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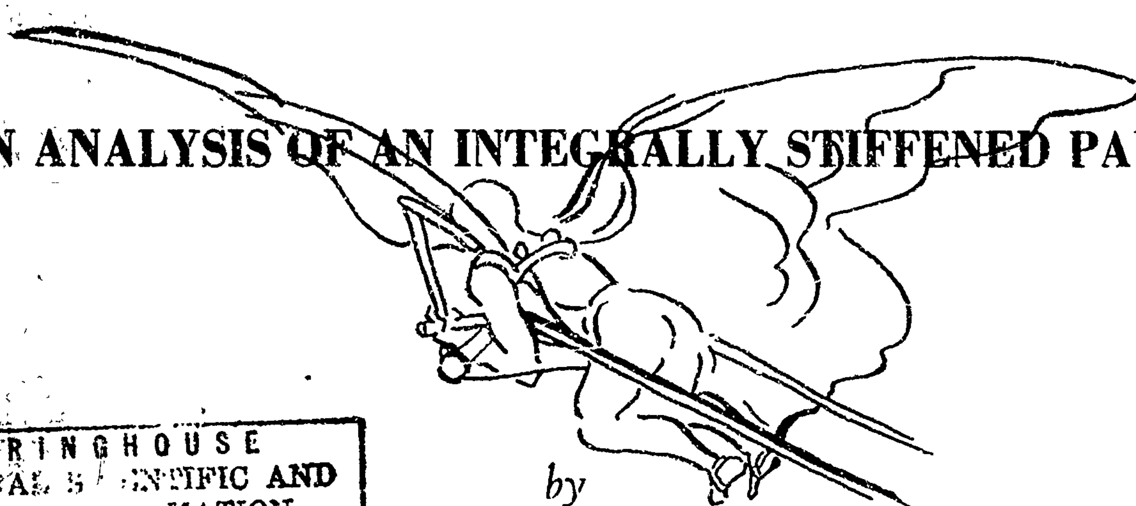
AN ANALYSIS OF AN INTEGRALLY STIFFENED PANEL



by

Everett L. Cook

Department of Aeronautical Engineering
School of Engineering
Wichita State University
October 1966

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Aeronautical Report 66-6

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INTRODUCTION

Matrix methods are used extensively in the analysis and design of aircraft structures. Although methods have been developed for both isothermal and thermal analyses, experimental verification of these methods for thin-wall structures is not available. The purpose of the investigation described in this report was:

1. To design and test on integrally stiffened rectangular shear panel (Figure 1),
2. To develop analytical procedures for shear panels subjected to thermo mechanical loading, and
3. To compare the analytical and experimental results to evaluate the accuracy of the analysis methods.

The panel has been designed and tested under mechanical loading. There is good agreement between the analytical and experimental results. It is also shown that any of the matrix methods of structural analysis investigated have the same order of accuracy when they are applied to the same mathematical model.

The body of the report presents a summary description of the methods of analysis which were investigated, the design of the test panel, the experimental program, and the analytical and experimental results. The details of the analysis methods and the digital computer programs developed to implement the use of these methods on the test panel are given in the appendices.

The thermal investigation has not been completed. The progress on the various phases of this investigation is presented in the appropriate sections of the body of the report. Appendix D presents the significant analytical results to date. This investigation is being continued as a thesis project by Mr. R.G. Merritt.

the analytical results which are compared with the experimental data. This program does not at present include provisions for a thermal analysis.

Both the force analysis and the displacement were performed with the Force/Displacement Program described in Appendix B. It is also shown in this appendix that not only do the force and displacement methods yield essentially identical results, but these results agree with those given by the direct stiffness method. This excellent agreement is, of course, directly related to the fact that identical mathematical models of the test panel were used for all three analyses. This means that the accuracy of the results of a matrix structural analysis depend only on the accuracy of the mathematical model and not on the method of analysis. In fact, Przemieniecki (Reference 6) suggests that if the substructure method is used, different parts of a structure may best be analyzed using different methods. A force method for calculating the displacements and internal forces due to initial strains is given in Appendix C. This will be expanded to include the displacement method for the thermal analysis.

TEST PANEL

The test panel is of monolithic construction and is symmetrical with respect to the plane of the web. Monolithic construction was selected because it eliminates both joint friction and joint thermal resistance; neither of which can be accurately accounted for in the analyses. Although most actual thin-wall structures are not symmetrical about the web plane, the analysis methods assume such symmetry. Therefore, this construction is ideally suited for obtaining experimental data for comparison with the analytical results.

The geometry of the test panel was dictated primarily by the results on the heat transfer study. Mr. D.E. Hull conducted this study and presented a paper on the results as a partial requirement for his undergraduate degree. Mr. Hull's report is given in Appendix D of this report. The purpose of this study was to obtain a geometry that would have nearly constant temperatures over the area of each stringer and over the length of each panel and, at the same time, a relatively large difference in temperature between the outboard stringer and the centerline of the panel. The geometry selected satisfies these requirements.

The only geometric considerations which influenced the design for mechanical loading were the overall length of the panel and the lug design. The panel length was limited to approximately twenty-four inches by the open height of the testing machine. The lugs were designed for a limit load of 18,000 pounds. However, the applied loads were limited to approximately 7,500 pounds, giving a factor of safety of 2.4.

The material selected for the test panel was 6061-T6 aluminum alloy plate. Aluminum alloy tool and jig plate was investigated and found to have not only low physical properties, but also a nonlinear stress-strain curve.

The panel was machined in the Wichita State University Engineering Machine Shop. The largest available mill was not large enough to allow milling one entire side without moving the panel. It was also necessary to fill the panels with water during the final cuts on the web to prevent thermal buckling. However, the desired thickness of the web was maintained within a tolerance of plus 0.0015- and minus 0.0005-inches, and the overall thickness was within plus 0.001- and minus 0.000-inches of the values specified in Figure 1.

TEST FIXTURES

The mechanical loading was performed in a 160,000 pound Tinus-Olsen Universal Testing Machine. The test panel is shown mounted in this machine in Figure 2. During the preliminary tests, it was discovered that a constant load could not be maintained with the loading system of the testing machine. It was then necessary to design a seperate loading system for the final tests. Two Blackhawk hydraulic cylinders with a total capacity of 28,000 pounds were mounted on the upper crosshead of the testing machine (Figure 3). A linkage was designed to transmit the load from the hydraulic cylinders, through the opening in the crosshead, to the whiffle-tree system. The load was applied and maintained with a hand-operated hydraulic pump.

The whiffle-tree systems are both shown in Figure 2 and the upper system is also shown in Figure 3. The upper system was designed to apply equal loads at all four of the lugs along the top of the panel. Its compact design was necessary because of the limited vertical clearance of the testing machine. Loads are applied only at the inboard stringers along the bottom of the panel.

The thermal tests will not be performed in the testing machine, but in a special fixture being fabricated for these tests. Two Conrow SNH strip heaters will be used to radiate heat to the edges of the outboard stringers. All other stringer and rib faces will be insulated. The web panels will be cooled by forced convection to produce the desired temperature distribution (Appendix D).

STRAIN GAGES

The strain gages used on the panel are Budd medium temperature (MT) foil strain gages. The axial gages on the stringers are Type C12-124-A and the rosettes are Type C12-124-R3C. These gages have a one-eight inch gage length and are recommended for temperatures up to 400°F for static loading conditions. The strain gage mounting pattern is shown in Figure 1.

The cement finally selected for bounding the strain gages was Bean BAP-1. Budd GA-50 was used for the gages installed prior to the preliminary tests. A previous experience had convinced the author that both the strain gages and the cement should be purchased from the same manufacturer. It was discovered subsequently to the preliminary tests that the cured cement contained microscopic bubbles. The manufacturer's sales representative was consulted and modifications to the installation procedure were made. When these modifications did not eliminate the bubbles, a new batch of cement was obtained. The results were the same. A technical representative of the Budd Company was finally consulted and it was learned that the GA-50 cement had been discontinued. The decision was then made to use the BAP-1 cement.

INSTRUMENTATION

A Budd-Datran 20-channel Automatic Strain Indicating System was originally proposed for strain gage data collection. A Clary digital printer owned by the University was to be used as the output device for the system. However, this printer could not be used without major modifications. This would have increased the total cost of the system to an amount well in excess of the budgeted amount. As a result, the decision was made to go to a manual system.

