ARPA ORDER NO. 189-1

MEMORANDUM
RM-5103-ARPA
OCTOBER 1966

SHORT-PERIOD PROPAGATION OF INFRASONIC WAVES FROM NUCLEAR EXPLOSIONS

W. C. Meecham

PREPARED FOR:
ADVANCED RESEARCH PROJECTS AGENCY

The RAND Corporation
SANTA MONICA • CALIFORNIA

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PREFACE

This Memorandum is part of RAND's continuing VELA Analysis Study for the Advanced Research Projects Agency. It discusses infrasonic waves with periods of less than a minute, as generated by nuclear explosions.

The Memorandum should be of interest to those involved in applied aspects of nuclear test detection, as well as to those interested in research on general characteristics of infrasonic waves.

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SUMMARY

One of the outstanding characteristics of acoustic signals received from nuclear explosions is their great time duration. This is true for long-wave periods (greater than a minute), for intermediate periods (about one minute), and for short periods (less than one minute). Ordinary dispersion cannot account fully for the effect in either of the shorter-period ranges.

This Memorandum suggests that for intermediate periods the extended signal duration may be due to refraction from large-scale weather fronts. The change in travel time due to horizontal refraction accounts for the signal duration. Short-period waves also have extended duration. It is shown that the very commonly occurring wind ducts, about 10 fps strength and several thousand feet thickness, can "split" the pulse hundreds of times. Such splitting into a transmitted and a reflected portion occurs for periods of about 10 to 20 sec for path lengths of thousands of kilometers. This splitting and the attendant delay may account for some of the short-period signal characteristics.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge the many extended and helpful discussions with P. Tamarkin, Advanced Research Projects Agency; T.F. Burke, The RAND Corporation; and J. Young, National Bureau of Standards.

CONTENTS

PREFACE iii

SUMMARY v

ACKNOWLEDGMENTS vii

LIST OF FIGURES xi

Section

I. INTRODUCTION 1

II. INTERMEDIATE-PERIOD PROPAGATION 4

III. SHORT-PERIOD PROPAGATION 11

IV. CONCLUSIONS 21

REFERENCES 22

LIST OF FIGURES

1. Average vertical variation of absolute temperature in the atmosphere 5

2. Comparison between dispersion curves for the 52 km and 300 km models 6

3. Frequency of occurrence of subducts of various intensities for the vertical components (temperature) and the west component 14

4. Frequency of occurrence of subducts of various thickness ... 15

5. Frequency of occurrence with height of vertical and west component subducts less than 10,000-ft thick 16

I. INTRODUCTION

This Memorandum is concerned with the propagation of infrasonic waves with periods of less than one minute and greater than one second. The source of disturbances of greatest interest here is a nuclear explosion, although many of the conclusions are applicable to other phenomena. Considerable attention has previously been given to long-period acoustic-gravity waves.(1-5) Such waves are best treated by the use of a normal mode description (as indeed is the case even for some periods shorter than a minute). Through extensive computations, it has been possible to reproduce many of the essential characteristics of long-period waves.(3) Because of the amplitude sensitivity of the various modes to atmospheric structure, it is difficult to calculate the expected pressure amplitude in detail. This difficulty is likely to remain for some time. There is some question about the usefulness of such a detailed calculation even if it could be made, since the result must change with daily weather, seasonal weather and geophysical path, as well as with other special conditions.

Thus, emphasis to date has been on long-period waves. Relatively little theoretical work has been done on disturbances observed with periods of less than one minute.(6) Here, the basic characteristics of such phenomena will be described first. The pressure amplitude of a nuclear explosion, observed at sufficient range to produce a linear disturbance (usually at such a range that the overpressure is less than one-tenth ambient pressure) is conveniently described by the expression(7)

$$p(t) = p_o \left(1 - \frac{t}{t_+}\right) e^{-t/t_+}, \quad t > 0 \qquad (1)$$

where $t_+ \equiv Y^{1/3} t_1$, $t_1 \approx 1$ sec and Y is the yield in kilotons (KT). The overpressure p_o is also proportional to $Y^{1/3}$. The Fourier Transform (FT) of this pressure signal is

$$P(\omega) = p_o \left(-i\omega\right) \left(t_+^{-1} - i\omega\right)^{-2} \tag{2}$$

The spectrum, obtained by taking the square of the absolute value of Eq. (2), has a maximum which occurs for one megaton (MT) at a period of about one minute. The first disturbance to arrive at ranges of thousands of kilometers has a period greater than a minute. After a time equal to about 5 percent of the acoustic travel time, an "acoustic" portion of the signal is observed with periods less than a minute. Ordinarily, this portion continues for well over an hour.(6) This part of the signal is subject to extreme fluctuation; sometimes it is not observed at all. It is suggested that the fluctuation may be due to high-altitude wind structure at about 50 km.

One of the effects requiring explanation is the duration of the signal both for intermediate and short periods. It was shown in an earlier report(5) that for long-period waves, periods greater than 60 sec, the dispersion is expected to stretch the signal duration to about 5 percent of the travel time, at least for the fundamental acoustic mode. Similar considerations should apply to the fundamental mode for periods less than a minute. For short-period waves, the method of ray acoustics can be applied, from which it will be seen that the signal is expected to last only a fraction of a percent of the travel time. Short-period signals are observed for much greater times. From ray tracing, a few short-period pulses spaced over a few minutes would be expected. Instead, there are dozens of such pulses lasting over an hour. It is suggested here that reflection and refraction from upper-atmosphere wind ducts and large-scale weather fronts account for these short-period phenomena.

Section II discusses intermediate-period propagation effects, and Section III treats short-period effects.

