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Navigation with High-Altitude Satellites: A Study of Ranging Errors

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LINCOLN LABORATORY

NAVIGATION WITH HIGH-ALTITUDE SATELLITES: A STUDY OF RANGING ERRORS

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ABSTRACT

In order to determine the position of a receiver on earth to within a fraction of a mile using high-altitude satellites, 10-meter accuracy in the receiver-satellite ranges or range differences is necessary. The factors that affect ranging systems attempting to achieve such accuracies are considered here rather than the detailed design of a ranging system. It is concluded that the effects of multipath propagation on ranging accuracy for receivers in aircraft must be experimentally studied prior to any judgment of the feasibility of such accuracies. In the absence of multipath, the satellite power and bandwidth requirements and ionospheric and tropospheric effects do not prohibit 10-meter range accuracy.

Accepted for the Air Force
Franklin C. Hudson
Chief, Lincoln Laboratory Office

NAVIGATION WITH HIGH-ALTITUDE SATELLITES:

A STUDY OF RANGING ERRORS

I. INTRODUCTION

Earth satellites have found a variety of useful applications in a relatively short period of time. One application for which satellites seem uniquely suited is that of accurate position determination and navigation. World-wide navigation using low-altitude satellites has already been shown to be feasible and an operational system is presently being contemplated.[1]

More recently, attention has focused on the use of high-altitude satellites for navigation. As an example, one possible scheme would have a number of satellites transmitting time signals which are generated independently by free-running clocks in each of the satellites. A user of the system could determine his position (3 coordinates) from (a) range difference measurements* for two pairs of satellites; (b) receiver height (relative to the earth's center); and (c) the satellite positions. In this scheme, the user is passive, i.e., he need not make any radio transmissions to navigate, and, furthermore, the user equipment need not include a very stable clock to measure the difference in range to two satellites. Synchronization of the satellite clocks could be accomplished by having a control station on the ground which provides each satellite with timing corrections relative to a master clock to be transmitted along with the actual timing signals in the

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* Difference in time of arrival of signals from two satellites.

form of digital data. Satellite coordinates could also be included as additional digital data in a similar manner involving control stations. A small number of control stations would be required to "see" all the satellites at synchronous altitudes.

There are apparent advantages in using high-altitude satellites for navigation. Position determination can be accomplished almost instantaneously and essentially continuously; i. e., the time required to obtain a position fix or begin another one is determined by the measurement time required for adequate ranging accuracy and the time to compute position. These features are very desirable for high-speed users such as jet aircraft. Some obvious disadvantages of high-altitude navigation satellites are that (a) several satellites are required for a position fix (whereas, a low-altitude system may require only a single satellite); and (b) the large path loss from synchronous altitude.

This report is concerned only with the problem of very accurate determination of satellite-user ranges (or range differences) in a high-altitude satellite navigation system such as that just described. An analysis of navigation errors* using high-altitude satellites has indicated that 10-meter range accuracies are desirable if position determinations to within a fraction of a mile are to be achieved. We consider here only the factors that would

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* Performed by F. C. Schweppe as part of a six-week feasibility study conducted at Lincoln Laboratory on navigation with high-altitude satellites. The participants in the study were T. J. Goblick, Jr., B. Reiffen, F. C. Schweppe, and R. Teoste. This report is an outgrowth of the original study of limitations on range measurements.

affect ranging systems which attempted to achieve 10-meter accuracies, rather than the detailed design of an optimum ranging system.

There are two basically different causes of errors in determining the distance between a satellite outside the atmosphere and an earth-based receiver by measurement of the travel time of radio signals transmitted by the satellite. First, errors in measuring the time of arrival of a signal at the receiver result from the random noise which masks the signal. Errors of this type depend upon such parameters as the radiated signal power, thermal noise level due to the sky background and receiver amplifier, and the shape of the signaling waveforms used, all of which are somewhat under the control of the system designer. The second cause of ranging errors is the propagation medium itself. Unknown or randomly varying propagation anomalies cause changes in transit time of radio waves propagating through the atmosphere. Errors of this type may be minimized by judicious choice of operating frequency.

In the following sections, we will discuss these sources of error in the context of the high-altitude satellite navigation concept. We, therefore, consider only satellites at synchronous altitude having earth-coverage antennas. The ground receiver, or navigation system user, must monitor more than one satellite simultaneously and must, therefore, use a non-directional antenna. Thus we take the receiver antenna to have a gain of -3 db (0-db gain with a 3-db polarization loss since it may be difficult to build a perfectly polarized omnidirectional antenna).

We do not consider the effects on ranging accuracy of receiver motion during the range measurements. We also do not consider the measurement of vehicle or receiver height as a ranging problem.

II. TIME DELAY ESTIMATION

Consider the problem of determining the time of arrival or time delay at a receiving station of a signal whose shape is known. If the signal transmitted has the form s(t), the waveform that the receiver observes consists simply of the transmitted signal delayed by the time τ that it takes for radio waves to travel from transmitter to receiver, and a masking noise, which we will take to be an additive white gaussian process with zero mean and (single-sided) spectral density N_o (watts/Hz) [correlation function $R_n(t, u) = \delta(t-u)N_o/2$]. The received waveform from which τ is to be determined is thus

$$r(t) = s(t-\tau) + n(t) \qquad , \; t_1 < t < t_2, \tag{1}$$

where the interval (t_1, t_2) includes as much of the received data as is actually available for processing. We have idealized the problem in that (a) we have not included any distortion of the transmitted signal by the propagating medium; (b) the amplitude of the signal is assumed to be known at the receiver; and (c) the statistics of the masking noise n(t) are completely known. Note that r(t) is a gaussian process with expected value

$$E[r(t)] = s(t-\tau) \tag{2}$$

and autocovariance

$$E\{[r(t) - s(t-\tau)][r(u) - s(u-\tau)]\} = E[n(t)\,n(u)] = \delta(t-u)N_o/2 \tag{3}$$

where $\delta(t-u)$ is a unit impulse function located at $t = u$. Thus the problem of determining τ can be viewed as the estimation of an unknown parameter of a gaussian random process, which is a standard statistical problem.[2, 3, 4]

Ranging Performance Criterion

Based on the observation of the received waveform $r(t)$, $t_1 < t < t_2$, which is a sample function of a gaussian process, we must arrive at an estimated value for the signal delay, denoted $\hat{\tau}$. Thus $\hat{\tau}$ is a random variable dependent on $r(t)$. A reasonable criterion by which to judge various estimation schemes is that of the mean squared error between the estimate $\hat{\tau}$ and the actual value of τ; i. e.,

$$\epsilon_\tau^2 = E(\tau - \hat{\tau})^2 . \tag{4}$$

If we have

$$E(\hat{\tau}) = \tau$$

then $\hat{\tau}$ is called an unbiased estimator. In this case, ϵ_τ^2 is just the variance of the estimate $\hat{\tau}$.

