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A NUMERICAL SOLUTION TO AN ABLATION PROBLEM WITH POSSIBLE LASER APPLICATIONS

by

James G. Faller

October 1966

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BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

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B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

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ABSTRACT

The problem of an ablating solid of limited extent under an applied non-uniform heat input is solved numerically, and the solution is compared in certain limiting cases with results from the exact analytical equations. Further, the usefulness of this model in predicting the depth of hole made in materials by a laser beam is explored.

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1. INTRODUCTION

The problem of an ablating* solid of limited extent under an applied non-uniform heat input is solved numerically, and the solution is compared in certain limiting cases with results from the exact analytical equations. Further, the usefulness of this model in predicting the depth of hole made in materials by a laser beam is explored.

Problems similar to the one considered have been formulated by numerous investigators,[1-8]** and some particular solutions exist. A numerical solution for this problem has been given by Landau[1] for the special case of a constant heat input to a semi-infinite solid. More recently Ready[4] reporting the effects of the absorption of laser radiation on metals introduced results from the ablation model for the case of a semi-infinite solid under a non-uniform heat input; but, he gave no details of the solution. In view of the very restrictive (specialized) treatment which this problem has received in the literature and because of its possible application to certain laser effects, we find sufficient justification in its reconsideration.

2. STATEMENT OF THE PROBLEM

A slab of thickness B initially at temperature $T_0g(x)$ is heated on one end with energy $Hf(t)$ over an area with dimensions large enough in relation to the final depth of heat penetration that heat flow becomes mathematically one-dimensional. The other end of the slab is insulated.

* *Ablation is generally associated with vaporization and sublimation, but the meaning has been extended to include liquefaction in which the liquid is removed as it is formed.*

** *Superscript numbers denote references which may be found on page 27.*

Given a pulse of sufficient power density and a long enough time, the surface of the slab will reach the phase-change temperature.* The material which has been transformed is removed immediately and the surface recedes into the solid. We are interested primarily in determining the thickness of material transformed with time.

We make the following definitions:

c = specific heat of material at constant pressure, (cal/g $^{\circ}$C)

ρ = density of material, (g/cm^3)

K = thermal conductivity of material, (cal/cm $^{\circ}$C sec)

κ = thermal diffusivity of material, equal to $K/c\rho$, (cm^2/sec)

L = latent heat of transformation of material, (cal/g)

$T_o g(x)$ = initial temperature distribution of material, ($^{\circ}$C); for $g(x) = 1$ the temperature is initially uniform throughout the material.

T_L = transformation temperature of material, ($^{\circ}$C)

$y(t)$ = thickness of material transformed, (cm)

$Hf(t)$ = heat input as a function of time where H is a given constant, (cal/cm^{2}sec); $f(t) = 1$ for a uniform heat input

B = initial thickness of material, (cm)

The following equations describe the process:

$$T_t = \kappa T_{xx} \qquad y(t) \le x < B,\ t > 0 \ , \tag{2.1}$$

$$T(x,0) = T_o g(x) < T_L \qquad 0 \le x \le B \ , \tag{2.2}$$

$$T_x(B,t) = 0 \ , \tag{2.3}$$

$$Hf(t) = -\,KT_x \qquad y(t) = 0 \ , \tag{2.4}$$

---

* *Whether the phase-change temperature is that for melting or vaporization will depend on the incident power density. At low densities only liquid will appear; whereas, at high densities the liquid formed prior to vaporization can be ignored because the latent heat of melting is much smaller than that of vaporization.*

$$Hf(t) = -KT_x + \rho L\frac{dy(t)}{dt} \quad , \tag{2.5}$$

$$\frac{dy(t)}{dt} \geq 0 \quad \text{for } T(y(t),t) = T_L \ , \tag{2.6}$$

$$\frac{dy(t)}{dt} = 0 \quad \text{for } T(y(t),t) < T_L \ . \tag{2.7}$$

Equation (2.1) may be recognized as the general parabolic relation of heat flow. The initial temperature distribution of the slab is given by (2.2), while (2.3) expresses the condition of no heat losses from the backface. Equation (2.5) is a statement relating the heat input to the rate of heat flow into the solid plus the rate of heat absorbed in the transformation. T_x is evaluated at the position of the boundary y(t). Before the phase-change temperature is reached there is no boundary motion, (2.7), so that (2.5) reduces to (2.4). Equation (2.6) states that when the surface reaches the temperature T_L there may or may not occur boundary motion; this will depend on whether the energy required for the phase-change is or is not absorbed at the surface.

