Utilization of the Reciprocity Theorem to Determine
The Near Field Air-to-Subsurface Propagation Formulas

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ABSTRACT

The electric and magnetic field components produced by vertical and horizontal dipoles (both electric and magnetic types) located at or above the surface of a plane, conducting, homogeneous earth are derived for the near field range. The height h of the transmitting antenna is $\geq 0^+$, while the depth z of the receiving antenna is $\leq 0^-$ (air-to-subsurface propagation). Ionospheric effects are neglected.

The derivations are based upon application of the reciprocity theorem to previously derived field components. It is observed that these equations reduce to well-known expressions when the horizontal separation (ρ) between the transmitting and receiving antennas is much greater than h.

ADMINISTRATIVE INFORMATION

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DEFINITION OF SYMBOLS USED

c $\cong 3 \times 10^8$ meters/second, velocity of light in free space

dA infinitesimal dipole area (meters^2)

$d\ell$ infinitesimal dipole length (meters)

E_ρ horizontal electric field component in the ρ direction (volts/meter)

E_ϕ horizontal electric field component in the ϕ direction (volts/meter)

E_z vertical electric field component (volts/meter)

h height ($h \geq 0^+$) or depth ($h \leq 0^-$) of transmitting antenna with respect to the earth's surface

HED horizontal electric dipole

HMD horizontal magnetic dipole

H_ρ horizontal magnetic field component in the ρ direction (amperes/meter)

H_ϕ horizontal magnetic field component in the ϕ direction (amperes/meter)

H_z vertical magnetic field component (amperes/meter)

$\vec{m}$ direction of magnetic dipole axis

0^+ an infinitesimal distance above the earth's surface

0^- an infinitesimal distance below the earth's surface

R $(\rho^2 + z^2)^{\frac{1}{2}}$

R_0 $[\rho^2 + (z - h)^2]^{\frac{1}{2}}$

R_1 $[\rho^2 + (z + h)^2]^{\frac{1}{2}}$

R' $(\rho^2 + h^2)^{\frac{1}{2}}$

u_o $(\lambda^2 + \gamma_o^2)^{\frac{1}{2}}$ (meters^{-1}) (air)

u_1 $(\lambda^2 + \gamma_1^2)^{\frac{1}{2}}$ (meters^{-1}) (earth)

VED vertical electric dipole

VMD vertical magnetic dipole

z height ($h \geq 0^+$) or depth ($h \leq 0^-$) of receiving antenna with respect to the earth's surface

γ_o $= (-\epsilon_o \mu_o \omega^2)^{\frac{1}{2}}$ upper half-space (free-space) propagation constant (meters^{-1})

γ_1 $= (i\sigma_1 \mu \omega - \epsilon \mu \omega^2)^{\frac{1}{2}} \cong (i\sigma_1 \mu_o \omega)^{\frac{1}{2}}$, lower half-space (earth) propagation constant (the displacement currents in the earth are assumed to be negligible) (meters^{-1})

δ $= (2/\omega\mu_o \sigma_1)^{\frac{1}{2}}$, skin depth in lower half space (earth)

ϵ_o $\cong 10^{-9}/36\pi$ farads/meter, permittivity of free space

λ dummy integration variable in the basic Sommerfeld integrals

λ_{air} $= c/f$, free-space wavelength

ρ $(x^2 + y^2)^{\frac{1}{2}}$ radial distance in a cylindrical coordinate system

σ_1 conductivity of the lower half space (earth) (mhos/meter)

ϕ $\tan^{-1} y/x$, azimuth angle in a cylindrical coordinate system

$\mu \cong \mu_o$ $= 4\pi \times 10^{-7}$ henries/meter, permeability of free space

ω $2\pi f$ radians/second, angular frequency

UTILIZATION OF THE RECIPROCITY THEOREM TO DETERMINE THE NEAR FIELD AIR-TO-SUBSURFACE PROPAGATION FORMULAS

INTRODUCTION

Bannister and Bannister and Hart have derived the near field (ρ comparable to λ_{nir}) subsurface-to-air electric and magnetic field components produced by vertical and horizontal dipole antennas (both electric and magnetic types).[1] These expressions, which are valid for $|\gamma_1 R| >> 1$ ($R >> \delta$), $h \leq 0^-, z \geq 0^+$, and $R >> |h|$ are listed in Tables 1 and 2. An additional restriction ($|\gamma_0^2 \rho/\gamma_1| << 1$) is required for the vertically polarized components (E_ρ, E_z, and H_ϕ). This limits the range to small "numerical distances",[2] although $|\gamma_0 \rho|$ may exceed unity (i.e., ρ may be $> \lambda_{nir}$).

In the present study, the author will derive the near field air-to-subsurface propagation formulas by utilizing the reciprocity theorem.

