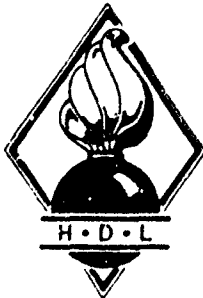AD

TR-1332

METHOD FOR EXTENSION OF DIELECTRIC CONSTANT AND LOSS MEASUREMENTS OF LIQUIDS TO 100 MHz WITH A FIXED-GEOMETRY SAMPLE HOLDER

by

Z. G. Sztankay

November 1966

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HARRY DIAMOND LABORATORIES

WASHINGTON. D.C. 20438

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ABSTRACT

A method for measuring the dielectric properties of liquid specimens on a Q-meter at frequencies up to 100 MHz was developed by (1) assuming an equivalent circuit for a cylindrically shaped liquid-sample holder, (2) determining relevant circuit parameters by experiment, and (3) deriving the necessary equations taking full account of the effects of sample-holder inductance.

Measurements made on two liquid samples are detailed and show that good accuracy is obtained when the standard liquid employed is a lossless dielectric with dielectric constant (K) near the K of the sample under test.

1.   INTRODUCTION

The effect of sample-holder inductance on dielectric constant and loss measurements in the megahertz frequency range was discussed in a previous report (ref 1). It was concluded in that report that the modified Hartshorn-Ward method in use with variable capacitance sample holders circumvents the errors due to the inductance. However, it was pointed out that with nonvariable sample holders one must correct for the inductance effects.

The liquid-sample holder used in this laboratory has no provision for changing capacitance. Thus one must measure the capacitance change used in the calculation for dielectric constant, K, by means of the change in the measuring-meter capacitance. At frequencies below 20 MHz this creates no problems; but at higher frequencies, where a Boonton Model 190-A Q-meter is used to indicate resonance, the sample-holder inductance affects the results. The loss calculation is also affected, since one cannot retune to the same capacitance values with and without sample.

2.   THEORY

2.1 Summary of Previous Results

The basic equations for the dielectric constant and loss tangent, derived earlier (ref 1), will be summarized in brief. The dielectric constant, K, is given by (ref 1, eq 13)

$$K = \frac{\Delta C_e}{\beta_v^2 C_A} + 1 \qquad (1)$$

where C_A is the air capacitance of the space in the sample holder that is filled with sample. The measured change in capacitance

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of the sample holder upon insertion of the sample, ΔC_e, is related, for small capacitance changes, to the actual change, ΔC_p, by (ref 1, eq 4)

$$\beta^2 \equiv \frac{\partial C_e}{\partial C_p} \sim \frac{\Delta C_e}{\Delta C_p} \tag{2}$$

The factor β, of course, is a function of the capacitance. For large capacitance changes, such as are usually involved in equation 1, this must be taken into consideration. Therefore we define β_v^2 as (ref 1 eq 12)

$$\beta_v^2 \equiv \frac{C_{ei} - C_{eo}}{C_{pi} - C_{po}} \tag{3}$$

where the subscripts o and i refer to the capacitance with the sample out and in, respectively. Once we know the sample-holder equations, we can equate β_v^2 to β^2 at a particular capacitance value. This value will be between C_{ei} and C_{eo} (and C_{pi} and C_{po}).

The effective capacitance C_e is influenced by the sample-holder inductance at high frequency. It also includes parts of the sample holder not accessible to the sample, while C_p is the actual capacitance of the part containing the sample.

Provided the effective conductance of the whole measuring circuit, G_R, may be written in the form of equation 15 of reference 1, the loss tangent may be calculated from (ref 1, eq 22)

$$\tan \delta_x = \frac{1}{\beta_i^2 \omega K C_A} \left[\frac{\omega C_R}{Q_o} \left(\frac{Q_o}{Q_i} - 1 \right) - \Delta \ (\text{other terms in } G_R) \right] \tag{4}$$

where the subscripts i and o refer to sample in and out, respectively; C_R is the total equivalent parallel resonant capacitance; G_R is the total equivalent parallel conductance; and Q is the Q of the equivalent parallel circuit. The second term in equation 4 refers to changes in G_R, upon inserting the sample, other than that due to the sample conductance. The resonant capacitance C_R is given by (ref 1, eq 14)

$$C_R = C_e + \alpha \, C_Q \tag{5}$$

where C_Q is the Q-meter tunable capacitance and α is its inductance correction factor (α is discussed in ref 2, and also defined in ref 1, eq 17).

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The dependence of equation 4 on C_R and the absolute value of Q_o (rather than just the ratio Q_o/Q_1) may be eliminated by relating them to the difference in capacitance of the two half-power points, as measured by the change in C_Q with sample out, $\Delta C_{Q_o\sqrt{2}}$ (ref 1). The equation for the loss tangent then becomes (ref 1, eq 27).

$$\tan \delta_x = \frac{1}{\beta_1^2 \omega K C_A} \left[-\frac{Q_o^2 \omega \left| \Delta C_{Q_o\sqrt{2}} \right|}{2} \left(\frac{Q_o}{Q_1} - 1 \right) - \Delta \text{ (other terms in } G_R) \right] \quad (6)$$

where the subscripts i and o on the parameters β and α indicate the capacitance values (with sample in and out, respectively) at which the parameters β (eq 2) and α (ref 1, eq 17) are to be taken.

2.2 Sample Holder Equivalent Circuit

To use the equations for the dielectric constant and loss, it is necessary to know the equivalent circuit of the sample holder. A drawing of the holder is shown in figure 1. The equivalent circuit chosen to represent the holder was one in which the inductance, conductance, resistance, and capacitance are evenly distributed throughout, as shown in figure 2. In the case of the capacitance, the total capacitance, C, is $C = C_p + C_n$, where C_n is the very small constant capacitance not influenced by the sample.

The circuit shown in figure 2 is also the equivalent circuit of a transmission line. The input impedance may thus be obtained from transmission line theory (ref 3). In our case it is more convenient to use its reciprocal, the admittance

$$Y_e = \frac{1}{Z_o} \left[\frac{Z_o \cosh \Gamma + Z_R \sinh \Gamma}{Z_R \cosh \Gamma + Z_o \sinh \Gamma} \right] \quad (7)$$

where Z_o, the characteristic impedance, equals $\sqrt{Z/Y}$, and Γ, related to the propagation constant of transmission line theory, is given by $\sqrt{ZY}$. Z is the total series impedance, $Z = R + j\omega L$, and Y is the total parallel admittance, $Y = G + j\omega C$. The parameters R, L, G, and C, like Z and Y, are defined here to be the total resistance, inductance, conductance, and capacitance for the sample holder rather than the per unit length quantities used in transmission line theory. The load impedance Z_R should be taken to be an open circuit, as this is most consistent with the physical situation. Thus equation 7 reduces to

conductor

insulator

Figure 1. Cross-sectional view of liquid sample holder.