Bound on Estimation Accuracy

Next we will find the minimum variance of any estimate for τ (the Cramer-Rao lower bound on the variance of an estimate[4]). In order to write a probability density for the received waveform $r(t)$, we first expand $r(t)$ in the Fourier series

$$r(t) = \sum_{i=1}^{\infty} r_i \varphi_i(t) \qquad , \; t_1 < t < t_2 , \tag{5}$$

where

$$r_i = \int_{t_1}^{t_2} r(t) \varphi_i(t) \, dt \tag{6}$$

and

$$\int_{t_1}^{t_2} \varphi_i(t) \varphi_j(t) \, dt = \delta_{ij} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases} \tag{7}$$

Using Eq. 1, we can write

$$r_i = s_i(\tau) + n_i \tag{8}$$

where

$$s_i(\tau) = \int_{t_1}^{t_2} s(t-\tau) \varphi_i(t) \, dt \tag{9}$$

and

$$n_i = \int_{t_1}^{t_2} n(t) \varphi_i(t) \, dt \tag{10}$$

If $s(t)$ is restricted in bandwidth by requiring that only a small fraction of its energy lies outside a specified range of frequencies, many of the coefficients $s_i(\tau)$ will be restricted to be very small. This implies that $s(t-\tau)$ can be represented by a truncated Fourier series

$$s(t-\tau) = \sum_{i=1}^{M} s_i(\tau) \varphi_i(t) \quad , \; t_1 < t < t_2 . \tag{11}$$

without significant error. Since $n(t)$ was taken to be white gaussian noise, the coefficients n_i are statistically independent regardless of the set of orthonormal functions $\{\varphi_i(t)\}$ that are used. Since $s_i(\tau) \approx 0$ for $i > M$, the coefficients n_i for $i > M$ are irrelevant to the problem of estimating τ since they do not yield any information about any coefficients for $i \le M$. Thus we may safely truncate the expansion (5) as

$$r(t) = \sum_{i=1}^{M} r_i \varphi_i(t) \quad , \; t_1 < t < t_2 . \tag{12}$$

Thus $r(t)$ is specified by the set of Fourier coefficients

$$\underline{r} = (r_1, r_2, \ldots, r_M) \tag{13}$$

for which we can write a probability density conditional on τ as

$$p(\underline{r}|\tau) = \prod_{i=1}^{M} \frac{\exp\{-[r_i - s_i(\tau)]^2/N_o\}}{(\pi N_o)^{1/2}} \tag{14}$$

Denoting our estimate as $\hat{\tau}(\underline{r})$ since it is a function of the observables r_i, the <u>bias</u> of the estimate is defined as

$$\psi(\tau) = \int [\hat{\tau}(\underline{r}) - \tau]\, p(\underline{r}|\tau)\, d\underline{r} \tag{15}$$

Differentiating with respect to τ, we get

$$\frac{d\psi(\tau)}{d\tau} = \psi'(\tau) = \int [\hat{\tau}(\underline{r}) - \tau] \frac{\partial p(\underline{r}|\tau)}{\partial \tau}\, d\underline{r} - 1. \tag{16}$$

Using

$$\frac{\partial \ln p(\underline{r}|\tau)}{\partial \tau} = \frac{1}{p(\underline{r}|\tau)} \frac{\partial p(\underline{r}|\tau)}{\partial \tau}$$

we get from the Schwarz inequality

$$1 + \psi'(\tau) = \int [\hat{\tau}(\underline{r}) - \tau] \frac{\partial \ln p(\underline{r}|\tau)}{\partial \tau} p(\underline{r}|\tau)\, d\underline{r}$$

$$= \int \{[\hat{\tau}(\underline{r}) - \tau] \sqrt{p(\underline{r}|\tau)}\} \{\frac{\partial \ln p(\underline{r}|\tau)}{\partial \tau} \sqrt{p(\underline{r}|\tau)}\} d\underline{r}$$

$$\leq \{\int [\hat{\tau}(\underline{r}) - \tau]^2 p(\underline{r}|\tau)\, d\underline{r}\}^{\frac{1}{2}} \{\int [\frac{\partial \ln p(\underline{r}|\tau)}{\partial \tau}] p(\underline{r}|\tau)\, d\underline{r}\}^{\frac{1}{2}}$$

and hence

$$\epsilon_\tau^2 \geq \frac{[1 + \psi'(\tau)]^2}{E[\frac{\partial \ln p(\underline{r}|\tau)}{\partial \tau}]^2} \tag{17}$$

This is a lower bound to the mean squared error for any kind of estimate of τ and is satisfied with equality if

$$\frac{\partial \ln p(\underline{r}|\tau)}{\partial \tau} = k(\tau)[\hat{\tau}(\underline{r}) - \tau] \tag{18}$$

where $k(\tau)$ is independent of $\hat{\tau}$.

Bound on Range Accuracy

A lower bound on the mean squared error with which the time of arrival of a signal can be estimated by any scheme can be found. First note that from Eqs. 7, 11, and 12,

$$\int_{t_1}^{t_2} [r(t) - s(t-\tau)]^2 \, dt = \sum_{i=1}^{M} [r_i - s_i(\tau)]^2 \tag{19}$$

Using this relation in Eq. 14 results in

$$\frac{\partial \ln p(\underline{r}|\tau)}{\partial \tau} = \frac{2}{N_o} \int_{t_1}^{t_2} [r(t) - s(t-\tau)] \frac{\partial s(t-\tau)}{\partial \tau} \, dt. \tag{20}$$

Since

$$\frac{\partial s(t-\tau)}{\partial \tau} = - \frac{\partial s(t-\tau)}{\partial t} \tag{21}$$

we use this together with Eqs. 1 and 3 to show that

$$E\left[\frac{\partial \ln p(\underline{r}|\tau)}{\partial \tau} \right]^2 = \frac{4}{N_o^2} \int_{t_1}^{t_2} dt \, \frac{\partial s(t-\tau)}{\partial t} \int_{t_1}^{t_2} du \, \frac{\partial s(u-\tau)}{\partial u} \, E[n(t)\, n(u)]$$

$$= \frac{2}{N_o} \int_{t_1}^{t_2} \left| \frac{\partial s(t-\tau)}{\partial t} \right|^2 dt \tag{22}$$

The Fourier transform of the signal $s(t-\tau)$ is

$$S(f) = \int_{t_1}^{t_2} s(t-\tau)\, e^{-j2\pi ft}\, dt\,, \quad (f \text{ in Hz}) \tag{23}$$

so that the transform of the signal $\frac{\partial s(t-\tau)}{\partial t}$ is obtained by integrating by parts, and is

$$\int_{t_1}^{t_2} \frac{\partial s(t-\tau)}{\partial t}\, e^{-j2\pi ft}\, dt = j2\pi f\, S(f). \tag{24}$$

From Parseval's theorem the received signal energy can be written as

$$E_s = \int_{t_1}^{t_2} s(t-\tau)^2\, dt = \int_{-\infty}^{\infty} |S(f)|^2\, df \tag{25}$$

and similarly,

$$\int_{t_1}^{t_2} \left|\frac{\partial s(t-\tau)}{\partial \tau}\right|^2 dt = (2\pi)^2 \int_{-\infty}^{\infty} f^2 |S(f)|^2\, dt = (2\pi B)^2 E_s \tag{26}$$

where

$$B^2 = \frac{\int_{-\infty}^{\infty} f^2 |S(f)|^2\, df}{\int_{-\infty}^{\infty} |S(f)|^2\, df} \tag{27}$$

can be interpreted as the mean squared bandwidth (in Hz^2) of the (normalized) spectrum of the ranging signal $s(t)$. Thus the mean squared ranging error can be lower bounded as

$$\epsilon_\tau^2 \geq \frac{[1+\psi'(\tau)]^2}{\frac{2E_s}{N_o}(2\pi B)^2} . \tag{28}$$

The parameters of interest regarding ranging accuracy appear to be the receiver signal-to-noise ratio E_s/N_o as well as the (mean squared) bandwidth of the ranging signal. However, it is not yet apparent that Eq. 28 represents a useful bound on ranging accuracy.

