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TURBULENT HEAT TRANSFER IN THE THERMAL ENTRANCE REGION OF A PERFECTLY INSULATED PIPE

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FLUID DYNAMICS FACILITIES RESEARCH LABORATORY

Project No. 7065

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OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

FOREWORD

This technical report was prepared by John W. Goresh and Robert G. Dunn, Fluid Dynamics Facilities Research Laboratory, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, on Project 7065, "Aerospace Simulation Techniques Research," under the direction of Mr. Elmer G. Johnson, Director of the Laboratory.

The authors wish to extend their gratitude to Henry E. Fettis and James C. Caslin of the Applied Mathematics Research Laboratory for their helpful suggestions in formulating the analytical expressions and adapting these to numerical computations. We also express our indebtedness to Mrs. Karen Thompson for her assistance in typing the report.

ABSTRACT

The problem considered is that of finding the thermal entrance or thermal mixing region of a pipe where the wall heat flux is zero along the length of the pipe. The fluid is assumed to enter the pipe with a non-uniform temperature profile and a fully developed turbulent velocity profile.

The approach is analogous to that introduced by Fettis for the solution of the uniform wall temperature problem.

The results for different radii tubes and Reynolds numbers are presented in graphs which show the adjustment of the temperature profile down the pipe. These results provide information concerning the <u>minimum</u> length mixing tube, since any wall losses will require a greater length.

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NOMENCLATURE

A_{mn}	defined by Equation 40
A_n	arbitrary constants
a,b,c	constants defined directly under Equation 15
C_p	specific heat
C_n	coefficients defined by Equation 30
D	pipe diameter
D_n	constants defined by Equation 12
exp	designates exponential
f(r)	function defined by Equation 43
$g_n(z)$	eigenfunctions for Equation 7
h	heat transfer coefficient
k	conductivity of the gas
k_w	conductivity of the gas at the wall
L_ℓ	mixing length $\left(\frac{x}{D}\right)_\ell$
Pr	Prandtl number $= \frac{C_p\mu}{k}$

p	parameter defined by Equation 27
q	total heat flux
q_r	radial heat flux
q_w	flux along the inner wall surface
R	pipe radius
R_o	Non-dimensionalizing pipe radius
r	radial coordinate
$\bar{r}$	non-dimensional radius $\frac{r}{R}$
Re	entrance Reynolds number
U_R	effective thermal conductance considered at the radius of the inner tube surface
u	velocity of the fluid stream in the direction of flow
V	average velocity
x	axial coordinate
$\frac{x}{D}$	dimensionless axial length
α, γ	constants defined directly under Equation 25
β	parameter which is proportional to the exponential decay factor for the temperature in the axial direction

Γ	gamma function
∂	indicates partial differentiation
θ	temperature difference between the local temperature and the temperature at the inner wall
θ_i	temperature at x = 0 and r = 0
θ_o	entrance temperature distribution
λ_n	eigenvalues for Equation 13
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density of the gas
$\sum$	refers to a summation
Φ_n	eigenfunctions for Equation 13
ω_n	eigenvalues for Equation 7

I. INTRODUCTION

The problem of heat transfer in fully established turbulent flow in cylindrical tubes has received considerable attention.[3-7] In all cases the flow Reynolds number is sufficiently large to justify the assumption of negligible axial conduction in the fluid. As a consequence, many of the mathematical investigations were reduced to solving an eigenvalue problem with the wall temperature taking the form of a step function. Once this was accomplished, other boundary conditions such as prescribed heat flux or prescribed wall temperature variation were included by the method of superposition. The case of a perfectly insulated tube wall is considered in this paper.

The fluid enters the pipe with a non-uniform temperature and a fully developed turbulent velocity profile. At succeeding axial stations, due to the radial conduction and turbulent mixing, the temperature will deviate from the entering profile until a uniform distribution is approximately achieved. Pipes with this length are used as mixing tubes and the adjustment or damping of the temperature profile is important in estimating the exit temperature profiles of shorter tubes.

The analysis is similar in the general mathematical approach to that presented by Latzko[1] and Fettis[2] for an isothermal wall. They assumed a one-seventh power velocity profile, a simplified eddy diffusivity and a Prandtl number of unity. However, Latzko[1] obtained crude approximations to the first three eigenvalues by using Legendre polynomials, while Fettis[2] obtained good estimates of the same three eigenvalues by the use of Jacobi polynomials.

II. STATEMENT OF THE PROBLEM

A schematic diagram showing the coordinate system is given in Figure 1. We shall consider the section of pipe to the right of $x = 0$, where the wall heat flux is equal to zero. The flow possesses a fully developed turbulent velocity profile and a selected entrance temperature profile at $x = 0$.

Subject to the limitations given below, the steady state energy equation is:

$$\rho C_p u \frac{\partial \theta}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r}(r q_r) \qquad (1)$$

where q_r is the radial heat flux, positive in the $+r$ direction.

In writing the energy equation (1), the usual basic assumptions are adopted:

a. The fluid properties are assumed constant.

b. Viscous dissipation is negligible.

c. Axial diffusion of heat is negligible compared to the axial convection.

d. The flow is hydrodynamically fully developed.

The statement of the problem is completed when the initial and boundary conditions are specified for the function $\theta(r,x)$.

At the inner surface of the pipe, we have:

$$k_w \frac{\partial\theta(r,x)}{\partial r}\bigg|_{r=R} = q_w = 0 \qquad (2)$$

where k_w and q_w represent the gas conductivity and flux at the wall, respectively.

The fluid temperature at the entrance (i.e., at $x = 0$) is given by the following:

$$\theta(r,0) = \theta_0 = \theta_i\left(1-\left(\frac{r}{R}\right)^2\right)^2 \qquad (2a)$$

Equations 1, 2 and 2a, and also the imposed condition that no infinite temperature exists, constitute the mathematical statement of the problem.

III. ANALYSIS

In a previous paper, Latzko's differential equation from Equation (1)

$$\frac{\partial}{\partial r}\left\{r\left(\frac{R^2-r^2}{2R}\right)^{6/7}\right\}\frac{\partial\theta}{\partial r}=pr\left\{1-\left(\frac{r}{R}\right)^2\right\}^{1/7}\frac{\partial\theta}{\partial x} \tag{3}$$

for convective heat transfer in fully developed turbulent flow with a velocity profile represented by the equation

$$u=8/7\,V\left\{1-\left(\frac{r}{R}\right)^2\right\}^{1/7} \tag{3a}$$

was solved subject to the following boundary condition at the wall:

$$q_r=U_r\,\theta \tag{4}$$

The present paper considers the same equation (3), but with the boundary condition q = 0 at the wall. This condition when the inlet temperature profile is constant leads to a trivial solution. In order to obtain a non-trivial solution, an initial temperature represented by the function

$$\theta=\theta_i\left(1-\bar{r}^2\right)^2=\theta_i\,z^{14} \tag{5}$$

is used, where $\bar{r}$ is defined as the non-dimensional radius, $\frac{r}{R}$, and

$$z = \left[1 - \left(\frac{r}{R}\right)^2\right]^{1/7}$$

A solution to Equation 3 can be written in the form

$$\theta = g(r)\ \exp(-\beta x) \tag{6}$$

and, since the fluid temperature and wall temperature asymptotically approach each other as the pipe length increases, β must be positive. When the solution as given by Equation 6, is inserted into Equation 3 and use is made of the non-dimensional radial transformation previously defined as z, the differential equation (3) becomes

$$\frac{d}{dz}\left(1 - z^7\right)\frac{dg}{dz} = -\omega z^7 g \tag{7}$$

where ω is defined as $\frac{49}{4}\beta^2$

A procedure analogous to that used by Fettis[2] for the isothermal wall problem was followed in obtaining a solution to equation 7 for this case, where the surface heat flux is zero. The appropriate boundary condition is:

$$\frac{\partial g}{\partial z} = 0 \text{ at } z = 0 \tag{8}$$

In addition, because of the singularity in Equation 7, i.e., when $z = 1$, it is necessary to require that the function g be finite at the centerline of the pipe.