The manual system selected consisted of 2 Budd Model P-350 Portable Strain Indicators and 13 Budd Model SB-1 10-channel Switch and Balance Units. This system was purchased and was used in the preliminary tests. Subsequently, Mr. D.E. Hull and Mr. Marvin Davidson, our wind tunnel director, modified one of the P-350 Strain Indicators so that it could be coupled, through a Dayton Instruments Model DI-6-54 digital converter, to an IBM 526 Printing Summary Punch. Two views of the system are shown in Figures 4 and 5. Manual switching and balancing is still necessary; however, the strain indicator readings are punched into cards-ten per card.

Each of the switch and balance units is connected to nine gage elements--the nine axial gages on one side of a stringer or three vertically aligned rosettes on a web panel. Due to the large number of gages, the load must be applied and released four times for a complete set of gage readings. To facilitate rapid and accurate switching between runs, each switch and balance is permanently wired to an Amphenol No. 26-190-32 connector and each set of nine gage elements is wired to a mating No. 26-159-32 connector.

It was originally proposed that the temperatures would be measured with standard thermocouples bonded to the test panel. The possibility of measuring the temperatures with a surface gage are now being investigated in order to avoid adding additional wires to the panel.

EXPERIMENTAL RESULTS

The mechanical loads were applied as shown in Figure 2. Four equal loads were applied at the nodes at the top of the panel. These were reacted by two equal loads at the inboard stringers at the bottom of the panel. The loads were applied in increments of 3,000-4,000 pounds up to a maximum load of approximately 15,000 pounds. The load was then decreased using approximately the same increments. Due to the difficulty in applying the load uniformly with the hand pump, no attempt was made to use precise increments; however, it was possible to maintain a constant load within plus or minus 100 pounds while a set of nine strain gages was being balanced and recorded.

The experimental results are shown in Figures 6-8, along with the analytical results of the direct stiffness analysis (Appendix A). The experimental points in these figures represent the strains in the vertical direction-- ε_{yy} in Tables A-3 through A-5. Strains are plotted, rather than stresses for two reasons:

1. The stresses at the location of a strain rosette cannot be calculated if any of the elements in the rosette are inoperative.

2. Since strain readings normally have only two or three significant figures, while at least eight significant figures are carried in the analysis, it is more accurate to manipulate the analytical results.

A large number of the strain gages were damaged beyond repair during the installation of the lead wires. The stringer gages were particularly easy to damage although brackets were used to support the panel while the lead wires were being installed. As a result the number of experimental points in the figures is fewer than is desirable. The solid circles represent the average strain for back-to-back gages, while the open cirles represent the strain at points where one of the back-to-back gages was inoperative.

Each of the plots in Figures 6-8 represents the strains along a horizontal line of strain gages. The locations are specified in the sketches on the figures. The ordinates in the figures are the microstrain per pound of load applied to each inboard stringer. The analytical results (Tables A-2 through A-5) are tabulated in essentially this form. The experimental results were reduced to this form by using the slope of a least-squares straight line through the data.

There is generally good agreement between the analytical and experimental results. The only explainable differences are those for the line of gages nearest the bottom of the panel. It appears that the loads are transfered from the stringers to the web more rapidly than the analytical solution can predict with the subelement size used here. It would be better, therefore, to have smaller subelements in this region. This is not possible with the present computer configuration.

CONCLUSIONS

The following conclusions can be drawn from the results of this investigation:

1. The mathematical model of the test panel used for the direct stiffness analysis was sufficiently accurate to provide good agreement between the analytical and experimental results.

2. The force and displacement methods could have been used with equally accurate results.

3. The derivation of the recurrence relationship for initial strains extends the recurrence method of Pestel and Leckie to include thermal loading.

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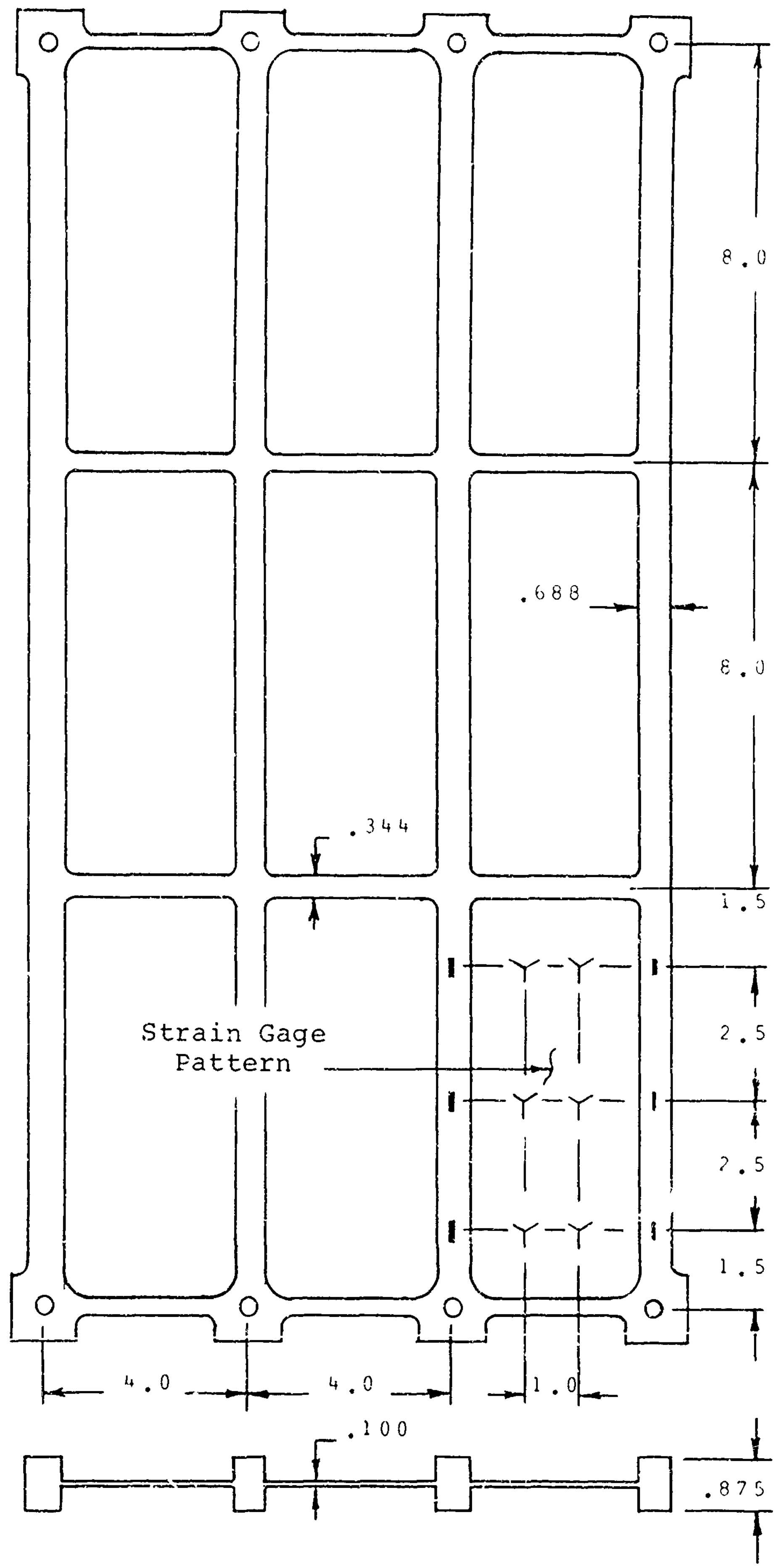


Figure 1. Test Panel Geometry.

