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A NON-LINEAR SHOCK WAVE REFLECTION THEORY

by

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Ray C. Makino

January 1967

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U. S. ARMY MATERIEL COMMAND
BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

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Ray C. Makino

Computing Laboratory

RDT&E Project No. 1P014501A14B

A B E R D E E N   P R O V I N G   G R O U N D, M A R Y L A N D

B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

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January 1967

A NON-LINEAR SHOCK WAVE REFLECTION THEORY

ABSTRACT

A one-dimensional theory of normal reflection of blast waves from walls is given. The method satisfies the initial and boundary conditions of the problem. It is shown how the entire reflected wave zone, including the reflected shock front and the pressure and impulse on the wall, can be calculated.

TABLE OF CONTENTS

LIST OF SYMBOLS

$*$ = denotes dimensional quantities

a = a constraint parameter on u

b = a constraint parameter on u

c^*_{oo} = sound velocity in undisturbed air

c^*_o = sound velocity in region traversed by incident shock

$$c_o = \frac{c^*_o}{c^*_{oo}}$$

c^* = sound velocity in region traversed by reflected shock

$$c = \frac{c^*}{c^*_{oo}}$$

E^*_{oo} = specific internal energy of undisturbed air

E^*_o = specific internal energy in region traversed by incident shock

$$E_o = \frac{E^*_o - E^*_{oo}}{c^{*2}_{oo}}$$

E^* = specific internal energy in region traversed by reflected shock

$$E = \frac{E^* - E^*_{oo}}{c^{*2}_{oo}}$$

M^* = mass of explosive

p^*_{oo} = total pressure of undisturbed air

p^*_o = total pressure in region traversed by incident shock

$$p_o = \frac{p^*_o}{p^*_{oo}}$$

p^* = total pressure in region traversed by reflected shock

LIST OF SYMBOLS (Contd)

$p = \frac{p^*}{p^*_{oo}}$

R^* = gas constant

S^*_{oo} = specific entropy of undisturbed air

S^*_o = specific entropy in region traversed by incident shock

$S_o = \frac{S^*_o - S^*_{oo}}{R^*}$

S^* = specific entropy in region traversed by reflected shock

$S = \frac{S^* - S^*_{oo}}{R^*}$

t^* = time

$t = \left(\frac{c^*_{oo}\ p^*_{oo}}{M^*}\right)^{1/3} t^*$

u^*_o = particle velocity in region traversed by incident shock

u^* = particle velocity in region traversed by reflected shock

$u = \frac{u^*}{c^*_{oo}}$

U^* = velocity of reflected shock

$U = \frac{U^*}{c^*_{oo}}$

x^* = linear distance

$x = \left(\frac{p^*_{oo}}{c^{*2}_{oo}\ M^*}\right)^{1/3} x^*$

γ = specific heat ratio of air

$\mu = \left(\frac{\gamma - 1}{\gamma + 1}\right)^{1/2}$

LIST OF SYMBOLS (Contd)

ρ^*_{oo} = density of undisturbed air

ρ^*_o = density in region traversed by incident shock

$\rho_o = \dfrac{\gamma\rho^*_o}{\rho^*_{oo}}$

ρ^* = density in region traversed by reflected shock

$\rho = \dfrac{\gamma\rho^*}{\rho^*_{oo}}$

INTRODUCTION

The study of damage to structures by blast waves requires analysis of the normally reflected pressure on the wall. To solve this problem the hydrodynamical equations of flow must be solved in the region between the wall and the reflected shock front. Since the system of non-linear partial differential equations of flow are presently solvable only by numerical methods that, when reliable, are somewhat cumbersome, and the possibility of exact analytical solution is remote, a method that simplifies the mathematics somewhat and is more amenable to analytic solution is desirable. Here, we examine a direction of simplification that reduces the system of equations for one-dimensional flow to a system of ordinary differential equations, by introduction of a constraint that satisfies the initial and wall conditions, as discussed by Makino.[1*]

Makino and Shear[2] have obtained a theory of reflected impulse at the wall by regarding each element of the incident wave to be individually reflected like the shock front, but this is a zero'th order approximation, not satisfying derivative conditions at the wall. Chang and Laporte[3] have obtained two theories of shock reflection. One is series expansion about the point of reflection, which, if truncated for practical purposes, may not fully satisfy the wall conditions. The other theory, which assumes the particle velocity to be zero all along the reflected shock line, may also not fully satisfy the wall conditions. Also, both theories are for the calculation of the reflected shock line only. In the theory we present here, we consider calculation of the entire reflected wave zone, from the shock front through interior points to points on the wall, such that wall conditions are satisfied through certain derivatives. However, it is probably most useful only in that phase of the wave exerting the greatest stress on the wall, which is the part of greatest interest for damage studies.