The equations can be manipulated more readily if the following transformations are made:

$$v(x,t) = \frac{T(x,t)}{T_L} \quad ,$$

$$\tau = \kappa t \ ,$$

$$N = \frac{L}{cT_L} \quad ,$$

$$p = \frac{H}{KT_L} \quad .$$

Equations (2.1) through (2.7) then become

$$v_\tau = v_{xx} \quad y(\tau) \leq x < B, \quad \tau > 0 \quad , \tag{2.8}$$

$$v(x,0) = v_o g(x) < 1 \quad 0 \leq x \leq B \ , \tag{2.9}$$

$$v_x(B,\tau) = 0 \quad , \tag{2.10}$$

$$pf(\tau) = -\, v_x \qquad y(\tau) = 0 \; , \tag{2.11}$$

$$pf(\tau) = -\, v_x + N \frac{dy(\tau)}{d\tau} \quad , \tag{2.12}$$

$$\frac{dy(\tau)}{d\tau} \geq 0 \quad \text{for } v(y(\tau),\tau) = 1 \; , \tag{2.13}$$

$$\frac{dy(\tau)}{d\tau} = 0 \quad \text{for } v(y(\tau),\tau) < 1 \; . \tag{2.14}$$

3. FINITE DIFFERENCES

We denote $v_{i,j}$ to be the value of v at the point i Δx at time j $\Delta\tau$ and $v_{i,j+1}$ the corresponding value at time $(j+1)\,\Delta\tau$. We further define according to practice $h = \Delta x$, $k = \Delta\tau$ and $r = \frac{k}{h^2}$. If the Crank-Nicholson differencing scheme[6] is selected, the time derivative can be approximated by

$$v_\tau = \frac{v_{i,j+1} - v_{i,j}}{k} + 0\left[k^2\right] \quad , \tag{3.1}$$

and the second order space derivative by

$$v_{xx} = \frac{1}{2h^2}\Big[\left(v_{i+1,j+1} + v_{i+1,j}\right) - 2\left(v_{i,j+1} + v_{i,j}\right) + \left(v_{i-1,j+1} + v_{i-1,j}\right)\Big] + 0\left[kh^2\right] \; , \tag{3.2}$$

where $0\left[\quad\right]$ is the truncation error. Substituting (3.1) and (3.2) in (2.8), introducing $r = \frac{k}{h^2}$ and rearranging we obtain

$$-\frac{r}{2}\,v_{i-1,j+1} + (1+r)v_{i,j+1} - \frac{r}{2}\,v_{i+1,j+1} = \frac{r}{2}\,v_{i-1,j} + (1-r)v_{i,j} + \frac{r}{2}\,v_{i+1,j} \; . \tag{3.3}$$

Equation (3.3) does not hold at the mathematical surface i = 0 since no account has been taken of the energy input to the material. This is dealt with by expanding $v_{i,j}$ in a Taylor series about the point (0,j+1) in both the space and the time directions. Thus

$$v_{1,j+1} = v_{0,j+1} + \left[\frac{\partial v}{\partial x}\right]_{0,j+1} \Delta x + (1/2) \left[\frac{\partial^2 v}{\partial x^2}\right]_{0,j+1} (\Delta x)^2 + 0\left[(\Delta x)^3\right] . \tag{3.4}$$

$$v_{0,j} = v_{0,j+1} - \left[\frac{\partial v}{\partial \tau}\right]_{0,j+1} \Delta\tau + \left[(\Delta\tau)^2\right] . \tag{3.5}$$

If $\left[\partial^2 v/\partial x^2\right]_{0,j+1}$ is replaced by $[\partial v/\partial\tau]_{0,j+1}$ in accordance with (2.8), then from (3.4), (3.5), and (2.11) the resulting equation becomes

$$(1+2r)v_{0,j+1} - 2rv_{1,j+1} = v_{0,j} + 2rhpf_{j+1} . \tag{3.6}$$

Equation (3.6) applies up to the time that the phase-change temperature is reached, i.e., so long as $y(\tau) = 0$.

Beyond this point account must be taken of the fact that the boundary no longer remains stationary. To avoid ambiguity in the use of the space coordinate i, a new coordinate w is defined such that the boundary will always lie at w or between w and w + 1. At the original surface w = 0. The index w, an integer, depends upon time, hence we denote this dependence by w_j. Initially, that is, at j = 0, $w_0 = 0$; however, at later times $0 \le w_j \le w_{j+1}$ and for each j the boundary lies between w_j and w_{j+1}. During boundary motion the temperature of the surface according to (2.13) is a known constant so that the first unknown temperature lies at the grid point w + 1 and is designated $v_{w+1,j+1}$. We expand about this point in both the time and space directions,

