MANPOWER ALLOCATION AND MATHEMATICAL PROGRAMMING

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1.   Manpower allocation problems are a special instance of resource
allocation problems--themselves specializations of general decision
problems.   In defining a decision problem, one specifies the available
courses of action, the consequences of these courses of action, the
objectives sought in dealing with the problem, and the consideration to
be given to random effects.   Dealing with a decision problem generally
involves four steps:

   (1) specifying a model--a set of assumed empirical relationships
       among variables,

   (2) stating a subset of decision variables--those variables the
       decision-maker is free to control,

   (3) specifying an objective function, or criterion, which is
       affected by the values of the decision variables, and

   (4) stating a procedure for analyzing the effect on the objective
       function of alternative values of the decision variables.

   A very significant portion of the general decision problem consists
of specifying a suitable model of the process--one, which in Karlin's
words, "...abstracts from reality in a way which preserves the essential
structure of the problem in such a way that its analysis affords insight
into both the original concrete situation, and other situations which

have the same formal structure."[2] Mathematical programming is simply
the mathematics of maximizing a function of several variables which are
subject to constraints. The field of constrained optimization is
broader and includes a variety of search procedures[3] and gradient
methods.[4] Mathematical programming provides a convenient and natural
conceptual framework with which to view many decision problems, and
can yield as a computational by-product a much greater variety of sub-
sidiary information than other optimization techniques' or purely de-
scriptive models.

There have been a few references to the use of mathematical program-
ming in manpower allocation,[5,6,7,8] but no systematic attempt to use
available tools is apparent. The failure to exploit mathematical
programming in manpower allocation problems is understandable given the
general lack of agreement on personnel cost or value measures, the
relative paucity of knowledge concerning empirical relationships be-
tween policy variables (such as re-enlistment bonuses) and resulting
actions (such as re-enlistments), and the only gradually increasing
availability of data relevant to manpower decisions. When problems have
been cast in this framework it has been either to exhibit the structure
of a solution, or to perform a specific study applicable to one specific
set of data. The advent of large-capacity computer systems with remote

[2] Karlin, S., Mathematical Methods and Theory in Games, Programming,
and Economics, Vol. I, Addison-Wesley, Reading, Mass., 1959.

[3] Wilde, D. J., Optimum Seeking Methods, Prentice-Hall, Englewood
Cliffs, New Jersey, 1964.

[4] Arrow, K. J., E. Hurwicz, and H. Uzawa, Studies in Linear and
Non-Linear Programming, Stanford, California, Stanford University Press,
1958.

[5] Kossack, C. F., and R. E. Beckwith, The Mathematics of Personnel
Utilization Models, WADC-TR-59-359, Lackland AFB, Texas, November 1959.

[6] Gorham, W., An Application of a Network Flow Model to Personnel
Planning, RM-2587, The RAND Corporation, June 1960.

[7] Kagen, H., Unpublished Notes on Rotation Models, Randolph AFB,
Texas, July 1966.

[8] Sorenson, R. C., "Logical Model Representing Flow in the U.S.
Army: Considerations Relative to Reduction in Turbulence," Technical
Research Note 156, U.S. Army Personnel Research Office, July 1965.

terminals and access to actual manpower and personnel data should
increase the use of mathematical programming techniques by decision-
makers, as well as analysts. The manpower planner should be able to
query on-line optimization routines which operate on timely data to
obtain initial schedules, courses of action, or allocation plans.

Aside from day to day use by the decision-maker, a great deal
of value accrues from formulating and analyzing a problem as a mathe-
matical programming problem. Retaining the same set of constraints,
one may experiment with various objective functions, or various
weightings of several objective functions to ascertain the sensitivity
of solutions to criteria. Through parametric and postoptimality
analyses one can determine how minor changes in data or coefficients
affect a solution. And the dual variables--or shadow prices--provide
valuable information on the importance of various constraining re-
lationships.