Solutions of Equation 7 which satisfy the above requirement and boundary condition will exist only for discrete values of ω and have the form

$$\theta = \sum_{n=0}^{n=\infty} D_n g_n(z) \exp\left(-\frac{\omega_n}{pR}\right)x \tag{9}$$

where $g(z)$ is the eigenfunction solution of Equation 7. The constants "D" must be determined to satisfy the initial condition $\theta(z,0) = \theta_0$, where θ_0 is prescribed entrance temperature profile. This may be accomplished by use of the orthogonality property of the eigenfunctions, $g_n(z)$, that is

$$\int_0^1 z^7 g_n(z) g_m(z)\, dz = 0 \tag{10}$$

when $m \neq n$. Setting,

$$\sum_{n=0}^{n=\infty} D_n g_n(z) = \theta_0 \tag{11}$$

we find that

$$D_n = \frac{\int_0^1 \theta_0 z^7 g_n(z)\,dz}{\int_0^1 z^7 g_n^2(z)\,dz} \tag{12}$$

Before obtaining the solution of Equation 7, we wish to consider the solution of the auxiliary equation

$$\frac{d}{dz}\left(1-z^7\right)\frac{dg}{dz} = -\lambda z^5 g \tag{13}$$

which is similar, but not identical, to Equation 7. With the change of variable $t=z^7$, Equation 13 becomes an equation of the hypergeometric type:

$$t(1-t)\frac{d^2g}{dt^2} + \left(\frac{6}{7} - \frac{13}{7}t\right)\frac{dg}{dt} + \frac{\lambda}{49}g = 0 \tag{14}$$

The details for obtaining the eigenvalues and eigenfunctions for Equation 14 are given in the Appendix.

Now assume that a solution of Equation 7 can be found in the form

$$g_n(z) = \sum_{n=0}^{N} D_n \;\Phi_n(z) \tag{15}$$

If Equation 15 is substituted into Equation 7, we obtain

$$\sum_{n=0}^{N} D_n \left[\frac{d}{dz}(1-z^7)\frac{d\Phi_n}{dz} + \omega_n z^7 \Phi_n \right] = 0 \tag{16}$$

Because the eigenfunctions Φ_n also satisfy the auxiliary Equation 13, Equation 16 becomes

$$\sum_{n=0}^{N} D_n \left[\lambda_n z^5 \Phi_n(z) - \omega_n z^7 \Phi_n(z) \right] = 0 \tag{17}$$

We can now make use of the Galerkin method to determine the D_n by requiring that the left side of Equation 17 be orthogonal to the Φ_n for $n = 0, 1, 2 \ldots N$, thus arriving at the following system of equations:

$$\sum_{n=0}^{N} D_n \left[\sigma_{mn} - \omega A_{mn} \right] = 0 \tag{18}$$

for $m = 0, 1 \ldots N$, with A_{mn} defined by the equation

$$A_{mn} = \int_0^1 z^7 \Phi_m(z) \Phi_n(z) dz \tag{19}$$

The characteristic equation of the system given by Equation 18 is the determinant

$$\sum_{n=0}^{N} D_n \left[\sigma_{mn} - \omega_n A_{mn}\right] = 0 \qquad (20)$$

The roots of Equation 20 give approximations to the first n eigenvalues of Equation 7, and the complete solution to Equation 3 is given by Equation 9.

In the present case, where the $\Phi_n(z)$ and therefore the $g_n(z)$ are expressed as polynomials, the coefficients D_n can be obtained explicitly provided the initial temperature distribution can also be described by a polynomial. For example, a suitable entrance temperature profile could be represented by the function

$$\theta_i f(\bar{r}) = \theta_i (1 - \bar{r}^2)^2 = \theta_i z^{14} \qquad (21)$$

By equating like powers of the function z^7, we obtain the following system of algebraic equations for the D_n:

$$\begin{aligned}
D_o \quad &+0.26688\ D_1 + 0.22691\ D_2 + 0.19413\ D_3 + 0.32980\ D_4 = 0 \\
&-0.31298\ D_1 - 0.68164\ D_2 - 0.98781\ D_3 - 5.42133\ D_4 = 0 \\
&-0.86258\ D_1 - 1.51668\ D_2 - 2.83335\ D_3 + 21.29747\ D_4 = 1 \qquad (22) \\
&+0.51278\ D_1 + 2.63729\ D_2 + 11.27409\ D_3 - 30.31206\ D_4 = 0 \\
&-0.18864\ D_1 - 0.36071\ D_2 - 7.95572\ D_3 + 14.22400\ D_4 = 0
\end{aligned}$$

Solving the last four of Equations 22 for the coefficients D_n, we obtain

$$\left.\begin{aligned} D_1 &= -1.19346 \\ D_2 &= 0.25817 \\ D_3 &= 0.061657 \\ D_4 &= 0.025205 \end{aligned}\right\} \tag{23}$$

Substituting these four values for the coefficients D_1 through D_4 into the first of Equations 22, we obtain for D_o the numerical value

$$D_o = 0.23965 \tag{24}$$

The complete solution as given by Equation 9 can now be written in the form

$$\begin{aligned} \frac{\theta}{\theta_i} = D_0 + D_1 g_1 \exp\left(\frac{\beta_1^2}{pR} x\right) + D_2 g_2 \\ \exp\left(-\frac{\beta_2^2}{pR} x\right) + D_3 g_3 \exp\left(-\frac{\beta_3^2}{pR} x\right) \\ + D_4 g_4 \exp\left(\frac{\beta_4^2}{pR} x\right) \end{aligned} \tag{25}$$

or

$$
\begin{aligned}
\frac{\theta}{\theta_i} = {} & 0.23965 - 1.9346 \exp\left(-\frac{\beta_1^2}{pR}x\right)\Big[0.26688 - 0.31298z^7 \\
& - 0.86258z^{14} + 0.51278z^{21} - 0.18864z^{28}\Big] \\
& + 0.25817 \exp\left(-\frac{\beta_2^2}{pR}x\right)\Big[0.22691 - 0.68164z^7 \\
& - 1.51668z^{14} + 2.63729z^{21} - 0.36071z^{28}\Big] \\
& + 0.06166 \exp\left(-\frac{\beta_3^2}{pR}x\right)\Big[0.19413 - 0.98781z^7 \\
& - 2.83335z^{14} + 11.27409z^{21} - 7.95572z^{28}\Big] \\
& + 0.025205 \exp\left(-\frac{\beta_4^2}{pR}x\right)\Big[0.32980 - 5.42133z^7 \\
& + 21.29747z^{14} - 30.31206z^{21} + 14.22400z^{28}\Big]
\end{aligned}
\tag{26}
$$

where

$$
p = \frac{8 \times 2^{\frac{5}{7}}}{7 \times 0.199} Re^{\frac{1}{4}} \tag{27}
$$

and R represents the inner wall radius.

IV. DISCUSSION AND CONCLUSIONS

The approach employed in obtaining a solution to Equation 3 for the specified boundary condition was analogous to that first used by Fettis[2] in solving the isothermal problem with a uniform entrance temperature profile. However, to obtain a non-trivial solution for the present case it was necessary to assume that the fluid entering the pipe had a non-uniform temperature profile. The eigenvalues ω_n and the eigenfunctions ϕ_n remain unchanged for any profile chosen, because neither depend on the initial temperature distribution. In the chosen numerical example, the initial temperature distribution was represented by the following equation:

$$\theta(\bar{r},o)=\theta_i f(\bar{r})=\theta_i(1-\bar{r}^2)^2 \tag{28}$$

where θ_i is the initial centerline temperature. The centerline temperature as a function of length-to-diameter ratio is presented in graphical form for several pipe diameters in Figure 2, and in Figure 3 the mixing lengths for various initial Reynolds numbers are shown. All the presented data are for a fixed mass flow, pressure and entrance centerline temperature. The results show that as the Reynolds number increases the mixing length increases. The adjustments of the temperature profile from a radial variation to a uniform distribution at succeeding axial stations are also presented in Figures 4 through 9. The variations of the exponential decay factor "β^2" with

Reynolds number are given in Table I.