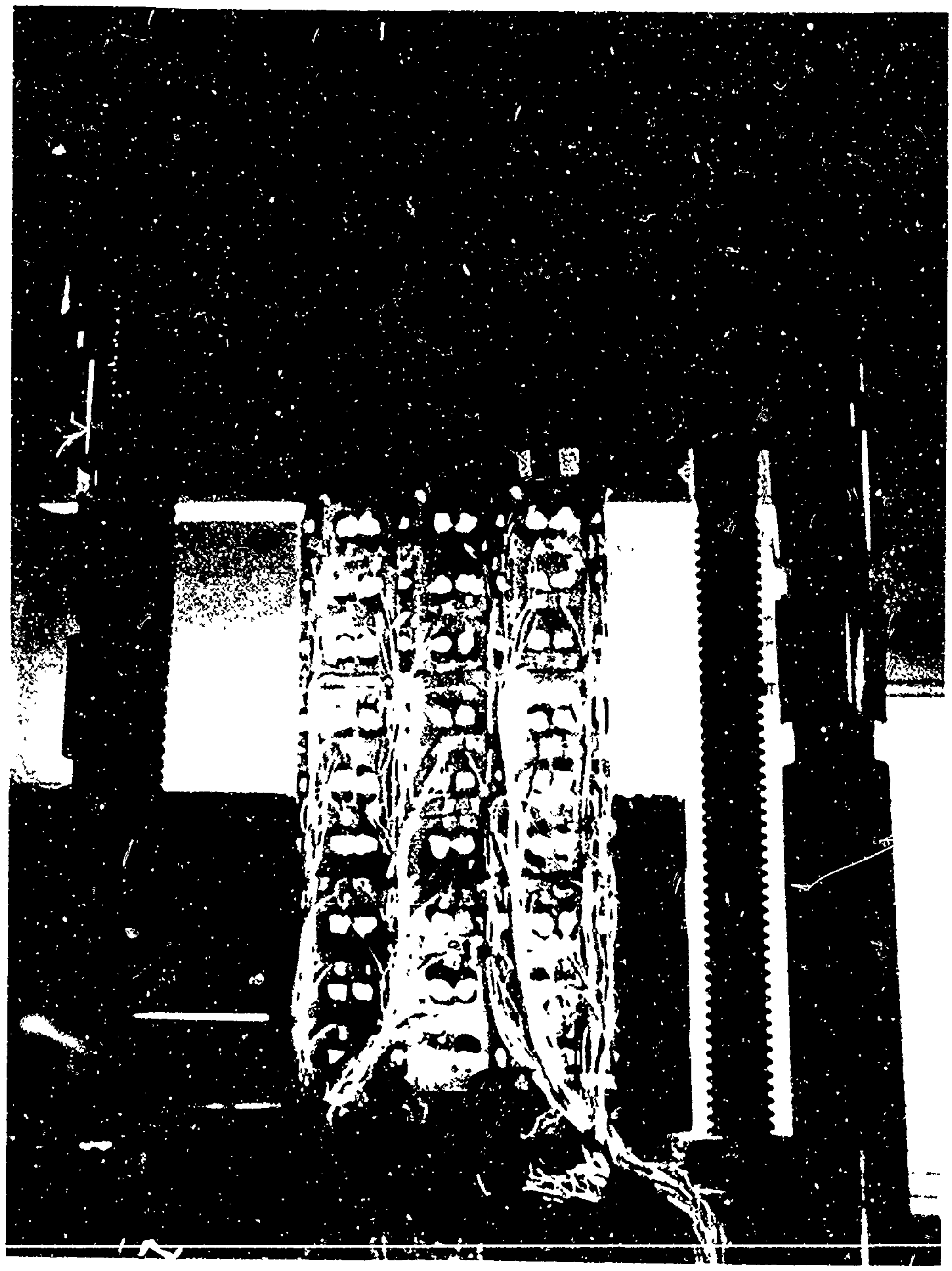


Figure 2. Test Panel Mounted in the Testing Machine

Figure 3. Loading System and Upper Whiffle-Tree

Figure 4. Strain Gage Data Collection System

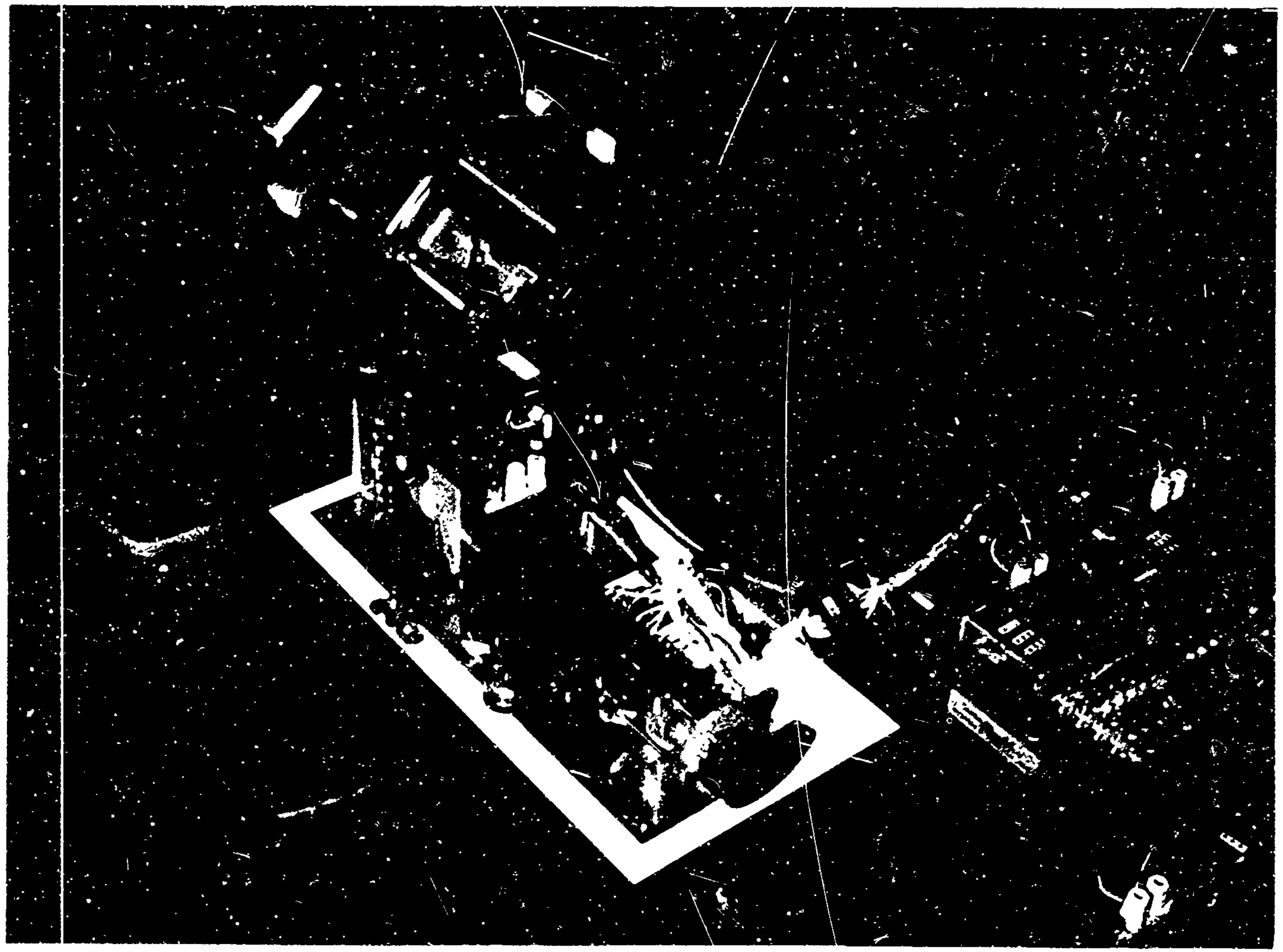


Figure 5. Modified Strain Indicator and Card Punch

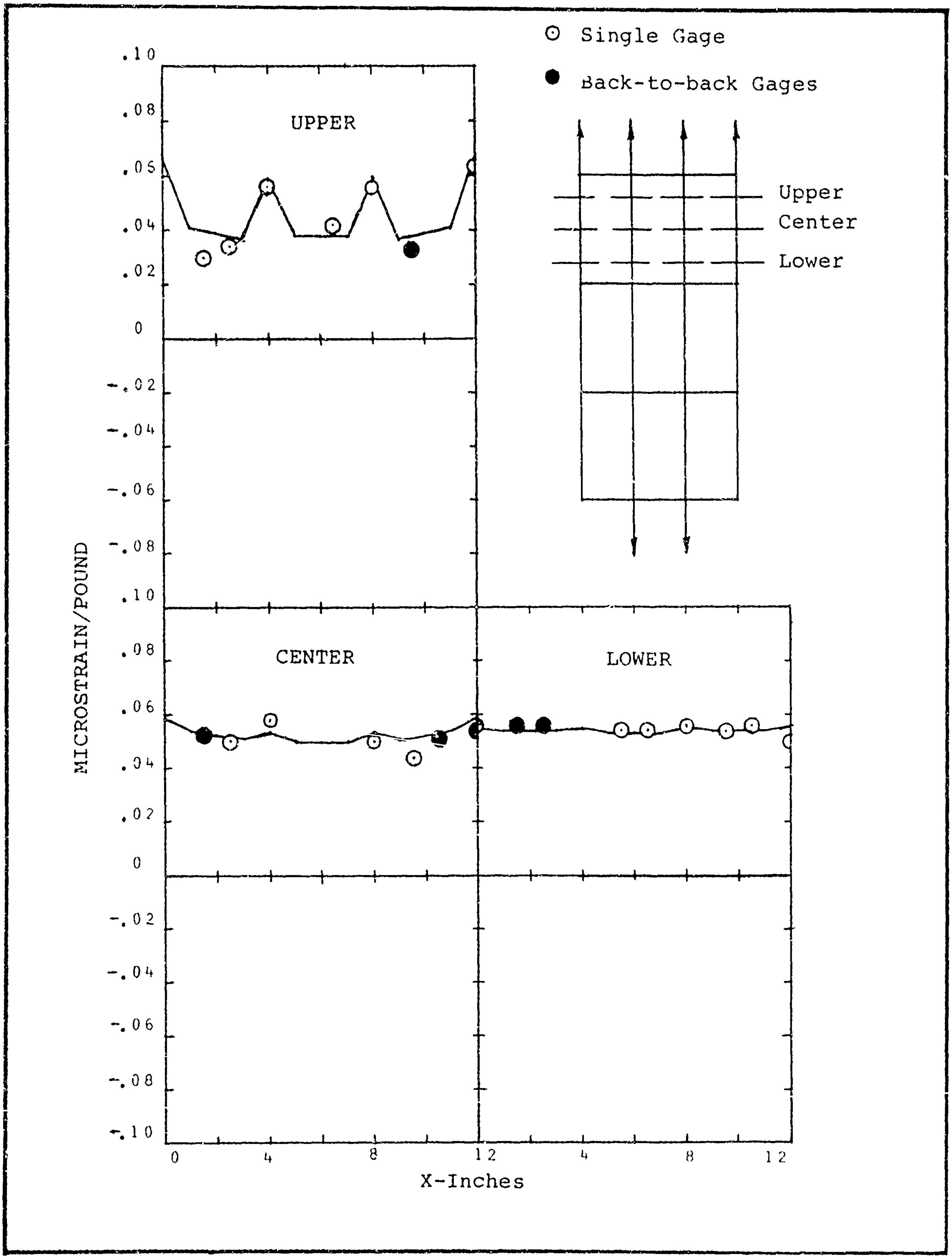


Figure 6. Analytical and Experimental Strains-Upper Panels

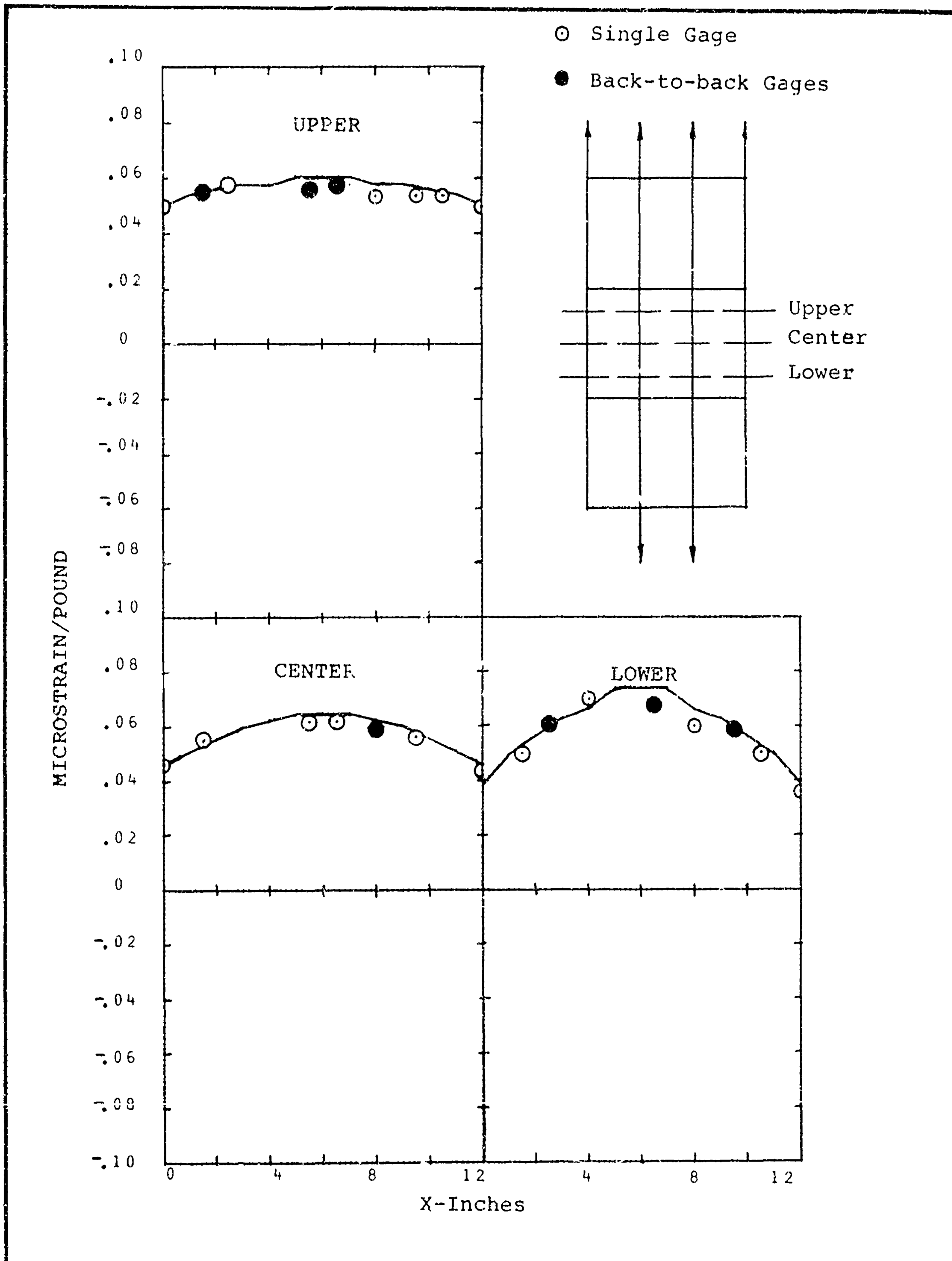


Figure 7. Analytical and Experimental Strains-Center Panels

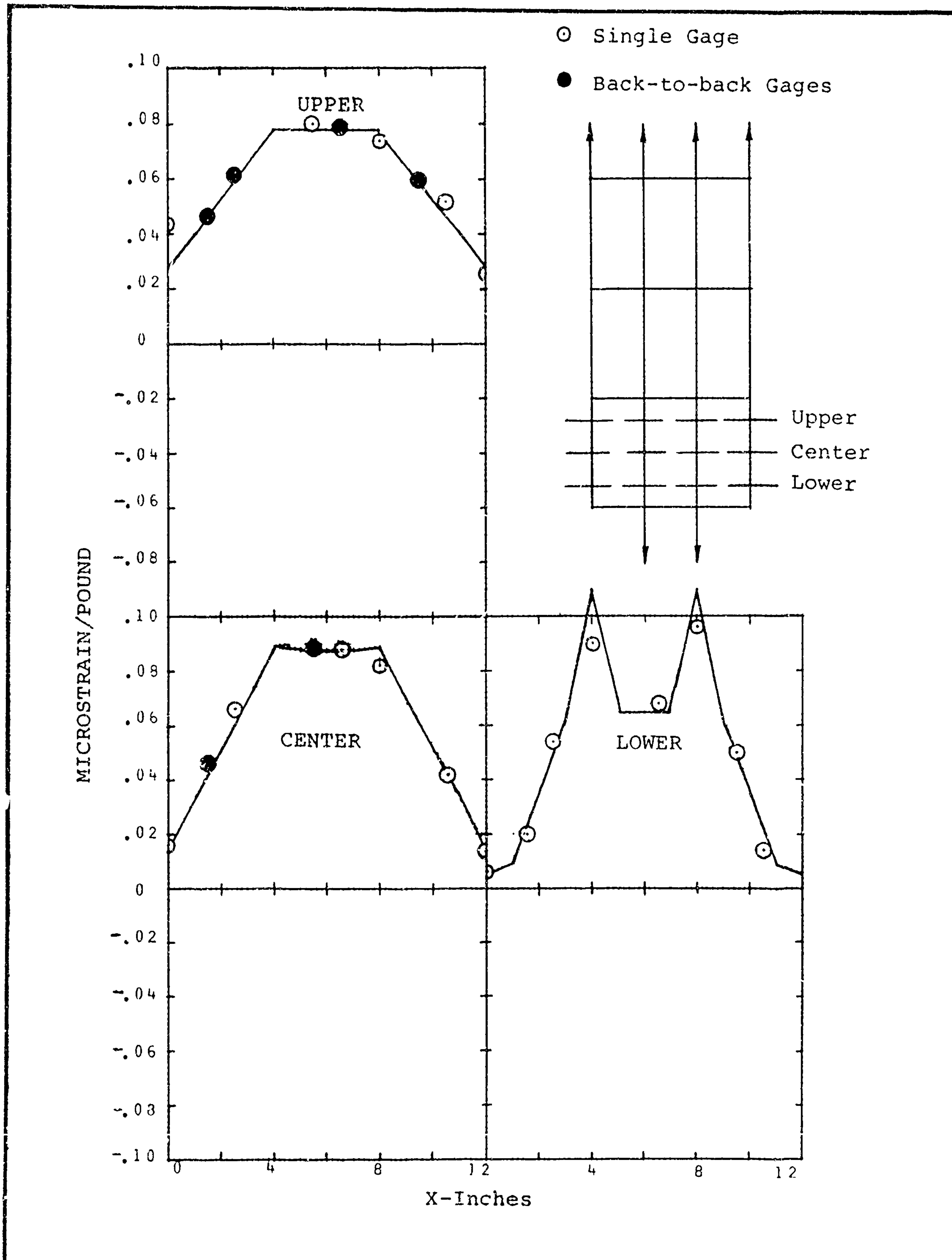


Figure 8. Analytical and Experimental Strains-Lower Panels

APPENDIX A

THE DIRECT STIFFNESS ANALYSIS PROGRAMS

The Direct Stiffness Analysis Programs, arbitrarily named DORA, were developed to analyze the test panel using the method presented by Turner, Clough, Martin, and Topp in Reference 7. There are four versions of the program; however, only the version (DORA IV) used to obtain the results to be compared with the force/displacement results (Appendix B) and the experimental data will be described in detail. The only inputs required are the geometry and physical properties of the structure and the external loads and constraints on the structure. The outputs of the program are the displacements of the nodes. An auxiliary program was written to calculate the strains in the stiffeners and the webs from the nodal displacements.

The direct stiffness method of Turner, Clough, Martin, and Topp is the simplest of all the methods of matrix structural analysis. It is based on the following:

1. The force-displacement relationship for each element in a structure can be expressed as

$$\{F\} = [K]\{u\} \tag{A-1}$$

where

$\{F\}$ = the column matrix of the nodal forces on the element. All nodal forces are included. In the displacement analysis of Appendix B, only a subset of independent forces are considered.

$\{u\}$ = the absolute nodal displacements of the element. In the displacement analysis only the displacements relative to a set of reference axes in the element are considered.

$[K]$ = the element stiffness matrix. This matrix, sometimes called the unreduced stiffness matrix, is singular. The reduced stiffness matrix used in a displacement analysis is derived from the unreduced stiffness matrix by the elimination of specified rows and columns.

2. The force-displacement relationship for each element in a structure, Equation (A-1), can be expanded to the order of the assembled structure stiffness matrix by adding rows and columns of zeros for the nodal displacements that are irrelevant for the element in question. These expanded element stiffness matrices can then be added to obtain the stiffness matrix of the assembled structure. The resulting force-displacement relationship is of exactly the same form as Equation (A-1), but the force vector represents the external forces at the nodes. Martin (Reference 4, Section 2.6) has an excellent discussion of the assembling of the stiffness matrix.

The method of expanding each element stiffness matrix to the order the structure stiffness matrix is not practical for machine computation. Even if only one element was considered at a time, the storage required would be twice that required for the structure stiffness matrix. It is relatively easy, however, to determine the position in the structure stiffness matrix for each element in the element stiffness matrix. Thus, when an element stiffness matrix has been obtained, its elements can be added into the proper position in the structure stiffness matrix. This latter method is used in all versions of DORA.

