Technical Report 194

CLIMATOLOGICAL PROBABILITY OF AIRCRAFT ICING

by

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PREFACE

This report has been prepared to assist AWS personnel in estimating the climatological probability of aircraft icing according to season, altitude, and area within the Northern Hemisphere. The procedure and data used to develop isopleth charts of icing probability are described.

These charts can be used in support of long-range planning. Information on short-range planning and day-to-day forecasting of icing conditions is contained in AWSM 105-39, "Forecasters' Guide on Aircraft Ic[illegible]

The assistance of Mr. Clarence B. Elam, Jr. is gratefully acknowledged.

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CLIMATOLOGICAL PROBABILITY OF AIRCRAFT ICING

SECTION A — INTRODUCTION

In 1957 the National Advisory Committee for Aeronautics [4] presented aircraft icing data which were obtained from various Air Weather Service reconnaissance flights. The data were collected at 700 mb and 500 mb. These data readily lent themselves to statistical analysis because of the manner in which they were collected. The reconnaissance aircraft flew every day, at the same levels and over the same routes. Furthermore, conditions conducive to icing were neither sought out nor avoided. An analysis of these data were provided by Appleman [1] in 1959. The results of this analysis are presented as graphs which depict the conditional probability of aircraft icing, given cloud amounts $\geq$ 6/10, as a function of altitude and temperature.

In 1964, Ingram and Gullion[1] presented an atlas which provided estimates of the percent frequency of potential icing conditions at various levels. Potential icing was considered to be the product of two factors — the probability of temperatures $\leq$ 0°C and the probability of having a cloud amount $\geq$ 6/10. It was subsequently realized that potential icing grossly overestimated actual icing frequency. Hq Air Weather Service then requested that the Environmental Technical Applications Center (ETAC) provide charts of actual icing probabilities based upon the best data available. This report was prepared to fulfill that request.

SECTION B — DEFINITION AND ASSUMPTIONS

Definition

The probability of aircraft icing at a specified altitude in the atmosphere is the chance that an aircraft at that specified altitude will in fact experience icing. Estimates of icing probabilities are based on climatological estimates of the parameters involved.

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[1] Ingram, D. M. and Gullion, J. L.: "Estimated Frequencies of Potential Icing Conditions at Specified Altitudes," Technical Report 182, Hq Air Weather Service (ETAC), 17 pp., September 1964 (superseded by AWSM 105-39, 22 August 1966).

Assumptions:

a. Probability of icing with cloud amounts less than 6/10 is negligible compared to that with cloud amounts $\geq$ 6/10. The condition that the cloud amount be equal to or greater than 6/10 is assumed necessary but not sufficient.

b. The range of temperatures necessary for icing extends from -2°C to -40°C. This report is prepared for conventional (reciprocating engine) aircraft and so it is assumed that the magnitude of dynamic heating (heating due to friction and adiabatic compression as the aircraft penetrates the air) is two Centigrade degrees. Furthermore, it is assumed that no supercooled water exists at temperatures lower than -40°C. The temperature range -2°C to -40°C is assumed necessary but not sufficient.

c. Weighted averages of means and variances of temperatures at constant-pressure levels can be used to estimate the same statistics at constant-height levels.

d. Frequencies of temperatures below -2°C at any specified altitude are assumed to be statistically independent of frequencies of cloud amounts $\geq$ 6/10 at the same altitude.

e. The mean cloud amount at an altitude is assumed to be equal to the probability of cloud amounts $\geq$ 6/10 at that same altitude. This assumption appears justified when one studies the shape of the cloud-amount distribution at any specified altitude. In most cases it is a U-shaped distribution; therefore, only a small error is introduced with this assumption.

SECTION C — PROCEDURE

Atmospheric temperature is a continuous function. The probability of aircraft icing at a specified altitude, location, and month is defined mathematically by the following:

$$(1) \qquad P(I) = \int_{-40^\circ C}^{-2^\circ C} F[I(T)|C \geq 6/10]\; P[C \geq 6/10]\; F[T]dT$$

where:

$F[I(T)|C \geq 6/10]$ = conditional probability density function of icing as a function of temperature given that the cloud amount is equal to or greater than 6/10.

$$(2) \qquad F[I(T)|C \geq 6/10] \neq \begin{cases} 1 \text{ for } -40^\circ C \leq T \leq -2^\circ C \\ 0 \text{ elsewhere} \end{cases}$$

Equation (2) implies that although the range of temperatures, -2°C to -40°C, and cloud amounts $\geq 6/10$ are necessary, they in themselves are not sufficient. Therefore, icing will not occur every time an aircraft is in clouds of amount $\geq 6/10$ and the temperature is between -2°C and -40°C. Other factors such as aircraft type, cloud type, liquid-water content, size distribution of cloud droplets, etc., play a part in determining those cases conducive to aircraft icing.

$P[C \geq 6/10]$ is the probability that the cloud amount is equal to or greater than 6/10. $F[T]$ is the probability density function of temperature. By definition:

$$(3) \qquad F[I(T)|C \geq 6/10] = \frac{F[I(T),\ C \geq 6/10]}{P[C \geq 6/10]}$$

thus,

$$(4) \qquad P(I) = \int_{-40^\circ C}^{-2^\circ C} F[I(T),\ C \geq 6/10]\ F[T]dT$$

For computational purposes, Equation (4) is replaced by the approximation:

$$(5) \qquad P(I) = \sum_{i=1}^{n} F[I(T),\ C \geq 6/10]_i\ F[T]_i\ \Delta T_i$$

where:

$F[I(T),\ C \geq 6/10]_i$ = probability of icing in temperature class interval ΔT_i with a cloud amount equal to or greater than 6/10

$F[T]_i$ = probability of temperatures in temperature class interval ΔT_i

Equation (5) is justified since:

(6)

$$\lim_{n \to \infty} \sum_{i=1}^{n} F[I(T),\ C \geq 6/10]_i\ F[T]_i\ \Delta T_i = \int_{-40^\circ C}^{-2^\circ C} F[I(T),\ C \geq 6/10]\ F[T]dT = P(I)$$

or,

(7)

$$\lim_{\Delta T_i \to 0} \sum_{i=1}^{n} F[I(T),\ C \geq 6/10]_i\ F[T]_i\ \Delta T_i = \int_{-40^\circ C}^{-2^\circ C} F[I(T),\ C \geq 6/10]\ F[T]dT = P(I)$$

For the purposes of this report, a temperature interval (ΔT_i) of two Centigrade degrees was used. Since temperatures between -2°C and -40°C are assumed to be statistically independent of the frequency of cloud amounts $\geq$ 6/10 at the same altitude, location, and month, we may write Equation (5) in the following manner:

(8) $$P(I) = P[C \geq 6/10] \sum_{i=1}^{n} F[I(T)|C \geq 6/10]_i\ F[T]_i\ \Delta T_i$$

From AWSM 105-39 ([2] page 33), Figure 12 presents a graphical representation of $F[I(T)|C \geq 6/10]$ for the 700- and 500-mb pressure surfaces. Figures 14 and 15 (AWSM 105-39, pp. 41-42) are an outgrowth of Figure 12. Presented in these two figures are:

$$\sum_{i=1}^{n} F[I(T)|C \geq 6/10]_i\ F[T]_i$$

for the 700- and 500-mb pressure surfaces. An inherent assumption in these two figures is that the temperatures at these levels are normally distributed, and as such, are completely defined by a mean and standard deviation.

Graphs similar to those presented in Figures 12, 14, and 15 (AWSM 105-39), were produced by ETAC for the 850-mb pressure surface. These graphs were prepared from pilot reports of icing, cloud cover as shown on weather-depiction charts, and 850-mb temperature analyses. The pilot reports and weather-depiction charts were used to determine the percent frequency of icing and no-icing occurrences in $\geq$ 6/10 clouds. The 850-mb charts provided a temperature for each of the cases. Based on this information a probability density function of icing in clouds $\geq$ 6/10 as a function of temperature at the 850-mb pressure surface was derived. Thus, the probability of icing, P(I), for a specified altitude, location, and month, can be determined from a knowledge of the mean temperature and corresponding standard deviation and probability of cloud amounts $\geq$ 6/10.

SECTION D — DISCUSSION OF DATA

Temperature Data

Temperature data from approximately 300 Northern Hemisphere stations were used in this report. Monthly mean temperatures and corresponding standard deviations for the 850-, 700-, and 500-mb pressure surfaces were available. At each altitude, seasonal values for the mean and standard deviation were derived as follows:

$$(9) \quad \bar{T}_{s_i} = 1/3 \sum_{n=1}^{3} \bar{T}_{n_i}$$

where:

$\bar{T}_{s_i}$ = mean seasonal temperature at altitude i

and

$\bar{T}_{n_i}$ = mean temperature for month n at altitude i

The winter season is defined as December, January, and February; spring as March, April, and May; summer as June, July, and August; and autumn as September, October, and November.

$$(10) \quad \sigma_{s_i} = \sqrt{1/3 \sum_{n=1}^{3} \sigma_{n_i}^2}$$

where:

σ_{s_i} = seasonal standard deviation of temperature at altitude i

and

σ_{n_i} = standard deviation of temperature for month n at altitude i

The months used for each season are the same as those specified above. The seasonal mean temperature and standard deviation at 5000 feet was assumed to be equal to those statistics at the 850-mb pressure surface. Similarly, 10,000-foot data were taken directly from the 700-mb data. The seasonal mean temperature for the 15,000-foot level was computed using the following weighted average:

$$(11) \quad \bar{T}_{s_{15,000}} = 0.4\ \bar{T}_{s_{700}} + 0.6\ \bar{T}_{s_{500}}$$

where:

$\bar{T}_{s_{15,000}}$ = mean seasonal temperature at 15,000 feet

$\bar{T}_{s_{700}}$ = mean seasonal temperature at 700 mb

and

$\bar{T}_{s_{500}}$ = mean seasonal temperature at 500 mb

The seasonal standard deviation of temperature for 15,000 feet was computed using the following equation:

$$(12) \quad \sigma_{s_{15,000}} = \sqrt{0.4\,\sigma^2_{s_{700}} + 0.6\,\sigma^2_{s_{500}}}$$

where:

$\sigma_{s_{15,000}}$ = seasonal standard deviation of temperature at 15,000 feet

$\sigma_{s_{700}}$ = seasonal standard deviation of temperature at 700 mb

and

$\sigma_{s_{500}}$ = seasonal standard deviation of temperature at 500 mb

The seasonal mean temperature at 20,000 feet was estimated by the following equation:

$$(13) \quad \bar{T}_{s_{20,000}} = \bar{T}_{s_{500}} + \Delta T$$

where:

$\bar{T}_{s_{20,000}}$ = mean seasonal temperature at 20,000 feet

$\bar{T}_{s_{500}}$ = mean seasonal temperature at 500 mb

and

ΔT = temperature difference between 500 mb and 20,000 feet taken from the ICAO Standard Atmosphere = -3.4°C

The seasonal standard deviation of temperature at 20,000 feet was assumed equal to that at 500 mb.