$$v_{w+s,j+1} = v_{w+1,j+1} - \left[\frac{\partial v}{\partial x}\right]_{w+1,j+1} (1-s)\,\Delta x + (1/2)\left[\frac{\partial^2 v}{\partial x^2}\right]_{w+1,j+1} (1-s)^2\Delta x^2 + 0\left[(\Delta x)^3\right] , \tag{3.7}$$

$$v_{w+1,j} = v_{w+1,j+1} - \left[\frac{\partial v}{\partial \tau}\right]_{w+1,j+1} \Delta\tau + 0\left[(\Delta\tau)^2\right] , \qquad (3.8)$$

$$\begin{aligned} v_{w+2,j+1} = v_{w+1,j+1} &+ \left[\frac{\partial v}{\partial x}\right]_{w+1,j+1} \Delta x \\ &+ (1/2)\left[\frac{\partial^2 v}{\partial x^2}\right]_{w+1,j+1} \Delta x^2 + 0\left[(\Delta x)^3\right]. \end{aligned} \qquad (3.9)$$

and where $0 \leq s \leq 1$. But at the surface we have $v_{w+s,j+1} = 1$. Further, if (3.7) and (3.9) are added after (3.9) has been multiplied through by (1-s) and (2.8) and (3.8) are substituted in that order in the resulting equation, we obtain

$$\begin{aligned} \left(2-s_{j+1}\right)\left(2r + 1-s_{j+1}\right) v_{w+1,j+1} - 2r\left(1-s_{j+1}\right) v_{w+2,j+1} = \\ \left(1-s_{j+1}\right)\left(2-s_{j+1}\right) v_{w+1,j} + 2r \cdot \end{aligned} \qquad (3.10)$$

For clarity the subscript j+1 has been attached to s to show the exact time at which this quantity is evaluated.

Next the position of the boundary $y(\tau)$ must be determined. From the preceding discussion it is clear that y may be expressed in terms of an integer w and a fraction s by

$$y_j = \left(w_j + s_j\right) h , \qquad (3.11a)$$

and

$$y_{j+1} = \left(w_{j+1} + s_{j+1}\right) h . \qquad (3.11b)$$

Subtracting (3.11a) from (3.11b) we obtain

$$y_{j+1} - y_j = \Delta y = h\left[\left(w_{j+1} - w_j\right) + \left(s_{j+1} - s_j\right)\right]. \qquad (3.12)$$

Since y_j, w_j and s_j are zero at the start of the transformation, there is no difficulty in keeping track of the boundary for subsequent times provided Δy can be calculated. This may be done from (2.12) which has the finite difference form

$$\frac{\Delta y}{k} = \left[\tilde{v}_x + pf_{j+1}\right] /N \quad , \tag{3.13}$$

where $\tilde{v}_x$ is the approximation of the space derivative v_x at the surface and may be evaluated by a three point Gregory-Newton forward difference approximation as

$$\begin{aligned} \tilde{v}_x = \Big[&- \left(5-2s_{j+1}\right) v_{w+1,j+1} \\ &+ 4\left(2 - s_{j+1}\right) v_{w+2,j+1} \\ &- \left(3 - 2s_{j+1}\right) v_{w+3,j+1} \Big] /2h \; . \end{aligned} \tag{3.14}$$

Equation (3.13) can be arranged more conveniently in terms of r giving

$$\frac{\Delta y}{h} = \frac{rh}{N} \left[\tilde{v}_x + pf_{j+1}\right] \quad . \tag{3.15}$$

One is confronted in (3.15) with the evaluation of s_{j+1} from temperatures which are not known and which themselves cannot be calculated without knowing s_{j+1}. To circumvent this problem an iterative procedure must be used.

Finally, we note that at the back surface (3.3) must be modified to take into account the boundary condition (2.10). Formally, this is done by extending the mesh one unit past the back face,* where the latter is indexed by $i = M = B/\Delta x$. The existence of a zero temperature gradient requires that $v_{M-1} = v_{M+1}$ which reduces (3.3) at the back face to

$$-rv_{M-1,j+1} + (1 + r)v_{M,j+1} = (1 - r)v_{M,j} + rv_{M-1,j} . \tag{3.16}$$

---

* *Creating a false boundary.*

4. CALCULATING PROCEDURE

4.1 General

The difference equations of Section 3 from which the temperatures are computed have the tridiagonal appearance (for each mesh row)

