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Abstract

This report presents the results of an analysis of data obtained from stereophotographs of the wave pattern in the wake of a ship model moving at constant speed ($V/\sqrt{gL} = 0.367$). The data were used to compute the wave resistance of the model following a procedure suggested by Eggers. Several methods of carrying out this procedure are discussed and the numerical results from two of them are compared with the residuary resistance obtained from a towing test. The wave resistance obtained by a least-squares method of analysis is quite close to the residuary resistance.

Introduction

The work reported here attempts to exploit data taken for another purpose by Sorensen [1966] for his dissertation. Sorensen was primarily interested in studying the wave pattern behind a ship moving along a canal. For this purpose he and Moffitt [1966] took stereoscopic photographs of the water surface behind a ship model in the Ship Towing Tank of the University of California. From these photographs Moffitt was able to read the wave height accurately to 0.15 mm. He prepared tables of the wave height at five different stations behind the ship at intervals of 0.0117 m. The location of the five stations and of the readings along each station are shown in Fig. 1. The values themselves are reproduced in Table 1 and the wave profiles are shown in Figs. 2 to 6.

The dimensions of the towing tank in which the photographs were taken are as follows:

breadth = 2 b = 8 ft., depth = 6 ft., length = 200 ft.

The still-water depth h at the time of the experiment was 5 ft.

The model speed when the photographs were taken was 4.7962 ft/sec. This corresponds to a ship Froude number $v/\sqrt{gL} = 0.367$ and also to a depth Froude number $v/\sqrt{gh} = 0.378$. The dimensions of the ship model are as follows:

Length = 5.333 ft., Beam = 0.667 ft.
Draft = 0.333 ft.
Displacement = 34.87 lbs.
Wetted surface area = 4.59 $ft.^2$

Although Sorensen was interested in the wave pattern as such, the same data could also be used for an investigation of the amount of wave energy being produced by the ship per unit time, or equivalently, of the wave resistance of the ship. Such an investigation seemed particularly timely inasmuch as there has been widespread interest since about 1960 in methods of measuring the wave resistance directly from wave profiles aft of the ship. The available profile data fitted naturally into a recently developed theory (Eggers, 1962) of wave-resistance determination from profile measurements. The original intention was to make several 'numerical experiments' with the data and then, if these worked out successfully, to use the computer programs with new data in order to test the procedure more thoroughly. This second step has been postponed. There is so much research in this problem being carried on simultaneously at various places that it seemed wiser to postpone further work until we could benefit from the insights to be gained from the other work.

Because of the somewhat tentative character of the work reported here, no attempt is made to provide an exhaustive bibliography and discussion of other work along the same lines. However, we wish to call attention especially to the work of Sharma (1963, 1964), Gadd and Hogben (1963) and Kobus (1965). There are in addition several papers in the International Symposium on Theoretical Wave Resistance, Ann Arbor, 1963 treating various aspects of the theory.

In the following sections of this report the results of the theory will first be summarized without proof. Thereafter the results of using the profile data in the theory and comparison with measurements by conventional means will be given. Finally some conclusions concerning the practical use of the method will be discussed.

Eggers' Theory

Let a ship, or ship model, be moving with constant velocity c along a rectangular canal of width $2b$ and still-water depth h. Let the z-axis be taken vertically upwards, the (x,z)-plane in the center of the canal, the (x,y)-plane in the undisturbed water surface, the positive x-direction in the direction of ship motion, and the coordinate system moving with the velocity of the ship. It will be convenient to suppose the (y,z)-plane to intersect the ship, say at its midsection. However, it is not necessary to assume the ship to be moving down the center of the canal, although, in fact, this is the situation we shall be concerned with later. We shall further suppose the water to be inviscid, and of density ρ, and its motion to be irrotational and steady in the coordinate system Oxyz. Hence there exists a velocity potential $\varphi(x,y,z)$ such that the absolute velocity of the water particles (i.e., the velocity relative to a fixed coordinate system) is given by

$$(u, v, w) = (\varphi_x, \varphi_y, \varphi_z),$$

Let the surface of the water be described by

$$z = \zeta(x,y)$$

Now consider some plane $x = x_0 < 0$ behind the ship. The wave resistance (there is no other resistance under our assumptions) of the ship is then given by the following 'exact' formula:

$$R = \frac{1}{2}\rho g \int_{-b}^{b} \zeta^2(x_0,y)\,dy + \frac{1}{2}\rho \int_{-b}^{b} dy \int_{-h}^{\zeta(x_0,y)} dz\,[-\varphi_x^2(x_0,y,z) + \varphi_y^2 + \varphi_z^2] \,. \tag{1}$$

The formula is 'exact' in the sense that no further approximations or assumptions beyond those already made are needed to derive it. However, insofar as all effects of viscosity and surface tension have been neglected in its derivation, it is not really exact. How exact it is, could be estimated by comparison of two experimental determinations of R . In one, R is determined by direct measurements of the ingredients of (1). In the other, it is also determined by the conventional method used in towing tanks in which the total resistance is measured with a dynamometer on the towing carriage and then decreased by an estimated value of the 'viscous' resistance. The difficulties attached to the estimation of the 'viscous' resistance are well known and will not be further discussed here. However, the lack of reliable values for the viscous resistance means that one cannot take the second measurement as a real standard of comparison.

Direct measurement of the various quantities entering into the integrals in (1) is not an easy task for it requires not only measurement of the surface profile at $x = x_0$, but also of the velocity components in the section $x = x_0$ at a sufficient number of points to allow an accurate determination of the second integral in (1). Eggers (1962) has proposed a way of avoiding this difficult measurement. He assumes that it is possible to take the section $x = x_0$ far enough aft of the ship so that the exact boundary conditions on the free surface may be well approximated by the usual linearized ones. This is, of course, a very reasonable assumption under most circumstances. However, since the wave energy cannot spread out as in the open ocean but must all be channeled down the canal, one can conceive of situations where the approximation may not be completely satisfactory. On the other hand, it is not likely to give misleading results in any practical situation and is almost certainly of less importance than the neglect of viscosity in (1). In any case, if one makes this assumption, one can show that for $x \leq x_0$ one may represent $\varphi(x,y,z)$ and $\zeta(x,y)$ as follows:

$$\varphi(x,y,z) = \sum_{\nu} \frac{g}{c\,k_\nu}\,(a_\nu \cos k_\nu x + b_\nu \sin k_\nu x)\cos\frac{\nu\pi}{2b}(y-b)\,\frac{\cosh\mu_\nu(z+h)}{\cosh\mu_\nu h} \qquad (2)$$

$$+ \sum_{\nu,n} c_{\nu n} e^{k_{\nu n}x} \cos\frac{\nu\pi}{2b}(y-b)\cos\mu_{\nu n}(z+h) \quad ,$$

$$\zeta(x,y) = \sum_{\nu} (-a_\nu \sin k_\nu x + b_\nu \cos k_\nu x)\cos\frac{\nu\pi}{2b}(y-b)$$

$$+ \sum_{\nu,n} c_{\nu n} k_{\nu n} \cos\mu_{\nu n} h\, e^{k_{\nu n}x} \cos\frac{\nu\pi}{2b}(y-b) \qquad (3)$$

Here μ_ν , $\nu = 0, 1, 2, \ldots$, are the positive solutions of the equations

$$\frac{c^2}{gh}\left[\mu h - \left(\frac{\nu\pi h}{2b}\right)^2 \frac{1}{\mu h}\right] = \tanh \mu h \quad , \qquad (4)$$

and $\mu_{\nu n}$, $\nu = 0, 1, \ldots$, $n = 0, 1, \ldots$, are the positive solutions of

$$\frac{c^2}{gh}\left[\mu h + \left(\frac{\nu\pi h}{2b}\right)^2 \frac{1}{\mu h}\right] = \tan \mu h \quad . \qquad (5)$$

If $c^2/gh > 1$, μ_0 does not exist, but μ_{00} does; if $c^2/gh < 1$, μ_{00} exists, but not μ_0 . Hence in the summations of (2) and (3), if the first one starts with $\nu = 0$ $(\nu = 1)$ the second one starts with $\nu = 1$ $(\nu = 0)$. In the second summation n starts with 0. The constants k_ν and $k_{\nu n}$ are defined by

$$k_\nu^2 = \mu_\nu^2 - \left(\frac{\nu\pi}{2b}\right)^2 ,$$

$$k_{\nu n}^2 = \mu_{\nu n}^2 + \left(\frac{\nu\pi}{2b}\right)^2 . \qquad (6)$$

Eggers has shown that substitution of (2) and (3) into (1) yields

$$R=\frac{1}{4}\rho g b\sum_{\nu}(a_\nu^2+b_\nu^2)\left[1-\frac{2\mu_\nu h+\sinh 2\mu_\nu h}{2\sinh 2\mu_\nu h}\;\frac{gh\tanh\mu_\nu h}{c^2\mu_\nu h}\right] . \quad (7)$$

Evidently R is independent of the coefficients $C_{\nu n}$ and depends only upon a_ν and b_ν through the combination $a_\nu^2 + b_\nu^2$ The practical advantage of (7) is that one can hope to determine the coefficients a_ν and b_ν from (3) by appropriate measurements of $\zeta(x,y)$, a quantity much more accessible to measurement than the velocity components in the whole vertical section $x = x_0$.

From an examination of (3) several possibilities suggest themselves. An attractive possibility is that one can find a value $x_0' < x_0 < 0$ such that for $x \leq x_0'$ the second summation in (3) can be neglected on account of the negative exponential factors. Let $x_1 < x_2 \leq x_0'$. Then, using the orthogonality of the functions $\left\{\cos\frac{\nu\pi}{2b}(y-b)\right\}$ in the interval $-b \leq y \leq b$ one may derive from (3) the equations

$$-\sin k_\nu x_i\, a_\nu + \cos k_\nu x_i\, b_\nu = \zeta_{\nu i} \quad , i = 1, 2, \ldots \qquad (8)$$

where

$$\zeta_{\nu i} = \frac{1}{b}\int_{-b}^{+b} \zeta(x_i, y)\cos\frac{\nu\pi}{2b}(y-b)\, dy .$$

From (8) one finds easily

$$a_\nu = \frac{\zeta_{\nu 1}\cos k_\nu x_2 - \zeta_{\nu 2}\cos k_\nu x_1}{\sin k_\nu (x_2 - x_1)} ,$$

$$b_\nu = \frac{\zeta_{\nu 1}\sin k_\nu x_2 - \zeta_{\nu 2}\cos k_\nu x_1}{\sin k_\nu (x_2 - x_1)} , \qquad (9)$$

$$a_\nu^2 + b_\nu^2 = \frac{1}{\sin^2 k_\nu(x_2 - x_1)}\left[\zeta_{\nu 1}^2 + \zeta_{\nu 2}^2 - 2\zeta_{\nu 1}\zeta_{\nu 2}\cos k_\nu(x_2 - x_1)\right] \quad (9)$$

The last formula in (9) is equivalent to one given by Eggers. Hence measurement of the wave profile along two sections perpendicular to the path of the ship and sufficiently far behind it should allow one to make the numerical quadratures indicated in (9) and thence to compute (7). One must be careful, however, to select $x_2 - x_1$ so that $k_\nu(x_2 - x_1)$ is not near $m\pi$, m = 0, 1, 2, ... , for although these singularities in the denominators are in principle compensated for in the numerators, they may, in fact, lead to difficulties in the numerical analysis.

In the present case there were available measurements of $\zeta(x,y)$ for five different values of x_i. According to the preceding paragraph, any pair of these should be sufficient to determine R. However, to use only two profiles when several are available would be to neglect a large amount of equally reliable data. In order to make use simultaneously of all the information available, the method of least squares may be used. This is explained in Appendix II. Suppose that profile measurements $\zeta(x_i, y)$ are available for $i = 1, \dots N$. Then, according to the method of least squares, the best determination of a_ν and b_ν is obtained by solving the following equations (derived in Appendix II):

$$\begin{aligned}
\left(\sum_i \sin k_\nu x_i \sin k_\nu x_i\right)a_\nu - \left(\sum_i \sin k_\nu x_i \cos k_\nu x_i\right)b_\nu &= -\sum_i \zeta_{\nu i}\sin k_\nu x_i\,, \\
-\left(\sum_i \sin k_\nu x_i \cos k_\nu x_i\right)a_\nu + \left(\sum_i \cos k_\nu x_i \cos k_\nu x_i\right)b_\nu &= \sum_i \zeta_{\nu i}\cos k_\nu x_i\,.
\end{aligned} \quad (10)$$

The solutions for a_ν and b_ν can be written as follows:

$$a_\nu = \frac{1}{\Delta} \sum_{i,j} J_{\nu j} \cos k_\nu x_i \sin k_\nu (x_i - x_j) \quad ,$$

(11)

$$b_\nu = \frac{1}{\Delta} \sum_{i,j} J_{\nu j} \sin k_\nu x_i \sin k_\nu (x_i - x_j) \quad ,$$

where

$$\Delta = \sum_{i,j} \cos k_\nu x_j \sin k_\nu x_i \sin k_\nu (x_i - x_j) = \sum_{\substack{i,j \\ i>j}} \sin^2 k_\nu (x_i - x_j)$$

Finally, one can show that

(12)

$$a_\nu^2 + b_\nu^2 = \frac{1}{\Delta} \sum_{i,j,k,l} J_{\nu j} J_{\nu l} \cos k_\nu (x_i - x_k) \sin k_\nu (x_i - x_j) \sin k_\nu (x_k - x_l)$$

which can be substituted directly into (7).