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* *Superscript numbers denote references which may be found on page 26.*

Ryzhov and Khristianovich[4] have developed a theory on the problem of two dimensional regular reflection, but the theory assumes isentropic flow and is therefore applicable to weak shocks only, and further, the boundary conditions are approximated. Shindiapin[5] has improved the theory with respect to boundary conditions, but has not extended the theory to non-isentropic flow.

While the so-called self-similar type solutions[6,7] can be extended to the reflection problem as an approximation, the choice of the similarity form to be assumed is made difficult by the strong influence of the wall and by the non-constancy of the quantities in front of the reflected shock.

The example considered here is for plane flow, or spherical flow at distances sufficiently far from the center of energy release that planar approximation suffices. It is shown how the flow parameters behind the reflected wave, in particular the pressure on the wall as a function of time and the impulse, can be obtained. For cylindrical and spherical flows, the same method is applicable if the reflecting surfaces have the corresponding symmetries.

FLOW EQUATIONS

The non-dimensionalized equations of flow describing the one-dimensional motion of air are, in Eulerian coordinates,[8] conservation of mass

(1a) $$p_t + up_x + \rho c^2 u_x = 0 \ ,$$

conservation of momentum

(1b) $$\frac{1}{\rho} p_x + u_t + uu_x = 0 \ ,$$

adiabaticity

(1c) $$S_t + uS_x = 0,$$

where t is the time with the non-dimensionalizing scaling factor (ambient sound speed times ambient pressure/mass of explosive)$^{1/3}$, x is the distance with the non-dimensionalizing scaling factor (ambient pressure/(ambient sound speed)^2/mass of explosive)$^{1/3}$, p is the pressure in units of the ambient pressure, u is the particle velocity in units of the ambient sound speed, c is the sound velocity in units of the ambient sound speed, ρ is the specific heat ratio γ (assumed constant) times the density in units of the ambient density, and S is the excess entropy over the ambient in units of the gas constant R.

This system of equations is supplemented by the equation of state, which, for illustrative purpose, we assume to be ideal:

(2a) $$\rho = \gamma p^{1/\gamma} \exp\left(- \frac{\gamma - 1}{\gamma} S\right),$$

(2b) $$E = \frac{1}{\gamma - 1} \frac{p}{\rho} - \frac{1}{\gamma(\gamma - 1)} \quad ,$$

where E is the non-dimensionalized energy.

The Eulerian coordinates x, t in the equations above are replaced by Lagrange coordinates. We define m to be the mass integral

(3) $$m = \int \rho(x,t)\, dx \quad ,$$

where the integration is performed on a constant t line starting from the wall. From this definition and from the continuity Equation (1a) in the form

(4) $$\rho_t = -(\rho u)_x \quad ,$$

we obtain

(5a) $$m_x = \rho \quad ,$$

(5b) $$m_t = -\rho u \quad ,$$

and also

$$x_t(m,t) = u. \tag{6}$$

Using Equations (5) and (2), we put Equation (1) in the form

$$\begin{Vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \gamma p\rho & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{Vmatrix} V = \begin{Vmatrix} 0 \\ 0 \\ 0 \end{Vmatrix} , \tag{7}$$

where $V \equiv \text{col} \| S_t, p_t, u_t, u_m, p_m, S_m \|$.

Differentiation of Equation (7) with respect to m and t gives

$$\begin{Vmatrix} 1 & 0 & 0 & 0 & 0 & \gamma p\rho & 0 & 0 & 0 \\ 0 & 1 & \gamma p\rho & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{Vmatrix} W = \begin{Vmatrix} \frac{\gamma + 1}{\gamma} p_t \left(\frac{p_m}{p} - \mu^2 S_m \right) \\ \frac{\gamma + 1}{\gamma} \frac{p_t^2}{p} \\ 0 \\ 0 \\ 0 \\ 0 \end{Vmatrix} \tag{8}$$

where

$$W \equiv \text{col} \| p_{mt}, p_{tt}, u_{mt}, u_{tt}, p_{mm}, u_{mm}, S_{mm}, S_{mt}, S_{tt} \| . \tag{9}$$

SHOCK CONDITIONS

The Rankine-Hugoniot conditions that must be satisfied across the shock front are,[7] in dimensionless form,

conservation of mass,

$$\rho(U - u) = \rho_o(U - u_o) , \tag{10a}$$

conservation of momentum,

(10b) $$\rho(U - u)^2 + p = \rho_o(U - u_o)^2 + p_o \ ,$$

conservation of energy,

(10c) $$\frac{1}{2}(U - u)^2 + E + \frac{p}{\rho} = \frac{1}{2}(U - u_o)^2 + E_o + \frac{p_o}{\rho_o} \ ,$$

where U is the shock velocity $\frac{dx}{dt}$.

