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INTERNAL CONVECTION EFFECTS IN THERMAL MODELS OF SPACE VEHICLES

Han M. Hsia and Jan A. van der Bliek

ARO, Inc.

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ARO, Inc.

FOREWORD

The research reported herein was done at the request of Headquarters, Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), under Program Element 61440514/8951.

The results of research presented were obtained by ARO, Inc. (a subsidiary of Sverdrup & Parcel and Associates, Inc.), contract operator of AEDC, AFSC, Arnold Air Force Station, Tennessee, under Contract AF 40(600)-1200. The analysis was carried out during the period from June to September, 1966, under ARO Project No. SA0412, and the manuscript was submitted for publication on November 21, 1966.

Dr. Han Min Hsia, Associate Professor, Department of Mechanical Engineering, North Dakota State University, Fargo, North Dakota, was employed by ARO, Inc., in the summer of 1966.

Mr. Donald C. Todd did the computer programming for the numerical calculations.

This technical report has been reviewed and is approved.

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ABSTRACT

The influence of gaseous convection inside a model placed in a space chamber with solar simulation was investigated by means of a numerical calculation. Thermal modeling relations were derived for a model with internal convection.

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NOMENCLATURE

A	Area, m^2
C	Ratio of thermal capacitances, $\frac{C_g M_g}{C_s M_s}$
C_s, C_g	Specific heat, joule/kg-°K
G_r	Grashof number, $\frac{g \ell^3 (T_s - T_g)}{\nu^2 T}$
g	Acceleration of gravity, m/sec^2
h	Heat-transfer coefficient, w/m^2-°K
k	Thermal conductivity, w/m-°K
ℓ	Reference length, m
M	Mass, kg

Nu	Nusselt number, $\frac{h\ell}{k}$
P	Dimensionless parameter, $\frac{\epsilon\sigma A_e}{h\ A_i}\ (T_E - T_o)^3$
Q	Dimensionless parameter, $\frac{T_o}{T_E - T_o}$
q	Heat flux, w/m^2
T	Temperature, °K
t	Time, sec
α	Absorbtivity
$\Delta\theta_s$	$(\theta_s)_{C=0} - (\theta_s)_{C>0}$ at $\theta = 0.8$
$\Delta\tau$	Dimensionless time interval
ϵ	Emissivity
θ	Dimensionless temperature, $\frac{T - T_o}{T_E - T_o}$
ν	Kinematic viscosity, m^2/sec
σ	Stefan-Boltzmann constant, $w/m^2\text{-}°K^4$
τ	Dimensionless time

SUBSCRIPTS

E	Final equilibrium condition
e	External
g	Gas
i	Internal
o	Initial
r	Reference
s	Solid, solar

SECTION I
INTRODUCTION

In thermal testing of space vehicles in a space simulation chamber there is the question of the influence of the gas (or liquid) in the spacecraft on the heat balance and the temperature-time history of the spacecraft. Also, if the model is a scaled version of the actual vehicle and if the gas significantly influences the temperature-time history, a scaling procedure must be devised to produce the proper convection effects. Tests carried out in a space chamber with gravity-induced internal convection do not simulate the free space conditions. It is therefore useful to determine the limiting conditions for which the internal convection can be neglected.

Although it is clear that the liquid in a thin-walled tank influences markedly the temperature-time history of such a tank when it is subjected to a periodic solar source in cold space, it is not immediately apparent when the heat capacity of a gas and the heat transmission through the gas are important.

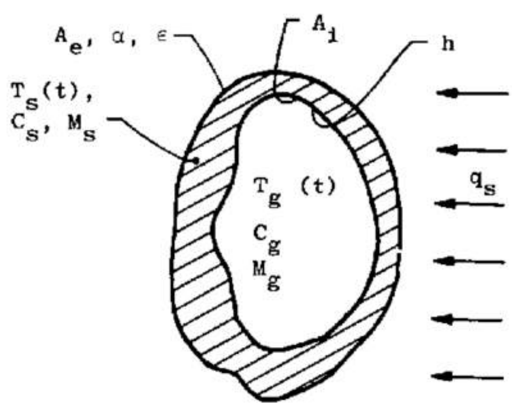
In this report the temperature-time history of an arbitrary shaped container, filled with a gas, is calculated. The container is placed in a cold (0°K) vacuum chamber and suddenly subjected to a simulated solar source maintained at one solar constant. This includes, for example, a simplified case of a space capsule carrying its own atmosphere.

The model used in the present calculations is very simple compared to many actual cases. However, it is believed that the results are useful as a guide in assessing the problem in practical cases.

