TECHNICAL REPORT 231-14

ON THE SURFACE WAVE PATTERN OF SUBMERGED BODIES ARRESTED IMPULSIVELY FROM UNIFORM MOTION

By

C. C. Hsu

November 1966

HYDRONAUTICS, incorporated
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NOTATION

f	Depth of submergence measured from the centerline of the body
F_λ	Function of θ as defined
F	Froude number based on depth
g	Gravitational acceleration
G	Function of κ, θ and τ as defined
ℓ	Distance between source and sink
M	Strength of point source
M_n	Function of t, source strength
M_n'	Function of t, source strength
M_1	Source strength
M_2	Sink strength
$R\ell$	Real part
R_n	Radial distance of source or sink, M_n
$\bar{R}_n$	Image of R_n
R_1	$\sqrt{x^2 + y^2}$
R_2	$\sqrt{(x+\ell)^2 + y^2}$

S	Surface
S'	Arbitrary surface enclosing singularities
t	Time
t_o	Initial reference time
U	Uniform free-stream speed
x,y,z	Rectangular Cartesian coordinates
x_n, y_n, z_n	Position of source or sink
δ_c	Angle between line of motion and straightline radiating from the origin
∂_1	$= \tan^{-1} (y/x)$
∂_2	$= \tan^{-1} \dfrac{y}{x + \ell}$
ζ	Surface elevation
ζ_o	Disturbance due to original motion
ζ_1	Disturbance caused by the stopping action
ζ_1	Initial unsteady surface disturbance
θ	Angle
θ_{λ_1}	$= - 1/4 \left[\cot \delta_\lambda - \sqrt{\cot^2\delta_\lambda - 8}\right]$
θ_{λ_2}	$= - 1/4 \left[\cot \delta_\lambda + \sqrt{\cot^2\delta_\lambda - 8}\right]$

κ	Dummy variable
κ_o	$= g/U^2$
Φ	Total velocity potential
Φ_o	Velocity potential due to the prescribed singularities
$\bar{\Phi}_o$	Image of Φ_o
$\tilde{\omega}$	$= x \cos \theta + y \sin \theta$
$\tilde{\omega}_1$	$= [x + U(t-\tau)] \cos \theta + y \sin \theta$
$\tilde{\omega}_2$	$= [x + \ell + U(t-\tau)] \cos \theta + y \sin \theta$
$\tilde{\omega}_n$	$= (x-x_n) \cos \theta + (y-y_n) \sin \theta$

ABSTRACT

In this report a theoretical study on the surface wave pattern of submerged bodies arrested impulsively from uniform motion has ~~been~~ carried out. Seen from above, the waves appear to pass through the suddenly stopped body, fading away in the foremost portion of the pattern. The extent of this disappearing wave front in advance of the stopped body is approximately equal to 1/2 Ut, where U is the constant speed which the body originally possessed, and t the time elapsed since the sudden alteration in motion took place. If the transient effect is negligible, i.e., for a sufficiently long time t, except for the disappearing wave front, the group of steady regular waves, produced by the initially moving submerged body, advances into still water within a V-shaped region as if no changes had taken place (~~see Figure 2~~). The front portion of this wave pattern becomes blunter as the elapsing time increases, due to the disappearing wave front. The location of the motionless body may be determined from the wave pattern by simple arithmetic calculations.

INTRODUCTION

When a small obstacle is moved forward through still water, the surface is covered with a wave pattern fixed relative to the obstacle. On the upstream side the wavelength is short and the force governing the disturbance is principally surface tension. On the downstream side the waves are longer and are governed principally by gravity. If at some instant a sudden alteration in the motion takes place, very interesting questions arise as to what form the surface disturbance will take in response to such abrupt changes. In this report the surface wave pattern of a submerged body arrested impulsively from uniform motion is studied. The analysis is based on the usual assumptions that the fluid is ideal and very large in extent and the disturbance is infinitesimal. It has been found possible to make calculations in this case, and the results illustrate various points of interest.

