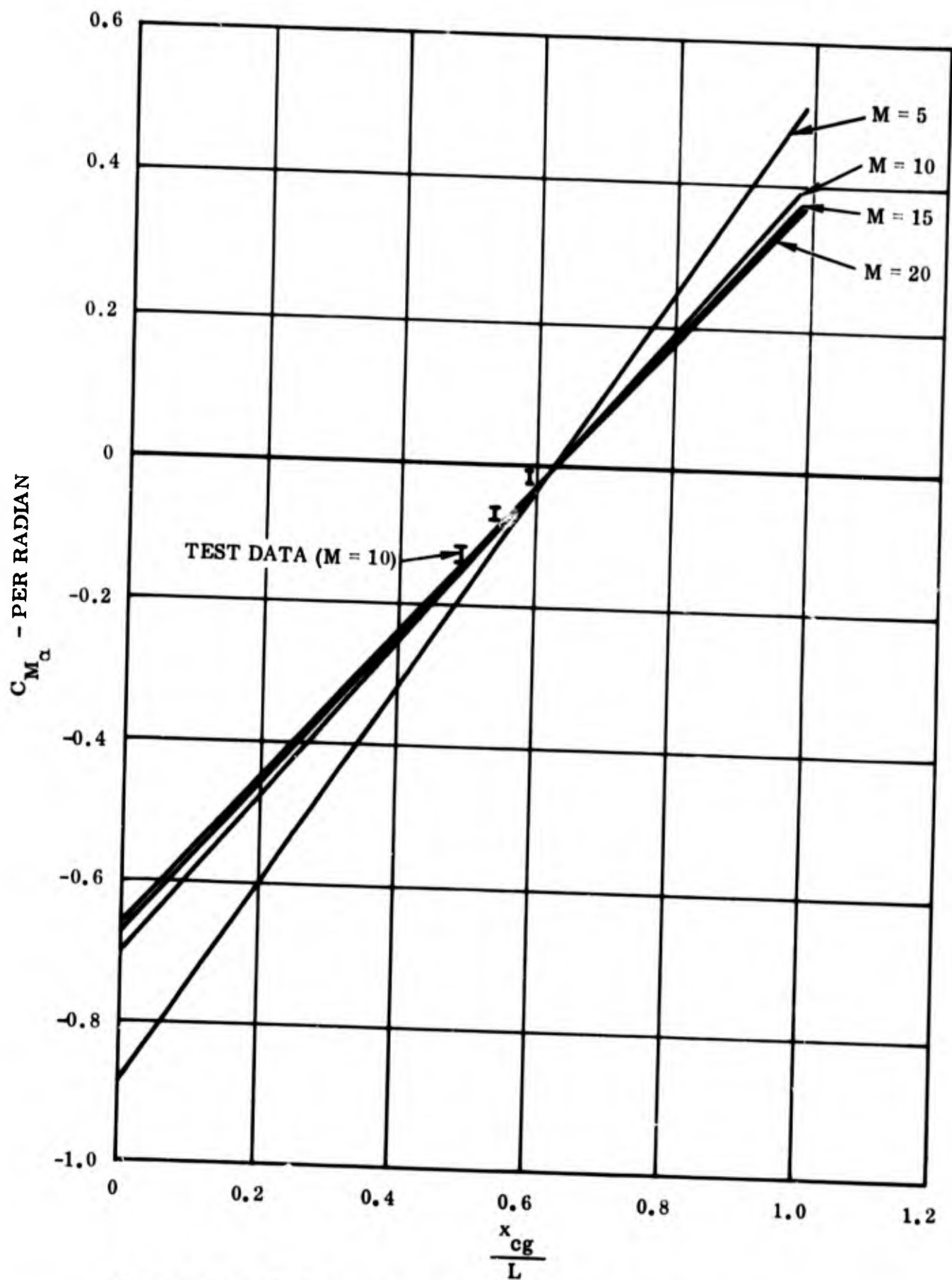


Figure 22. Static Moment Coefficient vs Center of Gravity Location for 10-Degree Sphere Cone at $R_N/R_B = 0.3$

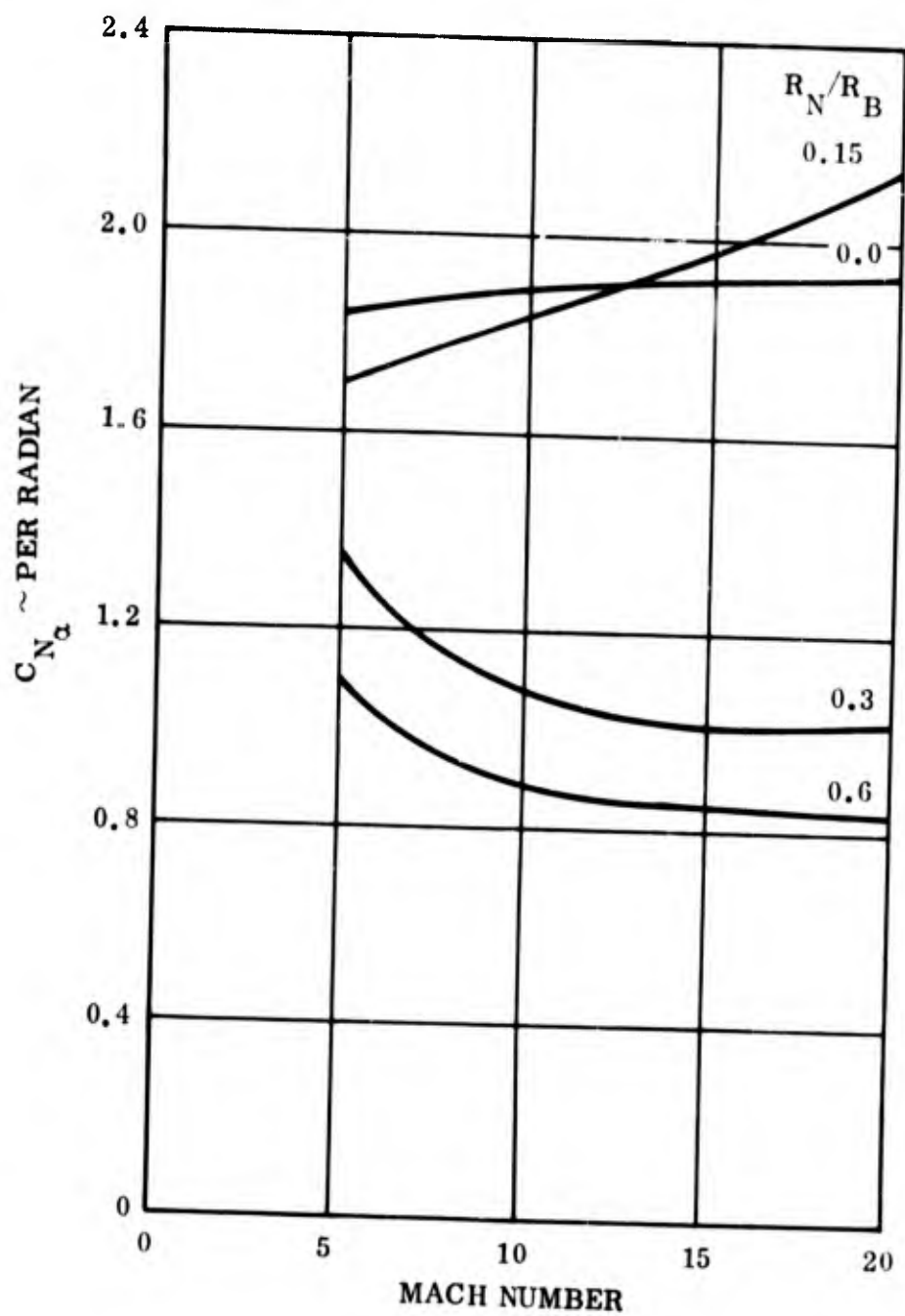


Figure 23. Static Force Coefficient vs Mach Number for 10-Degree Sphere Cone at $x_{cg}/L = 0.6$

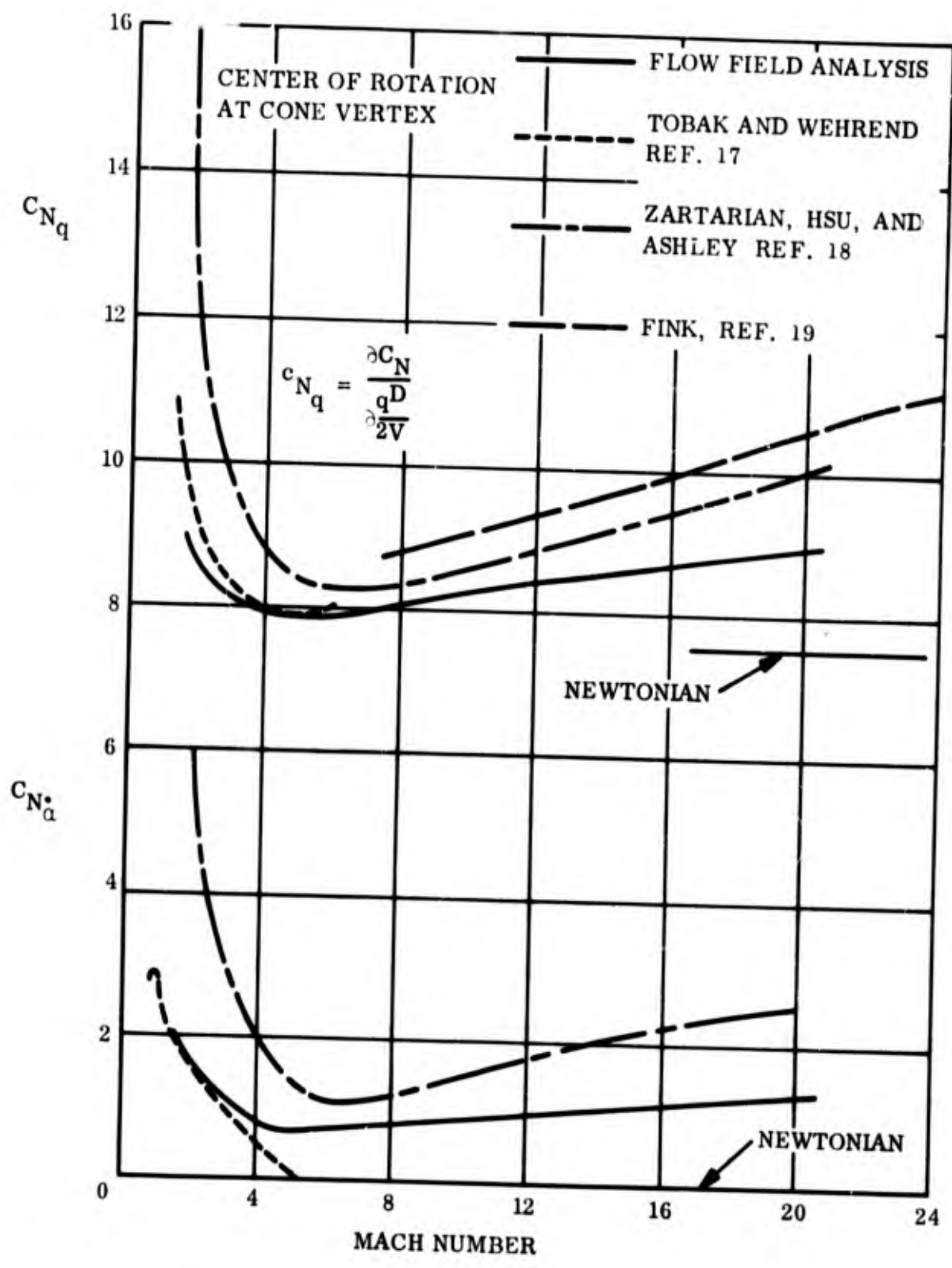


Figure 24. Comparison Of Theoretical Results
10-Degree Cone

9. CONCLUSIONS/SUMMARY

An unsteady flow field solution has been developed to enable the Research and Technology Division to compute static and dynamic stability coefficients for pointed and spherically blunted bodies of revolution (and analogous two-dimensional shapes). A small perturbation scheme has been used, together with a zero yaw steady state solution, to obtain the flow fields surrounding bodies undergoing small perturbations in motion about the zero yaw steady state condition. Force and moment coefficients have been obtained by integrating appropriate perturbations in pressure over the body surface.

Results of this flow field computation have been found to agree well with test data. Some numerical results of this solution are presented in Section 8. Figures 17, 18, 21 and 22 provide a comparison with test data obtained in Tunnel C, Arnold Engineering Development Center (reference 16).

Figure 24 shows a comparison of theoretical results for derivatives of the normal force coefficient, C_N , C_{N_q} and $C_{N_{\dot{\alpha}}}$ are shown separately in this figure, and the values for C_{N_q} are for a center of rotation at the vertex of the pointed cone. The results from the Flow Field analysis are compared with results from:

- 1) The potential theory due to Tobak and Wehrend, reference 17;
- 2) The shock-expansion theory due to Zartarian, Hsu and Ashley, reference 18;
- 3) The modified shock-expansion theory due to Fink, reference 19, and;
- 4) The Newtonian impact theory.

The Newtonian impact theory gives results which are considerably lower than the asymptotic values approached by the flow field results. They do not agree as well as might be expected based on comparison of the static stability derivatives.

The results from potential theory agree quite well with the flow field results at the lower Mach numbers, where the former theory is applicable. The only discrepancy occurs with C_{N_α} at Mach numbers above 3 or 4. C_{N_α} from the potential theory goes to the Newtonian value of zero at higher Mach numbers, while the flow field results show a value which increases with increasing Mach number and approaches a value on the order of 15% of C_{N_q} .

The results from shock expansion theory (reference 5) are in qualitative agreement with the flow field results and are in good quantitative agreement in the vicinity of Mach 8.

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APPENDIX I
A GUIDE FOR PREPARING INPUT
CARDS FOR THE FLOW FIELD COMPUTER PROGRAMS

The first part of this Appendix provides an outline of the terms which must appear on each input card for the pointed cone, transonic, zero-yaw supersonic and unsteady supersonic programs. The remainder of this Appendix is a more detailed guide to determining what value each term takes on, when carrying out an unsteady flow field solution.

The following terms must appear on the data cards for the pointed cone solution. *

<u>Card No.</u>	<u>Terms</u>	<u>No. of Cards</u>	<u>Comments</u>
1.	FFCASE (plus title)	1	
2.	IDPROG, KENSE (1 through 10)	1	
3.	TEST (1 through 10)	1	
4.	PUN, RHOUN, VUN, AMUN	1	
5.	DEL, RGI, ALT	1	
6.	OMBODY, DLTOM, NMAX	1	
7.	RIDEAL, GIDEAL, ZIDEAL, RHOO, RHOOD, GREAL	1	

The first character on each card must be either X or F. If the names of the terms on a card begin with I, J, K, L, M or N, the letter X must appear in column 1. Otherwise, F must be punched in column 1. On cards which contain both integers and non-integers, both letters must be used. For example, Card 6 will begin with the letter F, followed by OMBODY, comma, DLTOM, comma, XNMAX. No commas are to be used between the letters X and F and the number which immediately follows them. An asterisk must follow the last number on each card (except Card 1), unless a card ends in a series of zeros. The symbol \$ may be used in place of a series of zeros if no non-zero number appears to the right of the zeros. For instance Card 2 will usually be: X3, -1, \$.

The following terms must appear on the data cards for the zero yaw transonic

solution:*

<u>Card No.</u>	<u>Terms</u>	<u>No. of Cards</u>	<u>Comments</u>
1.	FFCASE (plus title)	1	
2.	IDPROG, KENSE (1 through 10)	1	
3.	NBTPW, NBTPR	1	Omit Card 3 if Kense(1) has been set equal to zero
4.	TEST (1 through 10)	1	
5.	PUN, RHOUN, VUN, AMUN	1	
6.	DEL, RGI, ALT	1	
7.	NUMPTS, NITMX, DLTRUN, TPREV, GPREV	1	
8.	RIDEAL, GIDEAL, ZIDEAL, RHOO, RHOOO, GREAL	1	
9A.	NEQU, NDEG (1 through NEQU)	1	Cards 9A through 9C describe shock.
9B.	A (1 through NDEG + 1))	NEQU	
9C.	RDIV (1 through (NEQU-1))	1	Omit Card 9C if NEQU < 2

*See footnote on page 206.

<u>Card No.</u>	<u>Terms</u>	<u>No. of Cards</u>	<u>Comments</u>
10A.	NCURVE, NTST (1 through NCURVE)	1	Cards 10A through 10C describe body
10B. (option 1)	A (1 through 12)	} NCURVE }	To use this option set NTST = - 1
10B. (option 2)	A (9 through 12)		To use this option set NTST = 0
10B. (option 3)	A (1 through 8), A(12)		To use this option set NTST = 1
10C.	CRV (1 through (NCURVE-1))	1	Omit Card 10C if NCURVE < 2
10D.	P (1 through NUMPTS)	1	
10E.	X (1 through NUMPTS)	1	

The following terms must appear on the data cards for the zero yaw supersonic solution*

<u>Card No.</u>	<u>Terms</u>	<u>No. of Cards</u>	<u>Comments</u>
1.	FFCASE (plus title)	1	
2.	IDPROG, KENSE (1 through 10)	1	

*See Footnote on page 206.

<u>Card No.</u>	<u>Terms</u>	<u>No. of Cards</u>	<u>Comments</u>
3.	NBTPW, NBTPR	1	
4.	TEST (1 through 10)	1	
5.	PUN, RHOUN, VUN, AMUN	1	
6.	DEL, RGI, ALT	1	
7.	NUMPTS, AKAPSH, AKDEL, AJFRZN	1	
8.	RIDEAL, GIDEAL, ZIDEAL, RHOO, RHOOO, GREAL	1	
9.	XEND, XTERM, MSTART, NTERM, NEXP	1	
10.	XEXP, REXP, THEXP, PEXP, NDIV	NEXP	Omit Card 10 when doing an unsteady solution.
11A.	NCURVE, NTST (1 through NCURVE)	1	Cards 11A through 11C describe body.
11B. (option 1)	A (1 through 12)		To use this option, set NTST = -1
11B. (option 2)	A (9 through 12)		To use this option, set NTST = 0

<u>Card No.</u>	<u>Terms</u>	<u>No. of Cards</u>	<u>Comments</u>
11B. (option 3)	A(1 through 8), A(12)		To use this option, set NTST = 1
11C.	CRV (1 through (NCURVE-1)	1	Omit Card 11C if NCURV < 2
12.	XC, RC, THC, RHOC, PC, \$	NUMPTS	The first card must give values at the shock, the last card values at the body.

The following terms must appear on the data cards for the unsteady supersonic solution. *

<u>Card No.</u>	<u>Terms</u>	<u>No. of Cards</u>	<u>Comments</u>
1.	FFCASE (plus title)	1	
2.	IDPROG, KENSE (1 through 10)	1	
3.	XEND, XTERM, ELP	1	
4.	MSTART, NUMJ, NMAX, NTERM, NT2, NT3, NT4, NT5	1	
5.	NBTPW, NBTPR	1	
6.	RIDEAL, GIDEAL, ZIDEAL, RHOO, RHO00, GREAL	1	

*See Footnote on page 206.

<u>Card No.</u>	<u>Terms</u>	<u>No. of Cards</u>	<u>Comments</u>
7.	NUMPTS, XO		Omit Card 7 for pointed bodies.
8.	DELX, RN, NRTST, NXINT, NT5	1	
9.	RTEST (1 through NRTST)	1	
10.	XINT (1 through NXINT)	1	
11.	XMAX		

A. TRANSONIC PROGRAM (ZERO-YAW)

Card 1 - The monitor FFCASE must appear on the first data card. * The remainder of this card (columns 7 through 72) may be used for any title the user desires to have printed out with the solution.

Card 2 - IDPROG identifies which link on the chain tape will be used. For the transonic solution, IDPROG must be set equal to 2. For the usual transonic solution to be used in obtaining an unsteady flow field solution, Kense (1) through Kense (10) should be set equal to zero.