Cloud Data

Cloud data for the 5000-, 10,000-, 15,000-, and 20,000-foot altitudes were computed by the method developed in AWSTR 186 [3]. Mean seasonal cloud amount data for 106 stations were utilized in this study. The method appearing in AWSTR 186 gives mean cloud amounts for specified levels. The mean cloud amount at an altitude is assumed equal to the probability of $\geq$ 6/10 cloud amount at that same altitude.

SECTION E — CHARTS

Charts of the conditional probability of icing, given a cloud amount $\geq$ 6/10, were constructed by combining the mean temperatures and corresponding standard deviations with the appropriate graph of $P[I(T)|C \geq 6/10]$. Grid values were then taken off the charts. Charts of $P[C \geq 6/10]$ were then drawn and grid values taken off. The resulting grid maps were then graphically multiplied. The resulting chart is the probability of aircraft icing at a given level.

An attempt has been made to ind··ate topographical areas of greater altitude than the altitude of the chart on which they appear. These appear as shaded sections on each map.

The accompanying charts provide estimates of the probability of aircraft icing at the 5000-, 10,000-, 15,000-, and 20,000-foot altitudes. Charts are presented for the winter, spring, summer, and autumn seasons.

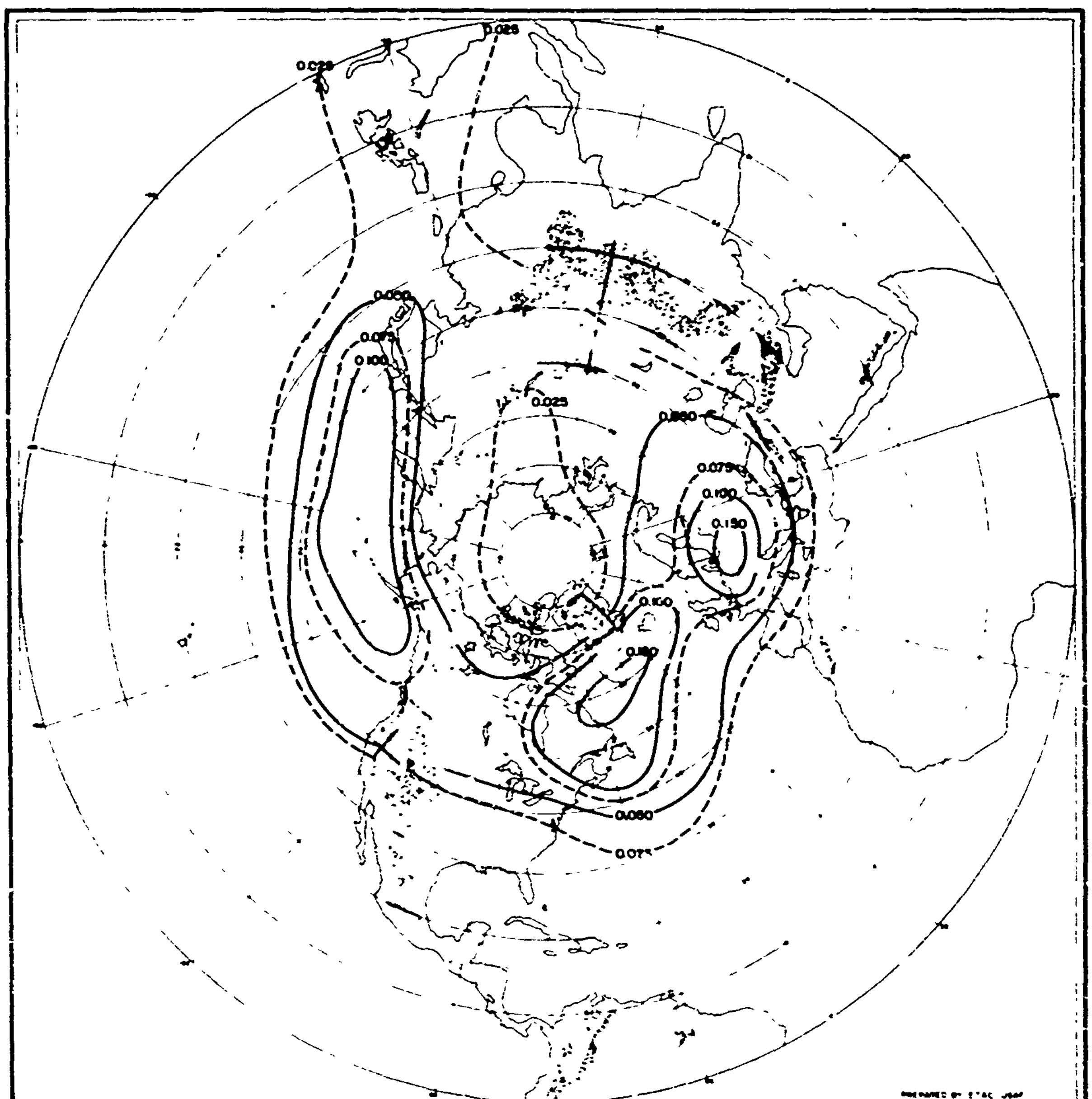
REFERENCES

[1] Appleman, H. S.: "Forecasting Aircraft Icing for Long-Distance Flights," 2d Weather Wing Forecasters' Bulletin No. C-14, Hq 2d Weather Wing, 5 pp., January 1959.

[2] AWSM 105-39: "Forecasters' Guide on Aircraft Icing," Hq Air Weather Service, 50 pp., 22 August 1966.

[3] McCabe, J. T.: "Estimating Mean Cloud and Climatological Probability of Cloud-Free Line-of-Sight," Technical Report 186, Hq Air Weather Service (ETAC), 26 pp., November 1965.

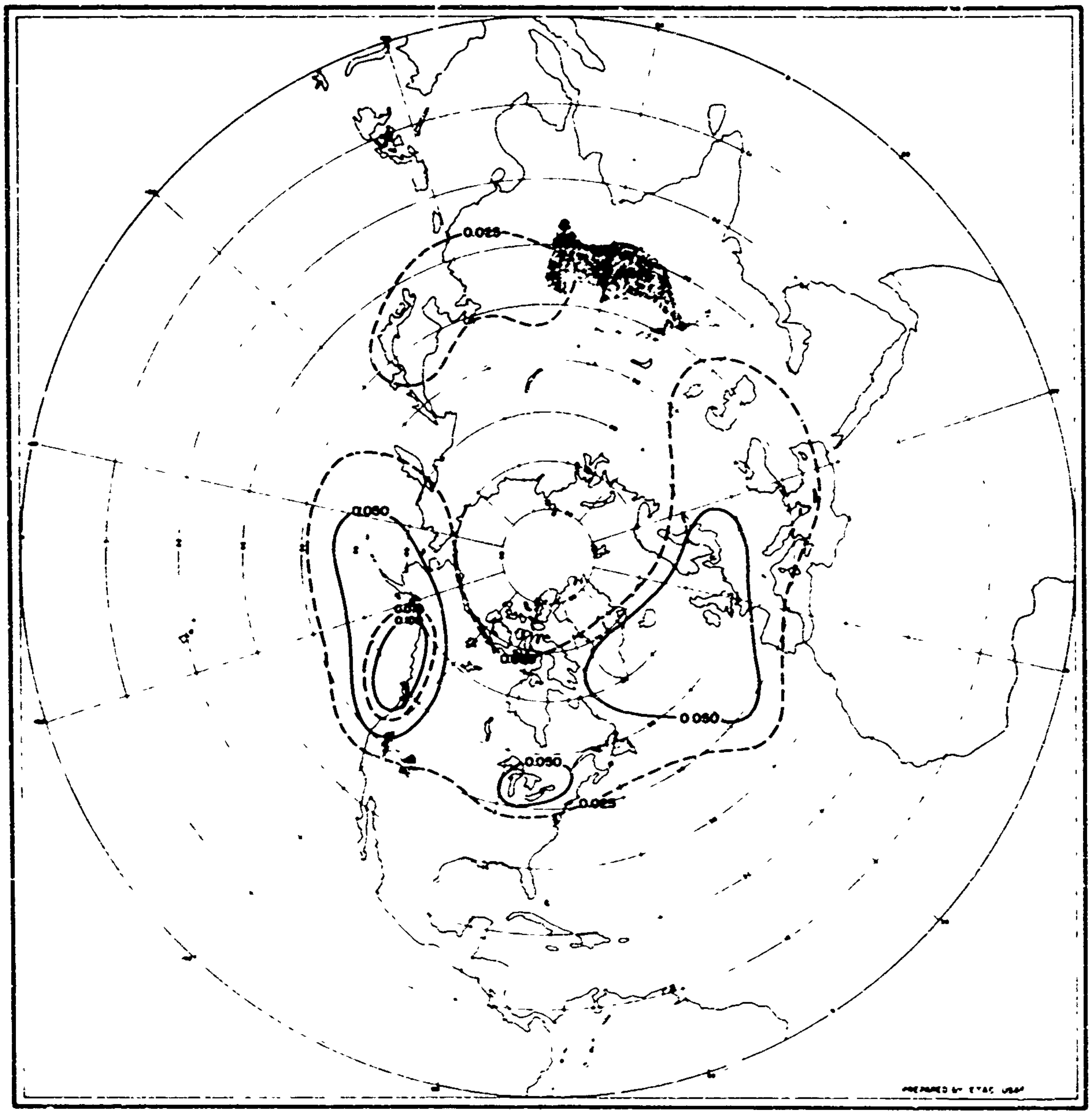
[4] Perkins, P., Lewis, W., and Mulholland, D.: "Statistical Study of Aircraft Icing Probabilities at the 700- and 500-Millibar Levels Over Ocean Areas in the Northern Hemisphere," Technical Note 3984, National Advisory Committee for Aeronautics, 31 pp., May 1957.