Maximum Likelihood Estimators

It is impossible to derive the best estimator for τ in the mean squared error sense in the general case. Instead, the maximum likelihood estimate (MLE) for τ (in the absence of an a priori probability density for τ) is commonly substituted with good reason (as will presently be shown); i. e., that value of $\hat{\tau}$ that maximizes $p(\underline{r}|\hat{\tau})$ is chosen as the estimate of τ. Thus the MLE satisfies

$$\frac{\partial \ln p(\underline{r}|\hat{\tau})}{\partial \hat{\tau}} = 0 \tag{29}$$

and using Eq. 19, we obtain an integral equation for $\hat{\tau}$ as

$$\int_{t_1}^{t_2} [r(t) - s(t-\hat{\tau})] \frac{\partial s(t-\hat{\tau})}{\partial \hat{\tau}} dt = 0 \tag{30}$$

Note that in the case of no noise masking $r(t)$, $r(t) = s(t-\tau)$ and $\hat{\tau} = \tau$ satisfies Eq. 30. In the case of very low noise power in the signal bandwidth, $p(\underline{r}|\hat{\tau})$ will have a narrow peak in the vicinity of τ. If the width of this peak is small enough, we may approximate $s(t-\tau)$ by the expansion

$$s(t-\tau) \approx s(t-\hat{\tau}) + (\tau-\hat{\tau}) \frac{\partial s(t-\hat{\tau})}{\partial \hat{\tau}} \tag{31}$$

allowing Eq. 1 to be rewritten

$$r(t) - s(t-\hat{\tau}) \approx (\tau-\hat{\tau}) \frac{\partial s(t-\hat{\tau})}{\partial \hat{\tau}} + n(t). \tag{32}$$

Using Eqs. 26 and 32 in 30 yields

$$\hat{\tau} \approx \tau + \frac{1}{(2\pi B)^2 E_s} \int_{t_1}^{t_2} n(t) \frac{\partial s(t-\hat{\tau})}{\partial \hat{\tau}} \, dt. \tag{33}$$

Since $n(t)$ was assumed to have zero mean,

$$E(\hat{\tau}) \approx \tau \tag{34}$$

and the MLE is unbiased for all τ in this so-called weak noise case. Therefore, $\psi'(\tau) \approx 0$. The variance of the estimate is thus

$$\epsilon_\tau^2 = E(\hat{\tau}-\tau)^2 \approx \frac{1}{(2\pi B)^4 E_s^2} \int_{t_1}^{t_2} dt \frac{\partial s(t-\hat{\tau})}{\partial \hat{\tau}} \int_{t_1}^{t_2} du \frac{\partial s(u-\hat{\tau})}{\partial \hat{\tau}} E[n(t)\, n(u)]$$

$$\approx \frac{1}{\frac{2E_s}{N_o}(2\pi B)^2} \qquad (35)$$

which shows that in this weak noise case the MLE of τ is <u>efficient</u>, in the sense that it achieves as small a variance as any estimate for τ.

III. IDEAL RANGING PERFORMANCE

Ideal Ranging Signal

If the ranging signal is restricted to the band of frequencies

$$f \leq W(\text{in Hz})^*$$

then the problem of finding the signal with maximum mean squared bandwidth (with normalized spectrum) is analogous to that of finding the distribution of a given amount of mass along a finite length rod to achieve the maximum moment of inertia. The optimum signal in this sense would have a spectrum S(f) whose area was concentrated at the edges of the allowable band and was zero elsewhere within the band, the resulting mean squared bandwidth being $B^2 = W^2$, of course. This is approximately achieved by a sinusoidal signal of frequency W and duration T seconds in which

$$W \gg \frac{1}{T} .$$

The sinusoid can thus be considered an ideal ranging signal in the sense that it has maximum mean squared bandwidth for a given spectral occupancy.

According to Eq. 35, a simple radio frequency (RF) carrier

$$s(t) = A \cos(2\pi f_o t + \phi_c)$$

can be located in time with great accuracy (if the parameters f_o and ϕ_o are accurately known at the receiver). It is not even necessary to do the range

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* All frequencies and bandwidths are expressed in Hz (cycles per second).

measurement at the RF frequency f_o since multiplication of the received signal by $2 \cos 2\pi(f_o - f_1)t$ gives

$$2\, s(t-\tau) \cos 2\pi(f_o - f_1)t = A\{\cos(2\pi f_1 t - 2\pi f_o \tau + \phi_o)$$

$$+ \cos[2\pi(2f_o - f_1)t - 2\pi f_o \tau + \phi_o)]$$

and bandpass filtering can then eliminate the term at frequency $2f_o - f_1$. The result of this transformation of the signal is

$$A \cos [2\pi f_1 (t - \frac{f_o}{f_1} \tau) + \phi_o] .$$

Calling $f_o \tau / f_1 = \eta$, the accuracy of the estimate of η is inversely proportional to f_1^2, but

$$\epsilon_\eta^2 = (\frac{f_o}{f_1})^2 \epsilon_\tau^2$$

and so ϵ_τ^2 is really independent of f_1.* This is a satisfying situation since it indicates that the range accuracy is inherently related only to the carrier frequency.

Resolving Ambiguities

There is another consideration with regard to ranging signals that we have not yet touched upon. That is the question of range ambiguities. Observation of a CW sine wave

$$\sin 2\pi W(t-\tau) = \sin(2\pi Wt - \phi)$$

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* This was pointed out by B. Reiffen.

cannot be used to determine the phase angle $\emptyset$ uniquely. There will always be an ambiguity of some multiple of 2π radians in the determination of $\emptyset$, implying that the time delay τ will always be ambiguous by multiples of the period of the sine wave. This effect implies that there are multiple solutions to Eq. 30, a question that we did not consider before.

A sine wave is still an efficient signal with which to achieve high range accuracy as long as a separate scheme is first used to reduce range uncertainty to less than one period of the sine wave, thus resolving the ambiguity problem by ruling out all but one of the many solutions of Eq. 30. Many practical ranging systems use this general scheme of a separate signal for coarse range determination to resolve ambiguities and a sine wave for the high resolution or fine range measurement. The range accuracy of such hybrid systems can still be computed from Eq. 35 by taking B and E_s to be the frequency and the energy of the fine-range sine wave respectively.