$$
\begin{aligned}
B_0 v_0 + C_0 v_1 &= d_0 \\
A_1 v_0 + B_1 v_1 + C_1 v_2 &= d_1 \\
0 v_0 + A_2 v_1 + B_2 v_2 + C_2 v_3 &= d_2 \\
&\vdots \\
A_{M-1} v_{M-2} + B_{M-1} v_{M-1} + C_{M-1} v_M &= d_{M-1} \\
A_M v_{M-1} + B_M v_M &= d_M ,
\end{aligned}
\tag{4.1}
$$

with zeros everywhere except on the main diagonal and on the two diagonals parallel to it on either side.[9] In (4.1) all known quantities have been lumped in d thereby eliminating the necessity of the further use of j. Through a Gaussian elimination process this system can be solved explicity for the unknown v's. The method is credited to L. H. Thomas,[10] but received wide attention in published form in an article by Bruce, Peaceman, Rachford, and Rice.[11] Expressed in a less cumbersome way[11] the system (4.1) becomes

$$B_0 v_0 + C_0 v_1 = d_0 , \qquad \text{(a)}$$

$$A_i v_{i-1} + B_i v_i + C_i v_{i+1} = d_i \qquad 1 \le i \le M - 1, \qquad \text{(4.2)(b)}$$

$$A_M v_{M-1} + B_M v_M = d_M , \qquad \text{(c)}$$

where the unknown temperatures corresponding to the $(j+1)^{st}$ step are given by

$$v_M = q_M$$

$$v_i = q_i - b_i v_{i+1} \qquad 0 \leq i \leq M-1 ,$$

with

$$q_0 = \frac{d_0}{B_0}$$

$$q_i = \frac{d_i - A_i q_{i-1}}{B_i - A_i b_{i-1}} \qquad 1 \leq i \leq M \tag{4.3}$$

$$b_0 = \frac{C_0}{B_0}$$

$$b_i = \frac{C_i}{B_i - A_i b_{i-1}} \qquad 1 \leq i \leq M-1 .$$

For convenience the calculations are carried out in two parts - Part I covering up to the time the phase-change temperature is reached and Part II beyond this point. In Part I (4.2) represents the system (3.6), (3.3) and (3.16); in Part II (4.2) represents the system (3.10), (3.3) and (3.16) with (3.15) and (3.12) being used in locating the boundary. Also in Part II, (4.2a) is more appropriately indexed as

$$B_{w+1} v_{w+1} + C_{w+1} v_{w+2} = d_{w+1} ,$$

with i being greater than or equal to w+2 in the remaining equations.

4.2 Detailed Procedure

Part I - Before Transformation

Using Equations (3.3), (3.6) and (3.16) computation continues until for some $j = j'$, $v_{0,j'+1} \geq 1$ and $v_{0,j'} < 1$.

To establish an accurate time for the beginning of the phase-change and as a baseline for Part II, the $(j'+1)$st = Jth row is re-determined using a smaller time step k' given by

$$k' = \left(\frac{1 - u_{0,j'}}{u_{0,j'+1} - u_{0,j'}}\right) k \quad ,$$

or equivalently

$$r' = \left(\frac{1 - u_{0,j'}}{u_{0,j'+1} - u_{0,j'}}\right) r \quad .$$

The time of the start of transformation is simply

$$\tau_m = j'k + k' \quad .$$

Part II - Boundary Begins to Move

As first step in Part II the boundary must be located. Since the temperatures at time $J + 1$ are yet to be determined, (3.15) cannot be used directly. We, therefore, approximate Δy by

$$\frac{\Delta y^0{}_{J+1}}{h} = \frac{r}{2N}\left[\left(-1.5 + 2v_{1,J} - .5v_{2,J}\right) + \left(hpf_{J+1}\right)\right] \cdot \qquad (4.4)$$

The first term inside the brackets may be recognized as the three-point Lagrange interpolation expanded in the neighborhood of the moving boundary with $w = s = 0$. Multiplying the brackets by 1/2 ensures the Δy is not over estimated. In the absence of any boundary displacement at time J the initial estimate of the position of the boundary is

$$y^0{}_{J+1} = \Delta y^0{}_{J+1} \quad . \qquad (4.5)$$

where the superscript designates the number of the iteration n, taken to be zero initially.

Further y^0_{J+1} is decomposed into its whole and fractional components,

$$y^0_{J+1} = h\left(w^0_{J+1} + s^0_{J+1}\right) \quad . \tag{4.6}$$

At this point (3.10), (3.3) and (3.16) are solved for all $v_{i,J+1}$. It should be emphasized that temperatures $v_{i,J}$ are not replaced by those at $v_{i,J+1}$ until the iteration for the (J+1) step has been completed.