The engineer is often required to design mixing tubes with varying degrees of wall heat losses. Thus, these calculations give the length of a perfectly insulated mixing tube for turbulent flow. This is the minimum length mixing tube, since any wall losses will require a greater length. Calculations for non-perfectly insulated pipes were presented in Reference 8. In the solution for non-perfectly insulated pipes, non-uniform entrance temperature profiles were not considered.

REFERENCES

1. Latzko, H., NACA TM 1068 (1944); original in German.

2. Fettis, H. E., "On the Eigenvalues of Latzko's Differential Equation," ZAMM, Vol. 37, Nos. 9, 10, (Sept./Oct. 1957).

3. Sleicher, C. A., Jr., and Tribus, M., "Heat Transfer in a Pipe with Turbulent Flow and Arbitrary Wall Temperature Distribution," Trans. ASME, Vol. 79, 1957.

4. Becker, H. L., "Heat Transfer in Turbulent Tube Flow," Applied Science Research, Section A, Vol. 6, 1957.

5. Sparrow, E. M., Hallman, T. M., and Siegel, R., "Turbulent Heat Transfer in the Thermal Entrance Region of a Pipe with Uniform Heat Flux," Applied Science Research, Vol. 7, 1957.

6. Deissler, R. G., and Elan, C. S., "Analytical and Experimental Investigation of Fully Developed Turbulent Flow of Air in a Smooth Tube with Heat Transfer with Variable Fluid Properties," NACA TN 2629 (1952).

7. Albrecht, P. H., and Churchhill, S. W., "The Thermal Entrance Region in Fully Developed Turbulent Flow," J. Amer. Inst. Chem. Engrs. 268-273 (1960).

8. Goresh, J. W., "Heat Transfer in Cylindrical Pipes with Fully Established Turbulent Flow and Exposed to a Uniform temperature Environment," Journal of Heat Transfer, August 1966.

APPENDIX A

The general solution to Equation 14 is

$$g = A\,F(a,b,c,t) + Bt^{1-c}\,F(a-c+1,b-c+1,2-c,t) \tag{A-1}$$

where

$$a+b = \frac{6}{7}$$

$$ab = -\frac{\lambda}{49}$$

and $c = \frac{6}{7}$

For the boundary condition given by Equation 8 to be satisfied at z = o, B must be zero. Therefore, Equation 15 now reduces to

$$g = A\,F(a,b,c,z^7) = A\,F\left[a,\left(\frac{6}{7} - a\right),\ \frac{6}{7},\ z^7\right] \tag{A-2}$$

But at z = 1, g must be finite; hence, the function F at z = 1 is

$$F(a,b,c,1) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\ ; \tag{A-3}$$

and if

$$b = \frac{6}{7} \text{ and } z = 1$$

the function F becomes

$$F(a, \frac{6}{7} - a, \frac{6}{7}, 1) = \frac{\Gamma(\frac{6}{7})\Gamma(\frac{6}{7} - a - \frac{6}{7} + a)}{\Gamma(\frac{6}{7} - a)\Gamma(a)} = \frac{\infty}{\Gamma(\frac{6}{7} - a)\Gamma(a)} \qquad \text{(A-4)}$$

Thus, for the particular values of b and c given above, the series for the function F(a,b,c,1) diverges unless it terminates. This requires that "a" must be zero or a negative integer, which leads to the following admissible values of "a":

$$a = 0, -1, -2, ----, -n \qquad \text{(A-5)}$$

yielding the corresponding eigenvalues

$$-\lambda_n = 49 \left(-n\right)\left(\frac{6}{7} + n\right)$$

where n = 0, 1, 2, 3 . . . . (A-6)

The resulting polynomial solutions are included in a more general class known as Jacobi polynomials and are defined by the equation

$$F_n(\alpha;\ \gamma;\ z^7) = F(-n,\ \frac{6}{7} + n;\ \frac{6}{7};\ z^7). \qquad \text{(A-7)}$$

For the present case

$$\left.\begin{aligned} a &= n, \text{ where } n = 0, 1, 2, 3 \ldots \\ \alpha &= \frac{6}{7} \\ \gamma &= \frac{6}{7} . \end{aligned}\right\} \tag{A-8}$$

Thus, the eigenfunctions of Equation 13 are given by the equation

$$A_n F_n\left(\frac{6}{7}; \frac{6}{7}; z^7\right) = A_n F_n \left(-n, \frac{6}{7} + n; \frac{6}{7}; z^7\right) \tag{A-9}$$

where the A_n are arbitrary constants.

The functions $F_n(z)$ are orthogonal with respect to z^5 as a weight factor:

$$\begin{aligned} &\int_o^1 z^5 F_m(z) F_n(z) dz = o \text{ for } m \neq n \\ &\int_o^1 z^5 F_n^2(z) dz = C_n^2 \text{ when } m = n \end{aligned} \tag{A-10}$$

In terms of the variable $t = z^7$, the orthogonality relation is

$$\int_o^1 t^{\gamma-1} (1-t)^{\alpha-\gamma} F_m F_n dt = o \text{ for } m = n, \tag{A-11}$$

where

$$\gamma = \frac{6}{7}$$

$$\alpha = \frac{6}{7}$$

and

$$\int_o^1 t^{\gamma-1}(1-t)^{\alpha-\gamma}\; F_n^2 dt$$

$$= \frac{\Gamma(\gamma)\; \Gamma\;(\alpha + 1-\gamma)}{\Gamma(\alpha)} \quad \frac{(\alpha + 1 -\gamma)_n}{(\alpha)_n\; (\gamma)_n} \quad \frac{n!}{2n} \tag{A-12}$$

It is easily verified that for the values α and γ given above, Equation 26 becomes

$$\int_o^1 z^5\; F^2 dz = \frac{(n!)^2}{7\left[\frac{6}{7}_n\right]^2 \left(\frac{6}{7} + 2n\right)} = C_n^2 \tag{A-13}$$

where

$(\alpha)_n = \alpha\; (\alpha + 1)\; (\alpha + 2)\ldots(\alpha + n-1)$ for n 1, 2, 3...

with $(\alpha)_o$ defined as

$$(\alpha)_o = 1.$$

For n = o, 1, 2, 3 and 4 Equation 27 yields

$$C_o = 0.4082$$

$$C_1 = 0.2609$$

$$C_2 = 0.2155$$

$$C_3 = 0.1904$$

$$C_4 = 0.1738 \qquad \text{(A-14)}$$

The first five Jacobi polynomials are easily obtained by expanding the series

$$F_n(\alpha;\ \gamma;\ t) = F(-n,\ \alpha + n;\ \gamma;\ t) = 1$$

$$+ \sum_{k=1}^{n} \binom{n}{k} \frac{(\alpha + n)\ (\alpha + n + 1)\ \ldots\ (\alpha + n + k - 1)}{\gamma(\gamma + 1)\ldots(\gamma + k-1)} z^k \qquad \text{(A-15)}$$

for n = 0, 1 + ...4 and $\gamma \neq 0, -1, \ldots -n + 1$.