If it is not convenient to measure the surface profile aft of x_0' , one must use the whole expression in (2). Suppose one measures the profile $J(x_i, y)$ at $N + 3$ sections $x_1 < x_2 < \cdot \cdot \cdot < x_{N+3} \leq x_0$. One may then derive from (3) the following equations:

$$- a_\nu \sin k_\nu x_i + b_\nu \cos k_\nu x_i + \sum_n c_{\nu n} k_{\nu n} \cos \mu_{\nu n} h \, e^{k_{\nu n} x_i} = \qquad (13)$$

$$\frac{1}{b} \int_{-b}^{b} J(x_i, y) \cos \frac{\nu\pi}{2b} (y - b) \, dy \quad , \qquad i = 1, 2 \ldots, N+3$$

If one terminates the summation in n with $n = N$, these equations will form, for each ν , $N+3$ linear equations in the N unknowns $a_\nu, b_\nu, c_{\nu 0}, c_{\nu 1}, \ldots c_{\nu N}$. We note here that if the velocity is subcritical, i.e., $c^2 < gh$ the coefficient of C_{oo} is zero. In this case the matrix of the coefficients for $\nu = 0$ takes the form

$$(\sin k_o x_i \quad \cos k_o x_i \quad 0 \quad k_{01} \cos \mu_{01} h \, e^{k_{01} x_i} \ldots k_{0N} \cos \mu_{0N} h \, e^{k_{0N} x_i}) \qquad (14)$$

Hence one may reduce the number of equations by one, say, $i = 1, \ldots, N+2$. On the other hand, if the velocity is supercritical, then μ_0 doesn't exist, and the coefficient matrix for $\nu = 0$ takes the form

$$(\quad 0 \qquad 0 \qquad k_{00} \cos \mu_{00} h \, e^{k_{00} x_i} \ldots k_{0N} \cos \mu_{0N} h \, e^{k_{0N} x_i}) \tag{15}$$

Here one evidently need take only $i = 1, \ldots, N+1$
In both cases for $\nu \geq 1$ one has the coefficient matrix

$$(\sin k_\nu x_i \quad \cos k_\nu x_i \quad k_{\nu 0} \cos \mu_{\nu 0} h \, e^{k_{\nu 0} x_i} \ldots \quad k_{\nu N} \cos \mu_{\nu N} h \, e^{k_{\nu N} x_i}) \tag{16}$$

Since, in fact, we have measured $\mathcal{J}(x_i, y)$ only at a finite number of values of y, say y_1, y_2, $\cdots$, y_M, it is also possible to treat (3) directly as a set of linear equations with right-hand side $\mathcal{J}(x_i, y_j)$, each value of the pair (i, j) giving one equation. The second summation in (3) will in this case be terminated at $\mu = MN - 3$ for $\nu \geq 1$ if $i = 1, \ldots, N$ and $j = 1, \ldots, M$. We shall give below some formulas which are useful for this approach.

Let us suppose that the exponential terms are negligible, that $\mathcal{J}$ can be represented with a_0, b_0, a_1, b_1, $\ldots$, a_P, b_P, and that $MN > 2P+2$. Since each pair (i, j) yields an equation in the a_ν and b_ν, there are more equations than unknowns. A 'best' solution in the sense of least squares, as explained in Appendix II, is given by the following set of equations:

$$\sum_{\nu=0}^{P} \Big[\sum_{i,j} \sin k_\mu x_i \sin k_\nu x_i \cos \frac{\mu\pi}{2b}(y_j - b) \cos \frac{\nu\pi}{2b}(y_j - b) \Big] a_\nu$$

$$- \Big[\sum_{i,j} \sin k_\mu x_i \cos k_\nu x_i \cos \frac{\mu\pi}{2b}(y_j - b) \cos \frac{\nu\pi}{2b}(y_j - b) \Big] b_\nu =$$

$$= -\sum_{i,j} \sin k_\mu x_i \cos\frac{\mu\pi}{2b}(y_j - b)\, \mathcal{J}(x_i, y_j) \quad , \tag{17}$$

$$\mu = 0, 1, \ldots, P \; ;$$

$$\sum_{\nu=0}^{P} -\Big[\sum_{i,j} \cos k_\mu x_i \sin k_\nu x_i \cos\frac{\mu\pi}{2b}(y_j - b) \cos\frac{\nu\pi}{2b}(y_j - b)\Big] a_\nu$$

$$+\Big[\sum_{i,j} \cos k_\mu x_i \cos k_\nu x_i \cos\frac{\mu\pi}{2b}(y_j - b) \cos\frac{\nu\pi}{2b}(y_j - b)\Big] b_\nu$$

$$= \sum_{i,j} \cos k_\mu x_i \cos\frac{\mu\pi}{2b}(y_j - b)\, \mathcal{J}(x_i, y_j) \, ,$$

$$\mu = 0, 1, \ldots P \, .$$

We note that in equations (17) all a_ν and b_ν are to be found simultaneously, instead of for one value of ν at a time as in (9) and (11). This entails solving P equations in P unknowns instead of two equations in two unknowns as in (8) and (10).

Numerical Calculations

The first use which was made of the wave-profile data was to calculate R according to formulas (9) and (7), i.e., under the assumption that the expnential terms in (3) are negligible. For this purpose a preliminary computation of μ_ν and k_ν is necessary. These are shown in Tables 2 and 3 for $\nu = 0$, 1, ..., 20. These are dimensional constants with dimensions $(\text{meter})^{-1}$. As is evident from (4) and (6), they are functions of the dimensionless variables c^2/gh and $h/2b$, whose values in the present case are $.378^2$ and .625, respectively.

In the computation of (9) one must decide which pair of sections to use. In order to test the consistency of the method, all combinations were tried. The results are shown in the table below. The value at an intersection x_i, x_j is $R \times 10^2/\rho g V$.

	x_1	x_2	x_3	x_4
x_2	0.831			
x_3	0.911	1.063		
x_4	1.162	0.883	0.795	
x_5	0.896	1.292	0.903	1.070

By the method of Least Squares 0.878

The discrepancy among the various values is, of course, intolerable if the method is to have any usefulness as a general procedure. It is easy to confirm that none of the values $k_\nu(x_i - x_j)$ are near a value of $n\pi$ for $n > 0$. However, the values are quite close to zero, especially when $|i-j| = 1$, and this may account for the large variation in this diagonal. If this is the chief cause for the dispersion in the results, then the accuracy should improve as $|i-j|$ increases. An error analysis given in Appendix I supports this conjecture. Unfortunately, the computed values shown in the table above do not seem to be converging as $|i-j|$ increases, although the variation along the diagonals $|i-j| = \text{const.}$ does seem to decrease. We assume that the value 0.896 associated with the pair x_1, x_5 is the most reliable one of this set.

In order not to lose the information available in the set of five measured profiles the value of R was also computed using the least-squares formulas in (10) - (12). This yielded a value

$$\frac{R}{\rho g V} \times 10^2 = 0.878$$

It is reasonable to assume that this is the most reliable estimate for the resistance coefficient which one can obtain from the given data and the underlying assumptions.

In order to have an independent determination of the wave resistance, the model was towed in the same towing tank where the photographs were taken and the total resistance measured with the towing dynamometer. The result is shown in Fig. 7. The frictional resistance was estimated by using the ITTC 1957 Line. This is also shown on Fig. 7. By subtracting the estimated frictional resistance from the total resistance, the usual "residuary resistance" R_r is obtained. This is plotted on Fig. 8 as a dimensionless coefficient $C_r = R_r/\rho g V$. On the same curve are plotted various values of the wave-resistance coefficient calculated by Eggers' method. Although one cannot identify the residuary resistance with any feeling of great confidence, one would expect the two not to be very far apart. The value which we have taken to be the most reliable value according to Eggers' method, 0.878, does not appear to be incompatible with the residuary resistance curve.

We have assumed up to now that the exponential terms in (3) were negligible. An attempt was also made to include these terms by solving equations (10) for

$a_\nu, b_\nu, c_{\nu 0}, c_{\nu 1}, c_{\nu 2}$.

The necessary values of $\mu_{\nu n}$ and $k_{\nu n}$ were computed and are also shown in Tables 2 and 3. Table 4 shows the computed values of a_ν, b_ν, and $M_{\nu n} \equiv c_{\nu n} k_{\nu n} \cos \mu_{\nu n} h$ for $n = 0,1,2$ and $\nu = 0$, 1, ..., 20. The table below shows $a_\nu^2 + b_\nu^2$ for $\nu = 0$, 1, ..., 6 computed by this method and by the earlier method in which the exponential terms are neglected (taking X_1 and X_5).

ν	$a_\nu^2 + b_\nu^2$ from (13)	$a_\nu^2 + b_\nu^2$ from (9)
0	0.485274	0.259565×10^{-3}
1	0.194135×10^{2}	0.251898×10^{-5}
2	0.593893×10	0.636358×10^{-4}
3	0.154821×10^{2}	0.279218×10^{-6}
4	0.121068×10^{2}	0.272192×10^{-6}
5	0.770153	0.789311×10^{-6}
6	0.957579	0.188503×10^{-4}

The values computed from (13) are obviously worthless. Although (13) makes use of all the information and (12) of only two sections, one must remember that we are also trying to extract more information, even though the extra information, namely $c_{\nu 0}$, c_{21}, c_{22}, does not enter into the determination of R. However, even though the $c_{\nu n}$ are not used later, it is obviously of importance to know whether or not the exponential terms can be neglected in the determination of a_ν and b_ν. The poor results in the present case are due to numerical error as we shall show below.

Table 5 shows the coefficent matrices for $\nu = 0$, 1, ... , 20, and also the values of $\mathcal{J}_{\nu i}$. An inspection of these matrices shows that the matrix elements $e^{k_{\nu n} x_i}$ are indeed negligible compared with $\sin k_\nu x_i$ and $\cos k_\nu x_i$. As a result, slight errors in the $\mathcal{J}_{\nu i}$ and round-off errors in the computation have led to completely misleading results. The only $M_{\nu n}$ which it might be feasible to look for are M_{01}, M_{10} M_{11}, M_{20}, and M_{21}. However, any serious attempt to evaluate the coefficients of the exponential terms should rely upon use of least-squares solutions together with a large number of profiles.

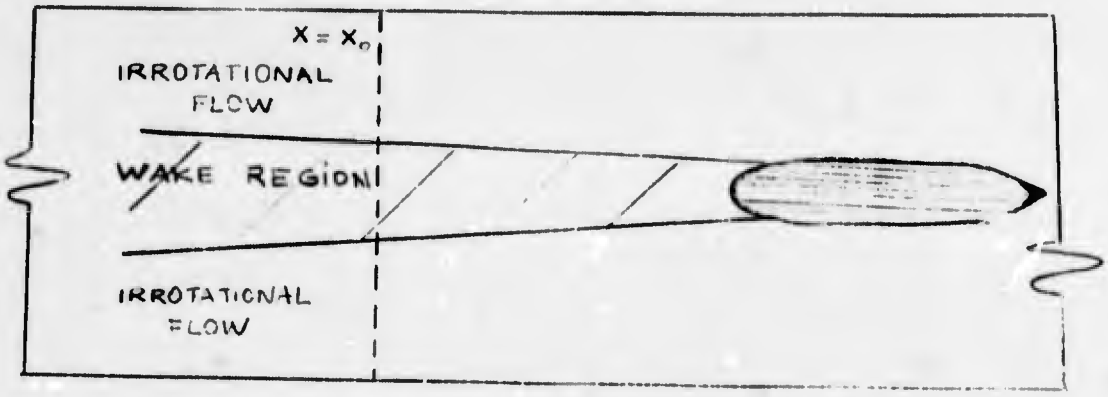
A preliminary attempt to use (17) indicated that quite a lot of computer time would be necessary. Consequently, the computations were not completed. However, since (17) supplies all the a_ν, b_ν at once, this may not, in fact, be an extravagant method of computation in comparison with (11). The matter will be examined later.

