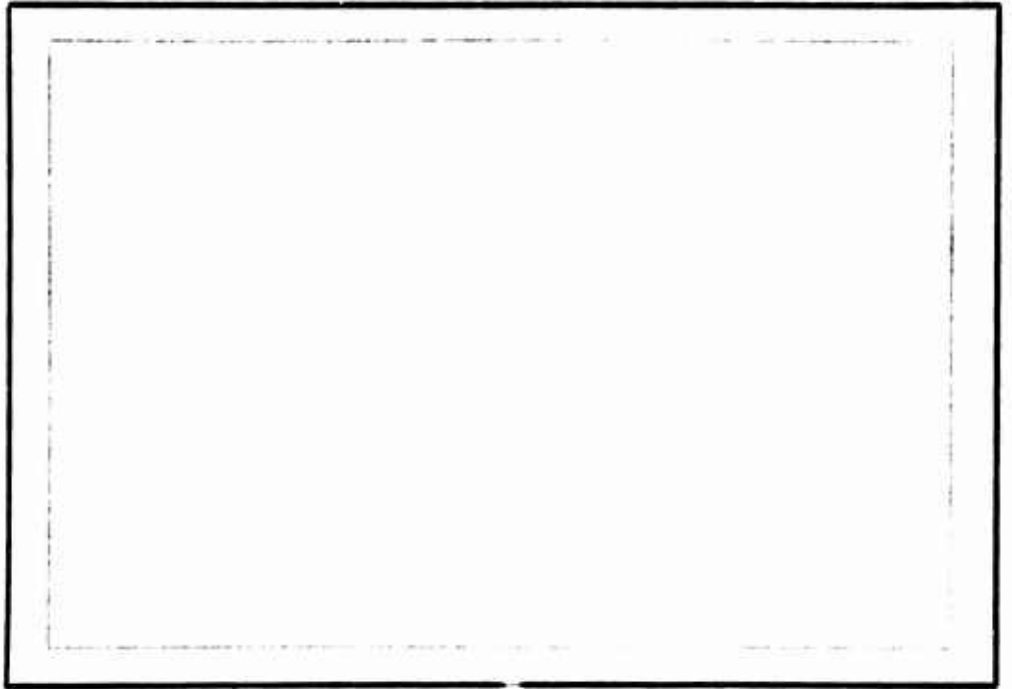AD 646221

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Behavior of Stiffened Plates,
Volume I: Analysis

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ABSTRACT

A compilation of techniques of analysis for ortho-
gonally stiffened, flat, rectangular plates, with
various combinations of loading and support conditions,
is presented. The specific types of plates considered
are sandwich, corrugated, rib-reinforced, and integrally
stiffened. Orthotropic plate theory is used as the
major form of solution throughout the report.

A complete bibliography, consisting of more than
three hundred references, is included.

DESCRIPTORS

Corrugations
Integral stiffening
Literature survey
Orthotropy
Plates
Rib-stiffening
Sandwich
Structural analysis

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INTRODUCTION

Historically, the first example of a stiffened plate
was a flat slab, reinforced by attached unidirectional
girders. Later, stiffening was provided by transverse,
as well as longitudinal, beams. Such configurations are
still prevalent in the construction of ships and bridges.
Concern about saving weight, while retaining strength,
in the design of aircraft resulted in the development
of sandwich construction, in which two thin plates which
carry tensile and compressive loads surround a light-
weight, but relatively thick, core designed to transmit
shear. Corrugated plates also have been a popular form
of construction for many years. However, the most recent
developments have been in the area of integrally stiffened
plates such as "waffle" and "dimple." The principle upon
which all of the latter are based is the removal of
material from the neutral axis of a flat plate by rolling,
pressing or punching, or upon the reduction of an origin-
ally flat thick plate by milling into a thin plate with
ribs.

It is interesting to note, from the curve shown
below, that interest in orthogonally stiffened plates
has been increasing steadily. This, in itself, demon-
strates a need for this survey of work in the field.

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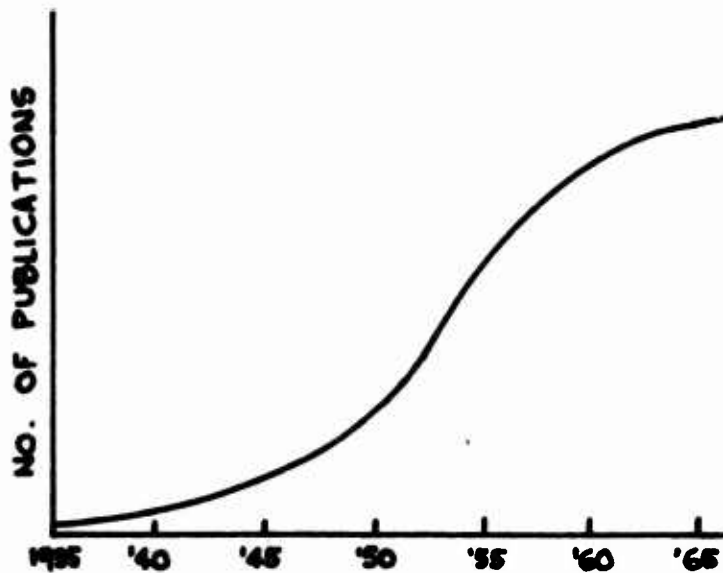
History of Interest in Stiffened Plates

A great deal of work in the area of structural analysis of orthogonally stiffened plates has been performed during the past twenty years. Many unique techniques of solution have been presented and a great number of specific problems have been solved. But, due to such a wide scattering of work in this area, it has been extremely difficult for the practicing engineer to find, in a reasonable amount of time, solutions to specific cases. For this reason, the writer felt that there was a definite need for consolidation of this research into a single report.

This study, therefore, represents a survey of the most common analytical techniques in the area of

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orthotropic plates. Within the body of the paper, methods of solution for sandwich, corrugated, rib-stiffened, and integrally stiffened flat, rectangular plates under static loading conditions are presented. In addition, a complete bibliography is provided for researchers who desire to study specific problems in more detail. Since the titles of most of the listed papers quite satisfactorily explain the contents of the papers, the bibliography should prove to be a useful tool.

Basically, there are three major techniques of analysis of orthogonally stiffened flat plates. The first method is based upon the replacement of the actual plate with a grid system of bars. The major flaw of this method of attack is the neglect of torsional rigidities resulting from interactions of the bars. A second technique is based upon energy concepts. This is rather straight-forward but errors are often intro- duced due to a failure to satisfy all boundary conditions. The third method of analysis is orthotropic plate theory, which provides continuity of the plate. Although the latter method requires rather difficult solutions for complex boundary conditions, this technique is the most general and most accurate. For this reason, orthotropic plate theory is employed throughout most of this report.

The characteristic equation is first developed and it is then applied to various plate configurations, loading cases, and support conditions. Finally, experimental techniques for determining orthogonality constants are discussed.

PART 1 - DIFFERENTIAL EQUATION OF DEFORMED PLATE

The derivation of the differential equation of the deformed orthotropic plate appears in many of the references (1, 6, 7, 37, 40, 41, 58, 97, etc.) cited here.

In the derivation, it is assumed that: the load acting on the plate is normal to the middle plane; planes normal to middle surface of undeformed plate remain normal after deformation; loads are reacted in shear and flexure; transverse shear deformation can be neglected (this constraint will later be removed in a refinement of the theory).

An element of the plate (Figure 1) is considered. The directions shown are considered positive (right-hand rule is used for moment vectors), and the coordinate axes are chosen to coincide with the principal axes of orthotropy.

1.1 Expressions for Stress and Strain

In order to evaluate the forces and moments of Figure 1, it is first necessary to write the expressions for the strains and the stresses. Letting the subscripts define directions, and using Hooke's Law:
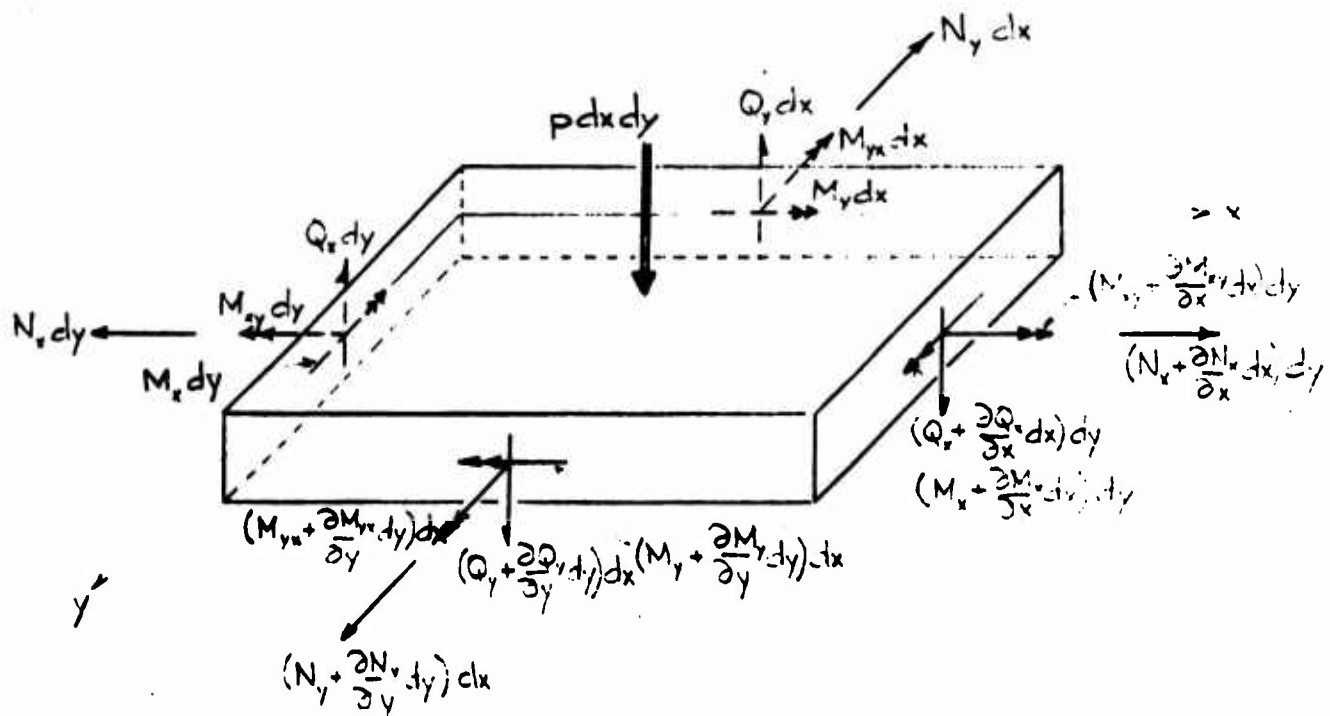
Figure 1
Element dx-dy (positive directions)

$$\epsilon_x = \frac{\sigma_x}{E_x} - \mu_{xy}\frac{\sigma_y}{E_y} \; ,$$

$$\epsilon_y = \frac{\sigma_y}{E_y} - \mu_{yx}\frac{\sigma_x}{E_x} \; , \qquad\qquad \Bigg\} \;\; (1)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G_{xy}} \; ,$$

where

$\epsilon, \gamma =$ normal and shearing strains, respectively,

$\sigma, \tau =$ normal and shearing stresses, respectively,

$E, G =$ moduli of elasticity and rigidity, respectively,

$\mu =$ Poisson's ratio.

Solving these expressions simultaneously gives the stress equations:

$$\sigma_x = \frac{E_x}{1-\mu_{xy}\mu_{yx}} (\epsilon_x + \mu_{yx}\epsilon_y) \; ,$$

$$\sigma_y = \frac{E_y}{1-\mu_{xy}\mu_{yx}} (e_y + \mu_{xy}\epsilon_x) \; , \qquad \Bigg\} \;\; (2)$$

$$\tau_{xy} = \gamma_{xy}G_{xy} \; .$$

It is generally convenient to write the strains in terms of simplified constants with matrix subscripts. Then, the expressions for strains (Eqs. (1)) become:

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \cdot \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \; , \qquad (3)$$

where

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$$S_{11} = 1/E_x \qquad\qquad S_{22} = 1/E_y$$

$$S_{12} = -\mu_{xy}/E_y \qquad\qquad S_{21} = -\mu_{yx}/E_x$$

$$S_{66} = 1/G_{xy} \; .$$

But, due to symmetry:

$$\frac{\mu_{xy}}{E_x} = \frac{\mu_{yx}}{E_y} \quad \longrightarrow \quad S_{12} = S_{21} \; , \qquad\qquad (4)$$

and there remain four elastic constants of the ortho-
tropic plate.

1.2 Geometry

Consider a plate element deformed as shown in
Figure 2.
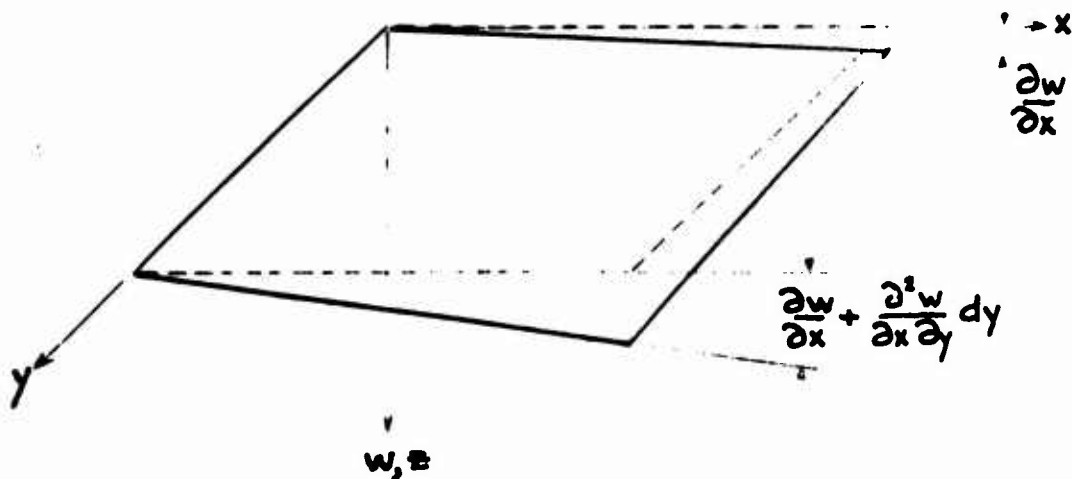
Figure 2
Deformed Element dx-dy

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The slope of the deformed surface,

$$m_x = \frac{\partial w}{\partial x} \qquad \left(m_y = \frac{\partial w}{\partial y}\right) . \qquad (5)$$

The curvature of the deformed surface,

$$\frac{1}{\rho_x} = \frac{\partial^2 w}{\partial x^2} \qquad \left(\frac{1}{\rho_y} = \frac{\partial^2 w}{\partial y^2}\right), \qquad (6)$$

where ρ is the radius of curvature. The "twist" of the surface,

$$\frac{1}{\rho_{xy}} = \frac{\partial^2 w}{\partial x \, \partial y} . \qquad (7)$$

If z is the normal distance from the middle surface of the plate to an investigated point, the strains can be written in terms of the geometry:

$$\left.\begin{aligned}
\epsilon_x &= -\frac{z}{\rho_x} = -z\,\frac{\partial^2 w}{\partial x^2} , \\
\epsilon_y &= -\frac{z}{\rho_y} = -z\,\frac{\partial^2 w}{\partial y^2} , \\
\gamma_{xy} &= \frac{2z}{\rho_{xy}} = 2z\,\frac{\partial^2 w}{\partial x \partial y} .
\end{aligned}\right\} \qquad (8)$$

1.3 Final Stress Expressions

The final stresses are evaluated from Eq. (3), by inverting the matrix of elastic constants:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix}^{-1} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}$$

$$= \frac{1}{S_{11}S_{22}-S_{12}^2} \begin{bmatrix} S_{22} & -S_{12} & 0 \\ -S_{12} & S_{11} & 0 \\ 0 & 0 & \frac{S_{11}S_{22}-S_{12}^2}{S_{66}} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} \quad . \tag{9}$$

Substituting the results of Eqs. (8) into Eq. (9):

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{z}{S_{11}S_{22}-S_{12}^2} \begin{bmatrix} S_{22} & -S_{12} & 0 \\ -S_{12} & S_{11} & 0 \\ 0 & 0 & \frac{S_{11}S_{22}-S_{12}^2}{S_{66}} \end{bmatrix} \begin{bmatrix} -\frac{\partial^2 w}{\partial x^2} \\ -\frac{\partial^2 w}{\partial y^2} \\ 2\frac{\partial^2 w}{\partial x \partial y} \end{bmatrix} \quad . \tag{10}$$

Thus, the stresses,

$$\sigma_x = -\frac{z}{S_{11}S_{22}-S_{12}^2}\left[S_{22}\frac{\partial^2 w}{\partial x^2} - S_{12}\frac{\partial^2 w}{\partial y^2}\right] \quad , \tag{11}$$

$$\sigma_y = -\frac{z}{S_{11}S_{22}-S_{12}^2}\left[S_{11}\frac{\partial^2 w}{\partial y^2} - S_{12}\frac{\partial^2 w}{\partial x^2}\right] \quad , \tag{12}$$

$$\tau_{xy} = \frac{2z}{S_{66}}\frac{\partial^2 w}{\partial x \partial y} \quad . \tag{13}$$

1.4 Bending and Twisting Moments

Recalling that z is the normal distance from the neutral surface, the stress couples can be evaluated from Eqs. (11), (12), and (13):

$$M_x = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_x \, z \, dz$$

$$= -\frac{t^3}{12(S_{11}S_{22} - S_{12}^2)} \left[S_{22} \frac{\partial^2 w}{\partial x^2} - S_{12} \frac{\partial^2 w}{\partial y^2} \right] ,$$

$$\left. \right\} \quad (14)$$

$$M_y = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_y \, z \, dz$$

$$= -\frac{t^3}{12(S_{11}S_{22} - S_{12}^2)} \left[S_{11} \frac{\partial^2 w}{\partial y^2} - S_{12} \frac{\partial^2 w}{\partial x^2} \right] ,$$

$$\left. \right\} \quad (15)$$

$$M_{xy} = \int_{-\frac{t}{2}}^{\frac{t}{2}} \tau_{xy} \, z \, dz$$

$$= \frac{t^3}{6 S_{66}} \frac{\partial^2 w}{\partial x \, \partial y} .$$

$$\left. \right\} \quad (16)$$

1.5 Rigidity Constants

The bending moment expressions can be rewritten in
a simplified form:

$$M_x = - D_x \left[\frac{\partial^2 w}{\partial x^2} + \mu_{xy} \frac{\partial^2 w}{\partial y^2} \right] , \qquad (17)$$

$$M_y = - D_y \left[\frac{\partial^2 w}{\partial y^2} + \mu_{yx} \frac{\partial^2 w}{\partial x^2} \right] , \qquad (18)$$

$$M_{xy} = D_R \frac{\partial^2 w}{\partial x \partial y} . \qquad (19)$$

where

$$D_x = \frac{E_x t^3}{12(1 - \mu_{xy}\mu_{yx})} \qquad (20)$$

= flexural rigidity in x-direction,

$$D_y = \frac{E_y t^3}{12(1 - \mu_{xy}\mu_{yx})} \qquad (21)$$

= flexural rigidity in y-direction,

$$D_R = \frac{G_{xy} t^3}{6} \qquad (22)$$

= torsional rigidity.

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It should be noted, at this point, that various systems for indexing of these rigidity constants are found in the literature. This variety of nomenclatures is extremely confusing due to the fact that a symbol may appear in several technical papers with a different meaning attached to it by each author. Most of the confusion arises from the assignment of symbols to the torsional rigidity constant. The symbol D_R has been chosen here because it does not appear in the reviewed literature, and, it is hoped, will help to avoid misunderstandings.

1.6 Shear Forces

With reference to Figure 1, the equation of equilibrium of forces in the z - direction:

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + P = 0. \tag{23}$$

The equation of equilibrium of moments about the x - axis:

$$\frac{\partial M_{xy}}{\partial x} - \frac{\partial M_y}{\partial y} + Q_y = 0. \tag{24}$$

From the equilibrium of moments about the y - axis:

$$\frac{\partial M_x}{\partial x} - \frac{\partial M_{xy}}{\partial y} - Q_x = 0 , \tag{25}$$

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after substitution of

$$M_{xy} = - M_{yx} \cdot \tag{26}$$

Substituting Eqs. (17), (18), and (19) into Eqs. (24)
and (25), and solving for the shears,

$$Q_x = - D_x \left[\frac{\partial^3 w}{\partial x^3} + \mu_{xy} \frac{\partial^3 w}{\partial x \partial y^2} \right] - D_k \frac{\partial^3 w}{\partial x \partial y^2} , \tag{27}$$

$$Q_y = - D_y \left[\frac{\partial^3 w}{\partial y^3} + \mu_{yx} \frac{\partial^3 w}{\partial x^2 \partial y} \right] - D_k \frac{\partial^3 w}{\partial x^2 \partial y} . \tag{28}$$

1.7 Differential Equation of Flexure

Substitution of Eqs. (24) and (25) makes possible
the writing of the equilibrium equation (23) in terms
of the moments and the load:

$$\frac{\partial^2 M_x}{\partial x^2} - 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = - P \cdot \tag{29}$$

Finally, from the substitution of Eqs. (17), (18) and
(19), the general differential equation of flexure of
the orthotropic plate is written:

$$D_x \frac{\partial^4 w}{\partial x^4} + \left[2 D_k + \mu_{xy} D_x + \mu_{yx} D_y \right] \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = P. \tag{30}$$

For simplicity, the final form of this equation is given
as:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2 D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = P , \tag{31}$$

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where

$$D_{xy} = D_R + \frac{1}{2}\left(\mu_{xy} D_x + \mu_{yx} D_y\right). \qquad (32)$$

1.8 Consideration of Shear Effects

It is recalled that one of the assumptions of the
above derivation was the negligible effect of trans-
verse shear. It must be recognized, however, that such
an assumption is not always permissible. In some plates,
such as sandwich (see Part 5), a low stiffness gives
great importance to the contribution of shear to
deflection.

For this reason, several investigators have con-
sidered this effect. Reissner (Refs. 78, 169) first
recognized this problem, and it has since been applied
to orthotropic plates by Crawford and Libove (Ref. 305),
Girkman and Beer (Ref. 21), Libove and Batdorf (Ref. 155),
March (Ref. 159), Medwadowski (Refs. 62, 63), Suchar
(Ref. 93), and Wang (Ref. 177), among others.

Using matrix notation consistent with that of
Eq. (3), the stress-strain relationship is given by:

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \cdot \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} \qquad (33)$$

Due to symmetry,

$$S_{12} = S_{21} \;,\quad S_{23} = S_{32} \;,\quad S_{13} = S_{31} \;,$$

and the elastic properties of the plate are character-
ized by nine elastic material constants. Thus, the
effect of transverse shear, together with that of
transverse normal stress, is considered.

Medwadowski (Ref. 62) applies nonlinear theory of
elasticity to the derivation of the characteristic
equation of the orthotropic plate. He formulates the
problem by using a system of sixteen simultaneous
equations -- Eq. (33), equilibrium equations, and
strain-displacement relationships. The problem is
reduced to a two-dimensional one by the introduction
of body-force resultants, X" and Y", and weighted
displacements, u, v, and w. With the introduction of
the Airy stress function, Φ, Medwadowski reduces the

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number of equations to the following four:

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + P_1 - P_2 + Z' - X'\frac{\partial w}{\partial x} - Y'\frac{\partial w}{\partial y} + Q \left.\right\}\text{(34)}$$
$$= 0,$$

$$Q_x + A_1\frac{\partial^2 Q_x}{\partial x^2} + A_2\frac{\partial^2 Q_x}{\partial y^2} = A_3\frac{\partial^3 w}{\partial x^3} + A_4\frac{\partial^3 w}{\partial x\partial y^2}$$
$$+ A_9\left(\frac{\partial P_1}{\partial x} - \frac{\partial P_2}{\partial x}\right) + A_{10}\frac{\partial Z'}{\partial x} + X'' + A_{34}\frac{\partial}{\partial x}\left(X'\frac{\partial w}{\partial x} + Y'\frac{\partial w}{\partial y} - Q\right), \left.\right\}\text{(35)}$$

$$Q_y + A_5\frac{\partial^2 Q_y}{\partial x^2} + A_6\frac{\partial^2 Q_y}{\partial y^2} = A_7\frac{\partial^3 w}{\partial x^2\partial y} + A_8\frac{\partial^3 w}{\partial y^3}$$
$$+ A_{11}\left(\frac{\partial P_1}{\partial y} - \frac{\partial P_2}{\partial y}\right) + A_{12}\frac{\partial Z'}{\partial y} + Y'' + A_{35}\frac{\partial}{\partial y}\left(X'\frac{\partial w}{\partial x} + Y'\frac{\partial w}{\partial y} - Q\right), \left.\right\}\text{(36)}$$

$$S_{22}\frac{\partial^4\Phi}{\partial x^4} + \left(2S_{12} + S_{66}\right)\frac{\partial^4\Phi}{\partial x^2\partial y^2} + S_{11}\frac{\partial^4\Phi}{\partial y^4}$$
$$= \left(\frac{\partial^2 w}{\partial x\partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(S_{12} + S_{22}\right)\frac{\partial^2 P}{\partial x^2} - \left(S_{11} + S_{12}\right)\frac{\partial^2 P}{\partial y^2}, \left.\right\}\text{(37)}$$

where

A_j = constant, depending on elastic properties and thickness of plate,

P_1, P_2 = loads acting on upper and lower faces of plate, respectively,

P = resultant body-force potential,

X', Y', Z', X'', Y'' = body-force resultants,

Φ = Airy stress functions,

$$Q = Q_x\frac{\partial^2 w}{\partial x^2} + 2Q_{xy}\frac{\partial^2 w}{\partial x\partial y} + Q_y\frac{\partial^2 w}{\partial y^2},$$

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and the remaining symbols have their normal meanings,
as used throughout this paper.

The effect of an elastic foundation at face
$z = t/2$ is considered, and a system of four partial
differential equations, governing the oscillations of
an orthotropic plate, is written. The system is then
linearized and reduced to a single partial differential
equation of sixth order through the choice of

$$F(x,y,z) \qquad = \text{stress function.}$$

The values of the constants contained in this fundamental
expression are then given, and the solution of the
equation is taken to be of the Lévy-type.

When the body-force terms and the effect of trans-
verse normal stress are neglected and elastic foundation
modulus is set equal to zero, Medwadowski's linearized
equations which govern the bending behavior reduce to
the less complex expressions of Libove and Batdorf
(Ref. 155).

Since the Libove-Batdorf paper uses a definition
of plate rigidities which is somewhat unique, no attempt
is made here to make these conform to the definitions
used in the previous derivations. Therefore, let

$$D_1 = -\frac{M_x}{\partial^2 w/\partial x^2} \quad , \qquad \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} (38)$$

$$D_3 = -\frac{M_y}{\partial^2 w/\partial y^2} \quad ,$$

analogous to the constants defined by Eqs. (17) and (18). The twisting rigidity retains its definition as given by Eq. (19):

$$D_R = \frac{M_{xy}}{\partial^2 w/\partial x \partial y} \quad .$$

In addition, Poisson's ratios are defined in terms of curvatures:

$$\nu_{yx} = -\frac{\partial^2 w/\partial y^2}{\partial^2 w/\partial x^2} \quad , \qquad \left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\} (39)$$

$$\nu_{xy} = -\frac{\partial^2 w/\partial x^2}{\partial^2 w/\partial y^2} \quad .$$

The refinement to the orthotropic plate theory, which was presented in the first part of this section, is introduced in the form of shear stiffnesses,

$$D_{Q_x} = \frac{Q_x}{\partial w/\partial x} \quad , \qquad \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} (40)$$

$$D_{Q_y} = \frac{Q_y}{\partial w / \partial y} \ . \qquad\qquad \left.\vphantom{\int}\right\} (40)$$

The bending moment equations which originally contained
flexure terms only (Eqs. (17), (18), and (19)) are now
refined by the consideration of the shear contribution:

$$M_x = \frac{D_1}{1 - \nu_{yx}\nu_{xy}}\left[\frac{\partial^2 w}{\partial x^2} - \frac{1}{D_{Q_x}}\frac{\partial Q_x}{\partial x} + \nu_{xy}\left(\frac{\partial^2 w}{\partial y^2} - \frac{1}{D_{Q_y}}\frac{\partial Q_y}{\partial y}\right)\right], \quad (41)$$

$$M_y = \frac{D_3}{1 - \nu_{yx}\nu_{xy}}\left[\frac{\partial^2 w}{\partial y^2} - \frac{1}{D_{Q_y}}\frac{\partial Q_y}{\partial y} + \nu_{yx}\left(\frac{\partial^2 w}{\partial x^2} - \frac{1}{D_{Q_x}}\frac{\partial Q_x}{\partial x}\right)\right], \quad (42)$$

$$M_{xy} = D_R\left[\frac{\partial^2 w}{\partial x \partial y} - \frac{1}{2 D_{Q_x}}\frac{\partial Q_x}{\partial y}\right] . \qquad\qquad (43)$$

Substituting the above equations into Eqs. (24) and
(25), and applying the equation of vertical equilibrium:

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = -P + N_x\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y\frac{\partial^2 w}{\partial y^2}, \quad (44)$$

three equations in w, Q_x, and Q_y are written:

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$$\left[N_x \frac{\partial^2}{\partial x^2} + 2N_{xy}\frac{\partial^2}{\partial x \partial y} + N_y\frac{\partial^2}{\partial y^2}\right]w + \left[\frac{\partial}{\partial x}\right]Q_x$$
$$+ \left[\frac{\partial}{\partial y}\right]Q_y = -P , \tag{45}$$

$$\left[-D_R \frac{\partial^3}{\partial x \partial y^2} - \frac{D_1}{1-\nu_{xy}\nu_{yx}}\left(\nu_{xy}\frac{\partial^3}{\partial x \partial y^2} + \frac{\partial^3}{\partial x^3}\right)\right]w$$
$$+ \left[\frac{1}{2}\frac{D_R}{D_{Q_x}}\frac{\partial^2}{\partial y^2} + \frac{D_1}{(1-\nu_{xy}\nu_{yx})D_{Q_x}}\frac{\partial^2}{\partial x^2} - 1\right]Q_x$$
$$+ \left[\frac{1}{2}\frac{D_R}{D_{Q_y}}\frac{\partial^2}{\partial x \partial y} + \frac{D_1\nu_{xy}}{(1-\nu_{xy}\nu_{yx})D_{Q_y}}\frac{\partial^2}{\partial x \partial y}\right]Q_y = 0, \tag{46}$$

$$\left[-D_R \frac{\partial^3}{\partial x^2 \partial y} - \frac{D_3}{1-\nu_{xy}\nu_{yx}}\left(\nu_{yx}\frac{\partial^3}{\partial x^2 \partial y} + \frac{\partial^3}{\partial y^3}\right)\right]w$$
$$+ \left[\frac{1}{2}\frac{D_R}{D_{Q_x}}\frac{\partial^2}{\partial x \partial y} + \frac{D_3\nu_{yx}}{(1-\nu_{xy}\nu_{yx})D_{Q_x}}\frac{\partial^2}{\partial x \partial y}\right]Q_x$$
$$+ \left[\frac{1}{2}\frac{D_R}{D_{Q_y}}\frac{\partial^2}{\partial x^2} + \frac{D_3}{(1-\nu_{xy}\nu_{yx})D_{Q_y}}\frac{\partial^2}{\partial y^2} - 1\right]Q_y = 0. \tag{47}$$

These equations are then solved for w, Q_x, and Q_y by means of determinants.

1.9 Summary

In general, orthotropic plate theory which omits
the effects of transverse shear gives good results for
orthotropic or orthogonally stiffened flat plates.
Due to the complexity of the problem, it is often
worthwhile to "trade off" a small degree of accuracy
for a great amount of computational labor. However,
the analyst must take care that these errors do not
become significant.

Summarizing the results of the bending theory of
orthotropic plates, the differential equation of flexure
is recalled:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = P \; ,$$

where

$$D_x = \frac{E_x t^3}{12(1 - \mu_{xy}\mu_{yx})} \; ,$$

$$D_y = \frac{E_y t^3}{12(1 - \mu_{xy}\mu_{yx})} \; ,$$

$$D_{xy} = \frac{G_{xy} t^3}{6} + \frac{1}{2}\left(\mu_{xy} D_x + \mu_{yx} D_y\right) .$$

The stresses, as given by Eqs. (11), (12), and (13),

$$\sigma_x = - \frac{z}{S_{11}S_{22}-S_{12}^2} \left[S_{22} \frac{\partial^2 w}{\partial x^2} - S_{12} \frac{\partial^2 w}{\partial y^2} \right] ,$$

$$\sigma_y = - \frac{z}{S_{11}S_{22}-S_{12}^2} \left[S_{11} \frac{\partial^2 w}{\partial y^2} - S_{12} \frac{\partial^2 w}{\partial x^2} \right] ,$$

$$\tau_{xy} = \frac{2z}{S_{66}} \frac{\partial^2 w}{\partial x \partial y} ,$$

where

$$S_{11} = \frac{1}{E_x} , \qquad S_{22} = \frac{1}{E_y} ,$$

$$S_{12} = - \frac{\mu_{xy}}{E_y} = - \frac{\mu_{yx}}{E_x} , \qquad S_{66} = \frac{1}{G_{xy}} .$$

The practical application of orthotropic plate theory depends upon the solution of the lateral deflection, w. This function is found by solving the fundamental equation (31) for a given set of boundary conditions. Then, it is possible to evaluate stresses, strains, moments, and shears. The application of the theory to specific cases of bending is considered in the following section.

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PART 2 - METHODS OF SOLUTION - TRANSVERSE LOADING

The solution of the basic differential equation (31) of the orthotropic plate,

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = P ,$$

is accomplished by one of the following methods:

2.1 Navier Solution

A double Fourier series solution of the form

$$f(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} , \qquad (48)$$

was used by a majority of investigators. Huber (Refs. 37 - 41) was the first to give his attention to orthogonally reinforced plates and his approach was based on the Navier-type function. Some of the other investigators to employ this method of attack were Csonka (Ref. 220), Giencke (Ref. 239), Heller (Ref. 309), Kaczhowski (Refs. 48, 49),   Nowacki (Ref. 71), Raville (Ref. 168), Robinson (Ref. 79), Schumann (Ref. 278), Soper (Ref. 90),   Timoshenko and Woinowsky-Krieger (Ref. 79), and Tolotti and Grioli (Ref. 290).

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The Navier solution is simple and straight-
forward, even for complex loading conditions. However,
the method becomes cumbersome in application due to its
relatively slow convergence property. It is most
undesirable, from a computational standpoint, when
higher derivatives of the deformation function, w,
are involved.

2.2 Lévy Solution

The Lévy-type solution is based on a single series
expression of the form:

$$ w = \sum_{m=1}^{\infty} Y_m \sin \frac{m\pi x}{a} , \tag{49} $$

where Y_m is dependent upon y and independent of x. This
technique, which is generally more efficient than the
above type, was employed by several investigators, among
them Ando (Ref. 209), Cornelius (Ref. 12), Hajek (Ref. 25),
Lekhnitsky (Ref. 58), Schade (Refs. 273-276), and
Timoshenko and Woinowsky-Krieger (Ref. 97).

2.3 Other Solutions

Most of the recent work has been concerned with
improving upon the computational labor involved in

solving the fundamental equation. Solutions have been submitted in many forms:

2.3.1 Maclaurin's Series

Rajappa and Reddy (Ref. 77) have applied Maclaurin's series to the problem of the simply supported rectangular plate and written the deformation equation:

$$w(x,y) = w(0,0) + \sum_n \left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} \right)^n \frac{w(0,0)}{n!} , \qquad (50)$$

with the origin of the coordinate system taken at the center of the plate.

2.3.2 Affine Transformation

Brilla (Ref. 8) makes use of the transformation,

$$x = x' , \qquad y = y' \sqrt[4]{\frac{D_y}{D_x}} , \qquad (51)$$

to solve this problem which satisfies the condition:

$$D_{xy} = \sqrt{D_x D_y} . \qquad (52)$$

The condition of Eq. (52) was recommended by Huber (Ref. 40) for reinforced concrete slabs, and is discussed by Timoshenko and Woinowsky-Krieger (Ref. 97).

2.3.3 Perturbation Technique

Vinson with Brull and Hess (Ref's. 34, 101-103) make use of the same transformation as Brilla (Eq. (41)) and then go on to obtain a perturbation expansion in terms of α for plates with rigidity ratios which satisfy the condition,

$$\frac{D_{xy}}{D_x} = \sqrt{\frac{D_y}{D_x}} \ . \tag{53}$$

The parameter, α, measures the deviation from this condition and is given by:

$$\alpha = 2\left(1 - \frac{D_{xy}}{\sqrt{D_x D_y}}\right) . \tag{54}$$

This expression, together with the transformation, changes the basic differential equation (31) to:

$$\nabla^4 w - \alpha \frac{\partial^4 w}{\partial \bar{x}^2 \partial \bar{y}^2} = \frac{P}{D_x} \ . \tag{55}$$

A comparison between this method and the standard techniques is made in Reference 101. It is shown that for the case of a uniformly loaded rectangular orthotropic plate on simple supports, the perturbation solution gives results of reasonable accuracy, while requiring considerably less computation time than the Navier and Lévy techniques.

2.3.4 Complex Variables

Mader (Ref. 61) considers the special condition specified by Eq. (52). Here, the basic differential

equation (31) is written:

$$D_x - 2D_{xy} m^2 + D_y m^4 = 0, \tag{56}$$

where

$$m = \pm \sqrt{\frac{D_{xy}}{D_y} \left[1 \pm \sqrt{1 - \frac{D_x D_y}{D_{xy}^2}} \right]} \quad .$$

The solution has the form:

$$\left. \begin{aligned} w &= \left[C_1 e^{\alpha(\theta + i\phi)y} + C_2 e^{-\alpha(\theta - i\phi)y} + C_3 e^{-\alpha(\theta + i\phi)y} \right. \\ &\quad \left. + C_4 e^{-\alpha(\theta - i\phi)y} \right] \sin ax \quad , \end{aligned} \right\} \tag{57}$$

where

$$\alpha = a \sqrt[4]{\frac{D_x}{D_y}} \quad ,$$

$$\beta = \pm \alpha (\theta \pm i\phi) = ma \quad .$$

Suchar (Ref. 93) introduces the complex variables,

$$\bar{z} = x + iy \qquad\qquad \bar{z} = x - iy , \tag{58}$$

and parameters suggested by Lekhnicky (Ref. 58),

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$$k_1 = \frac{1 + i\mu_1}{1 - i\mu_1} \qquad\qquad k_2 = \frac{1 + i\mu_2}{1 - i\mu_2}, \qquad (59)$$

to rewrite the basic differential equation of the orthotropic plate:

$$\left[k_1 \frac{\partial^2 W}{\partial z^2} - (1 - k_1\bar{k}_1)\frac{\partial^2 W}{\partial z \partial \bar{z}} + \bar{k}_1 \frac{\partial^2 W}{\partial \bar{z}^2}\right]\left[k_2 \frac{\partial^2 W}{\partial z^2} - (1 + k_2\bar{k}_2)\frac{\partial^2 W}{\partial z \partial \bar{z}} + \bar{k}_2 \frac{\partial^2 W}{\partial \bar{z}^2}\right]$$
$$= 0, \qquad (60)$$

in which

$$w(x,y) = W(z,\bar{z})$$

is a real solution of the characteristic equation.

In addition, Morkovin (Ref. 65) also employs a complex variable approach to the problem.

2.3.5 Energy Technique

A direct method of calculating stresses in orthotropic plates, without the usual intermediate step of calculating deflections, is presented by Coull (Ref. 13). The method of least work is employed.

The total strain energy due to bending is given as:

$$U = \frac{6}{t^3} \int_0^{2L} \int_{-c}^{c} \left[\left[A_{11}M_x^2 + A_{22}M_y^2 + A_{33}M_{xy}^2 + 2A_{12}M_x M_y \right] + \frac{t^2}{10} B_{11}\left(N_x^2 + N_y^2\right) \right] dy\, dx, \qquad (61)$$

where the constants,

$$A_{11} = \frac{E_y}{E_x E_y - E_{xy}^2} \qquad A_{22} = \frac{E_x}{E_x E_y - E_{xy}^2}$$

$$A_{12} = -\frac{E_{xy}}{E_x E_y - E_{xy}^2} \qquad A_{33} = \frac{1}{G_{xy}}$$

$$B_{11} = \frac{1}{G_\bullet}.$$

The plate considered is supported along, and parallel to, the y-axis, and it is free along the two remaining edges.

The moments and forces in the plate are expressed in terms of a single series, and two loading cases, symmetrical and antisymmetrical, are utilized. The solution involves an assumption of an n-th order polynomial for the bending moment M_x, which makes possible the writing of a set of n linear differential equations with constant coefficients.

2.3.6 Design Application

An interesting and useful set of tables for the design of orthotropic plates with various edge supports and under various loadings is given in a dual-language (English and German) book by Krug and Stein (Ref. 57).

The basic equation (31) is used,

$$D_x \frac{\partial^4 w}{\partial x^4} + 2 D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = P,$$

and the following notations and transformations are introduced:

$$x = \frac{D_{xy}}{\sqrt{D_x D_y}} \qquad\qquad \lambda = \frac{D_x}{D_y} \qquad\qquad (62)$$

$$\xi = \frac{x}{L} \qquad\qquad \eta = \frac{y}{L} \sqrt[4]{\lambda} \qquad\qquad (63)$$

$$L_\xi = \frac{L_x}{L} \qquad\qquad L_\eta = \frac{L_y}{L} \sqrt[4]{\lambda} \ . \qquad\qquad (64)$$

Using the above notation, the differential equation of the unloaded plate becomes

$$\frac{\partial^4 w}{\partial \xi^4} + 2 x \frac{\partial^4 w}{\partial \xi^2 \partial \eta^2} + \frac{\partial^4 w}{\partial \eta^4} = 0 . \qquad\qquad (65)$$

In order to reduce the number of parameters necessary to define a particular plate from three to two (x and e), the loading is referred to the ξ, η—coordinate system (Fig. 3) and the ratio,

$$e = \frac{L_y}{L_x} \sqrt[4]{\lambda} = \frac{L_\eta}{L_\xi} , \qquad\qquad (66)$$

is introduced.
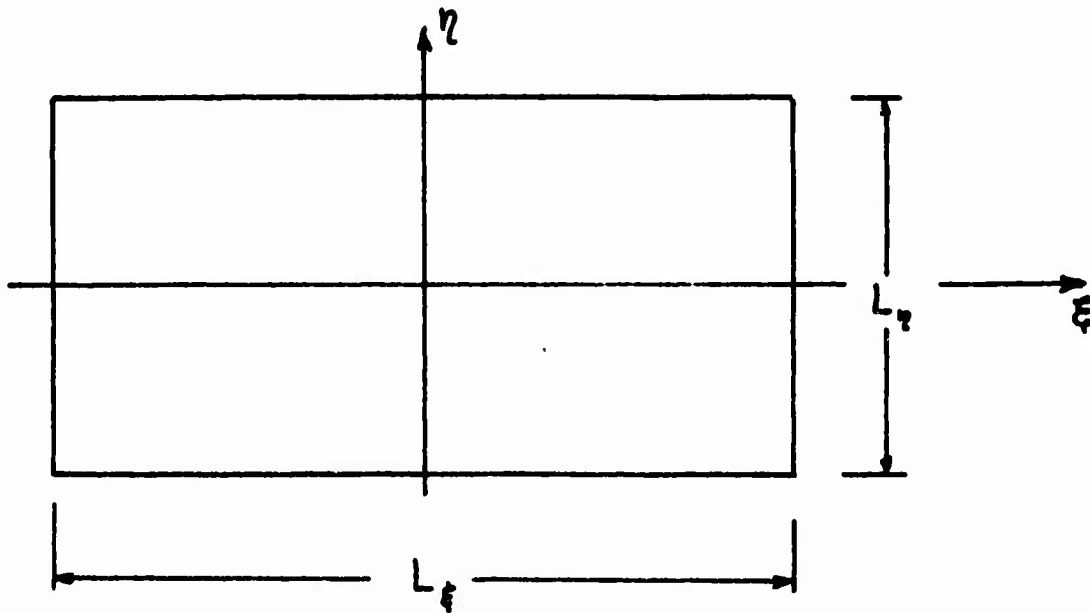
Figure 3
Transformed Plate

In the book, charts are drawn for the values,

$e = 2.00; 1.25; 1.00; 0.80; 0.50.$

$x = 0.80; 0.40; 0.$

The dimension L, from Eqs. (63) and (64) is chosen by the user of the influence surfaces, such that the length of the shorter side of the transformed plate becomes either 20, or 16, or 10, corresponding to the given e. Some results of this procedure are shown in the following chapter.

PART 3 - FLEXURAL BEHAVIOR

In this part, the bending problem, for various combinations of edge supports and loadings, is considered. The chapter is subdivided into sections according to the support conditions, with further sub-divisions made for different conditions of loading. In the accompanying drawings, the following notation is used for edge supports:

$\textcircled{C}$ = clamped,

$\textcircled{F}$ = free,

$\textcircled{S}$ = simply supported.

3.1 Simple Supports

The basic differential equation (31) of the ortho-tropic plate is recalled:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = P ,$$

where the loading term is a function of x and y,

$$P = f(x,y) , \tag{67}$$

and it can represent any type of transverse loading on the plate. The double Fourier series (Navier) form of this function is:

$$f(x,y) = \sum_{m}\sum_{n} A_{mn} \; \sin \frac{m\pi x}{a} \; \sin \frac{n\pi y}{b} \; , \qquad (68)$$

where

      m = number of sinusoidal half-waves in the x-direction,

      n = number of sinusoidal half-waves in the y-direction.

The total deflection of the plate shown in Figure 4 is then calculated as the sum of the partial deflections produced by the partial sinusoidal loadings of Eq. (68).
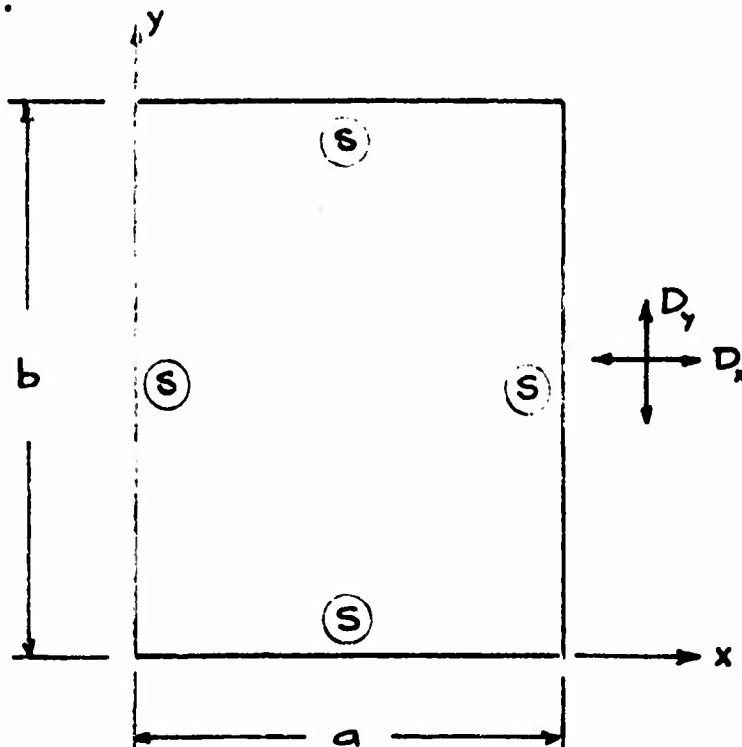
Figure 4
Simply Supported Plate

The general expression for the deformation of a simply supported rectangular orthotropic plate, in the Navier form, is given by:

$$w = \frac{1}{\pi^4} \sum_m \sum_n A_{mn} \frac{\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}}{D_x\left(\frac{m^4}{a^4}\right) + 2D_{xy}\left(\frac{m^2 n^2}{a^2 b^2}\right) + D_y\left(\frac{n^4}{b^4}\right)} \quad . \tag{69}$$

An alternate form of solution (discussed in Part 2) is the single-series expression attributed to Lévy. This solution is based upon the equation,

$$w = \sum_m Y_m \sin \frac{m\pi x}{a} \quad , \tag{70}$$

in which Y_m is a function of y alone and the b - edges $(x = 0, a)$ are simply supported.