$\frac{L}{n}$ $\frac{R}{n}$ $\frac{L}{n}$ $\frac{R}{n}$ $\frac{L}{n}$ $\frac{R}{n}$

$\frac{C}{n}$ $\frac{G}{n}$ $\frac{C}{n}$ $\frac{G}{n}$ $\frac{C}{n}$ $\frac{G}{n}$
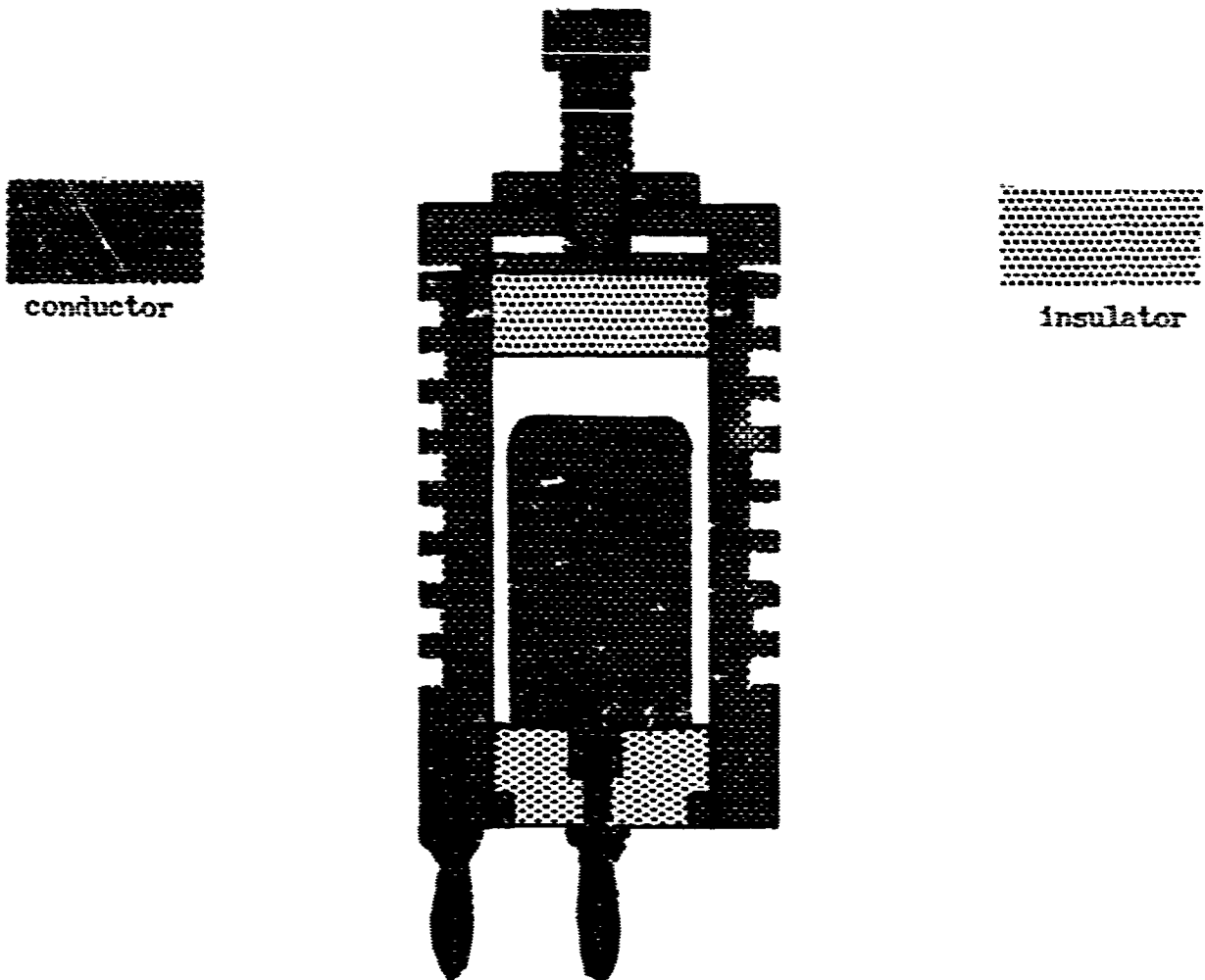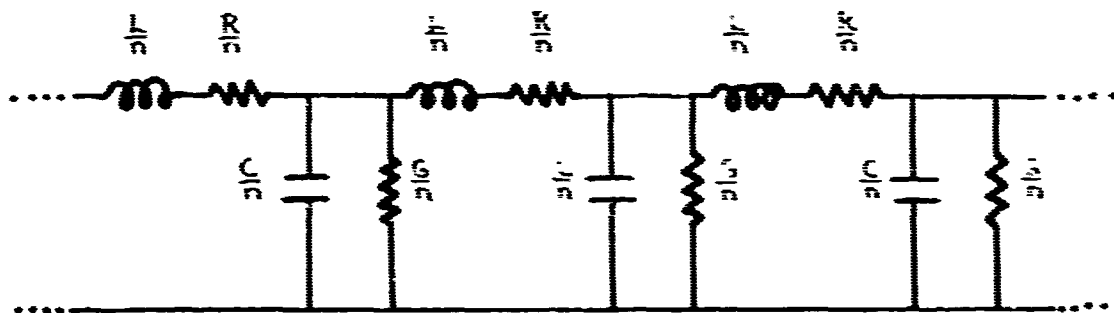
Figure 2. Assumed equivalent circuit of sample holder. There are n repeating sections, each having the circuit parameters C/n, G/n, L/n and R/n with n $\longrightarrow \infty$. The total capacitance, conductance, inductance and resistance equals C, G, L and R, respectively.

$$Y_e = \frac{1}{Z_o} \tanh \Gamma \qquad (8)$$

We may assume that the series resistance is small, $\omega L \gg R$, and that the parallel conductance is also small, $\omega C \gg G$. The former assumption is always good in the frequency range of interest (> 10 MHz), and the latter would come into question only for very lossy samples, as it is equivalent to the condition that $\tan \delta \ll 1$. With the above assumptions we obtain

$$\frac{1}{Z_o} \sim \sqrt{\frac{C}{L}} - j \frac{1}{2\omega} \sqrt{\frac{C}{L}} \left(\frac{G}{C} - \frac{R}{L}\right) \qquad (9)$$

and

$$\Gamma \sim j\omega\sqrt{LC} + \frac{\sqrt{LC}}{2}\left(\frac{G}{C} + \frac{R}{L}\right) \qquad (10)$$

The substitution of equations 9 and 10 into equation 8 yields

$$Y_e = j\sqrt{\frac{C}{L}} \tan \omega\sqrt{LC} - \frac{j\omega C}{4 \cos^2 \omega\sqrt{LC}} \left[\left(\frac{G}{\omega C}\right)^2 - \left(\frac{R}{\omega L}\right)^2\right]$$

$$+ \frac{G + RC/L}{2 \cos^2 \omega\sqrt{LC}} + \frac{1}{2\omega}\sqrt{\frac{C}{L}} \left(\frac{G}{C} - \frac{R}{L}\right) \tan \omega\sqrt{LC} \qquad (11)$$

Under the assumptions mentioned earlier, the second imaginary term is negligible compared with the first, at least at frequencies of interest. Changing the form of the first imaginary term slightly, and expanding the trigonometric functions in the real terms, keeping three terms in the expansions, and combining, we obtain

$$Y_e \sim j\omega C \frac{\tan \omega\sqrt{LC}}{\omega\sqrt{LC}} + G\left[1 + \frac{2}{3} \omega^2 LC + \frac{6}{15} (\omega^2 LC)^2 + \cdots\right]$$

$$+ \frac{R}{3} (\omega C)^2 \left[1 + \frac{4}{5} \omega^2 LC + \cdots\right] \qquad (12)$$