The stiffness matrix of the assembled structure is singular. It must be made nonsingular so that an inversion may be performed to obtain the displacements in terms of the external forces, i.e.,

$$\{u\} = [C]\{F\} \tag{A-2}$$

where $[C]$ = the flexibility matrix of the structure.

There are two practical procedures for obtaining the flexibility matrix:

1. The rows and columns of the stiffness matrix corresponding to the external constraints on the

structure can be eliminated. This nonsingular reduced matrix can then be inverted. Finally, the flexibility matrix can be obtained by expanding back to the original order and inserting zeros in the rows and columns corresponding to the constraints.

2. All but the diagonal elements in the rows and columns corresponding to the constraints can be set to zero and the diagonal elements can be set to unity. This matrix is nonsingular and can be inverted. Then the ones on the diagonal corresponding to the constraints can be set to zero.

The second procedure is used in DORA primarily because it is easier to program for arbitrarily specified constraints.

Once the flexibility matrix has been determined, the nodal displacements may be calculated from Equation (A-2). These displacements may then be substituted into the element force-displacement relationship, Equation (A-1), to obtain the nodal forces.

Several versions of the program were necessary because of the limited size of the available digital computer. An IBM 1620 with 40,000 positions of core storage and card input-output was used (all work prior to January, 1966, was done on the same machine with only 20,000 positions of core storage). The programs are all written in PDQ Fortran. This programming system was selected because it requires less core storage for the subroutines than do any of the IBM Fortran systems. It is also faster than the IBM Fortrans, but this is a secondary consideration. In the near future, a 1311 disk drive will be added to the computing system. The entire set of programs can then be combined and enlarged.

As mentioned above only DORA IV will be described in detail, however, the other versions can be summarized as follows:

DORA I

This program was written to demonstrate the feasibility of generating the assembled structure stiffness matrix as outlined above. Only structures with bar and/or beam elements can be analyzed with this program. These elements were chosen because there are many truss, continuous beam, and frame examples in the literature which provided results for checking and debugging the program. This program definitely proved the feasibility of the method. However, in order to make it a one-part program, the structure is limited to fourteen nodes.

DORA II

The program was broken down into parts and triangular and rectangular plate elements were added. The maximum number of nodes is limited to sixteen.

DORA III

Trapezoidal plate elements were added to the program and the beam elements were eliminated. The elimination of the beam elements reduced the number of degrees of freedom at each node from three to two. However, the part of the program which generated the flexibilities of the trapezoidal plate elements was very large, so that the maximum number of nodes is limited to twenty.

DORA IV

