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Satellite Celestial Navigation

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ABSTRACT

Satellite range and location data can be used to determine the location of an observer. This report discusses how the above may be done by using conventional celestial navigation methods.

In lieu of sighting with a marine sextant or a bubble octant to obtain an observed altitude (Ho), a receiver and a phase-measuring technique can be used to determine a parametric range distance between the satellite and the observer. Satellite height and range parameters can be reduced to obtain the value for an observed "virtual altitude' (VHo), which is then used in the same manner as the standard Ho measurement.

PROBLEM STATUS

This is a final report on one phase of the problem; work on other phases is continuing.

AUTHORIZATION

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SATELLITE CELESTIAL NAVIGATION

INTRODUCTION

Standard procedures for celestial navigation, that is, navigation by means of observations of celestial bodies, have been successfully used for some time. Fundamentally, the problem is the solution of an astronomical spherical triangle, the object being to determine one's longitude and latitude. Vertices of the navigational triangle are defined by the location of the nearer pole (either north or south), the subpoint (geographic position or ground position GP) of the celestial object, and the assumed position (AP) or dead-reckoning position (DR) of the observer.

First published in 1802, the "New American Practical Navigator" by Dr. Nathaniel Bowditch provided a comprehensive source of information in the presention of new and simplified formulas. Later, the title was altered by the U.S. Navy Hydrographic Office to the "American Practical Navigator", and it has been printed to provide more than 700,000 copies in about 70 editions since that time (1). Modern celestial navigation is said to have begun in the year 1837 when Captain Thomas Sumner discovered what has since been known as the Sumner Line, a line of position (LOP). Later, in 1875, Commander Marq Saint-Hilaire introduced the altitude difference method of determining the line of position (2). By the early part of the 20th century further refinements by various individuals, among them Admiral Ogura and Captain De Aquino, gave us transferred sets of the fundamental equations and also tables to solve the astronomical triangle by tabular methods. Tables by Dreisonstok (3), Ageton (4), Weems, the U.S. Works Progress Administration (5), and the British Admiralty Office (6) reduced the computation time of the navigator.

Using H.O. 9, the DR position, and a cosine-haversine method, an LOP in sea navigation can be plotted in 4 to 5 minutes (2). If an AP and either H.O. 208 or H.O. 211 is used, 2 to 3 minutes of computational work is required. The eight-volume set of H.O. 214 tables, in which each volume covers 10 degrees of latitude, will yield an LOP in about 1 minute using an AP. In less than 1 minute, the 18-volume set of H.O. 218 tables, in which each volume covers 5 degrees of latitude, can be used to obtain an LOP. However, in air navigation, after the observed altitude Ho is corrected to geocentric altitude and the AP is selected, the time to plot an LOP is about 5 minutes when using the Air Almanac and H.O. 214 or H.O. 218.

A range-measuring satellite can produce data which may be used in conjunction with any of the above existing methods. Satellite range and location data is indirectly substituted for a celestial body altitude measurement (Ho), declination (Dec.), and Greenwich Hour Angle (GHA) information associated with any instant of Greenwich Mean Time (GMT). The use of the Nautical Almanac remains unchanged (10). In lieu of sighting with a marine sextant or a bubble octant to obtain Ho, a radio receiver and a phase-measuring technique can be used to determine a parametric range distance between the satellite and the observer (7). Satellite height and range parameters can be reduced to obtain a value for the observed "virtual altitude" VHo, which is then used in the same manner as the standard Ho measurement.

OBSERVER ALTITUDE AND COALTITUDE

To a good approximation the earth can be considered as a sphere when using techniques of celestial navigation. In addition, a geocentric reference frame of angular measurement in latitude and longitude reduces position specification on earth to only two variables. A further simplification can be made by measuring length in nautical miles (n.mi.) since 1 n.mi. is approximately equal to the angular unit of 1 minute in arc on the surface of the earth.