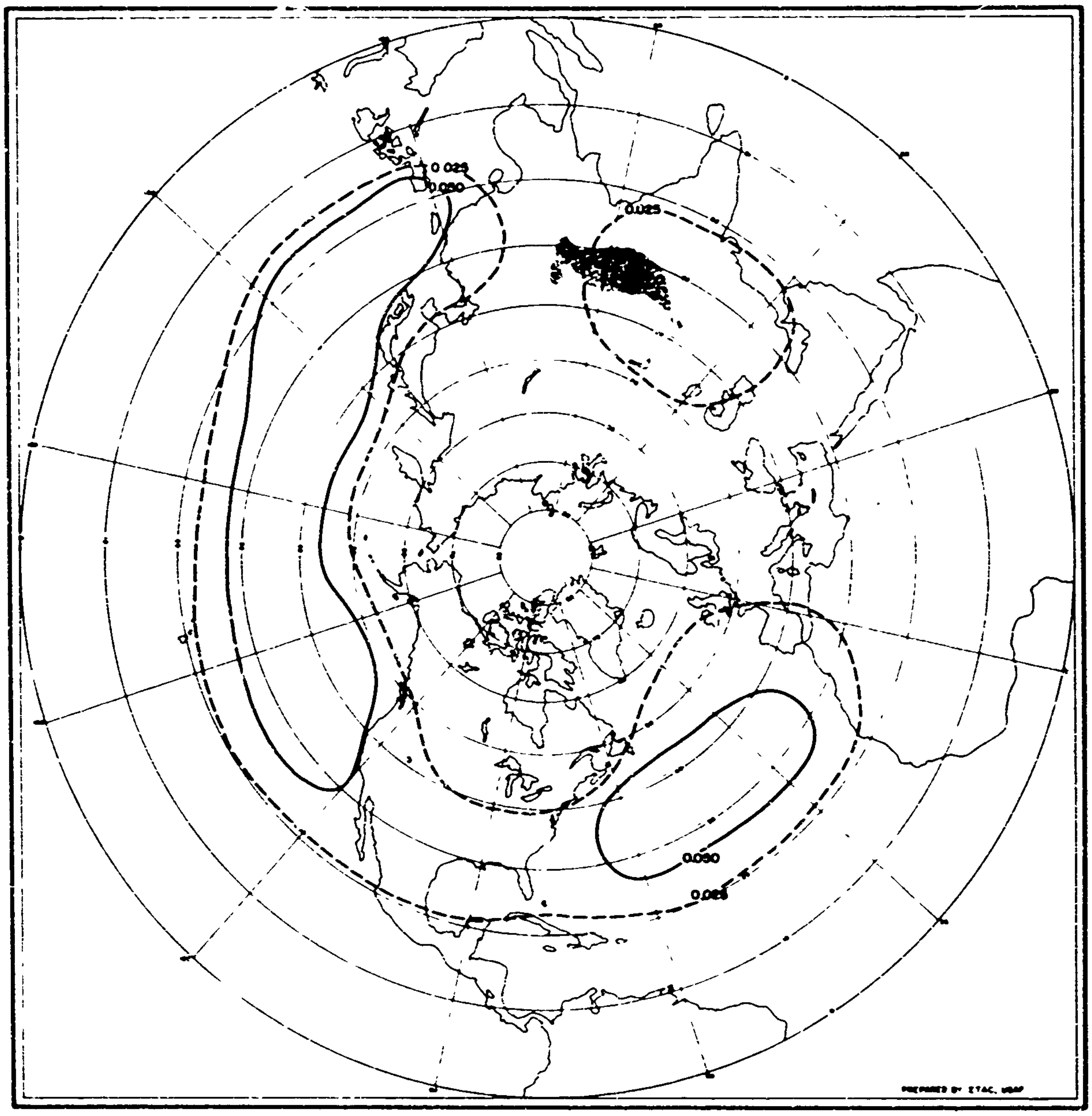
PROBABILITY OF ENCOUNTERING ICING CONDITIONS

WINTER, 5,000 FEET

FIGURE 1.



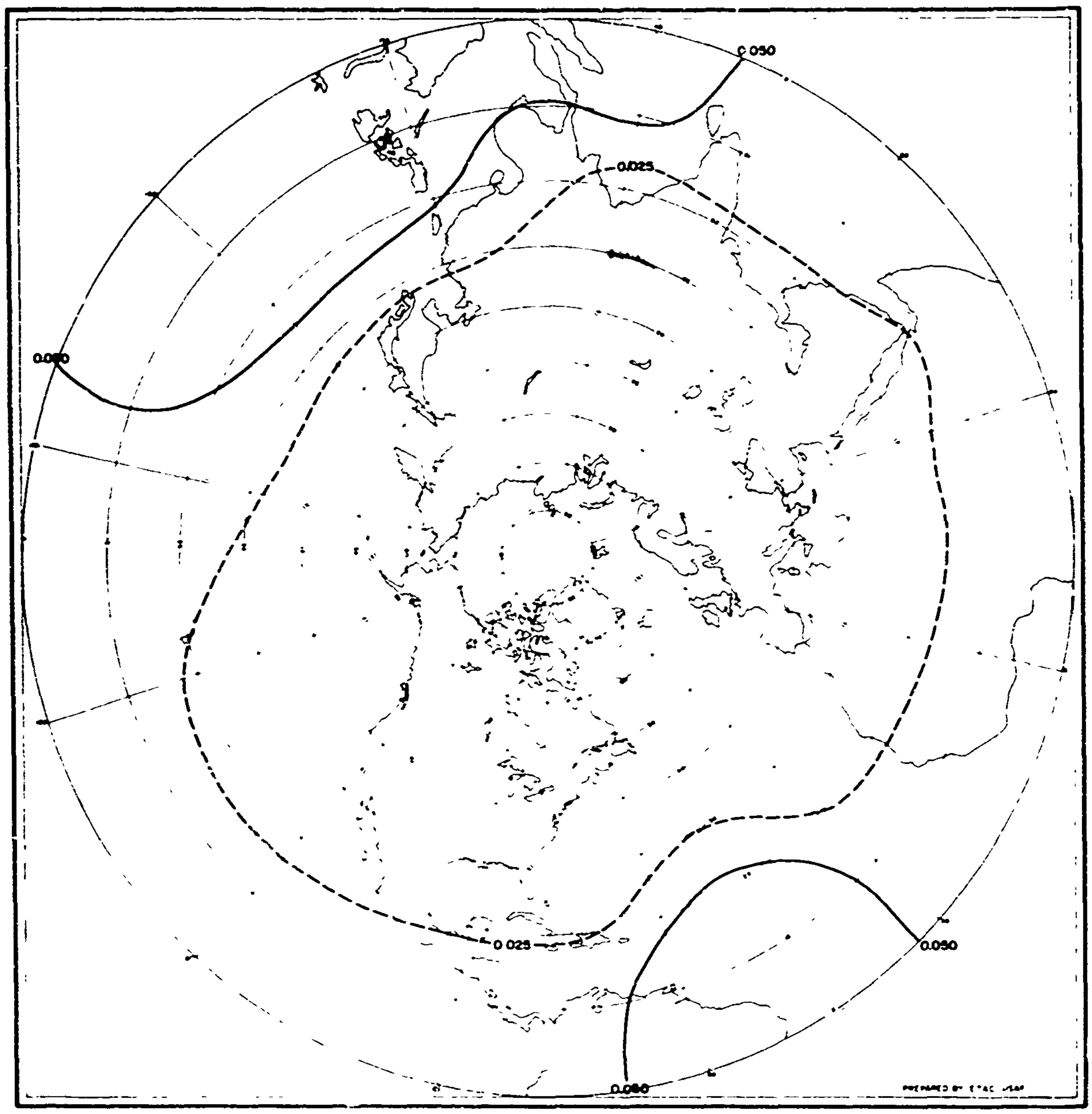
PROBABILITY OF ENCOUNTERING ICING CONDITIONS
WINTER, 10,000 FEET
FIGURE 2.