Both methods, in addition to any of the additional techniques described in Part 2, will be utilized here whenever they are applicable.

3.1.1 Uniformly Distributed Load over Entire Surface

The case of a simply supported plate, acted upon by a load, P_o, uniformly distributed over its entire surface is shown in Figure 5 and considered here.

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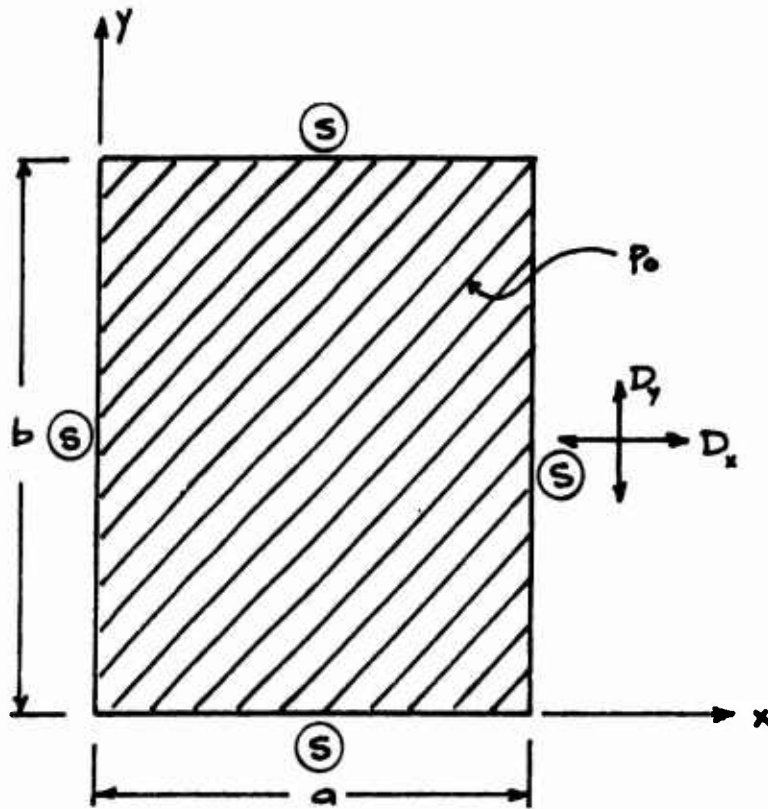
Figure 5
Simple Supports - Uniform Load

The Navier solution, as given in Reference 97, is first utilized. To evaluate the coefficient A_{mn} of Eq. (69), it is found from Eq. (68) that

$$A_{mn} = \frac{4}{ab} \int_0^a \int_0^b f(x,y) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \, dy \, dx . \qquad (71)$$

For the uniform loading,

$$f(x,y) = p_0 , \qquad (72)$$

and the coefficient thus becomes:

$$A_{mn} = \frac{4 P_0}{ab} \int_0^a \int_0^b SIN \frac{m\pi x}{a} SIN \frac{n\pi y}{b} dy dx$$

$$= \frac{16 P_0}{\pi^2 mn} \,. \qquad \Bigg\} \quad (73)$$

The substitution of Eq. (73) into Eq. (69) yields the deformation expression for a uniformly loaded, simply supported plate:

$$W = \frac{16 P_0}{\pi^6} \sum_m \sum_n \frac{SIN \frac{m\pi x}{a} SIN \frac{n\pi y}{b}}{mn \left(D_x \frac{m^4}{a^4} + 2D_{xy} \frac{m^2 n^2}{a^2 b^2} + D_y \frac{n^4}{b^4} \right)} \qquad (74)$$

Using the Lévy form of solution, Ando (Ref. 209) writes the deformation equation:

$$W = \frac{4 P_0 a^4}{D_x \pi^5} \sum_m \frac{(1 + \tilde{A}_m C + \tilde{B}_m S)}{m^5} SIN \frac{m\pi x}{a} \,, \qquad (75)$$

where

$$\tilde{A}_m = - \frac{C_0 + \frac{\xi}{\sqrt{1-\xi^2}} S_0}{SINH^2 \alpha_m t + cos^2 \alpha_m u} \,,$$

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$$\widetilde{B}_m = \frac{\frac{\zeta}{\sqrt{1-\zeta^2}} C_o - S_o}{\sinh^2 \alpha_m t + \cos^2 \alpha_m u} \quad ,$$

$$C = \cosh \frac{m\pi t}{a} Y \cos \frac{m\pi u}{a} Y \quad ,$$

$$S = \sinh \frac{m\pi t}{a} Y \sin \frac{m\pi u}{a} Y$$

$$t = \sqrt{\frac{\sqrt{D_x D_y} + D_{xy}}{2}} \quad ,$$

$$u = \sqrt{\frac{\sqrt{D_x D_y} - D_{xy}}{2}} \quad ,$$

$$\zeta = \frac{D_{xy}}{\sqrt{D_x D_y}} \quad ,$$

$$\alpha_m = \frac{m\pi}{a} Y \quad ,$$

O = subscript indicating initial condition.

The same problem is solved by Hess and Vinson (Ref. 34) using a perturbation technique. The basic equation is written in the form:

$$s^4 - 2 \frac{D_{xy}}{D_x} s^2 + \frac{D_y}{D_x} = 0 , \qquad (76)$$

which has the roots,

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$$s = \pm\sqrt{\frac{D_{xy}}{D_x} \pm \sqrt{\left(\frac{D_{xy}}{D_x}\right)^2 - \frac{D_y}{D_x}}} \quad . \tag{77}$$

There are, thus, three cases of solution, which are dependent upon the elastic properties of the ortho-tropic plate:

CASE I:

$$\left(\frac{D_{xy}}{D_x}\right)^2 > \frac{D_y}{D_x} \quad .$$

The deformation function for this case is given by:

$$w = \sum_{n=1,3,\ldots} \left\{ \frac{s_2^2}{s_1^2 - s_2^2} \left[\cosh s_1 \lambda_n x + \left(\frac{1 - \cosh s_1 \lambda_n a}{\sinh s_1 \lambda_n a} \right) \right.\right.$$
$$\left.\cdot \sinh s_1 \lambda_n x \right] + \frac{s_1^2}{s_2^2 - s_1^2} \left[\cosh s_2 \lambda_n x + \left(\frac{1 - \cosh s_2 \lambda_n a}{\sinh s_2 \lambda_n a} \right) \right.$$
$$\left.\left. \cdot \sinh s_2 \lambda_n x \right] + 1 \right\} \oint \sin \lambda_n y \quad . \tag{78}$$

CASE II:

$$\left(\frac{D_{xy}}{D_x}\right)^2 = \frac{D_y}{D_x} \quad .$$

$$w = \sum_{n=1,3,\ldots} \left\{ \left[-1 + \frac{\lambda_n S_2 x}{2}\left(\frac{1-\cosh\lambda_n S_2 a}{\sinh\lambda_n S_2 a}\right) \right] \cosh\lambda_n S_3 x \right.$$

$$- \left[\frac{\cosh\lambda_n S_2 a - 1}{\sinh^2\lambda_n S_3 a}\left(\frac{a\lambda_n S_3}{2} - \sinh\lambda_n S_3 a\right) \right.$$

$$\left. \left. - \frac{\lambda_n S_3 x}{2} \right] \sinh\lambda_n S_3 x \right\} \phi \, \sin\lambda_n x. \tag{79}$$

CASE III:

$$\left(\frac{D_{xy}}{D_x}\right)^2 < \frac{D_y}{D_x} .$$

$$w = \sum_{n=1,3,\ldots} \frac{4 p_0 b^4}{n^5 \pi^5 D_y} \left\{ \left[-\cos\lambda_n S_5 x + \frac{(\cos\lambda_n S_5 a - \cosh\lambda_n S_4 a)}{(\cosh^2\lambda_n S_4 a - \cos^2\lambda_n S_6 a)} \right.\right.$$

$$\left. \cdot \left(\sin\lambda_n S_5 a + \frac{S_4^2 - S_5^2}{2 S_4 S_5}\sinh\lambda_n S_4 a \right) \sin\lambda_n S_5 x \right] \cosh\lambda_n S_4 x$$

$$+ \left[\frac{(\cos\lambda_n S_5 a - \cosh\lambda_n S_4 a)}{(\cosh^2\lambda_n S_4 a - \cos^2\lambda_n S_6 a)} \left(-\sinh\lambda_n S_4 a + \frac{S_4^2 - S_5^2}{2 S_4 S_5}\sin\lambda_n S_6 a\right) \right.$$

$$\left.\left. \cdot \cos\lambda_n S_5 x + \frac{S_4^2 - S_5^2}{2 S_4 S_5}\sin\lambda_n S_6 x \right]\sinh\lambda_n S_4 x \right\} \sin\lambda_n y. \tag{80}$$

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In the above equations, the roots:

$$S_1 = \sqrt{\frac{D_{xy}}{D_x} + \sqrt{\left(\frac{D_{xy}}{D_x}\right)^2 - \frac{D_y}{D_x}}} \quad ,$$

$$S_2 = \sqrt{\frac{D_{xy}}{D_x} - \sqrt{\left(\frac{D_{xy}}{D_x}\right)^2 - \frac{D_y}{D_x}}} \quad ,$$

$$S_3 = \sqrt{\frac{D_{xy}}{D_x}} \quad ,$$

$$S_4 = \sqrt{\frac{1}{2}\left[\sqrt{\frac{D_y}{D_x}} + \frac{D_{xy}}{D_x}\right]} \quad ,$$

$$S_5 = \sqrt{\frac{1}{2}\left[\sqrt{\frac{D_y}{D_x}} - \frac{D_{xy}}{D_x}\right]} \quad .$$

The coordinates and elastic constants are taken according to Figure 5, and the parameter,

$$\lambda = \frac{\pi n}{b} \quad .$$

Using these equations as models, design curves are plotted using the dimensionless parameters,

$$\frac{w_{max} D_y}{P_o b^4} \quad , \quad \frac{M_{x(max)}}{P_o b^2} \quad , \quad \frac{M_{y(max)}}{P_o b^2} \quad ,$$

as functions of the three independent variables,

$$\frac{D_y}{D_x} \quad , \quad \frac{D_{xy}}{D_x} \quad , \quad \frac{b}{a} \quad .$$

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3.1.2 Uniformly Distributed Load over Part of Surface

A simply supported plate, acted upon by a uniformly distributed load over a portion of its surface (Fig. 6) is now considered.
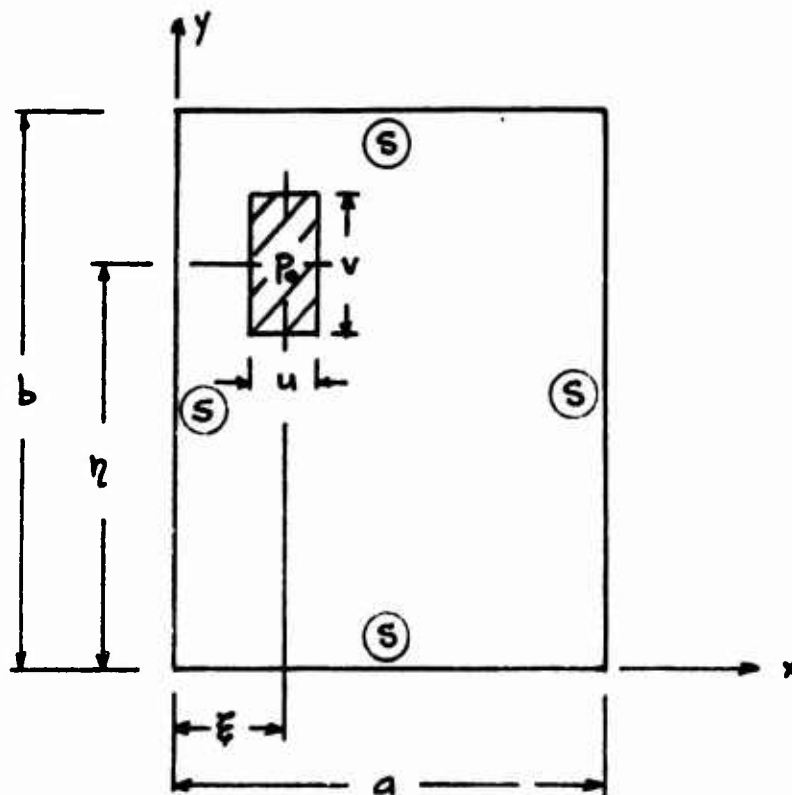
Figure 6
Simple Supports - Partial Load (I)

First, the Navier solution is applied. Recalling the equation giving the loading coefficient (Eq. (68)) and applying the proper conditions:

$$A_{mn} = \frac{4P_o}{ab} \int_{\xi - u/2}^{\xi + u/2} \int_{\eta - v/2}^{\eta + v/2} \sin \frac{m\pi x}{a} \, \sin \frac{n\pi y}{b} \, dy \, dx$$

$$= \frac{16 P_o}{\pi^2 mn} \sin \frac{m\pi \xi}{a} \sin \frac{n\pi \eta}{b} \sin \frac{m\pi u}{2a} \sin \frac{n\pi v}{2b} \, . \tag{81}$$

Substitution of this into Eq. (69) gives the deformation expression:

$$\boxed{W = \frac{16 P_o}{\pi^6} \sum_m \sum_n \frac{\sin \frac{m\pi \xi}{a} \sin \frac{n\pi \eta}{b} \sin \frac{m\pi u}{2a} \sin \frac{n\pi v}{2b} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}}{mn \left(D_x \frac{m^4}{a^4} + 2D_{xy} \frac{m^2 n^2}{a^2 b^2} + D_y \frac{n^4}{b^4} \right)}} \tag{82}$$

Next, the Lévy-type solution, as given by Ando (Ref. 209) is examined. The x-coordinate is changed (Fig. 7) to coincide with the center of the plate.
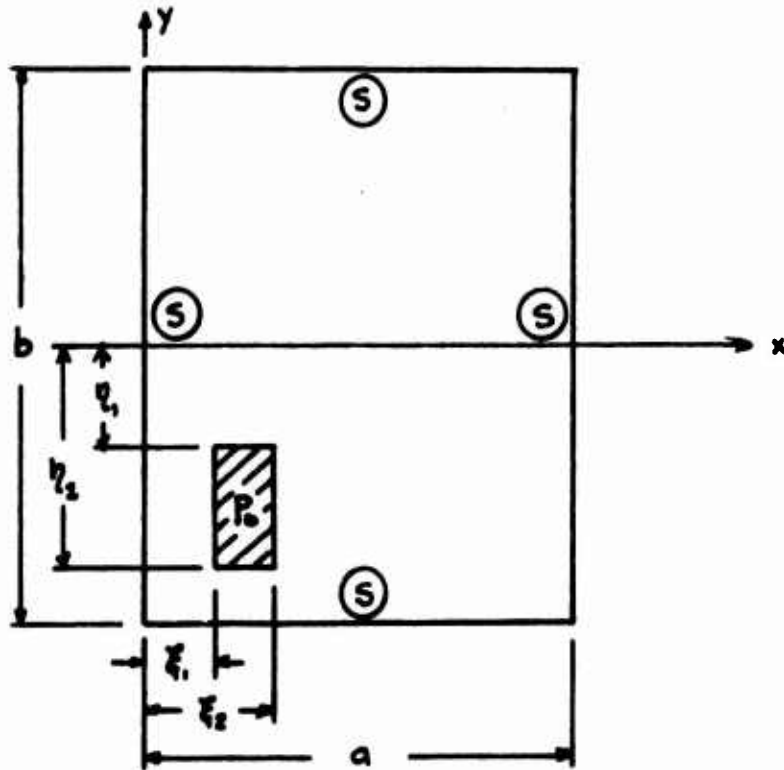
Figure 7
Simple Supports - Partial Load (II)

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Deflection expressions are written for three regions of the plate:

CASE I: $\eta_1 \leqq y \leqq \eta_2$:

$$
\begin{aligned}
w_{\mathrm{I}} = \sum_m \Big(A_m &+ C_1 \cosh \frac{m\pi\lambda}{a}y + C_2 \sinh \frac{m\pi\lambda}{a}y \\
&+ C_3 \cosh \frac{m\pi\bar{\lambda}}{a}y + C_4 \sinh \frac{m\pi\bar{\lambda}}{a}y \Big) \sin \frac{m\pi x}{a} \, .
\end{aligned}
\tag{83}
$$

CASE II: $\eta_2 \leqq y \leqq {}^b\!/_2$:

$$
\begin{aligned}
w_{\mathrm{II}} = \sum_m \Big(C_5 \cosh \frac{m\pi\lambda}{a}y &+ C_6 \sinh \frac{m\pi\lambda}{a}y \\
&+ C_7 \cosh \frac{m\pi\bar{\lambda}}{a}y + C_8 \sinh \frac{m\pi\bar{\lambda}}{a}y \Big) \sin \frac{m\pi x}{a} \, .
\end{aligned}
\tag{84}
$$

CASE III: $-{}^b\!/_2 \leqq y \leqq \eta_1$:

$$
\begin{aligned}
w_{\mathrm{III}} = \sum_m \Big(C_9 \cosh \frac{m\pi\lambda}{a}y &+ C_{10} \sinh \frac{m\pi\lambda}{a}y \\
&+ C_{11} \cosh \frac{m\pi\bar{\lambda}}{a}y + C_{12} \sinh \frac{m\pi\bar{\lambda}}{a}y \Big) \sin \frac{m\pi x}{a} \, .
\end{aligned}
\tag{85}
$$

The constants of the above equations are evaluated from boundary values:

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$$C_1 = \frac{A_m}{2}\left\{\frac{\bar{\lambda}^2}{\lambda^2-\bar{\lambda}^2}\left[\cosh\frac{m\pi\lambda}{a}\eta_1 + \cosh\frac{m\pi\lambda}{a}\eta_2\right.\right.$$
$$\left.\left. - \left(\sinh\frac{m\pi\lambda}{a}\eta_1 - \sinh\frac{m\pi\lambda}{a}\eta_2\right)\tanh\frac{m\pi\lambda}{a}y\right]\right\},$$

$$C_2 = \frac{A_m}{2}\left\{\frac{\bar{\lambda}^2}{\lambda^2-\bar{\lambda}^2}\left[\sinh\frac{m\pi\lambda}{a}\eta_1 + \sinh\frac{m\pi\lambda}{a}\eta_2\right.\right.$$
$$\left.\left. - \left(\cosh\frac{m\pi\lambda}{a}\eta_1 - \cosh\frac{m\pi\lambda}{a}\eta_2\right)\coth\frac{m\pi\lambda}{a}y\right]\right\},$$

$$C_3 = \frac{A_m}{2}\left\{\frac{\bar{\lambda}^2}{\lambda^2-\bar{\lambda}^2}\left[\cosh\frac{m\pi\bar{\lambda}}{a}\eta_1 + \cosh\frac{m\pi\bar{\lambda}}{a}\eta_2\right.\right.$$
$$\left.\left. - \left(\sinh\frac{m\pi\bar{\lambda}}{a}\eta_1 - \sinh\frac{m\pi\bar{\lambda}}{a}\eta_2\right)\tanh\frac{m\pi\bar{\lambda}}{a}y\right]\right\},$$

$$C_4 = \frac{A_m}{2}\left\{\frac{\bar{\lambda}^2}{\lambda^2-\bar{\lambda}^2}\left[\sinh\frac{m\pi\bar{\lambda}}{a}\eta_1 + \sinh\frac{m\pi\bar{\lambda}}{a}\eta_2\right.\right.$$
$$\left.\left. - \left(\cosh\frac{m\pi\bar{\lambda}}{a}\eta_1 - \cosh\frac{m\pi\bar{\lambda}}{a}\eta_2\right)\coth\frac{m\pi\bar{\lambda}}{a}y\right]\right\},$$

$$C_5 = \frac{A_m}{2}\left\{\frac{\bar{\lambda}^2}{\lambda^2-\bar{\lambda}^2}\left[\cosh\frac{m\pi\lambda}{a}\eta_1 - \cosh\frac{m\pi\lambda}{a}\eta_2\right.\right.$$
$$\left.\left. - \left(\sinh\frac{m\pi\lambda}{a}\eta_1 - \sinh\frac{m\pi\lambda}{a}\eta_2\right)\tanh\frac{m\pi\lambda}{a}y\right]\right\},$$

$$C_6 = \frac{A_m}{2}\left\{\frac{\bar{\lambda}^2}{\lambda^2-\bar{\lambda}^2}\left[\sinh\frac{m\pi\lambda}{a}\eta_1 - \sinh\frac{m\pi\lambda}{a}\eta_2\right.\right.$$
$$\left.\left. - \left(\cosh\frac{m\pi\lambda}{a}\eta_1 - \cosh\frac{m\pi\lambda}{a}\eta_2\right)\coth\frac{m\pi\lambda}{a}y\right]\right\},$$

$$C_7 = \frac{A_m}{2}\left\{\frac{\bar{\lambda}^2}{\lambda^2-\bar{\lambda}^2}\left[\cosh\frac{m\pi\bar{\lambda}}{a}\eta_1 - \cosh\frac{m\pi\bar{\lambda}}{a}\eta_2\right.\right.$$

$$- \left(\sinh \frac{m\pi\bar{\lambda}}{a}\eta_1 - \sinh \frac{m\pi\bar{\lambda}}{a}\eta_2 \right) \tanh \frac{m\pi\bar{\lambda}}{a}y \right] \Big\},$$

$$C_8 = \frac{A_m}{2} \left\{ \frac{\bar{\lambda}^2}{\lambda^2 - \bar{\lambda}^2} \left[\sinh \frac{m\pi\bar{\lambda}}{a}\eta_1 - \sinh \frac{m\pi\bar{\lambda}}{a}\eta_2 \right.\right.$$

$$\left.\left. - \left(\cosh \frac{m\pi\bar{\lambda}}{a}\eta_1 - \cosh \frac{m\pi\bar{\lambda}}{a}\eta_2 \right) \coth \frac{m\pi\bar{\lambda}}{a}y \right] \right\},$$

$$C_9 = \frac{A_m}{2} \left\{ \frac{\bar{\lambda}^2}{\lambda^2 - \bar{\lambda}^2} \left[-\cosh \frac{m\pi\lambda}{a}\eta_1 + \cosh \frac{m\pi\lambda}{a}\eta_2 \right.\right.$$

$$\left.\left. - \left(\sinh \frac{m\pi\lambda}{a}\eta_1 - \sinh \frac{m\pi\lambda}{a}\eta_2 \right) \tanh \frac{m\pi\lambda}{a}y \right] \right\},$$

$$C_{10} = \frac{A_m}{2} \left\{ \frac{\bar{\lambda}^2}{\lambda^2 - \bar{\lambda}^2} \left[-\sinh \frac{m\pi\lambda}{a}\eta_1 + \sinh \frac{m\pi\lambda}{a}\eta_2 \right.\right.$$

$$\left.\left. - \left(\cosh \frac{m\pi\lambda}{a}\eta_1 - \cosh \frac{m\pi\lambda}{a}\eta_2 \right) \coth \frac{m\pi\lambda}{a}y \right] \right\},$$

$$C_{11} = \frac{A_m}{2} \left\{ \frac{\bar{\lambda}^2}{\lambda^2 - \bar{\lambda}^2} \left[-\cosh \frac{m\pi\bar{\lambda}}{a}\eta_1 + \cosh \frac{m\pi\bar{\lambda}}{a}\eta_2 \right.\right.$$

$$\left.\left. - \left(\sinh \frac{m\pi\bar{\lambda}}{a}\eta_1 - \sinh \frac{m\pi\bar{\lambda}}{a}\eta_2 \right) \tanh \frac{m\pi\bar{\lambda}}{a}y \right] \right\},$$

$$C_{12} = \frac{A_m}{2} \left\{ \frac{\bar{\lambda}^2}{\lambda^2 - \bar{\lambda}^2} \left[-\sinh \frac{m\pi\bar{\lambda}}{a}\eta_1 + \sinh \frac{m\pi\bar{\lambda}}{a}\eta_2 \right.\right.$$

$$\left.\left. - \left(\cosh \frac{m\pi\bar{\lambda}}{a}\eta_1 - \cosh \frac{m\pi\bar{\lambda}}{a}\eta_2 \right) \coth \frac{m\pi\bar{\lambda}}{a}y \right] \right\},$$

$$\lambda = \sqrt{\frac{\sqrt{D_x D_y} + D_{xy}}{2}} + i \sqrt{\frac{\sqrt{D_x D_y} - D_{xy}}{2}} ,$$

$$\bar{\lambda} = \sqrt{\frac{\sqrt{D_x D_y} + D_{xy}}{2}} - i \sqrt{\frac{\sqrt{D_x D_y} - D_{xy}}{2}} ,$$

$$A_m = -\frac{2a^3 P_0}{D_x m^4 \pi^4} \int_{\xi_1}^{\xi_2} \sin \frac{m\pi\xi}{a} \, d\xi$$

$$= \frac{2a^4 P_0}{D_x m^5 \pi^5} \left(\cos \frac{m\pi\xi_1}{a} - \cos \frac{m\pi\xi_2}{a}\right).$$

In deriving the influence surfaces of their book of tables (Ref. 57), Krug and Stein make use of a transformation from the real structure (Fig. 8a) to a "new" structure (Fig. 8b).
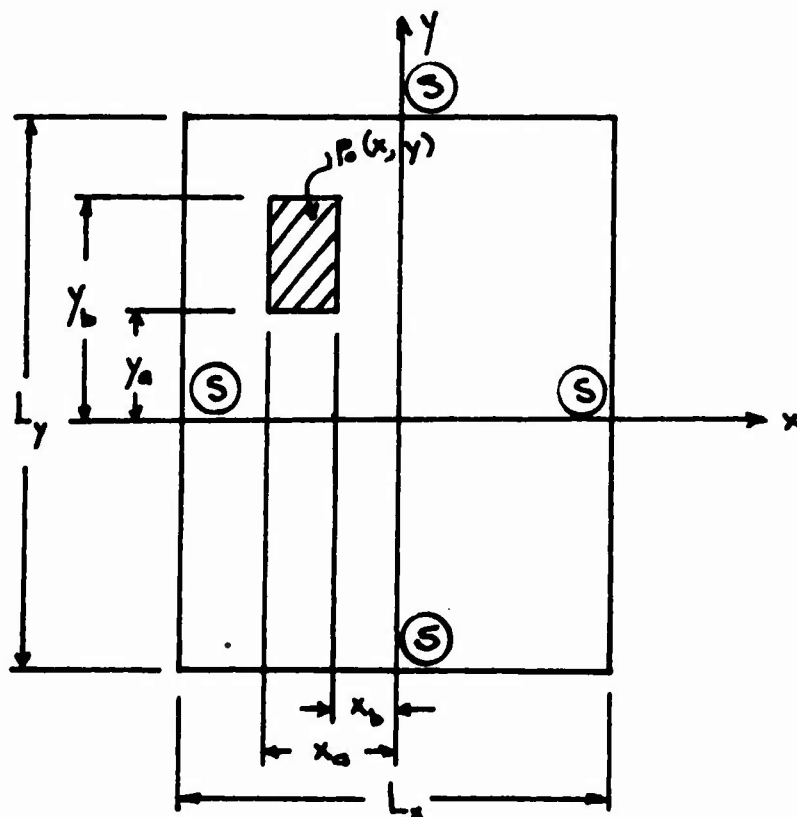
a) Real Structure
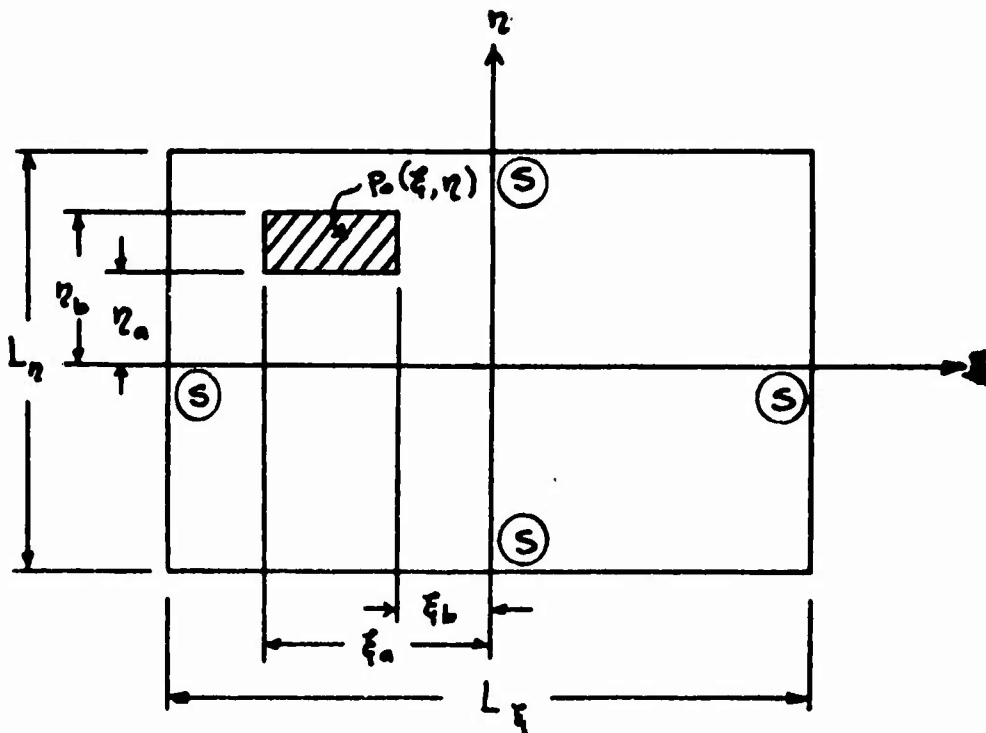
b) Transformed Structure

Figure 8
Simple Supports - Partial Load (III)

The method of transformation is given in Eqs. (62)
through (65), and the bending moments in the plate are:

$$
\left.
\begin{aligned}
M_x &= \frac{P_o L^2}{8\pi} \int_{\xi_a}^{\xi_b} \int_{\eta_a}^{\eta_b} M_\xi(\xi,\eta)\, d\xi\, d\eta \,, \\
M_y &= \frac{P_o L^2}{8\pi\sqrt{\lambda}} \int_{\xi_a}^{\xi_b} \int_{\eta_a}^{\eta_b} M_\eta(\xi,\eta)\, d\xi\, d\eta \,.
\end{aligned}
\right\} \quad (86)
$$

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Numerical values given by these integrals, for given
geometry and constants of orthotropy, are given in
the tables.

3.1.3 Concentrated Force

The next problem to be considered here is that of
the simply supported plate under the action of a con-
centrated load, P.

The writer applied the Navier form of solution to
this case in Reference 309. This is considered first.
(Figure 9).
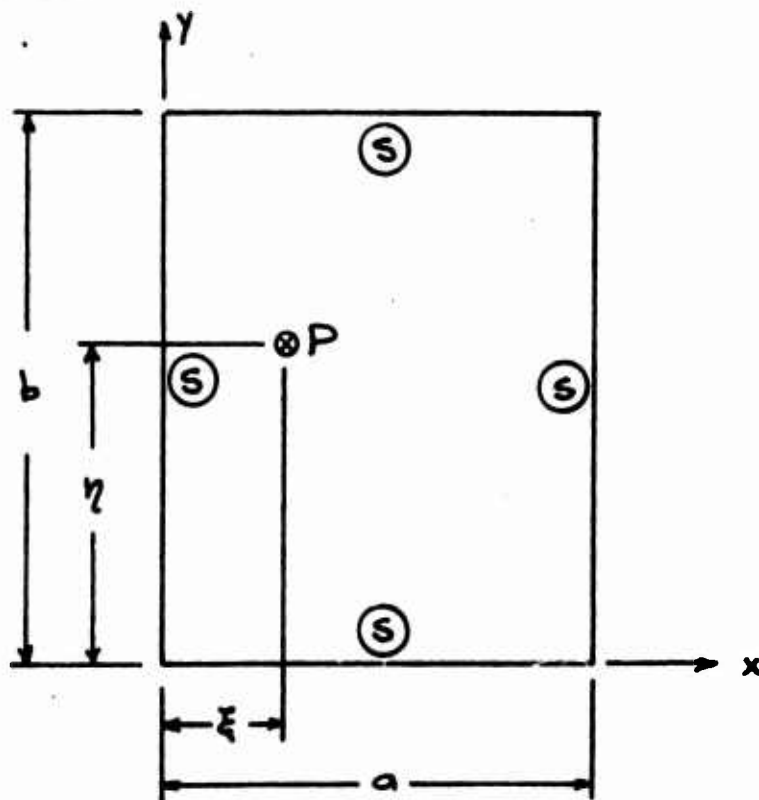
Figure 9
Simple Supports - Concentrated Load (I)

The load coefficient for the double-series solution, from Eq. (68), is written:

$$A_{mn} = \frac{4P}{ab} \sin \frac{m\pi\xi}{a} \sin \frac{n\pi\eta}{b} \, . \tag{87}$$

The deformation expression is derived from the substitution of this quantity into the general equation (69):

$$\boxed{w = \frac{4P}{ab\pi^4} \sum_m \sum_n \frac{\sin \frac{m\pi\xi}{a} \sin \frac{n\pi\eta}{b} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}}{D_x \frac{m^4}{a^4} + 2D_{xy} \frac{m^2 n^2}{a^2 b^2} + D_y \frac{n^4}{b^4}} \, .} \tag{88}$$

Ando (Ref. 209) solves the problem using the single series solution. Using a coordinate system chosen such that the x-axis is located at the center, rather than the edge, of the plate (Fig. 10), he writes two expressions for the deformation:

CASE I: $\eta \le y \le b/2$:

$$\boxed{\begin{aligned} w_{\mathrm{I}} = \sum_m \Big[& C_1 \cosh \frac{m\pi\lambda}{a} y + C_2 \sinh \frac{m\pi\lambda}{b} y \\ & + C_1 \cosh \frac{m\pi\bar{\lambda}}{a} y + C_2 \sinh \frac{m\pi\bar{\lambda}}{b} y \Big] \sin \frac{m\pi x}{a} \, , \end{aligned}} \tag{89}$$

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CASE II:

$$w_z = \sum_m \left[C_3 \cosh \frac{m\pi\lambda}{a}y + C_4 \sinh \frac{m\pi\lambda}{b}y \right. \\ \left. + C_3 \cosh \frac{m\pi\bar{\lambda}}{a}y + C_4 \sinh \frac{m\pi\bar{\lambda}}{b}y \right] \sin \frac{m\pi x}{a} \, . \tag{90}$$
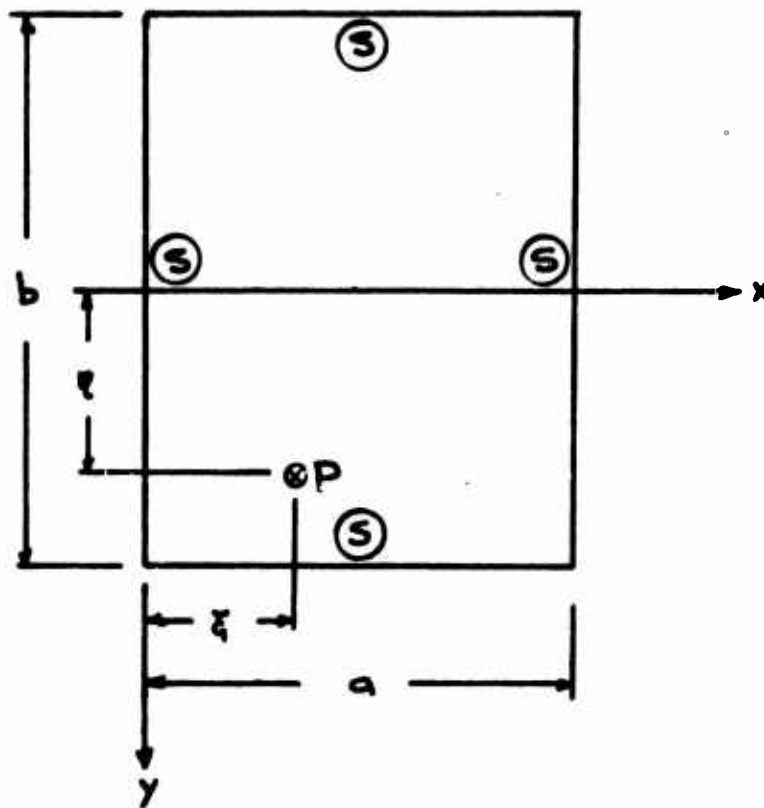
Figure 10
Simple Supports - Concentrated Load (II)

The constants of Eqs. (89) and (90) are the following:

$$C_1 = \frac{P a^2 \lambda \bar{\lambda}^2}{D_z m^3 \Pi^3 (\lambda^2 - \bar{\lambda}^2)} \left[- \sinh \frac{m \Pi \lambda \eta}{a} - \left(\cosh \frac{m \Pi \lambda \eta}{a} \cdot \tanh \frac{m \Pi \lambda y}{a} \right) \right] \sin \frac{m \Pi \xi}{a} \, ,$$

$$C_2 = \frac{P a^2 \lambda \bar{\lambda}^2}{D_z m^3 \Pi^3 (\lambda^2 - \bar{\lambda}^2)} \left[\cosh \frac{m \Pi \lambda \eta}{a} + \left(\sinh \frac{m \Pi \lambda \eta}{a} \cdot \coth \frac{m \Pi \lambda y}{a} \right) \right] \sin \frac{m \Pi \xi}{a} \, ,$$

$$C_3 = \frac{P a^2 \lambda \bar{\lambda}^2}{D_z m^3 \Pi^3 (\lambda^2 - \bar{\lambda}^2)} \left[\sinh \frac{m \Pi \lambda \eta}{a} - \left(\cosh \frac{m \Pi \lambda \eta}{a} \cdot \tanh \frac{m \Pi \lambda y}{a} \right) \right] \sin \frac{m \Pi \xi}{a} \, ,$$

$$C_4 = \frac{P a^2 \lambda \bar{\lambda}^2}{D_z m^3 \Pi^3 (\lambda^2 - \bar{\lambda}^2)} \left[- \cosh \frac{m \Pi \lambda \eta}{a} + \left(\sinh \frac{m \Pi \lambda \eta}{a} \cdot \coth \frac{m \Pi \lambda y}{a} \right) \right] \sin \frac{m \Pi \xi}{a} \, .$$

As in the previous case, Krug and Stein (Ref. 57) make use of the transformation of coordinates, as illustrated in Figures 11a and 11b.
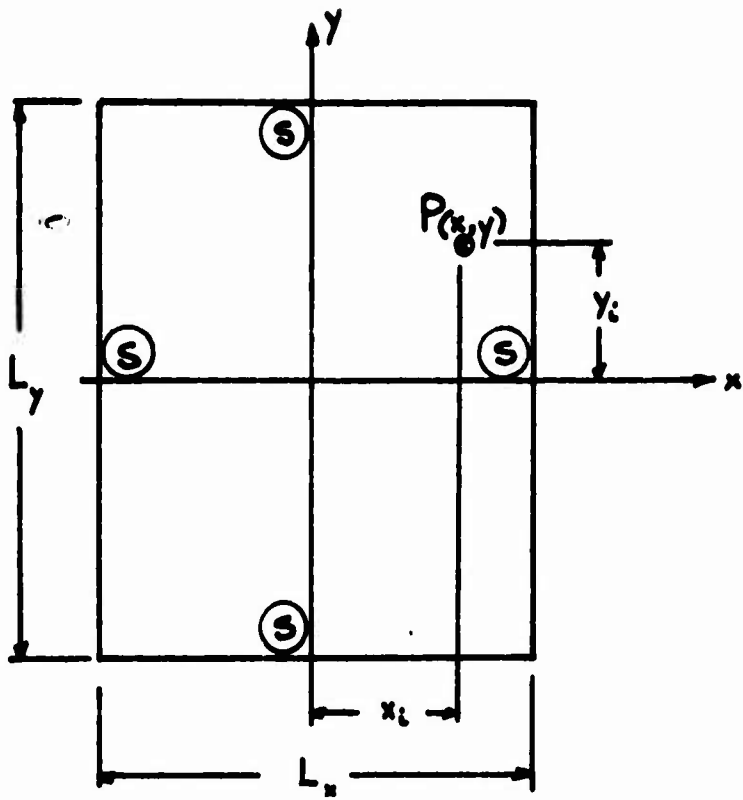
a)   Real Structure
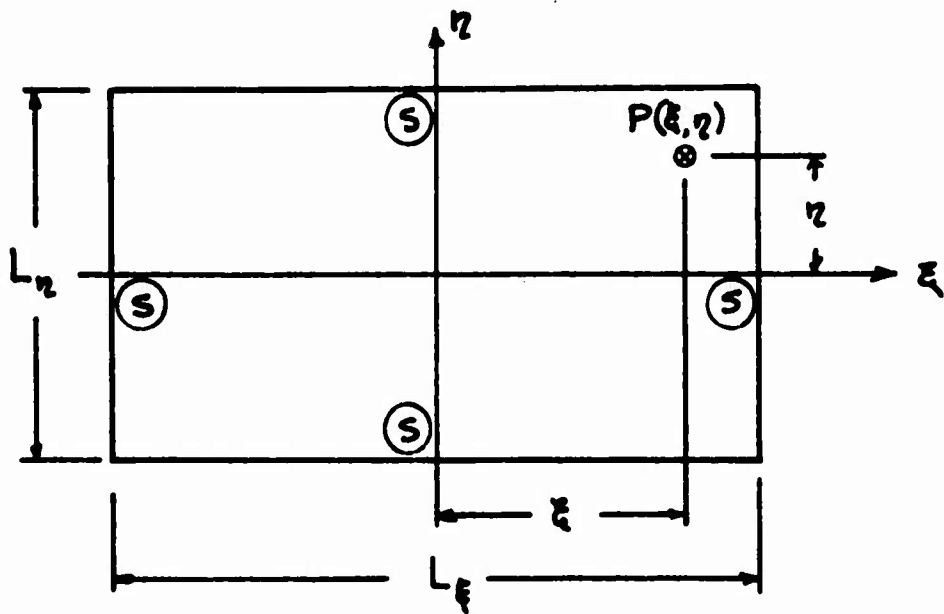
b)   Transformed Structure

Figure 11
Simple Supports - Concentrated Load (III)

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Using the transformations given by Eqs. (62) through (65), the moments are given:

$$M_x = \frac{1}{8\pi} \sqrt[4]{\frac{D_x}{D_y}} \; P(\xi,\eta) \; M_\xi(\xi,\eta) \; ,$$

$$M_y = \frac{1}{8\pi} \sqrt[4]{\frac{D_x}{D_y}} \; P(\xi,\eta) \; M_\eta(\xi,\eta) \; . \Bigg\} \quad (91)$$

Obviously, superposition is used for the calculation of deflections in the first two solutions, when more than one concentrated load is acting, and the latter solution (Eq. (91)) can be rewritten in the form:

$$M_x = \frac{1}{8\pi} \sqrt[4]{\frac{D_x}{D_y}} \sum_i P(\xi_i, \eta_i) \, M_\xi(\xi_i, \eta_i) \; . \quad (92)$$

3.1.4   Line Loading

In solving the problem of the simply supported plate under the action of a linear load parallel to the x-axis (Figure 12), Ando (Ref. 209) uses Eqs. (89) and (90) to give the deformation for the ranges $\eta \leq y \leq b/2$ and $y \leq \eta$, respectively. The constants are evaluated as follows:

$$C_1 = \frac{f_m \lambda \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)}\left[-\sinh \frac{m\pi\lambda\eta}{a} - \left(\cosh \frac{m\pi\lambda\eta}{a} \tanh \frac{m\pi\lambda y}{a} \right)\right] \; ,$$

$$C_2 = \frac{f_m \lambda \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} \left[\cosh \frac{m\pi\lambda}{a}y + \left(\sinh \frac{m\pi\lambda}{a}\eta \coth \frac{m\pi\lambda}{a}y \right) \right],$$

$$C_3 = \frac{f_m \lambda \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} \left[\sinh \frac{m\pi\lambda}{a}\eta - \left(\cosh \frac{m\pi\lambda}{a}\eta \tanh \frac{m\pi\lambda}{a}y \right) \right],$$

$$C_4 = \frac{f_m \lambda \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} \left[-\cosh \frac{m\pi\lambda}{a}\eta + \left(\sinh \frac{m\pi\lambda}{a}\eta \coth \frac{m\pi\lambda}{a}y \right) \right],$$

and

$$f_m = \frac{2a^2}{D_1 m^3 \pi^3} \int_{\xi_1}^{\xi_2} F(\xi) \sin \frac{m\pi\xi}{a} d\xi .$$

The load, $F(\xi)$ is represented as a single trigonometric series.

Should one consider the special case where the load acts on the x-axis ($\eta = 0$), the constants simplify to:

$$C_1 = -\frac{f_m \lambda \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} \tanh \frac{m\pi\lambda}{a}y = C_3 ,$$

$$C_2 = \frac{f_m \lambda \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} = -C_4 .$$
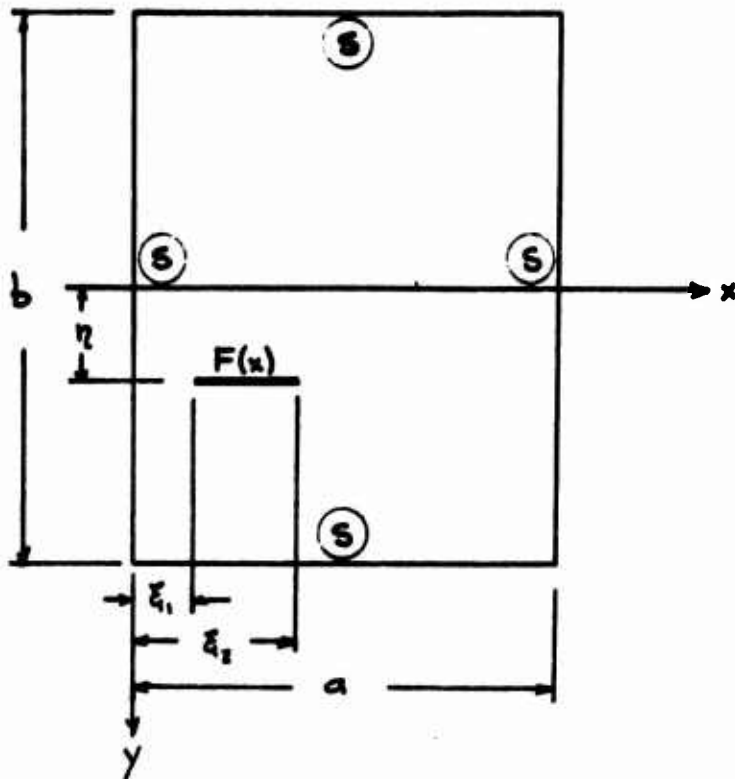
Figure 12
Simple Supports - Line Load (I)

A more general inclination of the line load is
considered by Krug and Stein (Ref. 57). As previously
described for other loadings, the coordinates are trans-
formed from those of Figure 13a to those of Figure 13b,
according to Eqs. (62) through (65).