Card 3 - NBTPW is the "logical" number of the tape drive on which the standard binary tape will be written (if Kense (1) is set equal to 1). A value of 26 is preset (this means tape drive B6). Any other tape drive may be used by setting

*The letters FFCASE must occupy columns 1 through 6 of the first card. No spaces should appear between them.

NBTPW equal to the appropriate "logical" number. * The term NBTPR has no meaning in this program, and should be set equal to zero.

Card 4 - Test (1) through test (5) are the convergence criteria used in various iterations in this program. If zeros are punched for these terms, certain preset values will be used in the solution. These preset values have been found to give good results in the past. It is recommended that they be used in production runs. Tests (6) through test (10) have no meaning in this program, and should be set equal to zero.

Card 5 - The free stream pressure and density, the flight velocity and Mach number, in that order, must be entered on Card 5. The units of the first three quantities are lb/ft^2 , slugs/ft^3 and ft/second , respectively.

Card 6 - The term DEL determines whether an axisymmetric or two-dimensional solution is to be carried out. (DEL = 0 means a two-dimensional solution; DEL = 1 means an axisymmetric solution). The real gas index, RGI, is set equal to 1 if a real gas solution is desired, equal to 0 if an ideal gas solution is desired. ALT is the altitude. It is merely printed out on the output sheet, and is not used in the computations.

Card 7 - NUMPTS is the number of points on the body surface, at which pressure and x will be provided as input (Cards 10D and 10E). An explanation of how the number of points and the pressures, etc., are obtained, is given in

*In the usual unsteady flow field solution, Kense (1) will be set equal to zero, card number 3 will be omitted, and no binary tape will be saved.

Appendix III of this report. NITMX is the maximum number of iterations which will be done in one of the iterative processes in this solution. A preset value of 5 will be obtained if NITMX is punched in as zero. It is recommended that this value be used in production runs. DLTRUN is the spacing between streamlines upstream of the shock. The value of DLTRUN determines on how many streamlines (between the body surface and shock wave) computations will be carried out. It is recommended that 40 to 60 streamlines be used. This can be done by setting DLTRUN equal to .015 for the first iterations, and adjusting it upward to decrease the number of streamlines or downward to increase it for following iterations. TPREV and GPREV are initial estimates of temperature and γ^* used in an iteration at the shock wave. Free stream temperature and 1.4 are the preset values. It is recommended that the preset values be used (by setting TPREV = GPREV = 0).

Card 8 - RIDEAL is the real gas constant for air, preset at 1716.5 ft lb/slug
R. GIDEAL is the ratio of specific heats used in ideal gas calculations, preset at 1.4. ZIDEAL is the compressibility factor used in ideal gas calculations, preset at 1.0. The remaining terms, RHOO, RHOOO and GREAL, are used in connection with the thermodynamic tables that have been provided with this solution. The preset values of these three terms should always be used (unless a different set of thermodynamic tables is provided in the future). The preset value of any of the above terms will be used, if zero is entered in its place on Card 8.

Card 9 - This "card" really consists of 2 to 6 cards, depending on how complicated a description of the shock wave shape is used. Some detail on how to choose a shock shape is given in Appendix III. They are:

1. Card 9A (one card) - NEQU is the number of equations of the form:

$X = A(1) + A(2)R + A(3)R^2 + \dots + A(11)R^{10}$, which will be used to describe the shock wave shape. * NDEG is the highest exponent of R which has a non-zero coefficient. One value of NDEG must be entered on this card for each equation, the first value corresponding to the equation which is valid at the axis of symmetry.

2. Card 9B (one card for each equation) - Each of these cards contains the coefficients A(1), A(2), etc. for one equation. The number of coefficients is always one greater than the value of NDEG which corresponds to that equation. The coefficients for the equation which is valid at the axis of symmetry, go on the first of these cards. Note that symmetry requires A(2) to be zero for this one equation.

3. Card 9C (one card if more than one equation is used-otherwise omit) - This card lists the R values at the points where the various equations are tangent to each other (RDIV), starting with the tangency point nearest the axis of symmetry. The number of values of RDIV is always one less than NEQU.

Card 10 - This "card" really consists of 4 to 10 cards (or more), depending on the complexity of the body shape and on the method chosen to express the input information at the body. **

*As many as six equations may be used, although usually only one is needed.

**The present discussion will be limited to the usual transonic solution to be used in obtaining an unsteady flow field solution. By this is meant the solution which results when Kense (1) through Kense (10) on card 2 are set equal to zero.

1. Card 10A (one card) - As many as six equations may be used to describe the body shape. NCURVE is the number which will actually be used. In the usual unsteady solution for a spherically blunted body, only one equation (that of a circle) will be needed. NTST indicates which of the coefficients in the equation:

$$r = \sum_{n=1}^6 A(n) \left[x - A(12) \right]^{(n-1)} + A(7) \left[x - A(12) \right]^{A(8)}$$

$$+ A(9) \sqrt{\left[A(10) \right]^2 - \left[x - A(11) - A(12) \right]^2}$$

will be punched on Card 10B. If NTST is set equal to minus 1, a number must appear on Card 10B for each of the 12 coefficients. * If NTST is set equal to zero, only A(9) through A(12) are to be punched on card 10B. The latter case is convenient for the spherically blunt-blunted body. One value of NTST must appear for each equation to be used.

2. Card 10B (one card for each equation) - This card must contain the twelve coefficients A(n) if NTST was set equal to -1, or the four coefficients A(9) through A(12) if NTST was set equal to zero. The latter case is equivalent to setting A(1) through A(8) equal to zero. The coefficients of the equation which is valid nearest the stagnation point must appear on the first of Cards 10B.

3. Card 10C (one card if more than one equation is used to describe the body, otherwise omit) - This card contains the x values (CRV) at the intersections of the

*Zero may of course be entered for any of these coefficients.

various equations used to describe the body surface. There will always be one less value of CRV than the number of equations used. In the usual unsteady flow field, only one curve is needed in the transonic region, and Card 10C will be omitted.

4. Card 10D - This card will contain the values of pressure at the body. The number of entries on this card must be equal to NUMPTS. (See Card 7.)

5. Card 10E - This card will contain x values corresponding to the points at which pressures are given on card 10D. Again, the number of entries on this card must be equal to NUMPTS.

Cards 10F through 10H will be used only if KENSE (3) is not set equal to zero. (See Card 2.) They will be omitted in the usual unsteady flow field solution, and, therefore, won't be discussed here.

B. POINTED CONE PROGRAM. (ZERO YAW AND UNSTEADY)

Card 1 - The monitor FFCASE must appear on the first data card. * The remainder of this card (columns 7 through 72) may be used for any title the user desires to have printed out with the solution.

Card 2 - IDPROG identifies which link on the chain tape will be used. For the pointed cone solution, IDPROG must be set equal to 3. Kense (1) determines whether or not the unsteady solution will be carried out, and whether or not a binary

*The letters FFCASE must occupy columns 1 through 6 of the first card. No spaces should appear between them.

tape is to be written (on tape drive A6). If an unsteady solution is to be done in the supersonic region, Kense (1) must be set equal to minus one. * If no solution is to be done in the supersonic region, Kense (1) should be set equal to plus one. Kense (2) through Kense (10) have no meaning in this program, and should be set equal to zero.

Card 3 - Test (1) through Test (5) are the convergence criteria used in various iterations in this program. If zeros are punched on Card 3 for these terms, certain preset values will be used in the solution. These preset values have been found to give good results in the past. It is recommended that they be used in production runs. Test (6) determines when a point is too close to the body surface for use as input in the supersonic programs (i. e. , when a point should be omitted from the binary tape which is written when Kense (1) is set equal to -1). Test (6) should be set equal to approximately .2 of the spacing DLTOM (See Card 6) converted to radians (e. g. set Test (6) equal to $.0035 \times \text{DLTOM}$)**. Tests (7) through (10) have no meaning in this program and should be set equal to zero.

*If Kense (1) is set equal to minus one, the solution will be written on a binary tape on tape drive A6. The tape should, of course, be retained for use in the unsteady solution in the supersonic region.

**Whenever a point is omitted from the binary tape, the perturbations at that point are also omitted from the pointed cone printout. The zero yaw values at that point, however, appear on the printout. Care must be taken to omit them when preparing input for the zero yaw supersonic solution.

Card 4 - The free stream pressure and density, the flight velocity and Mach number, in that order, must be entered on Card 4. The units of the first three quantities are lb/ft², slugs/ft³, and ft/second, respectively.

Card 5 - The term DEL determines whether an axisymmetric or two-dimensional solution is to be carried out. (DEL = 0 means a two-dimensional solution; DEL = 1 means an axisymmetric solution). The real gas index, RGI, is set equal to 1 if a real gas solution is desired, equal to 0 if an ideal gas solution is desired. ALT is the altitude. It is merely printed out on the output sheet, and is not used in the computations.

Card 6 - OMBODY is the cone half angle in degrees. DLTOM is the increment in angular position where results will be provided; i. e. results will be printed out at the shock wave, $\omega = \omega_s$, and at $\omega = \omega_s - \text{DLTOM}$, $\omega = \omega_s - 2 \times \text{DLTOM}$, etc. It is suggested that a value of DLTOM be obtained by estimating the shock angle (e. g., Chart 5 of NACA Report 1135), and then choosing DLTOM to give 10 to 15 increments between the body and the shock wave. NMAX determines how many of the coefficients in the series:

$$p = p_0 + p_{1,0} \alpha + p_{1,1} \left(\frac{L}{V_\infty}\right) \dot{\alpha} + p_{1,2} \left(\frac{L}{V_\infty}\right)^2 \ddot{\alpha} + \dots + p_{1,n} \alpha^n \left(\frac{L}{V_\infty}\right)^n$$

$$+ p_{2,0} q \left(\frac{L}{V_\infty}\right) + p_{2,1} \dot{q} \left(\frac{L}{V_\infty}\right)^2 + \dots + p_{2,n} q^n \left(\frac{L}{V_\infty}\right)^{n+1}$$

will be calculated. For example, if the term $C_{M\dot{\alpha}}$ is desired (but not $C_{M\ddot{\alpha}}$), NMAX should be set equal to 1.

Card 7 - RIDEAL is the real gas constant for air, preset at 1716.5 ft lb/slug
°R. GIDEAL is the ratio of specific heats used in ideal gas calculations, preset
at 1.4. ZIDEAL is the compressibility factor used in ideal gas calculations, pre-
set at 1.0. The remaining terms, RHOO, RHOOO and GREAL, are used in con-
nection with the thermodynamic tables that have been provided with this solution.
The preset values of these three terms should always be used (unless a different
set of thermodynamic tables is provided in the future). The preset value of any of
the above terms will be used, if zero is entered in its place on Card 7.

C. SUPERSONIC PROGRAM (ZERO YAW)

Card 1 - The monitor FFCASE must appear on the first data card. * The re-
mainder of this card (columns 7 through 72) may be used for any title the user
desires to have printed out with the solution.

Card 2 - IDPROG identifies which link on the chain tape will be used. For the
supersonic solution, IDPROG must be set equal to 5. For the usual supersonic
solution to be used in obtaining an unsteady solution for a spherically blunted body
(input line information is obtained from the Transonic solution), Kense (1) through
Kense (10) should be set equal to zero. For a pointed body (input line information
is obtained from the pointed cone solution), Kense (2) should be set equal to + 1,
and the remaining 9 Kenses should be set equal to zero. In some cases where a
supersonic solution has been carried to a certain point in the field, and the appropriate

*The letters FFCASE must occupy columns 1 through 6 of the first card. No
spaces should appear between them.

binary tape has been saved (see remarks on Card 3), the solution may be re-started where it was left off,* and carried on downstream from there. If this is the case, Kense (6) should be set equal to -1. The other Kenses should be treated as indicated above.

Card 3 - NBTPW is the "logical" number of the tape drive on which the standard binary tape will be written. It is preset to use tape drive B6 if NBTPW is set equal to zero. If the unsteady solution is to be used, this tape must be retained and used as input in that solution. NBTPR is the "logical" number of the tape drive on which the binary tape from the previous zero yaw supersonic solution must be placed if a "restart" run is being done (i. e. , if Kense (6) has been set equal to -1). The normal zero-yaw supersonic solution for use in obtaining an unsteady flow field solution will not be a restart run, and NBTPR should be set equal to zero. However, whenever the restart option is used, NBTPR must be assigned a different tape drive number than NBTPW (e. g. , set NBTPW equal to 26 and NBTPR to 16, then place a blank tape** on drive B6, and the tape with the beginning of the zero-yaw solution on drive A6***).

Card 4 - Test (1) through test (7) are the convergence criteria used in various iterations in this program. Test (5) and test (6) should each be set equal to 1E-05

*It may also be started up anywhere upstream of the point where it was left off.

See remarks on Card 9.

**This tape will then be input to the unsteady solution.

***See Section 7.1.3 of this report.

for the usual unsteady solution. All other terms on this card (Test (1) through Test (4) and Test (7) through Test (10)) should be set equal to zero in order to call for preset values.

Card 5 - The free stream pressure, density, flight velocity and Mach number, in that order, must be entered on Card 5. The units of the first three quantities are lb/ft^2 , slugs/ft^3 , and ft/second , respectively.

Card 6 - The term DEL determines whether an axisymmetric or two-dimensional solution is to be carried out. (DEL = 0 means a two-dimensional solution; DEL = 1 means an axisymmetric solution.) The real gas index, RGI, is set equal to 1 if a real gas solution is desired, equal to zero if an ideal gas solution is desired. ALT is the altitude. It is merely printed on the output sheet, and is not used in the computations.

Card 7 - NUMPTS is the number of points on the "starting" line* at which input information is given, AKAPSH is the shock angle (in radians) at the point where the "starting" line meets the shock wave. The terms AKDEL and AJFRZN should be set equal to zero in the usual (chemical equilibrium) calculations done with an unsteady flow field solution.

Card 8 - RIDEAL is the real gas constant for air, preset at 1716.5 ft lb/slug °R. GIDEAL is the ratio of specific heats used in ideal gas calculations, preset at 1.4. ZIDEAL is the compressibility factor used in ideal gas calculations, preset

*When doing a "restart" (Kense (6) = -1), NUMPTS should be the number of points on the original starting line.

at 1.0. The remaining terms, RHOO, RHO00 and GREAL are used in connection with the thermodynamic tables that have been provided with this solution. The preset values of all these terms should always be used (unless a different set of thermodynamic tables is provided in the future). The preset value of any of the above terms will be used, if zero is entered in its place on Card 8.

Card 9 - XEND is the x value at the end of the body. XTERM should always be set equal to zero. MSTART will be non-zero only in a restart run (in a run where Kense (6) has been set equal to -1). It specifies the M number of the last left hand characteristic line along which the results of the original run are to be used. Calculations along all characteristic lines downstream of it will be done as usual. NTERM should always be set equal to zero. NEXP is the number of expansion corners present on the body. For the unsteady flow field calculation it must be set equal to zero.

Card 10 - This "card" really consists of as many cards as there are expansion corners. It gives information about the expansion corners. For the unsteady flow field solution it must be omitted.

Card 11 - This "card" really consists of 4 to 10 cards (or more), depending on the complexity of the body shape.

1. Card 11A (one card) - As many as six equations may be used to describe the body shape. NCURVE is the number which will actually be used. NTST indicates which of the coefficients in the equation:*

*The origin of the x, r coordinate system should be placed at the stagnation point.

$$r = \sum_{n=1}^6 A(n) \left[x - A(12) \right]^{(n-1)} + A(7) \left[x - A(12) \right]^{A(8)}$$

$$+ A(9) \sqrt{\left[A(10) \right]^2 - \left[x - A(11) - A(12) \right]^2}$$

will be punched on Card 11B. If NTST is set equal to -1, a number must appear on Card 11B for each of the 12 coefficients. * If NTST is set equal to zero, only A(9) through A(12) are to be punched on Card 11B. The latter case is convenient for the spherically blunted body. One value of NTST must appear for each equation to be used.

2. Card 11B (one card for each equation) - This card must contain the twelve coefficients A(n) if NTST was set equal to -1, or the four coefficients A(9) through A(12) if NTST was set equal to zero. The latter case is equivalent to setting A(1) through A(8) equal to zero. The coefficients of the equation which is valid nearest the stagnation point must appear on the first of cards 11B.

3. Card 11C (one card if more than one equation is used to describe the body, otherwise omit) - This card contains the x-values (CRV) at the intersections of the various equations used to describe the body surface. There will always be one less value of CRV than the number of equations used.

*Zero may, of course, be entered for any of these coefficients.

Card 12 - There will be one Card 12 for each point on the "starting" line (NUMPTS). The first of these cards will provide data at the shock, the last one at the body. Each card will contain x, r, THC (flow angle in radians), RHOC (density in slugs/ft³), and PC (pressure in lb/ft²). The pressure entry on each card should be followed by a comma and a dollar sign. The information for Card 12 will come from the transonic or pointed cone solution. When doing a "restart", no card 12 is needed.

D. UNSTEADY SUPERSONIC PROGRAM

Card 1 - The monitor FFCASE must appear on the first data card. * The remainder of this card (columns 7 through 72) may be used for any title the user desires to have printed out with the solution.

Card 2 - IDPROG identifies which link on the chain tape will be used. For the unsteady supersonic solution, IDPROG must be set equal to 7. Kense (1) should be set equal to zero for a blunt body, and equal to -1 for a pointed body, Kense (2) should always be set equal to zero. If Kense (3) is set equal to zero, values of all the flow field parameters at all points where calculations were performed will be printed. If Kense (3) is set equal to 1, only force and moment coefficients will be printed. Kense (4) through Kense (10) have no meaning, and should be set equal to zero.

*The letters FFCASE must occupy columns 1 through 6 of the first data card. No spaces should appear between them.

Card 3 - XEND is the x value at the end of the body. The same XEND value should be used here as on Card 9 of the zero yaw supersonic solution. XTERM should always be set equal to zero. ELP is the length used in non-dimensionalizing α and q in part of the output (See Appendix III). It may be set equal to any desired length for the blunt body. For a pointed body, it must be compatible with the tape input from the pointed cone solution (i.e., it must be set equal to the tangent of the cone half angle which was used in the pointed cone solution).

Card 4 - MSTART is the M-number of the left running characteristic line on which calculations are to begin. Since unsteady flow field computations must start at least two left running characteristic lines downstream of the body point on the zero yaw starting line, a reasonable way to determine a value for MSTART is to find the M-value of the zero yaw starting line body point (from the zero yaw supersonic printout), and add 8 to it. NUMJ should always be set equal to 2. NMAX governs the point where the series:

$$p = p_0 + p_{1,0} \alpha + p_{1,1} \left(\frac{L}{V_\infty} \right) \dot{\alpha} + p_{1,2} \left(\frac{L}{V_\infty} \right)^2 \ddot{\alpha} + \dots$$

is to be truncated. It should always be set equal to 1 for a blunt body. For a pointed body it must be set equal to the value of NMAX used on Card 6 of the input to the pointed cone program. NTERM, NT2, NT3, NT4,

NT5 should always be set equal to zero.*

Card 5 - NBTPW and NBTPR govern tape assignments, and should always be set equal to zero in order to use the preset values.

Card 6 - The terms on this card have the same meaning as the corresponding terms on Card 8 of the zero yaw supersonic program. They should all be set equal to zero in order to use the preset values (unless an ideal gas with $\gamma \neq 1.4$ is being investigated).

Card 7 - This card is to be used in a blunt body solution only. It must be omitted when investigating a pointed body. For a blunt body NUMPTS must have the same value as on Card 7 of the zero yaw supersonic solution. X0 is the distance from the origin of the coordinate system to the center of the spherical nose. In the usual solution, the origin will be placed at the stagnation point, so that X0 will be equal to the nose radius.