Consider a straight line between the center of a celestial body and the center of the earth that would pierce the surface of the earth at the subpoint of the body. This subpoint directly below the celestial object is referred to as its geographical position or ground point GP. Within the limits of a practical mathematical solution we may regard all light rays, which either originate or are reflected from a celestial object to an observer on earth, as being parallel to that ray which defines the GP. The angle between the local tangential horizontal plane of the observer and a light ray to the celestial body is known as the observer altitude Ho. The plane in which the angle Ho lies is perpendicular to the horizon.

Although Ho is uniquely defined, observers can be located at other places on the earth and simultaneously measure the same angle. If the celestial object is viewed as being located on the axial line of an astronomically long right-circular cylinder, then the observer(s) will be located on the directrix formed by the intersection of the cylindrical wall and the earth. The acute angle between the horizon and the side of the cylinder is Ho. The directrix is a circle of equal altitude and a circle of equal altitude is a circle on the surface of the earth, at every point of which the altitude of a given celestial body is the same at a given instant. The further the observer is away from the GP the smaller will be the value of Ho. When the object is directly overhead at the observer's zenith, Ho is equal to 90 degrees. When the object appears to lie on the horizon, Ho is equal to zero degrees and the GP is a quarter of the distance around the earth from the observer.

Using the Ho measurement, the distance between the observer and the GP can now be determined. The angle at the center of the earth between the GP of the body and the position of the observer, that is, the arc of the great circle joining the two points, is known as coaltitude. It is equal to 90 degrees minus Ho. When the coaltitude is expressed in minutes of arc, it is equivalent to the distance expressed in nautical miles.

CHART LIMITATIONS

Mercator-type projections will be used to illustrate techniques in celestial navigation. There are other types of projections that can be employed to represent the earth on a plotting sheet; each has its own particular advantage. However, the Mercator is the most common type used in celestial navigation. Unless another type of chart is mentioned, it can be assumed that all plotting is done on a Mercator projection map. The particular type is the equatorial cylindrical orthomorphic projection. It is analogus to a projection on a cylindrical surface that is tangent to the equator of the earth. However, it is not perspective and the parallels cannot be located by geometrical projection, the latitude spacing being derived mathematically (1).

Expansion is the same in all directions and the angles are correctly shown since the projection is conformal. Great circles can be presented as rhumb lines and their directions can be measured directly on the chart. Distances also can be measured directly to a practical accuracy over a small spread in latitude. The latitude scale in the region of the plotting location is customarily used for measuring distances.

In the Mercator projection the longitude and latitude lines on the chart are orthogonal to each other. The meridial lines marking degrees of longitude are of equal spacing for any given chart. The associated lines of parallel for latitude are spaced according to the following formula from Bowditch (1):

$$M = a \ln 10 \log \tan \left(45^{\circ} + \frac{L}{2}\right) - a \left(e^2 \sin L + \frac{e^4}{3} \sin^3 L + \frac{e^6}{5} \sin^5 L + \ldots\right)$$

where

M = number of meridional parts between the equator and the given latitude
a = radius of the earth (in minutes of arc)
L = latitude
e = ellipticity of the earth

where, $e = \sqrt{2f - f^2}$
f = flattening factor of earth.

If the observer were to make sightings on three celestial objects whose GP's do not all lie on a great circle, the circles of equal altitude would mutually intersect each other in only one place. The point of intersection would be the location of the observer. Instead of different celestial bodies, the observer could use the same object by waiting until the earth had rotated sufficiently to give a different GP. This method also requires that the GP's are noncolinear wherein they do not all lie on a great circle. Two sightings would be sufficient if the observer could eliminate the ambiguity of two intersections by knowing his approximate location. One sighting for a solution would require that the observer have knowledge of either his longitude or latitude.

Any line on the surface of the earth upon which an observer is known to be located is called a line of position (LOP). A location which is determined by the intersection of LOP's is known as a fix. Depending upon the altitude of the celestial body, one of two different plotting methods can be used to represent the circle of equal altitude by an LOP.