The trapezoidal plate element was eliminated and the maximum number of nodes was increased to twenty-five. This is large enough for the analysis of the test panel. Since this is the version used for the analysis of the test panel, it will be described in more detail. The program is divided into six parts--Initialization, Bars, Triangular Plates, Rectangular Plates, and Matrix Inversion and Output. A program listing is given in Table A-1. The purpose and use of each part is as follows:

Initialization. The number of nodes, Young's Modulus, Poisson's Ratio, and the x- and y-coordinates of the nodes are read and stored. They are also repunched, with appropriate headings, for identification and debugging purposes. The size of the structure stiffness matrix is calculated and the elements of this matrix are set to zero.

Bars. The data cards for all of the bar elements in the structure are the input to this part of the program. If the structure does not contain bar elements, this part of the program is not loaded. Each bar is described on one card. The numbers of the two nodes which the bar connects, the area of the bar, and a control code must be specified. The control code for each card except the last is a zero. A one in the last card causes the next program part to be read after the last element has been processed.

The bar stiffness matrix was taken from Reference 7. The order has been changed so that the x- and y-displacements of the first node preceed those of the second node. There are no restrictions on the numerical order of the nodes, i.e., the number of the first node may be higher or lower than the number of the second node.

Triangular Plates. The input to the triangular plate program consists of cards containing the three node numbers, the thickness, and the control code (See "Bars"). This stiffness matrix was also taken from Reference 7. Rearrangement was not necessary. The nodes may be specified in any order.

Rectangular Plates. The numbers of the four nodes, the thickness, and the control code (See "Bars") must be specified for each rectangular plate. The order of the nodes is important! The required order is lower left, lower right, upper left, and upper right.

The rectangular plate stiffness matrix was taken from Reference 1, P. 50. The shear and direct stress stiffness matrices were combined and rearranged in the node order specified above. Argyris' stiffness matrix was chosen over the one given in Reference 7 because it is compatible with the trapezoidal plate stiffness matrix used in DORA III. Belirgen (Reference 2) has shown that either of these stiffness matrices gives satisfactory results for the type of structure being considered.

Matrix Inverse and Output. The primary purpose of this part of the program is to invert the stiffness matrix of the assembled structure. However, this matrix must first be made nonsingular. The constraints are specified with a data card for each constrained node. The data cards contain the node number; the x- and y-constraint codes, and the control code (See "Bars"). The constraint code is 1.0 for a constrained displacement and 0.0 for an unconstrained displacement. After each card is read, the row(s) and column(s) in the stiffness method corresponding to the constrained displacement(s) are modified by the method outlined previously. When all constraints have been imposed, the stiffness matrix is inverted to obtain the flexibility matrix.