RF Bandwidth Occupancy (Coarse-Fine Measurement Systems)

We face a dilemma in trying to relate range accuracy to RF bandwidth occupancy of the ranging signal since we have already noted that an unmodulated RF carrier can be used for the high resolution range measurement, which occupies essentially no bandwidth. In order to remove range ambiguities, a coarse range signal will have to be sent (at RF) and so there will be some bandwidth occupancy. Suppose the fine range measurement is made using the carrier of frequency f_o. The coarse range signal need only provide an estimate of time of arrival with enough accuracy to resolve the fine range ambiguities. The root mean squared (RMS) error of the coarse measurement, denoted σ_{cr}, can thus be taken as

$$\sigma_{cr} = \frac{\alpha}{f_o}, \quad 0 < \alpha < 1. \tag{36}$$

Therefore, for a given signal energy, the mean squared bandwidth of the coarse range signal should be (using Eq. 36 in Eq. 35),

$$B_{cr}^{2} = \frac{f_o^{2}}{\frac{2E_{cr}}{N_o} (2\pi\alpha)^2} \tag{37}$$

while the system accuracy is given by

$$\epsilon_{\tau}^{2} = \frac{1}{\frac{2E_o}{N_o} (2\pi f_o)^2} \tag{38}$$

where E_o and E_{cr} are the carrier and coarse range signal energies respectively. Combining these equations gives

$$B_{cr}^{2} = \frac{1}{(2\pi)^4 \frac{2E_o}{N_o} \frac{2E_{cr}}{N_o} \alpha^2 \epsilon_{\tau}^{2}} \tag{39}$$

In principle, we can find no objection to having the same signal serve as both the coarse and fine ranging signal. The fine range measurement could be made using the fine structure or carrier of the signal at RF or after translation to an intermediate frequency (IF). If the translated signal is narrowband, we can take

$$B^2 \approx f_o^{2}$$

for the fine measurement. The coarse measurement would effectively use the envelope or carrier modulation. If this modulation has mean squared bandwidth

B_{cr}^{2}, then the RF signal can occupy an RF bandwidth of no less than B_{cr} [and this bandwidth can be realized only if (a) the modulation is transmitted single-sideband, and (b) the actual occupied bandwidth of the modulating waveform is essentially B_{cr}, the minimum possible value]. In this case, we would have

$$E_o = E_{cr} = E_s. \tag{40}$$

Since we must have

$$\begin{aligned} E_o &\leq E_s \\ E_{cr} &\leq E_s \end{aligned} \tag{40}$$

use of Eq. 40 in Eq. 39 results in a formal lower bound on the required RF bandwidth for any ranging system as

$$W_{RF} \geq \frac{1}{(2\pi)^2 \frac{2E_s}{N_o} \alpha \epsilon_\tau} \tag{41}$$

The accuracy of the fine measurement would still be given by Eq. 38 (with $E_o = E_s$).

The above discussion is intended merely to point out that (a) the desired range accuracy alone does not imply anything about the occupied bandwidth W_{RF} of the ranging signal; (b) the a priori range uncertainty is related to the required RF bandwidth of the ranging signal; (c) a lower bound to W_{RF} was given for any ranging scheme with large enough a priori range uncertainty to necessitate a coarse range measurement. Study of some examples shows that for a given signal-to-noise ratio, the lower bound on W_{RF} of Eq. 41 can be approached well within a factor of 10 in certain situations.

In Table I some combinations of E_s/N_o and W_{RF} that satisfy Eq. 39 with equality have been computed for $\epsilon_\tau = 10^{-8}$ sec (3-meter RMS range error). The parameter α essentially controls the probability that the coarse measurement resolves the fine range ambiguities incorrectly, i. e., the probability of a grossly inaccurate or anomalous range determination. Reasonable values for α would be between 0. 5 and 0. 1, and for purposes of illustration we will choose $\alpha = 0.3$ (-5 db).

Table I

$\frac{E_s}{N_o}$ (db)	W_{RF} in KHz
6	100
16	10
26	1
36	0. 1

Double-Sideband (DSB) Amplitude Modulation

By way of illustrating a more conservative approach to obtaining the power-bandwidth requirements for ranging, we next discuss systems in which the ranging signal $s_o(t)$ is transmitted in the form

$$s(t) = s_o(t) \cos 2\pi f_o t$$

If the carrier parameters are all known and removal of the carrier (demodulation) can be accomplished without degrading the modulation (for ranging purposes), the range estimation can be done using the waveform $s_o(t)$. This can be accomplished in practice for a variety of modulations (for instance, those which allow carrier removal with a phase-locked loop). Thus if $s_o(t)$ has mean squared bandwidth B^2, then it must occupy a low-pass bandwidth of at least B Hz, which implies that the bandwidth occupied by s(t) satisfies

$$W_{DSB} \geq 2B.$$

Using this in Eq. 35 results in a bound on performance of systems which use only the double-sideband modulation for ranging. An example of such systems is a short-pulse radar system in which the pulse width governs the ranging accuracy. The bound on performance is

$$W_{DSB}^{2} \geq \frac{1}{\frac{2E_s}{N_o} \pi^2 \epsilon_\tau^2} \tag{42}$$

and this bound is plotted in Fig. 1 (again for $\epsilon_\tau = 10^{-8}$ sec). Denoting the average power of the received signal as

$$P_r = \frac{E_s}{T} \tag{43}$$

we can write

$$\frac{E_s}{N_o} = \frac{P_r}{N_o} T$$

which brings out explicitly the measurement time T as a parameter. In Fig. 1 the E_s/N_o axis is thus also labeled as P_r/N_o corresponding to a measurement time of 1 sec.