SECTION II
ANALYSIS

2.1 BASIC EQUATIONS

Consider an arbitrary shaped container of temperature T_s with a gas at temperature T_g. The container is placed in a cold (0°K) vacuum environment and it is subjected to parallel radiation, q_s, over its projected surface area, A_s, normal to the radiation direction. For simplicity the thermal conductivity of the



solid is assumed to be infinite. The energy balance of the solid is:

$$\alpha q_s A_s - \epsilon\sigma T_s{}^4 A_e - h(T_s - T_g)A_i = C_s M_s \frac{dT_s}{dt} \tag{1}$$

and the energy balance for the gas is:

$$h(T_s - T_g)A_i = C_g M_g \frac{dT_g}{dt} \tag{2}$$

The heat-transfer coefficient, h, is an average value over the temperature range considered.

Under steady-state conditions, Eqs. (1) and (2) give:

$$\alpha q_s A_s = \epsilon\sigma T_E{}^4 A_e \tag{3}$$

where T_E is the equilibrium temperature. Substitution of Eq. (3) in Eq. (1) gives:

$$\epsilon\sigma A_e(T_E{}^4 - T_s{}^4) - h(T_s - T_g)A_i = C_s M_s \frac{dT_s}{dt} \tag{4}$$

The following nondimensional parameters and variables are introduced:

$$\theta_s = \frac{T_s - T_{s,o}}{T_E - T_{s,o}} \qquad \theta_g = \frac{T_g - T_{g,o}}{T_E - T_{s,o}} \qquad \tau = \frac{hA_i}{C_s M_s}\, t$$

$$C = \frac{C_g M_g}{C_s M_s} \qquad P = \frac{\epsilon\sigma A_e}{h A_i}(T_E - T_o)^3 \qquad Q = \frac{T_o}{T_E - T_o}$$

We shall assume for the initial temperatures $T_{s,o} = T_{g,o} = T_o$, although no special difficulty is introduced when $T_{s,o} \neq T_{g,o}$. Substitution in Eqs. (2) and (4) gives the basic equations:

$$\theta_s - \theta_g = C \frac{d\theta_g}{d\tau} \tag{5}$$

$$P\left[(1 + Q)^4 - (\theta_s + Q)^4\right] - (\theta_s - \theta_g) = \frac{d\theta_s}{d\tau} \tag{6}$$

The boundary conditions are:

$$\tau = 0: \quad \theta_s = \theta_g = 0$$

$$\tau \to \infty: \quad \theta_s = \theta_g = 1$$

2.2 NUMERICAL METHOD

Equations (5) and (6) can be written in finite difference form:

$$\theta_{g,n} = \frac{\Delta\tau}{C}\,\theta_{s,n-1} + \left(1 - \frac{\Delta\tau}{C}\right)\theta_{g,n-1} \tag{7}$$

where $n \geq 1$ and $\theta_{g,o} = \theta_{s,o} = 0$

$$\theta_{s,n+1} = \theta_{s,n} + \Delta\tau \left\{ P\left[(1+Q)^4 - (\theta_{s,n} + Q)^4\right] - (\theta_{s,n} - \theta_{g,n}) \right\} \tag{8}$$

where $n \geq 0$ and $\theta_{g,o} = \theta_{s,o} = 0$

Equation (7) is a recursion formula for θ_g and can be written as:

$$\theta_{g,n} = \frac{\Delta\tau}{C}\left[\theta_{s,n-1} + \left(1 - \frac{\Delta\tau}{C}\right)\theta_{s,n-2} + \left(1 - \frac{\Delta\tau}{C}\right)^2 \theta_{s,n-3} + \ldots + \left(1 - \frac{\Delta\tau}{C}\right)^{n-2} \theta_{s,1}\right]$$

or

$$\theta_{g,n} = \frac{\Delta\tau}{C}\left[\theta_{s,n-1} + \sum_{j=2}^{j=n-1} \left(1 - \frac{\Delta\tau}{C}\right)^{j-1} \theta_{s,n-j}\right] \tag{9}$$

The convergence criterion for the series in Eq. (9) is:

$$\left|\left(1 - \frac{\Delta\tau}{C}\right)\frac{\theta_{s,n-j}}{\theta_{s,n-j-1}}\right| < 1$$

or also:

$$\left|\left(1 - \frac{\Delta\tau}{C}\right)\frac{\theta_{s,i}}{\theta_{s,i+1}}\right| < 1$$

where i is an arbitrary number in the regime of the series. This can also be written as:

$$1 - \frac{\theta_{s,i+1}}{\theta_{s,i}} < \frac{\Delta\tau}{C} < 1 + \frac{\theta_{s,i+1}}{\theta_{s,i}} \tag{10}$$

In the present problem

$$\frac{\theta_{s,i+1}}{\theta_{s,i}} > 1$$

and the series can be made to converge. The value of $\Delta\tau$ should be chosen such that the error in θ is as small as desired. For example, by choosing the first temperature increase, $\theta_{s,1} = 0.02$, the following criterion results from Eq. (8):

$$\Delta\tau = \frac{0.02}{P\left[(1+Q)^4 - Q^4\right]} \tag{11}$$

The value of $\Delta\tau$ is determined by Eqs. (10) and (11) and is dependent on C, P, and Q. For gases, even high pressure gases, the range of C was such that these conditions could be satisfied. However, it may not be possible to apply the same numerical technique to liquids in thin-walled containers.