GENERAL FORMULATION

It is well known that the fluid motion caused by a submerged obstacle may be taken to be that due to an appropriate distribution of singularities over or inside its surface. The total velocity potential for a submerged body moving through an ideal fluid of very large extent in irrotational motion may be expressed as

$$\Phi = \Phi_o - \bar{\Phi}_o + \frac{1}{\pi}\int_0^t d\tau \int_{-\pi}^{\pi} d\theta \int_0^{\infty} G(\kappa,\theta,\tau)e^{\kappa(z-i\tilde{\omega})}\sin\left\{\sqrt{g\kappa}(t-\tau)\right\}\sqrt{g\kappa}\,d\kappa \qquad [1]$$

where

- g is the gravitational acceleration,
- Φ_o is the velocity potential due to the prescribed singularities and depends mainly on body shape,
- $\bar{\Phi}_o$ is the image of Φ_o .

$$G(\kappa,\theta,\tau) = -\frac{1}{4\pi}\int_{S'}\left(\frac{\partial\phi_o}{\partial n} - \phi_o\frac{\partial}{\partial n}\right) e^{\kappa(z + i\tilde{\omega})} dS$$

S' is an arbitrary surface enclosing the singularities

x,y,z are the right-handed Cartesian coordinates and the origin is taken on the mean free surface with Oz vertically upwards, and

$\tilde{\omega} = x \cos\theta + y \sin\theta$.

The velocity potential ϕ_o due to an impulsive point source or sink at (x_n,y_n,z_n) started instantaneously at time t_o with a given velocity U and maintaining that speed is given by

$$\phi_o = \frac{M_n}{R_n} \tag{2}$$

where

$$M_n(t) = \begin{cases} 0 & t \leq t_o \\ M_n = \text{constant} & t > 0 \end{cases} \tag{3}$$

$$R_n = \sqrt{(x-x_n)^2 + (y-y_n)^2 + (z-z_n)^2}$$

t_o is the initial reference time.

The function G in this case is simply

$$G = M_n\, e^{\kappa[z_n + i(x_n \cos\theta + y_n \sin\theta)]} \tag{4}$$

The total velocity potential due to an impulsive point source or sink can then be shown to be

$$\phi = \frac{M_n}{R_n} - \frac{M_n}{\bar{R}_n} + \frac{1}{\pi}\int^{t} M_n(\tau)d\tau \int_{-\pi}^{\pi} d\theta \int_{0}^{\infty} e^{\kappa(z+z_n)} \cos \kappa\tilde{\omega}_n \sin\left\{\sqrt{g\kappa}(t-\tau)\right\}\sqrt{g\kappa}\, d\kappa \qquad [5]$$

where

$$\bar{R}_n = \sqrt{(x-x_n)^2 + (y-y_n)^2 + (z+z_n)^2}$$

$$\tilde{\omega}_n = (x-x_n)\cos\theta + (y-y_n)\sin\theta$$

Following the linearized dynamical condition at the free surface, the mean surface elevation may be expressed as

$$\zeta \approx \frac{1}{g}\frac{\partial\phi}{\partial t}\bigg|_{z=0}$$

$$= \frac{1}{g}\frac{\partial}{\partial t}\left(\phi_o - \bar{\phi}_o\right) + \frac{1}{\pi}\int^{t} d\tau \int_{-\pi}^{\pi} d\theta \int_{0}^{\infty} \kappa Q(k,\theta,\tau) e^{\kappa(z-i\tilde{\omega})} \cos\left\{\sqrt{g\kappa}(t-\tau)\right\} d\kappa \bigg| \text{ at } z=0 \qquad [6]$$

or

$$\zeta = R\ell \left\{ \frac{1}{\pi} \int^{t} M_n d\tau \int_{-\pi}^{\pi} d\theta \int_{0}^{\infty} \kappa \; {}^{\kappa(z_n+i\omega)} \cos \left\{ \sqrt{g\kappa}\,(t-\tau) \right\} d\kappa \right\}$$

at z=0 [7]

for motions due to a constant impulsive point source or sink with

$$\tilde{\omega} = [(x-x_n) + U(t-\tau)] \cos\theta + (y-y_n) \sin\theta$$

$R\ell$ denotes the real part.