II. INTERMEDIATE-PERIOD PROPAGATION

The characteristics of intermediate-period waves are very similar to those of long-period waves previously discussed.(5) Because temperature changes with altitude, the atmosphere can be regarded as a stratified medium for acoustic propagation. A typical average atmosphere is shown in Fig. 1. It is seen that there are two sound ducts where the velocity of sound is a minimum. Of these, the lower one is of primary importance for intermediate- and short-period infrasound, at least for large ranges. This is so because radiation penetrating into the upper duct will suffer viscous dissipation. For instance, in Ref. 8 it is seen that a 30-sec wave would show a reduction in pressure to about one-fourth when traveling to an altitude of 110 km and back.* In order to reach a range of 10,000 km, a wave would have to suffer many such attenuations, and the relatively strong signals observed at such ranges would not be possible. Expressed another way, the amplitude dependence on range would be entirely different from that observed. Accordingly, for periods of interest in this Memorandum, the atmospheric structure at altitudes above about 50 km will be neglected. Some characteristic dispersion curves for a 52-km atmosphere (whose properties are assumed constant above 52 km) and a 300 km atmosphere have been calculated in Ref. 1. These results are summarized in Fig. 2 where, as discussed in Refs. 1 and 5, it is seen

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*Of course, the correct way to calculate this is to include the attenuation in the normal mode treatment; however, this has not been done to date. It is thought that such a procedure would show that those modes which propagate primarily in the upper duct would, at higher frequencies, be heavily attenuated as described.

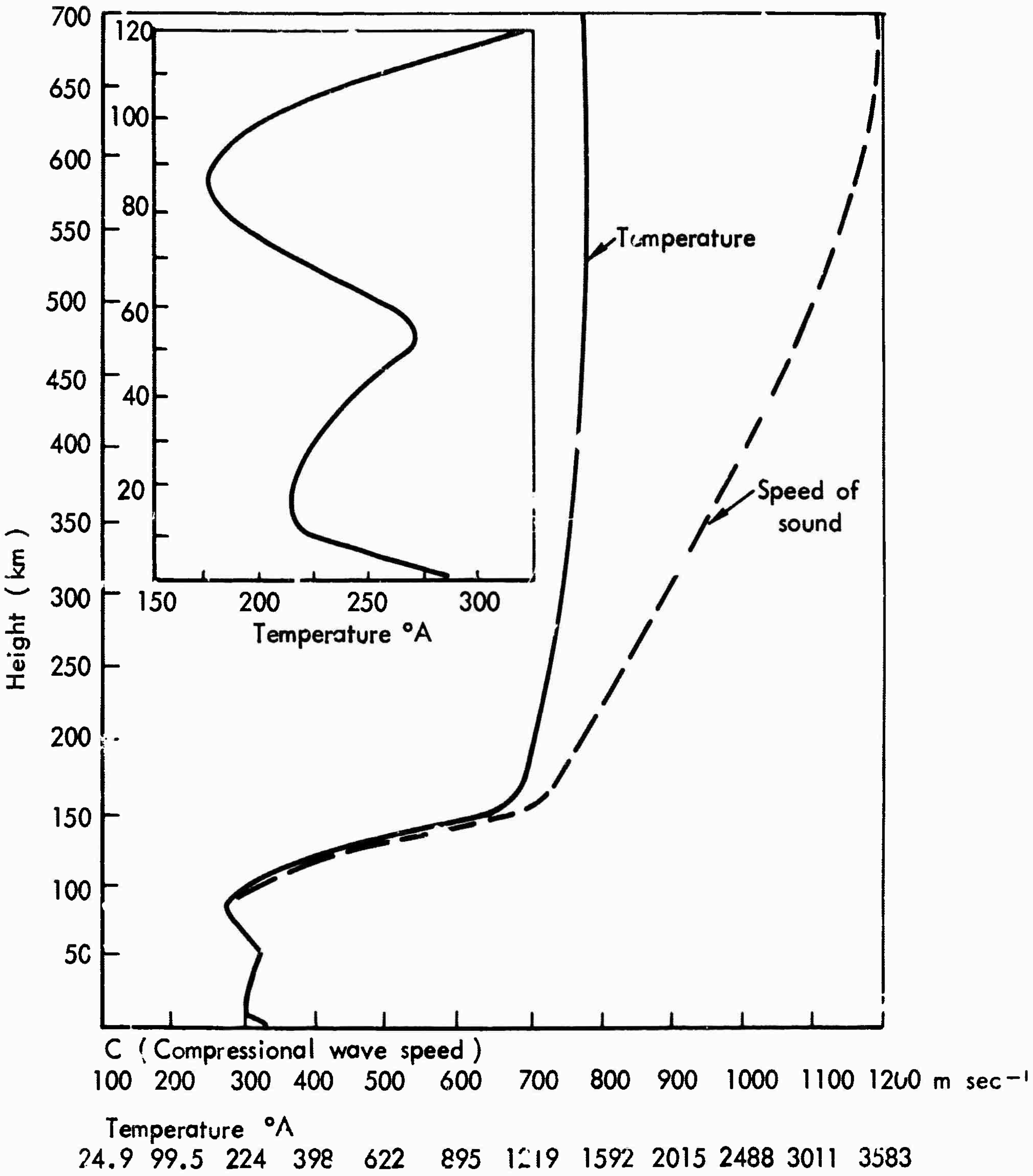


Fig. 1—Average vertical variation of absolute temperature in the atmosphere

estimated by the Committee on Space Research (solid curve) and related compressional wave speed (solid curve up to 74 km and broken curve above this level) in m sec^{-1} The details of the temperature distribution up to 120 km are given in the insert.