For n = o

$$F_o(\alpha, \gamma, t) = 1$$

n = 1

$$F_1(\alpha, \gamma, t) = 1 - 2.1667\ z^7$$

n = 2

$$F_2(\alpha, \gamma, t) = 1 - 6.6667\ z^7 + 6.9231\ z^{14} \tag{A-16}$$

n = 3

$$F_3(\alpha, \gamma, t) = 1 - 13.5000\ z^7 + 35.3077\ z^{14} - 24.1269\ z^{21}$$

n = 4

$$F_4(\alpha, \gamma, t) = 1 - 22.6691\ z^7 + 107.2429\ z^{14} - 171.5886\ z^{21}$$
$$+ 87.3830\ z^{28}$$

With the C_n and F_n thus obtained, we now proceed to obtain an approximate solution to Equation 7 by applying the Galerkin technique.

For this method, it is convenient to define a set of functions ϕ_n (z) such that

$$\phi_n(z) = \frac{F_n(z)}{C_n \sqrt{\lambda_n}} \qquad n = 1, 2, \ldots \tag{A-17}$$

with the property that

$$\lambda_n \int_o^1 z^5 \, \phi_m(z) \, \phi_n(z) \, dz = \delta_{mn} \tag{A-18}$$

where

$$\delta_{mn} \begin{cases} = 0 \text{ if } m \neq n \\ = 1 \text{ if } m = n \end{cases} \tag{A-19}$$

For n = o we shall define

$$\phi_o = \frac{F_o}{C_o} \, , \tag{A-20}$$

while the eigenfunctions $\phi_n(z)$ for n = 1, 2, 3 and 4 are computed by substitution of the λ_n, C_n and F_n directly into Equation 31. Listed below are the first five eigenfunctions:

$$\left.\begin{aligned} \phi_o &= 2.4495 \\ \phi_1 &= 0.4018 - 0.8706 \, z^7 \\ \phi_2 &= 0.2773 - 1.8488 \, z^7 + 1.9199 \, z^{14} \\ \phi_3 &= 0.2206 - 2.9776 \, z^7 + 7.7877 \, z^{14} - 5.3216^{21} \\ \phi_4 &= 0.1866 - 4.2293 \, z^7 + 20.0080 \, z^{14} - 32.0128 \, z^{21} \\ &\quad + 16.3028 \, z^{28} \end{aligned}\right\} \tag{A-21}$$