SECTION III
NUMERICAL RESULTS AND DISCUSSION

The temperatures of the solid (θ_s) and gas (θ_g) were calculated as a function of time (τ) with Eqs. (7) and (8). Numerical solutions were obtained for Q = 0.1, 0.3, amd 0.5 and P = 0.2, 0.4, and 0.6 with appropriate values of C, ranging from 0.02 to 0.4.

These values of Q correspond to T_E/T_o = 11, 4.33, and 3, respectively. The equilibrium temperature, T_E, is plotted versus α/ϵ in Fig. 1 for a flat plate (A_s/A_e = 1/2) and a sphere (A_s/A_e = 1/4), assuming radiation of one solar constant (q_s = 1395 w/m^2). The ratio α/ϵ ranges from a few tenths for light colored paints to over ten for polished zinc so that the regime of interest is T_E = 200 to 600°K. The values of Q correspond then to the range T_o = 60 to 200°K for the configurations in Fig. 1. In many cases T_o will be higher, which corresponds to a value of Q greater than 0.5. As will be seen, this would correspond to smaller convection effects.

A high value of P corresponds to a relatively small convective heat-transfer contribution as is apparent from the definition of P and Eq. (6). The value of h was estimated from the convective heat-transfer relation, Nu ~ $(Gr)^{1/4}$, where the proportionality constant was taken somewhat lower than for free external convection. For example, for a sphere of one meter diameter filled with air at from 1 to 100 atm at room temperature, it was estimated that h = 3 to 30 w/m^2-°K. If further ϵ = 1.0, A_e/A_i = 1, the range of P is from 0.05 to 0.5.

The quantity C is the ratio of thermal capacity of the gas to that of the enclosure. For air at one atmosphere enclosed in a spherical metal shell with a wall thickness of one hundredth of its diameter, C is of the order of 0.01.

Typical results of the numerical calculations are shown in Fig. 2. All other curves had the same character. The case of C = 0, no gas enclosed, was calculated separately and is shown in Fig. 2.

There are several ways in which the curves can be characterized. The method chosen here is shown in Fig. 3. The decrease in θ_s at θ_s = 0.8, caused by the presence of the gas at a given value of P and Q, was used as a measure of the effect of the gas on the temperature-time history of the solid. For all cases considered, this also corresponded closely to the maximum temperature deviation during the heating time. For the cases of Fig. 3, $\Delta\theta_s < 0.5C$. This certainly holds for Q > 0.5, which corresponds to

$$\frac{T_E}{T_o} = 1 + \frac{1}{Q} < 3$$

Most practical cases will satisfy the condition $\frac{T_E}{T_o} < 3$. Furthermore,

$$\Delta T = T_s - T_g = \frac{T_o}{Q} \Delta\theta_s$$

If, typically, $C < 0.01$ and $T_o < 300°K$ we find for $Q > 0.5$,

$$\Delta T < \frac{T_o}{Q} 0.5C \text{ or } \Delta T < 3°K$$

Therefore the major conclusion from Fig. 3 is that when $C << 0.1$ the effect of the gas on the temperature of the solid is very small and probably within the measuring accuracy in many tests. This is of course only true when P is greater than, say, 0.2. The convection heat-transfer coefficient, h, is proportional to the one-fourth power of the acceleration. In the zero gravity condition, the value of h will be determined by conduction through the gas only. It is expected that h for zero gravity is always smaller than in a space chamber, and that P will be larger, and therefore from Fig. 3, $\Delta\theta_s$ somewhat smaller than in a space chamber. However, since the thermal conductivity for gases is very low, it is expected that the time to reach equilibrium within the gas is longer when conduction is the only mode of heat transfer, which may make the present analysis subject to some inaccuracy.

SECTION IV
THERMAL SCALING

With the differential equations for the unknown temperatures T_s and T_g given in dimensionless form, the similarity parameters to be preserved in a scaled model test are given by the nondimensional coefficients in the equations. From Eqs. (5) and (6) it appears that these coefficients are C, P, and Q, and the characteristic time

$$t_r = \frac{C_s M_s}{h A_i}$$

is scaled accordingly. An alternate arrangement avoiding the presence of h in the characteristic time is given below.