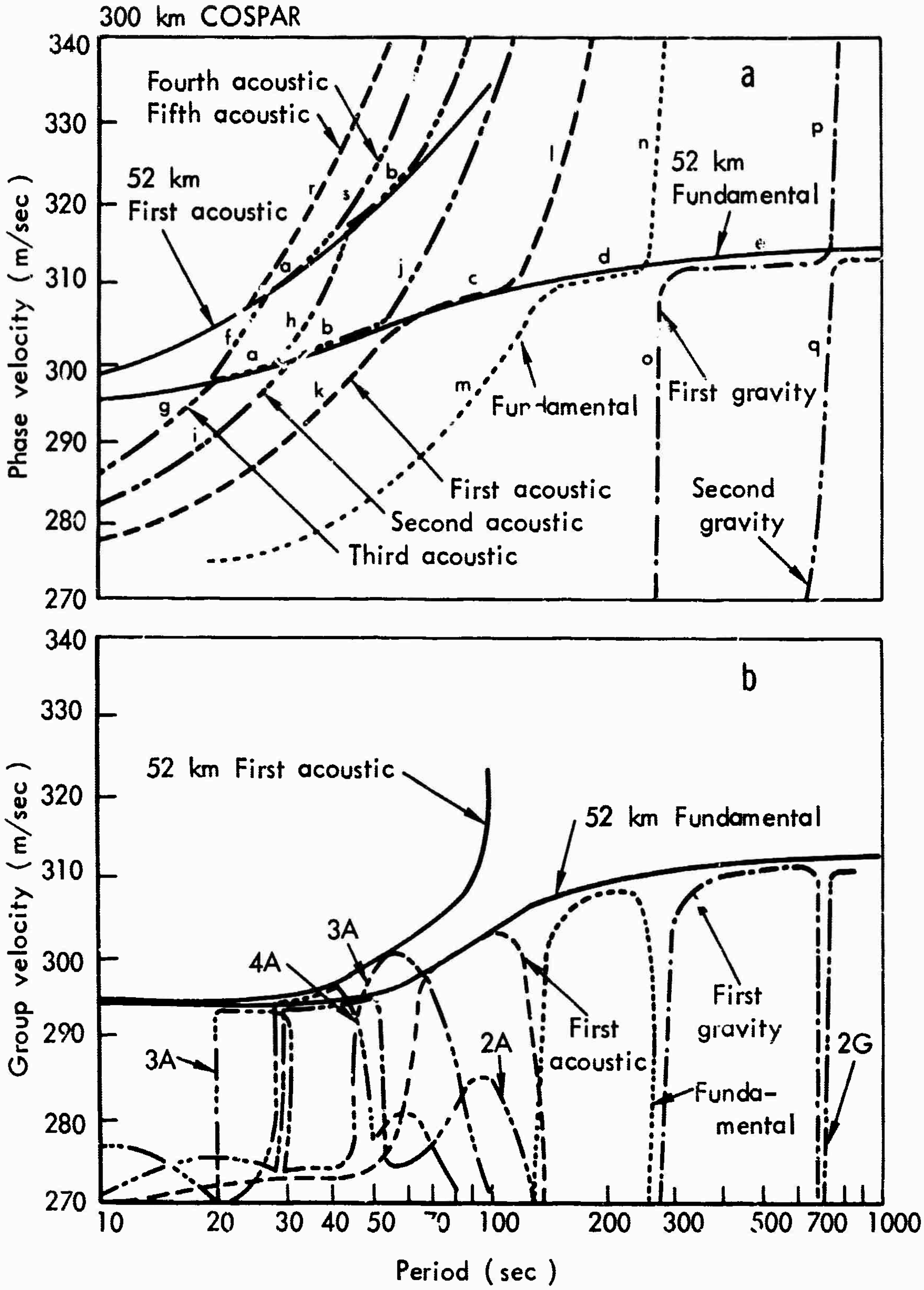


Fig. 2—Comparison between dispersion curves for the 52 km and 300 km models

showing that the quasi-horizontal portions of the phase and group velocity curves for the 300-km model coincide with the solutions for the 52-km model. a) phase velocity vs. period; b) group velocity vs. period. The small letters in the upper figure refer to points at which vertical profiles of kinetic energy were calculated and are shown in Fig. 8.

that the fundamental mode for the 52-km atmosphere consists of nearly horizontal portions of many different modes from the 300-km atmosphere.

Reference 5 gives the procedure for determining the amplitude and period of the signal received at range r and time t after the explosion. Essentially, a group velocity equal to r/t is desired; the period of the wave is determined by the intercept of this group velocity with the group velocity dispersion curve, for instance, for the fundamental mode. The amplitude is determined in part by the excitation functions, and it falls off inversely as the square root of the range--this is the major effect of the dispersion. The signal begins after a delay appropriate to a group velocity of about 315 mps (see Fig. 2) and ends after approximately 5 percent of the travel time. For a range of 10,000 km, this means that the signal should last about half an hour. In fact, the signal may last for an hour or two.(6) This effect may be due to refraction of the intermediate- and short-period signal from weather fronts.

The following sketch shows the top view of a ray refracted by a wind shear occurring over 10^3 km and attaining a wind speed of about 30 mps.

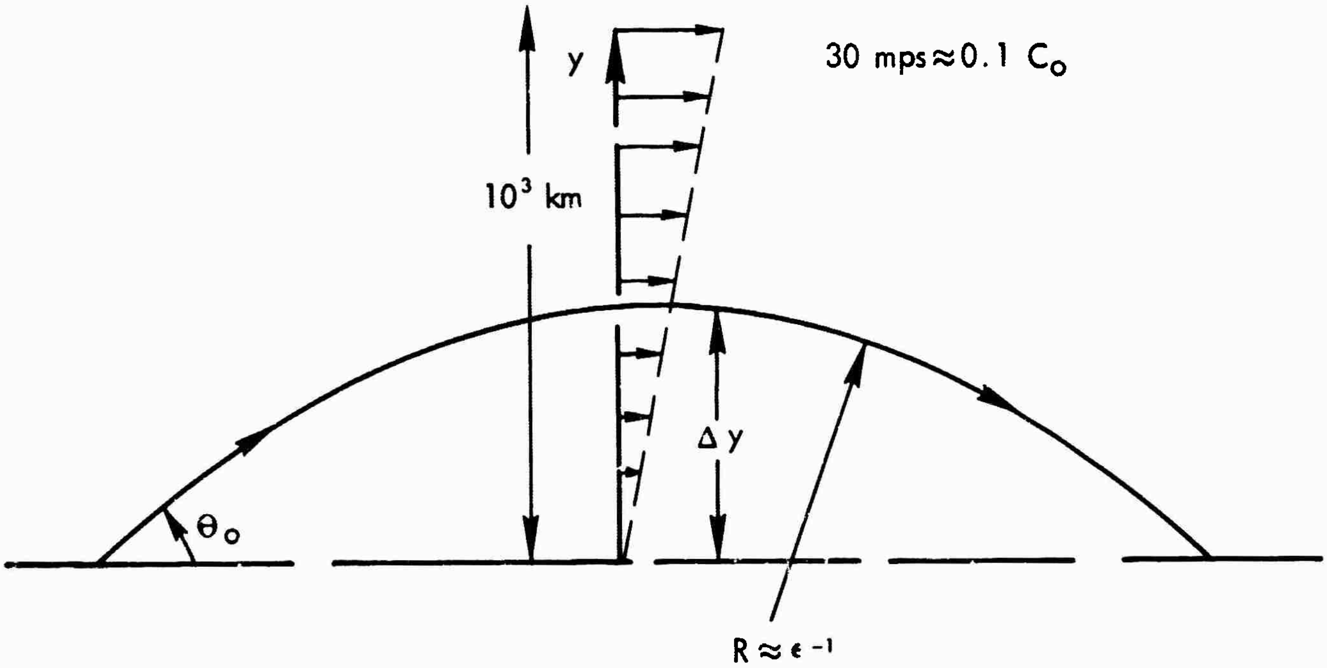
For the speed of sound, propagating downwind