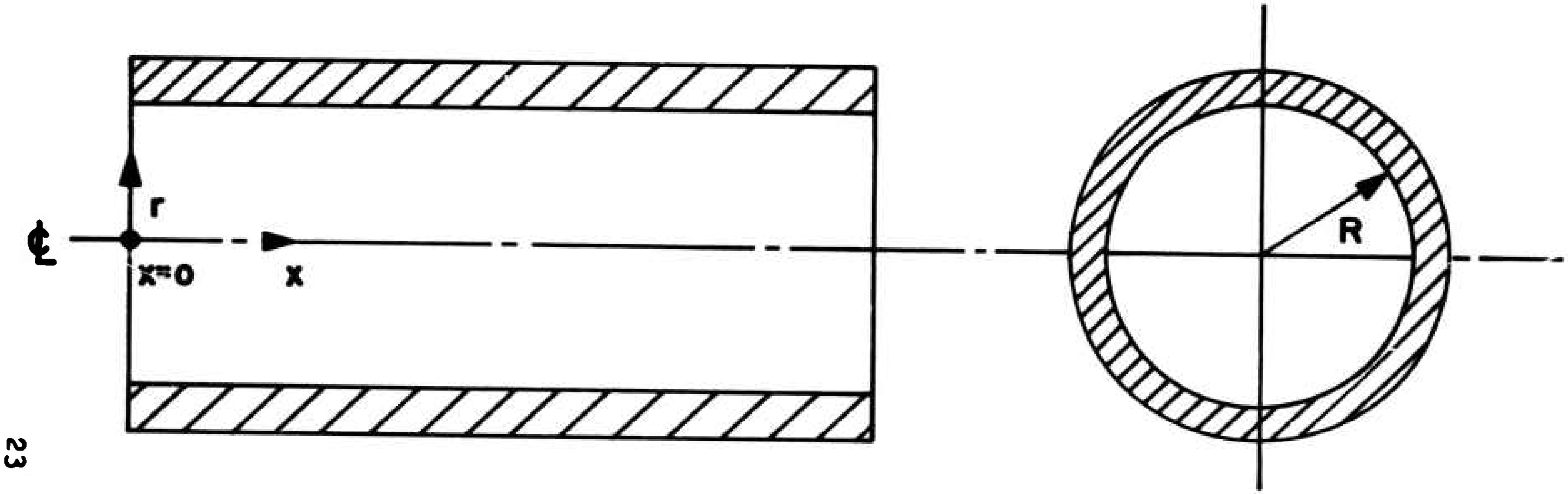

FIG. I SCHEMATIC DIAGRAM SHOWING THE COORDINATES USED

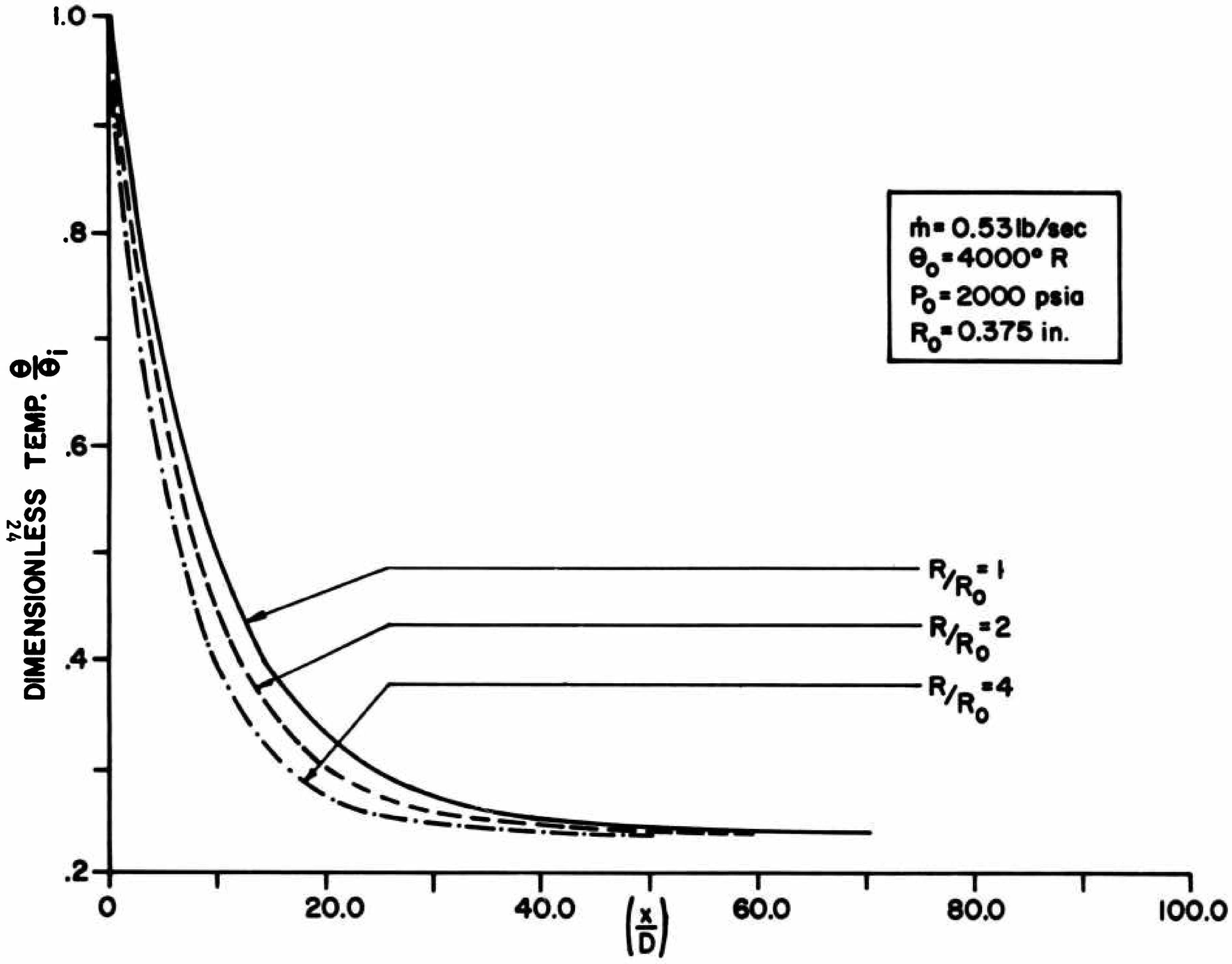

Fig 2 CENTERLINE TEMPERATURE VARIATION WITH NON-DIMENSIONAL AXIAL LENGTH

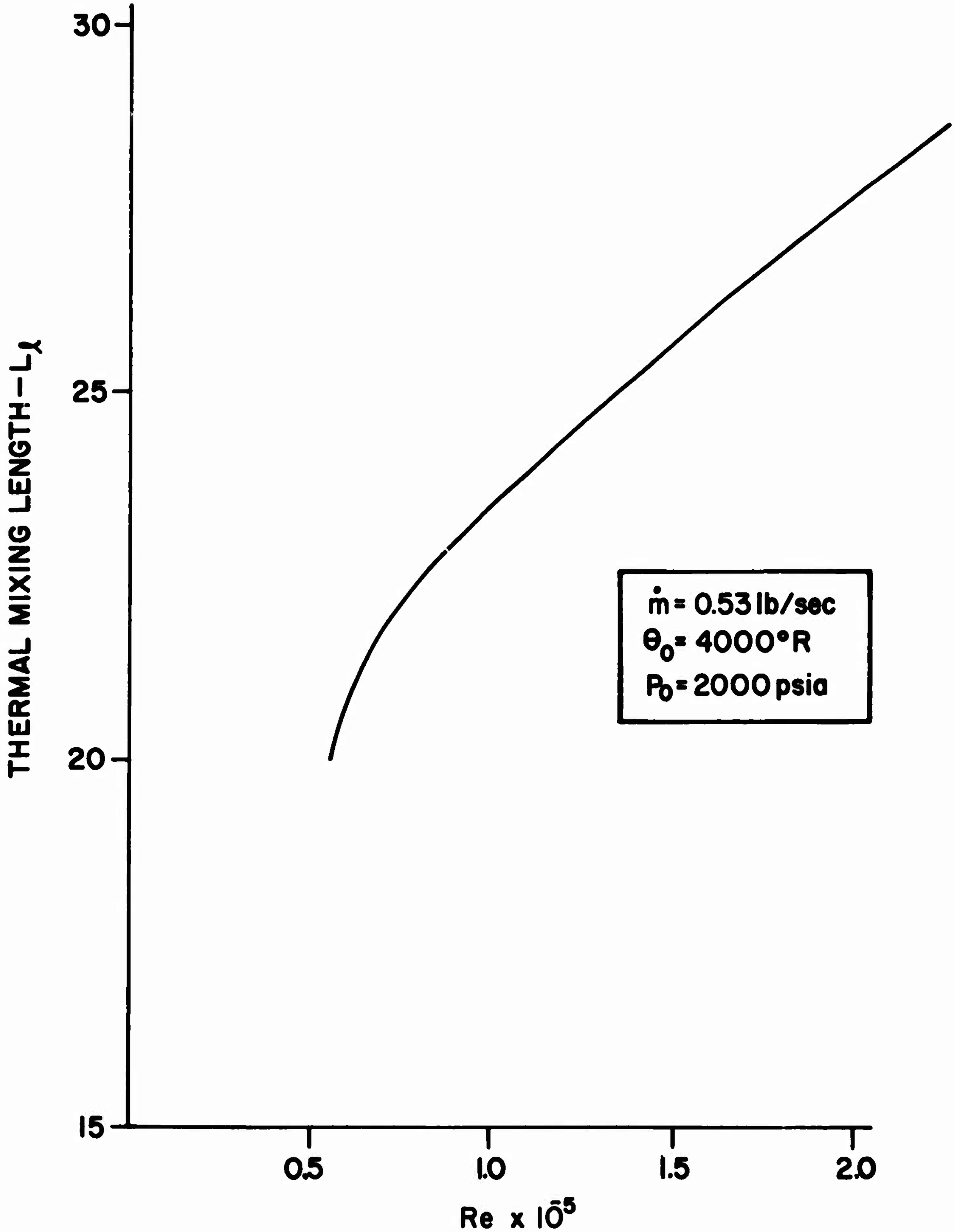

Fig. 3 VARIATION OF MIXING LENGTH WITH INITIAL REYNOLDS NUMBER

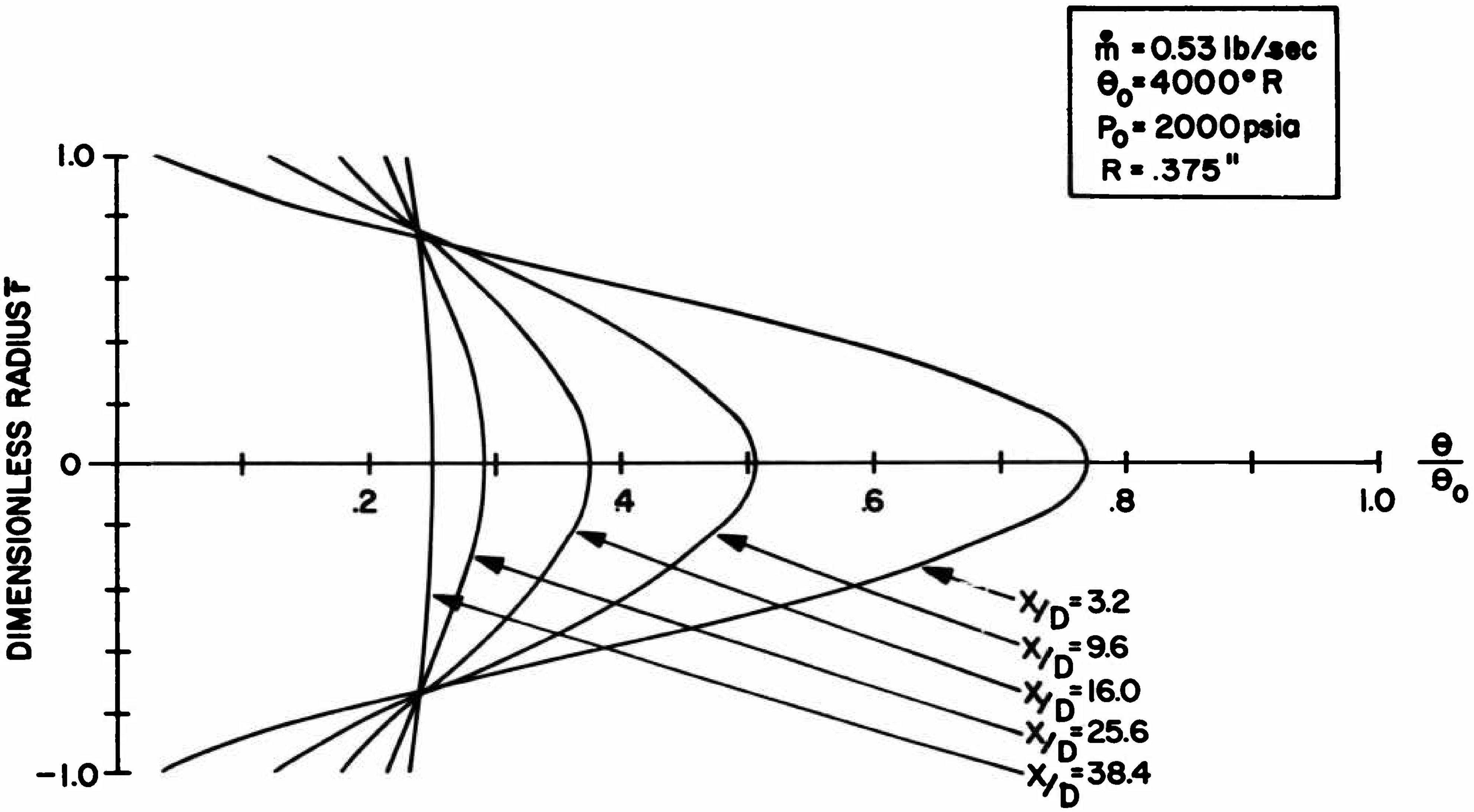

FIG. 4 RADIAL TEMPERATURE PROFILES AT DIFFERENT AXIAL POSITIONS

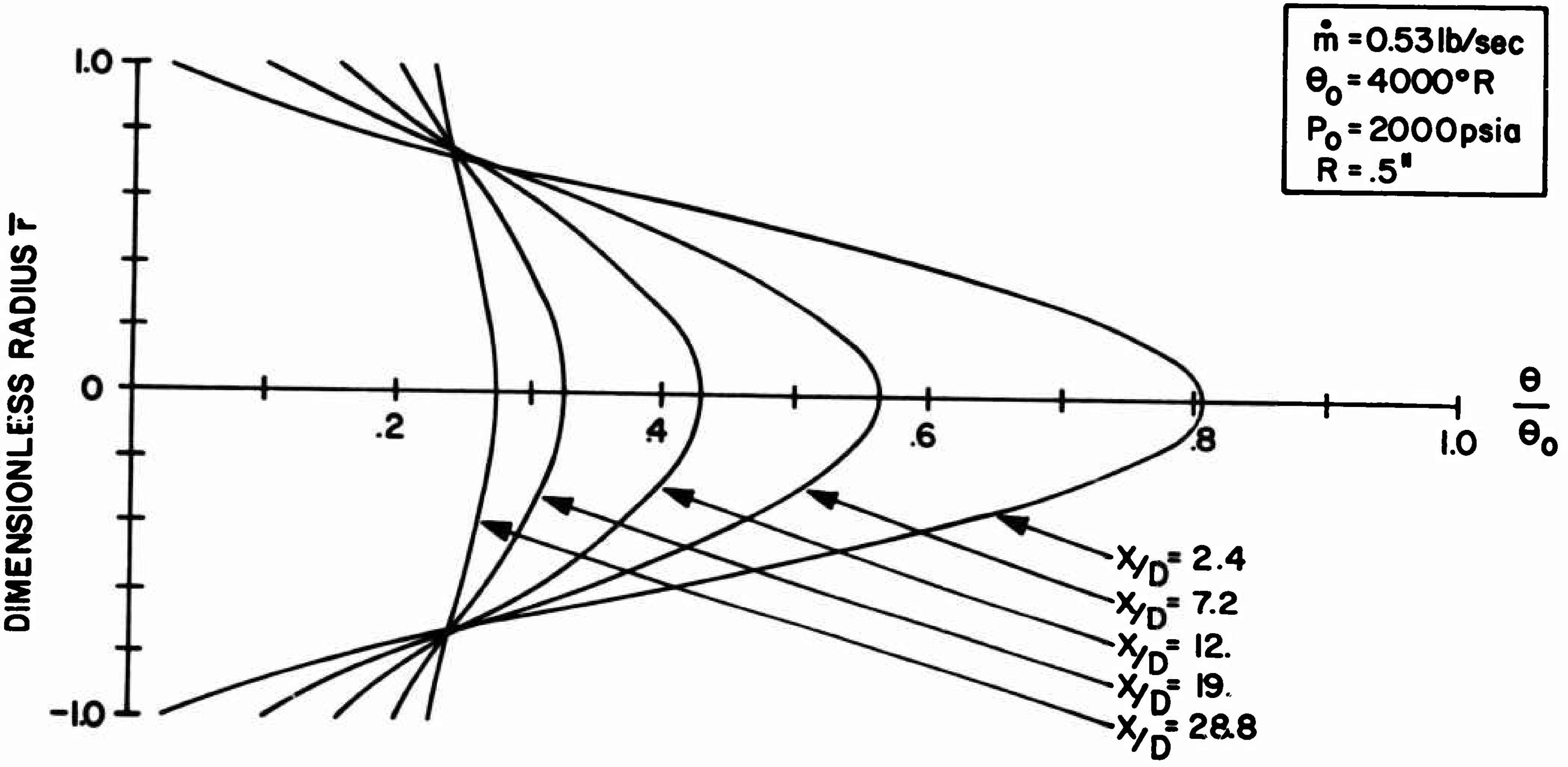

FIG. 5 RADIAL TEMPERATURE PROFILES AT DIFFERENT AXIAL POSITIONS

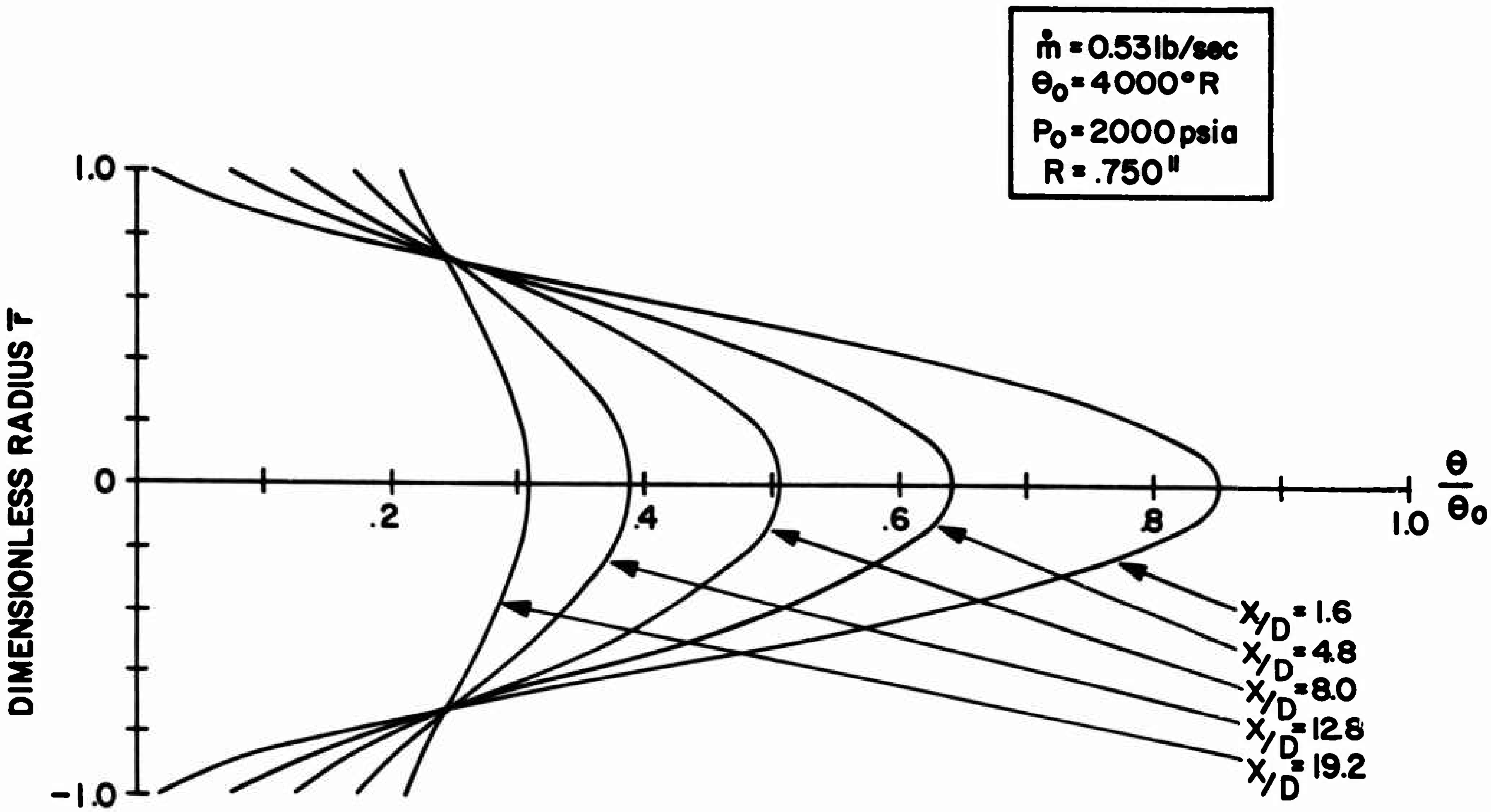

FIG. 6 RADIAL TEMPERATURE PROFILES AT DIFFERENT AXIAL POSITIONS

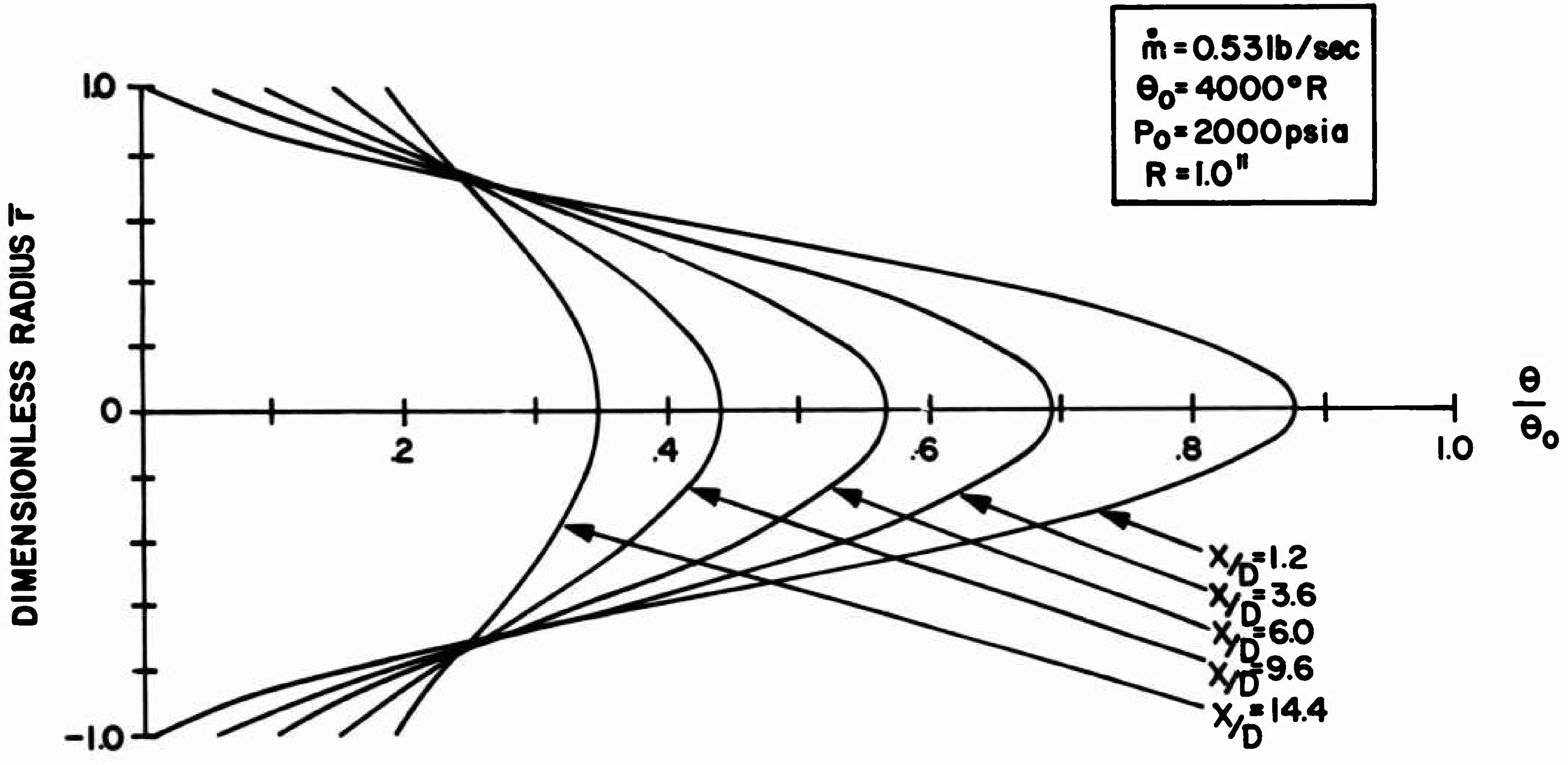

FIG. 7 RADIAL TEMPERATURE PROFILES AT DIFFERENT RADIAL POSITIONS

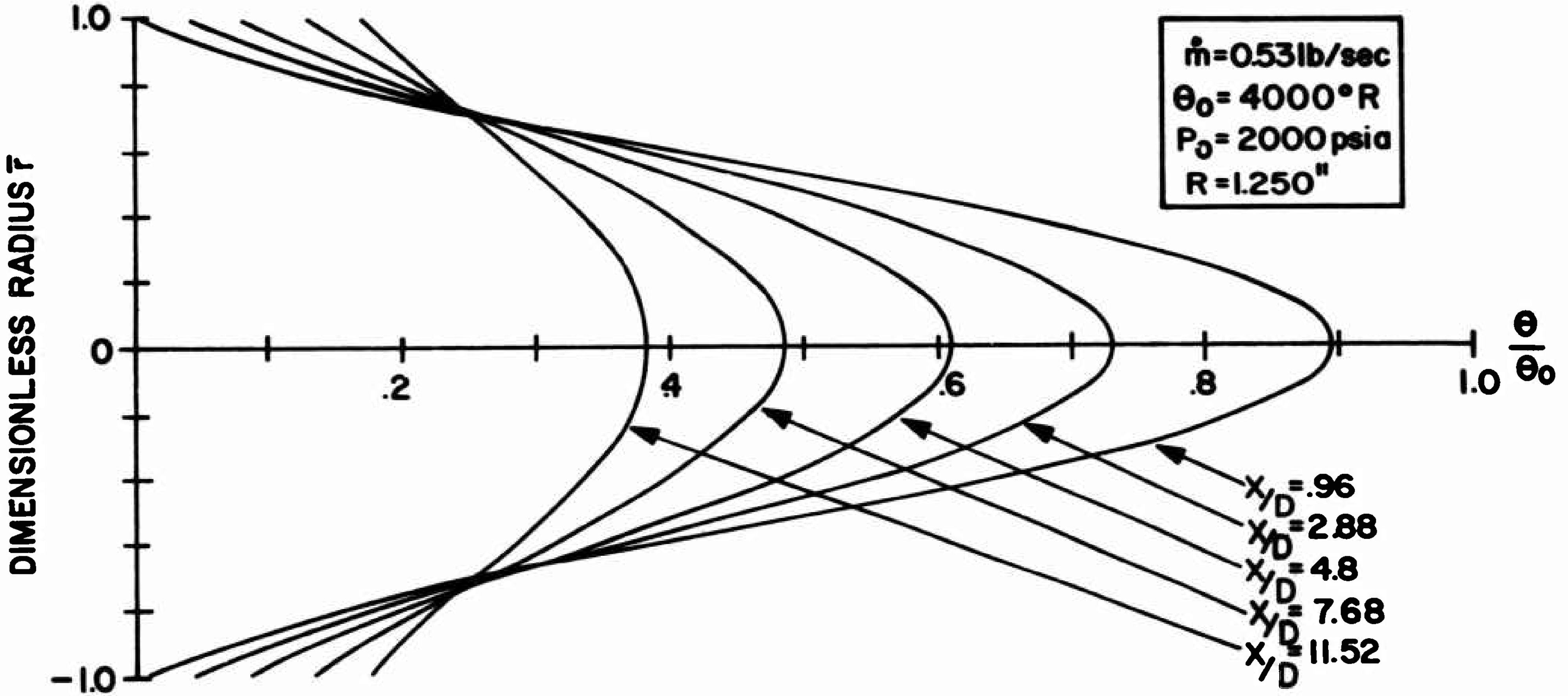

FIG. 8 RADIAL TEMPERATURE PROFILES AT DIFFERENT AXIAL POSITIONS