The equivalent parallel capacitance of the liquid sample holder is, therefore

$$C_e = C \frac{\tan \omega\sqrt{LC}}{\omega\sqrt{LC}} \qquad (13)$$

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or, upon expanding

$$C_e \sim C \left[1 + \frac{1}{3} \omega^2 LC + \frac{2}{15} (\omega^2 LC)^2 + \cdots \right] \qquad (14)$$

and the equivalent parallel conductance is

$$G_e \sim G \left[1 + \frac{2}{3}\omega^2 LC + \frac{6}{15} (\omega^2 LC)^2 + \cdots \right] + \frac{R}{3} (\omega C)^2 \left[1 + \frac{4}{5} \omega^2 LC + \cdots \right] \quad (15)$$

The factor β^2, defined by equation 2, is obtained by differentiating equation 14 with respect to C_p, with $C = C_p + C_n$:

$$\beta^2 \sim 1 + \frac{2}{3} \omega^2 LC + \frac{6}{15} (\omega^2 LC)^2 + \cdots \Big] \qquad (16)$$

This is precisely the term multiplying G in equation 15. Further, from equation 14, we may write

$$\left(\frac{C_e}{C}\right)^2 \sim 1 + \frac{2}{3} \omega^2 LC + \frac{17}{45} (\omega^2 LC)^2 + \cdots \qquad (17)$$

This is almost exactly equal to β^2, and at least approximately equal to the term multiplying $R/3(\omega C)^2$ in equation 15.

The equivalent parallel conductance of the sample holder may thus be written as

$$G_e \sim \beta^2 G + \frac{R}{3} (\omega C_e)^2 \qquad (18)$$

The total effective conductance of the measuring circuit is therefore in the form of equation 15 of reference 1, with $G_p \to G$, $G_n = R_p = 0$, and $R_p \to R/3$. (The last four terms in that equation are the Q-meter conductance terms.) Thus the loss tangent may be calculated from equation 4 or 6. Next, to find β_v (equation 3), we take the difference of equation 14 between $C_p = C_{pi}^v$ and $C_p = C_{po}$, remembering that $C = C_p + C_n$, and obtain

$$\frac{C_{ei} - C_{eo}}{C_{pi} - C_{po}}{}^2 = 1 + \frac{1}{3} \omega^2 L \left[\frac{C_{pi}^2 - C_{po}^2}{C_{pi} - C_{po}} + 2C_n \right]$$

$$+ \frac{2}{15} (\omega^2 L)^2 \left[\frac{C_{pi}^3 - C_{po}^3}{C_{pi} - C_{po}} + \frac{3C_n (C_{pi}^2 - C_{po}^2)}{C_{pi} - C_{po}} + 3C_n^2 \right] \quad (19)$$

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If we define C_{Av} by

$$C_{Av} \equiv \frac{C_i + C_o}{2} = \frac{C_{pi} + C_{po}}{2} + C_n \tag{20}$$

we find that equation 19 may be written exactly through the first two terms, and to a good approximation in the third term, as

$$\beta_v{}^2 \sim 1 + \frac{2}{3}\,\omega^2 L C_{Av} + \frac{6}{15}\,(\omega^2 L C_{Av})^2 + \cdots \tag{21}$$

Comparing equation 21 with equation 16, we see that $\beta_v{}^2 = \beta_{Av}{}^2$, the value of β^2 at $C = C_{Av}$.

To sum up the results of this section, we write down the following approximate relationships:

$$C_e \sim \beta C = \beta(C_p + C_n) \tag{22}$$

$$G_e \sim \beta^2 G + \frac{R}{3}\,(\omega C_e)^2 \tag{18}$$

$$K = \frac{\Delta C_e}{\beta_{Av}{}^2 C_A} + 1 \tag{23}$$

and

$$\beta \sim \frac{\tan \omega \sqrt{LC}}{\omega \sqrt{LC}}$$

$$\sim 1 + \frac{1}{3}\,\omega^2 L C + \frac{2}{15}\,(\omega^2 L C)^2 + \cdots \tag{24}$$

2.3 Discussion of Sample-Holder Equations

Theoretically, we should be able to use the equations obtained in the previous section to correct for the inductance error once we have found the value of L. However, we could still not find the correction factor to ΔC_e, $\beta_{Av}{}^2$, without first knowing C. Furthermore, the equivalent circuit is, of course, only approximate, so it is doubtful that the equation for β_{Av} is usable in such a direct way.

If very many measurements are to be made with the same liquid sample holder, the best procedure would be to find β as a function of C_e from equation 22, using standard liquids whose dielectric constants are known. We could then make plots of β

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versus C_e at the frequencies of interest, and refer to these plots when making corrections. It may well prove worthwhile to do this, but for the time being, an alternate method, outlined in the next section, was chosen.

Whatever method of finally correcting for the inductance errors is used, one very significant reduction in error may be made by using a standard liquid with dielectric constant known and close to that of the sample for the sample-out measurement. If κ is the dielectric constant of the standard, the dielectric constant of the unknown will be given by

$$K = \frac{\Delta C_e}{\beta_{Av}^2 C_A} + \kappa \tag{25}$$

instead of equation 23. If κ is close to K, the same percentage error in β_{Av}^2 will have much less significance than if equation 23, which is really the special case of equation 25 with $\kappa = 1$, is used.

2.4 Procedure for Calculating K

The dielectric constant is to be calculated from equation 25. In that equation, C_A and κ are known, and ΔC_e can be readily measured. Since C_R is constant, equation 5 yields

$$\Delta C_e = C_{ei} - C_{eo} = \alpha_o C_{Qo} - \alpha_i C_{Qi} \tag{26}$$

where the subscripts i and o refer to sample in and out (standard in), respectively. The function $\alpha(C_Q)$ may be obtained from the Q-meter manual (ref 2). A plot of α as a function of frequency and C_Q, taken from the manual , is shown in figure 3 of this report, however, for convenience. The equation for α is also given in figure 3. The function β_{Av}, however, is not known. If the change in capacitance ΔC_e is very small, we can use the β appropriate to the standard, β_o, as an approximation to β_{Av}. The factor β_o can be found from equation 22 with $C = C_o$ and $C_e = C_{eo}$. C_o is known and equation 5 gives C_{eo} once C_R has been determined (discussed later).

