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**DEPARTMENT OF CIVIL ENGINEERING
AND ENGINEERING MECHANICS**



**INFLUENCE DIAGRAMS FOR STRESSES AND DISPLACEMENTS
IN THREE-LAYER PAVEMENT SYSTEMS FOR AIRFIELDS**

PART II

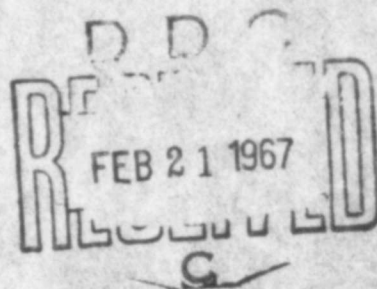
by

Donald M. Burmister

**DEPARTMENT of the NAVY
BUREAU of YARDS and DOCKS
WASHINGTON, D. C. 20390**

**Contract NBy 13009
Technical Report No. 2
DECEMBER 1966**

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ABSTRACT

PART II

PART II presents Influence Diagrams for Vertical Stresses, Shear Stresses, and Surface Deflections in Three Layer Pavement Systems, which are intended to provide the essential background and bases for evaluating the character and effectiveness of layered system reinforcing action. The vertical stress and shear stress transmission characteristics and critical regions, and the surface deflection performances are developed for nine three-layer concrete pavement systems for a range of layer moduli ratios, and ratios of layer 2 to layer 1 thickness. The effectiveness of reinforcing action, deflection performances, and shear stress performances in critical regions are treated and compared. Two layer and three layer thickness and layer moduli equivalences are developed and treated to illustrate methods for evaluating and for improving shear stress and deflection performances by modifications of layer moduli ratios and layer thicknesses. The major objectives are to develop a "feeling", intuition, and judgments regarding deflection and shear stress performances of layered pavements, and to develop relationships and methods for evaluating layer thickness and moduli ratios, and criteria for rational and effective design for multi-layer pavement system for airfields and for ensuring satisfactory pavement performances and long life.

SUBSEQUENT PARTS

PART III will present and illustrate the basic methods and procedures - (1) for evaluating vertical stresses and shear stresses and performances in critical depth regions of different two and three layer pavement systems and for establishing these critical regions for aircraft landing gear loading conditions; (2) for evaluating the deflection performances for these two and three layer systems; (3) for improving shear stress and deflection performances of these layered pavements systems by adjustments in layer thicknesses and in layer moduli ratios by the use of higher quality and strength characteristics of layer materials; and (4) for providing essential methods and procedures for layered system evaluations, equivalences, and comparisons, which are intended to lead to the formulation of significant design relationships and criteria for multi-layer pavement systems in order to ensure permanence, integrity, and long life of multi-layer pavement systems for airfields .

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PART II

INFLUENCE DIAGRAMS

FOR STRESSES AND DISPLACEMENTS

IN THREE-LAYER PAVEMENT SYSTEMS

FOR AIRFIELDS

The work of computations of Stresses and Displacements in a Three-Layer Pavement System was undertaken in the Department of Civil Engineering during the period from February 1960 through April 1962 under Contract NBy-13009 of The Department of The Navy, Bureau of Yards and Docks, Washington, D. C. 20390 with Columbia University, New York, N.Y. 10027. The programming of stresses and displacements in the Three-Layer System was done under Subcontract I by Computer Usage Company, Inc., 655 Madison Avenue, New York, N.Y. 10021. The numerical values of stresses and displacements were computed on the IBM 7090 at Eglin Air Force Base, Florida by Computer Usage Company during February and March 1962. The computing machine time was furnished by The United States Air Force under this Contract agreement. The tabulations of the computed output, covering the planned ranges of the Three-Layer System parameters were completed April 12, 1962, as follows:

- a) Tables of Influence Coefficients for the Three-Layered Soil Stress Problem, Volumes 1 to 26.
- b) Tables of Stresses and Displacements in Three-Layered Systems, Volumes 1 to 26.

STRESSES AND DEFLECTIONS IN A THREE-LAYER PAVEMENT SYSTEM FOR AIRFIELDS

INTRODUCTION

Investigations of the deflections of a layered pavement system and of the vertical and shear stresses imposed in the supporting layers by wheel loads of aircraft are essential aspects of pavement studies and design. Two and three layer system problems presented in 1943 [1] and 1945 [2] (numbers in brackets refer to a list of references) represent a closer agreement with actual stress and deflection performances of layered pavement systems. These layered system problems provides fundamental parametric relations and equations of physical laws that govern layered system performances. A basic understanding of layered system action and correct conceptions regarding stress-deflection responses and reinforcing action are essential.

The fundamental performance characteristics of three-layer pavement systems are treated in order to provide the essential background and effectiveness: (1) of the load spreading capacity; (2) of the stress reducing influences of the pavement reinforcing layers on the vertical stresses imposed in these layers and in the supporting subgrade soils; and (3) of the capacity of a layered pavement system to resist shear and tensile stresses in regions which are vulnerable to breakdown by shear deformation and bending.

It should be realized at the outset that investigations of stresses and deflections for the design of layered pavement systems are complex. It would be unrealistic and misleading to suppose that all an investigation demands is merely a facility in the use of stress influence charts. Much of present thinking and practice tends to be too matter-of-fact and unimaginative without giving due thought or study to the realm of validity of stress investiga-

tions, to the influences of geological, environmental, and structure conditions that control, and to the adequacy and reliability of the results of stress and deflection investigations. There is an essential need for a realistic, mature, and common sense approach and engineering imagination in making stress and deflection investigations for layered pavement systems. A major aspect of pavement studies is to raise the standards of excellence in practices and the conceptions of adequacy and reliability.

The major problems in stress and deflection investigations for layered pavement systems are to translate two-layer performances given in the Stress and Deflection Influence curves of Figs. 1 through 20 in Part I, Technical Report No. 1 of January 1965 under this Contract, and the three-layer performances in this present Part II Report into reliable predictions of Multi-Layer Pavement System performances. This requires a systematic and reliable evaluation of "layer equivalences". Comparative references are going to be made to the two-layer performances in Part I in order to build up on understanding and adequate bases for judgments. In view of these basic interrelationships, the Introduction on pages 2 to 7 and the Concepts and Principles in pages 8 and 9 of Part I should be reviewed, as a basic philosophy of approach.

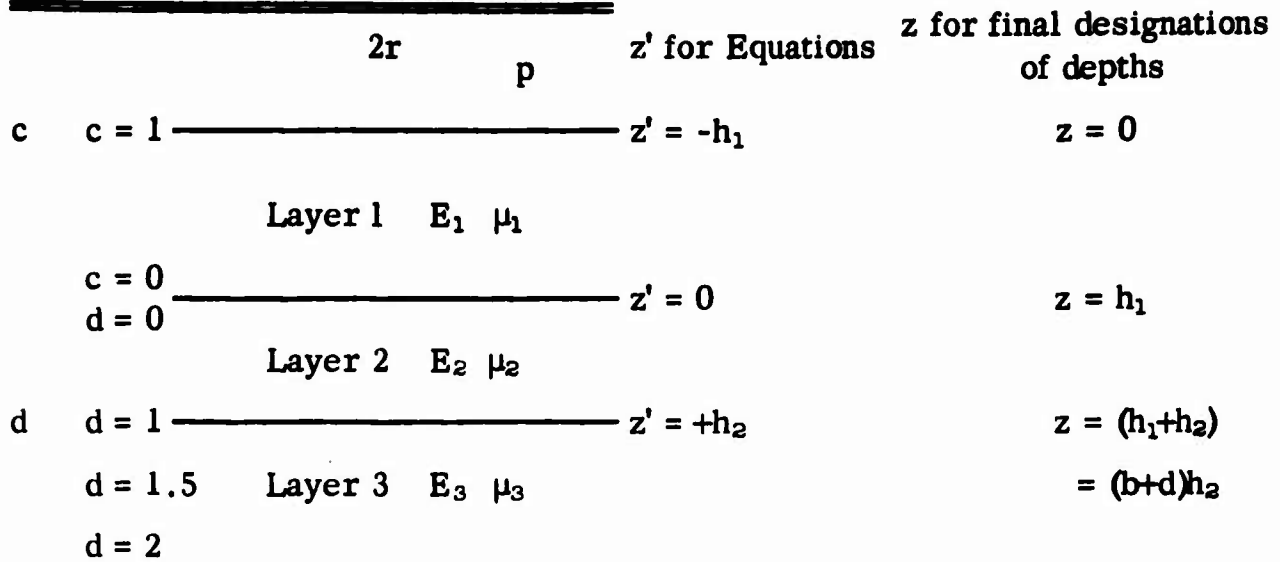
THREE LAYER SYSTEM CONDITIONS AND PARAMETERS

A multi-layer pavement system is illustrated in Fig. 1(a) of Part I, which is composed of reinforcing layers: asphalt pavement, base course, sub-base, and compacted subgrade layer. In Part II, a three-layer system is treated in Fig. 1 which is composed of reinforcing layers 1 and 2 and a subgrade layer 3.

The usual boundary conditions of the theory of elasticity for a semi-infinite mass loaded at the surface apply here. The surface at $z = 0$ is free of vertical and shear stresses outside of the load limits. At z and r equal to infinity, all stresses and displacements become equal to zero. The layers of the layered system within themselves are composed of homogeneous isotropic materials. The conditions of equilibrium of stresses and of compatibility of strains are satisfied in each layer of the layered system. In addition, the six continuity conditions of Eq. 1 for a three-layer system are satisfied across interfaces 1-2 and 2-3 between the layers, in order to insure continuity of transmission of stresses and strains across these interfaces. This means that the three layers of the pavement system work together as a structural unit without any slippages or loss of contact between layers. There are, however, discontinuities in the radial stresses, σ_r across interfaces 1-2 and 2-3, because with horizontal displacements - $u_1 = u_2$ at interface 1-2 and $u_2 = u_3$ at interface 2-3 in Eqs. 1, the magnitudes of the radial stresses, σ_r on either side of these interfaces must necessarily be governed by the respective moduli of the layers.

The parametric relations for the three layers are given in Eqs. 2. The range and intervals in the three-layer system parameters, which are given on page 7, were covered by systematic steps in the computations of the stress and deflection influence values.

THREE LAYER PAVEMENT SYSTEM



Complete Continuity at Interface (1-2) (1)

$$\sigma_{z1} = \sigma_{z2} \quad \tau_{rz1} = \tau_{rz2} \quad w_1 = w_2 \quad u_1 = u_2$$

and at Interface (2-3)

$$\sigma_{z2} = \sigma_{z3} \quad \tau_{rz2} = \tau_{rz3} \quad w_2 = w_3 \quad u_2 = u_3$$

Parametric Relations (2a)

Layer 1 $h_1 = bh_2 \quad \alpha = mh_2$
 $mh_1 = bmh_2 = b\alpha$
 $nz = cmh_1 = cbmh_2 = cb\alpha$

Note- z in Layer 1 has been inserted as minus (-) in Equations for Layer 1.

Layer 2 $h_1 = bh_2 \quad \alpha = mh_2$ (2b)
 $mh_1 = bmh_2 = b\alpha$
 $mz = dmh_2 = d\alpha$

Layer 3 $h_1 = bh_2 \quad \alpha = mh_2$ (2c)
 $mh_1 = bmh_2 = b\alpha$
 $mz = dmh_2 = d\alpha \quad d > 1$

FIG. 1 THREE-LAYER SYSTEM NOTATIONS, BASIC CONTINUITY CONDITIONS, AND BASIC LAYERED SYSTEM PARAMETERS.

THREE LAYER PARAMETERS

Ratio, r/h_2 of radial distances to thicknesses of Layer 2

Range of $r/h_2 =$	0	(0.02)	1.0	50 Values
	1.0	(0.1)	5.0	40
	5.0	(0.5)	25	40
				<u>130 Values</u>

Ratio, $h_1/h_2 = b$ of Layer 1 and Layer 2 Thicknesses

Range of b	0.125	0.25	0.50	1.0	10 Values
--------------	-------	------	------	-----	-----------

Depths z' at which Stresses and Displacements are Computed

Layer 1	w_1	$c =$	1.0	0.5	0	
	σ_{z1}			0.5	0	
	τ_{rz1}		0.75	0.5	0.25	0
Layer 2	w_2	$d =$			1.0	8 Values
	σ_{z2}			0.5		
	τ_{rz2}			0.5	0.25	1.0
Layer 3	σ_{z3}	$d =$	1.5	2	3	5
	τ_{rz3}		1.5	2		

Strength Ratios E_1/E_2 and E_2/E_3 for Layers 1, 2 and 3

.001	.002	.005	.01	.02	.05	1	2	5	10	20	50	100
200	300	500	1000	2000	5000	10000	20000	50000				

E_1/E_2 Combinations 22 Values
 E_2/E_3 Combinations 16 Values

26 Volumes of Tables

Poisson's Ratio, μ 4 Combinations

Layer 1 μ_1	0.2	0.4	0.2	0.4
Layer 2 μ_2	0.2	0.2	0.2	0.2
Layer 3 μ_3	0.2	0.2	0.4	0.4

THREE LAYER STRENGTH COEFFICIENTS

$$\text{Layers 1 \& 2} \quad k = \frac{E_2}{E_1} \frac{1+\mu_1}{1+\mu_2} \quad K = \frac{1 - k}{1+(3-4\mu_1)k} \quad (3a)$$

$$J = \frac{(3-4\mu_2) - (3-4\mu_1)k}{(3-4\mu_2)+k}$$

$$\text{Layers 2 \& 3} \quad n = \frac{E_3}{E_2} \frac{1+\mu_2}{1+\mu_3} \quad N = \frac{1 - n}{1+(3-4\mu_2)n} \quad (3b)$$

$$L = \frac{3-4\mu_3 - (3-4\mu_2)n}{(3-4\mu_3)}$$

THREE LAYER STRENGTH FUNCTIONS

$$[A] = 1 + \left[\frac{1-K}{1-J} JN + \frac{1-J}{1-K} KL \right] e^{-2\alpha} + \frac{1-J}{1-K} KN4\alpha^2 e^{-2\alpha} + JKLNe^{-4\alpha} \quad (4a)$$

$$[B] = J + \left[\frac{1-K}{1-J} J^2N + \frac{1-J}{1-K} J \right] e^{-2\alpha} + \frac{1-J}{1-K} N4\alpha^2 e^{-2\alpha} + JLNe^{-4\alpha} \quad (4b)$$

$$[C] = K + \left[\frac{1-K}{1-J} N + \frac{1-J}{1-K} K^2L \right] e^{-2\alpha} + \frac{1-J}{1-K} K^2N4\alpha^2 e^{-2\alpha} + KLNe^{-4\alpha} \quad (4c)$$

$$[D] = [1-JK]N2\alpha e^{-2\alpha} \quad (4d)$$

$$[E] = JK + \left[\frac{1-K}{1-J} JN + \frac{1-J}{1-K} KL \right] e^{-2\alpha} + \frac{1-J}{1-K} KN4\alpha^2 e^{-2\alpha} + LNe^{-4\alpha} \quad (4e)$$

THREE LAYER DENOMINATORS

$$D_3 = [A] - [B] e^{-2b\alpha} - [C] (1+4b^2\alpha^2) e^{-2b\alpha} - [D] 4b\alpha e^{-2b\alpha} + [E] e^{-4b\alpha} \quad (5a)$$

$$D_2 = [1 - (L + N + N4\alpha^2) e^{-2\alpha} + LNe^{-4\alpha}] \quad (5b)$$

LAYER 1 STRESSES AND DISPLACEMENTS

VERTICAL STRESS $\sigma_{z_1} = -p \frac{\theta}{2\pi} \int_0^{\infty} \frac{[N1-1]}{D_3} \frac{r}{h_2} J_1 (r\alpha/h_2) d\alpha$ (6)

$$\begin{aligned} [N1-1] = & + [A] [1+ (1-c) b\alpha] e^{-(1-c) b\alpha} \\ & - 0.5 [[B] + [C] (1+2\alpha) (1+2 cb\alpha)] e^{-(1+c) b\alpha} \\ & - 0.5 [[B] + [C] (1-2\alpha) (1-2 cb\alpha)] e^{-(3-c) b\alpha} \\ & - [D] [1+ (1+c) b\alpha] e^{-(1+c) b\alpha} - [1-(1+c) b\alpha] e^{-(3-c) b\alpha} \\ & + [E] (1-c) b\alpha e^{-(3+c) b\alpha} \end{aligned}$$

SHEARING STRESS $\tau_{rz_1} = -p [\sin \theta] \int_0^{\infty} \frac{1}{2\pi} \frac{[N2-1]}{D_3} \int_0^r J_1 (r\alpha/h_2) r dr d\alpha$ (7)

$$\begin{aligned} [N2-1] = & + [A] [(1-c) b\alpha] e^{-(1-c) b\alpha} \\ & + 0.5 [[B] - [C] (1+2b\alpha) (1-2cb\alpha)] e^{-(1+c) b\alpha} \\ & - 0.5 [[B] - [C] (1-2b\alpha) (1+2cb\alpha)] e^{-(3-c) b\alpha} \\ & + [D] [(1+c) b\alpha e^{-(1+c) b\alpha} - (1+c) b\alpha] e^{-(3-c) b\alpha} \\ & + [E] (1-c) b\alpha e^{-(3+c) b\alpha} \end{aligned}$$

DEFLECTION $w_1 = + 2 \frac{\theta}{2\pi} \frac{p r}{E_3} \int_0^{\infty} \frac{1+\mu_1}{2} \frac{E_3}{E_1} \frac{[N3-1]}{D_3} \frac{1}{\alpha} J_1 (r\alpha/h_2) d\alpha$ (8)

$$\begin{aligned} [N3-1] = & + [A] [2-2\mu_1 + (1+c) b\alpha] e^{-(1-c) b\alpha} \\ & + 0.5 [[B] + [C] (1+2b\alpha) (3-4\mu_1 + 2cb\alpha)] e^{-(1+c) b\alpha} \\ & - 0.5 [[B] + [C] (1-2b\alpha) (3-4\mu_1 - 2cb\alpha)] e^{-(3-c) b\alpha} \\ & + [D] [[2-2\mu_1 + (1+c) b\alpha] e^{-(1+c) b\alpha} + [2-2\mu_1 - (1+c) b\alpha] e^{-(3-c) b\alpha}] \\ & - [E] [[2-2\mu_1 - (1-c) b\alpha]] e^{-(3+c) b\alpha} \end{aligned}$$

LAYER 2 STRESSES AND DISPLACEMENTS

NUMERATOR BRACKETS FOR LAYER 2 AT INTERFACE [1-2]

$$\begin{aligned} [\sigma_z]_{1-2} = & + [A] [1+b\alpha] e^{-b\alpha} \\ & - 0.5 [[B] + [C] (1+2b\alpha)] e^{-b\alpha} \\ & - 0.5 [[B] + [C] (1-2b\alpha)] e^{-3b\alpha} \\ & - [D] [(1+b\alpha) e^{-b\alpha} - (1-b\alpha)] e^{-3b\alpha} \\ & + [E] [1-b\alpha] e^{-3b\alpha} \end{aligned} \quad (9)$$

$$\begin{aligned} [\tau_{rz}]_{1-2} = & + [A] [b\alpha] e^{-b\alpha} \\ & + 0.5 [[B] - [C] (1+2b\alpha)] e^{-b\alpha} \\ & - 0.5 [[B] - [C] (1-2b\alpha)] e^{-3b\alpha} \\ & - [D] [b\alpha e^{-b\alpha} - b\alpha e^{-3b\alpha}] \\ & - [E] [b\alpha] e^{-3b\alpha} \end{aligned} \quad (10)$$

DENOMINATORS FOR LAYER 2 & 3

$$D_3 = [[A] - [B] e^{-2b\alpha} - [C] (1+4b^2\alpha^2) - [D] 4b\alpha e^{-2b\alpha} + [E] e^{-4b\alpha}] \quad (11a)$$

$$D_2 = [1 - (L+N+N 4\alpha^2) e^{-2\alpha} + LN e^{-4\alpha}] \quad (11b)$$

VERTICAL STRESS

(12)

$$\sigma_{za} = -p \frac{\theta}{2\pi} \int_0^{\infty} \left[\frac{[\sigma_z]_{1-2}}{D_2 D_3} [N1-2a] - \frac{[\tau_{rz}]_{1-2}}{D_2 D_3} [N1-2b] \right] \frac{r}{h_2} J_1(r\alpha/h_2) d\alpha$$

$$\begin{aligned} [N1-2a] = & + [1+d\alpha] e^{-d\alpha} \\ & - 0.5 [L + N [1+(1-d)2\alpha] (1+2\alpha)] e^{-(2-d)\alpha} \\ & - 0.5 [L + N [1-(1-d)2\alpha] (1-2\alpha)] e^{-(2+d)\alpha} \\ & + L N [1-d\alpha] e^{-(4-d)\alpha} \end{aligned}$$

$$\begin{aligned} [N1-2b] = & + d\alpha e^{-d\alpha} \\ & - 0.5 [L - N [1+(1-d)2\alpha] (1-2\alpha)] e^{-(2-d)\alpha} \\ & + 0.5 [L - N [1-(1-d)2\alpha] (1+2\alpha)] e^{-(2+d)\alpha} \\ & + L N d\alpha e^{-(4-d)\alpha} \end{aligned}$$

SHEARING STRESS

(13)

$$\tau_{rza} = -p[\sin \theta] \int_0^{\infty} \frac{1}{2\pi} \left[\frac{[\sigma_z]_{1-2}}{D_2 D_3} [N2-2a] + \frac{[\tau_{rz}]_{1-2}}{D_2 D_3} [N2-2b] \right] \int_0^{r/h_2} \alpha J_1(r\alpha/h_2) \frac{r}{h_2} d(r/h_2) d\alpha$$

$$\begin{aligned} [N2-2a] = & + d\alpha e^{-d\alpha} \\ & + 0.5 [L - N [1-(1-d)2\alpha] (1+2\alpha)] e^{-(2-d)\alpha} \\ & - 0.5 [L - N [1+(1-d)2\alpha] (1-2\alpha)] e^{-(2+d)\alpha} \\ & + L N d\alpha e^{-(4+d)\alpha} \end{aligned}$$

$$\begin{aligned} [N2-2b] = & + (1-d\alpha) e^{-d\alpha} \\ & - 0.8 [L + N [1-(1-d)2\alpha] (1-2\alpha)] e^{-(2-d)\alpha} \\ & - 0.5 [L + N [1+(1-d)2\alpha] (1+2\alpha)] e^{-(2+d)\alpha} \\ & + L N (1+d\alpha) e^{-(4-d)\alpha} \end{aligned}$$

DEFLECTION

(14)

$$w_a = + \frac{2\theta}{2\pi} \frac{p r}{E_3} \int_0^\infty \frac{1+\mu_2}{2} \left[\frac{E_3}{E_2} \frac{[\sigma_z]_{1-2}}{D_2 D_3} [N3-2a] - \frac{[\tau_{rz}]_{1-2}}{D_2 D_3} [N3-2b] \right] \frac{1}{\alpha} J_1(r\alpha/h_2) d\alpha$$

$$\begin{aligned} [N3-2a] = & + (2-2\mu_2+d\alpha) e^{-d\alpha} \\ & + 0.5 \left[L + N [3-4\mu_2+(1-d)2\alpha] (1+2\alpha) \right] e^{-(2-d)\alpha} \\ & - 0.5 \left[L + N [3-4\mu_2 - (1-d)2\alpha] (1-2\alpha) \right] e^{-(2+d)\alpha} \\ & - L N (2-2\mu_2- d\alpha) e^{-(4-d)\alpha} \end{aligned}$$

$$\begin{aligned} [N3-2b] = & + (1-2\mu_2+d\alpha) e^{-d\alpha} \\ & + 0.5 \left[L - N [3-4\mu_2+ (1-d)2\alpha] (1-2\alpha) \right] e^{-(2-d)\alpha} \\ & + 0.5 \left[L - N [3-4\mu_2 - (1-d)2\alpha] (1+2\alpha) \right] e^{-(2+d)\alpha} \\ & + L N (1-2\mu_2 - d\alpha) e^{-(4-d)\alpha} \end{aligned}$$

LAYER 3 STRESSES AND DISPLACEMENTS

NUMERATOR BRACKETS FOR LAYER 3 AT INTERFACE [2-3]

$$[\sigma_z]_{2-3} = + [\sigma_z]_{1-2} \left[\begin{array}{l} + (1+\alpha) e^{-\alpha} \\ - 0.3[1+N(1+2\alpha)] e^{-\alpha} \\ - 0.5[L+N(1-2\alpha)] e^{-\alpha} \\ + LN(1-\alpha) e^{-3\alpha} \end{array} \right] \quad (15)$$

$$- [\tau_{rz}]_{1-2} \left[\begin{array}{l} + \alpha e^{-\alpha} \\ - 0.5[L-N(1-2\alpha)] e^{-\alpha} \\ + 0.5[L-N(1+2\alpha)] e^{-3\alpha} \\ + LN \alpha e^{-3\alpha} \end{array} \right]$$

$$[\tau_{rz}]_{2-3} = + [\sigma_z]_{1-2} \left[\begin{array}{l} + \alpha e^{-\alpha} \\ + 0.5[L-N(1+2\alpha)] e^{-\alpha} \\ - 0.5[L-N(1-2\alpha)] e^{-2\alpha} \\ + LN \alpha e^{-3\alpha} \end{array} \right] \quad (16)$$

$$[\tau_{rz}]_{1-2} \left[\begin{array}{l} + (1-\alpha) e^{-\alpha} \\ - 0.5[L+N(1-2\alpha)] e^{-\alpha} \\ - 0.5[L+N(1+2\alpha)] e^{-3\alpha} \\ + LN(1+\alpha) e^{-3\alpha} \end{array} \right]$$

VERTICAL STRESS

(17)

$$\sigma_{z_3} = -p \frac{\theta}{2\pi} \left[\int_0^{\infty} \frac{[\sigma_z]_2}{D_2 D_3} [N1-3a] - \frac{[\tau_{rz}]_{1-2}}{D_2 D_3} [N1-3b] \right] \frac{r}{h_2} J_1(r\alpha/h_2) d\alpha$$

$$[N1-3a] = + [1+(d-1)\alpha] e^{-(d-1)\alpha}$$

$$[N1-3b] = + [d-1]\alpha e^{-(d-1)\alpha}$$

SHEARING STRESSES

(18)

$$\tau_{rz_3} = -p [\sin \theta] \int_0^{\infty} \left[\frac{1}{2\pi} \frac{[\sigma_z]_{2-3}}{D_2 D_3} [N2-3a] + \frac{[\tau_{rz}]_{2-3}}{D_2 D_3} [N2-3b] \right] \int_0^{r/h_2} \alpha J_1(r\alpha/h_2) \frac{r}{h_2} dr d\alpha$$

$$[N2-3a] = (d-1)\alpha e^{-(d-1)\alpha}$$

$$[N2-3b] = [1-(d-1)\alpha] e^{-(d-1)\alpha}$$

DEFLECTION

(19)

$$w_3 = + \frac{2\theta}{2\pi} \frac{pr}{E_3} \int_0^{\infty} \frac{1+\mu_3}{2} \left[\frac{[\sigma_z]_{2-3}}{D_2 D_3} [N3-3a] - \frac{[\tau_{rz}]_{2-3}}{D_2 D_3} [N3-3b] \right] \frac{1}{\alpha} J_1(r\alpha/h_2) d\alpha$$

$$[N2-3a] = (d-1)\alpha e^{-(d-1)\alpha}$$

$$[N2-3b] = [1-(d-1)\alpha] e^{-(d-1)\alpha}$$

The following additional stress and displacement equations are given in order to have a complete set of two layer equations for reference. The integrations, $\int_0^r r dr$ are to be made by computational methods. The angle θ in these equations covers one quadrant only from 0° to 90° for circular diagram methods of evaluation for any shape, size, and position of loaded area.

