

MEMORANDUM

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OCTOBER 1966

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AN ANALYSIS OF  
MAJOR SCHEDULING TECHNIQUES  
IN THE DEFENSE SYSTEMS ENVIRONMENT

J. N. Holtz

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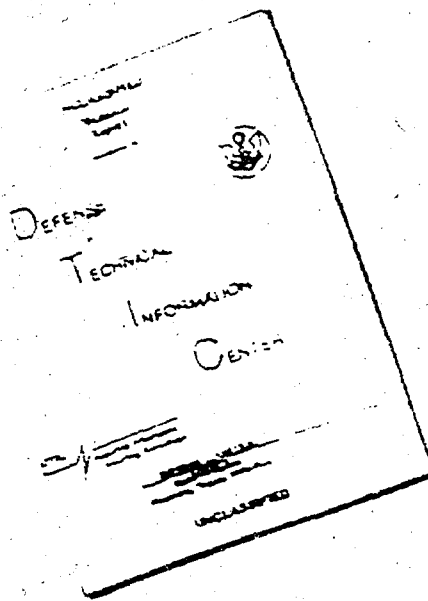
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J. N. Holtz

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PREFACE

The acquisition of a major new weapon system is a complex process involving numerous technologies, agencies, firms, and personnel. If the delivery schedule for a priority military program is to be attained, these diverse factors must be coordinated. Although formal scheduling techniques were in use over a half century ago, the shifting technological environment has made it necessary to adapt old techniques and develop new ones to manage the many factors involved in current weapon system development. Recently, defense requirements have stimulated the generation of numerous scheduling systems, each offering some promise for improving project management. This proliferation of techniques has in turn created pressures for their standardization.

This Memorandum surveys, compares, and evaluates the major scheduling techniques currently available to project management, and suggests areas for improvement. A comparison of existing techniques indicates that although some are relatively advanced, various aspects still require additional development and refinement.

The study should be of interest to scheduling departments ranging from first-line supervision in contractor organizations through system project offices and headquarters groups. It should also be useful in schools or training organizations that instruct personnel in the use of scheduling techniques. Hopefully, it will stimulate efforts to advance the state of the art in scheduling techniques, either by incremental improvements on existing techniques, or through development of substantially new systems.

### SUMMARY

✓ This Memorandum has three main objectives: (1) to describe simply and clearly the major characteristics and operating features of each of the more important scheduling techniques currently available to military management; (2) to compare and evaluate the techniques in terms of their applicability to the acquisition of weapon systems; and (3) to define areas for further research leading to improvement in the scheduling state of the art.

The nature of the systems acquisition environment, with its inherent complexities, is first examined in some detail, and criteria are established for comparing the various scheduling techniques. In describing each system, attention is paid to its appropriateness in the scheduling of both development and production activities. Since the newer and more unique scheduling requirements are generated in the development phase, they are illustrated by applying the essential features of each technique to a common hypothetical missile system development program.

Among the basic scheduling techniques are the

Gantt Chart,  
Milestone Chart,  
Line of Balance Technique,  
Critical Path Method, and the  
Program Evaluation and Review Technique (PERT).

In addition, variations of these have frequently been used by individual organizations for specific applications. The features of each of the basic techniques are described and compared in this Memorandum.

A discussion of the extent to which these techniques satisfy scheduling demands suggests certain areas where additional study is needed to develop a comprehensive and reasonably uniform system covering the total life cycle of a project. As might be expected, each of the techniques has its own most appropriate areas of application. Limitations in alternative applications range from minor to serious, depending on the application. Among the broader observations are the following:

(a) The network is particularly significant as a planning device because

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## I. INTRODUCTION

### THE WEAPON SYSTEM ACQUISITION ENVIRONMENT

The aerospace industry, faced with time deadlines and using sophisticated technology, requires scheduling techniques that are frequently more advanced than those of the more traditional commercially oriented firms. Consequently, the industry has devoted considerable effort in the past decade to advancing the scheduling state of the art. The devices discussed in this Memorandum, however, are not applicable solely to defense-oriented systems. Several are used by industrial firms on various commercial products, and these firms are increasingly adopting the more advanced techniques.

This Memorandum attempts to survey, compare, and evaluate the major scheduling techniques currently available to project management, and to suggest areas for further research that may lead to improving these techniques. To provide a framework for this analysis, the nature of the weapon system acquisition environment must be clearly understood. The following discussion describes several critical dimensions of this environment: the life cycle of a weapon system--its built-in uncertainties and dynamic character--the numerous firms involved in a given project, and the hierarchies of project management existing in corporations and agencies.\*

#### The Life Cycle of a Weapon System

Most, if not all, commercial products have a life cycle. Fad items--hula hoops, for example--have a very short life cycle. Other items--such as stoves or refrigerators--have a longer cycle. Each new product must be conceived, researched, designed, tested, produced, sold, and serve its function before it becomes obsolete.

Defense systems likewise have a life cycle, but their period of usefulness is limited by changing operational requirements and advances

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\* Readers already familiar with this environment may prefer to turn directly to the subsequent material.

in technology. This life cycle usually consists of several phases: (a) conceptual, (b) definition, (c) acquisition (including development and production), and (d) operation.

From a scheduling standpoint, perhaps the most significant characteristic of the life cycle is the change in the type of work performed in each phase. In the conceptual and definition phases, emphasis is on specifying the performance characteristics and hardware configurations that will eventually result for the system. Here the effort is primarily analytical, and activities are usually unique and varied.

In the development phase, the design, fabrication, and testing of a limited number of prototypes are usually the primary functions. Frequently, the vehicles used to test individual performance characteristics may be quite dissimilar. The activities in the development phase, although not highly repetitive, have reached the stage where enough information is available to permit the scheduling of resources to specific functions. In a large weapon system development, interactions among the activities are likely to be numerous, complex, and consequently, formidable to manage. A comprehensive scheduling system is therefore required to permit efficient management of the project.

When performance has been demonstrated by the prototypes, production operations usually follow. Contractors are required to produce quantities of the same item on a scale that on occasion approaches mass production. By this time, most of the design uncertainty has been overcome, and reasonably final production drawings exist for the components. It is thus possible to make detailed subdivision of production operations and to control the use of resources on these operations.

Eventually the completed systems, and spares, are turned over to the using commands--Strategic Air Command (SAC), Tactical Air Command (TAC), etc.--which are responsible for their deployment and operation until the systems become obsolete.

Managerial decisions affecting the project must be made throughout all phases of the life cycle. The diverse nature of the activities in each phase requires a variety of scheduling information. This Memorandum will attempt to determine whether any single scheduling technique is sufficiently versatile to be used throughout the entire life cycle of a project.

### Numerous Industrial Suppliers

The development of a new product frequently requires diverse technologies. An example is the recent commercial development of petrochemicals, which was accomplished by forming joint subsidiaries combining technologies adapted to petroleum and chemical firms. Yet the development of defense systems is substantially more complex than the development of most commercial products. The technologies required generally exceed the feasibly attainable capabilities of any one firm. Consequently, defense firms frequently form arrangements similar to a joint venture. The simplest arrangement involves the designation of one firm as a weapon system prime contractor, the other firms being affiliated with it as subcontractors.

Another common arrangement is where several large firms become associate contractors, each being responsible for developing a major segment of the weapon system. For example, one associate contractor is responsible for guidance, another for airframe, another for propulsion, etc. Frequently each associate contractor subcontracts a portion of his project to another firm; the subcontractor may sub-subcontract a smaller portion to yet another firm, etc. Such subcontracting frequently involves thousands of industrial firms in the system development effort.

A third arrangement is one similar to the associate contractor system but with the addition of an integrating contractor whose function is primarily to coordinate systems engineering and checkout for the entire weapon system.

Many governmental agencies often furnish personnel, facilities, or material to develop a system. Each industrial firm and governmental agency, in turn, has more than one level of internal management. The levels vary in number from firm to firm but range in scope from first-line supervision to top management. Consequently, for a significant weapon system there evolve a substantial number of managerial interrelationships. Each managerial group must be informed of plans and progress relating to its sphere of responsibility.

### Program Monitors

It is obvious that in this environment some group or agency should be responsible for management of the entire project. In the Air Force a System Project Office (SPO) is established in the appropriate division of the Air Force Systems Command (AFSC) to provide this function. The SPO is responsible for the project throughout the weapon acquisition phase. Upon completion and delivery of the hardware, the remaining responsibilities of the SPO are transferred to a weapon system manager in the Air Force Logistics Command (AFLC). Responsibility for operation of the weapon system in the field rests with one of the using commands (i.e., SAC, TAC, ADC, etc.). The SPO, in conjunction with AFLC and the training command (ATC), coordinates the planning for training and for the maintenance and supply which will be required in the operational phase of the system.

If many firms are to make portions of the system, some mechanism should exist to ensure that all components will mate (interface) and function properly in the completed system. The SPO has this responsibility and accomplishes it with technical support either from in-house systems engineering laboratories (those at Wright Field, for example) or from nonprofit engineering concerns.\*

In defense contracting, the industrial firms deal with only one consumer, the Government, and more specifically with the program manager designated by the Department of Defense. The importance of national defense, coupled with this monopsony (one buyer) situation, naturally leads the Government to take a very active interest in the progress of the system. The SPO is primarily responsible for directing the program, while AFSC, Headquarters USAF, and the Office of the Secretary of Defense (OSD) are also involved in reviewing its progress. In addition, the Bureau of the Budget, Congressional committees, and even the President may become involved in a particular program from time to time.

Again, it is essential that the information systems used for analyzing program status be capable of directing pertinent information to each of the appropriate agencies and individuals concerned.

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\*The MITRE Corporation, Aerospace Corporation, etc.

### Dynamic Nature of the Environment

To be useful in this environment a scheduling system also must be responsive to extensive changes in the projects. The project life cycle generally lasts a period of several years; frequently, development effort alone will require four or five years. A mix of various weapon systems is necessary to accomplish the objectives of national defense. From time to time the assessment of the threat to our national security may be modified, which in turn may alter the relative priority of a given project in this mix or affect the amount of funds allocated over time to the project. These factors often result in either an accelerated schedule or a program "stretchout."

Likewise, general technological advances and experience on a specific project frequently lead to design changes that affect the project schedule. The scheduling system must respond to these changes if it is to be useful to management.

### CRITERIA FOR COMPARISON OF ALTERNATIVE SCHEDULING TECHNIQUES

It is difficult, if not impossible, to prepare a quantitative assessment of the utility of a particular scheduling technique. It is possible, however, to isolate features that are desirable and then to assess the extent to which these features are satisfied. Although, conceptually, it is possible to assign weights to each feature and thereby construct an index of relative usefulness, this additional step, being inherently subjective, will be left to the reader.

The following criteria are not intended to be comprehensive but are sufficiently basic to be helpful in estimating the strengths and weaknesses of each technique. The discussion in the subsequent sections should indicate the usefulness of these criteria in assessing various systems.

1. Validity. The information contained in the system and presented to the appropriate levels of management should reflect genuine progress. For example, suppose a guidance system is required to keep a missile on course, and a gyroscope is an integral component of this

guidance system. If the gyroscope is improperly designed, a bias will be introduced into the measurement of spatial relationships. Measurements used in the guidance system will be invalid, that is, they will not reflect the true state of affairs.

2. Reliability. The data contained in the system should be consistent regardless of who obtains them or when they are obtained. In the above example, suppose that the gyroscope were properly designed, and thus capable of providing a valid measurement of attitude, but that electrical pulses, external to the gyroscope, frequently altered its motion and generated inconsistent readings. Readings used by the guidance system would then be unreliable. Relating this example to scheduling techniques, the system may be well designed, and consequently valid, yet subject to error because of weaknesses in data collection, and therefore unreliable. Or the reverse, that is, reliable yet invalid results also are possible.

3. Implementation. A large number of personnel are likely to be involved in furnishing inputs to and using outputs from a scheduling system. Thus the technique should be easy to explain and understand, and simple to operate.

4. Universality of Project Coverage. Ideally, one scheduling system should be sufficient from beginning to end of a project life cycle. All levels of management should be able to use the information in the system, and all relevant factors to be controlled should be encompassed by the one system.

5. Sensitivity Testing (Simulation). Since management decision-making involves selecting one course of action out of alternative possible courses, it is desirable to assess the scheduling implications of these alternatives. A system that enables management to simulate the impacts of alternative courses of action can facilitate the selection process and lead to better decisions concerning the project.

6. Forecasting. One purpose of collecting data is to assess the probability of accomplishing future tasks. Some scheduling systems are oriented more explicitly toward longer term operations than others.

7. Updating. Program decisions in a dynamic environment must be based on current data. The scheduling system should be capable of incorporating rapidly, and with ease, information on project progress.

8. Flexibility. A desirable feature in a scheduling technique is its ability to adapt easily to changes in the project. This feature is closely related to a simulation capability. The system must be flexible if simulation of alternatives is to be possible, but a system may be flexible without emphasizing simulation potential.

9. Cost. The scheduling system should provide the required information at the lowest cost. Cost is a difficult factor to measure for several reasons. First, scheduling costs are not usually uniformly recorded by industry and government, probably because the functions attributable to collection of data in support of the system vary among contractors. Also, total scheduling costs are needed to compare techniques. In a Gantt system, for example, time standards are as much a part of the cost as is chart preparation, yet this factor frequently is not included in estimates of schedule cost.

Second, systems that are the most useful in terms of the above criteria generally involve greater cost. Consequently, the appropriate cost statistic is not total dollar cost, but rather cost per unit of utility, or benefit. This cannot as yet be precisely measured.

Finally, cost is largely a function of the size of the program, and implementation of each system involves both fixed and variable costs. Thus, techniques with high fixed costs tend to be relatively less expensive in large-scale applications and relatively more expensive in small projects.

#### MISSILE SYSTEM DEVELOPMENT EXAMPLE

A hypothetical missile system has been selected to facilitate a comparison of alternative scheduling techniques for the development phase of a project. Although the example is greatly abbreviated, it will suffice to demonstrate the major characteristics of each technique. Various nonstandard illustrations are used in describing applications to production processes.

Table 1 contains all the basic data--events, activities, and time estimates--needed to compare the scheduling techniques for the missile system development example. The discussion in the various sections throughout the Memorandum will draw upon this table.



Table 1

## DATA FOR THE MISSILE SYSTEM DEVELOPMENT EXAMPLE

Event No.	Events	Activity No.	Activities	Estimated Time Required (weeks)		
				Optimistic	Most Likely	Pessimistic
1	Begin project	1-2	Assemble maintenance equipment fabrication facilities	1	3	6
2	Start maintenance equipment fabrication	1-3	Assemble training facilities for operating personnel	3	4	5
3 <sup>a</sup>	Start training of operating personnel	1-4	Assemble ground equipment fabrication facilities	0.1	0.2	0.5
4	Start ground equipment fabrication	1-5	Assemble installation and check out equipment fabrication facilities	0.1	0.2	0.5
5	Start installation and check out equipment fabrication	1-6	Assemble missile erection equipment fabrication facilities	1	3	6
6	Start missile erection equipment fabrication	1-7	Assemble missile transportation vehicle fabrication facilities	5	7	8
7	Start missile transportation vehicle fabrication	1-8	Assemble missile fabrication facilities	0.1	0.2	0.5
8	Start missile transportation vehicle fabrication	1-9	Assemble emplacement fabrication facilities	4	5	6
9	Start emplacement equipment fabrication	1-10	Assemble training facilities for maintenance personnel	1	2	3
10 <sup>b</sup>	Start training of maintenance personnel	1-11	Assemble site construction facilities	3	4	5
11	Start site construction	2-12	Fabricate maintenance equipment	14	19	26
12	Maintenance equipment fabrication completed	3-13	Train operating personnel	16	19	20
13	Training of operating personnel completed	4-21	Fabricate ground equipment	16	19	21
14	Installation and checkout equipment fabrication completed	5-14	Fabricate installation and checkout equipment	4	6	7
15 <sup>c</sup>	Missile erection equipment fabrication completed	6-15	Fabricate missile erection equipment	2	3	5
16 <sup>d</sup>	Missile transportation vehicle fabrication completed	7-16	Fabricate missile transportation vehicle	8	9	12
17	Missile functional test completed	8-17	Fabricate missile	27	30	34
18	Missile fabrication completed	9-19	Fabricate emplacement equipment	26	28	31
19	Emplacement equipment fabrication completed	10-29	Train maintenance personnel	8	9	10
20	Preliminary check out of installation and checkout equipment completed	11-30	Construct launch site	18	21	25
21	Ground equipment fabrication completed	12-25	Transport maintenance equipment to site	0.1	0.2	0.5
22	Installation and checkout equipment on dock at site	13-23	Transport operating personnel to site	0.1	0.2	0.5
23	Operating personnel at site	14-20	Test installation and checkout equipment	5	7	8
24	Ground equipment on dock at site	15-26	Transport missile erection equipment to site	0.1	0.2	0.5
25	Maintenance equipment on dock at site	16-27	Deliver missile transportation vehicle to site	0.1	0.2	0.5
26	Missile erection equipment on dock at site	17-18	Correct deficiencies in missile	8	10	11
27	Missile on dock at site	18-27	Transport missile on missile transportation vehicle to site	0.1	0.2	0.5
28 <sup>e</sup>	Emplacement equipment on dock at site	19-28	Transport emplacement equipment to site	0.2	0.4	1.0
29	Maintenance personnel at site	20-22	Transport installation and checkout equipment to site	0.1	0.2	0.5
30	Site construction completed	21-24	Transport ground equipment to site	0.1	0.2	0.5
31	Installation of maintenance equipment completed	22-32	Install checkout equipment at site	0.1	0.2	0.5
32	Installation of ground equipment completed	23-32	Operating personnel install ground equipment	0.1	0.3	1.0
33	Missile installation completed	24-32	Ground equipment installed at site	0.1	0.2	0.5
34	Launch site completed	25-31	Install maintenance equipment	1	2	3.5
35	First operational unit completed	26-33	Install missile erection equipment at site	0.2	0.4	1.0
		27-33	Install missile at site	0.1	0.2	0.5
		28-33	Utilize emplacement equipment to install missile	0.6	1	2.0
		29-33	Perform maintenance operations on missile site	0.1	0.2	0.5
		30-34	Check out site construction	0.1	0.2	0.5
		31-34	Check out installed maintenance equipment	1	2	3.5
		32-34	Check out installation of ground equipment	0.1	0.2	0.5
		33-34	Check out missile installation	16	24	34
		34-35	Transfer responsibility for operational unit to using command	0.1	0.2	0.5

NOTE: Footnotes a through e indicate interdependencies between events. The meaning of interdependency is explained on page 9.

<sup>a</sup> Must be completed before event No. 20.

<sup>b</sup> Must be completed before event No. 20.

<sup>c</sup> Must be completed before event no. 17.

<sup>d</sup> Must be completed before event No. 17.

<sup>e</sup> Must be completed before event No. 34.

Project status is measured by the accomplishment of events representing significant points of partial completion of a project. Activities, on the other hand, occur over a time horizon. Each activity is defined by a starting and an ending event. Resources are consumed by activities rather than events. Decisions made by project management may alter the levels and qualities of resources applied to activities. Estimates of the time required to accomplish each activity are given in Table 1. These estimates are indicated as "optimistic," "most likely," and "pessimistic,"\* and serve as the schedule data for the example.

Generally, the events and activities required to complete a component or subsystem are dependent upon the results of the preceding activities in that subsystem. Frequently, information generated through performance on an activity in one subsystem also is essential to the definition and performance of activities in a different subsystem. For example, information concerning the size, weight, etc., of a missile must be obtained from the missile design before the launching equipment can be designed and fabricated. In general, fabrication of launching equipment is separate from fabrication of the missile except for this information requirement. This relationship makes the activities interdependent. Such interdependencies must be considered in scheduling projects. The relevant interdependencies are identified in footnotes to Table 1.

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\*The meaning of optimistic, most likely, and pessimistic times is explained in Section V.