Formulating a mathematical programming problem forces explicit
definition of the constraints operating on the system, and the type of
criteria one will consider. One must concentrate on the empirical re-
lationships among the variables, and on the type of data required to
perform the analysis. Often this calls into question existing policies
and goals in an objective way not otherwise possible. Practically,
computational economies can result from casting problems as mathematical
programming problems because of the existence of computing algorithms
and routines in most computing centers. Standard capacitated transpor-
tation algorithms and linear programming routines are almost always
available, averting a good deal of programming effort. Many problems
are of course non-linear, either in objective function or constraints,
but in many instances the problem may be approximated by a linear
programming formulation over the relevant range. Several such linear
approximations and parametrizations of the coefficients and constraints
can yield insight into the behavior and sensitivity of the solution,
and are certainly more valuable than failure to perform the analysis
because of the recognized non-linearities of the original problem.

Recognizing that a particular manpower allocation problem can be
viewed as a mathematical problem only begins the task of analysis.

Important practical barriers remain. The foremost may be the conceptual definition of the technological or objective function coefficients. In considering the movement of Air Force personnel between grades, specialties, and skill levels, an obvious characterization is a Markov process, and the mathematical programming tool which comes to mind is the optimization of some cost or value criterion within the structure of the Markov process, while satisfying some set of requirement constraints. But what is the value or cost of a given person in a given state? Or what is the cost or value of having a person transition between two states? Subsistence, training, and budget costs enter, but so does the relative importance of having a particular capability within the Air Force. Even given the definition of the various coefficients, estimates of numerical values may still be difficult to obtain in the manpower and personnel area. It has only been in the last four or five years that personnel records have systematically been kept in a form usable by computer systems. Cost and productivity figures are inherently difficult to generate because of the existence of joint products, joint usage, variable productivity of human beings, and the many mixes of labor, capital, and organizational structures existing. The cost and effort involved in obtaining data may be considerable. Also, a realistic formulation of an actual problem may be prohibitively large for current computer systems. This consideration shaped the Rotation Base Model which will later be described.

In general, manpower allocation problems appear to involve a natural decomposition by specialty, or groups of specialties, and this may lead to computationally feasible problems. Even if problems are excessively large, a combination of heuristic and formal analysis can usually reduce the problem to tractable size. While the time and cost in personnel to formulate a model, obtain data, construct and check out the working model, and begin to use the results is not to be underestimated, one of the derivative benefits in working with mathematical programming models has always been the increase in knowledge and expertise of the analysis team. This makes it important that model design, construction, and use be coordinated closely with the agency which will ultimately use the model or solutions. The danger of an

analysis effort separated from the user's staff is the "Not Invented Here" syndrome.

2.   In evaluating the effects on the Air Force of the recent OSD Civilianization Program, we became aware of the "rotation base requirement" problem. This requires specifying the number of military personnel in each grade, Air Force Specialty Code (AFSC), and skill level required in the United States (ConUS) to support overseas military personnel requirements. The ConUS base must be sufficiently large to allow personnel to serve reasonable long tours in the United States between tours in overseas areas. While methods were available to estimate this rotation base,[9] we thought it desirable to construct a more accurate estimating tool.

We initially simplified the problem by creating aggregates of AFSC's, grades, and skill levels--for example all Communications/ Electronics personnel at the journeyman skill level, in the grades of Airman First Class and Staff Sergeant--and denoted these as an "assignment class." We then aggregated all geographic assignments of the same type and tour length as "tour areas." This type distinction permits differentiation between a twelve-month tour in, say, South Vietnam, and a twelve-month tour in Dakar. We assumed a static situation with requirements and personnel policies known and unchanging. The model was to determine a set of personnel rotation and training assignments which met requirements at all bases, and which used the minimum number of military personnel.

We conceive the rotation and assignment process as being a network of nodes and arcs. Each node is an assignment class in a particular tour area--for example, Communications/Electronics personnel in ConUS. Each arc connects two nodes and represents a possible assignment action. The basic requirement forcing personnel flow within the system is the requirement that personnel rotate out of all overseas bases at the end of their tours, and that all requirements be continuously met.

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[9]Kagen, op. cit.