When ΔC is large, we cannot equate β_{Av} to β_o. With some approximations, however, we find β_{Av} from β_o. First, to find C_{Av}, for use in the correction term,

$$C_{Av} \sim \frac{C_{eAv}}{\beta_o} = \frac{1}{2}\left(\frac{C_{ei}}{\beta_o} + C_o\right) \tag{27}$$
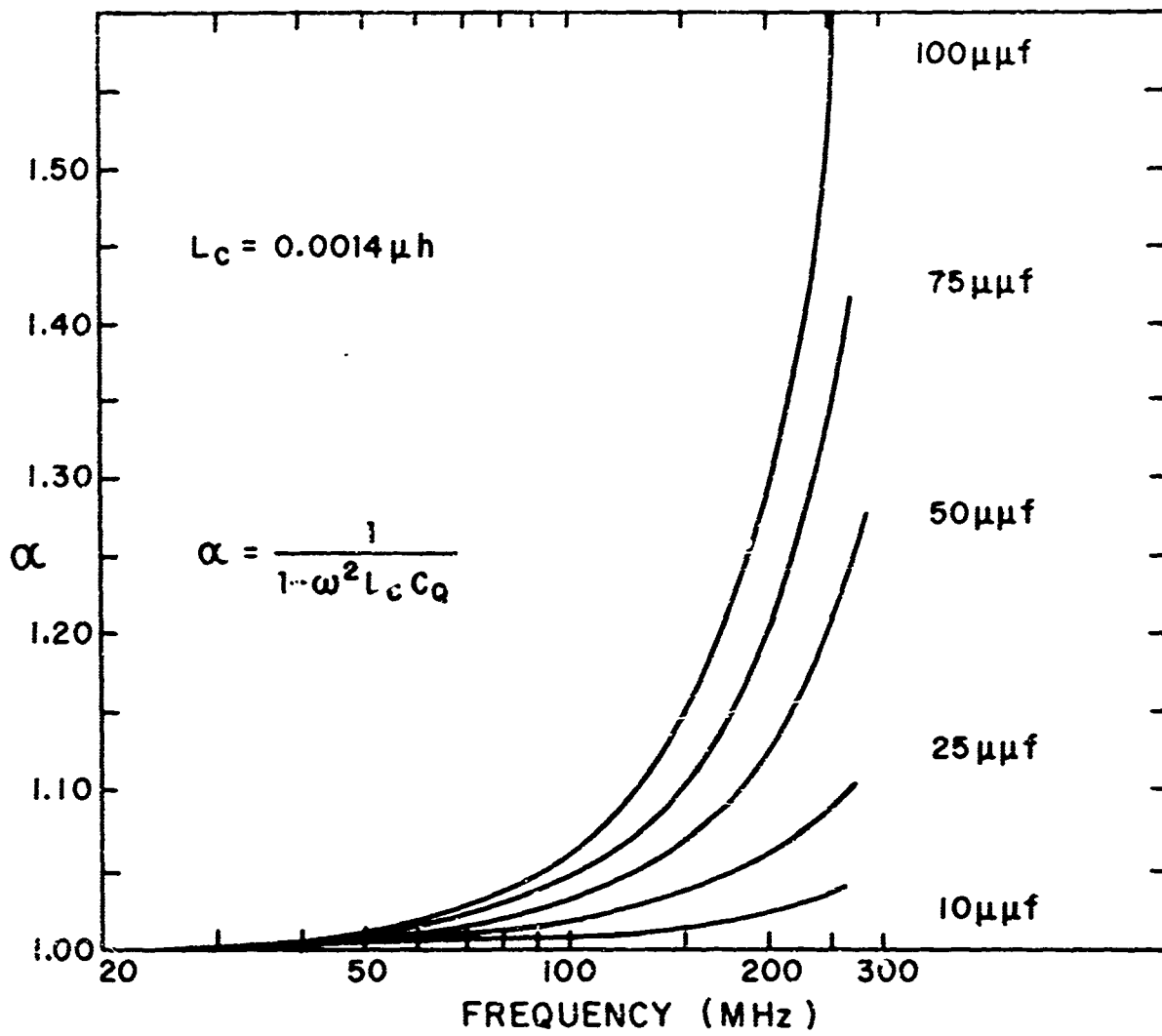
Figure 3. α as a function of frequency (from ref 6 p 16).

The figure shows α plotted against FREQUENCY (MHz) on a logarithmic scale from 20 to 300. Curves are labeled $100\mu\mu f$, $75\mu\mu f$, $50\mu\mu f$, $25\mu\mu f$, and $10\mu\mu f$.

$$L_c = 0.0014\,\mu h$$

$$\alpha = \frac{1}{1 - \omega^2 L_c C_Q}$$

All the terms on the right in equation 27 are known or measurable. Second, we note that β_o^2 and β_{Av}^2 are given by equation 16 with $C = C_o$ and $C = C_{Av}$, respectively. We see from that equation that, approximately,

$$\beta_{Av}^2 \sim \left[1 + (\beta_o - 1)\,\frac{C_{Av}}{C_o}\right]^2 \tag{28}$$

Equation 28 is incorrect only in the third term of β_{Av}, as

$$1 + (\beta_o - 1)\,\frac{C_{Av}}{C_o} = 1 + \frac{1}{3}\,\omega^2 L C_{Av} + \frac{2}{15}\,(\omega^2 L)^2 C_{Av} C_o + \cdots \tag{29}$$

By examining equations 27 and 29, we see that if $C_i > C_o$ the first approximation causes an overcorrection as C_{Av} comes out too large (since in this case $\beta_o < \beta_i$), while the second one caused an under correction, as in the third term of β_{Av}^2, C_o, which is in this case less than C_{Av}, appears in place of C_{Av}. If $C_i < C_o$, the reverse takes place. Thus the two errors have a partially cancelling effect.

For calculation purposes, we may eliminate some of the steps by substituting C_{eo}/C_o for β_o in equation 27, and the result into equation 28. After rearranging, we find that

$$\beta_{Av}^2 = \frac{1}{4}\left[1 + \frac{C_{eo} + C_{ei}}{C_o} - \frac{C_{ei}}{C_{eo}}\right]^2 \tag{30}$$

Equation 30 may then be used directly in equation 25 for calculation of the dielectric constant.

2.5 Procedure for Calculating Tan δ

The loss tangent may be calculated from either equation 4 or 6. For either equation, we need to know β_i, but that is now easily calculated. First, we obtain C_i from

$$C_i = C_o + \frac{\Delta C_e}{\beta_{Av}^2} \tag{31}$$

Thus β_i is given by

$$\beta_i = \frac{C_{ei}}{C_i} = \frac{C_{ei}}{C_o + \Delta C_e / \beta_{Av}^2} \tag{32}$$

All quantities on the right in equation 32 are known.

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Next we must consider the subtractive term in the
tan δ_x equations. We must, therefore, write down the total
parallel conductance, including the measuring meter conduc-
tances. With the Boonton Model 190-A Q-meter, or a meter with
a similar equivalent circuit, the total conductance is

$$G_R = \beta^2 G + \frac{R}{3}(\omega C_e)^2 + \alpha^2 G_v + \alpha^2(\omega C_Q)^2 R_c + G_d + (\omega C_R)^2 R_m \qquad (33)$$

The first two terms are from equation 18, and the last four are
the Q-meter conductance terms. The latter are given in refer-
ence 2, and their method of derivation is discussed in reference
1. The terminology differs only in that the combination of $R + R_1$
in the manual is here (and in ref 1) labeled R_m, and the C of
the manual is here (and in ref 1) called C_Q.