Concluding Remarks

As has been mentioned in the earlier discussion, any attempt to assess the usefulness of Eggers' formula is handicapped by the lack of any reliable standard of comparison. However, insofar as the residuary resistance can be considered as such, the agreement between the residuary resistance and the best value obtainable from Eggers' method is quite good for the single Froude number for which profile measurements were available.

Future experiments and their analysis can profitably be directed at two targets. For one of these the assumptions underlying Eggers' formula should be accepted and effort should be concentrated on improving the measuring and computing techniques. In particular, preliminary computations of the numbers k_ν and $k_{\nu n}$ for a useful spread of Froude numbers would allow one to predict how far behind the ship one should go before starting the profile measurements in order to avoid influence of the exponential terms, and how far apart the profiles should be in order to minimize the effect of measurement errors. Experiments should then be planned with this information in mind. The lack of a reliable standard of comparison for the results can in some measure be compensated for by tests of the internal consistency of the results.

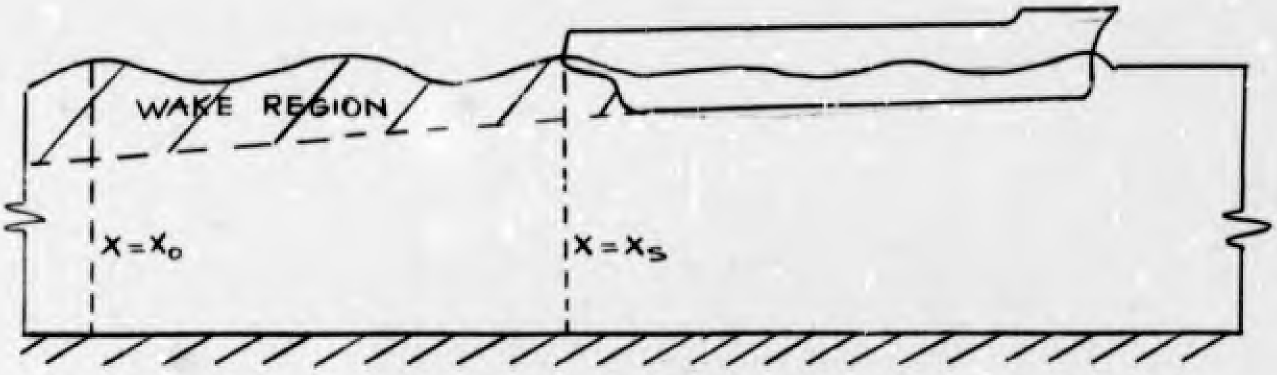
The other target should be the underlying assumption that the motion is irrotational astern of the ship. This assumption, together with linearization, shows up specifically in the assumption that $\varphi(x,y,z)$ can be represented by (2) in a region $x \leq x_0 < 0$. In the wake region of the ship the motion is certainly not irrotational. Furthermore, even if on conjectures, as has been done by several persons, that there is a region of irrotational flow outside the wake, as indicated on the sketch below, it will not be the case that the velocity



potential φ can be represented in this region by (2). A different representation must be used. It has also been suggested that φ be extended harmonically from the 'irrotational region' into the wake region and that the resistance computed from the extended φ be used to define a 'wave resistance' for the flow that actually occurs. Although the harmonically extended φ will be continuous on the plane $y = 0$, its derivative φ_y will in general be discontinuous, so that the resistance is no longer given by (2). Fortunately, the modification is easy, and is similar to one occurring in the theory of wings of finite length. It is as follows:

$$R = \frac{1}{2}\rho g \int_{-b}^{b} \zeta^2(x_0,y)\,dy + \frac{1}{2}\rho \int_{-b}^{b} dy \int_{-h}^{\zeta(x_0,y)} dz\,[-\varphi_x^2(x_0,y,z) + \varphi_y^2 + \varphi_z^2]$$
$$- \rho \int_{x_0}^{x_s} dx \int_{-h}^{\zeta(x,0)} dz\, \varphi_x(x,0,z)\,[\varphi_y(x,+0,z) - \varphi_y(x,-0,z)], \qquad (18)$$

where x_s is the x-coordinate of the stern of the ship and $\zeta(x,y)$ in the wake region is not the measured wave surface, but instead that determined from the harmonically extended φ. The last integral over the plane $y = 0$ should properly be taken over the shaded region shown below.



With this expression for R and the new representation for one must now derive the analogue of (7) and then devise a means of deriving the new representation from measurements of $\zeta(x,y)$ in the 'irrotational region'. It does not seem likely that transverse cuts $\zeta(x_i,y)$ can be used for this. Finally it should be emphasized that this procedure does not really 'take account of viscosity' in any fundamental sense. It is simply a procedure for exploiting as far as possible the theory of irrotational flow of an inviscid fluid.

Appendix I. Error Analysis

We shall place the present problem in a more general setting. Suppose we wish to determine unknowns $x_1, \dots, x_n$ from the set of equations

$$\sum_{j=1}^{n} a_{ij} x_j = c_i \quad , \qquad i = 1, \dots n, \tag{I.1}$$

but suppose also that the constants c_i are determined only up to some value ϵ_i. The error ϵ_i may take on any value between $-\epsilon$ and $+\epsilon > 0$. We wish to find the effect upon the of this amount of indeterminacy in the c_i. We consider then the equations

$$\sum_{j=1}^{n} a_{ij} x_j = c_i + \epsilon_i \quad , \qquad i = 1, \dots, n. \tag{I.2}$$

In matrix notation we may write (I.1) and (I.2) in the form

$$A\underset{\sim}{x} = \underset{\sim}{c} \quad , \qquad A\underset{\sim}{x} = \underset{\sim}{c} + \underset{\sim}{\epsilon} \tag{I.3}$$

If we assume that an inverse exists, the solutions are given, respectively, by

$$\underset{\sim}{x} = A^{-1}\underset{\sim}{c} \qquad \text{and} \qquad \underset{\sim}{x} = A^{-1}\underset{\sim}{c} + A^{-1}\underset{\sim}{\epsilon} \tag{I.4}$$

The error in $\underset{\sim}{x}$ evidently depends linearly upon the errors in $\underset{\sim}{c}$. However, how much ϵ is magnified depends upon the character of the matrix A^{-1}.

Let us suppose that we are more interested in the error in the length of the vector $\underset{\sim}{x}$ than in the components themselves. Consider then the error in the length squared:

$$\begin{aligned}(\underset{\sim}{x},\underset{\sim}{x}) - (A^{-1}\underset{\sim}{c}, A^{-1}\underset{\sim}{c}) &= (A^{-1}\underset{\sim}{c} + A^{-1}\underset{\sim}{\epsilon}, A^{-1}\underset{\sim}{c} + A^{-1}\underset{\sim}{\epsilon}) - (A^{-1}\underset{\sim}{c}, A^{-1}\underset{\sim}{c}) \\ &= 2(A^{-1}\underset{\sim}{c}, A^{-1}\underset{\sim}{\epsilon}) + (A^{-1}\underset{\sim}{\epsilon}, A^{-1}\underset{\sim}{\epsilon}) \\ &= 2(A^{-1^T}A^{-1}\underset{\sim}{c}, \underset{\sim}{\epsilon}) + O(\epsilon^2) \\ &= 2\, a_{ij}^{-1}\, a_{ik}^{-1}\, c_j\, \epsilon_k + O(\epsilon^2)\end{aligned} \tag{I.5}$$

Here we have used the following standard notation:

$$(\underset{\sim}{x},\underset{\sim}{y}) = \sum_{i=1}^{n} x_i y_i \quad . \tag{I.6}$$

Evidently the error in (x,x) is still linear in ϵ , but how much it is magnified depends upon $A^{-1^T}A^{-1}$. (A superscript A^T denotes the transposed matrix.)

Let us now apply these considerations to the present problem. Our equations are of the form

$$-\sin k_\nu x_i a_\nu + \cos k_\nu x_i b_\nu = J_{\nu i} \ , \tag{I.7}$$

where i enumerates the measured profile sections, say $i = 0, \ldots, m$ (in the present case $m = 5$) and

$$J_{\nu i} = \frac{1}{b}\int_{-b}^{b} J(x_i, y) \cos\frac{\nu\pi}{2b}(y-b)\, dy \tag{I.8}$$

If we consider any two sections, i and j , then the matrix A corresponding to the analysis above is

$$A = \begin{pmatrix} -\sin k_\nu x_i & \cos k_\nu x_i \\ -\sin k_\nu x_j & \cos k_\nu x_j \end{pmatrix} \tag{I.9}$$

It is then easy to compute $A^{-1} = (a_{ij}^{(-1)})$ and $A^{-1^T}A^{-1} = (\sum_i a_{ij}^{(-1)} a_{ik}^{(-1)})$

They are as follows

$$A^{-1} = \frac{1}{\sin^2 k_\nu (x_i - x_j)} \begin{pmatrix} \cos k_\nu x_j & -\cos k_\nu x_i \\ \sin k_\nu x_j & -\sin k_\nu x_i \end{pmatrix} , \qquad (I.10)$$

$$A^{-1^T} A^{-1} = \frac{1}{\sin^4 k_\nu (x_i - x_j)} \begin{pmatrix} \cos 2k_\nu x_j & -\cos k_\nu (x_i + x_j) \\ -\cos k_\nu (x_i + x_j) & \cos 2k_\nu x_i \end{pmatrix} \qquad (I.11)$$

It is now evident from (I.10) and (I.11) that the errors in determining the $J_{\nu i}$ will be multiplied by $1/\sin^2 k_\nu (x_i - x_j)$ in determining a_ν and b_ν , and by $1/\sin^4 k_\nu (x_i - x_j)$ in determining $a_\nu^2 + b_\nu^2$. The best choice of sections x_i and x_j will clearly be one which makes $|\sin k_\nu (x_i - x_j)|$ as large as possible. In the present situation where k_ν varies from 4.6 to 11 as ν goes from 0 to 20, a value of $|x_i - x_j|$ near 0.15 m would appear to have been a better choice than the value 0.0175 m actually used. Although it is not necessary to use the same pair of sections for each value of ν , it is convenient to do this in order to reduce the number of profiles to be measured.

Appendix II. Least-Squares Fit

As in Appendix I, we shall begin by formulating a general problem and then specializing it to the problem at hand. Let us suppose that we have a set of equations

$$\sum_{j=1}^{n} a_{ij} x_j = c_i \quad , \quad i = 1, \ldots, m \tag{II.1}$$

with more equations than unknowns, i.e., $m > n$. If the equations were consistent, we would only need to select n of them and solve for the x_i . However, if the c_i are determined by experiment, they will not be quite consistent and each set of n equations will give a different set of x_i's . If there is no special reason to favor some values of the c_i over others, one would like to find a method for using all the available equations in determining the The method with the best theoretical justification is the method of least squares. In this method one seeks the solution which minimizes the quantity

$$D = \sum_{i=1}^{m} [\sum_{j=1}^{n} a_{ij} x_j - c_i]^2 \tag{II.2}$$

A straightforward calculation shows that D will be a minimum if the x_i satisfy

$$\sum_{j=1}^{n} (\sum_{i=1}^{m} a_{ik} a_{ij}) x_j = \sum_{i=1}^{m} a_{ik} c_i \quad , \quad k = 1, \ldots, n \tag{II.3}$$

In matrix notation, the equation

$$A \underset{\sim}{x} = \underset{\sim}{c} \tag{II.3}$$

where $\underset{\sim}{x}$ is an n-vector and $\underset{\sim}{c}$ an m-vector, has been replaced by

$$A^T A \underset{\sim}{x} = A^T \underset{\sim}{c} \tag{II.4}$$

In order to apply this to the set of equations (I.7), we must construct the matrix $A^T A$ and the vector $A^T \underset{\sim}{c}$ where in this case $n = 2$, but m is the number of profile sections. It is easy to confirm that

$$A^T A = \begin{pmatrix} \sum_i \sin k_y x_i \sin k_y x_i & -\sum_i \sin k_y x_i \cos k_y x_i \\ -\sum_i \sin k_y x_i \cos k_y x_i & \sum_i \cos k_y x_i \cos k_y x_i \end{pmatrix} \qquad \text{(II.5)}$$

and

$$A^T \underset{\sim}{c} = \left(-\sum_i J_{yi} \sin k_y x_i \,,\, \sum_i J_{yi} \cos k_y x_i \right) \quad . \qquad \text{(II.6)}$$

Equation (II.4) is then just (10). The application to (3) in order to obtain (17) is also straightforward.