Card 8 - At the end of the printout for the unsteady supersonic program, a Table of C_{N_α} , C_{M_α} , $(C_{M_\alpha} + C_{M_q})$ and $(C_{N_\alpha} + C_{N_q})$ vs X_{cg}/L is provided for bodies with any desired R_N/R_B ratio, or bodies of any length desired. Cards 8 through 11 determine how many tables (and how many lines in each table) are to be printed out. DELX is the increment in X_{cg}/L at which the information is to be

* There is one exception to this rule. Occasionally the zero yaw supersonic solution will not converge at a shock point, and the solution downstream of the right running characteristic line which passes through that shock point will not be valid. In that case NTERM should be set equal to the N-value of the right hand characteristic line which emanates from the shock point immediately upstream of the bad shock point.

printed. R_N is the nose radius of the (blunt) body. For a pointed body, zero should be used. NRTST is the number of R_N/R_B ratios for which tables are to be provided. A maximum of ten is permitted. NXINT is the number of body lengths for which tables are to be provided. A maximum of ten is permitted. * NT5 is a tape assignment which should be set equal to zero in order to use the preset value. In no case should NT5 on Card 8 be given a different value than it was given on Card 4.

Card 9 - RTEST are the values of R_N/R_B for which tables are desired. The number of entries on this card must be equal to NRTST. The largest value of RTEST must appear first, followed by the next largest, etc. If NRTST on Card 8 has been set equal to zero, F\$ should appear on Card 9.

Card 10 - XINT are the values of x for which tables are desired. ** The number of entries on this card must be equal to NXINT. The smallest value of XINT must appear first, followed by the next smallest, etc. If NXINT on Card 8 has been set equal to zero, F\$ should appear on Card 10.

Card 11 - XMAX is the largest value of x_{cg}/L desired in the tables. Only one term will appear on this card. All the tables in a given computer run will then terminate at the same value of x_{cg}/L .

*The option to specify body lengths rather than R_N/R_B , is provided to permit calculations for pointed cones where R_N/R_B is always zero, and for bodies with cylindrical sections where R_N/R_B is ambiguous.

**If the origin of the coordinate system has been placed at the stagnation point, the values of XINT will be equal to the body lengths of interest.

Summary of Tape Assignments - Since as many as 7 special tape drives may be needed in the unsteady flow field solution, a summary of tape assignments is given below. It is assumed that NBTPW, NBTPR, NT2, NT3, NT4 and NT5 have been set equal to zero.

<u>Tape drive</u>	<u>Description of Tape</u>
A4	Chain Tape
B7	Atmospheric Tables (real gas solution only)
A6	Output of pointed cone program (pointed body only)
A6	Scratch tape (blunt body only)
A7, A8, B5	Scratch Tapes
B6	Output of zero yaw supersonic program.

APPENDIX II

A GUIDE TO READING THE COMPUTER PRINTOUTS

The meaning of most of the information which appears in the various computer printouts is self evident. This guide provides information on some areas (such as the units used) where some additional comment is necessary. It is assumed that the user will have the various printouts on hand when reading this guide.

A. ZERO YAW SOLUTIONS - GENERAL

The following units are used in all zero yaw solutions:

RHO = density	- slugs/ft ³
P = pressure	- lb/ft ²
V = VEL = velocity	- ft/sec
angles	- radians (unless marked otherwise)
(Th = THETA = flow direction, SIGMAC = shock angle, both measured from the axial)	
T = TEMP = TEMPERATURE	- degrees Rankine
S/R = entropy	- non-dimensional (S/R) where R is the gas constant
PSI = stream function	- slugs/ft ² sec (e. g. upstream of the shock wave the stream function is defined

$\psi = \left(\frac{r}{2}\right)^{\delta} r \rho_{\infty} V_{\infty}$, where r is non-dimensionalized against any reference length chosen by the user).

X and R = axial and radial coordinates

non-dimensionalized against a reference length chosen by the operator (in preparing the coefficients of the body shape).

The first two pages of all three zero yaw solutions (transonic, pointed cone and supersonic) show the title and values of the constants RIDEAL, GIDEAL, ZIDEAL, RHOO, RHOOO, GREAL. (See Cards 1 and 8, Appendix I.) The remainder of the printout for each solution must be explained separately.

1. The Zero Yaw Transonic Solution

The third page of the transonic solution indicates the coefficients of the shock equations which were used (Card 9, Appendix I). The first column represents the curve closest to the axis.

The fourth page prints the free stream conditions (which were also a part of the input). Altitude is in any units the user chose when punching Card 6, Appendix I.

The fifth page prints the body curve coefficients punched on Card 10A and B (Appendix I). If more than one curve is used, the X and R values at the intersections are also printed, as well as the flow direction at each intersection, computed from the upstream and downstream body curves.

The printout of the solution begins on page 6. The paragraph headed $M = 1$ represents the body streamline. The next paragraph ($M = 2$) represents the next streamline

from the body, etc. The line $N = 0$ in each paragraph provides the flow field parameters immediately inside the shock wave on each streamline. The lines $N = 1$, $N = 2$ etc, provide the flow field solution at the intersections of the streamlines with the normals (i. e. with lines emanating from the various points at the body where pressures were given (Cards 10D and E, Appendix I), and drawn in such a way that they are normal to each streamline they cross). Since entropy and stream function are constant along streamlines in the shock layer, a single value of each is printed at the top of each paragraph. The shock angle at the point where the streamline crosses the shock wave is also printed at the top of each paragraph.

Interspersed with the paragraphs of information on streamlines, are the "continuity points". These are the points where the normals intersect the shock wave. At these points, a comparison is made of the pressures and X values at the shock computed from the assumed shock wave shape with the corresponding values computed from the pressure distribution (at the body) and the conservation laws. The headings XS, RS, and PS refer to values computed from the given shock shape, while XC, RC and PC refer to the values computed through use of the conservation laws. The same information on the continuity points is repeated near the end of the printout, under the heading "Summary of Continuity Points". This summary is useful in the process of converging the transonic solution. (See Appendix IV.)

The final part of the transonic printout is a "Summary of Normals" which emanate from the body at the two points* immediately downstream of the sonic point. This

* Points where pressure was provided on Card 10D (Appendix I).

summary of normals is convenient in preparing input for the zero yaw supersonic solution. All information in this summary also appears in the earlier part of the printout.

2. The Zero Yaw Supersonic Solution

The third page of the supersonic solution prints out the body curve coefficients which were punched on Cards 11A and B (Appendix I). If more than one curve is used, the X and R values at the intersections are also printed, as well as the flow direction at each intersection, computed from the upstream and downstream body curves.

The free stream conditions appear on page 4. Altitude is in any units chosen by the user when punching Card 6.

The next six pages provide a tabulation of properties downstream of an oblique shock against shock angle for the given free stream conditions. The shock angle is given in degrees in column 1 and in radians in column 2.

The eleventh page is headed "Test to Check Conservation Laws". If all the (internal) iterations in the program converged properly, all numbers on this page (except those in the first column) will be equal to 1.0 to at least five significant places.

Page 12 prints the information provided on input Card 12, and provides some additional flow field values computed from that input. Scanning this page (particularly the Mach number and entropy columns) for "smooth" variations, generally provides an excellent check for input errors on Card 12.

The printout of the solution begins on page 13. The first paragraph is a repetition of page 12*. The following paragraphs provide the flow field parameters along left running Mach lines, each paragraph providing data downstream of the preceding paragraph. The shock angle on each Mach line appears at the end of the paragraph. The values MC (column 1) are constant along left running Mach lines, and NC (column 2) are constant along right running Mach lines. The last column in each paragraph (NITER) indicates the numbers of iterative cycles performed in the computation. It has no physical significance.

At the end of the printout are two summaries, which provide information at the body surface and at the shock wave. In the "Summary of Body Data" the following conventions are used:

$$CP = \text{pressure coefficient} = \frac{P - P_{\infty}}{1/2 \rho_{\infty} V_{\infty}^2}$$

CPMAX = stagnation point pressure coefficient

$$CX = \text{axial force coefficient} = \frac{F_x}{1/2 \rho_{\infty} V_{\infty}^2 A_{\text{base}}}$$

PT = stagnation point pressure in lb/ft²

The "Summary of Shock" data provides the shock shape, and is self explanatory.

When the restart option is used (See Card 2, Appendix D), the sequence of the information on the first few pages of the output will be somewhat different (e.g. the values

* This repetition occurs only when a pointed body is being investigated. It is not there for a blunt body.

downstream of an oblique shock will not print out). Basically, however, the output will be the same as that described above, so that the user will have no difficulty in recognizing it.

3. The Zero Yaw Pointed Cone Solution

Page 3 of the pointed cone solution prints the free stream conditions.

The next two or three pages provide a history of the iterative process used in obtaining the solution. The number of pages used in this Section of the output depends on the number of iterative cycles performed before reaching convergence. The iterative process consists of choosing a shock angle and computing the corresponding cone (body) angle, then choosing a new shock angle and repeating the process until the computed body angle is equal to the desired value. For each step of the iteration the shock angle and body angle, as well as the flow field properties just downstream of the shock and at the body are printed*. In general, this Section of the printout will not be of interest to the user.

The next page after this history, prints the zero yaw solution along conical surfaces (at angle Ω)**, with vertex at the body vertex. The top line of the printout provides values immediately downstream of the shock wave, the bottom line provides values at the body surface, and the remaining lines provide values elsewhere in the shock layer. The X and R values are along a left running Mach line emanating from the body at $x=1$.

* The values printed in the columns marked X and R have no significance.

** Note that for a pointed cone the flow field values are constant along these conical surfaces.

It is this tabulation which must be used as input to the zero yaw supersonic program (when the pointed cone represents the nose of a more complex body).

The remainder of the printout is the unsteady flow solution for a pointed cone, and will be discussed in Appendix II. B.

B. THE UNSTEADY SOLUTIONS - GENERAL

Printouts of unsteady flow field solutions will result from two programs: the pointed cone program, and the unsteady supersonic program. It should be noted that the normalizing factors are not the same in the two programs, nor are the velocity components taken in the same coordinate systems. The printouts are described separately below.

1. The Unsteady Pointed Cone Program

The unsteady flow field solution for the pointed cone is performed by the same program as the zero yaw pointed cone solution, and the results are printed out immediately after the zero yaw results. The j and n combination is indicated at the top of each page. The force and moment coefficients which appear in the next line are defined by:

$$\text{Force Coeff} = \frac{\left[F_{\text{Normal}} / \frac{1}{2} \rho_{\infty} V_{\infty}^2 A_{\text{Base}} \right]}{\left[\alpha_{j,n} \left(\frac{D_{\text{Base}}}{2V_{\infty}} \right)^m \right]}$$

$$\text{and Moment Coeff} = \frac{\left[M_z / \frac{1}{2} \rho_{\infty} V_{\infty}^2 D_{\text{Base}} A_{\text{Base}} \right]}{\left[\alpha_{j,n} \left(\frac{D_{\text{Base}}}{2V_{\infty}} \right)^m \right]}$$

where: $m = n$ for $j = 1$

$= n+1$ for $j = 2$

and: $\alpha_{1,0} = \alpha$, $\alpha_{1,1} = \dot{\alpha}$, $\alpha_{1,2} = \ddot{\alpha}$, etc.

$\alpha_{2,0} = q$, $\alpha_{2,1} = \dot{q}$, $\alpha_{2,2} = \ddot{q}$, etc.

All results are for rotation about the body vertex.

The term DELTA which appears on the next line is used in the denominator in several divisions carried out by the program. If it ever is equal to zero, it will serve as an error signal. Otherwise, it is of no interest to the user. OMEGA PRIME is the perturbation in shock angle. The normalization used in this term, as well as in all the perturbation terms printed below it, is indicated in the typical expansion

$$\begin{aligned} w_s = & w_{s_0} + w_{s_{1,0}} \alpha \cos \varphi + w_{s_{1,1}} \left(\frac{R}{V_\infty} \right) \dot{\alpha} \cos \varphi + w_{s_{1,2}} \left(\frac{R}{V_\infty} \right)^2 \ddot{\alpha} \cos \varphi + \dots \\ & + w_{s_{2,0}} \left(\frac{R}{V_\infty} \right) \cos \varphi q + w_{s_{2,1}} \left(\frac{R}{V_\infty} \right)^2 \cos \varphi \dot{q} + w_{s_{2,2}} \left(\frac{R}{V_\infty} \right)^3 \cos \varphi \ddot{q} \end{aligned}$$

$$\text{where } R = \sqrt{x^2 + r^2}$$

It should be noted that the velocity components U, V and W are in the R, θ and φ directions respectively, in the spherical coordinate system shown in Figure 3, Section 4. HP is the perturbation in stagnation enthalpy.

The final page of the pointed cone printout summarizes the force coefficients (N) and moment coefficients (M). The usual force and moment coefficients (for rotation

about the body vertex) are found in this summary for $j = 1$ (α) and $J = 2$ (q). The values included for $j = 3$ are defined:

$$N_{3,n} = N_{1,n} / \tan \theta_{\text{Body}}, \quad M_{3,n} = -M_{1,n} / \tan \theta_{\text{Body}}$$

they will not be of interest to most users.

2. The Unsteady Supersonic Program

The unsteady supersonic program printout begins with the title and with RIDEAL, GIDEAL, etc, like the zero yaw solutions. This is followed by the starting line information. In the case of the blunt body, this information is computed in the unsteady flow program (using the zero yaw solution as input). The location of this starting line is determined by the choice of MSTART (Card 7, Appendix D). In the pointed body case, the starting line information represents the input from the pointed cone solution (on binary tape), after conversion to the cylindrical coordinate system (Figure 9, Section 4), and after a change in non-dimensionalizing constants. Note that each point on the starting line (as well as each point elsewhere in the field) requires as many lines of output as there are j, n combinations. The starting line tabulation can be distinguished from the data that follows it, by the absence of the columns headed MC and NC.

The major part of the printout consists of paragraphs*, the first line of which prints the value of j and n , x and R at a point on the body, force and moment (about

* There are $j \times n$ paragraphs for each left running characteristic line.

$x = x_0$), the force and moment coefficients and center of pressure location*. The next line provides the perturbation in shock radius and in the tangent to the shock angle ($dR_s/dX = \tan \sigma$). The remainder of each paragraph provides the perturbations in the flow field parameters along a left running characteristic line. It is possible to print out only the first line of each of these paragraphs by setting Kense (3) = 1 (Card 2, Appendix D).

The information of greatest interest to most users of this solution, appears at the end of the printout. It is tabulated in two forms. The first is headed "Summary of Body Data", and the location of the reference axis for that tabulation is indicated. This tabulation simply summarizes** C_{n_α} , C_{M_α} , $(C_{N_\alpha} + C_{N_q})$ and $(C_{M_\alpha} + C_{M_q})$; center of pressure location, and the nose to base radius ratio (for a body with base at the given x value).

* These coefficients, etc. are those which would exist if the body ended at the given x value. In the solution for the blunt body, several characteristic lines stretch from the starting line to the shock, and do not include a body point. For these characteristic lines, the first line of the paragraph will give only the j and n values, but no values for force and moment coefficients, etc.

** The non-dimensionalizing length in the summary is not the same as the non-dimensionalizing length in the body of the printout. See the definitions at the end of this Section.

The second tabulation is interspersed between the lines of the first. It is headed* "Summary of Force and Moment Coefficients for $R_N/R_B = \dots$." In it, $C_{N\alpha}$, $C_{M\alpha}$, $(C_{N\dot{\alpha}} + C_{Nq})$ and $(C_{M\dot{\alpha}} + C_{Mq})$ are tabulated against center of gravity, for the radius ratio or body length indicated in the heading. The user has the option of printing this tabulation for as many as ten radius ratios and ten body lengths (See Cards 8, 9, 10 Appendix I).

The coefficients printed in the unsteady supersonic flow field program are defined as follows:

In the body of the printout

$$CN(1,0) = C_{N\alpha} = \frac{\partial (F_N/A_{Base} q_\infty)}{\partial \alpha} \quad \text{per radian}$$

$$CM(1,0) = C_{M\alpha} = \frac{\partial (M_Z/D_{Base} A_{Base} q_\infty)}{\partial \alpha}$$

$$CN(2,0) + CN(1,1) = C_{Nq} + C_{N\dot{\alpha}}$$

$$= \frac{\partial (F_N/A_{Base} q_\infty)}{\partial \left(\frac{q ELP}{V_\infty} \right)} + \frac{\partial (F_N/A_{Base} q_\infty)}{\partial \left(\frac{\dot{\alpha} ELP}{V_\infty} \right)}$$

* If the Table is for a requested body length, the heading will Be "Summary of Force and Moment Coefficients for XINT = ...".

$$CM(2,0) + CM(1,1) = C_{M_q} + C_{M_\alpha}$$

$$= \frac{\partial \left(\frac{M_Z / A_{Base} D_{Base} q_\infty}{\left(\frac{q}{V_\infty} \right) ELP} \right)}{\partial \left(\frac{q}{V_\infty} \right)} + \frac{\lambda \left(\frac{M_Z / A_{Base} D_{Base} q_\infty}{\left(\frac{\dot{\alpha}}{V_\infty} \right) ELP} \right)}{\partial \left(\frac{\dot{\alpha}}{V_\infty} \right)}$$

In the two tabulations at the end of the printout

$$CN = C_{N_\alpha} = \frac{\partial \left(\frac{F_N / A_{Base} q_\infty}{\partial \alpha} \right)}{\partial \alpha} \quad \text{per radian}$$

$$CM = C_{M_\alpha} = \frac{\partial \left(\frac{M_Z / A_{Base} L q_\infty}{\partial \alpha} \right)}{\partial \alpha}$$

$$\begin{aligned} (CNA + CNQ) &= \left(C_{N_\alpha} + C_{N_q} \right) \\ &= \frac{\partial \left(\frac{F_N / A_{Base} q_\infty}{\left(\frac{\dot{\alpha} L}{2 V_\infty} \right)} \right)}{\partial \left(\frac{\dot{\alpha} L}{2 V_\infty} \right)} + \frac{\lambda \left(\frac{F_N / A_{Base} q_\infty}{\left(\frac{q L}{2 V_\infty} \right)} \right)}{\partial \left(\frac{q L}{2 V_\infty} \right)} \end{aligned}$$

$$\begin{aligned} (CMA + CMQ) &= \left(C_{M_\alpha} + C_{M_q} \right) \\ &= \frac{\partial \left(\frac{M_Z / A_{Base} L q_\infty}{\lambda \left(\frac{\dot{\alpha} L}{2 V_\infty} \right)} \right)}{\partial \left(\frac{\dot{\alpha} L}{2 V_\infty} \right)} + \frac{\partial \left(\frac{M_Z / A_{Base} L q_\infty}{\partial \left(\frac{q L}{2 V_\infty} \right)} \right)}{\partial \left(\frac{q L}{2 V_\infty} \right)} \end{aligned}$$

where: L = body length

and q_∞ = free stream dynamic pressure.

APPENDIX III

CAPABILITIES AND LIMITATIONS OF THE UNSTEADY FLOW FIELD SOLUTION IN COMPUTING STATIC AND DYNAMIC FORCE AND MOMENT COEFFICIENTS

This Appendix presents a brief list of the capabilities and limitations which should be kept in mind when using the unsteady flow field solution. The limitations on shape listed are those encountered in obtaining an unsteady (and small yaw) solution, and do not necessarily apply when only a zero yaw solution is needed.

1. Capabilities

- Body Shapes:
1. Spherically blunted axisymmetric bodies, with no sharp expansion or compression corners. The afterbody must be tangent to the spherical nose downstream of the sonic line.
 2. Two-dimensional bodies with a right circular cylinder (axis perpendicular to flow) nose. The afterbody must be tangent to the cylindrical nose downstream of the sonic line.
 3. Axisymmetric pointed bodies with no sharp expansion or compression corners.
 4. Two-dimensional pointed bodies may be approximated by using a blunted two-dimensional body with very small r_N/r_B .