The bending moment expressions are then given for
the general case of variable loading, $\mathfrak{f}$:

$$M_x = \frac{1}{8\pi} \sqrt[4]{\frac{D_x}{D_y}} \sqrt{1 + \frac{\tan^2\phi}{\sqrt{(D_x/D_y)}}} (\cos\phi) L \int_{\beta_a}^{\beta_b} f(\beta) M_\xi(\beta)\, d\beta, \qquad (93)$$

$$M_y = \frac{1}{8\pi} \sqrt[4]{\frac{D_x}{L}} \sqrt{1 \perp \frac{\tan^2\phi}{D_x/D_y}} (\cos\phi) L \int_{\beta_a}^{\beta_b} f(\beta) M_\eta(\beta)\, d\beta. \qquad (94)$$
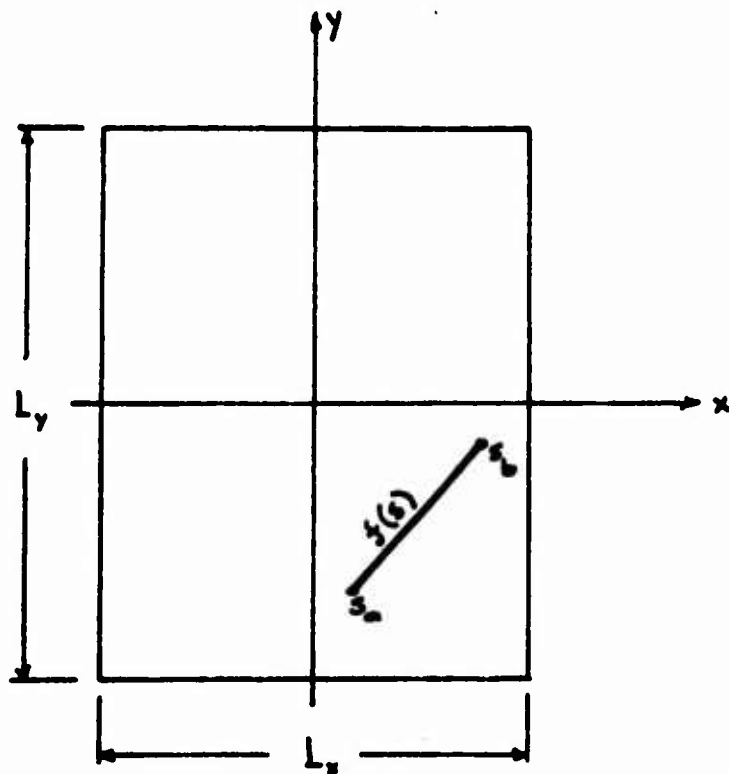
a) Real Structure
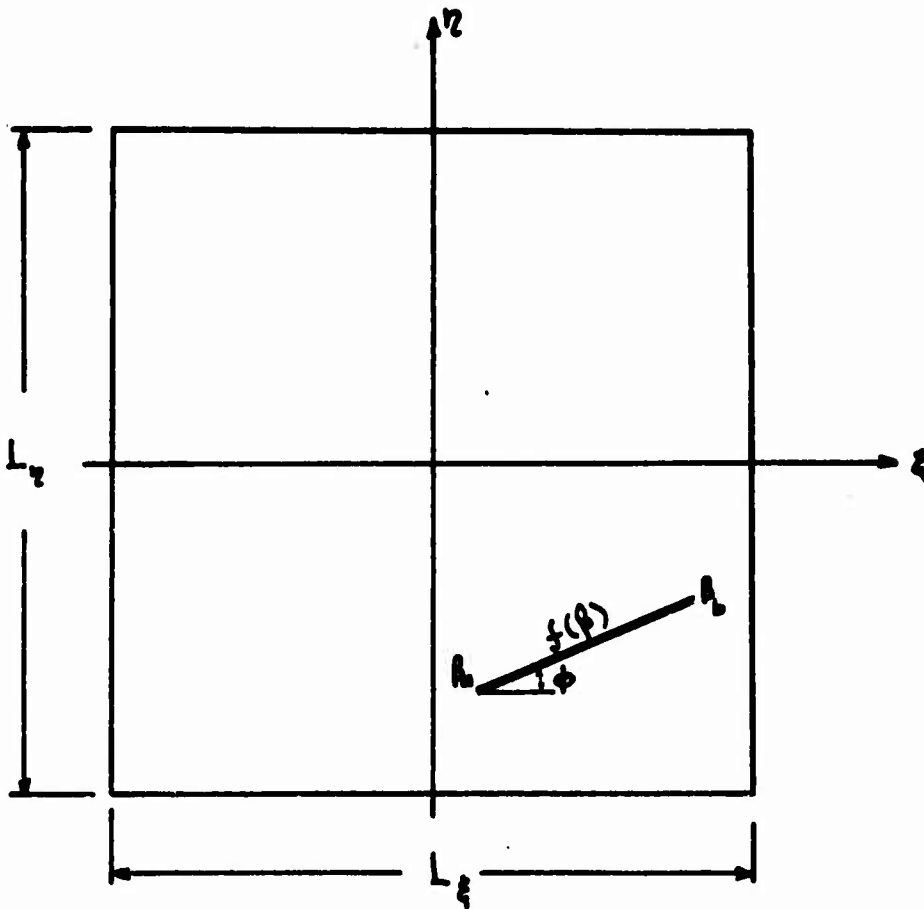
b) Transformed Structure

Figure 13
Simple Supports - Line Load (II)

For the special case of a load uniformly distributed along a line, of course, $f(\beta) = f$, and it is placed in front of the integral sign.

3.1.5 Moment Distributed Along a Line

The problem of a simply supported plate under the action of a uniform moment distributed along the $y = b$ edge (Fig. 14) is considered by Vinson and Brull (Ref. 103).
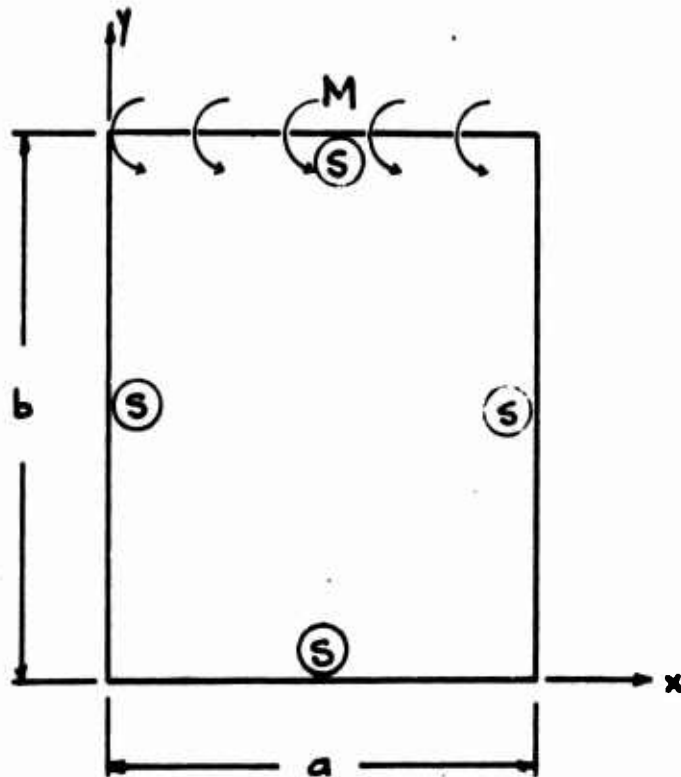
Figure 14
Simple Supported - Edge Moment

Using the transformations,

$$\bar{y} = y \sqrt[4]{\frac{D_x}{D_y}} \quad , \quad \bar{b} = b \sqrt[4]{\frac{D_x}{D_y}} \; , \tag{95}$$

the boundary conditions are:

$$
\left.
\begin{aligned}
& w(x,0) = w(x,\bar{b}) = w(0,\bar{y}) = w(a,\bar{y}) = 0, \\[4pt]
& \left.\frac{\partial^2 w}{\partial x^2}\right|_{(0,\bar{y})} = \left.\frac{\partial^2 w}{\partial x^2}\right|_{(a,\bar{y})} = \left.\frac{\partial^2 w}{\partial y^2}\right|_{(x,0)} = 0, \\[4pt]
& M(x,\bar{b}) = M.
\end{aligned}
\right\} \tag{96}
$$

If the perturbation solution is limited to cases where the rigidity ratios (D_y/D_x) and (D_{xy}/D_x) are nearly equal, the basic equation (31) of the orthotropic plate is rewritten:

$$\frac{\partial^4 w}{\partial x^4} - \alpha \frac{\partial^4 w}{\partial x^2 \partial \bar{y}^2} + \frac{\partial^4 w}{\partial \bar{y}^4} = \frac{P}{D_x} \; , \tag{97}$$

where

$$\alpha = 2\left(1 - \frac{D_{xy}}{\sqrt{D_x D_y}}\right) = 2\left(1 - \frac{(D_{xy}/D_x)}{\sqrt{(D_y/D_x)}}\right).$$

The solution is taken as a series in powers of α:

$$w = \sum_{n=0}^{\infty} \phi_n(x,\bar{y}) \, \alpha^n \; , \tag{98}$$

which makes Eq. (97) become:

$$\nabla^4 \phi_0 + \sum_{n=1}^{\infty}\left[\nabla^4 \phi_n - \frac{\partial^4 \phi_{n-1}}{\partial x^2 \partial \bar{y}^2}\right]\alpha^n = \frac{P}{D_x} \; , \tag{99}$$

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where the operator,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial \bar{y}^2} \ .$$

Since the curvature vanishes along edge $y = \bar{b}$, the moment,

$$M(x,\bar{b}) = M = -D_y \frac{\partial^2 w}{\partial \bar{y}^2}(x,\bar{b}) \ , \qquad (100)$$

and by substituting Eq. (98) into Eq. (100),

$$\left. \begin{aligned} \frac{\partial^2 w}{\partial \bar{y}^2}(x,\bar{b}) &= -\frac{M}{D_y} \\ &= \frac{\partial^2 \Phi_0(x,\bar{b})}{\partial \bar{y}^2} + \frac{\partial^2 \Phi_1(x,\bar{b})}{\partial \bar{y}^2}\alpha + \frac{\partial^2 \Phi_2(x,\bar{b})}{\partial \bar{y}^2}\alpha^2 + ... \end{aligned} \right\} (101)$$

The governing equation for the first term of the solution is

$$D_x \nabla^4 \Phi_0(x,\bar{y}) = 0 , \qquad (102)$$

and the boundary conditions are:

$$\left. \begin{aligned} \Phi_0 &= 0 & \text{ALL EDGES} \\ \frac{\partial^2 \Phi_0}{\partial x^2} &= 0 & x = 0,a \\ \frac{\partial^2 \Phi_0}{\partial \bar{y}^2} &= 0 & y = 0 \\ \frac{\partial^2 \Phi_0}{\partial \bar{y}^2} &= -\frac{M}{D_y} & y = b. \end{aligned} \right\} (103)$$

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The solution is taken in the form:

$$\Phi_o = \Psi_o(x,\bar{y}) + f(\bar{y}) M ,$$

(104)

where Ψ_o satisfies homogeneous boundary conditions and

$$f(\bar{y}) = -\frac{1}{6D_y}\left[\frac{\bar{y}^3}{\bar{b}} - \bar{b}\bar{y}\right].$$

(105)

From previous conditions, the function,

$$\Psi_o(x,\bar{y}) = \sum_{m=1}^{\infty}\left[(C_1 + C_2 x)\ \cosh\lambda_m x + (C_3 + C_4 x)\ \sinh\lambda_m x\right]\sin\lambda_m\bar{y},$$

where the constants,

$$C_1 = \frac{2M\bar{b}^2(-1)^m}{D_y\, m^3\pi^3} ,$$

$$C_2 = -\frac{\lambda_m M\bar{b}^2(-1)^m}{D_y\, m^3\pi^3}\ \frac{1-\cosh\lambda_m a}{\sinh\lambda_m a} ,$$

$$C_3 = -\frac{M\bar{b}^2(-1)^m}{D_y\, m^3\pi^3}\ \frac{(1-\cosh\lambda_m a)(\lambda_m a - 2\sinh\lambda_m a)}{\sinh^2\lambda_m a} ,$$

$$C_4 = -\frac{\lambda_m M\bar{b}^2(-1)^m}{D_y\, m^3\pi^3} ,$$

and

$$\lambda_m = \frac{m\pi}{\bar{b}} .$$

The second term is taken in the form,

$$\Phi_1(x,\bar{y}) = \sum_{m=1}^{\infty} \Theta_m(x) \sin \lambda_m y \ ,$$

where the functions $\Theta_m(x)$ are evaluated by:

$$\left.\begin{aligned}
\Theta_m(x) &= \left(C_5 + C_6 x + A_0 x^2 + A_1 x^3\right) \cosh \lambda_m a \\
&\quad + \left(C_7 + C_8 x + A_2 x^2 + A_3 x^3\right) \sinh \lambda_m a \ ,
\end{aligned}\right\} (107)$$

where the constants,

$$C_5 = 0 \ ,$$

$$\begin{aligned}
C_6 = {} & 2 A_0 a - \frac{3 A_1 a}{\lambda_m}\left(\lambda_m a + \frac{\cosh \lambda_m a}{\sinh \lambda_m a}\right) \\
& - \frac{A_2}{\lambda_m}\left[1 + \frac{2 \lambda_m a \cosh \lambda_m a}{\sinh \lambda_m a}\right. \\
& \left. - \frac{3 A_3}{\lambda_m}\left(1 + \frac{\lambda_m a \cosh \lambda_m a}{\sinh \lambda_m a}\right)\right] \ ,
\end{aligned}$$

$$\begin{aligned}
C_7 = {} & \frac{A_0 a}{\lambda_m \sinh \lambda_m a}\left(\lambda_m a \cosh \lambda_m a + \sinh \lambda_m a\right) \\
& + \frac{A_1 a^2 \cosh \lambda_m a}{\lambda_m \sinh^2 \lambda_m a}\left(2 \lambda_m a \sinh \lambda_m a + 3 \cosh \lambda_m a\right) \\
& + \frac{A_2 a}{\lambda_m \sinh^2 \lambda_m a}\left(\cosh \lambda_m a \sinh \lambda_m a + 2 \lambda_m a + \lambda_m a \sinh^2 \lambda_m a\right) \\
& + \frac{A_3 a^2}{\lambda_m \sinh^2 \lambda_m a}\left(3 \cosh \lambda_m a \sinh \lambda_m a + 3 \lambda_m a\right. \\
& \left. + 2 \lambda_m a \sinh^2 \lambda_m a\right) \ ,
\end{aligned}$$

$$C_8 = - \frac{A_0}{\lambda_m} \ ,$$

and

$$A_0 = - \frac{(C_1 \lambda_m + C_4) \lambda_m}{8} \, ,$$

$$A_1 = - \frac{C_2 \lambda_m^2}{24} \, ,$$

$$A_2 = - \frac{(C_3 \lambda_m + C_2) \lambda_m}{8} \, ,$$

$$A_3 = - \frac{C_4 \lambda_m^2}{24} \, .$$

The evaluation of these terms is necessary before the deformation equation (98) can be solved.

A more general case of a non-uniform moment distributed along any line parallel to the x-axis (Fig. 15) is considered by Ando (Ref. 209).
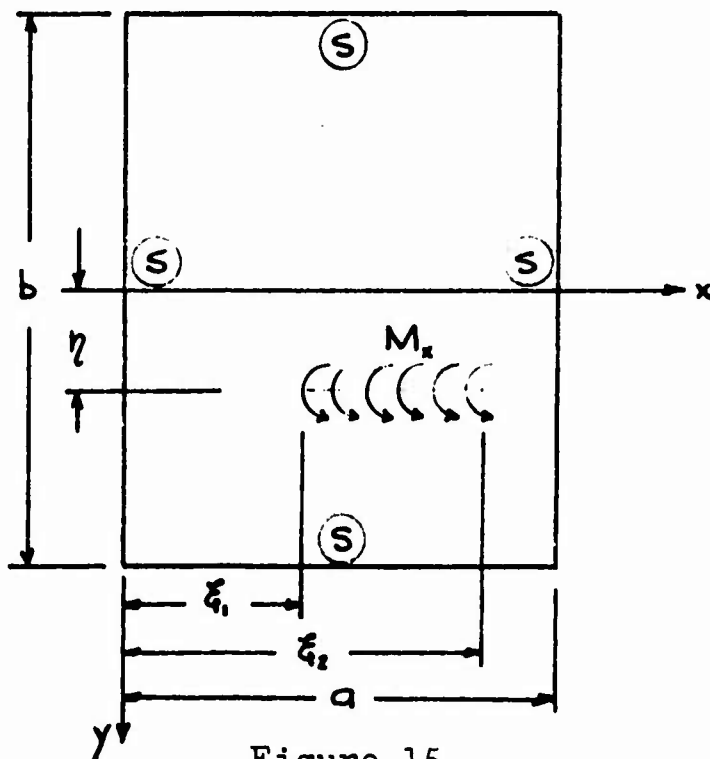
Figure 15
Simple Supports - Line Moment

The problem is solved by using the solution of Part 3.1.4 for a line loading. If a linear load, such as that of Figure 12, acts on the plate, and another load of the same intensity, parallel and very near to it is allowed to act simultaneously in the opposite direction, a line couple is created. This is shown in Figure 16.
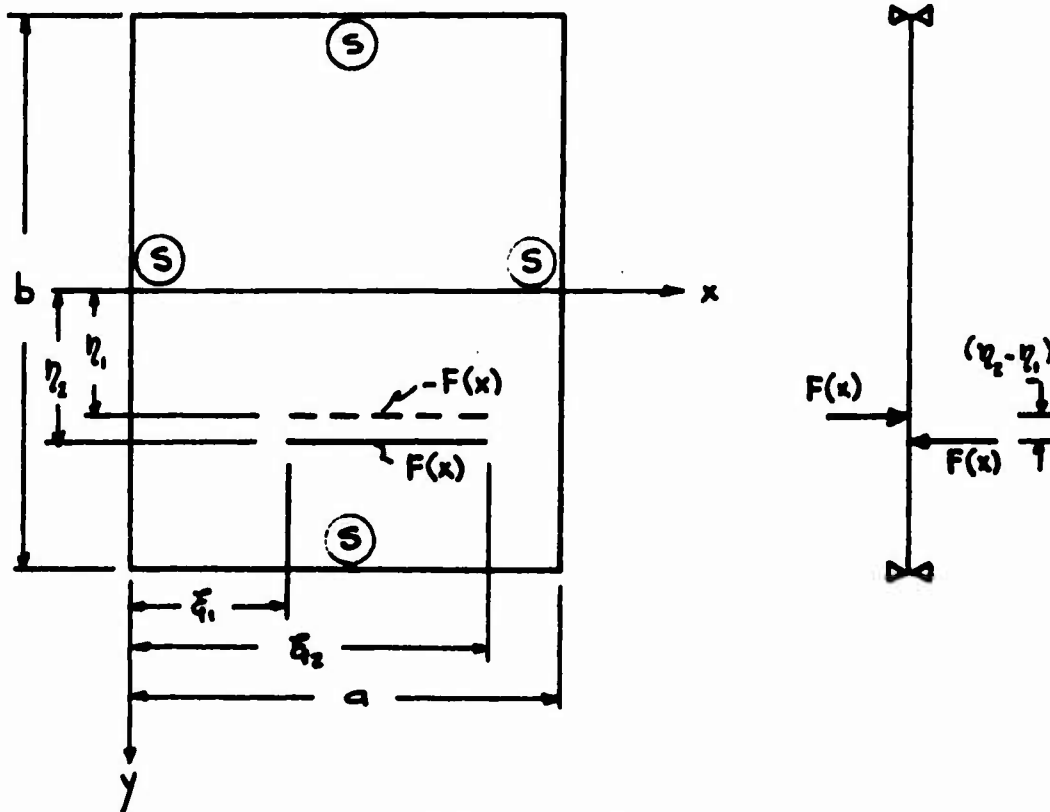
Figure 16
Simple Supports - Line-Load Couple

Thus, as

$$(\eta_2 - \eta_1) \longrightarrow 0,$$

the loading of Figure 16 approaches that of Figure 15, and

$$M(x) = \lim_{(\eta_2 - \eta_1) \to 0} F(x) \ (\eta_2 - \eta_1) . \tag{108}$$

Thus, the deformation for two regions is again given by Eqs. (89) and (90), and the constants for this loading condition are:

$$C_1 = \frac{E_m \lambda^2 \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} \left[-\cosh \frac{m\pi\lambda}{a}\eta - \left(\sinh \frac{m\pi\lambda}{a}\eta \right. \right.$$
$$\left. \left. \cdot \tanh \frac{m\pi\lambda}{a}y \right) \right] ,$$

$$C_2 = \frac{E_m \lambda^2 \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} \left[\sinh \frac{m\pi\lambda}{a}\eta + \left(\cosh \frac{m\pi\lambda}{a}\eta \right. \right.$$
$$\left. \left. \cdot \coth \frac{m\pi\lambda}{a}y \right) \right] ,$$

$$C_3 = \frac{E_m \lambda^2 \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} \left[\cosh \frac{m\pi\lambda}{a}\eta - \left(\sinh \frac{m\pi\lambda}{a}\eta \right. \right.$$
$$\left. \left. \cdot \tanh \frac{m\pi\lambda}{a}y \right) \right] ,$$

$$C_4 = \frac{E_m \lambda^2 \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} \left[-\sinh \frac{m\pi\lambda}{a}\eta + \left(\cosh \frac{m\pi\lambda}{a}\eta \right. \right.$$
$$\left. \left. \cdot \coth \frac{m\pi\lambda}{a}y \right) \right] ,$$

and the loading term,

$$E_m = \frac{2a}{D_2 \, m^2 \pi^2} \int_{\xi_1}^{\xi_2} M(\xi) \sin \frac{m\pi\xi}{a} \, d\xi \ .$$

The problem is greatly simplified, of course, when the line of the moment coincides with the x-axis. Then, the constants become:

$$C_1 = -\frac{E_m \, \lambda^2 \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} = C_3 \ ,$$

$$C_2 = \frac{E_m \, \lambda^2 \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} = -C_4 \ .$$

3.2 - 2 Opposite Edges Clamped and the Other Two Simply Supported

Since the problem of an orthotropic rectangular plate having two opposite sides built in and the two remaining sides simply supported is of some practical importance, it deserves discussion here. Several loading conditions are considered and the method of Ando (Ref. 209) is utilized.

3.2.1 Uniformly Distributed Load

A plate with a uniformly distributed loading
acting over the entire length, b, and over a portion
of the length, a, is considered (see Figure 17).
The notation Ⓢ is again used to indicate simple
supports, and Ⓒ designates clamped edges.
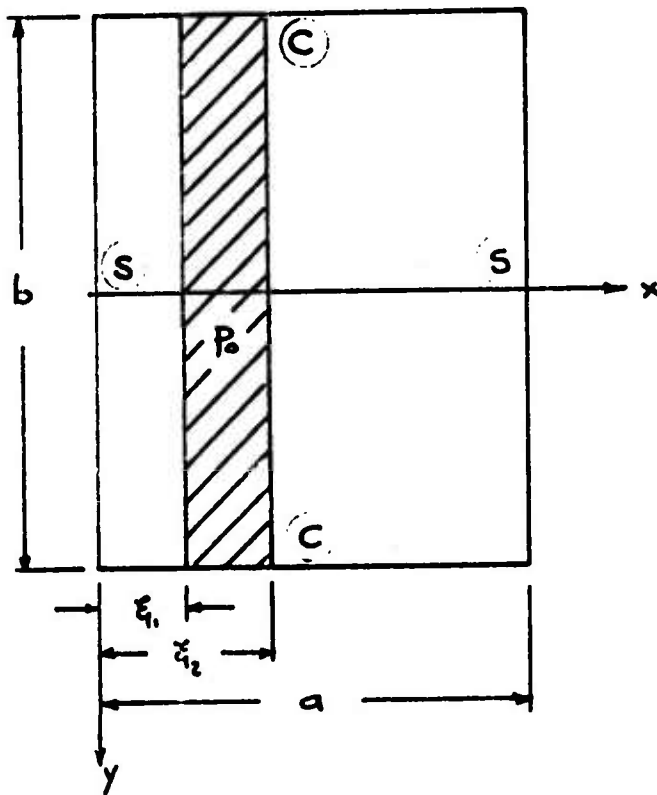
Figure 17
Ⓢ - Ⓒ - Ⓢ - Ⓒ - Uniform Load

The deformation equations (83), (84), and (85) are again used as the basic expressions. Due to the boundary conditions:

$$\left.\begin{array}{l} w_2 = 0 = \dfrac{\partial w_2}{\partial y} \quad \text{AT} \quad y = \dfrac{b}{2}, \\[2em] w_3 = 0 = \dfrac{\partial w_3}{\partial y} \quad \text{AT} \quad y = -\dfrac{b}{2}, \end{array}\right\} \quad (109)$$

and because of the simplification of the loading (relative to that shown in Figure 7), the constants become:

$$C_1 = C_5 = C_9 = \frac{A_m \bar{\lambda} \,\text{SINH}\, \frac{m\pi\bar{\lambda}}{a}y}{\lambda \cosh\frac{m\pi\bar{\lambda}}{a}y \,\text{SINH}\,\frac{m\pi\lambda}{a}y - \bar{\lambda}\cosh\frac{m\pi\lambda}{a}y\,\text{SINH}\,\frac{m\pi\bar{\lambda}}{a}y},$$

$$C_2 = C_4 = C_6 = C_8 = C_{10} = C_{12} = 0,$$

$$C_3 = C_7 = C_{11} = \frac{A_m \lambda \,\text{SINH}\, \frac{m\pi\lambda}{a}y}{\bar{\lambda}\cosh\frac{m\pi\lambda}{a}y\,\text{SINH}\,\frac{m\pi\bar{\lambda}}{a}y - \lambda\cosh\frac{m\pi\bar{\lambda}}{a}y\,\text{SINH}\,\frac{m\pi\lambda}{a}y}.$$

These values are substituted into Eqs. (83), (84), and (85) and deflections are found in any of the three regions. For the case of a uniform loading over the entire surface, $\xi_1 = 0$ and $\xi_2 = a$, in the evaluation of the load parameter,

$$A_m = -\frac{2a^3 P_e}{D_x m^4 \pi^4} \int_{\xi_1}^{\xi_2} \sin \frac{m\pi\xi}{a} \, d\xi$$

$$= \frac{2a^4 P_e}{D_x m^5 \pi^5} \left(\cos \frac{m\pi\xi_1}{a} - \cos \frac{m\pi\xi_2}{a} \right).$$

3.2.2 Concentrated Force

Next, the orthotropic rectangular plate of Figure 10 is considered. Here, however, the edge conditions are:

$$x = 0,\ a \quad \longrightarrow \quad \text{simply supported,}$$

$$y = \overset{+}{\underset{-}{}} b/2 \quad \longrightarrow \quad \text{clamped.}$$

Eqs. (89) and (90) are used to describe the deformed condition, and the constants for these boundary conditions become:

$$C_1 = \frac{P a^2 \lambda \bar{\lambda}}{D_x m^3 \pi^3 (\lambda^2 - \bar{\lambda}^2)} \left[-\bar{\lambda} \sinh \frac{m\pi\lambda\eta}{a} + \bar{\lambda} \gamma_1 \cosh \frac{m\pi\lambda\eta}{a} \right.$$

$$\left. - \lambda \gamma_2 \cosh \frac{m\pi\lambda\eta}{a} \right] \sin \frac{m\pi\xi}{a},$$

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$$C_2 = \frac{P a^2 \lambda \bar{\lambda}}{D_2 m^3 \Pi^3 (\lambda^2 - \bar{\lambda}^2)} \left[\bar{\lambda} \cosh \frac{m \Pi \lambda}{a} \eta + \bar{\lambda} \delta_1 \sinh \frac{m \Pi \lambda}{a} \eta \right.$$

$$\left. - \lambda \delta_2 \sinh \frac{m \Pi \lambda}{a} \eta \right] \sin \frac{m \Pi \xi}{a} ,$$

$$C_3 = \frac{P a^2 \lambda \bar{\lambda}}{D_2 m^3 \Pi^3 (\lambda^2 - \bar{\lambda}^2)} \left[\bar{\lambda} \sinh \frac{m \Pi \lambda}{a} \eta + \bar{\lambda} \gamma_1 \cosh \frac{m \Pi \lambda}{a} \eta \right.$$

$$\left. - \lambda \gamma_2 \cosh \frac{m \Pi \lambda}{a} \eta \right] \sin \frac{m \Pi \xi}{a} ,$$

$$C_4 = \frac{P a^2 \lambda \bar{\lambda}}{D_2 m^3 \Pi^3 (\lambda^2 - \bar{\lambda}^2)} \left[- \bar{\lambda} \cosh \frac{m \Pi \lambda}{a} \eta + \bar{\lambda} \delta_1 \sinh \frac{m \Pi \lambda}{a} \eta \right.$$

$$\left. - \lambda \delta_2 \sinh \frac{m \Pi \lambda}{a} \eta \right] \sin \frac{m \Pi \xi}{a} ,$$

where

$$\gamma_1 = \frac{\lambda \cosh \frac{m \Pi \lambda}{a} \gamma \cosh \frac{m \Pi \bar{\lambda}}{a} \gamma - \bar{\lambda} \sinh \frac{m \Pi \lambda}{a} \gamma \sinh \frac{m \Pi \bar{\lambda}}{a} \gamma}{\bar{\lambda} \cosh \frac{m \Pi \lambda}{a} \gamma \sinh \frac{m \Pi \bar{\lambda}}{a} \gamma - \lambda \cosh \frac{m \Pi \bar{\lambda}}{a} \gamma \sinh \frac{m \Pi \lambda}{a} \gamma} ,$$

$$\gamma_2 = \frac{\bar{\lambda}}{\bar{\lambda} \cosh \frac{m \Pi \lambda}{a} \gamma \sinh \frac{m \Pi \bar{\lambda}}{a} \gamma - \lambda \cosh \frac{m \Pi \bar{\lambda}}{a} \gamma \sinh \frac{m \Pi \lambda}{a} \gamma} ,$$

$$\delta_1 = \frac{\bar{\lambda} \cosh \frac{m \Pi \lambda}{a} \gamma \cosh \frac{m \Pi \bar{\lambda}}{a} \gamma - \lambda \sinh \frac{m \Pi \lambda}{a} \gamma \sinh \frac{m \Pi \bar{\lambda}}{a} \gamma}{\bar{\lambda} \cosh \frac{m \Pi \lambda}{a} \gamma \sinh \frac{m \Pi \lambda}{a} \gamma - \lambda \cosh \frac{m \Pi \lambda}{a} \gamma \sinh \frac{m \Pi \bar{\lambda}}{a} \gamma} ,$$

$$\delta_2 = \frac{\bar{\lambda}}{\bar{\lambda} \cosh \frac{m \Pi \bar{\lambda}}{a} \gamma \sinh \frac{m \Pi \lambda}{a} \gamma - \lambda \cosh \frac{m \Pi \lambda}{a} \gamma \sinh \frac{m \Pi \bar{\lambda}}{a} \gamma} .$$

In the special case of the load P acting on the x-axis,

$$\eta = 0 ,$$

and the constants become:

$$C_1 = \frac{P a^2 \lambda \bar{\lambda}}{D_x m^3 \pi^3 (\lambda^2 - \bar{\lambda}^2)} (\bar{\lambda} \gamma_1 - \lambda \gamma_2) \sin \frac{m \pi \xi}{a} = C_3 ,$$

$$C_2 = \frac{P a^2 \lambda \bar{\lambda}}{D_x m^3 \pi^3 (\lambda^2 - \bar{\lambda}^2)} \sin \frac{m \pi \xi}{a} = - C_4 .$$

When the concentrated load is located at the center of the plate,

$$\eta = 0 , \qquad \xi = \frac{a}{2} ,$$

and the constants are simplified further:

$$C_1 = \frac{(-1)^{\frac{m-1}{2}} P a^2 \lambda \bar{\lambda}}{D_x m^3 \pi^3 (\lambda^2 - \bar{\lambda}^2)} (\bar{\lambda} \gamma_1 - \lambda \gamma_2) = C_3 ,$$

$$C_2 = \frac{(-1)^{\frac{m-1}{2}} P a^2 \lambda \bar{\lambda}}{D_x m^3 \pi^3 (\lambda^2 - \bar{\lambda}^2)} = - C_4 .$$

$$(m = 1, 3, 5, \ldots)$$

3.2.3 Line Loading

The plate illustrated in Figure 12 is considered
next. The edge conditions are:

$$x = 0,a \longrightarrow \text{simply supported,}$$
$$y = \pm b/2 \longrightarrow \text{clamped.}$$

The deformation equations (89) and (90) are again
used to give the forms of the deflected surface for the
two regions of y. For the boundary conditions con-
sidered here, the constants become:

$$C_1 = \frac{f_m \lambda \bar{\lambda}}{2(\lambda^2 - \bar{\lambda}^2)} \left[-\bar{\lambda} \sinh \frac{m\pi\lambda}{a}\gamma + \bar{\lambda}\gamma_1 \cosh \frac{m\pi\lambda}{a}\gamma \right.$$
$$\left. - \lambda\gamma_2 \cosh \frac{m\pi\lambda}{a}\gamma \right],$$

$$C_2 = \frac{f_m \lambda \bar{\lambda}}{2(\lambda^2 - \bar{\lambda}^2)} \left[\bar{\lambda} \cosh \frac{m\pi\lambda}{a}\gamma + \bar{\lambda}\delta_1 \sinh \frac{m\pi\lambda}{a}\gamma \right.$$
$$\left. - \lambda\delta_2 \sinh \frac{m\pi\lambda}{a}\gamma \right],$$

$$C_3 = \frac{f_m \lambda \bar{\lambda}}{2(\lambda^2 - \bar{\lambda}^2)} \left[\bar{\lambda} \sinh \frac{m\pi\lambda}{a}\gamma + \bar{\lambda}\gamma_1 \cosh \frac{m\pi\lambda}{a}\gamma \right.$$
$$\left. - \lambda\gamma_2 \cosh \frac{m\pi\lambda}{a}\gamma \right],$$

$$C_4 = \frac{f_m \lambda \bar{\lambda}}{2(\lambda^2 - \bar{\lambda}^2)} \left[-\bar{\lambda} \cosh \frac{m\pi\lambda}{a}\gamma + \bar{\lambda}\delta_1 \sinh \frac{m\pi\lambda}{a}\gamma \right.$$
$$\left. - \lambda\delta_2 \sinh \frac{m\pi\lambda}{a}\gamma \right],$$

where f_m is defined in Paragraph 3.1.4 and the parameters γ and δ are given in the preceding paragraph.

For the particular case of the line load F being applied along the x-axis,

$$\eta = 0,$$

and the constants simplify to:

$$C_1 = \frac{f_m \lambda \bar{\lambda}}{2(\lambda^2 - \bar{\lambda}^2)} (\bar{\lambda}\gamma_1 - \lambda\gamma_2) = C_3 \, ,$$

$$C_2 = \frac{f_m \lambda \bar{\lambda}}{2(\lambda^2 - \bar{\lambda}^2)} = -C_4 \, .$$

3.3 Infinitely Long Plate

An infinitely long plate with the long sides simply supported is now considered. Since the ratio of b/a is very large, the use of the single series, Lévy-type, solution is justified and the deformation is taken in the form of Eq. (49):

$$w = \sum_m Y_m \sin \frac{m\pi x}{a} \, ,$$

and Y_m satisfies the condition:

$$D_y Y_m^{iv} - 2D_{xy} \frac{m^2 \pi^2}{a^2} Y_m^{ii} + D_x \frac{m^4 \pi^4}{a^4} Y_m = 0 . \qquad \text{(110)}$$

The roots of the characteristic equation have four forms:

$$\text{roots} = \pm \frac{m \pi}{a} \sqrt{\frac{D_{xy}}{D_y} \pm \sqrt{\frac{D_{xy}^2}{D_y^2} - \frac{D_x}{D_y}}} . \qquad \text{(111)}$$

Using the parameters,

$$\lambda = \sqrt[4]{\frac{D_y}{D_x}} \qquad , \qquad \mu = \frac{D_{xy}}{\sqrt{D_x D_y}} ,$$

three cases (as discussed in Paragraph 3.1.1) must be considered:

$$\text{Case I} \quad : \quad \mu > 1 \quad \rightsquigarrow \quad \left(\frac{D_{xy}}{D_x}\right)^2 > \frac{D_y}{D_x} ,$$

$$\text{Case II} \quad : \quad \mu = 1 \quad \rightsquigarrow \quad \left(\frac{D_{xy}}{D_x}\right)^2 = \frac{D_y}{D_x} ,$$

$$\text{Case III} : \quad \mu < 1 \quad \rightsquigarrow \quad \left(\frac{D_{xy}}{D_x}\right)^2 < \frac{D_y}{D_x} .$$

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3.3.1 Line Loading

Let the line load, q, act along the x-axis of an infinitely long plate, as shown in Figure 18. The deformation will be taken in the single series form, as shown by Timoshenko and Woinowsky-Krieger (Ref. 97).
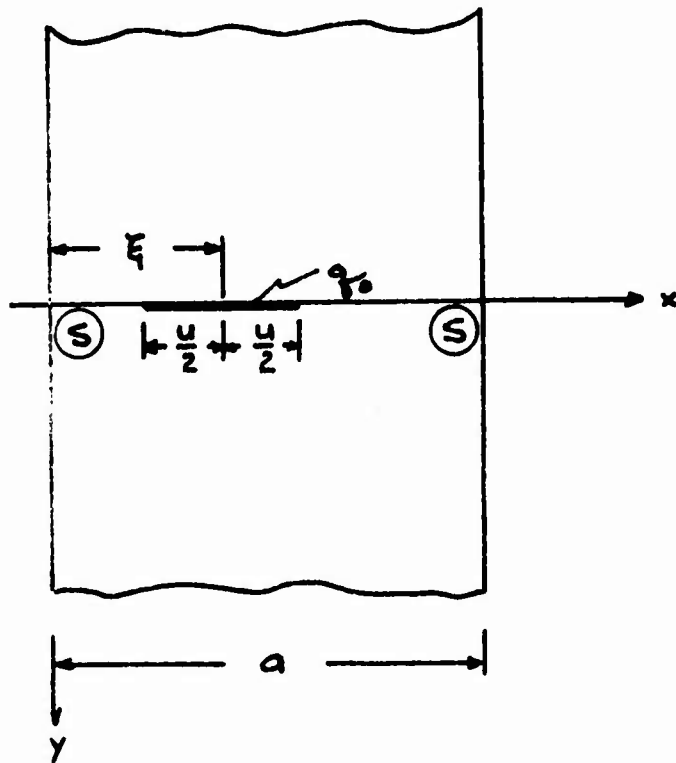
Figure 18
Infinite Plate - Line Load

For Case I, all the roots of Eq. (111) are real and the deformation,

$$ w_{\textrm{I}} = \frac{q_0 a^4}{2\pi^4 D_x(\alpha^2 - \beta^2)} \sum_m \frac{1}{m^5}\left(\alpha e^{-\frac{my}{\alpha}} - \beta e^{-\frac{my}{\beta}}\right) \sin\frac{m\pi x}{a} . \qquad (112) $$

For Case II, there are two double roots, and the deformation,

$$ w_{\textrm{II}} = \frac{q_0 a^3}{\pi^4 D_x} \sum_m \frac{1}{m^4}\left(1 + \frac{m\pi y}{a}\right) e^{-\frac{m\pi y}{a}} \sin\frac{m\pi x}{a} . \qquad (113) $$

In the third case, the deformation has the form,

$$ w_{\textrm{III}} = \frac{q_0 a^2}{4\pi^2 \sqrt{D_x D_y}} \sum_m \frac{1}{m^3}\left(\alpha' \sin\frac{my}{\alpha'} + \beta' \cos\frac{my}{\alpha'}\right) e^{-\frac{m x}{\beta'}} \sin\frac{m\pi x}{a} . \qquad (114) $$

The following notation was introduced in the above expressions for the sake of brevity:

$$ \alpha = \frac{a\lambda}{\pi}\sqrt{\mu + \sqrt{\mu^2 - 1}} \quad , $$

$$ \beta = \frac{a\lambda}{\pi}\sqrt{\mu - \sqrt{\mu^2 - 1}} \quad , $$

$$ \alpha' = \frac{a\lambda}{\pi}\sqrt{\frac{2}{1-\mu}} \quad , $$

$$ \beta' = \frac{a\lambda}{\pi}\sqrt{\frac{2}{1+\mu}} \quad , $$

and q_0 is the load per unit length.

3.3.2 Concentrated Force

Now, a concentrated load, P, is applied at
$(x = \xi$, $y = 0)$ of Figure 18. Then, the deformation
expressions for the three cases of elastic properties
become:

$$W_I = \frac{P a^3}{\pi^4 D_x (\alpha^2 - \beta^2)} \sum_m \frac{1}{m^3}\left(\alpha e^{-\frac{m\pi y}{a}} - \beta e^{-\frac{m\pi y}{\beta}}\right) \sin \frac{m\pi\xi}{a} \sin \frac{m\pi x}{a}, \tag{115}$$

$$W_I = \frac{P a^2}{2\pi^3 \lambda D_x} \sum_m \frac{1}{m^3}\left(1 + \frac{m\pi y}{a\lambda}\right) e^{-\frac{m\pi y}{\beta}} \sin \frac{m\pi\xi}{a} \sin \frac{m\pi x}{a}, \tag{116}$$

$$W_{II} = \frac{P a}{2\pi^3 \sqrt{D_x D_y}} \sum_m \frac{1}{m^3}\left(\alpha' \sin \frac{m y}{\alpha'} + \beta' \cos \frac{m y}{\alpha'}\right) e^{-\frac{m\pi y}{\beta}} \sin \frac{m\pi\xi}{a} \sin \frac{m\pi x}{a} \tag{117}$$

Nowacki, in Reference 71, obtains a solution in
closed form for the bending mements due to a load of
this type.

3.4 All Edges Clamped

The difficulty presented by the use of fixed
supports in the theory of plates is many times greater
than that encountered in beam theory. The usual method
of solution is superposition. Generally, the deformation
is found for a simply supported plate under the actual

loading, and it is added to the deformation due to distributed moments along the edges.

Any of the previously discussed techniques can be applied to solving this problem by superposition. Since Ando's method has been used rather extensively in this chapter, it will again be applied here.

The problem is divided into two parts: 1.) Simple supports at $x = \pm a/2$ edges, fixed supports at $y = \pm b/2$ edges, and under actual loading (Fig. 19a); 2.) Fixed supports on $y = \pm b/2$ edges and moment distributed along $x = \pm a/2$ edges (Fig. 19b). The combined effect of the two parts is the clamped plate shown in Figure 19c.
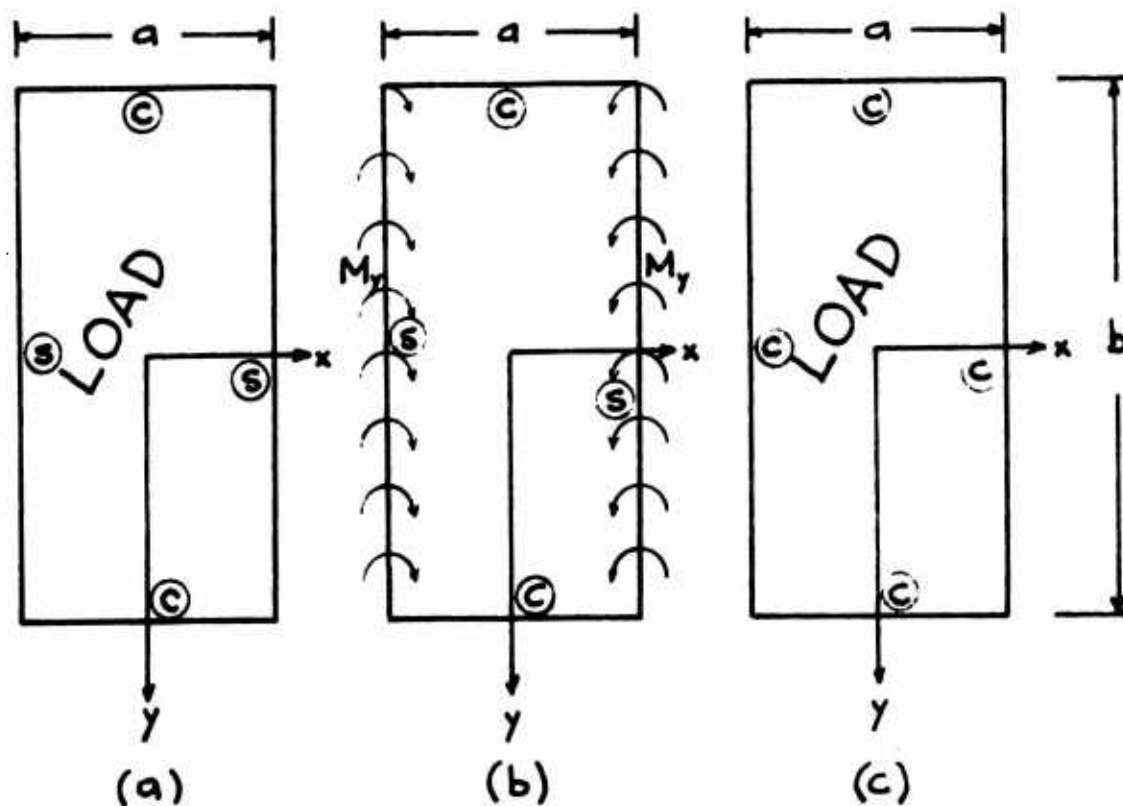
Figure 19
Clamped Plate by Superposition

The total deformation, w, is composed of two terms
corresponding to the above breakdown:

$$W = W_a + W_b \; , \qquad\qquad (118)$$

where

W_a = deflection of plate of Figure 19a under actual loading,

W_b = deflection of plate of Figure 19b.

The edge moment expression is taken as:

$$M_y = \sum_m E_m \sin \frac{m\pi y}{b} \; , \qquad\qquad (119)$$

where E_m defines its intensity.