Substitution of $\theta_g = \frac{T_g}{T_r}$, $\theta_s = \frac{T_s}{T_r}$ and $\tau = \frac{t}{t_r}$ in Eqs. (2) and (4) gives, after rearranging:

$$\theta_s - \theta_g = \frac{C_g M_g}{h A_i t_r} \frac{d\theta_g}{d\tau}$$

or also:

$$\theta_s - \theta_g = \left(\frac{C_g M_g}{C_s M_s}\right)\left(\frac{\epsilon\sigma A_e T_r^3}{hA_i}\right)\left(\frac{C_s M_s}{\epsilon\sigma A_e T_r^3 t_r}\right)\frac{d\theta_g}{d\tau} \qquad (12)$$

and

$$\left(\frac{\epsilon\sigma A_e T_r^3}{hA_i}\right)\left[\left(\frac{T_E}{T_r}\right)^4 - \theta_s^4\right] - (\theta_s - \theta_g) = \left(\frac{C_s M_s}{\epsilon\sigma A_e T_r^3 t_r}\right)\left(\frac{\epsilon\sigma A_e T_r^3}{hA_i}\right)\frac{d\theta_s}{d\tau} \qquad (13)$$

Apparently similarity between prototype and model is assured when the following parameters are preserved:

$$\left(\frac{C_g M_g}{C_s M_s}\right), \left(\frac{\epsilon\sigma A_e T_r^3}{hA_i}\right), \left(\frac{C_s M_s}{\epsilon\sigma A_e T_r^3 t_r}\right), \text{ and} \left(\frac{T_E}{T_r}\right)$$

where

$$T_E = \sqrt[4]{\frac{\alpha q_s A_s}{\epsilon\sigma A_e}}$$

In practical cases the variation of q_s in a space chamber is limited. Also, it is difficult to change ϵ and α at will so that temperature and material preservation (at least to the extent of its radiative properties) are desirable. Preservation of the external geometry gives

$$\left(\frac{A_s}{A_e}\right)_{\text{model}} = \left(\frac{A_s}{A_e}\right)_{\text{prototype}}$$

If $T_r = T_E$ in both cases, the scaling parameters become:

$$\left(\frac{C_g M_g}{C_s M_s}\right), \left(\frac{A_e}{hA_i}\right), \text{ and} \left(\frac{C_s M_s}{A_e t_r}\right)$$

Since it is expected that under space conditions h is much smaller than under laboratory conditions, the internal area, A_i, has to be reduced accordingly for the model test or h has to be reduced by placing internal low conductivity partitions in the model.

The problem of scaling the thermal masses with temperature and material preservation was discussed by Adkins*; similar procedures would be applicable here.

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*D. L. Adkins. "Scaling of Transient Temperature Distributions of Simple Bodies in a Space Chamber." Thermophysics and Temperature Control of Spacecraft and Entry Vehicles. Academic Press, New York, 1966, G. B. Heller, editor.

Finally it is noted that the time for corresponding temperatures between model and prototype is given by:

$$(t)_{model} = \frac{(C_s M_s / A_e)_{model}}{(C_s M_s / A_e)_{prototype}} (t)_{prototype}$$

SECTION V
CONCLUSIONS

The influence of internal convection in a model in a space chamber on the temperature of the model was investigated with a numerical computation.

It was found that for gases with total thermal capacitance below one percent of the thermal capacitance of the test vehicle, the influence of convection is negligible under "normal" conditions ($Q > 0.5$ or $T_E/T_O < 3$). Charts are presented showing the effect of convection.

Thermal modeling rules for transient heating with internal convection were derived. For simulating space conditions, the internal convection can be reduced by partitioning the model internally.