In actually dealing with the problem of rotation and reassignment, the Air Force recognizes that some specialties are capable of being utilized immediately in other jobs, and that with some cross-training, new skills can be acquired. Furthermore, assignment classes which are surplus in ConUS can be cross-trained and rotated overseas with a shorter ConUS tour than assignment classes which are required in ConUS. We constructed the model to recognize this "surplus" labor pool, and to take advantage of cross-training opportunities wherever cross-training seemed to offer a manpower saving. Within the model an arc (assignment possibility) connects every pair of assignment classes between which personnel can move with cross-training. Training is not free in man-power terms--it ties up military personnel. For instance, if 100 men per month are entered into a two-month aircraft mechanic course, the total manpower pool must allow for the 200 men who are continuously in training.

It is the sum of all personnel in training and all personnel serving at all bases that the model seeks to minimize. This manpower expression is composed of three parts--personnel in training, personnel assigned to meet requirements at bases, and personnel excess to ConUS requirements but necessary in ConUS to support the rotation flow. If personnel are available at a ConUS node in excess of requirements at that node, the model allows those personnel to be entered into training pipelines for assignments to other AFSC's without requiring additional manpower to fill the pipelines. It is the "free" substitutability of some classes of personnel that make manpower economies possible within this model and within the Air Force.

The abstracted rotation problem is thus to minimize total manpower at all nodes, i, and over all arcs (i, j) subject to the requirement that personnel at overseas bases be returned at a rate dependent on the overseas tour length. This is a simple problem conceptually and we de-cided to include about 50 assignment classes and 20 tour areas--1000 nodes. Any attempt to leave this problem in the form of a standard linear programming problem rapidly generates about 2000 equations and possibly 3000 variables--well beyond the computational capability of standard linear programming routines. By using a bounded variable

algorithm, it might be possible to reduce the problem to 1000 con-
straints--still formidable. We therefore chose to handle the network
as a network--although a complex and clumsy one--and used a capacitated
transportation algorithm as the computational mechanism. The Fulkerson
out-of-kilter algorithm[10] is an efficient method for finding a minimum
cost circulation in a capacitated network, and it is the algorithm
used. Due to the summation constraints on the system, and the ne-
cessity to distinguish between surplus personnel and required person-
nel, the network becomes quite complex, and the number of arcs pro-
liferates furiously. However, we are able to deal with the typical 50
assignment class by 20 tour area problem quite easily and without ex-
cessive computer time. This is the final functional criterion by which
we judge this first Rotation Base Requirements Model.

It might appear that having formulated the problem mathematically,
and having decided to use the out-of-kilter algorithm, the task was
finished. But to construct a fairly complex network by hand is a
tedious procedure. As in most such modeling-programming efforts, the
primary tasks were constructing logical routines to translate relevant
inputs in standard form--lists of requirements by AFSC, by base, for
instance--into the network description viewed by the out-of-kilter
algorithm. A post processor, or set of report generators, was con-
structed to take the results of the out-of-kilter computations--
expressed only as raw flows--and interpret these into assignment and
rotation information meaningful to a manpower planner. The task of
computer programming and checkout required four months of a skilled
mathematician-programmer's efforts. The initial model has been briefed
and distributed to various interested offices of the Air Force. The
model is written entirely in Fortran IV, and as soon as it is fully
documented, there should be no difficulty in making it available to
other interested users.

During its construction and use we have become aware of other
factors which affect the rotation base question, and we are con-
structing a new model, RBM-II, which will consider accession and loss

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[10]Fulkerson, D. R., An Out-of-Kilter Method for Minimal Cost Flow
Problems, The RAND Corporation, Paper P-1825, Jan. 18, 1960.

rates, training school capacities, and training costs. These features will be grafted onto the already complex network, but we see no alternative way to treat a problem of this magnitude.

As an example of the use of the model, consider the following hypothetical situation. Suppose we have the following requirements in three assignment classes in two tour areas.