From this, and from previous considerations, the
deformations:

$$\left. \begin{aligned} W_a &= \sum_m \left[(-1)^{\frac{m-1}{2}} \left(A_m + C_1 \cosh \frac{m\pi \lambda}{a} y + C_2 \cosh \frac{m\pi \bar{\lambda}}{a} y \right) \right. \\ &\quad \cdot \cos \frac{m\pi x}{a} \; , \end{aligned} \right\} (120)$$

$$\left. \begin{aligned} W_b &= \sum_m \left[\left(C_3 \cosh \frac{m\pi \lambda}{a} y + C_4 \cosh \frac{m\pi \bar{\lambda}}{a} y \right) \cos \frac{m\pi x}{a} \right. \\ &\quad + \left. \left(C_5 \cosh \frac{m\pi \lambda}{a} x + C_6 \cosh \frac{m\pi \bar{\lambda}}{a} x \right) \cos \frac{m\pi}{b} y \right] . \end{aligned} \right\} (121)$$

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Evaluating boundary conditions, the total deformation becomes:

$$
\begin{aligned}
W = \sum_m \Big\{ & (-1)^{\frac{m-1}{2}} A_m + \Big[(-1)^{\frac{m-1}{2}} C_1 + C_3 \Big] \cosh \frac{m\pi\lambda}{a} y \\
& + \Big[(-1)^{\frac{m-1}{2}} C_2 + C_4 \Big] \cosh \frac{m\pi\bar\lambda}{a} y \Big\} \cos \frac{m\pi x}{a} \\
& + \sum_m \Big(C_5 \cosh \frac{m\pi\lambda x}{a} + C_6 \cosh \frac{m\pi\bar\lambda}{a} y \Big) \cos \frac{m\pi y}{b} .
\end{aligned}
\tag{122}
$$

The constants of Eq. (122) have the following values:

$$
C_1 = - \frac{A_m \bar\lambda \sinh \frac{m\pi\bar\lambda}{a} y}{\bar\lambda \cosh \frac{m\pi\lambda}{a} y \sinh \frac{m\pi\lambda}{a} y - \lambda \cosh \frac{m\pi\bar\lambda}{a} y \sinh \frac{m\pi\lambda}{a} y} ,
$$

$$
C_2 = - \frac{A_m \lambda \sinh \frac{m\pi\lambda}{a} y}{\lambda \cosh \frac{m\pi\bar\lambda}{a} y \sinh \frac{m\pi\lambda}{a} y - \bar\lambda \cosh \frac{m\pi\lambda}{a} y \sinh \frac{m\pi\bar\lambda}{a} y} ,
$$

$$
\begin{aligned}
C_3 = & \frac{4ab \cosh \frac{m\pi\bar\lambda}{a} y}{\bar\lambda \cosh \frac{m\pi\lambda}{a} y \sinh \frac{m\pi\lambda}{a} y - \lambda \cosh \frac{m\pi\bar\lambda}{a} y \sinh \frac{m\pi\lambda}{a} y} \sum_n \Big\{ \Big[\frac{(-1)^{\frac{n-1}{2}}(-1)^{\frac{m-1}{2}}_m E_m}{D_2 \, n^4 \pi^5} \Big] \\
& \cdot \Big[\frac{1}{(\lambda^2-\bar\lambda^2)\big[(\frac{a}{b})^2\lambda^2+(\frac{m}{n})^2\big]} + \frac{1}{(\lambda^2-\bar\lambda^2)\big[(\frac{a}{b})^2\bar\lambda^2+(\frac{m}{n})^2\big]} \Big] \Big\} ,
\end{aligned}
$$

$$
\begin{aligned}
C_4 = & \frac{4ab \cosh \frac{m\pi\lambda}{a} y}{\lambda \cosh \frac{m\pi\bar\lambda}{a} y \sinh \frac{m\pi\lambda}{a} y - \bar\lambda \cosh \frac{m\pi\lambda}{a} y \sinh \frac{m\pi\lambda}{a} y} \sum_n \Big\{ \Big[\frac{(-1)^{\frac{n-1}{4}}(-1)^{\frac{m-1}{2}} E_m}{D_2 \, n^4 \pi^5} \Big] \\
& \cdot \Big[\frac{1}{(\lambda^2-\bar\lambda^2)\big[(\frac{a}{b})^2\lambda^2+(\frac{m}{n})^2\big]} + \frac{1}{(\lambda^2-\bar\lambda^2)\big[(\frac{a}{b})^2\bar\lambda^2+(\frac{m}{n})^2\big]} \Big] \Big\} ,
\end{aligned}
$$

$$
C_5 = \frac{b^2 E_m}{D_2 \, m^2 \pi^2 (\lambda^2-\bar\lambda^2) \cosh \frac{m\pi\lambda}{a} y} ,
$$

$$
C_6 = \frac{b^2 E_m}{D_2 \, m^2 \pi^2 (\lambda^2-\bar\lambda^2) \cosh \frac{m\pi\bar\lambda}{a} y} .
$$

$$
\left. \right\} \tag{123}
$$

The constant, A_m, is evaluated for each particular
loading condition:

a.) Uniformly distributed load (p_o):

$$A_m = \frac{4 p_o a^4}{D_x m^5 \pi^5} . \tag{124}$$

b.) Concentrated load (P - at distance ξ from left
y-edge):

$$A_m = \frac{2 P a^3}{D_x m^4 \pi^4} \sin \frac{m \pi \xi}{a} . \tag{125}$$

c.) Hydrostatic pressure (maximum p_o - varies along
x-axis):

$$A_m = \frac{2 (-1)^{m+1} p_o a^4}{D_x m^5 \pi^5} . \tag{126}$$

3.5 Summary

Most of the available solutions to the problem
of bending of rectangular orthotropic plates require
the calculation of the deformation, w, as an inter-
mediate step prior to the computation of desired
stresses. Regardless of the method used to arrive at
the deformation function, the expressions given in
Part 1 are applied in the determination of stresses,
forces, and moments.

According to Eqs. (27) and (28), the vertical
shears,

$$Q_x = -D_x \left[\frac{\partial^3 w}{\partial x^3} + \mu_{xy} \frac{\partial^3 w}{\partial x \partial y^2} \right] - D_k \frac{\partial^3 w}{\partial x \partial y^2} \ ,$$

$$Q_y = -D_y \left[\frac{\partial^3 w}{\partial y^3} + \mu_{yx} \frac{\partial^3 w}{\partial x^2 \partial y} \right] - D_k \frac{\partial^3 w}{\partial x^2 \partial y} \ .$$

The moments, from Eqs. (14), (15), and (16) are given by:

$$M_x = -\frac{t^3}{12(S_{11}S_{22} - S_{12}^2)} \left[S_{22} \frac{\partial^2 w}{\partial x^2} - S_{12} \frac{\partial^2 w}{\partial y^2} \right] \ ,$$

$$M_y = -\frac{t^3}{12(S_{11}S_{22} - S_{12}^2)} \left[S_{11} \frac{\partial^2 w}{\partial y^2} - S_{12} \frac{\partial^2 w}{\partial x^2} \right] \ ,$$

$$M_{xy} = \frac{t^3}{6 S_{66}} \frac{\partial^2 w}{\partial x \partial y} \ .$$

The bending and shear stresses, respectively, are given by Eqs. (11), (12), and (13) as:

$$\sigma_x = -\frac{z}{S_{11}S_{22} - S_{12}^2} \left[S_{22} \frac{\partial^2 w}{\partial x^2} - S_{12} \frac{\partial^2 w}{\partial y^2} \right] \ ,$$

$$\sigma_y = -\frac{z}{S_{11}S_{22} - S_{12}^2} \left[S_{11} \frac{\partial^2 w}{\partial y^2} - S_{12} \frac{\partial^2 w}{\partial x^2} \right] \ ,$$

$$\tau_{xy} = \frac{2z}{S_{66}} \frac{\partial^2 w}{\partial x \partial y} \ .$$

The meanings of the parameters used in the above expressions, and the coordinate system in which they are oriented, are given in Part 1 of this report.

PART 4 - ELASTIC STABILITY

A great deal of research has been devoted to the problem of the stability of various types of stiffened plates, with reliance upon orthotropic plate theory. More progress has been made in buckling theory than in bending theory for two reasons: First, the buckling problem of aircraft wings has been of greater interest than the bending problem, and, second, the solution of the former is considerably simpler than that of transverse bending.

Some of the authors whose work is of importance due to either original contribution or summary of available methods in the general area of buckling of orthotropic plates are the following: Barg (Ref. 5), Gerard (Ref. 235), Heck and Ebner (Ref. 30), Hu (Ref. 36), Huber (Ref. 37), Kollbrunner and Meister (Ref. 53), Lekhnitski (Ref. 58), Nowacki (Ref. 71), Pflüger (Ref. 73), Pochop (Ref. 266), Radok (Ref. 267), Sawczuk (Ref. 82), Schmit and Kicher (Ref. 317), Schultz (Ref. 83), Seide (Ref. 280), Seydel (Refs. 84 and 282), Shuleshko (Ref. 86), Sokolowski (Ref. 284), Strasser (Ref. 285), Thielemann (Ref. 95), Timoshenko (Ref. 289), Timoshenko and Gere (Ref. 96), Wilde (Ref. 105), Wittrick (Ref. 107), and Yusuff (Refs. 111 and 300).

Among the contributors to special problems are:
Feldman (Ref. 232), who presents a study of square
orthotropic plates; Caldwell (Ref. 214), who considers
the special case of a plate in edge compression with
reactions provided by edge shear; Rockey and Cook
(Refs. 269 and 270) and Symonds (Ref. 287), who solve
the shear buckling problem; Floor and Burgerhout
(Ref. 233), Hayashi (Ref. 27), and Wang and Zuckerberg
(Ref. 294), all of whom investigate the post-buckling
behavior of orthotropic plates; Klitchieff (Ref. 250),
who limits his study to plates with longitudinal
reinforcement.

The contributors to the buckling of sandwich
plates are not mentioned here. A summary of their
work is contained in the chapter concerning the behavior
of sandwich construction.

This chapter contains only the orthotropic
buckling theory. It should be noted immediately that
this theory cannot be applied indiscriminately to all
stiffened plates. For this reason, limitations are
discussed first.

4.1 Limitations

Since orthotropic plate theory is based upon
average flexural and twisting rigidities (Eqs. (20),
(21), (22)), it can be stated, in general, that the
theory can be safely applied to any plate with a large
number of equal and equidistant stiffeners parallel to
one of the edges (Ref. 96).

In addition, Gerard and Becker (Ref. 236) show
by test results that orthotropic theory may be used
in the following cases: a.) plate in compression with
three or more stiffeners; b.) plate in shear with any
number of longitudinal stiffeners; c.) any plate with
transverse stiffeners which has a low or high value of

$$\frac{EI}{bD} ,$$

where

EI = stiffness of stiffener,

b = width of plate,

D = rigidity of plate.

4.2 General Equation

With reference to the derivation of Eq. (31), if
the transverse load, p, is now replaced by in-plane
forces, N_x, N_y, and N_{xy}, the differential equation of

the deformed plate is written:

$$
\left.
\begin{aligned}
&D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} \\
&+ N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} = 0 \ .
\end{aligned}
\right\} \quad (127)
$$

Or, in terms of the stresses, the last three terms of Eq. (127) can be written:

$$
\dots + t \left[\sigma_x \frac{\partial^2 w}{\partial x^2} + 2 T_{xy} \frac{\partial^2 w}{\partial x \partial y} + \sigma_y \frac{\partial^2 w}{\partial y^2} \right], \qquad (128)
$$

where t is the thickness of the orthotropic plate, and the rigidities are defined by Eqs. (20), (21), (22), and (32).

Buckling due to the compressive loads per unit length, N_x and N_y, due to shear per unit length, N_{xy}, and due to combined loadings will be considered here.

4.3 Simple Supports

The orthotropic plate with simple supports along all edges (Fig. 20), is first considered. The following paragraphs give solutions for various loading conditions.
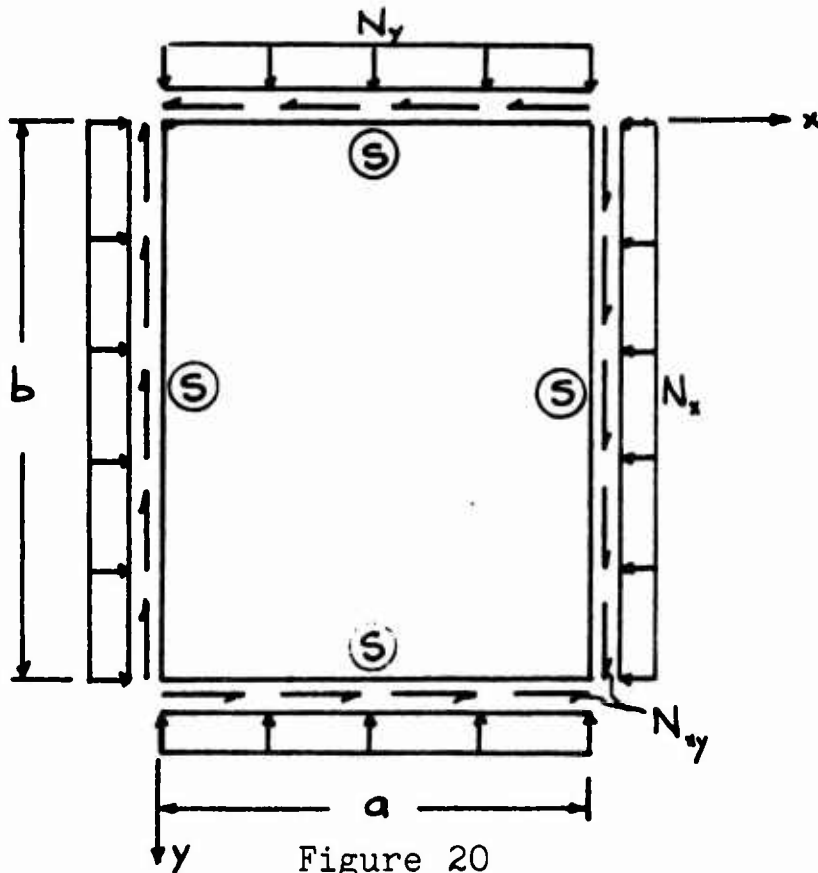
Figure 20
Simply Supported Plate

4.3.1 Uniaxial Compression

Let the plate of Figure 20 be loaded uniaxially
by the compressive load N_x alone. Then, Eq. (128)
becomes:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + t\sigma_x \frac{\partial^2 w}{\partial x^2} = 0. \quad (129)$$

The deflected surface can be taken in the form of a
double series,

$$W = \sum_m \sum_n {}' W_o \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} . \qquad (130)$$

But, it is known that the critical condition exists
when m = 1, n = 1, and the deformation,

$$W = W_o \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} . \qquad (131)$$

Substituting this into Eq. (129) and solving for the
stress,

$$\sigma_x = \frac{\pi^2}{b^2 t} \left[D_x \frac{b^2}{a^2} + 2 D_{xy} + D_y \frac{a^2}{b^2} \right] . \qquad (132)$$

The smallest value for the critical stress is obtained
when

$$\frac{a}{b} = \sqrt[4]{\frac{D_x}{D_y}} , \qquad (133)$$

and the critical buckling stress, as given by Timoshenko
and Gere (Ref. 96),

$$\sigma_{CR} = \frac{2\pi^2}{b^2 t} \left[\sqrt{D_x D_y} + D_{xy} \right] \qquad (134)$$

Lundquist and Stowell (Ref. 253) give the buckling
stress, for the condition of Eq. (133), as

$$\sigma_{CR} = \frac{\pi^2 t^2}{12 b^2} \left[2 \sqrt{D_x D_y} + \mu_{xy} D_x + \mu_{yx} D_y + 4G \right] . \qquad (135)$$

Wittrick, in Reference 107, gives an explicit solution to the buckling problem. Using Eq. (130) as the equation of the deformed plate, he takes as the solution,

$$k = \left[\frac{(a/b)_e}{m} + \frac{m}{(a/b)_e} \right]^2 , \qquad (136)$$

where

$$k = \text{buckling coefficient,}$$

$$(a/b)_e = \text{effective aspect ratio.}$$

The buckling coefficient is of the same type as that used for isotropic plates and can be applied if the effective aspect ratio,

$$\left(\frac{a}{b} \right)_e = \frac{a}{b} \sqrt[4]{\frac{D_y}{D_x}} , \qquad (137)$$

is employed.

A plot of Eq. (136) is given in Figure 21. From this, the coefficient, k, is evaluated for a given value of $(a/b)_e$. This quantity is then substituted into the expression for the critical buckling load per unit length,

$$\left(N_x\right)_{cR} = \frac{\pi^2 \sqrt{D_x D_y}}{b^2} \left[k - 2 + \frac{D_R}{\sqrt{D_x D_y}} \right] . \tag{138}$$

4.3.2 Biaxial Compression

The plate of Figure 20 is now loaded by compressive loads in both directions. Thus, Eq. (127) becomes:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2 D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} = 0. \tag{139}$$

For this loading condition, Wittrick (Ref. 107) applies the lower curve of Figure 21, but the effective aspect ratio must be modified to take into account the load N_y. Thus, the effective aspect ratio used for this case,

$$\left(\frac{a}{b}\right)_e = \frac{a}{b} \left[\frac{D_y}{D_x} \left(1 - \frac{b^2 N_y}{\pi^2 D_y} \right) \right]^{\frac{1}{4}} , \tag{140}$$

is first evaluated. The buckling coefficient, k, is found from Figure 21, and the following expression is used to complete the analysis:
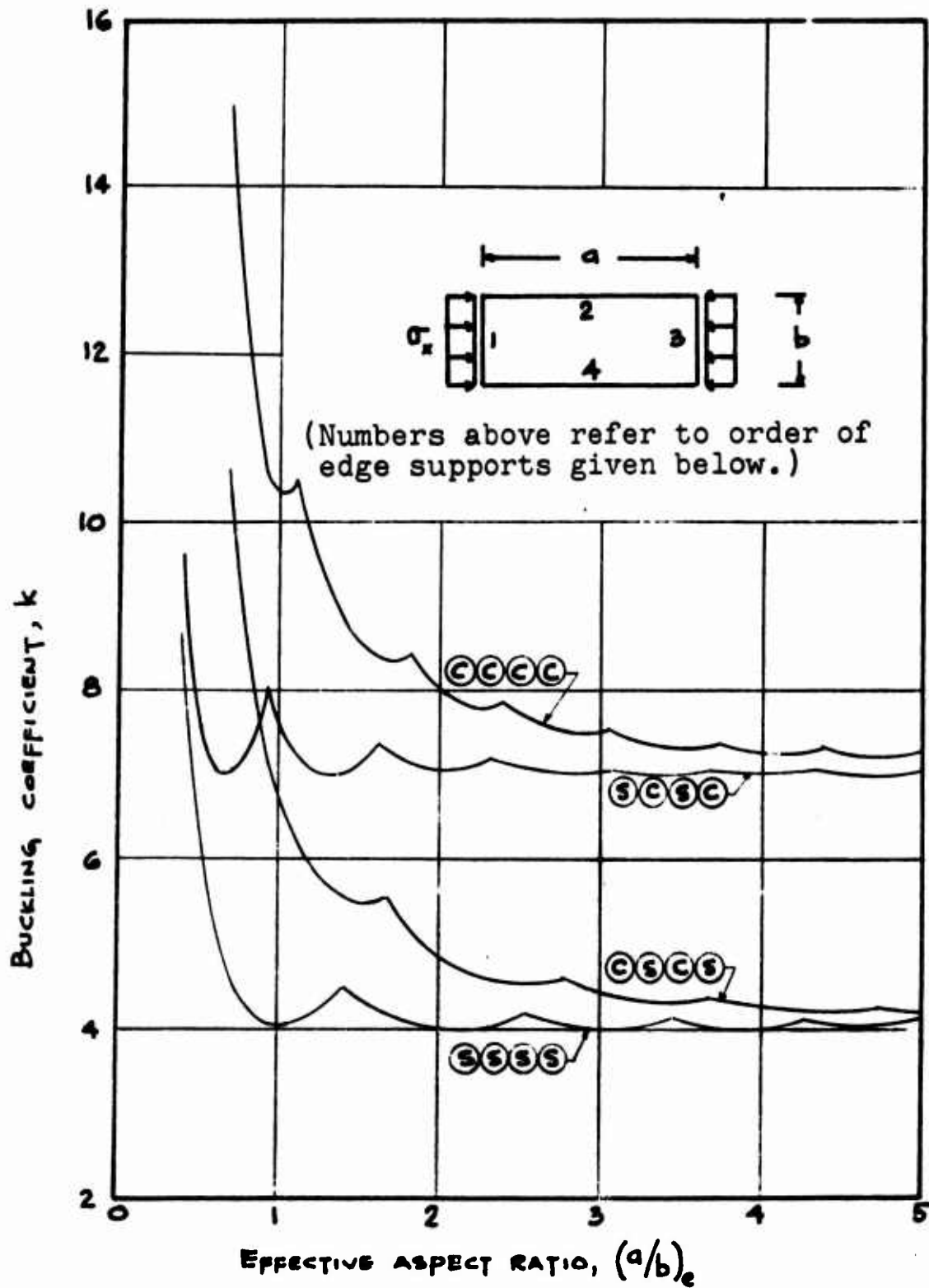
Figure 21
Buckling Coefficients for Uniaxially Loaded Plates
(from Reference 107)

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$$k = 2 + \frac{(b^2 N_x / \pi^2) - D_s}{\sqrt{D_x D_y} \left(1 - \frac{b^2 N_y}{\pi^2 D_y}\right)} \qquad . \qquad \text{(141)}$$

4.3.3 Edge Shear

When the plate shown in Figure 20 is acted upon by edge shears alone, the basic equation (127) becomes:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2 D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y} = 0 . \quad \text{(142)}$$

Seydel (Ref. 84) also makes use of the isotropic-type buckling coefficient to give the expression for the critical shear force per unit length,

$$(N_{xy})_{cr} = k_s \frac{\pi^2 \sqrt[4]{D_x D_y^3}}{b^2} \qquad . \qquad \text{(143)}$$

The shear buckling coefficient, k_s, is dependent upon the effective aspect ratio of Eq. (137) and upon the parameter $D_x / 2 \sqrt{D_x D_y}$. The coefficient is obtained from Figure 22a (which is given by Gerard and Becker in Reference 236), and it is substituted into Eq. (143).

A similar result is given by Heck and Ebner (Ref. 30) in the form:

$$(N_{xy})_{cr} = C_s \frac{\sqrt[4]{D_x D_y^3}}{\left(\frac{b}{2}\right)^2} \quad , \tag{144}$$

where C_s is a coefficient dependent on the parameters,

$$\Theta = \frac{\sqrt{D_x D_y}}{D_{xy}} \quad , \qquad \beta = \frac{1}{(a/b)_e} = \frac{b}{a}\sqrt[4]{\frac{D_x}{D_y}} \quad . \tag{145}$$

For ranges

$$0 \leq \beta \leq 1 \quad , \quad \Theta \geq 1 \; ,$$

the curves in Figure 23 apply, and C_s is found and substituted into Eq. (144).
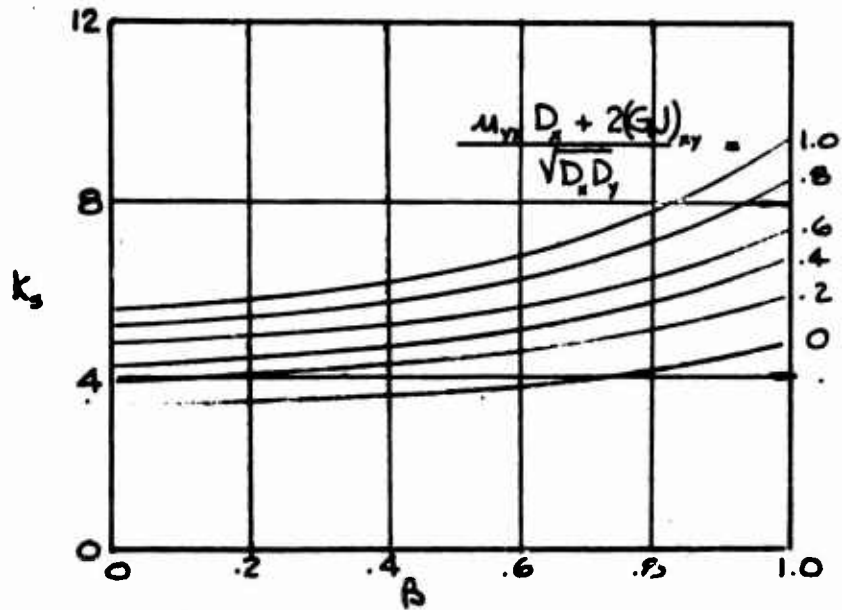
a.) Simply Supported Edges
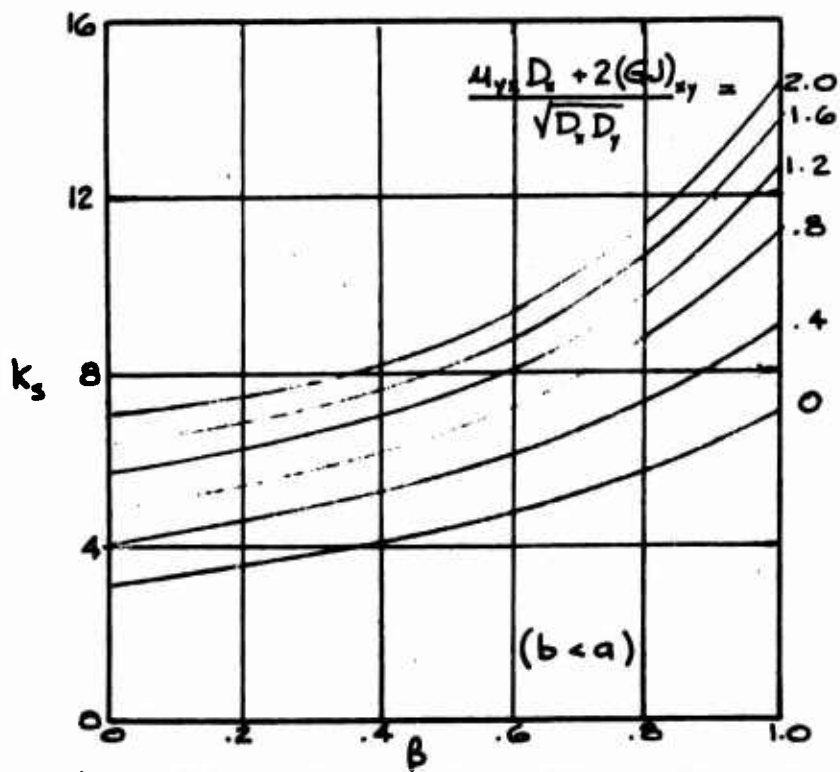
b.) x-Edges Clamped, y-Edges Hinged

Figure 22
Shear Buckling Coefficients
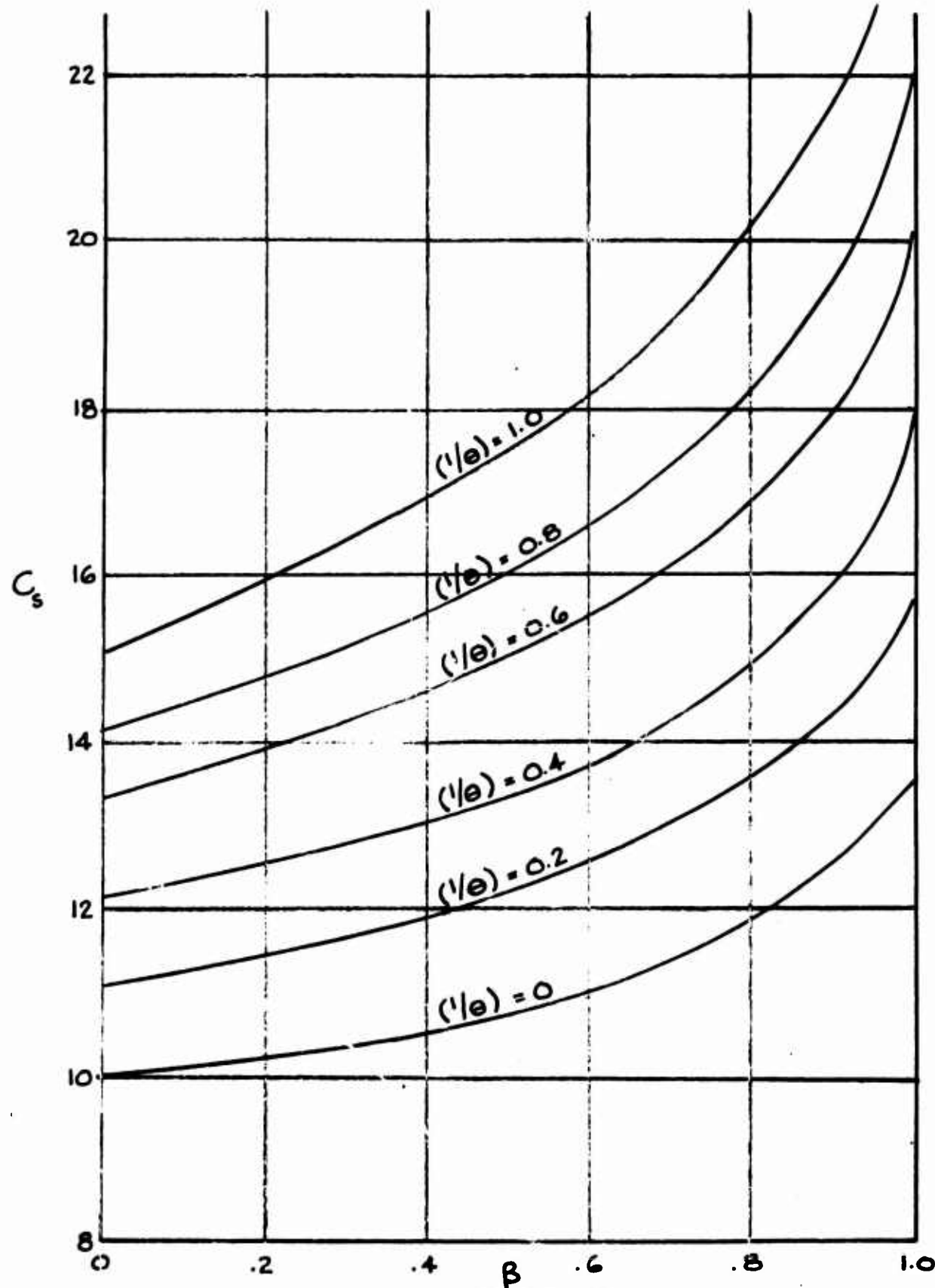
Figure 23
Shear Buckling Parameter

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4.4 Two Edges Clamped, Two Edges Simply Supported

The rectangular orthotropic plate with two
opposite edges clamped and the other two edges
simply supported (Figure 24) is now considered.
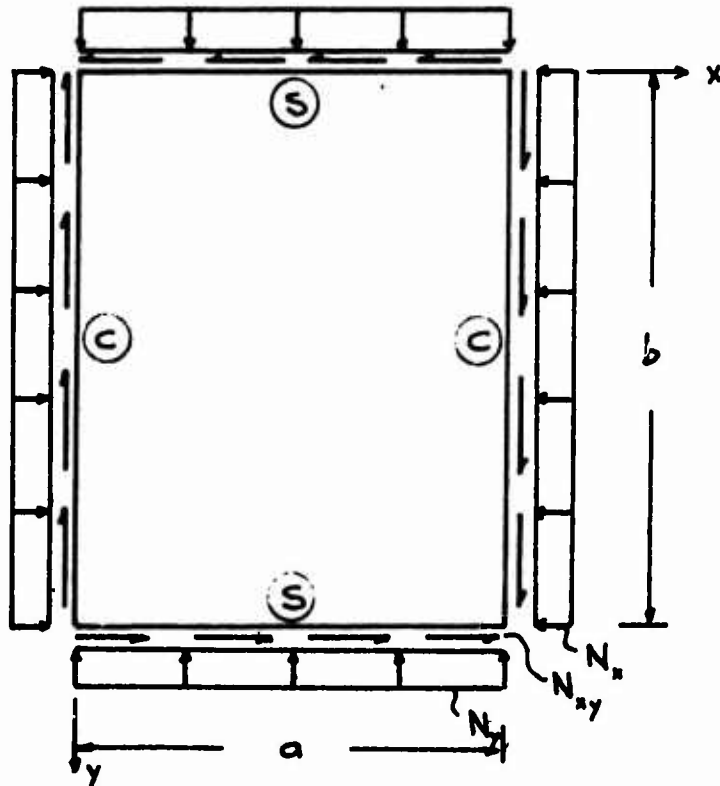
Figure 24
Buckling of Ⓒ - Ⓢ - Ⓒ - Ⓢ Plate

4.4.1 Clamped Edges Loaded; Others Unloaded

When N_x acts, and $N_y = N_{xy} = 0$, on the plate of
Figure 24, Wittrick (Ref. 107) gives the solution as:

$$(N_x)_{cr} = \frac{\pi^2 \sqrt{D_x D_y}}{b^2} \left[k - 2 + \frac{D_{xy}}{\sqrt{D_x D_y}} \right], \qquad (146)$$

where **k** is the coefficient from Figure 21 for a particular value of $(a/b)_e$ given by Eq. (137).

4.4.2 Simply Supported Edges Loaded; Others Unloaded

When N_y acts, and $N_x = N_{xy} = 0$, on the plate of Figure 24, the solution of Wittrick is given in the form:

$$(N_{xy})_{cr} = \frac{\pi^2 \sqrt{D_x D_y}}{b^2} \left[k - 2.4 + \frac{1.2 D_{xy}}{\sqrt{D_x D_y}} \right], \qquad (147)$$

and **k** is available from Figure 21 for the effective aspect ratio given by Eq. (137).

Heck and Ebner (Ref. 30) give as the solution for the case of $D_x > D_y$,

$$(N_{xy})_{cr} = \frac{2\pi^2}{b^2} \left[2D_x \frac{b^2}{a^2} + D_{xy} \right]. \qquad (148)$$

Lundquist and Stowell (Ref. 253) show that the critical buckling stress for this case is

$$\sigma_{cr} = \frac{\pi^2 t^2}{12 b^2}\left[4.554\sqrt{D_x D_y} + 1.237(\mu_{xy} D_x + \mu_{yx} D_y) + 4.943 G\right] ,\qquad (149)$$

for the minimum condition,

$$\frac{a}{b} = .664 \sqrt[4]{\frac{D_x}{D_y}} .$$

4.4.3 Biaxial Compression

When N_x and N_y act on the plate of Figure 24 and $N_{xy} = 0$, Wittrick uses the curves of Figure 21, but the aspect ratio given by Eq. (140) must be applied. The solution is obtained from Eq. (141).

4.4.4 Edge Shear

As in paragraph 4.3.3, the critical shear force per unit length for the plate of Figure 24, when $N_x = N_y = 0$, is given by

$$\boxed{(N_{xy})_{cr} = k_s \frac{\pi^2 \sqrt[4]{D_x D_y^3}}{b^2} ,}$$

where the shear buckling coefficient is available from Figure 22b (given by Timoshenko and Woinowsky-Krieger in Reference (97) for plates whose edges obey the inequality,

$$a > b.$$

4.5 All Edges Clamped

A plate whose four edges are clamped (Figure 25) is considered.
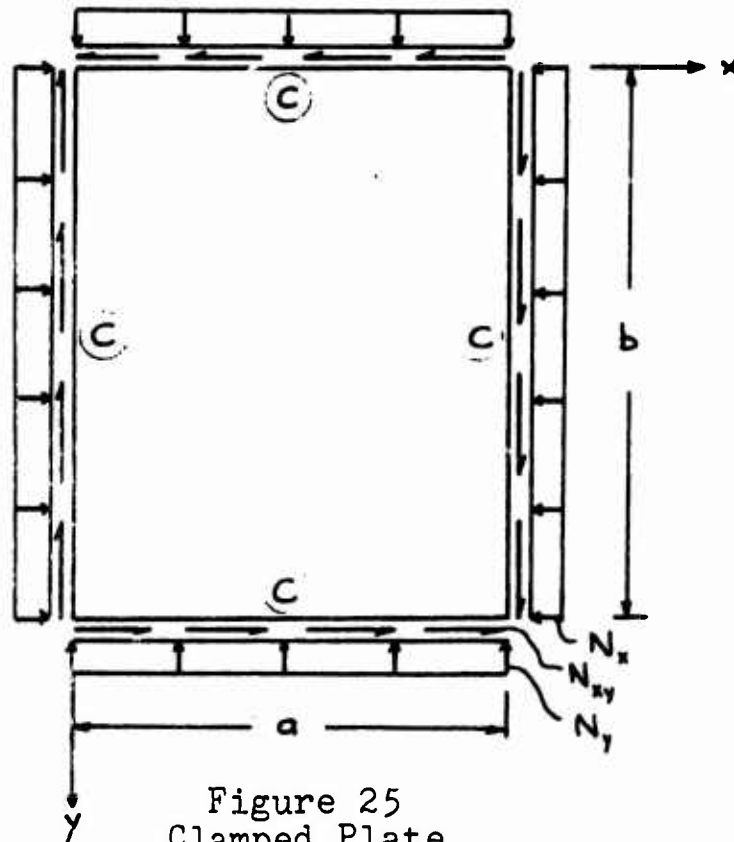
Figure 25
Clamped Plate

4.5.1 Uniaxial Compression

When N_x is acting and $N_y = N_{xy} = 0$, the critical load on the plate of Figure 25 is given by Wittrick (Ref. 107) as

$$\boxed{(N_{xy})_{cR} = \frac{\pi^2 \sqrt{D_x D_y}}{b^2} \left[k - 2.46 + \frac{1.23 \, D_{\ell}}{\sqrt{D_x D_y}} \right] \,,}$$ (150)

and the buckling coefficient is given by Figure 21 for the effective aspect ratio,

$$\left(\frac{a}{b}\right)_e = \frac{a}{b} \sqrt[4]{\frac{D_y}{D_x}} \,.$$

4.5.2 Biaxial Compression

Timoshenko and Gere (Ref. 96) solve the problem of a biaxially compressed isotropic plate with clamped edges, assuming the deformation function as

$$w = \frac{\delta}{4} \left(1 - \cos \frac{2\pi x}{a}\right)\left(1 - \cos \frac{2\pi y}{b}\right) \,,$$ (151)

and using strain energy to calculate the critical buckling stress. A similar analysis can be performed for the orthotropic plate.

4.6 Other Buckling Cases

Several other cases of loadings and edge conditions are considered in this section.

4.6.1 Combined Shear and Compression

Sandorff (Ref. 315) investigates the buckling of a plate under the action of a compressive load and shear (Figure 26).
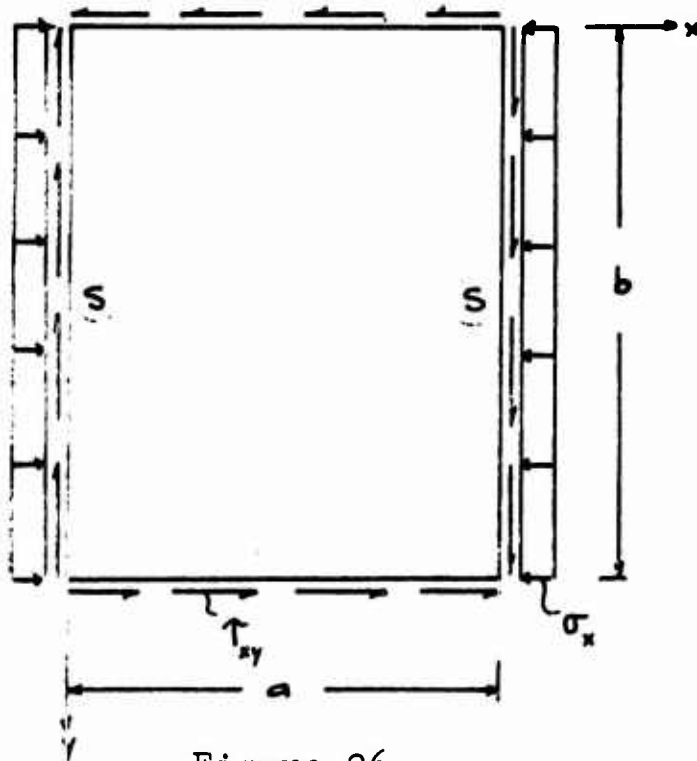
Figure 26
Combined Loading

The assumption is made that b is infinite and an interaction curve (Figure 27) is presented. The parameters of this curve are stress ratios. That is,

$$R_c = \frac{(\sigma_x)_{\text{APPLIED}}}{(\sigma_x)_{\text{ALLOWABLE}}} \quad , \qquad R_s = \frac{(\tau_{xy})_{\text{APPLIED}}}{(\tau_{xy})_{\text{ALLOWABLE}}} \quad . \qquad (152)$$
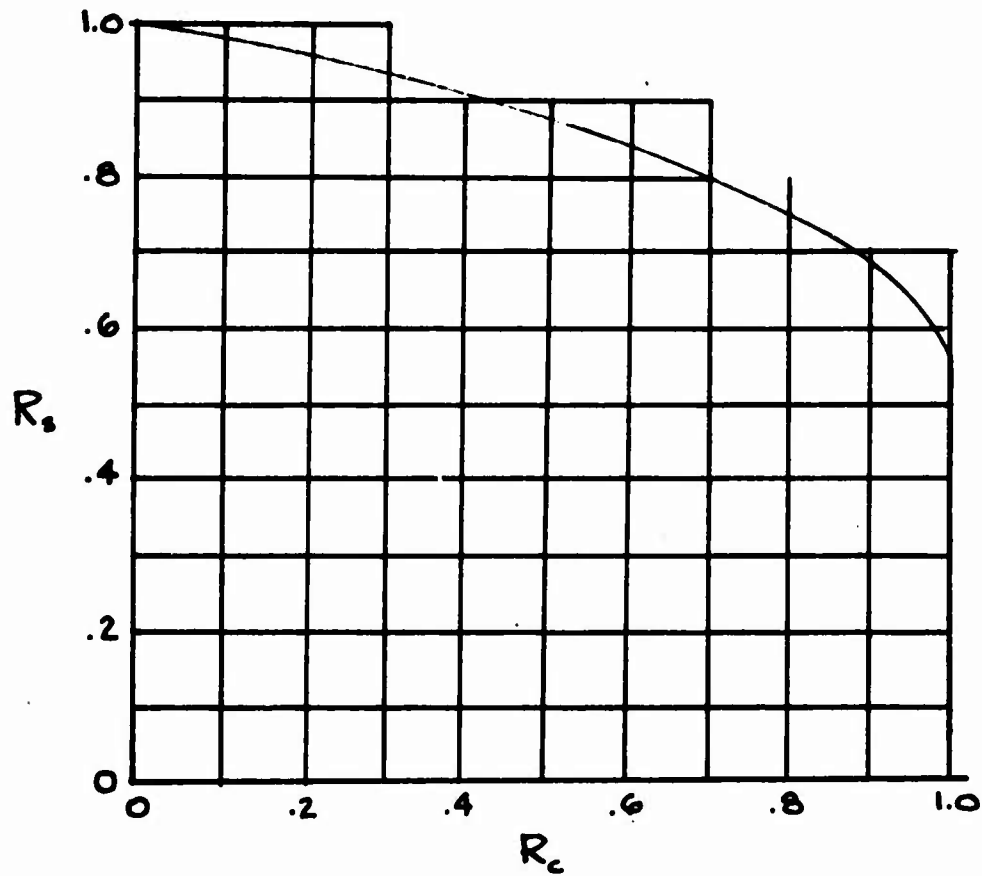
Figure 27
Interaction—Shear and Compression

4.6.2 Loaded Edges Simply Supported; Other Edges Clamped and Free

An implicit solution for the case shown in Figure 28 is presented by Lundquist and Stowell (Ref. 253) and discussed by the writer (Ref. 309).
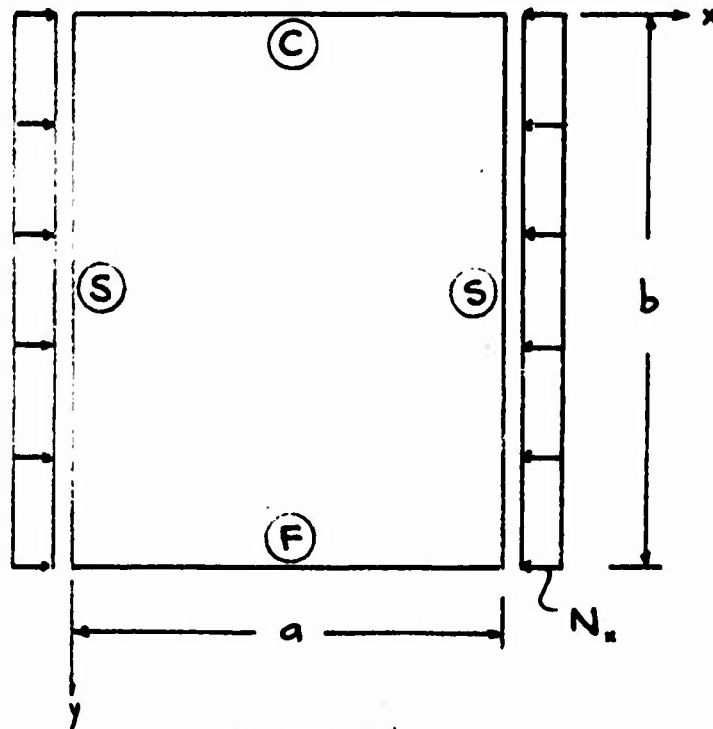
Figure 28
Buckling of (S) - (C) - (S) - (F) Plate

The critical buckling stress is given by:

$$\sigma_{cx} = \left(\frac{t}{b}\right)^2 \left[.769 \sqrt{D_x D_y} - .270 (\mu_{xy} D_x + \mu_{yx} D_y) + 1.712 G \right], \qquad (153)$$

for the condition

$$\frac{a}{b} = 1.46 \sqrt[4]{\frac{D_x}{D_y}} \quad .$$

4.6.3 Three Edges Simply Supported; One Edge Free

When the clamped edge of Figure 28 is replaced
by a simply supported edge, the critical buckling stress
of Eq. (154) simplifies to:

$$\sigma_{cr} = \left(\frac{t}{b}\right)^2 \left[\frac{\pi D_x}{12}\left(\frac{b}{a}\right)^2 + G\right] , \qquad (154)$$

for the same limiting condition.

PART 5 - SANDWICH PLATES

The name "sandwich plate" describes a composite plate which consists of two thin sheets--faces-- and a relatively thick core. The faces are generally of aluminum alloy or fiberglass, while the core can be a light solid material such as plastic and wood or a thin metal sheet formed into a cellular configuration ("honeycomb"). The basic principle of sandwich construction is the reliance upon the outer faces to carry end loads and upon the core to transmit shear and keep the faces a constant distance apart.

Ordinarily, the modulus of elasticity of the core material is extremely small (about 0.1%) in comparison to that of the faces. Thus, the core provides very little resistance to bending. But, because its modulus of rigidity is also very low, the core experiences appreciable shearing deformations. Thus, shearing deformations must be considered in the analysis. In addition, since the faces or the core, or both, may have orthogonally anisotropic stretching properties, the composite plate may well be orthogonally anisotropic in its bending properties. Thus, in many cases, the theory

which should be used to analyze sandwich plates is orthotropic plate theory, with consideration of shearing deformations (see Chapter 1).

The amount of research performed in the field of sandwich construction is overwhelming. For this reason, the bibliography of this report, pertaining to this area, is by no means complete. For specialized and more detailed bibliographies of sandwich-plate literature, see References 131, 135, and 137.

Probably the most important of the papers regarding the theory of sandwich plates are those of Reissner (Ref. 169), Hoff (Refs. 139, 140), and Libove and Batdorf (Ref. 155). Other authors who have made significant contributions in the investigation of the bending and the buckling of sandwich plates are: Aleksandrova (Ref. 113), Anderson (Ref. 116), Anderson and Updegraff (Ref. 117), Bijlaard (Ref. 120), Cheng (Ref. 125), Dundrova (Ref. 127), Eringen (Ref. 128), Goodier (Ref. 132), Guest and Solvey (Ref. 136), Hoff and Mautner (Ref. 142), Hopkins and Pearson (Ref. 143), Horvay (Ref. 144), Hubka, Dow and Seide (Ref. 145), Kimel (Ref. 149), Lewis (Ref. 154), Libove and Hubka

(Ref. 156), March and Smith (Ref. 161), Mushtari (Ref. 163), Norris (Ref. 164), Raville (Ref. 168), Seide (Refs. 171, and 172), Seide and Stowell (Ref. 173), Thurston (Ref. 176), Weikel and Kobayashi (Ref. 178), Williams (Ref. 181), Yen, Salerno and Hoff (Ref. 184), Yusuff (Refs. 190, 191), and Zahn and Cheng (Ref. 192).

Burns (Ref. 123) and Burns and Skogh (Refs. 212, 213) present minimum weight analyses of sandwich plates. Kuenzi (Ref. 152) presents design criteria. Keer and Lazan (Ref. 148), Mindlin (Ref. 162), and Yu (Refs. 185-189) consider dynamic behavior. Manufacture of sandwich plates is discussed by Lowy and Jaffee (Ref. 157). References 126 and 170 are of considerable value to the designer as handbooks.

This chapter is intended as a brief summary of some of the highlights of the small- and large-deflection theories and local and gross instability of sandwich plates from several of the papers listed in Section B of the Bibliography.

5.1 Flexure

The analysis of sandwich plates in bending
follows essentially the same procedure as that discussed
in Part 3. However, as was stated previously, the
effect of transverse shear deformations must be incor-
porated into the analysis.

5.1.1 Small Deflections

Williams (Ref. 181), Hopkins and Pearson (Ref.
143), and March (Refs. 158-161) were among the first
investigators to ac.ount for shear deformations by
assuming that two adjacent vertical strips remain
vertical after surface displacement, and that it is
the shear between the strips which limits the pene-
tration of the sinusoidal face displacement of the
interface of core and facing. Thus, the vertical dis-
placement of the face is taken in the form,

$$v_f = A_0 \sin \frac{m\pi x}{L} , \qquad (155)$$

while the displacement of the core in the same direction
is given by

$$v_c = v_f e^{-k\theta} . \qquad (156)$$

The constant, k, is given by

$$k = \frac{(G_{xz})_c}{(E_z)_c} \left(\frac{m\pi}{L}\right)^2,$$
(157)

where

c = subscript referring to core,

E_z = modulus of elasticity in z-direction,

G_{xz} = shear modulus in xz-plane,

L = panel length,

m = number of half-waves in x-direction,

x = longitudinal coordinate,

z = coordinate through thickness.