## II. GANTT AND MILESTONE CHARTS

### GANTT TECHNIQUE

The Gantt technique was the first formal scheduling system to be used by management.\* The cornerstone of the technique is the Gantt chart, which is basically a bar chart showing planned and actual performance for those resources that management desires to control. In addition, major factors that create variance (i.e., overproduction or underproduction) are coded and depicted on the chart.

### Application to Production Operations

The Gantt chart was designed for, and is most successfully applied to, highly repetitive production operations. Normally, it assumes that time standards are available for each operation and that the objective of management is to obtain "normal" output from each major resource employed, especially labor and machinery. If, for example, it has been established that an average of 60 seconds (including personal time)\*\* is required for a "typical" worker to assemble a cigarette lighter,

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\* Developed by Henry L. Gantt in the late 1800s, the technique was based on the scientific management approach of Frederick W. Taylor. Prior to the twentieth century, management of productive operations was loosely organized. Few standards existed by which performance could be gauged. In the 1880s, Taylor altered the process of management by attempting to substitute "scientific management" for "opinions" and "hunches" based on little factual data.

This "scientific method" involved identifying tasks and subtasks to be performed in the productive operations of the plant. The subtasks were refined into elementary work movements, which were "timed" to determine how much time each movement should require under normal working conditions if performed by a "typical" operator. The elementary operations were then assigned to an operator and their accumulated times became a standard by which the operator's performance was measured. The variance, if any, between work planned for the day, week, etc., and work completed for the period was analyzed to determine the factors responsible for underperformance (or overperformance), so that corrective action could be prescribed.

Gantt met Taylor in 1887 and became actively involved in the scientific management movement. Gantt made numerous contributions to management philosophy, but he is remembered primarily for his graphic technique, which he devised to display data required for scheduling purposes.

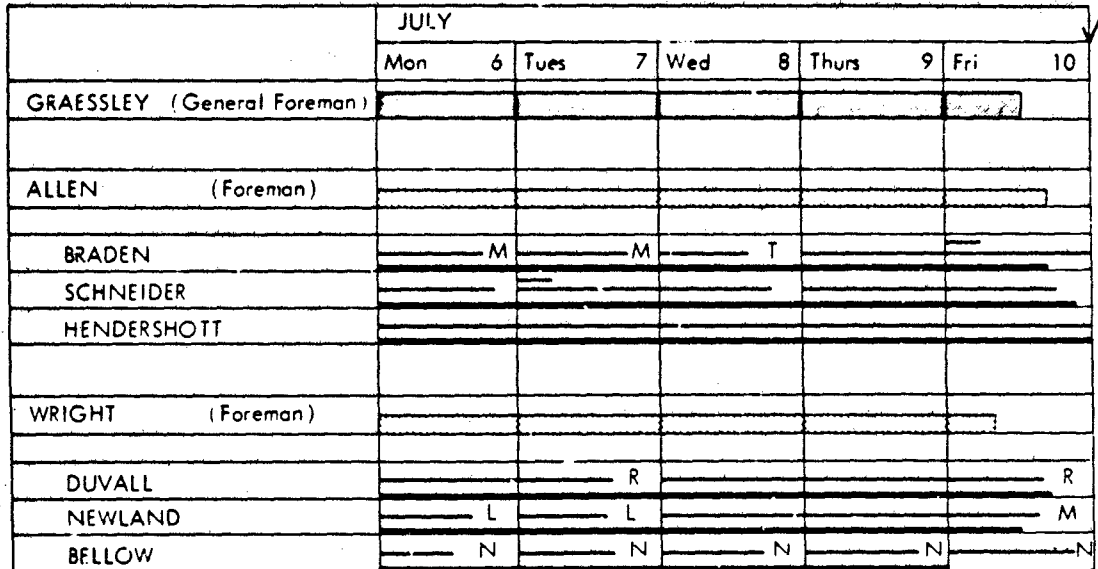
\*\* An allowance for coffee breaks, wash room, etc.

then each man assigned to that task should be scheduled to assemble 60 per hour and he should meet this quota. Reasons for underperformance should be established.

A similar example can be given for machinery. If a drilling machine is rated as requiring 30 seconds to drill six holes in a two-barrel carburetor, then that machine should be scheduled to perform this function on 120 carburetors per hour. Again, reasons for any variation in performance should be established.

The Gantt charts applicable to these two types of production operation are called "man-loading" and "machine-loading," respectively. An example of a man-loading chart is given in Fig. 1. The machine-loading chart is similar, except that machine time rather than man time is scheduled. The chart shown in Fig. 1 provides the following information:

- o The "✓" indicates that the chart was based on actual production through Friday, July 10.
- o The space shown for each day represents the output scheduled for that day. The thin line indicates the output actually produced by the worker for the day. In the example, Mr. Braden failed to produce his scheduled output on Monday, Tuesday, and Wednesday. His underproduction on Monday and Tuesday was due to material troubles (M) and Wednesday's underproduction was traced to tool troubles (T). On Thursday, Braden met his scheduled output, and on Friday he exceeded it. The overproduction on Friday is indicated by a second thin line.
- o Braden's performance for the entire week is shown as a heavy, solid line immediately beneath the thin lines representing his daily performance. It can be readily seen that his cumulative output for the week was less than scheduled. Each worker's performance is analyzed in a similar way.
- o Because the foreman is responsible for the output of those working under him, the chart records the scheduled output of his combined work force. In the example, the shaded line opposite his name indicates that Mr. Allen did not meet the scheduled output for the week. The reasons for this underperformance can be traced to specific employees on specific days.



#### LEGEND

- A. The ordinate (y axis) comprises a discrete listing of the names of employees in a department. The abscissa (x axis) represents a time horizon.
- B. Other characteristics
  1. | | Width of daily space represents amount of work that should be done in a day.
  2. ——— Amount of work actually done in a day.
  3. - - - - - Time taken on work on which no estimate is available.
  4. ——— Weekly total of operator. Solid line for estimated work; broken line for time spent on work not estimated.
  5. [ ] Weekly total for group of operators.
  6. [ ] Weekly total for department.
  7. Reasons for falling behind:
    - A = Absent
    - N = New operator
    - L = Slow operator
    - R = Repairs needed
    - T = Tool trouble
    - M = Material trouble
    - Y = Lot smaller than estimated

Fig. 1 — Gantt man-loading chart

o The general foreman is responsible for the overall production of the department and thus the row opposite his name represents the scheduled output for the entire department. In the example the solid bar indicates that the output of the department did not meet the week's scheduled production. Consequently, the factors responsible for the poor performance and the areas in which they occurred will need to be determined.

In a similar manner, the work performance of several departments can be combined on a single chart to show aggregative accomplishment. Charts can also be prepared for various managerial levels so that performance can be depicted and responsibility traced throughout the organization. The graphs are normally maintained on a daily basis to provide up-to-date control.

Frequently, even in production operations, workers perform tasks for which there are no time standards, such as tool repair, housekeeping, etc. The amount of time spent on such tasks is usually represented by a dashed line. This type of effort is not indicated in Fig. 1, but the line is identified in the legend.

Gantt charts need not be organized along departmental lines only. For example, instead of showing quantity of output for one department, the chart could depict the progress of various departments striving simultaneously toward completion of a component or some other appropriate unit. This latter type of chart is more appropriate for prototype development and testing. Its application is discussed below.

#### Application to Development Operations

To demonstrate the application of a Gantt chart to nonrepetitive operations we will use the hypothetical missile system development example presented on page 7. A schedule of planned activities (taken from Table 1) is shown in Fig. 2.\*

In constructing such a schedule, it is important to keep in mind that when activities must be performed in series, they cannot be

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\* Activities with a most likely time of less than 1.0 week add little to the illustration at this point and are omitted, reducing the number of activities listed from 43 to 22.

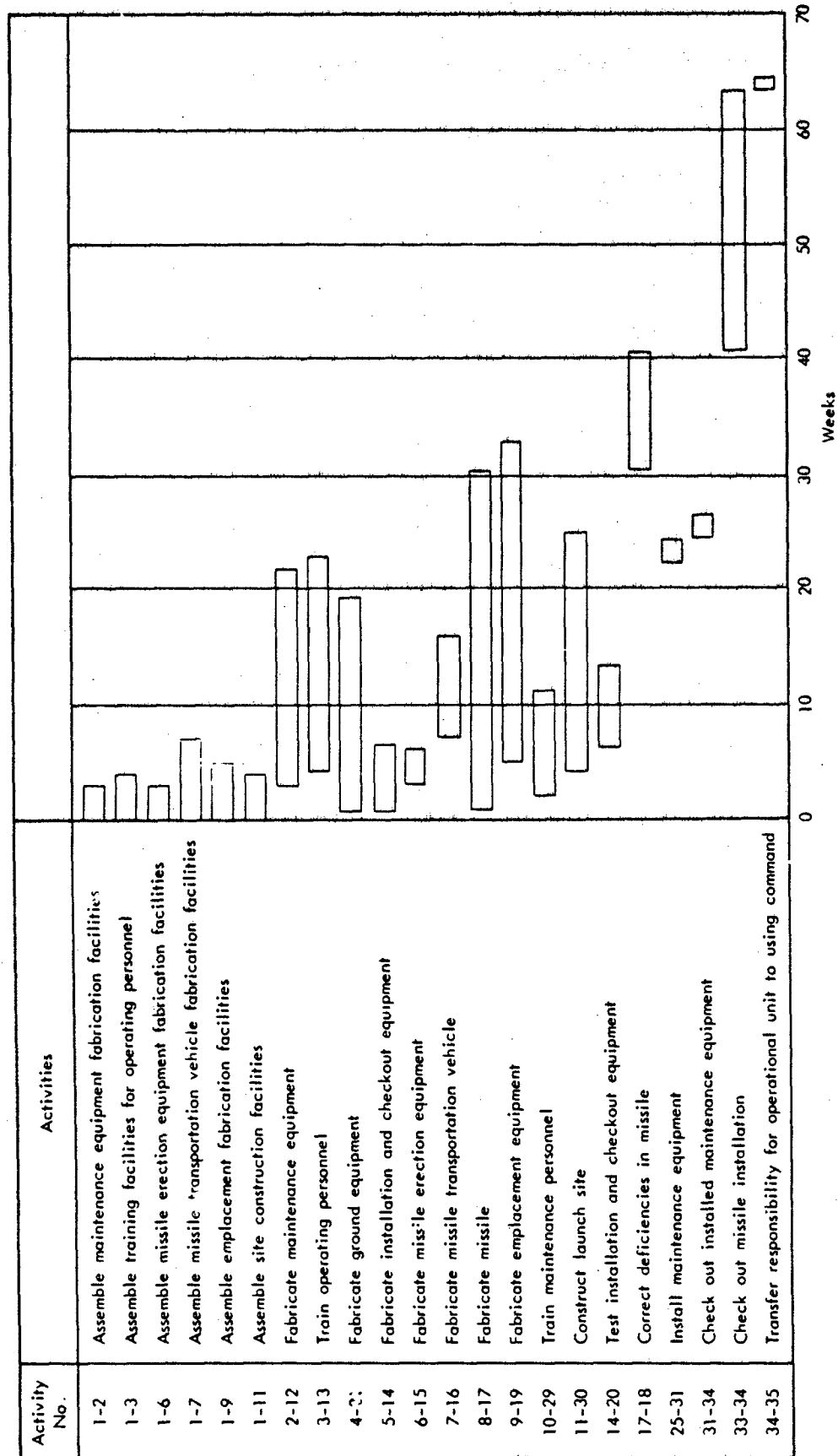


Fig.2—Gantt chart showing plan for missile system development

scheduled to begin before their predecessors are completed.\* Assuming available resources and a desire to complete all activities as soon as possible, the tendency would be to schedule each activity at its earliest start time,\*\* i.e., as soon as the prior activity is scheduled to be completed. Only certain "critical" activities need be scheduled in this fashion; most others can be delayed as long as the scheduled completion of the project is not jeopardized.\*\*\*

Unfortunately, the degree of flexibility which exists in scheduling a project cannot be readily ascertained through the use of the Gantt charts because relationships among activities in a project are not clearly revealed. For example, in Fig. 2 activities 2-12 (fabricate maintenance equipment), 3-13 (train operating personnel), and 3-17 (fabricate missile) are all scheduled to be completed before activity 17-18 (correct deficiencies in missile) is scheduled to begin. That activities 3-17 and 17-18 are in series, i.e., have a formal predecessor-successor relationship, is not revealed by the chart.

Figure 3 is a typical Gantt chart used by management to control activities after the schedule is completely prepared and actual operations are under way. The chart assumes the project has been in operation for 20 weeks and is scheduled for completion in an additional 40 weeks.

The chart indicates that activity 9-19 (fabricate emplacement equipment) and activity 11-30 (construct launch site) are, respectively, four weeks and one week ahead of schedule. However, activities 2-12 (fabricate maintenance equipment) and 4-21 (fabricate ground equipment) are, respectively, two and three weeks behind schedule. On the basis of the information in Fig. 3, it is not obvious whether the project will

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\* Managers do occasionally assign resources to portions of later activities in a series before earlier activities are completed.

\*\* This term was not formally introduced into the scheduling literature until the critical path technique evolved. However, since it simplifies the description, it is used here in explaining the basis for construction of the Gantt chart.

\*\*\* In subsequent discussion of scheduling techniques, such latter points are called latest start times, and the flexibility in scheduling certain activities are termed "float" or "slack."



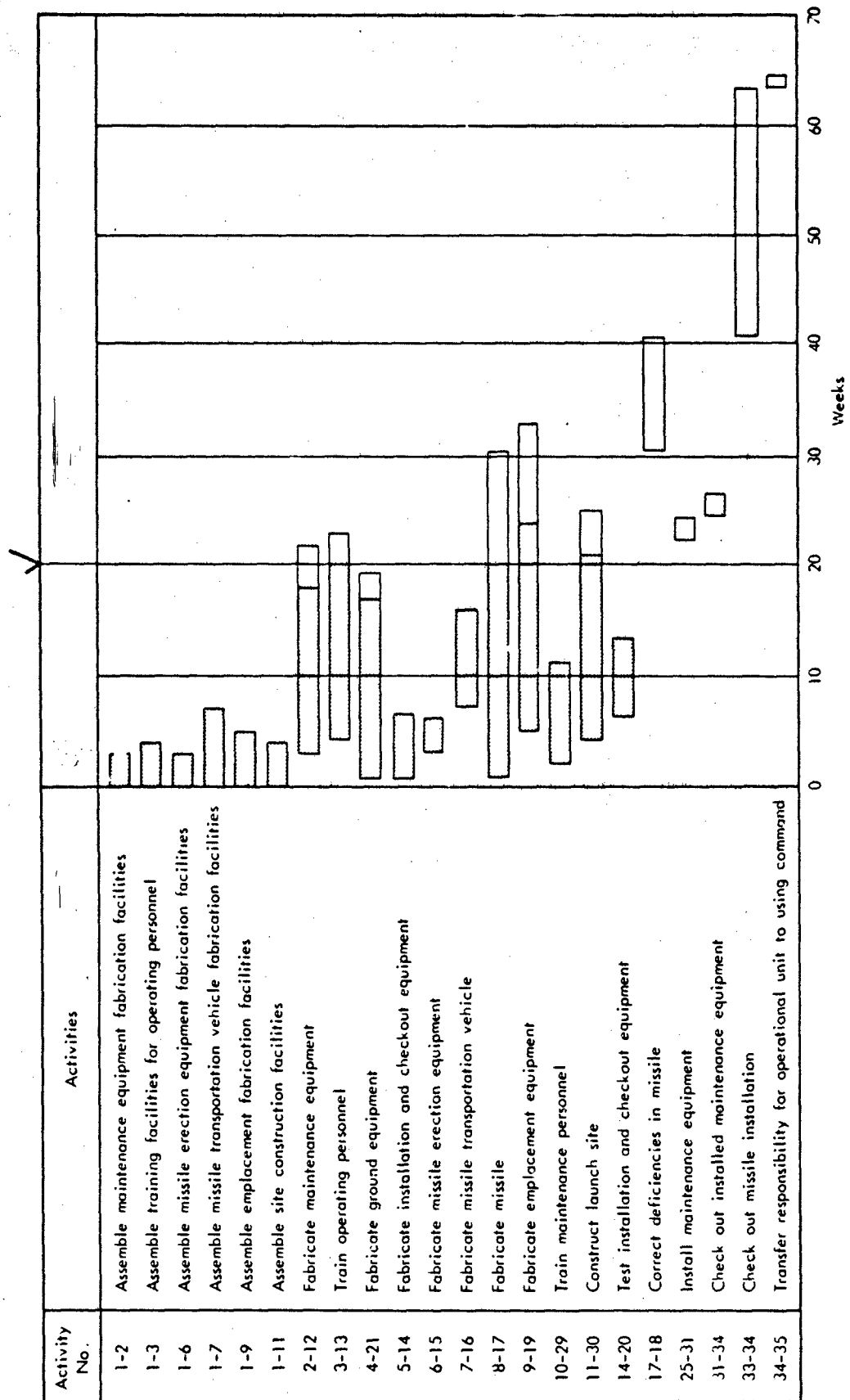


Fig.3—Gantt chart showing progress against plan

be completed on schedule. Actually it is possible to complete the fabrication of maintenance equipment and the fabrication of ground equipment as late as the 60th and 64th week, respectively, and still complete the project on schedule.\* Since the chart does not provide this information, it is necessary to use other techniques to establish interrelationships and to compute the earliest start and latest completion dates for each activity. A Gantt chart incorporating all of this information would be too cluttered to be easily read and understood.

A Gantt chart based on earliest start times combined with a transparent overlay based on latest completion times would provide more of the information useful for scheduling but would still not depict the interrelationships existing among activities.

The Gantt technique was devised originally for use by first-line supervision on repetitive production operations. It is an excellent tool for this type of operation because (1) good estimates of normal production times can be obtained when work is performed repetitively; and (2) production responsibility of first-line supervision is normally limited to a few operations. Thus, significant interrelationships, if any, are obvious at this level. The complex interrelationships evolve when information on many facets of an overall project must be presented to higher levels of management. The large amount of detailed information accumulated at the foreman level must then be compiled and summarized into fewer activities.

The more important strengths and weaknesses of the Gantt technique are summarized in Table 2.

#### MILESTONE TECHNIQUE

The milestone scheduling system is based largely on the same principles as the Gantt system but the technique of displaying project status differs. The milestone system is usually applied to development

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\*The method for computation of latest completion dates is given in Table 7 in Sec. IV.

Table 2

GANTT TECHNIQUE--STRENGTHS AND WEAKNESSES

Criteria	Strengths	Weaknesses
1. Validity	Good in production operations. Because of short time duration of each measured operation, only small errors in measurement are likely to occur.	No explicit technique for depicting interrelationships, which are especially important in development.
2. Reliability	Simplicity of system affords some reliability.	Frequently unreliable, especially in development stage, because judgment of estimator may change over time. Numerous estimates in a large project, each with some unreliability, may lead to errors in judging status.
3. Implementation	Easiest of all systems in some respects because it is well understood. (System implies existence of time standards.)	Quite difficult to implement for the control of operations in development phase, where time standards do not ordinarily exist and must be developed.
4. Universality of project coverage	Can comprehensively cover a given phase of a life cycle. Effective at the resource or input level of control.	Less useful in definition and development phases of life cycle.
5. Sensitivity testing (simulation)		No significant capability.
6. Forecasting	In production operations, good technique to assess ability to meet schedule on a given activity if based on good time standards.	Weak in forecasting ability to meet schedule when interrelationships among activities are involved.
7. Updating	Easy to update graphs weekly, etc., if no major program changes.	
8. Flexibility		If significant program changes occur frequently, numerous charts must be completely reconstructed.
9. Cost	Data gathering and processing relatively inexpensive. Display can be inexpensive if existing charts can be updated and if inexpensive materials are used.	The graph tends to be inflexible. Program changes require new graphs which are time consuming and costly. Frequently expensive display devices are used.