LAYER 1 STRESSES AND DISPLACEMENTS

HORIZONTAL STRESS, σ_{x_1} (20)

$$\begin{aligned}
 & - \frac{p}{2\pi h_2^2} \int_0^\infty d\alpha \int_0^r r dr \int_0^\theta \left[\frac{[N4-1]}{D_3} \alpha J_0(r\alpha/h_2) - \frac{[N4-1]}{D_3} \frac{h_2}{r} J_1(r\alpha/h_2) + \right. \\
 & \quad \left. + \frac{[N5-1]}{D_3} \alpha J_0(r\alpha/h_2) \right]_1 \cos^2 \theta d\theta + \left[\frac{[N4-1]}{D_3} \frac{h_2}{r} J_1(r\alpha/h_2) + \right. \\
 & \quad \left. \frac{[N5-1]}{D_3} \alpha J_0(r\alpha/h_2) \right]_2 \sin^2 \theta d\theta
 \end{aligned}$$

HORIZONTAL STRESS, σ_{y_1} (21)

$$- \frac{p}{2\pi h_2^2} \int_0^\infty d\alpha \int_0^r r dr \int_0^\theta \left[[\text{Ditto}]_1 \sin^2 \theta d\theta + [\text{Ditto}]_2 \cos^2 \theta d\theta \right]$$

SHEAR STRESS, $\tau_{xy_1} = \tau_{yx_1}$ (22)

$$- \frac{p}{2\pi h_2^2} \int_0^\infty d\alpha \int_0^r r dr \int_0^\theta \left[\frac{[N4-1]}{D_3} \alpha J_0(r\alpha/h_2) - 2 \frac{[N4-1]}{D_3} \frac{h_2}{r} J_1(r\alpha/h_2) \right] \times \sin \theta \cos \theta d\theta$$

HORIZONTAL DISPLACEMENT, u_1 (23)

$$- \frac{p}{2\pi h_2^2} \frac{pr}{E_3} \int_0^\theta d\theta \int_0^r r dr \int_0^\infty \frac{1+\mu_1}{2} \frac{E_3}{E_1} \frac{[N4-1]}{D_3} J_1(r\alpha/h_2) d\alpha$$

LAYER 1 STRESSES AND DISPLACEMENTS --Continued

LAYER 1 NUMERATOR BRACKETS

(24)

$$\begin{aligned} [N4-1] = & + [A] [1-2\mu_1-(1-c)b\alpha] e^{-(1-c)b\alpha} \\ & + 0.5 [[B] - [C](1+b\alpha)(3-4\mu_1-cb\alpha)] e^{-(1+c)b\alpha} \\ & + 0.5 [[B] - [C](1-b\alpha)(3-4\mu_1+cb\alpha)] e^{-(3-c)b\alpha} \\ & - [D] [[1-2\mu_1-(1+c)b\alpha] e^{-(1+c)b\alpha} - [1-2\mu_1+(1+c)b\alpha] e^{-(3-c)b\alpha}] \\ & + [E] [[1-2\mu_1+(1-c)b\alpha] e^{-(3+c)b\alpha}] \end{aligned}$$

$$\begin{aligned} [N5-1] = & + [A] 2\mu_1 e^{-(1+c)b\alpha} - [C] 2\mu_1 (1+b\alpha) e^{-(1+c)b\alpha} \\ & - [C] 2\mu_1 (1-b\alpha) e^{-(3-c)b\alpha} + [E] 2\mu_1 e^{-(3+c)b\alpha} \\ & - [D] 2\mu_1 [e^{-(1+c)b\alpha} - e^{-(1-c)b\alpha}] \end{aligned}$$

LAYER 2 STRESSES AND DISPLACEMENTS

HORIZONTAL STRESS σ_{x_2} and σ_{x_2}

SHEAR STRESS τ_{xy_2} and τ_{yx_2}

HORIZONTAL DISPLACEMENT u_2

Substitute for Numerator Brackets [N4-1] and [N5-1] in Equations 20, 21, 22, 23, and 24 the following Numerator Brackets with $[\sigma_z]_{1-2}$ and $[\tau_{rz}]_{1-2}$ given by Equations 9 and 10 for Layer 2 Stresses and Displacements.

$$\frac{[N4-1]}{D_3} \rightarrow \left[\frac{[\sigma_z]_{1-2}}{D_2 D_3} [N4-2a] + \frac{[\tau_{rz}]_{1-2}}{D_2 D_3} [N4-2b] \right] \quad (25)$$

$$\frac{[N5-1]}{D_3} \rightarrow \left[\frac{[\sigma_z]_{1-2}}{L_2 D_2} [N5-2a] + \frac{[\tau_{rz}]_{1-2}}{D_2 D_3} [N5-2b] \right]$$

LAYER 2 NUMERATOR BRACKETS (26)

$$\begin{aligned} [N4-2a] = & + (1-2\mu_2 - d\alpha) e^{-d\alpha} \\ & + 0.5 \left[L-N [3-4\mu_2-2(1-d)\alpha] (3+2\alpha) \right] e^{-(2-d)\alpha} \\ & + 0.5 \left[L-N [3-4\mu_2+2(1-d)\alpha] (1-2\alpha) \right] e^{-(2+d)\alpha} \\ & + L N [2-2\mu_2+d\alpha] e^{-(4-d)\alpha} \end{aligned}$$

$$\begin{aligned} [N4-2b] = & + (2-2\mu_2 - d\alpha) e^{-d\alpha} \\ & + 0.5 \left[L + N [3-4\mu_1 - 2(1-d)\alpha](1-2\alpha) \right] e^{-(2-d)\alpha} \\ & - 0.5 \left[L + N [3-4\mu_2+2(1-d)\alpha](1+2\alpha) \right] e^{-(2+d)\alpha} \\ & - L N [2-2\mu_2 + d\alpha] e^{-(4-d)\alpha} \end{aligned}$$

LAYER 2 STRESSES AND DISPLACEMENTS--Continued

LAYER 2 NUMERATOR BRACKETS

$$\begin{aligned} [\text{N5-2a}] = & + 2\mu_2 e^{-d\alpha} - N2\mu_2(1+2\alpha) e^{-(2-d)\alpha} & (27) \\ & - N 2\mu_2 (1-2\alpha) e^{-(2+d)\alpha} + LN 2\mu_2 e^{-(4-d)\alpha} \end{aligned}$$

$$\begin{aligned} [\text{N5-2b}] = & + 2\mu_2 e^{-d\alpha} + N2\mu_2(1-2\alpha) e^{-(2-d)\alpha} \\ & - N 2\mu_2 (1+2\alpha) e^{-(2+d)\alpha} - LN 2\mu_2 e^{-(4-d)\alpha} \end{aligned}$$

LAYER 3 STRESSES AND DISPLACEMENTS

HORIZONTAL STRESSES

$$\sigma_{x_3} \text{ and } \sigma_{y_3}$$

SHEAR STRESSES

$$\tau_{xy_3} \text{ and } \tau_{yx_3}$$

HORIZONTAL DISPLACEMENTS

$$u_3$$

Substitute for Numerator Brackets [N4-1] and [N5-1] in Equations 20, 21, 22, 23 and 24. The following Numerator Brackets with $[\sigma_z]_{2-3}$ and $[\tau_{rz}]_{2-3}$ given by Equations 15 and 16 for Layer 3 Stresses and Displacements.

$$\frac{[N4-1]}{D_3} \rightarrow \left[\frac{[\sigma_z]_{2-3}}{D_2 D_3} [N4-3a] + \frac{[\tau_{rz}]_{2-3}}{D_2 D_3} [N4-3b] \right] \quad (28)$$

$$\frac{[N5-1]}{D_3} \rightarrow \left[\frac{[\sigma_z]_{2-3}}{D_2 D_3} [N5-3a] + \frac{[\tau_{rz}]_{2-3}}{D_2 D_3} [N5-3b] \right]$$

$$\frac{[N4-3a]}{D_3} = - [1-2\mu_3 - (d-1)\alpha] e^{-(d-1)\alpha} \quad (29)$$

$$\frac{[N4-3a]}{D_3} = + [2-2\mu_3 - (d-1)\alpha] e^{-(d-1)\alpha}$$

$$\frac{[N5-3a]}{D_3} = - 2\mu_3 e^{-(d-1)\alpha}$$

$$[N5-3b] = + 2\mu_3 e^{-(d-1)\alpha}$$

CHARACTER OF THREE LAYER SYSTEM STRESS AND DEFLECTION EQUATIONS

The three layer stress and displacement Eqs. 3 to 29 reveal in their systematic relationships the dependence of stresses and deflections upon the basic three layer parameters on pages 7 and 8, namely: the ratio, r/h_2 of radial distance to thicknesses of layer 2; the ratio, h_1/h_2 of layer 1 and layer 2 thicknesses; the depth - thickness ratio, z/h ; and the three layer strength coefficients - K, J, N, and L of Eqs. 3, and three layer strength functions [A] to [E] of Eqs. 4. The computations of stresses and deflections cover the ranges of the three layer parameters on page 7 for r/h_2 , z/h_2 , E_1/E_2 , and E_2/E_3 , and Poisson's ratio combinations.

In order to facilitate the computations of stresses and deflections, the parametric relations of Eqs. 2 on page 6 were used, in which all parametric relations are referenced to the thickness, h_2 of layer 2. The depth-thickness ratio, z/h is referenced in Fig. 1 to $z' = 0$ at interface 1-2 through the subsidiary relations: $z' = + d h_2$ for layers 2 and 3 continuously. The true depths in the three layer system for tabulations of computed stress and deflection influence values are referenced to $z = 0$ at the top surface of layer 1, through the relations: $z = (1-c) b h_2$ for layer 1, and $z = (1+d) h_2$ for layers 2 and 3. Since the thickness, h_2 of layer 2 was used as the basic reference throughout these computations, the basic three layer parameter, r/h_2 is defined as the ratio of all radial distances, r in all layers to the thickness, h_2 of layer 2. The thicknesses, h_1 of layer 1 are taken into account by the relation $h_1 = b h_2$ in Eq. 2a. The ratios, r/h_2 can be converted, for purposes of analyzing and evaluating layered system performances, to ratios, r/h_1 by the relations - $[r/h_1] = [r/bh_2] = [r/h_2] \times [1/b]$.

The three layer performances, with regard to stress transmission characteristics, magnitude and distribution of stresses imposed in the three layers by surface loads, and the deflection responses are governed by the complex interacting influences of the fundamental three layer parameters discussed above, in Equations 6 through 29. The deflection performances of a three layer system with regard to reinforcing action, stiffness, and load spreading capacity of reinforcing layer 1 and 2 are governed by the settlement or deflection coefficient, F_w , which is derived from Eq. 8, on page 9 for deflections at the surface of layer 1 for $z = 0$, or $c = 1$ in $z = (1-c) b h_2$ after performing the integrations.

$$\text{Deflection at Surface, } z = 0 \qquad w = C p r F_w / E_3 \qquad (8a)$$

Where the parametric relation, $F_w = f_w [E_1/E_2, E_2/E_3, \mu_1, \mu_2, \mu_3, r/h_2, z/h]$

$$\text{Deflection at Interface (1-2), } z = h_1 \qquad w = C p r F_{w(1-2)} / E_3 \qquad (8b)$$

The deflection coefficient, F_w expresses not only the controlling interacting influences of all these three layer parameters, but also must reflect the actual influences: (a) of the preconditioning and prestressing of the three layer system during construction, and (b) of the restraints and shear strength continuity incorporated in layer interfaces 1-2 and 2-3 and throughout layers 1 and 2.

The stress and deflection equations reveal the systematic form of the five lines of each equation, and the fundamental nature of the physical parametric relations of Eqs. 6 to 29 that must exist among the three layer parameters, which govern stress and deflection performances in systematic, inter-related, and consistent patterns. The Three Layer common denominators,

D_3 and D_2 of Eq. 5 insure the continuity of stresses and displacements across interfaces 1-2 and 2-3.

The reinforcing action, stiffness, load spreading capacity, and stress reducing capacity of reinforcing layer 1 and 2 on stresses imposed in subgrade layer 3 are of principal concern in evaluating deflection, bending, and shear-deformation performances of layered systems. These fundamental performance characteristics of three-layer systems are treated in considerable detail in graphical presentations, in order to provide essential bases for understanding, intuitive thinking, evaluation, and judgments regarding their real character and effectiveness over the full range of the three layer parameter given on page 7 for vertical stresses, σ_z ; shear stresses, τ_{rz} , and deflections, w for which computations of influence values have been completed under this contract. In addition, direct comparison will be made between two layer performances in Part I and three layer performances in the present Part II. Also, bases for evaluating "layer equivalences" will be treated.

PART II-A

VERTICAL STRESS, SHEAR STRESS, and DEFLECTION CHARACTERISTICS and PERFORMANCES of TWO and THREE LAYER CONCRETE PAVEMENT SYSTEMS .

The character, distribution, and magnitude of vertical stresses and shear stresses throughout the layers of layered pavement systems and of surface deflections are of principal concern in studying, comparing, and evaluating the stress transmission characteristics and performances, the effectiveness and permanence of reinforcing action, the load spreading capacity, and the shear deformation and flexing capacity of layered pavement systems . They provide the essential bases for judging the adequacy of pavement design . When these pavement characteristics and performances are fully visualized and known, their complex interacting influences become significant and can be studied and evaluated on rational, logical, and systematic bases and by appropriate design procedures .

The vertical stress, shear stress, and deflection characteristics and performances of layered pavement systems are "ordered" phenomena, not haphazard phenomena . Their complex interacting influences are therefore also ordered, and they can be evaluated as major aspects of layered pavement system design . The major evaluation problem is not empirical correlations, but rather is that of establishing significant and realistic parametric relationships, which are revealed by the layered pavement system theory, but modified by dimensionless coefficients obtained from evaluations of actual field performances of layered pavement systems .

It then becomes possible to delineate regions on layered pavement systems of critical vertical and shear stress values and of critical flexing action with respect to single and dual wheel and landing gear loadings into pavement surfaces . Hence the necessary adjustment can be made with regard to number of layers, layer thicknesses, and layer strength properties in order to minimize

or to eliminate destructive shear deformation and flexing action influences, and to satisfy the essential design requirements and to achieve fully satisfactory pavement performances and long life .

Thorough knowledge and understanding of, and clear insight into, the fundamental stress transmission characteristics and stress performances and deflection performances of three layer pavement systems in comparison with those of two layer systems are essential for pavement studies, evaluations, and designs. A series of graph have been prepared in sequence in order to bring out and to permit visualization of their complex interacting influences on overall performances , as summarized in Table 1.

Table 1. Systematic Series of Figures for Two Layer and Three Layer Concrete Pavement Systems and Ranges of Values of Parameters .

a) Values of Parameters Common to All Figures .

Effective radius of bearing area, $r_e = 10''$

Layer thicknesses - $h_1 = 8''$ Concrete

$h_2 = 0, 8'',$ and $16''$ Base Course

Depths in layered system - $z = n h_1$. $n = 0.5, 1, 1.5, 2, 3, 4,$ and 5

Poison's Ratio of Layers - $\mu_1 = \mu_2 = \mu_3 = 0.2$

Layered system radius - thickness ratios - $r/h_1 = 0$ to 4 .

$$r_e/h_1 = 10''/8'' = 1.25$$

b) Figure Group Capital Letter Designations- Characteristics and Performances

Group A- Vertical Stress, σ_z

Group B- Shear Stress, τ_{rz}

Group C- Surface Deflection, w

Table 1--Continued-

c) Strength Properties of Layers expressed by Moduli and Moduli Ratios .

Figures	2	3	4	5	6	7	8	9
E_1/E_2	20	20	20	20	50	50	50	50
E_2/E_3	20	50	100	200	10	20	50	100
E_1 Concrete	3,000,000 - 3m				3,000,000 - 3m			
E_2 Base	150,000 - 150th.				150,000 - 150th.			
E_3 Subgrade	7500	3000	1500	750	6000	3000	1200	600
E_1/E_3	400	1000	2000	4000	500	1000	2500	5000

First of all, each letter group A, B, and C of Figures 2 to 9 are given together as a unit in order to compare, to bring out and to evaluate - (1) for Group A and B their stress transmission characteristics and performances and the effectiveness of reinforcing action, load spreading capacity, and stress reducing capacity of the layered pavement systems, as influenced and controlled by layer thicknesses, r/h_1 ratios, and layer moduli ratios; and (2) for Group C the deflection performances, the effectiveness and permanence of reinforcing action and the shear deformation and flexing capacity of layered pavement systems. And second, each group of Figs. 2A, 2B, and 2C to Figs. 9A, 9B, and 9C are treated with regard to the complex interaction influences on layered pavement system performances, particularly with regard to destructive shear deformations and flexing action influences under wheel loadings.

Group A, Vertical Stresses in Figs. 2A to 9A

In each Fig. 2 to 9, influence curves of vertical stresses, σ_z/p are plotted against the layered system parameter, r/h_1 in the lower portion of the figure for the three pavement systems- ($h_1 = 8''$, $1/b = 0$, $h_2 = 0$) ($h_1 = 8''$, $1/b = 1$, $h_2 = 8''$), and ($h_1 = 8''$, $1/b = 2$, $h_2 = 16''$) and at depths, $z = n h_1$ in the layers of the pavement systems as follows:

Thickness Designation- $1/b$	0	1	2
Layer 2 Thickness	$h_2 = 0$	$h_2 = h_1$	$h_2 = 2h_1$
Center of Layer 1	$z = 0.5h_1$	$z = 0.5h_1$	$z = 0.5h_1$
Interface 1 - 2	$z = 1.0h_1$	$z = 1.0h_1$	$z = 1.0h_1$
Center of Layer 2	(Depth, $z = 1.5h_1$)	$z = 1.5h_1$	$z = 2.0h_1$
Interface (2 - 3)	(Depth, $z = 2.0h_1$)	$z = 2.0h_1$	$z = 3.0h_1$
Depths		$z = 3.0h_1$	$z = 4.0h_1$

These depths in the layered systems are given in the pavement designations on the right-hand side of the figures of vertical stress influence curves, pages 27 to 34.

A study and comparison of the vertical stress influence curves in the lower portions of Figs. 2A to 9A disclose the character of the vertical stress performances of layered pavement systems and their effectiveness of reinforcing action and load spreading capacity in reducing the magnitudes of vertical stresses imposed on the top of the subgrade layer, and hence the protection afforded by the reinforcing layers to the subgrade layer. These figures show that magnitudes of vertical stress influence coefficients, σ_z/p imposed on the top of the subgrade layer are governed by the two layer and three layer parameters- E_1/E_2 , E_2/E_3 , z/h_1 and particularly r/h_1 . It is clear from Figs. 2A to 9A that for constant values of E_1/E_2 , E_2/E_3 , and z/h_1 the effectiveness of reinforcing action decreases markedly with increase in r/h_1 values. Hence it must be realized that the effectiveness of reinforcing action of a given