Restrictions in plotting the circles of equal altitude by using only Ho and GP information occur in trying to represent the earth on a two-dimensional surface. A spherical surface for plotting would be too large to give the accuracy which is generally required for navigation. In any type of projection scheme for a sphere to be represented on a two-dimension surface certain distortion effects develop. A large circle of equal altitude from a low-altitude observation would appear to have the profile of an egg when plotted on a Mercator projection chart. However, as the altitude increases, the radius becomes smaller and the shape has a more circular appearance. There is no exact boundary which defines a high-altitude observation. The value of 87 degrees is the altitude limit selected by the U.S. Naval Academy and will be the value used in this discussion (8). For high-altitude observations the plotting of the circle of equal altitude as an LOP is accomplished in the following simple two-step method:

1. locate the GP of the observed body,
2. using the GP as a center point, draw a circle whose radius is equal to the coaltitude of the celestial body expressed in minutes of arc.

When the altitude of the celestial body is below 87 degrees, a different type of solution is generally required. Since a small segment of a circle, whose radius is very large compared to the arc, may be approximated by a straight line, we can estimate the LOP of an observer. In the region of the observer's location an LOP may be drawn perpendicular to the coaltitude line between the GP and the observer. By knowing the azimuthal bearing of celestial object at the observer's location, a portion of the coaltitude line can be drawn and the perpendicular LOP laid. In this way a smaller chart, which does not require the plotting of the GP, may be used. However, it now becomes necessary to determine azimuth and to correct for the approximate location (either DR or AP) of the observer. The navigational triangle is used to solve for these quantities. One side, the coaltitude, is known from the Ho measurement.

LINE OF POSITION CORRECTIONS

Within the inherent accuracy of the measuring instrument, the Ho measurement provides the observer with an exact determination of the distance that he is from the GP. Now, let us take a different approach. If the observer were to make an estimate of his location, it would be possible for him to mathematically compute an altitude value for a given celestial body. Instead of the observed altitude Ho, this value is called the computed altitude Hc. Of course, only by chance would Hc equal Ho because in estimating his location (either DR or AP) the possibility for error was introduced.

Two probable sources of errors can be identified. One occurs when Hc differs from Ho, even though the computed azimuth bearing at the estimated position for the GP is identical to the actual azimuth bearing for the same GP at the observer's location. This altitude error can be corrected. The other source of error develops when the computed azimuth at the estimated position for the GP does not agree with what would be the actual azimuth of the s? GP at the observer's location. The error that is produced by this second situation cannot always be corrected. Its existence is independent of whether or not Ho agrees with Hc. Since a determination of the observer's position requires the intersection of at least two LOP's, the true azimuth bearing of the GP at his location is not immediately known. Therefore, the azimuthal discrepancies occur "after the fact", and consequently the navigator might elect to choose an iterative type of graphical solution for maximum plotting accuracy.

First, in the equal azimuth case, if the estimated location is on the correctly oriented coaltitude line (observer to GP) but either further or closer to the GP than it should be, the value of Hc will be, respectively, smaller or greater than Ho. The correct intersection of the LOP and the coaltitude line can be found by a simple procedure:

1. Determine the absolute difference between Ho and Hc in minutes of arc, which is equivalent to nautical miles. This value is called altitude difference, abbreviated "a".

2. If Ho is more than Hc, move the LOP along the coaltitude line from the estimated position toward the GP by an amount equal to "a". This translation is known as "aT", i.e., altitude difference toward (the GP).

3. If Ho is less than Hc, accordingly move the LOP in the opposite direction. This translation is known as "aA", i.e., altitude difference away (from the GP). Note: A useful memory aid in recalling the above steps is the oriental-sounding "Ho Mo To" for Ho More, Toward (8).

Second, in the unequal azimuth case, let the estimated location not lie on the properly oriented coaltitude line (observer to GP) but rather on an estimated orientation of the coaltitude line (estimated location to GP). In this case the LOP corrected by the aT or aA adjustment would not necessarily contain the position of the observer. This discrepancy occurs because a straight-line approximation of the LOP was introduced when, in reality, the true LOP is actually the segment of a circle of equal altitude.