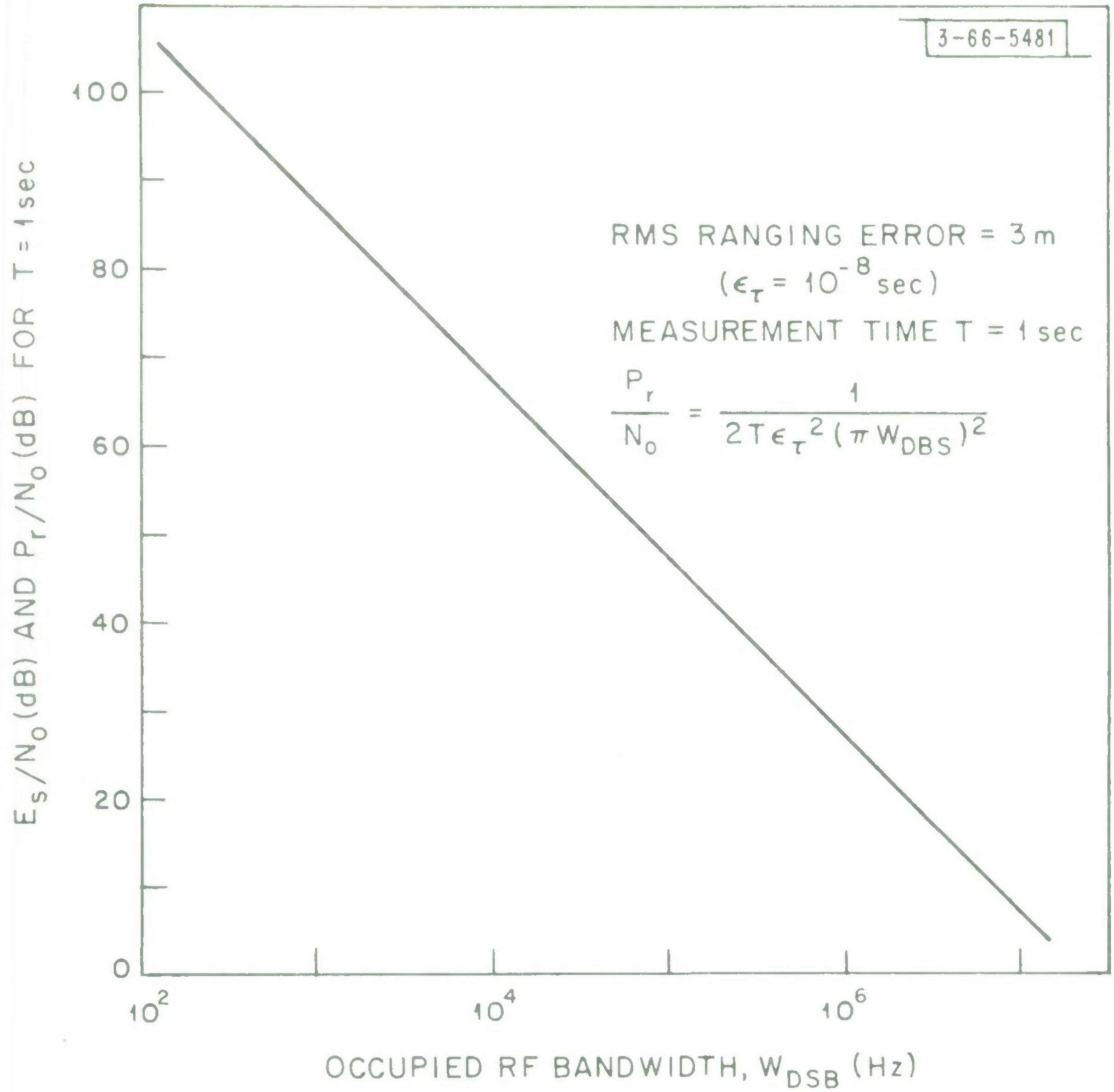

Fig. 1. Ideal DSB-AM ranging system trading curve for bandwidth vs signal-to-noise ratio.

Note the radical difference in the required signal-to-noise ratio-bandwidth combinations between the ideal DSB system and those given in Table I for the same range accuracy. The DSB requirements are conservative in technology, although they still correspond to ideal performance in the sense that we took $W_{DSB} = 2B$. The bound on occupied bandwidth in a DSB-AM ranging system can be approached well within a factor of 10, for a given signal-to-noise ratio and a large a priori range uncertainty. Even this margin does not lead to prohibitive satellite power levels and bandwidths. The performance indicated in Table 1 may perhaps be approached as closely with further work on the type of signal processing required.

Satellite Power Requirements

Having calculated the minimum required signal-to-noise ratio $\frac{E_s}{N_o} = \frac{P_r T}{N_o}$ at the receiver corresponding to a given signal bandwidth for a DSB-AM ranging system, we now translate this into satellite transmitter power requirements. As an example calculation, the signal power P_r received by an antenna with gain $G_{rec} = -3\,\text{dB}$ from a satellite at synchronous altitude with an earth coverage antenna (gain $G_{sat} = 18\,\text{dB}$) at an operating frequency of 300 MHz is

$$\begin{aligned} P_r\,(\text{dBw}) &= P_{sat}\, G_{sat}\, G_{path}\, G_{rec} \\ &= P_{sat}\,(\text{dBw}) + 18\text{ dB} - 174\text{ dB} - 3\text{ dB} \\ &= P_{sat}\,(\text{dBw}) - 159\text{ dB} \end{aligned} \tag{44}$$

where P_{sat} (dBw) is the satellite radiated power expressed in decibels re 1 watt. The path gain varies inversely as the square of the operating frequency, so P_r can be regarded as a function of frequency for a fixed P_{sat}.

The noise power density N_o is due to sky background noise and receiver noise, both of which are dependent on operating frequency. The noise temperature of cosmic noise in the direction of the galactic plane is plotted as a function of frequency in Fig. 2. Also shown in this figure are the noise temperature of an uncooled parametric front end[5] and the sum of the sky and parametric receiver noise temperatures. The right-hand scale of Fig. 2 translates these noise temperatures to noise power densities.

Returning to our example calculation of P_{sat}, the receiver noise power density is seen from Fig. 2 to be

$$N_o = -200 \text{ dBw (re 1 Hz)}$$

at the assumed operating frequency of 300 MHz. If an RF bandwidth occupancy of 10^5 Hz is chosen, Fig. 1 indicates that we must have

$$\frac{P_r}{N_o} \geq 47 \text{ dB}$$

corresponding to a measurement time T = 1 second. Therefore, using Eq. 44,

$$\frac{P_r}{N_o} \text{ (dB)} = P_{sat} \text{ (dBw)} - N_o \text{ (dBw)} - 159 \text{ dB}$$

and so

$$P_{sat} \geq 47 \text{ dB} - 200 \text{ dBw} + 159 \text{ dB} = 6 \text{ dBw (4 watts)}.$$