The four antennas considered — VED, VMD, HED, and HMD — are situated at a height h with respect to a cylindrical coordinate system (ρ, ϕ, z) and are assumed to carry a constant current I. The VED and HED are oriented in the z and x directions, respectively, and the axes of the VMD and HMD are oriented in the z and y directions, respectively. The various dipole orientations for the situation $h = 0^+$ are shown in Fig. 1. The plane, conducting, homogeneous earth occupies the lower half space (z < 0) while the air occupies the upper half space (z > 0). Meter-Kilogram-Second (MKS) units are employed and a time factor of $e^{i\omega t}$ is assumed.

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[1] P.R. Bannister ''Surface to Surface and Subsurface to Air Propagation - Quasi-Static and Near Field Ranges,'' paper presented at the AGARD/NATO symposium on Subsurface Communications, Paris, France, 25-29 April 1966; P.R. Bannister and W.C. Hart, The Near Fields of Subsurface Electric Dipole Antennas, USL Report No. 728, 4 March 1966; and P.R. Bannister and W.C. Hart, The Near Fields of Subsurface Magnetic Dipole Antennas, USL Report No. 729, 7 March 1966.

[2] K.A. Norton, ''The Propagation of Radio Waves Over the Surface of the Earth and in the Upper Atmosphere,'' Proceedings of the I.R.E., vol. 25, no. 9, September 1937, pp. 1203-1236.

Table 1

SUBSURFACE-TO-AIR PROPAGATION FORMULAS FOR THE NEAR-FIELD RANGE

Dipole Type	E_ρ	E_ϕ	E_z
VED	$\dfrac{Id\ell\, e^{\gamma_1 b}}{2\pi\sigma_1}\dfrac{\rho e^{-\gamma_0 R}}{R^5}\left[z(3+3\gamma_0 R) - \dfrac{\gamma_0^2}{\gamma_1}R^2(1+\gamma_0 R)\right]$	0	$-\dfrac{Id\ell\, e^{\gamma_1 b}}{2\pi\sigma_1}\dfrac{e^{-\gamma_0 R}}{R^3}\left[1+\gamma_0 R+\gamma_0^2\rho^2 - \dfrac{3z^2}{R^2}(1+\gamma_0 R)\right]$
VMD	0	$-\dfrac{IdA\, e^{\gamma_1 b}}{2\pi\sigma_1}\dfrac{\rho e^{-\gamma_0 R}}{R^5}\left[\left(3+3\gamma_1 z-\dfrac{15z^2}{R^2}\right)(1+\gamma_0 R)+\left(1+\gamma_1 z-\dfrac{6z^2}{R^2}\right)(\gamma_0 R)^2\right]$	0
HED	$\dfrac{Id\ell\cos\phi\, e^{\gamma_1 b}}{2\pi\sigma_1}\dfrac{e^{-\gamma_0 R}}{R^3}\left[\left(1-\gamma_1 z\right)(1+\gamma_0 R)+\gamma_0^2 R^2\right]$	$\dfrac{Id\ell\sin\phi\, e^{\gamma_1 b}}{2\pi\sigma_1}\dfrac{e^{-\gamma_0 R}}{R^3}\left[\left(2+\gamma_1 z-\dfrac{3z^2}{R^2}\right)(1+\gamma_0 R)\right]$	$\dfrac{i\mu_0\omega Id\ell\cos\phi\, e^{\gamma_1 b}}{2\pi\gamma_1}\dfrac{\rho e^{-\gamma_0 R}}{R^3}(1+\gamma_0 R)$
HMD	$\dfrac{i\mu_0\omega IdA\cos\phi\, e^{\gamma_1 b}}{2\pi\gamma_1}\dfrac{e^{-\gamma_0 R}}{R^3}\left[\left(1-\gamma_1 z\right)(1+\gamma_0 R)+\gamma_0^2 R^2\right]$	$\dfrac{i\mu_0\omega IdA\sin\phi\, e^{\gamma_1 b}}{2\pi\gamma_1}\dfrac{e^{-\gamma_0 R}}{R^3}\left[\left(2+\gamma_1 z-\dfrac{3z^2}{R^2}\right)(1+\gamma_0 R)\right]$	$\dfrac{i\mu_0\omega IdA\cos\phi\, e^{\gamma_1 b}}{2\pi}\dfrac{e^{-\gamma_0 R}}{R^3}\rho(1+\gamma_0 R)$

Table 1 (Cont'd)