Using s^0_{J+1} and the appropriate $v_{i,J+1}$'s a new estimate of the boundary position is obtained from (3.15). This then becomes Δy^1_{J+1} from which y^1_{J+1} is derived. As an improved estimate of the boundary position the n and (n+1) iterative values of y_{J+1} are combined and a new s_{J+1} and w_{J+1} are found,

$$\frac{y^0_{J+1} + y^1_{J+1}}{2} = h\left(w^1_{J+1} + s^1_{J+1}\right) \; . \tag{4.7}$$

Then the system (4.2) is again solved and the process is repeated a fixed number of times or until some test is satisfied. We have chosen the test that the absolute value of the ratio of the difference between two successive values of y to their average be less than some constant ϕ and write

$$\left| \frac{y^n - y^{n-1}}{\left(y^n + y^{n-1}\right)/2} \right| \leq \phi \; , \tag{4.8}$$

where $0 < \phi \ll 1$.

For an approximation to y in the succeeding time step J + 2, a linear change of the boundary with time is assumed,

$$y^0_{J+2} = y_{J+1} + \Delta y_{J+1} \quad , \tag{4.9}$$

where y_{J+1} and Δy_{J+1} represent values of the quantities obtained in the final iteration of the (J+1)st step. The above procedure commencing with (4.6) is then repeated. Computation continues until $\tau \geq \tau_{max}$, where τ_{max} is the duration of the input pulse.

5. ERROR ANALYSIS

With assurance of stability for independent h's and k's, error analysis is one of the prime considerations of a numerical method. This subject is currently under extensive study, and most discussions found in the literature are of a qualitative nature.[9] Two main sources of error generally encountered are present here and are (a) the truncation error arising from the approximation of the analytical equations by finite differences, and (b) the round-off error due to use of only a finite number of decimal places in the arithmetic operations and due to the fact that the iterative solution is only continued until there is no change out to a certain decimal place. These errors are oppositely influenced by the interval size h. Decreasing h decreases the truncation error but in general increases the round-off error.

An example will best serve to check the accuracy of the present solution. For a semi-infinite slab and a constant heat input the time required for the surface to reach the phase-change temperature can be compared to the exact analytical solution.[12] These times are shown in Table I using physical data for aluminum undergoing a phase-change at the melting temperature. It is readily seen that with decreasing h at constant r the approximate time that the surface reaches the melting temperature more nearly approaches that of the exact solution. The error in the approximate solution is reduced by the square of the factor by which h is decreased. Thus if h is diminished by a factor of 8, e.g., in going from h = .01 to h = .00125, the error will decline by a factor of 64.

TABLE I

Time of the start of melting for a semi-infinite solid as determined by (a) the exact analytical solution* and (b) the approximate solution of this paper for varying h and constant r = 0.4. Based on physical data of Al.**

Exact Solution time, sec	Approximate Solution time, sec	h	% Error
$.2082 \times 10^{-3}$	$.2475 \times 10^{-3}$	.01	19
	$.2180 \times 10^{-3}$	.005	4.7
	$.2107 \times 10^{-3}$	.0025	1.2
	$.2088 \times 10^{-3}$	.00125	.3

* *See Reference 12.*

** $K = .480$, $c = .214$, $\rho = 2.70$, $L = 94.5$, $T_o = 20$, $T_L = 660$, $H = 2.07 \times 10^4$, $B = 1.0$, $f(\tau) = 1.0$.

In Figure 1 the results of Table I are extended into Part II involving the moving boundary. Here, however, it is no longer possible to calculate an exact solution and the approximate results must be compared to the steady state solution (1) given by

$$y = \frac{H\left(t - t_L\right)}{\rho\left[L + c\left(T_L - T_o\right)\right]} \quad , \tag{5.1}$$

where in addition to the quantities already defined t is the duration of the pulse and t_L is the time required to reach the transformation temperature.

Equation (5.1) gives an upper limit to the penetration of the boundary, which in the time $t = .24 \times 10^{-2}$ sec is 0.0725 cm. Even for the least favorable choice in h the solution is reasonable. Except in the very short time after transformation has begun, the boundary motion is essentially linear with time.

It is not clear from the preceding discussion how one, short of trial and error, goes about making an appropriate selection of h. Time and accuracy must be simultaneously considered and some optimum found between the two. Rather than select h first it is more meaningful to choose k first and then find the desired h by $h = (k/r)^{1/2}$.* This is necessary because any choice in h no matter how small might not be applicable due to the short duration of an input pulse. For a nanosecond long input pulse as encountered in the q-switched laser mode an h of say .00125 would yield a k of 0.625×10^{-6} for $r = 0.4$ which exceeds the entire length of the pulse. In choosing a k one may be guided by a rule of thumb in which the total time of the pulse is divided into about one thousand k's i.e., max j = 1000. Then a back calculation can be made to find an approximate h which can also be divided into B to give some whole number M.