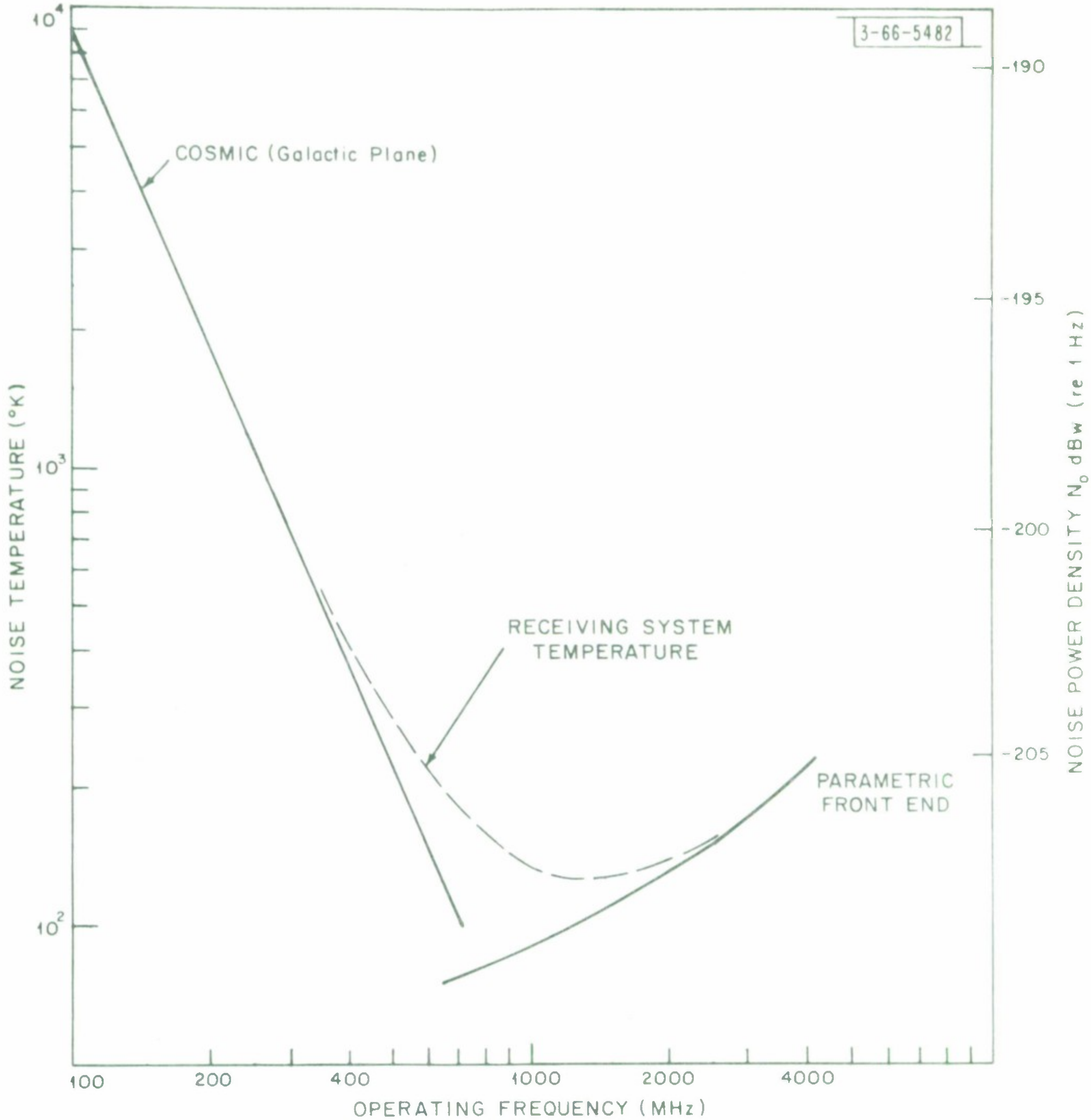

Fig. 2. Thermal noise environment of ranging system.

Four watts of radiated satellite power are required to achieve an RMS ranging error of 3 meters at an operating frequency of 300 MHz. At a frequency of 1200 MHz, the path gain decreases by 12 dB and N_o is about -208 dBw, a decrease of 8 dB (see Fig. 2), implying that $P_{sat} \geq 6$ dB + 12 dB − 8 dB = 10 dBw (10 watts). By scaling path gain in this manner and using Fig. 2, the required P_{sat} can be obtained for all operating frequencies of interest corresponding to

$$\frac{P_r}{N_o} = 47 \text{ dB}$$

This curve has been plotted in Fig. 3. The fact that this value of P_r/N_o corresponds to $W_{DSB} = 10^5$ Hz in an ideal DSB ranging system is also indicated on the curve. By using Fig. 1 to obtain the required P_r/N_o for other values of the ideal system bandwidth, curves of P_{sat} can be calculated for various ideal DSB ranging system combinations of E_s/N_o and W_{DSB}. Such curves also appear in Fig. 3 and it should be noted that they apply to any (non-ideal) DSB ranging system that requires the indicated P_r/N_o (but perhaps a larger, non-ideal bandwidth).

For a given P_r/N_o, a minimum of satellite power is required for operating frequencies in the vicinity of 300 MHz, but this is a very broad minimum. The curves rise for low frequencies because the cosmic noise increase requires greater satellite power. The curves rise again for high frequencies because of the increasing N_o and the decreasing aperture of the receiving antenna with increasing frequency. However, operating frequencies well above 300 MHz are actually desirable in order to avoid frequency-dependent ionospheric propagation anomalies.

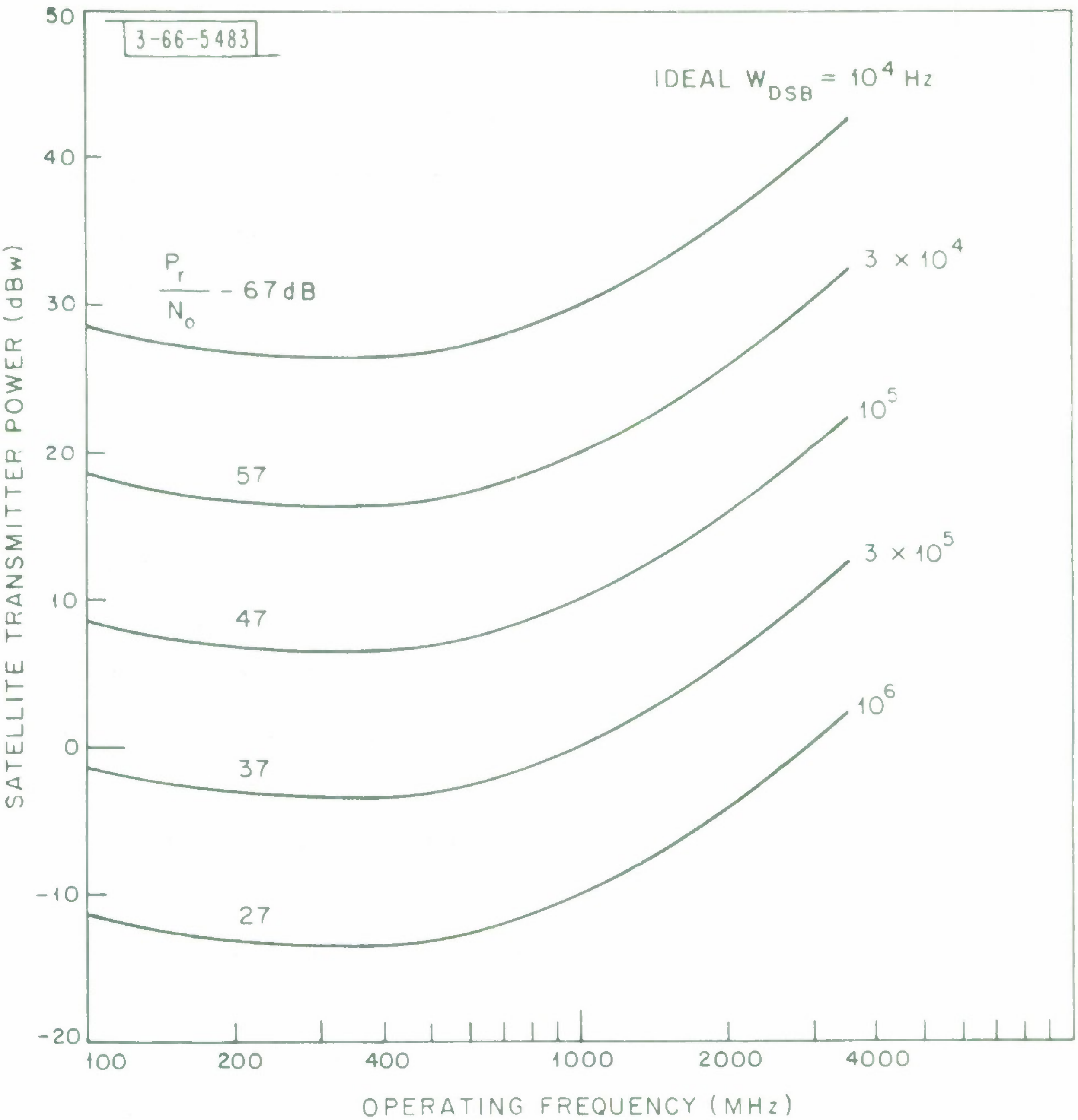

Fig. 3. Satellite transmitter power required for "Ideal" DSB ranging system vs operating frequency.