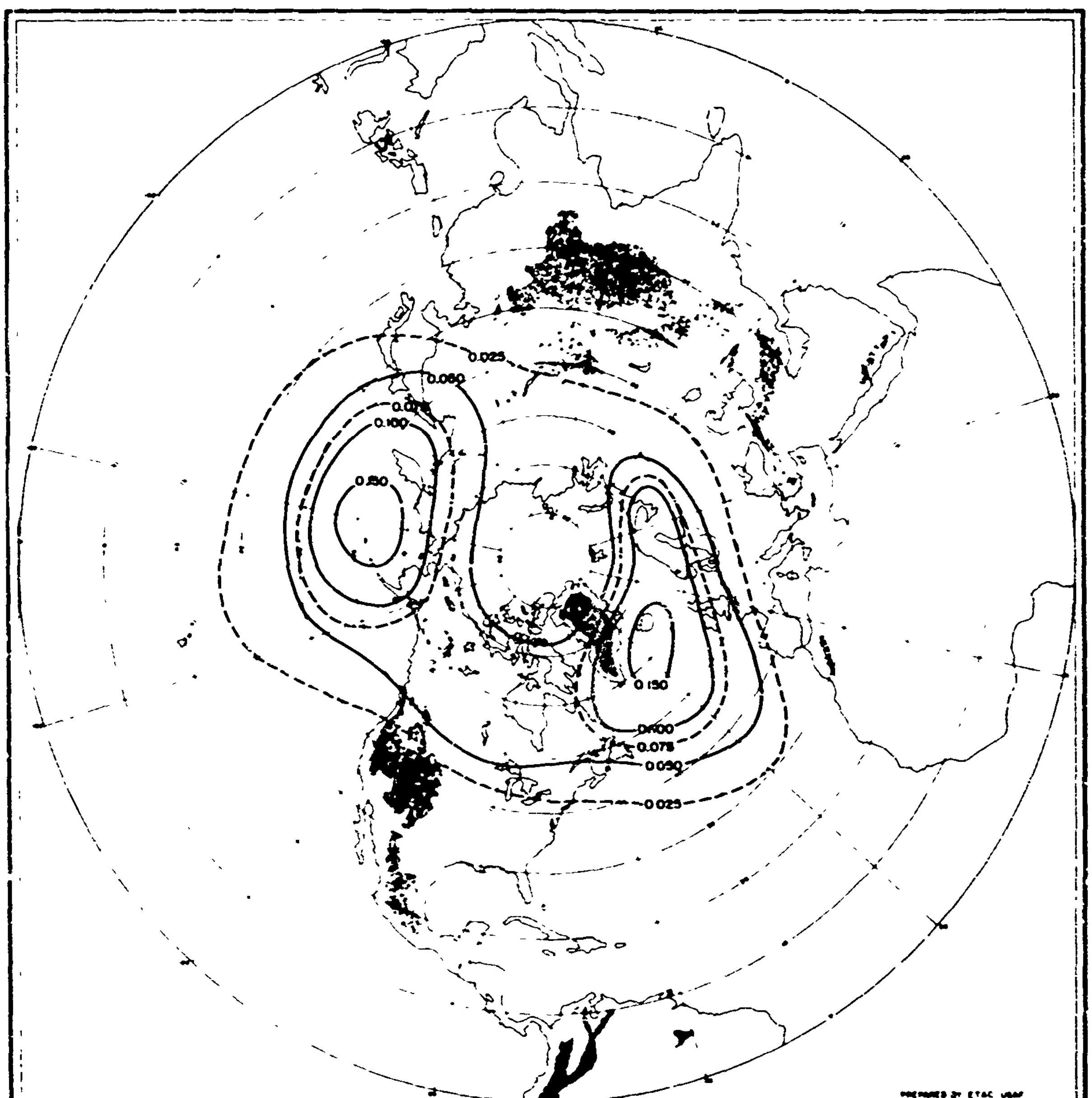
PROBABILITY OF ENCOUNTERING ICING CONDITIONS

WINTER, 15,000 FEET

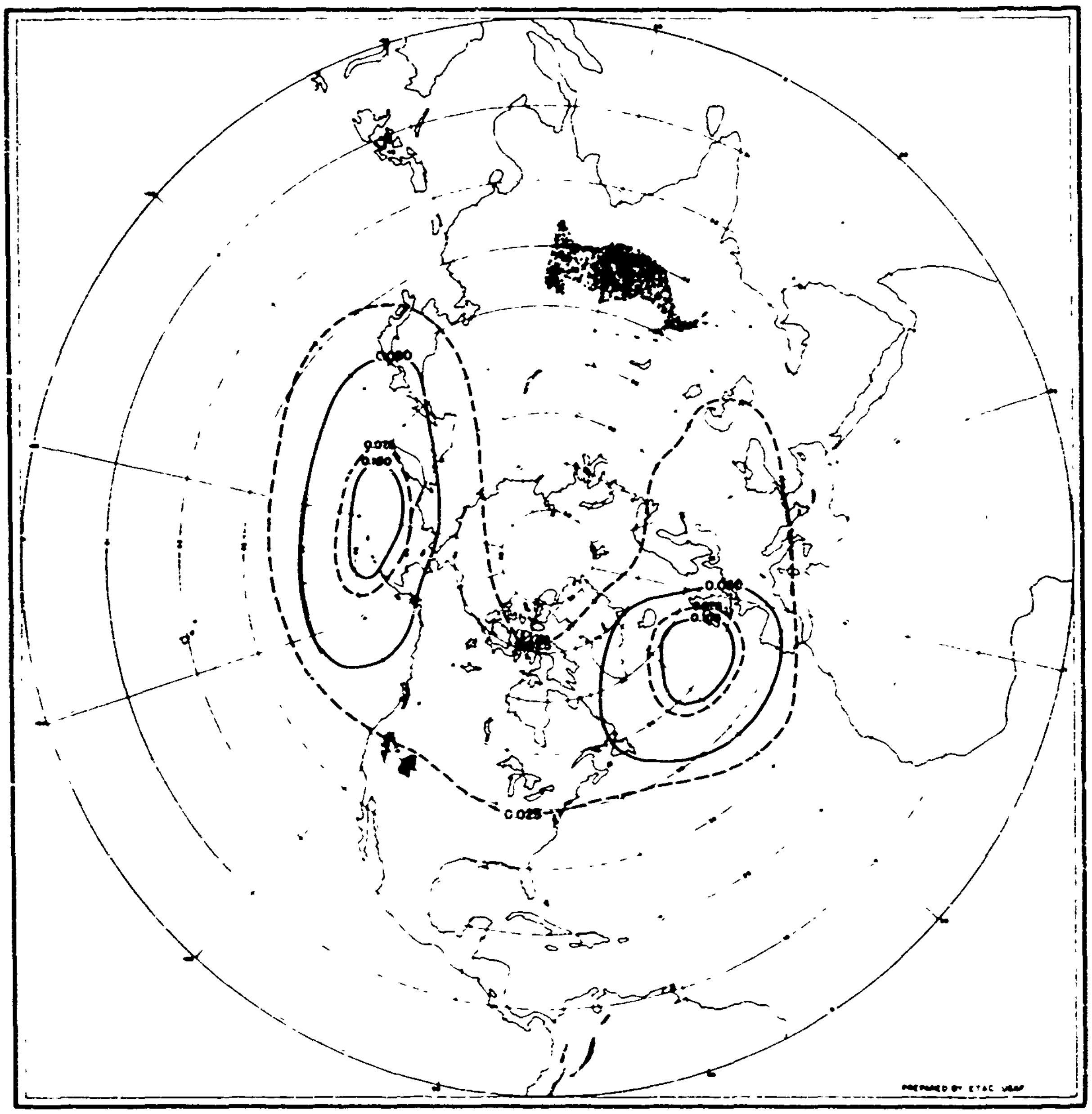
FIGURE 3.



PROBABILITY OF ENCOUNTERING ICING CONDITIONS
WINTER, 20,000 FEET
FIGURE 4.