Gas: Any of the above shapes may be investigated using either real air, or an ideal gas with any desired specific heat ratio, γ

Altitude: Solutions may be carried out at any altitude where the assumption of continuum is valid. They may also be carried out at any (continuum) free stream conditions which might exist in a test facility.

Flight Velocity Real gas solutions have been carried out at flight velocities ranging from 4000 feet/second to 35,000 feet/second. It is possible to go to still higher velocities in some cases. (Ideal gas solutions can be carried out at much higher velocities.) Some difficulties should be expected in obtaining solutions on bodies which are very long compared to the nose diameter (blunt bodies) or to the conical nose (pointed bodies), particularly at low Mach numbers.

Outputs: 1. The quantities computed for blunted bodies (axisymmetric or) two-dimensional are:

$$C_{N_{\alpha}}, (C_{N_{\alpha}} + C_{N_q})$$

$$C_{M_{\alpha}}, (D_{M_{\alpha}} + C_{M_q})$$

2. The quantities computed for pointed axisymmetric bodies are:

$$C_{N_{\alpha}}, C_{N_{\alpha}^{\cdot}}, C_{N_{\alpha}^{\cdot\cdot}}, C_{N_{\alpha}^{\cdot\cdot\cdot}}, \dots$$

$$C_{M_{\alpha}}, C_{M_{\alpha}^{\cdot}}, C_{M_{\alpha}^{\cdot\cdot}}, C_{M_{\alpha}^{\cdot\cdot\cdot}}, \dots$$

$$C_{N_q}, C_{N_{\dot{q}}}, C_{N_{\ddot{q}}}, \dots$$

$$C_{M_q}, C_{M_{\dot{q}}}, C_{M_{\ddot{q}}}, \dots$$

3. In addition, the flow field variables and the perturbations in them (due to the unsteady motion) are computed at a large number of points in the flow field.

2. Limitations

Body Shapes: The unsteady solution cannot, at present, handle expansion corners or secondary shocks. It has been used successfully in a case where an expansion corner was approximated by an arc tangent to both of the surfaces which intersect in the (expansion) corner.

Timing:

1. The zero yaw transonic solution requires from 5 to 20 iterations on the input values (i. e., inputs must be submitted to the computer up to 20 times, waiting in each case for the output from one iteration to be returned before submitting the next input). The user should not expect to complete a zero yaw transonic solution in less than two weeks. This difficulty is usually overcome by making use of the zero yaw transonic solutions which have been solved during the past six years (usually an old solution is available adequately close to the desired free stream conditions).

2. The usual unsteady flow field solution is carried out in three steps:

- a. Zero yaw transonic solution or pointed cone solution
(depending on whether the body is pointed or blunt nosed)
- b. Zero yaw supersonic solution
- c. Unsteady supersonic solution

Output from step a. must be available in order to go on to step b. It is desirable (but not usually essential) to inspect the output of step b. before submitting the input for step c. If inputs for steps b and c are submitted to the computer simultaneously, the machine operator must be given special instructions for handling of magnetic tapes.

3. The pointed cone solution and the transonic solution (one iteration) can each be expected to use approximately one minute of computer time per run. The zero yaw supersonic solution and the unsteady supersonic solution, however, can be expected to take five to twenty minutes per run. The following factors tend to require longer running times:

- a. Real gas calculations (as opposed to ideal gas)
- b. Finer mesh grids
- c. Bodies which are long compared to the nose diameter (blunted bodies), or to the conical nose (pointed bodies)

- d. Retaining many terms in the expansion for force coefficients and moment coefficients (e. g., $C_N = C_{N_\alpha} \alpha + C_{N_{\dot{\alpha}}} \dot{\alpha} + \dots + C_{N_q} q + C_{N_{\dot{q}}} \dot{q} + \dots$).

This last factor applies to the unsteady flow field solution only.

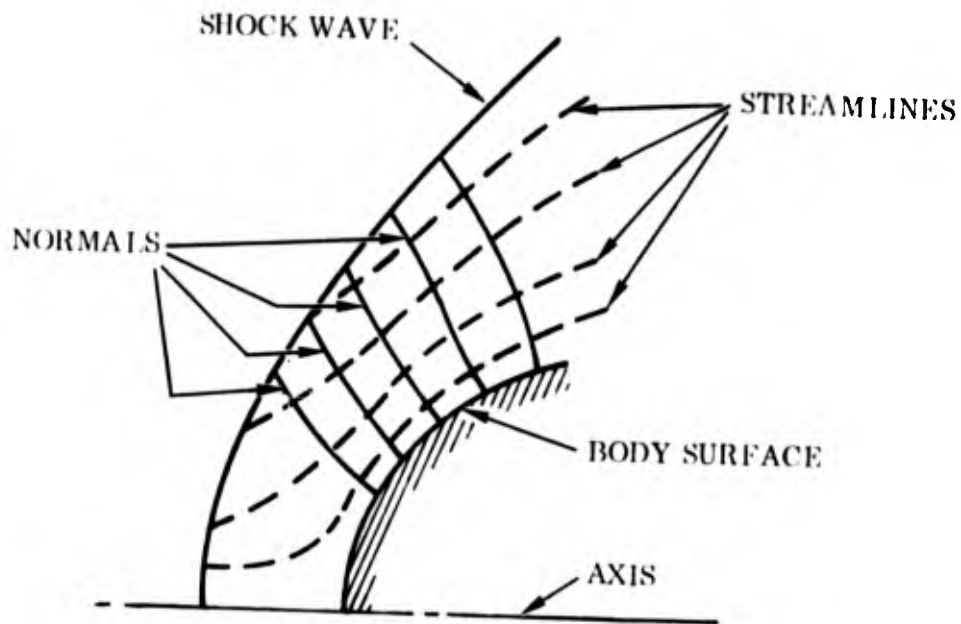
APPENDIX IV

GUIDE FOR CONVERGING THE ZERO YAW TRANSONIC SOLUTION

The zero yaw transonic solution requires the user to participate in an iterative process in the sense that he must make a first estimate of both the shock shape and the pressure distribution at the body; and then make use of the information provided by the computer solution to make repeatedly better estimates of these values, i. e. , to approach convergence. A method for making these estimates will be indicated in this Appendix.

The initial estimates are probably the easiest to make. They can be based on any approximate knowledge of the flow field the user may have. For instance, a Newtonian pressure distribution could be used at the body surface. It is, however, suggested that the available solutions given in Appendix V be used for this purpose. A good first estimate of the shock shape can be gotten simply by using the shock shape from an available solution which was carried out at free stream conditions which are similar to the free stream conditions for the new solution. A first estimate of pressure distribution at the body should be obtained by scaling the pressure distribution from the same available solution, by the ratio of free stream pressures.

Obtaining improved estimates from the previous iteration is a much more complex matter. Agreement between the estimated shock shape and the computed shock points is, of course, a necessary condition, and a rule of thumb has been developed for improving this agreement. Before stating this rule of thumb, it is necessary to explain the coordinate grid on which the transonic solution is carried out. The grid consists of approximately 40 to 60 streamlines, and eight to ten lines which are normal to the



streamlines where they intersect. One of these normals emanates from each point on the body where a pressure was given. A shock point ("continuity" point) is computed at the other end of each normal. When a converged solution has been achieved the pressure (and other flow field parameters) at the continuity points, and the location of these points, agree well with the corresponding values computed from the assumed shock shape. The following rule of thumb has been useful in obtaining this agreement:*

If the pressure at the body surface is raised at any normal, the pressure at the continuity point on that normal will also rise, and

- a. The continuity point will move away from the body if the normal is primarily upstream of the sonic line, or
- b. The continuity point will move toward the body if the normal is primarily downstream of the sonic line.

*This rule was established by R. H. Edsall and Mrs. S. F. Hill.

The reverse will happen if pressure at the body surface is lowered. This rule obviously is useful only if both pressure (at the continuity points) and location of the continuity points dictate the same change in pressure at the body. It is, in effect, a rule for adjusting the continuity points to make them "fit" the assumed shock.

The alternate possibility is to adjust the shock to make it "fit" the continuity points. Obviously, steepening the shock will raise the pressure computed downstream of it and making the shock less steep will have the opposite effect. The effect a change in shock shape will have on agreement in location between the continuity points and the shock shape is also apparent. *

Whether a better solution will result from changing the shock shape or the pressure distribution, will become more apparent as the user gains experience with the program. Generally, it is better to change one or the other at any step in the iteration, not both simultaneously.

The preceding discussion applies primarily to a situation where the result of the previous iteration is a smooth, complete solution (i. e. , the streamlines converge smoothly as they approach the sonic line and then diverge; and the solution has continued until a continuity point is computed at the shock end of each normal). If the estimate of the shock shape or of the pressure distribution is not a good one, it is possible that the result not be smooth, or that the solution may not go to completion.

*While a small change in shock shape will have some effect on the location and pressure at the continuity points, that effect is much less pronounced than the effect on the pressure which is computed directly from the assumed shock shape (by the Rankine Hugoniot equations).

When this happens, it is necessary to combine an understanding of the programming of this solution with an understanding of flow field characteristics, in order to determine what needs to be done. No more specific directions can be provided.

APPENDIX V

SUMMARY OF AVAILABLE ZERO YAW TRANSONIC SOLUTIONS

This Appendix contains Figure 25 which is a graphical index of the altitudes and velocities for which solutions are included. The remaining pages in this Appendix provide the input information needed for supersonic solutions (upper part of each page), and the shock shape and pressure distribution which were used as input to the converged transonic solution (lower part of each page).

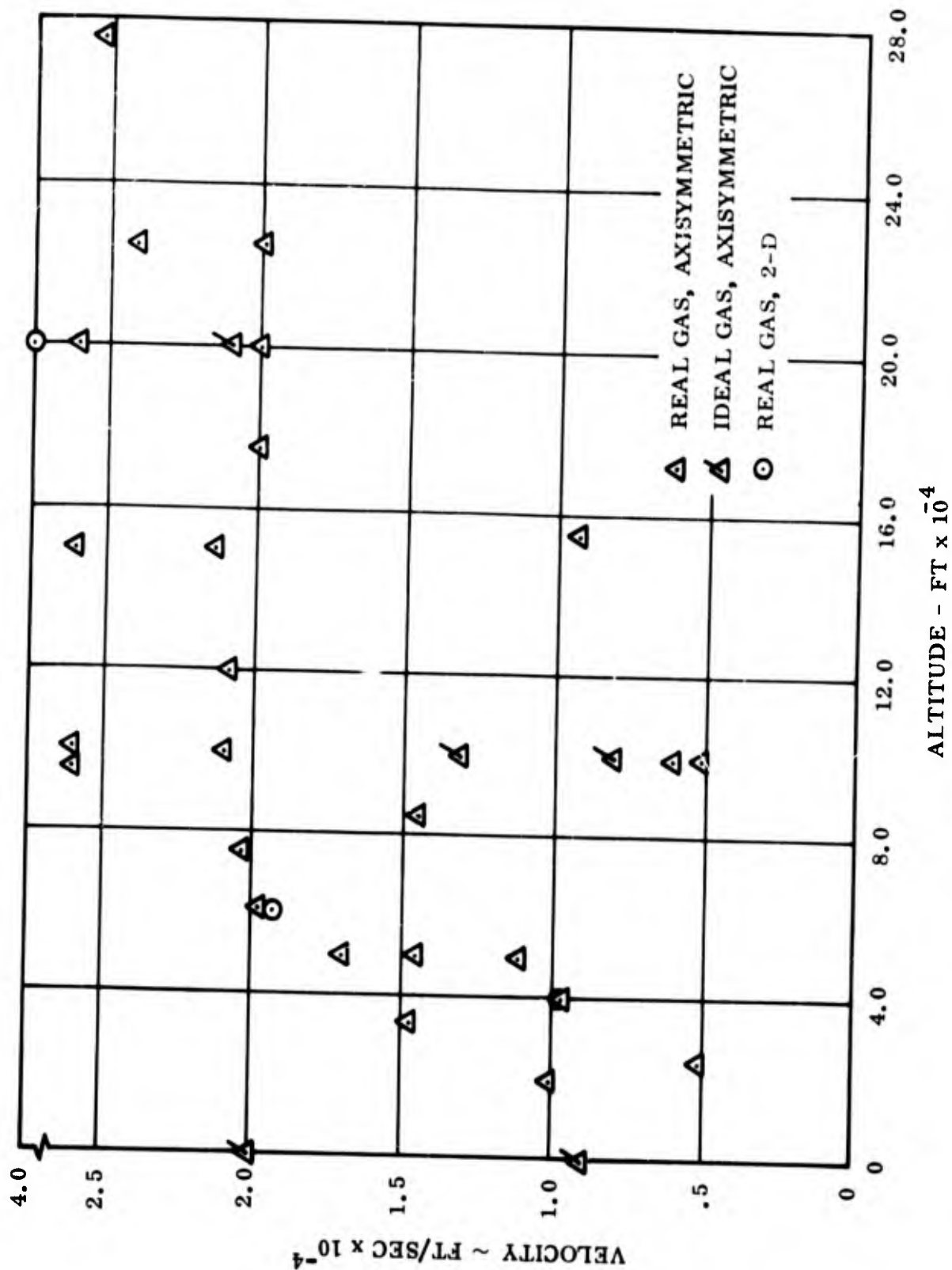


Figure 25. Available Transonic Solutions

$M_\infty = 7.7$
 Shock Angle = .97853
 Test Cell

Real Gas
 Two-Dimensional

	x	r	θ	ρ	P
F.03447,	.47282,	.8353,	.12989E-4,	166.62,	\$
F.03570,	.47030,	.8351,	.12520E-4,	165.20,	\$
F.04000,	.46810,	.8345,	.12200E-4,	164.00,	\$
F.04500,	.46350,	.8332,	.11600E-4,	161.80,	\$
F.05000,	.45910,	.8314,	.11080E-4,	159.70,	\$
F.05500,	.45460,	.8290,	.10600E-4,	157.70,	\$
F.06000,	.45010,	.8262,	.10225E-4,	155.80,	\$
F.06500,	.44550,	.8230,	.99100E-5,	154.00,	\$
F.07000,	.44100,	.8194,	.96600E-5,	152.35,	\$
F.07500,	.43645,	.8152,	.94500E-5,	150.70,	\$
F.08000,	.43160,	.8110,	.92750E-5,	149.20,	\$
F.08500,	.42690,	.8072,	.91250E-5,	147.70,	\$
F.09000,	.42200,	.8038,	.89900E-5,	146.20,	\$
F.09500,	.41725,	.8010,	.88200E-5,	144.75,	\$
F.10000,	.41240,	.7985,	.87500E-5,	143.30,	\$
F.10500,	.40740,	.7961,	.86500E-5,	141.75,	\$
F.11000,	.40240,	.7939,	.85500E-5,	140.20,	\$
F.11500,	.39750,	.7918,	.84400E-5,	138.55,	\$
F.12000,	.39260,	.7899,	.83250E-5,	136.80,	\$
F.12500,	.38760,	.7877,	.82300E-5,	135.10,	\$
F.13000,	.38270,	.7855,	.81250E-5,	133.10,	\$
F.13500,	.37750,	.7832,	.80100E-5,	131.00,	\$
F.14000,	.37250,	.7805,	.78400E-5,	128.80,	\$
F.14500,	.36750,	.7775,	.77600E-5,	126.45,	\$
F.15000,	.36250,	.7732,	.76300E-5,	124.00,	\$
F.15267,	.35967,	.7679,	.75612E-5,	122.57,	\$

Shock Equation	Input	x	P
A(1) = -.10445148	$P_\infty = 3.168$.009187	241.00
A(3) = .42868578	$\rho_\infty = 1.26E-6$.019369	231.40
A(5) = 1.5644642	$V_\infty = 14190.0$.033210	217.97
A(7) = -5.6965067		.050603	202.25
A(9) = 12.413087		.071416	183.74
A(11) = -10.805891		.095492	163.00
All Other Coefficients = 0		.122645	141.80
		.152671	122.57
		.185340	103.95
		.220405	85.50
		.257595	68.00

$M_\infty = 10$
 Shock Angle = 1.0368
 Altitude = 40,000 Feet

Real Gas
 Two-Dimensional

	x	r	θ	ρ	P
F.02065,	.56853,	.8014,	.41882E-2,	35892,	\$
F.02450,	.56471,	.7970,	.41077E-2,	35468,	\$
F.03059,	.55868,	.7910,	.39890E-2,	34838,	\$
F.03469,	.55461,	.7875,	.39163E-2,	34440,	\$
F.04128,	.54802,	.7830,	.38086E-2,	33856,	\$
F.04585,	.54342,	.7793,	.37442E-2,	33510,	\$
F.05297,	.53618,	.7747,	.36492E-2,	32995,	\$
F.05821,	.53080,	.7709,	.35853E-2,	32630,	\$
F.06590,	.52284,	.7654,	.34968E-2,	32114,	\$
F.07104,	.51748,	.7618,	.34424E-2,	31780,	\$
F.07930,	.50877,	.7561,	.33591E-2,	31254,	\$
F.08734,	.50020,	.7508,	.32846E-2,	30751,	\$
F.09190,	.49529,	.7479,	.32441E-2,	30468,	\$
F.10102,	.48541,	.7423,	.31667E-2,	29902,	\$
F.10609,	.47986,	.7394,	.31255E-2,	29586,	\$
F.11572,	.46924,	.7340,	.30496E-2,	28975,	\$
F.12199,	.46227,	.7306,	.30014E-2,	28566,	\$
F.13211,	.45091,	.7252,	.29249E-2,	27880,	\$
F.14248,	.43915,	.7194,	.28457E-2,	27128,	\$
F.15304,	.42701,	.7128,	.27626E-2,	26292,	\$
F.16375,	.41453,	.7049,	.26731E-2,	25348,	\$
F.17454,	.40172,	.6946,	.25740E-2,	24265,	\$
F.18534,	.38857,	.6807,	.24606E-2,	23000,	\$

Shock Equation

A(1) = -.146

A(3) = .520

All Other Coefficients = 0

Input

$P_\infty = 393.12$

$\rho_\infty = .58727E-2$

$V_\infty = 9680.8$

x	P
.009186	49300
.019369	47400
.033210	44800
.050603	41400
.071416	38200
.095492	34200
.122645	30300
.152671	27000
.185340	23000
.220405	19500

$M_\infty = 20$
 Shock Angle = 1.0574
 Altitude = 60,430 Feet

Real Gas
 Two-Dimensional

	x	r	A	ρ	P
F.03952,	.46990,	.8885,	.22889E-2,	56703,	\$
F.04412,	.46581,	.8670,	.22122E-2,	55694,	\$
F.04672,	.46348,	.8600,	.21724E-2,	55155,	\$
F.65168,	.45901,	.8470,	.21032E-2,	54183,	\$
F.05471,	.45625,	.8410,	.20650E-2,	53629,	\$
F.06021,	.45119,	.8310,	.20023E-2,	52691,	\$
F.06373,	.44793,	.8260,	.19660E-2,	52127,	\$
F.06985,	.44220,	.8170,	.19094E-2,	51206,	\$
F.07374,	.43851,	.8130,	.18769E-2,	50654,	\$
F.08053,	.43202,	.8049,	.18255E-2,	49742,	\$
F.08486,	.42784,	.8013,	.17959E-2,	49189,	\$
F.09238,	.42052,	.7956,	.17494E-2,	48271,	\$
F.09778,	.41520,	.7920,	.17188E-2,	47634,	\$
F.10607,	.40698,	.7872,	.16757E-2,	46681,	\$
F.11479,	.39826,	.7832,	.16341E-2,	45685,	\$
F.12389,	.38909,	.7798,	.15931E-2,	44623,	\$
F.13330,	.37953,	.7766,	.15517E-2,	43465,	\$
F.14293,	.36970,	.7730,	.15086E-2,	42173,	\$
F.15267,	.35967,	.7679,	.14619E-2,	40700,	\$