SUBSURFACE-TO-AIR PROPAGATION FORMULAS FOR THE NEAR-FIELD RANGE

Dipole Type	H_ρ	H_ϕ	H_z
VED	0	$\dfrac{Id\ell}{2\pi}\dfrac{\gamma_o^2}{\gamma_1^2}e^{\gamma_1 b}\dfrac{\rho}{R^3}(1+\gamma_o R)e^{-\gamma_o R}$	0
VMD	$\dfrac{IdA\,e^{\gamma_1 b}}{2\pi\gamma_1}\dfrac{\rho\,e^{-\gamma_o R}}{R^5}\left[\left(3-\dfrac{15z^2}{R^2}\right)(1+\gamma_o R)+\left(1-\dfrac{6z^2}{R^2}\right)(\gamma_o R)^2-\dfrac{z^2}{R^2}(\gamma_o R)^3\right]$	0	$\dfrac{IdA\,e^{\gamma_1 b}}{2\pi\gamma_1^2}\dfrac{e^{-\gamma_o R}}{R^5}\left\{\left[9(1+\gamma_1 z)-\dfrac{15z^2}{R^2}(6+\gamma_1 z)+\dfrac{105z^4}{R^4}\right](1+\gamma_o R)+\left[4(1+\gamma_1 z)-\dfrac{6z^2}{R^2}\left(\dfrac{39}{6}+\gamma_1 z\right)+\dfrac{45z^4}{R^4}\right](\gamma_o R)^2-\left[(1+\gamma_1 z)-\dfrac{z^2}{R^2}(9+\gamma_1 z)+\dfrac{10z^4}{R^4}\right](\gamma_o R)^3\right\}$
HED	$\dfrac{Id\ell\sin\phi\,e^{\gamma_1 b}}{2\pi\gamma_1}\dfrac{e^{-\gamma_o R}}{R^3}\left[\left(2-\dfrac{3z^2}{R^2}\right)(1+\gamma_o R)-\gamma_o^2 z^2\right]$	$-\dfrac{Id\ell\cos\phi\,e^{\gamma_1 b}}{2\pi\gamma_1}\dfrac{e^{-\gamma_o R}}{R^3}\left[1+\gamma_o R+\gamma_o^2 R^2\right]$	$\dfrac{Id\ell\sin\phi\,e^{\gamma_1 b}}{2\pi\gamma_1^2}\dfrac{e^{-\gamma_o R}}{R^4}\left(\dfrac{\rho}{R}\right)\left[\left(3+3\gamma_1 z\right)(1+\gamma_o R)+\left(1+\gamma_1 z-\dfrac{6z^2}{R^2}\right)(\gamma_o R)^2-\dfrac{15z^2}{R^2}\right]$
HMD	$\dfrac{IdA\sin\phi\,e^{\gamma_1 b}}{2\pi}\dfrac{e^{-\gamma_o R}}{R^3}\left\{\left(2-\dfrac{3z^2}{R^2}\right)(1+\gamma_o R)-\gamma_o^2 z^2\right\}$	$-\dfrac{IdA\cos\phi\,e^{\gamma_1 b}}{2\pi}\dfrac{e^{-\gamma_o R}}{R^3}\left(1+\gamma_o R+\gamma_o^2 R^2\right)$	$\dfrac{IdA\sin\phi\,\rho\,e^{\gamma_1 b}\,e^{-\gamma_o R}}{2\pi\gamma_1 R^5}\left[\left(3+3\gamma_1 z-15z^2/R^2\right)(1+\gamma_o R)+(1+\gamma_1 z-6z^2/R^2)(\gamma_o R)^2\right]$

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Table 2

SUBSURFACE-TO-AIR PROPAGATION FORMULAS FOR THE NEAR-FIELD RANGE WITH $\rho >> z$

Dipole Type	E_ρ	E_ϕ	E_z
VED	$-\dfrac{Idl}{2\pi\sigma_1}\dfrac{e^{\gamma_1 b}}{\rho^4}\left[\left(\dfrac{\gamma_0^2}{\gamma_1}\right)(1+\gamma_0\rho)\rho^2 - z(3+3\gamma_0\rho+\gamma_0^2\rho^2)\right]e^{-\gamma_0\rho}$	0	$-\dfrac{Idl}{2\pi\sigma_1}e^{\gamma_1 b}\dfrac{e^{-\gamma_0\rho}}{\rho^3}(1+\gamma_0\rho+\gamma_0^2\rho^2)$
VMD	0	$-\dfrac{IdA\,e^{\gamma_1 b}}{2\pi\sigma_1}\dfrac{e^{-\gamma_0\rho}}{\rho^4}(3+3\gamma_0\rho+\gamma_0^2\rho^2)(1+\gamma_1 z)$	0
HED	$\dfrac{Idl\cos\phi\,e^{\gamma_1 b}}{2\pi\sigma_1}\dfrac{e^{-\gamma_0\rho}}{\rho^3}[(1-\gamma_1 z)(1+\gamma_0\rho)+\gamma_0^2\rho^2]$	$\dfrac{Idl\sin\phi\,e^{\gamma_1 b}}{2\pi\sigma_1}\dfrac{e^{-\gamma_0\rho}}{\rho^3}[(2+\gamma_1 z)(1+\gamma_0\rho)]$	$\dfrac{i\mu_0\omega Idl\cos\phi\,e^{\gamma_1 b}}{2\pi\gamma_1}\dfrac{e^{-\gamma_0\rho}}{\rho^2}[1+\gamma_0\rho]$
HMD	$\dfrac{i\mu_0\omega IdA\cos\phi\,e^{\gamma_1 b}}{2\pi\gamma_1}\dfrac{e^{-\gamma_0\rho}}{\rho^3}[(1-\gamma_1 z)(1+\gamma_0\rho)+\gamma_0^2\rho^2]$	$\dfrac{i\mu_0\omega IdA\sin\phi\,e^{\gamma_1 b}}{2\pi\gamma_1}\dfrac{e^{-\gamma_0\rho}}{\rho^3}[(2+\gamma_1 z)(1+\gamma_0\rho)]$	$\dfrac{i\mu_0\omega IdA\cos\phi\,e^{\gamma_1 b}}{2\pi}\dfrac{e^{-\gamma_0\rho}}{\rho^2}(1+\gamma_0\rho)$