If we assume that the sample-holder conductance is zero
with the sample out, and that we may neglect the small change
due to the difference between α_o and α_i in the already small
G_v term, and if we remember that C_R does not change, we see that

$$\Delta \text{ (other terms in } G_R) = \frac{R}{3}\omega^2(C_{ei}{}^2 - C_{eo}{}^2) + R_c \omega^2(\alpha_i{}^2 C_{Qi}{}^2 - \alpha_o{}^2 C_{Qo}{}^2) \qquad (34)$$

All terms on the right-hand side are known except for R, which
must be determined by experiment. This is best done by making
measurements, first with the sample holder removed (denoted by
subscript o) and subsequently with the empty sample holder, or
sample holder with a lossless dielectric, in the circuit (sub-
script i). In this case tan $\delta_x = C_{eo} = 0$; therefore equations
4 and 34 yield

$$\frac{R}{3} - \frac{1}{C_{ei}{}^2} \left[\frac{C_R}{\omega Q_o} \left(\frac{Q_o}{Q_i} - 1 \right) + R_c(\alpha_o{}^2 C_{Qo}{}^2 - \alpha_i{}^2 C_{Qi}{}^2) \right] \qquad (35)$$

The meter contact resistance R_c is obtained from the manual (ref 2)
as

$$R_c \sim 0.0004 \sqrt{F_{MHz}} \ \Omega \qquad (36)$$

when F_{MHz} is the frequency in megahertz. Once equation 35 has
been used to obtain R as a function of frequency for a sample
holder, this value of R may be used in equation 34 and equation
4 or 6 to find tan δ_x for a sample.

As with the K measurement, we may reduce the error
by filling the sample holder with a standard dielectric for
the sample-out measurement. If the standard has a K similar
to the unknown, C_{ei} will not be too different from C_{eo} (and
C_{Qi} from C_{Qo}) and, hence, the magnitude of the correction
terms will be reduced. Presumably one would use the same
standard for this calculation as for the K calculation, as
then no additional measurement need be performed. This im-
poses one further requirement for the standard—that it be
lossless.

2.6 Determination of C_R

The total resonant capacitance of the circuit, C_R,
is needed in both the dielectric constant and loss tangent cal-
culation. In the K calculation, however, C_R is needed only in
the calculation for the inductance correction term β_{Av}. Spe-
cifically, equation 5 is used to calculate the effective capac-
itances, and it involves C_R. Equation 5 itself affords the
best way of calculating C_R for this purpose, by determining C_Q
with the sample holder not attached (i.e., with $C_e = 0$). With
C_R determined in this manner, the effective capacitance, C_e,
is really measured as the difference between two C_Q readings,
suitably corrected by α.

However, because of the limited range of the C_Q dial,
and the large sample and standard effective capacitances, es-
pecially at high frequencies, it is not always possible to
resonate with the coil only at the same frequency as with the
sample and standard. In this case, one should resonate with coil
only at a frequency, ω_c, as close as possible to the measure-
ment frequency, and find the equivalent-circuit resonant in-
ductance in parallel with C_R and G_R, L_R (mostly due to the par-
ticular work coil being used, but including L_i, defined in the
manual) from

$$L_R = \frac{1}{\omega_c^2 C_Q} - L_c \tag{37}$$

The internal inductance correction is taken care of by the sub-
traction of L_c, which is given in the manual ($L_c \sim 0.0014\ \mu h$)
(ref 2). The resonant capacitance at the measurement frequency
is then obtained from

$$C_R = \frac{1}{\omega^2 L_R} \tag{38}$$

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In the loss calculation, C_R is an integral p. of the calculation. However, as discussed earlier, it may be related to the width of the resonance curve, and thus eliminated from the equation, along with Q_o. It is mainly on the basis of eliminating Q_o, rather than the absolute value of C_R, that it is recommended that the loss be calculated using equation 6 instead of equation 4. However, it should be noted that this equation gives tan δ_x mainly as a function of capacitance differences, rather than absolute values, as well as voltage ratios. The absolute values do appear in the correction term, given by equation 34, in which the C_e's are already known, as they were determined for the dielectric constant calculation.

3. EXPERIMENTAL PROCEDURE AND RESULTS

3.1 Validity of Equations

Table I gives low-frequency and effective capacitances measured to check the validity of the proposed sample-holder equations. Data were taken at various frequencies for the empty sample holder and holder filled with benzene. Benzene is essentially lossless and has a constant K for frequencies up to and beyond those used in these measurements.

Capacitance bridge measurements were made at 100 kHz to determine the true capacitances C_o and C_i of the empty and benzene-filled sample holder, respectively. The Q-meter was used for high-frequency determinations, at 22, 50, 78, 85, and 92 MHz, of C_{eo} and C_{ei}. These quantities were calculated using equation 5, with C_R being determined by resonating the meter with the sample holder not attached. For the 92-MHz point, equations 37 and 38 had to be used. For that reason the results at that frequency are less precise.

Quantities calculated from table I are given in table II. The ratio C_{eo}/C_o gave β_o and allowed the calculation of L from equation 24. The expanded form of that equation was found convenient in this case, as it converges rapidly for small C. The results for L differ somewhat for the different frequencies, with an average value of L = 0.050 μH. The differences may well be explained by the inaccuracies of the absolute capacitance measurements with the Q-meter. The value of L obtained for a given frequency was then used, along with C_i, to calculate β_i from equation 24, this time using the form of the equation involving the tangent. This gave a calculated C_{ei} that could be compared with the experimentally determined C_{ei}. In all cases, the experimental C_{ei} was higher than the predicted one. This suggests that L, as determined from data for the empty sample holder, does not apply exactly to the benzene-filled sample holder.

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TABLE I.—MEASURED CAPACITANCES (pF) IN CHECK OF EQUIVALENT CIRCUIT

f, MHz	22	50	78	85	92
C_o	26.4	26.4	26.4	26.4	26.4
C_i	56.2	56.2	56.2	56.2	56.2
C_{eo}	26.4	27.3	29.7	31.2	31
C_{ei}	57.4	63.7	82.5	---	98

TABLE II.—CALCULATED QUANTITIES FROM EQUIVALENT CIRCUIT AND TABLE I

f, MHz	22	50	78	85	92
β_o	-	1.034	1.125	1.182	1.17
L, μH	-	0.039	0.052	0.060	0.05
β_i	-	1.08	1.33	-	1.53
C_{ei}, pF	-	60.7	74.9	-	85.7

TABLE III.—EFFECTIVE CAPACITANCES (pF) CALCULATED FROM LOW-FRE-
QUENCY DATA OF TABLE I AND CIRCUIT EQUATIONS WITH
$L = 0.060$ μH

f, MHz	22	50	78	85	92
C_{ei}	57.4	63.4	78.7	85.6	94.7
C_{eo}	26.7	27.9	30.4	31.3	32.3

If a value of $L = 0.060$ µH is chosen, the predicted C_{ei}'s, table III, are quite close to the experimental values. The value of 0.060 µH is not very far from the actual average value of 0.050 µH. In fact, the experimental value at 85 MHz was 0.060 µH (unfortunately, no benzene data were taken at that frequency). The higher value of L, however, when used in equation 14 with C_o to calculate the predicted C_{eo}'s, results in a little too high a value of the C_{eo}'s.

It should be remembered in assessing the results that these comparisons are between capacitances that differ by more than a factor of two. The inexactness of the corrections are probably due to the approximate applicability of the equivalent circuit, which led to the circuit equations. The proposed correction method for samples is based on the same equations, but comparisons are to be made on capacitances that are relatively close to each other. The results confirm that the method should not be used except with a standard that does not differ widely in dielectric constant from the sample.