Shock Equation	Input	x	P
A(1) = -.0970	$P_\infty = 147.31$.019369	71400
A(3) = .60	$\rho_\infty = .22E-3$.033210	67500
All Other Coefficients = 0	$V_\infty = 19,362$.050603	63000
		.071416	57900
		.095492	52400
		.122645	46500
		.152671	40700
		.185340	35200
		.220405	29900

$M_\infty = 30$ Real Gas
 Shock Angle = 1.0315 Two-Dimensional
 Altitude = 275,000 Feet

F.05326, .38484, .9346, .32795E-6, 8.5476, \$
 F.05450, .38391, .9320, .32087E-6, 8.4606, \$
 F.05645, .38245, .9290, .31074E-6, 8.3335, \$
 F.05791, .38135, .9259, .30378E-6, 8.2398, \$
 F.06009, .37970, .9221, .29438E-6, 8.1069, \$
 F.06242, .37793, .9181, .28555E-6, 7.9772, \$
 F.06424, .37654, .9150, .27959E-6, 7.8843, \$
 F.06685, .37452, .9107, .27191E-6, 7.7608, \$
 F.06897, .37286, .9072, .26626E-6, 7.6687, \$
 F.07193, .37054, .9024, .25938E-6, 7.5510, \$
 F.07510, .36803, .8975, .25289E-6, 7.4366, \$
 F.07787, .36581, .8933, .24809E-6, 7.3452, \$
 F.08147, .36289, .8882, .24249E-6, 7.2367, \$
 F.08496, .36004, .8835, .23779E-6, 7.1407, \$
 F.08908, .35665, .8785, .23296E-6, 7.0373, \$
 F.09347, .35298, .8737, .22842E-6, 6.9359, \$
 F.09639, .35054, .8710, .22581E-6, 6.8725, \$
 F.10123, .34644, .8670, .22181E-6, 6.7722, \$
 F.10633, .34210, .8636, .21802E-6, 6.6708, \$
 F.11164, .33754, .8607, .21444E-6, 6.5665, \$
 F.11710, .33283, .8580, .21091E-6, 6.4572, \$
 F.12265, .32803, .8552, .20731E-6, 6.3400, \$

Shock Equation

$A(1) = -.06$
 $A(3) = .7775$

All Other Coefficients = 0

Input

$P_\infty = .0098$
 $\rho_\infty = .1909E-7$
 $V_\infty = 25,395$

x

P

.009186 11.280
 .019363 10.820
 .033210 10.120
 .050603 9.230
 .071416 8.270
 .095492 7.250
 .122645 6.340
 .152671 5.510
 .185340 4.670

$M_\infty = 38,5097$ Real Gas
 Shock Angle = .9228 Two-Dimensional
 Altitude = 200,000 Feet

	x	r	θ	ρ	P
F.09515,	.35234,	.8398,	.98167E-5,	591.04,	\$
F.09658,	.35108,	.8399,	.91875E-5,	575.82,	\$
F.09827,	.34960,	.8400,	.85721E-5,	560.77,	\$
F.10000,	.35807,	.8401,	.80669E-5,	548.68,	\$
F.10186,	.34643,	.8402,	.76277E-5,	537.42,	\$
F.10291,	.34549,	.8403,	.74225E-5,	531.74,	\$
F.10486,	.34376,	.8410,	.70859E-5,	522.15,	\$
F.10608,	.34268,	.8417,	.69070E-5,	516.73,	\$
F.10810,	.34088,	.8422,	.66501E-5,	508.60,	\$
F.11012,	.33909,	.8435,	.64292E-5,	501.34,	\$
F.11145,	.33791,	.8442,	.63072E-5,	496.98,	\$
F.11341,	.33616,	.8448,	.61429E-5,	491.09,	\$
F.11472,	.33500,	.8454,	.60456E-5,	487.47,	\$
F.11652,	.33340,	.8467,	.59247E-5,	482.88,	\$
F.11817,	.33195,	.8483,	.58236E-5,	479.02,	\$
F.11918,	.33106,	.8495,	.57698E-5,	476.82,	\$
F.12048,	.32992,	.8513,	.57024E-5,	474.13,	\$
F.12107,	.32940,	.8523,	.56726E-5,	472.96,	\$
F.12193,	.32865,	.8538,	.56314E-5,	471.31,	\$
F.12247,	.32819,	.8548,	.56061E-5,	470.33,	\$
F.12265,	.32803,	.8552,	.55973E-5,	470.00,	\$

Shock Equation	Input	x	P
A(1) = -.028	$P_\infty = .4715$.002739	930.00
A(3) = .835645	$\rho_\infty = .681180E-6$.009186	896.00
A(5) = .638545	$V_\infty = 40,000$.019369	857.00
A(7) = 4.68223		.033210	802.00
A(9) = -20.7557		.050603	735.00
A(11) = 18.9403		.071416	650.00
All Other Coefficients = 0		.095492	560.00
		.122645	470.00
		.152671	385.00
		.185340	311.50
		.220405	242.50
		.257595	187.00

$M_\infty = 3.5$ Ideal Gas
 Shock Angle = .94796 Three- Dimensional
 Test Cell

	x	r	θ	ρ	P
F.09081	.51841	.5886	.92591E-2	19544	\$
F.09250	.51580	.5890	.91900E-2	19425	\$
F.09500	.51200	.5897	.90900E-2	19275	\$
F.10000	.50490	.5920	.89050E-2	18990	\$
F.10500	.49750	.5950	.87200E-2	18710	\$
F.11000	.49000	.5988	.85500E-2	18450	\$
F.11500	.48280	.6035	.83900E-2	18225	\$
F.12000	.47550	.6092	.82350E-2	17990	\$
F.12500	.46830	.6150	.80850E-2	17750	\$
F.13000	.46100	.6209	.79400E-2	17500	\$
F.13500	.45400	.6265	.77900E-2	17240	\$
F.14000	.44700	.6322	.76450E-2	16950	\$
F.14500	.44040	.6379	.75000E-2	16675	\$
F.15000	.43350	.6435	.73500E-2	16380	\$
F.15500	.42700	.6492	.72000E-2	16060	\$
F.16000	.42070	.6550	.70400E-2	15725	\$
F.16500	.41440	.6607	.68700E-2	15375	\$
F.17000	.40770	.6664	.67000E-2	15000	\$
F.17500	.40150	.6721	.65150E-2	14610	\$
F.18000	.39500	.6775	.63200E-2	14190	\$
F.18250	.39190	.6794	.62200E-2	13950	\$
F.18534	.38857	.6807	.61105E-2	13705	\$

Shock Equation	Input	x	P
A(1) = -.09700622	$P_\infty = 2109.5$.019369	31864
A(3) = .79591112	$\rho_\infty = 2.498E-3$.033210	29945
A(5) = -.65590844	$V_\infty = 3804.6$.050603	27581
A(7) = .47566722		.071416	24926
A(9) = 5.8931018		.095492	22099
A(11) = -.11875398		.122645	19324
All Other Coefficients = 0		.152671	16446
		.185340	13705
		.220405	11306
		.257595	9182

$M_\infty = 5$
 Shock Angle = .9593450
 Altitude - 100,000 Feet

Real Gas
 Three-Dimensional

x	r	θ	ρ	P
F.07896,	.44647,	.6722,	.15514E-3,	452.66, \$
F.08100,	.44390,	.6725,	.15380E-3,	449.60, \$
F.08500,	.43880,	.6733,	.15075E-3,	443.60, \$
F.08900,	.43370,	.6745,	.14750E-3,	437.50, \$
F.09300,	.42860,	.6760,	.14400E-3,	431.60, \$
F.09700,	.42360,	.6782,	.14060E-3,	426.30, \$
F.10100,	.41860,	.6815,	.13740E-3,	421.40, \$
F.10500,	.41360,	.6864,	.13425E-3,	416.30, \$
F.10900,	.40880,	.6915,	.13130E-3,	411.00, \$
F.11300,	.40400,	.6970,	.12890E-3,	406.00, \$
F.11700,	.39940,	.7026,	.12650E-3,	400.70, \$
F.12100,	.39460,	.7083,	.12410E-3,	395.50, \$
F.12500,	.39000,	.7146,	.12190E-3,	390.00, \$
F.12900,	.38550,	.7216,	.11960E-3,	384.40, \$
F.13300,	.38090,	.7286,	.11740E-3,	378.75, \$
F.13700,	.37650,	.7360,	.11500E-3,	373.00, \$
F.14100,	.37220,	.7437,	.11275E-3,	367.00, \$
F.14500,	.36780,	.7520,	.11050E-3,	360.50, \$
F.14900,	.36350,	.6701,	.10810E-3,	354.10, \$
F.15267,	.35967,	.7679,	.10598E-3,	348.00, \$

Snock Equation

Snock Equation	Input	x	P
A(1) = -.074132599	$P_\infty = 23.085$.009187	718.50
A(3) = .73981921	$\rho_\infty = 3.2114E-5$.019369	686.00
A(5) = -.55422108	$V_\infty = 5016.0$.033210	635.00
A(7) = 8.4003366		.050603	577.00
A(9) = -31.186169		.071416	524.00
A(11) = 33.588193		.095492	467.46
All Other Coefficients = 0		.122646	406.00
		.152671	348.00
		.185340	297.50
		.220405	250.00

$M_\infty = 6$
 Shock Angle = .96271
 Altitude = 100,000 Feet

Real Gas
 Three-Dimensional

x	r	θ	ρ	P
F.08387	.43880	.7008	.17214E-3	660.61, \$
F.08600	.43625	.7004	.16950E-3	654.00, \$
F.09000	.43145	.6995	.16500E-3	642.60, \$
F.09400	.42650	.6988	.16050E-3	632.50, \$
F.09800	.42170	.6986	.15610E-3	623.20, \$
F.10200	.41685	.6990	.15220E-3	614.50, \$
F.10600	.41210	.7004	.14900E-3	605.60, \$
F.11000	.40725	.7031	.14610E-3	597.00, \$
F.11400	.40250	.7075	.14300E-3	588.00, \$
F.11800	.39790	.7124	.13960E-3	579.50, \$
F.12200	.39325	.7176	.13625E-3	571.00, \$
F.12600	.38875	.7231	.13300E-3	562.40, \$
F.13000	.38440	.7286	.13010E-3	553.50, \$
F.13400	.37980	.7345	.12725E-3	544.50, \$
F.13800	.37550	.7407	.12425E-3	535.00, \$
F.14200	.37100	.7475	.12150E-3	526.00, \$
F.14600	.36675	.7547	.11850E-3	516.80, \$
F.15000	.36250	.7625	.11580E-3	507.50, \$
F.15267	.35967	.7679	.11407E-3	501.00, \$

Shock Equation	Input	x	P
A(1) = -.065719962	$P_\infty = 23.085$.009187	1032.00
A(3) = .7791446	$\rho_\infty = 3.2114E-5$.019369	986.16
A(5) = .41018938	$V_\infty = 6019.2$.033210	915.00
A(7) = 3.7485976		.050603	839.50
A(9) = -9.8680425		.071416	760.00
A(11) = 6.1913265		.095492	670.00
All Other Coefficients = 0		.122646	580.00
		.152671	501.00
		.185340	421.87
		.220405	352.85

$M_\infty = 8$ Ideal Gas
 Shock Angle = .94912 Three-Dimensional
 Sea Level

x	r	θ	ρ	P
F.08560	.43650	.6946	.12754E-1	104060, \$
F.08740	.43440	.6947	.12600E-1	103230, \$
F.08920	.43210	.6948	.12440E-1	102500, \$
F.09120	.42975	.6951	.12275E-1	101700, \$
F.09480	.42540	.6960	.12000E-1	100500, \$
F.09860	.42070	.6970	.11715E-1	99300, \$
F.10240	.41625	.6982	.11435E-1	98000, \$
F.10600	.41180	.7002	.11160E-1	96800, \$
F.10980	.40730	.7031	.10870E-1	95500, \$
F.11360	.40280	.7073	.10580E-1	94050, \$
F.11720	.39870	.7115	.10315E-1	92800, \$
F.12100	.39430	.7162	.10020E-1	91400, \$
F.12480	.39000	.7212	.97350E-2	90000, \$
F.12840	.38610	.7264	.94750E-2	88650, \$
F.13220	.38150	.7323	.92100E-2	87920, \$
F.13600	.37740	.7383	.89750E-2	85800, \$
F.13960	.37350	.7453	.87650E-2	84500, \$
F.14340	.36950	.7520	.85600E-2	83150, \$
F.14720	.36550	.7586	.83600E-2	81650, \$
F.14900	.36350	.7618	.82750E-2	81000, \$
F.15080	.36170	.7647	.81900E-2	80350, \$
F.15270	.35970	.7679	.81100E-2	79500, \$

Shock Equation	Input	x	P
A(1) = -.0697619	$P_\infty = 2116.2$.0027390	171500
A(3) = .63076	$\rho_\infty = 2.3769E-3$.0091863	166370
A(5) = 3.3759	$V_\infty = 8931.2$.0193690	157840
A(7) = -22.369		.0332097	146000
A(9) = 65.1042		.0506030	132400
A(11) = -70.2076		.0714163	117930
All Other Coefficients = 0		.0954917	103400
		.1226453	91530
		.1526707	79500
		.1853397	67000
		.2204050	54200
		.2575950	42500

$M_\infty = 10$ Real Gas
 Shock Angle = .9732098 Three-Dimensional
 Altitude = 20,000 Feet

x	r	θ	ρ	P
F.07098	.37793	.7863	.91053E-2	83869, \$
F.07280	.37600	.7863	.90400E-2	83050, \$
F.07440	.37440	.7863	.89550E-2	82350, \$
F.07800	.37060	.7863	.86950E-2	80760, \$
F.08140	.36710	.7864	.84300E-2	79280, \$
F.08480	.36360	.7869	.81550E-2	77800, \$
F.08820	.36025	.7880	.78900E-2	76450, \$
F.09160	.35690	.7915	.76400E-2	75150, \$
F.09500	.35355	.7968	.74000E-2	73940, \$
F.09840	.35040	.8029	.71900E-2	72830, \$
F.10200	.34690	.8096	.69800E-2	71770, \$
F.10540	.34375	.8164	.68100E-2	70820, \$
F.10880	.34050	.8235	.66600E-2	69950, \$
F.11220	.33740	.8315	.65350E-2	69060, \$
F.11580	.33400	.8407	.64150E-2	68150, \$
F.11920	.33100	.8497	.63100E-2	67320, \$
F.12100	.32956	.8545	.62600E-2	66900, \$
F.12265	.32803	.8552	.62096E-2	66500, \$

Shock Equation

- A(1) = -.046719
- A(3) = .564527
- A(5) = 4.19503
- A(7) = -19.7608
- A(9) = 27.071
- A(11) = 4.729

All Other Coefficients = 0

Input

- $P_\infty = 972.5$
- $\rho_\infty = .12664E-2$
- $V_\infty = 10,369.0$

x	P
.0027390	119850
.0091863	119110
.0193690	115770
.0332097	108400
.0506030	98550
.0714163	87150
.0954917	76000
.1226453	66500
.1526707	59650
.1853397	53000
.2204050	47000
.2575950	41500

$M_\infty = 10$ Real Gas
 Shock Angle = 0.98914 Three-Dimensional
 Altitude = 40,000 Feet

x	r	θ	ρ	P
F.06849,	.38025,	.7730,	.40676E-2,	33267,\$
F.07000,	.37850,	.7725,	.40100E-2,	33055,\$
F.07400,	.37440,	.7726,	.38900E-2,	32535,\$
F.07800,	.37030,	.7757,	.37700E-2,	32050,\$
F.08200,	.36630,	.7804,	.36600E-2,	31610,\$
F.08600,	.36230,	.7855,	.35550E-2,	31185,\$
F.09000,	.35830,	.7910,	.34550E-2,	30750,\$
F.09400,	.35450,	.7967,	.33600E-2,	30355,\$
F.09800,	.35070,	.8030,	.32700E-2,	29970,\$
F.10200,	.34670,	.8102,	.31880E-2,	29575,\$
F.10600,	.34300,	.8177,	.31100E-2,	29200,\$
F.11000,	.33925,	.8265,	.30450E-2,	28835,\$
F.11400,	.33575,	.8360,	.29840E-2,	28450,\$
F.11800,	.33210,	.8458,	.29280E-2,	28065,\$
F.12264,	.32803,	.8552,	.28645E-2,	27620,\$

Shock Equation	Input	x	P
A(1) = -.04928334	$P_\infty = 393.12$.002739	50400
A(3) = .63243631	$\rho_\infty = 5.8727E-4$.009186	49000
A(5) = 2.9626095	$V_\infty = 9680.8$.019369	46600
A(7) = -17.932815		.033210	43700
A(9) = 50.928433		.050603	39900
A(11) = -54.532132		.071416	35712
All Other Coefficients = 0		.095492	31445
		.122645	27620
		.152671	23500
		.185340	19400

$M_\infty = 11.46$ Real Gas
 Shock Angle = .97788 Three-Dimensional
 Altitude=50,429 Feet

x	r	θ	ρ	P
F.07304	.37576	.7838	.26923E-2	26423, \$
F.07400	.37475	.7841	.26790E-2	26275, \$
F.07600	.37275	.7850	.26400E-2	26040, \$
F.07800	.37075	.7862	.25860E-2	25795, \$
F.08000	.36875	.7876	.25340E-2	25550, \$
F.08200	.36680	.7892	.24840E-2	25310, \$
F.08400	.36485	.7910	.24350E-2	25095, \$
F.08600	.36290	.7928	.23900E-2	24875, \$
F.08800	.36095	.7947	.23480E-2	24680, \$
F.09000	.35896	.7967	.23090E-2	24480, \$
F.09200	.35700	.7989	.22725E-2	24300, \$
F.09400	.35510	.8012	.22375E-2	24100, \$
F.09600	.35310	.8036	.22020E-2	23925, \$
F.09800	.35120	.8062	.21700E-2	23750, \$
F.10000	.34930	.8090	.21400E-2	23575, \$
F.10200	.34737	.8119	.21110E-2	23410, \$
F.10400	.34550	.8150	.20860E-2	23250, \$
F.10600	.34360	.8182	.20625E-2	23100, \$
F.10800	.34180	.8217	.20380E-2	22925, \$
F.11000	.33990	.8255	.20150E-2	22775, \$
F.11200	.33800	.8294	.19940E-2	22625, \$
F.11400	.33610	.8335	.19725E-2	22460, \$
F.11600	.33425	.8380	.19520E-2	22300, \$
F.11800	.33240	.8428	.19325E-2	22150, \$
F.12000	.33050	.8480	.19140E-2	21980, \$
F.12100	.32960	.8508	.19030E-2	21900, \$
F.12265	.32803	.8552	.18883E-2	21768, \$

Shock Equation	Input	x	P
A(1) = -.045543984	$P_\infty = 238.67$.002739	40650
A(3) = .67726273	$\rho_\infty = .35653E-3$.009187	39420
A(5) = 1.3225792	$V_\infty = 11,097$.019369	37500
A(7) = .35915569		.033210	35011
A(9) = -18.290546		.050603	31960
A(11) = 31.481946		.071416	28596
All Other Coefficients = 0		.095492	25074
		.122646	21768
		.152671	18600
		.185340	15500
		.220405	12500