More general and rigorous treatments are given by Libove and Batdorf (Ref. 155) and Reissner (Ref. 169). Most of the work which follows chronologically is based on the principles cited in these papers.

The Libove-Batdorf paper (see Paragraph 1.8) develops a theory which applies to orthotropic or isotropic plates with homogeneous or nonhomogeneous cores. The expressions derived therein are Eqs. (45), (46) and (47). Assuming that N_x, N_y, and N_{xy} are

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constant throughout the plate, these equations are
solved for the deflection and the shears:

$$
\boxed{
\begin{aligned}
[D]\, w &= -[M]\, p, \\
[D]\, Q_x &= -[N]\, p, \\
[D]\, Q_y &= -[P]\, p.
\end{aligned}
}
$$

(158)

and the differential operators are defined in the
following manner:

$$
\begin{aligned}
[D] =&\; \frac{D_R D_1}{2 D_{e_y}} \cdot \frac{\partial^6}{\partial x^6}
+ \left[\frac{D_R D_1}{2 D_{e_x}} + \frac{D_1 D_3 - \frac{D_R D_2 v_{xy}}{2} - \frac{D_R D_2 v_{yx}}{2}}{D_{e_y}} \right] \frac{\partial^6}{\partial x^4 \partial y^2} \\
&+ \left[\frac{D_R D_3}{2 D_{e_y}} + \frac{D_1 D_3 - \frac{D_R D_2 v_{xy}}{2} - \frac{D_R D_2 v_{yx}}{2}}{D_{e_x}} \right] \cdot \frac{\partial^6}{\partial x^2 \partial y^4} \\
&+ \frac{D_R D_3}{2 D_{e_x}} \cdot \frac{\partial^6}{\partial y^6} - D_1 \frac{\partial^4}{\partial x^4} - \left[2 D_R (1 - v_{xy} v_{yx}) + D_1 v_{xy} + D_3 v_{yx} \right] \\
&\cdot \frac{\partial^4}{\partial x^2 \partial y^2} - D_3 \frac{\partial^4}{\partial y^4} + \frac{D_R D_1}{2 D_{e_x} D_{e_y}} \left[N_x \frac{\partial^6}{\partial x^6} + N_y \frac{\partial^6}{\partial x^4 \partial y^2} + 2 N_{xy} \frac{\partial^6}{\partial x^5 \partial y} \right] \\
&+ \left[\frac{D_1 D_3 - \frac{D_R D_2 v_{xy}}{2} - \frac{D_R D_2 v_{yx}}{2}}{D_{e_x} D_{e_y}} \right] \left[N_x \frac{\partial^6}{\partial x^4 \partial y^2} + N_y \frac{\partial^6}{\partial x^2 \partial y^4} + 2 N_{xy} \frac{\partial^6}{\partial x^3 \partial y^3} \right] \\
&+ \frac{D_R D_3}{2 D_{e_x} D_{e_y}} \left[N_x \frac{\partial^6}{\partial x^2 \partial y^4} + N_y \frac{\partial^6}{\partial y^6} + 2 N_{xy} \frac{\partial^6}{\partial x \partial y^5} \right] \\
&- \left[\frac{D_R (1 - v_{xy} v_{yx})}{2 D_{e_y}} + \frac{D_1}{D_{e_x}} \right] \left[N_x \frac{\partial^4}{\partial x^4} + N_y \frac{\partial^4}{\partial x^2 \partial y^2} + 2 N_{xy} \frac{\partial^4}{\partial x^3 \partial y} \right] \\
&- \left[\frac{D_R (1 - v_{xy} v_{yx})}{2 D_{e_x}} + \frac{D_3}{D_{e_y}} \right] \left[N_x \frac{\partial^4}{\partial x^2 \partial y^2} + N_y \frac{\partial^4}{\partial y^4} + 2 N_{xy} \frac{\partial^4}{\partial x \partial y^3} \right] \\
&+ (1 - v_{xy} v_{yx}) \left[N_x \frac{\partial^2}{\partial x^2} + N_y \frac{\partial^2}{\partial y^2} + 2 N_{xy} \frac{\partial^2}{\partial x \partial y} \right].
\end{aligned}
$$

$$[M] = \frac{D_a D_1}{2 D_{q_x} D_{q_y}} \cdot \frac{\partial^4}{\partial x^4} + \left[\frac{D_1 D_3 - \frac{D_a D_1 v_{xy}}{2} - \frac{D_a D_3 v_{yx}}{2}}{D_{q_x} D_{q_y}}\right]\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{D_a D_3}{2 D_{q_x} D_{q_y}}\cdot\frac{\partial^4}{\partial y^4}$$

$$-\left[\frac{D_a(1-v_{xy}v_{yx})}{2 D_{q_x}} + \frac{D_3}{D_{q_y}}\right]\frac{\partial^2}{\partial y^2} - \left[\frac{D_a(1-v_{xy}v_{yx})}{2 D_{q_y}} + \frac{D_1}{D_{q_x}}\right]\frac{\partial^2}{\partial x^2} + (1 - v_{xy}v_{yx}),$$

$$[N] = \frac{D_a D_1}{2 D_{q_y}}\cdot\frac{\partial^5}{\partial x^5} + \left[\frac{D_1 D_3 - \frac{D_a D_1 v_{xy}}{2} - \frac{D_a D_3 v_{yx}}{2}}{D_{q_y}}\right]\frac{\partial^5}{\partial x^3 \partial y^2} + \frac{D_a D_3}{2 D_{q_y}}\cdot\frac{\partial^5}{\partial x \partial y^4}$$

$$- D_1 \frac{\partial^3}{\partial x^3} - \left[D_a(1-v_{xy}v_{yx}) + D_1 v_{xy}\right]\frac{\partial^3}{\partial x \partial y^2},$$

$$[P] = \frac{D_a D_3}{2 D_{q_x}}\cdot\frac{\partial^5}{\partial x^4 \partial y} + \left[\frac{D_1 D_3 - \frac{D_a D_1 v_{xy}}{2} - \frac{D_a D_3 v_{yx}}{2}}{D_{q_x}}\right]\frac{\partial^5}{\partial x^2 \partial y^3} + \frac{D_a D_3}{2 D_{q_x}}\cdot\frac{\partial^5}{\partial y^5}$$

$$- D_3 \frac{\partial^3}{\partial y^3} - \left[D_a(1-v_{xy}v_{yx}) + D_3 v_{yx}\right]\frac{\partial^3}{\partial x^2 \partial y}.$$

The elastic constants which appear in the above equations are defined by Eqs. (20), (21), (22), (38), (39), and (40). A particular problem, thus, is solved by using Eqs. (158) for a given set of boundary conditions. At a free y-edge:

$$M_x = M_{xy} = Q_x = 0. \tag{159}$$

For a simply supported y-edge, with all points[*] along boundary prevented from moving parallel to edge:

$$w = M_y = \frac{Q_y}{D_{Q_y}} = 0. \tag{160}$$

For a simply supported y-edge, with all points free to move parallel to edge:

$$w = M_x = M_{xy} = 0. \tag{161}$$

For a clamped y-edge, with movement parallel to edge prevented:

$$w = \left(\frac{\partial w}{\partial x} - \frac{Q_x}{D_{Q_x}} \right) = \frac{Q_y}{D_{Q_y}} = 0. \tag{162}$$

For a clamped y-edge, with all points[*] free to move parallel to edge:

$$w = \left(\frac{\partial w}{\partial x} - \frac{Q_x}{D_{Q_x}} \right) = M_{xy} = 0. \tag{163}$$

Particular results for the bending of sandwich plates under uniform and concentrated loads are analyzed by Yen, Gunturkun, and Pohle (Ref. 183). However, both the core and the faces are considered isotropic and, therefore, will not be discussed here.

---

[*]Except those in middle surface.

The same is true of Kuenzi's (Ref. 152) report on design criteria.

Raville (Ref. 168) presents a study of simply supported rectangular sandwich plates, with isotropic facings and orthotropic core, acted upon by a uniformly distributed load, p. The expression for the maximum deflection (at the center of the plate) is given by a double Fourier series:

$$W_{MAX} = \frac{16\,p\,a^4(1-\mu^2)}{\pi^6 EI} \sum_m \sum_n (-1)^{\frac{m+n}{2}-1}$$
$$\cdot \left\{ \frac{1+[m^2+(\frac{1-\mu}{2})n^2\rho^2]S_x+[(\frac{1-\mu}{2})m^2+n^2\rho^2]S_y+[(\frac{1-\mu}{2})(m^2+n^2\rho^2)^2]S_xS_y}{mn\,(m^2+n^2\rho^2)^2\,[1+(\frac{1-\mu}{2})(m^2S_y+n^2\rho^2S_x)]} \right\} \tag{164}$$

where

$$S_x = \frac{\pi^2 E_c\, t_1 t_2}{G_{xz}\,a^2\,(1-\mu^2)(t_1+t_2)} \quad ,$$

$$S_y = \frac{\pi^2 E_c\, t_1 t_2}{G_{yz}\,a^2\,(1-\mu^2)(t_1+t_2)} \quad ,$$

$$I = \frac{t_1 t_2}{t_1+t_2}\left(c + \frac{t_1+t_2}{2}\right)^2 \quad ,$$

$$m, n = \text{odd integers},$$

$$E \quad = \text{Young's modulus of facings},$$

$$\mu \quad = \text{Poisson's ratio of facings},$$

$$\rho \quad = a/b,$$

while a,b and the coordinate axes are consistent
with their definitions throughout this report.

In further refinements of small deflection
theory, Hoff (Ref. 139) presents a variational
approach which is also presented, in a generalized
fashion, by Eringen (Ref. 128). Both papers are
limited to sandwiches consisting of facings and core
with isotropic properties.

Cheng (Ref. 125) considers the problem of bend-
ing of a plate with isotropic facings, but orthotropic
core. The basic equation for bending is written:

$$\left[1 - D_{yy} \frac{\partial^2}{\partial x^2} - D_{xx} \frac{\partial^2}{\partial y^2} \right] \Delta \Delta w$$

$$= \frac{1}{D} \left[\left(1 - D_2 \frac{\partial^2}{\partial x^2} - D_1 \frac{\partial^2}{\partial y^2} \right) P + \frac{2 D_{xx} D_{yy}}{1 - \mu} \Delta \Delta P \right], \qquad (165)$$

in which

$$D = \frac{Et(h+t)^2}{2(1-\mu^2)} \quad ,$$

$$D_{xx} = \frac{(1-\mu)D}{2hG_{xz}} \quad ,$$

$$D_{yy} = \frac{(1-\mu)D}{2hG_{yz}} \quad ,$$

$$D_1 = D_{xx} + \frac{2D_{yy}}{1-\mu} \quad ,$$

$$D_2 = D_{yy} + \frac{2D_{xz}}{1-\mu} \quad ,$$

$h =$ thickness of core,

$t =$ thickness of facings,

$E =$ Young's modulus of facings,

$\mu =$ Poisson's ratio of facings,

$$\Delta = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \quad .$$

The homogeneous solution of Eq. (165) satisfies

$$\left[1 - D_{yy}\frac{\partial^2}{\partial x^2} - D_{xx}\frac{\partial^2}{\partial y^2}\right]\Delta\Delta w = 0 \quad . \qquad (166)$$

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This form is used to solve the problem of a simply supported plate under a uniformly distributed load (Raville's solution is given elsewhere in this section). Since the load intensity, p, is constant, the right hand side of Eq. (165) reduces to p/D, and its particular solution is taken in the form:

$$
\left.
\begin{aligned}
W_1 &= \frac{P}{24D}\left(x^4 - 2ax^3 + a^3 x\right) \\
&= \frac{4pa^4}{\pi^5 D}\sum_m \frac{1}{m^5}\, \sin\frac{m\pi x}{a}\,.
\end{aligned}
\right\} \quad (167)
$$

The maximum deflection, at the center of the plate, is then given by:

$$
\boxed{\;W_{max} = \frac{4pa^4}{\pi^5 D}\sum_m \frac{(-1)^{\frac{m-1}{2}}}{m^5}\left[1 - \frac{2 + \alpha_m \phi_m \tanh\alpha_m}{2\cosh\alpha_m}\right],\;}
\qquad (168)
$$

where

$$
\alpha_m = \frac{m\pi b}{2a}\,,
$$

$$
\phi_m = \frac{\left[2\mu D_{xx} - (1+\mu)D_{yy}\right]\alpha_m^2 + \dfrac{\mu D_{xx}\alpha_m^2}{(1-\mu)(b/2)^2}}{(1+\mu)(D_{xx}-D_{yy})\alpha_m^2 + \dfrac{(D_{yy}-\mu D_{xx})\alpha_m^2}{(1-\mu)(b/2)^2}}\,.
$$

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5.1.2 Large Deflections

Generally speaking, the study of large deflections
of sandwich plates is an extension, through the
inclusion of shear effects, of von Karman's nonlinear
theory of elasticity. Such a study is quite important
because, since the advent of high strength alloy steels
and titanium alloys, it is possible to construct a
sandwich consisting of a soft core and extremely thin
facings.

Reissner (Ref. 169) considers a plate with
isotropic facings and core, and uses the simplifying
assumption that stresses in the core, whose vectors lie
in the xy-plane (see Fig. 1), and the variation of
face stresses over the thickness of the faces, can both
be neglected. In other words, the face sheets act
as membranes, while the core resists transverse shear
and normal stress. Introducing an Airy stress function,
F, two simultaneous differential equations are written
to describe the behavior of an isotropic sandwich plate:

$$\Delta\Delta F = 2tE\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right)\right], \qquad (169)$$

The nonlinear theory of a symmetrically loaded sandwich beam is considered by Zahm and Cheng in Reference 192, based on the assumptions that the exact load distribution is known and can be represented by a Fourier series and that the core material is anti-plane.

5.2 Elastic Stability

Buckling of sandwich plates is divided here into two parts, local instability and general (gross) buckling.

5.2.1 Local Buckling

5.2.1.1 Face Wrinkling

Several investigators consider local crinkling of the faces in a plate consisting of isotropic facings and an orthotropic core. Among those who originated such studies are Williams (Ref. 181), Hoff and Mautner (Refs. 141, 142), and Goodier (Ref. 132). A relatively simple formula for the critical compressive load (applied in x-direction) per unit

width of panel is given by Williams:

$$(N_x)_{cr} = 1.7t \left[1 + \frac{h(E_s)_c}{2tE}\right] \sqrt[3]{G_{xz}(E_s)_c E}$$
$$\cdot \left[\frac{1}{1 + .29\left[\frac{G_{xz}}{E}\right]^{2/3} \left[\frac{(E_s)_c}{G_{xz}}\right]^{1/6} \frac{(E_z)_c}{G_{xz}}}\right] ,$$

(172)

where the subscript, c, denotes "core", the coordinates
are those of Figure 1, and all other symbols are as
defined in Part 5.1.

Goodier and Hsu (Ref. 133) investigate face
wrinkling in a non-sinusoidal mode and derive a lower
critical load. Yusuff (Refs. 190, 191) considers
initial imperfections and, for the plate shown in
Figure 29, derives equations for critical stress, based
on local wrinkling, of a plate with isotropic faces
and core.
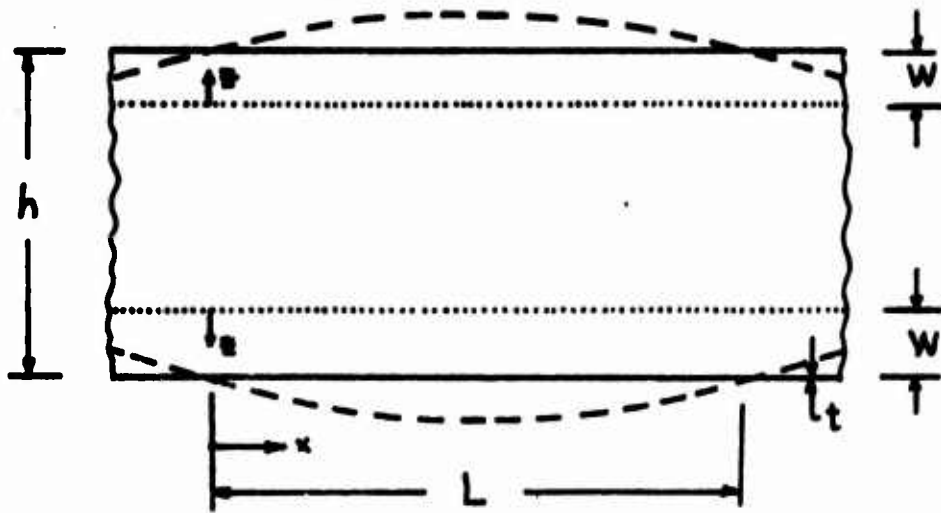
Figure 29

Wrinkling of Facing

W is the "zone of displacement",

$$W = .72t \sqrt[3]{\frac{E E_c}{G_{as}}} .$$
(173)

For a failure of the core by tension or compression,
the critical stress,

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$$\sigma_{cr} = \frac{\sqrt{\dfrac{2EE_c t}{3h}}}{1 + \dfrac{2E_c}{t}\dfrac{A_o}{F}} \quad , \qquad (t < 2W)$$

$$\sigma_{cr} = \frac{.96\sqrt[3]{EE_c G_{xz}}}{1 + \dfrac{E_c}{W}\dfrac{A_o}{F}} \quad . \qquad (t > 2W)$$

(174)

For core failure due to shear,

$$\sigma_{cr} = \frac{.96\sqrt[3]{EE_c G_{xz}}}{1 + \dfrac{\pi G_{xz}}{L_{cr}}\dfrac{A_o}{F}} \quad .$$

(175)

The ratio, A_o/F, is known as the "waviness parameter"
and is given in Figure 30. The curve given in this
figure has been confirmed by various tests, among
them those of Harris and Crisman (Ref. 138).
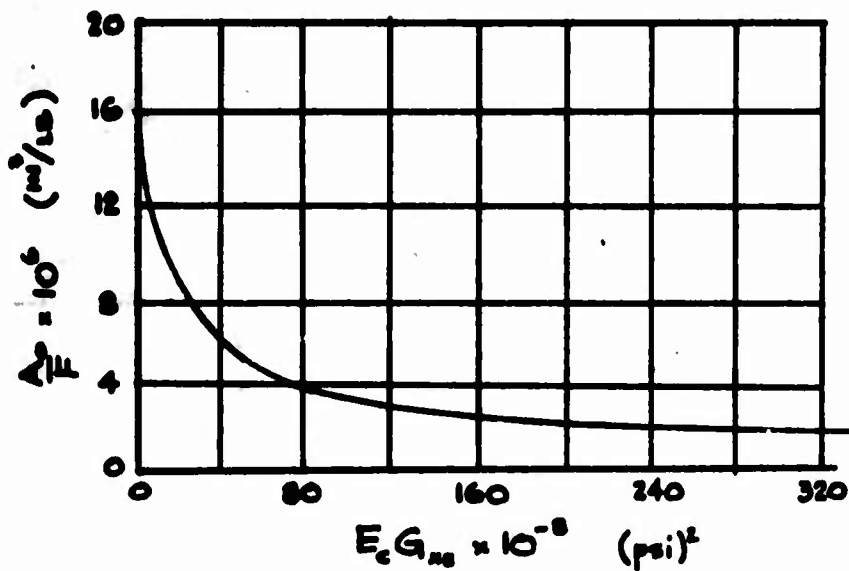
Figure 30
Waviness Parameter versus Core Moduli

5.2.1.2 Intracellular Buckling

A second form of local instability is that of
"dimpling" or intracellular buckling. This failure, of
course, is dependent upon the core configuration.

Anderson and Updegraff (Ref. 117) investigated
the buckling of truss-core sandwich panels experimentally.

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$$D \Delta\Delta w = \left[1 - \frac{thE}{2(1-\mu^2)G_c}\Delta\right]\left[P + \frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2}\right.$$

$$\left. - 2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right], \tag{170}$$

where G_c is the shear modulus of the core and the definitions of all other terms are given in Eq. (165).

These equations (127 and 128), together with the boundary conditions, determine the functions F and w. For the solution of such differential equations, the reader should refer to page 425 of Reference 96.

The large deflections of a sandwich plate with orthotropic core between isotropic face sheets are considered by Alwan (Refs. 114, 115). Here, the solution is given by two differential equations, the first of which is Eq. (169) and the second,

$$\left[1 - D_{yy}\frac{\partial^2}{\partial x^2} - D_{xx}\frac{\partial^2}{\partial y^2}\right]\Delta\Delta w = \frac{1}{D}\left[1 - \left(D_{yy} + \frac{2D_{xx}}{1-\mu}\right)\frac{\partial^2}{\partial x^2}\right.$$

$$\left. - \left(D_{xx} + \frac{2D_{yy}}{1-\mu}\right)\frac{\partial^2}{\partial y^2} + \frac{2D_{xx}D_{yy}}{1-\mu}\Delta\Delta\right]\left[P + \frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2}\right.$$

$$\left. - 2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right]. \tag{171}$$

Writing the critical buckling stress of the faces in the intracellular mode,

$$\sigma_{cr} = \frac{k\pi^2 E_c}{12(1-\mu_c^2)} \left(\frac{t}{b}\right)^2 , \tag{176}$$

where the symbols are defined in Figure 31 and in Part 5.1. The values of the buckling coefficient, k, are given in Figure 32.
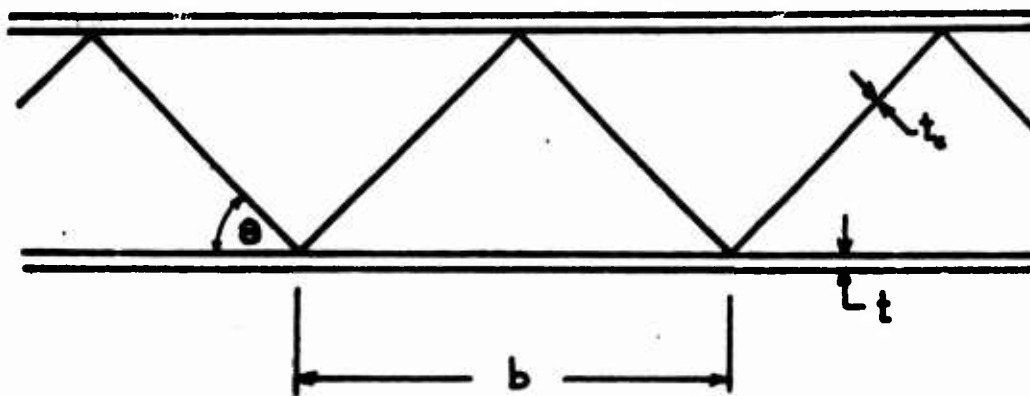
Figure 31

Truss-Core Sandwich
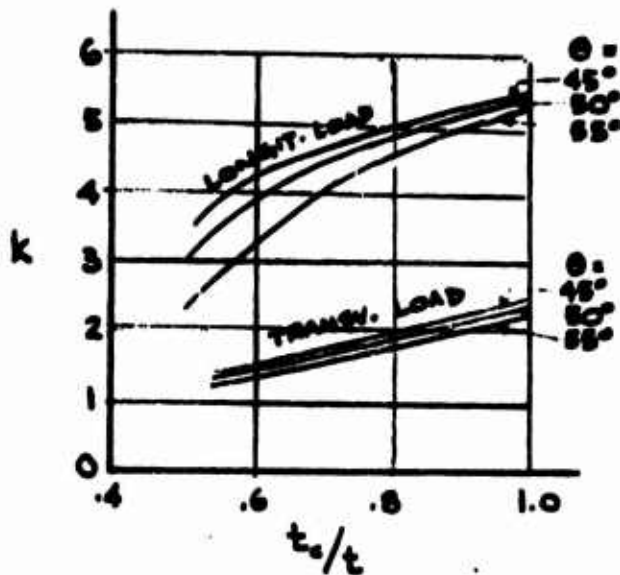
Figure 32

Truss-Core Buckling Coefficient

A design curve, for honecombs, based on intra-
cellular buckling, is given on page C12.8 of Reference
122. When uniaxial compression is applied in the
direction of the ribbon,

$$\frac{\sigma_{cc}}{\eta} = .75\, E\, \left(\frac{t}{s}\right)^{\frac{1}{2}} , \tag{177}$$

where η is the plasticity reduction factor,

$$\eta = \frac{2E_T}{E + E_T} ,$$

and E_T is the tangent modulus of the facings. The
geometry of the honeycomb is given in Figure 33.

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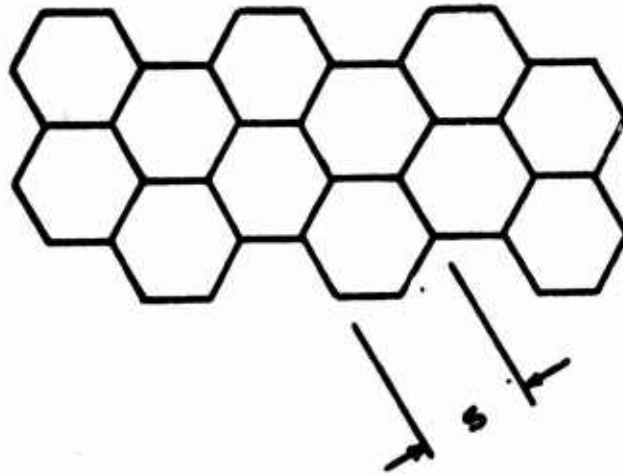
Figure 33

Honeycomb

5.2.1.3 Shear Crimping

The third form of local instability is excessive transverse shear stress (shear crimping). Such a failure may occur when the sandwich panel is not initially perfectly flat.

The problem is considered by Boller and Norris
(Ref. 121) and Bruhn (Ref. 122). The latter gives
the design equation,

$$\frac{T_{cr}}{\eta} = .6E \left(\frac{t}{s}\right)^{4/2} ,$$

(178)

with terms defined in the same manner as those of
Eq. (177).

5.2.2 General Buckling

The general buckling of sandwich plates has been
investigated by several authors for many types of end
conditions and loadings.

5.2.2.1 Simply Supported Edges

Buckling of simply supported sandwich plates is
discussed by Gerard and Becker (Ref. 236), Hoff
(Refs. 139, 140), Kimel (Ref. 149), March and Smith
(Ref. 161), and Seide and Stowell (Ref. 173).

For the isotropic sandwich plate in uniaxial compression, the formula for the critical buckling stress is given by Hoff:

$$\sigma_{cR} = CF \, \sigma_{cR}^{F} \quad . \tag{179}$$

σ_{cR}^{F} is the buckling stress of the unconnected faces,

$$\sigma_{cR}^{F} = \frac{\pi^2 E \, t^2}{3(1-\mu^2)L} \quad ,$$

F is the form factor,

$$F = 1 + 3\left(1 + \frac{h}{t}\right)^2 \, .$$

and C is a reduction factor for the buckling of the composite panel. Its values are given by the curve in Figure 34, for various parameters,

$$R = \frac{G_{HL}}{F \, \sigma_{cR}^{F}} \quad .$$
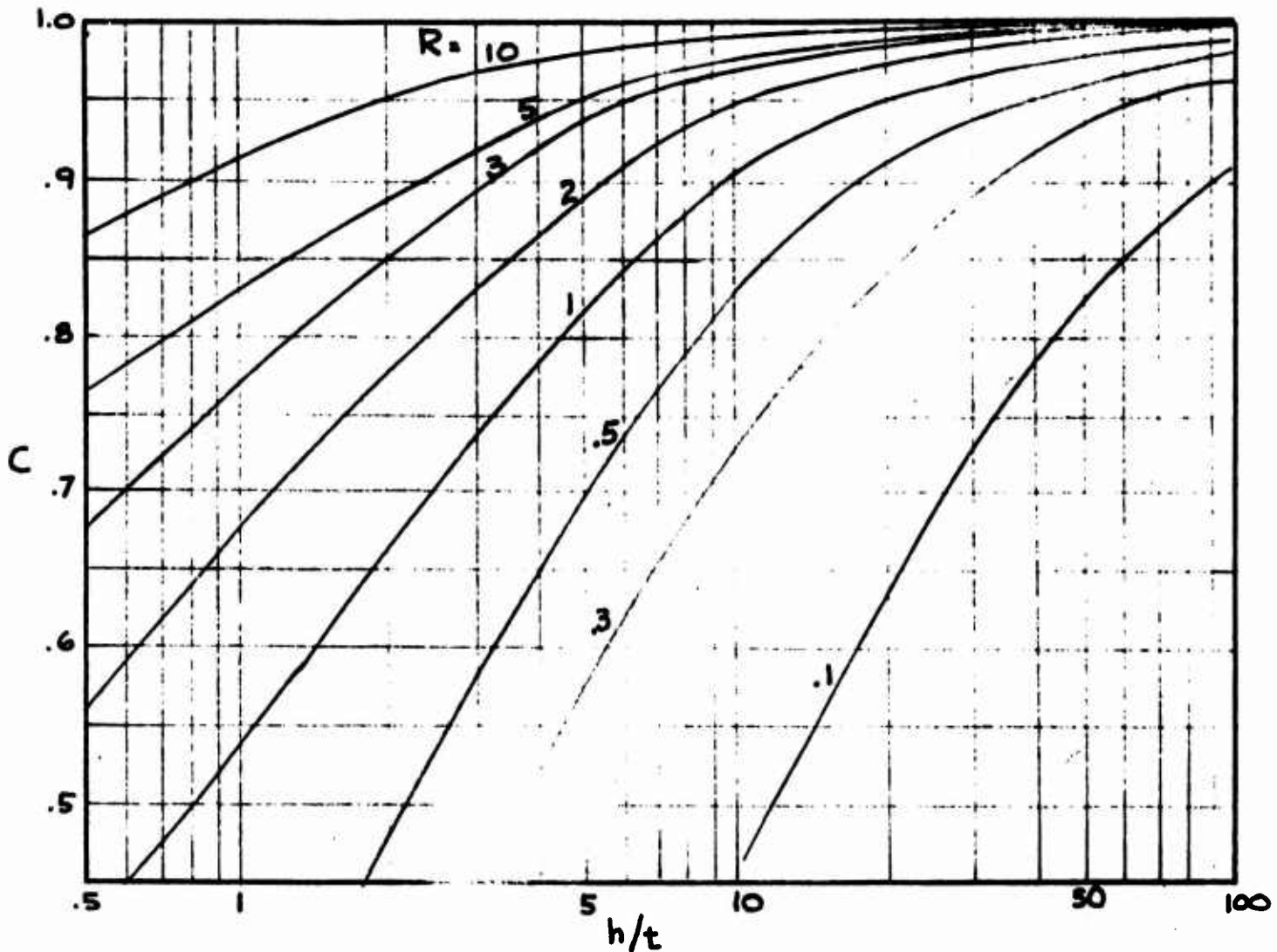
Figure 34

Reduction Factor - Simple Supports

Kimel, in Reference 149, performs a theoretical
analysis of the simply supported plate under the
combination of edgewise bending and compression on
opposite edges. The sandwich considered is made up
of isotropic facings and an orthotropic core. The
results of this analysis are quite lengthy and will not
be presented here.

A uniaxially compressed sandwich with an orthotropic core is discussed by March and Smith (Ref. 161). With the load applied along the a-edges, the critical buckling load,

$$\left(N_y\right)_{cr} = \frac{\pi^2}{a^2}\sqrt{D_1 D_2}\left[\frac{b^2}{n^2 a^2}\sqrt{\frac{D_1}{D_2}} + \frac{n^2 a^2}{b^2}\sqrt{\frac{D_2}{D_1}} + 2\,\Omega\right],$$ (180)

where

$$\Omega = \left(D_1 D_2\right)^{-\frac{1}{2}} \int_{-\left(\frac{h}{2}+t\right)}^{\left(\frac{h}{2}+t\right)} \left[\frac{E_y \mu_{yx} + 2\left(1 - \mu_{xy}\mu_{yx}\right)G_{xy}}{1 - \mu_{xy}\mu_{yx}}\right] z^2\, dz ,$$

$$D_1 = \int_{-\left(\frac{h}{2}+t\right)}^{\left(\frac{h}{2}+t\right)} \frac{E_x}{1 - \mu_{xy}\mu_{yx}}\, z^2\, dz ,$$

$$D_2 = \int_{-\left(\frac{h}{2}+t\right)}^{\left(\frac{h}{2}+t\right)} \frac{E_y}{1 - \mu_{xy}\mu_{yx}}\, z^2\, dz ,$$

n = number of half-waves in x-direction.

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A family of curves, for various values of n, must
be constructed in order to make Eq. (180) suitable
for design.

5.2.2.2 Loaded Edges Pinned, Other Clamped

This case is considered by Yen, Salerno, and
Hoff (Ref. 184). A close approximation to the true
value of the buckling load is obtained by establishing
a lower bound by Leggett's method and ˠn upper bound
by Galerkin's approach. The derived expressions are
extremely complex, but a diagram, well suited for
design, is presented.

A Metalite sandwich (isotropic facings, ortho-
tropic core) is studied by Seide (Ref. 171). In this
paper, the flexural rigidity of the faces is neglected,
thus underestimating the shear stiffness of the core
(discussed by Bijlaard in Reference 120). Using a
combined result of Seide and Bijlaard (modification
of former's method by including face rigidities), the
critical buckling load along the pinned edges,

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$$(N_x)_{cR} = K_o \, \eta \, \frac{\pi^2 D_\tau}{b^2} \left[\frac{D_p}{D_\tau} + \phi \right] . \tag{181}$$

where

K_o = buckling coefficient from Figure 35,

η = plasticity factor (see 5.2.1.2),

$D_\tau = \dfrac{Et\,(h+t)^2}{2\,(1-\mu^2)}$,

$D_p = \dfrac{Et^3}{6\,(1-\mu^2)}$,

$\phi = \dfrac{\alpha}{\alpha + \eta r K_o}$,

$r = \dfrac{(\pi^2 D_\tau / b^2 G_{xz})}{[h/(h+t)^2]}$,

$\alpha = 1 + \dfrac{a^2}{b^2 m}$,

and all other terms are those used with consistency throughout this paper.

March and Smith (Ref. 161) take the x-edges as loaded and simply supported and derive the expression for the critical buckling load of a sandwich with an orthotropic core as:

$$(N_y)_{cr} = \frac{16\pi^2}{3a^2} \left[D_1 \frac{b^2}{n^2 a^2} + \frac{3}{16} D_2 \frac{n^2 a^2}{b^2} + \frac{\Omega}{2} \right],$$

(182)

where all terms are defined in Eq. (180).

5.2.2.3 Loaded Edges Clamped, Others Pinned

The solution to the problem of the x-edges
loaded and clamped and the y-edges simply supported is
given by March and Smith (Ref. 161) for the sandwich
with an orthotropic core:

$$(N_y)_{cr} = K_{cr} \frac{12\sqrt{D_1 D_2}}{a^2},$$

(183)

where, for one half-wave,

$$k_{cr} = \frac{\pi^2}{48}\left[3\frac{b^2}{a^2}\sqrt{\frac{D_1}{D_2}} + 16\frac{a^2}{b^2}\sqrt{\frac{D_2}{D_1}} + 8\,\Omega\right],$$

for two half-waves,

$$k_{cr} = \frac{\pi^2}{60}\left[\frac{b^2}{a^2}\sqrt{\frac{D_1}{D_2}} + 41\frac{a^2}{b^2}\sqrt{\frac{D_2}{D_1}} + 10\,\Omega\right],$$

for three half-waves,

$$k_{cr} = \frac{\pi^2}{120}\left[\frac{b^2}{a^2}\sqrt{\frac{D_1}{D_2}} + 136\frac{a^2}{b^2}\sqrt{\frac{D_2}{D_1}} + 20\,\Omega\right],$$

for four half-waves,

$$k_{cr} = \frac{\pi^2}{240}\left[\frac{b^2}{a^2}\sqrt{\frac{D_1}{D_2}} + 353\frac{a^2}{b^2}\sqrt{\frac{D_2}{D_1}} + 34\,\Omega\right],$$

and all terms are defined in Eq. (180).

5.2.2.4 All Edges Clamped

March and Smith (Ref. 161) solve the problem of the clamped sandwich plate with isotropic facings

and orthotropic core. The critical buckling load
along the x-edge,

$$(N_Y)_{CR} = K_{CR} \; \frac{12 \sqrt{D_1 D_2}}{a^2} \quad , \tag{184}$$

in which the buckling coefficient for one half-wave,

$$k_{CR} = \frac{\pi^2}{9} \left[3 \frac{b^2}{a^2} \sqrt{\frac{D_1}{D_2}} + 3 \frac{a^2}{b^2} \sqrt{\frac{D_2}{D_1}} + 2 \Omega \right] ,$$

for two half-waves,

$$k_{CR} = \frac{\pi^2}{180} \left[16 \frac{b^2}{a^2} \sqrt{\frac{D_1}{D_2}} + 123 \frac{a^2}{b^2} \sqrt{\frac{D_2}{D_1}} + 40 \Omega \right] ,$$

for three half-waves,

$$k_{CR} = \frac{\pi^2}{45} \left[2 \frac{b^2}{a^2} \sqrt{\frac{D_1}{D_2}} + 51 \frac{a^2}{b^2} \sqrt{\frac{D_2}{D_1}} + 10 \Omega \right] ,$$

for four half-waves,

$$k_{CR} = \frac{\pi^2}{612} \left[16 \frac{b^2}{a^2} \sqrt{\frac{D_1}{D_2}} + 1059 \frac{a^2}{b^2} \sqrt{\frac{D_2}{D_1}} + 136 \Omega \right] ,$$

and all terms are defined in Eq. (180).
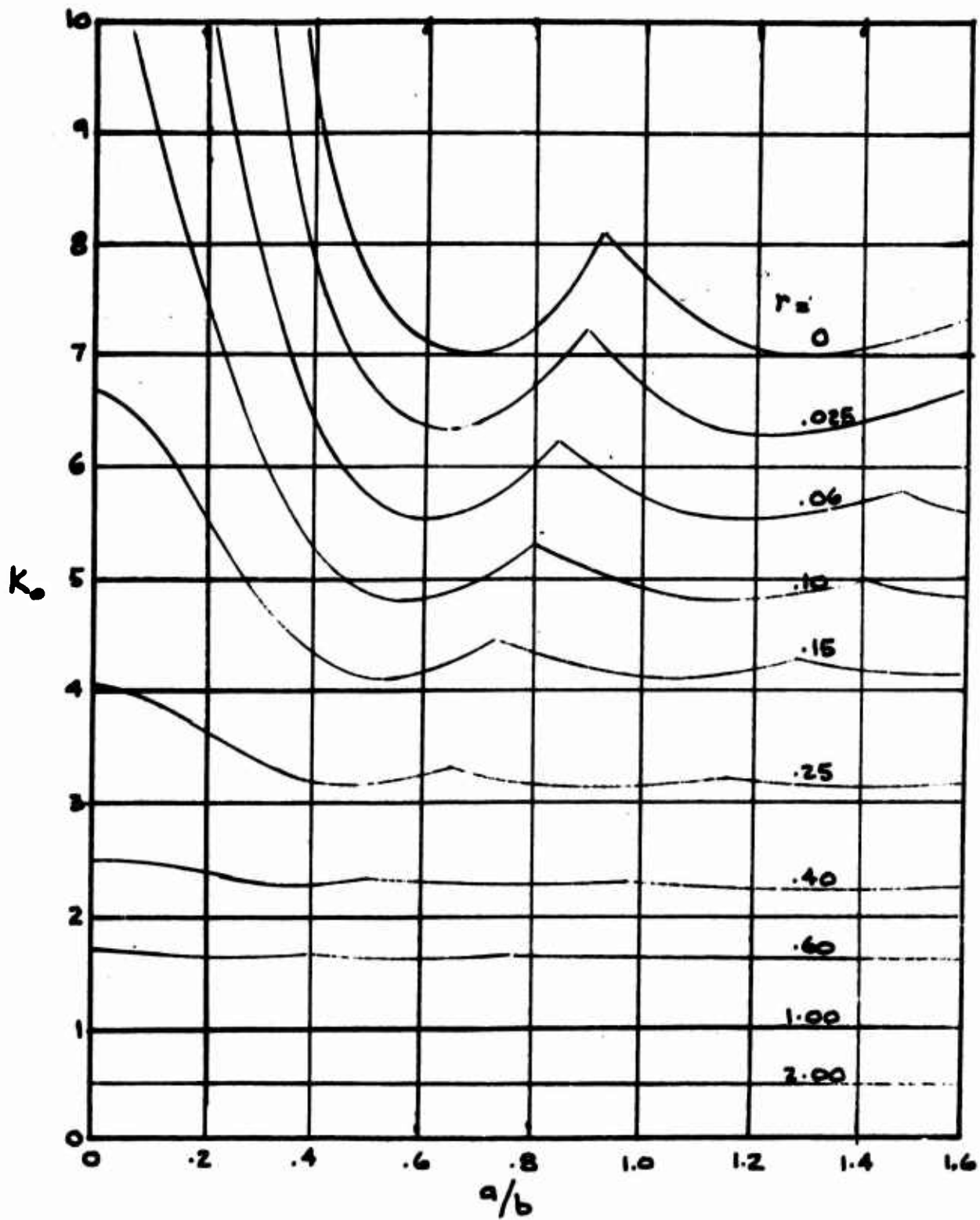
Figure 35

Buckling Coefficients for Eq. (181)~ $\mu = 1/3$

This problem is also solved by Thurston (Ref. 176). The buckling load is given by:

$$(N_x)_{cr} = K' \frac{\pi^2 Et (h+t)^2}{2a^2 (1-\mu^2)} , \tag{185}$$

in which K' is a buckling coefficient given by Figure 36. In the Figure, S is the sandwich parameter,

$$S = \frac{D_o \pi^2}{a^2 G_{xz} h} , \tag{186}$$

where

$$D_o = \frac{Et (h+t)^2}{2 (1-\mu^2)} .$$
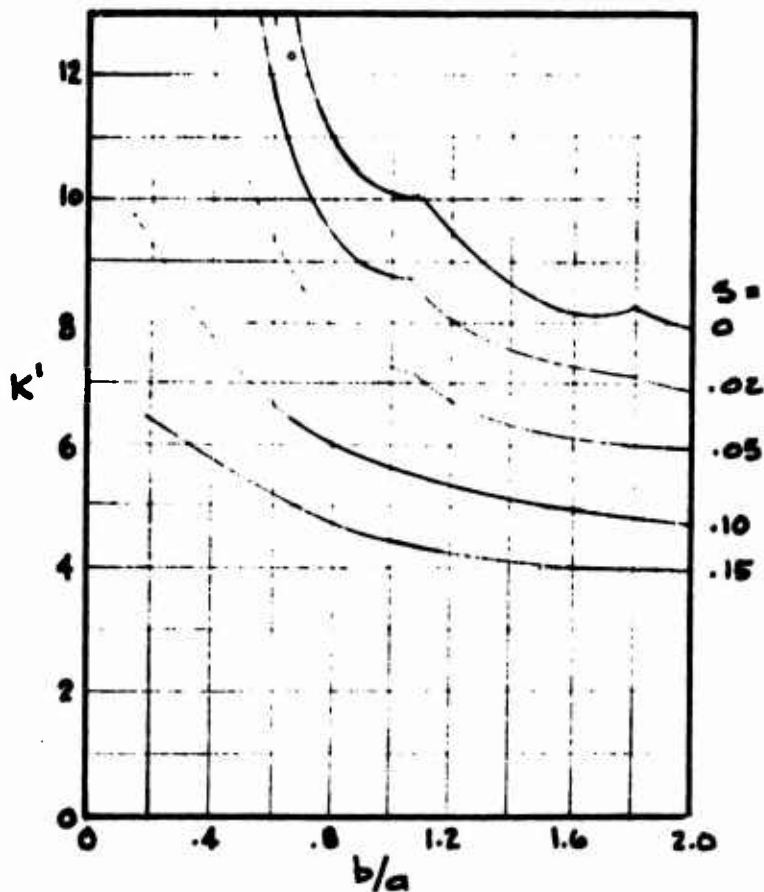
Figure 36

Buckling Coefficients for Eq. (185)

5.3 Summary

Only the most common cases of loading and end
supports have been considered in this chapter. For
other combinations of end conditions and loads, the
reader is advised to search the titles of Section B
of the Bibliography.

PART 6 - CORRUGATED PLATES

Corrugated plates were first introduced as a mode of construction in England in the late 1920's. Their first use was in the form of tank and sub-division bulkheads in ships. Corrugated sheet steel was later used in the construction of aircraft wings and fuselages. In recent years, corrugated sheet attached to a flat plate (generally called "corrugation-stiffened panel") has been used in the forming of skin of high-speed flight vehicles such as the X-15. A more recent advance has been the adoption of corrugated plate for use as core in an all-metal sandwich.

6.1 Flexure

The flexure of corrugated plates of various configurations has been discussed by several authors. Among these, the following are the most prominent: Andreeva (Ref. 193), Caldwell (Refs. 196, 197), Shibuya (Ref. 205), Stroud (Ref. 335), Platus and Uchiyama (Ref. 204), Bergman and Reissner (Ref. 195), and Timoshenko and Woinowsky-Krieger (Ref. 97).

The discussion which follows is divided into two parts. First, a single corrugated plate is considered, and, second, a built-up configuration of corrugated

and flat plates is discussed.

6.1.1 Single Sheets

The bending of single corrugated sheet is taken up by Timoshenko and Woinowsky-Krieger (Ref. 97). For a sinusoidal configuration (Fig. 37), given by

$$ z = f \sin \frac{\pi x}{L} , \qquad (187) $$

the orthotropic constants are:

$$ D_x = \frac{L}{s} \frac{Et^3}{12(1-\mu^2)} , \qquad (188) $$

$$ D_y = EI , \qquad (189) $$

$$ D_{xy} = \frac{s}{L} \frac{Et^3}{12(1+\mu)} , \qquad (190) $$

in which

E = modulus of elasticity of material,

μ = Poisson's ratio of material,

s = arc length of half-wave.

Figure 37
Sinusoidal Corrugation

From the geometry, the arc length and the moment of
inertia are approximately given by

$$s = L \left[1 + \frac{\pi^2 f^2}{4 L^2} \right] , \tag{191}$$

$$I = \frac{f^2 t}{2} \left[1 - \frac{.81}{1 + 2.5 \left(\frac{f}{2L} \right)^2} \right] . \tag{192}$$

The orthotropic plate theory, discussed in Part 3, is
than applied for the solution of a particular bending
problem.

Corrugations of a different geometry (Fig. 38)
are considered by Caldwell (Ref. 196). The author
takes into account the effect of inelastic behavior.
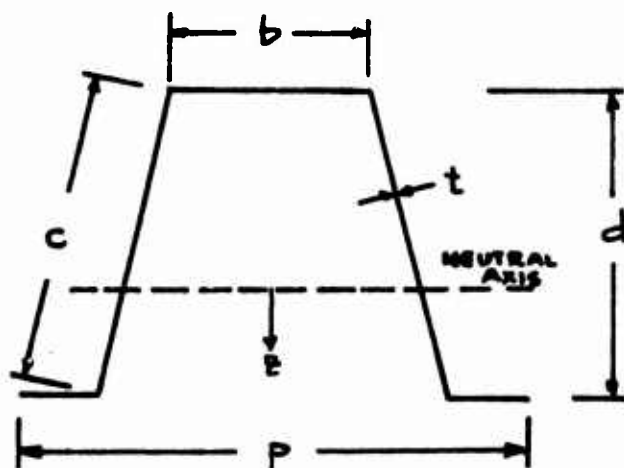
Figure 38
"Hat-type" Corrugation

Experiments show that elastic stresses and deformations can be calculated from ordinary bending theory using a moment of inertia,

$$I = \frac{td^2}{6}(3b+c),$$

(193)

and a section modulus,

$$Z = \frac{td}{3}(3b+c).$$

(194)

Assuming that the strains are large, the ultimate bending strength of the panel is given by:

$$M_{u.T.} = \frac{td}{2}(2b+c)\sigma_Y,$$

(195)

where σ_Y is the yield strength of the material.