Imagine now the fluid motion to have been generated instantaneously at time t_o by a constant impulsive point source or sink and to be arrested instantaneously at a later time t_1. The impulsive point source or sink in this case may be described as

$$M_n(t) = \begin{cases} 0 & t \leq t_o \\ M_n = \text{constant} & t_o < t < t_1 \\ 0 & t \geq t_1 \end{cases} \qquad [8]$$

Since the problem in the present study is assumed to be linear, condition [8] may be rewritten as the sum of

$$M_n(t) = \begin{cases} 0 & t \le t_o \\ M_n = \text{constant} & t > t_o \end{cases} \qquad [9]$$

and

$$M_n'(t) = \begin{cases} 0 & t \le t_1 \\ -M_n = \text{constant} & t > t_1 \end{cases} \qquad [10]$$

The corresponding mean free-surface elevations can be expressed as

$$\zeta = R\ell \left\{ \frac{1}{\pi} \int_{t_o}^{t} M_n d\tau \int_{-\pi}^{\pi} d\theta \int_{o}^{\infty} \kappa \; e^{\kappa(z+i\tilde{\omega})} \cos\left\{\sqrt{g\kappa}(t-\tau)\right\} d\kappa \right.$$

$$\left. + \frac{1}{\pi} \int_{t_1}^{t} M_n' d\tau \int_{-\pi}^{\pi} d\theta \int_{o}^{\infty} \kappa \; e^{\kappa(z_n+i\tilde{\omega})} \cos\left\{\sqrt{g\kappa}(t-\tau)\right\} d\kappa \right\}$$

$$= \zeta_o + \zeta_1 \qquad [11]$$

where

ζ_o is the disturbances due to the original motion

ζ_1 represents the disturbances caused by the stopping action.

Expression [11] is quite general and can be readily extended to wave motion caused by submerged bodies.

Consider the special case of a Rankine ovoid submerged in water at a given depth f (Figure 1), suddenly started from rest with a given velocity U at time t_o and maintaining that speed until t_1 at which the body is instantaneously brought to rest. For a first approximation the fluid motion resulting from the starting (or stopping) action may be taken to be that due to a point source (or sink) and a point sink (or source) of constant strength M distribution at the point (0,0,-f) and $(0,\ell,f)$, respectively. The mean surface elevations in this case may be shown to be

$$\zeta = \zeta_o + \zeta_1 \qquad [12]$$

where

$$\zeta_o = R\ell \left\{ \frac{1}{\pi} \int_{t_o}^{t} M d\tau \int_{-\pi}^{\pi} d\theta \int_{o}^{\infty} \kappa \, e^{-\kappa f} \left(e^{i\kappa\tilde{\omega}_1} - e^{i\kappa\tilde{\omega}_2} \right) \cos\left\{ \sqrt{g\kappa}\,(t-\tau) \right\} d\kappa \right\} \qquad [13]$$

$$\zeta_1 = -R\ell\left\{\frac{1}{\pi}\int_{t_1}^{t} Md\tau \int_{-\pi}^{\pi} d\theta \int_{0}^{\infty} \kappa\ e^{-\kappa f}\left(e^{i\kappa\tilde{\omega}_1} - e^{i\kappa\tilde{\omega}_2}\right)\cos\left\{\sqrt{g\kappa}(t-\tau)\right\} d\kappa\right\} \qquad [14]$$

respectively, with

$$\tilde{\omega}_1 = [x + U(t-\tau)]\cos\theta + y\sin\theta$$

$$\tilde{\omega}_2 = [x + \ell + U(t-\tau)]\cos\theta + y\sin\theta$$

RESULTS AND DISCUSSIONS

In this section the surface wave pattern of a Rankine body arrested impulsively from uniform motion is to be discussed in some detail. The time t_o and t_1 in the previous formulation are taken in this case as negative infinity and zero, respectively. The part ζ_o in Equation [12] then represents the steady-state surface waves which travel, in a group, only downstream of the body and is contained within the straightlines radiating from the origin, each making with the line of motion an angle, δ_c, approximately $19^{o}28'$. Far downstream ζ_o may be expressed simply as

$$\zeta_o = 4\kappa_o \sum_{\lambda=1}^{2} \frac{M_\lambda}{U} \sec^3\theta_{\lambda_1} \exp(-\kappa_o f \sec^2\theta_{\lambda_1}) \sqrt{\frac{2\pi}{\kappa_o R_\lambda \left|F_\lambda''(\theta_{\lambda_1})\right|}} \cos\left(\kappa_o R_\lambda F_\lambda(\theta_{\lambda_1}) + \frac{\pi}{4}\right)$$

$$+ 4\kappa_o \sum_{\lambda=1}^{2} \frac{M_\lambda}{U} \sec^3\theta_{\lambda_2} \exp(-\kappa_o f \sec^2\theta_{\lambda_2}) \sqrt{\frac{2\pi}{\kappa_o R_\lambda \left|F_\lambda''(\theta_{\lambda_2})\right|}} \cos\left(\kappa_o R_\lambda F_\lambda(\theta_{\lambda_2}) - \frac{\pi}{4}\right)$$