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Table 2 (Cont'd)

SUBSURFACE-TO-AIR PROPAGATION FORMULAS FOR THE NEAR-FIELD RANGE WITH $\rho >> z$

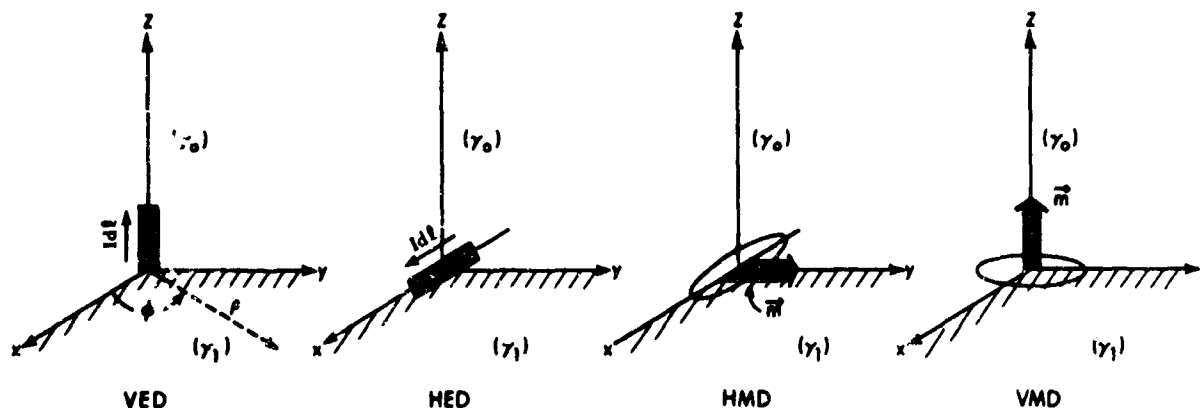| Dipole Type	H_ρ	H_ϕ	H_z
VED	0	$\dfrac{Idl}{2\pi\rho^2}\dfrac{\gamma_0^2}{\gamma_1^2}(1+\gamma_0\rho)e^{\gamma_1 h}e^{-\gamma_0\rho}$	0
VMD	$-\dfrac{IdA\,e^{\gamma_1 h}\,e^{-\gamma_0\rho}}{2\pi\gamma_1}\dfrac{}{\rho^4}(3+3\gamma_0\rho+\gamma_0^2\rho^2)$	0	$-\dfrac{IdA\,e^{\gamma_1 h}}{2\pi\gamma_1^2}\dfrac{e^{-\gamma_0\rho}}{\rho^3}$ $(9+9\gamma_0\rho+4\gamma_0^2\rho^2+\gamma_0^3\rho^3)(1+\gamma_1 z)$
HED	$\dfrac{Idl\,\sin\phi\,e^{\gamma_1 h}}{\pi\gamma_1}\dfrac{e^{-\gamma_0\rho}}{\rho^3}[1+\gamma_0\rho]$	$-\dfrac{Idl\,\cos\phi\,e^{\gamma_1 h}}{2\pi\gamma_1}\dfrac{e^{-\gamma_0\rho}}{\rho^3}$ $[1+\gamma_0\rho+\gamma_0^2\rho^2]$	$\dfrac{Idl\,\sin\phi\,e^{\gamma_1 h}}{2\pi\gamma_1^2}\dfrac{e^{-\gamma_0\rho}}{\rho^4}$ $[(1+\gamma_1 z)(3+3\gamma_0\rho+\gamma_0^2\rho^2)]$
HMD	$\dfrac{IdA\,\sin\phi\,e^{\gamma_1 h}\,e^{-\gamma_0\rho}}{2\pi}\dfrac{}{\rho^3}[2(1+\gamma_0\rho)]$	$-\dfrac{IdA\,\cos\phi\,e^{\gamma_1 h}}{2\pi}\dfrac{e^{-\gamma_0\rho}}{\rho^3}$ $[1+\gamma_0\rho+\gamma_0^2\rho^2]$	$\dfrac{IdA\,\sin\phi\,e^{\gamma_1 h}}{2\pi\gamma_1}\dfrac{e^{-\gamma_0\rho}}{\rho^4}$ $[(3+3\gamma_0\rho+\gamma_0^2\rho^2)(1+\gamma_1 z)]$