**NOTE:** Recall that this table is intended only as a summary of certain qualitative information on the relative usefulness of the scheduling technique. As indicated previously, a more formal quantitative evaluation of the extent to which the criteria are met was considered infeasible in this study.

projects and is frequently used at several of the higher-management levels, for example, corporate, SPO, AFSC, and Hq USAF.

A milestone represents an important event along the path to project completion. All milestones are not equally significant. The most significant are termed "major milestones" usually representing the completion of an important group of activities. (Also, events of lesser significance are often called "footstones" and "inch stones" at least in conversation if not in the formal literature.) In reality, of course, there are many gradations of importance.

Events that are designated as milestones vary from system to system. Attempts are currently being made to establish milestones common to all programs, especially within major systems. For example, events such as "Contractor Selected," "Equipment Delivered," and "Final Acceptance Inspection Completed" are common to all systems, while "Aircraft Flyaway" is common to all aircraft systems, but not to missile systems. It is anticipated that milestone standardization, if successful, will be of significant help to program monitors in comprehending the status of the program, as well as in comparing progress on various programs.

#### Milestone Chart

Systems management requirements currently specify that schedule data be furnished in milestone form by the System Project Office (SPO) and various contractors. In the planning phase, milestones are established for the total life cycle of the program. Major milestones are included in a comprehensive development plan, i.e., the System Package Program.\* Progress in accordance with the plan usually is reported for two time periods: (1) milestones scheduled to occur in the current fiscal year and (2) milestones scheduled to be completed during the current month.

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\* Described in System Program Documentation, Air Force Regulation 375-4, Department of the Air Force, Washington, D.C., Nov. 25, 1963. Progress information is reported in accordance with a procedure sometimes referred to as the Rainbow Reporting System. When initiated the Rainbow System required status information on cost, manpower, facilities, and technical performance, as well as schedule information. The system was called Rainbow because each type of information required was described on a card of a designated color, the assembled package being not unlike a rainbow.

A chart showing selected milestones for our hypothetical missile system is presented in Fig. 4. The milestones are designated by their event number as given in Table 1 and are for the current year. The project status is shown as of April 30, 1966. On that date five milestones had been completed on schedule. The milestones for event 16 (missile transportation vehicle fabrication completed) was completed two months behind schedule. Also, it was anticipated that event 19 (emplacement equipment fabrication completed) would not be completed by August as scheduled, but would lag a month; thus it should be re-scheduled to be completed in September. The remaining milestones are expected to be completed on schedule.

#### Collection and Reliability of Data

The method of collecting and organizing data is similar to that for the Gantt technique. Only the graphic presentation is different.

Date of chart April 30, 1966

Event No.	Milestones	1965			1966												1967		
		O	N	D	J	F	M	A	M	J	J	A	S	O	N	D	J	F	M
12	Maintenance equipment fabrication completed			◆															
13	Training of operating personnel completed			◆															
14	Installation and checkout equipment fabrication completed				◆														
15	Missile erection equipment fabrication completed			◆															
16	Missile transportation vehicle fabrication completed			◆			◆												
18	Missile fabrication completed										◆								
19	Emplacement equipment fabrication completed											◆	◆						
20	Preliminary check out of installation and checkout equipment completed						◆												
21	Ground equipment fabrication completed							◆	◆										
30	Site construction completed									◆									
33	Missile installation completed																		◆
35	First operational unit completed																		◆

#### LEGEND

- ◆ Action completed on schedule (completed action)
- ◆ Action not completed on schedule (actual slippage)
- ◇ Anticipated delayed accomplishment of future action (anticipated slippage)
- ◆ Scheduled (or rescheduled) action

Fig.4—Milestone chart applied to missile project

Accordingly, the strengths and weaknesses of the milestone technique are very similar to those summarized in Table 2 for the Gantt technique. The milestone reporting system can be automated with relative ease. Data on changes in status can be read into a computer, which prints the required format depicting progress on the appropriate milestones. This innovation tends to reduce the costs of the system and also to improve the timeliness of the data.

### III. THE LINE OF BALANCE TECHNIQUE (LOB)

#### APPLICATION TO PRODUCTION OPERATIONS

The line of balance technique (LOB) was developed to improve scheduling and status reporting in an ongoing production process. Essentially the technique consists of four elements:

1. The objective,
2. The program or production plan,
3. Measurement of progress, and
4. The line of balance.

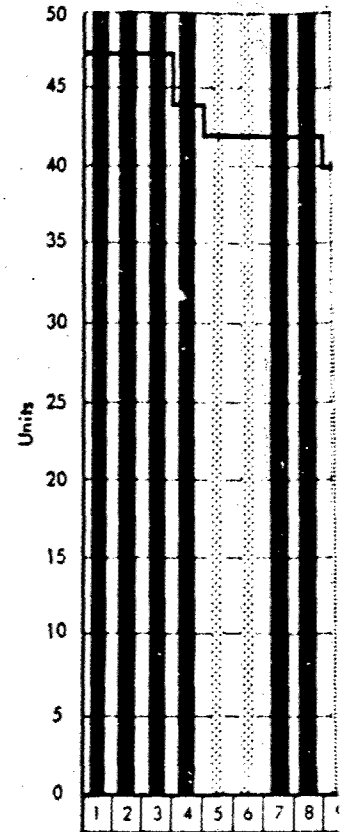
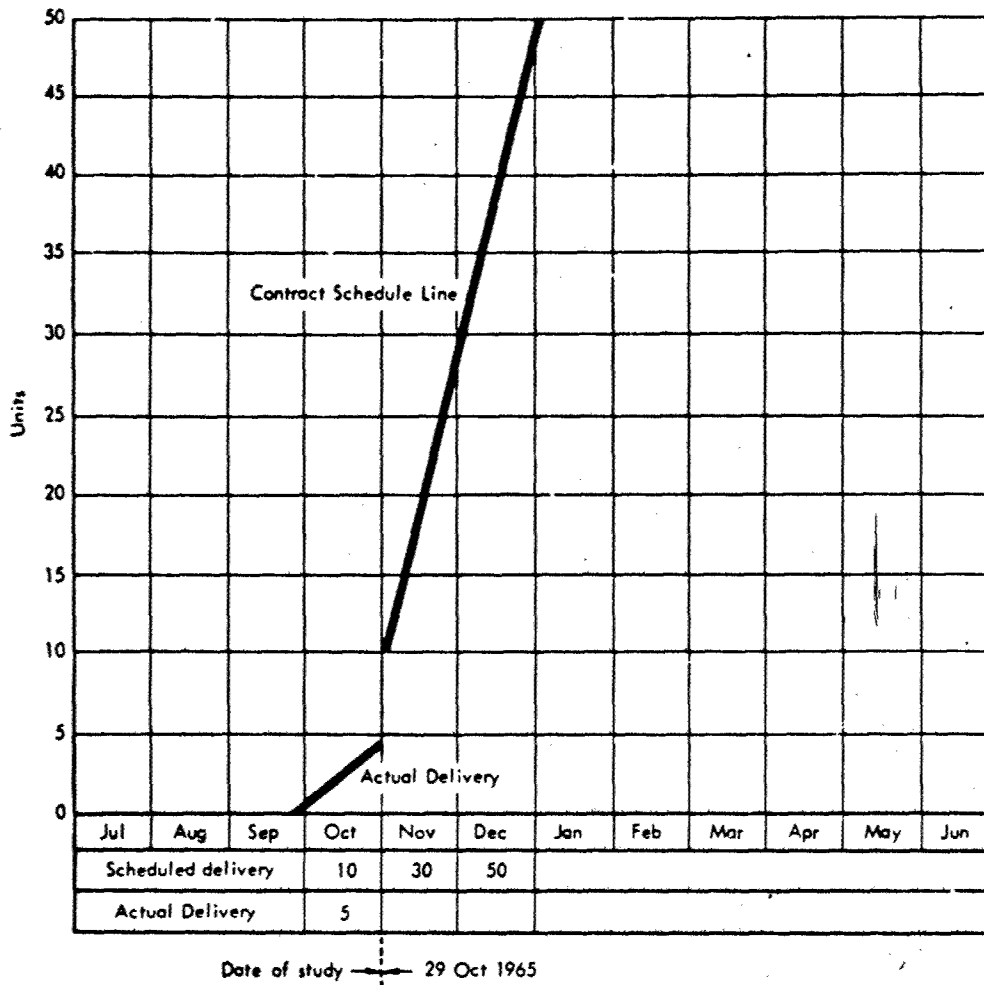
#### The Objective

The first step in scheduling production is to obtain the contract delivery schedule. The objective of the production operation is to meet a schedule based on cumulative deliveries. Figure 5(a) illustrates this objective as used in LOB. The chart shows the cumulative number of units scheduled to be delivered and the dates of delivery. The contract schedule line represents the cumulative quantity of units scheduled to be delivered over time.

#### The Program

The second step is to chart the program. The program, also called the production plan, comprises the stages in the producer's planned production process and consists, essentially, of key manufacturing and assembly operations sequenced in a logical production scheme over the time period required to complete. A sample program is presented in Fig. 5(b). Time is shown in working days remaining until each unit can be completed. Symbols and color schemes can be used to depict different types of activity, such as assembly, machining, purchasing of materials, etc.

(a) The Objective (cumulative delivery schedule)



(b) The Program or Project

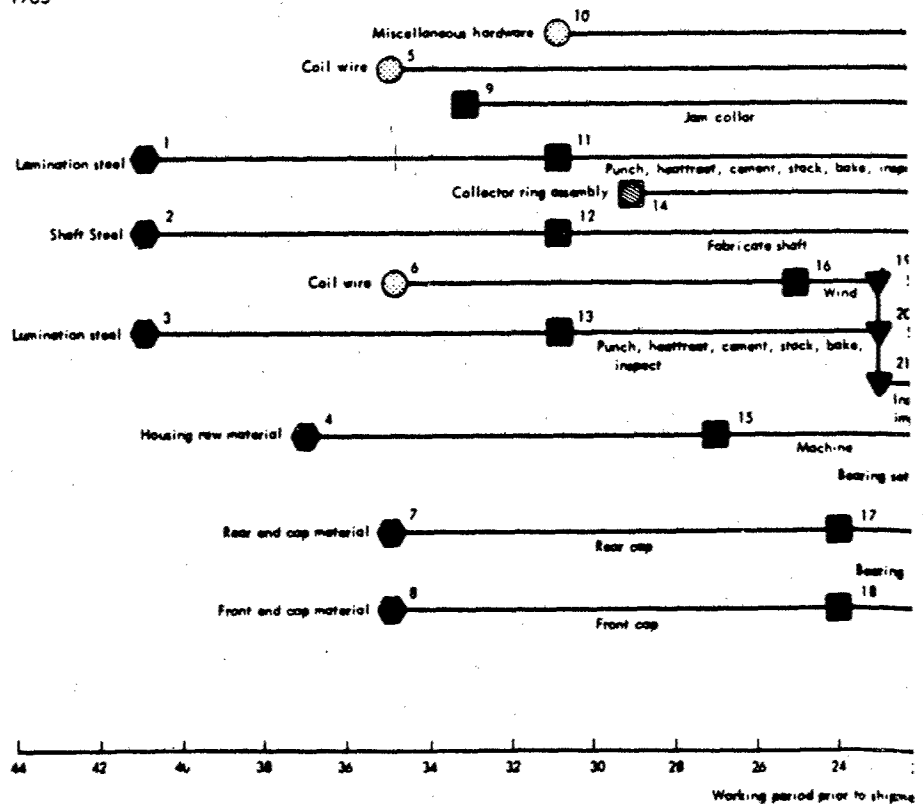
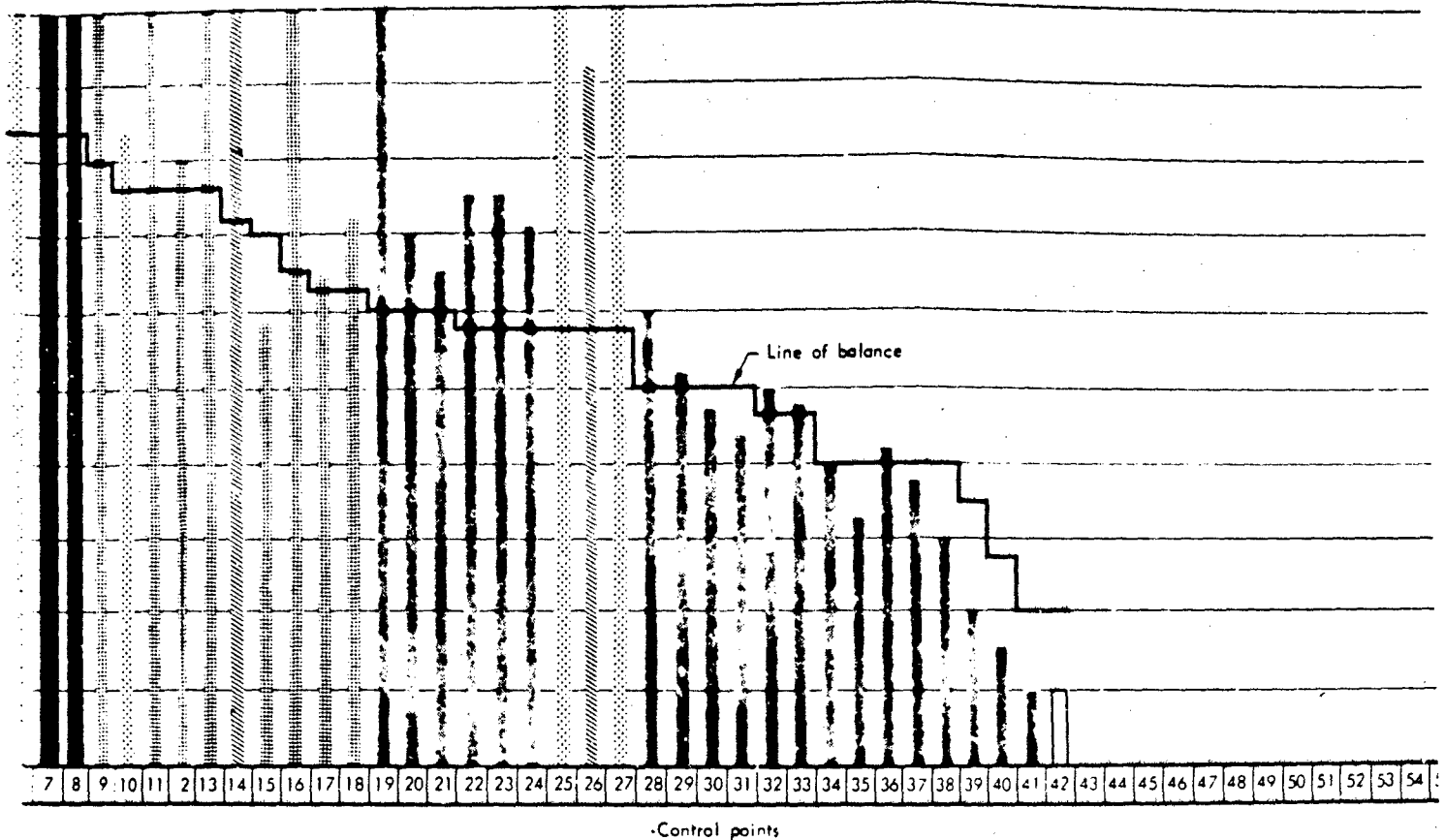


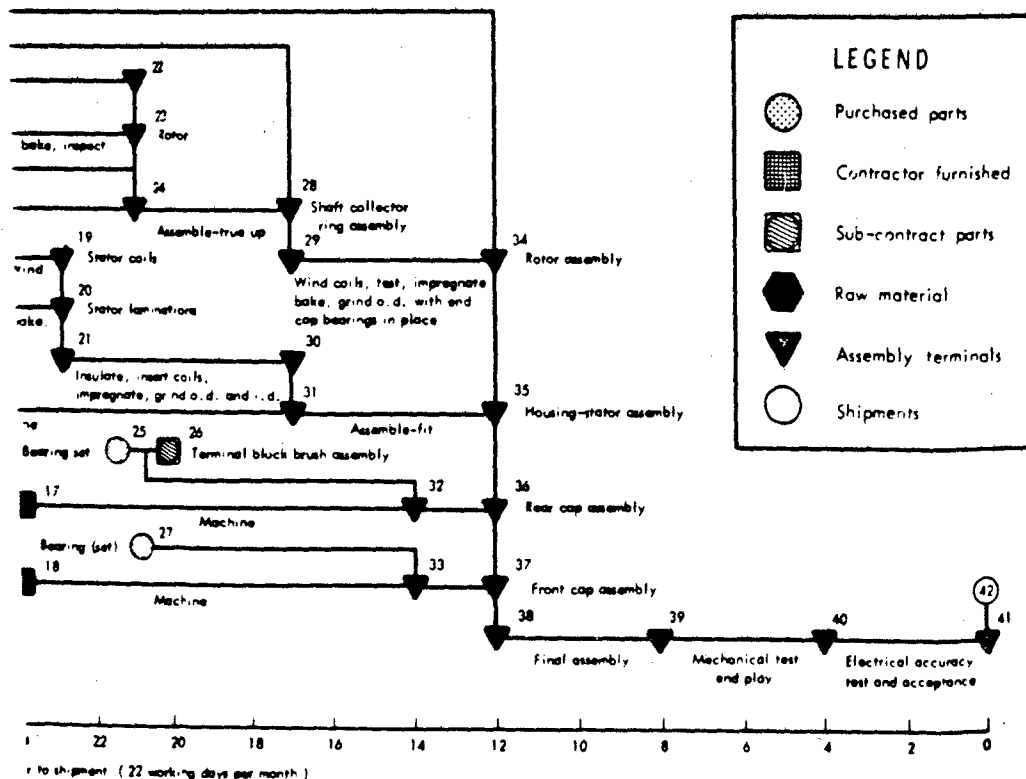
Fig.5—The Line of



(c) Program Progress Data and Line of Balance (showing progress through major control points)



m or Production Plan (showing major operations and control points)



ne of Balance Technique \*

### Measurement of Progress

To illustrate the control function, let us assume that production has been in progress for a month. We are then able to measure the status of the components (units) in the various stages of completion.

Program progress data are obtained by taking a physical inventory of the quantities of materials, parts, or sub-assemblies that have passed through a series of control points in the production plan. The data are then plotted on a bar chart illustrated by Fig. 5(c). For example, if control point 15 in chart (b) were selected, the inventory might reveal that 29 units were completed on that date and hence 29 would be shown on the bar chart, which thus represents actual production progress.\*

### Line of Balance

The last step is to construct the line of balance, which represents the number of units that should pass through each control point at a given date if management can reasonably expect the objective, i.e., the delivery schedule, to be met.

The line of balance is constructed in the following manner:

1. Select a particular control point, for example, 15.\*\*
2. From the production plan (Fig. 5(b)) determine the number of days required to complete a unit from the control point to the end of the production plan (i.e., 27 days).
3. Using this number determine the date the units should be completed. (October 29 plus 27 working days is December 3.)
4. Find the point corresponding to this completion date (December 3) on the contract schedule line and ascertain the number of units (35) that should be completed on that date if the delivery schedule is to be met.

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\*The legend also utilizes shading in parts (b) and (c) to indicate the type of material or function involved. This assists in identifying general areas of responsibility.

\*\*Actually one would probably start with the last control point (42) and work back through the project. For our purposes here control point 15 is of special interest in illustrating the usefulness of the technique.

5. Draw a line on the production progress chart (Fig. 5(c)) at that level (35 units) and over the control point (15).
6. Repeat this procedure for each control point and connect the horizontal lines over the control points. The resulting line is the line of balance. It indicates the quantities of units that should have passed through each control point on the date of the study (October 30) if the delivery schedule is to be met.

The production progress chart shows the status of a program at a given point in time. Thus management can determine at a glance how actual progress compares with planned progress. Where actual progress lags planned progress, the variance can be traced to the individual control point(s).