$M_\infty = 13$ Real Gas
 Shock Angle = .96954 Three Dimensional
 Altitude = 100,000 Feet

	x	r	θ	ρ	P
F.08082,	.36736,	.8098,	.29066E-3,	3326.8,	\$
F.08200,	.36620,	.8098,	.28695E-3,	3295.0,	\$
F.08400,	.36430,	.8100,	.28000E-3,	3242.0,	\$
F.08600,	.36235,	.8101,	.27325E-3,	3194.0,	\$
F.08800,	.36045,	.8104,	.26675E-3,	3194.0,	\$
F.09000,	.35858,	.8107,	.26075E-3,	3108.0,	\$
F.09200,	.35660,	.8112,	.25500E-3,	3071.0,	\$
F.09400,	.35470,	.8119,	.24940E-3,	3035.0,	\$
F.09600,	.35280,	.8127,	.24395E-3,	3000.0,	\$
F.09800,	.35092,	.8139,	.23880E-3,	2967.5,	\$
F.10000,	.34900,	.8152,	.23400E-3,	2936.0,	\$
F.10200,	.34715,	.8170,	.22925E-3,	2907.5,	\$
F.10400,	.34530,	.8192,	.22510E-3,	2880.0,	\$
F.10600,	.34340,	.8218,	.22125E-3,	2855.0,	\$
F.10800,	.34155,	.8247,	.21775E-3,	2830.0,	\$
F.11000,	.33970,	.8278,	.21440E-3,	2807.5,	\$
F.11200,	.33785,	.8313,	.21125E-3,	2785.0,	\$
F.11400,	.33600,	.8350,	.20840E-3,	2762.5,	\$
F.11600,	.33420,	.8390,	.20580E-3,	2741.0,	\$
F.11800,	.33240,	.8433,	.20340E-3,	2720.0,	\$
F.12000,	.33050,	.8480,	.20100E-3,	2700.0,	\$
F.12200,	.32865,	.8535,	.19875E-3,	2679.0,	\$
F.12265,	.32803,	.8552,	.19811E-3,	2672.0,	\$

Shock Equation	Input	x	P
A(1) = -.035654124	$P_\infty = 23.085$.002739	5040.7
A(3) = .63308299	$\rho_\infty = 3.2114E-5$.009187	4860.0
A(5) = 3.7249846	$V_\infty = 13,041.6$.019369	4653.7
A(7) = -21.642120		.033210	4364.1
A(9) = 61.998513		.050603	3973.9
A(11) = -73.963013		.071416	3529.9
All Other Coefficients = 0		.095492	3071.7
		.122646	2672.0
		.152671	2310.3
		.185340	1972.2
		.220405	1660.0

$M_\infty = 15$ Real Gas
 Shock Angle = .9538400 Three-Dimensional
 Altitude = 50,000 Feet

	x	r	θ	ρ	P
F.08007,	.36830,	.7992,	.32914E-2,	45650,	\$
F.08100,	.36730,	.8016,	.32400E-2,	45270,	\$
F.08200,	.36640,	.8035,	.31950E-2,	44920,	\$
F.08400,	.36450,	.8068,	.30940E-2,	44240,	\$
F.08600,	.36260,	.8096,	.30050E-2,	43550,	\$
F.08800,	.36075,	.8119,	.29250E-2,	42900,	\$
F.09000,	.35850,	.8138,	.28450E-2,	42330,	\$
F.09200,	.35690,	.8155,	.27750E-2,	41800,	\$
F.09400,	.35505,	.8170,	.27080E-2,	41300,	\$
F.09600,	.35320,	.8185,	.26430E-2,	40860,	\$
F.09800,	.35130,	.8202,	.25800E-2,	40430,	\$
F.10000,	.34945,	.8220,	.25270E-2,	40000,	\$
F.10200,	.34760,	.8238,	.24750E-2,	39600,	\$
F.10400,	.34570,	.8258,	.24290E-2,	39200,	\$
F.10600,	.34385,	.8280,	.23830E-2,	38825,	\$
F.10800,	.34210,	.8304,	.23410E-2,	38470,	\$
F.11000,	.34025,	.8330,	.23050E-2,	38125,	\$
F.11200,	.33835,	.8360,	.22700E-2,	37770,	\$
F.11400,	.33640,	.8394,	.22320E-2,	37425,	\$
F.11600,	.33430,	.8428,	.21980E-2,	37090,	\$
F.11800,	.33240,	.8463,	.21620E-2,	36750,	\$
F.12000,	.33050,	.8500,	.21300E-2,	36450,	\$
F.12140,	.32920,	.8527,	.21080E-2,	36240,	\$
F.12264,	.32803,	.8552,	.20910E-2,	36051,	\$

Shock Equation	Input	x	P
A(1) = -.038794354	$P_\infty = 243.61$.002739	71220
A(3) = .81766091	$\rho_\infty = 3.6391E-4$.009186	68854
A(5) = .22304873	$V_\infty = 14,521$.019369	65398
A(7) = 5.8552453		.033210	61156
A(9) = -31.560219		.050603	55482
A(11) = 40.547567		.071416	49584
All Other Coefficients = 0		.095492	42932
		.122645	36051
		.152671	30616
		.185340	26324
		.220405	22237
		.257595	18600

$M_\infty = 15$ Real Gas
 Shock Angle = .9642800 Three-Dimensional
 Altitude = 85,000 Feet

	x	r	θ	ρ	P
F.08214,	.36533,	.8147,	.66754E-3,	8993.8,	\$
F.08360,	.36395,	.8137,	.65400E-3,	8878.0,	\$
F.08480,	.36285,	.8133,	.64400E-3,	8788.0,	\$
F.08760,	.36015,	.8130,	.61900E-3,	8585.0,	\$
F.09020,	.35770,	.8135,	.59700E-3,	8421.0,	\$
F.09300,	.35550,	.8149,	.57450E-3,	8260.0,	\$
F.09560,	.35260,	.8167,	.55400E-3,	8120.0,	\$
F.09840,	.34995,	.8192,	.53450E-3,	7980.0,	\$
F.10100,	.34755,	.8217,	.51750E-3,	7864.0,	\$
F.10380,	.34492,	.8246,	.50000E-3,	7744.0,	\$
F.10640,	.34260,	.8280,	.48500E-3,	7649.0,	\$
F.10920,	.34000,	.8320,	.47100E-3,	7550.0,	\$
F.11180,	.33765,	.8361,	.45950E-3,	7460.0,	\$
F.11460,	.33515,	.8410,	.44900E-3,	7375.0,	\$
F.11720,	.33280,	.8455,	.44100E-3,	7297.0,	\$
F.12000,	.33040,	.8505,	.43200E-3,	7215.0,	\$
F.12120,	.32930,	.8576,	.42900E-3,	7180.0,	\$
F.12265,	.32803,	.8552,	.42440E-3,	7140.0,	\$

Shock Equation	Input	x	P
A(1) = -.03943039	$P_\infty = 47.0$.0027390	13930
A(3) = .8668105	$\rho_\infty = 0.698E-4$.0091863	13460
A(5) = .1154097	$V_\infty = 14,565$.0193690	12750
A(7) = 2.158591		.0332097	11850
A(9) = 4.130238		.0506030	10760
A(11) = -52.30871		.0714163	9580
All Other Coefficients = 0		.0954917	8330
		.1226453	7140
		.1526707	6100
		.1853397	5160

$M_\infty = 17.56$ Real Gas
 Shock Angle = .9520200 Three-Dimensional
 Altitude = 50,000 Feet

x	r	θ	ρ	P
F.08173,	.36556,	.8082,	.35283E-2,	62829, \$
F.08300,	.36430,	.8091,	.34550E-2,	62100, \$
F.08420,	.36305,	.8102,	.33775E-2,	61380, \$
F.08720,	.36040,	.8126,	.32200E-2,	59990, \$
F.09000,	.35795,	.8150,	.30700E-2,	58690, \$
F.09260,	.35535,	.8174,	.29400E-2,	57580, \$
F.09540,	.35270,	.8197,	.28180E-2,	56500, \$
F.09900,	.35030,	.8221,	.27125E-2,	55600, \$
F.10080,	.34770,	.8247,	.26150E-2,	54700, \$
F.10360,	.34510,	.8275,	.25300E-2,	53850, \$
F.10620,	.34270,	.8302,	.24600E-2,	53150, \$
F.10900,	.34020,	.8339,	.23950E-2,	52450, \$
F.11180,	.33760,	.8380,	.23375E-2,	51780, \$
F.11440,	.33525,	.8422,	.22890E-2,	51200, \$
F.11720,	.33275,	.8467,	.22400E-2,	50620, \$
F.12000,	.33030,	.8511,	.21960E-2,	50500, \$
F.12120,	.32930,	.8530,	.21800E-2,	49800, \$
F.12265,	.32803,	.8552,	.21587E-2,	49496, \$

Shock Equation	Input	x	P
A(1) = -.0386601	$P_\infty = 243.61$.0027390	97750
A(3) = .78635600	$\rho_\infty = 3.6391E-4$.0091863	94500
A(5) = 1.0005400	$V_\infty = 17,000$.0193690	89741
A(7) = 2.8262		.0332097	83924
A(9) = -31.3266		.0506030	76130
A(11) = 49.6393		.0714163	67397
All Other Coefficients = 0		.0954917	58298
		.1226453	49496
		.1526707	42066
		.1853397	36566
		.2204050	30948
		.2575950	26100

$M_\infty = 18$
 Shock Angle = .9875700
 Sea Level

Ideal Gas
 Three-Dimensional

x	r	θ	ρ	P
F.05854,	.39274,	.7349,	.13952E-1,	556960,\$
F.06000,	.39120,	.7350,	.13770E-1,	553500,\$
F.06200,	.38890,	.7357,	.13530E-1,	548900,\$
F.06600,	.38450,	.7375,	.13080E-1,	540500,\$
F.07000,	.38000,	.7408,	.12700E-1,	534200,\$
F.07400,	.37550,	.7460,	.12330E-1,	528500,\$
F.07800,	.37140,	.7522,	.12000E-1,	523400,\$
F.08200,	.36700,	.7595,	.11690E-1,	518000,\$
F.08600,	.36300,	.7677,	.11410E-1,	512900,\$
F.09000,	.35900,	.7745,	.11150E-1,	507600,\$
F.09400,	.35480,	.7827,	.10800E-1,	502500,\$
F.09800,	.35100,	.7917,	.10670E-1,	497000,\$
F.10200,	.34700,	.8013,	.10445E-1,	491500,\$
F.10600,	.34320,	.8110,	.10245E-1,	485600,\$
F.11000,	.33950,	.8217,	.10045E-1,	479700,\$
F.11400,	.33580,	.8327,	.98500E-2,	473500,\$
F.11800,	.33210,	.8440,	.96600E-2,	466750,\$
F.12000,	.33030,	.8490,	.95700E-2,	463400,\$
F.12200,	.32850,	.8537,	.94800E-2,	459700,\$
F.12264,	.32803,	.8552,	.94428E-2,	458650,\$

Shock Equation

- A(1) = -.061114073
- A(3) = .73766679
- A(5) = .048917737
- A(7) = 4.6635489
- A(9) = -21.582338
- A(11) = 24.521790

Input

- $P_\infty = 2116.2$
- $\rho_\infty = 2.3769E-3$
- $V_\infty = 20,095.2$

x

P

- .009186 835000
- .019369 791000
- .033210 735000
- .050603 675470
- .071416 604700
- .095492 535140
- .122645 458650
- .152671 385550
- .185340 321500

All Other Coefficients = 0

$M_\infty = 18.1$ Real Gas
 Shock Angle = .95576 Three-Dimensional
 Altitude 175,000 Feet

x	r	θ	ρ	P
F.08945	.35778	.8369	.16746E-4	346.780, \$
F.09040	.35690	.8369	.16450E-4	343.400, \$
F.09160	.35580	.8370	.16100E-4	339.100, \$
F.09360	.35400	.8370	.15540E-4	332.600, \$
F.09580	.35200	.8370	.14960E-4	326.100, \$
F.09780	.35020	.8371	.14480E-4	320.600, \$
F.09980	.34840	.8373	.14030E-4	315.500, \$
F.10200	.34640	.8375	.13580E-4	310.250, \$
F.10400	.34460	.8379	.13225E-4	306.000, \$
F.10600	.34280	.8383	.12890E-4	302.000, \$
F.10820	.34080	.8390	.12550E-4	297.900, \$
F.11020	.33900	.8401	.12280E-4	294.500, \$
F.11220	.33720	.8419	.12050E-4	291.400, \$
F.11420	.33525	.8445	.11800E-4	288.400, \$
F.11640	.33350	.8472	.11610E-4	285.800, \$
F.11840	.33170	.8506	.11450E-4	283.500, \$
F.12060	.32980	.8540	.11275E-4	281.000, \$
F.12160	.32895	.8547	.11210E-4	279.900, \$
F.12265	.32803	.8552	.11141E-4	278.840, \$

Shock Equation	Input	x	P
A(1) = -.02820980	$P_\infty = 1.2334$.0027390	523.97
A(3) = .7464340	$\rho_\infty = .14123E-5$.0091863	512.84
A(5) = 2.949250	$V_\infty = 20,000$.0193690	492.23
A(7) = -21.32760		.0332097	460.79
A(9) = 94.07350		.0506030	419.64
A(11) = -188.7800		.0714163	372.41
All Other Other Coefficients = 0		.0954917	324.41
		.1226453	278.84
		.1526707	236.78
		.1853397	196.57

$M_\infty = 19.25$ Real Gas
 Shock Angle = .94404 Three Dimensional
 Altitude = 200,000 Feet

x	r	θ	ρ	P
F.08988,	.35707,	.8303,	.73945E-5,	147.660,\$
F.09100,	.35610,	.8310,	.72950E-5,	145.600,\$
F.09200,	.35520,	.8316,	.71400E-5,	143.850,\$
F.09400,	.35340,	.8328,	.68450E-5,	140.550,\$
F.09600,	.35160,	.8340,	.66050E-5,	137.700,\$
F.09800,	.34985,	.8352,	.63750E-5,	135.100,\$
F.10000,	.34805,	.8364,	.61650E-5,	132.600,\$
F.10220,	.34610,	.8379,	.59600E-5,	130.250,\$
F.10420,	.34435,	.8392,	.57950E-5,	128.400,\$
F.10620,	.34250,	.8406,	.56500E-5,	126.600,\$
F.10820,	.34075,	.8420,	.55150E-5,	125.000,\$
F.11040,	.33880,	.8437,	.53800E-5,	123.350,\$
F.11240,	.33700,	.8454,	.52700E-5,	122.000,\$
F.11440,	.33525,	.8471,	.51750E-5,	120.800,\$
F.11660,	.33330,	.8495,	.50800E-5,	119.850,\$
F.11860,	.33150,	.8520,	.50100E-5,	118.400,\$
F.12060,	.32980,	.8541,	.49400E-5,	117.500,\$
F.12160,	.32895,	.8547,	.49100E-5,	117.050,\$
F.12265,	.32803,	.8552,	.48791E-5,	116.620,\$

Shock Equation

A(1)	=	-.0272035
A(3)	=	.775645
A(5)	=	1.56676
A(7)	=	-2.63521
A(9)	=	.593935
A(11)	=	-28.3261

Input

P_∞	=	.47151
ρ_∞	=	.6118E-6
V_∞	=	20,000

x

P

.002739	232.39
.0091863	223.01
.019369	211.16
.0332097	197.89
.0506030	183.13
.0714163	162.51
.0954917	139.44
.1226453	116.62
.1526707	97.18
.1853397	80.96

All Other Coefficients = 0

$M_\infty = 19.52$

Real Gas

Shock Angle = .95119

Three-Dimensional

Altitude = 150,000 Feet

	x	r	θ	ρ	P
F.08833,	.35876,	.8296,	.40887E-4,	994.57,	\$
F.08940,	.35780,	.8300,	.40150E-4,	984.40,	\$
F.09060,	.35670,	.8305,	.39300E-4,	973.00,	\$
F.09300,	.35450,	.8314,	.37700E-4,	952.00,	\$
F.09520,	.35255,	.8323,	.36310E-4,	934.40,	\$
F.09740,	.35050,	.8332,	.35050E-4,	918.00,	\$
F.09980,	.34840,	.8343,	.33800E-4,	902.00,	\$
F.10200,	.34635,	.8353,	.32790E-4,	888.00,	\$
F.10440,	.34425,	.8366,	.31840E-4,	874.00,	\$
F.10660,	.34225,	.8379,	.31050E-4,	863.00,	\$
F.10900,	.34010,	.8395,	.30300E-4,	852.10,	\$
F.11120,	.33810,	.8412,	.29690E-4,	842.00,	\$
F.11340,	.33615,	.8434,	.29175E-4,	831.00,	\$
F.11580,	.33405,	.8464,	.28640E-4,	824.50,	\$
F.11800,	.33210,	.8501,	.28200E-4,	816.60,	\$
F.12040,	.32995,	.8539,	.27750E-4,	808.80,	\$
F.12140,	.32910,	.8547,	.27600E-4,	805.50,	\$
F.12265,	.32803,	.8552,	.27397E-4,	802.00,	\$

Shock Equation	Input	x	P
A(1) = -.028473534	$P_\infty = 3.0597$.002739	1535.1
A(3) = .76620454	$\rho_\infty = 3.5642E-6$.009186	1486.8
A(5) = 1.3474288	$V_\infty = 21,400$.019369	1425.9
A(7) = 1.9468755		.033210	1334.0
A(9) = -16.885667		.050603	1218.3
A(11) = 27.864783		.071416	1084.6
All Other Coefficients = 0		.095492	943.21
		.122645	802.00
		.152671	675.75
		.185340	555.00

$M_\infty = 20$
 Shock Angle = .951
 Altitude = 60,430 Feet

Real Gas
 Three-Dimensional

	x	r	θ	ρ	P
F.08408,	.36347,	.8349,	.22413E-2,	51485,	\$
F.08540,	.36225,	.8345,	.22040E-2,	50850,	\$
F.08680,	.36090,	.8340,	.21600E-2,	50190,	\$
F.08920,	.35865,	.8332,	.20875E-2,	49150,	\$
F.09180,	.35620,	.8326,	.20120E-2,	48180,	\$
F.09440,	.35375,	.8321,	.19390E-2,	47325,	\$
F.09700,	.35130,	.8317,	.18690E-2,	46500,	\$
F.09960,	.34890,	.8316,	.18050E-2,	45750,	\$
F.10200,	.34670,	.8319,	.17500E-2,	45125,	\$
F.10500,	.34390,	.8325,	.16910E-2,	44425,	\$
F.10720,	.34180,	.8334,	.16545E-2,	43950,	\$
F.10980,	.33950,	.8350,	.16150E-2,	43450,	\$
F.11240,	.33710,	.8376,	.15800E-2,	42975,	\$
F.11500,	.33470,	.8412,	.15510E-2,	42525,	\$
F.11760,	.33240,	.8455,	.15250E-2,	42100,	\$
F.12000,	.33030,	.8500,	.15025E-2,	41740,	\$
F.12140,	.32905,	.8527,	.14910E-2,	41520,	\$
F.12265,	.32803,	.8552,	.14816E-2,	41333,	\$

Shock Equation

A(1) =	-.03248550
A(3) =	.694825
A(5) =	4.279610
A(7) =	-28.2064
A(9) =	89.5846
A(11) =	-115.997

Input

$P_\infty =$	147.31
$\rho_\infty =$	2.2E-4
$V_\infty =$	19,361.6

x P

.0027390	74948
.0091863	73657
.0193690	70799
.0332097	66237
.0506030	60054
.0714163	53160
.0954917	46921
.1226453	41333
.1526707	35233
.1853397	30100

All Other Coefficients = 0

$M_\infty = 20$ Real Gas
 Shock Angle = .9504800 Three-Dimensional
 Altitude 120,000 Feet

x	r	θ	ρ	P
F.08810	.35930	.8281	.14453E-3	3336.8, \$
F.08900	.35850	.8286	.14275E-3	3307.0, \$
F.09100	.35665	.8302	.13775E-3	3249.0, \$
F.09300	.35483	.8312	.13250E-3	3189.5, \$
F.09500	.35300	.8317	.12795E-3	3135.0, \$
F.09700	.35115	.8322	.12375E-3	3086.0, \$
F.09900	.34930	.8329	.12000E-3	3040.0, \$
F.10100	.34750	.8335	.11665E-3	3000.0, \$
F.10300	.34565	.8344	.11355E-3	2962.5, \$
F.10500	.34385	.8356	.11100E-3	2927.0, \$
F.10700	.34205	.8367	.10850E-3	2894.0, \$
F.10900	.34025	.8382	.10600E-3	2862.5, \$
F.11100	.33850	.8400	.10425E-3	2834.0, \$
F.11300	.33670	.8422	.10250E-3	2805.0, \$
F.11500	.33490	.8447	.10095E-3	2780.0, \$
F.11700	.33315	.8476	.99350E-4	2758.0, \$
F.11900	.33137	.8512	.98000E-4	2734.5, \$
F.12100	.32950	.8545	.96850E-4	2712.0, \$
F.12264	.32803	.8552	.95800E-4	2693.2, \$