APPENDIX
ILLUSTRATIONS

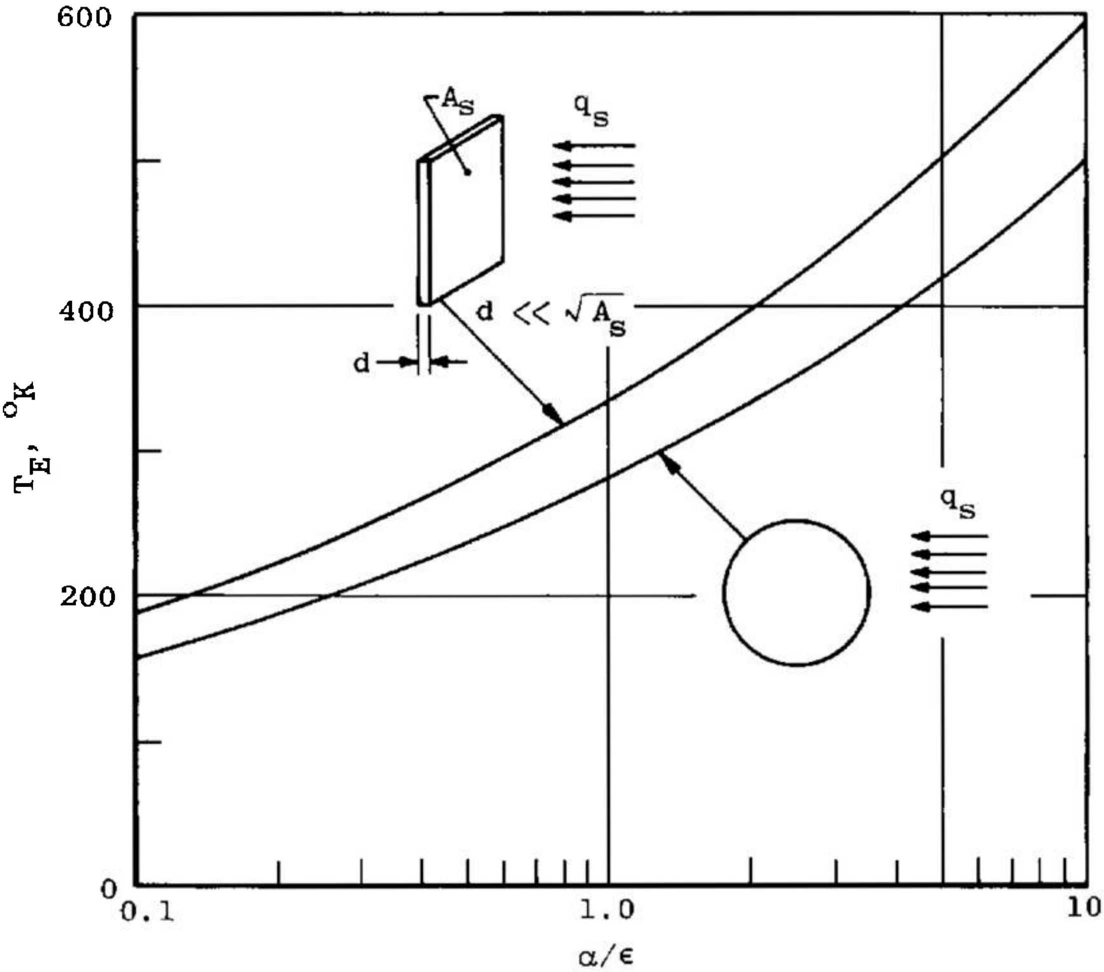

Fig. 1 Equilibrium Temperature of Bodies in Cold Space, Exposed to Solar Radiation