Table 1

MANPOWER REQUIREMENTS

| | Tour Area | |
Assignment Class	ConUS	Overseas
Missile Maintenance	75,000	0
Communications/Electronics	15,000	8,000
Aircraft Maintenance	0	10,000

With no cross-training, given a 48-month ConUS tour, and a 12-month overseas tour, the total ConUS manpower requirement to support this rotational situation is 147,216 men. If we permit cross-training according to Table 2, we would intutitively expect to use the large supply personnel assignment class 1 (missile maintenance) to supply the requirements of assignment class 3 overseas.

Table 2

CROSS-TRAINING TIMES (MONTHS)

| | To | | |
From	Missile Maint.	Comm./Elec.	Aircraft Maint.
Missile Maint.	0	2	1
Comm./Elec.	4	0	1
Aircraft Maint.	1	2	0

The optimal rotation pattern calculated by the Rotation Base Model is shown in Fig. 1.

ConUS   Overseas

Missile Maint.

351
351

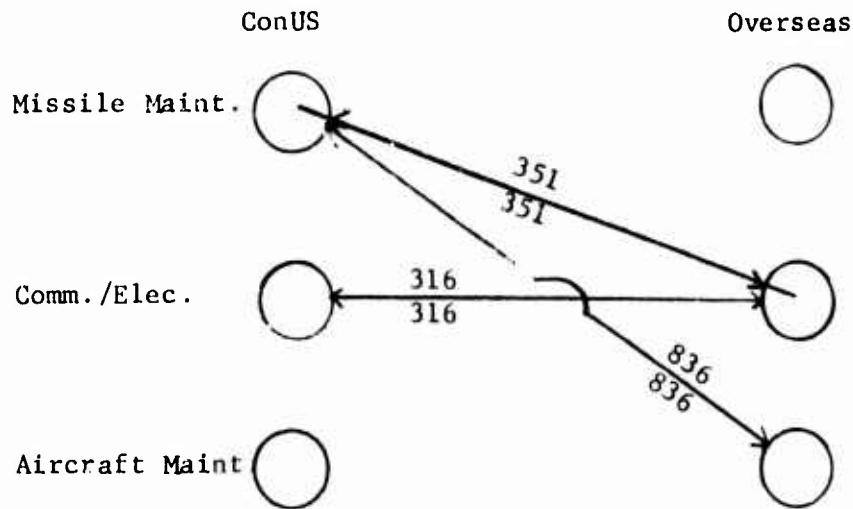Comm./Elec.   316
316

836
836

Aircraft Maint

Fig. 1 -- Monthly Personnel Reassignments

In this rotation pattern there are 3514 personnel in training, 75,072 on duty at ConUS assignment class 1, and 15,168 on duty at ConUS assignment class 2, a total required rotation base of 93,754 compared to 147,216 in the original independent rotation base requirement.[11]

Assume that a policy decision is made to extend all tours for personnel in this overseas tour area from 12 months to 16 months. The new optimal flow pattern is shown in Fig. 2.

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[11] Requirements are not met exactly due to the truncation which occurs in the integer arithmetic of the model.
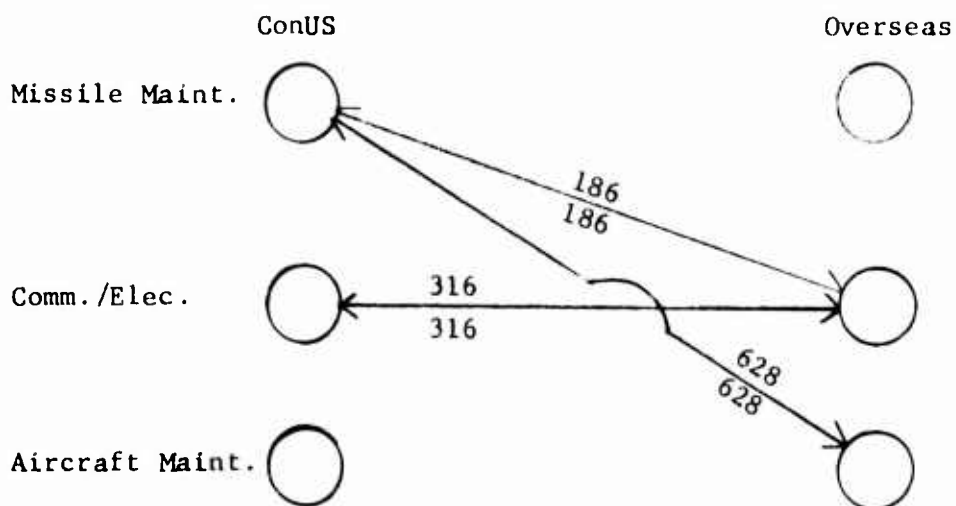
Fig. 2 -- Personnel Flow Pattern With 16-Month Overseas Tour