Shock Equation	Input	x	P
A(1) = -.028473534	$P_\infty = 9.8372$.002739	5155.5
A(3) = .76620454	$\rho_\infty = 1.2697E-5$.019186	4993.3
A(5) = 1.3474288	$V_\infty = 20,830$.019369	4788.8
A(7) = 1.9468755		.033210	4480.1
A(9) = -16.885667		.050603	4091.5
A(11) = 27.864783		.071416	3642.4
All Other Coefficients = 0		.095492	3167.5
		.122645	2613.2
		.152671	2269.2
		.185340	1863.6

$M_\infty = 20$ Real Gas
 Shock Angle = .9734600 Three-Dimensional
 Altitude = 200,000 Feet

x	r	θ	ρ	P
F.09255,	.35501,	.8602,	.78322E-5,	168.33,\$
F.09360,	.35410,	.8585,	.73900E-5,	165.95,\$
F.09460,	.35315,	.8566,	.75500E-5,	163.60,\$
F.09660,	.35135,	.8531,	.72800E-5,	159.40,\$
F.09860,	.34955,	.8497,	.70100E-5,	155.50,\$
F.10060,	.34770,	.8467,	.67450E-5,	151.95,\$
F.10260,	.34590,	.8440,	.65150E-5,	148.85,\$
F.10460,	.34410,	.8418,	.63000E-5,	146.00,\$
F.10660,	.34230,	.8402,	.61100E-5,	143.30,\$
F.10880,	.34050,	.8397,	.59400E-5,	140.90,\$
F.11060,	.33875,	.8405,	.57900E-5,	138.85,\$
F.11260,	.33700,	.8425,	.56450E-5,	137.05,\$
F.11460,	.33520,	.8445,	.55100E-5,	135.40,\$
F.11660,	.33340,	.8470,	.54000E-5,	134.00,\$
F.11860,	.33160,	.8493,	.53000E-5,	132.80,\$
F.12060,	.32980,	.8525,	.52100E-5,	131.70,\$
F.12160,	.32898,	.8538,	.51700E-5,	131.20,\$
F.12265,	.32803,	.8552,	.51248E-5,	130.70,\$

Shock Equation	Input	x	P
A(1) = -.0641084	$P_\infty = .47518$.0027390	249.70
A(3) = .865146	$\rho_\infty = 6.0583E-7$.0091863	242.00
A(5) = -1.14338	$V_\infty = 20,958.0$.0196390	230.70
A(7) = 10.2636		.0332097	215.00
A(9) = -32.7591		.0506030	194.50
A(11) = 26.3273		.0714163	174.55
All Other Coefficients = 0		.0954917	152.60
		.1226453	130.70
		.1526707	114.48
		.1853397	103.75

$M_\infty = 20$ Ideal Gas
 Shock Angle = .95712 Three-Dimensional
 Altitude = 200,000 Feet

	x	r	θ	ρ	P
F.08901,	.43181,	.7206,	.35682E-5,	148.13,	\$
F.09100,	.42949,	.7200,	.35120E-5,	146.50,	\$
F.09300,	.42700,	.7196,	.34550E-5,	144.85,	\$
F.09700,	.42220,	.7184,	.33350E-5,	141.60,	\$
F.10100,	.41750,	.7182,	.32150E-5,	138.60,	\$
F.10500,	.41265,	.7180,	.30940E-5,	135.70,	\$
F.10900,	.40795,	.7180,	.29780E-5,	133.00,	\$
F.11300,	.40325,	.7186,	.28730E-5,	130.35,	\$
F.11700,	.39850,	.7200,	.27740E-5,	127.70,	\$
F.12100,	.39395,	.7225,	.26850E-5,	125.30,	\$
F.12500,	.38950,	.7268,	.26000E-5,	122.90,	\$
F.12900,	.38500,	.7325,	.25250E-5,	120.45,	\$
F.13300,	.38060,	.7384,	.24550E-5,	118.15,	\$
F.13700,	.37625,	.7445,	.23870E-5,	115.90,	\$
F.14100,	.37190,	.7510,	.23220E-5,	115.60,	\$
F.14500,	.36750,	.7575,	.22550E-5,	111.40,	\$
F.14900,	.36345,	.7646,	.21920E-5,	109.10,	\$
F.15100,	.36140,	.7679,	.21600E-5,	108.00,	\$
F.15267,	.35967,	.7679,	.21338E-5,	107.00,	\$

Shock Equation

A(1) =	-.06410840
A(3) =	.8651460
A(5) =	-1.143380
A(7) =	10.26360
A(9) =	-32.75910
A(11) =	26.32730

All Other Coefficients = 0

Input	x	P
$P_\infty =$.47518	.0027390 242.75
$\rho_\infty =$	6.0583E-7	.0091863 234.41
$V_\infty =$	20,958.0	.0193690 221.60
		.0332097 204.91
		.0506030 186.56
		.0714163 167.09
		.0954917 148.17
		.1226453 127.03
		.1526707 107.00
		.1853397 89.20

$M_\infty = 20.7$ Real Gas
 Shock Angle = .94767 Three-Dimensional
 Altitude = 225,000 Feet

x	r	θ	ρ	P
F.09126,	.35625,	.8382,	.29852E-5,	57.429, \$
F.09240,	.35520,	.8382,	.28900E-5,	56.775, \$
F.09340,	.35430,	.8382,	.28200E-5,	56.200, \$
F.09540,	.35245,	.8384,	.27130E-5,	55.100, \$
F.09760,	.35050,	.8386,	.26180E-5,	53.960, \$
F.09960,	.34870,	.8388,	.25360E-5,	52.980, \$
F.10180,	.34670,	.8392,	.24500E-5,	51.990, \$
F.10380,	.34490,	.8397,	.23850E-5,	51.150, \$
F.10600,	.34295,	.8404,	.23150E-5,	50.325, \$
F.10800,	.34120,	.8412,	.22550E-5,	49.650, \$
F.11000,	.33940,	.8422,	.22025E-5,	49.040, \$
F.11220,	.33745,	.8437,	.21525E-5,	48.400, \$
F.11420,	.33570,	.8455,	.21125E-5,	47.875, \$
F.11640,	.33370,	.8481,	.20740E-5,	47.375, \$
F.11840,	.33190,	.8508,	.20410E-5,	46.930, \$
F.12060,	.32990,	.8536,	.20075E-5,	46.475, \$
F.12160,	.32900,	.8545,	.19930E-5,	46.285, \$
F.12264,	.32803,	.8552,	.19794E-5,	46.100, \$

Shock Equation

Shock Equation	Input	x	P
A(1) = -.0255625	$P_\infty = .15719$.002739	88.750
A(3) = .792294	$\rho_\infty = .23571$.009186	85.875
A(5) = 1.93497	$V_\infty = 20,000$.019369	82.127
A(7) = -12.9126		.033210	76.893
A(9) = 57.7919		.050603	70.800
A(11) = -99.9061		.071416	63.500
		.095492	54.000
		.122645	46.100
		.152671	39.100
		.185340	32.600
		.220405	26.500
		.257595	20.600

All Other Coefficients = 0

$M_\infty = 20.933$ Real Gas
 Shock Angle = .9493600 Three-Dimensional
 Altitude = 100,000 Feet

x	r	θ	ρ	P
F.08745,	.36023,	.8240,	.35581E-3,	8537.3, \$
F.08900,	.35880,	.8252,	.34650E-3,	8420.0, \$
F.09100,	.35695,	.8268,	.33500E-3,	8282.0, \$
F.09300,	.35510,	.8282,	.32300E-3,	8155.0, \$
F.09500,	.35330,	.8295,	.31350E-3,	8025.0, \$
F.09700,	.35145,	.8307,	.30370E-3,	7905.0, \$
F.09900,	.34965,	.8317,	.29500E-3,	7800.0, \$
F.10200,	.34690,	.8328,	.28344E-3,	7647.2, \$
F.10600,	.34327,	.8350,	.27072E-3,	7469.8, \$
F.11000,	.33964,	.8381,	.26010E-3,	7314.0, \$
F.11300,	.33700,	.8413,	.25370E-3,	7215.0, \$
F.11500,	.33518,	.8439,	.24938E-3,	7145.1, \$
F.11700,	.33400,	.8469,	.24600E-3,	7085.0, \$
F.11900,	.33165,	.8505,	.24225E-3,	7025.1, \$
F.12100,	.32960,	.8538,	.23900E-3,	6965.0, \$
F.12264,	.32803,	.8552,	.23616E-3,	6916.9, \$

Shock Equation	Input	x	P
A(1) = -.029720581	$P_\infty = 23.085$.002739	13325.0
A(3) = .7741153	$\rho_\infty = 3.2114E-5$.009186	12870.0
A(5) = 1.1871262	$V_\infty = 21,000$.019369	12291.0
A(7) = .2720463		.033210	11498.0
A(9) = -13.399047		.050603	10501.0
A(11) = 19.744798		.071416	9348.0
		.095492	8128.9
		.122645	6916.9
		.152671	5822.8
		.185340	4781.7
		.220405	3850.0
		.257595	3050.0

All Other Coefficients = 0

$M_\infty = 23.716$ Real Gas
 Shock Angle = .94568 Three-Dimensional
 Altitude = 150,000 Feet

	x	r	θ	ρ	P
F.09404,	.35358,	.8470,	.49899E-4,	1474.6,	\$
F.09500,	.35270,	.8445,	.48800E-4,	1458.0,	\$
F.09700,	.35090,	.8421,	.46650E-4,	1426.0,	\$
F.09900,	.34910,	.8409,	.44700E-4,	1393.5,	\$
F.10100,	.34730,	.8404,	.42800E-4,	1363.0,	\$
F.10300,	.34550,	.8404,	.41200E-4,	1336.0,	\$
F.10500,	.34370,	.8406,	.39650E-4,	1311.0,	\$
F.10700,	.34190,	.8410,	.38350E-4,	1290.0,	\$
F.10900,	.34010,	.8416,	.37150E-4,	1271.5,	\$
F.11100,	.33840,	.8426,	.36100E-4,	1254.0,	\$
F.11300,	.33660,	.8439,	.35200E-4,	1240.0,	\$
F.11500,	.33485,	.8458,	.34450E-4,	1226.5,	\$
F.11700,	.33310,	.8485,	.33800E-4,	1214.0,	\$
F.11900,	.33130,	.8516,	.33230E-4,	1204.0,	\$
F.12100,	.32950,	.8543,	.32700E-4,	1194.0,	\$
F.12264,	.32803,	.8552,	.32416E-4,	1186.2,	\$

Shock Equation	Input	x	P
A(1) = -.02430903	$P_\infty = 3.0597$.002739	2269.70
A(3) = .68683618	$\rho_\infty = 3.5642E-6$.009186	2198.30
A(5) = 4.5451286	$V_\infty = 26,000$.019369	2108.30
A(7) = -32.063228		.033210	1972.40
A(9) = 113.92778		.050603	1801.40
A(11) = -157.14619		.071416	1603.80
All Other Coefficients = 0		.095492	1394.90
		.122645	1186.20
		.152671	999.58
		.185340	821.08

$M_\infty = 25$ Real Gas
 Shock Angle = .94102 Three-Dimensional
 Altitude = 225,000

x	r	θ	ρ	P
F.09527	.35237	.8483	.34798E-5	83.903, \$
F.09600	.35170	.8479	.34375E-5	83.100, \$
F.09800	.34990	.8470	.32550E-5	81.050, \$
F.10000	.34810	.8463	.30950E-5	79.150, \$
F.10200	.34630	.8458	.29550E-5	77.350, \$
F.10400	.34460	.8455	.28350E-5	75.800, \$
F.10600	.34280	.8453	.27300E-5	74.400, \$
F.10800	.34100	.8456	.26500E-5	73.200, \$
F.11000	.33930	.8459	.25750E-5	72.100, \$
F.11200	.33750	.8465	.25080E-5	71.100, \$
F.11400	.33580	.8476	.24500E-5	70.200, \$
F.11600	.33400	.8491	.23980E-5	69.450, \$
F.11800	.33220	.8512	.23500E-5	68.700, \$
F.12000	.33045	.8533	.23150E-5	68.100, \$
F.12200	.32870	.8548	.22750E-5	67.550, \$
F.12264	.32803	.8552	.22625E-5	67.321, \$

Shock Equation	Input	x	P
A(1) = -.022431390	$P_\infty = .15719$.002739	130.500
A(3) = .85812356	$\rho_\infty = .23571E-6$.009186	125.730
A(5) = .014637381	$V_\infty = 24156.2$.019369	120.240
A(7) = 10.910602		.033210	112.570
A(9) = -52.590469		.050603	103.030
A(11) = 73.959399		.071416	92.174
All Other Coefficients = 0		.095492	79.670
		.122645	67.321
		.152671	56.888
		.185340	47.000
		.220405	37.050
		.257595	28.200

$M_\infty = 25.031$
 Shock Angle = .9329000
 Altitude = 200,000 Feet

Real Gas
 Three-Dimensional

x	r	θ	ρ	P
F.09526	.35216	.8433	.91906E-5	249.610, \$
F.09600	.35160	.8434	.89800E-5	247.100, \$
F.09800	.34980	.8439	.85100E-5	241.400, \$
F.10000	.34800	.8444	.80750E-5	235.600, \$
F.10200	.34620	.8449	.76950E-5	230.400, \$
F.10400	.34440	.8455	.73750E-5	225.900, \$
F.10600	.34270	.8461	.71000E-5	221.800, \$
F.10800	.34090	.8466	.68700E-5	218.200, \$
F.11000	.33910	.8472	.66700E-5	215.050, \$
F.11200	.33740	.8478	.64900E-5	212.100, \$
F.11400	.33560	.8486	.63400E-5	209.500, \$
F.11600	.33380	.8499	.62050E-5	207.200, \$
F.11800	.33210	.8518	.60800E-5	205.100, \$
F.12000	.33030	.8541	.57750E-5	203.150, \$
F.12200	.32860	.8551	.58750E-5	201.350, \$
F.12264	.32803	.8552	.58460E-5	200.870, \$

Shock Equation

- A(1) = -.022431390
- A(3) = .85812356
- A(5) = .014637381
- A(7) = 10.910602
- A(9) = -52.590469
- A(11) = 73.959399

All Other Coefficients = 0

Input	x	P
$P_\infty = .47151$.002739	392.29
$\rho_\infty = 6.118E-7$.009186	379.50
$V_\infty = 26,000$.019369	361.45
	.033210	338.39
	.050603	309.71
	.071416	277.08
	.095492	238.75
	.122645	200.87
	.152671	166.36
	.185340	134.00

$M_\infty = 25.917$ Real Gas
 Shock Angle = .94351 Three-Dimensional
 Altitude = 100,000 Feet

x	r	θ	ρ	P
F.09125,	.35646,	.8366,	.41298E-3,	13149, \$
F.09300,	.35485,	.8365,	.39750E-3,	12920, \$
F.09500,	.35305,	.8364,	.38050E-3,	12665, \$
F.09700,	.35120,	.8364,	.36490E-3,	12420, \$
F.09900,	.34940,	.8364,	.35080E-3,	12200, \$
F.10100,	.34755,	.8365,	.33860E-3,	12000, \$
F.10300,	.34575,	.8368,	.32780E-3,	11825, \$
F.10500,	.34392,	.8374,	.31800E-3,	11660, \$
F.10700,	.34214,	.8383,	.30960E-3,	11500, \$
F.10900,	.34040,	.8394,	.30200E-3,	11370, \$
F.11100,	.33860,	.8408,	.27550E-3,	11240, \$
F.11300,	.33680,	.8427,	.28950E-3,	11120, \$
F.11500,	.33508,	.8450,	.28450E-3,	11020, \$
F.11700,	.33320,	.8478,	.27950E-3,	10915, \$
F.11900,	.33140,	.8511,	.27520E-3,	10815, \$
F.12100,	.32960,	.8540,	.27080E-3,	10720, \$
F.12265,	.32803,	.8552,	.26725E-3,	10646, \$

Shock Equation	Input	x	P
A(1) = -.025465172	$P_\infty = 73.784$.002739	20520
A(3) = .79267501	$\rho_\infty = 1.1022E-4$.009187	19819
A(5) = .58500589	$V_\infty = 26,000$.019369	18926
A(7) = 6.1853521		.033210	17704
A(9) = -33.088087		.050603	16168
A(11) = 40.844931		.071416	14392
All Other Coefficients = 0		.095492	12513
		.122646	10646
		.152671	8960
		.185340	7356
		.220405	5920
		.257595	4687

$M_\infty = 26.857$ Real Gas
 Shock Angle = .94235 Three-Dimensional
 Altitude = 75,000 Feet

x	r	θ	ρ	P
F.08989,	.35780,	.8302,	.13469E-2,	44851, \$
F.09100,	.35680,	.8306,	.13170E-2,	44400, \$
F.09300,	.35492,	.8313,	.12640E-2,	43575, \$
F.09500,	.35310,	.8320,	.12110E-2,	42775, \$
F.09700,	.35130,	.8328,	.11660E-2,	42040, \$
F.09900,	.34950,	.8336,	.11240E-2,	41375, \$
F.10100,	.34770,	.8343,	.10880E-2,	40775, \$
F.10300,	.34580,	.8351,	.10575E-2,	40220, \$
F.10500,	.34400,	.8361,	.10285E-2,	39695, \$
F.10700,	.34220,	.8371,	.10030E-2,	39200, \$
F.10900,	.34040,	.8384,	.98100E-3,	38775, \$
F.11100,	.33865,	.8401,	.96100E-3,	38350, \$
F.11300,	.33690,	.8420,	.94200E-3,	37950, \$
F.11500,	.33510,	.8445,	.92550E-3,	37600, \$
F.11700,	.33330,	.8474,	.91000E-3,	37250, \$
F.11900,	.33148,	.8508,	.89600E-3,	36940, \$
F.12100,	.32960,	.8538,	.88200E-3,	36600, \$
F.12265,	.32803,	.8552,	.87173E-3,	36352, \$

Shock Equation	Input	x	P
A(1) = -.02725	$P_\infty = 73.784$.002739	70228
A(3) = .79267501	$\rho_\infty = 1.1022E-4$.009187	67875
A(5) = .58500589	$V_\infty = 26,000$.019369	64816
A(7) = 6.1853521		.033210	60630
A(9) = -33.088187		.050603	55370
A(11) = 40.84493		.071416	49288
		.095492	42852
		.122646	36352
		.152672	30508
		.185340	24994
		.220485	19909
		.257595	14800

All Other Coefficients = 0

$M_\infty = 30$ Real Gas
 Shock Angle = .93288 Three-Dimensional
 Altitude = 275,000 Feet

	x	r	θ	ρ	P
F.09719,	.35046,	.8529,	.32142E-6,	7.483,	\$
F.09800,	.34975,	.8526,	.31180E-6,	7.390,	\$
F.10000,	.34800,	.8519,	.28900E-6,	7.175,	\$
F.10200,	.34625,	.8512,	.27000E-6,	6.980,	\$
F.10400,	.34448,	.8506,	.25480E-6,	6.816,	\$
F.10600,	.34274,	.8499,	.24310E-6,	6.670,	\$
F.10800,	.34097,	.8494,	.23350E-6,	6.542,	\$
F.11000,	.33920,	.8491,	.22550E-6,	6.430,	\$
F.11200,	.33744,	.8492,	.21870E-6,	6.338,	\$
F.11400,	.33567,	.8496,	.21300E-6,	6.255,	\$
F.11600,	.33390,	.8505,	.20850E-6,	6.179,	\$
F.11800,	.33218,	.8521,	.20450E-6,	6.110,	\$
F.12000,	.33042,	.8538,	.20100E-6,	6.050,	\$
F.12200,	.32860,	.8549,	.19770E-6,	5.998,	\$
F.12264,	.32803,	.8552,	.19680E-6,	5.982,	\$