6.1.2 Built-up Panels

Stroud (Ref. 335) calculates the elastic constants of a corrugated plate attached on one side to a flat plate (Fig. 39).
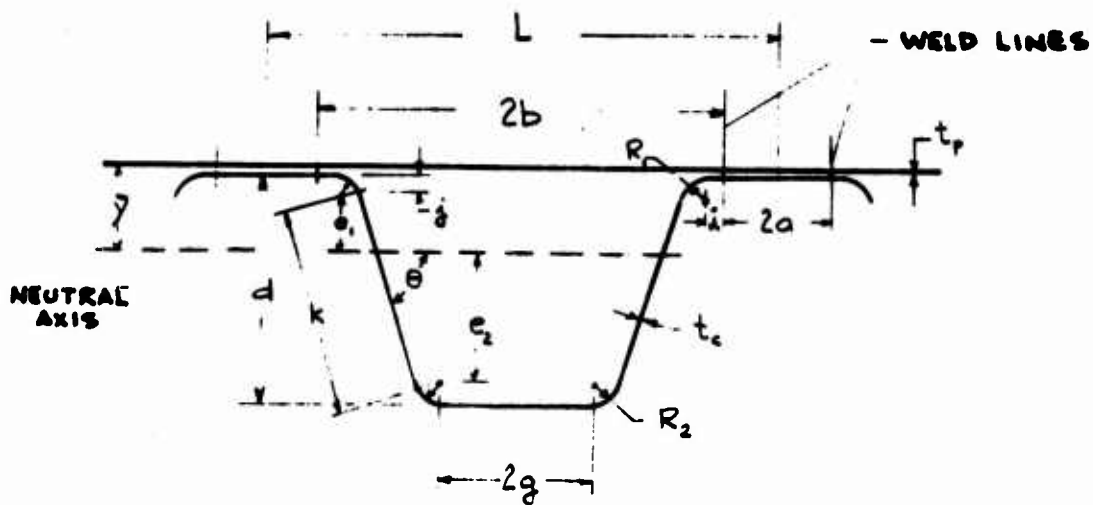
Figure 39
Corrugation-Stiffened Plate

The differential equation of this orthotropic plate
is written in a form slightly different from that of
Eq. (31):

$$\frac{D_1}{1-\mu_{xy}\mu_{yx}}\frac{\partial^4 w}{\partial x^4} + 2\left[D_3 + \frac{\mu_{xy}D_1}{1-\mu_{xy}\mu_{yx}}\right]\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{D_2}{1-\mu_{xy}\mu_{yx}}\frac{\partial^4 w}{\partial y^4} = p \cdot \quad (196)$$

D_1 is the bending stiffness in a direction parallel
to the troughs ("into the paper" - - - Fig. 39),

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and, with E being the modulus of elasticity of the
material, its value is:

$$
\begin{aligned}
D_1 = \frac{2E}{L} \Bigg[& \frac{(a+b)t_p^3}{12} + \left(\bar{y} - \frac{t_p}{2}\right)^2 (a+b)t_p + \frac{(a+i)t_c^3}{12} \\
& + (e_1 + R_1)^2 (a+i)t_c + t_c R_1^3 \left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right) + 2t_c R_1^2 e_1 \sin\theta \\
& + e_1^2 R_1 \theta t_c + \frac{t_c}{12} k^3 \sin^2\theta + \frac{kt_c^3}{12} \cos^2\theta \\
& + \left(t_p + \frac{t_c}{2} + j + \frac{k}{2}\sin\theta - \bar{y}\right) kt_c + t_c R_2^3 \left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right) \\
& + 2t_c e_2 R_2^2 \sin\theta + e_2^2 R_2 \theta t_c + \frac{gt_c^3}{12} + (e_2 + R_2)^2 t_c g \Bigg],
\end{aligned}
\tag{197}
$$

in which the location of the neutral axis is given by

$$
\bar{y} = \frac{
\left[\begin{aligned}
& \frac{t_p^2}{2}(a+b) + \left(t_p + \frac{t_c}{2}\right)(a+i)t_c + \left(t_p + \frac{t_c}{2} + R_1 - \frac{R_1 \sin\theta}{\theta}\right) R_1 \theta t_c \\
& + \left(t_p + \frac{t_c}{2} + j + \frac{n-j}{2}\right) kt_c + \left(t_p + \frac{t_c}{2} + d - R_2 + \frac{R_2 \sin\theta}{\theta}\right) R_2 \theta t_c \\
& + \left(t_p + \frac{t_c}{2} + d\right) g t_c
\end{aligned}\right]
}{
t_p(a+b) + (a+i)t_c + R_1 \theta t_c + kt_c + R_2 \theta t_c + g t_c
} .
$$

The bending stiffness in a direction normal to the
trough axis,

$$
D_2 = \frac{a+b}{\dfrac{a}{D_2''} + \dfrac{b}{D_2'}} ,
\tag{198}
$$

where

$$D_2' = \left\{ \left[\frac{bEt_c^3}{12(1-\mu^3)} \quad \frac{1}{i + (R_1+R_2)\theta + k + g - \ldots} \right. \right.$$

$$\overline{-H[R_1^2(\theta - \sin\theta) + jk + \frac{k^2}{2}\sin\theta + \ldots}$$

$$\overline{+ R_2(n\theta - R_2\theta\cos\theta + R_2\sin\theta) + gd]}$$

$$\left. \left. + \frac{Et_c^3}{12(1-\mu^2)} \right] (1 - \mu_{xy}\mu_{yx}) \right\} ,$$

$$D_2'' = \frac{E(t_f + t_c)^3}{12(1-\mu^3)} (1 - \mu_{xy}\mu_{yx}) ,$$

$$H = \frac{R_1^2(\theta - \sin\theta) + jk + \frac{k^2}{2}\sin\theta + \ldots}{R_1^3(\frac{3\theta}{2} - 2\sin\theta + \sin\frac{2\theta}{4}) + j^2k + \ldots}$$

$$\frac{+ R_2(n\theta - R_2\theta\cos\theta + R_2\sin\theta) + gd \ldots}{+ jk^2\sin\theta + \frac{k^3}{3}\sin^2\theta + R_2 n^2\theta - \ldots}$$

$$\overline{- 2n R_2^2 \theta\cos\theta + 2n R_2^2\sin\theta + \ldots}$$

$$\overline{+ \frac{R_2^3}{2}(3\theta - 3\sin\theta\cos\theta - 2\theta\sin^2\theta) + gd^2} \quad .$$

The twisting stiffness, with G being the shear modulus
of the material,

$$D_3 = \frac{\frac{G}{2}\left\{b(t_1+t_2)+d\left[b+g-\iota+(R_2-R_1)\tan\frac{\phi}{2}+2(R_1^2-R_2^2)\tan\frac{\phi}{2}+(R_2^2-R_1^2)\theta\right]\right\}}{\frac{b}{t_1}+\frac{1}{t_2}\left[\iota+\theta(R_1+R_2)+k+g\right]} \qquad (199)$$

The bending moments are evaluated by substituting the above values (in which μ is the Poisson's ratio of the material) into

$$\left.\begin{aligned}
M_x &= -\frac{D_1}{1-\mu_{xy}\mu_{yx}}\left[\frac{\partial^2 w}{\partial x^2}+\mu_{xy}\frac{\partial^2 w}{\partial y^2}\right], \\
M_y &= -\frac{D_2}{1-\mu_{xy}\mu_{yx}}\left[\frac{\partial^2 w}{\partial y^2}+\mu_{yx}\frac{\partial^2 w}{\partial x^2}\right], \\
M_{xy} &= D_3\frac{\partial^2 w}{\partial x\partial y}.
\end{aligned}\right\} \qquad (200)$$

The same problem is considered by Wempner and McKinley (Ref. 180) and similar results are derived for plates connected by one weld-line (a = 0 in Fig. 39).

6.2 Elastic Stability

The buckling of corrugated plates has been studied by various investigators. Among these are Andreeva (Ref. 193), Ashwell (Ref. 194), Libove and Hubka (Ref. 156), Plaineveaux (Ref. 203), Stroud (Ref. 326), Seydel (Ref. 84), Timoshenko and Gere (Ref. 96), Wempner

and McKinley (Ref. 180), and Dean (Ref. 198).

This portion will again be divided into two phases: the first dealing with single corrugated sheets and the second with panels consisting of flat plates attached to corrugated sheets.

6.2.1 Single Sheets

Since consideration of bending requirements alone would dictate the design of a thin plate with deep troughs, it is necessary to consider limitations imposed upon such a design by <u>local instability</u> of the component "walls". Caldwell (Ref. 196) discusses this problem, taking into account the interaction between webs and flanges and referring all symbols to the configuration of Figure 38.

The critical buckling stress, based on local instability (and, thus, independent of panel length and edge supports), is given by:

$$\sigma_{cr} = k_c^2 \; \frac{D}{t c^2} \; , \tag{201}$$

where D is a material property, $Et^3/12(1-\mu^2)$, and K_c is a constant dependent upon the dimensions of the corrugation (see Figure 38 and 40).

Timoshenko and Gere (Ref. 96) discuss the work of Bergmann and Reissner (Ref. 195) and Seydel (Ref. 84) for the critical buckling shear load, and give the expression:

$$(N_{xy})_{cR} = 4 C_s \frac{\sqrt[4]{D_x D_y^3}}{b^2} , \qquad (202)$$

where C_s is the buckling coefficient given in Figure 23. The equation (202) is that given in paragraph 4.3.3 and all definitions are given there.
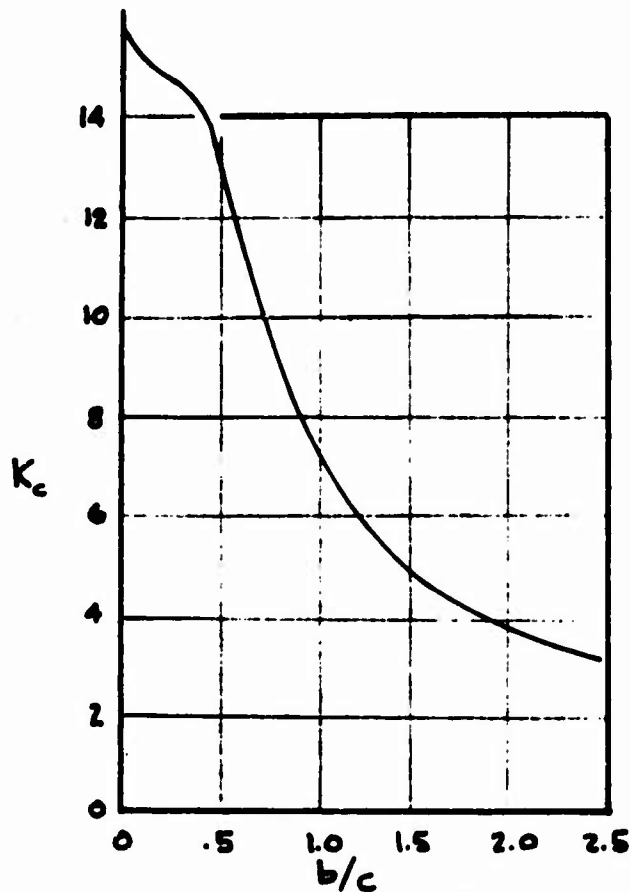
Figure 40
Buckling Coefficient of Eq. (201)

6.2.2 Built-up Panels

The formulas derived by Stroud (Ref. 335) can be
extended to the buckling case be rewriting the differ-
ential equation (196) such that it includes edge
loadings:

$$\frac{D_1}{1-\mu_{xy}\mu_{yx}} \frac{\partial^4 w}{\partial x^4} + 2\left[D_3 + \frac{\mu_{xy} D_1}{1-\mu_{xy}\mu_{yx}}\right]\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{D_2}{1-\mu_{xy}\mu_{yx}} \frac{\partial^4 w}{\partial y^4}$$

$$= N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2}.$$

Thus, if the orthotropic constants defined by Eqs.
(197), (198), and (199) are substituted into Eq. (203),
the solutions given in Part 4 can be applied to par-
ticular problems.

This problem is also discussed by Wempner and
McKinley (Ref. 180) and theoretical results are com-
pared with experimental data.

6.3 Summary

The increase in load-carrying ability accomplished
by constructing corrugations parallel to the loading
and the high flexural rigidity of corrugated plates are
well recognized. However, plates of this type have
several disadvantages: a) i.e. strength is increased

in one direction, while it is decreased in the direction normal to it; b) weight per unit projected area is greater than that for flat plates; c) workability of corrugated sheet is limited.

For these reasons, various investigators have explored other configurations which can be formed as easily as corrugations. Smith and Gray (Ref. 320) discuss a method for redistributing metal in flat sheets for the purpose of obtaining maximum strength-weight ratios and rigidity-weight ratios. Farmer and Spangler (Ref. 308) investigate a pattern of "calottes", or hemispheres, pressed into a flat sheet in a random pattern. This writer (Ref. 309) and Tewes (Ref. 321) present a study of plates stiffened by "dimples" (L-shaped depressions pressed into the sheet in an interlocking manner). "Waffle beading", forming of intersecting sinusoidally shaped beads in a sheet metal panel, is discussed by Rogge in Reference 314.

PART 7 - RIB-REINFORCED PLATES

The bending and buckling of flat plates stiffened
by longitudinal or transverse ribs, or both, is a
subject which has received much attention in the
literature. Early studies were concerned with multi-
piece skin-and-stiffener combinations, while more
recent investigations are generally confined to in-
tegrally-stiffened structures. Both types are con-
sidered in this chapter.

7.1 Flexure

Among the many investigators concerned with the
bending of rib-stiffened plates are the following:
Borcz (Ref. 326), Chapman and Slatford (Ref. 217),
Floor and Burgerhout (Ref. 233), Giencke (Refs. 237-
239), Giuliani (Ref. 240), Holmes (Ref. 244), Huber
(Ref. 40), Jaeger and Hendry (Ref. 246), Lekhnitsky
(Ref. 58), Richart, Newmark and Siess (Ref. 268),
Schade (Refs. 273-276), Schumann (Refs. 278, 279),
Terazawa and Tarada (Ref. 288), Timoshenko and
Woinowsky-Krieger (Ref. 97), Tolotti and Grioli
(Ref. 290), and Yamaki (Ref, 298).

The bending of orthogonally stiffened plates
(to which this study is limited) can be analyzed by
means of orthotropic plate theory, as described in
Parts 1 and 2. The elastic constants of the govern-
ing differential equation (31) must be determined
experimentally, according to the methods outlined in
the following chapter. Then, deformations, stresses,
etc., can be evaluated. Orthotropic constants for
several cases of stiffening are discussed here.

Lekhnitsky (Ref. 58) considers a plate with
equally spaced one-directional stiffeners symmetrical
with respect to the midplane of the plate (Figure 41).
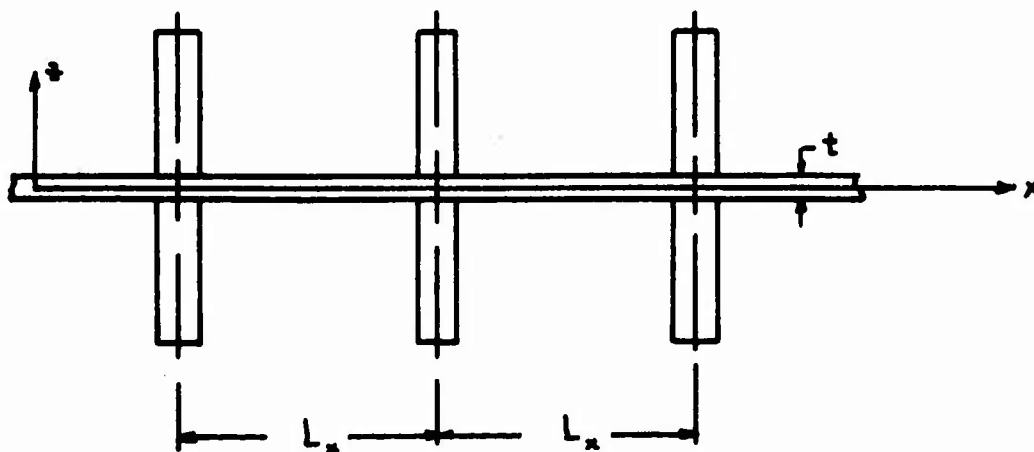
Figure 41
Stiffened Plate Governed by Eqs. (204)

The constants of the characteristic equation (31) are:

$$D_x = D_{xy} = \frac{E_p t^3}{12(1-\mu^2)},$$

$$D_y = \frac{E_p t^3}{12(1-\mu^2)} + \frac{E_s (I_y)_s}{L_x},$$

(205)

where subscripts p and s refer to properties of plate and stiffener, respectively.

If the plate of Figure 41 is now reinforced by another set of equal stiffeners in the y-direction, a distance L_y apart,

$$D_x = \frac{E_p t^3}{12(1-\mu^2)} + \frac{E_s (I_x)_s}{L_y},$$

$$D_y = \frac{E_p t^3}{12(1-\mu^2)} + \frac{E_s (I_y)_s}{L_x},$$

(206)

$$D_{xy} = \frac{E_p t^3}{12(1-\mu^2)},$$

where

$E_p, E_s =$ moduli of elasticity of plate and stiffeners, respectively,

$\mu =$ Poisson's ratio of plate material,

$(I_x)_s =$ moment of inertia of one stiffener, parallel to x-axis,

$(I_y)_s =$ moment of inertia of one stiffener, parallel to y-axis.

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The constants for a plate integrally reinforced by equal and equidistant ribs are derived by Pflügger (Ref. 264) and Timoshenko and Woinowsky-Krieger (Ref. 97). The latter give the following expressions for the plate shown in Figure 42:
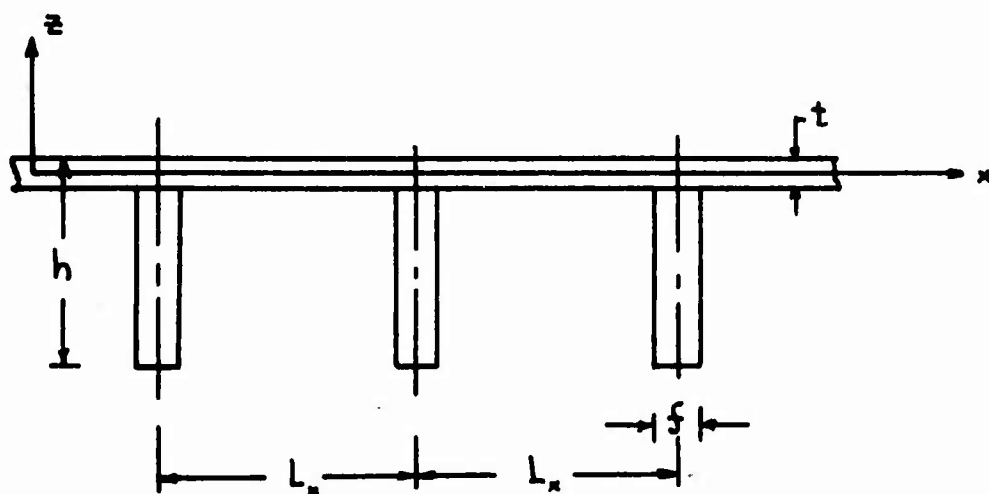
Figure 42
Integrally Stiffened Plates Governed by Eqs. (206)

$$D_x = \frac{E L_x t^3}{12(L_x - f + \alpha^3 f)} \quad ,$$

$$D_y = \frac{E I_1}{L_x} \quad ,$$

$$D_{xy} = 2 D_k' + \frac{C}{2 L_x} \quad ,$$

$$\left. \right\} \text{(206)}$$

where

I_1 = moment of inertia of one T-section of width L_x,

C = torsional rigidity of one rib,

D_k' = torsional rigidity of plate without ribs,

$\alpha = t/d$.

In addition to solutions which make use of orthotropic theory, in which the interaction between plate and stiffener is generally neglected, several writers presented more sophisticated direct methods of attack.

One of the early investigators, Schade (Ref. 276), considers a rectangular plate with longitudinal and transverse stiffeners, and with the two y-edges clamped and the x-edges free, acted upon by a uniformly distributed load, p (Figure 43).

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The deformation expression, taken as a single

series, has the form:

$$w = \frac{4 p b^4 s_y}{\pi^6 E I_y} \sum_n \frac{1}{n^6} \sin \frac{n\pi y}{b} \left\{ 1 - \frac{1}{\sinh \alpha a + \frac{\alpha}{\beta} \sin \beta a} \right.$$
$$\cdot \left[\sinh \alpha (a-x) \cos \beta x + \frac{\alpha}{\beta} \cosh \alpha (a-x) \sin \beta a \right.$$
$$\left. \left. + \sinh \alpha x \cos \beta (a-x) + \frac{\alpha}{\beta} \cosh \alpha x \sin \beta (a-x) \right] \right\} \ , \qquad (207)$$

where

$$\alpha = \frac{n}{b} \frac{\pi}{\sqrt{2}} \sqrt[4]{\frac{\lambda_y}{\lambda_x}} \sqrt{1+\eta} \ ,$$

$$\beta = \frac{\sqrt{1-\eta}}{\sqrt{1+\eta}} \alpha \ ,$$

$$\eta = \frac{\lambda_s}{\sqrt{\lambda_x \lambda_y}} \ ,$$

$$\lambda_x = \frac{I_x}{s_x} + \frac{2(I_m - I_x)}{b} = \text{ equivalent moment of inertia}$$
of longitudinal stiffener,

$$\lambda_s = \frac{\lambda_{sx} + \lambda_{sy}}{2} = \text{ average unit moment of inertia}$$
of plate alone,

I_x = moment of inertia of longitudinal
stiffener and effective skin,

I_y = moment of inertia of transverse
stiffener and effective skin,

I_m = moment of inertia of central
longitudinal stiffener and effective
skin,

$$\lambda_y = \frac{I_y}{s_y} = \text{ unit moment of inertia of plate}$$
alone.
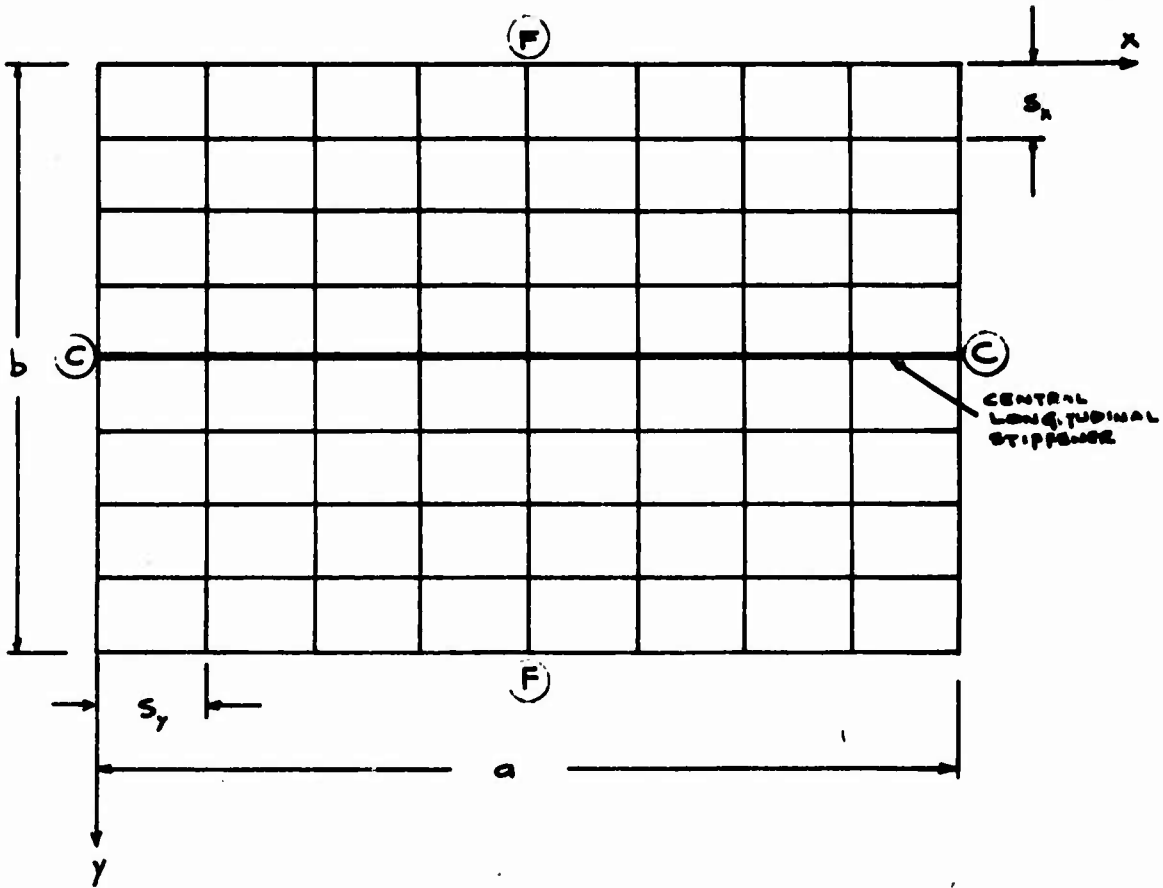
Figure 43
Stiffened Plate Governed by Eq. (207)

A numerical procedure for the direct calculation
of stresses is presented by Kempner (Ref. 248). Holmes
(Ref. 244) discusses the effect of shear lag upon the
bending of stiffened plates.

7.2 Elastic Stability

The problem of buckling of stiffened plates is
extremely important to the designers of aircraft
because of the rigid requirement of aerodynamic
smoothness of wing surfaces. For this reason, this
area has been investigated by a great number of
researchers. Among the more important contributors
are the following: Burns and Skogh (Ref. 212),
Caldwell (Ref. 214), Dow, Hickman and Rosen (Ref. 228),
Gerard (Refs. 234, 235), Gerard and Becker (Ref. 236),
Irving and Mullineux (Ref. 245), Klitchieff (Ref. 250),
Nowacki (Ref. 262), Pflüger (Ref. 264), Pochop (Ref. 266),
Radok (Ref. 267), Rockey and Cook (Refs. 269, 270),
Rosenhaupt (Ref. 271), Seide (Ref. 280), Seydel (Ref. 282),
Sokolowski (Ref. 284), Strasser (Ref. 285), Timoshenko
(Ref. 289), Timoshenko and Gere (Ref. 96), Tsuiji
(Ref. 291), Wang and Zuckerberg (Ref. 294), and Yusuff
(Ref. 300).

When orthotropic theory applies, the deflections
of the stiffened plate are governed by Eq. (127):

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy}\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4}$$
$$+ N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} = 0,$$

and the values of the constants D_x, D_y, and D_{xy} are given by Eqs. (204) or (205) or (206) or can be determined by experimental means, using methods outlined in the next part. The expressions derived in Part 4 can then be used to solve particular problems.

A very high percentage of papers dealing with this area are experimental in nature, and, thus, solutions are generally given in the forms of curves. The extremely large amount of information available renders impossible a summary of basic expressions and data in this report. Since the literature contains data on many different configurations of stiffened panels, it is impossible to present a concise guide here.

For this reason, the analyst is referred to the bibliography of this paper. The titles of most papers adequately describe the specialized topics with which they deal. For excellent summaries, Handbook of Structural Stability, Parts V and VII (Refs. 235 and 236), are particularly recommended. Bruhn's well-known reference (Ref. 122) also contains a great deal of information dealing with the buckling of stiffened panels.

7.3 Summary

The flat plate reinforced by either longitudinal
or transverse stringers, or both, remains an extremely
important problem in the design of ships, flight
vehicles, and bridges. Only the static behavior has
been discussed here. However, there are several in-
vestigators who have also concerned themselves with the
dynamic problem. Among these are: Feldman (Ref. 232),
Greenspon (Ref. 23), and McElman, Mikulas and Stein
(Ref. 256).

PART 8 - DETERMINATION OF ELASTIC CONSTANTS

In the previous chapters, use is made of ortho-
tropic plate theory to solve various bending and
buckling problems. In order that this theory may be
applied, it is first necessary to determine the various
elastic constants of orthotropy. Let us review the
characteristic differential equation for a general
loading:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + P$$
$$+ N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} = 0 .$$

In this expression, the shear deformation effects are
neglected. Since such a simplification is permissible
in many cases, the determination of the three constants,
D_x, D_y, and D_{xy}, will first be considered.

In the more refined case, where shear deformations
are accounted for (see Paragraph 1.8) three additional
constants must be evaluated. The methods of evaluation
of all of these values are discussed in this part.

8.1 Bending Stiffness

The original scheme for measuring the bending
stiffness of orthotropic plates is given by

Bergsträsser (Ref. 325). The same method is then
discussed by Thielemann in 1945, and later by Hearmon
and Adams (Ref. 29), Hoppmann et al (Refs. 329-332),
and Tsai and Springer (Ref. 338).

First, the relationship between stresses and
strains, Eq. (3), is recalled:

$$
\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \cdot \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}
$$

Since $S_{12} = S_{21}$, there are four elastic constants to
be determined in the case where shearing deformations
are neglected.

For the determination of the bending constants:

$$
S_{11} = \frac{1}{E_x} \ , \quad S_{22} = \frac{1}{E_y} \ ,
$$

$$
S_{12} = S_{21} = \frac{-\mu_{xy}}{E_y} = \frac{-\mu_{yx}}{E_x} \ ,
$$

the plate is oriented and supported as shown in Figure
44.
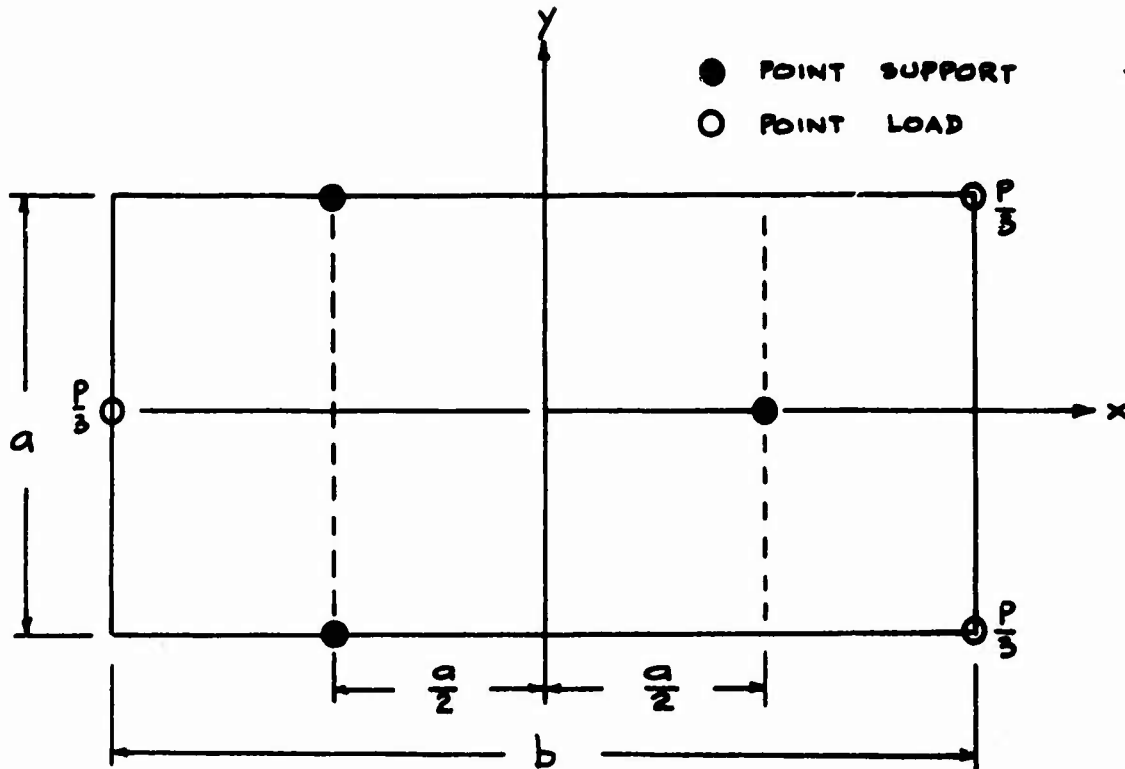
Figure 44
Test for Bending Stiffness (I)

Thus, a uniformly distributed bending moment, M_x, is applied to the plate. Therefore, the deformation at any point (x, y),

$$w = \frac{6M_x}{t^3}\left(s_{11}x^2 + s_{12}y^2\right) + Ax + By + C , \qquad (208)$$

where A, B, and C are constants. The moment in terms of the total applied load, P,

$$M_x = \frac{P}{2a}\left(\frac{b}{2} - \frac{a}{2}\right). \tag{209}$$

From this, the deflection,

$$w = \frac{3P}{at^3}\left(\frac{b}{2} - \frac{a}{2}\right)(s_{11}x^2 + s_{12}y^2) + Ax + By + C. \tag{210}$$

It is now necessary to make two deflection measurements in order to evaluate the constants S_{11} and S_{12}. These two measurements are at

$$x = 0, \ y = 0,$$
$$x = 0, \ y = \frac{a}{2}.$$

It is known that the deflection is zero at the supports, given by the coordinates:

$$x = \frac{a}{2}, \quad y = 0,$$
$$x = -\frac{a}{2}, \quad y = \frac{a}{2},$$
$$x = -\frac{a}{2}, \quad y = -\frac{a}{2}.$$

The evaluation of Eq. (210) for these five conditions gives five equations and five unknowns (S_{11}, S_{12}, A, B, C) which are then solved.

The same procedure is followed for the determination of S_{22} and S_{21}, and, thus, the values of the Young's moduli, E_x and E_y, and the Poisson-type constants, M_{xy} and M_{yx}, are known. Thus, the orthotropic stiffnesses, from Eqs. (20) and (21),

$$D_x = \frac{E_x t^3}{12(1-\mu_{xy}\mu_{yx})} \quad ,$$

$$D_y = \frac{E_y t^3}{12(1-\mu_{xy}\mu_{yx})} \quad .$$

Another scheme for determining the bending rigidity is presented by Stroud (Ref. 335) and slightly modified by this writer in an unpublished report. The rectangular plate is loaded such that the region between supports is in pure bending (Figure 45).
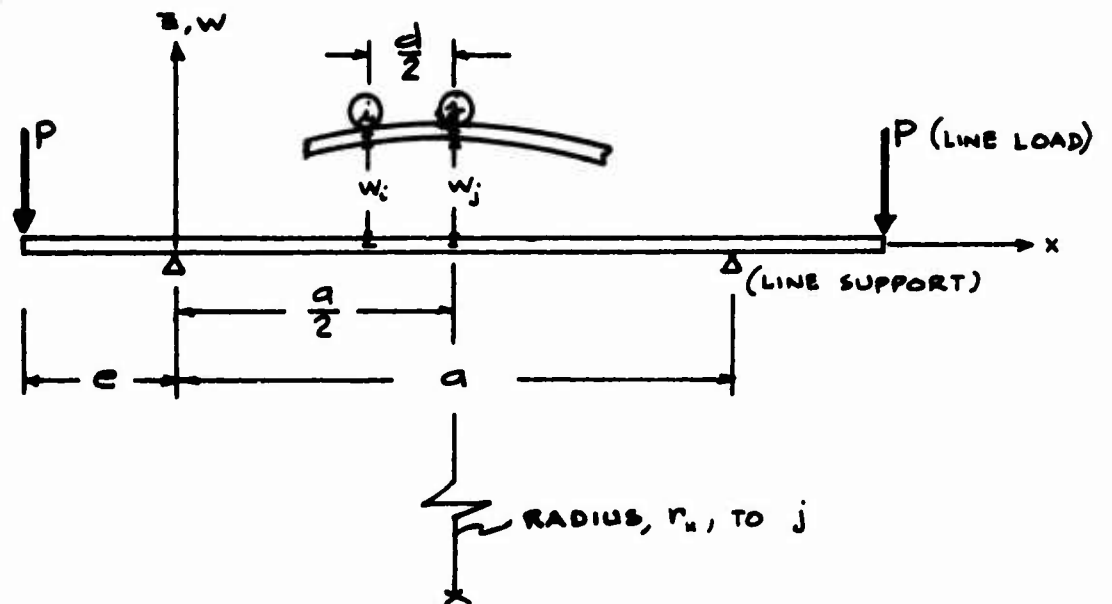
Figure 45
Test for Bending Stiffness (II)

For the shape of the deflected curve,

$$w = Kx^2 .$$ (211)

Differentiating,

$$\frac{\partial^2 w}{\partial x^2} = 2k \quad , \quad \frac{\partial^2 w}{\partial y^2} = \frac{\partial^2 w}{\partial x \partial y} = 0 .$$

Thus, the bending moment, from Eq. (17),

$$M_x = -2k D_x .$$ (212)

The radius of curvature between points i and j is
found from the geometry:

$$r = \frac{1}{\left(\frac{\partial^2 w}{\partial x^2}\right)} \cdot \frac{1}{2k} = \frac{4(w_j - w_i)^2 + d^2}{4(w_j - w_i)} \quad . \tag{213}$$

Substituting Eq. (213) into Eq. (212), writing the moment in terms of the load, and solving for the stiffness:

$$D_x = \frac{P_e \left[4(w_j - w_i)^2 + d^2\right]}{16(w_j - w_i)b} \quad . \tag{214}$$

The same procedure is followed for D_y.

8.2   Twisting Stiffness

The loading condition required for the determination of the twist term, D_{xy}, is that of uniformly distributed edge twisting couples. This is accomplished rather simply through the set-up shown in Figure 46, which is described by several writers, among them Timoshenko and Woinowsky-Krieger (Ref. 97), Hoppman et al (Refs. 329-332, 340), Adams (Ref. 29), Stroud (Ref. 335), and Tsai and Springer (Ref. 338).

As is illustrated in Figure 46, a square plate is subjected to concentrated loading at one pair of diagonally opposed corners and supported by point

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supports (perhaps ball bearings) at the other two
corners.
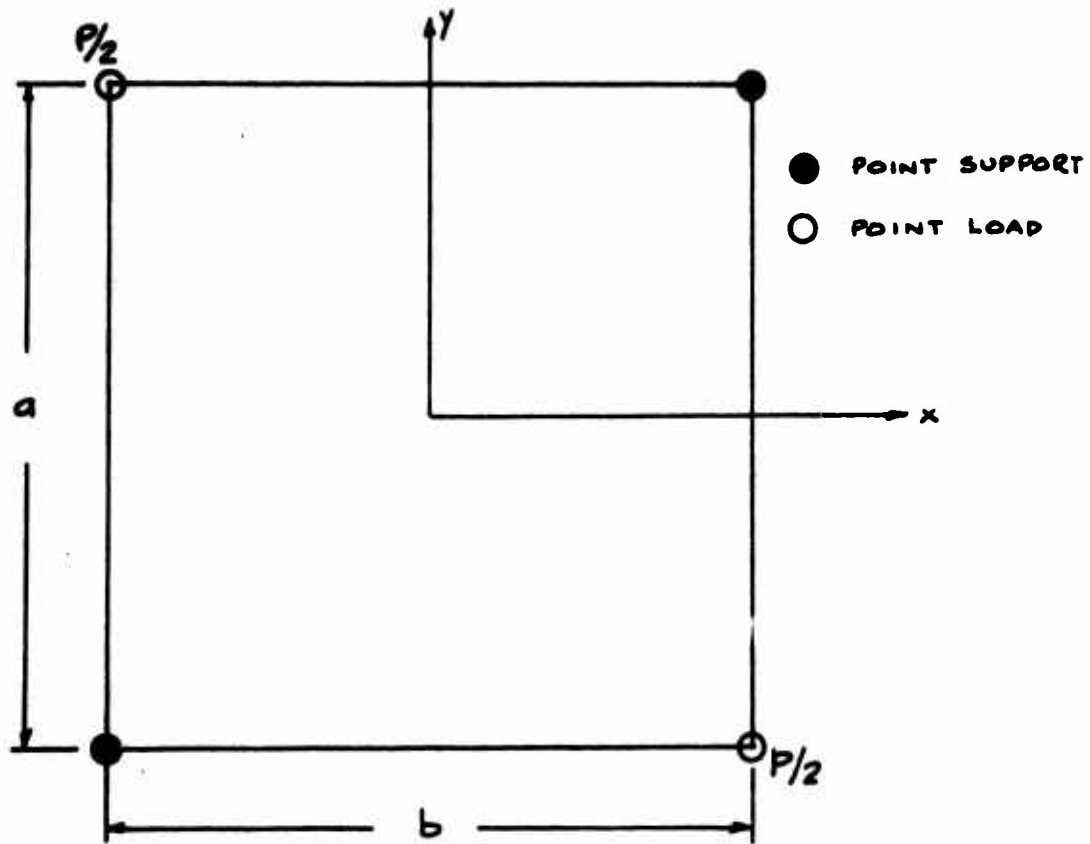
Figure 46
Test for Twisting Stiffness

The theory given by Timoshenko and Woinowsky-Krieger
(Ref. 97) shows that the concentrated loads, P/2,
acting at each of the unsupported corners, produce a
torsional moment, M_{xy}, per unit length along each side
of

$$M_{xy} = \frac{P}{4} . \tag{215}$$

The deformation is given by

$$
\begin{aligned}
w &= \frac{6M_{xy}}{t^3} s_{66} xy + Ax + By + C \\
&= \frac{3P}{2t^3} s_{66} xy + Ax + By + C .
\end{aligned}
\tag{216}
$$

Since the supports are at the same height and the deflection at the supports is zero $(w = 0)$, the constants A, B, C can be determined. Only one measurement, at

$$
x = 0, \quad y = 0
$$

is required to solve Eq. (216) for S_{66}. Finally, the relationships,

$$
s_{66} = \frac{1}{G_{xy}} ,
$$

$$
D_{xy} = \frac{G_{xy} t^3}{6} + \frac{1}{2} \left(\mu_{xy} D_x + \mu_{yx} D_y \right) ,
$$

are used to find the twisting stiffness.

The same testing procedure can be used for a more direct method of calculation. Since the edges of the twisted plate are straight, the plate is in a condition of pure twist with no shears Q_x and Q_y present. Thus, if the deflection, δ, of one of the unsupported corners $(a/2, -a/2)$ or $(-a/2, a/2)$ is measured, its relationship to the twist is given by the simple relationship,

$$
\frac{\partial^2 w}{\partial x \partial y} = \frac{2\delta}{a^2} .
\tag{217}
$$

From Eq. (19), the torsional rigidity,

$$D_R = \frac{G_{xy} t^3}{6} = \frac{M_{xy}}{\dfrac{\partial^2 w}{\partial x \partial y}} \; .$$

From this and Eqs. (217) and (215),

$$D_R = \frac{P a^2}{8 \delta} = \frac{G_{xy} t^3}{6} \; . \tag{218}$$

This is substituted into

$$D_{xy} = \frac{G_{xy} t^3}{6} + \frac{1}{2} \left(\mu_{xy} D_x + \mu_{yx} D_y \right),$$

to get the torsional stiffness.

Great care must be exercised in performing the twist tests. It must be remembered that the edges are to be kept straight. In testing patterns such as corrugations or dimples, care must be taken to allow each cell of the edge cross-section in the xz- and yz- planes to rotate the same amount. The satisfaction of these two requirements may require stiffening of the edges in such a way that the torsional rigidity of the stiffeners is very low while their bending rigidity is high (thin plates), and perhaps filling the edge cells

in such a way that they can be attached to the stiffeners.

Becker and Gerard (Ref. 323) suggest a somewhat different experimental technique in that a cantilevered plate is twisted. But, of course, the twisted plate has the same shape as that discussed above and the method of calculating the constant is the same.

8.3 Shear Stiffness

When the shear deformations of the orthotropic plate are accounted for (Paragraph 1.8), the shear stiffnesses, D_{Q_x} and D_{Q_y}, must be determined. Libove and Batdorf (Ref. 155) and Libove and Hubka (Ref. 156) discuss a method for this calculation.

The transverse shear stiffness, D_{Q_x}, is found by loading the plate as a beam by a uniformly distributed load (Fig. 47).
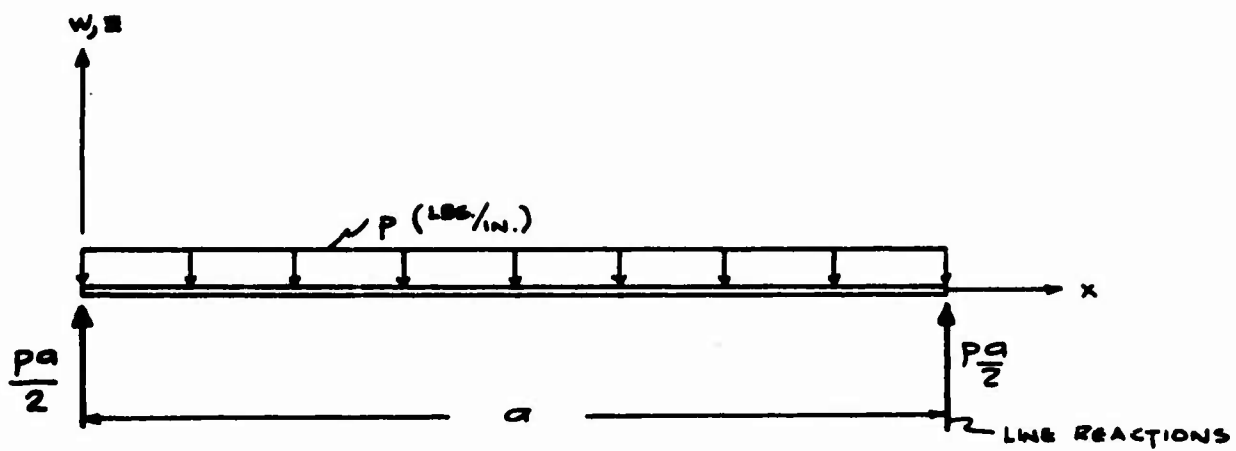
Figure 47
Test for Shear Stiffness

The bending moment at any point,

$$M_x = \frac{1}{b}\left(\frac{pax}{2} - \frac{px^2}{2}\right) \cdot \qquad (219)$$

The rate of change of transverse shear,

$$\frac{\partial Q_x}{\partial x} = -\frac{p}{b} \cdot \qquad (220)$$

The curvature, $\partial^2 w/\partial x^2$, is determined from deflection
measurements along the x-axis. Then, the shear stiff-
ness is found from:

$$
\left.
\begin{aligned}
D_{Q_x} &= \dfrac{\dfrac{\partial Q_x}{\partial x}}{\dfrac{\partial^2 w}{\partial x^2} + \dfrac{M_x}{D_x}} \\[4mm]
&= -\dfrac{\dfrac{P}{b}}{\dfrac{\partial^2 w}{\partial x^2} + \dfrac{1}{D_x b}\left(\dfrac{Pax}{2} - \dfrac{Px^2}{2}\right)} \quad .
\end{aligned}
\right\} \tag{221}
$$

The shear stiffness, D_{Q_y} , is obviously determined
in the same manner.