NAVIGATIONAL TRIANGLE

Through the assumption of an estimated location of the observer, the information necessary for the solution of a spherical triangle has been acquired. By solving this navigational triangle we obtain the values for Hc and azimuth of the GP at the estimated location of the observer. Before getting into the solution of the triangle, a few definitions (8) will prove helpful:

1. Navigational triangle ... is that spherical triangle on the surface of the earth formed by the great circles connecting the nearer pole, the position of the observer, and the GP of the given celestial body.

2. Nearer pole ... is the pole on the surface of the earth nearer to the position of the observer; it is the north pole for an observer in north latitude and the south pole for an observer in south latitude. When considered to be a point on the celestial sphere, it is called the "elevated pole".

3. Same name .. is a case in which the altitude of the observer and the declination of a celestial body are of the same name, both north or both south.

4. Contrary name ... is a case in which the latitude of the observer and the declination of the celestial body are of opposite name, i.e., one north and the other south.

5. Coaltitude ... is one side of the navigation triangle. It may be considered the angle at the center of the earth between the GP of the body and the position of the observer, or the arc of the great circle joining the two points. It is equal to 90° minus the observed altitude.

6. Colatitude ... is another side of the triangle. It may be considered the angle at the center of the earth between the position of the observer and the nearer pole, or the arc of the great circle joining the two points. It is equal to 90° minus the latitude of the observer.

7. Polar distance ... is the remaining side of the navigational triangle. It may be considered the angle at the center of the earth between the GP of the body and the nearer pole, or the arc of the great circle joining the two points. It is equal to 90° minus the declination of the celestial body if the latitude of the observer and declination are of the "same" name, and to 90° plus the declination if they are of "contrary" name.

8. Meridian angle (t) ... is one angle of the navigational triangle. It is the angle at the nearer pole measured from the meridian of the observer toward the meridian of the GP. It is suffixed E (east) or W (west) as the GP is east or west of the observer.

9. Azimuth angle (Az) ... is another angle of the triangle. It is the angle at the observer, measured from his meridian (in the direction of the nearer pole) toward the great circle joining the observer and the GP. It is prefixed N (north) or S (south) as the nearer pole is north or south of the observer, and suffixed E (east) or W (west) as the GP is east or west of the observer.

10. Azimuth (Zn) ... is the true direction of the celestial body or its GP, measured from 000° at north clockwise to 360°. It is obtained by converting Az, by means of its labels.

11. Assumed latitude (aL) ... is the latitude of the observer's assumed position.

12. Assumed longitude (aλ) ... is the longitude of the observer's assumed position.

13. Tabulated altitude (ht) ... is the computed altitude for the exact values of entering arguments given in H.O. 214 tables (9). After correcting ht for the AP interpolation, ht becomes Hc.

Equations which may be used for the solution of the navigational triangle are:

$$Hc = \sin^{-1}[\sin(GP\ lat.)\ \sin(aL) + \cos(GP\ lat.)\ \cos(aL)\ \cos(a\lambda - GP\ long.)]$$

$$Az = \cos^{-1}\left[\frac{\sin(GP\ lat.) - \sin(Hc)\ \sin(aL)}{\cos(Hc)\ \sin(aL)}\right]$$

where, $-90^\circ \leq \text{latitude} \leq 90^\circ$

$-180^\circ \leq \text{longitude} \leq 180^\circ$

and, $-90^\circ \leq Hc \leq 90^\circ$

$0^\circ \leq Az \leq 180^\circ$

Negative values of Hc will be explained later. At this time the lower limit of Hc may be considered as zero. In the H.O. 214 tables (9) the lower limit is 5° 00'.0.

Rather than burden the navigator with a large amount of calculations, the simpler approach is to solve the navigational triangle through the use of tables. The tables selected to be used for illustrating celestial navigation techniques are the H.O. 214 tables contained in nine volumes. These tables are comprised of tabulated solutions of the navigational triangle, so arranged as to yield computed altitude, Hc, and azimuth angle Az by inspection. The tables are applicable equally to observations of the sun, moon, planets, and navigational stars, whether observed in north or south latitude (9).