$$C = C_o (1 + \epsilon y)$$

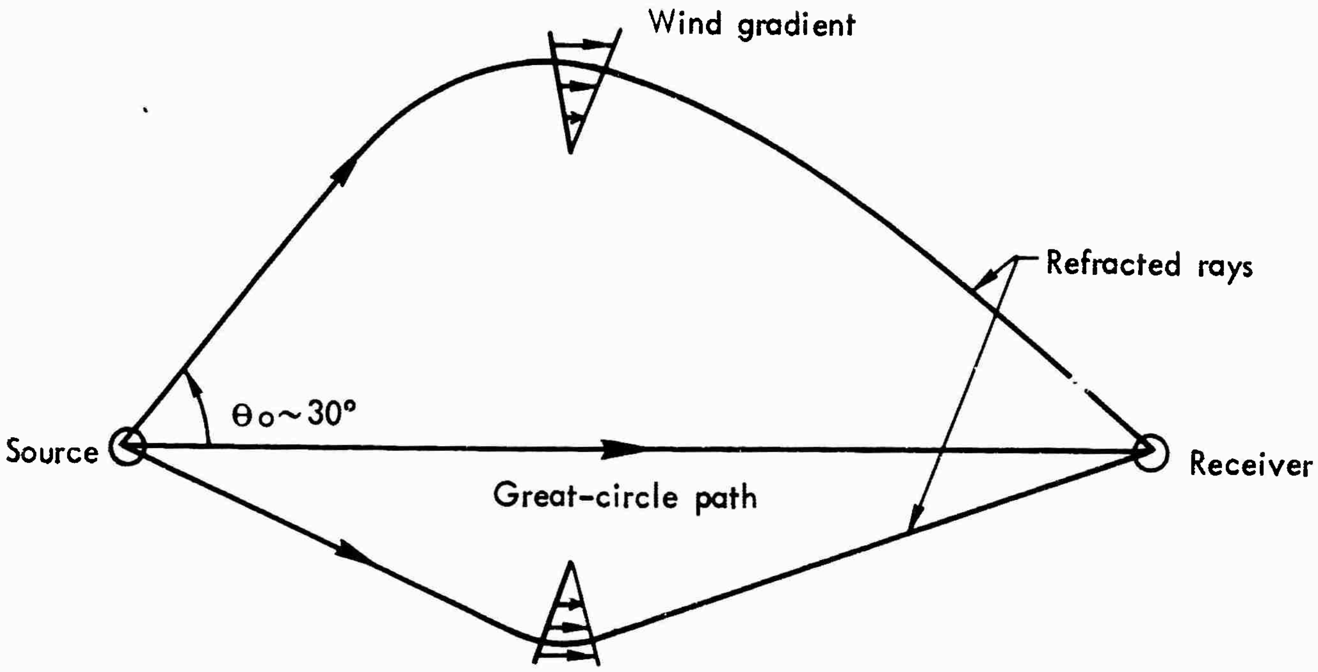
with $\epsilon = 10^{-4}$ km^{-1}. For small angles this gives a radius of curvature of 10^4 km. The largest angle θ_o trapped by this gradient is given by

$$\Delta y = 10^3 \text{ km} = \frac{1}{2} 10^4 \theta_M^2$$

or $\theta_M \approx 30^\circ$. The curved ray takes less time than a straight ray would. The fractional change in travel time is of order $\theta_o^2 \approx \frac{1}{5}$.



In a more realistic physical situation, a number of such horizontal wind gradients might be expected to be placed in regions off the great-circle path from source to receiver. Then, at a range of 15,000 km, a first arrival would occur along the great circle, with delayed arrivals coming from weather front wind gradients. These later arrivals would be off-angle and could have time delays of about 20 percent of the travel time. It should be emphasized that the travel time for a refracted ray is a local minimum in a single wind gradient. However, for several gradients (see sketch on next page), the travel time for such refracted rays is, in general, greater.



Of course, for ranges of 10,000 km or more, there might be several such refractions, each with its appropriate delay time. These arrivals could well overlap and in such a way to give an apparently continuous signal for one to several hours.[*] One characteristic of such a model is that as time of reception increases, azimuth angle at which reception occurs changes by appreciable amounts. This direction of reception could, of course, be determined using a modest array. Furthermore, it is expected that as signal delay time increases, amplitude and frequency of the appropriate normal mode would change (dispersion causes low frequencies to arrive first for each refracted path). Power spectrum functions, taken for

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[*] The skip distance, i.e., distance traveled between reflections at the earth, is about 300 km in the lower duct. Thus, in 10,000 km of travel, over 30 skips are required. If a ray encounters a weather front with fair probability in one skip, it will do so several times in many skips.

signals with different delays, should show such effects. However, available power spectrum analyses are not yet sufficiently complete to confirm this conjecture.