The loads on the structure are specified similarly to the constraints. A card is read for each node with at least one external load. The card contains the node number, the x- and y-loads, and the control code (See "Bars"). Both loads must be specified although one of them may be zero. The loads are placed in the load vector [F]; and after the last card is read, the displacements are calculated using Equation (A-2). The displacements are then punched into cards. The x- and y-displacements (u and v) of each node, including the constrained nodes, are punched.

The stiffness matrix is replaced by the flexibility matrix during the inversion; therefore, the stiffness matrix must be reassembled in order to study the effects of different external constraints on a structure. This will not be necessary when the disk drive becomes available since the stiffness matrix may be stored on the disk and recalled when needed. The effects of any number of loading conditions can be studied by generating a new load vector, i.e., by reading in a new set of load cards.

Examples

Four panel configurations were analyzed using DORA IV (Figure A-1). Since the test panel was loaded symmetrically, only one half of the panel is analyzed. The roller constraints along the centerline of the panel maintain the symmetry of lateral displacement, but allow longitudinal displacements. The constraint on the lower edge represents the load applied with the lower whiffle tree.

Panel Configuration I is also analyzed by the force and displacement methods and the results are compared (Appendix B). The results are given in Table A-2. The other three configurations could not be analyzed with the Force/Displacement Program due to the large number of internal forces and deformations.

Panel Configurations II-IV were analyzed to obtain stiffener and web strains for comparison with the experimental data. Due to the size limitations of the computer, it was not possible to use one configuration with small enough subelements to provide strains which would compare favorably with the experimental data. Therefore, Configurations II, III, and IV were used to obtain strains in the upper, center, and lower panels, respectively. The vertical dimensions for the smaller subelements (Nodes 1-16) were selected so that the line of strain gages was along the horizontal centerline of the elements. The results are shown in Tables A-3 through A-5.

The present versions of DORA do not have provisions for calculating stresses or strains. A special program was written to calculate the axial strains in the stiffener elements and the direct and shear strains at the center of the web elements. A listing of this program is given in Table A-6.

The method used here for obtaining the strains for comparison with the experimental data yields satisfactory results and could undoubtedly be used successfully with most other types of structures. However, a substructure method, such as the one presented in Reference 6, would certainly yield more accurate results. The capability has not, as yet, been incorporated into DORA.