TABLES OF COMPUTED ALTITUDE AND AZIMUTH

Three arguments are used to enter H.O. 214 tables (Ref. 9): the observer's AP latitude L, the GP declination (Dec.), and the meridian angle t between the AP and GP longitude lines. There are nine volumes that comprise the set of tables; each volume covers 10° of latitude L. Within each volume, the computed values of ht and Az are listed for intervals of 1° of L, 30' of Dec. up to 29°, and at selected intervals beyond that, and 1° of t (8). The minimum value of Hc is 5° 00'.0. In the tables, the altitude ht is abbreviated Alt., meridian angle t is abbreviated H.A. (hour angle), and latitude L is abbreviated Lat. For a comprehensive discussion in the use of the tables, Refs. 1 and 8 are quite helpful. Reference 9 contains illustrated examples of celestial navigation procedures.

It is necessary to select an AP for the observer in such a manner as to facilitate direct entry into the tables. The assumed longitude aλ and assumed latitude aL are selected to provide whole number degrees for L and t. Depending on the sign of L and Dec., the base values will be found in one of two categories of Dec. values, "Same" or "contrary", as previously mentioned. The column in which the base values are contained is indicated by the tabulated Dec. value that is closest to the Dec. of the GP. In the preparation of determining the three arguments (L, Dec., and t) it is helpful to label them with directional signs. The quantities L and Dec. are prefixed N or S according to the hemisphere in which they are measured. The value of t is suffixed E or W according to the direction in which it is measured. A few examples are:

Example 1

Parameter	Latitude	Longitude
DR (known)	N 33° 04.1'	107° 18.4' W
GP (known)	N 28° 02.6'	115° 23.6' W
AP	N 33° 00.0'	107° 23.6' W

L = N 33°, Dec. = N 28° 00.0', t = 8° W

Example 2

Parameter	Latitude	Longitude
DR (known)	N 33° 04.1'	107° 18.4' W
GP (known)	N 45° 10.5'	101° 55.0' W
AP	N 33° 00.0'	107° 55.0' W

L = N 33°, Dec. = N 45° 00.0', t = 6° E

Example 3

Parameter	Latitude	Longitude
DR (known)	N 10° 41.3'	8° 10.4' W
GP (known)	S 44° 13.6'	16° 05.8' E
AP	N 10° 00.0'	7° 54.2' W

L = N 10°, Dec. = S 45° 00.0', t = 24°E

After the AP has been selected in order to provide the three arguments, the tabular information can be obtained. The Δd entry represents the change in altitude due to a change of 1.0' of arc of declination (9). The Δt entry represents the change in altitude due to a change of 1.0' of arc of meridian angle. When working from a DR position, a ΔL value is used. The ΔL values are located in a special table in the back of each volume. The ΔL increment represents the change in altitude due to a change of 1.0' of arc of latitude. The use of a DR position method and the ΔL value will be covered later. All delta (Δ) values are used for purposes of interpolation and are given a sign, + or -, according to adjacent tabular information.

To extract the base values from the tables the following procedure can be used:

1. According to the value of L, select the applicable volume.
2. Use L to determine the correct section of the volume to be used. Each section is marked by an indexed tab for L.
3. Chose the "Same" listing of values to obtain information if L and Dec. have the same name, i.e., both lie in the same hemisphere; otherwise, chose "Contrary".
4. From the "Same" or "Contrary" listing, locate the Dec. column value that is closest to the Dec. of the celestial body.
5. From the H.A. column locate the corresponding t value.
6. Record the base values that are referenced by the Dec. column and t row. If Δd and Δt do not have a decimal point in the value, record the value without any decimal change.
7. Determine the sign of the Δd and Δt values in the following manner:

   a. Δd ... By comparing ht for the entering Dec. with the values of ht for adjoining tabulated Dec. values, determine whether ht is increasing or decreasing as the tabulated Dec. approaches the exact Dec. If ht is increasing, preface Δd with a plus sign, and minus if ht is decreasing.

   b. Δt ... By comparing ht for the entering t with the values of ht for the adjoining tabulated t values, determine whether ht is increasing or decreasing as the tabulated t approaches the exact t. If ht is increasing, preface Δt with a plus sign, and minus if ht is decreasing.