These conditions simplified by Equation (2) give

(11a) $$u = u(p;u_o,p_o,S_o) = u_o + (p - p_o)\left[\frac{2}{\gamma(\gamma + 1)}\,\frac{\exp\left(\frac{\gamma - 1}{\gamma}S_o\right)}{p_o^{1/\gamma}(p + \mu^2 p_o)}\right]^{1/2} ,$$

(11b) $$S = S(p;u_o,p_o,S_o) = S_o + \frac{\gamma}{\gamma - 1}\,\ell n\left(\frac{p}{p_o}\right)^{1/\gamma}\left(\frac{\mu^2 p + p_o}{p + \mu^2 p_o}\right) ,$$

(11c) $$U = U(p;u_o,p_o,S_o) = u_o + \left[\frac{(p + \mu^2 p_o)\exp\left(\frac{\gamma - 1}{\gamma}S_o\right)}{(1 - \mu^2)\,p_o^{1/\gamma}}\right]^{1/2} .$$

Let $D = \frac{\partial}{\partial t} + \rho(U - u)\frac{\partial}{\partial m}$ be the differential operator in the direction of the shock line. Implicit differentiation of u and S along the reflected shock line gives

(12a)
$$-\frac{\partial u}{\partial p} p_t + u_t + \rho(U - u)u_m - \rho(U - u)\frac{\partial u}{\partial p} p_m = F_1 ,$$

$$F_1 \equiv \frac{\partial u}{\partial p_o} Dp_o + \frac{\partial u}{\partial u_o} Du_o + \frac{\partial u}{\partial S_o} DS_o ,$$

(12b)
$$-\frac{\partial S}{\partial p} p_t - \rho(U - u)\frac{\partial S}{\partial p} p_m + \rho(U - u)S_m = F_2 ,$$

$$F_2 \equiv \frac{\partial S}{\partial p_o} Dp_o + \frac{\partial S}{\partial u_o} Du_o + \frac{\partial S}{\partial S_o} DS_o ,$$

where the shock path is given by

(13)
$$Dm = \rho(U - u) .$$

The quantities , identified by subscript o, that result from the incident wave we assume to be known as functions of m and t. From Equation (11), the first partial derivatives with respect to the arguments p, u_o, p_o, S_o, become

(14a)
$$\frac{\partial u}{\partial p} = \frac{(u - u_o)}{(p - p_o)} \frac{\left[p + (1 + 2\mu^2)p_o\right]}{2(p + \mu^2 p_o)} ,$$

(14b)
$$\frac{\partial u}{\partial u_o} = 1 ,$$

(14c)
$$\frac{\partial u}{\partial S_o} = \frac{\gamma - 1}{2\gamma}(u - u_o) ,$$

(14d) $$\frac{\partial u}{\partial p_o} = \frac{-(u - u_o)\left[p^2 + (3\gamma - 2)pp_o + (\gamma - 1)\mu^2 p_o^2\right]}{2\gamma\, p_o(p - p_o)(p + \mu^2 p_o)} ,$$

(14e) $$\frac{\partial S}{\partial p} = \frac{1}{\gamma + 1} \frac{(p - p_o)^2}{p(p + \mu^2 p_o)(\mu^2 p + p_o)} ,$$

(14f) $$\frac{\partial S}{\partial u_o} = 0 ,$$

(14g) $$\frac{\partial S}{\partial S_o} = 1 ,$$

(14h) $$\frac{\partial S}{\partial p_o} = \frac{-1}{\gamma + 1} \frac{(p - p_o)^2}{p_o(p + \mu^2 p_o)(\mu^2 p + p_o)} .$$