Assumptions:

1. Synchronous altitude satellite with earth coverage antenna
2. Parametric receiver front end
3. Receiver antenna gain — 3 dB
4. Measurement time 1 second
5. RMS range error 3 meters

IV. ATMOSPHERIC PROPAGATION ANOMALIES

Propagation anomalies introduced by the atmosphere can be corrected out to the extent that they are predictable. Of course, random anomalies cannot be so removed and thus represent fundamental limitations imposed on ground-based receivers that attempt to make very precise measurements of signal parameters.

The Ionosphere

The inhomogeneities of the ionosphere result in a refractive bending of radio waves which is frequency dependent. This bending of the propagation path results in a difference between the direction of arrival of radio waves at the receiver and the actual direction of the transmitter. This effect does not concern us. Ionospheric refraction also results in a propagation path length which is different from the actual distance between transmitter and receiver. This effect is the one with which we are concerned in measuring time of arrival of signals from exo-atmospheric satellites.

Millman[6] has computed the range error due to ionospheric refraction for a model ionosphere. His results are shown in Fig. 4 for an exo-atmospheric transmitter and an earth-based receiver. Note the difference in range errors due to the different elevation angles (angle above the horizon) of the transmitter. This variation with elevation angle is swamped by the day-night variation of the electron density in the ionosphere, which would be difficult to predict. This data indicates that frequencies above 1000 MHz would allow overall range accuracies of the order of 10 meters RMS even if ionospheric refraction is ignored.

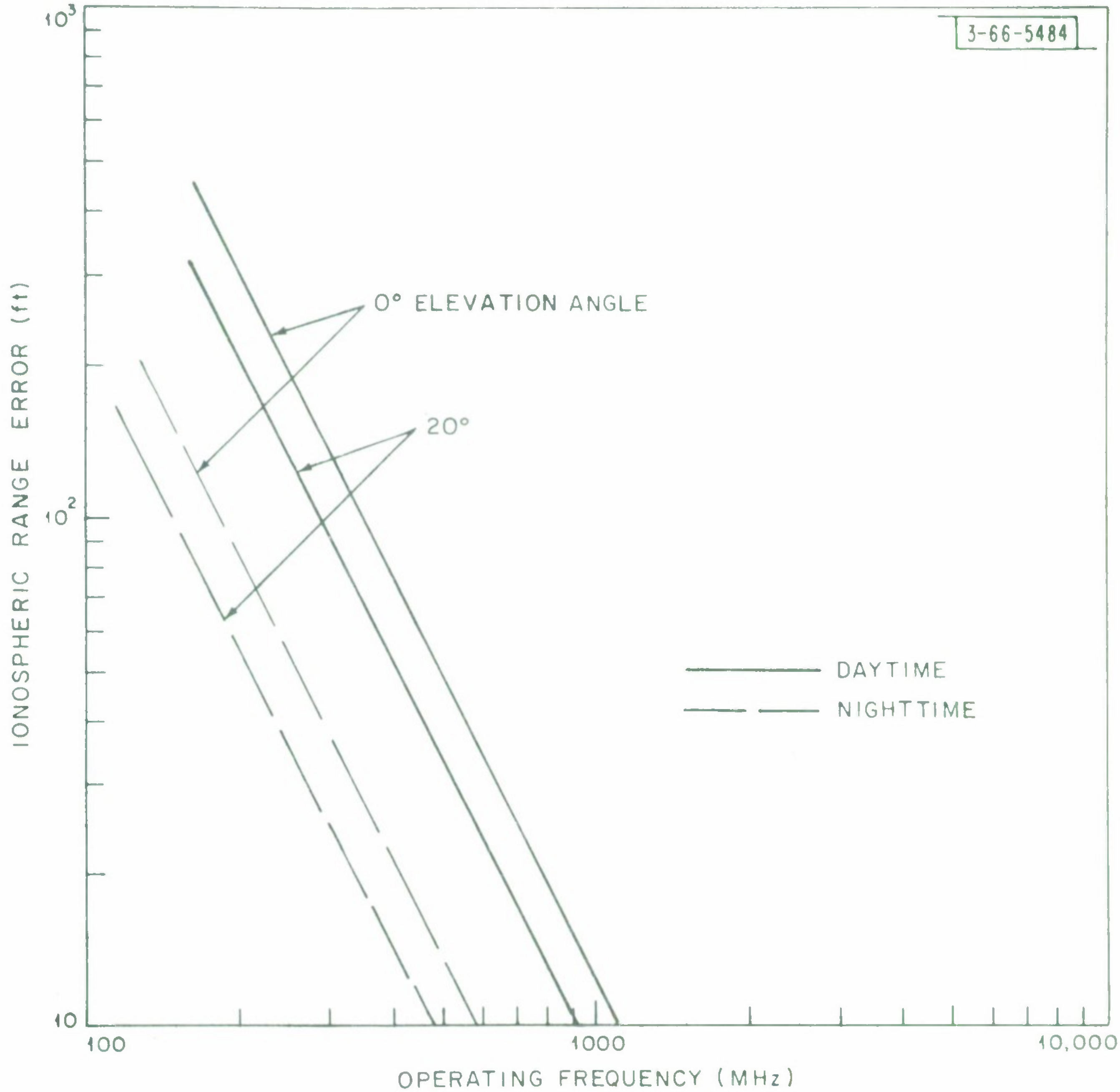

Fig. 4. Range errors due to ionospheric refraction for a one-way path from a high-altitude satellite to the earth's surface.

The Troposphere

Millman[6] has also calculated the range errors due to tropospheric refraction for a standard atmosphere for conditions of 0% and 100% humidity (see Fig. 5). These errors depend on air temperature and humidity but are independent of operating frequency. If corrections for humidity and temperature are not made, the errors due to these causes could be considered as random ones. For elevation angles below 4°, the errors due to neglecting humidity are greater than 10 feet. Additional errors due to neglecting temperature are of the order of 15 to 20% of the humidity errors. Thus, if simple range corrections are made for tropospheric refraction (ignoring temperature and humidity corrections), the elevation angles at which satellites are usable for navigation (i. e., resulting in range errors of the order of 10 meters RMS) is restricted to be above 8°.

Millman also points out that refraction is a function of air density and, hence, most of the range error is due to radio path refraction at low altitude. High-flying aircraft, therefore, would have to modify their tropo-refraction correction or, perhaps, might even be able to ignore it.