---

* *In implicit methods stability is ensured for all $r > 0$.* [*Ref. 9, p.339*].

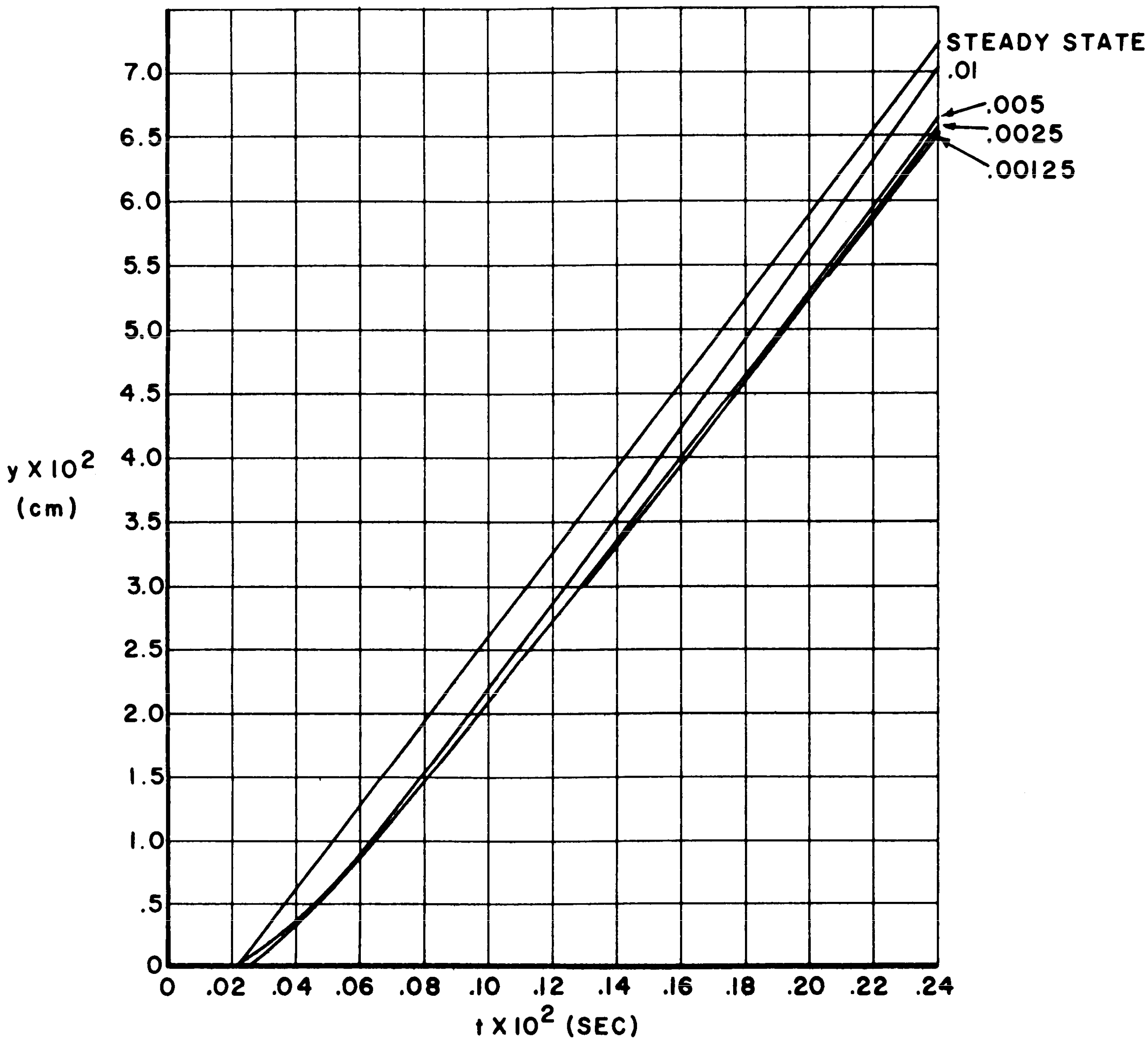

FIG. 1.-THE DEPENDENCE OF THE POSITION OF THE BOUNDARY y ON TIME t FOR CONSTANT $r = .4$ AND h OF .01, .005, .0025, .00125.