For the appropriate normal modes to be activated, it is essential that the excitation function not vanish at the source altitude or the receiver altitude (frequently at the ground). From Refs. 1 and 3, it is seen that this is not a trivial requirement. In Fig. 2, for instance, it is seen that the excitation function (or to be exact, the energy density function) vanishes at the ground for the fundamental mode (52-km atmosphere) for periods less than about 38 sec. Thus, an important determining factor is the excitation function behavior at the altitudes of interest.

III. SHORT-PERIOD PROPAGATION

For periods less than a minute, it is appropriate to use ray acoustics for vertical refraction to describe signal characteristics. These short-period waves will, of course, be horizontally refracted, as are the long-period waves described in Section II. One effect observed in the data is the occurrence of dozens of pulses, similar to the initial pulse of the explosion itself, spread over an hour. Ordinary ray tracing also suggests that many pulses will occur, although not as many as are observed; they would, however, only last for a few minutes. It is thought that these pulses and their extended duration can be accounted for by diffraction effects caused by the vertical wind structure, to be discussed in this section. It seems that horizontal refraction alone cannot explain all of the signal duration and pulse splitting effects.*

In order to obtain a clearer idea of the ray propagation characteristics, consider a ray tracing result obtained by Burke.** He uses an approximation to the average temperature structure in Fig. 1, an assumed height of burst (HOB) of about 4 km, a range of about 10,000 km, and a receiving ground station. The temperature structure is such that no trapped rays touch the ground, so the "received"

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*It might at first be thought that more complicated horizontal wind structure than that discussed in Section II (first sketch) would explain the complicated short-period pulse structure which is observed without resorting to a discussion of the vertical wind structure. However, the horizontal scale of the wind structure is very much greater than that of the vertical structure (see Ref. 9) and this is of less importance for the short-period pulse structure; and second, extreme vertical wind irregularities actually occur and will surely have a complicating effect on the received signal, as discussed in this Memorandum.

**Personal communication from T.F. Burke of The RAND Corporation.

rays arrive about 4 km above the ground, diffraction effects being assumed to provide a signal. (A slightly different atmosphere would have given rays intersecting the ground.) The first arrival occurs 547 min after burst. Later arrivals are given in Table 1.

Table 1

RAY CHARACTERISTICS FOR ICAO STANDARD ATMOSPHERE

HOB = 4 km, Range = 9800 km

Initial angle from horizontal (deg)	Relative arrival time (sec)
- 16.0	0
+ 16.3	3.88
- 13.8	105
+ 14.1	109
- 11.3	201
+ 11.7	205
- 8.38	290
+ 8.69	293
- 3.83	371
+ 4.13	373

The arrivals come in pairs consisting of about equal and opposite initial angles at the burst position, 4 km above the ground. An important point to notice, as Burke points out, is that the total spread in signal is little more than 6 min. However, short-period pulses are observed arriving throughout the duration (over an hour) of the intermediate-period waves. It is suggested that this added signal duration is due to vertical inhomogeneities, i.e., wind ducts. It seems possible that these ducts split the short-period pulses with fair probability. The ducts occur frequently enough that this pulse splitting occurs over and over again, generating many pulses after traveling a considerable range. It will be seen that the duration time for such a split signal is much greater than that for the rays for an ICAO atmosphere (6 min) as shown in Table 1.

Atmospheric patterns of vertical wind structure must first be considered in order to discuss this effect. Useful reviews of these characteristics have been published,(9-10) and some of the results are summarized in Figs. 3 through 5, taken from Ref. 9. Figure 3 shows that ducts stronger than 5 fps are predominantly due to wind. Furthermore, the typical wind duct has an intensity of 10 to 20 fps. Figure 4 shows that most ducts (0.8) are less than 10,000-ft thick, although about 0.1 have thicknesses between 10,000 and 20,000 ft. Frequency of occurrence with height is presented in Fig. 5, where it is seen that ducts less than 10,000-ft thick occur about 20 percent of the time (within a given 10,000-ft interval) at altitudes between 20 and 30 km. A similar result holds for thicker ducts, those between 10,000- and 20,000-ft thick. Ducts occur between 40 and 50 km about one-fourth to one-half as often as at their altitudes of maximum occurrence (between 20 and 30 km). There is some doubt about the validity of the temperature (vertical) duct characteristics shown in Fig. 5.

In order to examine the effect of these ducts on sound propagation, consider the following reflection problem (see sketch on p. 17).