TABLE 1. THE READING VALUE OF THE WAVE HEIGHT WH
WAVE HEIGHT IN METERS = WH x 3.5×10^{-3}

ROW / COLUMN	1	2	3	4	5
1	5.0	4.8	4.3	3.9	3.4
2	5.0	4.8	4.3	3.9	3.4
3	4.9	4.6	4.2	3.7	3.2
4	4.7	4.3	3.9	3.4	2.8
5	4.4	4.1	3.7	3.2	2.5
6	3.9	3.6	3.3	2.8	2.3
7	3.3	3.2	2.7	2.4	1.9
8	2.8	2.6	2.2	2.0	1.7
9	2.3	1.9	1.7	1.4	1.1
10	2.2	1.1	1.2	0.8	0.3
11	1.5	1.3	0.8	0.4	0.0
12	1.0	0.7	0.5	0.3	0.1
13	0.7	0.5	0.3	0.3	0.1
14	0.5	0.4	0.2	0.4	0.2
15	0.5	0.5	0.4	0.5	0.6
16	0.5	0.4	0.7	0.8	1.0
17	0.5	0.7	1.1	1.4	1.6
18	1.0	1.2	1.5	1.9	2.2
19	1.3	1.8	2.1	2.6	2.9
20	2.2	2.6	2.9	3.3	3.7
21	2.9	3.2	3.4	3.7	4.1
22	3.4	3.5	3.6	4.2	3.8
23	3.6	3.7	3.7	3.6	3.9
24	3.6	3.5	3.4	3.2	3.1
25	3.3	3.2	3.1	3.1	2.8
26	3.1	3.2	3.1	2.8	2.8
27	3.2	3.1	3.1	2.9	2.8
28	3.2	3.1	3.1	2.9	2.6
29	3.2	3.1	2.9	2.9	2.5
30	3.2	3.2	2.9	2.7	2.3
31	3.3	3.1	2.9	2.5	1.9
32	3.2	3.0	2.5	2.1	1.5
33	2.9	2.5	1.8	1.5	0.8
34	2.5	2.0	1.4	0.9	0.1
35	1.9	1.3	0.8	0.4	-0.4
36	1.3	0.9	0.5	0.0	-0.4
37	0.9	0.6	0.3	-0.2	-0.5
38	0.7	0.3	0.2	-0.4	-0.7
39	0.6	0.1	-0.2	-0.7	-1.0
40	0.2	-0.4	-0.8	-1.2	-1.7
41	-0.3	-0.8	-1.2	-1.6	-2.0
42	-0.9	-1.2	-1.6	-1.9	-2.1
43	-1.2	-1.5	-1.8	-1.8	-2.1
44	-1.2	-1.3	-1.5	-1.6	-1.9
45	-1.1	-1.2	-1.3	-1.4	-1.4
46	-0.9	-0.8	-0.9	-0.9	-0.8
47	-0.2	-0.2	-0.2	-0.6	-0.7
48	-0.1	-0.4	-0.7	-1.1	-1.5
49	-0.5	-1.0	-1.4	-1.6	-2.0
50	-0.9	-1.3	-1.6	-1.7	-1.8
51	-1.0	-1.2	-1.5	-1.6	-1.7
52	-1.0	-1.2	-1.3	-1.5	-1.6
53	-1.0	-1.1	-1.5	-1.6	-1.7
54	-1.0	-1.2	-1.4	-1.5	-1.8
55	-1.0	-1.3	-1.4	-1.5	-1.8
56	-1.2	-1.4	-1.6	-1.8	-2.0
57	-1.2	-1.4	-1.6	-1.7	-1.8
58	-1.2	-1.4	-1.4	-1.7	-1.8
59	-1.2	-1.3	-1.5	-1.8	-1.8
60	-1.2	-1.5	-1.6	-2.0	-2.2
61	-1.2	-1.5	-1.7	-1.9	-2.1
62	-1.3	-1.4	-1.6	-1.9	-2.1
63	-1.3	-1.4	-1.6	-1.7	-2.1
64	-1.2	-1.3	-1.6	-1.7	-2.0
65	-1.2	-1.3	-1.6	-1.7	-2.0
66	-1.2	-1.4	-1.6	-1.8	-1.9
67	-1.1	-1.4	-1.5	-1.9	-1.9
68	-1.3	-1.5	-1.6	-2.0	-2.0
69	-1.3	-1.4	-1.5	-1.9	-2.0
70	-1.3	-1.5	-1.6	-1.8	-1.9
71	-1.3	-1.5	-1.6	-1.8	-1.9
72	-1.2	-1.4	-1.7	-1.8	-2.0
73	-1.3	-1.4	-1.7	-1.7	-1.9
74	-1.2	-1.4	-1.6	-1.7	-1.8
75	-1.2	-1.5	-1.6	-1.8	-1.8
76	-1.3	-1.5	-1.7	-1.7	-2.0
77	-1.3	-1.5	-1.7	-1.7	-2.0
78	-1.3	-1.5	-1.6	-1.7	-1.9
79	-1.3	-1.6	-1.6	-1.9	-2.0
80	-1.3	-1.6	-1.7	-2.0	-2.1
81	-1.4	-1.6	-1.7	-1.9	-2.1
82	-1.3	-1.6	-1.7	-1.9	-2.1
83	-1.3	-1.6	-1.7	-2.0	-2.1
84	-1.3	-1.6	-1.7	-1.9	-2.1
85	-1.2	-1.6	-1.7	-1.9	-2.1
86	-1.2	-1.4	-1.5	-1.9	-2.1
87	-1.1	-1.3	-1.5	-1.7	-1.8
88	-1.0	-1.3	-1.6	-1.7	-1.8
89	-1.0	-1.3	-1.5	-1.9	-2.0
90	-0.7	-0.7	-1.1	-1.2	-1.4
91	-0.6	-0.9	-0.8	-0.9	-1.1
92	-1.1	-1.1	-1.2	-1.1	-1.2
93	-1.4	-1.5	-1.6	-1.5	-1.6
94	-1.7	-2.0	-2.1	-2.0	-2.1
95	-1.9	-1.9	-2.2	-2.3	-2.5
96	-1.5	-1.7	-2.0	-2.2	-2.5
97	-1.3	-1.5	-1.9	-2.2	-2.3
98	-1.0	-1.3	-1.6	-1.9	-2.2
99	-0.5	-1.0	-1.3	-1.9	-1.9
100	-0.2	-0.6	-2.0	-1.3	-1.7
101	0.2	-0.1	-1.5	-0.9	-1.3
102	0.5	0.3	-0.2	-0.4	-0.8
103	0.6	0.5	0.1	-0.1	-0.4
104	0.6	0.5	0.1	-0.1	-0.3
105	0.6	0.5	0.1	-0.1	-0.3
106	1.0	0.7	0.2	-0.1	-0.4
107	1.5	1.3	0.8	0.4	-0.1
108	2.2	1.8	1.4	1.1	0.4
109	2.5	2.3	1.9	1.4	1.7
110	2.4	2.4	2.1	1.9	1.6
111	2.3	2.2	2.2	2.1	1.8
112	2.1	1.9	1.9	2.1	1.8
113	1.8	1.8	1.8	1.8	1.7
114	1.8	1.9	1.8	1.8	1.7
115	1.9	2.0	1.8	1.8	1.7
116	2.2	2.3	2.0	1.9	1.9
117	2.2	2.4	2.3	2.2	2.2
118	2.2	2.3	2.4	2.6	2.6
119	1.7	2.0	2.1	2.4	2.6
120	1.1	1.7	1.9	1.9	2.3
121	0.7	1.3	1.2	1.4	1.8
122	0.2	0.5	0.8	1.3	1.3
123	-0.1	0.1	0.5	0.7	0.7
124	-0.1	-0.1	0.1	0.3	0.4
125	-0.3	-0.4	-0.1	-0.1	0.0
126	-0.5	-0.4	-0.3	-0.3	0.0
127	-0.3	-0.3	-0.3	-0.3	-0.2
128	-0.1	-0.2	-0.2	-0.2	-0.3
129	-0.2	0.0	0.1	-0.1	-0.2
130	0.9	0.7	0.3	0.2	0.2
131	1.3	1.0	0.8	0.5	0.4
132	1.8	1.5	1.2	0.9	0.9
133	2.3	1.9	1.7	1.4	1.1
134	2.8	2.6	2.2	2.0	1.7
135	3.3	3.2	2.7	2.4	1.9
136	3.9	3.6	3.3	2.8	2.3
137	4.4	4.1	3.7	3.2	2.5
138	4.7	4.3	3.9	3.4	2.8
139	4.9	4.6	4.2	3.7	3.2
140	5.0	4.8	4.3	3.9	3.4
141	5.0	4.8	4.3	3.9	3.4

TABLE 2.

ν	μ_{ν}	$\mu_{\nu 0}$	$\mu_{\nu 1}$	$\mu_{\nu 2}$
0.0	0.45880000E 01	0.00000000E-38	0.23749800E 01	0.46422000E 01
1.0	0.49250376E 01	0.47053000E 00	0.24544800E 01	0.46680700E 01
2.0	0.57439461E 01	0.74098000E 00	0.26150200E 01	0.47323800E 01
3.0	0.67886349E 01	0.86775000E 00	0.27567200E 01	0.48089900E 01
4.0	0.79350286E 01	0.92952000E 00	0.28554300E 01	0.48792200E 01
5.0	0.91321657E 01	0.96267000E 00	0.29204900E 01	0.49363200E 01
6.0	0.10357477E 02	0.98213000E 00	0.29636800E 01	0.49804400E 01
7.0	0.11599840E 02	0.99440000E 00	0.29931300E 01	0.50139800E 01
8.0	0.12853238E 02	0.10025900E 01	0.30138500E 01	0.50395000E 01
9.0	0.14114161E 02	0.10083200E 01	0.30288600E 01	0.50590800E 01
10.0	0.15380433E 02	0.10124700E 01	0.30400300E 01	0.50742900E 01
11.0	0.16650640E 02	0.10155700E 01	0.30485300E 01	0.50862600E 01
12.0	0.17923822E 02	0.10179500E 01	0.30551400E 01	0.50958200E 01
13.0	0.19199307E 02	0.10198100E 01	0.30603800E 01	0.51035400E 01
14.0	0.20476611E 02	0.10212900E 01	0.30645900E 01	0.51098500E 01
15.0	0.21755375E 02	0.10224900E 01	0.30680200E 01	0.51150600E 01
16.0	0.23035329E 02	0.10234800E 01	0.30708500E 01	0.51194200E 01
17.0	0.24316266E 02	0.10243000E 01	0.30732200E 01	0.51230900E 01
18.0	0.25598023E 02	0.10249800E 01	0.30752100E 01	0.51262000E 01
19.0	0.26880473E 02	0.10255700E 01	0.30769100E 01	0.51288700E 01
20.0	0.28163512E 02	0.10260600E 01	0.30783700E 01	0.51311700E 01

TABLE 3

ν	k_{ν}	$k_{\nu 0}$	$k_{\nu 1}$	$k_{\nu 2}$
0.0	0.45880000E 01	0.00000000E-38	0.23749800E 01	0.46422000E 01
1.0	0.47535326E 01	0.13716127E 01	0.27720741E 01	0.48426027E 01
2.0	0.51335392E 01	0.26811832E 01	0.36712425E 01	0.53884239E 01
3.0	0.55808831E 01	0.39613504E 01	0.47475059E 01	0.61697400E 01
4.0	0.60337311E 01	0.52366760E 01	0.58917102E 01	0.70968694E 01
5.0	0.64728955E 01	0.65134329E 01	0.70730006E 01	0.81157458E 01
6.0	0.68934827E 01	0.77924199E 01	0.82789267E 01	0.91957605E 01
7.0	0.72952085E 01	0.90733156E 01	0.95023710E 01	0.10318732E 02
8.0	0.76792353E 01	0.10355687E 02	0.10738639E 02	0.11473083E 02
9.0	0.80470968E 01	0.11639178E 02	0.11984480E 02	0.12651010E 02
10.0	0.84003230E 01	0.12923521E 02	0.13237601E 02	0.13847047E 02
11.0	0.87403167E 01	0.14208521E 02	0.14496352E 02	0.15057248E 02
12.0	0.90683239E 01	0.15494035E 02	0.15759530E 02	0.16278707E 02
13.0	0.93854367E 01	0.16779958E 02	0.17026242E 02	0.17509229E 02
14.0	0.96926101E 01	0.18066210E 02	0.18295809E 02	0.18747146E 02
15.0	0.99906785E 01	0.19352730E 02	0.19567714E 02	0.19991161E 02
16.0	0.10280374E 02	0.20639472E 02	0.20841555E 02	0.21240263E 02
17.0	0.10562340E 02	0.21926398E 02	0.22117017E 02	0.22493639E 02
18.0	0.10837146E 02	0.23213480E 02	0.23393844E 02	0.23750642E 02
19.0	0.11105296E 02	0.24500694E 02	0.24671838E 02	0.25010748E 02
20.0	0.11367242E 02	0.25788020E 02	0.25950830E 02	0.26273525E 02

TABLE 4.