The total manpower in training and in the rotation base is 92,447, a reduction of 1307 due to the tour length extension.

Assume again that in the original hypothetical example one desired to know the total manpower saving generated by a reduction of 4000 in assignment class 3 overseas. The revised flow pattern is shown in Fig. 3 and indicates a saving of 620 personnel in the training pipeline.
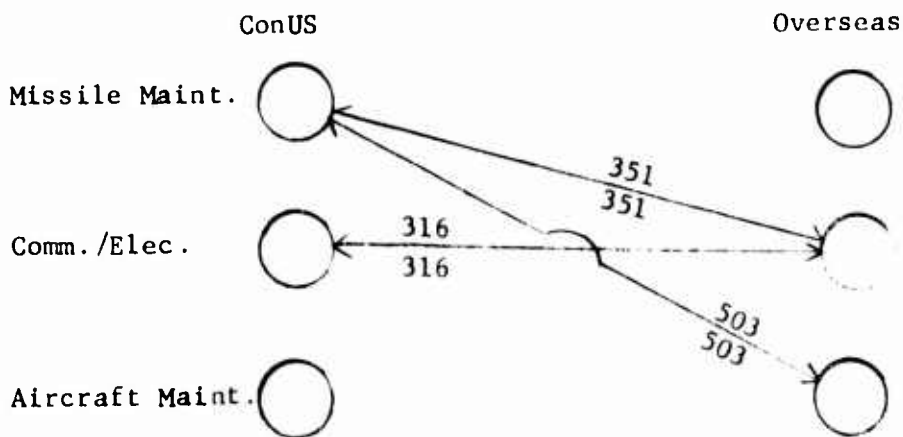

Fig. 3 -- Personnel Flow Pattern With
Manpower Reduction of 4000 Overseas

The Rotation Base Model as it presently exists is more a re-
fined estimating tool than a cross-training scheduling algorithm. This
is due to the assumption of a system in equilibrium and the absence
of dollar costs and training school capacities. We have found it
somewhat difficult to obtain estimates of some of the parameters re-
quired both for this model and for its successor. Such things as the
average ConUS tour length served in a given AFSC when that AFSC is in
short supply, and what that AFSC is surplus, are not usually recorded,
and the military personnel data files are still fairly cumbersome to
access and manipulate for purposes of estimation.

We feel, however, that none of the problems we have mentioned--con-
ceptual definitions of cost or value coefficients, computational feasi-
bility, or data availability--are insurmountable. The insights provided
by the exercise of this model in particular, and programming models in
general, can be most valuable to the using agency and the decision-
maker.

3. In the remainder of this paper we have briefly described some
generalizations of the rotation situation which lead to familiar and
pervasive manpower and personnel assignment problems. We have indicated
how these problems may be formulated in a mathematical programming
context, the type of information available, and the analyses which can
be performed. More detailed treatment of these problems is given in
the previously cited RAND Memorandum which is forthcoming.

A. A more realistic treatment of the rotation base problem must
recognize accession and loss rates, dollar costs of training, cross-
training, and rotation, capacity limitations of the training establish-
ment, and the fact that instructor requirements may depend on student
loads. One method of dealing with this problem is by framing it as a
linear programming problem.