When the input energies are very large as in the case of the laser the time taken for the material to reach the transformation temperature is much shorter (sometimes by a factor of a thousand of more) than the duration of the entire pulse. It is, therefore, necessary to use different grid sizes for Parts I and II. If a single grid size is chosen say on the basis of the high resolution required in Part I the computational times required for Part II would become prohibitive. If on the other hand, the choice of grid sizes is made on the basis of the total pulse, the approximation of the time of the phase-change will contain a ridiculously large error. A rough idea for the onset time of the transformation may be obtained from an analytical solution of the heat equation. On this basis a reasonable selection of k and hence h may be made for Part I. The h selected in this manner may be so fine, however, that for the particular thickness of solid B, M will become an exceedingly large number. Insofar as the heat flow is concerned, only a small fraction of the total M points in the solid may register any change in temperature in the short times considered, the solid behaving as semi-infinite beyond a certain depth. Thus by either reducing B or equivalently reducing M by some arbitrary factor the times for calculation in Part I do not become prohibitive. When Part II is entered a much larger h may be used. To avoid the possibility of error the final temperature v_M may be printed as part of the output at the ends of Parts I and II. If v_M is the same as the initial temperature of the solid then the slab has remained semi-infinite throughout the length of the calculations.

6. AN EXAMPLE FOR LASER-INDUCED DAMAGE

In suggesting that the present problem may apply to laser-induced damage in materials we are cognizant of the pitfalls involved. The interaction of the laser with the target is a complex phenomenon. Several elements contribute to this complexity. It is not certain whether the plasma created in front of the target remains transparent to the beam throughout the duration of the pulse. It may be a safe assumption to make at low incident power densities but not at high incident power densities such as those delivered by the laser in the q-switched mode. In addition to the plasma the target emits globules of transformed material which may further act to absorb the incoming beam. Another important consideration is the reflectance of the beam by the target which may be a strong function of the depth of hole, i.e., in the initial stages of heating and penetration highly reflecting materials such as aluminum may absorb only a small portion of the incoming radiation, while at a later stage or as the hole deepens, the entire incoming beam may be absorbed by being trapped in the hole. The model, furthermore, makes no allowance for the possibility of superheating or for the co-presence of vapor and liquid. Any attempt, however, to construct a mathematical model of laser damage which takes all the preceding factors into account would prove an enormously difficult task because a clear understanding of their relative significance is currently lacking.

In Table II and Figure 2, data are shown comparing the depth of hole predicted to that actually observed in Al of semi-infinite thickness. Case 1 in Table II is for a constant input energy, whereas Case 2 if for the variable pulse shown at the top of Figure 2. The lower portion of Figure 2 is a plot of the position of the boundary with time for this variable pulse with the maximum y being the entry in Table II.

In cases 1 and 2 the transformation is assumed to occur at the temperature of vaporization, the possibility of melting being totally ignored. Ignoring melting when vaporization is involved is not a serious error since the heat of vaporization and melting are so vastly different, the former being more than twenty-seven fold greater than the latter in Al.

The discrepancy, assuming full absorption of the energy of the beam by the target, is quite large. A second calculation has been done for each case at about half of this energy on the basis that much of the incident pulse is reflected. Specimens of Al treated with #300

TABLE II

DEPTH OF HOLE PRODUCED IN Al BY LASER PULSE

Case	Description of Pulse			
	Power Density joules/cm^2-sec	Duration sec	Calculated cm	Observed* cm
1	8.33×10^{6}	$.6 \times 10^{-3}$	1.43×10^{-1}	$.78 \times 10^{-1}$
	4.16×10^{6}	$.6 \times 10^{-3}$	$.695 \times 10^{-1}$	
2	1×10^{9} (peak)+	44×10^{-9}	6.05×10^{-4}	3.6×10^{-4}
	$.5 \times 10^{9}$	44×10^{-9}	2.87×10^{-4}	