Fig. 1 - Dipole Orientations when h = 0⁺

RECIPROCITY THEOREM

The reciprocity theore..n (applicable to dipoles in the presence of any linear media) states that the voltage V_2 induced in antenna 2 by current I_1 of antenna 1 is the same as the voltage V_1 induced in antenna 1 by an identical current I_2 flowing in antenna 2. (For further details, see Carson,[3] Galejs,[4] or Wait.[5]) Application of this theorem, which utilizes the geometry in Fig. 1, results in

$$E_z^{HM} [z, h] = i \omega \mu_0 H_\phi^{VE} \cos \phi \frac{dA}{dl} [h, z],$$

(1)

where

$[a, b]$ = [height or depth of transmitting dipole, height or depth of
receiving dipole] ;

[3] J.R. Carson, ''Reciprocal Theorems in Radio Communication,'' Proceedings of the I.R.E.,vol. 17, no. 6, June 1929, pp. 952-956.

[4] J. Galejs, ''Excitation of VLF and ELF Radio Waves by a Horizontal Magnetic Dipole,'' Radio Science, Journal of Research, National Bureau of Standards, vol. 65D, no. 3 May-June 1961, pp. 305-311.

[5] J.R. Wait, Electromagnetic Waves in Stratified Media, Pergamon Press, Oxford, 1962, pp. 168 - 174.

$$H_z^{HM}[z, h] = -H_\rho^{VM} \sin\phi \frac{dA^{HM}}{dA^{VM}}[h, z];\qquad(2)$$

$$E_z^{HE}[z, h] = -E_\rho^{VE} \cos\phi \frac{dl^{HE}}{dl^{VE}}[h, z];\qquad(3)$$

$$H_z^{HE}[z, h] = \frac{-1}{i\,\omega\,\mu_0} E_\phi^{VM} \sin\phi \frac{dl}{dA}[h, z];\qquad(4)$$

$$H_\rho^{HE}[z, h] = \frac{1}{i\,\omega\,\mu_0} E_\phi^{HM} \frac{dl}{dA}[h, z];\qquad(5)$$

$$H_\phi^{HE}[z, h] = \frac{-1}{i\,\omega\,\mu_0} E_\rho^{HM} \frac{dl}{dA}[h, z];\qquad(6)$$

and

$$E_z^{VE}, H_z^{VM}, H_\rho^{HM}, H_\phi^{HM}, E_\rho^{HE}, E_\phi^{HE}[z, h] = E_z^{VE}, H_z^{VM}, H_\rho^{HM}, H_\phi^{HM}, E_\rho^{HE}, E_\phi^{HE}[h, z].\qquad(7)$$

DERIVATION OF THE NEAR FIELD ELECTRIC AND MAGNETIC FIELD COMPONENTS

By employing the results listed in Tables 1 and 2 and Eqs. (1) through (7), the near field air-to-subsurface field component expressions, which are valid for $|\gamma_1 R'| >> 1$ (i.e., $R' >> \delta$), $h \geq 0^+$, $z \leq 0^-$, and $R' >> |z|$, can be determined. The additional restriction ($|\gamma_0^2 \rho/\gamma_1| << 1$) is required for the vertically polarized components. Also, ionospheric effects are neglected. These field component expressions are as follows:

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for the vertical electric dipole,

$$E_\rho \cong \frac{-I\,d\,l\,i\,\mu_0\,\omega\,e^{\gamma_1 z}}{2\,\pi\,\gamma_1}\;\frac{\rho\,e^{-\gamma_0 R'}}{(R')^3}\,(1+\gamma_0\,R')\,, \tag{8}$$

$$E_z \cong \frac{-I\,d\,l\,e^{\gamma_1 z}}{2\,\pi\,\sigma_1}\;\frac{e^{-\gamma_0 R'}}{(R')^3}\left[\,1+\gamma_0\,R'+\gamma_0^2\,\rho^2-\frac{3\,h^2}{(R')^2}\,(1+\gamma_0\,R')\,\right], \tag{9}$$

and

$$H_\phi \cong \frac{I\,d\,l\,e^{\gamma_1 z}}{2\,\pi}\;\frac{\rho\,e^{-\gamma_0 R'}}{(R')^3}\,(1+\gamma_0\,R')\,; \tag{10}$$

for the vertical magnetic dipole,

$$E_\phi \cong \frac{-I\,d\,A\,e^{\gamma_1 z}}{2\,\pi\,\sigma_1}\;\frac{e^{-\gamma_0 R'}}{(R')^4}\left(\frac{\rho}{R'}\right)\left[\left(3+3\,\gamma_1\,h-\frac{15\,h^2}{(R')^2}\right)(1+\gamma_0\,R')\right.$$
$$\left.+\left(1+\gamma_1\,h-\frac{6\,h^2}{(R')^2}\right)(\gamma_0\,R')^2\right], \tag{11}$$