We might take as an objective function the total cost of enlisting,
training, rotating, and separating the personnel force required to
meet requirements in all tour areas. The rotation system must satisfy
certain constraints. Movement must occur from each overseas tour area
at a rate determined by the number of personnel in that area and the
area tour length. Each period personnel are lost to the service from

every assignment class in every area at a known rate. Training schools leading to each assignment have a known capacity. We may assume that the number of instructors required in each training establishment is a linear function of the number of personnel being trained in that establishment. This problem can be formulated as choosing the rotation flows to minimize the total cost subject to the conditions listed--a linear programming problem. In addition to the obvious parametric analyses possible, the dual variables of the rotation flow constraints provide an indication of the tour areas which are particularly demanding in the rotational system.

B. Assume that personnel requirements by specialty and location are changing significantly each year. A steady state formulation may then not be appropriate. If, for instance, we know requirements by assignment class and tour area for each of five successive years, the problem may be viewed as a dynamic linear programming problem. We might use as a criterion the minimization of the cost of creating the manpower pool for the entire five years, where the decision variables are the cross-training flows and new accessions. The constraints affecting the system include the condition that requirements be met in all specialties in all periods, and the conditions relating the availability of specialists in one period to the number and type of specialists available the previous period. One might use this formulation to ask how much it would be worth in dollars to increase the retention rate in a particular specialty.

C. Suppose that requirements in the multi-period case are not known precisely, but that the probability distribution of the individual requirements can be estimated. Such problems can be dealt with in two ways.[12] A decision can be made for one period at a time, deferring later decisions until early requirement information is available. Solutions to sequential formulations of this type result in "strategies" rather than solutions--statements as to how the decision-maker should behave when more information becomes available to him.

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[12]Hadley, G., Nonlinear and Dynamic Programming, Addison-Wesley Reading, Massachusetts, 1964.

In planning for several years ahead one may have to solve the problem non-sequentially, or "once-and-for-all." In one situation of this type only the set of requirements will be random variables. In many industrial situations a penalty cost can be set on failure to meet requirements in any period, and this penalty cost can be entered into an appropriate objective function. If the resources provided are greater than the actual requirement in some period, there is once again a penalty. In the case of programming military personnel it is difficult to set a dollar cost or an effectiveness cost on personnel shortages, or on the effort required to reprogram resources into positions with shortages. An alternative method avoids the necessity of specifying penalty costs, but instead specifies some arbitrary probability with which each requirement constraint must be met. Solving the problem with a constraint involving an arbitrarily selected probability imputes a cost to that constraint in the optimal solution, which the decision-maker may not care to impute. It is therefore preferable to explicitly include penalty costs in the objective function. This can lead to a non-linear programming problem which has a separable concave objective function.

D. In an organization as large as the Air Force the allocation of manpower resources among major commands is a significant decision problem. It involves determining the distribution of contract service dollars, civil service authorizations, and military personnel authorizations in such a way that each command is able to perform its mission, and so that the overall effectiveness of the Air Force is maximized. Considered as a single large Air Force problem, the manpower re - lationships affecting the commands are fairly independent, while a few constraints affect all commands simultaneously. This type of structure is called a block diagonal problem.

There are several decomposition algorithms in existence which could be applied to this problem, but the most interesting is a recent algorithm by Zschau.[13] This algorithm appears to be the natural

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[13] Zschau, E., "A Primal Decomposition Algorithm for Linear Programming," Working Paper 91, Graduate School of Business, Stanford University, Stanford, California, January 1967.

computational analogue of bargaining in a decentralized structure, and seems sure to have a great deal of utility as a conceptual as well as a computational tool.

Using Zschau's approach, consider that the problem as seen by Headquarters USAF is maximization of overall effectiveness. The Air Force must utilize manpower resources which are of two types--those which are possessed solely by commands and those which are shared by all commands. An example of the former might be Depot Maintenance personnel, or R&D scientists, while Air Policemen are examples of the latter.

Each command faces the problem of using resources allocated to it in a way that maximizes its own effectiveness, while Headquarters must allocate common resources among the commands so that Air Force total effectiveness is maximized. Zschau shows conditions under which this problem is a concave programming problem and develops an algorithm to solve it, which seems to be a natural one for the analysis of allocation problems in large organizations.