* *See Reference 4.*

\+ *See Figure 2 for pulse shape.*

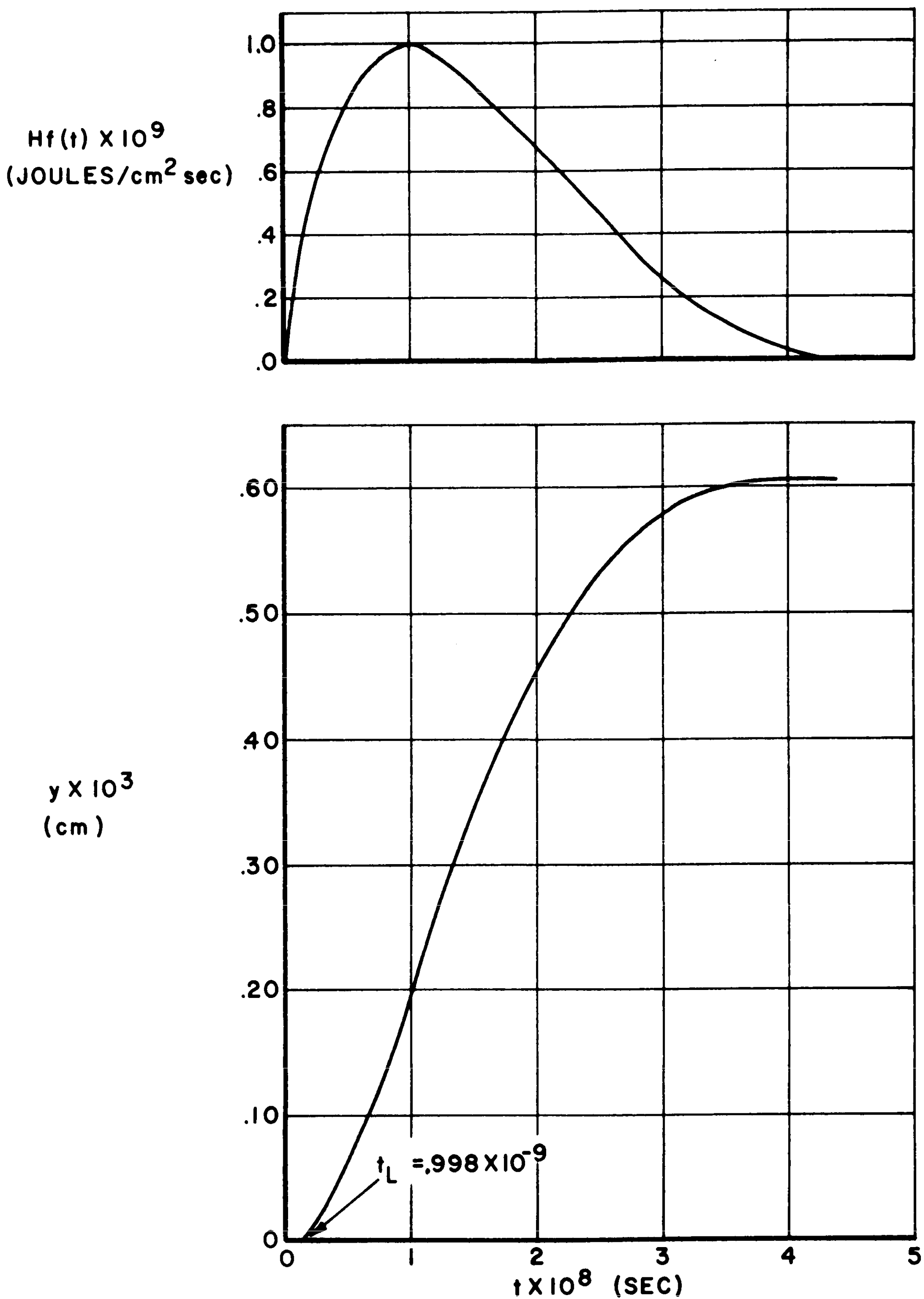

FIG. 2.- BOUNDARY MOTION y AS A FUNCTION OF THE TIME t FOR INPUT PULSE Hf(t) (SHOWN ON TOP GRAPH). MATERIAL IS Al AT VAPOR TEMPERATURE.

grit emery paper have been found in this laboratory to reflect up to 60 per cent of the incident light at room temperature. While large reflectance is a reasonable assumption during the heating-up, it is not known whether it will remain at or near the initial level for the entire length of the pulse. Better agreement with the experiment results when making the assumption of a constant reflectance throughout the pulse duration.

7. CONCLUSIONS

An implicit numerical scheme has been used successfully to solve the heat conduction problem for an ablating solid. The method introduces simplicity and accuracy in the calculations and allows for economic use of computer time. Further, the solution is valid for a variable input energy and sample thickness, features which make it especially suitable in applications related to the laser damage of materials.

In the example given for laser-induced damage to Aluminum we have purposely avoided manipulating our input energies etc., so as to obtain a better agreement with experimental observations. Published data of laser damage to targets is scarce and incomplete. Information concerning surface preparation, the spread in the measured input energies and depths, and the shape of the holes produced has never been set down for the same samples. In certain cases one cannot be certain whether a semi-infinite slab was used as reported or if the conditions for one-dimensional heat flow were truly satisfied. From the limited experimental observations considered, it is difficult to determine how widely the proposed model can be applied to predicting laser damage to targets.

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JAMES G. FALLER

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The problem of an ablating solid of limited extent under an applied non-uniform heat input is solved numerically, and the solution is compared in certain limiting cases with results from the exact analytical equations. Further, the usefulness of this model in predicting the depth of hole made in materials by a laser beam is explored.

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