3.2 Determination of R

The sample-holder series resistance R was determined using equation 35. Measurements were made of the circuit Q's and meter capacitances needed to resonate with the sample holder not attached, with it attached but empty, and with it attached and filled with benzene. The Q-meter readings were corrected for internal effects as described in the manual (ref 2) and the precursor of this report (ref 1, equation 16). Frequencies used were 22, 50, 78, and 85 MHz.

R was calculated at each frequency, first with the empty-sample-holder Q's and C_Q's (subscript i) being compared with the sample-holder-removed Q's and C_Q's (subscript o). The results are shown in figure 4.

The resistances are plotted against the square root of the frequency because of the expected linear relationship, due to the skin effect. The large scatter of the points is caused by subtraction of two similar numbers in both terms of equation 35. This is especially true in the first term, as Q_o/Q_i is very close to unity. especially at the lower-frequency points, which are therefore quite unreliable.

Putting benzene, which is essentially lossless in this frequency range, into the sample holder alleviated this problem. With benzene, Q_i and C_{Qi} were considerably reduced, so that the subtractive terms in equation 35 became relatively smaller compared with the additive terms. It should be mentioned parenthetically, however, that the reduction in Q_i, which is the circuit Q, is not due to loss in the benzene, but rather to the increase in the capacitance C_{ei}.

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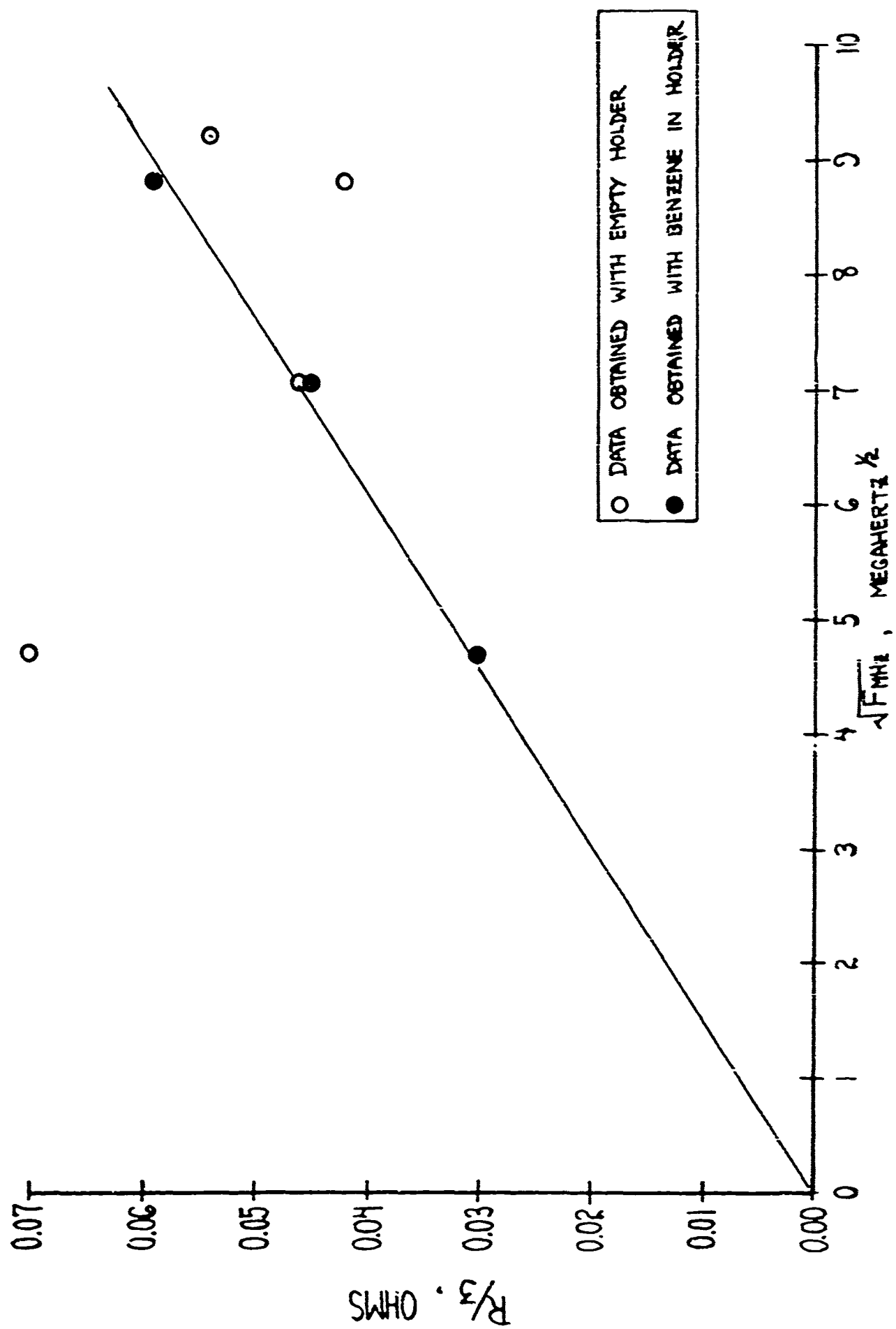Figure 4. Plotting one-third of the total sample holder series resistance as a function of the square root of the frequency.

○ DATA OBTAINED WITH EMPTY HOLDER

● DATA OBTAINED WITH BENZENE IN HOLDER

$\sqrt{F_{MHz}}$, MEGAHERTZ $^{1/2}$

R/3 , OHMS

The results obtained with benzene are also plotted against the square root of the frequency in figure 4. The points fall very nicely on a straight line through the origin, showing the skin effect. If F_{MHz} represents the frequency in megahertz, then R may be written as

$$\frac{R}{3} = 0.007 \sqrt{F_{MHz}} \ \Omega \tag{39}$$

Figure 4 would seem to justify having one more significant figure in this expression (if we consider only the points with benzene in the holder), but if we remember the approximate nature of the equation used in calculating the R's, we see that one significant figure is all that is justified. By comparing with equation 36, it is seen that the sample-holder resistance is considerably higher than the Q-meter resistance, so that the R_c term in equation 34 is usually negligible. This, however, would not be true if the conditions of measurement were such that the C_Q's were considerably higher than the C_e's.

Equations 39 and 36 may be used in equation 34, and the latter substituted into equation 6 (or equation 4). The final form of t   tan δ equation using the capacitance variation method for C_R/Q_o (i.e., equation 6) then becomes

$$\tan \delta_x = \frac{1}{\beta_i^2 KC_A} \left[\frac{\alpha_o^2 |LC_{Q_o}|}{2} \right]^{1/2} \left(\frac{Q_o}{Q_i} - 1 \right) - 4 \times 10^{-8} F_{MHz}^{\frac{3}{2}} (C_{ei}^2 - C_{eo}^2$$

$$- 2 \times 10^{-9} F_{MHz}^{\frac{3}{2}} (\alpha_i^2 C_{Qi}^2 - \alpha_o^2 C_{Qo}^2) \tag{40}$$

The capacitances in this equation must be in picofarads, and F_{MHz} is the frequency in megahertz. The last term, as mentioned, is usually negligible.

3.3 Results with Two Samples

Dielectric constant and loss calculations using these methods have been made on several liquids. The calculation of two samples, α-methylstyrene and poly-α-methylstyrene, at 50 and 78 MHz, will be discussed, and the results compared with low-frequency measurements. Data with very similar samples were taken up to 90 MHz, with similar results, but those data were not taken by the author of this report and will therefore not be discussed here.