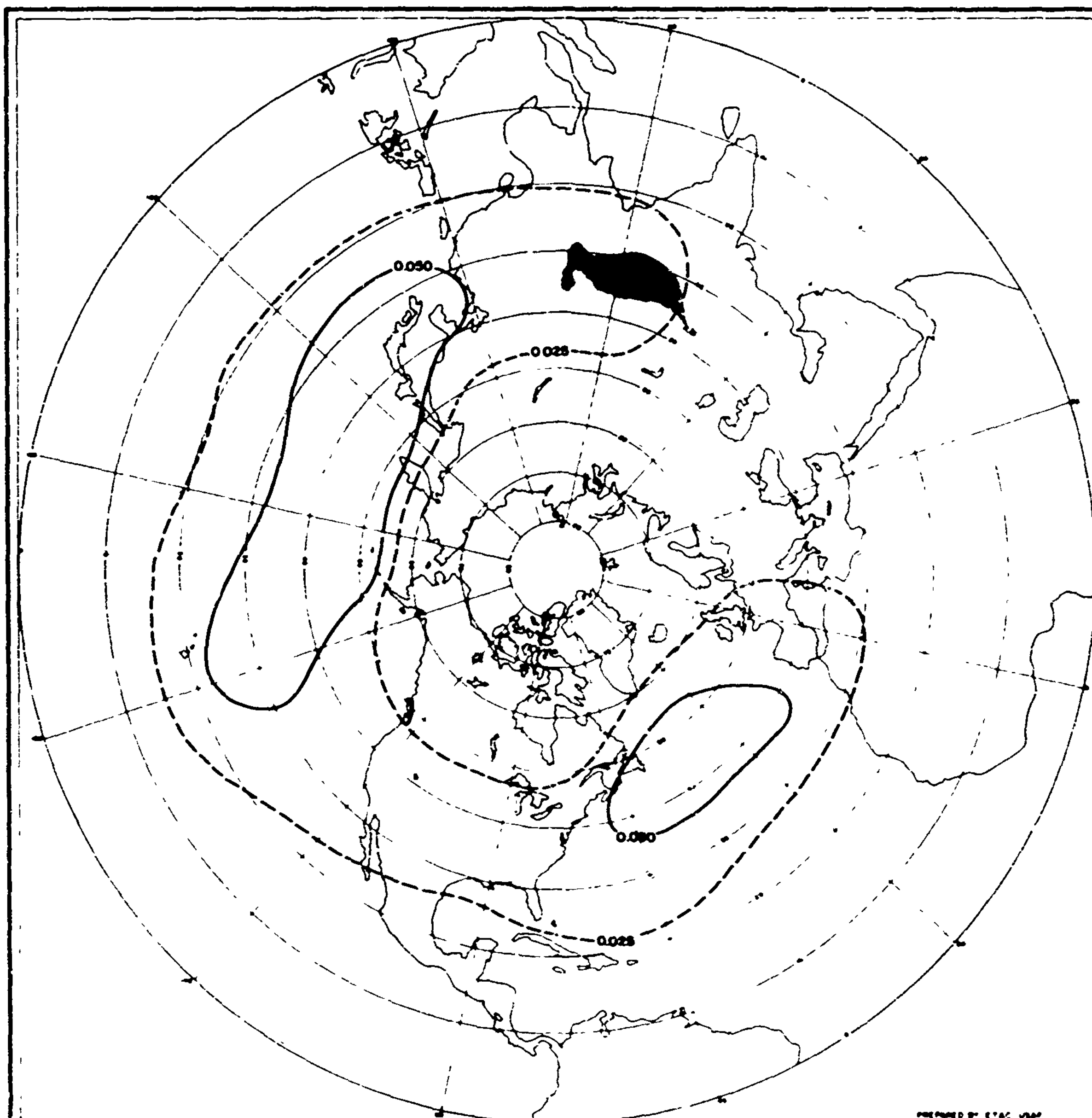
PROBABILITY OF ENCOUNTERING ICING CONDITIONS
SPRING, 5,000 FEET
FIGURE 5



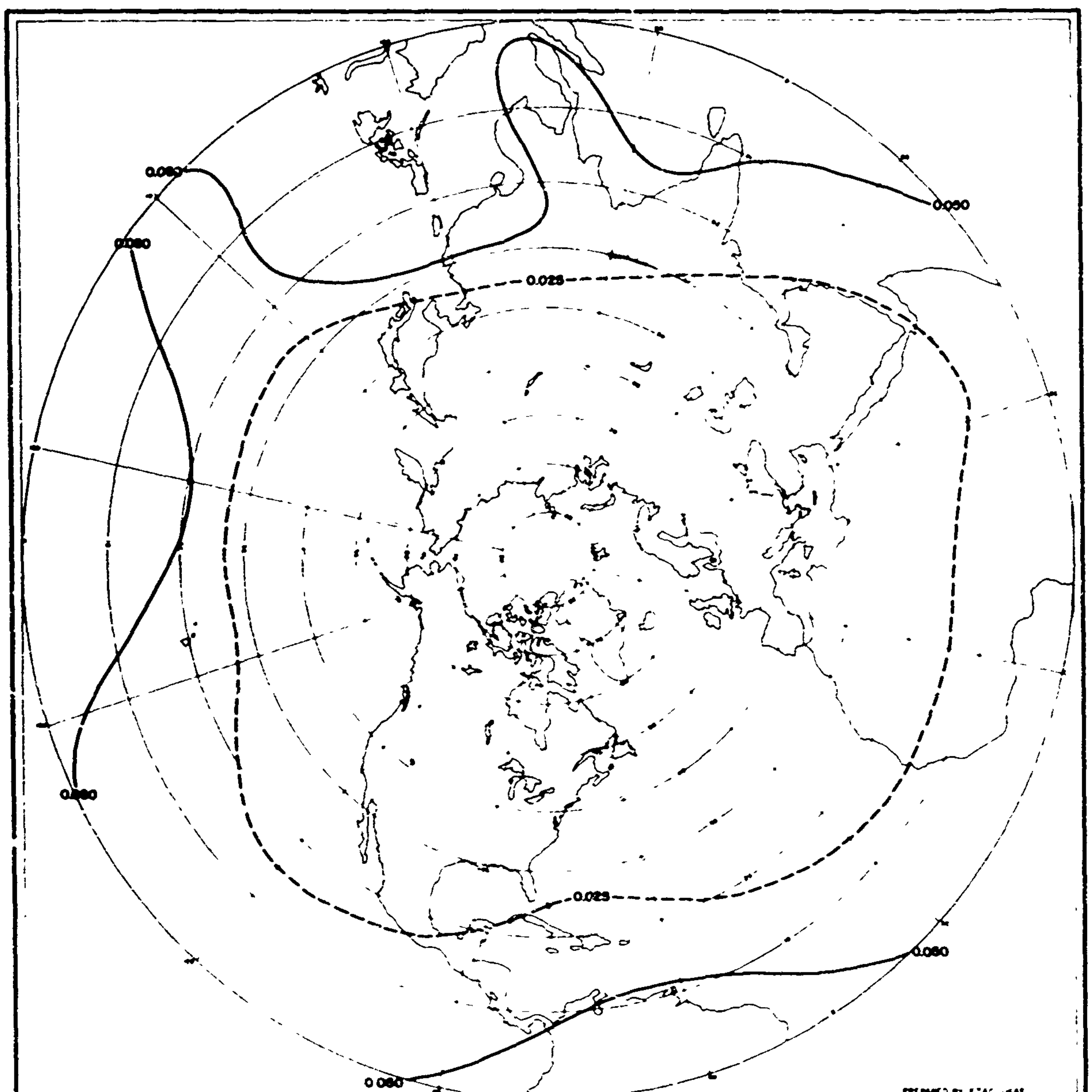
PROBABILITY OF ENCOUNTERING ICING CONDITIONS

SPRING, 10,000 FEET

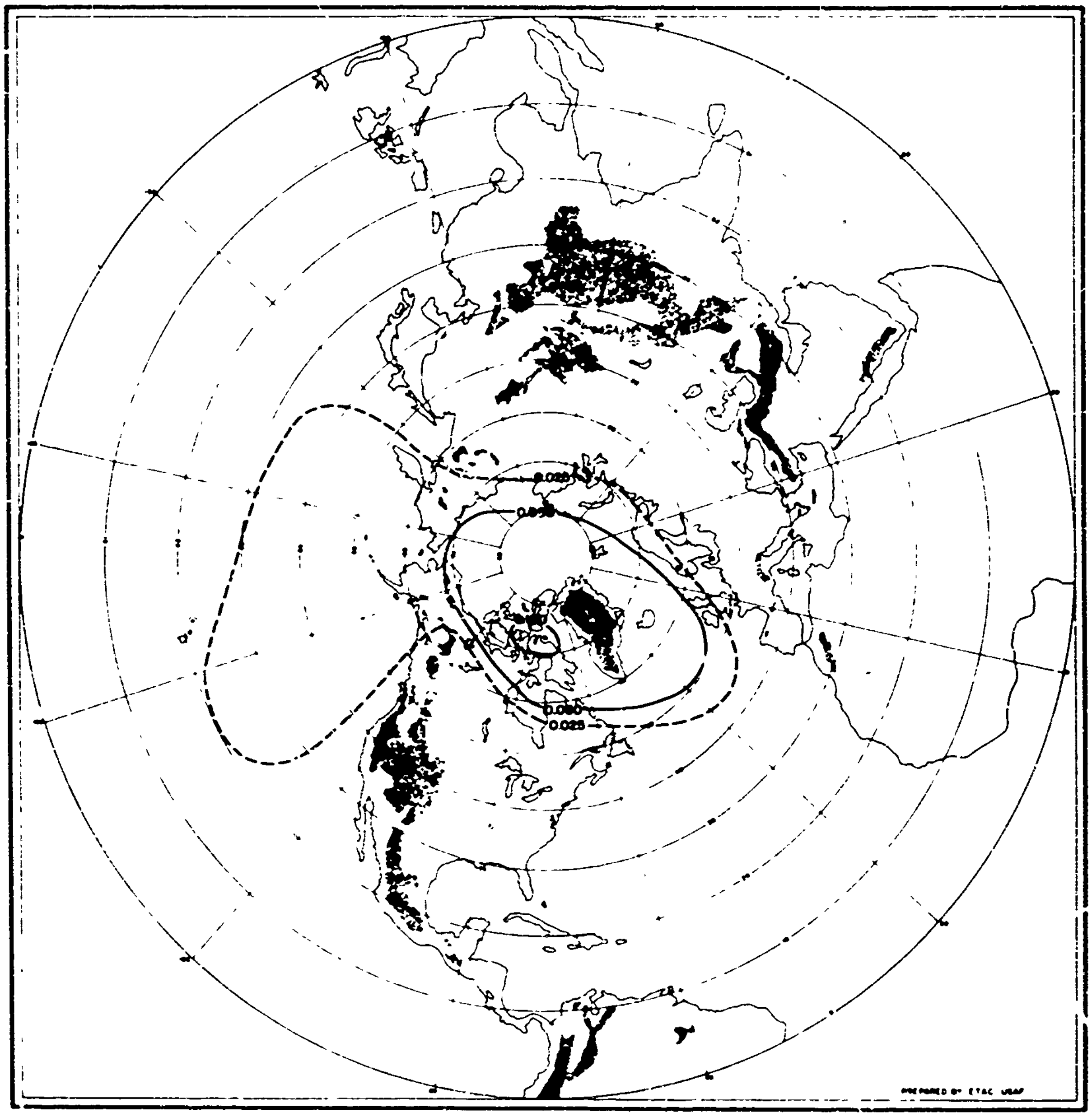
FIGURE 6.