PART 9 - BIBLIOGRAPHY

A listing of references pertaining to the analysis
and design of orthogonally anisotropic plates follows.
The references are arranged by subject matter in the
following manner:

Section A: Orthotropic Plate Theory
Section B: Sandwich Plates
Section C: Corrugated Plates
Section D: Externally Stiffened Plates
Section E: Integrally Stiffened Plates
Section F: Experimental Determination of
            Orthotropic Plate Characteristics
Section G: Optimization and Synthesis in Design
            of Stiffened Plates

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SECTION A -- ORTHOTROPIC PLATE THEORY
****

1. AKITA,Y. + N.ANDO *BENDING THEORY OF ORTHOTROPIC PLATES AND ITS APPLICATION
   TO THE ANALYSIS OF SHIP STRUCTURES* (JAP.) SEMPAKU, VOL.31, NO.9, 1958

2. AMBARTSUMIAN,S.A. *ON THE THEORY OF BENDING OF ANISOTROPIC PLATES AND SHALLOW
   SHELLS* APPLIED MATHEMATICAL MECHANICS, VOL.24, NO.2,P.500, 1960

3. AMBARTSUMIAN,S.A. *THEORY OF ANISOTROPIC SHELLS* MOSCOW/STATE PUBLISHING
   HOUSE, 1961 (NASA TECH.TRANSL. F-118,MAY 1964)

4. BARES,R. + P.MACHAN. *CLOSED ANALYSIS OF ARTIFICIAL ORTHOTROPIC PLATES*
   (CZECH) ROZPRAVY,CESKOSLOVENSKA AKADEMIE VED, VOL.72, NO.4, 1962

5. PARG,Y.A. *DETERMINATION OF CRITICAL LOAD OF ORTHOTROPIC AND ISOTROPIC PLATES
   AND BEAMS* (UKRANIAN) PRIKLADNAYA MEKHANIKA, VOL.5, NO.1, P.18, 1959

6. BRILLA,J. *ANISOTROPIC PLATES* (CZECH) BRATISLAVA/VYDAVATELSTVO SLOVENSKEJ
   AKADEMIE VIED, 1958

7. BRILLA,J. *SOME MIXED BOUNDARY CONDITIONS OF ANISOTROPIC PLATES* (GER)
   ZEITSCHRIFT FUR ANGEWANDTE MATHEMATIK UND MECHANIK, VOL.40, NO.10/11, P.472,
   OCT/NOV 1960

8. BRILLA,J. *USE OF AFFINE TRANSFURMAIIUNS FOR SOLVING ORTHOTROPIC PLATES*(CZECH)
   STAVEBNICKY CASOPIS, VOL.8, NO.1, P.JK, 1960

9. CHANG,J. *A NEW APPROACH TO SHORT SPAN BRIDGE DESIGN--ORTHOTROPIC PLATE*
   PROCEEDINGS OF 3RD ANNUAL BRIDGE ENGINEERING CONF.,COLORADO STATE UNIV., 1962

10. CONWAY,H.D. *SOME PROBLEMS OF ORTHOTROPIC PLANE STRESS* JOURNAL OF APPLIED
    MECHANICS, VOL.75, P.72, 1953

11. CONWAY,H.D. *STRESS DISTRIBUTIONS IN ORTHOTROPIC STRIPS* JOURNAL OF APPLIED
    MECHANICS, VOL.77, P.353, 1955

12. CORNELIUS,W. *CALCULATION OF SURFACE LOAD-CARRYING CAPACITY THROUGH THE THEORY
    OF ORTHOTROPIC PLATES* (GER) STAHLBAU, VOL.21, NO.2, 1952

13. COULL,A. *DIRECT STRESS ANALYSIS OF ORTHOTROPIC BRIDGE SLABS* ASCE JOURNAL OF
    THE STRUCTURAL DIVISION, VOL.90, NO.ST2, APR 1964

14. COULL,A. *THE STRESS ANALYSIS OF ORTHOTROPIC BRIDGE SLABS* QUARTERLY JOURNAL
    OF MECHANICS AND APPLIED MATHEMATICS, VOL.17, NO.4, P.437, NOV 1964

15. DAS,Y.C. *BUCKLING OF RECTANGULAR ORTHOTROPIC PLATES* APPLIED SCIENTIFIC
    RESEARCH, VOL.AXI, 1963

16. EWELL,W.W., S.OKUBO, + J.I.ABRAMS, *DEFLECTIONS IN GRIDWORKS AND SLABS* TRANS-
    ACTIONS OF ASCE, VOL.117, 1952

17. FREYER,R. *ON THE STABILITY OF ANISOTROPIC PLATES* (GER) TECH.BERICHTE DER
    Z.W.B., VOL.11, NO.7, P.231, JUL 1944

18. GALLAGHER.R.H. + R.D.HUFF. *DERIVATION OF THE FORCE-DISPLACEMENT PROPERTIES OF
TRIANGULAR AND QUADRILATERAL ORTHOTROPIC PLATES IN PLANE STRESS AND BENDING*
BELL AEROSPACE CORP.. REPT.D2114-950005. JAN 1964

19. GATEWOOD.B.E. *APPLIED AND THERMAL STRESSES IN STIFFENED NONISOTROPIC PLATES
WITH VARIABLE THICKNESS* AIAA 6TH STRUCTURES AND MATERIALS CONFERENCE. P.12.
APR 1965

20. GEDGILI.H.S. *BENDING OF ORTHOTROPIC RECTANGULAR PLATES. SIMPLY SUPPORTED
ALONG TWO EDGES. BY LATERAL LOADS* (FR) PUBLICATION OF THE INTERNATIONAL
ASSOCIATION OF BRIDGE AND STRUCTURAL ENGINEERING. VOL.11. P.111. 1951

21. GIRKMANN.K. + R.BEER. *APPLICATION OF THE EXACT REISSNER THEORY TO ORTHOTROPIC
PLATES* (GER) OSTERREICHISCHES INGENIEUR-ARCHIV. VOL.12. NO.1/2. P.101. NOV
1958

22. GLADWELL.G.M.L. *UNIQUENESS THEOREM FOR THIN CLAMPED ANISOTROPIC PLATES*
MATHEMATIKA. VOL.4. NO.1. P.70. JUNE 1957

23. GREENSPON.J.E. *AN APPROXIMATE METHOD FOR OBTAINING THE FREQUENCIES. DEFLEC-
TIONS. AND STRESSES IN SANDWICH AND CROSS-STIFFENED RECTANGULAR PLATES* J.G.
ENGINEERING ASSOCIATES. TR NO.1. 1960

24. GREENSPON.J.E. *PLASTIC DEFORMATION OF UNSTIFFENED AND STIFFENED RECTANGULAR
PLATES* DAVID TAYLOR MODEL BASIN. TECH.REPT.NO.3. FEB 1964

25. HAJEK.J. *AN APPLICATION OF THE SUMMATION METHOD TO THE ANALYSIS OF PLATES*
(CZECH) STAVEBNICKY CASOPIS. VOL.4. P.213. 1962

26. HAYASHI.K. *ELASTIC CONSTANTS AND RIGIDITY OF ORTHOTROPIC PLATES* (JAP)
JOURNAL OF THE AERONAUTICAL SOCIETY OF JAPAN. VOL.8. NO.79. 1941

27. HAYASHI.K. *EQUATION OF DEFLECTION OF ORTHOTROPIC PLATES AND THE EFFECTIVENESS
OF RECTANGULAR PLATES AFTER COMPRESSIVE BUCKLING* (JAP) JOURNAL OF THE AERONAU
TICAL SOCIETY OF JAPAN. VOL.9. NO.83. 1942

28. HEARMON.R.F.S. *THE ELASTIC CONSTANTS OF ANISOTROPIC MATERIALS* REVIEWS OF
MODERN PHYSICS. VOL.18. P.409. 1946

29. HEARMON.R.F.S. + E.H.ADAMS. *THE BENDING AND TWISTING OF ANISOTROPIC PLATES*
BRITISH JOURNAL OF APPLIED PHYSICS. VOL.3. P.150. MAY 1952

30. HECK.O.S. + H.EBNER. *METHODS AND FORMULAS FOR CALCULATING THE STRENGTH OF
PLATE AND SHELL CONSTRUCTIONS AS USED IN AIRPLANE DESIGN* NACA TM-785. 1936

31. HENDRY.A.W. + L.G.JAEGER. *THE ANALYSIS OF GRID FRAMEWORKS AND RELATED STRUCT-
URES* ENGLEWOOD CLIFFS/PRENTICE-HALL. 1959

32. HERZOG.M. *COMPUTATION OF ANISOTROPIC PLATES WITH UNIFORMLY DISTRIBUTED LOAD
ACCORDING TO THE STRIP METHOD* (GER) SCHWEIZERISCHE BAUZEITUNG. VOL.72. NO.32.
P.457. AUG 1954

33. HESS.T.E. *DESIGN OF ORTHOTROPIC BEAMS AND PLATES UNDER UNIFORM LATERAL PRES-

SURE* GENERAL ELECTRIC CO., MISSILE AND SPACE VEHICLE DEPT., MEMO 123-78, 1959

34. HESS,T.E. + J.R.VINSON. *BEHAVIOR OF ORTHOTROPIC STRUCTURES. FINAL REPORT*
    GENERAL ELECTRIC CO., MISSILE AND SPACE VEHICLE DEPT., DOCUMENT 61SD46, MAR1961

35. HOMBERG,H. + J.WEINMEISTER. *INFLUENCE SURFACES FOR GRIDWORKS* (GER) BERLIN,
    1956

36. HU,L.W. *LOAD-CARRYING CAPACITIES FOR RECTANGULAR PLATES OF ANISOTROPIC
    MATERIALS* AIR FORCE REPT.AFOSR- 65-0314, JAN 1965

37. HUBER,M.T. *ON THE ELASTIC THEORY OF ANISOTROPIC PLATES* (GER) STEPHEN TIMO-
    SHENKO 60TH ANNIVERSARY VOLUME, NEW YORK/MACMILLAN, 1938

38. HUBER,M.T. *SEVERAL APPLICATIONS OF THE BENDING THEORY OF ORTHOTROPIC PLATES*
    (GER) ZEITSCHRIFT FUR ANGEWANDTE MATHEMATIK UND MECHANIK. VOL.6, NO.3 1926

39. HUBER,M.T. *STATIC PROBLEMS OF TECHNICALLY IMPORTANT ORTHOTROPIC PLATES* (GER)
    WARSAW/GEBETHNER AND WOLFF, 1929

40. HUBER,M.T. *THE FOUNDATION FOR A RATIONAL CALCULATION OF REINFORCED CONCRETE
    WITH CROSS-WISE REINFORCING* (GER) ZEITSCHRIFTDES OSTERREICHISCHEN INGENIEUR
    UND ARCHITEKTEN-VEREINES, VOL.66, NO.30, P.557, 1914

41. HUBER,M.T. *THEORY OF PLATES* (POL) LWOW, 1921

42. HUFFINGTON,N.J. *THEORETICAL DETERMINATION OF RIGIDITY PROPERTIES OF
    ORTHOGONALLY STIFFENED PLATES* JOURNAL OF APPLIED MECHANICS, VOL.23, NO.1, P.
    15, MAR 1956

43. IKEDA,K. *PLANE STRESS OF ORTHOTROPIC PLATES* (JAP) JOURNAL OF THE AERO-
    NAUTICAL SOCIETY OF JAPAN, VOL.9, NO.90, 1942

44. ILLING,E. *THE BENDING OF THIN ANISOTROPIC PLATES* QUARTERLY JOURNAL OF AP-
    PLIED MATHEMATICS, VOL.5, PART 1, P.12, MAR 1952

45. IWINSKI,T. + J.NOWINSKI. *THE PROBLEM OF LARGE DEFLECTIONS OF ORTHOTROPIC
    PLATES* (POL) ARCHIWUM MACHANIKI STOSOWANEJ, VOL.9, NO.5, P.593, 1957

46. JAVOR,T. *THE ANALYSIS OF SKEW ORTHOTROPIC PLATES BY THE SINGULARITY METHOD*
    (CZECH) STAVEBNICKY CASOPIS, VOL.9,NO.4,P.195, 1961

47. KACZKOWSKI,Z. *ON ORTHOTROPIC RECTANGULAR PLATES WITH FREE EDGES* ARCHIVUM
    MECHANIKI STOSOWANEJ, VOL.7, NO.4, P.457, 1955

48. KACZKOWSKI,Z. *ORTHOTROPIC RECTANGULAR PLATES WITH ARBITRARY BOUNDARY CONDI-
    TIONS* ARCHIVUM MECHANIKI STOSOWANEJ, VOL.8, NO.2, P.179, 1956

49. KACZKOWSKI,Z. *RECTANGULAR ORTHOTROPIC THIN PLATES WITH ARBITRARY BOUNDARY
    CONDITIONS* ARCHIVUM MECHANIKI STOSOWANEJ, VOL.10, NO.4, P.525, 1958

50. KAFKA,V. *STRESS AND DEFORMATION OF A HOMOGENEOUS, TRANSVERSELY ISOTROPIC
    HALFSPACE UNDER UNIFORMLY DISTRIBUTED RECTANGULAR LOAD* (CZECH) APLIKACE
    MATEMATIKY, VOL.7, NO.1, P.37, 1962

51. KAO,J.-S. *LIMIT ANALYSIS OF ORTHOTROPIC PLATES* PH.D.DISSERTATION, NORTH-
WESTERN UNIVERSITY, 1964

52. KLIMOV,S.A. *CALCULATION OF STRUCTURALLY ORTHOTROPIC PLATES FOR FLEXURE* (RUSS)
REF.ZHURNAL MEKHANIKY, NO.8, 1965

53. KOLLBRUNNER,C.F. + M.MEISTER. *BUCKLING, THEORY AND COMPUTATION OF PLATES*
(GER) BERLIN/SPRINGER/VERLAG, 1958

54. KOROLEV,V.I. *ON THE CALCULATION OF STIFFENED PLATES AND SHELLS* (RUSS) IN-
ZHENERNII SBORNIK,AKADEMIIA NAUK,USSR, VOL.26, P.21, 1958

55. KOVARIK,V. *RECTANGULAR ORTHOTROPIC PLATES* (CZECH) STAVEBNICKY CASOPIS, VOL.
9, NO.1, P.49, 1961

56. KOVARIK,V. *THE ANALYSIS OF SKEW ORTHOTROPIC PLATES BY VARIATIONAL METHODS
OF SUCCESSIVE APPROXIMATIONS* (CZECH) STAVEBNICKY CASOPIS, VOL.10, NO.3, P.167
1962

57. KRUG,S. + P.STEIN. *INFLUENCE SURFACES OF ORTHOGONAL ANISOTROPIC PLATES*
BERLIN/SPRINGER-VERLAG, 1961

58. LEKHNITSKY,S.G. *ANISOTROPIC PLATES* (RUSS) MOSCOW/GOSUD.IZDAT.TEKH.-TEOR.LIT.
1957

59. LEKHNITSKY,S.G. *THEORY OF ELASTICITY OF AN ANISOTROPIC ELASTIC BODY* (ENGLISH
TRANSLATION BY P.FERN) SAN FRANCISCO/HOLDEN-DAY, 1963

60. LIGHTFOOT,E. *A GRID FRAMEWORK ANALOGY FOR LATERALLY LOADED PLATES* INTER-
NATIONAL JOURNAL OF MECHANICAL SCIENCES, VOL.6, P.201, 1964

61. MADER,F.W. *CALCULATION OF ORTHOTROPIC PLATE SUBJECTED TO DISTRIBUTED LOAD,
BOUNDARY BENDING MOMENTS AND BOUNDARY DEFLECTIONS* (GER) STAHLBAU, VOL.26,
P.131, 1957

62. MEDWADOWSKI,S.J. *A REFINED THEORY OF ELASTIC, ORTHOTROPIC PLATES* JOURNAL OF
APPLIED MECHANICS, VOL.25, NO.4, P.437, DEC 1958

63. MEDWADOWSKI,S.J. *ON A REFINED THEORY OF ELASTIC ORTHOTROPIC PLATES* PH.D.
DISSERTATION,UNIV.OF CALIFORNIA,BERKELEY, JUNE 1956

64. MELKONIAN,A.P. + A.A.KHACHATRIAN. *STABILITY OF RECTANGULAR TRANSVERSELY
ISOTROPIC PLATES* (RUSS) PRIKLADNAIA MEKHANIKA, VOL.11, NO.2, P.29, 1966

65. MORKOVIN,V. *ON THE DEFLECTION OF ANISOTROPIC THIN PLATES* QUARTERLY OF AP-
PLIED MATHEMATICS, VOL I, P.116, 1943

66. MORLEY,L.S.D. *BENDING OF RECTANGULAR ORTHOTROPIC PLATES UNDER CONCENTRATED
LOAD WITH TWO OPPOSITE EDGES SIMPLY SUPPORTED* ROYAL AIRCRAFT ESTABLISHMENT,
REPT.TR-65189, AUG 1963

67. MOSSAKOWSKI,J. *SINGULAR SOLUTIONS IN THE THEORY OF ORTHOTROPIC PLATES*
ARCHIVUM MECHANIKI STOSOWANEJ, VOL.6, NO.3, 1954

68. MOSSAKOWSKI.J. *SINGULAR SOLUTIONS OF ANISOTROPIC PLATES* ARCHIVUM MECHANIKI
    STOSOWANEJ. VOL.7. NO.1. 1955

69. NARUOKA.M. *BENDING THEORY OF ORTHOTROPIC PLATES AND ITS APPLICATION* (JAP)
    JOURNAL OF THE SOCIETY OF CIVIL ENGINEERS.JAPAN. VOL.31. NO.10. 1956

70. NORRIS.C.B. *STRENGTH OF ORTHOTROPIC MATERIALS SUBJECTED TO COMBINED STRESSES*
    FOREST PRODUCTS LAB.. REPT.1816

71. NOWACKI.W. *A CONTRIBUTION TO THE THEORY OF ORTHOTROPIC PLATES* ACTA TECHNICA.
    ACADEMIAE SCIENTIARUM.BUDAPEST. VOL.8. NO.1/2. P.109. 1954

72. PANC.V. *GENERALIZED THEORY OF LOW SHEAR-STRENGTH ORTHOTROPIC PLATES OF
    CONSTANT THICKNESS* (GER) ACTA TECHNICA.CSAV. VOL.10. NO.5. P.527. 1965

73. PFLUGER.A. *THE BUCKLING OF ANISOTROPIC RECTANGULAR PLATES* MCDONNELL AIRCRAFT
    CORP.. REPT.1537. OCT 1949

74. PLANTEMA.F.J. + P.J.SEVENHUYSEN. *BENDING OF ORTHOTROPIC PLATES UNDER TRANS-
    VERSE LOADING. AND THE ANALOGY WITH BEAMS ON AN ELASTIC LAYER* WISSENSCHAFT-
    LICHE GESELLSCHAFT FUR LUFT- UND RAUMFAHRT. EUROPAISCHER LUFTFAHRTKONGRESS.
    6TH. MUNICH. SEPT 1965

75. RAJAPPA.N.R. *ON THE BENDING OF RECTANGULAR ORTHOTROPIC PLATES WITH SMALL
    INITIAL CURVATURE* JOURNAL OF THE ROYAL AERONAUTICAL SOCIETY. VOL.68. NO.640.
    P.280. APR 1964

76. RAJAPPA.N.R. *ON THE DEFLECTION OF RECTANGULAR ORTHOTROPIC PLATES UNDER LATERAL
    LOADS* JOURNAL OF THE ROYAL AERONAUTICAL SOCIETY. VOL.68. NO.643. P.483. JULY
    1964

77. RAJAPPA.N.R. + D.V.REDDY. *ANALYSIS OF AN ORTHOTROPIC PLATE BY MACLAURIN'S
    SERIES* JOURNAL OF THE ROYAL AERONAUTICAL SOCIETY. VOL.67. P.126. FEB 1963

78. REISSNER.E. *ON THE THEORY OF BENDING OF ELASTIC PLATES* JOURNAL OF MATHE-
    MATICS AND PHYSICS. VOL.23. P.184. 1944

79. ROBINSON.J.R. *THE BUCKLING AND BENDING OF ORTHOTROPIC SANDWICH PANELS WITH ALL
    EDGES SIMPLY SUPPORTED* AERONAUTICAL QUARTERLY. VOL.6. NO.2. P.125. 1955

80. SARKISIAN.V.S. + L.A.MOVSISIAN. *METHOD FOR DETERMINING CRITICAL LOADS OF
    ANISOTROPIC PLATES* (RUSS) INZHENERSKYI ZHURNAL. VOL.5. NO.4. P.777. 1965

81. SAWCZUK.A. *ON THE THEORY OF ANISOTROPIC PLASTIC PLATES AND SHELLS* ARCHIVUM
    MECHANIKI STOSOWANEJ. VOL.13. NO.3. P.355. 1961

82. SAWCZUK.A. *SOME PROBLEMS OF LOAD-CARRYING CAPACITIES OF ORTHOTROPIC AND NON-
    HOMOGENEOUS PLATES* ARCHIVUM MECHANIKI STOSOWANEJ. VOL.8. NO.4. P.549. 1956

83. SCHULTZ.H.G. *LOAD-CARRYING CAPACITY OF ORTHOTROPIC PLATES LOADED IN COMPRES-
    SION* (GER) STAHLBAU. VOL.33. NO.4. P.123. APR 1964

84. SEYDEL.E. *ON THE BUCKLING OF RECTANGULAR ISOTROPIC OR ORTHOGONALLY ANISO-

TROPIC PLATES UNDER COMPRESSIVE STRESS* (GER)   INGENIEUR-ARCHIV. VOL.4. P.169.
1933

85. SHOW-WEN.Y. + H.KEH-CHIH.   *AN APPROXIMATE THEORY OF ELASTIC ORTHOTROPIC PLATES
WITH TRANSVERSE SHEAR DEFORMATIONS* (CHINESE)   ACTA MECHANICA SINICA. VOL.6.
NO.4. DEC 1963

86. SHULESHKO.P.   *A REDUCTION METHOD FOR BUCKLING PROBLEMS OF ORTHOTROPIC PLATES*
AERONAUTICS QUARTERLY. VOL.8. PART 2. P.145. MAY 1957

87. SMITH.C.B.   *SOME NEW TYPES OF ORTHOTROPIC PLATES LAMINATED OF ORTHOTROPIC
MATERIALS*   JOURNAL OF APPLIED MECHANICS. VOL.20. NO.2. P.286. JUNE 1953

88. SOKOLNIKOFF.I.S.   *APPROXIMATE METHODS OF SOLUTION OF TWO-DIMENSIONAL PROBLEMS
IN ANISOTROPIC ELASTICITY*   PROCEEDINGS OF THE 3RD SYMPOSIUM IN APPLIED
MATHEMATICS OF THE AMERICAN MATHEMATICAL SOCIETY. 1949

89. SOLECKI.R.   *GENERAL SOLUTION FOR A THIN ORTHOTROPIC RECTANGULAR PLATE*
BULLETIN DE L'ACADEMIA POLONAISE DES SCIENCES.WARSAW. VOL.8. NO.8. P.399. 1960

90. SOPER.W.G.   *LARGE DEFLECTION OF STIFFENED PLATES*   JOURNAL OF APPLIED MECHANICS
VOL.25. NO.4. P.444. DEC 1958

91. SUCHAR.M.   *COMPUTATION BY MEANS OF POLYNOMIALS OF INFLUENCE SURFACES FOR
ANISOTROPIC PLATES WITH FINITE DIMENSIONS*   ARCHIVUM MECHANIKI STOSOWANEJ. VOL.
10. NO.5. P.615. 1958

92. SUCHAR.M.   *GENERAL FORM OF THE SURFACE OF DEFLECTION OF A THIN ANISOTROPIC
PLATE IN A MULTICONNECTED REGION*   BULLETIN DE L'ACADEMIA POLONAISE DES
SCIENCES.WARSAW. VOL.8. NO.2. P.69. 1960

93. SUCHAR.M.   *ON SINGULAR SOLUTIONS IN THE THEORY OF ANISOTROPIC PLATES*   BULLETIN
DE L'ACADEMIE POLONAISE DES SCIENCES.WARSAW. VOL.12. NO.1. 1964

94. TARAPOREWALLA.K.J.   *DESIGN OF GRID AND DIAGRID SYSTEMS ON THE ANALOGY OF
DESIGN OF PLATES*   STRUCTURAL ENGINEER. VOL.36. NO.4. P.121. APR 1958

95. THIELEMANN.W.   *BUCKLING OF ANISOTROPIC PLATE STRIPS* (GER)   DEUTSCHE VERSUCH-
SANSTALT LUFTFAHRT. REPT. 16. JUNE 1956

96. TIMOSHENKO.S. + J.M.GERE.   *THEORY OF ELASTIC STABILITY*   NEW YORK/MC GRAW-HILL.
P.403. 1961

97. TIMOSHENKO.S. + S.WOINOWSKY-KRIEGER.   *THEORY OF PLATES AND SHELLS*   NEW YORK/
MC GRAW-HILL. P.364. 1959

98. TOPFER.H.-J.   *UNIQUENESS AND PRACTICAL CALCULATION OF SOLUTIONS OF BOUNDARY-
VALUE PROBLEMS IN THE THEORY OF ORTHOGONALLY ANISOTROPIC PLATES* (GER)   ZEIT-
SCHRIFT FUR ANGEWANDTE MATHEMATIK UND MECHANIK. VOL.42. NO.4/5. APR/MAY 1962

99. TRENKS.K.   *CONTRIBUTION TO THE CALCULATION OF ORTHOGONAL ANISOTROPIC RECT-
ANGULAR PLATES* (GER)   BAUINGENIEUR. VOL.29. NO.10. P.372. 1954

100. TROMBSKI.M.   *BENDING OF TWO COUPLED ORTHOTROPIC PLATE STRIPS OF DIFFERING

RIGIDITY UNDER THE EFFECT OF UNIFORMLY DISTRIBUTED LOADS* (POL) MECHANIKA,
NO.15, P.23, 1965

101. VINSON,J.R. *NEW TECHNIQUES OF SOLUTION FOR PROBLEMS IN THE THEORY OF ORTHO-
TROPIC PLATES* PH.D.DISSERTATION,UNIV.OF PENNSYLVANIA, 1961

102. VINSON,J.R. + M.A.BRULL, *APPROXIMATE SOLUTIONS FOR RECTANGULAR ORTHOTROPIC
PLATES* UNIV.OF PENNSYLVANIA,TOWNE SCHOOL, TECH.REPT. 1, CONTRACT NONR 551
(44), 1964

103. VINSON,J.R. + M.A.BRULL, *NEW TECHNIQUES OF SOLUTION FOR PROBLEMS IN THE
THEORY OF ORTHOTROPIC PLATES* PERGAMON/PROCEEDINGS OF FOURTH U.S. NATIONAL
CONGRESS,BERKELEY, P.817, 1962

104. VITOLS,V., R.J.CLIFTON + T.AU, *ANALYSIS OF COMPOSITE BEAM BRIDGES BY ORTHO-
TROPIC PLATE THEORY* ASCE JOURNAL OF THE STRUCTURAL DIVISION, VOL.89, NO.ST4,
P.71, AUG 1963

105. WILDE,P. *RECTANGULAR ANISOTROPIC PLATE WITH CLAMPED EDGES* ARCHIVUM MECHANIKI
STOSOWANEJ, VOL.12, NO.2, P.241, 1960

106. WILDE,P. *THE GENERAL SOLUTION FOR A RECTANGULAR ORTHOTROPIC PLATE EXPRESSED
BY A DOUBLE TRIGONOMETRIC SERIES* ARCHIVUM MECHANIKI STOSOWANEJ, VOL.10, NO.
5, P.746, 1958

107. WITTRICK,W.H. *CORRELATION BETWEEN SOME STABILITY PROBLEMS FOR ORTHOTROPIC
AND ISOTROPIC PLATES UNDER BIAXIAL AND UNIAXIAL DIRECT STRESS* AERONAUTICAL
QUARTERLY, VOL.4, PART1, P.83, AUG 1952

108. WOINOWSKY-KRIEGER,S. *ON THE DEFLECTION OF ORTHOTROPIC PLATE STRIPS BY MEANS
OF CONCENTRATED LOADS* (GER) INGENIEUR-ARCHIV, VOL.25, NO.2, P.90, 1957

109. YAMANA,M. *ON THE ELASTIC STABILITY OF AEROPLANE STRUCTURES* (JAP) JOURNAL
OF THE FACULTY OF ENGINEERING,TOKYO UNIV., VOL.20, NO.8, P.163, 1933

110. YETTRAM,A.L. + H.M.HUSAIN, *GRID-FRAMEWORK METHOD FOR PLATES IN FLEXURE*
ASCE JOURNAL OF THE ENGINEERING MECHANICS DIVISION, VOL.91, NO.EM3, JUNE 1965

111. YUSUFF,S. *LARGE DEFLECTION THEORY FOR ORTHOTROPIC RECTANGULAR PLATES SUB-
JECTED TO EDGE COMPRESSION* JOURNAL OF APPLIED MECHANICS, VOL.19, NO.4, P.446,
DEC 1952

112. ZIENKIEWICZ,O.C. + Y.K.CHEUNG, *THE FINITE ELEMENT METHOD FOR ANALYSIS OF
ELASTIC ISOTROPIC AND ORTHOTROPIC SLABS* PROCEEDINGS OF THE INSTITUTION OF
CIVIL ENGINEERS, VOL.28, PAPER 6726, P.471, AUG 1964

SECTION B -- SANDWICH PLATES
****

113. ALEKSANDROVA,A.Y.(ED.) *PROBLEMS IN THE ANALYSIS OF ELEMENTS OF AVIATION CON-
     STRUCTION* (RUSS) MOSCOW/STATE PUBLISHING HOUSE OF DEFENSE INDUSTRY. 1959

114. ALWAN,A.M. *LARGE AND SMALL DEFLECTION ANALYSES APPLIED TO RECTANGULAR SAND-
     WICH PLATES WITH ORTHOTROPIC CORES* PH.D. THESIS. UNIV.OF WISCONSIN. 1963

115. ALWAN,A.M. *LARGE DEFLECTION OF SANDWICH PLATES WITH ORTHOTROPIC CORES* AIAA
     JOURNAL. VOL.2, NO.10,, P.1820, OCT 1964

116. ANDERSON,M.S. *OPTIMUM PROPORTIONS OF TRUSS-CORE AND WEB-CORE SANDWICH PLATES
     LOADED IN COMPRESSION* NASA TN D-98, SEPT 1959

117. ANDERSON,M.S. + R.G.UPDEGRAFF. *SOME RESEARCH RESULTS ON SANDWICH STRUCTURES*
     NACA TN-4009, JUNE 1957

118. BENSON,A.S. + J.MAYERS. *GENERAL INSTABILITY AND FACE WRINKLING OF SANDWICH
     PLATES--UNIFIED THEORY AND APPLICATIONS* AIAA,3RD AEROSPACE SCIENCES MEETING,
     PAPER 66-138, JAN 1966

119. BIJLAARD,P.P. *ON THE OPTIMUM DISTRIBUTION OF MATERIAL IN SANDWICH PLATES IN
     THEIR PLANE* PROCEEDINGS OF THE 1ST U.S.NATIONAL CONGRESS OF APPLIED MECHAN-
     ICS, P.373, 1952

120. BIJLAARD,P.P. *STABILITY OF SANDWICH PLATES* JOURNAL OF AERONAUTICAL SCIENCES,
     VOL.16, NO.9, P.573, SEPT 1949

121. BOLLER,K.H. + C.B.NORRIS. *EFFECT OF SHEAR STRENGTH ON MAXIMUM LOADS OF SAND-
     WICH COLUMNS* FOREST PRODUCTS LAB., REPT NO.1815, MAR 1955

122. BRUHN,E.F. *ANALYSIS AND DESIGN OF FLIGHT VEHICLE STRUCTURES* CINCINNATI/
     TRI-STATE OFFSET, 1965

123. BURNS,A.B. *MINIMUM WEIGHT ANALYSES FOR HONEYCOMB SANDWICH PLATES AND SHELLS*
     LOCKHEED MISSILES AND SPACE CO., REPT.NO.2-60-64-31, JULY 1964

124. CHANG,C.C., I.K.EBCIOGLU + C.H.HAIGHT. *GENERAL STABILITY ANALYSIS OF
     ORTHOTROPIC SANDWICH PANELS FOR FOUR DIFFERENT BOUNDARY CONDITIONS* (GER)
     ZEITSCHRIFT FUR ANGEWANDTE MATHEMATIK UND MECHANIK. VOL.42, NO.9, SEPT 1962

125. CHENG,S. *ON THE THEORY OF BENDING OF SANDWICH PLATES* 4TH U.S. NATIONAL
     CONGRESS OF APPLIED MECHANICS, P.511, 1962

126. *COMPOSITE CONSTRUCTION FOR FLIGHT VEHICLES,PARTS I,II,III* MILITARY HANDBOOK
     23, DEPT. OF DEFENSE

127. DUNDROVA,V., V.KOVARIK, + P.SLAPAK. *THEORY OF SANDWICH PLATES* (GER) ACTA
     TECHNICA,PRAGUE, VOL.9, NO.3, P.281, 1964

128. ERINGEN,A.C. *BENDING AND BUCKLING OF RECTANGULAR SANDWICH PLATES* PROCEEDINGS
     OF THE 1ST U.S.NATIONAL CONGRESS OF APPLIED MECHANICS, P.381, 1952

187

129. FAN.T.C.   *FACE WRINKLING MODE OF BUCKLING OF SANDWICH PANELS* RAND CORP..
     REPT.P-3202. AUG 1965

130. FLUGGE.W.   *THE OPTIMUM PROBLEM OF THE SANDWICH PLATE*   JOURNAL OF APPLIED
     MECHANICS. VOL.19. P.104. 1952

131. FOSS.J.I.   *FOR THE SPACE AGE. A BIBLIOGRAPHY OF SANDWICH PLATES AND SHELLS*
     DOUGLAS AIRCRAFT CO.. REPT.SM-42883. 1962

132. GOODIER.J.N.   *CYLINDRICAL BUCKLING OF SANDWICH PLATES*   JOURNAL OF APPLIED
     MECHANICS. VOL.13. P.253. 1946

133. GOODIER.J.N. + HSU.   *NONSINUSOIDAL BUCKLING MODES OF SANDWICH PLATES*   JOURNAL
     OF AERONAUTICAL SCIENCES. VOL.21. NO.8. P.525. 1954

134. GOODIER.J.N. + I.M.NEOU.   *THE EVALUATION OF THEORETICAL CRITICAL COMPRESSION
     IN SANDWICH PLATES*   JOURNAL OF AERONAUTICAL SCIENCES. VOL.18. P.649. 1951

135. GRAZIANO.E.E.   *ALL-METAL SANDWICHES EXCEPT HONEYCOMB-CORE. AN ANNONATED
     BIBLIOGRAPHY*   LOCKHEED AIRCRAFT CO.. REPT.3-77-62-5/SB-62-8. 1962

136. GUEST.J. + J.SOLVEY.   *ELASTIC STABILITY OF RECTANGULAR SANDWICH PLATES UNDER
     BIAXIAL COMPRESSION*   AERONAUTICAL RESEARCH LAB..MELBOURNE.AUSTRALIA. REPT.
     SM-251. MAY 1957

137. HABIP.L.M.   *A SURVEY OF MODERN DEVELOPMENTS IN THE ANALYSIS OF SANDWICH
     STRUCTURES*   APPLIED MECHANICS REVIEWS. VOL.18. NO.2. P.93. FEB 1965

138. HARRIS.B.J. + W.C.CRISMAN.   *FACE-WRINKLING MODE OF BUCKLING OF SANDWICH PANELS*
     ASCE JOURNAL OF ENGINEERING MECHANICS. VOL.91. NO.EM3. P.93. JUNE 1965

139. HOFF.N.J.   *BENDING AND BUCKLING OF RECTANGULAR SANDWICH PLATES* NACA TN-2225.
     1950

140. HOFF.N.J.   *THE BUCKLING OF SANDWICH STRUCTURAL ELEMENTS*   SHERMAN FAIRCHILD
     FUEL.FUND PAPER. INSTITUTE OF AERONAUTICAL SCIENCES. PREPRINT 165. P.13. 1948

141. HOFF.N.J. + S.E.MAUTNER.   *BENDING AND BUCKLING OF SANDWICH BEAMS*   JOURNAL OF
     AERONAUTICAL SCIENCES. VOL.15. P.707. DEC 1948

142. HOFF.N.J. + S.E.MAUTNER.   *THE BUCKLING OF SANDWICH-TYPE PANELS*   JOURNAL OF
     AERONAUTICAL SCIENCES. VOL.12. NO.3. P.285. JULY 1945

143. HOPKINS.H.G. + S.PEARSON.   *THE BEHAVIOR OF FLAT SANDWICH PANELS UNDER UNIFORM
     TRANSVERSE LOADING*   ROYAL AIRCRAFT ESTABLISHMENT. REPT.SME 3277. 1944

144. HORVAY.G.   *BENDING OF HONEYCOMBS AND PERFORATED PLATES*   JOURNAL OF APPLIED
     MECHANICS. VOL.19. P.122. 1952

145. HUBKA.R.E.. N.F.DOW. + P.SEIDE.   *RELATIVE STRUCTURAL EFFICIENCIES OF FLAT
     BALSA-CORE SANDWICH AND STIFFENED-PANEL CONSTRUCTION*   NACA TN-2514. 1951

146. JOHNSON.J.E.JR. + J.W.SEMONIAN.   *A STUDY OF THE EFFICIENCY OF HIGH-STRENGTH.
     STEEL. CULLULAR-CORE SANDWICH PLATES IN COMPRESSION*   NACA TN-3751. SEP 1956

189

147. KAECHELE,L.E. *MINIMUM WEIGHT DESIGN OF SANDWICH PANELS* RAND CORP., REPT. RM-1895, MAR 1957

148. KEER,L. + B.J.LAZAN. *DAMPING AND FATIGUE PROPERTIES OF SANDWICH CONFIGURA-TIONS IN FLEXURE* ASD, TR-61-646, 1961

149. KIMEL,W.R. *ELASTIC BUCKLING OF A SIMPLY SUPPORTED RECTANGULAR SANDWICH PANEL SUBJECTED TO COMBINED EDGEWISE BENDING AND COMPRESSION* FOREST PRODUCTS LAB., REPT.1857, SEPT 1956

150. KOVARIK,V. *FINITE-DEFLECTION THEORY OF SANDWICH PLATES* ARCHIVUM MECHANIKI STOSOWANEJ, VOL.17, NO.4, P.363, 1965

151. KUENZI,E.W. *MINIMUM WEIGHT STRUCTURAL SANDWICH* FOREST PRODUCTS LAB., REPT. FPL-086, OCT 1965

152. KUENZI,E.W. *STRUCTURAL SANDWICH DESIGN CRITERIA* FOREST PRODUCTS LAB., REPT. 2161, OCT 1959

153. KUENZI,E.W., W.S.ERICKSEN, + J.J.ZAHN. *SHEAR STABILITY OF FLAT PANELS OF SANDWICH CONSTRUCTION* FOREST PRODUCTS LAB., REPT. 1560, MAY 1962

154. LEWIS,W.C. *DEFLECTION AND STRESSES IN A UNIFORMLY LOADED, SIMPLY SUPPORTED, RECTANGULAR SANDWICH PLATE--EXPERIMENTAL VERIFICATION AND THEORY* FOREST PRODUCTS LAB., REPT.1847A, DEC 1956

155. LIBOVE,C. + S.B.BATDORF. *A GENERAL SMALL-DEFLECTION THEORY FOR FLAT SANDWICH PLATES* NACA REPT. NO.899, 1948

156. LIBOVE,C. + R.E.HUBKA. *ELASTIC CONSTANTS FOR CORRUGATED-CORE SANDWICH PLATES* NACA TN-2289, 1951

157. LOWY,M.J. + R.I.JAFFEE. *ON THE DEVELOPMENT OF LOW-COST, FORMABLE, ALL-METAL SANDWICH PANELS WITH CORRUGATED CORES* AEROSPACE ENGINEERING, VOL.20, NO.11, P.14, NOV 1961

158. MARCH,H.W. *FLAT PLATES OF PLYWOOD UNDER UNIFORM OR CONCENTRATED LOADS* FOREST PRODUCTS LAB., REPT.1312, MAR 1942

159. MARCH,H.W. *STRESS-STRAIN RELATIONS IN WOOD AND PLYWOOD CONSIDERED AS ORTHO-TROPIC MATERIALS* FOREST PRODUCTS LAB., REPT.1503, FEB 1944

160. MARCH,H.W. *SUMMARY OF FORMULAS FOR FLAT PLATES OF PLYWOOD UNDER UNIFORM OR CONCENTRATED LOADS* FOREST PRODUCTS LAB., REPT.1300, OCT 1941

161. MARCH,H.W. + C.B.SMITH *BUCKLING LOADS OF FLAT SANDWICH PANELS IN COMPRESSION, VARIOUS TYPES OF EDGE CONDITIONS* FOREST PRODUCTS LAB., REPT.1525, MAR 1956

162. MINDLIN,R.D. *FLEXURAL VIBRATIONS OF ELASTIC SANDWICH PLATES* COLUMBIA UNIV., ONR TR NO.35, 1959

163. MUSHTARI,K.M. *APPLICABILITY OF DIFFERENT THEORIES OF SANDWICH PLATES* (RUSS) IZVESTIYA AKADEMII NAUK,SSSR, OTDELENIE TEKHNICHESKIKH NAUK, NO.6, P.163, 1960

164. NORRIS.C.B. *COMPRESSIVE BUCKLING CURVES FOR FLAT SANDWICH PANELS WITH
     ISOTROPIC FACINGS AND ISOTROPIC AND ORTHOTROPIC CORES* FOREST PRODUCTS LAB.,
     REPT.1854, JAN 1958

165. OLSEN.H. + F.REINITZHUBER, *TWO-LAYERED PLATES* (GER) BERLIN, 1950

166. PENZIEN.J. + T.DIDRIKSSON, *EFFECTIVE SHEAR MODULUS OF HONEYCOMB CELLULAR
     STRUCTURE* AIAA JOURNAL, VOL.2, NO.3, P.531, MAR 1964

167. PLANTEMA.F.J. *SANDWICH CONSTRUCTION--THE BENDING AND BUCKLING OF SANDWICH
     BEAMS, PLATES, AND SHELLS* NEW YORK/JOHN WILEY + SONS, 1966

168. RAVILLE.M.E. *DEFLECTION AND STRESSES IN A UNIFORMLY LOADED, SIMPLY SUPPORTED
     RECTANGULAR SANDWICH PLATE* FOREST PRODUCTS LAB., REPT.1847, SEPT 1962

169. REISSNER.E. *FINITE DEFLECTIONS OF SANDWICH PLATES* JOURNAL OF AERONAUTICAL
     SCIENCES, VOL.15, NO.7, P.435, JULY 1948

170. *SANDWICH CONSTRUCTION FOR AIRCRAFT, PARTS I AND II* FOREST PRODUCTS LAB.,
     /NC-23 PANEL, U.S.GOVT.PRINTING OFFICE, 1955

171. SEIDE.P. *COMPRESSIVE BUCKLING OF FLAT RECTANGULAR METALITE TYPE SANDWICH
     PLATES WITH SIMPLY SUPPORTED LOADED EDGES AND CLAMPED UNLOADED EDGES* NACA TN-
     1886, MAY 1949

172. SEIDE.P. *SHEAR BUCKLING OF INFINITELY LONG SIMPLY SUPPORTED METALITE TYPE
     SANDWICH PLATES* NACA TN-1910, 1949

173. SEIDE.P. + E.Z.STOWELL, *ELASTIC AND PLASTIC BUCKLING OF SIMPLY SUPPORTED
     SOLID-CORE SANDWICH PLATES IN COMPRESSION* NACA REPORT 967, 1950

174. SMITH.R.C.T. *BUCKLING OF FLAT PLYWOOD PLATES IN COMPRESSION* AUSTRALIAN
     REPORT ACA-12, 1944

175. STEVENS.G.H. *COMPRESSIVE AND SHEAR PROPERTIES OF TWO CONFIGURATIONS OF
     SANDWICH CORES OF CORRUGATED FOIL* FOREST PRODUCTS LAB., REPT.1889, DEC 1962

176. THURSTON.G.A. *BENDING AND BUCKLING OF CLAMPED SANDWICH PLATES* JOURNAL OF
     AERONAUTICAL SCIENCES, VOL.24, NO.6, P.407, JUNE 1957

177. WANG.C.-T. *PRINCIPLE AND APPLICATION OF COMPLEMENTARY ENERGY METHOD FOR THIN
     HOMOGENEOUS AND SANDWICH PLATES AND SHELLS WITH FINITE DEFLECTIONS* NACA
     TN-2620, 1952

178. WEIKEL.R.C. + A.S.KOBAYASHI, *ON THE LOCAL ELASTIC STABILITY OF HONEYCOMB
     FACE PLATE SUBJECTED TO UNIAXIAL COMPRESSION* JOURNAL OF AERONAUTICAL SCIENCES
     VOL.26, P.672, 1959

179. WEMPNER.G.A. + J.L.BAYLOR, *GENERAL THEORY OF SANDWICH PLATES WITH DISSIMILAR
     FACINGS* INTERNATIONAL JOURNAL OF SOLIDS AND STRUCTURES, VOL.1, NO.2, MAY 1965

180. WEMPNER.G.A. + J.W.MC KINLEY, *CORRUGATED OPEN-FACE SANDWICH PLATE* AEROSPACE
     ENGINEERING, P.56, DEC 1962

190

181. WILLIAMS.D. ET AL. *FLAT SANDWICH PANELS UNDER COMPRESSIVE END LOADS* ROYAL
AIRCRAFT ESTABLISHMENT. REPT.AD-3174. 1941

182. WITTRICK.W.H. *A THEORETICAL ANALYSIS OF THE EFFICIENCY OFSANDWICH CONSTRUC-
TION UNDER COMPRESSIVE END LOAD* AERONAUTICAL RESEARCH COUNCIL RM-2016.
APR 1945

183. YEN.K.T.. S.GUNTURKUN + F.V.POHLE. *DEFLECTIONS OF A SIMPLY SUPPORTED RECT-
ANGULAR SANDWICH PLATE SUBJECTED TO TRANSVERSE LOADS* NACA TN-2581. DEC 1951

184. YEN.K.T.. V.L.SALERNO + N.J.HOFF. *BUCKLING OF RECTANGULAR SANDWICH PLATES
SUBJECTED TO EDGEWISE COMPRESSION WITH LOADED EDGES SIMPLY SUPPORTED AND UN-
LOADED EDGES CLAMPED* NACA TN-2556. JAN 1952

185. YU.Y.-Y. *A NEW THEORY OF SANDWICH PLATES. GENERAL CASE* POLYTECHNIC INST. OF
BROOKLYN. CONTRACT AFOSR TN-59-1163. 1959

186. YU.Y.-Y. *DAMPING OF FLEXURAL VIBRATIONS OF SANDWICH PLATES* JOURNAL OF AERO-
NAUTICAL SCIENCES. VOL.29. P.790. 1962

187. YU.Y.-Y. *FLEXURAL VIBRATIONS OF ELASTIC SANDWICH PLATES* JOURNAL OF AERO-
NAUTICAL SCIENCES. VOL.27. P.272. 1960

188. YU.Y.-Y. *SIMPLIFIED VIBRATION ANALYSIS OF ELASTIC SANDWICH PLATES* JOURNAL
OF AERONAUTICAL SCIENCES. VOL.27. P.894. 1960

189. YU.Y.-Y. + N.REN. *DAMPING PARAMETERS OF LAYERED PLATES AND SHELLS* AIR FORCE
REPT.64-2087. APR 1964

190. YUSUFF.S. *FACE WRINKLING AND CORE STRENGTH IN SANDWICH CONSTRUCTION* JOURNAL
OF THE ROYAL AERONAUTICAL SOCIETY. VOL.64. P.164. 1960

191. YUSUFF.S. *THEORY OF WRINKLING IN SANDWICH CONSTRUCTION* JOURNAL OF THE
ROYAL AERONAUTICAL SOCIETY. VOL.59. P.30. 1955

192. ZAHN.J.J. + S.CHENG. *EDGEWISE COMPRESSIVE BUCKLING OF FLAT SANDWICH PANELS.
LOADED ENDS SIMPLY SUPPORTED AND SIDES SUPPORTED BY BEAMS* FOREST PRODUCTS
LAB.. REPT.019. FEB 1964

SECTION C -- CORRUGATED PLATES
****

193. ANDREEVA,L.E. *CALCULATION OF THE PROPERTIES OF CORRUGATED MEMBRANES* (RUSS)
     PRIBOROSTROENIE, NO.3, P.11, 1956

194. ASHWELL,D.G. *THE STABILITY IN BENDING OF SLIGHTLY CORRUGATED PLATES* JOURNAL
     OF THE ROYAL AERONAUTICAL SOCIETY, VOL.56, NO.502, P.782, OCT 1952