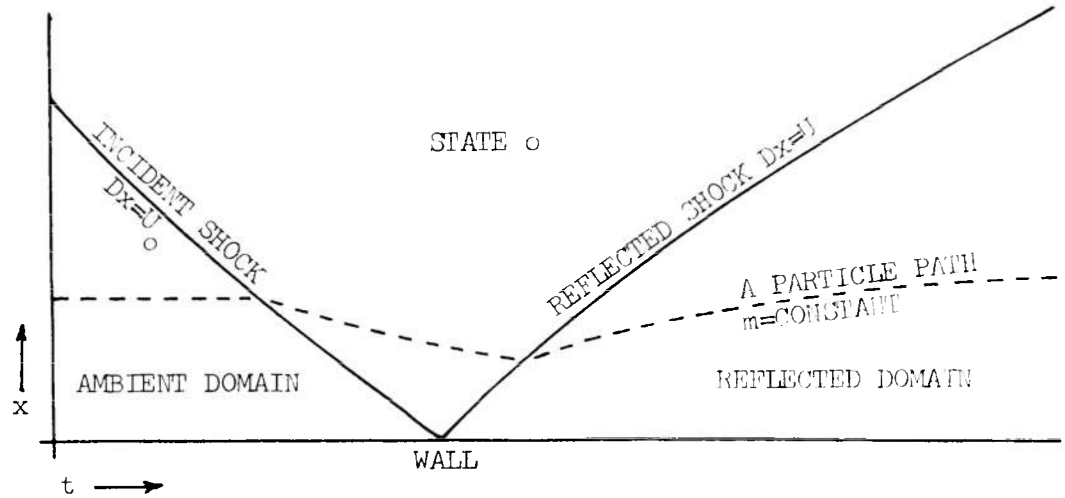
CONSTRAINTS

For simplicity, it is assumed that the reflecting wall is at $m = x = 0$ (see Figure). At the wall the particle velocity must satisfy the conditions

(15) $$\left(\frac{\partial^n u(x,t)}{\partial t^n}\right)_{x=0} = 0, \qquad n = 0, 1, 2, \ldots .$$

From this result and the ideal gas law (2a), we can show by taking higher derivatives of (1a) and (1b) that u must satisfy also the condition

(16) $$\left(\frac{\partial^2 u(x,t)}{\partial x^2}\right)_{x=0} = 0.$$



We now impose some constraints on the flow. There is an infinity of ways of doing this. The choice will depend on the nature of the ambient conditions and the incident wave. For blast waves in still air, in the neighborhood of the wall at all times and also in the neighborhood of the asymptotic shock, we expect the particle velocity to vary slowly with x, and so we choose

$$u = a(t)x + b(t)x^3 \quad , \tag{17}$$

where a(t) and b(t) are functions of t only. This expression satisfies both boundary conditions (15) and (16), and also permits the initial and shock conditions to be satisfied.

For subsequent purpose, we differentiate Equation (17) twice with respect to m:

$$u_m = \frac{u + 2bx^3}{\rho x} \quad , \tag{18a}$$

$$u_{mm} = \frac{6bx}{\rho^2} - \frac{u_m}{\gamma p} [p_m - (\gamma - 1) \; pS_m] \quad . \tag{18b}$$

DETERMINATION OF REFLECTION DOMAIN

Equations (18a), (12), and (7) are six equations involving the six components of V. Solving for V, we obtain

$$\text{(19)} \qquad V = \left\| \begin{matrix} 0 \\[2ex] - H\{\rho(U-u)\frac{\partial u}{\partial p} + 1\} \\[2ex] -G - H\frac{\partial u}{\partial p} \\[2ex] \frac{H}{\gamma p \rho}\{\rho(U-u)\frac{\partial u}{\partial p} + 1\} \\[2ex] G + H\frac{\partial u}{\partial p} \\[2ex] \frac{1}{\rho(U-u)}\left[F_2 + \frac{\partial S}{\partial p}\{G\rho(U-u) - H\}\right] \end{matrix} \right\|$$

where

$$G = \frac{(u + 2bx^3)(U-u) - xF_1}{x\{\rho(U-u)\frac{\partial u}{\partial p} + 1\}} \quad ,$$

$$H = \frac{\gamma p(u + 2bx^3)}{x\{\rho(U-u)\frac{\partial u}{\partial p} + 1\}} \quad .$$

By differentiating Equation (12) along the reflected shock line, we have

$$\text{(20a)} \qquad -2\frac{\partial u}{\partial p}\rho(U-u)\, p_{mt} - \frac{\partial u}{\partial p}\, p_{tt} + 2\rho(U-u)\, u_{mt}$$
$$+ u_{tt} - \rho^2(U-u)^2\frac{\partial u}{\partial p}\, p_{mm} + \rho^2(U-u)^2\, u_{mm} = F_3 \ ,$$