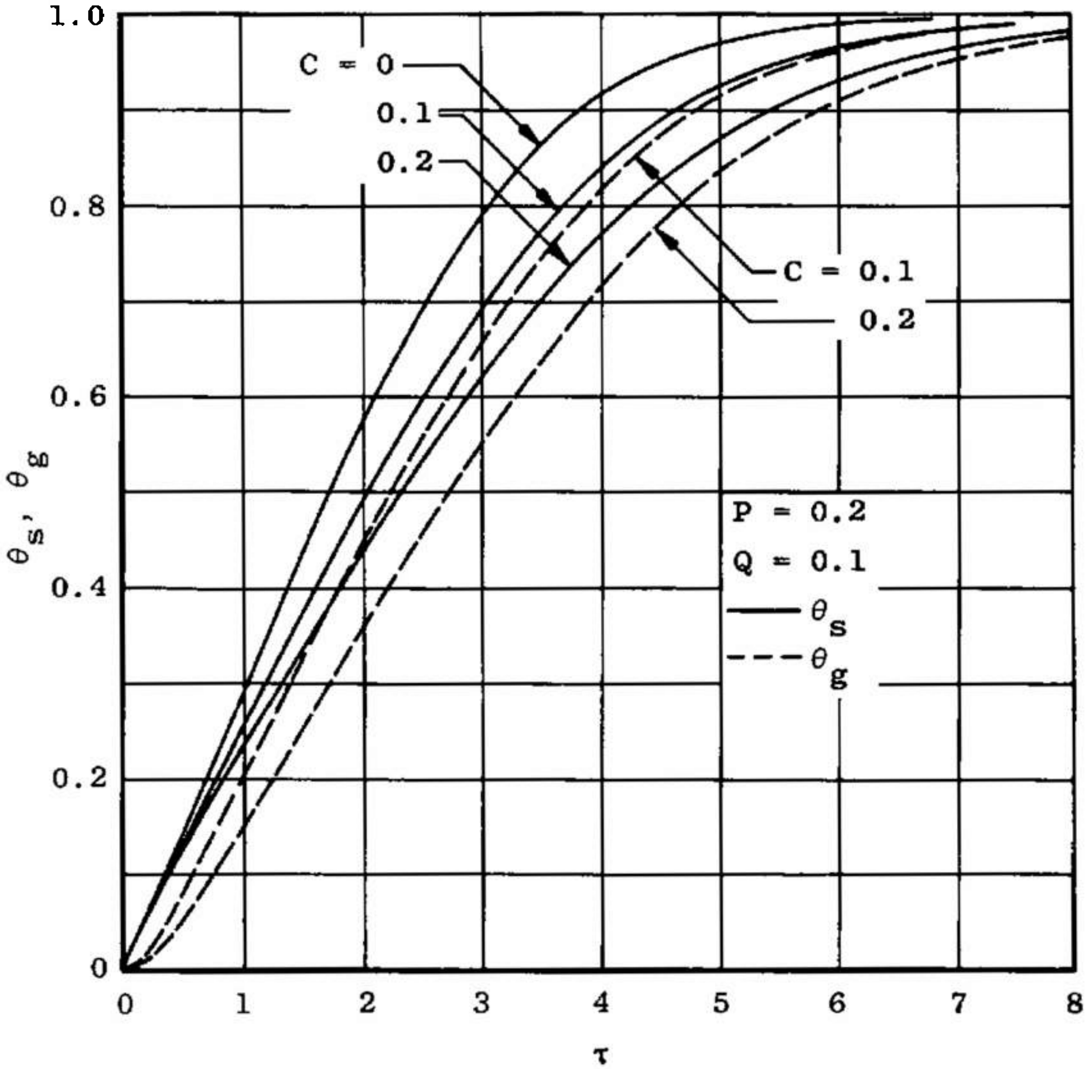

Fig. 2 Temperature of Solid and Gas as a Function of Time

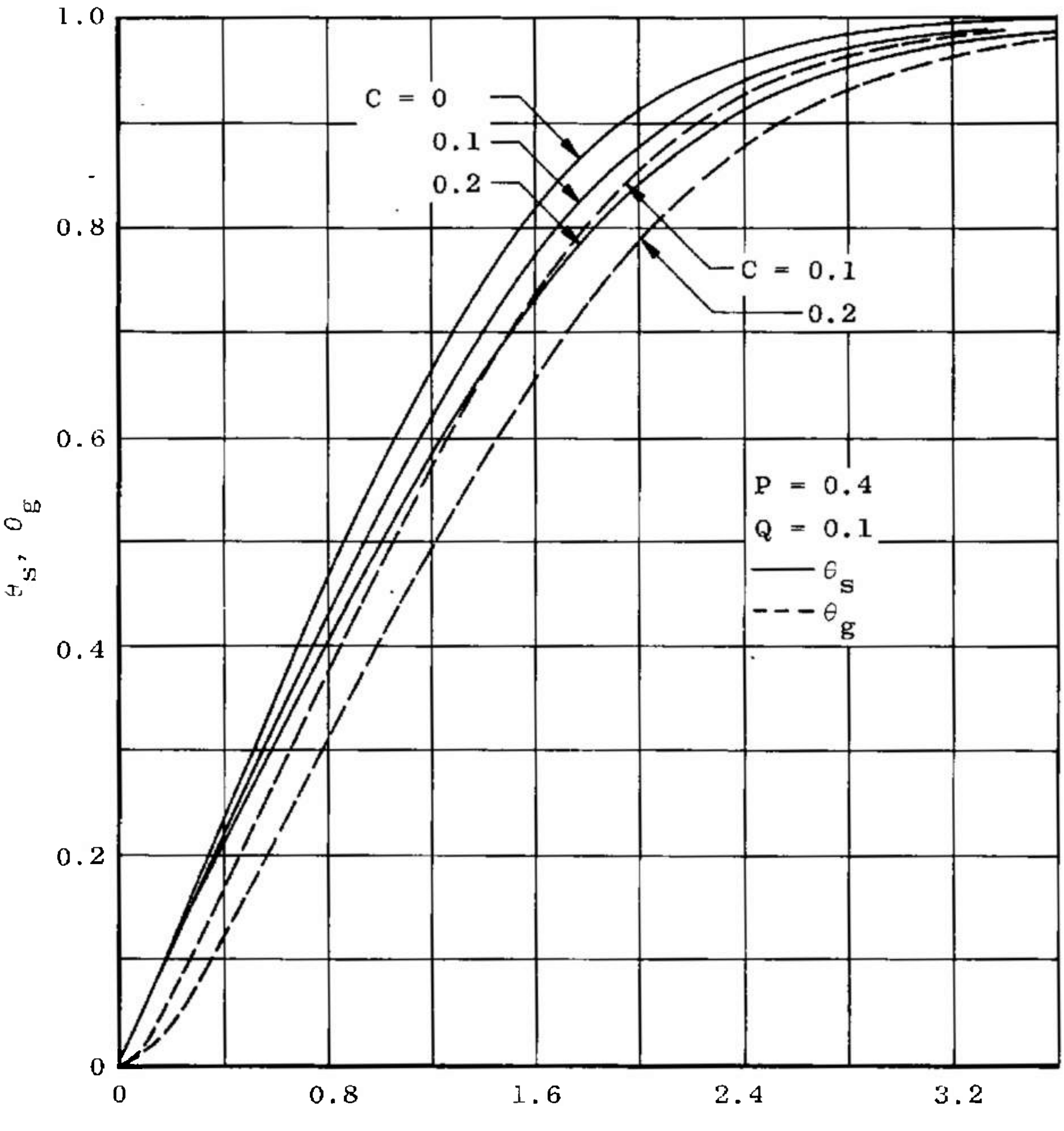