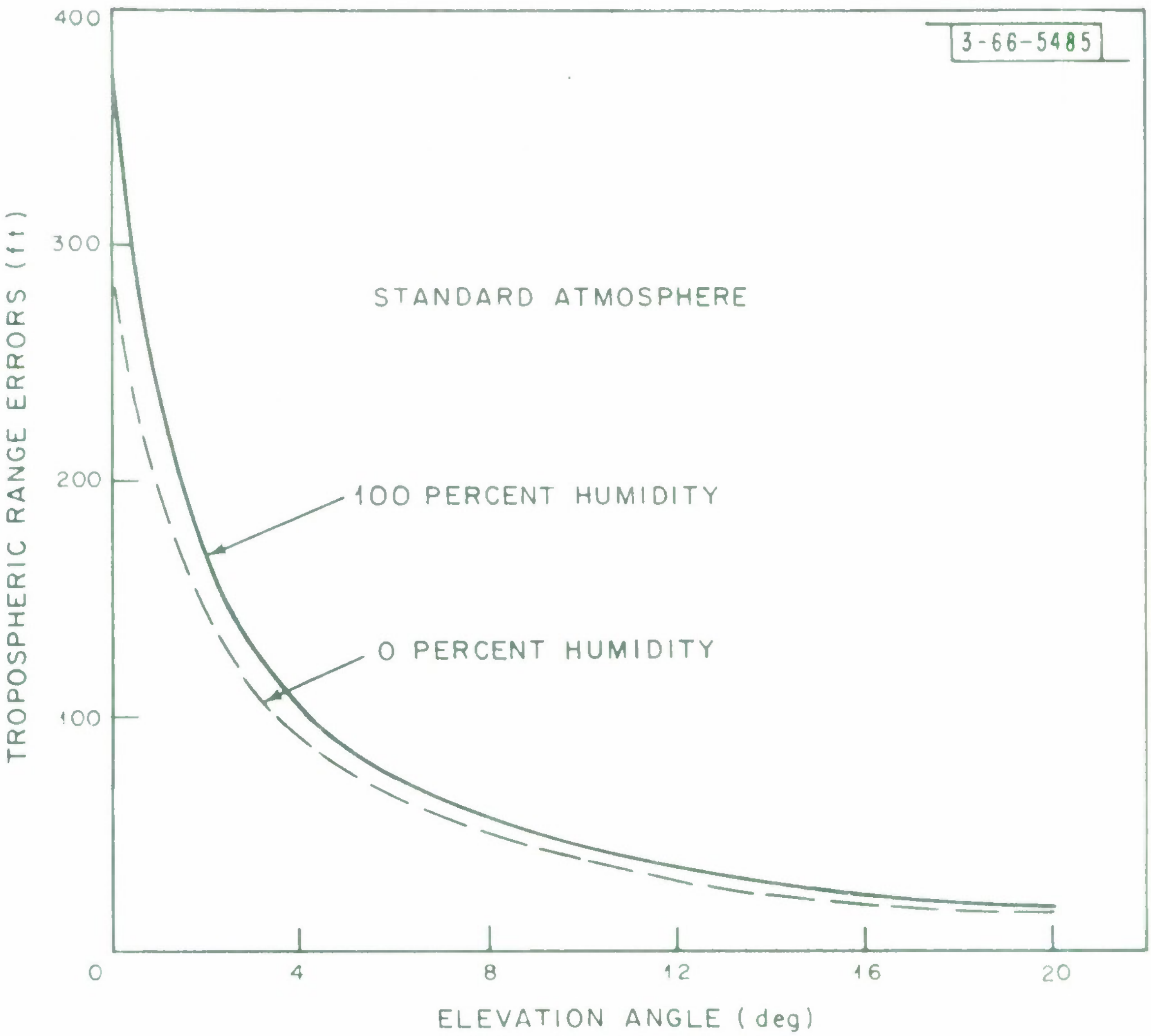

Fig. 5. Range errors due to tropospheric refraction for a one-way path from a high-altitude satellite to the earth's surface.

V. MULTIPATH

A receiving antenna with a wide-angle pattern, perhaps a hemispherical pattern, is needed to receive the timing signals from three satellites simultaneously, especially since the satellites cannot be clumped in one part of the sky for accurate position determination. Such an antenna installed on an aircraft may also receive the satellite signals reflected from the earth (multipath propagation) with deleterious effects on ranging accuracy. Severe multipath propagation of satellite signals has been observed at an operating frequency of 229 MHz in aircraft flying over water and smooth terrain.[9] Studies are presently being conducted to determine its effect on digital communication links to aircraft via satellites. The much more precise measurements required of a satellite navigation receiver will be more seriously affected than a digital communication link. Hence, more refined experiments will be needed to determine the effects of multipath on ranging accuracy.

The angle between the waves arriving at the receiver via the direct path and the earth-reflected paths decreases as the satellite elevation angle decreases. Also the reflections are expected to be strongest for low-incidence angles. An ideal up-looking antenna with a hemispherical pattern would not be sensitive to signals other than those arriving directly from a satellite. A practical antenna would have to have a sharp-edged pattern to attenuate the multipath signals well for satellites at low elevation angles. This may be difficult to achieve in a practical antenna, especially in the UHF band. Multipath considerations may, therefore, also restrict the use of satellites at low elevation angles.

At higher frequencies (having wavelengths of the order of a foot or less), even an ocean surface would appear to be a "rough scatterer" of radio waves. One would expect that reflected signals will be incoherent, thus simply contributing to the receiver noise level masking the direct path signals. The effect of incoherent multipath would be much less serious than specular or coherent reflections. The antenna design problems may also be more readily solved at the higher frequencies. Multipath considerations, therefore, seem to favor the use of operating frequencies above the UHF band for several reasons.

The effects of multipath on ranging signals can be experimentally studied rather conveniently if a directional antenna is available to provide a multipath-free reference signal from which the effects of satellite motion and satellite clock instability can be determined. Such a reference signal can then be used with a down-looking antenna to study characteristics of the multipath, such as strength, coherence, and bandwidth. The effects of multipath on ranging accuracy could also be directly determined for an experimental receiving antenna with such a reference signal by simply comparing the ranges measured at the two antennas.

Since many high-accuracy ranging systems do the fine range measurement on a CW sine wave, any reasonably stable satellite oscillator could provide some useful data with the combination of receiving antennas described above. Navy navigation satellites could provide some data at 150 and 400 MHz. An experimental satellite transmitting timing signals from a very stable clock is currently being contemplated. Such a satellite should be able to provide (with suitable ground equipment) almost all the necessary information at its operating frequency (probably in the UHF band). There is a need for measurements in the 800- to 2000-MHz range, especially since this appears to be the most promising one for navigation. If a satellite-borne clock is not available in this frequency range, signals transponded by a satellite may suffice. Such features as switched antennas in the satellite must be avoided however.

VI. SUMMARY

1. It appears that the main uncertainty concerning the accurate measurement of range to high-altitude satellites is the effect of multipath reception. Experimental investigations are necessary to resolve the issue before a firm statement can be made with regard to the limitations on range accuracy for a high-altitude satellite navigation system.

2. Ionospheric and tropospheric refraction do not seriously affect range accuracy for operating frequencies above 1000 MHz and satellites at least 8° above the horizon (Figs. 4 and 5).

3. Satellite transmitter power levels and bandwidths consistent with a range accuracy of 3 meters RMS are not prohibitive (Fig. 3).

4. Further work on ranging signal design is needed, taking into account the complexity of satellite and user equipment and sensitivity to multipath, as well as satellite power and bandwidth requirements.

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13. ABSTRACT

In order to determine the position of a receiver on earth to within a fraction of a mile using high-altitude satellites, 10-meter accuracy in the receiver-satellite ranges or range differences is necessary. The factors that affect ranging systems attempting to achieve such accuracies are considered here rather than the detailed design of a ranging system. It is concluded that the effects of multipath propagation on ranging accuracy for receivers in aircraft must be experimentally studied prior to any judgment of the feasibility of such accuracies. In the absence of multipath, the satellite power and bandwidth requirements and ionospheric and tropospheric effects do not prohibit 10-meter range accuracy.

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