The data and calculated quantities are given in table
IV. The first section gives the Q-meter capacitance measure-
ments, corrected for the internal inductance; the effective capac-
itances calculated from them; the effective capacitance change,
ΔC_e; and the low-frequency capacitance of the sample holder with
the standard (benzene) measured on a capacitance bridge at 100
kHz. The next section gives the necessary quantities in the K
determination, calculated as outlined earlier. The long method
was used, instead of equation 30, to more clearly indicate the
magnitudes of the terms involved. Using a value of 23.5 pF for
C_A, also determined earlier at low frequency, and $K = 2.28$ for
the dielectric constant of benzene at $25°C$, the results for K
are 2.58 and 2.54 for the monomer and polymer, respectively,
at both frequencies.

The parameters of the loss calculations are given in
the third section of table IV. The value of β_4 was calculated
from equation 32. The Q's listed in the table have been cor-
rected for the internal errors (ref 2). The next entry of the
table gives the total loss term; and this is followed by the
two correction terms. The total loss term in this case was
calculated using C_R/Q_0 directly (i.e., equation 4 instead of 6
was used). The second correction term is negligible in all
these cases, but the first is substantial, especially for the
lower loss monomer. The tan δ_x's for the two frequencies are
substantially the same, but differ considerably for the two
samples.

The results are plotted as a function of frequency in
figure 5, along with results obtained at 22 MHz with the Q-meter
(these were not outlined in table IV as the corrections at that
frequency are quite small), and lower-frequency points using a
capacitance bridge. The results show, as was expected, that the
monomer has a constant K and tan δ up to 10^8 Hz, while the K of
the polymer shows a drop, with a corresponding peak in the loss.
The high-frequency results are quite consistent with the low-
frequency measurements. This is true despite the inductance
corrections which are quite large, especially at 78 MHz.

4. CONCLUSIONS

It is concluded that the procedures for calculating the di-
electric constant and loss that have been outlined give reasonably
good results up to ~100 MHz. Of course, the accuracy of K will
depend on how close the dielectric constants of the standard and
sample are. We can always reduce the error by finding, or mixing,
a standard whose dielectric constant is closer to that of the sam-
ple. For the loss, results will be reasonable for 0.1 > tan δ
> 0.001. The upper limit originates from assumptions made in

TABLE IV.—OUTLINE FOR DIELECTRIC CONSTANT AND LOSS TANGENT CALCULATIONS FOR α-METHYLSTYRENE AND POLY-α-METHYLSTYRENE

| Relevant equation	Parameter	Applicable specimen	Monomer	Polymer	Monomer	Polymer
	F, MHz	sample & standard	50	50	78	78
	C_R, pF	no holder	87.1	87.1	110.5	110.5
	$\alpha_o C_{Qo}$, pF	standard	23.4	23.4	27.9	27.9
	$\alpha_1 C_{Q1}$, pF	sample	14.2	15.5	11.4	13.9
	C_o, pF	standard	56.3	56.3	56.3	56.3
(5)	C_{eo}, pF		63.7	63.7	82.6	82.6
(5)	C_{e1}, pF		72.9	71.6	99.1	96.6
(26)	ΔC_e, pF		9.2	7.9	16.5	14.0
(22)	β_o		1.13	1.13	1.47	1.47
(27)	C_{Av}, pF		60.4	59.8	61.9	61.0
(28)	β_{Av}^2		1.30	1.30	2.30	2.28
	ΔC_o, pF		7.08	6.08	7.17	6.14
	C_A, pF		23.5	23.5	23.5	23.5
	$\Delta C / C_A$		0.30	0.26	0.30	0.26
	κ	standard	2.28	2.28	2.28	2.28
(25)	κ	sample	2.58	2.54	2.58	2.54
(31)	C_1, pF		63.4	62.4	63.5	62.4
(32)	β_1		1.15	1.15	1.56	1.55
	β_1^2		1.32	1.32	2.44	2.39
	Q_o	standard	226	226	194	194
	Q_1	sample	185	145	138	114
	$\dfrac{C_R}{Q_o}\left(\dfrac{Q_o}{Q_1} - 1\right)$, pF		0.085	0.22	0.23	0.40
	$\omega \dfrac{R}{3}(C_{e1}^2 - C_{eo}^2)$, pF		0.02	0.02	0.08	0.07
	$\omega R_c(\alpha_1^2 C_{Q1}^2 - \alpha_o^2 C_{Qo}^2)$, pF		-0.0003	-0.0003	-0.001	-0.001
	$\tan \delta_x (\beta_1^2 \kappa C_A)$, pF		0.065	0.20	0.15	0.33
	$\beta_1^2 \kappa C_A$, pF		80.2	78.7	147.8	142.9
(40)	$\tan \delta_x$	sample	0.0008	0.0025	0.0010	0.0023
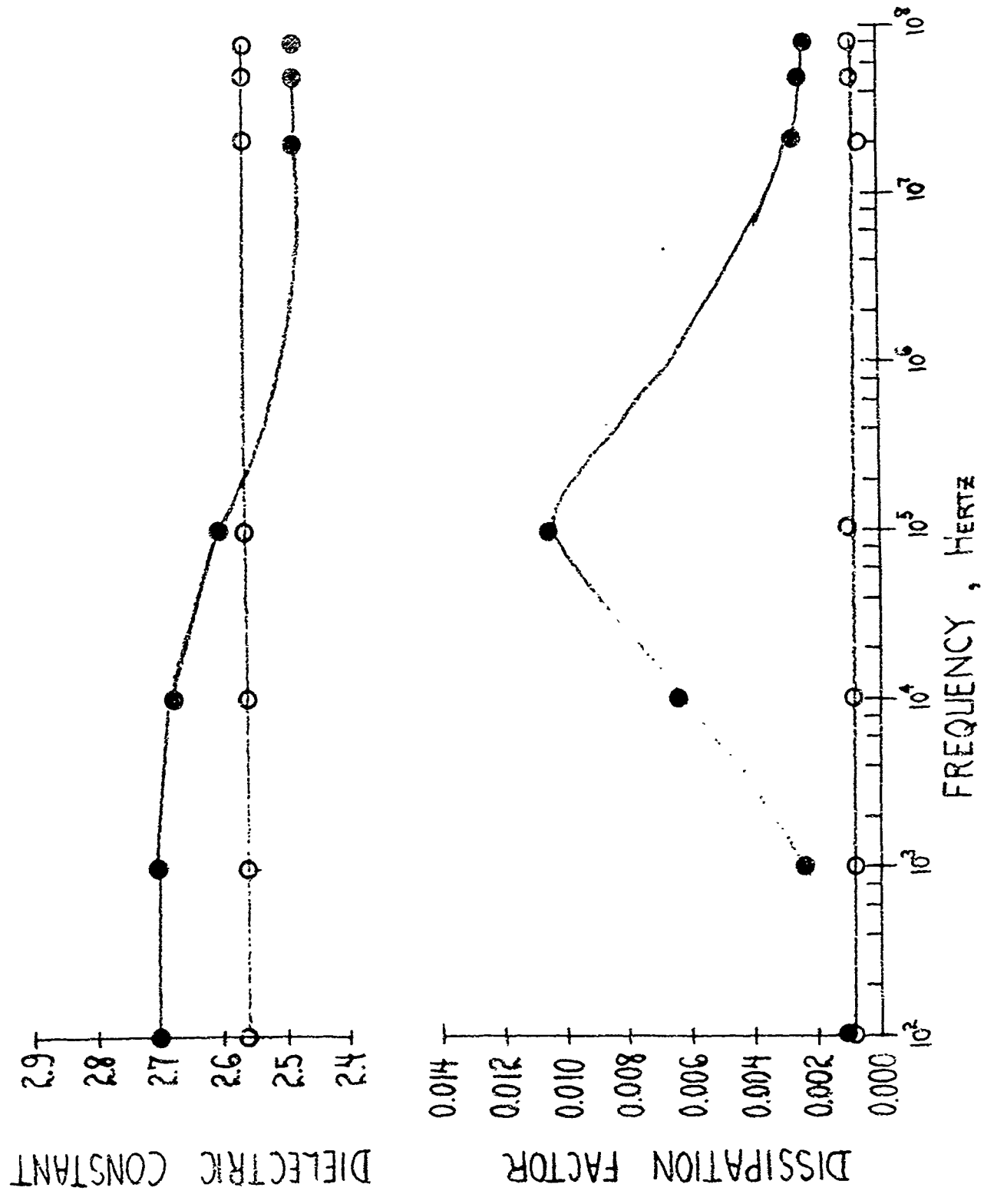
Figure 5. Dielectric constant and dissipation factor as a function of frequency for α-methylstyrene (O) and poly-α-methyl-styrene (●).