In the example described above, it is evident that without management action the delivery schedule will not be met because several control points, including the last one, are behind schedule. By using both the production plan and the program progress chart, one can begin at the end control point (42) and trace back through the series to find the source of the delay. Working backward, we see that control point 37 is a critical point of delay. If 37 were on schedule, then it is quite likely that all the succeeding control points would be on schedule. In trying to determine why 37 is behind schedule, we see that control points 35, 31, and 30 are also behind schedule. Control point 35, however, is in series with 31 and is presumably held up because 31 is not on schedule, which in turn is held up because control point 30 is not on schedule. We note that the control points preceding operation 30 are on schedule and therefore assume that the difficulty probably lies within operation 30 itself. The initial difficulty, however, lies in the sequence of activities preceding operation 31, so that 31 is behind schedule because 15 is behind schedule. Thus control point 15 is the bottleneck. It is reasonable to assume that with more management surveillance, and perhaps with more resources devoted to operations 15 and 30, operation 31 will be on schedule, and as a result so will 35, 37, 38, 39, 40, 41, and 42.

### APPLICATION TO DEVELOPMENT OPERATIONS

Although LOB has been widely applied to production operations at the prime and associate contractor level, a variant of this technique can be used in the development stage of a weapon system where only one complete system, or a small number of complete systems, is to be produced. In this case, control of the quantity of items through a given point is not relevant as it is in production operations. Instead, monitoring of progress is directed toward major events, that is, the completion of significant activities in the development process. In our discussion, we assume the development of a single unit using the hypothetical missile system described in Sec. I.

As applied to the development phase, the four elements of the technique are essentially the same as those for production scheduling and control, but their composition is altered.

#### The Objective

Instead of scheduling many units, the delivery schedule is based on the production of a single unit or on a limited number of units. The objectives chart will thus show the required percent completion of individual activities, rather than number of systems through each control point. Figure 6 illustrates this possible adaptation of LOB to the hypothetical development project.\* Supporting data are given in Tables 3 and 4.

The scheduled starting date of the component begins in the appropriate week at a point on the abscissa representing zero percent completion. The scheduled completion date of each activity is represented in the appropriate week at a point on the abscissa which represents 100 percent completion. A straight line is drawn between these two points. This straight line assumes that the same rate of progress will occur throughout the activity period. If the scheduler has reason to doubt that progress will proceed at a constant rate, the line can be drawn in any shape that management feels will correctly depict the expected progress.

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\* The list of activities has been condensed for purposes of illustration.

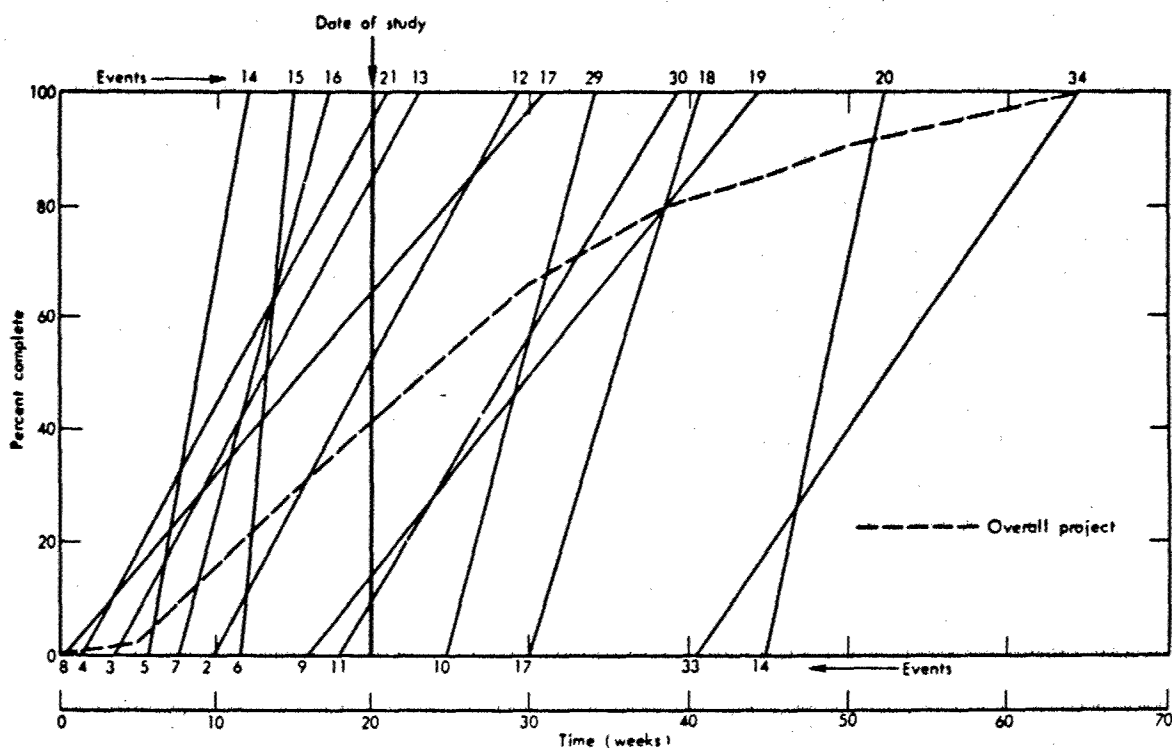


Fig.6—LOB prototype development objectives chart

Table 3

SUPPORTING DATA FOR FIG. 6

Activity No.	Activities	Estimated Activity Time (weeks)	Scheduled Dates	
			Start	Complete
2-12	Fabricate maintenance equipment	19	10	29
3-13	Train operating personnel	19	4	23
4-21	Fabricate ground equipment	19	2	21
5-14	Fabricate installation and checkout equipment	6	6	12
6-15	Fabricate missile erection equipment	3	12	15
7-16	Fabricate missile transportation vehicle	9	8	17
8-17	Fabricate missile	30	0.2	30.2
9-19	Fabricate emplacement equipment	28	16	44
10-29	Train maintenance personnel	9	25	34
11-30	Construct launch site	21	18	35
14-20	Test installation and checkout equipment	7	45	52
17-18	Correct deficiencies in missile	10	30.2	40.2
33-34	Check out missile installation	24	40.6	64.6
Total .....		204	---	---

Table 4

DATA FOR OVERALL PROJECT OBJECTIVES CURVE

Time Period (Identified by Final Week)	Estimated Activity-Weeks Required During Period	Cumulative Activity-Weeks to Date	Percent of Planned Completion <sup>a</sup>
0	0	0	0
5	9	9	4.4
10	21	30	14.7
15	30	60	29.4
20	28	88	43.1
25	24	112	54.9
30	24	136	66.6
35	19	155	76.0
40	14	169	82.8
45	9	178	87.2
50	10	188	92.1
55	7	195	95.6
60	5	200	98.0
65	4	204	100.0

<sup>a</sup>Information in this column is basis for dotted line in Fig. 6.

Using the data in Table 4, an overall project objectives curve can be constructed as follows:

1. Summarize the weeks estimated to complete each activity and thus obtain the total activity-weeks of effort to be involved during each incremental time period. (Computations were made for five-week intervals in the example.)
2. Compute the cumulative activity-weeks of planned effort through the end of each time period.
3. Compute the ratio of (1) over (2) for each time period. This ratio is the percent of the project planned to be completed at the respective points. The line connecting these points is the overall project objectives curve. The completion date of the last activity should coincide with the completion date of the overall project.

### The Development Plan

A flow chart showing the development plan of the hypothetical missile system is given in Fig. 7. Procedurally, the development plan

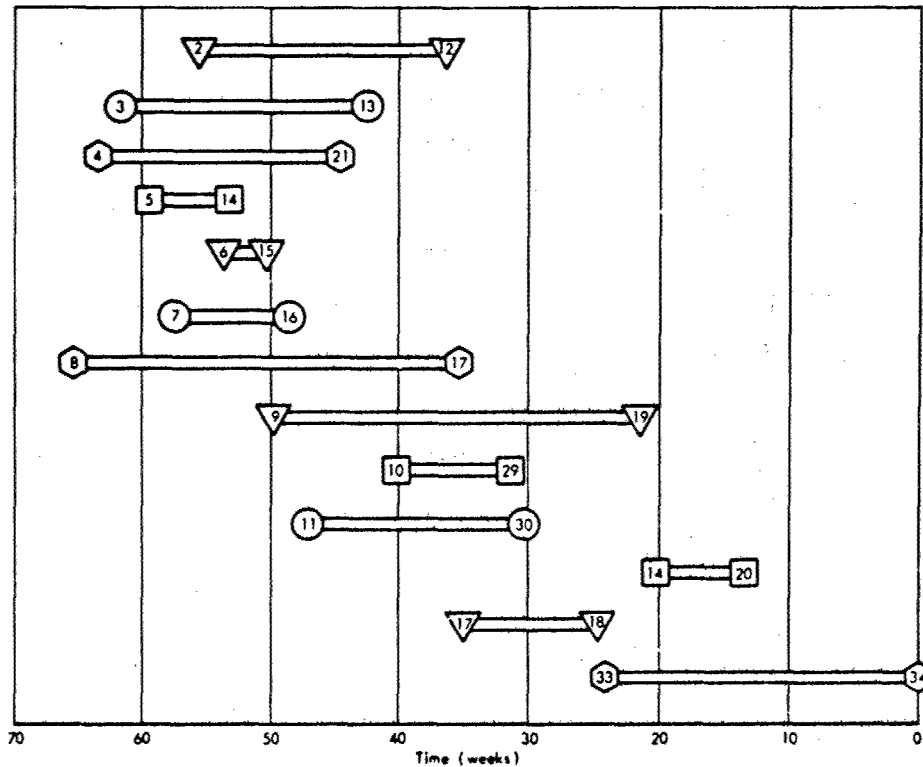


Fig. 7—LOB prototype development plan chart

chart is taken as a control point for the progress chart (see Fig. 8 on page 31). The development plan chart in our example does not show connections between the activities because only 13 activities out of the 34 given in Table 1 are included. If all 34 were shown, the activities would follow in sequence to the completed missile system.

### Determination of Progress

There is no technique available to determine true overall program status where considerable uncertainty exists concerning completion dates. The original estimated time to complete an activity, the length of time devoted to it to date and the current physical state of completion all may be known. However, the actual time required to complete

it is not known and must be estimated by the responsible project engineer. The LOB technique for approximating the status of the program is as follows:

$$\text{Percent completion} = 1 - \frac{d}{A},$$

where d = the number of weeks required to complete  
a particular activity,

A = the gross number of weeks originally estimated for the entire project.

As an example, suppose that the time originally required to complete the development phase was 10 weeks, that 8 weeks have already elapsed, and that the current estimate of the time to completion is 4 weeks. According to the LOB formula, the development phase is  $100(1 - (4/10)) = 60$  percent complete.

Two alternative techniques could also be used to estimate percent completion. For example, if it now appears that the total time required for the development phase is 12 weeks, when 4 weeks remain to completion one can consider that the development is actually 8/12 or 67 percent complete, and not 60 percent complete as revealed by the LOB formula.

A second alternative would be to place the 8 actual weeks of effort over the original time estimate (10 weeks); this would indicate that the phase was 80 percent complete.

While the major reference material on LOB discusses the second alternative, it selects the basic LOB technique as the preferable one because "while the prescribed method requires one additional mathematical step, it helps compensate for inaccuracies in the initial estimate of time required for the entire phase."\* However, in some respects the first alternative appears to be the most realistic because it is based on current information rather than on the original estimate.

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\* Line of Balance Technology, op. cit., p. 19.



On the other hand, it is obvious that no simple algorithm alone can be expected to solve the problem of precisely determining the actual percent completion of a complex project.

The procedure recommended in the LOB technique is applied to our hypothetical missile system in Table 5, and the program progress is shown in Fig. 8. (Control points are the ending events for the activities.)

To determine total project status, sum the estimated weeks required to complete each activity (d), and divide by the total number of

Table 5  
SUPPORTING COMPUTATIONS FOR FIG. 8  
(Percent completion: 20th week)

Activity No.	Activities	d	A	1 - (d/A)
2-12	Fabricate maintenance equipment	12	19	37
3-13	Train operating personnel	4	19	79
4-21	Fabricate ground equipment	4	19	79
5-14	Fabricate installation and checkout equipment	0	6	100
6-15	Fabricate missile erection equipment	0	3	100
7-16	Fabricate missile transportation vehicle	0	9	100
8-17	Fabricate missile	11	30	63
9-19	Fabricate emplacement equipment	24	28	14
10-29	Train maintenance personnel	9	9	0
11-30	Construct launch site	17	21	19
14-20	Test installation and checkout equipment	7	7	0
17-18	Correct deficiencies in missile	10	10	0
33-34	Check out missile installation	24	24	0
Total .....		122	204	40

weeks originally estimated to be required for the entire project (A). This gives the percentage not completed ( $d \div A$ ). Subtract the percentage not completed from 100 percent, and the result is the percentage of the total project completed [ $1 - (d/A)$ ].

In the example (Table 5), the total activity-weeks originally estimated were 204. In the 20th week of the project, it is estimated

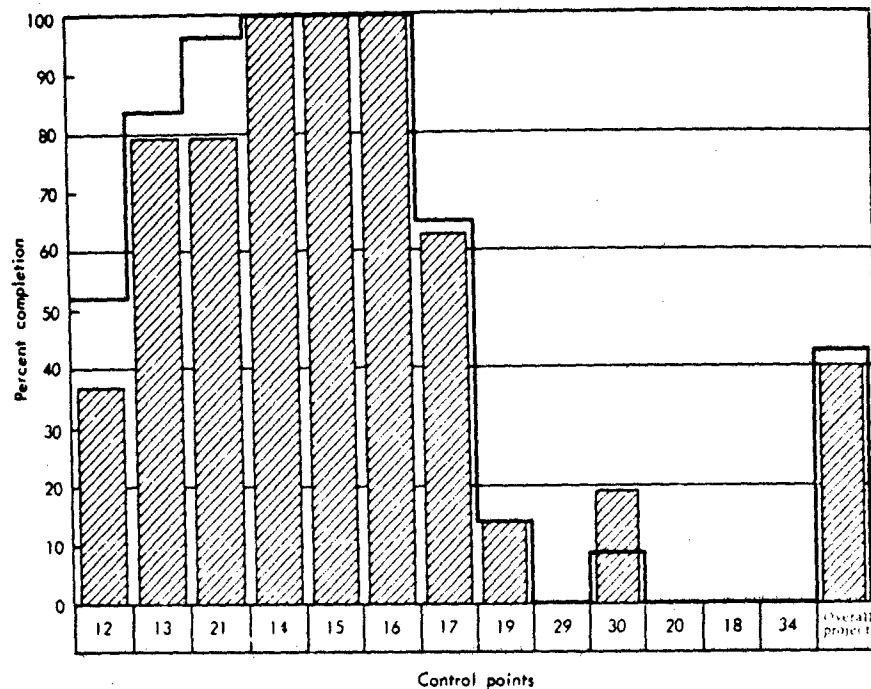


Fig. 8—LOB prototype development phase progress chart

that 122 activity-weeks will be needed to complete the project. Accordingly, the estimated percentage of the overall project completed is  $1 - (122/204) = 40$  percent.

Although the LOB technique does not provide any sophisticated way of guiding personnel in the process of estimating time remaining to complete a project, one method frequently used by schedulers is to divide a major phase into a number of individual technical tasks and then relate the number completed to the total. However, such a method has the limitation of assuming that all tasks are of equal difficulty. An alternative, of course, is for the estimator to draw more generally on his own experience in determining estimated time to completion.

#### The Line of Balance

An additional step is necessary to complete the analysis of program progress. That step is "striking the LOB." On the objectives chart (Fig. 6), construct a vertical line perpendicular to the abscissa at the date of the study. This vertical line will intersect several, if not all, of the percent completion lines for the individual events

at a point representing their currently scheduled completion status. Then draw a horizontal line at the percent completion point on the progress chart (Fig. 8), above the respective events. Thus, both the scheduled status and the actual status of the events and of the overall project are shown for the date of the study. Notice that in the development phase, the line of balance does not necessarily descend continuously in a stepwise fashion as it must in the production plan.

#### EVALUATION OF LOB TECHNIQUE

The LOB technique, like the Gantt technique, was originally designed for production operations. The Gantt technique focused on providing management with information relating to the efficient utilization of resources. Machine and manpower inputs to the production process were emphasized. On the other hand, the LOB technique is product oriented. Its information centers on the extent to which the planned production of a quantity of items is actually being realized. It is not directly concerned with the efficient utilization of resources. Its key usefulness is that bottlenecks in the production process are emphasized. Management must then take appropriate action, generally increasing the level of resources at these bottlenecks. Consequently, Gantt and LOB are complementary techniques.

The LOB technique has some applicability in prototype development when a limited number of components, or operations, are to be controlled. The LOB development plan chart is capable of depicting interrelationships, although seldom is the effort made to include all such relationships.

The LOB technique has several limitations. The inability to precisely state the percent completion of components is one area that can lead to weakened managerial control of the project.

In addition, if management wishes to examine the impact of alternative approaches to overcoming a bottleneck, the LOB affords no simulation capability for this purpose. The determination of the time to complete a component is left up to the judgment of an engineer, and LOB is silent as to how this estimate should be made. Consequently, inconsistencies occur and reliability is impaired. Finally, the technique is rather inflexible. If there is a change in the development

plan, the entire chart system may need to be reconstructed; the updating of program progress requires extensive chart changes. Table 6 further identifies the strengths and weaknesses of the LOB technique.

Table 6

LOB TECHNIQUE--STRENGTHS AND WEAKNESSES

Criteria	Strengths	Weaknesses
1. Validity	Uncertainties surrounding completion times in production operations are minimal; consequently LOB affords management a sound technique for judging status of operations.	Uncertainties encountered in the development phase impair judgment on actual project status. The techniques for estimation of percent completion can lead to erroneous decisions concerning project development.
2. Reliability	Compares favorably with Gantt technique.	
3. Implementation	Only slightly more difficult to comprehend and to implement than Gantt technique.	
4. Universality of project coverage	Capable of covering a system life cycle.	Does not emphasize resource allocation directly.
5. Sensitivity testing (simulation)		No significant capability for simulating alternative courses of action.
6. Forecasting	Depicts status of project well in production stage and can forecast whether or not schedule will be met.	Offers no technique to handle uncertainty in development phase.
7. Updating		Considerable clerical effort required to update graphs.
8. Flexibility		Inflexible. When major program changes occur, the entire set of graphs must be redrawn.
9. Cost	Data gathering and computations can be handled routinely. Expense is moderate and largely for clerical personnel and chart materials.	Charts require frequent reconstruction, which is time-consuming.

NOTE: Recall that this table is intended only as a summary of certain qualitative information on the relative usefulness of the scheduling technique. As indicated previously, a more formal quantitative evaluation of the extent to which the criteria are met was considered infeasible in this study.