FIG. 2A DISTRIBUTION OF VERTICAL STRESSES IN TWO AND THREE LAYER PAVEMENT SYSTEMS. 8 INCH CONCRETE. 20/20

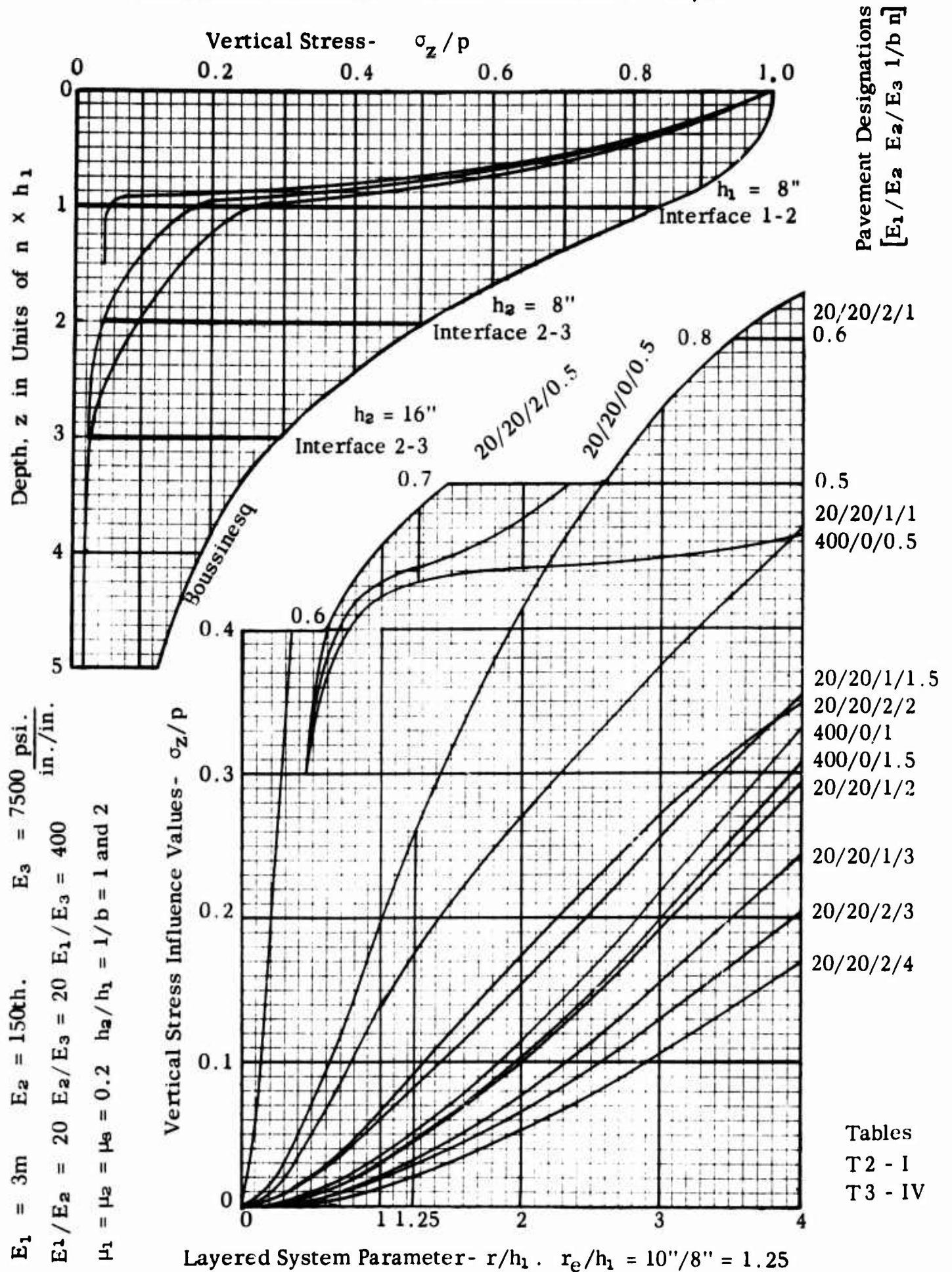


FIG. 3A DISTRIBUTION OF VERTICAL STRESSES IN TWO AND THREE LAYER PAVEMENT SYSTEMS. 8 INCH CONCRETE. 20/50

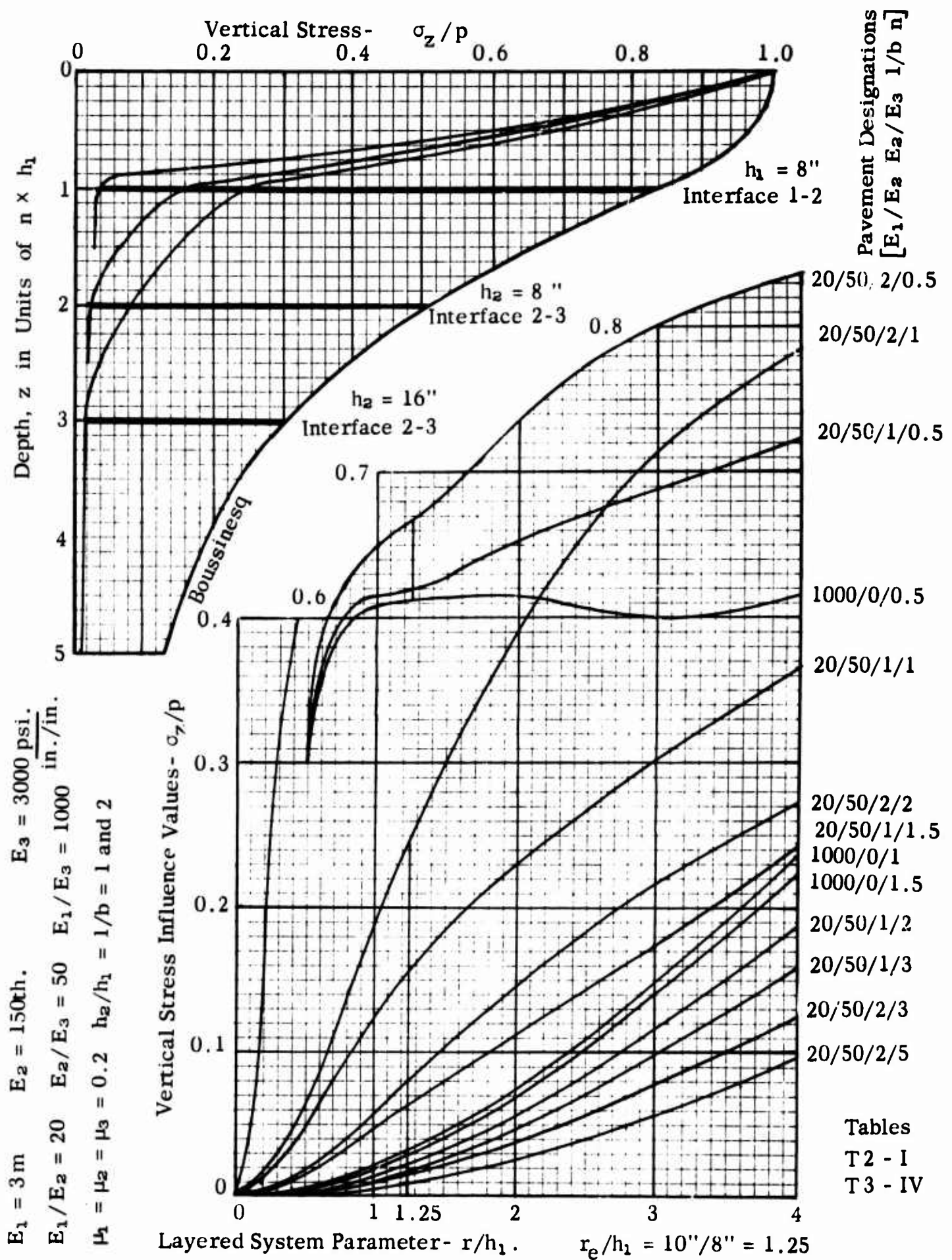


FIG. 4A DISTRIBUTION OF VERTICAL STRESSES IN TWO AND THREE LAYER PAVEMENT SYSTEMS. 8 INCH CONCRETE. 20/100

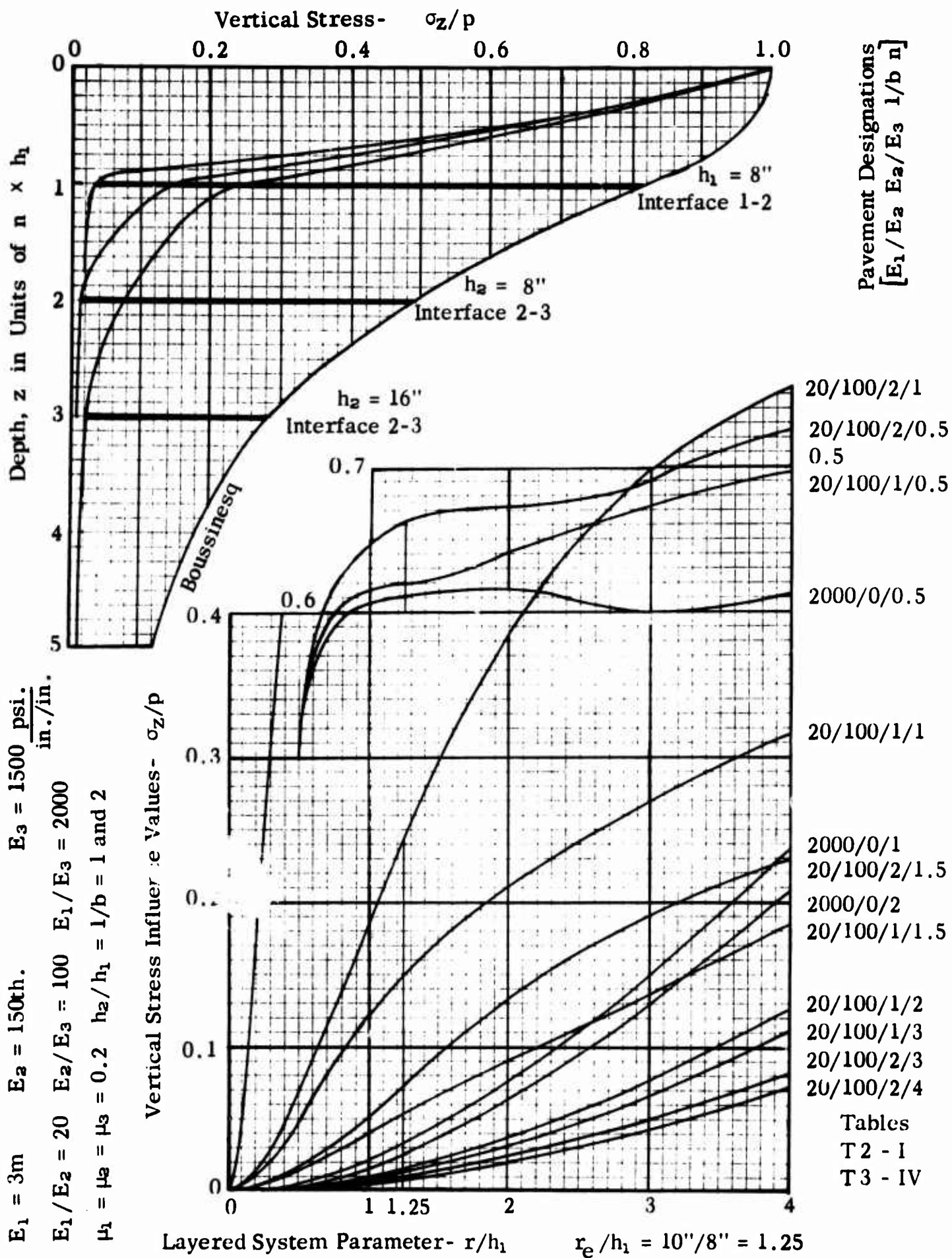


FIG. 5 A DISTRIBUTION OF VERTICAL STRESSES IN TWO AND THREE LAYER PAVEMENT SYSTEMS. 8 INCH CONCRETE. 20/200

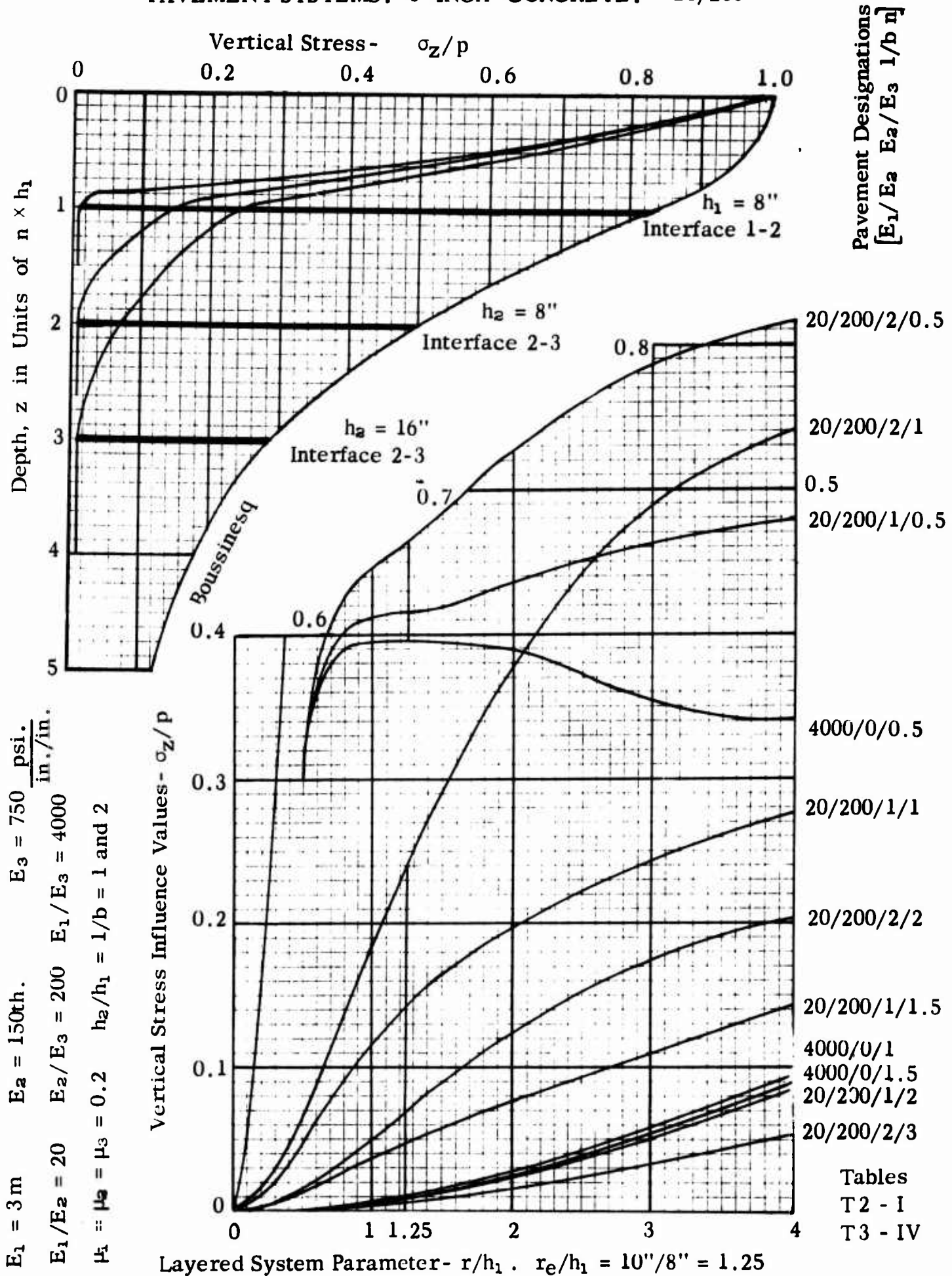


FIG. 6A DISTRIBUTION OF VERTICAL STRESSES IN TWO AND THREE LAYER PAVEMENT SYSTEMS. 8 INCH CONCRETE. 50/10

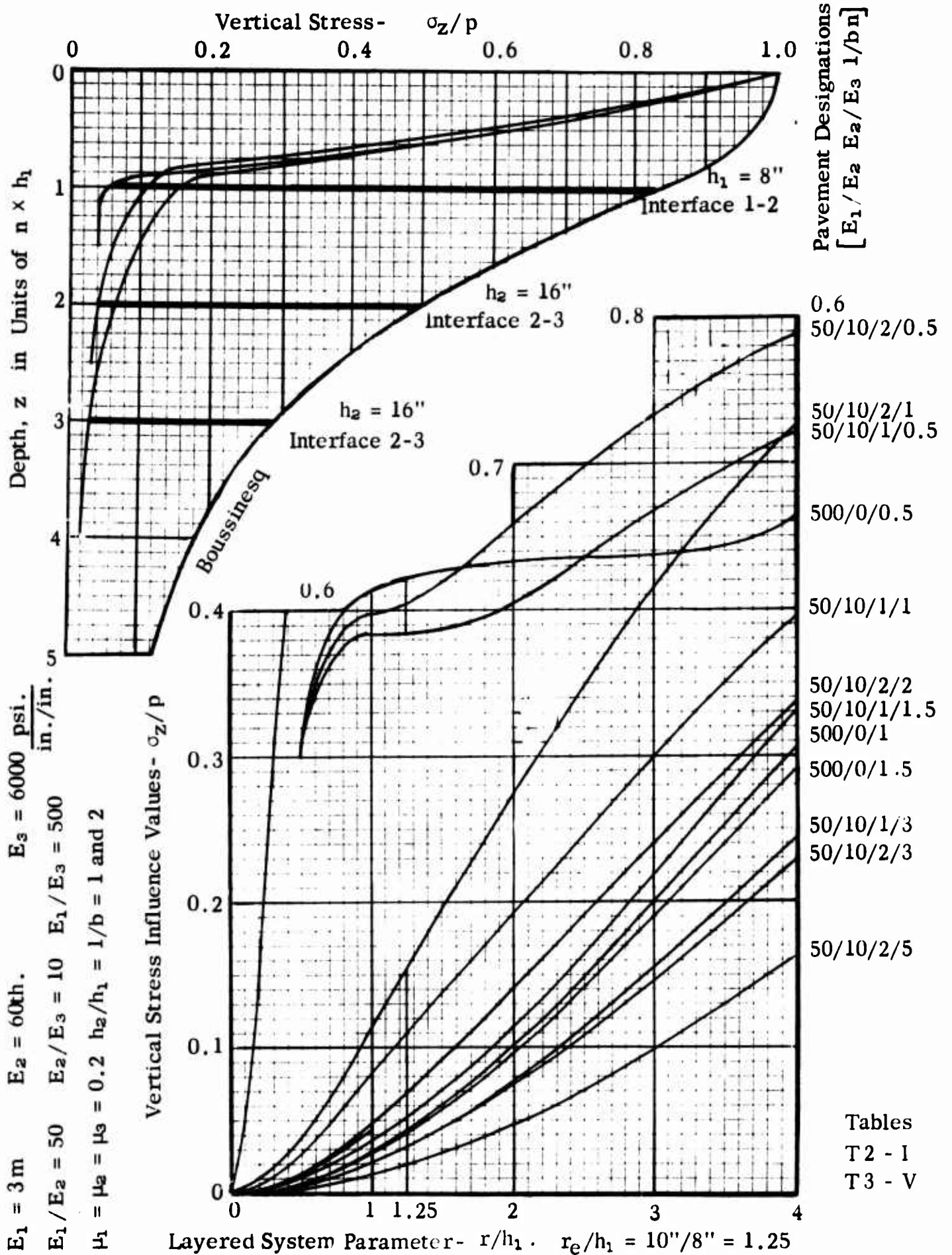


FIG. 7A DISTRIBUTION OF VERTICAL STRESSES IN TWO AND THREE LAYER PAVEMENT SYSTEMS. 8 INCH CONCRETE. 50/20

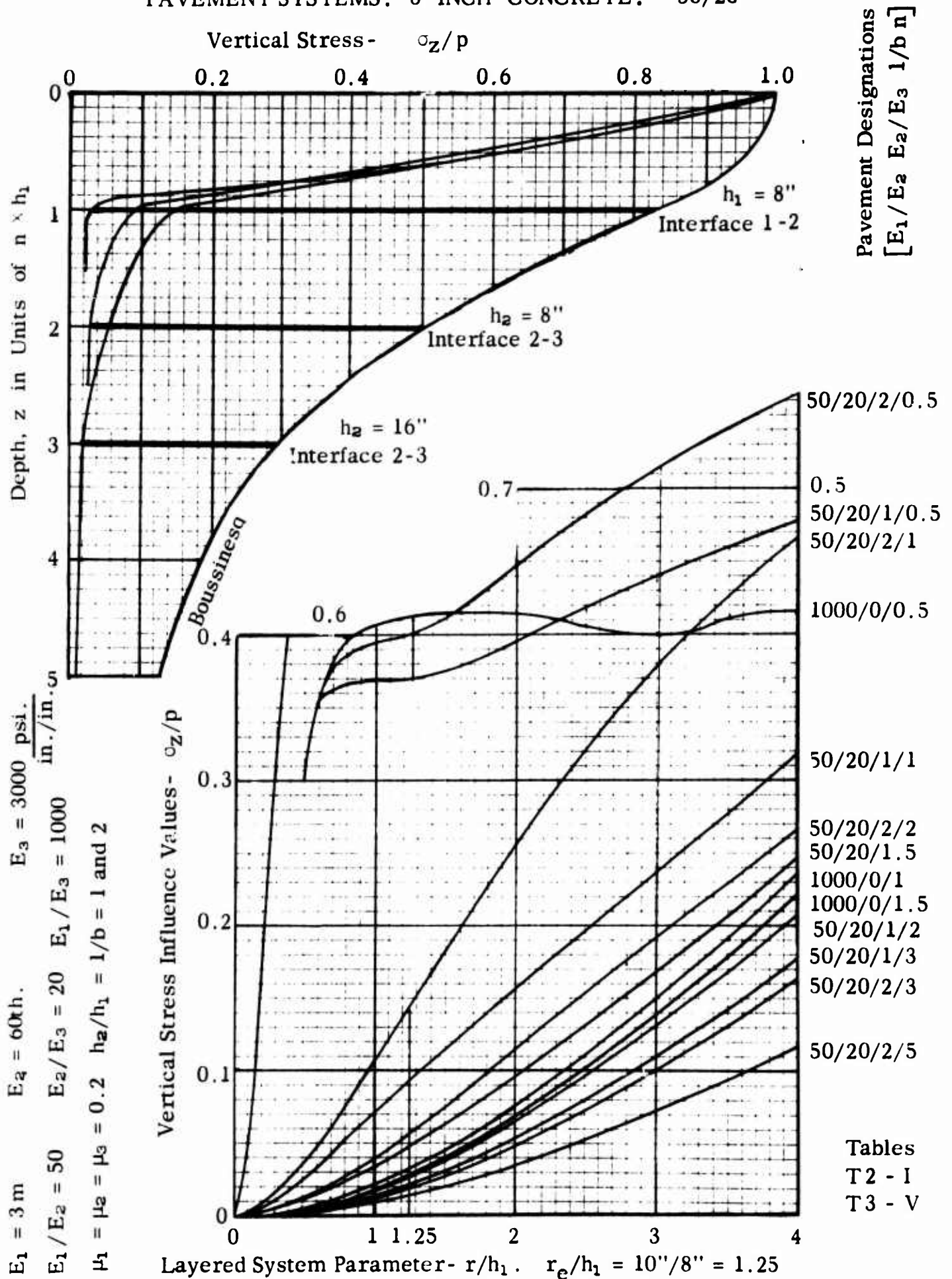


FIG. 8A DISTRIBUTION OF VERTICAL STRESSES IN TWO AND THREE LAYER PAVEMENT SYSTEMS. 8 INCH CONCRETE. 50/50

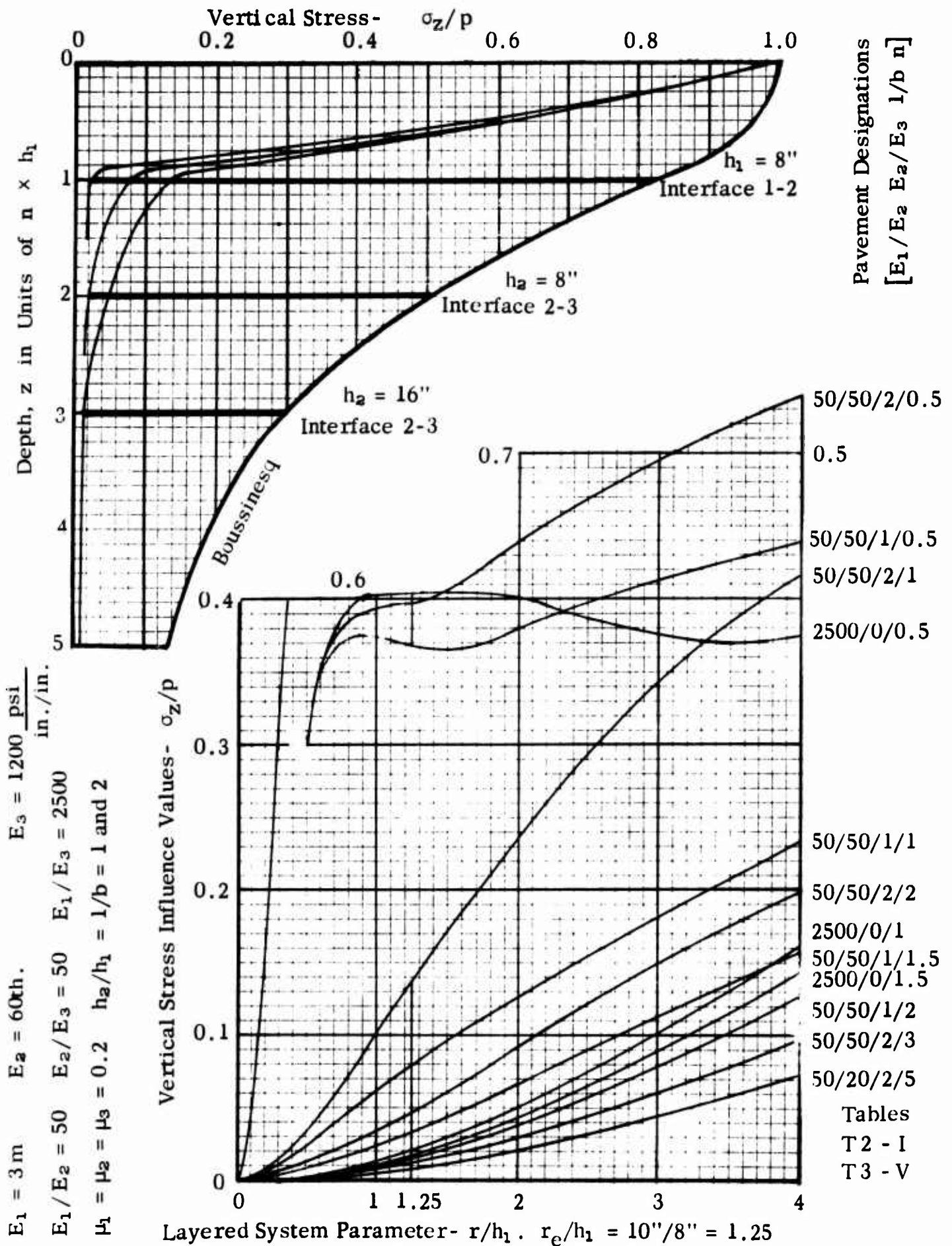
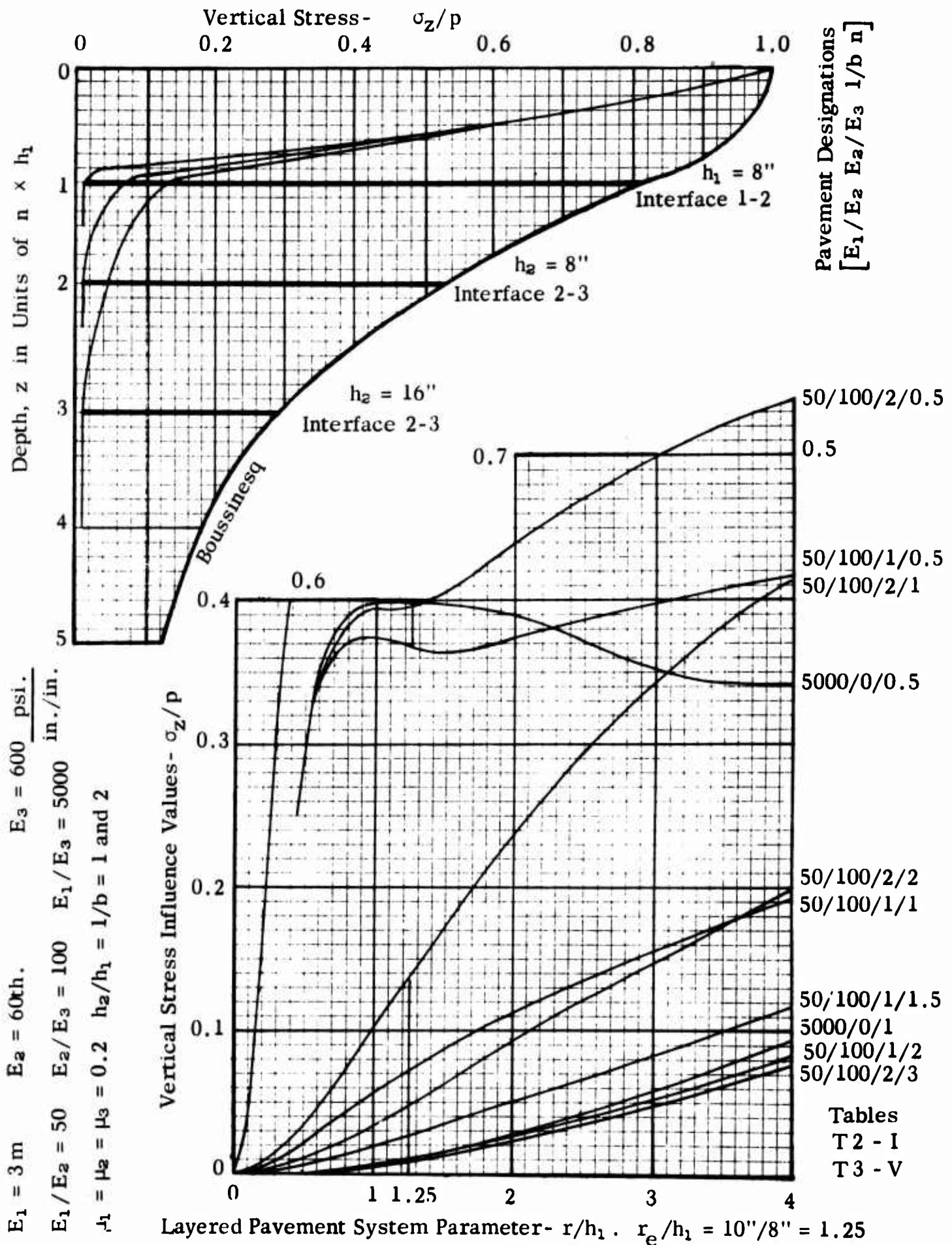


FIG. 9A DISTRIBUTION OF VERTICAL STRESSES IN TWO AND THREE LAYER PAVEMENT SYSTEMS. 8 INCH CONCRETE. 50/100



designed and constructed layered pavement system is not an inherent constant, but is controlled and affected adversely by increases in radius of bearing area through the governing σ_z/p and r/h_1 relations in Figs. 2A to 9A. These effectiveness relationships are illustrated in Table 2 for r/h_1 values of 1.00, 1.25, and 2.00 for the layered pavement systems of Figs. 2A to 9A. Table 2 also shows that an increase in effectiveness can definitely be achieved by design in three ways: (1) by the use of higher strength, E_1/E_2 and E_2/E_3 materials in the base course layer 2 for a constant strength and constant thickness of concrete Layer 1; (2) by an increase in thickness of the base course layer 2, and (3) by a combination of both of the above methods.