(20b) $$- 2\frac{\partial S}{\partial p}\rho(U - u)\, p_{mt} - \frac{\partial S}{\partial p} p_{tt} - \frac{\partial S}{\partial p}\rho^2(U - u)^2\, p_{mm}$$

$$+ \rho^2(U - u)^2\, S_{mm} = F_4 ,$$

where

(21a) $$F_3 \equiv DF_1 + Dp\, D\left[\frac{\partial u}{\partial p}\right] - \left[u_m - p_m \frac{\partial u}{\partial p}\right] D\, [\rho(U - u)] ,$$

(21b) $$F_4 \equiv DF_2 + \left[\frac{\partial S}{\partial p} p_m - S_m\right] D\, [\rho(U - u)] + Dp\left[\frac{\partial S}{\partial p}\right] .$$

Equations (20), (18b), and (8) are nine equations involving the nine components of W. Solving for W gives

(22) $$W = B^{-1}\, Y ,$$

where

(23a) $$Y = \left\| \begin{array}{c} \frac{\gamma + 1}{\gamma} p_t \left(\frac{p_m}{p} - \mu^2 S_m\right) \\ \frac{\gamma + 1}{\gamma} \frac{p_t^2}{p} \\ 0 \\ 0 \\ F_3 \\ \frac{6bx}{\rho^2} - \frac{\gamma + 1}{\gamma} u_m \left(\frac{1}{\gamma + 1} \frac{p_m}{p} - \mu^2 S_m\right) \\ F_4 \\ 0 \\ 0 \end{array} \right\| ,$$

(23b) $$B^{-1} = \begin{Vmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{Vmatrix} ,$$

(23c) $$\xi = [\rho(U - u)^2 + \gamma p] \frac{\partial u}{\partial p} + 2(U - u) ,$$

(23d) $$B_{11} = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ \frac{-\gamma p\left[2\rho(U - u) \frac{\partial u}{\partial p} + 1\right]}{\xi} & \frac{\rho(U - u)^2 \frac{\partial u}{\partial p} + 2(U - u)}{\xi} & \frac{-\gamma p \rho^2 (U - u)^2 \frac{\partial u}{\partial p}}{\xi} & \frac{\gamma p}{\xi} \\ \frac{2\rho(U - u)\frac{\partial u}{\partial p} + 1}{\rho \xi} & \frac{1}{\rho\xi}\left(\frac{\partial u}{\partial p}\right) & \frac{\rho(U - u)^2 \frac{\partial u}{\partial p}}{\xi} & -\frac{1}{\rho\xi} \\ -1 & 0 & 0 & 1 \end{Vmatrix} ,$$

(23e) $$B_{12} = \begin{Vmatrix} 0 & -\gamma p \rho & 0 & 0 & 0 \\ -\frac{\gamma p}{\xi} & \frac{\gamma p \rho\left[\rho(U - u)^2 + \gamma p + 2\gamma p \rho(U - u) \frac{\partial u}{\partial p}\right]}{\xi} & 0 & 0 & 0 \\ \frac{1}{\rho\xi} & \frac{-\left[\rho(U - u)^2 + \gamma p + 2\gamma p \rho(U - u)\frac{\partial u}{\partial p}\right]}{\xi} & 0 & 0 & 0 \\ 0 & \gamma p \rho & 0 & 0 & 0 \end{Vmatrix} ,$$

$$
B_{21} = \left\| \begin{array}{cccc}
\dfrac{-\left[2\rho(U-u)\dfrac{\partial u}{\partial p} + 1\right]}{\rho\xi} & -\dfrac{1}{\rho\xi}\dfrac{\partial u}{\partial p} & \dfrac{\gamma p \dfrac{\partial u}{\partial p} + 2(U-u)}{\xi} & \dfrac{1}{\rho\xi} \\
0 & 0 & 0 & 0 \\
\dfrac{\left[3\rho(U-u)^2 - \gamma p\right]\dfrac{\partial S}{\partial p}}{\rho^2(U-u)^2\ \xi} & \dfrac{2\ \dfrac{\partial S}{\partial p}}{\rho^2(U-u)\xi} & \dfrac{2(U-u)\ \dfrac{\partial S}{\partial p}}{\xi} & \dfrac{\left[\rho(U-u)^2 + \gamma p\right]\ \dfrac{\partial S}{\partial p}}{\rho^2(U-u)^2\ \xi} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array} \right\| , \tag{23f}
$$