#### IV. THE CRITICAL PATH METHOD (CPM)

##### APPLICATION OF CPM<sup>\*</sup>

The critical path method (CPM) was the first technique designed specifically for complex, one-of-a-kind operations. Although initially used to plan and control the construction of facilities, it applies equally well to development of new weapon systems and is designed to interrelate diverse activities and explicitly depict important interdependencies. The construction of a chemical plant, for example, requires coordination of numerous functions and activities. A well-coordinated construction schedule can shorten the project by months and thereby significantly reduce project costs. The CPM technique utilizes a network approach and a limited time-cost trade-off capability for organizing data on these types of interactions. Accordingly, the basic elements in CPM are:

1. The flow diagram or network,
2. Critical time paths ,
3. Float (scheduling leeway), and
4. The time-cost function.

##### Network

The development of a network or flow diagram that embraces all events and activities and explicitly recognizes major known interdependencies among activities is an important element in the CPM. It is based on the following simple concepts:

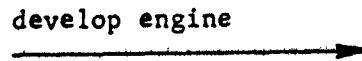
1. An activity (or job) is depicted by an arrow:



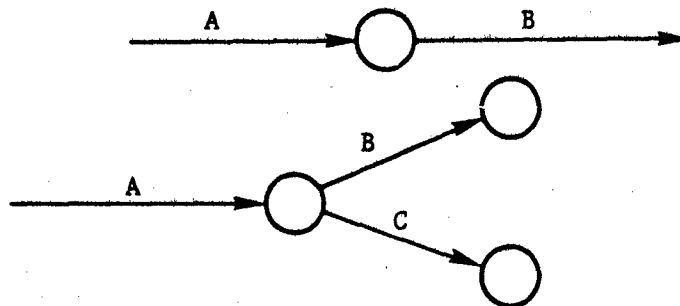
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<sup>\*</sup>The basic development is attributable to M. R. Walker who was with the Engineering Service Division of E. I. DuPont De Nemours & Company Inc., and J. E. Kelley, Jr., Remington Rand Univac (now Sperry Rand Corporation).

2. Each arrow is identified by an activity description:



3. A sequence of activities is indicated by linking arrows:

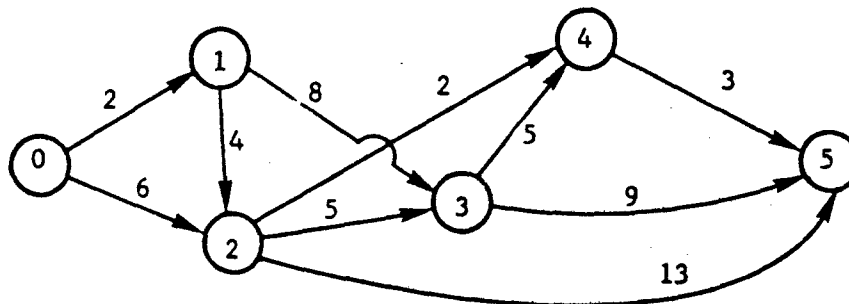


4. Events link activities:

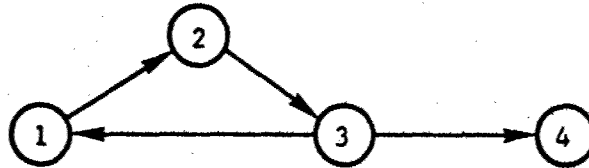


An event occurs at a point in time and signifies either the start or completion of an activity.

5. A grouping of activities and events forms a network. Networks may be either activity- or event-oriented. In activity-oriented networks, the activities (arrows) are labeled; in event-oriented networks, the events (circles, or other symbols) are labeled:



There are certain rules to follow in constructing a network; e.g., no looping is allowed:



Looping indicates not only that event 1 must be completed before event 2, and event 2 before event 3, but also that event 3 must be completed before event 1. It is logically not possible to require the start of a preceding event that depends on completion of a succeeding event.

6. The length of an arrow has no significance; it merely identifies the direction of work flow. Also, time estimates which are secured for activities represent elapsed or flow time and are not identified--at least initially--with calendar dates.\*

#### The Critical Time Paths

In a complex project, involving multiple activities and events, sequences or paths of activities can be identified. These paths vary in length according to the time required to accomplish the component activities. The path or paths requiring the longest time are called the critical paths. When a critical path has been determined, management is advised to devote resources to those activities along this path in an effort to reduce the time requirement and thus shorten the overall program. Of course, as one critical path is shortened, another eventually becomes critical.

#### Float

Some leeway exists in scheduling activities not on a critical path. This leeway is called float. The technique for determining float is as follows:

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\*The distinction between flow time and calendar (or scheduled) time will be clarified further under the subsequent section on the PERT system.

Starting at the beginning of the network, determine the earliest occurrence time for each event in the program. Since the first event (which has no preceding activities) must occur before any succeeding activities can begin, assign it an earliest occurrence time (ES) of zero. Add to this time the duration of the activity leading to the next event; this yields the ES for that succeeding event. If several activities lead to a given event, then its ES is the highest value obtained by adding the duration of each predecessor activity to the ES of the activity's beginning event. Thus when an event is a part of two or more paths, the longest path to the event must be completed before any subsequent activities can be started. Continue the process until the final event has been reached; its ES becomes the earliest completion time for the project.

To determine the latest occurrence time (LC) for each event, begin with the time estimate for the completed project, obtained from the ES procedure above, and assign this as the LC for the final event. Then subtract from this the time duration of the immediate predecessor activity to obtain the LC for the activity's beginning event. If an event has several succeeding activities, its LC is taken as the smallest value obtained by subtracting the duration of each of these activities from the LC of its ending event. In this manner calculate the LC for each event, starting at the end of the network and working backward along activity paths until the beginning event is reached, which will have  $LC = 0$ .

If for each event both the earliest and the latest occurrence time are available, the float or leeway in scheduling each event can be readily calculated. Those events and activities with zero float are necessarily on the critical path.

The actual procedure for computing float is as follows: Let  $i$  = an event signifying the origin of an activity, let  $j$  = an event signifying the termination of the activity, and  $Y_{ij}$  = the activity time duration. Note that an activity's earliest start time ( $ES_{ij}$ ) equals  $ES_i$ , the earliest occurrence time of event  $i$ ; and the activity's latest completion time ( $LC_{ij}$ ) equals  $LC_j$ , the latest occurrence time of event  $j$ .

Construct a matrix by entering the  $Y_{ij}$  for each activity in the proper cell. For example, using the network shown in item 5 above, a matrix can be constructed as follows:



ES						
0	0	2	6	-	-	-
2	1	-	4	8	-	-
6	2	-	-	5	2	13
11	3	-	-	-	5	9
16	4	-	-	-	-	3
20						

0	2	6	11	17	20	LC
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Computing Earliest Occurrence Time. The procedure for computing earliest occurrence time (ES) is as follows:

1. Enter a zero in the first cell of the ES column, which represents the starting time of the project.
2. Add the corresponding values of  $Y_{ij}$  to the ES values column by column. In our example,  $ES_0 = 0$  and  $Y_{01} = 2$ ; hence  $0 + 2 = 2$ , and we enter 2 in the ES column below the zero, indicating that 2 weeks are required before the activities immediately after event 1 can be started.
3. Continue this procedure for each column. For example, the values in column 2 of the matrix are 6 weeks and 4 weeks. The corresponding values in the ES column are 0 and 2 weeks. Adding  $6 + 0 = 6$  and  $4 + 2 = 6$ , we see that by either path it will be 6 weeks before event 2 can occur. Consequently, we enter 6 in the ES column opposite event 2.
4. Where different times result from this summation process, select the longest time (path) and enter that number in the ES column. For example, column 3 of the matrix has  $Y_{ij}$  values of 8 and 5; the corresponding ES values are 2 and 6. By adding  $8 + 2 = 10$  and  $5 + 6 = 11$ , we see that 11 is the longest time path and place it in the ES column.

Computing Latest Occurrence Time. The procedure for computing latest occurrence time (LC) is as follows:

1. Enter the longest time path in the project (i.e., 20 weeks, taken from the last cell in the ES column) in the last cell of the LC row.

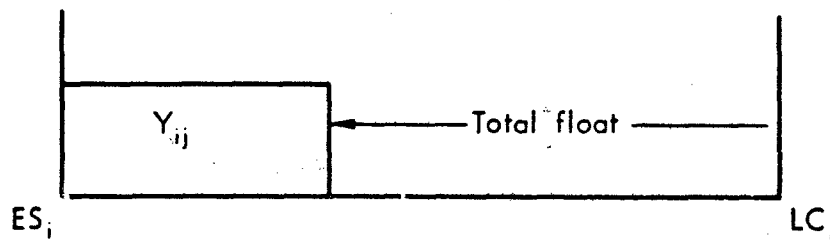
2. Subtract the corresponding values of  $Y_{ij}$  from the LC values row by row. In our example,  $LC_5 = 20$ , and  $Y_{45} = 3$ ; hence  $20 - 3 = 17$ , and we enter 17 in the LC row to the left of the 20 weeks. This means that event 4 must occur by the seventeenth week if the project is to be completed in 20 weeks. Continue this procedure for each row.

3. Where different times result from the subtraction process, select the shortest time (path) and enter that number in the LS row. For example, row 3 of the matrix has  $Y_{ij}$  values of 5 and 9; the corresponding LC values are 17 and 20. By subtracting  $17 - 5 = 12$  and  $20 - 9 = 11$ , we see that the shortest time path is 11 and enter that number in the LC row.

4. The last entry in the LC row should be a zero, corresponding to the zero in the first cell of the ES column.

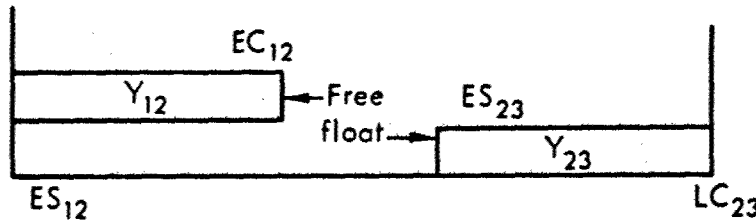
Identifying Events on the Critical Path. Every event that has an equal ES and LC time is on the critical path. In our example, event 1 has an ES of 2 and an LC of 2; hence it is on the critical path. Event 4 has an ES of 16 and an LC of 17; hence it is not on the critical path. Accordingly, the critical path includes events 0, 1, 2, 3, and 5.

Identifying Total Float. Total float for an activity is the amount of time available for an activity less the amount of estimated time required to complete the activity. In our example, total float for an activity equals  $(LC_j - ES_i) - Y_{ij}$ . Thus, for event 3,  $LC_3 = 11$ ;  $ES_1 = 2$ ;  $Y_{13} = 8$ ; hence  $(11 - 2) - 8 = 1$  week of float.

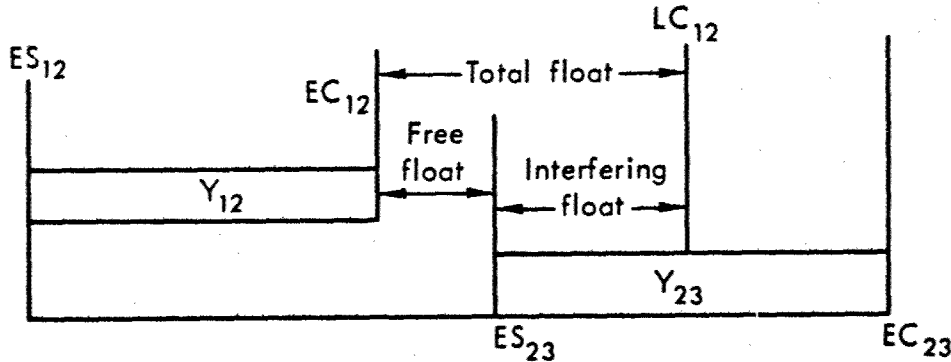


Other Types of Float: Free, Interfering, and Independent. It may be desirable to know how much a preceding activity may be delayed (if at all) without interfering with the earliest start of the succeeding activity. This is called free float. At this point, it is necessary

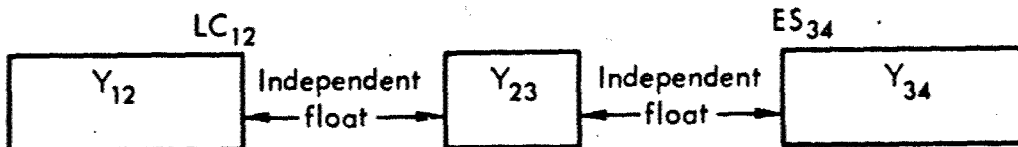
to introduce data on an activity's completion time ( $EC_{ij}$ ).  $EC_{ij}$  is derived by adding the estimated time required for an activity ( $Y_{ij}$ ) to the activity's earliest start time ( $ES_{ij}$ ). To compute free float: Let  $ES_{12}$ ,  $EC_{12}$ ,  $LC_{12}$ , and  $Y_{12}$  apply to the preceding activity and let  $ES_{23}$ ,  $EC_{23}$ ,  $LC_{23}$ , and  $Y_{23}$  apply to the succeeding activity. Then  $ES_{23} - (EC_{12} + Y_{12}) = \text{free float for activity 1-2}$ .



Interfering float is total float minus free float. The concept also can be presented in a diagram. For example, any delay in activity 1-2 beyond the ES date of activity 2-3 will delay or interfere with activity 2-3. Hence, part of the total float for activity 1-2 is free float ( $ES_{23} - EC_{12}$ ) and the remainder is interfering float ( $LC_{12} - ES_{23}$ ).



Independent float is computed as  $ES_{34} - LC_{12} - Y_{23}$ . For example, if all activities prior to activity 2-3 are completed by the  $LC_{12}$  date, and all activities succeeding activity 2-3 are started at the  $ES_{34}$  date, then  $ES_{34} - LC_{12}$  is the amount of time available to perform activity 2-3. Subtracting the actual time required to perform the activity from the available time gives the independent float; i.e., the activity can be displaced forward or backward within this time interval without interfering with any other event.



### Time-Cost Function

The contribution to system management embodied in the CPM does not end with the time parameter. It also provides a technique to aid management in making time-cost trade-off decisions.

The technique is quite simple, requiring only four estimates: (1) normal activity time, (2) normal activity costs, (3) activity times on a "crash" basis, and (4) cost on a "crash" basis. These estimates are based on the principle of the time-cost curve, as illustrated in Fig. 9.

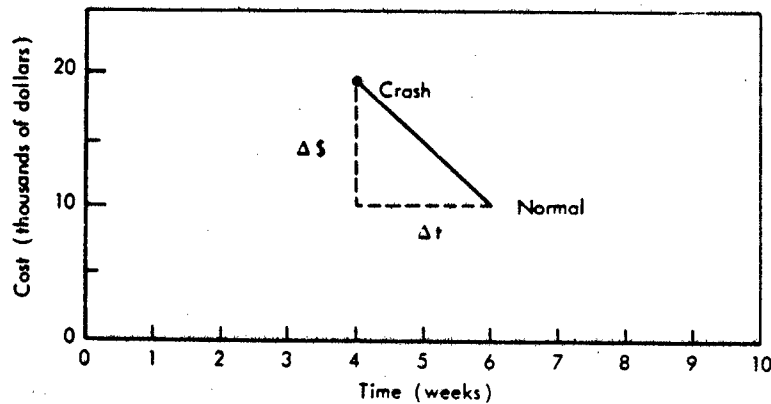


Fig.9—CPM time-cost trade-off

In this example, the normal activity time estimate would be six weeks and the cost estimate would be \$10,000. On a crash basis, the activity time would be four weeks and the cost \$20,000. A simple assumption would be that cost and time are related inversely and linearly (i.e., for each reduction in time there will be a corresponding increment of added cost). For example, according to Fig. 9, shortening the time by one week (from six to five) would cost \$5,000. The decision-maker can compare the costs of shortening the schedule by allocating additional resources to an activity (or activities) on the critical path for which marginal cost is less than for any other activity. Thus the time required on any path can be shortened at least cost. Assumptions other than an inverse linear relationship can also be introduced by properly reflecting them in the shape of the time-cost curve.

The task of calculating these time-cost trade-offs can be quite formidable to accomplish manually if the project becomes even moderately complex. A computer program assuming linear time-cost relationships

has been developed that will automatically schedule the project for the least cost activities. This computer routine requires at least the two time-cost data points--i.e., assuming normal and crash programs for each activity. Non-linear assumptions are more difficult to treat in large projects.

It is not the purpose of this Memorandum to explore time-cost relationships; however, this mechanism is usually considered a component of CPM and should be mentioned when comparing CPM with PERT.\*

#### APPLICATION TO THE MODEL

The CPM can be applied to the hypothetical missile system described in Sec. II. Figure 10 represents the planned sequence of activities in network form. The numbered circles correspond to the events in Table 1. Note that interdependencies are depicted in the network. For example, event 3 must occur before event 20 can be completed. Such interdependencies can be readily ascertained from the CPM network but would not be clearly evident in a tree diagram or in a Gantt or milestone chart.

It could be argued that engineers responsible for development are usually aware of these interrelationships when the Gantt chart is used, and nothing is gained by the network presentation. This may indeed be the case in simple or small-scale projects. However, when a number of managers are involved in planning and measuring the progress of a complex system, they may not be aware of the effect of interdependencies beyond their immediate sphere of interest. It is possible that a subcontractor may be well aware of those relationships within his control, and yet not realize that his schedule is in jeopardy because another department will not be able to deliver its portion of the project on time, or, conversely, that a component he is developing may, if not

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\* Several scheduling techniques have since been expanded to incorporate cost considerations, e.g., PERT-Cost, RAMPS, SPAR, etc. See especially PERT-Cost System Description Manual, Vol. 3, U.S. Air Force, December 1963; Jack Moshman, Jacob Johnson, and Madalyn Larsen, RAMPS--A Technique for Resource Allocation and Multi-Project Scheduling, Proceedings 1963 Spring Joint Computer Conference; and J. D. Wiest, The Scheduling of Large Projects with Limited Resources, Research Memorandum No. 113, Graduate School of Industrial Administration, Carnegie Institute of Technology, Pittsburgh, 1963.

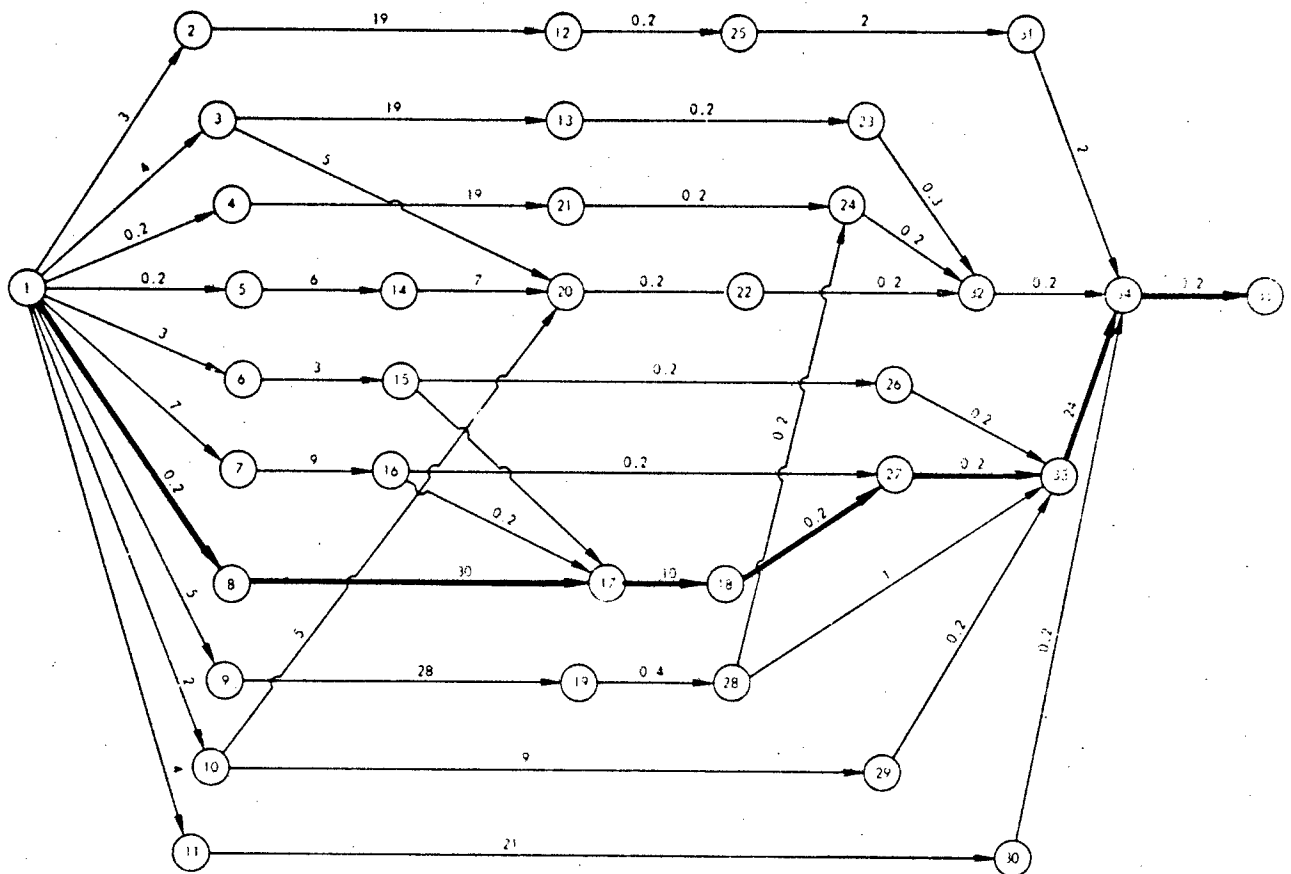


Fig.10—CPM network applied to hypothetical missile system

completed on time, retard another subcontractor and hence the entire project. In making the relationships explicit, the network serves as a communications device to ensure that all parties concerned are aware of the overall plan and their responsibilities in view of the plan.