Table 2. Effectiveness of Reinforcing Action of Two Layer and Three Layer Systems in Reducing Vertical Stresses Imposed on the Top of the Subgrade Layer at Interface 1-2 or Interface 2-3, as Indicated by the Values of the Vertical Stress Influence Coefficient, σ_z/p for the $[E_1/E_2 - E_2/E_3 - 1/b - n]$ Values for the Two Layer and Three Layer Systems of Figs. 2A to 9A and Values of r/h_1 of 1.0, 1.25 and 2.0 for Comparative Purposes. Increase in Effectiveness is Indicated by a Decrease in σ_z/p .

r/h_1	Two Layer $h_2 = 0$						Three Layer $h_2 = h_1$						Three Layer $h_2 = 2h_1$							
	400/0/1	1000/0/1	2000/0/1	4000/0/1	20/20/1/2	20/50/1/2	20/100/1/2	20/200/1/2	20/20/2/3	20/50/2/3	20/100/2/3	20/200/2/3	20/20/2/3	20/50/2/3	20/100/2/3	20/200/2/3	20/20/2/3	20/50/2/3	20/100/2/3	20/200/2/3
1.00	.035	.022	.017	.007	.030	.015	.010	.003	.019	.010	.005	.003	.029	.015	.010	.006	.065	.038	.024	.015
1.25	.050	.032	.025	.012	.045	.022	.015	.009	.029	.015	.010	.006	.029	.015	.010	.006	.065	.038	.024	.015
2.00	.112	.073	.062	.027	.100	.055	.037	.023	.065	.038	.024	.015	.065	.038	.024	.015	.065	.038	.024	.015
	500/0/1	1000/0/1	2500/0/1	5000/0/1	50/10/1/2	50/20/1/2	20/20/1/2	20/100/1/2	20/10/2/3	20/20/2/3	50/50/2/3	20/100/2/3	20/10/2/3	20/20/2/3	50/50/2/3	20/100/2/3	20/10/2/3	20/20/2/3	50/50/2/3	20/100/2/3
1.00	.028	.022	.013	.006	.028	.017	.007	.006	.022	.011	.008	.004	.022	.011	.008	.004	.022	.011	.008	.004
1.25	.043	.032	.021	.011	.091	.027	.015	.011	.032	.019	.011	.008	.032	.019	.011	.008	.032	.019	.011	.008
2.00	.102	.073	.051	.025	.095	.066	.038	.025	.074	.049	.030	.032	.074	.049	.030	.032	.074	.049	.030	.032

In the design of layered pavement systems and in evaluations of existing pavements for different loading conditions, equivalence of different layered systems with regard to vertical stress imposed on the top of the subgrade layer become an important consideration. Equivalent two and three layer systems with regard to reinforcing action are indicated by constant σ_z/p values on horizontal lines in Fig. 2A to 9A and by corresponding r/h_1 values and $[E_1/E_2 - E_2/E_3 - 1/b]$ values at the subgrade interface. These equivalents are given in Table 3 for comparative purposes by the different moduli ratios and corresponding r/h_1 value combinations, taking for these comparative purposes the weakest two layer system with $E_1/E_2 = 400$ of Fig. 2A, as the basis. The increase in r/h_1 values means the larger corresponding wheel loads or tire sizes could be supported on the pavement systems for the same magnitude of imposed σ_z/p stress on the top of the subgrade layer.

Table 3. Equivalent Two Layer and Three Layer Pavement Systems for Constant $\sigma_z/p = 0.5$ Values at the Subgrade Interface for the Different Layered Systems of Figs. 2A to 9A, as Indicated by r/h_1 Values.

Two Layer, $h_2 = 0$	400/0/1	1000/0/1	2000/0/1	4000/0/1
	500/0/1		2500/0/1	5000/0/1
r/h_1	1.25	1.6	1.9	2.75
Three Layer, $h_2 = h_1$	20/20/2/3	20/50/1/2	20/100/1/2	20/200/1/2
r/h_1	1.35	1.90	2.35	3.00
	50/10/1/2	50/20/1/2	50/50/1/2	20/100/1/2
r/h_1	1.50	1.75	2.30	2.90
Three Layer, $h_2 = 2h_1$	20/20/2/3	20/50/2/3	20/100/1/2	20/200/2/3
r/h_1	1.60	2.30	3.00	3.80
	50/10/2/3	50/20/2/3	20/20/2/3	50/100/2/3
r/h_1	1.60	2.05	2.70	3.10

The effect on the three layer vertical stress influence coefficients, σ_z/p

of a change in Poisson's ratio from a value of $\mu_3 = 0.2$ for a sand subgrade to a value of $\mu_3 = 0.4$ for a clay subgrade is very small, being negligible for r/h_1 values less than 2.0 and decreasing the vertical stress influence coefficient by about 1.0 to 2.0 percent at r/h_1 equal to 4.0.

The vertical stress influence coefficients, σ_z/p for the given layer thicknesses and $r/h_1 = 1.25$ obtained from the influence curve graph were used to plot the distribution with depth of the vertical stress, σ_z/p in the upper portions of Figs. 2A to 9A through the layered pavement systems for $h_2 = 0$, $h_2 = h_1$, and $h_2 = 2h_1$. Each graph illustrates the characteristic vertical stress distribution produced by the three variations in thickness of base course layer 2 and by the particular moduli ratios of the layers composing the layered system, as given by the pavement designation - $[E_1/E_2 - E_2/E_3 - 1/b]$.

While the general forms of the vertical stress distribution curves are similar, there are significant difference in each graph, which result directly from the changes in thickness of base course layer 2, $h_2 = 0$, $h_2 = h_1$, and $h_2 = 2h_1$. First, there is a decrease in vertical stress imposed at the top of the subgrade layer with increase in thickness of base layer 2 in all stress distribution graph of Figs. 2A to 9A. Second, for base course layer 2 thickness, $h_2 = 0$ concrete reinforcing layer 1 must take up within itself all of the vertical stress load by a reduction from the applied tire pressure, p to the vertical stress value imposed at the top of the subgrade layer, which results in a maximum vertical stress gradient, $-\partial\sigma_z/\partial z$ through strong reinforcing layer 1. This creates an adverse vertical stress condition within reinforcing layer 1, as discussed under the shear stresses imposed in layered pavement systems. With increase in base layer 2 thickness to $h_2 = h_1$ and $h_2 = 2h_1$, the base layer favorably absorbs within itself an increasing proportion of the imposed vertical stress load, thus favorably relieving the stronger concrete reinforcing layer 1 of a considerable portion of its vertical stress load.

By a comparison of Figs. 2A, to 5A having moduli ratios of $[E_1/E_2 - E_2/E_3]$ equal to 20/20, 20/50, 20/100, and 20/200 with Figs. 6A to 9A having moduli ratios of 50/10, 50/20, 50/50, and 50/100 it is clearly seen that the vertical stress reduction in concrete layer 1 is much more favorable for $E_1/E_2 = 20$ than for $E_1/E_2 = 50$, because base course layer 2 absorbs within itself a larger proportion of the vertical stress loading and thus favorable relieves stronger concrete reinforcing layer 1. Furthermore it is clearly evident the $E_1/E_2 = 20$ base course provides much greater support to the concrete layer 1, as evidenced by a comparison vertical stresses at interface 1-2 between concrete layer 1 and base course layer 2 and imposed on the top of base course layer 2. Thus reinforcing layers 1 and 2 in Figs. 2A to 5A act together more effectively in developing the necessary support for aircraft wheel loads and more effective reinforcing action. An important principle of layered system pavement design can be stated, namely- that greater effectiveness of reinforcing action of pavement systems can be achieved by selecting and using smaller E_1/E_2 jumps (20 versus 50) between concrete layer 1 and base course layer 2 and by increasing the thickness of base course layer 2 to meet design requirements with regard to shear stresses imposed in concrete layer 1 and to deflections of the pavement system.

These facts have most important implications. In all Figs. 2A to 9A, the vertical stresses imposed on the subgrade are very low and full subgrade protection is assured. But it follows that, because of the stiffness and strong reinforcing action of concrete layer 1, the subgrade does not and can not develop really effective support for a concrete slab by deflection, when laid directly on the subgrade with $h_2 = 0$. Hence each 25' by 25' slab panel must support a wheel load by "slab action" deflections, and the concrete slab must absorb within itself these imposed vertical stress and shear stress conditions. This means that fully effective dowel reinforcing between concrete slab panels is essential. Figs. 6A to 9A show that a base course of the strength and quality

of $E_1/E_2 = 50$ for this concrete layer 1 with the larger modulus "jump" between concrete layer 1 and base course layer 2 and hence having a relatively lower base course modulus itself, is practically ineffective in providing support to concrete layer 1 within the deflection performance of this three layer pavement system. This is a well-recognized condition in practice. The basic problem of design of such a layered pavement system is either: (1) To use higher strength and quality base course materials with a smaller E_1/E_2 jump from layer 1 to layer 2 and to ensure the desired better reinforcing and deflection performances by higher specifications standards and especially by adequately supervised excellence of construction to meet the higher specification standards; or (2) to use thinner fully reinforced concrete slabs or pre-stressed concrete which can safely deflect sufficiently to engage and to mobilized greater subgrade support within allowable vertical stresses imposed at the top of the subgrade layer and shear stresses imposed in the reinforcing concrete layer 1. In Figs. 2A to 5A, a base course modulus of at least 100,000 to 150,000 psi./(in./in.) would be required to meet these design conditions effectively. Figs. 6A to 9A show that a base course modulus of 60,000 psi./(in./in.) is relatively ineffective in providing support.

Group B, Shear Stresses in Figs. 2B to 9B

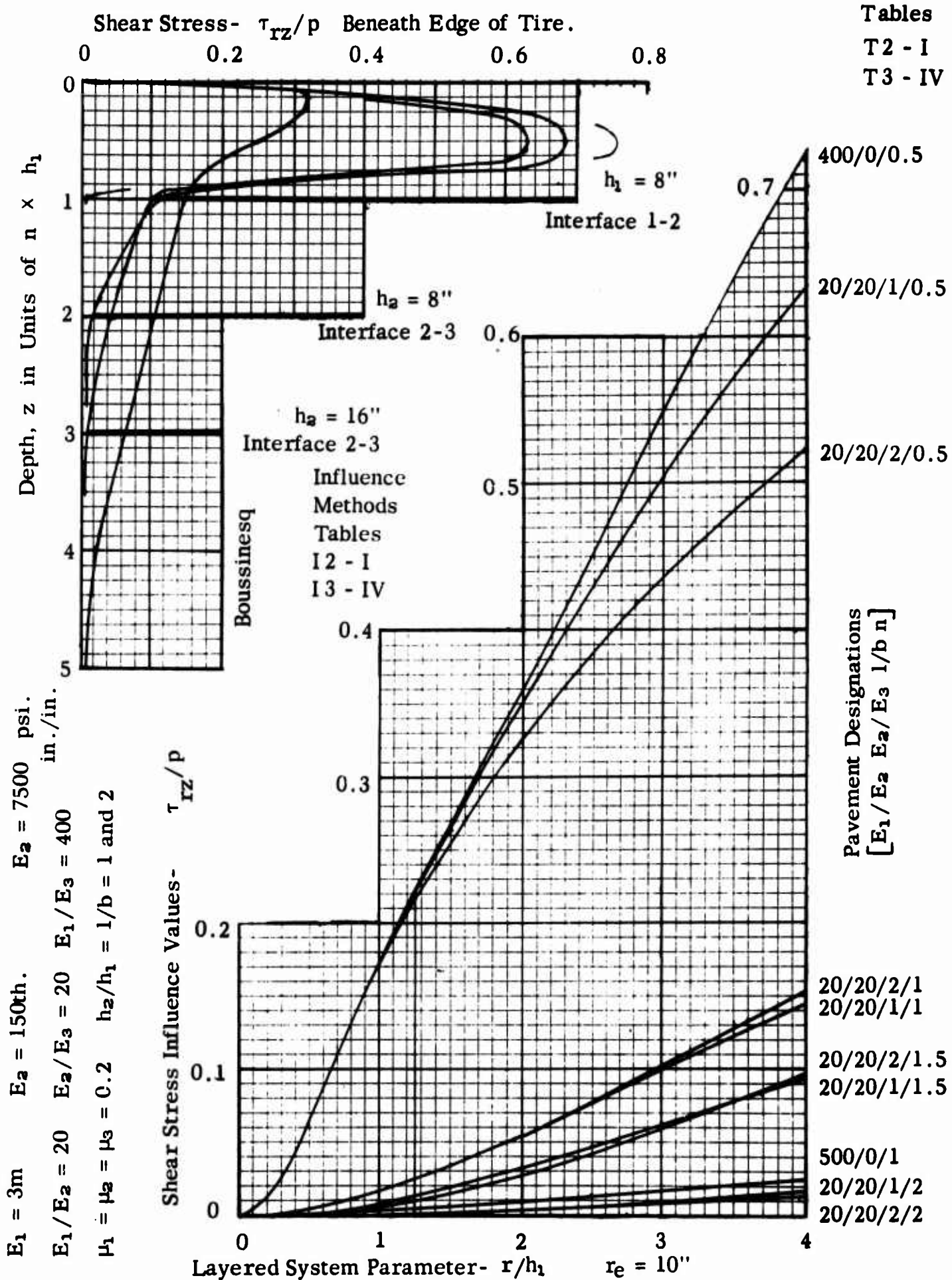
The character, magnitude, and distribution of shear stresses, τ_{rz} imposed by wheel loads in layers 1, 2, and 3 of two and three layer concrete pavement systems are of principal concern, because they make it possible to delineate regions of critical shear stresses at different locations with respect to single and dual wheel and landing gear loadings applied on a pavement surface. The regions of high shear stresses, which exceed some critical values with respect to mobilizable sustained shear strengths, may become the most vulnerable regions of adverse and excessive shear deformations, and hence of final breakdown of a layered pavement structure. It therefore becomes of major importance to delineate and to evaluate such critical shear stress conditions.

In each Fig. 2B to 9B influence curves of τ_{rz}/p are plotted against the layered system parameter, r/h_1 in the lower portion of the figures for the three pavement systems - ($h_1 = 8''$, $1/b = 0$, $h_2 = 0$), ($h_1 = 8''$, $1/b = 1$, $h_2 = 8''$), and ($h_1 = 8''$, $1/b = 2$, $h_2 = 16''$) and at depths $z = nh_1$ in the layers of the pavement systems as follows:

Pavement Designation	E_1/E_3 [$E_1/E_2 - E_2/E_3 - 1/b = 1$]	[$E_1/E_2 - E_2/E_3 - 1/b = 2$]	
Layer 2 Thickness	$h_1 = 0$	$h_2 = h_1$	$h_2 = 2h_1$
Center of Layer 1	$z = 0.5h_1$	$z = 0.5h_1$	$z = 0.5h_1$
Interface (1 - 2)	$z = 1.0h_1$	$z = 1.0h_1$	$z = 1.0h_1$
Center of Layer 2	$z = 1.5h_1$	$z = 1.5h_1$	$z = 2.0h_1$
Interface (2 - 3)	$z = 2.0h_1$	$z = 2.0h_1$	$z = 3.0h_1$
Depths		$z = 3.0h_1$	$z = 4.0h_1$

These depths in the layered system are given in the pavement designations on the right-hand side of the figures of shear stress influence curves in the lower portions of Figs. 2B to 9B in pages 42 to 49.

FIG. 2B DISTRIBUTION OF SHEAR STRESSES IN TWO AND THREE LAYER PAVEMENT SYSTEMS. 8 INCH CONCRETE. 20/20



$E_1 = 3m$ $E_2 = 150th.$ $E_2 = 7500$ psi.
 $E_1/E_2 = 20$ $E_2/E_3 = 20$ $E_1/E_3 = 400$ in./in.
 $\mu_1 = \mu_2 = \mu_3 = 0.2$ $h_2/h_1 = 1/b = 1$ and 2

FIG. 3B DISTRIBUTION OF SHEAR STRESSES IN TWO AND THREE LAYER PAVEMENT SYSTEMS. 8 INCH CONCRETE. 20/50

Tables
T2 - I
T3 - IV

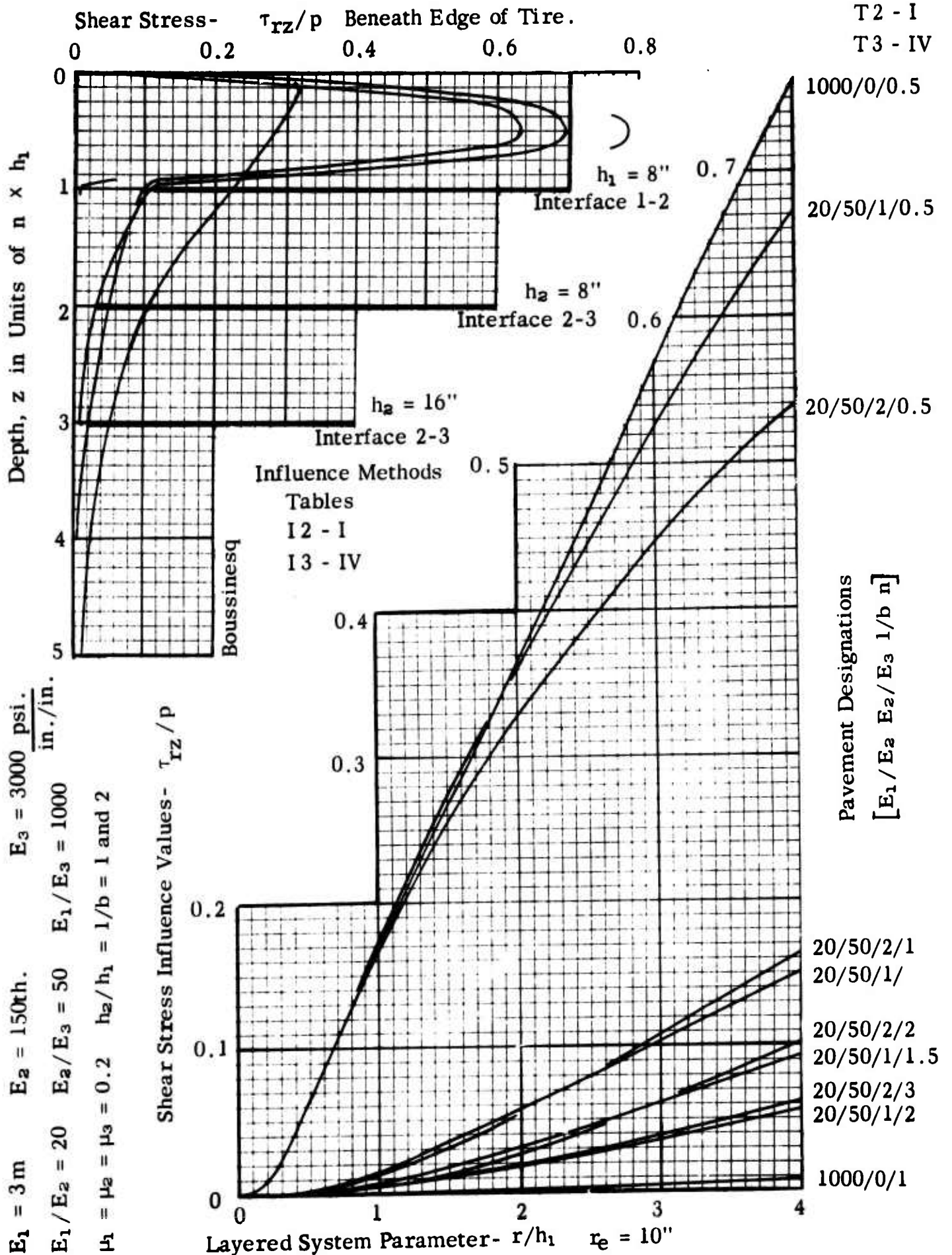
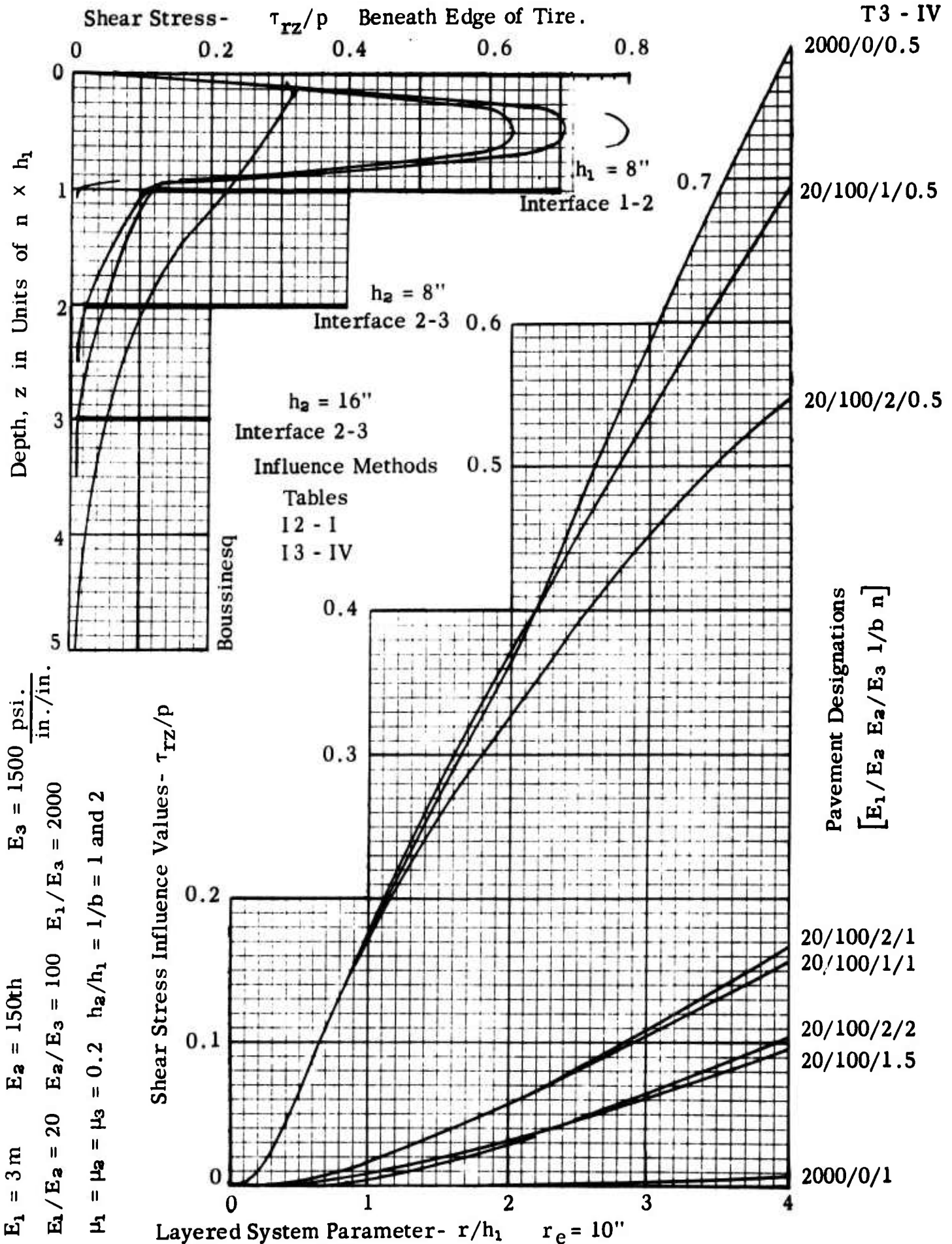