$$H_\rho \cong \frac{-I\,d\,A\,e^{\gamma_1 z}}{2\,\pi\,\gamma_1}\;\frac{\rho\,e^{-\gamma_0 R'}}{(R')^5}\left[\left(3+3\,\gamma_1\,h-\frac{15\,h^2}{(R')^2}\right)(1+\gamma_0\,R')\right.$$
$$\left.+\left(1+\gamma_1\,h-\frac{6\,h^2}{(R')^2}\right)(\gamma_0\,R')^2\right], \tag{12}$$

and

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$$H_z \cong \frac{-I\,d\,A\,e^{\gamma_1 z}}{2\pi\gamma_1^2}\,\frac{e^{-\gamma_0 R'}}{(R')^5}\left\{\left[9\,(1+\gamma_1 h)-\frac{15\,h^2}{(R')^2}(6+\gamma_1 h)\right.\right.$$

$$\left.+\frac{105\,h^4}{(R')^4}\right]\left[1+\gamma_0 R'\right]+\left[4\,(1+\gamma_1 h)-\frac{6\,h^2}{(R')^2}\left(\frac{39}{6}+\gamma_1 h\right)\right. \tag{13}$$

$$\left.\left.+\frac{45\,h^4}{(R')^4}\right](\gamma_0 R')^2+\left[1+\gamma_1 h-\frac{h^2}{(R')^2}(9+\gamma_1 h)+\frac{10\,h^4}{(R')^4}\right](\gamma_0 R')^3\right\};$$

for the horizontal electric dipole,

$$E_\rho \cong \frac{I\,d\,l\cos\phi\,e^{\gamma_1 z}}{2\pi\sigma_1}\,\frac{e^{-\gamma_0 R'}}{(R')^3}\left[(1-\gamma_1 h)(1+\gamma_0 R')+(\gamma_0 R')^2\right], \tag{14}$$

$$E_\phi \cong \frac{I\,d\,l\sin\phi\,e^{\gamma_1 z}}{2\pi\sigma_1}\,\frac{e^{-\gamma_0 R'}}{(R')^3}\left[\left(2+\gamma_1 h-\frac{3\,h^2}{(R')^2}\right)(1+\gamma_0 R')\right], \tag{15}$$

$$E_z \cong \frac{-I\,d\,l\cos\phi\,e^{\gamma_1 z}}{2\pi\sigma_1}\,\frac{\rho\,e^{-\gamma_0 R'}}{(R')^5}\left[h\left(3+3\,\gamma_0 R'+(\gamma_0 R')^2\right)\right.$$

$$\left.-\frac{\gamma_0^2\,(R')^2}{\gamma_1}(1+\gamma_0 R')\right], \tag{16}$$

$$H_\rho \cong \frac{I\,d\,l\sin\phi\,e^{\gamma_1 z}}{2\pi\gamma_1}\,\frac{e^{-\gamma_0 R'}}{(R')^3}\left[\left(2+\gamma_1 h-\frac{3\,h^2}{(R')^2}\right)(1+\gamma_0 R')\right] \tag{17}$$

$$H_\phi \cong \frac{-I\,d\,l\cos\phi\,e^{\gamma_1 z}}{2\pi\gamma_1}\,\frac{e^{-\gamma_0 R'}}{(R')^3}\left[(1-\gamma_1 h)(1+\gamma_0 R')+(\gamma_0 R')^2\right], \tag{18}$$

and

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$$H_z \cong \frac{I\,d\,l \sin\phi\, e^{\gamma_1 z}}{2\pi\,\gamma_1{}^2}\ \frac{\rho\,e^{-\gamma_0 R'}}{(R')^5} \left[\left(3 + 3\,\gamma_1\,h - \frac{15\,h^2}{(R')^2}\right)(1 + \gamma_0\,R')\right.$$

$$\left.+ \left(1 + \gamma_1\,h - \frac{6\,h^2}{(R')^2}\right)(\gamma_0\,R')^2\right]\ ; \tag{19}$$

and for the horizontal magnetic dipole,

$$E_\rho \cong \frac{I\,d\,A\,i\,\mu_0\,\omega \cos\phi\, e^{\gamma_1 z}}{2\pi\,\gamma_1}\ \frac{e^{-\gamma_0 R'}}{(R')^3}\left[1 + \gamma_0\,R' + (\gamma_0\,R')^2\right]\ , \tag{20}$$

$$E_\phi \cong \frac{I\,d\,A\,i\,\mu_0\,\omega \sin\phi\, e^{\gamma_1 z}}{2\pi\,\gamma_1}\ \frac{e^{-\gamma_0 R'}}{(R')^3}\left[\left(2 - \frac{3\,h^2}{(R')^2}\right)(1 + \gamma_0\,R') - \gamma_0{}^2\,h^2\right]\ , \tag{21}$$