The problem of keeping the planning and control information system attuned to actual development operations is common to all managerial techniques. A major advantage of the network-type presentation is that it enables the manager to cumulate the activity times along a given path to determine the total estimated time per path. The longest time path is the critical path. In Fig. 10, for example, the longest time path is 64.8 weeks and is composed of events 1, 8, 17, 18, 27, 33, 34, and 35. All other paths are estimated to be completed in

less time. If the tasks are scheduled to take the estimated time, then all paths other than this critical path contain float. Any path on which estimated completion time is greater than, or equal to, the time remaining before a scheduled project completion date is called critical. Hence, there may exist the most critical path, the second most critical, etc. In Fig. 10, events 1, 9, 19, 28, 33, 34, and 35 would comprise the second most critical path (58.6 weeks).

Table 7 presents the matrix of task times needed to compute the ES and LC times for each activity in our hypothetical missile system. For example, activity 14-20 has an earliest start time of 6.2 weeks from the beginning of the project. It must be completed in the 64th week or the activity will "float" the succeeding activity beyond the project completion date. Thus we can compute the various float concepts for activity 14-20:\*

$$\begin{aligned}
 1. \text{ Total float} &= (LC_{20} - ES_{14}) - Y_{14-20} \\
 &= (64 - 6.2) - 7 = 50.8 \\
 2. \text{ Free float} &= ES_{20-22} - (ES_{14-20} + Y_{14-20}) \\
 &= 13.2 - (6.2 + 7) = 0. \\
 3. \text{ Independent float} &= (ES_{20-22} - LC_{5-14}) - Y_{14-20} \\
 &= (13.2 - 57) - 7 = -50.8
 \end{aligned}$$

1. Total float. Assuming that there are no project changes, and that activity 14-20 is started at the earliest possible date and completed at the earliest possible time, 50.8 weeks will elapse before activity 20-22 will have to be started. Consequently, freedom exists to allocate resources to other more critical tasks up to a maximum of 50.8 weeks before the scheduled completion of activity 14-20 is jeopardized.

2. Free float. If activity 20-22 were to start on the earliest possible date, no freedom would exist to allocate resources to other

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\* For clarity, and to be consistent with the explanation on pages 40-41, LC and ES values are identified by the appropriate activity designator which in turn is composed of both the starting and ending event numbers. Technically, however, ES can be fully defined by the starting event and LC by the ending event number.

Table 7  
CPM MATRIX SHOWING EARLIEST START (ES) AND  
LATEST COMPLETION (LC) TIMES FOR ILLUSTRATIVE EXAMPLE

ES	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
LC	0	3	4	0.2	0.2	3	7	0.2	5	2	4	19	19	6	3	9	30	28	5	7	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.4	0.2	0.2	0.2	0.2	0.2	0.2	0.2
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0	41.4	44.9	45	51	26.8	21	0.2	11.2	31.4	43.4	60.4	63.9	57	29.8	30	30.2	40.2	39.2	64	64	64.2	64.1	64.2	60.6	40.2	40.4	39.6	40.4	62.6	64.4	62.6	64.4	40.6	64.6	64.8
---	------	------	----	----	------	----	-----	------	------	------	------	------	----	------	----	------	------	------	----	----	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------



tasks. In other words, there would be no free float in the scheduling of activity 14-20. As mentioned previously, total float can be subdivided into free float and interfering float. Interfering float would delay (interfere with) the start of the subsequent activity (20-22) beyond the earliest start date. In the above example, all of the float for activity 14-20 is interfering.\*

3. Independent float. In the illustration a negative value (-50.8) was obtained and therefore there is no independent float. This negative statistic does indicate that there would be no time available to perform activity 14-20 if the prior activity (5-14) were delayed until its latest completion date and if scheduling the subsequent activity (20-22) on its earliest start date was contemplated. In fact, the latest completion date for activity 5-14 significantly postdates the earliest start date for activity 20-22. This, of course, is of no real concern here because the earliest start date of activity 20-22 (also identified by  $LC_{20}$ ) can be delayed substantially without jeopardizing project completion. In other words, total float exists, but independent float, being a very restrictive concept, does not in this case.

#### EVALUATION OF CPM

The network concept of CPM is an excellent device for explicitly depicting significant interrelationships among events. The flow of all activities is on paper so that those concerned can analyze the work plan and approve or disapprove it. Communication of planned activity is thus facilitated.

Since time estimates lead to the determination of a critical path, the attention of management is focused on the activities along the path so that resources can be applied to them, perhaps by reallocation from other activities where float exists.

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\* If these resources were allocated, it would be at the cost of delaying the start of task 20-22. This may, nevertheless, be a wise decision since activity 20-22 may be delayed a maximum of 50.8 weeks and the project can still be on schedule.

One criticism of CPM is that emphasis on critical path activities may obscure the fact that some activities on a second path may be very close to being critical and would become so with slight changes in values. However, this possibility can be alleviated by determining the first most critical path, the second most critical path, etc., and then determining the critical activities within this broader context.

The time-cost function, although not fully implemented in actual systems, can provide trade-off information on the relative cost of reducing scheduled time in various activities. This trade-off feature linking cost and schedule is beyond the scope of this study but nevertheless is an important element of the CPM method.

CPM does not provide a capability for handling schedule uncertainty. For example, the development of a component may involve a major engineering improvement, and there may be considerable uncertainty regarding the time required for its accomplishment. In CPM, the responsible individual must provide management with his single best estimate of the time requirement. He may not reflect his uncertainty in terms of a range of estimates. The single value is incorporated into the network and the critical path determined. If the estimate is in error, then the critical path may be incorrectly drawn.

The strengths and weaknesses of CPM are summarized in Table 8.

Table 8

CPM TECHNIQUES--STRENGTHS AND WEAKNESSES

Criteria	Strengths	Weaknesses
1. Validity		No formula is provided to estimate time to completion; consequently, the technique is as valid as the estimator. The margin of error is generally less in construction than in development.
2. Reliability		Numerous estimates in a large project, each with some unreliability may lead to significant errors in judging project status.
3. Implementation		Relatively difficult to explain, especially if the various concepts of float are utilized.
4. Universality of project coverage	Very good for single-shot activities, such as construction or development projects.	Weak in the production phase of a weapon life cycle. The technique is not well adapted to scheduling production quantities.
5. Sensitivity testing (simulation)	Excellent for simulating alternative plans, especially when coupled with the time-cost aspect.	
6. Forecasting	Strongly oriented to forecasting ability to accomplish future events on schedule.	
7. Updating	Good capability. Activities are clearly identified and time estimates can be obtained as needed.	
8. Flexibility	Portions of the network can be easily changed to reflect program changes.	
9. Cost		Considerable data are required to use CPM as both a planning and status reporting tool and a computer is almost invariably required. Therefore, the cost outlay can be fairly extensive.

NOTE: Recall that this table is intended only as a summary of certain qualitative information on the relative usefulness of the scheduling technique. As indicated previously, a more formal quantitative evaluation of the extent to which the criteria are met was considered infeasible in this study.

## V. PROGRAM EVALUATION AND REVIEW TECHNIQUE (PERT)

### PERT METHODOLOGY

The program evaluation and review technique (PERT)\* was formulated at approximately the same time as the critical path method (CPM). Like CPM, PERT is designed for scheduling activities in the development phase and is not directly suitable for application to repetitive production operations. Both CPM and PERT are based on the network concept; both identify a critical path; both isolate float or slack. CPM, however, pioneered simple time-cost trade-off relationships. PERT, on the other hand, used a more sophisticated approach to the problem of treating schedule uncertainties.

Since the events, activities, and network concepts embodied in PERT are the same as those described for CPM, our discussion of PERT will cover only the major differences between the two techniques.

### The PERT Planning Phase: Estimated Time

It is essential in the PERT planning process to secure estimates of the amount of time required to complete each activity. PERT recommends that three estimates be obtained rather than a single point estimate:

1. Optimistic time, a, (only 1 percent of the time would the activity be completed more quickly),\*\*
2. Most likely time, m, (mode),
3. Pessimistic time, b, (only 1 percent of the time would more time be required).\*\*

This estimating method has the following advantages. First, estimators usually make more valid estimates if they can express the extent of their uncertainty. Range-of-time estimates are more realistic

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\*PERT was developed by C. E. Clark, W. Fazar, D. G. Malcolm, and J. H. Roseboom, working with the management consulting firm of Booz, Allen and Hamilton, the Navy Bureau of Ordnance, and Lockheed Corporation.

\*\*This 1 percent requirement is frequently relaxed in practice.

and informative than a single point estimate. They are particularly worthwhile assuming that the burden of preparation does not become excessive.

Second, a single point estimate is likely to be the mode. In estimating activity time, the mean is generally considered a more representative statistic than the mode. It more nearly represents all possible values in the time distribution because it is based on all the information relative to the distribution, rather than being merely the most frequent single estimate.

The beta (B) distribution is used in the PERT estimation process.\* A formula approximating the mean of the distribution, called the expected time ( $t_e$ ), can be derived based on the three time estimates and the beta distribution. For example:

$$t_e = \frac{a + 4m + b}{6}$$

Letting  $a = 5$  months,  $m = 7$  months, and  $b = 15$  months, we obtain

$$\frac{a + 4m + b}{6} = \frac{5 + 4(7) + 15}{6} = \frac{48}{6} = 8 \text{ months.}$$

Note that the midpoint of the range is  $(15 + 5) \div 2 = 10$  months. The mode is 7 months. The mean ( $t_e$ ) is 8 months. The mean lies one-third of the distance from the mode to the midpoint of the range.

### The Critical Path

After the expected time has been determined for each activity in the network, it is possible to compute the critical path, which is simply the longest path of expected times in the network. When more than one time path leads into an event, the longest time path leading into that event establishes the expected time for the event.

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\* The beta distribution has two interesting characteristics: (1) The range precisely equals six standard deviations (i.e., the "tails" of the distribution do not approach infinity), and (2) using the PERT approximation, the mean of the distribution lies one-third of the distance from the mode to the midpoint of the range. Also, in practice the skewness of activities tends to be toward the right.

### Calendar Time

The scheduler is now ready to schedule the start and the completion of each activity, based on the expected time estimates. Several concepts have been developed to aid management in monitoring progress and allocating resources to the activities. The first is that of the earliest expected occurrence date of an event ( $T_E$ ). Normally, the start of a project is associated with a specific calendar date, and then the elapsed time for an activity is added to that date to determine the calendar date of the next event. This procedure is followed for every event on the PERT network. In working from the start to the end of the project--i.e., the forward pass--the expected earliest occurrence dates for each event can be determined.

After the earliest completion date has been established for the end item in the project, the latest allowable occurrence date ( $T_L$ ) for each event can be determined by proceeding backward--the backward pass--from the earliest completion date,<sup>\*</sup> or from a promised due date, and subtracting expected times. The  $T_L$  represents the latest date that an event can occur and not jeopardize the project completion date.<sup>\*\*</sup>

It is now possible to determine the amount of slack in the project. Slack is the time flexibility available to management in scheduling resources to a given activity and is defined as  $T_L - T_E$ . If  $T_L$  is later than  $T_E$ , then positive slack exists and management has some freedom in scheduling the event. If  $T_L$  is earlier than  $T_E$ , negative slack exists and completion of the project is in jeopardy. The path with the most negative slack, or the least positive slack if there is no negative slack, is necessarily the longest time path--the critical path.

Negative slack should not exist in the planning phase of a project. If the  $T_E$  and  $T_L$  were computed as described above, negative slack could not exist, and the critical path would contain, at a minimum,

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<sup>\*</sup> If the earliest completion date is used as the project completion date (due date) in performing the backward pass, earliest and latest completion dates will be identical for events on the critical path.

<sup>\*\*</sup>  $T_E$  and  $T_L$  correspond to the ES and LC measures for events in Chapter IV. A different symbol has been selected to emphasize that the event occurrence times in the PERT model are probabilistic measures--the sum of expected activity duration times.

zero slack. Frequently, however, under pressure from the customer or in eagerness to obtain a contract, a contractor will agree to complete a project in less time than is indicated by the preliminary estimates.\* This directed completion date is then entered on the calendar as the scheduled completion date ( $T_S$ ) and, in the backward pass, new  $T_L$  dates are computed based on the directed date. Consequently, negative slack may exist.

Negative slack must be remedied in one of two ways if management expects to complete the project on schedule. First a portion of the resources can be withdrawn from noncritical activities and allotted to critical activities. This, of course, implies that such resources--i.e., skills, equipment, or facilities--are transferable. Second, management can increase the overall level of resources devoted to the project.

From a project management standpoint, an ideal situation would exist if there were zero slack on all activities. Adequate resources would then be optimally allocated, given the completion date of the program.\*\*

After analysis of possible trade-offs of resources, acceptable scheduled completion dates ( $T_S$ ) can be determined and the activities scheduled. As one might suspect, in general  $T_S$  should occur between  $T_E$  and  $T_L$ .

#### The PERT Operating Phase

The acceptance by management of the  $T_S$  means the acceptance of a plan of action and the end of the initial PERT planning phase. The authorization of work to be performed as scheduled begins the PERT operating phase. Essentially, this phase involves reporting program status and acting on this information. The following information is reported during the operating phase:

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\* It also should be noted that such estimates usually are made with a specified level of funding in mind and are subject to modification if the anticipated funding level is revised.

\*\* This assumes that the cost-time relationship for individual activities is a continuously decreasing function to the right, as illustrated in part in Fig. 9.

1. Completed activities and their completion dates.
2. Changes in activity time estimates.
3. Changes in schedule.
4. Event and activity additions and deletions.

Input data are prepared and computer printouts of status are distributed periodically (generally every two weeks) to the appropriate levels of management.

The PERT-Time system cycle, with the interrelationships between the planning and the operating phases, is shown in Fig. 11.

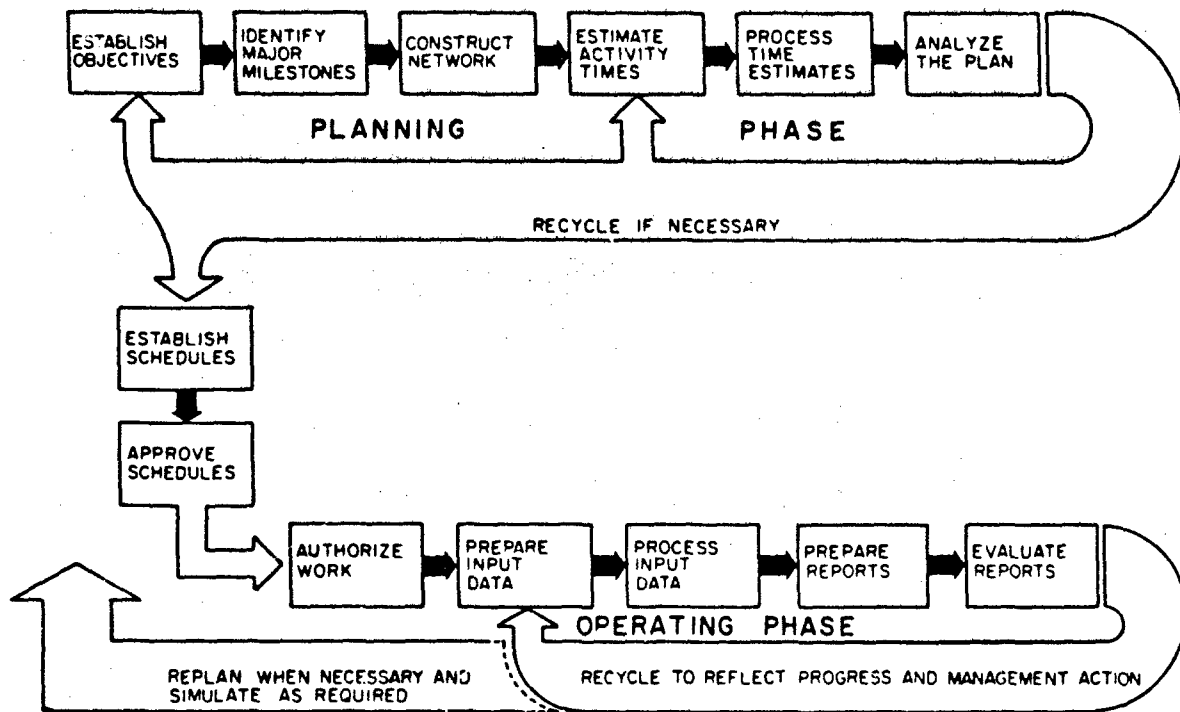


Fig. 11—PERT: Planning and operating phases\*

Various types of data are contained in the PERT system. Thus far, the most important use of operating data for control purposes appears to be through the analysis of slack. The amount of slack

\*Taken from PERT-Time System Description Manual, Vol. 1, U.S. Air Force, September 1963, p. V-3.



(often negative) is charted periodically so that management can follow the trend from week to week. Normally, with good control the amount of negative slack on a path should decrease over time. This decrease is generally attributable either to greater management attention to activities on that path or, as described below, to the lessening of uncertainty concerning completion times.

#### The Standard Deviation ( $\sigma$ ) of an Activity

By using three time estimates for each activity, the scheduler can apply probability theory in determining uncertainty in scheduling activities. Assuming that the beta distribution is a valid representation of the distribution of the estimates, the standard deviation for an activity can be approximated by the following equation:

$$\sigma = \frac{b - a}{6},$$

thus the range ( $b - a$ ) is six times the standard deviation.

To illustrate the standard deviation of an activity, let  $a = 10$  months,  $m = 13$  months, and  $b = 16$  months. Then

$$t_e = \frac{10 + 4(13) + 16}{6} = 13; \quad \sigma = \frac{(16 - 10)}{6} = 1.$$

A common interpretation of this statistic is that a 67 percent chance (67 out of 100 times) exists that the activity will be completed within one standard deviation (12th and 14th months); a 95 percent chance between  $2\sigma$  of the mean (11th and 15th months); and a 99 percent chance between  $3\sigma$  of the mean (10th and 16th months). This interpretation is misleading because the above applies to a normal, and not to a beta distribution. Depending on the skewness of the beta distribution one  $\sigma$  from the mean may contain considerably more or less than 67 percent of the observations. The  $\sigma$  has no inherent meaning in quantifying uncertainty for an individual activity; however it is used to compute the  $\sigma$  of an event which is described in the following subsection.

Probability of Meeting Scheduled Date  
or of Having Positive Slack

The probability of meeting the scheduled date or, alternatively, of having positive slack can be determined by using the concepts of slack and standard deviation of an activity. While this probability statistic can be computed for any event in the project, the ending event is used in the following example.