8. Label Az to provide a reference for the conversion to Zn which takes place later. Prefix Az with the sign (N or S) or L; suffix Az with the sign of t (E or W).

Arguments from the three previous examples will be used to illustrate the extracted base values:

Example	L	Dec.	t	ht	Δd	Δt	Az
4[α]	N 33°	N 28° 00.0'	8° W	81° 29.4'	+.61	-.71	N 123.9° W
5[β]	N 33°	N 45° 00.0'	6° E	77° 08.1'	-.92	-.30	N 019.4°E
6[γ]	N 10°	S 45° 00.0'	24° E	30° 53.3'	+.88	-.34	N 160.4° E

[α]See example 1.
[β]See example 2.
[γ]See example 3.

There exists a third tabular correction factor, namely ΔL. Its derivation and use will be covered in the discussion of the method that requires this value. The procedures for determining the base value are applicable to the three methods of solving the navigational triangle. The "Δd only" method is most frequently used. Its simplicity provides for rapid navigation, and the accuracy is sufficient for general deep-sea navigational requirements.

THE "Δd only" METHOD

As an illustration of the method, the information in examples 1 and 4 are used. Solution for the LOP can be accomplished by the following steps:

1. Estimate the DR location, select an observable celestial body that will give Ho, and chose an AP. From this information solve for t:

DR lat.	N 33° 04.1'	DR lon.	107° 18.4' W
GP lat. [Dec.]	N 28° 02.6'	GP lon.	115° 23.6' W
AP lat. [aL]	N 33° 00.0'	AP lon. [aλ]	107° 23.6' W
		t	8° W

2. Calculate the value for d diff. (i.e., Declination difference) as follows:

$$\text{d diff.} = |\text{Exact Dec.} - \text{Table Dec.}|$$

3. Use the three "boxed" arguments to enter the tables in H.O. 214 and extract base values for ht, Δd, and Az. All base values except ht should be signed.

4. To determine the d correction, which is

$$(\text{d diff.})(\pm\ \Delta d) = \text{d correction},$$

the multiplication tables that are printed on the inside back cover and facing page of each volume of the H.O. 214 set can be used.

5. According to the prefix and suffix of Az determine Zn.

6. Correct the value of ht by the d correction to obtain Hc.

7. Determine the absolute difference between Ho and Hc to obtain "a". Label the difference aA or aT.

8. Outlined below is a convenient form of listing the above information. The base value names are "boxed" for ease of location.

d diff. .... 02.6'	$\boxed{\Delta d}$ .... +.61	d correction	+ 1.6'	-
$\boxed{Az}$ ..... N 123.9° W				
Zn ..... 236.1°				
$\boxed{ht}$ ..... 81° 29.4'				
Corr. .. +1.6'				
Hc ..... 81° 31.0'				
Ho ..... 81° 5.2' (known)				
a ...... 25.8 A nautical miles				

The sight must be plotted from the AP as follows (9):

Latitude ... The whole degree with which the tables were entered, i.e., aL.
Longitude .. The longitude which was assumed in finding the meridian angle in whole degrees, i.e., aλ.

THE "Δd and Δt" METHOD

Everything in the "Δd only" method is applicable to the "Δd and Δt" method. The same examples are used. The steps are:

1. Same as Step 1, "Δd only".

2. Same as Step 2, "Δd only". In addition, claculate the value for t diff. (i.e., meridian angle difference) as follows:

$$t \text{ diff.} = \left| \text{DR lon.} - a\lambda \right|$$

3. Same as Step 3, "Δd only". In addition, extract Δt and give it a sign.

4. Same as Step 4, "Δd only". In addition, to determine the t correction, which is

$$(t \text{ diff.})(\pm \Delta t) = t \text{ correction},$$

the multiplication tables that are printed on the inside back cover and facing page of each volume of the H.O. 214 set can be used.