[15]

where

$$\kappa_o = \frac{g}{U^2} \qquad M_2 = -M_1 = -M$$

$$R_1 = \sqrt{x^2 + y^2} \qquad R_2 = \sqrt{(x + \ell)^2 + y^2}$$

$$\delta_1 = \tan^{-1}\left(\frac{y}{x}\right) \qquad \delta_2 = \tan^{-1}\frac{y}{x + \ell}$$

$$F_\lambda(\theta) = \sec^2\theta \cos(\theta - \delta_\lambda)$$

$$\theta_{\lambda_1} = -\frac{1}{4}\left[\cot\delta_\lambda - \sqrt{\cot^2\delta_\lambda - 8}\right]$$

$$\theta_{\lambda_2} = -\frac{1}{4}\left[\cot\delta_\lambda + \sqrt{\cot^2\delta_\lambda - 8}\right]$$

The part ζ_1 represents the surface disturbances due to the impulsive stopping action which may be expressed in terms of an impulsive sink and source as

$$\zeta_1 = - R\ell \left\{ \frac{1}{\pi} \int_0^t M d\tau \int_{-\pi}^{\pi} d\theta \int_0^\infty \kappa \, e^{-\kappa f} \left(e^{i\kappa\tilde{\omega}_1} - e^{i\kappa\tilde{\omega}_2} \right) \cos\left\{ \sqrt{g\kappa}(t-\tau) \right\} d\kappa \right\} \qquad [16]$$

It has been shown in Reference 1 that the resulting disturbance due to an impulsive source or sink may be separated into three parts:

(i) A steady local disturbance which travels both upstream and downstream and diminishes as the distance from the body increases.

(ii) A group of steady regular waves which travel downstream from the origin only as far as x = 1/2 Ut with group velocity 1/2 U.

(iii) A time-dependent circular disturbance (ζ_1) which travels both upstream and downstream and diminishes rapidly as time increases.

Since the steady disturbances expressed in Equation [16] cancel the corresponding part of ζ_o, the resulting surface elevation ζ may be shown to be

$$\zeta = \begin{cases} \zeta_1(t) & \text{for } x > 0 \text{ and } R \leq \tfrac{1}{2}Ut\cos\delta,\ x < 0 \\ \zeta_o + \zeta_1(t) & R > \tfrac{1}{2}Ut\cos\delta,\ x < 0 \end{cases} \qquad [17]$$

If the time t is sufficiently long Equation [17] is approximately

$$\zeta \approx \begin{cases} 0 & \text{for } x > 0 \text{ and } R \leq \tfrac{1}{2}Ut\cos\delta,\ x < 0 \\ \zeta_o & R > \tfrac{1}{2}Ut\cos\delta,\ x < 0 \end{cases} \qquad [18]$$

The resulting surface wave pattern is shown in Figure 2. It seems that, except for the fading wave front, the group of steady waves developed by the original motion of the body will advance into still water unblemished. The extent of the disappearing wave front is, of course, dependent upon the values of U and t. The propagating wave front is seen to become blunter as t increases. It is also interesting to note that the position of the arrested body can easily be located by simple geometric or arithmetic manipulations.

Although the foregoing discussions are based on the surface wave motion due to the action of simple Rankine bodies, similar results are expected to prevail for wave motion due to arbitrary bodies, except at very low speeds in which case the effect of capillarity may become important.

REFERENCE

1. Hsu, C. C., "On the Surface Pattern of Submerged Bodies Started From Rest," HYDRONAUTICS, Incorporated Technical Report 231-7, April 1965.