$$E_z \cong \frac{I\,d\,A\,i\,\mu_0\,\omega \cos\phi\, e^{\gamma_1 z}}{2\pi\,\gamma_1}\left(\frac{\gamma_0{}^2\,\rho}{\gamma_1}\right)\frac{e^{-\gamma_0 R'}}{(R')^3}\,(1 + \gamma_0\,R')\ , \tag{22}$$

$$H_\rho \cong \frac{I\,d\,A \sin\phi\, e^{\gamma_1 z}}{2\pi}\ \frac{e^{-\gamma_0 R'}}{(R')^3}\left[\left(2 - \frac{3\,h^2}{(R')^2}\right)(1 + \gamma_0\,R') - \gamma_0{}^2\,h^2\right]\ , \tag{23}$$

$$H_\phi \cong \frac{-\,I\,d\,A \cos\phi\, e^{\gamma_1 z}}{2\pi}\ \frac{e^{-\gamma_0 R'}}{(R')^3}\left[1 + \gamma_0\,R' + (\gamma_0\,R')^2\right]\ , \tag{24}$$

and

$$H_z \cong \frac{I\,d\,A \sin\phi\, e^{\gamma_1 z}}{2\pi\,\gamma_1}\ \frac{\rho\,e^{-\gamma_0 R'}}{(R')^5}\left[\left(3 - \frac{15\,h^2}{(R')^2}\right)(1 + \gamma_0\,R')\right.$$

$$\left.+ \left(1 - \frac{6\,h^2}{(R')^2}\right)(\gamma_0\,R')^2 - \frac{h^2}{(R')^2}(\gamma_0\,R')^3\right]\ . \tag{25}$$

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When $|\gamma_0 R'| << 1$, all these expressions reduce to the quasi-near range results derived by Bannister.[6] Furthermore, when $\rho >> h$, these expressions reduce to the following well-known results:

for the vertical electric dipole when $\rho >> h$,

$$E_\rho \cong \frac{-i\mu_0 \omega I\,dl\,e^{\gamma_1 z}}{2\pi\gamma_1}\ \frac{e^{-\gamma_0 \rho}}{\rho^2}(1 + \gamma_0\rho)\,, \tag{26}$$

$$E_z \cong \frac{-I\,dl\,e^{\gamma_1 z}}{2\pi\sigma_1}\ \frac{e^{-\gamma_0 \rho}}{\rho^3}(1 + \gamma_0\rho + \gamma_0^2\rho^2)\,, \tag{27}$$

and

$$H_\phi \cong \frac{I\,dl\,e^{\gamma_1 z}}{2\pi}\ \frac{e^{-\gamma_0 \rho}}{\rho^2}(1 + \gamma_0\rho)\,; \tag{28}$$

for the vertical magnetic dipole when $\rho >> h$,

$$E_\phi \cong \frac{-I\,dA\,e^{\gamma_1 z}}{2\pi\sigma_1}\ \frac{e^{-\gamma_0 \rho}}{\rho^4}[(1 + \gamma_1 h)(3 + 3\gamma_0\rho + \gamma_0^2\rho^2)]\,, \tag{29}$$

$$H_\rho \cong \frac{-I\,dA\,e^{\gamma_1 z}}{2\pi\gamma_1}\ \frac{e^{-\gamma_0 \rho}}{\rho^4}[(1 + \gamma_1 h)(3 + 3\gamma_0\rho + \gamma_0^2\rho^2)]\,, \tag{30}$$

and

$$H_z \cong \frac{-I\,dA\,e^{\gamma_1 z}}{2\pi\gamma_1^2}\ \frac{e^{-\gamma_0 \rho}}{\rho^5}[(1 + \gamma_1 h)(9 + 9\gamma_0\rho + 4\gamma_0^2\rho^2 + \gamma_0^3\rho^3)]\,; \tag{31}$$

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[6] P.R. Bannister, ''The Quasi-Near Fields of Dipole Antennas,'' a paper being prepared for submission to the IEEE PGAP.

for the horizontal electric dipole when $\rho >> h$,

$$E_\rho \cong \frac{I\,d\,l \cos\phi\, e^{\gamma_1 z}}{2\pi\sigma_1}\; \frac{e^{-\gamma_0 \rho}}{\rho^3}\,[(1 - \gamma_1 h)(1 + \gamma_0 \rho) + \gamma_0^2 \rho^2], \tag{32}$$

$$E_\phi \cong \frac{I\,d\,l \sin\phi\, e^{\gamma_1 z}}{2\pi\sigma_1}\; \frac{e^{-\gamma_0 \rho}}{\rho^3}\,[(2 + \gamma_1 h)(1 + \gamma_0 \rho)], \tag{33}$$

$$E_z \cong \frac{-I\,d\,l \cos\phi\, e^{\gamma_1 z}}{2\pi\sigma_1}\; \frac{e^{-\gamma_0 \rho}}{\rho^4}\left[h(3 + 3\gamma_0 \rho + \gamma_0^2 \rho^2) - \frac{\gamma_0^2 \rho^2}{\gamma_1}(1 + \gamma_0 \rho)\right], \tag{34}$$