195. BERGMAN,S. + H.REISSNER, *ON THE BENDING OF CORRUGATED SHEET-STEEL STRIPS
     UNDER TANGENTIAL STRESS* (GER) ZEITSCHRIFT FUR METALLKUNDE, VOL.20, P.475,
     1929

196. CALDWELL,J.B. *BENDING STRENGTH OF CORRUGATED PLATE* ENGINEERING, VOL.174,
     NO.4528, P.609, NOV 1952

197. CALDWELL,J.B. *THE STRENGTH OF CORRUGATED PLATING FOR SHIPS BULKHEADS**
     QUARTERLY TRANSACTIONS OF THE INSTITUTION OF NAVAL ARCHITECTS,LONDON, VOL.97,
     NO.3, P.495, JULY 1955

198. DEAN,W.R. *THE ELASTIC STABILITY OF A CORRUGATED PLATE* PROCEEDINGS OF THE
     ROYAL SOCIETY,LONDON, VOL.111, P.144, 1926

199. HARINX,J.A. *DESIGN OF CORRUGATED DIAPHRAGMS* ASME ANNUAL MEETING, PAPER
     55-A-112, NOV.1955

200. HARRIS,L.A. + R.R.AUELMANN, *STABILITY OF FLAT, SIMPLY SUPPORTED, CORRUGATED-
     CORE SANDWICH PLATES UNDER COMBINED LOADS* JOURNAL OF AEROSPACE SCIENCES, VOL.
     27, P.525, JULY 1960

201. LEQUEUX,P. *STRUCTURES OF CORRUGATED STAINLESS STEEL* (FR) ASSOCIATION FRANC-
     AISE DES INGENIEURS ET TECHNICIENS DE L'AERONAUTIQUE ET DE L'ESPACE,7TH, 1965

202. OKUBO,H. *THE INFLUENCE OF THE FORM OF CORRUGATION UPON THE STRENGTH OF A
     CORRUGATED PLATE, PART I* REP.INST.OF HIGH SPEED MECHANICS, TOHOKU UNIV.,
     VOL.1 P.69, 1951

203. PLAINEVAUX,J.E. *OPTIMUM PROFILE OF CORRUGATED PLATES AND PILE TANKS*
     ACADEMIE ROYALE DE BELGIQUE BULLETIN DE LA CLASSE SCIENCES, VOL.40, NO.9, P.
     962, 1954

204. PLATUS,D.L. + S.UCHIYAMA, *VERY-LARGE-DEFLECTION BEHAVIOR OF CORRUGATED STRIPS*
     AIAA JOURNAL, VOL.3, NO.8, P.1549, AUG 1965

205. SHIBUYA,I. *SOME NOTES ON CORRUGATED PLATES* (JAP) JOURNAL OF THE AERONAUT-
     ICAL SOCIETY OF JAPAN, VOL.7, NO.61, 1940

206. TORVIK,P.J. + B.J.LAZAN, *A CORRUGATED ADDITION FOR INCREASED DAMPING IN
     FLEXURE* WRIGHT-PAT.AIR FORCE BASE,A.F.MATERIALS LAB., AFML-TR-64-373, JAN
     1965

207. WEBER,C. *ON THE EFFECTIVE WIDTH OF A BIAXIALLY CORRUGATED STEEL PLATE RE-
     INFORCED BY STIFFENERS* (GER) BAUINGENIEUR, VOL.28, NO.3, P.172, AND NO.5,
     P.172, 1953

SECTION D -- EXTERNALLY STIFFENED PLATES
****

208. ALLEN.D.N.D. + R.T.SEVERN *COMPOSITE ACTION OF BEAMS AND SLABS UNDER TRANS-
     VERSE LOADING.PART III. A RECTANGULAR SLAB WITH REPEATED SPANWISE BEAMS WHOSE
     FLEXURAL STIFFNESS VARIES ACROSS THE SPAN* STRUCTURAL ENGINEER. VOL.40.NO.6.
     JUNE 1962

209. ANDO.N. *ON THE STRENGTH OF ORTHOGONALLY STIFFENED PLATES (1ST REPORT. THEORET-
     ICAL SOLUTION OF ORTHOTROPIC PLATE SUBJECTED TO BENDING)* REPORT OF THE TRANSP
     ORTATION TECHNICAL RESEARCH INSTITUTE.TOKYO. NO.48. MAR 1962

210. ANDO.N. *ON THE STRENGTH OF ORTHOGONALLY STIFFENED PLATES (2ND REPORT. EXPERI-
     MENTAL STUDIES ON BENDING OF ORTHOGONALLY STIFFENED PLATE AND COMPARISON WITH
     BENDING THEORY OF ORTHOTROFIC PLATE)* REPORT OF THE TRANSPORTATION TECHNICAL
     RESEARCH INSTITUTE. NO.53. NOV 1962

211. AU.T. + J.C.L.CHANG. *FUNDAMENTALS OF ORTHOTROPIC PLATE DESIGN* AISC ENGINEER-
     ING JOURNAL. VOL.1. NO.2. P.34. APR 1964

212. BURNS.A.B. + J.SKOGH. *COMBINED LOADS MINIMUM WEIGHT ANALYSIS OF STIFFENED
     PLATES AND SHELLS* AIAA JOURNAL OF SPACECRAFT AND ROCKETS. VOL.3. P.235.
     FEB 1966

213. BURNS.A.B. + J.SKOGH. *MINIMUM WEIGHT ANALYSIS OF STIFFENED PLATES AND SHELLS
     SUBJECTED TO COMBINED LOADS* LOCKHEED MISSILES AND SPACE CO.. REPT.NO.
     2-60-64-29. JUNE 1964

214. CALDWELL.J.B. *ELASTIC INSTABILITY OF STIFFENED SHEET UNDER COMPRESSION REACTED
     BY SHEAR* JOURNAL OF THE ROYAL AERONAUTICAL SOCIETY. VOL.63. NO.582. P.366.
     JUNE 1959

215. CHANG.J. *ORTHOTROPIC-PLATE CONSTRUCTION FOR SHORT-SPAN BRIDGES* CIVIL ENGINE-
     ERING. P.53. DEC 1961

216. CHANG.J. *ORTHOTROPIC PLATE DECK SHOWS MERIT IN SHORT SPAN HIGHWAY BRIDGES*
     MODERN WELDED STRUCTURES. VOL.1. P.71. JAN 1964

217. CHAPMAN.J.C. + J.E.SLATFORD. *BENDING OF PLATING WITH WIDELY SPACED STIF-
     FENERS* INSTITUTION OF MECHANICAL ENGINEERS. PREPRINT. NOV 1956

218. COX.H.L. + J.R.RIDDELL. *BUCKLING OF A LONGITUDINALLY STIFFENED FLAT PANEL*
     BRITISH A.R.C..REPT.NO.11374. APR 1948

219. CRAWFORD.R.F. *MINIMUM WEIGHT POTENTIAL FOR STIFFENED PLATES AND SHELLS* AIAA
     JOURNAL. VOL.1. NO.4. P.879. APR 1963

220. CSONKA.P. *APPLICATION OF THE THEORY OF ORTHOTROPIC PLATES TO OBTAIN RESULTS OF
     LOADING TESTS ON REINFORCED CONCRETE RIB SLABS* (GER) BAUTECHNIK. VOL.35.
     NO.11. P.436. NOV 1958

221. *DESIGN MANUAL FOR ORTHOTROPIC STEEL PLATE DECK BRIDGES* AMERICAN INSTITUTE OF
     STEEL CONSTRUCTION. N.Y.. NOV 1962

222. DOW,N.F. *DESIGN CHARTS FOR LONGITUDINALLY STIFFENED WING COMPRESSION PANELS*
SOCIETY OF AUTOMOTIVE ENGINEERS, QUARTERLY TRANSACTIONS, VOL.3, P.122, 1949

223. DOW,N.F. + W.A.HICKMAN, *COMPARISON OF THE STRUCTURAL EFFICIENCY OF PANELS
HAVING STRAIGHT-WEB AND CURVED-WEB Y-SECTION STIFFENERS* NACA TN-1787,
JAN 1949

224. DOW,N.F. + W.A.HICKMAN, *DESIGN CHARTS FOR FLAT COMPRESSION PANELS HAVING
LONGITUDINAL EXTRUDED Y-SECTION STIFFENERS AND COMPARISON WITH PANELS HAVING
FORMED Z-SECTION STIFFENERS* NACA TN-1389, AUG 1947

225. DOW,N.F. + W.A.HICKMAN, *DIRECT-READING DESIGN CHARTS FOR 24S-T3 ALUMINUM-
ALLOY FLAT COMPRESSION PANELS HAVING LONGITUDINAL FORMED HAT-SECTION STIFFENERS
AND COMPARISONS WITH PANELS HAVING Z-SECTION STIFFENERS* NACA TN-2792, MAR
1952

226. DOW,N.F. + W.A.HICKMAN, *DIRECT-READING DESIGN CHARTS FOR 75S-T ALUMINUM-ALLOY
FLAT COMPRESSION PANELS HAVING LONGITUDINAL STRAIGHT-WEB Y-SECTION STIFFENERS*
NACA TN-1640, AUG 1948

227. DOW,N.F., W.A.HICKMAN + H.L.MC CRACKEN, *COMPRESSIVE-STRENGTH COMPARISONS OF
PANELS HAVING ALUMINUM-ALLOY SHEET AND STIFFENERS WITH PANELS HAVING MAGNESIUM-
ALLOY SHEET AND ALUMINUM-ALLOY STIFFENERS* NACA TN-1274, 1947

228. DOW,N.F., W.A.HICKMAN, + B.W.ROSEN, *DATA ON THE COMPRESSIVE STRENGTH OF SKIN-
STRINGER PANELS OF VARIOUS MATERIALS* NACA TN-3064, JAN 1954

229. DOW,N.F., R.E.HUBKA + W.M.ROBERTS, *DIRECT-READING DESIGN CHARTS FOR 24S-T
ALUMINUM ALLOY FLAT COMPRESSION PANELS HAVING LONGITUDINAL STRAIGHT-WEB Y-
SECTION STIFFENERS* NACA TN-1777, JAN 1949

230. DOW,N.F. + A.S.KEEVIL,JR., *DIRECT-READING DESIGN CHARTS FOR 24S-T ALUMINUM-
ALLOY FLAT COMPRESSION PANELS HAVING LONGITUDINAL FORMED Z-SECTION STIFFENERS*
NACA TN-1778, JAN 1949

231. EVANS,J.H. *A STRUCTURAL ANALYSIS AND DESIGN INTEGRATION WITH APPLICATION TO
THE MIDSHIP SECTION CHARACTERISTICS OF TRANSVERSELY FRAMED SHIPS* TRANSACTIONS
OF S.N.A.M.E., VOL.66, P.244, 1958

232. FELDMAN,M.R. *STABILITY OF ORTHOTROPIC PLATES WITH STEP-WISE VARIABLE
STIFFNESS* (RUSS) SOVIET PHYSICS-DOKLADY(AMERICAN INST.OF PHYSICS), VOL.53,
NO.4, P.353, OCT 1961

233. FLOOR,W.K.G. + T.J.BURGERHOUT, *EVALUATION OF THE THEORY ON THE POST-BUCKLING
BEHAVIOUR OF STIFFENED, FLAT, RECTANGULAR PLATES SUBJECTED TO SHEAR AND
NORMAL LOADS* NATIONAL LUCHT LAB.,AMSTERDAM, REPT.S770, 1951

234. GERARD,G. *EFFICIENT APPLICATIONS OF STRINGER PANEL AND MULTICELL WING CON-
STRUCTION* JOURNAL OF AERONAUTICAL SCIENCES, VOL.16, P.35, 1949

235. GERARD,G. *HANDBOOK OF STRUCTURAL STABILITY--PART V, COMPRESSIVE STRENGTH OF
FLAT STIFFENED PANELS* NACA TN-3785, AUG 1957

236. GERARD,G. + H.BECKER, *HANDBOOK OF STRUCTURAL STABILITY--PART VII,STRENGTH OF

THIN-WING CONSTRUCTION* NASA TN-D-162, SEPT 1959

237. GIENCKE,E. *ANALYSIS OF RIBBED PLATES, PARTS 1 AND 2* (GER) STAHLBAU, VOL.29,
     NO.1, P.1, JAN 1960, AND VOL.29, NO.2, P.47, FEB 1960

238. GIENCKE,E. *BASIC EQUATIONS FOR ORTHOTROPIC PLATES WITH ECCENTRIC STIFFENERS*
     (GER) STAHLBAU, VOL.24, P.128, 1955

239. GIENCKE,E. *FUNDAMENTAL EQUATIONS FOR ORTHOTROPIC PLATE WITH ECCENTRIC
     STIFFENERS* (GER) STAHLBAU, VOL.26, NO.6, P.128, JUNE 1955

240. GIULIANI,G.C. *SLAB REINFORCED BY A SET OF EQUIDISTANT RIBS* (ITAL) INGEGNERE,
     VOL.35, NO.10, P.922, OCT 1961

241. GOMZA,A. + P.SEIDE, *MINIMUM WEIGHT DESIGN OF SIMPLY SUPPORTED TRANSVERSELY
     STIFFENED PLATES UNDER COMPRESSION* NACA TN-1710, 1948

242. HARLANDER,L.A. *OPTIMUM PLATE-STIFFENER ARRANGEMENT FOR VARIOUS TYPES OF
     LOADING* JOURNAL OF SHIP RESEARCH, VOL.4, P.49, 1960

243. HICKMAN,W.A. + N.F.DOW. *DIRECT-READING DESIGN CHARTS FOR 75S-T6 ALUMINUM-
     ALLOY FLAT COMPRESSION PANELS HAVING LONGITUDINAL EXTRUDED Z-SECTION STIF-
     FENERS* NACA TN-2435, FEB 1952

244. HOLMES,M. *STIFFENED PLATING UNDER TRANSVERSE LOAD* QUARTERLY JOURNAL OF
     MECHANICS AND APPLIED MATHEMATICS, VOL.12, PART 4, 1959

245. IRVING,J. + N.MULLINEUX. *RECTANGULAR PLATES WITH STRINGERS AND RIBS* JOURNAL
     OF AERONAUTICAL SCIENCES, VOL.21, NO.12, P.847, DEC 1954

246. JAEGER,L.G. + A.W.HENDRY. *THE ANALYSIS OF INTERCONNECTED BRIDGE GIRDERS BY THE
     DISTRIBUTION OF HARMONICS* THE STRUCTURAL ENGINEER, VOL.34, P.241, 1956

247. JOJIC,K.V. *THE BUCKLING OF A REINFORCED RECTANGULAR PLATE* 9TH CONGRESS OF
     INTERNATIONAL APPLIED MECHANICS,UNIV. OF BRUSSELS, VOL.7, P.56, 1957

248. KEMPNER,J. *APPLICATION OF NUMERICAL PROCEDURE TO THE STRESS ANALYSIS OF
     STRINGER-REINFORCED PANELS* NACA ARR L5C09A, 1945

249. KENNEDY,J.B. + M.W.HUGGINS, *SERIES SOLUTION OF SKEWED STIFFENED PLATES* ASCE
     JOURNAL OF THE ENGINEERING MECHANICS DIVISION, VOL.90, NO.EM1, FEB 1964

250. KLITCHIEFF,J.M. *ON THE STABILITY OF PLATES REINFORCED BY LONGITUDINAL RIBS*
     JOURNAL OF APPLIED MECHANICS, VOL.18, NO.4, P.364, DEC 1951

251. KONDO,J. + A.OSTAPENKO. *TEST ON LONGITUDINALLY STIFFENED PLATE PANELS WITH
     FIXED ENDS. STRAIN AND DIAL GAGE READINGS* LEHIGH UNIV. FRITZ ENGINEERING LAB.
     REPT. 248.12, JULY 1964

252. KOSOFSKY,F. *ECCENTRICALLY STIFFENED ORTHOTROPIC PLATE, WITH VARIOUS BOUNDARY
     CONDITIONS* PH.D.DISSERTATION, CARNEGIE INSTITUTE OF TECHNOLOGY, DEPT. OF C.E.
     1964

253. LUNDQUIST,E.E. + E.Z.STOWELL. *CRITICAL COMPRESSIVE STRESS FOR OUTSTANDING

FLANGES*   NACA TN-734, 1942

254. MASSONET,C.   *ORTHOTROPIC PLATES AND ORTHOTROPIC CYLINDRICAL SHELLS WITH ASYM-
     METRIC RIBS* (FR) PUBLICATIONS OF THE INTERNATIONAL ASSOCIATION FOR BRIDGE AND
     STRUCTURAL ENGINEERING,ZURICH, VOL.19, P.201, 1959

255. MAZURKIEWICZ,Z.   *BENDING AND BUCKLING OF RECTANGULAR PLATE REINFORCED
     TRANSVERSELY BY RIBS WITH VARIABLE RIGIDITIES* BULLETIN ACAD.POLONAISE SCI.,
     VOL.10, NO.8, 1962

256. MC ELMAN,J.A., M.M.MIKULAS + M.STEIN, *STATIC AND DYNAMIC EFFECTS ON ECCENTRIC
     STIFFENING OF PLATES AND CYLINDRICAL SHELLS*   AIAA 2ND ANNUAL MEETING, SAN
     FRANCISCO, PAPER 65-370, JULY 1965

257. MICHAEL,D.   *THE STRUCTURAL ACTION OF MULTI-STRINGER SHEAR PANELS*   INTER-
     NATIONAL JOURNAL OF MECHANICAL SCIENCE, VOL.7, NO.8, AUG 1965

258. MICKS,W.R.   *A METHOD OF ESTIMATING THE COMPRESSIVE STRENGTH OF OPTIMUM SHEET-
     STIFFENER PANELS FOR ARBITRARY MATERIAL PROPERTIES, SKIN THICKNESS AND STIF-
     FENER SHAPES* JOURNAL OF AERONAUTICAL SCIENCES, VOL.20, P.705, 1953

259. MILEK,W.A.JR.   *AISC ORTHOTROPIC PLATE DESIGN MANUAL* AISC ENGINEERING JOURNAL,
     VOL.1, NO.2, P.40, APR 1964

260. MUCKLE,W.   *BULKHEADS WITH VERTICAL STIFFENERS AND HORIZONTAL GIRDERS* TRANS-
     ACTIONS OF THE NORTHEAST COAST INSTITUTION OF ENGINEERS AND SHIPBUILDERS, VOL.
     79, NO.3, JAN 1963

261. NEWMARK,N.M.   *A DISTRIBUTION PROCEDURE FOR THE ANALYSIS OF SLABS CONTINUOUS
     OVER FLEXIBLE BEAMS* UNIV.OF ILLINOIS ENGINEERING EXPERIMENT STATION, BULLETIN
     NO.304

262. NOWACKI,W.   *THE STABILITY OF RECTANGULAR PLATES WITH RIBS* BULLETIN ACADEMIE
     POLONAISE SCIENCE, VOL.2, NO.2, P.85, 1954

263. PETERSON,F.F., E.S.JOHNSON + E.N.JACOBS, *ORTHOGONALLY STIFFENED STEEL PLATES*
     STANFORD UNIV.MECANICAL ENGR.LAB., NAVY CONTRACT N6-ONR-25513, 1949

264. PFLUGER,A.   *BUCKLING PROBLEM OF ORTHOTROPIC PLATES WITH HOLLOW STIFFENERS*
     (GER) ZEITSCHRIFT FUR FLUGWISENSCHAFTEN, VOL.5, NO.6, P.178, JUNE 1957

265. PFLUGER,A.   *ORTHOTROPIC PLATES WITH HOLLOW STIFFENERS* (GER) OSTERREICHISCHES
     INGENIEUR-ARCHIV, VOL.2, P.199, 1955

266. POCHOP,F.   *ON THE STABILITY OF LONG, EQUIDISTANTLY, TRANSVERSELY STIFFENED
     RECTANGULAR PLATES* (GER) OSTERREICHISCHES INGENIEUR-ARCHIV,WIEN, VOL.6, NO.5,
     P.387, 1952

267. RADOK,J.R.M.   *THE STABILITY OF STIFFENED PLATES AND SHELLS* GRONINGER,
     HOLLAND/NOORDHOFF, 1955

268. RICHART,F.E., N.M.NEWMARK + C.P.SIESS, *HIGHWAY BRIDGE FLOORS* UNIV. OF
     ILLINOIS ENGINEERING EXPERIMENT STATION, BULLETIN 25

269. ROCKEY.K.C. + I.T.COOK. *INFLUENCE OF THE TORSIONAL RIGIDITY OF TRANSVERSE STIFFENERS UPON THE SHEAR BUCKLING OF STIFFENED PLATES* AERONAUTICAL QUARTERLY VOL.15, NO.2, P.198, MAY 1964

270. ROCKEY.K.C. + I.T.COOK. *SHEAR BUCKLING OF CLAMPED AND SIMPLY-SUPPORTED INFINITELY LONG PLATES REINFORCED BY TRANSVERSE STIFFENERS AND A CENTRAL LONG-ITUDINAL STIFFENE #* AERONAUTICAL QUARTERLY, VOL.13, NO.2, P.95, MAY 1962

271. ROSENHAUPT.S. *CALCULATION OF SLABS WITH LATERAL STIFFENERS* (GER) BAUTECHNIK, VOL.41, NO.2, P.48, FEB 1964

272. ROTH.C.E. + D.LIU. *COMPUTER APPLICATIONS TO THE STRUCTURAL DESIGN OF A COMBINATION FRAMED MIDSHIP SECTION* M.I.T. DEPT.OF NAVAL ARCHITECTURE + MARINE ENGR.. REPT.NO.66-1, 1966

273. SCHADE.H.A. *APPLICATION OF ORTHOTROPIC PLATE THEORY TO SHIP BOTTOM STRUCTURE* PROCEEDINGS OF THE 5TH INTERNATIONAL CONGRESS OF APPLIED MECHANICS.WILEY, P.140, 1938

274. SCHADE.H.A. *BENDING THEORY OF SHIP BOTTOM STRUCTURE* TRANSACTIONS OF SNAME, VOL.46, P.176, 1938

275. SCHADE.H.A. *DESIGN CURVES FOR CROSS-STIFFENED PLATING UNDER UNIFORM BENDING LOAD* TRASACTIONS OF SNAME, VOL.49, 1941

276. SCHADE.H.A. *THE ORTHOGONALLY STIFFENED PLATE UNDER UNIFORM LATERAL LOAD* JOURNAL OF APPLIED MECHANICS, VOL.7, NO.4, P.143, 1940

277. SCHUETTE.E.H. *CHARTS FOR THE MINIMUM-WEIGHT DESIGN OF 24S-T ALUMINUM-ALLOY FLAT COMPRESSION PANELS WITH LONGITUDINAL Z-SECTION STIFFENERS* NACA TR-827, 1945

278. SCHUMANN.H. *ANALYSIS OF ORTHOGONALLY ANISOTROPIC RECTANGULAR STEEL PLATES CONSISTING OF A PLANE SHEET METAL COVER AND IN ADDITION WITH ONE SIDE HAVING DISCONTINUOUSLY ARRANGED TRIANGULAR HOLLOW RIBS WITH TORSIONAL STIFFNESS* (GER) STAHLBAU, VOL.32, NO.6, P.166, JUNE 1963

279. SCHUMANN.H. *ANALYSIS OF ORTHOGONAL OR ANISOTROPIC RECTANGULAR PLATES WITH CONSIDERATION OF DISCONTINUOUS ARRANGEMENT OF RIBS* (GER) STAHLBAU, VOL.29, NO 10, P.302, OCT 1960

280. SEIDE.P. *THE EFFECT OF LONGITUDINAL STIFFENERS LOCATED ON ONE SIDE OF A PLATE ON THE COMPRESSIVE BUCKLING STRESS OF THE PLATE-STIFFENER COMBINATION* NACA TN-2837, JAN 1953

281. SEIDE.P. + M.STEIN. *COMPRESSIVE BUCKLING OF SIMPLY SUPPORTED PLATES WITH LONGITUDINAL STIFFENERS* NACA TN-1825, 1949

282. SEYDEL.E. *CONTRIBUTION TO THE PROBLEM OF BUCKLING OF STIFFENED PLATES DUE TO COMPRESSIVE STRESS* (GER) LUFTFAHRTFORSCHUNG, BAND 8, P.71, JULY 1930

283. SIESS.C.P., N.M.NEWMARK, ET AL., *STUDIES OF SLAB AND BEAM HIGHWAY BRIDGES* PARTS I AND II, UNIV. OF ILLINOIS BULLETINS 363,375,396,405,416

197

284. SOKOLOWSKI.M. *THE STABILITY OF AN ORTHOTROPIC INFINITELY LONG PLATE WITH
     RIBS* ARCHIVUM MECHANIKI STOSOWANEJ, VOL.8, NO.4, P.507, 1956

285. STRASSER.A. *BUCKLING OF STIFFENED PLATES* (GER) OSTERREICHISCHES INGENIEUR
     ARCHI , VOL.7, NO.3, P.262, 1953

286. SUJATA.H.L. *THE EFFECTIVE PLATE WIDTH OF LONGITUDINALLY STIFFENED PLATES AT
     PLASTIC BUCKLING* JOURNAL OF AEROSPACE SCIENCES, VOL.29, NO.3, MAR 1962

287. SYMONDS.M.F. *MINIMUM WEIGHT DESIGN OF A SIMPLY SUPPORTED TRANSVERSELY STIF-
     FENED PLATE LOADED IN SHEAR* JOURNAL OF AERONAUTICAL SCIENCES, VOL.23, NO.7,
     P.685, JULY 1956

288. TERAZAWA.K. + Y.TERADA, *THE MUTUAL INTERFERENCE BETWEEN LONGITUDINAL AND
     TRANSVERSE MEMBERS IN SHIP STRUCTURES* (JAP) JOURNAL OF THE SOCIETY OF KANSAI
     NAVAL ARCHITECTS, NO.73, 1954

289. TIMOSHENKO.S. *ON THE STABILITY OF STIFFENED PLATES* U.S.EXPERIMENTAL MODEL
     BASIN.WASHINGTON NAVY YARD, REPT.402, 1935

290. TOLOTTI.C. + G.GRIOLI, *ON THE CALCULATION OF PLATES WITH RIBS* (ITAL)
     GIORNALE DEL GENIO CIVILE, VOL.87, P.275, JUNE 1949

291. TSUIJI.T. *STRENGTH OF LONGITUDINALLY STIFFENED PLATE PANELS WITH LARGE B/T*
     LEHIGH UNIV.INSTITUTE OF RESEARCH.FRITZ ENGR.LAB.REPT. 248.14, JUNE 1965

292. WAH.T.(EDITOR) *A GUIDE FOR THE ANALYSIS OF SHIP STRUCTURES* U.S.DEPT.OF
     COMMERCE, OFFICE OF TECHNICAL SERVICES, 1960

293. WAH.T. *INVESTIGATION OF STIFFENED STEEL PLATES* SOUTHWEST RESEARCH INSTIT-
     UTE, CONTRACT NOBS-92130, APRIL 1966

294. WANG.C.-T. + H.ZUCKERBERG: *INVESTIGATION OF STRESS DISTRIBUTION IN RECT-
     ANGULAR PLATES WITH LONGITUDINAL STIFFENERS UNDER AXIAL COMPRESSION AFTER
     BUCKLING* NACA TN-2671, APR 1952

295. WILDE.P. *AN ORTHOTROPIC PLATE WITH THIN-WALLED ASYMMETRIC RIBS* (POL)
     ROZPRAWY INZINIERSKIE, VOL.7, NO.3, P.275, 1959

296. WOLCHUK.R. *ORTHOTROPIC PLATE DESIGN FOR STEEL BRIDGES* CIVIL ENGINEERING,
     P.38, FEB 1959

297. WOLCHUK.R. *STEEL DECK BRIDGES WITH LONG RIB SPANS* CIVIL ENGINEERING, P.53,
     FEB 1964

298. YAMAKI.N. *BENDING OF STIFFENED PLATES UNDER UNIFORM NORMAL PRESSURE* REPORT
     OF THE INSTITUTE OF HIGH-SPEED MECHANICS.TOHOKU UNIV., REPT.137, VOL.14, P.115,
     1962/63

299. YOUNGER.D.G. *THE DEVELOPMENT OF OPTIMUM DESIGN ENVELOPE CURVES FOR SHEET-
     STIFFENER COMPRESSION PANELS HAVING HAT, Z, OR Y-SECTION STIFFENERS FOR
     VARIOUS MATERIALS AND TEMPERATURE ENVIRONMENTS* PROCEEDINGS OF THE IST MID-
     WESTERN CONFERENCE ON SOLID MECHANICS, P.5, APR 1953

300. YUSUFF.S. *BUCKLING PHENOMENA OF STIFFENED PANELS* JOURNAL OF AEROSPACE
     SCIENCES. VOL.25. P.507, AUG 1958

SECTION E -- INTEGRALLY STIFFENED PLATES
****

301. CATCHPOLE,E.J. *THE OPTIMUM DESIGN OF COMPRESSION SURFACES HAVING UNFLANGED
     INTEGRAL STIFFENERS*   JOURNAL OF THE ROYAL AERONAUTICAL SOCIETY, VOL.58, P.756,
     NOV 1954

302. CONTINI,R. *INTEGRALLY STIFFENED ALUMINUM EXTRUSION*   PRODUCT ENGINEERING,
     VOL.22, P.129,,DEC 1951

303. CRAMP,E.E., + R.C.JUSTICE, *ANALYSIS OF INTEGRALLY-STIFFENED SKIN STRUCTURES*
     THE BOEING CO.,AERO-SPACE DIV., REPT.D5-13235, MAR 1966

304. CRAWFORD,R.F. *A THEORY FOR THE ELASTIC DEFLECTIONS OF PLATES INTEGRALLY
     STIFFENED ON ONE SIDE*   NACA TN-3646, APR 1956

305. CRAWFORD,R.F. + C.LIBOVE, *SHEARING EFFECTIVENESS OF INTEGRAL STIFFENING*
     NACA TN-3443, JUNE 1955

306. DOW,F., L.R.LEVIN, + J.L.TROUTMAN, *ELASTIC BUCKLING UNDER COMBINED STRESSES
     OF FLAT PLATES WITH INTEGRAL WAFFLE-LIKE STIFFENING* NACA TN-3059, 1954

307 DOW,N.F., C.LIBOVE, + R.E.HUBKA, *FORMULAS FOR THE ELASTIC CONSTANTS OF PLATES
     WITH INTEGRAL WAFFLE-LIKE STIFFENING* NACA REPORT 1195, 1954

308. FARMER,G.D.,JR., + W.B.SPANGLER, *INVESTIGATION OF CALOTTAN SHEET STIFFENING
     PROCESS* U.S.ARMY RESEARCH AND DEVELOPMENT LABS., FT.BELVOIR, REPT. 1727-TR,
     OCT 1962

309. HELLER,C.O. *STRESSES AND DEFORMATIONS OF DIMPLED PLATES* MARINE ENGINEERING
     LAB., R + D REPT.245/64, AUG 1964

310. JOHNSTON,G.S. + R.B.LANTZ, *EFFICIENCY OF WAFFLE GRID PANELS UNDER COMPRESSIVE
     LOADING* AIAA JOURNAL OF AIRCRAFT, VOL.2, NO.5, P.437, SEP/OCT 1965

311. KEEN,E.D. *INTEGRAL CONSTRUCTION--ITS APPLICATION TO AIRCRAFT DESIGN AND ITS
     EFFECT ON PRODUCTION METHODS* JOURNAL OF THE ROYAL AERONAUTICAL SOCIETY, VOL.
     57, NO.508, P.215, APR 1953

312. LEGG,K.L.C. *INTEGRAL CONSTRUCTION--A SURVEY AND AN EXPERIMENT* JOURNAL OF THE
     ROYAL AERONAUTICAL SOCIETY, VOL.58, NO.523, P.485, JULY 1954

313. PFEFFER,F. *REINFORCED CONCRETE WAFFLE PLATES* (GER) OSTERREICHISCHES
     INGENIEUR ZEITSCHRIFT, VOL.7, NO.10, P.323, OCT 1964

314. ROEGE,B. *WAFFLE BEADING FOR STIFFENING SHEET METAL PANELS* PRODUCT ENGINEER-
     ING, VOL.26, NO.8, P.171, AUG 1955

315. SANDORFF,P.E. *DESIGN OF STRUCTURAL MODELS WITH APPLICATION TO STIFFENED
     PANELS UNDER COMBINED SHEAR AND COMPRESSION* JOURNAL OF AERONAUTICAL SCIENCES,
     VOL.23, NO.7, P.623, JULY 1956

316. SANDORFF,P.E. + G.W.PAPEN, *INTEGRALLY STIFFENED STRUCTURES* AERONAUTICAL
     ENGINEERING REVIEW, VOL.9, NO.2, P.30, FEB 1950

317. SCHMIT,L.A. + T.P.KICHER. *STRUCTURAL SYNTHESIS OF SYMMETRIC WAFFLE PLATE*
     NASA TN-D-1691, 1962

318 SCHMIT,L.A., T.P.KICHER + W.M.MORROW. *STRUCTURAL SYNTHESIS CAPABILITY FOR
    INTEGRALLY STIFFENED WAFFLE PLATES* AIAA JOURNAL, VOL.1, NO.12, P.2820, DEC
    1963

319. SCHMIT,L.A.,+ W.M.MORROW. *METHODS OF WAFFLE PLATE SYNTHESIS* CASE INST.OF
     TECHNOLOGY, EDC REPT. 2-63-3, JUNE 1963

320. SMITH,R.S. + H.GRAY. *DESIGN-STRENGTHENED MATERIALS* JOURNAL OF AERONAUTICAL
     SCIENCES, VOL. , P.213, JULY 1943

321. TEWES,W.A. *INITIAL PHASE OF LIGHTWEIGHT STRUCTURE DEVELOPMENT, THE INVESTIGA-
     TION OF L-SHAPED, BRASS, DIMPLED SHEETS* MARINE ENGINEERING LAB., REPT. 1/65,
     JAN 1965

322. ZAHORSKI,A. *EFFECT OF MATERIAL DISTRIBUTION ON STRENGTH OF PANELS* JOURNAL OF
     AERONAUTICAL SCIENCES, VOL.11, NO.3, 1944

SECTION F -- EXPERIMENTAL DETERMINATION OF ORTHOTROPIC-PLATE CHARACTERISTICS
****

323. BECKER,H. + G.GERARD. *MEASUREMENT OF TORSIONAL RIGIDITY OF STIFFENED PLATES*
NEW YORK UNIV., TECH. REPT. SM-61-14, OCT 1961

324. BECKETT,R.E., R.J.DOHRMAN, + K.D.IVES. *AN EXPERIMENTAL METHOD FOR DETERMINING
THE ELASTIC CONSTANTS OF ORTHOGONALLY STIFFENED PLATES* EXPERIMENTAL MECHANICS
PERGAMON PRESS, 1963

325. BERGSTRASSER,M. *DETERMINATION OF TWO ELASTIC CONSTANTS OF PLATES* (GER)
ZEITSCHRIFT FUR TECHNISCHE PHYSIK, VOL.8, P.355, 1927

326. BORCZ,A. *PLATES WITH STIFFENING BEAMS* (POL) ROZPRAWY INZYNIERSKIE, VOL.6, NO
3, P.351, 1958

327. CLIFTON,R.J., J.C.L.CHANG + T.AU. *ANALYSIS OF ORTHOTROPIC PLATE BRIDGES*
ASCE JOURNAL OF THE STRUCTURAL DIVISION, VOL.89, NO.ST5, OCT 1963

328. HESS,T.E. *ELASTIC CONSTANTS FOR ORTHOTROPIC STRUCTURES* GENERAL ELECTRIC CO.,
MISSILE AND SPACE VEHICLE DEPT., MEMO 125-019, 1960

329. HOPPMANN,W.H.II. *BENDING OF ORTHOGONALLY STIFFENED PLATES* JOURNAL OF AP-
PLIED MECHANICS, VOL.22, NO.2, P.267, 1955

330. HOPPMANN,W.H.II. *ELASTIC COMPLIANCES OF ORTHOGONALLY STIFFENED PLATES* PRO-
CEEDINGS OF THE SOCIETY FOR EXPERIMENTAL STRESS ANALYSIS, VOL.14, NO.1, P.137,
1956

331. HOPPMANN,W.H.II, N.J.HUFFINGTON + L.S.MAGNESS. A STUDY OF ORTHOGONALLY STIF-
FENED PLATES* JOURNAL OF APPLIED MECHANICS, VOL.23, NO.3, P.343, 1956

332. HOPPMANN,W.H.II + W.G.SOPER. *NOTE ON THE BOUNDARY CONDITIONS FOR BENDING
EXPERIMENTS WITH BARS AND PLATES* JOURNAL OF APPLIED MECHANICS, VOL.23, P.
646, 1956

334. HUFFINGTON,N.J. + R.N.SCHUMACHER. *TWISTING OF ORTHOGONALLY STIFFENED PLATES*
MARTIN CO. RESEARCH REPT. 44, AUG 1963

335. KUENZI,E.W. *METHODS FOR DETERMINING THE ELASTIC CONSTANTS OF NON-METALLIC
MATERIALS* SYMPOSIUM ON DETERMINATION OF ELASTIC CONSTANTS,ASTM, P.31, 1952

336. STROUD,W.J. *ELASTIC CONSTANTS FOR BENDING AND TWISTING OF CORRUGATION-STIF-
FENED PANELS* NASA TR R-166, DEC 1963

327. TENNYSON,R.C. *AN EXPERIMENTAL INVESTIGATION OF THE BEHAVIOR OF STIFFENED
PLATES IN AXIAL COMPRESSION* UNIV.OF TORONTO,INSTITUTE OF ASTROPHYSICS, UTIA
TECH.NOTE 57, SEPT 1961

328. THOMAS,F.G. + A.SHORT. *A LABORATORY INVESTIGATION OF SOME BRIDGE DECK
SYSTEMS* PROCEEDINGS OF THE INSTITUTION OF CIVIL ENGINEERS, VOL.:, PART I,
P.125, 1952

329. TSAI,S.W. *EXPERIMENTAL DETERMINATION OF THE ELASTIC BEHAVIOR OF ORTHOTROPIC

PLATES* TRANSACTIONS OF ASME,JOURNAL OF ENGINEERING FOR INDUSTRY, VOL. ,P.315
AUG 1965

330. TSAI,S.W. + G.S.SPRINGER, *THE DETERMINATION OF MODULI OF ANISOTROPIC PLATES*,
JOURNAL OF APPLIED MECHANICS, VOL.85, PART 3, P.467, SEPT 1963

331. WITT,R.K., W.H.HOPPMANN II + R.S.BUXBAUM, *DETERMINATION OF ELASTIC CONSTANTS
CF ORTHOTROPIC MATERIALS WITH SPECIFIC REFERENCE TO LAMINATES* ASTM BULLETIN
NO.194, P.53, DEC 1953

SECTION G -- OPTIMIZATION AND SYNTHESIS IN DESIGN OF STIFFENED PLATES
.***

116 ANDERSON.A.S. *OPTIMUM PROPORTIONS OF TRUSS-CORE AND WEB-CORE SANDWICH PLATES
    LOADED IN COMPRESSION*   NASA TN D-98, SEPT 1959

119. BIJLAARD.P.P. *ON THE OPTIMUM DISTRIBUTION OF MATERIAL IN SANDWICH PLATES IN
    THEIR PLANE* PROCEEDINGS OF THE 1ST U.S.NATIONAL CONGRESS OF APPLIED MECHAN-
    ICS, P.373, 1952

123 BURNS.A.B. *MINIMUM WEIGHT ANALYSES FOR HONEYCOMB SANDWICH PLATES AND SHELLS*
    LOCKHEED MISSILES AND SPACE CO.. REPT.NO.2-60-64-31, JULY 1964

212.BURNS.A.B. + J.SKOGH. *COMBINED LOADS MINIMUM WEIGHT ANALYSIS OF STIFFENED
    PLATES AND SHELLS* AIAA JOURNAL OF SPACECRAFT AND ROCKETS, VOL.3, P.235,
    FEB 1966

213 BURNS.A.B. + J.SKOGH. *MINIMUM WEIGHT ANALYSIS OF STIFFENED PLATES AND SHELLS
    SUBJECTED TO COMBINED LOADS* LOCKHEED MISSILES AND SPACE CO.. REPT.NO.
    2-60-64-29, JUNE 1964

301. CATCHPOLE.E.J. *THE OPTIMUM DESIGN OF COMPRESSION SURFACES HAVING UNFLANGED
    INTEGRAL STIFFENERS* JOURNAL OF THE ROYAL AERONAUTICAL SOCIETY, VOL.58, P.756,
    NOV 1954

214. CRAWFORD.R.F. *MINIMUM WEIGHT POTENTIAL FOR STIFFENED PLATES AND SHELLS* AIAA
    JOURNAL, VOL.1, NO.4, P.879, APR 1963

332 *DEVELOPMENT OF A PROCEDURE FOR AUTOMATED SYNTHESIS OF MINIMUM WEIGHT
    STRUCTURES* AIR FORCE FLIGHT DYNAMICS LAB..WRIGHT-PAT.A.F.BASE,FDL-TDR-64-141,
    OCT 1964

333 EVANS.J.H. + D.KHOUSHY. *OPTIMIZED DESIGN OF MIDSHIP SECTION STRUCTURE* TRANS-
    ACTIONS OF S.N.A.M.E.. VOL.71, P.144, 1963

334 FARRAR.D.J. *THE DESIGN OF COMPRESSION STRUCTURES FOR MINIMUM WEIGHT* JOURNAL
    OF THE ROYAL AERONAUTICAL SOCIETY, VOL.53, P.1041, NOV 1949

130. FLUGGE.W. *THE OPTIMUM PROBLEM OF THE SANDWICH PLATE* JOURNAL OF APPLIED
    MECHANICS, VOL.19, P.104, 1952

335 GELLATLY.R.A., R.H.GALLAGHER + W.A.LUBERACKI. *DEVELOPMENT OF A PROCEDURE FOR
    AUTOMATED SYNTHESIS OF MINIMUM WEIGHT STRUCTURES* AIR FORCE FLIGHT DYNAMICS
    LAB..WRIGHT-PAT.AIR FORCE BASE, REPT.FDL-TDR-64-141, OCT 1964

336. GERARD.G. *MINIMUM WEIGHT ANALYSIS OF COMPRESSION STRUCTURES* NEW YORK/NEW
    YORK UNIV.PRESS, 1956

337 GERARD.G. *MINIMUM WEIGHT ANALYSIS OF ORTHOTROPIC PLATES UNDER COMPRESSIVE
    LOADING* JOURNAL OF AEROSPACE SCIENCES, VOL.27, NO.1, P.21, JAN 1960

338 GERARD.G. *OPTIMUM STRUCTURAL DESIGN CONCEPTS FOR AEROSPACE VEHICLES* AIAA
    JOURNAL OF SPACECRAFT AND ROCKETS, VOL.3, P.5, JAN 1966

241.[*] GOMZA.A. + P.SEIDE. *MINIMUM WEIGHT DESIGN OF SIMPLY SUPPORTED TRANSVERSELY STIFFENED PLATES UNDER COMPRESSION* NACA TN-1710. 1948

242.[*] HARLANDER.L.A. *OPTIMUM PLATE-STIFFENER ARRANGEMENT FOR VARIOUS TYPES OF LOADING* JOURNAL OF SHIP RESEARCH. VOL.4. P.49. 1960

147.[*] KAECHELE.L.E. *MINIMUM WEIGHT DESIGN OF SANDWICH PANELS* RAND CORP.. REPT. RM-1895. MAR 1957

354. KHOUSHY.D. *WEIGHT STRENGTH ANALYSIS OF CARGO SHIPS' STRUCTURES* M.I.T.DEPT. OF NAVAL ARCHITECTURE + MARINE ENGR.. PH.D.DISSERTATION. 1962

151.[*] KUENZI.E.W. *MINIMUM WEIGHT STRUCTURAL SANDWICH* FOREST PRODUCTS LAB.. REPT. FPL-086. OCT 1965

205.[*] PLAINEVAUX.J.E. *OPTIMUM PROFILE OF CORRUGATED PLATES AND PILE TANKS* ACADEMIE ROYALE DE BELGIQUE BULLETIN DE LA CLASSE SCIENCES. VOL.40. NO.9. P. 962. 1954

317.[*] SCHMIT.L.A. + T.P.KICHER. *STRUCTURAL SYNTHESIS OF SYMMETRIC WAFFLE PLATE* NASA TN-D-1691. 1962

318.[*] SCHMIT.L.A.. T.P.KICHER + W.M.MORROW. *STRUCTURAL SYNTHESIS CAPABILITY FOR INTEGRALLY STIFFENED WAFFLE PLATES* AIAA JOURNAL. VOL.1. NO.12. P.2820. DEC 1963

319.[*] SCHMIT.L.A.. + W.M.MORROW. *METHODS OF WAFFLE PLATE SYNTHESIS* CASE INST.OF TECHNOLOGY. EDC REPT. 2-63-3. JUNE 1963

277.[*] SCHUETTE.E.H. *CHARTS FOR THE MINIMUM-WEIGHT DESIGN OF 24S-T ALUMINUM-ALLOY FLAT COMPRESSION PANELS WITH LONGITUDINAL Z-SECTION STIFFENERS* NACA TR-827. 1945

340. SHANLEY.F.R. *WEIGHT-STRENGTH ANALYSIS OF AIRCRAFT STRUCTURES* NEW YORK/DOVER. 1960

287.[*] SYMONDS.M.F. *MINIMUM WEIGHT DESIGN OF A SIMPLY SUPPORTED TRANSVERSELY STIF-FENED PLATE LOADED IN SHEAR* JOURNAL OF AERONAUTICAL SCIENCES. VOL.23. NO.7. P.685. JULY 1956

294.[*] YOUNGER.D.G. *THE DEVELOPMENT OF OPTIMUM DESIGN ENVELOPE CURVES FOR SHEET-STIFFENER COMPRESSION PANELS HAVING HAT. Z. OR Y-SECTION STIFFENERS FOR VARIOUS MATERIALS AND TEMPERATURE ENVIRONMENTS* PROCEEDINGS OF THE 1ST MID-WESTERN CONFERENCE ON SOLID MECHANICS. P.5. APR 1953

* Chronologically listed elsewhere in bibliography.

CONCLUSION

The expressions given in this report should enable
the analyst to calculate the stresses and deformations
of flat plates of various configurations -- sandwich,
corrugated, rib-reinforced, integrally stiffened -- due
to static loading conditions. Various loading possi-
bilities, in combination with a variety of end support
conditions, are described.

Perhaps the greater contribution of this report is
the inclusion of the most complete list of references
of work in the area of orthotropic plates. The bibli-
ography of Part 9 is divided according to subject matter
in such a manner that a researcher in this field will
find it convenient in locating publications pertinent
to his work.

A second report, which will be more suitable for
design, will be published in the near future. In this
future publication, the author will present the infor-
mation concerning orthotropic plates in such a manner
that the designer will be able to make rational com-
parisons between the various configurations of stiffened
plates, based upon some standard criterion, such as
weight or cost. This work is presently taking place,
and it will be presented in a report titled "Orthogonally
Stiffened Plates: A Design Handbook."

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13. ABSTRACT

A compilation of techniques of analysis for orthogonally stiffened, flat, rectangular plates, with various combinations of loading and support conditions, is presented. The specific types of plates considered are sandwich, corrugated, rib-reinforced, and integrally stiffened. Orthotropic plate theory is used as the major form of solution throughout the report.

A complete bibliography, consisting of more than three hundred references, is included.

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14 Key Words	Link A		Link B		Link C	
	Role	Wt	Role	Wt	Role	Wt
Airplane panels						
Anisotropy						
Honeycomb cores						
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Panels (structural)						
Sandwich panels						
Sheets						
Stresses						
Structural properties						
Structures						