FIG. 4B DISTRIBUTION OF SHEAR STRESSES IN TWO AND THREE LAYER PAVEMENT SYSTEMS. 8 INCH CONCRETE. 20/100

Tables
T2 - I
T3 - IV



$E_1 = 3m$ $E_2 = 150th$ $E_3 = 1500 \frac{psi.}{in./in.}$
 $E_1/E_2 = 20$ $E_2/E_3 = 100$ $E_1/E_3 = 2000$
 $\mu_1 = \mu_2 = \mu_3 = 0.2$ $h_2/h_1 = 1/b = 1$ and 2

FIG. 5B DISTRIBUTION OF SHEAR STRESSES IN TWO AND THREE LAYER PAVEMENT SYSTEMS. 8 INCH CONCRETE. 20/200

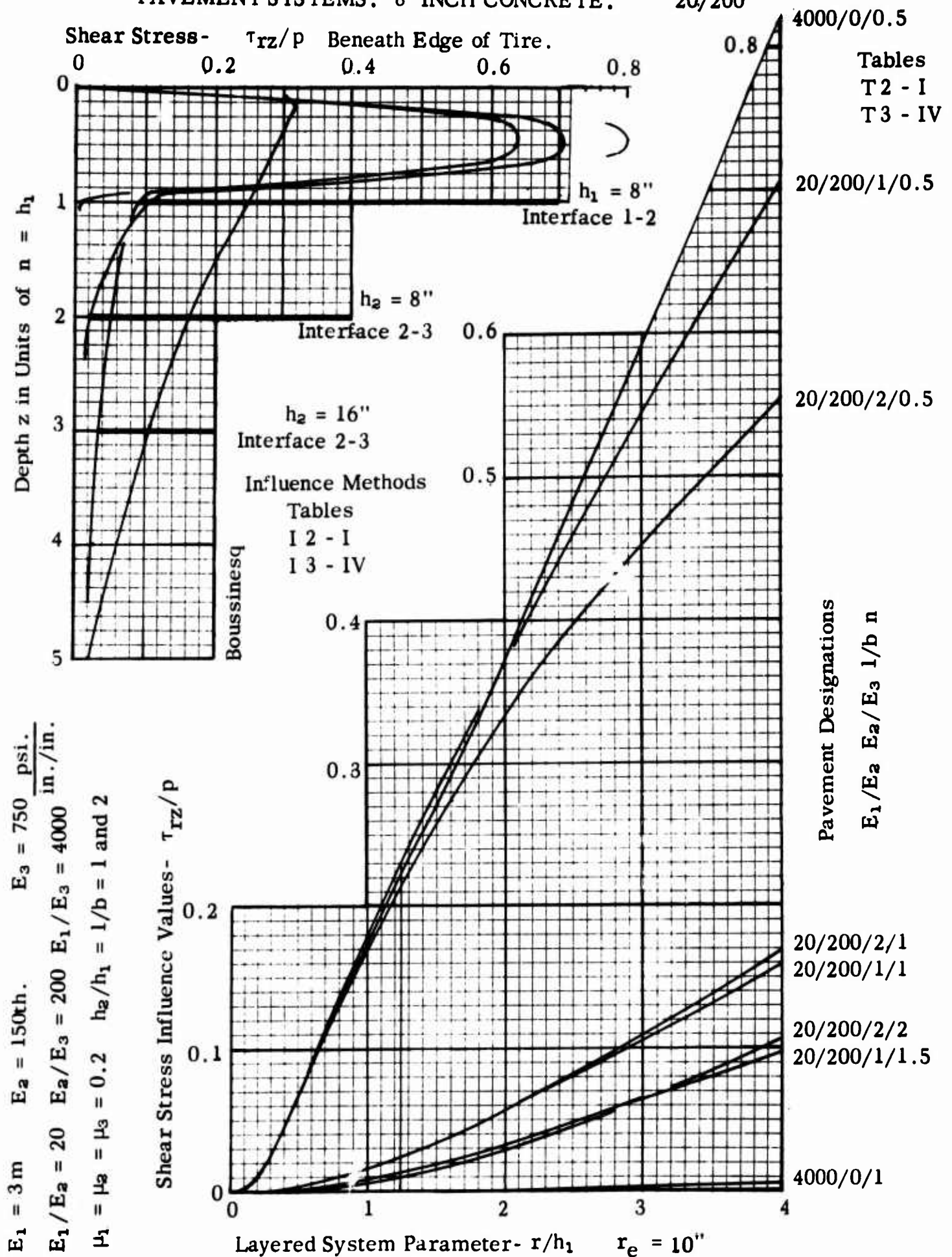


FIG. 6B DISTRIBUTION OF SHEAR STRESSES IN TWO AND THREE LAYER PAVEMENT SYSTEMS. 8 INCH CONCRETE. 50/10

Tables
T2 - I
T3 - V

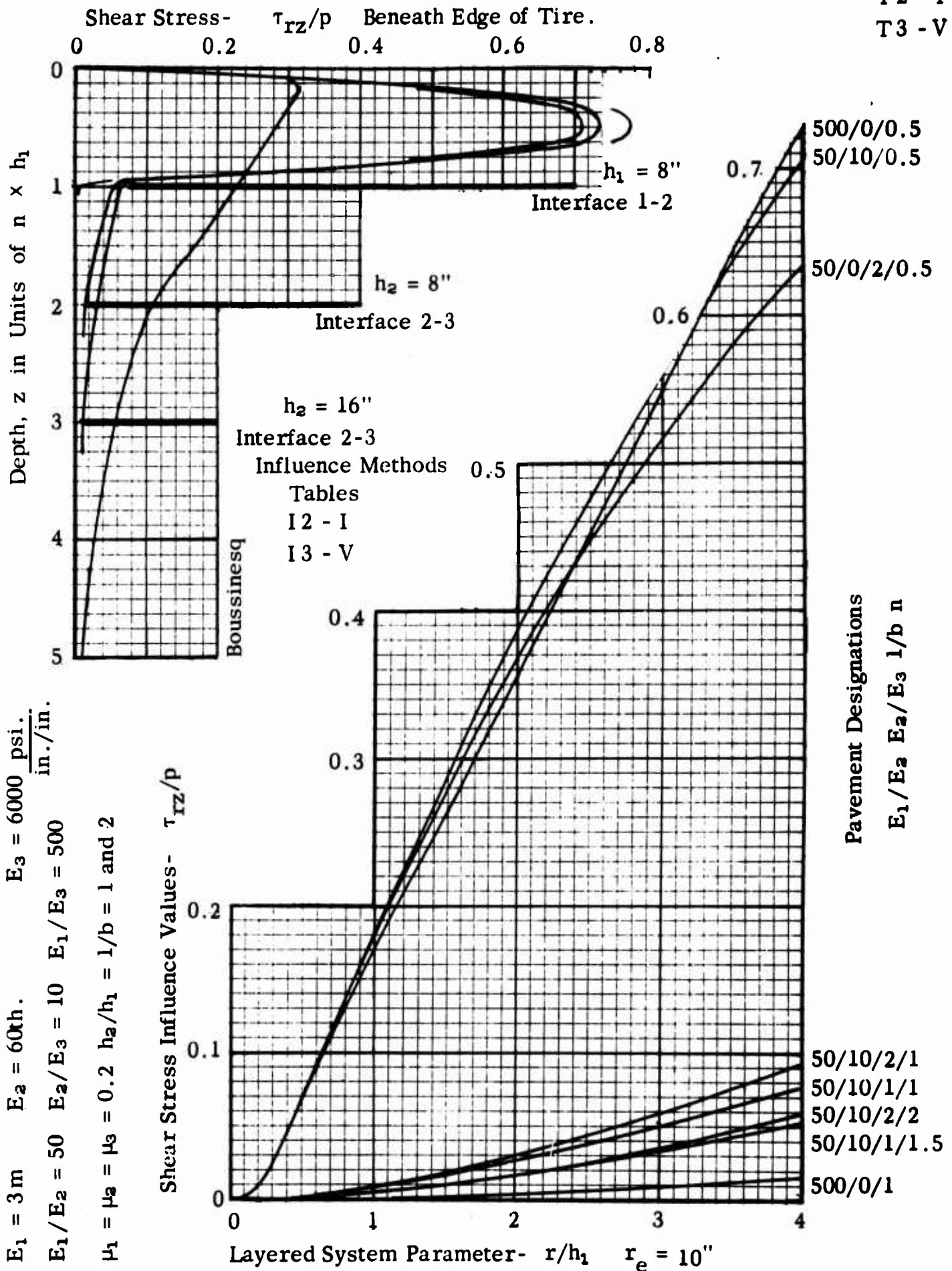


FIG. 7B DISTRIBUTION OF SHEAR STRESSES IN TWO AND THREE LAYER PAVEMENT SYSTEMS. 8 INCH CONCRETE. 50/20

Tables
T2 - I
T3 - V

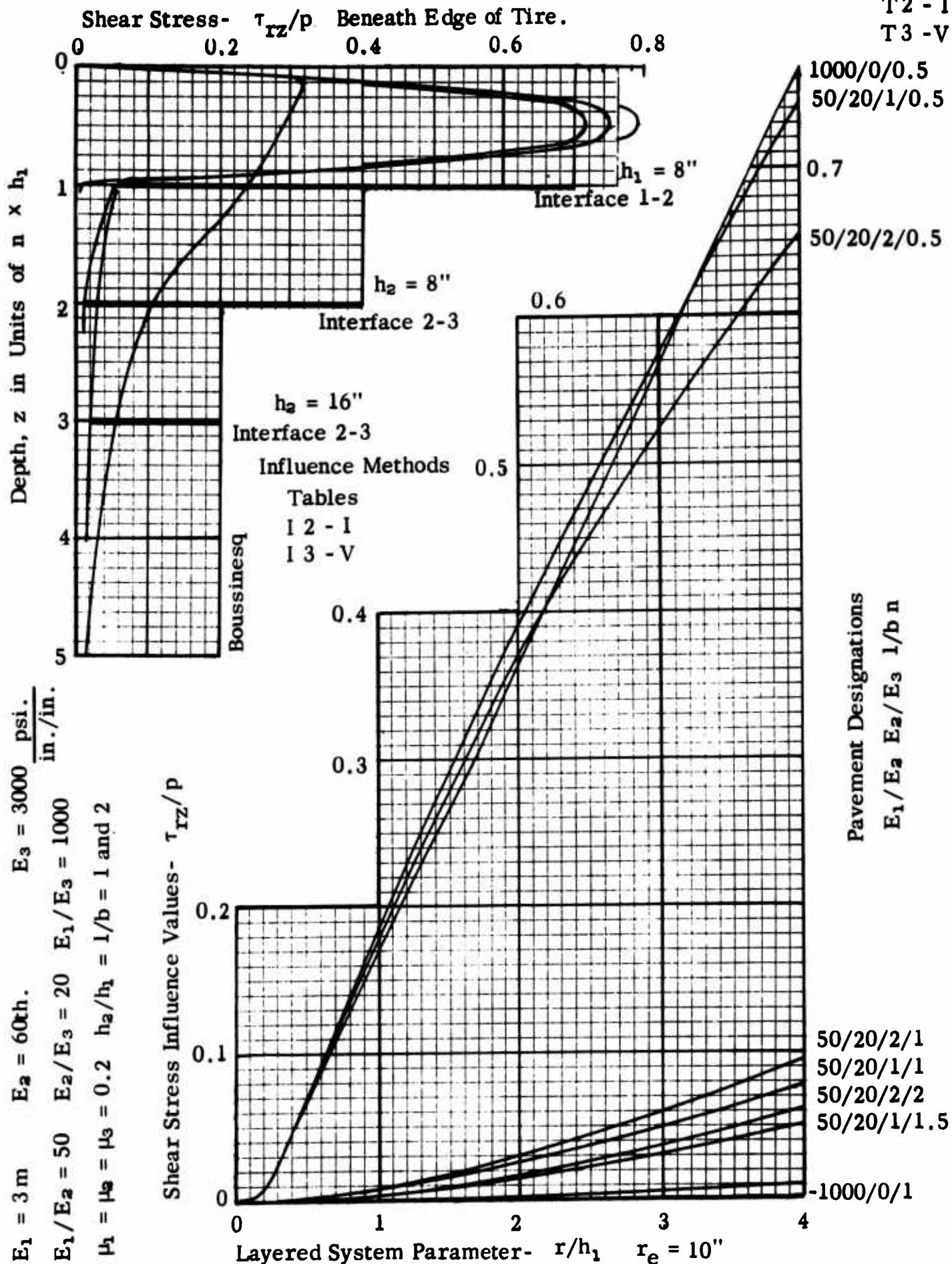


FIG. 8B DISTRIBUTION OF SHEAR STRESSES IN TWO AND THREE LAYER PAVEMENT SYSTEMS. 8 INCH CONCRETE. 50/50

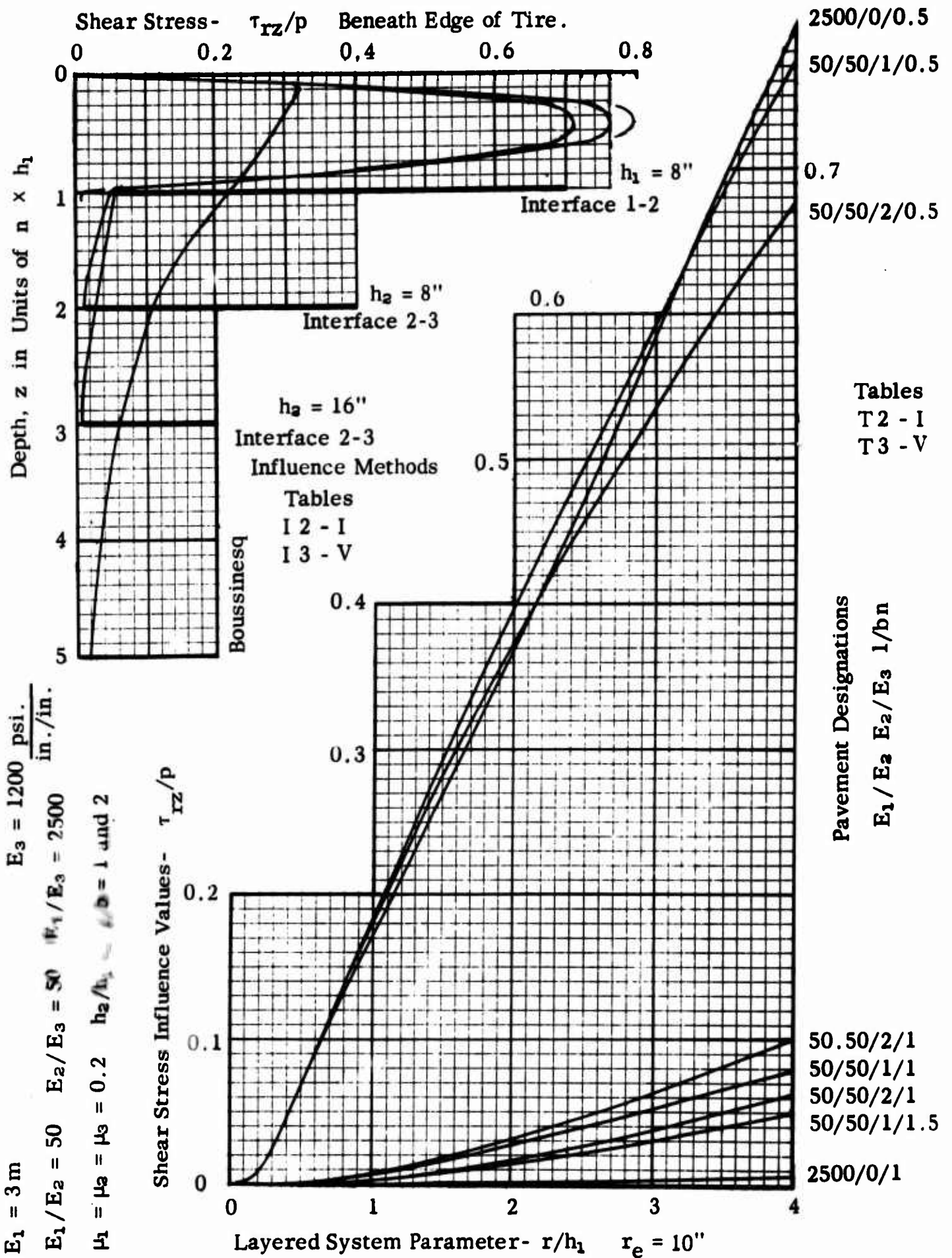
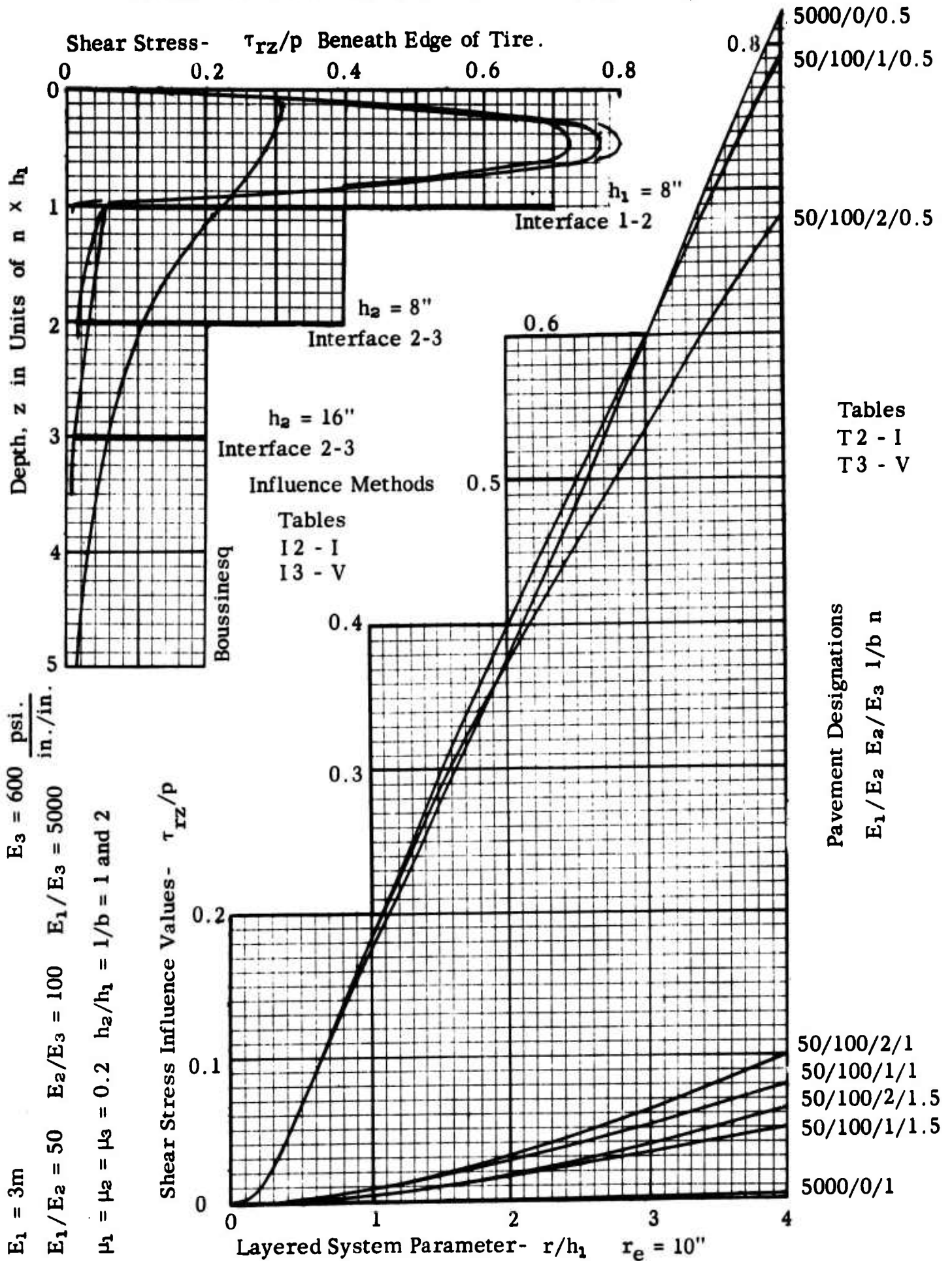


FIG. 9B DISTRIBUTION OF SHEAR STRESSES IN TWO AND THREE LAYER PAVEMENT SYSTEMS. 8 INCH CONCRETE. 50/100



A study and comparison of the shear stress influence curves in lower portions of Figs. 2B to 9B disclose the character of the shear stress transmission characteristics and performances of layered pavement systems and their effectiveness of reinforcing action and load spreading capacity in reducing the magnitudes of shear stresses imposed in the upper portions of the subgrade layer. These figures show that the magnitudes of shear stress influence coefficients, τ_{rz} are governed by the two layer and three layer parameters - E_1/E_2 , E_2/E_3 , z/h_1 and particularly r/h_1 . It is clear from Figs. 2B to 9B for constant values of E_1/E_2 , E_2/E_3 , and $z/h_1 = n$ that effectiveness of reinforcing action decreases markedly with increase in r/h_1 values, either constant h_1 and increasing r or constant r decreasing h_1 . Hence it must be realized that the effectiveness of reinforcing action of a given designed and constructed pavement system is not an inherent constant, but is controlled and affected adversely by increases in radius of bearing area through the governing σ_z/p and r/h_1 relations in Figs. 2B to 9B on pages 42 to 50. A comparison of the characteristic patterns of vertical stress influences curves in the lower portions of Figures 2A to 9A on pages 27 to 34 with the characteristic but different pattern of shear stress influence curves in the lower portions of Figs. 2B to 9B on pages 42 to 50 reveals a marked difference shear stress transmission characteristics through the layers of layered pavement systems. The vertical stress influence curves in Figs. 2A to 9A are rather evenly distributed with regard to depth in a layered pavement system. On the other hand the shear stress influence curves in Figs. 2B to 9B have very high shear stress influence values at the center of reinforcing layer 1, are zero at the surface of layer 1 and characteristically drop markedly to low values at and below interface 1-2. This means that the reinforcing layer 1 affords a very marked shear stress protection to the top of the base course layer and especially to the top of the subgrade layer.

The shear stresses, τ_{rz} at the surface of a pavement system, $z = 0$ must be equal to zero, as a boundary condition. The computed shear stress

influence coefficients at depth of z equal to $0.25h_1$, $0.5h_1$, $0.75h_1$, and $1.0h_1$ show that the maximum shear stress occurs at the center or mid-depth, $z = 0.5h_1$ of reinforcing layer 1. In the subgrade layer 3, the shear stresses decrease to comparatively very small values. It is clear from comparisons of Figs. 2B to 5B having a modulus ratio, $E_1/E_2 = 20$ with Figs. 6B to 9B having a modulus ratio, $E_1/E_2 = 50$ that the maximum shear stresses at the center of layer 1, $z = 0.5h_1$ are less for $E_1/E_2 = 20$ than for $E_1/E_2 = 50$, and especially for $h_2 = 2h_1$ ($1/b = 2$). This is an important and favorable shear stress transmission characteristic of layered systems from the standpoint of design methods for improving critical shear stress conditions and shear deformation performances in layered pavement systems.

It is also evident for both layer moduli ratios, $E_1/E_2 = 20$ and $E_1/E_2 = 50$ that the shear stresses decrease to comparatively low values in base course layer 2. In subgrade layer 3 the shear stresses decrease to low values, which is a favorable shear stress transmission characteristic of layered pavement system, showing the extent of favorable protection of the subgrade layer, which is the weakest layer, against excessive shear stresses imposed by aircraft wheel loads. It is important to note the adverse increase in shear stress influence coefficients in Figs. 2B to 9B at all depths with increase in r/h_1 , either (1) by an increase in radius, r of bearing area for constant layer 1 thickness, h_1 such as may be caused by increase in aircraft loads and hence increase in tire sizes, or (2) by a decrease in layer thickness, h_1 for constant radius of bearing, which is a design condition.

It becomes evident that the critical depth region for shear stresses in a layered pavement is at the center or mid-depth of layer, $z = 0.5h_1$. Therefore, as long as the reinforcing action and competence of strong reinforcing layer 1 are fully active, the shear stresses imposed in the weaker subgrade layer just on the under side of the subgrade layer interface can not become critical unless

these imposed shear stresses exceed the mobilizable, sustained shear strength of the subgrade material. Since these shear stresses decrease with increase in reinforcing action and competence of the reinforcing layer, proper design of a layered pavement system can provide fully adequate shear protection for the subgrade layer.