ν	a_{ν}	b_{ν}	$M_{\nu 0}$	$M_{\nu 1}$	$M_{\nu 2}$
0.0	0.691886E 00	0.910394E-01	0.000000E-38	0.417910E 03	-0.481362E 05
1.0	0.952089E-01	-0.440505E 01	0.775974E 03	-0.268996E 05	0.860308E 06
2.0	0.121019E 01	0.211527E 01	-0.184574E 05	0.241164E 06	-0.334562E 07
3.0	0.380516E 01	0.100141E 01	-0.105805E 07	0.880889E 07	-0.649300E 08
4.0	-0.288066E 01	0.195156E 01	0.303454E 08	-0.190887E 09	0.891243E 09
5.0	0.859870E-01	-0.873361E 00	-0.240118E 09	0.123493E 10	-0.410355E 10
6.0	-0.200369E 00	-0.235818E 00	-0.245025E 10	0.107549E 11	-0.272014E 11
7.0	0.772435E 00	0.256456E-01	0.174384E 12	-0.676976E 12	0.138143E 13
8.0	-0.271227E 00	0.245405E 00	-0.226723E 13	0.796540E 13	-0.136097E 14
9.0	0.457231E-02	-0.677482E-01	0.107284E 14	-0.346821E 14	0.509751E 14
10.0	-0.498186E-01	-0.521648E-01	0.346954E 15	-0.105046E 16	0.136971E 16
11.0	0.904605E-01	0.143840E-01	-0.103896E 17	0.296752E 17	-0.347326E 17
12.0	0.732265E-01	-0.497503E-01	-0.266701E 18	0.726205E 18	-0.777356E 18
13.0	-0.628848E-01	0.170837E 00	0.131321E 20	-0.342436E 20	0.337813E 20
14.0	-0.951872E-02	-0.191473E-01	-0.434131E 20	0.109312E 21	-0.100913E 21
15.0	0.570153E-01	0.402941E-01	0.302050E 22	-0.735181E 22	0.636075E 22
16.0	-0.170114E-02	0.612049E-03	-0.143121E 22	0.338852E 22	-0.278034E 22
17.0	-0.138907E-01	0.122985E-01	-0.519361E 24	0.119885E 25	-0.936741E 24
18.0	0.368940E-03	-0.648810E-02	0.569021E 25	-0.128567E 26	0.963859E 25
19.0	0.938165E-02	0.330016E-01	-0.463987E 27	0.102475E 28	-0.735022E 27
20.0	-0.263402E-02	-0.236663E-02	0.974068E 27	-0.211259E 28	0.146268E 28

TABLE 5.

ν	SECTION NUMBER	$-a_\nu \sin k_\nu x$	$b_\nu \cos k_\nu x$	$M_{\nu 0} e^{k_{\nu 0} x}$	$M_{\nu 1} e^{k_{\nu 1} x}$	$M_{\nu 2} e^{k_{\nu 2} x}$	$\frac{2}{\ell}\int_0^\ell \zeta \cos\frac{2\nu\pi}{\ell}(y-\ell)\,dy$
0.0	1.0	-0.869993E 00	-0.520494E 00	0.000000E 01	0.450267E-02	0.259068E-04	0.674025E-02
0.0	2.0	-0.840108E 00	-0.559136E 00	0.000000E 01	0.431330E-02	0.234454E-04	0.344486E-02
0.0	3.0	-0.922111E 00	-0.386926E 00	0.000000E 01	0.414356E-02	0.220217E-04	0.223473E-02
0.0	4.0	-0.959171E 00	-0.411722E 00	0.000000E 01	0.397485E-02	0.203034E-04	0.112962E-02
0.0	5.0	-0.977114E 00	-0.245108E 00	0.000000E 01	0.381404E-02	0.187192E-04	-0.320872E-03
1.0	1.0	-0.984612E 00	-0.180295E 00	0.441387E-01	0.182644E-02	0.166215E-04	0.125573E-02
1.0	2.0	-0.995102E 00	-0.979646E-01	0.430918E-01	0.173497E-02	0.150872E-04	0.111193E-02
1.0	3.0	-0.999880E 00	-0.149134E-01	0.429694E-01	0.165577E-02	0.139617E-04	0.113179E-02
1.0	4.0	-0.997670E 00	0.692198E-01	0.410720E-01	0.157734E-02	0.127750E-04	0.102229E-02
1.0	5.0	-0.988652E 00	0.150881E 00	0.409979E-01	0.150267E-02	0.117992E-04	0.959866E-03
2.0	1.0	-0.775639E 00	0.631299E 00	0.226455E-02	0.235906E-03	0.674384E-05	0.759324E-02
2.0	2.0	-0.715776E 00	0.698342E 00	0.214072E-02	0.221227E-03	0.431695E-05	0.745171E-02
2.0	3.0	-0.650244E 00	0.759732E 00	0.204250E-02	0.207469E-03	0.332849E-05	0.794565E-02
2.0	4.0	-0.572654E 00	0.819935E 00	0.194897E-02	0.194551E-03	0.357494E-05	0.903249E-02
2.0	5.0	-0.503998E 00	0.863705E 00	0.185964E-02	0.182445E-03	0.325326E-05	0.735712E-02
3.0	1.0	0.129771E 00	0.991544E 00	0.121029E-03	0.203879E-04	0.891999E-06	-0.242401E-04
3.0	2.0	0.225849E 00	0.974165E 00	0.113763E-03	0.187624E-04	0.719918E-06	0.225505E-04
3.0	3.0	0.319756E 00	0.947501E 00	0.106144E-03	0.172666E-04	0.646237E-06	-0.491421E-04
3.0	4.0	0.410621E 00	0.911996E 00	0.993344E-04	0.158990E-04	0.580098E-06	0.147929E-03
3.0	5.0	0.497677E 00	0.867421E 00	0.924016E-04	0.146242E-04	0.520727E-06	0.174819E-03
4.0	1.0	0.916550E 00	0.399102E 00	0.669977E-05	0.149964E-05	0.973097E-07	-0.495923E-03
4.0	2.0	0.953865E 00	0.300139E 00	0.611308E-05	0.135174E-05	0.859437E-07	-0.575940E-03
4.0	3.0	0.990216E 00	0.137932E 00	0.557777E-05	0.122836E-05	0.759061E-07	-0.402980E-03
4.0	4.0	0.995617E 00	0.935211E-01	0.508934E-05	0.110800E-05	0.670409E-07	-0.590451E-03
4.0	5.0	0.999299E 00	-0.119421E-01	0.464347E-05	0.999450E-06	0.592110E-07	-0.521075E-03
5.0	1.0	0.831860E 00	-0.555256E 00	0.366943E-06	0.102734E-05	0.958255E-08	-0.711496E-03
5.0	2.0	0.763587E 00	-0.645705E 00	0.327613E-06	0.907776E-07	0.831382E-08	-0.543903E-03
5.0	3.0	0.685707E 00	-0.727378E 00	0.292141E-06	0.802099E-07	0.721306E-08	-0.595743E-03
5.0	4.0	0.599038E 00	-0.800721E 00	0.260669E-06	0.708707E-07	0.625894E-08	-0.500727E-03
5.0	5.0	0.504690E 00	-0.863309E 00	0.232597E-06	0.626197E-07	0.542947E-08	-0.406787E-03
6.0	1.0	0.252674E-01	-0.999680E 00	0.199956E-07	0.661982E-08	0.821117E-09	-0.462449E-03
6.0	2.0	-0.952014E-01	-0.995458E 00	0.174466E-07	0.571919E-08	0.699963E-09	-0.985120E-03
6.0	3.0	-0.214307E 00	-0.976766E 00	0.152225E-07	0.494781E-08	0.595151E-09	-0.154599E-02
6.0	4.0	-0.330296E 00	-0.943877E 00	0.132820E-07	0.428344E-08	0.506685E-09	-0.196954E-02
6.0	5.0	-0.441486E 00	-0.897268E 00	0.115888E-07	0.370415E-08	0.431769E-09	-0.241288E-02
7.0	1.0	-0.776212E 00	-0.630471E 00	0.108488E-08	0.409763E-09	0.638098E-10	-0.948485E-03
7.0	2.0	-0.859167E 00	-0.526514E 00	0.925603E-09	0.346137E-09	0.532677E-10	-0.927230E-03
7.0	3.0	-0.910283E 00	-0.413986E 00	0.789797E-09	0.293109E-09	0.444572E-10	-0.755506E-03
7.0	4.0	-0.955584E 00	-0.294720E 00	0.673764E-09	0.248204E-09	0.371207E-10	-0.902844E-03
7.0	5.0	-0.985341E 00	-0.170655E 00	0.574842E-09	0.210179E-09	0.309979E-10	-0.834274E-03
8.0	1.0	-0.981720E 00	0.190332E 00	0.596644E-10	0.245477E-10	0.461706E-11	0.306614E-02
8.0	2.0	-0.947367E 00	0.320150E 00	0.489407E-10	0.203421E-10	0.377718E-11	0.259001E-02
8.0	3.0	-0.895931E 00	0.444193E 00	0.408286E-10	0.168570E-10	0.309009E-11	0.197936E-02
8.0	4.0	-0.829339E 00	0.560227E 00	0.340612E-10	0.139690E-10	0.252798E-11	0.125765E-02
8.0	5.0	-0.745809E 00	0.666159E 00	0.284155E-10	0.115758E-10	0.206812E-11	0.493484E-03
9.0	1.0	-0.516202E 00	0.856467E 00	0.316417E-11	0.144243E-11	0.316628E-12	-0.315672E-03
9.0	2.0	-0.390879E 00	0.920442E 00	0.258107E-11	0.116953E-11	0.253747E-12	-0.375466E-03
9.0	3.0	-0.257817E 00	0.966194E 00	0.210543E-11	0.949269E-12	0.203353E-12	-0.471174E-03
9.0	4.0	-0.119651E 00	0.992816E 00	0.171744E-11	0.769853E-12	0.162968E-12	-0.618274E-03
9.0	5.0	0.298845E-01	0.999782E 00	0.140095E-11	0.623390E-12	0.130592E-12	-0.717041E-03
10.0	1.0	0.258220E 00	0.966086E 00	0.170335E-12	0.833655E-13	0.208373E-13	0.390198E-02
10.0	2.0	0.396943E 00	0.917843E 00	0.135857E-12	0.661269E-13	0.163532E-13	0.372892E-02
10.0	3.0	0.527105E 00	0.849800E 00	0.108358E-12	0.524530E-13	0.128340E-13	0.361972E-02
10.0	4.0	0.665895E 00	0.763426E 00	0.864254E-13	0.416067E-13	0.100722E-13	0.329634E-02
10.0	5.0	0.750753E 00	0.660583E 00	0.689319E-13	0.330031E-13	0.790465E-14	0.297126E-02
11.0	1.0	0.859691E 00	0.510914E 00	0.915584E-14	0.475680E-14	0.132782E-14	0.465170E-03
11.0	2.0	0.927482E 00	0.373968E 00	0.714021E-14	0.369097E-14	0.102024E-14	0.420303E-03
11.0	3.0	0.973616E 00	0.228192E 00	0.556832E-14	0.286395E-14	0.783905E-15	0.289162E-03
11.0	4.0	0.997017E 00	0.771984E-01	0.434247E-14	0.222224E-14	0.602319E-15	0.322456E-03
11.0	5.0	0.997137E 00	-0.756182E-01	0.338649E-14	0.172432E-14	0.462795E-15	0.287584E-03
12.0	1.0	0.978013E 00	-0.208542E 00	0.491568E-15	0.268701E-15	0.824733E-16	0.161188E-02
12.0	2.0	0.932768E 00	-0.360478E 00	0.374824E-15	0.203937E-15	0.620787E-16	0.194350E-02
12.0	3.0	0.864080E 00	-0.503354E 00	0.285805E-15	0.154782E-15	0.466521E-16	0.205857E-02
12.0	4.0	0.773677E 00	-0.633580E 00	0.217928E-15	0.117475E-15	0.350874E-16	0.222346E-02
12.0	5.0	0.664831E 00	-0.747883E 00	0.166171E-15	0.891601E-16	0.263895E-16	0.235399E-02
13.0	1.0	0.535619E 00	-0.802525E 00	0.263674E-16	0.150569E-16	0.501802E-17	0.367801E-03
13.0	2.0	0.457370E 00	-0.889277E 00	0.196579E-16	0.111772E-16	0.369369E-17	0.403156E-03
13.0	3.0	0.305811E 00	-0.952092E 00	0.146557E-16	0.829716E-17	0.271986E-17	0.591326E-03
13.0	4.0	0.146021E 00	-0.989281E 00	0.109264E-16	0.615925E-17	0.200131E-17	0.511127E-03
13.0	5.0	-0.176994E-01	-0.999843E 00	0.814603E-17	0.457220E-17	0.147313E-17	0.578800E-03
14.0	1.0	-0.595043E-01	-0.998228E 00	0.141377E-17	0.838255E-18	0.300224E-18	0.195202E-03
14.0	2.0	-0.227160E 00	-0.973858E 00	0.103019E-17	0.608591E-18	0.216255E-18	0.392097E-03
14.0	3.0	-0.388295E 00	-0.921535E 00	0.750954E-18	0.441859E-18	0.155770E-18	0.427272E-03
14.0	4.0	-0.538285E 00	-0.842764E 00	0.547402E-18	0.320792E-18	0.112203E-18	0.576984E-03
14.0	5.0	-0.672826E 00	-0.739801E 00	0.399026E-18	0.232902E-18	0.808205E-19	0.753215E-03
15.0	1.0	-0.672547E 00	-0.740054E 00	0.757037E-19	0.464204E-19	0.177147E-19	-0.937351E-04
15.0	2.0	-0.791324E 00	-0.611785E 00	0.539552E-19	0.329603E-19	0.124853E-19	-0.916524E-04
15.0	3.0	-0.885284E 00	-0.464961E 00	0.384547E-19	0.234031E-19	0.879960E-20	0.426213E-04
15.0	4.0	-0.952747E 00	-0.303764E 00	0.274073E-19	0.166171E-19	0.620193E-20	0.115187E-03
15.0	5.0	-0.991062E 00	-0.133405E 00	0.195336E-19	0.117988E-19	0.437110E-20	0.217846E-03
16.0	1.0	-0.994884E 00	-0.173216E 00	0.405313E-20	0.255934E-20	0.103323E-20	0.336995E-03
16.0	2.0	-0.999983E 00	0.581196E-02	0.282441E-20	0.177717E-20	0.712472E-21	0.449928E-03
16.0	3.0	-0.982804E 00	0.184653E 00	0.196818E-20	0.123404E-20	0.491291E-21	0.534445E-03
16.0	4.0	-0.933900E 00	0.357533E 00	0.137152E-20	0.856902E-21	0.338774E-21	0.603956E-03
16.0	5.0	-0.854851E 00	0.519973E 00	0.955736E-21	0.595021E-21	0.233604E-21	0.656638E-03
17.0	1.0	-0.892752E 00	0.450548E 00	0.216911E-21	0.140587E-21	0.596810E-22	0.278781E-03
17.0	2.0	-0.794739E 00	0.606952E 00	0.147787E-21	0.954672E-22	0.402607E-22	0.249272E-03
17.0	3.0	-0.669649E 00	0.742679E 00	0.100692E-21	0.648279E-22	0.271598E-22	0.126319E-03
17.0	4.0	-0.521745E 00	0.853101E 00	0.686040E-22	0.440219E-22	0.183219E-22	0.160030E-03
17.0	5.0	-0.355065E 00	0.934461E 00	0.467418E-22	0.298934E-22	0.123599E-22	0.214196E-03
18.0	1.0	-0.460213E 00	0.887809E 00	0.116043E-22	0.769866E-23	0.341905E-23	-0.263479E-03
18.0	2.0	-0.234659E 00	0.958648E 00	0.773025E-23	0.511233E-23	0.225623E-23	-0.157301E-03
18.0	3.0	-0.987731E-01	0.995110E 00	0.514953E-23	0.339486E-23	0.148993E-23	0.470362E-04
18.0	4.0	0.905910E-01	0.995888E 00	0.343037E-23	0.225437E-23	0.982178E-24	0.103247E-03
18.0	5.0	0.276707E 00	0.960954E 00	0.228515E-23	0.149702E-23	0.648623E-24	0.410254E-04
19.0	1.0	0.141427E 00	0.991326E 00	0.620621E-24	0.420467E-24	0.194497E-24	0.188297E-03
19.0	2.0	0.229396E 00	0.947284E 00	0.404219E-24	0.273134E-24	0.125544E-24	0.373135E-03
19.0	3.0	0.497409E 00	0.867574E 00	0.263274E-24	0.177392E-24	0.810422E-25	0.322966E-03
19.0	4.0	0.655493E 00	0.755201E 00	0.171476E-24	0.115134E-24	0.523149E-25	0.494015E-03
19.0	5.0	0.788999E 00	0.614404E 00	0.111684E-24	0.747442E-25	0.337707E-25	0.526196E-03
20.0	1.0	0.665180E 00	0.746683E 00	0.431845E-25	0.229123E-25	0.109960E-25	-0.615614E-03
20.0	2.0	0.792619E 00	0.600507E 00	0.211314E-25	0.145490E-25	0.694306E-26	-0.657436E-03
20.0	3.0	0.907621E 00	0.430646E 00	0.136566E-25	0.923852E-26	0.434397E-26	-0.620335E-03
20.0	4.0	0.969826E 00	0.244000E 00	0.856925E-26	0.584641E-26	0.276312E-26	-0.574094E-03
20.0	5.0	0.998879E 00	0.473776E-01	0.545695E-26	0.372514E-26	0.174784E-26	-0.497793E-03