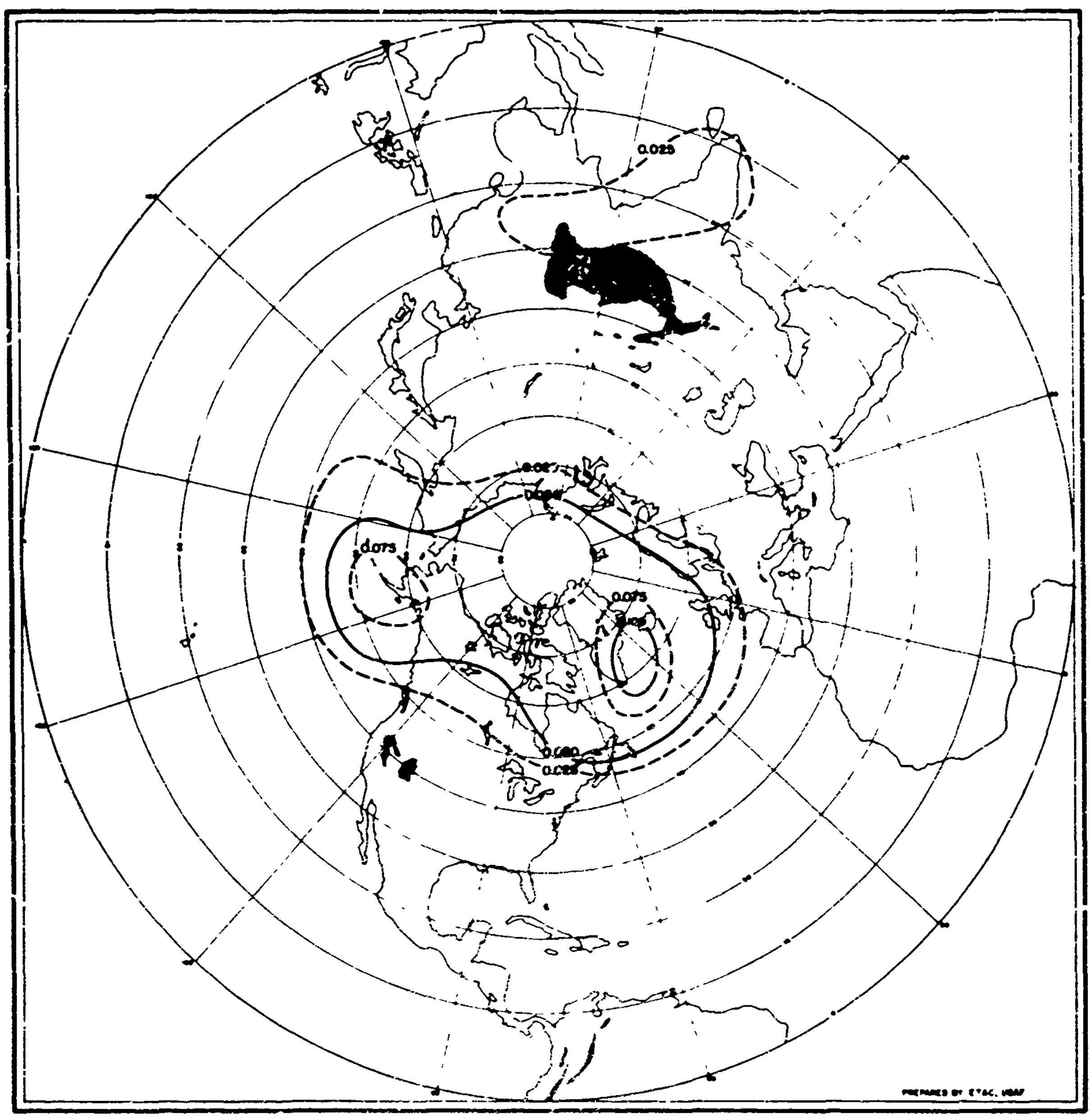
PROBABILITY OF ENCOUNTERING ICING CONDITIONS
SPRING, 15,000 FEET
FIGURE 7.



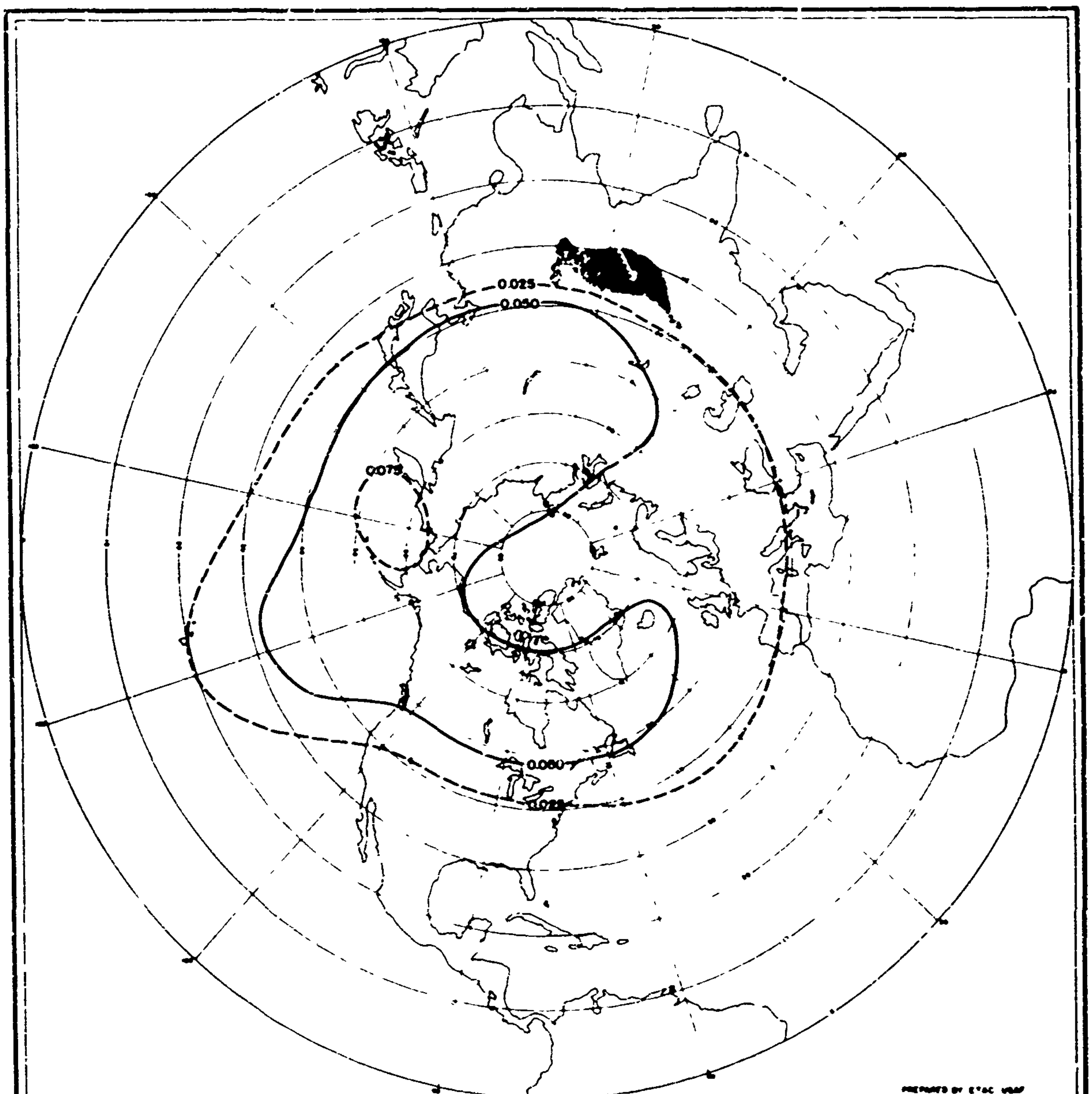
PROBABILITY OF ENCOUNTERING ICING CONDITIONS
SPRING, 20,000 FEET
FIGURE 8.