In the upper graphs of Figs. 2B to 9B the distribution of shear stresses, τ_{rz}/p are plotted through the depth of the layered pavement system for $h_2 = 0$, $h_2 = h_1$ and $h_2 = 2h_1$, which illustrate the characteristic shear stress transmission through the reinforcing layers and upper part of the subgrade layer. The curves of distribution of shear stresses imposed in the layered pavement system can not be obtained directly from the shear stress influence curves at the bottom of the figures as done for the vertical stresses in Figs. 2A to 9A beneath the center line of the loaded area at the loaded area at the surface of the pavement. The vertical stresses in Equation 6, page 9, are directly additive with respect to the horizontal angle, θ , and the vertical stress influence curves in the lower graphs of Figs. 2A to 9A can be used directly to determine the magnitude of the vertical stresses, σ_z/p at different depths by taking the influence values at r/h_1 equal to 1.25 for this case from the appropriate influence curve of depth designated by $[l/b, n]$. These vertical stress distribution curves are plotted in the upper graphs of Figs. 2A to 9A. The shear stresses, τ_{rz}/p , on the other hand, are vector quantities through the term- $[\sin \theta]$ in Equation 7. Hence the shear stresses, τ_{rz}/p directly beneath the center line of the loaded area is zero at all depths, z . Special stress influence methods of analysis are required for determining the location of maximum shear stresses with respect to a single equivalent circular tire footprint. These special shear stress influence methods are developed and illustrated in the Report of Part III. By trial methods of tire locations with respect to the point at and below which the shear stresses are to be computed, for example- at distances from the center of the tire to the shear stress point of: 0, $r/4$, $r/2$, $3r/4$, $r = 1$ at tire edge, $5r/4$, $6r/4$, etc., it was

determined that the maximum shear stress, $\tau_{rz}/p(\max)$ always occurred for a single tire beneath the edge of the tire. The distribution of the shear stresses was then determined at this location at depths in the layered pavement system of $z = h_1/4, h_1/2, 3h_1/4$, and thereafter at the center of layer 2, and at the interface 2-3 with subgrade, etc. The Boussinesq shear stress distribution in a homogeneous earth mass was computed and is given for comparison.

The distribution of shear stresses, τ_{rz}/p with depth beneath the edge of the equivalent circular tire loading in the layered pavement systems are given in the upper diagrams of Figs. 2B to 9B. It becomes immediately evident that the maximum shear stress is imposed at the center of the strong concrete reinforcing layer 1, beneath the edge of the tire loading. Therefore the most critical shear stress zone in layered pavement systems for a single tire loading becomes known. This maximum shear stress is much higher than the Boussinesq value for a uniform soil deposit, being more than twice as great for the moduli ratios of these layered pavement systems of Figs. 2B to 9B.

The shear stress distributions in layered systems are characteristic of the reinforcing action and stiffness of strong reinforcing layer 1. For a concrete pavement laid directly on the subgrade, the concrete layer 1 takes practically all of the shear load within layer 1, and an insignificant shear stress, τ_{rz}/p is imposed on the weaker subgrade layer. For layer moduli ratios - $[E_1/E_2 - E_2/E_3]$ of 20/20 to 20/200 in Figs. 2B to 5B base course layer 2 characteristically takes more shear stress load and provides more relief of high shear stress loads in layer 1, than for the layer moduli ratios 50/10 to 50/100 in Figs. 6B to 9B. It is clearly evident that a stronger base course with a smaller E_1/E_2 to E_2/E_3 jump is considerably more competent and effective in taking a larger share of the shear load and thereby in reducing the maximum shear stress imposed at the center of reinforcing layer 1. It is also clearly evident that the thicker base course layer (twice as thick) considerably relieves

and reduces the high shear stress intensity imposed at the center of reinforcing layer 1. Thus important design considerations are made clear.

A most significant aspect of the layered system vertical stress, σ_z/p distributions in the upper diagram in Figs. 2A to 9A is the marked vertical stress reduction and hence negative stress gradient, $-\partial\sigma_z/\partial z$ through reinforcing layer 1, which is much greater than for the Boussinesq stress. The negative vertical stress gradient in layer 1 increases principally with E_1/E_2 and decreases with increase in thickness of layer 2, but is only slightly affected by the moduli ratio, E_2/E_3 , as is evident in Figs. 2A to 9A. The only mechanism by which such a high negative vertical stress gradient can exist and can be maintained in two and three layer systems is by the presence of an equally high positive shear stress gradient through reinforcing layer 1 in accordance with the well-known stress equilibrium condition of the theory of elasticity, expressed as stress gradients-

$$\frac{\partial\sigma_z}{\partial z} + \frac{\partial\tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} = 0 \quad (30)$$

As a consequence of the increase in reinforcing action and stiffness of reinforcing layer 1 with increase in the moduli ratio, E_1/E_2 in a layered system, the shear stresses accordingly must build up in the reinforcing layer 1 and hence must become more critical. The character and critical nature of this shear stress build-up and distribution in reinforcing layer 1 are illustrated by comparing Figs. 2B to 5B with moduli ratios- $[E_1/E_2 - E_2/E_3]$ of 20/20 to 20/200 with Figs. 6B to 9B with moduli ratios - 50/10 to 50/100, where the maximum shear stress occurs at the mid-depth, or center of reinforcing layer 1 beneath the edge of a single tire loading. It is to be noted especially that there is a significant decrease in this maximum shear stress imposed in reinforcing layer 1, as a result of increase in thickness of base course layer 2 and that the effectiveness and competence of base course layer 2 to take a consider-

able part of the shear stress load increases with smaller E_1/E_2 jumps between reinforcing layer 1 and base course layer 2 for E_1/E_2 equal to 20 in contrast to 50. These relationships become important design considerations in improving the shear stress competence and performances of layered pavement systems.

It is evident in Fig. 2B to 9B for all the cases given that the magnitude of the shear stresses, τ_{rz}/p imposed on the subgrade layer are very low indeed in comparison with the Boussinesq shear stresses for a concrete reinforcing layer 1 and that they can not become critical for the subgrade layer unless the subgrade modulus itself is extremely low and less than 600 psi./in. per in.). It is also clear that the subgrade is fully protected in Fig. 2A to 9A and Figs. 2B to 9B is fully protected against both high vertical stresses and high shear stresses, as long as the reinforcing action, load-spreading capacity, and stiffness of reinforcing layers 1 and 2 continue to be fully effective and competent. Only a serious deterioration and break-down of reinforcing layers 1 and 2, particularly strong layer 1 could alter this favorable situation adversely, in which case the moduli, E_1 and E_2 of the concrete and base course layer 1 and 2 would markedly decrease in magnitude with respect to the subgrade modulus, and hence the shear stresses would decrease in reinforcing layers 1 and 2 and very adversely increase in the subgrade layer toward the Boussinesq curve values in the upper diagrams of Figs. 2B to 9B.

In the lower diagrams of shear stress influence curves in Figs. 2B to 9B imposed shear stresses, τ_{rz}/p are markedly a function of the two and three layer parameter r/h_1 . Shear stresses increase markedly and even adversely, either with increase in effective radius of tire foot-print area for a constant thickness of concrete reinforcing layer 1, which can occur with change in airport runway usage toward higher aircraft loads, larger tire sizes, and increase in tire pressures, or with decrease in thickness of reinforcing concrete layer 1 for a constant radius of tire area, which is a design consideration. It is seen that

the shear stress influence curves at mid-depth or center of concrete reinforcing layer 1 are bunched closely together and yield high shear stress values for the two and three layer systems of Figs. 2B to 9B. It is also seen in quite marked contrast to the vertical stress influence curves of Figs. 2A to 9A that the shear stress influence curves are bunched closely together at the bottom of the shear stress influence diagrams and yield comparatively very low shear stress values at interfaces 1-2 with $n = 1$ ($z = nh_1$), and at the subgrade interfaces 2-3 for two and three layer systems with $n = 1, 2,$ and 3 for $h_2/h_1 = 0, 1,$ and $2,$ and $z = h_1, 2h_1$ and $3h_1,$ respectively. This is evidence of the shear stress protection afforded by the strong reinforcing action and stiffness of concrete layer 1 and of the contributions of base course layer 2 for its given variations in thickness. It is evident, however, that the subgrade protection decrease somewhat with increase in effective tire radius for the same thicknesses of concrete reinforcing layer 1 and of base course layer 2, and decreases with decrease in thicknesses of layers 1 and 2 for a constant effective tire radius.

The governing concepts and principles of C 1 through C 7, given in pages 8 and 9 of the January 1965 Technical Report No. 1, Part I, apply here in evaluating and judging the conditions that control here. In addition, Concepts C 8 and C 9 may be stated here.

C 8- As a consequence of the vertical stress and associated shear stress gradients of Eq. 30, the critical and adverse character of the shear stresses imposed at the mid-depth or center of strong reinforcing layer 1 beneath the edge of tire foot-print area and of the ratio, τ_{rz}/σ_z increases with increase in E_1/E_2 and $r/h_1,$ either increase in radius for constant thickness of layer 1, $h_1,$ or with decrease in thickness, h_1 for constant radius, $r;$ but the critical character of the shear stresses decreases favorably with smaller E_1/E_2 jumps between reinforcing layer 1 and base course layer 2 and with increase in thickness of base course layer 2.

C 9- The basic shear stress-strain and vertical stress-strain relation of Eq. 31, govern shear and vertical performances of layered pavement systems, as follows.

$$\tau_{rz} = [\partial w / \partial r + \partial u / \partial z] \times E / (1 + \mu)$$

$$E \partial w / \partial z = [\sigma_z - \mu \sigma_r - \mu \sigma_\theta] \quad (31)$$

These equations show clearly the maximum shear stresses and vertical stresses imposed at the critical mid-depth region of strong reinforcing layer 1 beneath the edge of a tire foot-print area are essentially deflection-dependent.

C 10- Shear stresses can not be imposed in reinforcing layer 1 without first of all having appreciable shear deformations in layer 1 caused by the vertical deflection of the layered system under wheel loads, the quantity, $\partial w / \partial r$ really representing the deflection curvature of the layered pavement system under the wheel load.

These associated shear stress, vertical stress, and deflection relations in layered systems through Concepts C 1 to C 10, give special point to the necessity of incorporating high prestress, mechanical bonding, and shear strength continuity throughout a layered pavement system by the conditioning and prestressing influences of systematic heavy rolling. Such reinforcing base course, subbase, and subgrade layers are much superior in their deflection performances and supporting values, than would be the case if densified by vibration methods alone to the same required relative densities, but without any prestressing and keying action by heavy rolling. But shear strength on the critical mid-depth plane of reinforcing layer 1 can not be mobilized without first of having some slight shear deformations in reinforcing layer 1 caused by deflections of a layered system under wheel loads.

For granular base course and subbase materials of low coherence, the potential horizontal shear strength, $S(\max)$ mobilizable by deflection on horizontal planes at the critical mid-depth of reinforcing layer 1 is given approximately by

the following equation:

$$S(\text{max.}) = [\sigma_z + h_1 \gamma/2 + p_N] \tan \phi / F.S. > \tau_{zx} \quad (32)$$

where $h_1 \gamma/2$ is the half thickness of layer 1 times its unit weight above the critical mid-depth; σ_z is the vertical stress imposed on the plane beneath the edge of the loaded tire foot-print; p_N is the effective influences of the prestress, keying action, and shear strength continuity on this plane; τ_{zx} is the maximum shear stress imposed on this mid-depth plane beneath the edge of the loaded tire foot-print; and F.S. is a suitable factor of safety to ensure long life against a shear deformation breakdown of reinforcing layer 1 under repeated loads. Base course and subbase materials of high quality and maximum compaction should possess a sustained angle of friction, ϕ of 45° or greater.

The use of multi-layer pavement systems, increase in thickness of the effective combined reinforcing layers, and of smaller jumps in layer moduli ratios- E_1/E_2 , E_2/E_3 , etc., between layers provide the most effective and practical methods for controlling and limiting surface deflections, and hence for reducing shear stresses imposed in the critical shear deformation breakdown values under the action of repeated wheel loadings of traffic and service conditions. It is evident that the competence of layered pavement systems depends upon the deflection performances and reinforcing action of reinforcing layer 1, which becomes the criterion for adequacy of pavement system design.

A comparative study of the shear stress magnitudes and distribution in reinforcing layers 1 and 2 and the respective shear loads in these layers brings out clearly the greater effectiveness of overall shear performances of the layered pavement systems of Figs. 2B to 5B than Fig. 6B to 9B. For the same tire loading conditions, the maximum shear stress in the critical mid-depth region of reinforcing layer 1 and the shear load are favorably lower for the pavement systems with lower moduli ratios, E_1/E_2 in Figs. 2B to 5B than for the pave-

ment systems with higher moduli ratios, E_1/E_2 in Figs. 6B to 9B. Also, the shear load capacity is accordingly higher and more favorable in base course layer 2 in Figs. 2B to 5B than in Figs. 6B to 9B. Furthermore, it is evident that doubling the thickness of base course layer 2 in Figs. 6B to 9B does not increase the shear stress effectiveness equal to that of Figs. 2B to 5B, respectively. Thus important layered pavement system design considerations are established.

The basic design problem now is to establish by condition surveys and reevaluations of existing layered pavement system, having known traffic conditions (wheel load repetitions), specifically with regard to their present deflection performance in order to establish by a properly conducted series of load tests their layer moduli ratios and the moduli of the individual layers of the pavement systems. By layered system methods of analysis and evaluation, as outlined in Part III of Technical Reports, the necessary shear strength criteria could then be established for the design of layered pavement systems for different aircraft wheel loadings, volume of traffic and number of repetitions aircraft loadings, and of the anticipated life of airport layered pavement systems.

If an actual breakdown of the reinforcing action of layer 1 starts as a result of excessive shear deformations, then the layer 1 modulus, E_1 must decrease and the modulus ratio, E_1/E_2 must also decrease. Then the deflection of reinforcing layer 1 and base course layer 2 must increase accordingly, thus increasing shear deformations in the weakened layered system. More shear load is transferred to the base course layer 2 and subgrade layer 3 and the shear stresses must increase in these layers accordingly toward the higher Boussinesq values. The final phase of the breakdown in this new deeper critical region is due to excessive lateral plastic displacements in the subgrade layer 3, which results in the final local failure and destruction of a pavement system after the effectiveness of reinforcing action and E_1/E_2 values have been destroyed.

Thus it becomes clearly evident from a comparative study of the shear stress magnitudes and distributions of Figs. 2B to 5B with Figs. 6B to 9B that for the same tire loading conditions the overall shear performances: (1) of lower shear load carried by reinforcing layer 1; (2) lower maximum shear stresses in the critical mid-depth region of layer 1; and (3) of higher shear load capacity of base course layer 2 for the moduli ratios of Figs. 2B to 5B are considerably more favorable than for the moduli ratios of Figs. 6B to 9B, which result in a higher shear load, higher maximum shear stresses in the critical mid-depth region of layer 1 and the relative ineffectiveness of base course layer 2 to take shear load. Thus important layered pavement system design considerations are established.

The basic design problem now is to establish by comprehensive condition surveys and reevaluations of present deflection performances of existing layered systems specifically with regard to their deflection and shear stress performances in order to determine their present layer moduli ratios and the individual layer moduli from a program of load tests, which would effectively establish the necessary shear strength criteria in relation to design maximum aircraft loadings on airport pavements.

Group C. Pavement Deflections in Figs. 2C to 9C.

It is evident from the figures and discussions for vertical stresses and shear stresses imposed by aircraft wheel loads in layered pavement systems that definite information on the stress transmission characteristics, stress performances, and magnitudes of vertical and shear stresses induced in critical regions of layered pavement systems are indispensable for adequate design, in order to ensure satisfactory performances and long life. The vertical stress and shear stress transmission characteristics and stress performances of layered pavement systems: (1) are predetermined by the layer moduli ratios and layer thickness ratios; (2) are adversely affected by increase in radius of bearing area in service as a result of increase in size and loads of aircrafts through the governing influences of the basic layered system parameter, r/h_1 (r/bh_2); (3) are definitely interrelated by Eq. 30, page 54; and (4) are definitely deflection-dependent through Eqs. 3, page 57.

The deflection performances of the two and three layer pavement systems, paralleling the vertical stress and shear stress performances of Figs. 2A to 9A and Figs. 2B to 9B, respectively, are given in Figs. 2C to 9C. In each figure, a series of deflection influence curves of the surface deflections ($Z = 0$ at top of layer 1) at the center of a uniformly loaded equivalent circular area are given, plotting the surface deflection coefficient- $F_w = w_c E_2 / (C p r) = f [h_1/r, h_1/h_2, E_1/E_2, E_2/E_3, \mu_1, \mu_2, \mu_3)$ against the basic layered system parameter, h_1/r or bh_2/r . These deflection influence curves form systematic and characteristic patterns with regard to forms, curvatures, relative spacings, and regions of greatest influences on layered pavement deflection performances. The light line deflection curves cover a range of two layer moduli ratios, E_1/E_3 for a concrete reinforcing layer 1 laid directly on subgrade layer 3 (base course layer- $h_2 = 0$) and they form the fundamental bases for comparison of the effectiveness of two layer and three layer pavement systems with regard to deflection performances, where major consideration should now be given to control of shear stress performances.

Also it is most important to note that the three-layer deflection curves conform systematically with the two-layer deflection curves. This conformity, which should be expected from the nature of the deflection influence coefficient, F_w , as it approaches the limiting values for $h_2 = 0$ or $h_1 = 0$, or $E_1/E_2 = 1.0$. This conformity of two-layer and three-layer deflection curves has most important implications with regard to pavement performance evaluations and equivalences of two and three layer systems with identical deflection influence coefficients, but different layer moduli ratios and h_1/r values. These layered system equivalences form the bases for pavement evaluation and design studies and investigations. In each Fig. 2C to 5C and Fig. 6C to 9C are given two heavy line deflection curves for the respective three-layer pavement systems, as follows, in accordance with the pavement system designations of Table 1(b) on pages 24 and 25.

<u>Figures</u>	<u>Layered Pavement System Designations</u>		
	<u>Two Layer</u>	<u>Three Layer</u>	
2C	400/0	20/20/1	20/20/2
3C	1000/0	20/50/1	20/50/2
4C	2000/0	20/100/1	20/100/2
5C	4000/0	20/200/1	20/200/2
6C	500/0	50/10/1	50/10/2
7C	1000/0	50/20/1	50/20/2
8C	2500/0	50/50/1	50/50/2
9C	5000/0	50/100/1	50/100/2

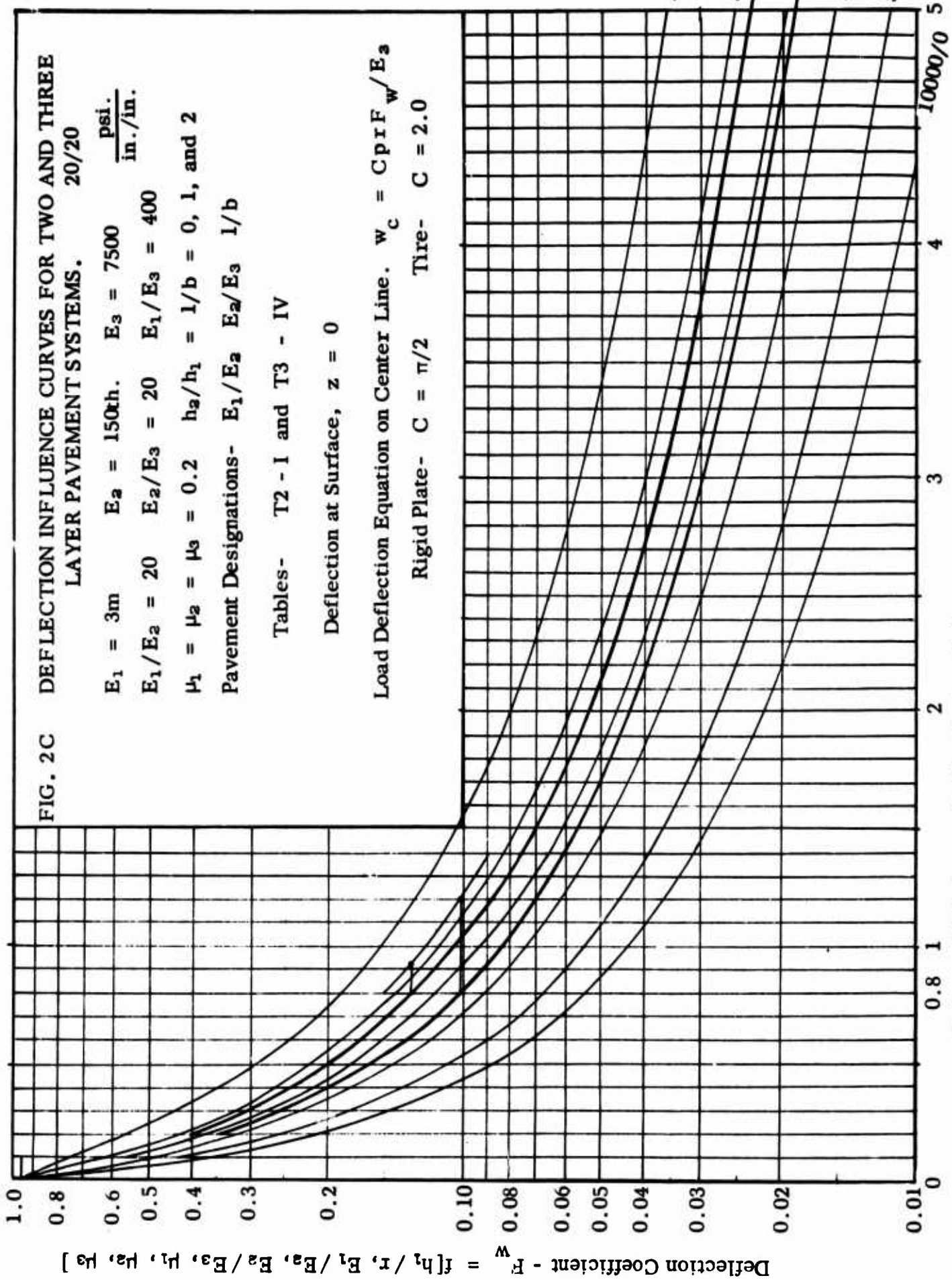


FIG. 2C DEFLECTION INFLUENCE CURVES FOR TWO AND THREE LAYER PAVEMENT SYSTEMS. 20/20

$E_1 = 3m$ $E_2 = 150th$ $E_3 = 7500 \frac{\text{psi.}}{\text{in./in.}}$
 $E_1/E_2 = 20$ $E_2/E_3 = 20$ $E_1/E_3 = 400$
 $\mu_1 = \mu_2 = \mu_3 = 0.2$ $h_a/h_1 = l/b = 0, 1, \text{ and } 2$
 Pavement Designations - E_1/E_2 E_2/E_3 l/b

Tables - T2 - I and T3 - IV

Deflection at Surface, $z = 0$

Load Deflection Equation on Center Line. $w_c = CprF_w/E_3$

Rigid Plate - $C = \pi/2$ Tire - $C = 2.0$

Pavement Designations
 $[E_1/E_2 - E_2/E_3 - l/b]$

$h_1 = 8''$ Concrete
 $h_2 = 8'' \text{ \& } 16''$ B. E.
 $r_e = 10''$ $h_1/r_e = 0.8$

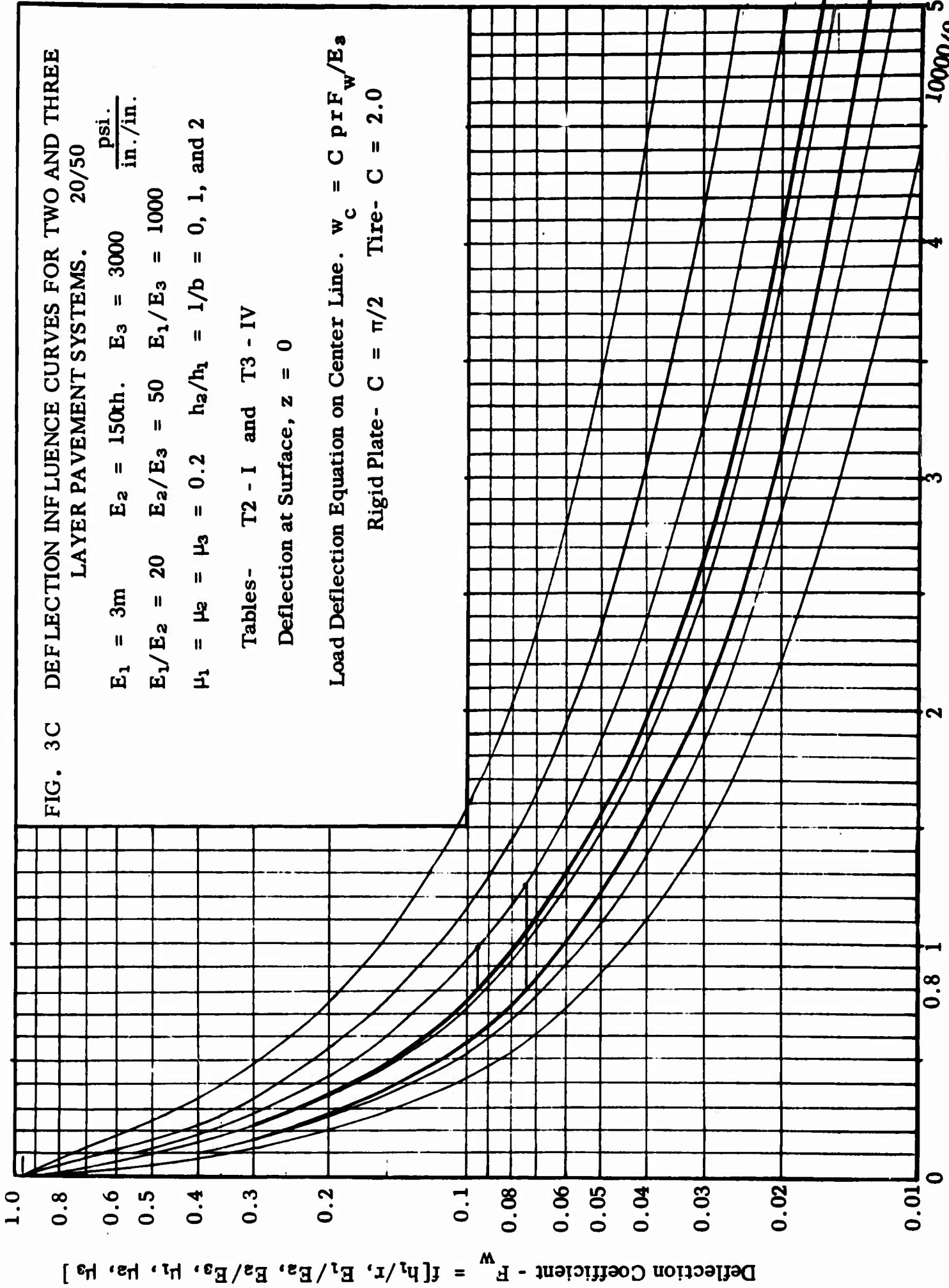


FIG. 3C DEFLECTION INFLUENCE CURVES FOR TWO AND THREE LAYER PAVEMENT SYSTEMS. 20/50

$E_1 = 3m$ $E_2 = 150th.$ $E_3 = 3000$ $\frac{\text{psi.}}{\text{in./in.}}$
 $E_1/E_2 = 20$ $E_2/E_3 = 50$ $E_1/E_3 = 1000$
 $\mu_1 = \mu_2 = \mu_3 = 0.2$ $h_2/h_1 = 1/b = 0, 1, \text{ and } 2$