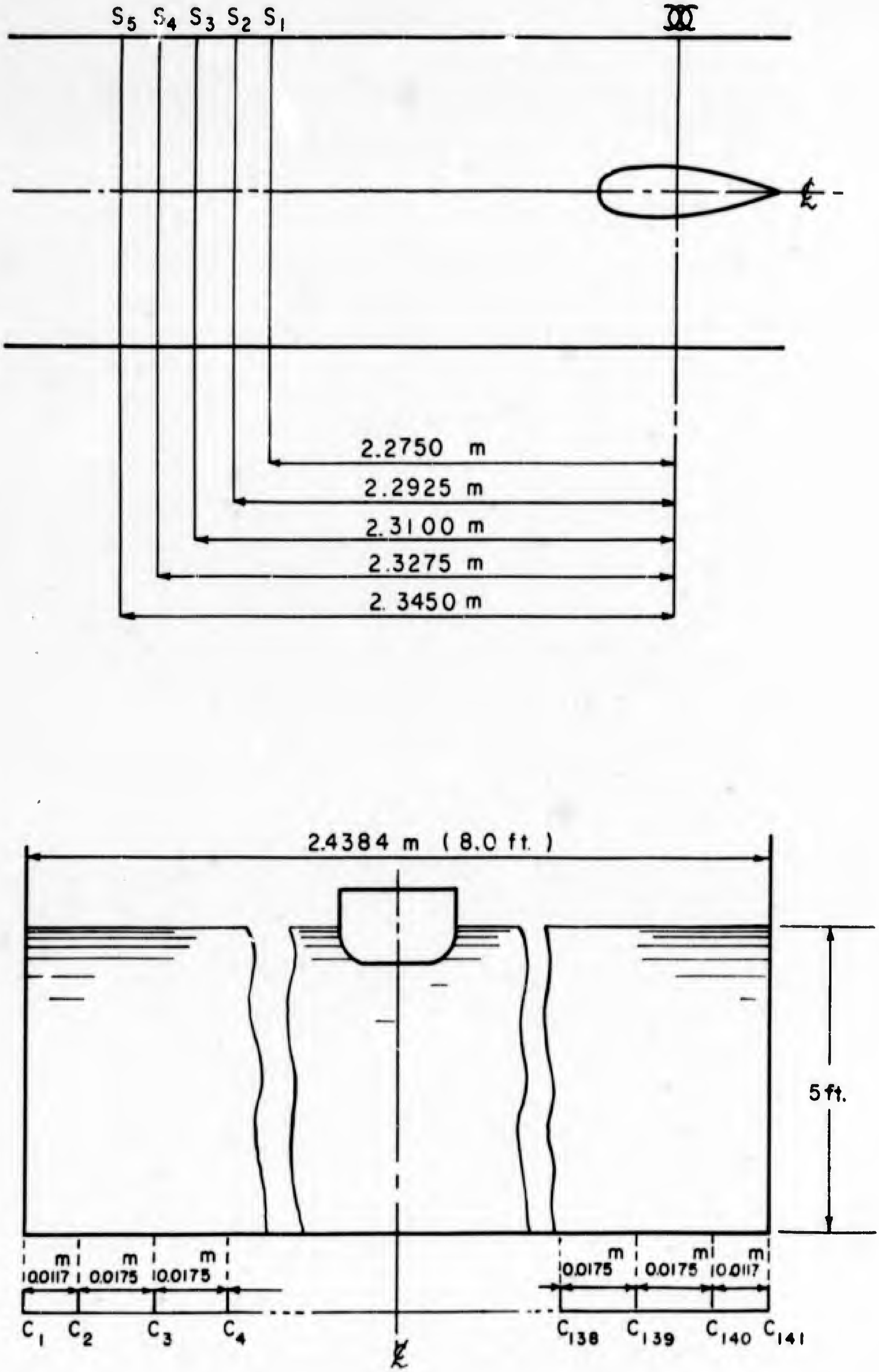

S5 S4 S3 S2 S1
2.2750 m
2.2925 m
2.3100 m
2.3275 m
2.3450 m
2.4384 m (8.0 ft.)
5 ft.
m 0.0117
m 0.0175
m 0.0175
C1 C2 C3 C4
C138 C139 C140 C141