5. Same as Step 5, "Δd only".

6. Correct the value of ht by the d correction and t correction to obtain Hc.

7. Same as Step 7, "Δd only".

8. An expanded outlined, as compared to the "Δd only" outline, is given to serve as a form of listing the information:

d diff.	02.6'	Δd	+.61
t diff.	05.2'	Δt	-.71

	+	-
d correction	1.6'	
t correction		3.7'
Sum	1.6'	3.7'
Correction	-2.1'	

Az	N 123.9° W
Zn	263.1°

ht	81° 29.4'
Corr	-2.1'
Hc	81° 27.3'
Ho	81° 5.2' (known)
a	22.1 A nautical miles

The sight must be plotted from the following position (9):

Latitude ... The whole degree with which the tables were entered, i.e., aL.
Longitude ... The DR longitude, i.e., DR lon.

THE "Δd, Δt, and ΔL" METHOD

Procedures in both of the previously described methods are used in the "Δd, Δt, and ΔL" method. The information in examples 1 and 4 are used to show the technique in this solution. The steps are as follows:

1. Same as Step 1, "Δd and Δt" method.

2. Same as Step 2, "Δd and Δt" method. In addition, calculate the value for L diff. (i.e., latitude difference) as follows:

$$\text{L diff.} = \left| \text{DR lat.} - \text{aL} \right| .$$

3. Same as Step 3, "Δd and Δt" method. In addition, give ΔL a sign. This is done according to the following:

if Az > 90° and DR lat. > aL, then ΔL sign = -
if Az > 90° and DR lat. < aL, then ΔL sign = +
if Az < 90° and DR lat. > aL, then ΔL sign = +
if Az < 90° and DR lat. < aL, then ΔL sign = -

4. To determine the L correction, which is,

$$\text{L correction} = (\Delta\text{L value})(\text{L diff.})$$

where, $$\Delta\text{L value} = \text{"}\Delta\text{L sign"} \left| \cos \text{Az} \right|$$

the tables that are printed near the back of each H.C. 214 volume (just before the Δd and Δt tables) can be used. The table values are the product of the absolute ΔL value times L diff. in minutes. The entering arguments are Az and L diff.

5. Same as Step 5, "Δd only" or "Δd and Δt" method.

6. Correct the value of ht by the d correction, t correction, and L correction to obtain Hc.

7. Same as Step 7, "Δd only" or "Δd and Δt" method.

8. A complete form listing the information is given:

DR lat.	N 33° 04.1'	DR lon.	107° 18.4' W
GP lat. [Dec.]	N 28° 02.6'	GP lon.	115° 23.6' W
AP lat. [aL]	N 33° 00.0'	AP lon. [λ] ..	107° 23.6' W
		t	8° W

					+	−
d diff.	02.6'	Δd	+.61	d correction	1.6'	
t diff.	05.2'	Δt	−.71	t correction		3.7'
L diff.	04.1'	ΔL sign (+)..(-)		L correction		2.3'
				Sum	1.6'	6.0'
				Correction	-4.4'	

Az .... N 123.9° W
Zn .... 263.1°

ht .... 81° 29.4'
Corr .... -4.4'
Hc .... 81° 25.0'
Ho .... 81° 5.2' (known)
a .... 19.8 A nautical miles

The sight is plotted from the following DR position (9):

Latitude ... DR lat.
Longitude ... DR lon.

VIRTUAL OBSERVER ALTITUDE

By using a range-measuring satellite, the distance between the observer and the satellite can be obtained. With this information it is not necessary to restrict the observer to be located on the surface of the earth/water. He may now be at a position below, on, or above the mean surface of the earth. For nonsurface locations it is necessary to know his deviation in terms of depth or height. The analogy of height deviation in celestial navigation is "dip". When using a marine sextant in celestial navigation, a dip correction is made for the depression of the visible horizon caused by the observer's height-of-eye above the mean surface of the sea.

In the case of the observer being above the surface of the earth, his height deviation may be greater, the same, or less than the height of the satellite. Therefore, the method is adaptable to a rudimentary form of aerospace navigation inasmuch as it would give terrestrial longitude and latitude position. However, the spacial location is but the correction of the height deviation. The accuracy of such a method is contingent upon the relative motion of the satellites and the observer in terms of their geocentric referenced x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$, and t specification.