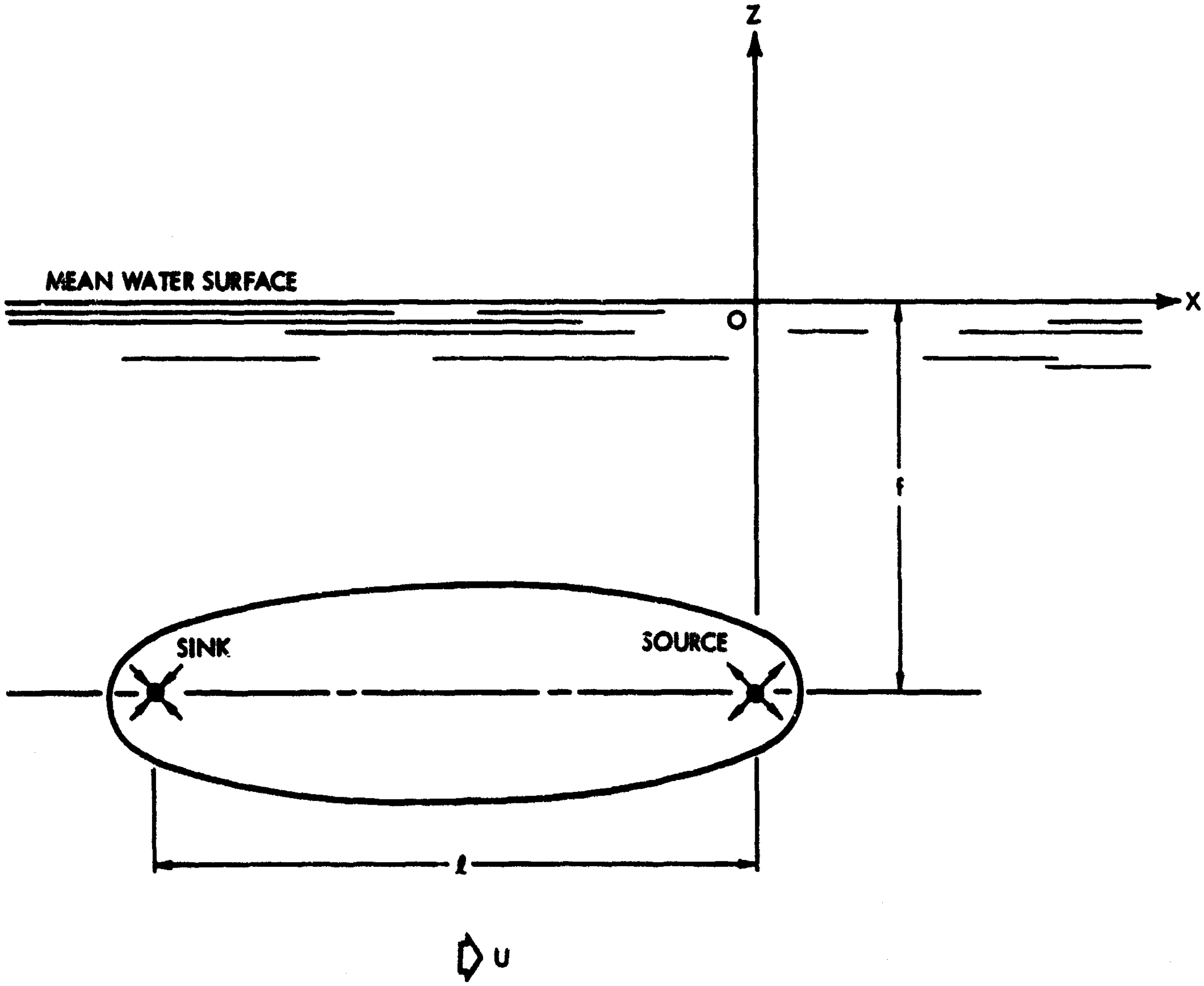

FIGURE 1 - DEFINITION SKETCH

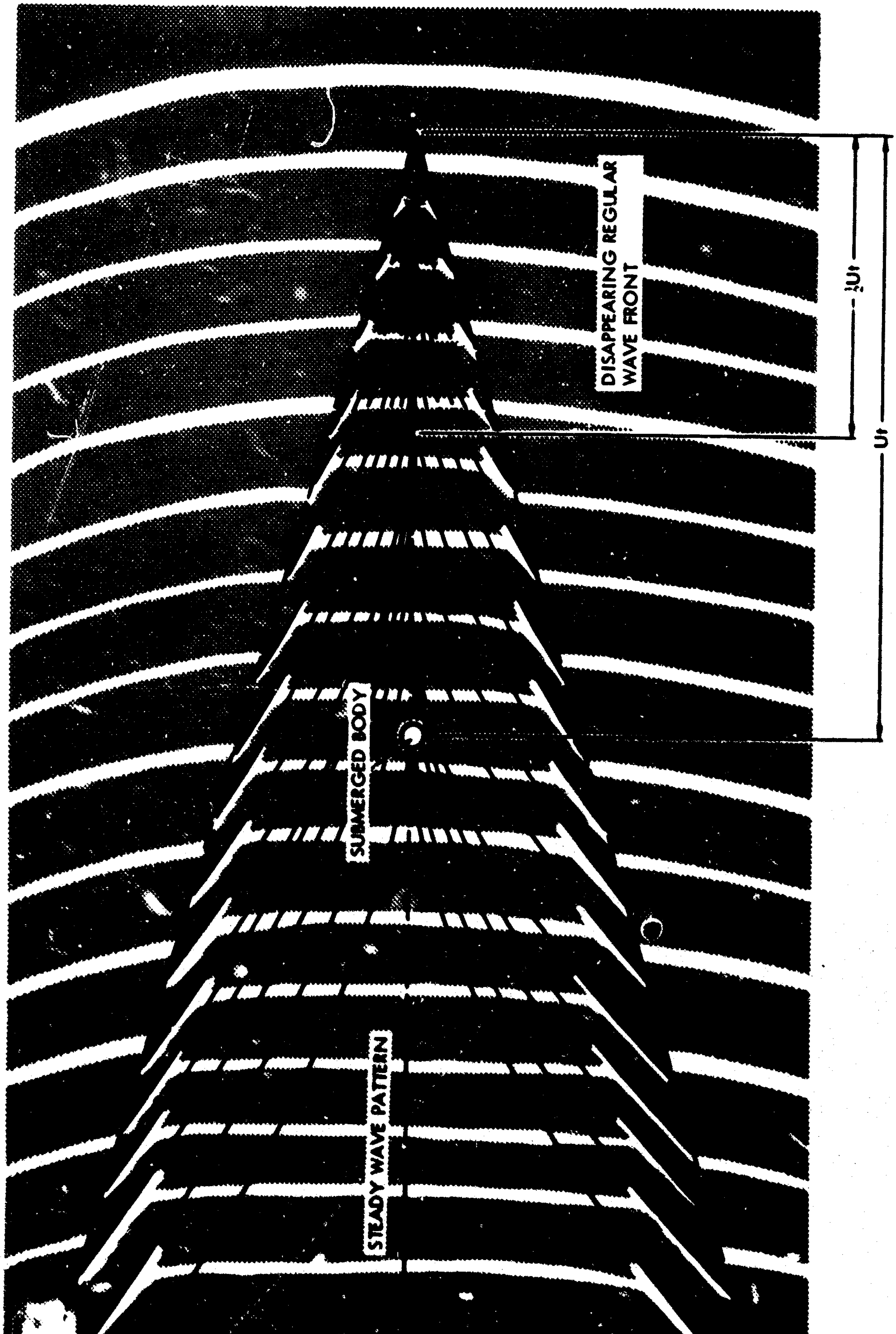

FIGURE 2 - SURFACE WAVE PATTERN OF A SUBMERGED BODY ARRESTED IMPULSIVELY FROM UNIFORM MOTION

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In this report a theoretical study on the surface wave pattern of submerged bodies arrested impulsively from uniform motion has been carried out. Seen from above, the waves appear to pass through the suddenly stopped body, fading away in the foremost portion of the pattern. The extent of this disappearing wave front in advance of the stopped body is approximately equal to 1/2 Ut, where U is the constant speed which the body originally possessed, and t the time elapsed since the sudden alteration in motion took place. If the transient effect is negligible, i.e., for a sufficiently long time t, except for the disappearing wave front, the group of steady regular waves, produced by the initially moving submerged body, advances into still water within a V-shaped region as if no changes had taken place (see Figure 2). The front portion of this wave pattern becomes blunter as the elapsing time increases, due to the disappearing wave front. The location of the motionless body may be determined from the wave pattern by simple arithmetic calculations.

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