FIG. 1

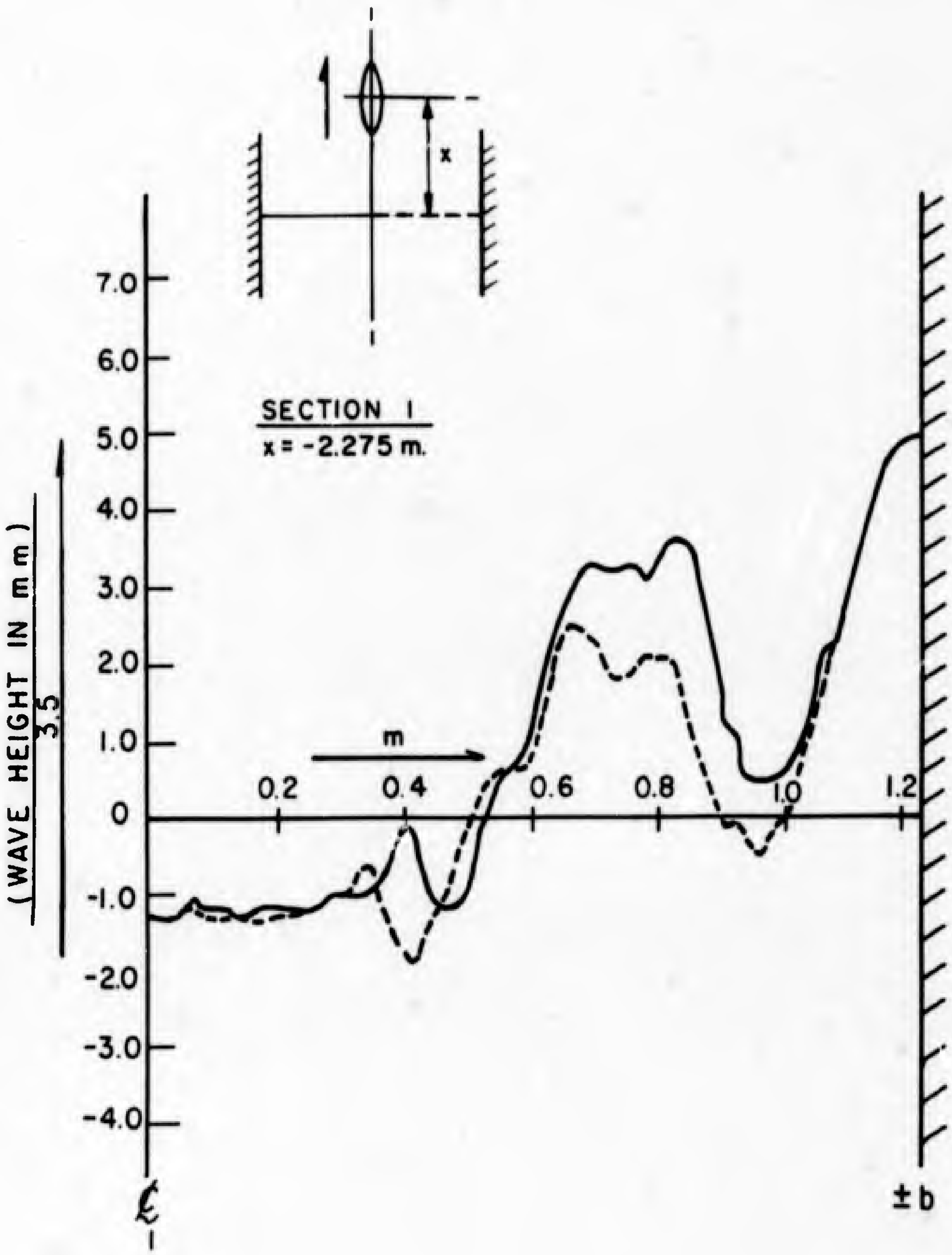


FIG.2 WAVE HEIGHT

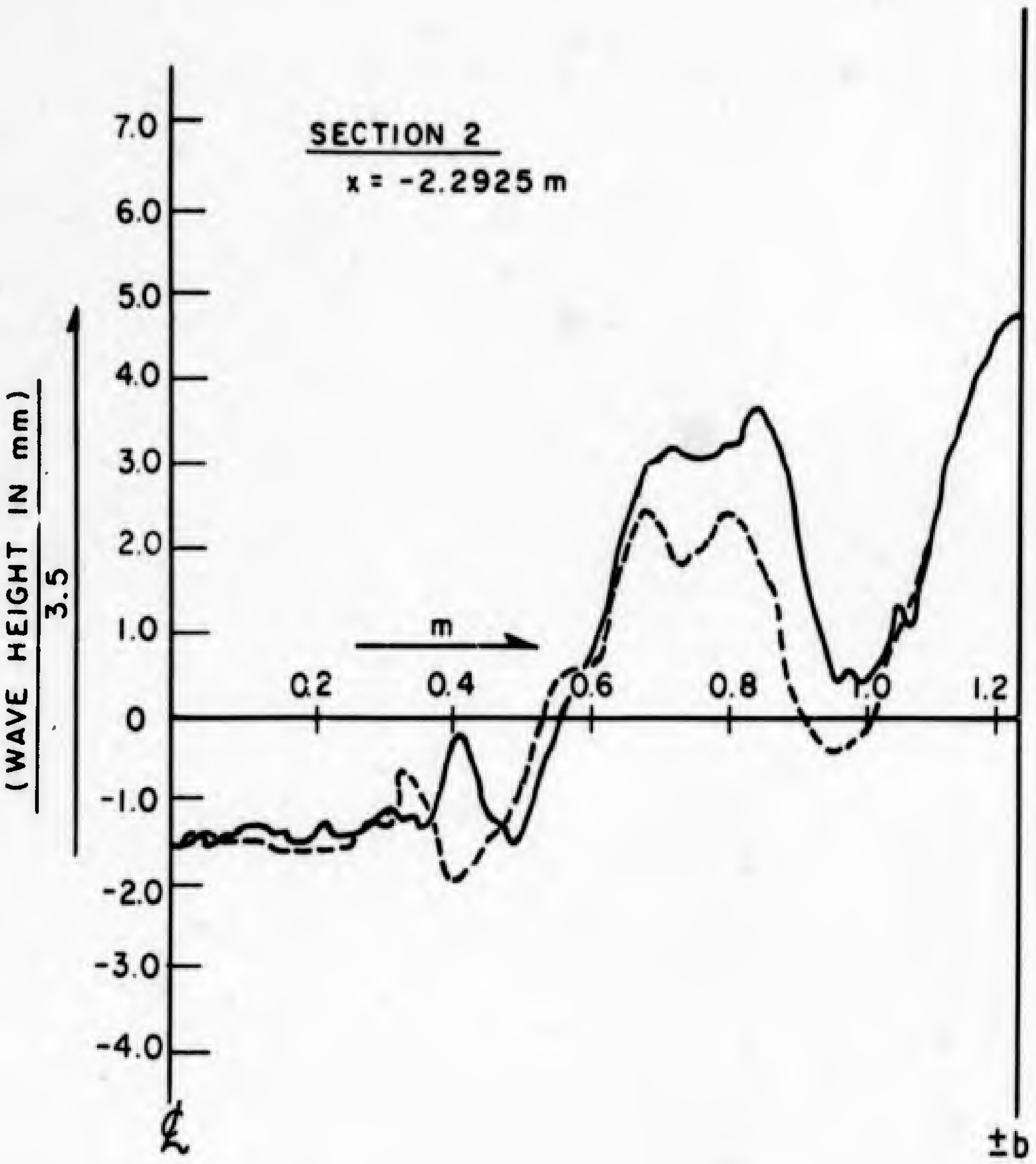


FIG.3 WAVE LENGTH

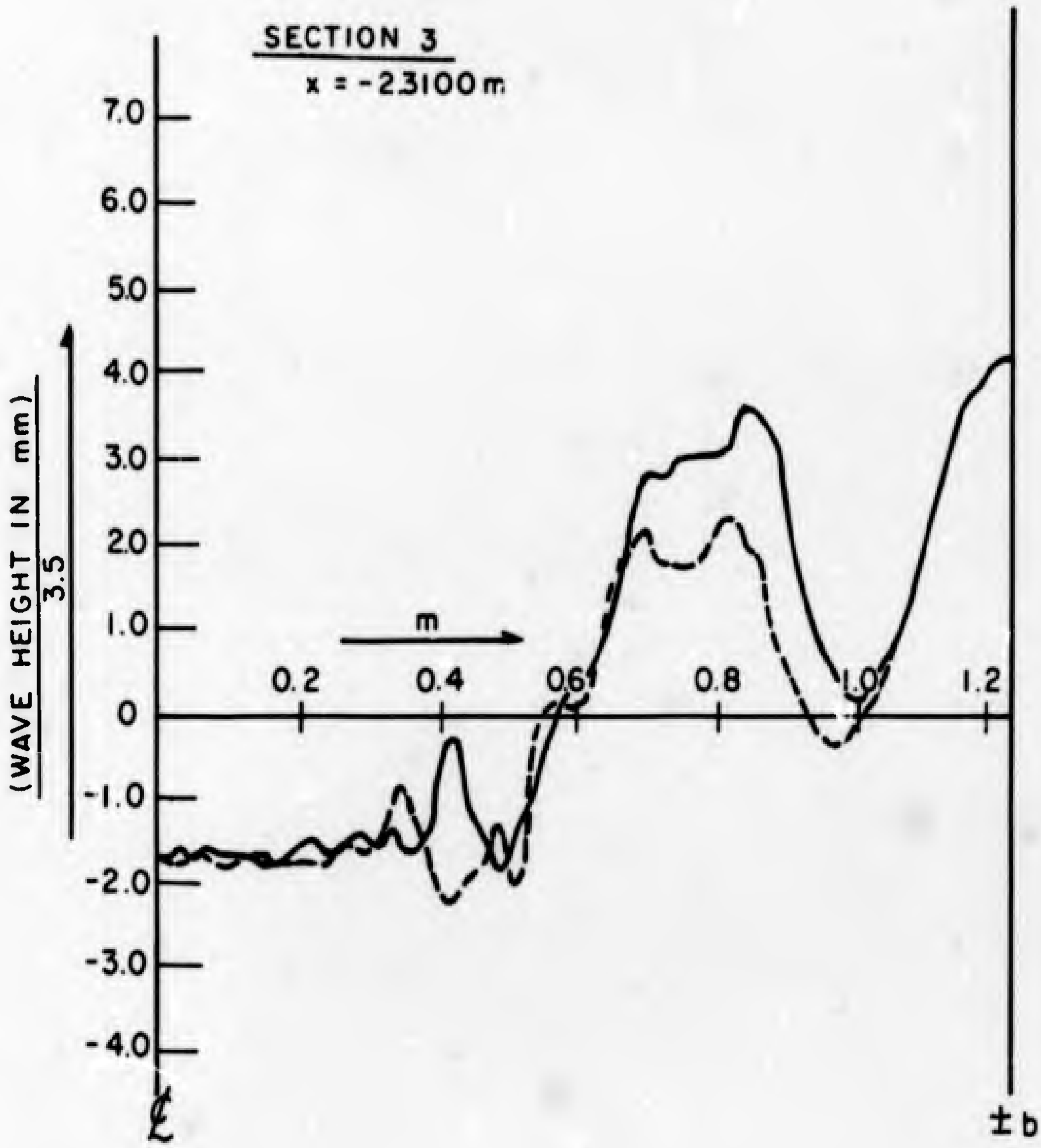


FIG.4 WAVE HEIGHT

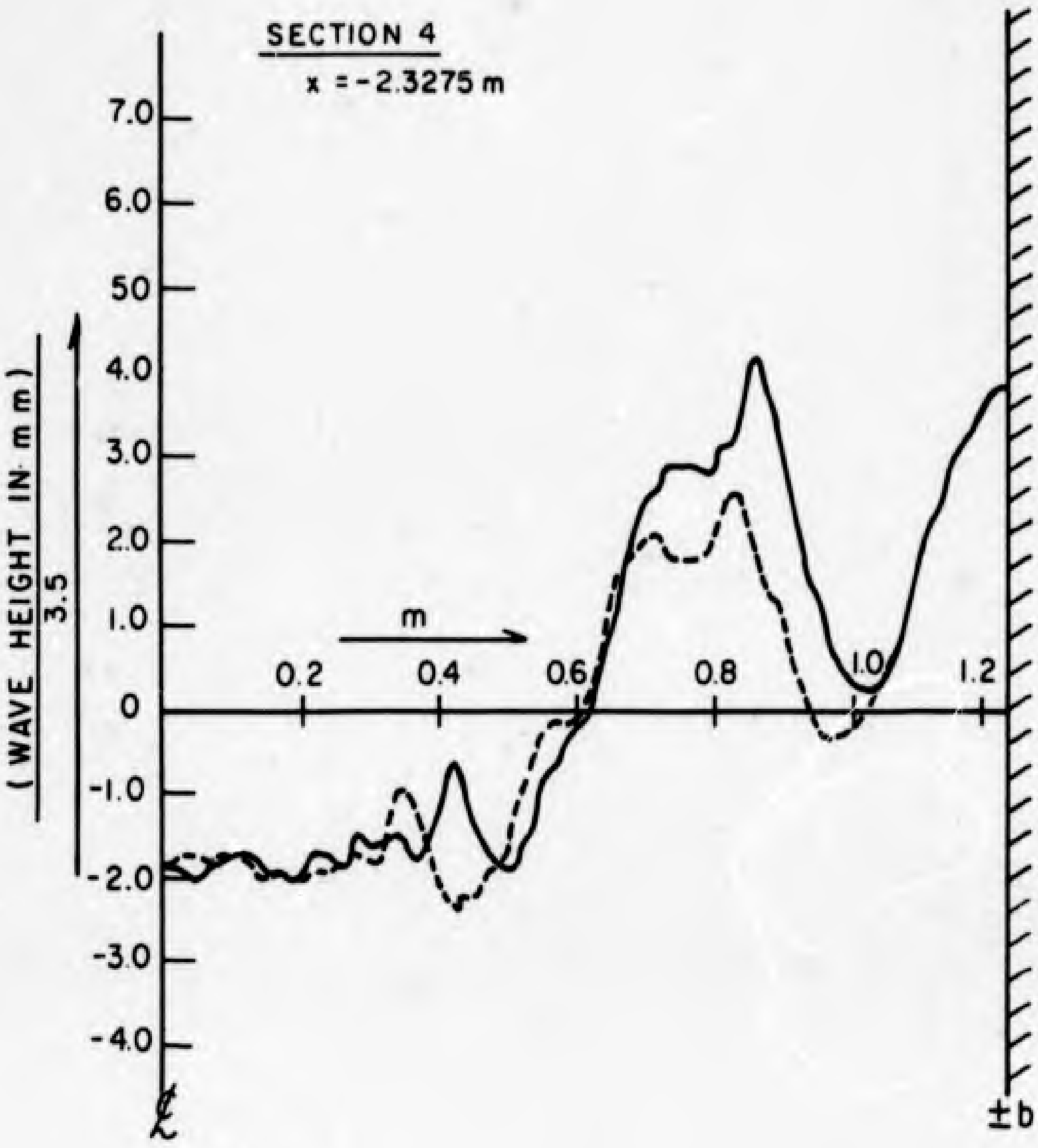


FIG.5 WAVE HEIGHT

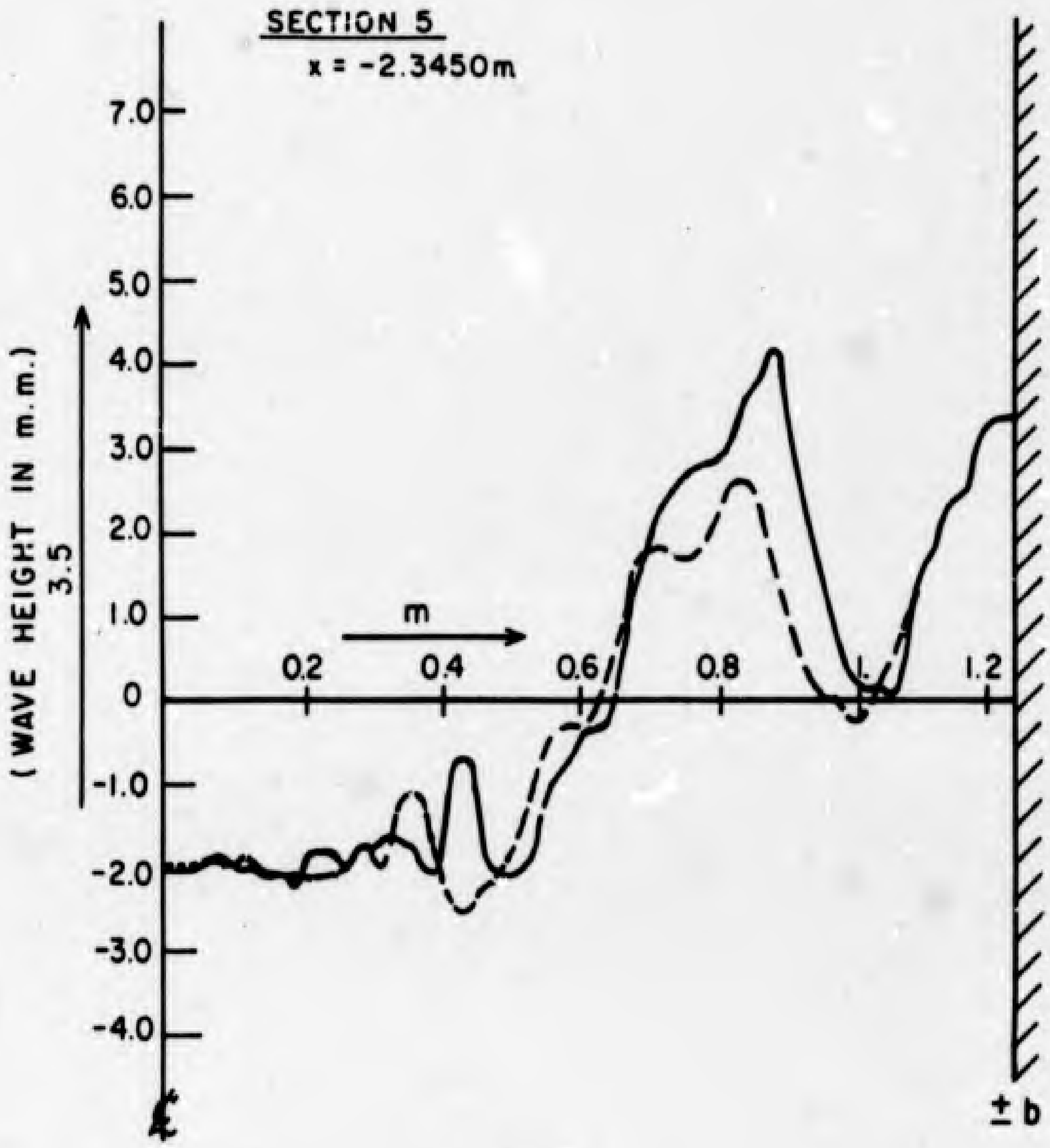


FIG. 6 WAVE HEIGHT

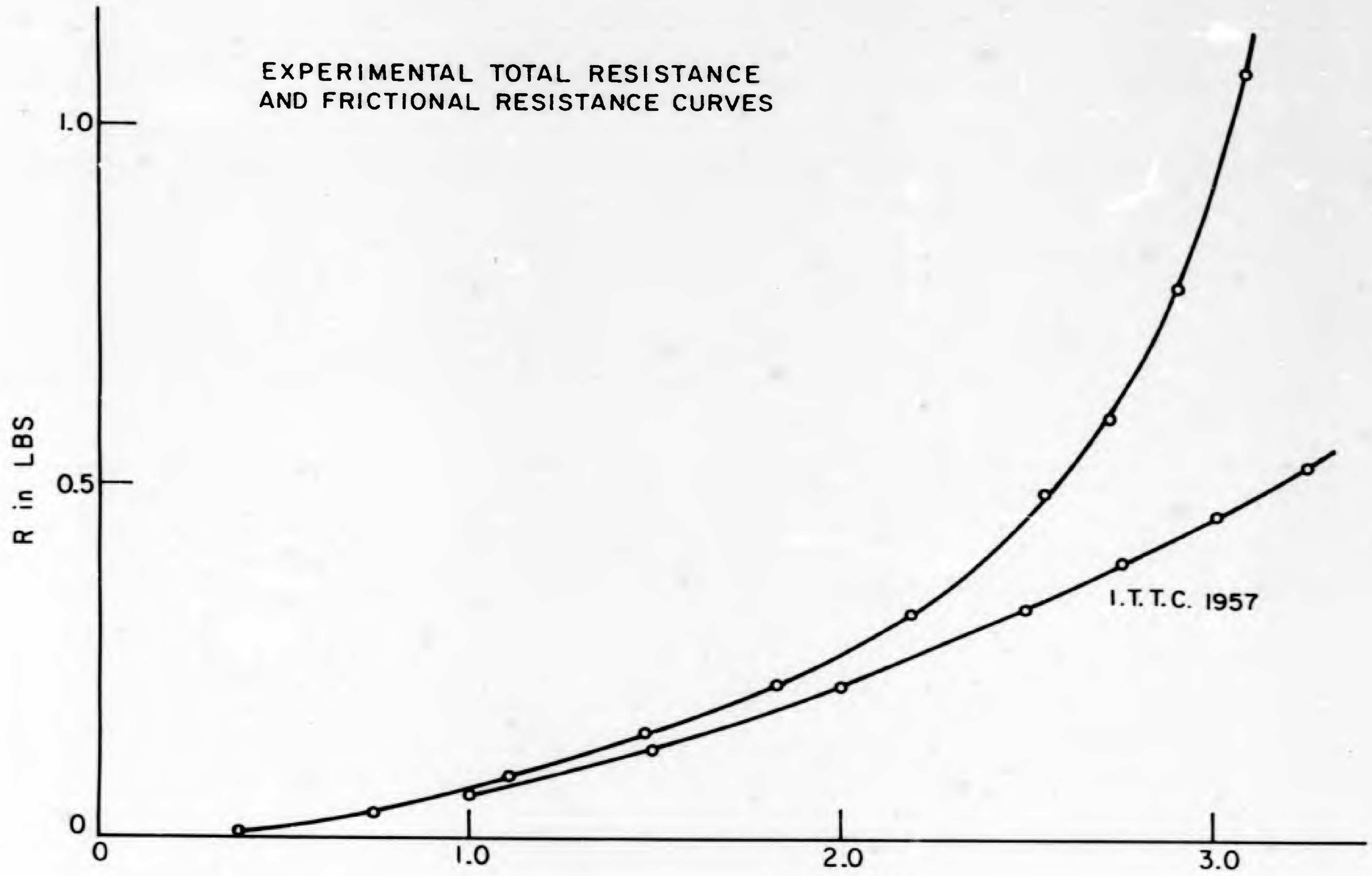