First, compute the length of the longest path leading into any specific event ( $T_E$ ), and then compute the standard deviation of that event's earliest occurrence time ( $\sigma T_E$ ). The  $\sigma T_E$  is defined as the square root of the sum of the activity standard deviations squared ( $\sigma^2$ ), i.e., the variances of the activities lying on the longest time path leading into that event. Here the event time (path length) is generally assumed to be normally distributed,\* not beta distributed. The probability of meeting the scheduled date can then be determined by using tables of the areas under the normal curve.\*\* The formula for this normalized statistic is

$$\frac{T_S - T_E}{\sigma T_E}.$$

If we assume that an event is scheduled to be completed in 10 months, but the earliest expected date is 12 months from now, and the standard deviation of that event is 1 month, then

$$Z = \frac{T_S - T_E}{\sigma T_E} = \frac{10 - 12}{1} = \frac{-2}{1} = -2.$$

From tables of the area under a normal curve we find that a Z of minus two (-2) is associated with 0.0228 of the total area under the curve;

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\*Through invoking the central limit theorem. Also, the assumption must be made that the activities are independent. This assumption has been challenged on numerous occasions, since many activities are interconnected and also frequently appear on more than one path in a network.

\*\* Tables of areas under the normal curve can be found in virtually every basic statistics textbook. They are also included in the PERT-Time System Description Manual, Appendix B.

in other words, only two times out of 100 would management expect to complete that event on schedule. Since this represents a small probability, it is clear that the program schedule is in difficulty.

Unfortunately, management usually does not employ this probability measure because of a feeling that too much uncertainty exists in the entire estimation and planning process for this statistical calculation to have meaning. It also appears that management in general is not familiar with probability theory.

Actually, similar information can be presented to management without using formal probability theory. The scheduling section in the Dynasoar (X-20) Program Office derived an interesting surrogate for such probability statistics. A "recovery ratio" was computed that was simply a ratio of the negative slack to the length of the critical path. For example, path A in a project may require 20 weeks to perform and contain 5 weeks of negative slack. Path B may require 3 weeks and have 1 week of negative slack. Slack calculations alone would indicate that Path A was most critical (i.e.,  $5 > 1$ ). The recovery ratios for path A would be  $(5 \div 20)$ ,  $1/4$ , or 0.25; for path B they would be  $(1 \div 3)$ ,  $1/3$ , or 0.33. This would indicate that in reality path B is more critical than path A, since only 3 weeks would remain to pick up 1 week of negative slack, whereas on A, 20 weeks are available to pick up 5 weeks of negative slack. The recovery ratio is easier to compute and to understand than the probability distribution, and is worth serious consideration by management.

#### Types of PERT Networks

The fact that various levels of management and numerous interrelationships among firms, agencies, and military offices that are involved in weapon system acquisition was brought out in Sec. I of this Memorandum. In such an environment, with its variety of demands, a single network often will not suffice. Accordingly, variations have been evolved to handle various aspects of the planning and control process.

### 1. Detailed and Operating Level Networks

Generally, each prime or associate contractor constructs and uses a network that covers his individual sphere of program responsibility. If a portion of the project is subcontracted to another firm that subcontractor in turn may be required to construct and use a network for his portion of the project. These networks are constructed in considerable detail and frequently comprehend even relatively minor activities and events.\* Such networks are utilized by operating managements and are termed operating networks, or detailed networks. In addition, since they often cover only a fragment of a project, NASA has referred to them as fragnets (fragmentary networks).

### 2. Integrated Project Networks

The detailed operating networks prepared by the separate firms and agencies may be combined or integrated, generally at the SPO level, into one comprehensive network encompassing all events in the entire project. Although perhaps not directly involved in detailed operations, the SPO can exercise management surveillance over the progress of the entire project through use of this integrated network.

### 3. Condensed or Summary Networks

Generally, detailed networks contain too much operating data for top project management or other interested parties (i.e., DOD, Headquarters USAF, etc.) monitoring the progress of the program on a more aggregative basis. To accomplish this, a summary, or condensed network is constructed which eliminates much of the detail, yet retains the events of major significance. Such networks frequently are displayed in project control offices.

Accurate translations of activity time estimates must be made when the operating networks are either integrated or condensed. The integration and condensation processes involve identifying, recording, coordinating and storing interface events.\*\* Various computer routines

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\* It is evident that the level of detail may vary among contractors.

\*\* An interface event signals the transfer of responsibility, end items, or information from one part of the project effort to another.

are being developed to accomplish this complex and vital task. The relationship among these various forms of networks is indicated in Fig. 12. This diagram depicts condensation of networks prior to network integration. Either condensation or integration can occur first depending on the requirements of the levels of program management.

Information usually is abstracted from the condensed network and forwarded to agencies above the SPO in milestone form. The current procedure of selecting information from the networks and listing this information in line item or narrative form appears to have limitations. One relatively simple improvement would utilize the network concept and its relevant information on interrelationships at top project levels. Perhaps this could be accomplished by including a requirement that summary networks be incorporated in the System Package Program<sup>\*</sup> and that project progress be reported against these networks.

#### APPLICATION OF PERT TO HYPOTHETICAL MISSILE SYSTEM

The PERT network shown in Fig. 13 for our hypothetical missile system is a summary network. The events and activities are identical to those used in the network illustrating the critical path technique in Sec. IV, and the same "rules" are followed. In each case the work flow envisioned at the time is the same, and thus the networks are the same. However, in the PERT network, three time estimates are used for each activity: optimistic, most likely, and pessimistic. These estimates (taken from Table 1) are displayed in red on Fig. 13 to emphasize the major difference between CPM and PERT.

The expected times as derived from the three time estimates in PERT frequently will differ somewhat from the single time estimates recorded in CPM. As described above, the mean is conceptually a more comprehensive measure incorporating the extreme (optimistic and pessimistic) values and in this sense reflects the range of uncertainty revealed by the three time estimates. The mean used in PERT will vary

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<sup>\*</sup>As mentioned previously, the System Package Program is the basic management document for major weapon systems programs. (See also p. 19.)

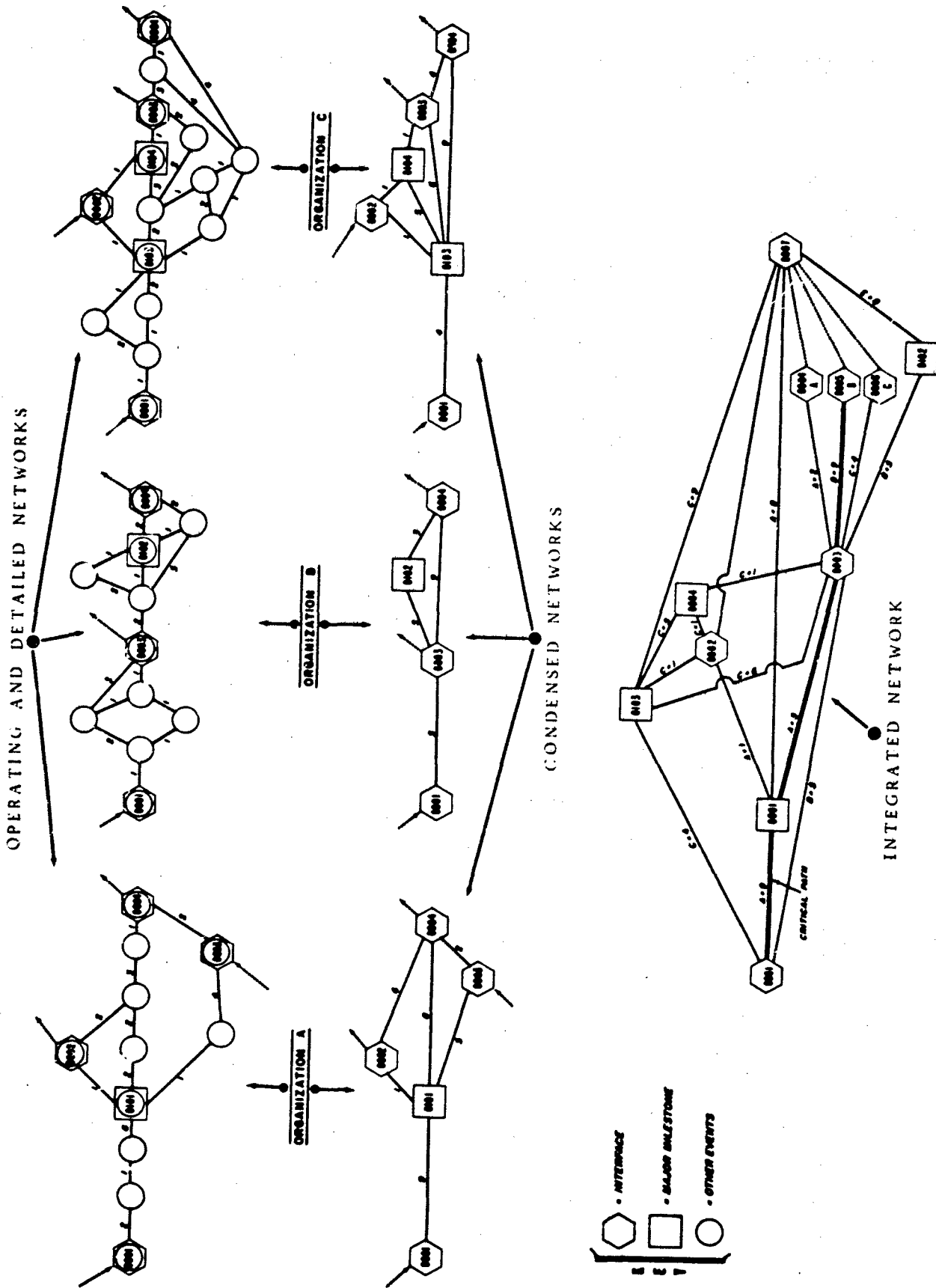


Fig. 12—PERT: Relationships among networks \*

\* Adapted from Planning and Control Techniques and Procedures (PCT), Headquarters, U.S. Army Materiel Command, AMC Regulation 11-16, Vol.2, August 1963, Fig.11-3-5.

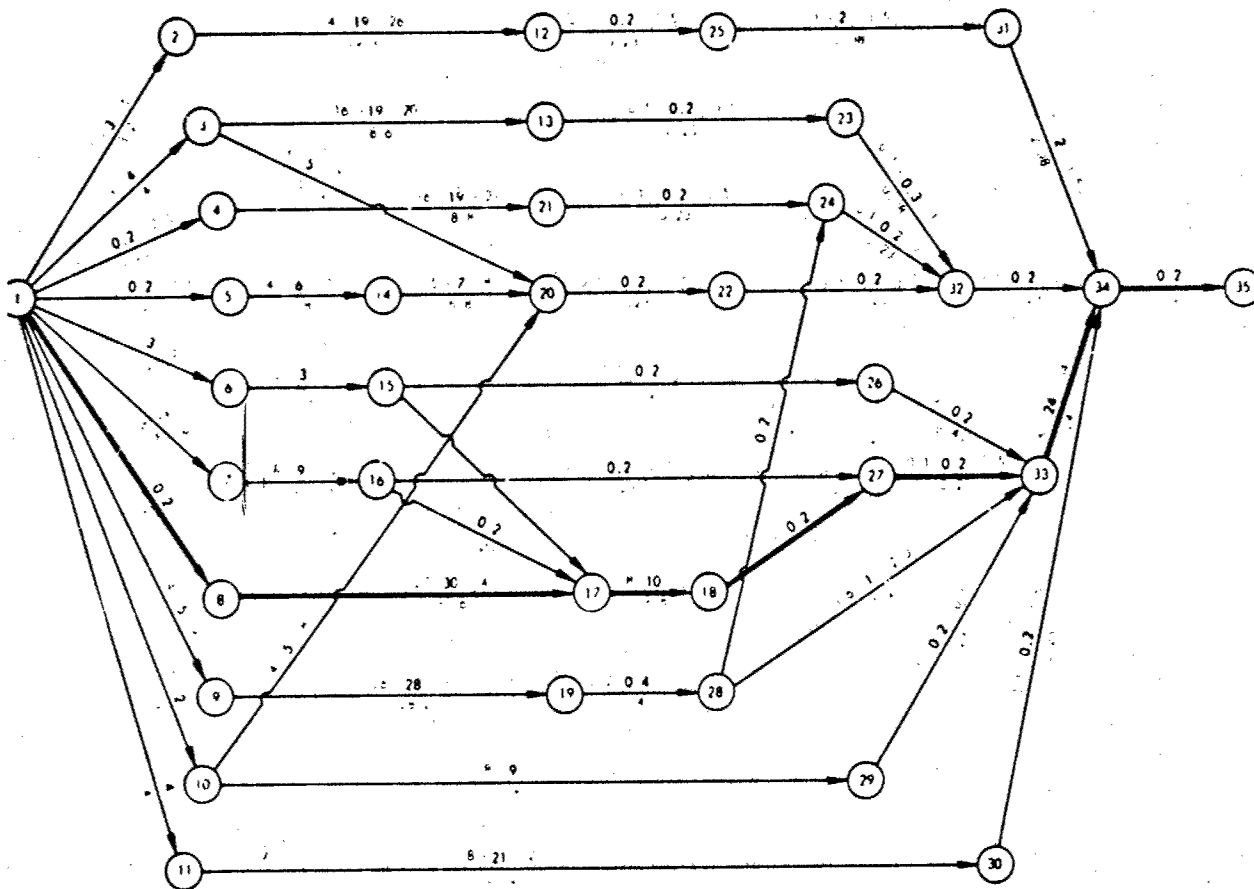


Fig.13—PERT network applied to hypothetical missile system

from the mode used in CPM in those instances when the time interval between the mode and the optimistic time differs from the interval between the mode and the pessimistic time estimates. The mean and mode will coincide where there is equal uncertainty in positive and negative directions, since the mode will bisect the range between the optimistic and pessimistic times.

Once the expected time for each event has been computed, the techniques are identical in their method of identifying the critical path. In our missile system example, the critical path for PERT is identical with that for CPM (i.e., events 1, 8, 17, 18, 27, 33, 34, and 35). However, the time to complete the project has been lengthened from 64.8 weeks to 68.6 weeks. This difference is the result of the fact that

in the estimates for individual activities prepared using the PERT technique, the range between the optimistic and expected times was less than the range between the pessimistic and expected times.

In this simple example, the difference of 3.8 weeks in a project of 69 weeks' duration is only moderately important. It would be difficult to state clearly which estimate (PERT or CPM) was the more accurate. The PERT technique is more sophisticated in that it does attempt to deal with uncertainty. However, various mathematicians have questioned certain of the simplifying assumptions used in the PERT estimation process.\*

If, in scheduling the activities for our hypothetical missile system, we assume a directed date of 65 weeks from now, that becomes the scheduled date for completion of the project. If the length of the critical path is 68.6 weeks, we have negative slack of 3.6 weeks. Because of the uncertainty inherent in the activities as shown by the estimation intervals, management may require some "feeling" for the likelihood of the project's being completed on schedule. Using the concept of zero slack, this likelihood can be ascertained by computing (1) the standard deviations of the activities on the critical path ( $\sigma$ ), (2) the standard deviation of the final event ( $\sigma_E$ ), (3) the Z statistic, and, finally, (4) the probability of positive slack. These computations are given in Table 9.

Note that the probability is only 0.25. A gambling management might be content to proceed and see what develops, but most managements would probably require at the very least a 50 percent chance of meeting schedule. This would then call either for shifting resources from non-critical events to critical events or for employing a higher level of resources. If positive slack exists, management frequently is content to assume that the project will be completed on schedule.

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\*An analysis of the significance of these arguments is found in K.R. MacCrimmon and C. A. Ryavec, An Analytical Study of the PERT Assumptions, The RAND Corporation, RM-3408, December 1962.



Table 9

COMPUTATIONS REQUIRED FOR PROBABILITY OF POSITIVE SLACK

(1) Standard deviation and variance of critical path activities:

Event Number on Critical Path	Standard Deviation of an Activity $[(b - a)/6]$	Variance of an Activity ( $\sigma^2$ )
8	$\frac{0.5 - 0.1}{6} = 0.07$	0.0049
17	$\frac{54 - 27}{6} = 4.5$	20.2500
18	$\frac{11 - 8}{6} = 0.5$	0.2500
27	$\frac{0.5 - 0.1}{6} = 0.07$	0.0049
33	$\frac{0.5 - 0.1}{6} = 0.07$	0.0049
34	$\frac{34 - 16}{6} = 3.0$	9.0000
35	$\frac{0.5 - 0.1}{6} = 0.07$	0.0049
Total variance along the critical path .... 29.5196 or 29.52		

(2) Standard deviation for the final event =  $\sqrt{29.52}$  or 5.43.

$$(3) Z = \frac{T_S - T_E}{\sigma_{T_E}} = \frac{65 - 68.6}{5.43} = \frac{-3.6}{5.43} = -0.662.$$

(4) Probability of positive slack. Referring to tables of the area under the normal curve, the Z statistic corresponding to the number -0.662 is approximately 0.25. This means that only 25 times out of 100 could management expect to complete the project on schedule.

EVALUATION OF PERT

Since its formulation, PERT has been received both favorably and unfavorably. Those who favor it recognize it as a good planning tool. Others feel that it has been offered as a panacea for all scheduling problems. Still others think that the technique is "basically nothing

new." PERT, however, does offer several concepts not previously incorporated in scheduling techniques.

Unfortunately, PERT, as conceived by its developers, has never been applied in total to any major system. In particular, the three time estimates and the probability computations have never had a thorough test throughout a full project cycle. Perhaps the most complete attempt was the use of the three time estimates on the Dynasoar program, but that program was cancelled before completion.

Obviously, it is difficult to make a satisfactory comparison between CPM and PERT if the factors unique to PERT--its three time estimates and use of probability theory--are not implemented. Since use of the beta distribution in the PERT technique has been attacked by mathematicians, and engineers have been reluctant to make the three time estimates because they believe them to be too time-consuming, the probability calculations have usually been abandoned, perhaps justifiably. However, any new system that is to be used by numerous firms requires time to implement. Perhaps PERT should be implemented a portion at a time. Further study might indicate that in most cases expected time estimates do not vary significantly from single point estimates and therefore multiple estimates are not justified in view of the added inconvenience and cost. On the other hand, the problem of dealing with uncertainties in estimates remains. This issue is as yet unresolved.

At first, PERT had no cost-estimating capability. Now the network and critical path features of PERT-Time have proved their worth, and attempts are being made to extend the concept to the cost and reliability aspects of project management. The first full-scale application of the PERT-Cost technique was made on the TFX program. It is important to note that the PERT-type network provides a common framework for incorporating these other factors, and thus PERT provides the basis for a more completely integrated management system.

PERT has earned widespread acceptance in industry and government, and undoubtedly will be the dominant scheduling system for major development programs for some time to come, especially since attempts are being made to integrate it with companion techniques for planning and control of cost. In addition, it appears likely that a related

effort will be made to utilize it in the planning and control of technical performance.

Some of the strengths and weaknesses of PERT are summarized in Table 10.

Table 10

PERT TECHNIQUE--STRENGTHS AND WEAKNESSES

Criteria	Strengths	Weaknesses
1. Validity	PERT, like CPM, is capable of depicting work sequence. The use of three time estimates should make it more valid than any other technique.	
2. Reliability		On the other hand, securing three time estimates for each activity requires more information which would tend to introduce additional error.
3. Implementation		The complete PERT system is quite complex and therefore difficult to implement.
4. Universality of project coverage	Very strong in development phase.	Requires adaptation for application to production operations
5. Sensitivity testing (simulation)	Since PERT is usually mechanized, it has good potential for simulating the impact of various resource allocations on the schedule, or the various ways of sequencing work.	
6. Forecasting	PERT is strongly oriented to forecasting the ability to accomplish future events on schedule.	
7. Updating	Activities are clearly identified and elapsed times can be obtained as needed.	Estimation of activity times is quite time-consuming, and calculation of expected times requires use of a computer.
8. Flexibility	As the project changes over time, the network and new time estimates can be readily adjusted to reflect changes, especially if present experimental efforts on automatic plotting of networks are successful.	
9. Cost		More data and more computations are required than in any other system; hence the system is more costly.