$$
B_{22} = \left\| \begin{array}{ccccc}
-\dfrac{1}{\rho\xi} & \dfrac{\left[\rho(U-u)^2 + \gamma p + 2\gamma p\rho(U-u)\ \dfrac{\partial u}{\partial p}\right]}{\xi} & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
\dfrac{-\left[\rho(U-u)^2 + \gamma p\right]\dfrac{\partial S}{\partial p}}{\rho^2(U-u)^2\ \ \xi} & \dfrac{\left[\rho(U-u)^2 - \gamma p\right]^2\ \dfrac{\partial S}{\partial p}}{\rho(U-u)^2\ \xi} & \dfrac{1}{\rho^2(U-u)^2} & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{array} \right\| , \tag{23g}
$$

Solving Equation (18a) for b(t), there is obtained

$$(24) \qquad b(t) = \frac{\rho x \, u_m - u}{2x^3} \quad .$$

Differentiating this expression with respect to t along the reflected shock line, there follows

$$(25) \qquad Db = \left(\frac{\rho x u_m - u}{2x^3}\right)_t + \rho\left(U - u\right)\left(\frac{\rho x u_m - u}{2x^3}\right)_m \equiv F_5 \quad ,$$

Equations (2a), (11), (13), (25), together with the implicit derivative

$$(26) \qquad Dp = p_t + \rho(U - u)\, p_m \quad ,$$

give the system of ordinary differential equations

$$(27) \quad \begin{Vmatrix} D & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & D & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & D & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & D & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{Vmatrix} \begin{Vmatrix} p \\ b \\ x \\ m \\ u \\ S \\ \rho \end{Vmatrix} = \begin{Vmatrix} p_t + \rho(U - u)\, p_m \\ F_5 \\ U(p;u_o,p_o,S_o) \\ \rho(U - u) \\ u(p;u_o,p_o,S_o) \\ S(p;u_o,p_o,S_o) \\ \rho(p,S) \end{Vmatrix} \quad .$$

Using (19) and (22), we can solve this system.

The solutions of this system give b(t) as a function of time; a(t) is then determined by

$$(28) \qquad a(t) = \frac{u - b(t)x^3}{x} \quad .$$

The flow behind the reflected shock can now be calculated as solutions of ordinary differential equations, with each point on the calculated shock line serving as an initial point. x is given by

$$x_t = u = a(t)x + b(t)x^3 \quad , \tag{29}$$

which integrates along the m = constant line into

$$x = x(m) \exp\left\{\int_{t(m)}^{t} a(t)dt\right\}\left[-\int_{t(m)}^{t} 2x^2(m)b(t) \exp\left\{\int_{t(m)}^{t} 2a(\tau)d\tau\right\}dt + 1\right]^{-1/2} , \tag{30}$$

where x(m), t(m) is a point on the shock line. This value of x substituted into Equation (29) gives u as a function of m, t. The pressure is then obtained from Equation (7):

$$p_t = -\gamma p\rho u_m = -\gamma p[a(t) + 3b(t)x^2] \quad , \tag{31}$$

which integrates into

$$p = p(m) \exp\left\{-\int_{t(m)}^{t} \gamma[a(t) + 3b(t)x^2]dt\right\} . \tag{32}$$

On this m = constant line, the entropy S = S(m) is constant and is given on the shock line. The density then follows from Equation (2).

On the wall the pressure simplifies to

$$p(0,t) = p(0) \exp\left\{-\int_{t(0)}^{t} \gamma\, a(t)dt\right\} . \tag{33}$$

The positive impulse imparted to the wall we take to be

$$(34) \qquad \text{Pos. impulse} = \int_{t(0)}^{\bar{t}} \gamma \left[p(0) \exp\left\{ -\int_{t(0)}^{t} a(t)dt \right\} - 1 \right] dt \ ,$$

where $\bar{t}$ is the time at which $p(0,t) = 1$.

RALPH E. SHEAR RAY C. MAKINO

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9. See page 123 of Reference 8.

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13. ABSTRACT

A one-dimensional theory of normal reflection of blast waves from walls is given. The method satisfies the initial and boundary conditions of the problem. It is shown how the entire reflected wave zone, including the reflected shock front and the pressure and impulse on the wall, can be calculated.

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