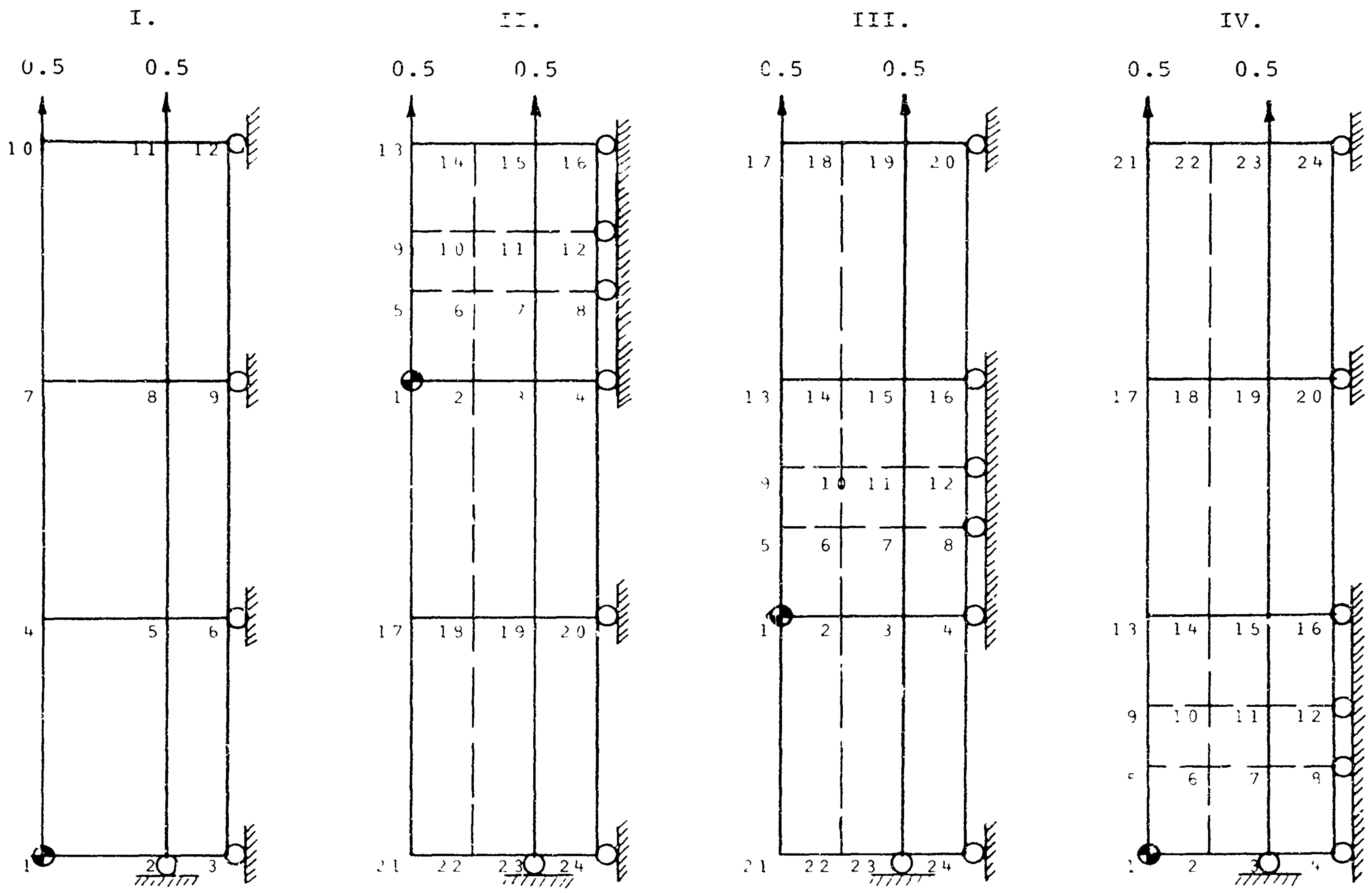


Figure A-1 Panel Configurations for Direct Stiffness Analysis

TABLE A-1

DORA PROGRAM LISTING

```
C  C
C     DORA IV - BARS AND PLATES (EXCEPT TRAPEZOIDAL)
C
C     INITIALIZATION - 25 NODES - 2 DEGREES OF FREEDOM
C  ************************************************
C
C     E.L. COOK - MAY 1966
C
      COMMON NO,NOX,E,V,STIFF(50,50),X(25),Y(25)
C
C     PUNCH AND PRINT DORA IV HEADING
C
      CONTROL 102
      PUNCH 944
      PUNCH 999
      PRINT 944
C
C     PRINT PROGRAM SWITCH 2 OFF MESSAGE
C
C     IF SENSE SWITCH 2 IS *ON*, THE MESSAGES AT THE END OF THE PROGRAM
C      ARE BYPASSED.
C
      CONTROL 102
      PRINT 950
C
C     PUNCH NODES, MODULUS, AND RATIO HEADINGS
C
      PUNCH 999
      PUNCH 955
      PUNCH 956
      PUNCH 999
C
C     READ NUMBER OF NODES, YOUNG*S MODULUS, AND POISSON*S RATIO
C
   10 READ 901,NO,E,V
      PUNCH 901,NO,E,V
C
C     PUNCH COORDINATE HEADING
C
      PUNCH 999
      PUNCH 999
      PUNCH 960
      PUNCH 999
C
C     READ NODE NUMBER, X COORDINATE, AND Y COORDINATE.
C
      DO 20 J=1,NO
      READ 902,I,X(I),Y(I)
   20 PUNCH 902,I,X(I),Y(I)
C
      NOX=2*NO
      DO 25 I=1,NOX
      DO 25 J=1,NOX
   25 STIFF(I,J)=0.0
C
      ALINK=LINK(1.0)
```

```
  901 FORMAT(I3,2E16.8)
  902 FORMAT(13,2E16.8)
  944 FORMAT(42H DORA 1V - 25 NODES - 2 DEGREES OF FREEDOM)
  950 FORMAT(26H TURN PROGRAM SWITCH 2 OFF)
  955 FORMAT(5HNO OF,3X,8H YOUNG*S,7X,10H POISSON*S)
  956 FORMAT(5HNODES,3X,8H MODULUS,9X,6H RATIO)
  960 FORMAT(5H NODE,6X,2H X,14X,2H Y)
  999 FORMAT(1H )
C
      END
```

```
C  C                              TABLE A-I (CONTINUED)
C
C      DORA IV - BARS AND PLATES (EXCEPT TRAPEZOIDAL)
C
C      BARS - 25 NODES - 2 DEGREES OF FREEDOM
C   ***************************************
C
C      E.L. COOK - MAY 1966
C
      COMMON NO,NOX,E,V,STIFF(50,50),X(25),Y(25)
      DIMENSION ESTIF(4,4)
C
C      PUNCH HEADINGS
C
      PUNCH 999
      PUNCH 999
      PUNCH 901
      PUNCH 999
      PUNCH 902
      PUNCH 999
C
C      READ THE NUMBERS OF THE TWO END NODES, THE AREA, AND THE CONTROL
C       CODE.  THE CONTROL CODE IS 0 FO ALL MEMBERS EXCEPT THE LAST ONE,
C       FOR WHICH IT IS 1.
C
   30 READ 903,I,J,AR,KK
      PUNCH 903,I,J,AR
C
      XJI=X(J)-X(I)
      YJI=Y(J)-Y(I)
      SJI=SQRTF(XJI*XJI+YJI*YJI)
C     A=LAMBDA AND U=MU
      A=XJI/SJI
      ASQ=A*A
      U=YJI/SJI
      USQ=U*U
      AU=A*U
      AEL=AR*E/SJI
      ESTIF(1,1)=AEL*ASQ
      ESTIF(1,2)=AEL*AU
      ESTIF(1,3)=-ESTIF(1,1)
      ESTIF(1,4)=-ESTIF(1,2)
      ESTIF(2,2)=AEL*USQ
      ESTIF(2,3)=ESTIF(1,4)
      ESTIF(2,4)=-ESTIF(2,2)
      ESTIF(3,3)=ESTIF(1,1)
      ESTIF(3,4)=ESTIF(1,2)
      ESTIF(4,4)=ESTIF(2,2)
      DO 40 K=1,4
      DO 40 L=K,4
   40 ESTIF(L,K)=ESTIF(K,L)
      DO 50 K=1,2
      KI=K+2*(I-1)
      KJ=K+2*(J-1)
```

```
C     C
      DO 50 L=1,2
      LI=L+2*(I-1)
      LJ=L+2*(J-1)
      STIFF(KI,LI)=STIFF(KI,LI)+ESTIF(K,L)
      STIFF(KI,LJ)=STIFF(KI,LJ)+ESTIF(K,L+2)
      STIFF(KJ,LI)=STIFF(KJ,LI)+ESTIF(K+2,L)
   50 STIFF(KJ,LJ)=STIFF(KJ,LJ)+ESTIF(K+2,L+2)
      IF(KK)890,30,60
C
   60 ALINK=LINK(1,0)
C
  890 STOP
C
  901 FORMAT(4HBARS)
  902 FORMAT(6H NODES,6X,5H AREA)
  903 FORMAT(2I3,E16.8,I3)
  999 FORMAT(1H )
C
      END
```

```
C   C                          TABLE A-I (CONTINUED)

C       DORA IV - BARS AND PLATES (EXCEPT TRAPEZOIDAL)
C
C       TRIANGULAR PLATES - 25 NODES - 2 DEGREES OF FREEDOM
C   ****************************************************************
C
C       E.L. COOK - MAY 1966
C
        COMMON NO,NOX,E,V,STIFF(50,50),X(25),Y(25)
        DIMENSION ESTIF(6,6)
C
        NDOF=2
C
C       PUNCH HEADINGS
C
        PUNCH 999
        PUNCH 999
        PUNCH 901
        PUNCH 999
        PUNCH 902
        PUNCH 999
C
C       READ THREE NODE NUMBERS, THE THICKNESS, AND THE CONTROL CODE
C
   80   READ 906,I,J,K,T,KK
        PUNCH 906,I,J,K,T
C
        XJI=X(J)-X(I)
        XKJ=X(K)-X(J)
        XIK=X(I)-X(K)
        YJI=Y(J)-Y(I)
        YKJ=Y(K)-Y(J)
        YIK=Y(I)-Y(K)
        ET=0.5*E*T/((1.0-V**2)*ABSF(XJI*Y(K)+XIK*Y(J)+XKJ*Y(I)))
        V1=0.5*(1.0-V)
        V2=0.5*(1.0+V)
        ESTIF(1,1)=V1*XKJ*XKJ+YKJ*YKJ
        ESTIF(1,2)=-V2*XKJ*YKJ
        ESTIF(1,3)=V1*XKJ*XIK+YKJ*YIK
        ESTIF(1,4)=-V1*XKJ*YIK-V*XIK*YKJ
        ESTIF(1,5)=V1*XJI*XKJ+YJI*YKJ
        ESTIF(1,6)=-V1*XKJ*YJI-V*XJI*YKJ
        ESTIF(2,2)=XKJ*XKJ+V1*YKJ*YKJ
        ESTIF(2,3)=-V1*XIK*YKJ-V*XKJ*YIK
        ESTIF(2,4)=XKJ*XIK+V1*YKJ*YIK
        ESTIF(2,5)=-V1*XJI*YKJ-V*XKJ*YJI
        ESTIF(2,6)=XJI*XKJ+V1*YJI*YKJ
        ESTIF(3,3)=V1*XIK*XIK+YIK*YIK
        ESTIF(3,4)=-V2*XIK*YIK
        ESTIF(3,5)=V1*XJI*XIK+YJI*YIK
        ESTIF(3,6)=-V1*XIK*YJI-V*XJI*YIK
        ESTIF(4,4)=XIK*XIK+V1*YIK*YIK
        ESTIF(4,5)=-V1*XJI*YIK-V*XIK*YJI
        ESTIF(4,6)=XJI*XIK+V1*YJI*YIK
        ESTIF(5,5)=V1*XJI*XJI+YJI*YJI
        ESTIF(5,6)=-V2*XJI*YJI
        ESTIF(6,6)=XJI*XJI+V1*YJI*YJI
```

```
      DO 85 L=1,6
      DO 85 M=L,6
      ESTIF(L,M)=ET*ESTIF(L,M)
   85 ESTIF(M,L)=ESTIF(L,M)
      DO 90 L=1,2
      LI=L+NDOF*(I-1)
      LJ=L+NDOF*(J-1)
      LK=L+NDOF*(K-1)
      DO 90 M=1,2
      MI=M+NDOF*(I-1)
      MJ=M+NDOF*(J-1)
      MK=M+NDOF*(K-1)
      STIFF(LI,MI)=STIFF(LI,MI)+ESTIF(L,M)
      STIFF(LI,MJ)=STIFF(LI,MJ)+ESTIF(L,M+2)
      STIFF(LI,MK)=STIFF(LI,MK)+ESTIF(L,M+4)
      STIFF(LJ,MI)=STIFF(LJ,MI)+ESTIF(L+2,M)
      STIFF(LJ,MJ)=STIFF(LJ,MJ)+ESTIF(L+2,M+2)
      STIFF(LJ,MK)=STIFF(LJ,MK)+ESTIF(L+2,M+4)
      STIFF(LK,MI)=STIFF(LK,MI)+ESTIF(L+4,M)
      STIFF(LK,MJ)=STIFF(LK,MJ)+ESTIF(L+4,M+2)
   90 STIFF(LK,MK)=STIFF(LK,MK)+ESTIF(L+4,M+4)