NOTE: Recall that this table is intended only as a summary of certain qualitative information on the relative usefulness of the scheduling technique. As indicated previously a more formal quantitative evaluation of the extent to which the criteria are met was considered infeasible in this study.

## VI. AREAS FOR FURTHER RESEARCH

Immediately following the development of CPM and PERT, many scheduling systems, often identified by ingenious acronyms, were developed by industrial firms and governmental agencies. Most were attempts to link time and cost information by relating both types of data to the same network events. Several of the systems were developed by individual corporations to conform to their special planning and control requirements and were documented for use by other interested firms.

This recent stimulus to innovate in the area of planning and control techniques is largely attributable to the development of PERT and CPM and to the availability of high-speed computers. The importance of these variations should not be minimized, for they contribute toward improvements in existing scheduling systems.

The following are some aspects of schedule planning and control that offer potential for further development or implementation in meeting management requirements for information:

1. Greater use of data available in existing scheduling systems.
2. Use of networks in the selection of alternatives.
3. Integration of cost and performance elements with scheduling techniques.
4. Development of a technique for identifying and processing interfaces.
5. Simplification of scheduling techniques.
6. Extension of network concept to top-level management.

### GREATER USE OF DATA AVAILABLE IN EXISTING SCHEDULING SYSTEMS

Management has, in general, made substantial use of the information in the available scheduling techniques. In several instances, however, these techniques could be made even more useful. For example, at present no use is made of the concept of slack in the Gantt or milestone techniques when planning developmental activity. It would be very simple to compute the latest start date for an activity in addition to

its earliest start date. This simple extension of the data, perhaps presented on a transparent overlay over the control chart of a given date, would enable management to assess how critical schedule slippage is on each milestone.

Likewise, the data contained in the PERT system really never have been fully utilized. As indicated previously, the three time estimates have never been consistently collected throughout any major weapon system program, with the result that the use of probability applications have been entirely lost to management. This is indeed unfortunate since probabilistic approaches represent the best formal techniques available to quantify the uncertainty inherent in forecasting the future completion times for a project. PERT, without probability, in essence "degenerates" into the critical path method.

If three time estimates were obtained for a major system, statistics similar to the recovery ratio described in Sec. V could be computed. Although not as refined and sophisticated as the probability computations, the recovery ratio would provide a better measure of uncertainty than is presently obtainable and thus should lead to more appropriate decisionmaking.

A well-documented case history of the application of the complete PERT technique to a major system would provide a basis for evaluating certain features of the PERT technique, such as the applicability of the beta distribution. Modifications resulting from such an evaluation would probably improve PERT and lead to more complete implementation on projects where this technique is applicable.

#### USE OF NETWORKS IN THE SELECTION OF ALTERNATIVES

Management is continually faced with alternative courses of action in system development. At least two major types of decisions recur throughout the life cycle of a major system: (1) the selection of a component design from alternative designs; and (2) the proper mix of resources to develop a component at minimum cost. The network is a very valuable aid in this decisionmaking process.

When more than one design is feasible, preparation of networks depicting the development paths for each alternative can be of considerable assistance in choosing the most suitable course of action.

Given a network of the activities involved in developing a certain component, management can secure estimates of time required to complete the component, assuming varying amounts and qualities of resources for individual events. The least cost allocation can be selected, based on an acceptable level of quality and a time constraint. Alternatively, given quality and cost limitations, the approach requiring the shortest period of time can be selected.

These are examples of sensitivity analysis, i. e., of testing the sensitivity of time to various component designs or resource mixes. It is difficult for management to make these decisions by intuition alone. Simulation of alternatives provides a quantitative basis for selecting a preferred approach. In general, increased emphasis should be devoted to this process of identifying and analyzing feasible alternatives.

The PERT technique permits simulation of alternatives if management takes time to develop the networks and obtain the estimates. Further research is desirable in scheduling for operations with limited resources. For example, a forecast of activities based on presently available resources may indicate that certain scheduled dates are unattainable. Two or more activities may require a limited resource--for example, unique engineering skills--concurrently. Since the impact of this resource limitation may not have been properly identified initially, these activities will be completed later than originally estimated.

#### INTEGRATION OF COST AND PERFORMANCE PARAMETERS WITH SCHEDULING TECHNIQUES

Scheduling of resources to activities has cost and performance as well as schedule implications. Frequently, the more resources applied the shorter the time to completion, but also the greater the total cost. Or, the higher the caliber of resource the shorter the time to design technically complex projects and perhaps the higher the performance of the resulting system. It follows, then, that decisionmaking

would be improved if information were interrelated on the quantity and quality of resources and the probability of attaining or surpassing the desired performance specifications.

The critical path method offers a limited capability for time-cost trade-offs. Recently, PERT/Cost has been developed establishing work packages, comprehending groups of activities the completion of which usually are identified by major events. Estimation can then be made on the cost to complete a work package as well as on the time required to attain the event. Thus, time and cost can be interrelated.

Some research is being directed toward delineation of performance specifications for components and subsystems based on an overall configuration index directly relatable with cost work packages and scheduled events. Estimates of the probability of attaining the performance specifications also can be made as the project progresses.

Each of these techniques has merit and when appropriately integrated into a comprehensive information system, a much improved basis for planning and controlling projects should evolve. The network concept appears to provide a useful framework for such a system. As familiarity is gained with the operational aspects of PERT/Cost, any weak features should be overcome and suitable cost-estimating techniques should evolve. Considerable research remains before the technical performance features become operational at a detailed level and additional effort will be required to develop and integrate these techniques into a single system.

#### DEVELOPMENT OF A TECHNIQUE FOR IDENTIFYING AND PROCESSING INTERFACES

The interface problem has always plagued schedulers in constructing networks. A technical interface occurs where one item in a project mates with another item; or when information on the project is exchanged between engineers or managers; or when responsibility is transferred from one organization to another. There are countless such interfaces in a large program, varying in level of importance in their effect on project control. The most significant interfaces are placed on a network and become interface events--i.e., events that signal the transfer



of responsibility, end items, or information from one part of the program to another.

Several thousand network interfaces may occur during the acquisition of a large program. Failure to recognize significant interfaces will result in loss of information and can distort schedule information considerably. This distortion and loss of information is illustrated in Fig. 14. Starting at the top of the figure, panel A depicts a simple network for firms 1 and 2. The critical path for events under the control of firm 1 is A-B-C and is 12 weeks long. For the events under firm 2's control, the critical path is D-E-F and is 16 weeks long.

Panel B shows the two networks integrated into one network. Note the dependency of event B in firm 1 upon completion of event D in firm 2. This dependency, plus the addition of starting event (I) and ending event (J), lengthens the critical path to 28 weeks and has events in both firms lying on the critical path.

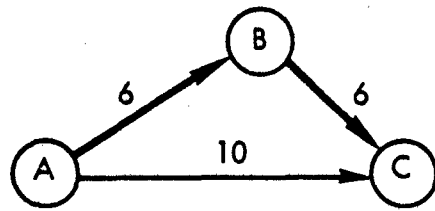
Panel C is a summary network for the integrated network in panel B. Failure to recognize the interface between event D and event B resulted in a critical path of 21 weeks; in addition, all events on the critical path are within firm 2.

Panel D is a condensed network with the interrelationships properly summarized. Note that the critical path is 28 weeks, the same as in the integrated network in panel B.

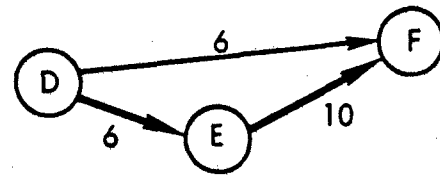
Interface events permit separate networks to be integrated into one, complete network, and detailed networks to be summarized into display networks. If interfaces are properly selected and coordinated, loss of schedule information at the various levels of project control will be minimized.

Most firms that use networks to schedule large projects have evolved their own summarization and integration techniques, and each firm has its preferred technique for handling interfaces. However, there seems to be no complete documentation that describes a technique for the proper identification and processing of all significant interfaces. A process, including a computer routine, must be established to select, record, coordinate, and store information on these interfaces.

Panel A: Networks for individual firms

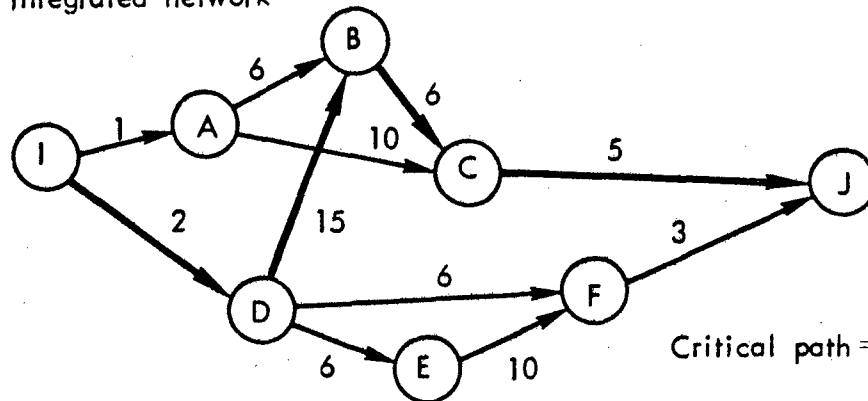


Firm No.1



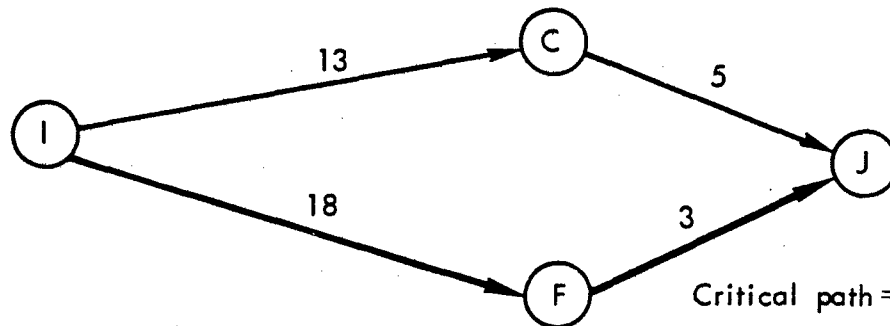
Firm No.2

Panel B: Integrated network



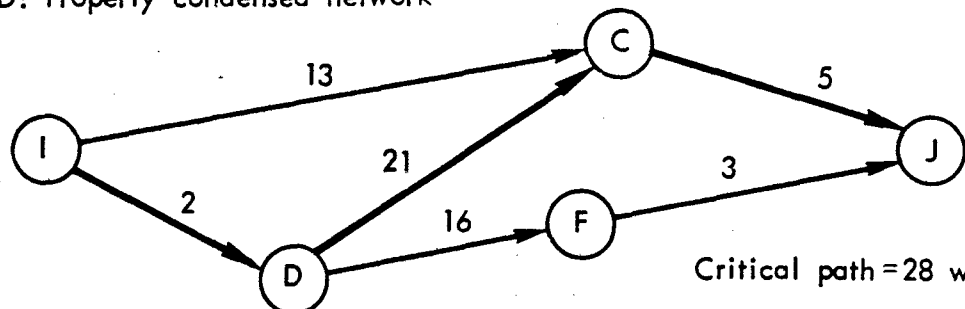
Critical path = 28 weeks

Panel C: Condensed network distorted because ignored inter-firm interface D-B



Critical path = 21 weeks

Panel D: Properly condensed network



Critical path = 28 weeks

Fig.14—Network integration and condensation

A more basic problem as yet unsolved is the establishment of criteria to identify the interfaces that should appear on a network. Interfaces must be identified and then defined to the satisfaction of the parties concerned. Definition of an interface assumes importance where completion of an event signals the transfer of responsibility from one department or firm to another. Disputes that frequently arise over whether the tasks comprising the interface have in fact been completed can be minimized if the criteria for interface completion are agreed upon in advance. The process of assembling responsible parties to identify and define project interfaces is, in itself, a significant communication problem.

In brief, further research is needed to establish criteria for interface definition and identification. Additionally, documentation of the various computer techniques to automatically select, record, coordinate, and store information would be desirable, perhaps leading to development of one satisfactory standard technique. Hopefully, such developments would minimize, or eliminate, distortion and loss of information throughout the family of project networks.

#### SIMPLIFICATION OF SCHEDULING TECHNIQUES

It is interesting to observe how scheduling techniques actually have been used in the aerospace environment. It appears that frequently decisionmakers have selected various systems in a heterogeneous fashion from the inventory of scheduling techniques. Figure 15 shows the areas where these techniques are used over the life cycle of a project. Note that Gantt charts tend to be used for controlling resources at the operating levels of production management; that milestone charts are used by top-level management; and that PERT or CPM is used by middle-level management for the development phase and LOB for production operations.

In view of the types of activities and varied resources (manpower, facilities, equipment, etc.) to be controlled, it appears that at present no one scheduling technique will serve in place of all the others. In fact, except for the similarities of PERT and CPM, the scheduling techniques seem to be complementary rather than substitutes for each other.

MANAGEMENT LEVEL	PHASE	
	DEVELOPMENT	PRODUCTION
HEADQUARTERS USAF AFSC	MILESTONES	
SPO CORPORATE LEVEL	PERT/CPM MILESTONES	LOB GANTT
OPERATING LEVEL	PERT/CPM GANTT	LOB GANTT

Fig.15—Typical use of scheduling techniques over a project life cycle

This does not imply that the current scheduling techniques are ideal. Indeed, a major purpose of this Memorandum has been to show the weaknesses of the techniques and to suggest areas where further research might overcome their limitations. Obviously the two approaches possible at present are to continue to improve and extend existing systems and to develop substantially new systems.

An attempt to extend and simplify existing scheduling techniques has been made by the U.S. Army Materiel Command with a technique termed PERT/LOB.\* As the title implies, this technique does not involve radically new concepts but is a synthesis of existing ones. However, an ingenious configuration of existing concepts can itself be an innovation of considerable usefulness.

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\* For a reasonably complete description of this technique, see Planning and Control Techniques and Procedures (PCT), Headquarters, U.S. Army Materiel Command, AMCR 11-16, August 1963.

During the development phase, most activities are undertaken only once, whereas in the production phase numerous activities must be performed repeatedly to complete the delivery schedule. PERT/LOB is an attempt to combine both development and production scheduling systems. It is designed primarily for use when items are to be produced in batches, such as test hardware in the development phase.

PERT/LOB modifies the network scheduling concept by combining both single-shot and repeated activities on the same network. Mechanically, this involves the introduction of the symbols  $\otimes$  and  $\rightarrow$  for repetitive events and activities, respectively. The standard PERT network symbols are retained for the one-time events and activities. The network thus shows the nature of all the activities and events.

In PERT/LOB, control is directed toward batch-type operations for production. Repeated activities are charted on a graph that shows the number of batches and the scheduled beginning and ending times for each batch.\* This chart, unique to PERT/LOB, is called the Repetitive Activity Input/Output Plan (RAI/OP) and is basically a Gantt chart for each activity (see Fig. 16). From the time estimates one can compute the earliest start date ( $T_E$ ) and the latest start date ( $T_L$ ) for each batch shown on the chart. Consequently slack can be computed and displayed graphically, either directly on the chart or on overlays (as suggested in Sec. III).

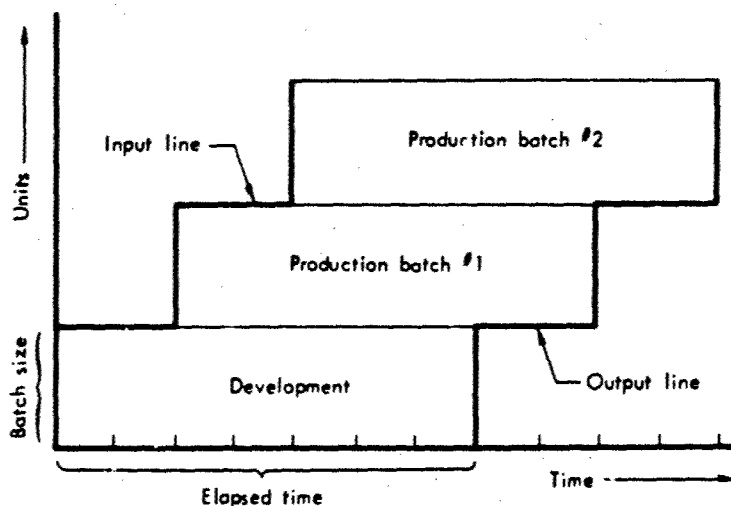


Fig. 16—PERT/LOB repetitive activity input/output plan

\*As production proceeds, shorter activity times can be introduced to reflect learning curve effects.

A further minor modification in scheduling technique involves the construction of the line of balance on the program progress chart. In the traditional LOB technique for continuous production of individual units, the line of balance indicates the quantity of units that should have passed through various control points if the delivery schedule is to be met. In PERT/LOB, the line of balance is constructed to indicate the number of batches that should be completed. Since a batch generally contains more than one unit, this method results in a more conservative line of balance than the traditional method. In PERT/LOB the entire batch must be past the control point to be on schedule. For example, if there were 10 units in the batch and 7 units had been completed, the batch would be behind schedule, whereas if only units were being controlled, the completion of 7 units might indicate that the schedule would be met without undue difficulty.

Finally, since PERT/LOB is based on a work breakdown structure, work packages can be costed; and where batches are produced, cost estimates for each batch can be employed.

In brief, PERT/LOB is a conglomerate technique embodying numerous concepts entirely within the state of the art in scheduling. It provides a framework for planning and control of the development and production phases of a major system, and also permits the combining of schedule and cost considerations. Although not representing a real advance in the state of the art, it is a step in the proper direction. Further experimentation along these lines should be encouraged.

#### EXTENSION OF NETWORK CONCEPT TO TOP-LEVEL MANAGEMENT

Networks are not currently employed at the lowest levels of activity nor at the highest levels of project monitoring. There are valid reasons for not using them at the lower levels since the Gantt chart is more applicable to the control of resource inputs (machines, labor, etc.). However, assuming that the networks can be adequately summarized, there is no valid reason for not extending the network concept to top-level management, who presently rely on milestone charts. In the System Package Program (SPP) in particular the milestones could be related to

each other in network form and could, with little effort, be made to depict major interdependencies in the project. It would also be possible to show slack for each milestone, and perhaps the probability of zero slack. If networks were not feasible for certain projects at the SPP level, at least the analysis of slack and other pertinent information could be integrated into the milestone chart system.

#### NEED FOR NEW SCHEDULING CONCEPTS AND TECHNIQUES

Each of the scheduling techniques discussed in this Memorandum contained new concepts that were evolved to meet a perceived need. A need now exists for a reasonably standardized technique capable of encompassing the entire weapon system. There is every reason to be optimistic that new concepts and applications--perhaps beyond networks--will continue to evolve to meet this need.

This Memorandum has been directed toward defining the multi-faceted environment of weapon systems acquisition and explaining the major scheduling systems developed to date. Analysis of these scheduling techniques has highlighted their strengths and weaknesses, which, in turn, has led to suggested areas where further research is needed. Admittedly, a bias exists for preferring theoretically superior information systems, in spite of the fact that formidable problems frequently exist in their wide-scale implementation. In the future, advanced scheduling techniques will undoubtedly evolve. Currently, the wise selection and adaption of techniques, with full knowledge of their strengths and limitations, should lead to improved scheduling and, consequently, to increased effectiveness in meeting delivery dates and in the efficient use of resources.

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ABSTRACT  A survey of the major scheduling techniques currently available to project management. The nature of the weapon systems acquisition environment with its inherent problems is examined in detail. Several scheduling devices (the Gantt chart, milestone chart, line-of-balance technique, critical-path method, and PERT) evaluated and contrasted through a hypothetical weapon system. The extent to which existing techniques meet scheduling demands and areas for additional study also discussed.		II. KEY WORDS  Defense systems Scheduling Military planning	

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