Depending on whether the observer is below or above the mean surface of the earth, his DR or AP would be above (zenith) or below (nadir). Consider the following two geometrical forms at the location of the observer: (a) a plane which is perpendicular to the observer's zenith or nadir line (when the observer is on earth, it is his local tangential horizon plane); (b) a line parallel to the ray that defines the GP of the satellite which is regarded as a celestial body. The angle between these two forms is defined as the observer's virtual altitude VHo; it lies in the plane which contains the GP line of the satellite and the line at the location of the observer that is parallel to the GP line.

To determine position by means of celestial navigation techniques, use VHo exactly as Ho. The value for VHo can be determined as follows:

$$VHo = \sin^{-1}\left[\frac{(R+d)^2 + (R+H)^2 - r^2}{2(R+d)\ (R+H)}\right]$$

where, r = range distance between the observer and the satellite
d = deviation of the observer in depth or height
R = mean radius of the earth
H = height of the satellite
and, the sign of d is positive when the observer is located above the surface of the earth; otherwise, it is negative.

When VHo or Hc is negative, the terrestial position of the observer is located on the rear hemisphere whose equatorial plane is perpendicular to the GP line of the satellite. For positive values of altitude, the converse is true and the observer would be on the forward hemisphere. In normal circumstances a negative altitude would occur only for elevated positions such as those in aerospace applications, and then only when the coaltitude angle exceeds 90 degrees.

Calculation and assigned direction of the absolute altitude difference "a" is the same as before regardless of the sign of VHo and Hc.

POLAR REGIONS

Azimuthal lines in celestial navigation are essentially great circles.

Both the celestial body and the observer (or his assumed position) lie on the plane defined by the great circle which contains their positions. In the Mercator projection a rhumb line was used to represent the azimuthal line. Although this loxodromic line provides sufficient accuracy in the lower latitudes, its use in the polar regions no longer approximates the azimuthal line of the great circle within the required accuracy. There is no well-defined boundary which can be used as a standard to indicate the polar regions. For general purposes, the navigator may consider the polar regions as extending from the geographical poles of earth to a latitude of 70 degrees (1).

Chosing a type of chart which will yield the best plotting accuracy in the polar regions becomes more involved than selecting the previous equatorial cylindrical orthomorphic projection. However, to illustrate a method in determining position, the polar azimuthal equidistant projection is used when the observer is within 5° of the pole (1). In this projection a plane is tangent at the pole, the meridians appear as straight lines intersecting at the pole, and the paralles of latitude appear as equally spaced concentric circles. The distance scale along any meridian is constant.

The AP or DR of the observer is selected to be the pole. When this is the case, Az = GP lon. and Hc = GP lat., and the previous LOP techniques are then used. Regardless of the type of projection that is selected for celestial navigation plotting, VHo is used in the exact same manner as Ho.

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Since the intended purpose of this paper was to illustrate how satellite range data may be used in conventional celestial navigation methods, standard books in navigation were selected as references. The material covered in the references was liberally used throughout the discussion.

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APPENDIX
PROCEDURE FORM

DR lat. ....

GP lat. [Dec.] ...
AP lat. [aL] .....

d diff. ... Δd ....
t diff. ... Δt ....
L diff. ... ΔL sign (+) ..

Az ....
Zn ....

ht ....
Corr. ......______
Hc ....
Ho ....______
a ....

DR lon. ....

GP lon. ....
AP lon. [aλ] .....______
t ....

	+	-
d corr. ...		
t corr. ...		
L corr. ...		
Sum		
Corr		

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13. ABSTRACT

Satellite range and location data can be used to determine the location of an observer. This report discusses how the above may be done by using conventional celestial navigation methods.

In lieu of sighting with a marine sextant or a bubble octant to obtain an observed altitude (Ho), a receiver and a phase-measuring technique can be used to determine a parametric range distance between the satellite and the observer. Satellite height and range parameters can be reduced to obtain the value for an observed "virtual altitude" (VHo), which is then used in the same manner as the standard Ho measurement.

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