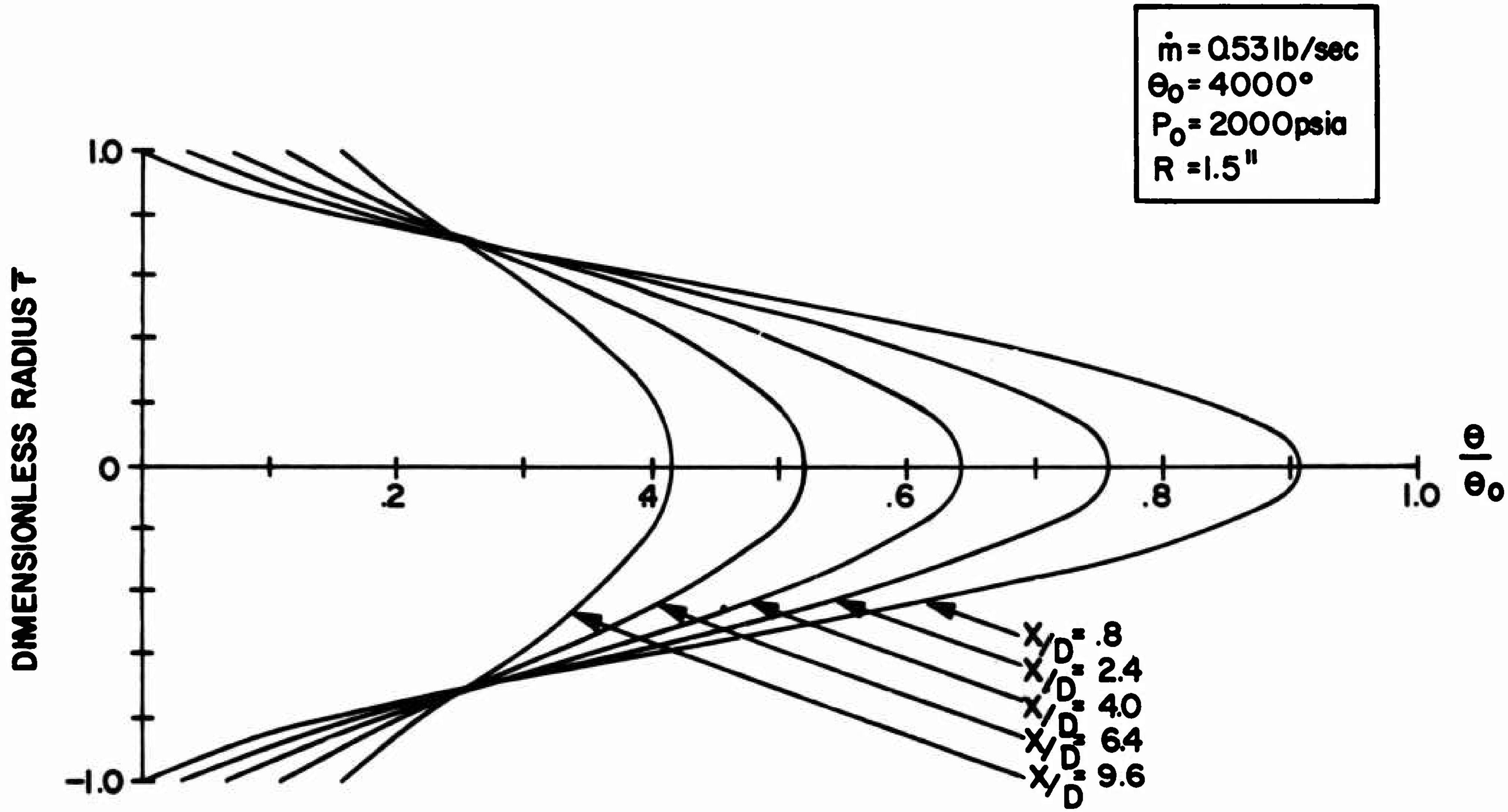

FIG. 9 RADIAL TEMPERATURE PROFILES AT DIFFERENT AXIAL POSITIONS

TABLE I.

VARIATION OF THE EXPONENTIAL DECAY
FACTOR WITH REYNOLDS NUMBER

R (inches)	Re x 10^5	β_n^2
0.375	2.24939	1.63577 5.04350 10.20157 20.37003
0.500	1.68704	1.31831 4.06469 8.22172 16.41676
0.625	1.34964	1.11516 3.43831 6.95473 13.88689
0.750	1.12470	0.97263 2.99888 6.06589 12.11209
0.875	0.96403	0.86644 2.67146 5.40361 10.78969
1.000	0.84352	0.78387 2.41688 4.88867 9.76147
1.125	0.74980	0.71760 2.21254 4.47534 8.93615
1.250	0.67482	0.66308 2.04443 4.13531 8.25720

TABLE I. (Continued)

R (inches)	Re x 10 ^{5}	β^2_n
1.375	0.613	0.61733 1.90339 3.85002 7.68755
1.500	0.562	0.57833 1.78315 3.60680 7.20189

ERRATA

ARL 66-0178 September 1966

TURBULENT HEAT TRANSFER IN THE THERMAL REGION OF A PERFECTLY INSULATED PIPE