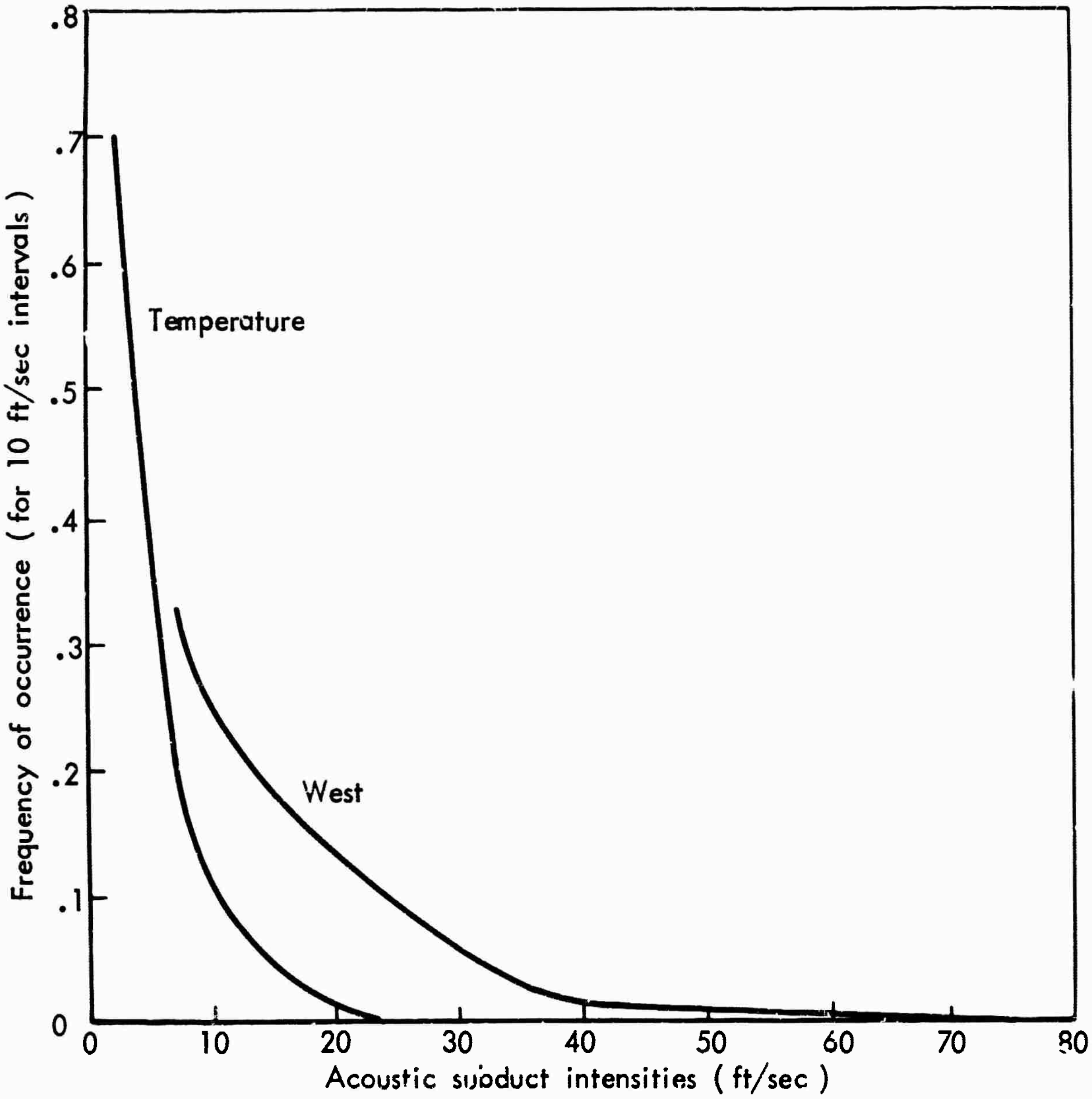


Fig. 3—Frequency of occurrence of subducts of various intensities for the vertical components (temperature) and the west component