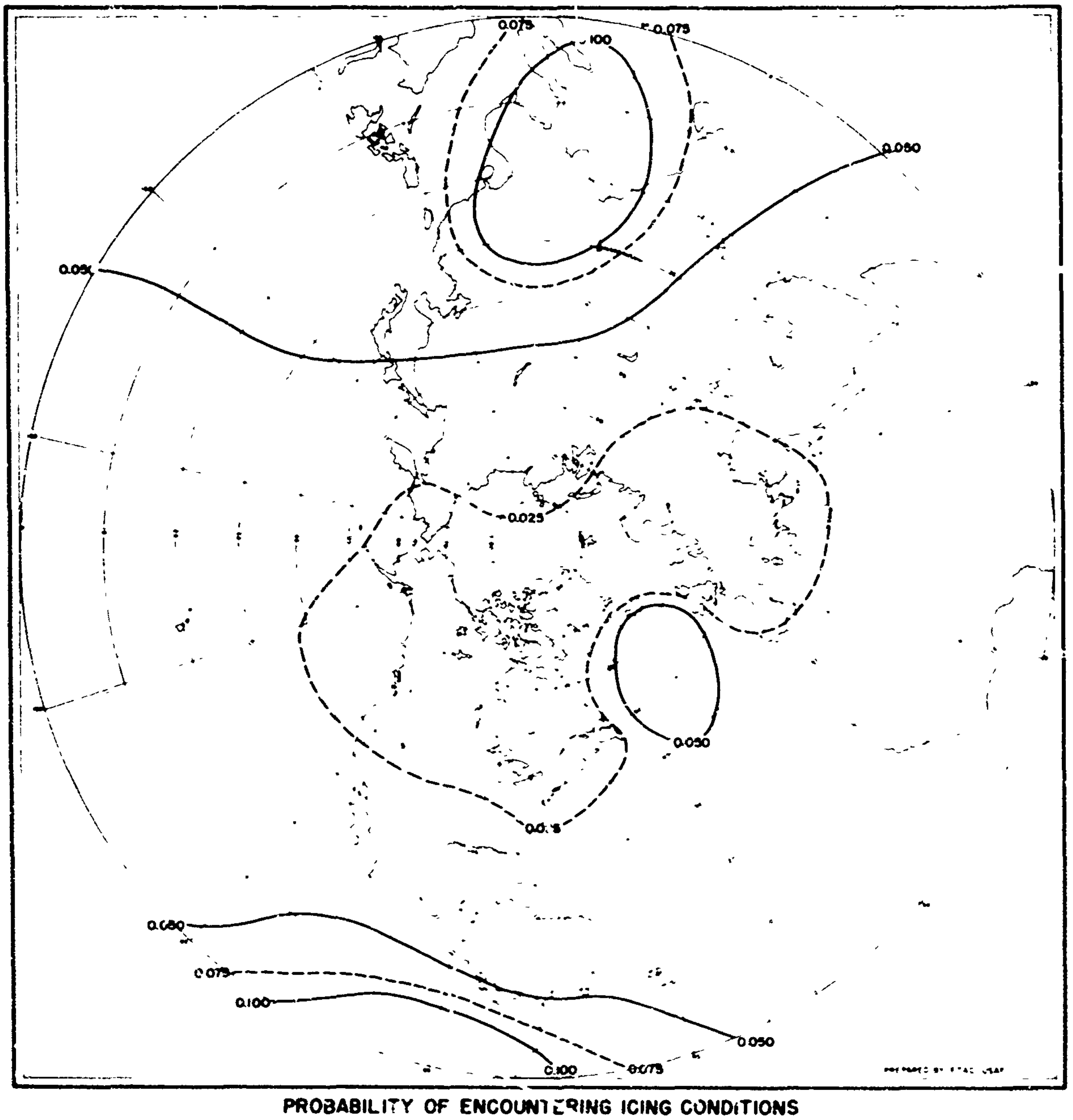
PROBABILITY OF ENCOUNTERING ICING CONDITIONS
SUMMER, 5,000 FEET
FIGURE 9.



PROBABILITY OF ENCOUNTERING ICING CONDITIONS
SUMMER, 10,000 FEET
FIGURE 10.



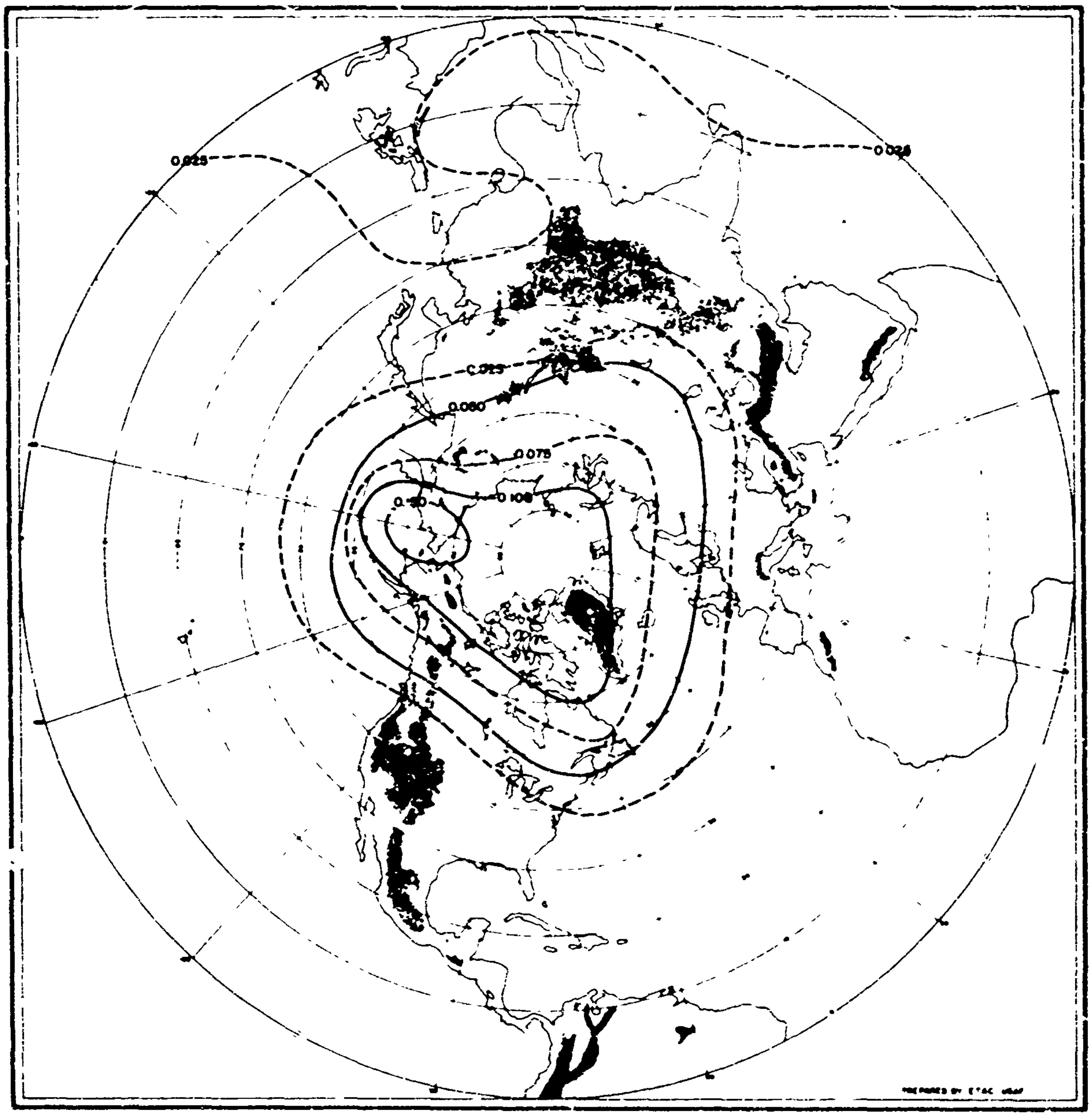
PROBABILITY OF ENCOUNTERING ICING CONDITIONS
SUMMER, 15,000 FEET
FIGURE 11.



PROBABILITY OF ENCOUNTERING ICING CONDITIONS

SUMMER, 20,000 FEET

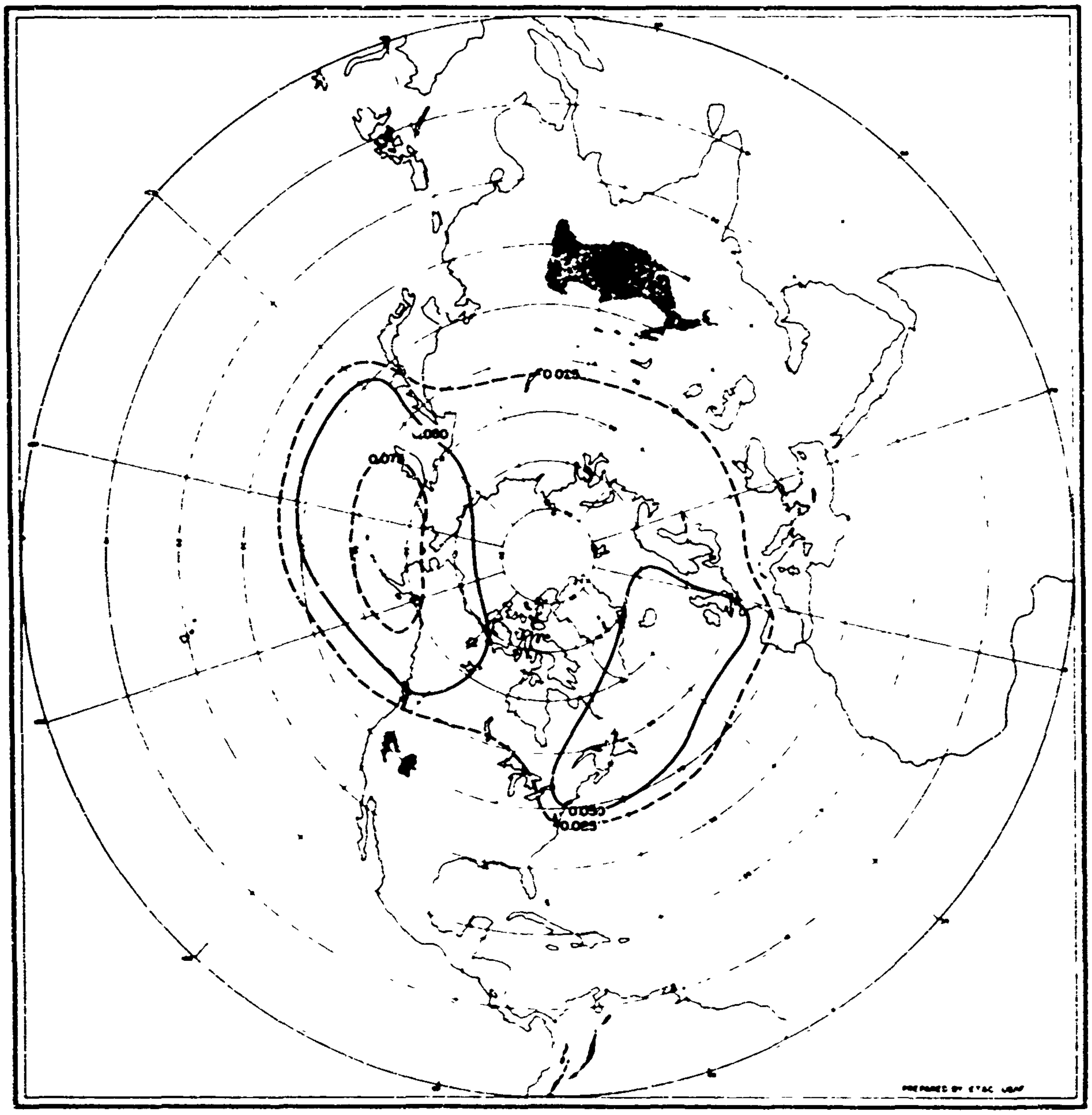
FIGURE 12.