Numbered equations as shown on listed pages should be changed as follows:

Equation No.	Page No.	Change
15	8	$g_n(z)=\sum_{n=0}^{N} B_n \Phi_n(z)$
16	9	$\sum_{n=o}^{N} B_n \left[\frac{d}{dz}(1-z^7)\frac{d\Phi_n}{dz} \omega_n z^7 \Phi_n\right] = 0$
17	9	$\sum_{n=o}^{N} B_n \left[\lambda_n z^5 \Phi_n(z)-\omega_n z^7 \Phi_n(z)\right] = 0$
18	9	$\sum_{n=o}^{N} B_n \left[\sigma_{mn}-\omega A_{mn}\right] = 0$
20	10	$\left\lvert\sigma_{mn}-\omega_n A_{mn}\right\rvert = 0$

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4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
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13. ABSTRACT

The problem considered is that of finding the thermal entrance or thermal mixing region of a pipe where the wall heat flux is zero along the length of the pipe. The fluid is assumed to enter the pipe with a non-uniform temperature profile and a fully developed turbulent velocity profile.

The approach is analogous to that introduced by Fettis for the solution of the uniform wall temperature problem.

The results for different radii tubes and Reynolds numbers are presented in graphs which show the adjustment of the temperature profile down the pipe. These results provide information concerning the minimum length mixing tube, since any wall losses will require a greater length.

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14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Turbulence Thermal Entrance Region Zero Wall Heat Flux Heat Transfer in Cylinders Minimum Length Mixing Tube Fully Developed Hydrodynamic Conditions Steady State Conditions Non-Uniform Entrance Temperature Profile						

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