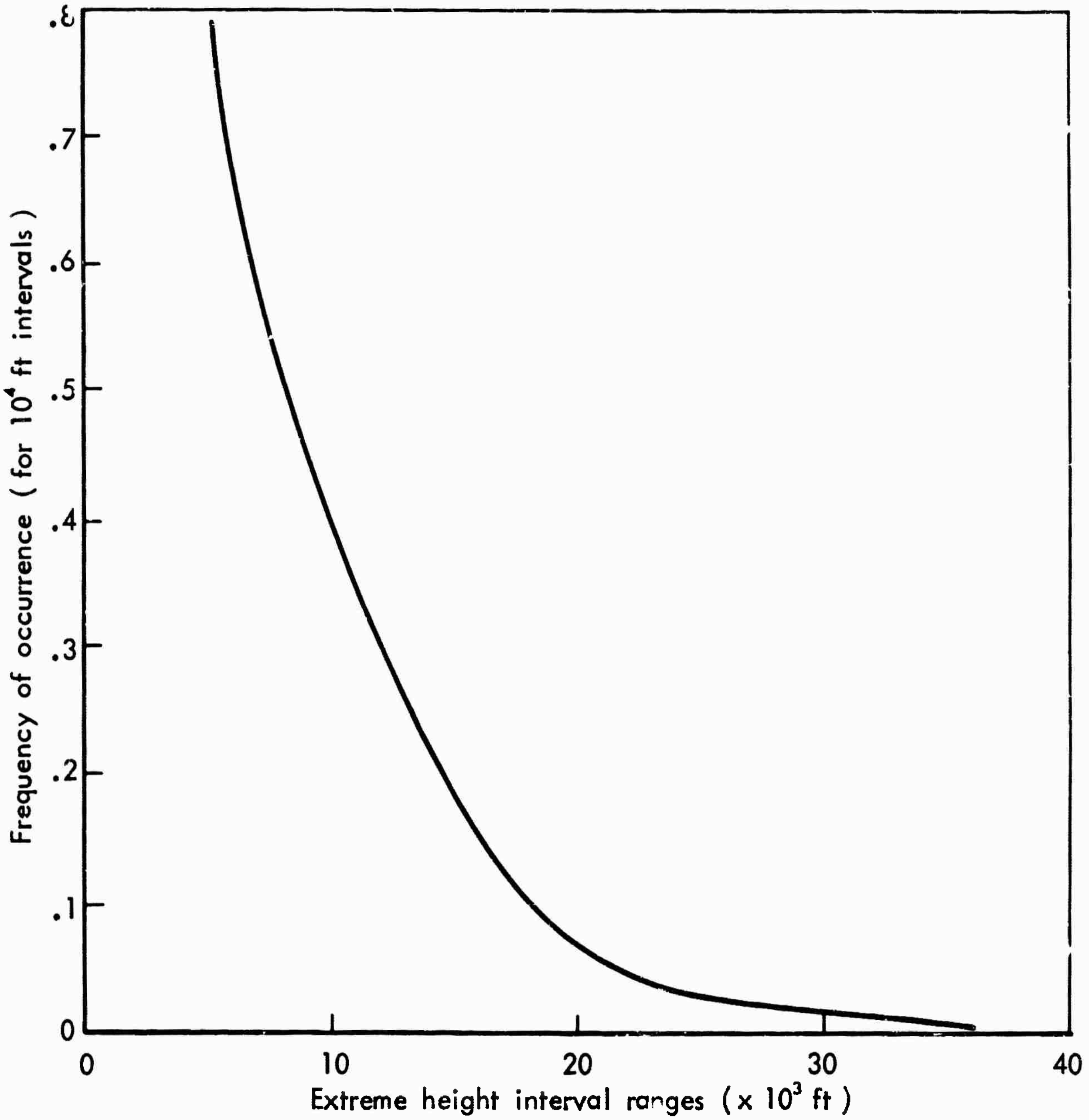


Fig.4—Frequency of occurrence of subducts of various thicknesses

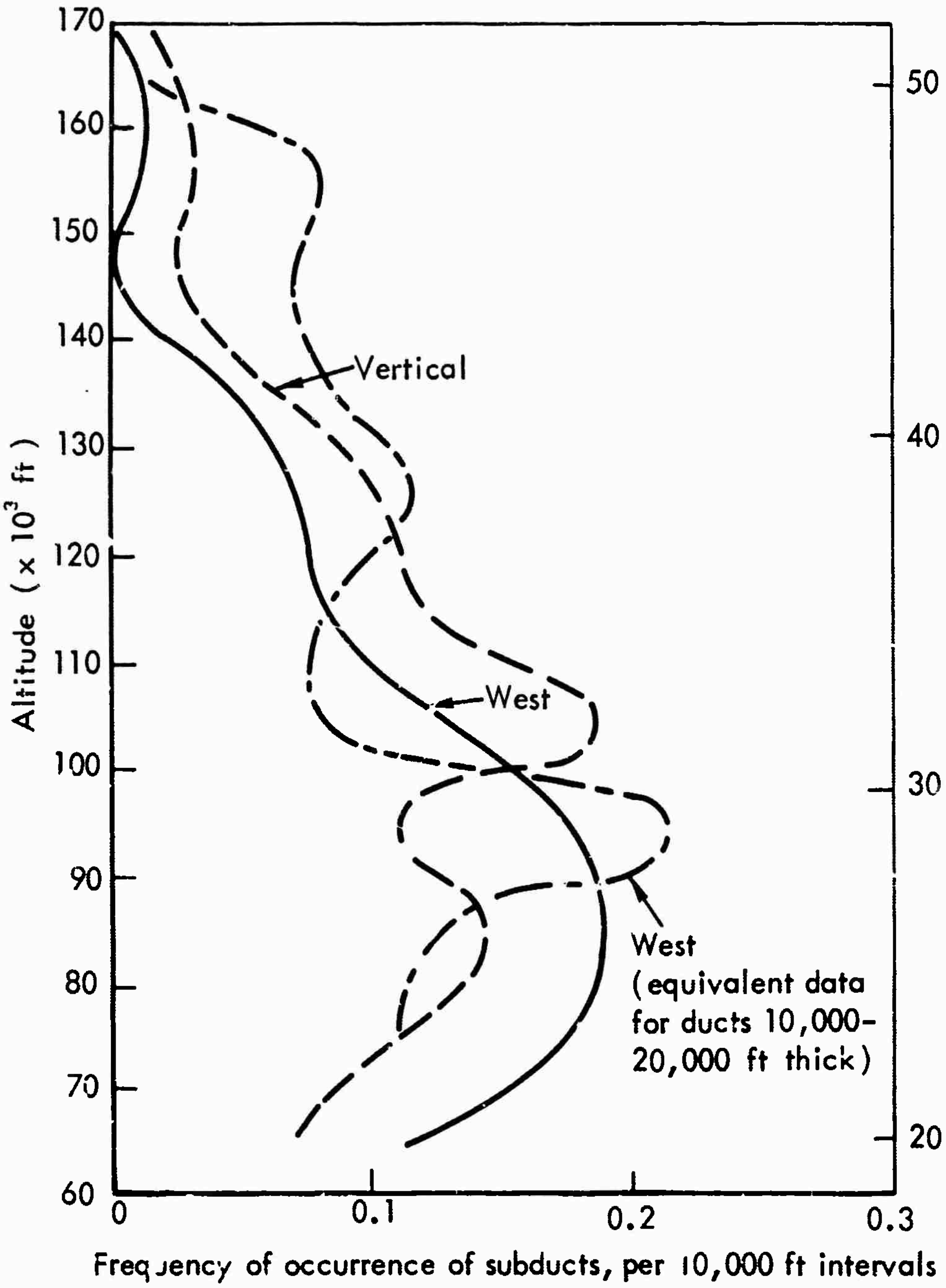
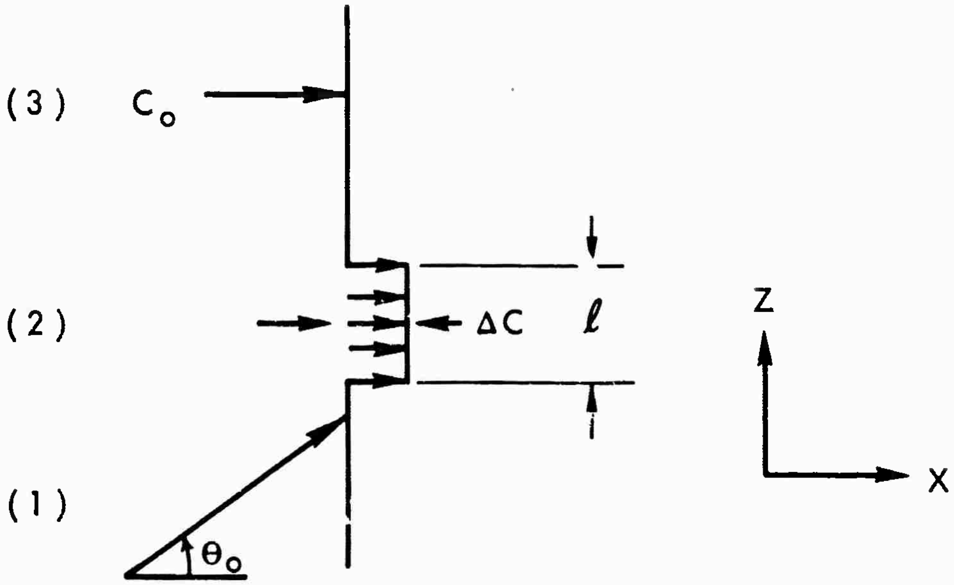


Fig.5—Frequency of occurrence with height of vertical and west component subducts less than 10,000-ft thick