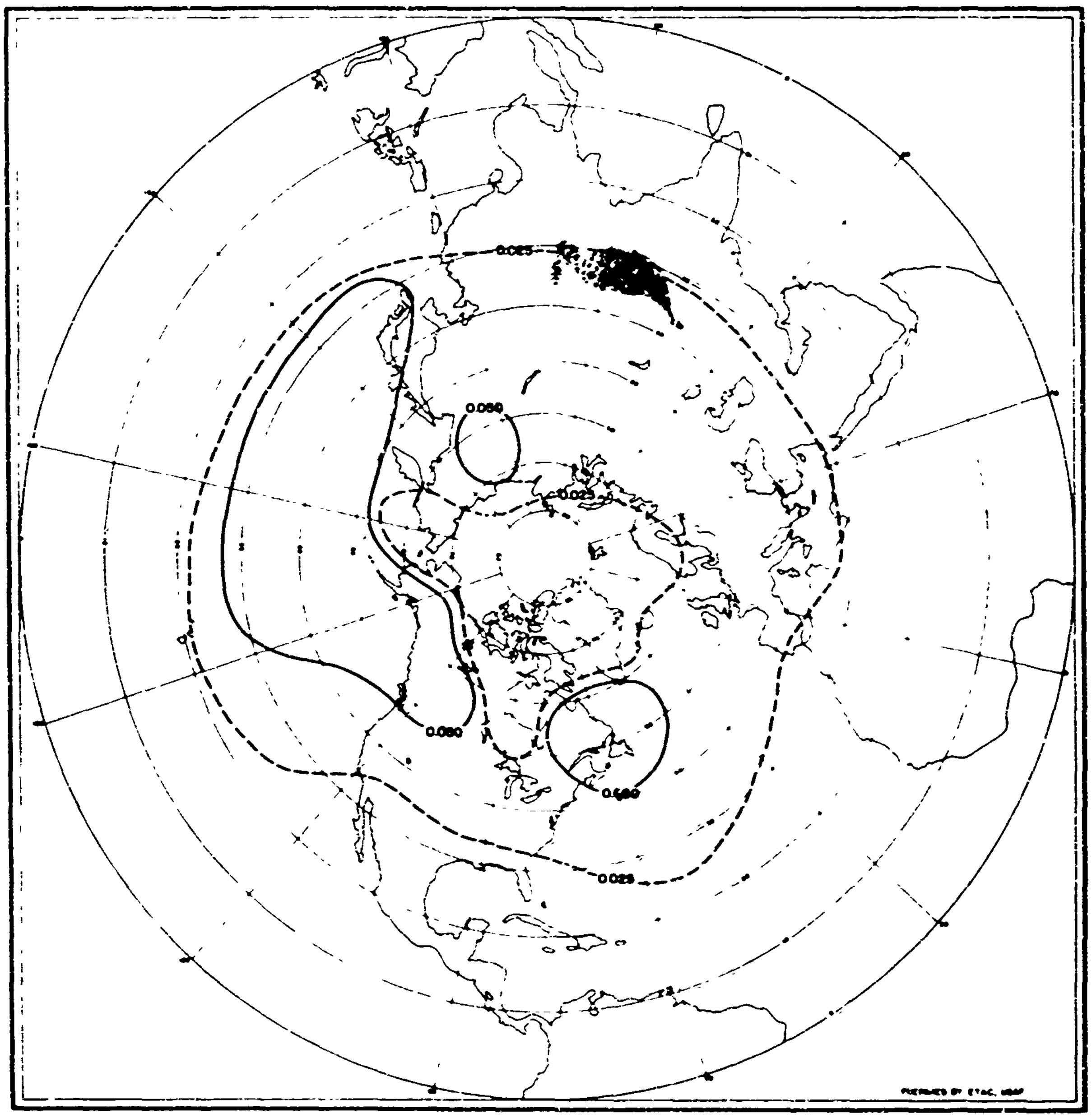
PROBABILITY OF ENCOUNTERING ICING CONDITIONS

AUTUMN, 5,000 FEET

FIGURE 13



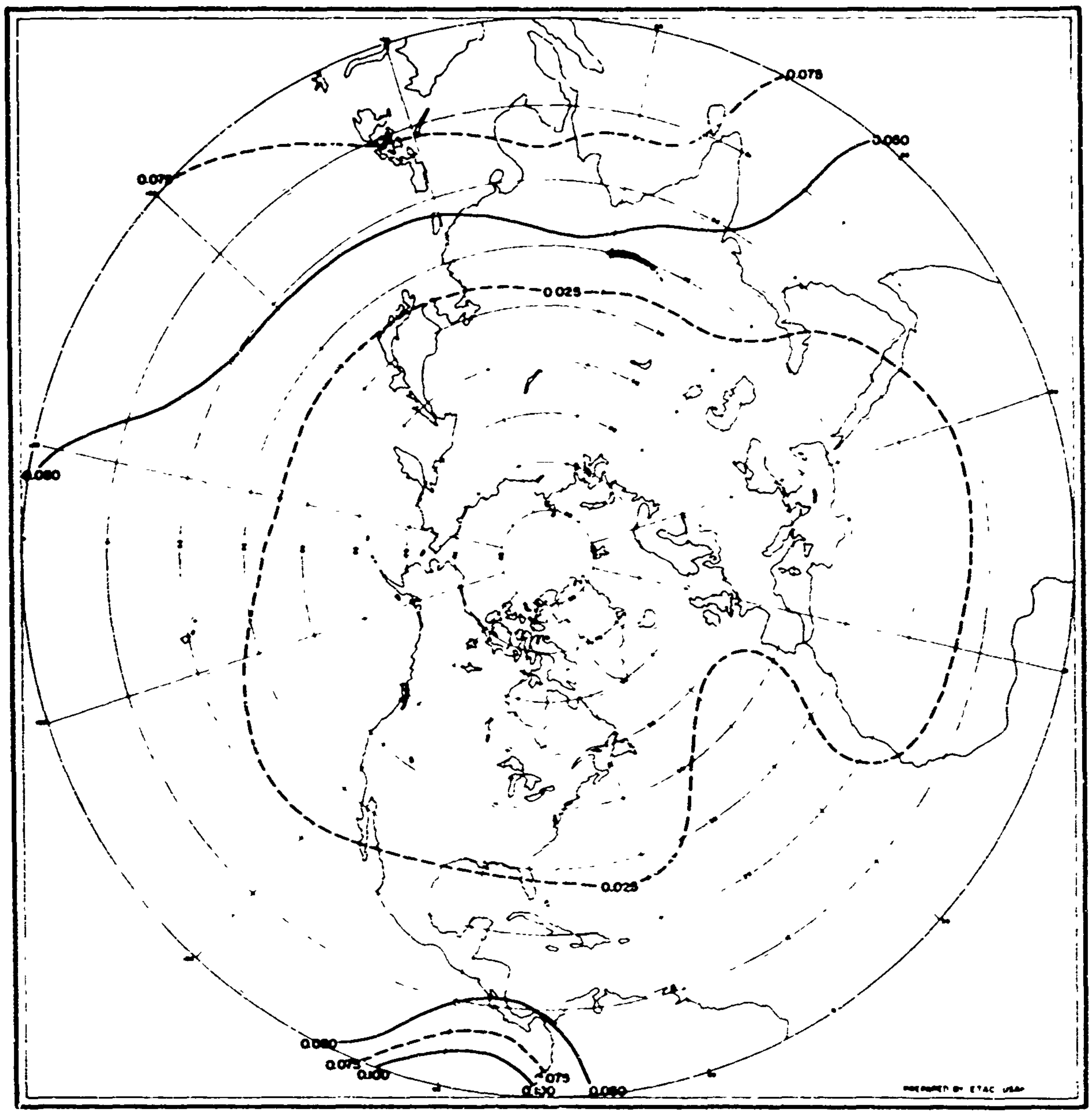
PROBABILITY OF ENCOUNTERING ICING CONDITIONS
AUTUMN, 10,000 FEET
FIGURE 14



PROBABILITY OF ENCOUNTERING ICING CONDITIONS

A.JTUMN, 15,000 FEET

FIGURE 15.



PROBABILITY OF ENCOUNTERING ICING CONDITIONS

AUTUMN, 20,000 FEET

FIGURE 16.

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13. ABSTRACT

This report presents seasonal Northern Hemisphere isopleth charts depicting the climatological probability of aircraft icing at 5,000, 10,000, 15,000, and 20,000 feet. It also describes the method used in determining these probabilities. A discussion of the data employed is also given.