$$H_\rho \cong \frac{I\,d\,l \sin\phi\, e^{\gamma_1 z}}{2\pi\gamma_1}\; \frac{e^{-\gamma_0 \rho}}{\rho^3}\,[(2 + \gamma_1 h)(1 + \gamma_0 \rho)], \tag{35}$$

$$H_\phi \cong \frac{-I\,d\,l \cos\phi\, e^{\gamma_1 z}}{2\pi\gamma_1}\; \frac{e^{-\gamma_0 \rho}}{\rho^3}\,[(1 - \gamma_1 h)(1 + \gamma_0 \rho) + \gamma_0^2 \rho^2], \tag{36}$$

and

$$H_z \cong \frac{I\,d\,l \sin\phi\, e^{\gamma_1 z}}{2\pi\gamma_1^2}\; \frac{e^{-\gamma_0 \rho}}{\rho^4}\,[(1 + \gamma_1 h)(3 + 3\gamma_0 \rho + \gamma_0^2 \rho^2)]; \tag{37}$$

and for the horizontal magnetic dipole when $\rho >> h$,

$$E_\rho \cong \frac{I\,d\,A\,i\,\mu_0\,\omega \cos\phi\, e^{\gamma_1 z}}{2\pi\gamma_1}\; \frac{e^{-\gamma_0 \rho}}{\rho^3}\,(1 + \gamma_0 \rho + \gamma_0^2 \rho^2), \tag{38}$$

$$E_\phi \cong \frac{I\,d\,A\,i\,\mu_0\,\omega \sin\phi\, e^{\gamma_1 z}}{\pi\gamma_1}\; \frac{e^{-\gamma_0 \rho}}{\rho^3}\,(1 + \gamma_0 \rho), \tag{39}$$

$$E_z \cong \frac{I\,d\,A\,i\,\mu_0\,\omega \cos\phi\, e^{\gamma_1 z}}{2\pi\gamma_1}\left(\frac{\gamma_0^2}{\gamma_1}\right)\frac{e^{-\gamma_0 \rho}}{\rho^2}\,(1 + \gamma_0 \rho). \tag{40}$$

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$$H_\rho \cong \frac{I\,d\,A\,\sin\phi\,e^{\gamma_1 z}}{\pi} \cdot \frac{e^{-\gamma_0 \rho}}{\rho^3}\,(1 + \gamma_0 \rho)\,, \tag{41}$$

$$H_\phi \cong \frac{-I\,d\,A\,\cos\phi\,e^{\gamma_1 z}}{2\pi} \cdot \frac{e^{-\gamma_0 \rho}}{\rho^3}\,(1 + \gamma_0 \rho + \gamma_0^2 \rho^2)\,, \tag{42}$$

and

$$H_z \cong \frac{I\,d\,A\,\sin\phi\,e^{\gamma_1 z}}{2\pi\gamma_1} \cdot \frac{e^{-\gamma_0 \rho}}{\rho^4}\,(3 + 3\gamma_0 \rho + \gamma_0^2 \rho^2)\,. \tag{43}$$

When $\rho \gg h$, only the VMD and HED field component expressions have transmitting antenna height gain factors. The height gain factor is defined as the ratio of the field strength (at some depth z) when the transmitting antenna is at a height h to that when the transmitting antenna is a height $h = 0^+$. Some numerical results for the various height gain factors when $|\gamma_0 \rho| \ll 1$ are given by Bannister and Hart.[7] (Note that z must be replaced by h and Eqs. (1) through (7) must be employed in order to apply the results obtained by Bannister and Hart to the air-to-subsurface propagation case.)

CONCLUSIONS

The air-to-subsurface electric and magnetic field components produced by vertical and horizontal dipoles located above the surface of a plane, conducting, homogeneous earth have been derived for the near field range. Ionospheric effects have been neglected. When $\rho \gg h$, these field component expressions reduce to well-known expressions. In addition, when $|\gamma_0 R'| \ll 1$, they reduce to the quasi-near range formulas.

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[7] P.R. Bannister and W.C. Hart, The Quasi-Static Fields of Dipole Antennas - Part II, USL Report No. 719, 8 February 1966; Part III, USL Report No. 720, 23 February 1966.

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13. ABSTRACT

The electric and magnetic field components produced by vertical and
horizontal dipoles (both electric and magnetic types) located at or above the
surface of a plane, conducting, homogeneous earth are derived for the near
field range. The height h of the transmitting antenna is $>0^+$, while the depth
z of the receiving antenna is $<0^-$ (air-to-subsurface propagation). Ionospheric
effects are neglected

The derivations are based upon application of the reciprocity theorem
to previously derived field components. It is observed that these equations
reduce to well-known expressions when the horizontal separation (ρ) between
the transmitting and receiving antennas is much greater than h.

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14. KEY WORDS	LINK A		LINK B		LINK C	
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