EXPERIMENTAL TOTAL RESISTANCE
AND FRICTIONAL RESISTANCE CURVES
R in LBS
1.0
0.5
0
0
1.0
2.0
3.0
I.T.T.C. 1957
V in KNOTS

FIG. 7

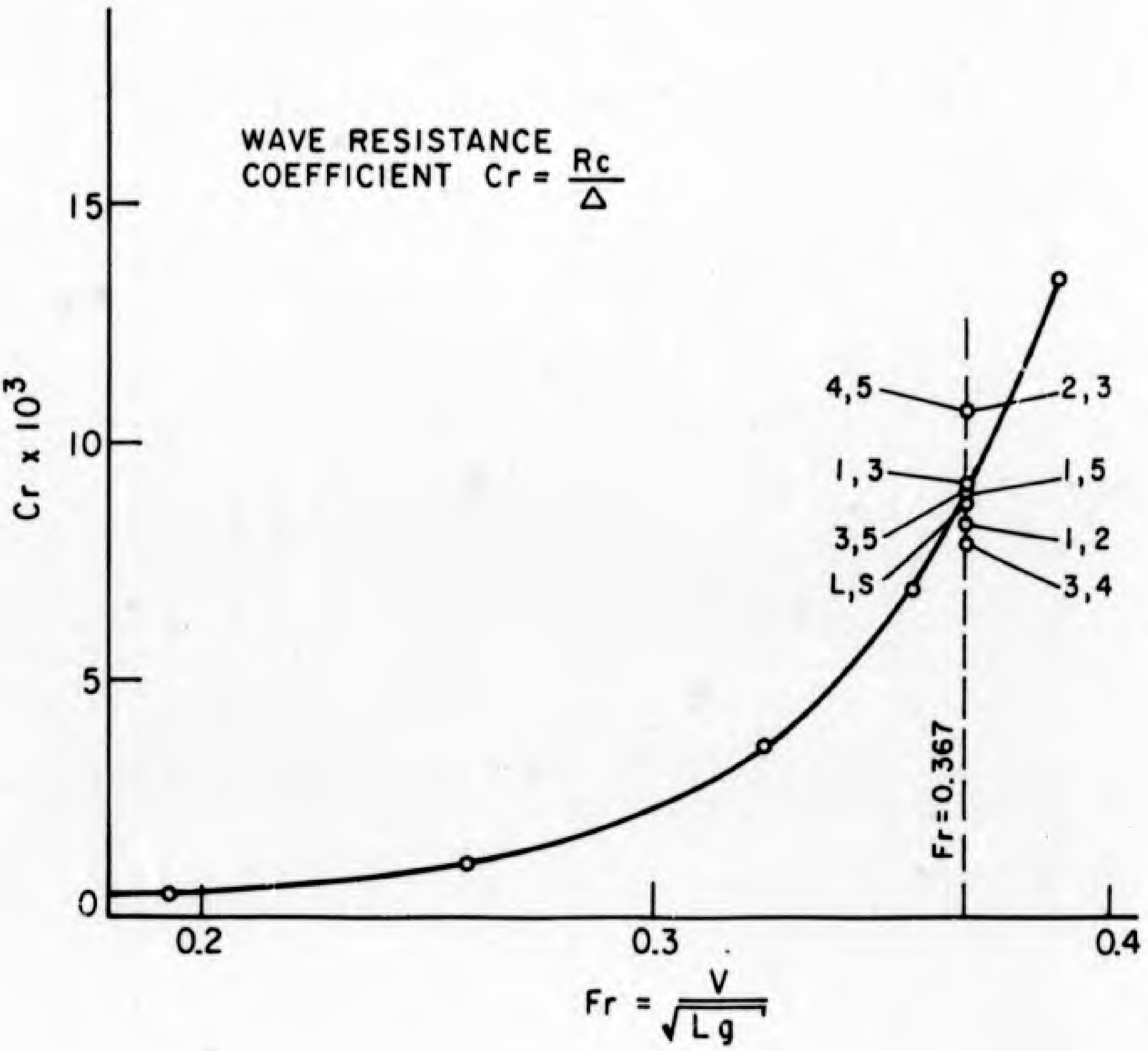

WAVE RESISTANCE
COEFFICIENT Cr = Rc/Δ
Cr x 10³
15
10
5
0
0.2
0.3
0.4
Fr = V/√Lg
Fr=0.367
4,5
2,3
1,3
1,5
3,5
1,2
L,S
3,4

FIG. 8

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