deriving the equivalent circuit, and the lower limit comes from
the size of the sample holder correction term. The latter limit
is very approximate, as it depends on the circuit capacitances.
Whether one can assign an actual number to the loss tangent, or
only make the statement that it is smaller than a certain number,
will depend on the individual circumstances. The loss calcu-
lation also includes the large multiplicative β_i^2 correction. It
is, therefore, considerably less accurate than the dielectric
constant determination.

It should be possible to make measurements at somewhat higher
frequencies, perhaps in the 100-to-200-MHz range, by using smaller
capacitance sample holders, as the correction term goes as powers
of $1/3\omega^2 LC$. Some experimentation on this has been done. It would,
however, be difficult to go to still higher frequencies with this
technique. An obvious upper frequency limit is, of course, reached
when the expansions used no longer converge, but the method should
certainly be abandoned before that limit is reached. One should
judge, in each instance, from the size of β, how much, if any, sig-
nificance should be attached to the results.

5. PROCEDURE FOR HIGH-FREQUENCY MEASUREMENTS

To make a measurement with the liquid sample holder at fre-
quencies greater than 20 MHz, use Q-meter 190-A and follow the
procedure outlined below.

First, choose a standard liquid with negligible loss compared
with that of the sample and a known, constant-with-frequency, dielec-
tric constant as close to that of the sample as is available.
(For low K samples this standard dielectric may be air.) If K
is not known even approximately, various standards may have to be
tried. Then measure the following quantities:

C_{QR} - the resonant C_Q with the sample holder not attached
(in rare cases, this may be obtained only at a fre-
quency different from the measuring frequency—
then see discussion by equations 37 and 38).

C_{Qo} - The resonant C_Q with the standard in the holder.

Q_o - the resonant circuit Q with the standard (unless ex-
treme accuracy is desired, and seems justified, the
Q's may be taken directly as the meter readings for
frequencies up to 100 MHz, as only their ratio is
used; if deemed advisable, correct as described in
manual.)

C_{Qo+} — the C_Q above C_{Qo} where $Q = Q_o/\sqrt{2}$

C_{Qo-} — the C_Q below C_{Qo} where $Q = Q_o/\sqrt{2}$

C_{Qi} — the resonant C_Q with the sample in the holder

Q_i — the resonant circuit Q with the sample

Next, obtain from the manual, or figure 3 of this report:

α_R — the internal correction factor (for L_c) with $C_Q = C_{QR}$

α_o — the internal correction factor with $C_Q = C_{Qo}$

α_i — the internal correction factor with $C_Q = C_{Qi}$

Now calculate:

$$C_R = \alpha_R C_{QR}$$

$$C_{eo} = C_R - \alpha_o C_{Qo}$$

$$C_{ei} = C_R - \alpha_i C_{Qi}$$

$$\Delta C_e = C_{ei} - C_{eo}$$

$$\Delta C_{Qo\frac{1}{2}} = C_{Qo+} - C_{Qo-}$$

Further, obtain from notebook, references, or low-frequency measurement:

C_o — low-frequency capacitance of sample holder with standard

C_A — air capacitance of portion of holder that is accessible to the liquids

κ — dielectric constant of the standard

Now calculate:

$$\beta^2_{Av} = \frac{1}{4}\left[1 + \frac{C_{eo} + C_{ei}}{C_o} - \frac{C_{ei}}{C_{eo}}\right]^2$$

$$K = \frac{\Delta C_e}{\beta^2_{Av} C_A} + \kappa$$

$$\beta_i = \frac{C_{ei}}{C_o + \Delta C_e/\beta^2_{Av}}$$

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$$\tan \delta_x = \frac{1}{\beta_1^2 K C_A} \left[\frac{\alpha_c^2 \left| \Delta C_{Qo} \frac{1}{2} \right|}{2} \left(\frac{Q_o}{Q_1} - 1 \right) - 4 \times 10^{-9} F_{MHz}^{\frac{3}{2}} (C_{ei}^2 - C_{eo}^3) \right.$$

$$\left. - 2 \times 10^{-9} F_{MHz}^{\frac{3}{2}} (\alpha_1^2 C_{Qi}^2 - \alpha_o^2 C_{Qo}^2) \right]$$

In the last equation, note that:

F_{MHz} - frequency of measurement in megahertz

All capacitances in picofarads

The quantity $\dfrac{\alpha_o^2 \left| \Delta C_{Qo} \frac{1}{2} \right|}{2}$ may be replaced by $\dfrac{C_R}{Q_o}$, if desired (see page 7).

The following simplifications can sometimes be made:

(1) The last term in the tan δ calculation should always be dropped unless the C_Q's are considerably larger than the C_e's.

(2) At the lower frequencies the internal inductance corrections, described in the manual, may be omitted (i.e., $\alpha_R = \alpha_o = \alpha_1 = 1$).

6. REFERENCES

(1) Z. G. Sztankay, HDL Report TR-1329 dated 31 October 1966.

(2) Instruction Manual for the Model 190-A Q-Meter, Boonton Radio Corporation, Boonton, New Jersey.

(3) J. D. Ryder, Networks, Lines and Fields, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1955.

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13. ABSTRACT

A method for measuring the dielectric properties of liquid specimens on a Q-meter at frequencies up to 100 MHz was developed by (1) assuming an equivalent circuit for a cylindrically shaped liquid-sample holder, (2) determining relevant circuit parameters by experiment, and (3) deriving the necessary equations taking full account of the effects of sample-holder inductance.

Measurements made on two liquid samples are detailed and show that good accuracy is obtained when the standard liquid employed is a lossless dielectric with dielectric constant (K) near the K of the sample under test.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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Permittivity						
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