Shock Equation	Input	x	P
A(1) = -.020016780	$P_\infty = 9.771E-3$.002739	11.660
A(3) = .83564530	$\rho_\infty = 1.909E-8$.009186	11.276
A(5) = .63854486	$V_\infty = 25,395$.019369	10.784
A(7) = 4.6822268		.033210	10.096
A(9) = -20.755696		.050603	9.2394
A(11) = 18.940326		.071416	8.2654
All Other Coefficients = 0		.095492	7.1268
		.122645	5.9819
		.152671	4.9975
		.185340	4.1300
		.220405	3.2500
		.257595	2.4600

$M_\infty = 35.885$ Real Gas
 Shock Angle = .92413 Three-Dimensional
 Altitude = 100,000 Feet

	x	r	θ	ρ	P
F.09270,	.35474,	.8348,	.47485E-3,	24732.7,	\$
F.09340,	.35415,	.8350,	.46520E-3,	24560.0,	\$
F.09400,	.35350,	.8351,	.45350E-3,	24345.0,	\$
F.09600,	.35174,	.8357,	.42015E-3,	23752.0,	\$
F.09800,	.34995,	.8363,	.39200E-3,	23215.0,	\$
F.10000,	.34815,	.8369,	.36750E-3,	22740.0,	\$
F.10200,	.34635,	.8375,	.34650E-3,	22325.0,	\$
F.10400,	.34455,	.8382,	.33050E-3,	21960.0,	\$
F.10600,	.34280,	.8389,	.31600E-3,	21635.0,	\$
F.10800,	.34095,	.8398,	.30450E-3,	21335.0,	\$
F.11000,	.33920,	.8410,	.29450E-3,	21075.0,	\$
F.11200,	.33735,	.8425,	.28600E-3,	20825.0,	\$
F.11400,	.33560,	.8444,	.27850E-3,	20605.0,	\$
F.11600,	.33380,	.8469,	.27200E-3,	20400.0,	\$
F.11800,	.33200,	.8499,	.26600E-3,	20210.0,	\$
F.12000,	.33025,	.8532,	.26050E-3,	20035.0,	\$
F.12200,	.32850,	.8548,	.25575E-3,	19860.0,	\$
F.12265,	.32803,	.8552,	.25423E-3,	19819.0,	\$

Shock Equation	Input	x	P
A(1) = -.02800	$P_\infty = 23.085$.002739	39326
A(3) = .835645	$\rho_\infty = 3.2114E-5$.009187	37916
A(5) = .638545	$V_\infty = 36,000$.019369	36222
A(7) = .468223		.033210	33934
A(9) = -20.7557		.050603	31027
A(11) = 18.9403		.071416	27480
		.095492	23677
		.122646	19819
		.152671	16344
		.185340	13150
		.220405	10260
		.257595	7900

All Other Coefficients = 0

APPENDIX VI

DERIVATION OF THE GOVERNING EQUATIONS FOR THE FLOW FIELD PERTURBATIONS

The governing equations for the perturbations in the supersonic region of the flow field are given in Section 4. 4. 2. These equations are derived from Equations (4. 6) through (4. 9) by specializing to the $\vec{j}, \vec{n}, \vec{\phi}$ coordinate system of Section 4. 4. 2 and making use of the perturbation forms (e. g. , Equation (5. 1)). The details of the derivation of Equation (4. 63) will be given in this appendix. The remaining equations of Section 4. 4. 2 can be obtained through a similar process.

The continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{V} = 0 \tag{4. 6}$$

can be specialized to the $\vec{j}, \vec{n}, \vec{\phi}$ coordinate system of Section 4. 4. 2 by using Equation (4. 6. 2) to conclude that:

$$\nabla \cdot \rho \vec{V} = \nabla \cdot (\rho V \vec{j} + \delta \rho w \vec{\phi}) \tag{A. 1}$$

$$= \rho V \nabla \cdot \vec{j} + \vec{j} \cdot \nabla (\rho V) + \delta \rho w \nabla \cdot \vec{\phi} + \delta \vec{\phi} \cdot \nabla (\rho w). \tag{A. 1}$$

The vectors \vec{j} and \vec{n} have been defined as

$$\left. \begin{aligned} \vec{j} &= \vec{x} \cos \theta + \vec{r} \sin \theta \\ \vec{n} &= -\vec{x} \sin \theta + \vec{r} \cos \theta \end{aligned} \right\} \tag{4. 60}$$

so that

$$\begin{aligned}
 \nabla \cdot \vec{J} &= \frac{\partial(\cos \theta)}{\partial x} + \left(\frac{1}{r}\right) \delta \frac{\partial [(r) \delta \sin \theta]}{\partial r} \\
 &= -\sin \theta \frac{\partial \theta}{\partial x} + \cos \theta \frac{\partial \theta}{\partial r} + \frac{\delta \sin \theta}{r} \\
 &= \frac{\partial \theta}{\partial n} + \frac{\delta \sin \theta}{r}
 \end{aligned}
 \tag{A-2}$$

and

$$\begin{aligned}
 \vec{J} \cdot \nabla (\rho V) &= \cos \theta \frac{\partial (\rho V)}{\partial x} + \sin \theta \frac{\partial (\rho V)}{\partial r} \\
 &= \frac{\partial (\rho V)}{\partial J}
 \end{aligned}$$

where $\frac{\partial}{\partial J}$ and $\frac{\partial}{\partial n}$ represent derivatives in the \vec{J} and \vec{n} directions.

Similarly

$$\delta \rho w \nabla \cdot \vec{\varphi} = 0$$

and

$$\delta \vec{\varphi} \cdot \nabla (\rho w) = \delta \left(\frac{1}{r}\right) \left(\frac{\partial (\rho w)}{\partial \varphi}\right)$$

Equations (A-2) and (A-3) substituted in Equation (A-1) result in:

$$\nabla \cdot \rho \vec{V} = \rho V \frac{\partial \theta}{\partial n} + \frac{\delta \rho V \sin \theta}{r} + \frac{\partial (\rho V)}{\partial J} + \frac{\delta}{r} \frac{\partial (\rho w)}{\partial \varphi}
 \tag{A-4}$$

and Equation (4.6) becomes:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho V)}{\partial \delta} + \rho V \frac{\partial \theta}{\partial n} + \frac{\delta}{r} \frac{\partial (\rho w)}{\partial \varphi} + \frac{\delta \rho V \sin \theta}{r} = 0 \quad (\text{A-5})$$

The perturbation forms (e. g. Equation (5.1)) can now be substituted in Equation (A-5).

The perturbation forms will be written in the following notation:

$$\rho = \rho_0 + \sum_{j=1}^2 \sum_{n=0}^{\infty} \rho_{j,n} \left(\frac{L}{V\infty} \right)^m \alpha_{j,n} \quad (\text{A-6})$$

where

$$\begin{aligned} m &= n && \text{when } j = 1 \\ &= n + 1 && \text{when } j = 2, \end{aligned}$$

$$\alpha_{j,n} = \frac{d^n \alpha_j}{dt^n}$$

and

$$\rho_{j,n} = \bar{\rho}_{j,n} \cos(\varphi\delta).$$

When Equation (A-6) and similar expressions* for V , θ and w are substituted in Equation (A-5), the following equation results (neglecting higher order terms in the perturbations $\alpha_{j,n}$):

* It should be noted that $w_0 = 0$, and that $\sin(\varphi\delta)$ appears in the expression for $w_{j,n}$ in place of $\cos(\varphi\delta)$.

$$\begin{aligned}
& \sum_{j=1}^2 \sum_{n=0}^{\infty} \rho_{j,n} \left(\frac{L}{V_{\infty}} \right)^m \alpha_{j,n+1} + \frac{\partial (\rho_0 V_0)}{\partial \beta_0} + \frac{\partial (\rho_0 V_0)}{\partial n_0} \sum_{j=1}^2 \sum_{n=0}^{\infty} \theta_{j,n} \left(\frac{L}{V_{\infty}} \right)^m \alpha_{j,n} \\
& + \sum_{j=1}^2 \sum_{n=0}^{\infty} \frac{\partial (\rho_0 V_{j,n} + V_0 \rho_{j,n})}{\partial \beta_0} \left(\frac{L}{V_{\infty}} \right)^m \alpha_{j,n} + \rho_0 V_0 \frac{\partial \theta_0}{\partial n_0} \\
& + \frac{\partial \theta_0}{\partial n_0} \sum_{j=1}^2 \sum_{n=0}^{\infty} (\rho_0 V_{j,n} + V_0 \rho_{j,n}) \left(\frac{L}{V_{\infty}} \right)^m \alpha_{j,n} \\
& - \rho_0 V_0 \frac{\partial \theta_0}{\partial \beta_0} \sum_{j=1}^2 \sum_{n=0}^{\infty} \theta_{j,n} \left(\frac{L}{V_{\infty}} \right)^m \alpha_{j,n} + \rho_0 V_0 \sum_{j=1}^2 \sum_{n=0}^{\infty} \frac{\partial \theta_{j,n}}{\partial n_0} \left(\frac{L}{V_{\infty}} \right)^m \alpha_{j,n} \\
& + \frac{\delta}{r} \sum_{j=1}^2 \sum_{n=0}^{\infty} \frac{\partial (\rho_0 w_{j,n})}{\partial \varphi} \left(\frac{L}{V_{\infty}} \right)^m \alpha_{j,n} + \frac{\delta \rho_0 V_0 \sin \theta_0}{r} \\
& + \frac{\delta \rho_0 V_0 \cos \theta_0}{r} \sum_{j=1}^2 \sum_{n=0}^{\infty} \theta_{j,n} \left(\frac{L}{V_{\infty}} \right)^m \alpha_{j,n} \\
& + \frac{\delta \sin \theta_0}{r} \sum_{j=1}^2 \sum_{n=0}^{\infty} (\rho_0 V_{j,n} + V_0 \rho_{j,n}) \left(\frac{L}{V_{\infty}} \right)^m \alpha_{j,n} = 0 \quad (A-7)*
\end{aligned}$$

* An explanation of the difference between $\partial/\partial \beta_0$ and $d/d \beta_0$ is given at the end of this appendix.

However, we note that Equation (3.1) implies that the zero yaw terms in Equation (A-7) add up to zero, i. e. that:

$$\frac{\partial (\rho_0 V_0)}{\partial \beta_0} + \rho_0 V_0 \frac{\partial \theta_0}{\partial n_0} + \frac{\delta \rho_0 V_0 \sin \theta_0}{r} = 0 \quad (\text{A-8})$$

We also note that the various perturbation motions $\alpha_{j,n}$ are independent of each other at any instant, so that Equation (A-7) implies an infinite number of equations, each containing only one of the motions $\alpha_{j,n}$. These facts permit us to drop the summation notation, and to divide each term in the equation by the quantity:

$$\alpha_{j,n} \left(\frac{L}{V_\infty} \right)^m .$$

The result is:

$$\begin{aligned} (1 - \delta_{0,n}) \rho_{j,n-1} \left(\frac{V_\infty}{L} \right) + \theta_{j,r} \frac{\partial (\rho_0 V_0)}{\partial n_0} + \frac{\partial (\rho_0 V_{j,n} + V_0 \rho_{j,n})}{\partial \beta_0} \\ + (\rho_0 V_{j,n} + V_0 \rho_{j,n}) \frac{\partial \theta_0}{\partial n_0} - \rho_0 V_0 \theta_{j,n} \frac{\partial \theta_0}{\partial \beta_0} + \rho_0 V_0 \frac{\partial \theta_{j,n}}{\partial n_0} \\ + \frac{\delta}{r} \frac{\partial (\rho_0 w_{j,n})}{\partial \varphi} + \frac{\delta \rho_0 V_0 (\cos \theta_0) \theta_{j,n}}{r} \\ + \frac{\delta (\sin \theta_0)}{r} (\rho_0 V_{j,n} + V_0 \rho_{j,n}) = 0 \end{aligned}$$

The above equation is seen to be Equation (4.64).

In obtaining the first term of Equation (4.64) from the first term of Equation (A-7), use was made of the following identity:

$$\sum_{j=1}^2 \sum_{n=0}^{\infty} \rho_{j,n} \left(\frac{L}{V_{\infty}} \right)^m \alpha_{j,n+1} = \sum_{j=1}^2 \sum_{n=1}^{\infty+1} \rho_{j,n-1} \left(\frac{L}{V_{\infty}} \right)^{m-1} \alpha_{j,n}$$

DERIVATIVES WITH RESPECT TO \mathcal{L} AND n

When taking derivatives with respect to distance along the streamlines or the lines normal to them, care must be taken to differentiate between the streamlines and normals of the perturbed field, and those of the zero yaw steady state field. In Equations (A-1) through (A-5) the derivatives were taken in the perturbed field. In carrying out the solution, however, it is more convenient to deal with a grid consisting of the zero yaw streamlines and normals. It is therefore necessary to express the derivatives $\partial F / \partial \mathcal{L}$ and $\partial F / \partial n$ (where F can represent any variable, e.g. ρ as given in Equation (A-6)) in terms of $\partial F / \partial \mathcal{L}_0$ and $\partial F / \partial n_0$, where the subscript zero signifies derivatives along the zero yaw grid.

In obtaining the desired expressions, it is convenient to make use of the $x \cdot r$ coordinate system by noting that the differential distances in the $\vec{\mathcal{L}}$ and \vec{n} directions are given by:

$$d\mathcal{L} = (\cos \theta) dx + (\sin \theta) dr$$

$$dn = -(\sin \theta) dx + (\cos \theta) dr$$

so that

$$\frac{\partial F}{\partial l} = (\cos \theta) \frac{\partial F}{\partial x} + (\sin \theta) \frac{\partial F}{\partial r}$$

and

$$\frac{\partial F}{\partial n} = -(\sin \theta) \frac{\partial F}{\partial x} + (\cos \theta) \frac{\partial F}{\partial r}$$

The variable θ can be expanded in the perturbation form given by Equation (A-6), so that

$$\begin{aligned} \frac{\partial F}{\partial l} = & \left[\cos \left(\theta_0 + \sum_{j=1}^2 \sum_{n=0}^{\infty} \theta_{j,n} \left(\frac{L}{V_{\infty}} \right)^{ln} \alpha_{j,n} \right) \right] \frac{\partial F}{\partial x} \\ & + \left[\sin \left(\theta_0 + \sum_{j=1}^2 \sum_{n=0}^{\infty} \theta_{j,n} \left(\frac{L}{V_{\infty}} \right)^m \alpha_{j,n} \right) \right] \frac{\partial F}{\partial r} \end{aligned}$$

and

$$\begin{aligned} \frac{\partial F}{\partial n} = & - \left[\sin \left(\theta_0 + \sum_{j=1}^2 \sum_{n=0}^{\infty} \theta_{j,n} \left(\frac{L}{V_{\infty}} \right)^m \alpha_{j,n} \right) \right] \frac{\partial F}{\partial x} \\ & + \left[\cos \left(\theta_0 + \sum_{j=1}^2 \sum_{n=0}^{\infty} \theta_{j,n} \left(\frac{L}{V_{\infty}} \right)^m \alpha_{j,n} \right) \right] \frac{\partial F}{\partial r} \end{aligned}$$

Applications of the well known trigonometric identities for the sine and cosine of the sum of two angles yields*:

$$\begin{aligned} \frac{\partial F}{\partial \theta} &= (\cos \theta_0) \frac{\partial F}{\partial x} - (\sin \theta_0) \frac{\partial F}{\partial r} \sum_{j=1}^2 \sum_{n=0}^{\infty} \theta_{j,n} \left(\frac{L}{V_{\infty}}\right)^m \alpha_{j,n} \\ &+ (\sin \theta_0) \frac{\partial F}{\partial r} + (\cos \theta_0) \frac{\partial F}{\partial x} \sum_{j=1}^2 \sum_{n=0}^{\infty} \theta_{j,n} \left(\frac{L}{V_{\infty}}\right)^m \alpha_{j,n} \end{aligned}$$

and

$$\begin{aligned} \frac{\partial F}{\partial n} &= -(\sin \theta_0) \frac{\partial F}{\partial x} - (\cos \theta_0) \frac{\partial F}{\partial r} \sum_{j=1}^2 \sum_{n=0}^{\infty} \theta_{j,n} \left(\frac{L}{V_{\infty}}\right)^m \alpha_{j,n} \\ &+ (\cos \theta_0) \frac{\partial F}{\partial r} - (\sin \theta_0) \frac{\partial F}{\partial x} \sum_{j=1}^2 \sum_{n=0}^{\infty} \theta_{j,n} \left(\frac{L}{V_{\infty}}\right)^m \alpha_{j,n} \end{aligned}$$

* The approximations $\cos \left(\sum_{j=1}^2 \sum_{n=0}^{\infty} \theta_{j,n} \left(\frac{L}{V_{\infty}}\right)^m \alpha_{j,n} \right) = 1.0$ and

$\sin \left(\sum_{j=1}^2 \sum_{n=0}^{\infty} \theta_{j,n} \left(\frac{L}{V_{\infty}}\right)^m \alpha_{j,n} \right) = \sum_{j=1}^2 \sum_{n=0}^{\infty} \theta_{j,n} \left(\frac{L}{V_{\infty}}\right)^m \alpha_{j,n}$ have

also been used.

The differential lengths $d\mathcal{J}$ and dn (when used without subscripts) signify distance along streamlines and normals in the perturbed flow. If we use the subscript zero to signify the zero yaw steady state field, we note that

$$\frac{\partial F}{\partial \mathcal{J}_0} = (\cos \theta_0) \frac{\partial F}{\partial x} + (\sin \theta_0) \frac{\partial F}{\partial r}$$

and

$$\frac{\partial F}{\partial n_0} = -(\sin \theta_0) \frac{\partial F}{\partial x} + (\cos \theta_0) \frac{\partial F}{\partial r}$$

Using the expressions for $\partial F/\partial \mathcal{J}_0$ and $\partial F/\partial n_0$ in the preceding expressions for $\partial F/\partial \mathcal{J}$ and $\partial F/\partial n$ yields:

$$\frac{\partial F}{\partial \mathcal{J}} = \frac{\partial F}{\partial \mathcal{J}_0} + \frac{\partial F}{\partial n_0} \sum_{j=1}^2 \sum_{n=0}^{\infty} \theta_{j,n} \left(\frac{L}{V^{\infty}} \right)^m \alpha_{j,n}$$

and

$$\frac{\partial F}{\partial n} = \frac{\partial F}{\partial n_0} - \frac{\partial F}{\partial \mathcal{S}_0} \sum_{j=1}^2 \sum_{n=0}^{\infty} \theta_{j,n} \left(\frac{L}{V^{\infty}} \right)^m \alpha_{j,n}$$

These expressions have been used in Equation (A-7).

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13. ABSTRACT <p>A method of computing static and dynamic force and moment coefficients has been prepared for the Research and Technology Division. The method provides for a complete inviscid flow field solution to be computed on the IBM 7094 machine. The solution provides values of all pertinent flow field parameters at a large number of points in the shock layer surrounding a pointed or spherically blunted body of revolution (or the analogous two-dimensional shapes) in supersonic or hypersonic flight. Perturbations (of each flow field parameter) due to angle of attack, rate of change of angle of attack, and pitch velocity are also computed. The force and moment coefficients are obtained by integrating the appropriate perturbations in pressure over the body surface.</p> <p>The above flow field capabilities have been delivered to the Research and Technology Division in the form of magnetic tapes. A sample solution was demonstrated on the Wright-Patterson AFB computer installation on June 30-July 1, 1964. The analysis and programming of this solution is described in this report.</p>		

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