Fig. 2 Continued

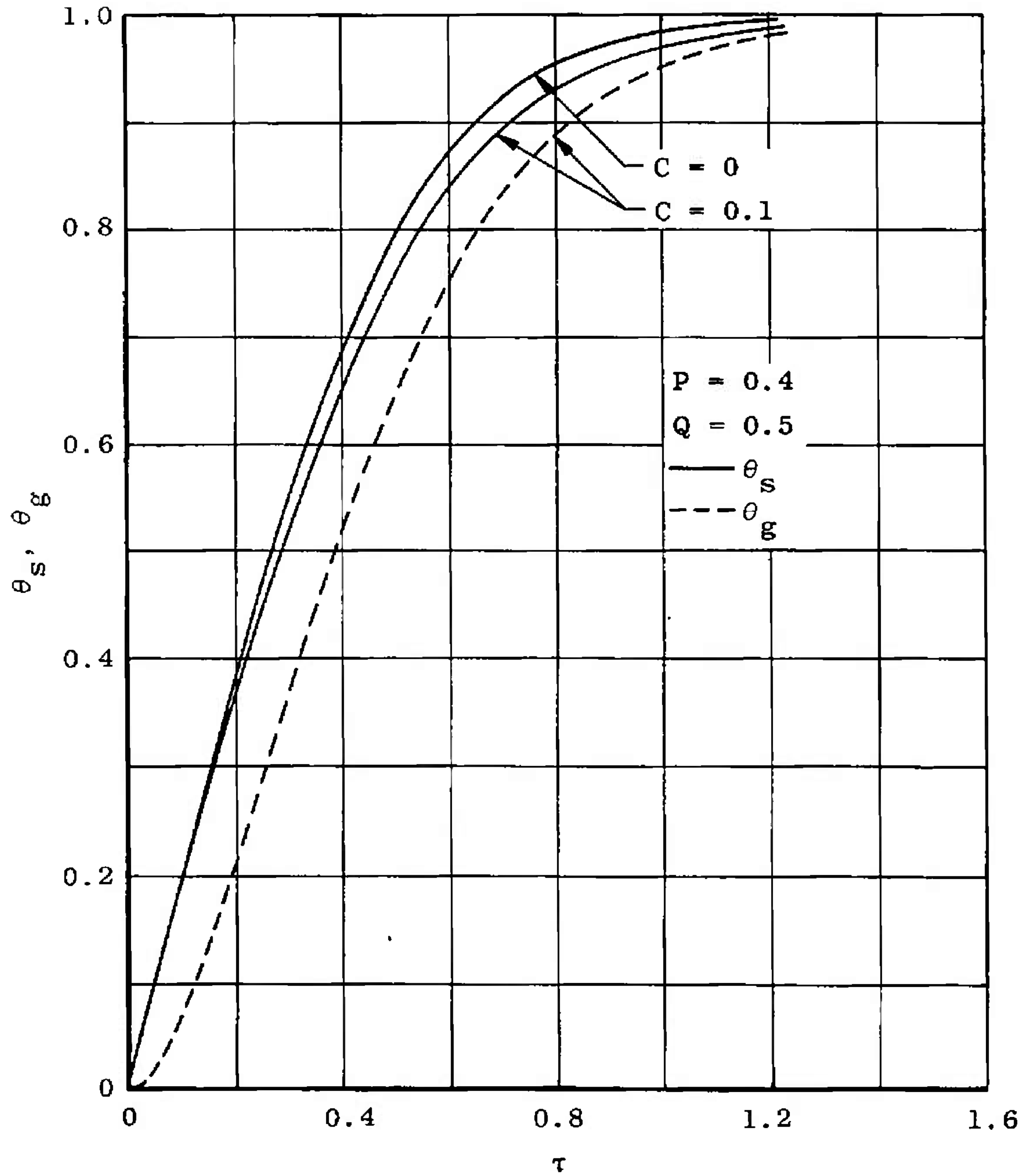

Fig. 2 Concluded

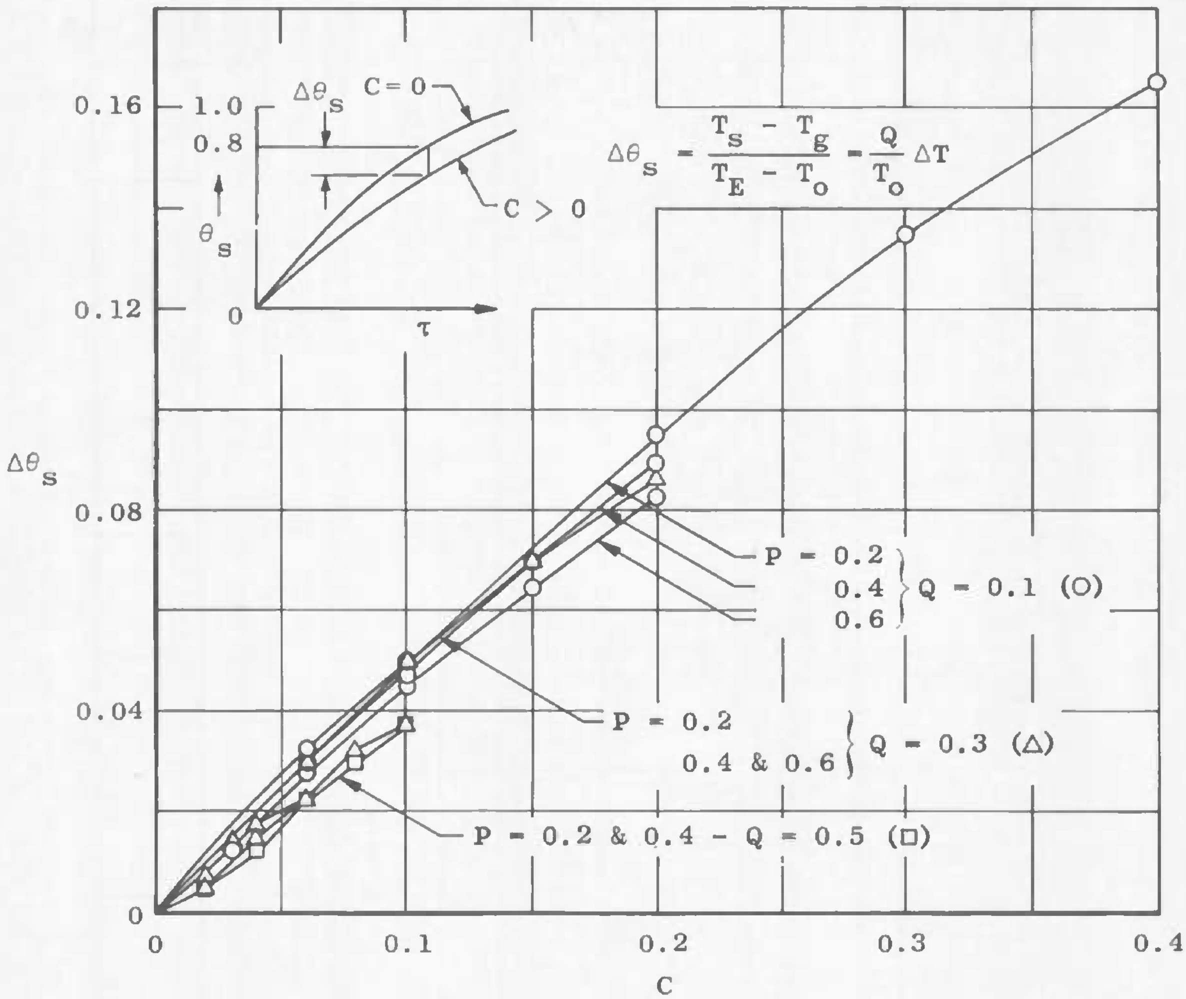

Fig. 3 Effect of Gas on Temperature of Solid during Transient Conditions

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