A sound wave of unit pressure amplitude is incident, making angle θ_o with the horizontal, from the region (1) shown in the sketch upon the wind duct of thickness ℓ and strength Δc. It is assumed that the fluid density is constant and that the incident wave has angular frequency ω. The transmission coefficient into region (3) is

$$A_3 = 2\left\{2\cos\left(k_2\,\ell\,\sin\theta_2\right) + i\left[r_{12} + r_{12}^{-1}\right]\sin\left(k_2\,\ell\,\sin\theta_2\right)\right\}^{-1} \tag{5}$$

with

$$r_{12} \equiv k_o \sin\theta_o \Big/ \left(k_2 \sin\theta_2\right),\quad k_2 \equiv \omega\Big/\left(C_o + \Delta c\right),\quad k_o \equiv \omega/C_o \tag{6}$$

and

$$\frac{\cos\theta_2}{\cos\theta_o} = \frac{k_o}{k_2} \tag{7}$$

restricted to the small-angle situation of greatest interest,

($\theta_o \ll 1$). The critical angle (waves incident with angles less than this totally reflected if ℓ is large) is given by

$$\theta_c \simeq \sqrt{2\Delta c/C_o} \tag{8}$$

For $\theta_o \sim \theta_c$

$$\text{Fraction of energy reflected} \sim \left(k_o\ \ell\right)^2 \tag{9}$$

For incident angles somewhat greater than the critical angle, almost all of the energy is transmitted; for incident angles somewhat less than the critical angle, almost all of the energy is reflected.

Consider a typical case:

frequency = 0.05 sec^{-1}
C_o = 300 mps
ℓ = 3 km
Δc = 5 mps

$$\text{Fraction of energy reflected} \sim 0.3 \tag{10}$$

The critical angle is

$$\theta_o \cong 0.2 \text{ rad} \approx 10^{\circ} \tag{11}$$

For periods within a factor of two or three of 20 sec, the pulse is split and a fraction of the energy is transmitted, with the remainder of the energy being reflected. This is so for the duct of strength $\Delta c \approx 5$ mps (a value typically found) and thickness 3 km. Consider the probability of occurrence of such a duct. From Fig. 5, it is seen that a duct of thickness zero to about 3 km has a probability of occurrence of about 0.1 per 10,000 ft in the altitude range of 30 to 50 km. (In order to find incident angles of 5 to 10 deg, the upper portion of the ray path must be examined, i.e., in the 30- to 50-km range.) Using these figures, the probability of finding such a duct in the given altitude range is about 0.6. In Fig. 5, it is seen that the probability of finding a duct of thickness 3 to 6 km is about the same. Such thicker ducts are seen from Eq. (9) to split periods twice as long as the 20-sec period of the example. It is concluded that the portion of the pulse with period near 20 sec will usually be split once during each vertical transit through the atmosphere. Thus, in the ray example treated in Table 1, after 30 skip distances, thousands of partial pulses might be expected with periods of the order of 20 sec. Even after allowing for some overestimation, it is evident that the commonly occurring wind ducts can greatly complicate the short-period portions of the sound pulse. From Eq. (9) and the example in Table 1, it is seen that intermediate periods are relatively unaffected by these wind ducts.

Consider now the time duration of the pulse splitting. From Table 1, for instance, it is seen that the first arrival has an average speed of 299 mps; later arrivals are slower by up to one percent. Consider the extreme case of a ray reflected from a wind duct located at the atmospheric sound-velocity minimum, about 14 km, each time it arrives at this altitude (see Fig. 1). Its average speed will be about the average of the sound speed from the ground to 14 km; this is approximately 315 mps. Thus, such a multiple-reflected ray will be speeded up by about 5 percent. At the range of 10,000 km used in Table 1, this gives an extreme signal duration of about half an hour. Thus, in this case the short-period signal duration is about the same as the duration due to dispersion of the intermediate periods, as described in Section II. This short-period duration is in accord with the observations,(6) when the gross weather front refractions described in Section II are considered. (They contribute several overlapping records of the type discussed in this section.)

For shorter periods, the upper-atmospheric wind structure is at least important as the temperature structure in determining propagation characteristics.

IV. CONCLUSIONS

It has been noted that one of the outstanding characteristics of the signal is its great duration. For intermediate periods, the signal lasts longer than simple dispersion would predict, at least for the fundamental mode. It is believed that this is because the received signal is comprised of refractions from large-scale weather fronts (due to horizontal variation in either the temperature or wind structure or both). These refractions are shown to be probable on the basis of known meteorology; the extra length of path traveled can easily account for the observed signal delay. In addition to the duration of the intermediate-period signal, it is observed that short-period signals also have an extended duration, much greater than the few minutes which would be expected from simple ray acoustics. It is felt that this signal delay is due to pulse splitting brought about by the commonly occurring wind ducts. From meteorological measurements, it is shown that for a range of 10,000 km, part of the pulse, with period of about 10 sec, could be split dozens of times. Experimentally determined data, in fact, show the occurrence of a great many such short-period pulses.

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DOCUMENT CONTROL DATA

1 ORIGINATING ACTIVITY

THE RAND CORPORATION

2a. REPORT SECURITY CLASSIFICATION

UNCLASSIFIED

2b. GROUP

3. REPORT TITLE

SHORT-PERIOD PROPAGATION OF INFRASONIC WAVES FROM NUCLEAR EXPLOSIONS

4. AUTHOR(S) (Last name, first name, initial)

Meecham, W. C.

5. REPORT DATE October 1966	6a. TOTAL No. OF PAGES 33	6b. No. OF REFS. 10
7. CONTRACT OR GRANT No. SD-79	8. ORIGINATOR'S REPORT No. RM-5103-ARPA	

9a. AVAILABILITY/LIMITATION NOTICES

DDC 1

9b. SPONSORING AGENCY

Advanced Research Projects Agency

10. ABSTRACT

A possible explanation is given for the great time duration of intermediate-period (about one minute) and short-period (less than one minute) acoustic-gravity waves received from nuclear explosions. It is suggested that the signal delay for intermediate periods may be due to refraction from large-scale weather fronts, and that the signal delay for short periods may be caused by commonly occurring wind ducts.

11. KEY WORDS

Wave propagation
Signals
Nuclear radiation
Detection
Nuclear blasts
Weather