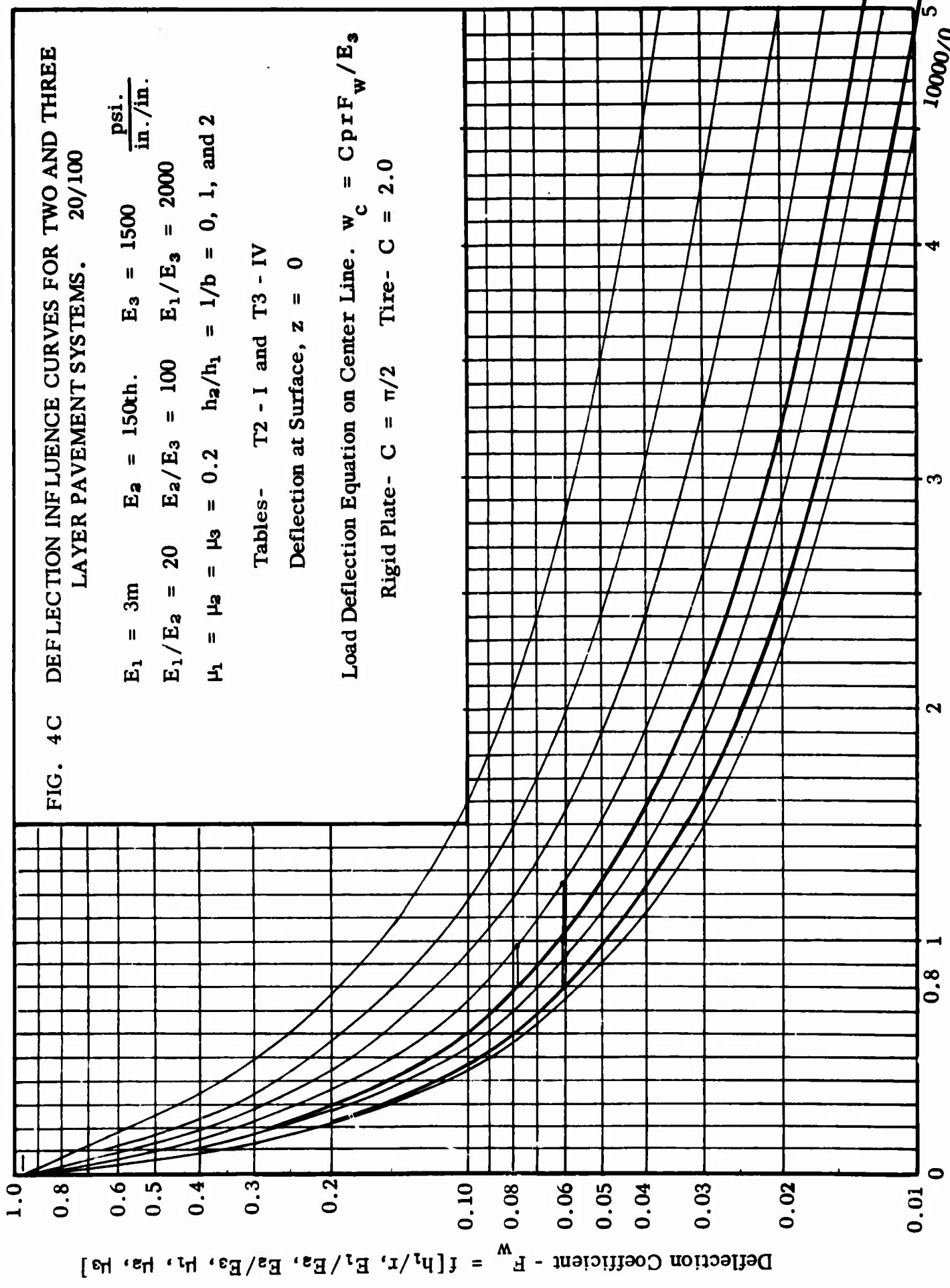
Tables- T2 - I and T3 - IV

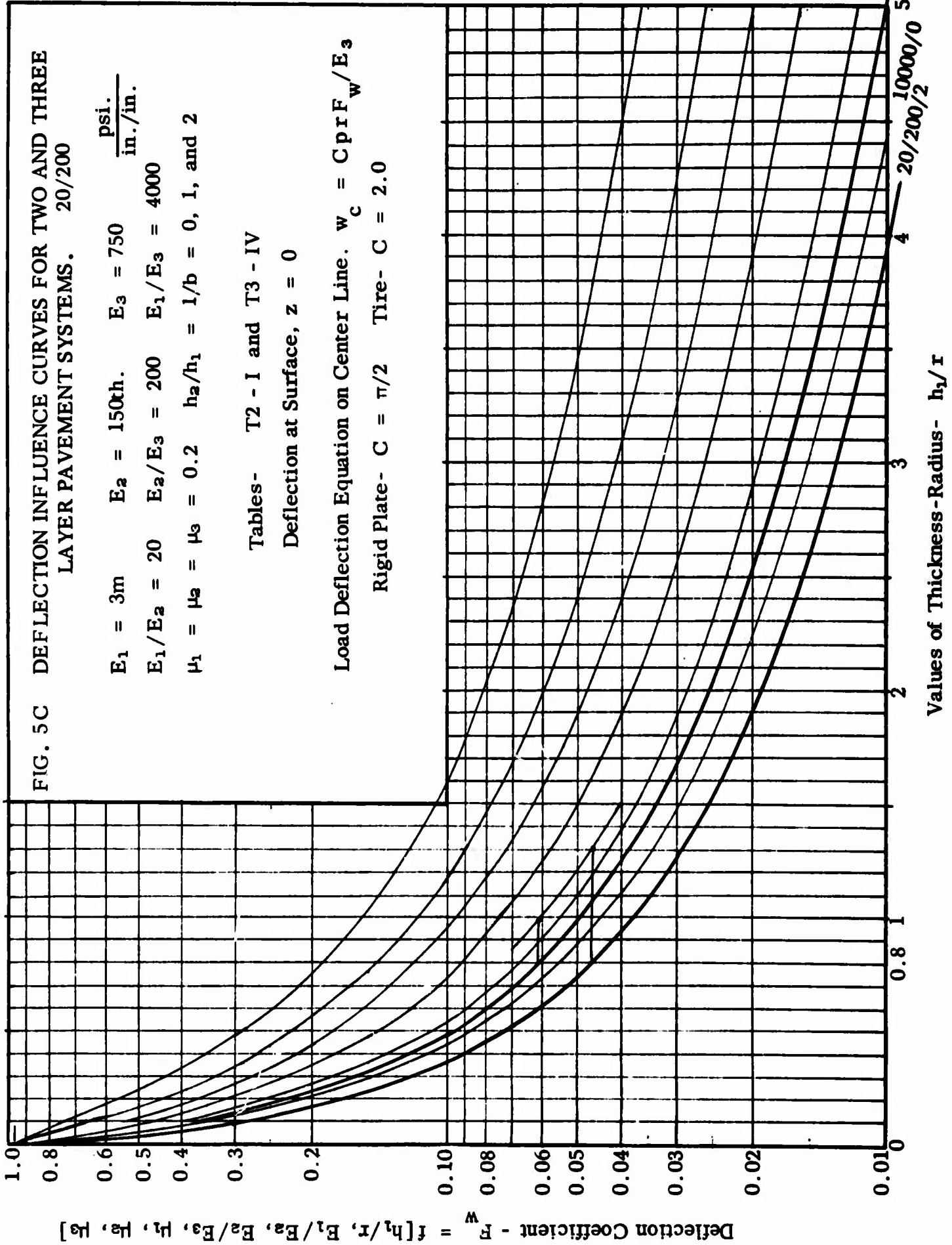
Deflection at Surface, $z = 0$

Load Deflection Equation on Center Line. $w_c = C p r F_w / E_3$

Rigid Plate- $C = \pi/2$ Tire- $C = 2.0$

$h_1 = 8''$ Concrete
 $h_2 = 8'' \text{ \& } 16''$ Base
 $[E_1/E_2 - E_2/E_3 - 1/b]$
 $r_e = 10''$ $h_1/r_e = 0.8$





Pavement Designations
 $[E_1/E_2 - E_2/E_3 - 1/b]$

$h_1 = 8''$ Concrete
 $h_2 = 8''$ & 16'' Base
 $r_e = 10''$ $h_1/r_e = 0.8$

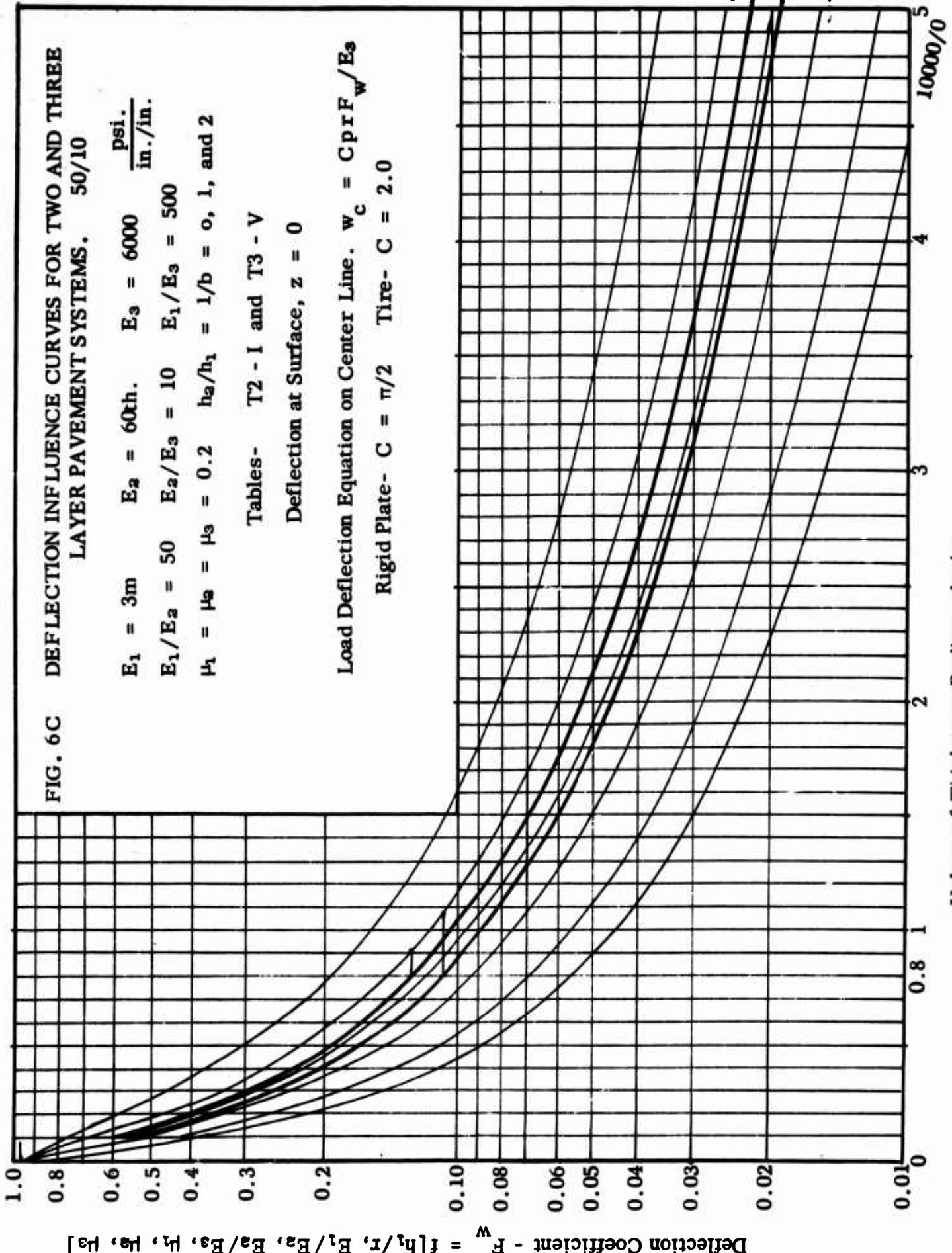


FIG. 6C DEFLECTION INFLUENCE CURVES FOR TWO AND THREE LAYER PAVEMENT SYSTEMS. 50/10

$E_1 = 3m$ $E_2 = 60th.$ $E_3 = 6000$ $\frac{\text{psi.}}{\text{in./in.}}$
 $E_1/E_2 = 50$ $E_2/E_3 = 10$ $E_1/E_3 = 500$
 $\mu_1 = \mu_2 = \mu_3 = 0.2$ $h_2/h_1 = 1/b = 0, 1, \text{ and } 2$

Tables - T2 - 1 and T3 - V
 Deflection at Surface, $z = 0$

Load Deflection Equation on Center Line. $w_c = CprF_w/E_3$
 Rigid Plate - $C = \pi/2$ Tire - $C = 2.0$

Pavement Designations
 $h_1 = 8''$ Concrete
 $h_2 = 8''$ & 16'' Base
 $r_e = 10''$ $h_1/r_e = 0.8$

Deflection Coefficient - $F_w = f[h_1/r, E_1/E_2, E_2/E_3, \mu_1, \mu_2, \mu_3]$

Values of Thickness-Radius - h_1/r

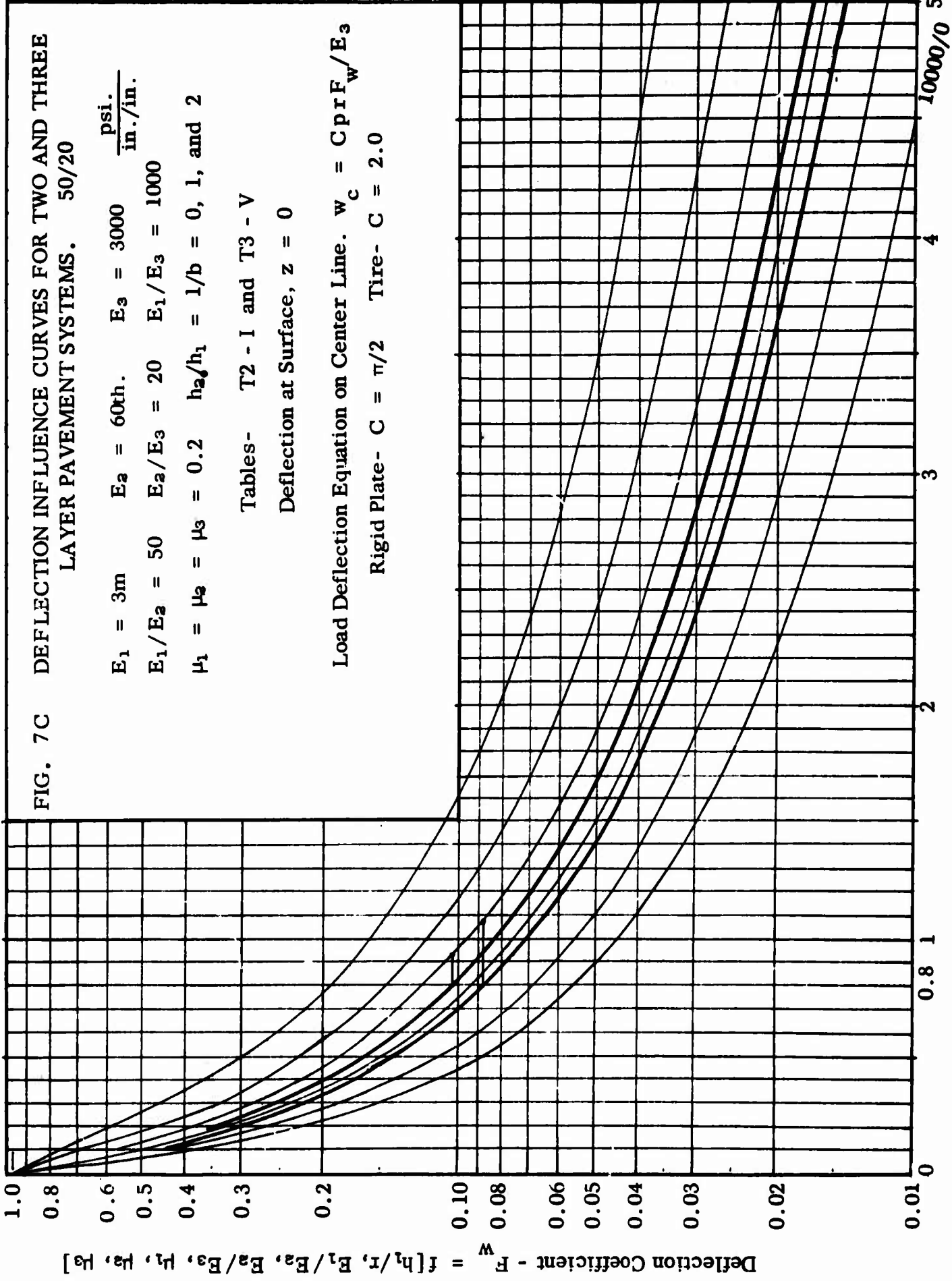


FIG. 7C DEFLECTION INFLUENCE CURVES FOR TWO AND THREE LAYER PAVEMENT SYSTEMS. 50/20

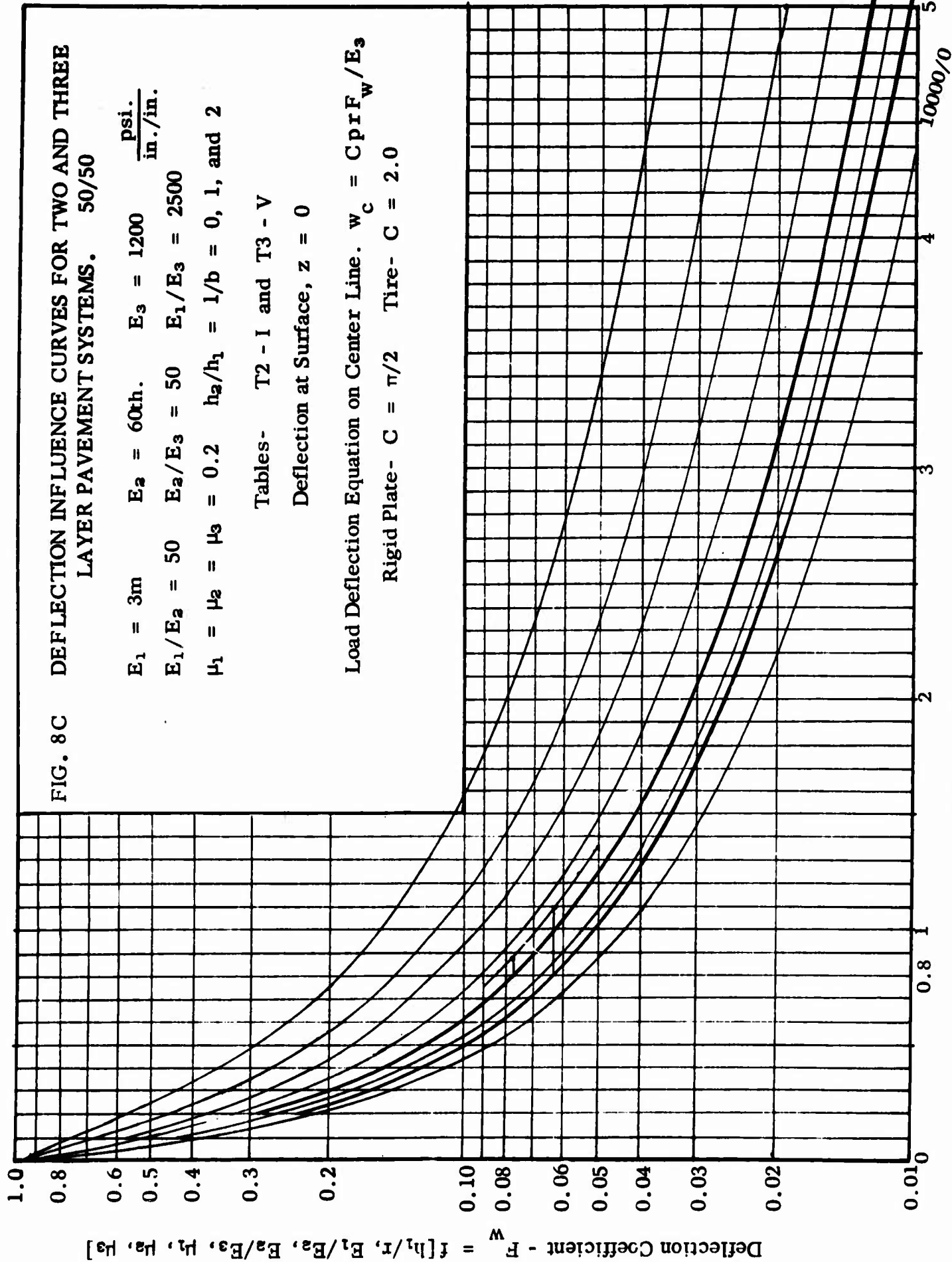
$E_1 = 3m$ $E_2 = 60th.$ $E_3 = 3000$ $\frac{\text{psi.}}{\text{in./in.}}$
 $E_1/E_2 = 50$ $E_2/E_3 = 20$ $E_1/E_3 = 1000$
 $\mu_1 = \mu_2 = \mu_3 = 0.2$ $h_2/h_1 = 1/b = 0, 1, \text{ and } 2$

Tables- T2 - I and T3 - V
 Deflection at Surface, $z = 0$

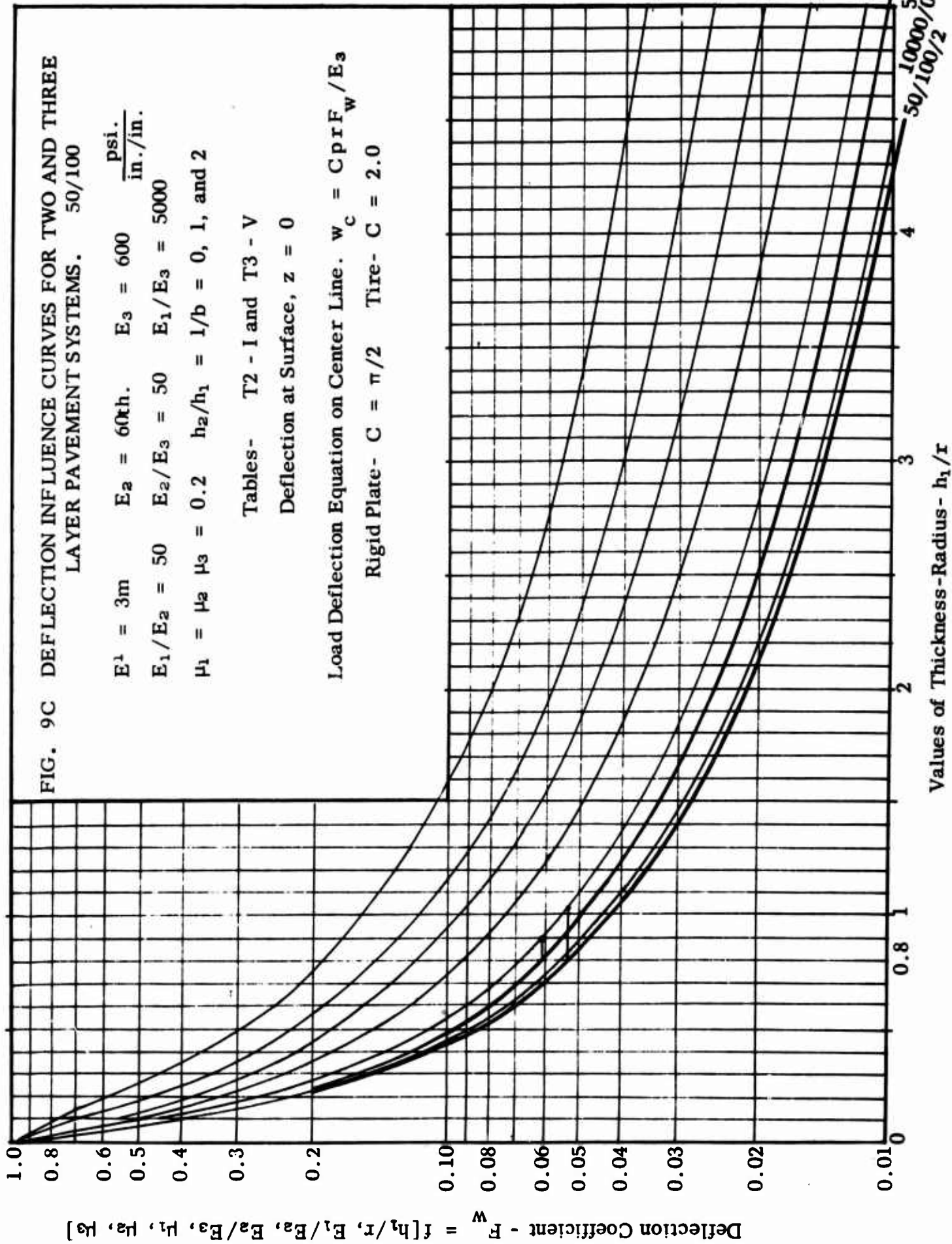
Load Deflection Equation on Center Line. $w_c = CprF_w/E_3$
 Rigid Plate- $C = \pi/2$ Tire- $C = 2.0$

$h_1 = 8''$ Concrete
 $h_2 = 8'' \text{ \& } 16''$ Base
 $r_e = 10''$ $h_1/r_e = 0.8$
 Pavement Designations
 $[E_1/E_2 - E_2/E_3 - 1/b]$

Values of Thickness - Radius - h_1/r



Pavement Designations
 $[E_1/E_2 - E_2/E_3 - 1/b]$
 $h_1 = 8''$ Concrete
 $h_2 = 8'' \text{ \& } 16''$ Base
 $r_e = 10''$ $h_1/r_e = 0.8$



Pavement Designations
 $[E_1/E_2 - E_2/E_3 - 1/b]$
 $h_1 = 8''$ Concrete
 $h_2 = 8''$ & 16'' Base
 $r_e = 10''$ $h_1/r_e = 0.8$

The effectiveness of the reinforcing action, the stiffness, and the deflection performances of layered pavement systems are disclosed in the deflection influence curves of Figs. 2C to 9C by the value of the deflection influence coefficient, F_w for any selected values of the basic layered system parameters, h_1/r and E_1/E_3 for the two layer systems with $h_2 = 0$, and $[E_1/E_2 - E_2/E_3 - 1/b]$ for three layer systems. It is evident that the deflection coefficient, F_w plays a governing and dominant role in the deflections performances of layered pavement systems through the surface deflection equation- $w_c = C p r F_w / E_3$, deflection being directly proportional to F_w . The upper limit or maximum value of $F_w = 1.0$ for which $E_1/E_2 = 1.0$, or a non-layered, homogeneous soil mass. Figs. 2C to 9C show that the dimensionless deflection coefficient, F_w is dependent upon and governed by the basic layered system dimensionless parametric ratios, expressed functionally by the expression- $F_w = f_w [E_1/E_2, E_2/E_3, h_1/r, h_2/h_1, \mu_1, \mu_2, \mu_3]$.

The systematic and characteristic patterns of deflection influence curves of Figs. 2C to 9C reveal the nature of the dependence of deflection performances of layered pavement systems upon the basic parameters, h_1/r , h_2/h_1 and the layer moduli ratios. For a constant h_1/r value, the value of the deflection coefficient, F_w decreases substantially with increase in the layer moduli ratios downward through the pattern of deflection influence curves, thereby decreasing deflection responses of a layered system. This gives special point to the fact that effective improvement in deflection performances for a constant thickness of reinforcing layers can be achieved by the selection and use of higher strength and quality of layer materials, and particularly by actual constructional excellence in the field in order to attain the full potential strength properties, E_1/E_2 and E_2/E_3 of these materials.

It is also evident that the effectiveness of reinforcing action and deflection performances of layered pavement systems are strongly influenced by

h_1/r and h_2/h_1 values. For constant values of E_1/E_2 for a two layer system and of radius bearing area, the greatest improvement of two layer system reinforcing action, stiffness, and deflection performances can be achieved by increase in thickness, h_1 of reinforcing layer 1 in the region of h_1/r less than 1.0, where the deflection influence curves are steepest. These relationships are of fundamental importance as practical and effective means for controlling and limiting high shear stresses in the critical mid-depth regions of reinforcing layer 1 beneath the edge of a tire loading. For h_1/r values greater than 1.0, the improvement in deflection performances with increase in thickness of reinforcing layer 1 becomes considerably less effective, because the deflection influence curves flatten out considerably.

Also, it is most important to note for a constant E_1/E_2 value and thickness, h_1 of reinforcing layer 1, that the effectiveness of a given layered pavement system decreases considerably and adversely with increase in radius, r of the bearing, as indicated by the adverse increase in F_w , especially for h_1/r values less than 1.0. This reveals definitely that the reinforcing action of a constructed layered pavement system is not a constant quantity, but is adversely affected subsequent to construction by a change in service conditions to aircrafts with larger tire sizes (larger r) and heavier loads. Pavement design should take full cognizance of this practically certain eventuality.

The spacings of these curves are systematic and very closely approximate a logarithmic scale for h_1/r greater than about 0.5. This means that E_1/E_2 curves can be quite accurately interpolated between the curves noted. The log scale of F_w is marked on a strip of paper, laid over the curves, and inclined in one direction or the other to match closely three points, for example- 2, 5, and 10; 10, 20, 50, etc., at a number of locations, so that a smooth interpolated curve may be drawn for intermediate E_1/E_2 values.

Basic Preliminary Pavement Evaluation Approach.

The above general considerations revealed by and obtained from the vertical stress transmission characteristics and performances of Figs. 2A to 9A, from the shear stress transmission characteristics, performances, and critical regions of Figs. 2B to 9B, and from the center-line surface deflection performances of Figs. 2C to 9C for a range of two and three layer pavement systems are important as first guides in pavement studies and evaluations. The second basic step is to evaluate and directly to compare the center-line surface deflection performances, and the vertical stress and shear stress performances in critical regions induced by typical single tire loadings on a factual basis for selected, specific, known, or estimated limiting conditions, that may govern layered pavement system performances, such as aircraft loads, subgrade moduli in prepared subgrades in excavations and embankments, and potential layer moduli and shear strength properties for concrete and asphalt reinforcing pavement layer 1, and for selected, high quality base course (and eventually subbase layer 3) materials for reinforcing layer 2.

In order to illustrate this basic evaluation approach, data and information was obtained from the deflection performances of Figs. 2C to 9C, shear stress performances in critical regions and competence of base layer 2 to take shear load in Figs. 2B to 9B, and vertical stress performances and competence of base layer 2 to take vertical stress load in Figs. 2A to 9A are used for the following governing conditions for this study:

- a) Concrete layer 1 modulus, $E_1 = 3,000,000$ psi. per in./in.
- b) Range and values of the Base layer 2 modulus, E_2 , as fixed for this study, are given by the modulus ratio, E_1/E_2 for layers 1 and 2 in the respective Figs. 2 to 9.
- c) Range and values of the Subgrade layer 3 modulus, E_3 , as fixed for this study, are given by the modulus ratios, E_2/E_3 for layers 2 and 3, or E_1/E_3 for layers 1 and 3 in the respective Figs. 2 to 9.
- d) Thickness of concrete layer 1, $h_1 = 8''$, thickness of base layer 2,

$h_2 = h_1 = 8''$ and $h_2 = 2h_1 = 16''$ (where $bh_2 = h_1$).

e) Radius of equivalent circular tire loading, $r_e = 10''$. Basic radius-thickness layered system ratio- $r_e/h_1 = 1.25$, or $h_1/r_e = 0.80$.

f) Average contact pressure for this study, 100 psi.

g) Deflection, $w = C_p r F_w / E_3 = 2 \times 100 \times 10 F_w / E_3 = 2000 F_w / E_3$.

The comparative findings of this second basic step are given in Table 4 for study and evaluations, with regard to deflection performances.

Table 4. Comparative Estimated Deflections and Maximum Shear Stresses for Two and Three Layer Pavement Systems of Figs. 2C to 9C and Figs. 2B to 9B with varying Subgrade Moduli and Layer Moduli, as Bases for Design Evaluations and Judgments of Effectiveness the Layer Pavement Systems.

1	2	3	4	5	6	7	8	9	10
Pavement	E_3	F_w	w inches	$\tau(\max)$ psi	Pavement	E_3	F_w	w inches	$\tau(\max)$ psi
Layer Moduli ratios.									
500/0	6000	.140	.047**	76					
1000/0	3000	.102	.068*	79					
2000/0	1500	.091	.121	70					
5000/0	600	.066	.220	80					
20/20/1	7500	.135	.036**	68	20/20/2	7500	.102	.027**	63
20/50/1	3000	.095	.063*	69	20/50/2	3000	.073	.049*	63
20/100/1	1500	.078	.104	71	20/100/2	1500	.06	.080	63
20/200/1	750	.061	.163	71	20/200/2	750	.046	.123	64
50/10/1	6000	.130	.043**	74	50/10/2	6000	.110	.037**	71
50/20/1	3000	.102	.068*	75	50/20/2	3000	.088	.066*	71
50/50/1	1200	.078	.130	76	50/50/2	1200	.063	.105	71
50/100/1	600	.061	.202	76	50/100/2	600	.052	.170	72

Note- Acceptable Deflection Values are limited here, for example to, 0.05** and 0.07* inches.

The basic layered system parametric relations provide suitable and adequate bases for pavement system evaluations and design. Design inherently involves special problems which first must be visualized, and second must be treated and evaluated on an individual basis in order representatively and adequately "to fit and to tailor" design and construction control to existing and/or modified environmental conditions and to anticipated service conditions. The principal problems are: (a) to limit pavement deflections and accumulated permanent settlements under long-term repeated loadings to non-objectionable values; (b) to ensure the permanence, integrity, and continuity of the pavement structure against deterioration and breakdown under repeated wheel loads; and (c) to increase the life of the pavement structure, giving due consideration to environmental and eventual service conditions that prevail and control.

Design always deals with multi-layer pavement systems. The design of multi-layer pavement systems adequately to satisfy all environmental and service requirements involves: first, a determination of the number of layers required, principally to limit shear stresses imposed in critical pavement regions to well below shear breakdown values under repeated loadings by using smaller "jumps" in E_1/E_2 values between layers 1, 2, 3 etc; second, the selection and use of high quality and high strength materials for layers 1 and 2, and determinations of their effective working E-values, as constructed in-place; and third, the evaluation of the thickness requirements for these layers in order fully to attain these design objectives.

In the WASHO Road Test Reports [28, 29] (Numbers of Appended References), in The Hybla Valley Test Project [32], and in The AASHO Road Test Reports [35 to 37] considerable data has been presented on construction and compaction control, and on uniformity of construction with regard to in-place moisture contents and compacted densities of the subgrade, subbase, and base course layers of the layered pavement systems. These Test Pavement Systems

were considered to represent good or above average construction. In marked contrast, my evaluations of plate load test data for The WASHO Road Test Sections and The Hybla Valley Test Project, [3, pp. 48-53], [5, 449-453, Figs. 6-7], [40, pp. 166-171, Figs. 8-9] have disclosed that the spread of in-place layer moduli is much too large among the different test sections of each of these projects to be considered acceptable, as good construction. The Load Test Data for The WASHO Road Test have been analyzed and evaluated by the principles and methods of layered pavement system analyses and evaluations and the range of layer moduli in psi. in./in. are given [low-average-high] with the number of load tests noted as follows, in Table 5.

Table 5. Range of Layer E-Values [Low-Average-High] Evaluation for the Test Sections of The WASHO Road Test.

<u>Layer</u>	<u>Lower Range of E-Values</u>	<u>Higher Range of E-Values</u>
Compacted Subgrade	10 Tests [5200-5800-6400] Acceptable	3 Tests [7,000-10,200-11,000] Excellent
Pit-run gravel Subbase	6 Tests [8,000-11,000-14,000] Unacceptable	5 Tests [20,000-22,000-26,000] Acceptable
4" A C - 2" B Pavement	5 Tests [60,000-70,000-80,000] Only Fair	6 Tests [100,000-120,000-160,000] Excellent
2" A C - 4" B Pavement	6 Tests [40,000-62,000-80,000] Only Fair	6 Tests [80,000-92,000-110,000] Acceptable

In Table 6 are given tentative quality ratings for layer moduli compiled from evaluations of plate load bearing tests, in order to provide some basis and guide for judgments, regarding what may be considered acceptable for good in-place construction. Such base course and subbase quality ratings can be established and expanded by carefully conducted load bearing tests during construction or from reevaluation surveys.

Table 6. Tentative Quality Ratings for Layer Moduli, E in psi. in./in. for Crushed Stone Base Courses and Gravel Subbase Courses, Compiled from Evaluation of Plate Load Bearing Tests.

Crushed Stone Base Course

- B-1 $E_b = 100,000$ - best quality, maximum compaction and keying
- B-2 $E_b = 75,000$ - best quality, good compaction and keying
- B-3 $E_b = 50,000$ - good quality, good compaction and keying

Gravel Subbases

- B-4 $E_{sb} = 30,000$ - well-graded, maximum compaction and keying
- B-5 $E_{sb} = 20,000$ - run-of-bank, good compaction

Since design of multi-layer pavements adequately to meet the essential requirements of satisfactory service and long life involves: first, the determination of the number of layers; second, the selection of satisfactory high quality, high strength layer materials and corresponding E-values; and third, the evaluations of the thickness requirements for these layers, then some "yard stick of equivalences" of layer moduli and layer thicknesses becomes essential for testing and evaluating the problems of multi-layer pavement design.

C. 11- The concept of equivalences of layered systems for different layer moduli and layer thicknesses with regard to identical reinforcing action and deflection performances is defined by a horizontal line drawn on the deflection influence curves of Figs. 2C to 9C for a selected, constant reference value of the deflection coefficient, F_w : (1) which intercepts a number of E_1/E_2 deflection influence curves; (2) which yields a series of layer materials having different E_1/E_2 values referenced to the same subgrade modulus, E_2 for a two layer system or E_3 for a three layer system; and (3) which have associated h_1/r values for thickness equivalences for a constant bearing radius, r at each intercepted E_1/E_2 influence curve. The reference value of F_w cannot be taken indiscriminantly, but must be either that used in the evaluations, or a modified

value for which the estimated or computed deflections are within satisfactory limits.

In each Fig. 2C to 9C two short horizontal equivalence lines are drawn at the appropriate deflection coefficient, F_w value defined by the intersection of the vertical line of $h_1/r_e = 0.8$ with the two respective heavy line three layer deflection influence curve, designated in the right-hand margin of each figure. This horizontal equivalence line is extended toward the right to an intersection with a thin line two layer deflection influence curve or an interpolated curve, having the appropriate E_1/E_3 value noted in the tabulation of defining layer moduli ratio designations given in each Fig. 2C to 9C. This intersection of the horizontal equivalence line with the appropriate E_1/E_3 curve defines a new $(h_1/r)_e$ value, which now establishes the equivalent thickness - $(h_1/r)_e \times (r_e = 10'') = (h_1)_e$ of concrete layer 1 for a two layer system having the same E_1 - value of layer 1 and the same E_3 - value of the subgrade modulus. Since by this horizontal line construction, the deflection coefficient, F_w and the concrete layer 1 and subgrade layer 3 moduli are identical, the original three layer system and the equivalent two layer system have the same deflection performances, which is the essential evaluation and design consideration sought for. However, these two layered systems will not have the same vertical stress and shear stress performances, since the new layer 1 thickness is now increased and the layer 2 thickness becomes zero. The principal fact of importance is that a rational and extremely useful basis for deflection performance equivalences has now been established between two and three layer systems, either for the evaluation of load-bearing tests, or for building up a multi-layer pavement equivalent to tried two and three layer systems. In Table 7 are given the two layer equivalences for the three layer systems in Figs. 2C to 9C and in Table 4 for comparisons of equivalent layer 1 thicknesses.

Table 7. Evaluation of Two Layer Equivalences for the Three Layer Pavement Systems of Figs. 2C to 9C with Essential Data for Equivalences Tabulated $(h_1)_e = (h_1/r)_e \times r_e$, versus $h_1 = 8''$ for $r_e = 10''$.

1 E_1/E_3	2 E_3	3 Pavement	4 F_w	5 $(h_1)_e$	6 Pavement	7 F_w	8 $(h_1)_e$
400	7500	20/20/1	.135	9.2	20/20/2	.102	12.0
1000	3000	20/50/1	.095	9.8	20/50/2	.073	12.5
2000	1500	20/100/1	.078	9.8	20/100/2	.060	12.6
4000	750	20/200/1	.061	9.9	20/200/2	.046	13.0
500	6000	50/10/1	.130	9.0	50/10/2	.110	10.7
1000	3000	50/20/1	.102	9.2	50/20/2	.088	10.8
2500	1200	50/50/1	.078	9.0	50/50/2	.063	10.9
5000	600	50/100/1	.061	9.0	50/100/2	.052	10.2

An important aspect of pavement system design is to test out the deflection performances and the shear stresses imposed at the critical mid-depth of pavement layer 1, beneath the edge tire loadings, with regard to the influences of increase in tire size (r_e), of increase in tire pressure, p , and of the required changes or adjustments in layer thicknesses, as bases for making final design judgments and decisions to meet established criteria for deflection and shear performances, or to modify these criteria in the light of pavement evaluation surveys. In these cases, it is necessary in the evaluations to use directly the basic deflection equation- $w = C p r F_w/E_3$, in conjunction with the deflection influence diagrams of Figs. 2C to 9C. For example, if a pavement deflection performance equal to $w = 0.050$ inches was adopted as the criteria, the deflection coefficient, F_w and the two and three layer thicknesses for the pavement systems of Fig. 4 would accordingly be modified, as evaluated in Table 8, columns 8 and 9 for new equivalent thickness requirements, $(h_1)_c$ and $(h_2)_e$. Where these new thickness requirements become excessive, as

Table 8. Adjustments in Layer Thickness Required to Meet a Deflection Performance Requirement of 0.50 inches for Three Layer Pavements of Table 4.

1 Fig.	2 Pavement	3 E ₃	4 F _w	5 w = 2000 F _w E ₃	Thickness Requirements for w = 0.050 inches				9 (h ₂) _e	Approximate τ(max), Layer 1
					6 F _w = .050 E ₃ 2000	7 h ₁ /r	8 (h ₁) _e	10		
2C	20/20/1 20/20/2	7500	.135 .102	.036* .027*	.167 .167	.63 .48	6.3 4.8	6.3 9.6	86 105	
3C	20/50/1 20/50/2	3000	.095 .073	.063 .049	.075 .075	1.05 .78	10.5 7.8	10.5 15.6	53 65	
4C	20/100/1 20/100/2	1500	.078 .060	.104 .080	.0375 .0375	1.68 1.32	16.8 13.2	16.8 26.4	34 38	
5C	20/200/1 20/200/2	750	.061 .046	.163 .123	.01875 .01875	2.65 2.00	26.5 20.0	26.5 NG 40.0 NG	21 25	
6C	50/10/1 50/10/2	6000	.130 .110	.043* .037*	.150 .150	.67 .55	6.7 5.5	6.7 11.0	88 103	
7C	50/20/1 50/20/2	3000	.102 .088	.068 .066	.075 .075	1.10 .93	11.0 9.3	11.0 18.6	55 61	
8C	50/50/1 50/50/2	1200	.078 .063	.130 .105	.030 .030	2.07 1.72	20.7 17.2	20.7 NG 34.4 NG	29 33	
9C	50/100/1 50/100/2	600	.061 .052	.202 .170	.015 .015	2.62 2.12	26.2 21.2	26.2 NG 42.4 NG	22 27	

noted by the designation- NG, the corresponding moduli ratios, E_1/E_2 in the pavement designations is considered to be too inefficient and too ineffective and hence the base course material is of too poor quality for the given subgrade moduli, E_3 less than about 1200 psi. per in./in. The effectiveness of these pavement systems could be considerably improved by adequately compacting the upper 12 to 18 inches of the subgrade soils, thus producing an effective four-layer pavement system and permitting a reduction in base course thickness.

The adjusted equivalent thickness $(h_1)_e$ reinforcing layer 1 from an original 8 inches to the values in column 8 will now cause a change in the shear stresses imposed at the critical mid-depth region of layer 1 from the maximum values shown in Figs. 2B to 9B and given in columns 5 and 10 of Table 4. Since the total shear load is identical for all of the pavement systems of both Table 4 and Table 8, and since the major part of the shear load is carried by reinforcing layer 1, and approximate and conservative estimate can be made for the maximum shear stresses imposed at the critical mid-depth region of reinforcing layer 1 by the following relations.

Approximate estimates of the maximum shear stresses are given in column 10 of Table 8, which are imposed at the critical mid-depth region of equivalent reinforcing layer 1 having an adjusted equivalent thickness, $(h_1)_e$ in column 8 to meet the deflection performance requirements of 0.05 inches for the three layer pavement systems of Figs. 2C to 9C. The following approximations were used in making these maximum shear stress estimates for these preliminary pavement evaluation studies.

(1) The shear load carried by the original reinforcing layer 1 of thickness, $h_1 = 8''$ and by the adjusted equivalent reinforcing layer 1 of thickness, $(h_1)_e$ of column 8, Table 8, have approximate parabolic distributions.

(2) For the comparative purposes of these estimates, the proportions of the total shear load carried by original reinforcing layer 1 of thickness, h_1 and by

the adjusted equivalent- reinforcing layer 1 of thickness, $(h_1)_e$ are considered to be approximately equal.

(3) Then on the basis of these approximations:

$$\frac{2}{3} \times 8'' \times \tau(\max) \text{ (Table 4, Col. 5 and 10)} = \frac{2}{3} \times (h_1)_e \times \tau(\max)_e \text{ (Table 8, Col. 10) or } \tau(\max)_e = \tau(\max) \times 8/(h_1)_e$$

It is evident in Table 10 that where the original deflections in Table 4, Columns 4 and 9 were less than the adjusted deflection of 0.05 inches in Table 8 that the adjusted layer 1 thickness- $(h_1)_e$ would be less than the original layer 1 thickness, h_1 of 8'', and hence the maximum shear stresses in layer 1 would be larger, as noted by the asterisks (four values only) in column 10 than the maximum shear in columns 5 and 10 of Table 4. For all other adjusted equivalent layered system in Table 8, of column 8, the maximum shear stresses are reduced and improved by required adjustments for a reduction in deflection performances to 0.05 inches. Therefore, a major problem is to establish by reevaluation surveys of layered pavement systems in service a reliable and valid maximum shear stress criterion in the critical mid-depth region of reinforcing layer, which takes full account of repeated loading effects on layered pavement systems. These approximate maximum shear stress estimates serve a very useful purpose for preliminary multi-layer pavement evaluation studies for delineating a possible satisfactory range of layer thicknesses and a range of layer qualities and strength properties, as indicated by layer moduli and moduli ratios.

In addition, evaluations of the tensile stress performances of reinforcing layer 1 with regard to their critical and destructive character under repeated flexing action of tire loadings; (1) at the bottom of reinforcing layer 1 beneath the center-line of the tire footprint area, (2) at the surface of reinforcing layer 1 between dual tire loadings; and (3) at the surface of reinforcing layer 1 at about

a tire width outside of the tire footprint area .

Once, a multi-layer pavement system design has been narrowed down to a few good possibilities by the above first approximation methods, then more exact information on shear stress, tensile stress, and deflection performances can be evaluated by the layered system methods and procedures of analysis and evaluation, which are given and illustrated in PART III of this final report . The pavement system design can then be finalized on a rational, factual and scientific basis .

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13. ABSTRACT
The report **PART II**
PART II presents Influence Diagrams for Vertical Stresses, Shear Stresses, and Surface Deflections in Three-Layer Pavement Systems, which are intended to provide the essential background and bases for evaluating the character and effectiveness of layered system reinforcing action. The vertical stress and shear stress transmission characteristics and critical regions, and the surface deflection performances are developed for nine three-layer concrete pavement systems for a range of layer moduli ratios, and ratios of layer 2 to layer 1 thickness. The effectiveness of reinforcing action, deflection performances, and shear stress performances in critical regions are treated and compared. Two-layer and three-layer thickness and layer moduli equivalences are developed and treated to illustrate methods for evaluating and for improving shear stress and deflection performance by modifications of layer moduli ratios and layer thicknesses. The major objectives are to develop a "feeling", intuition, and judgments regarding deflection and shear stress performances of layered pavements, and to develop relationships and methods for evaluating layer thickness and moduli ratios, and criteria for rational and effective design for multi-layer pavement system for airfields and for ensuring satisfactory pavement performances and long life.
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