C
      IF(KK)890,80,100
C
  100 ALINK=LINK(1,0)
C
  890 STOP
C
  901 FORMAT(17HTRIANGULAR PLATES)
  902 FORMAT(7H  NODES,6X,10H THICKNESS)
  906 FORMAT(3I3,E16.8,I3)
  999 FORMAT(1H )
C
      END
```

```
C  C                          TABLE A-1 (CONTINUED)
C
C     DORA IV - BARS AND PLATES (EXCEPT TRAPEZOIDAL)
C
C     RECTANGULAR PLATES - 25 NODES - 2 DEGREES OF FREEDOM
C  ********************************************************
C
C     E.L. COOK - MAY 1966
C
      COMMON NO,NOX,E,V,STIFF(50,50),X(25),Y(25)
      DIMENSION ESTIF(8,8)
C
C     PUNCH HEADINGS
C
      PUNCH 999
      PUNCH 999
      PUNCH 901
      PUNCH 999
      PUNCH 902
      PUNCH 999
C
C     READ THE NUMBERS OF THE FOUR NODES, THE THICKNESS, AND THE CONTROL
C      CODE.  THE NODE NUMBERS MUST BE IN THE FOLLOWING ORDER - LOWER
C      LEFT, LOWER RIGHT, UPPER LEFT, AND UPPER RIGHT.  THE CONTROL CODE
C      IS 0 FOR ALL CARDS EXCEPT THE LAST ONE, FOR WHICH IT IS 1.
C
   60 READ 905,I,J,K,L,T,KK
      PUNCH 905,I,J,K,L,T
C
      AM=(X(J)-X(I))/(Y(K)-Y(I))
      ET=E*T/(24.0*(1.0-V**2))
      A1=ET*8.0/AM
      B1=ET*8.0*AM
      C1=ET*6.0*V
      A2=ET*4.0*AM*(1.0-V)
      B2=ET*4.0*(1.0-V)/AM
      C2=ET*3.0*(1.0-V)
      ESTIF(1,1)=A1+A2
      ESTIF(1,2)=C1+C2
      ESTIF(1,3)=-A1+0.5*A2
      ESTIF(1,4)=C1-C2
      ESTIF(1,5)=0.5*A1-A2
      ESTIF(1,6)=-ESTIF(1,4)
      ESTIF(1,7)=-0.5*ESTIF(1,1)
      ESTIF(1,8)=-ESTIF(1,2)
      ESTIF(2,2)=B1+B2
      ESTIF(2,3)=ESTIF(1,6)
      ESTIF(2,4)=0.5*B1-B2
      ESTIF(2,5)=ESTIF(1,4)
      ESTIF(2,6)=-B1+0.5*B2
      ESTIF(2,7)=ESTIF(1,8)
      ESTIF(2,8)=-0.5*ESTIF(2,2)
      ESTIF(3,3)=ESTIF(1,1)
      ESTIF(3,4)=ESTIF(1,8)
      ESTIF(3,5)=ESTIF(1,7)
      ESTIF(3,6)=ESTIF(1,2)
      ESTIF(3,7)=ESTIF(1,5)
      ESTIF(3,8)=ESTIF(1,4)
```

```
      ESTIF(4,4)=ESTIF(2,2)
      ESTIF(4,5)=ESTIF(1,2)
      ESTIF(4,6)=ESTIF(2,8)
      ESTIF(4,7)=ESTIF(1,6)
      ESTIF(4,8)=ESTIF(2,6)
      ESTIF(5,5)=ESTIF(1,1)
      ESTIF(5,6)=ESTIF(1,8)
      ESTIF(5,7)=ESTIF(1,3)
      ESTIF(5,8)=ESTIF(1,6)
      ESTIF(6,6)=ESTIF(2,2)
      ESTIF(6,7)=ESTIF(1,4)
      ESTIF(6,8)=ESTIF(2,4)
      ESTIF(7,7)=ESTIF(1,1)
      ESTIF(7,8)=ESTIF(1,2)
      ESTIF(8,8)=ESTIF(2,2)
      DO 65 M=1,8
      DO 65 N=M,8
   65 ESTIF(N,M)=ESTIF(M,N)
      DO 70 M=1,2
      MI=M+2*(I-1)
      MJ=M+2*(J-1)
      MK=M+2*(K-1)
      ML=M+2*(L-1)
      DO 70 N=1,2
      NI=N+2*(I-1)
      NJ=N+2*(J-1)
      NK=N+2*(K-1)
      NL=N+2*(L-1)
      STIFF(MI,NI)=STIFF(MI,NI)+ESTIF(M,N)
      STIFF(MI,NJ)=STIFF(MI,NJ)+ESTIF(M,N+2)
      STIFF(MI,NK)=STIFF(MI,NK)+ESTIF(M,N+4)
      STIFF(MI,NL)=STIFF(MI,NL)+ESTIF(M,N+6)
      STIFF(MJ,NI)=STIFF(MJ,NI)+ESTIF(M+2,N)
      STIFF(MJ,NJ)=STIFF(MJ,NJ)+ESTIF(M+2,N+2)
      STIFF(MJ,NK)=STIFF(MJ,NK)+ESTIF(M+2,N+4)
      STIFF(MJ,NL)=STIFF(MJ,NL)+ESTIF(M+2,N+6)
      STIFF(MK,NI)=STIFF(MK,NI)+ESTIF(M+4,N)
      STIFF(MK,NJ)=STIFF(MK,NJ)+ESTIF(M+4,N+2)
      STIFF(MK,NK)=STIFF(MK,NK)+ESTIF(M+4,N+4)
      STIFF(MK,NL)=STIFF(MK,NL)+ESTIF(M+4,N+6)
      STIFF(ML,NI)=STIFF(ML,NI)+ESTIF(M+6,N)
      STIFF(ML,NJ)=STIFF(ML,NJ)+ESTIF(M+6,N+2)
      STIFF(ML,NK)=STIFF(ML,NK)+ESTIF(M+6,N+4)
   70 STIFF(ML,NL)=STIFF(ML,NL)+ESTIF(M+6,N+6)
C
      IF(KK)890,60,100
C
  100 ALINK=LINK(1,0)
C
  890 STOP
C
  901 FORMAT(18HRECTANGULAR PLATES)
  902 FORMAT(8H   NODES,8X,10H THICKNESS)
  905 FORMAT(4I3,E16.8,I3)
  999 FORMAT(1H )
C
      END
```

```
C  C                          TABLE A-I (CONTINUED)
C
C     DORA IV - BARS AND PLATES (EXCEPT TRAPEZOIDAL)
C
C     MATRIX INVERSE AND OUTPUT - 25 NODES - 2 DEGREES OF FREEDOM
C  ***************************************************************
C
C     E.L. COOK - MAY 1966
C
      COMMON NO,NOX,E,V,STIFF(50,50),X(25),Y(25)
      DIMENSION P(50),D(50)
C
      NO2=2*NO
C
C     PUNCH CONSTRAINT HEADINGS
C
      PUNCH 999
      PUNCH 999
      PUNCH 901
      PUNCH 999
      PUNCH 902
      PUNCH 999
C
C     READ THE NUMBER OF A NODE WITH  AT LEAST ONE CONSTRAINT, THE X AND
C      Y CONSTRAINT CODES, AND THE CONTROL CODE.  THE CONSTRAINT CODE IS
C      1.0 FOR CONSTRAINED DISPLACEMENTS AND 0.0 FOR UNCONSTRAINED
C      DISPLACEMENTS.  THE CONTROL CODES ARE THE SAME AS IN PART 2.
C
  130 READ 904,I,F1,F2,KK
      PUNCH 904,I,F1,F2
C
      J=2*(I-1)
      IF(F1)890,160,140
  140 DO 150 K=1,NO2
      STIFF(J+1,K)=0.0
  150 STIFF(K,J+1)=0.0
      STIFF(J+1,J+1)=1.0
  160 IF(F2)890,220,170
  170 DO 180 K=1,NO2
      STIFF(J+2,K)=0.0
  180 STIFF(K,J+2)=0.0
      STIFF(J+2,J+2)=1.0
  220 IF(KK)890,130,230
  230 CONTINUE
C
  600 DO 604 I=1,NO2
      STORE=STIFF(I,I)
      STIFF(I,I)=1.0
      DO 601 J=1,NO2
  601 STIFF(I,J)=STIFF(I,J)/STORE
      DO 604 K=1,NO2
      IF(K-I)602,604,602
  602 STORE=STIFF(K,I)
      STIFF(K,I)=0.0
      DO 603 J=1,NO2
  603 STIFF(K,J)=STIFF(K,J)-STORE*STIFF(I,J)
  604 CONTINUE
```

```
C   C
  235 DO 236 I=1,NO2
  236 P(I)=0.0
C
C     PUNCH LOAD HEADINGS
C
      PUNCH 999
      PUNCH 999
      PUNCH 903
      PUNCH 999
      PUNCH 902
      PUNCH 999
C
C     READ THE NUMBER OF A NODE WITH AN EXTERNAL LOAD. THE X AND Y
C      LOADS, AND THE CONTROL CODE.  TWO LOADS MUST BE SPECIFIED,
C      ALTHOUGH ONE OF THEM MAY BE ZERO. THE CONTROL CODES ARE THE SAME
C      AS IN PART 2.
C
  238 READ 904,I,P1,P2,KK
      PUNCH 904,I,P1,P2
C
      J=2*(I-1)
      P(J+1)=P1
      P(J+2)=P2
      IF(KK)890,238,240
  240 CONTINUE
C
      DO 250 I=1,NO2
      D(I)=0.0
      DO 250 J=1,NO2
  250 D(I)=D(I)+STIFF(I,J)*P(J)
C
C     PUNCH DISPLACEMENT HEADINGS
C
      PUNCH 999
      PUNCH 999
      PUNCH 905
      PUNCH 999
      PUNCH 906
      PUNCH 999
C
C     DISPLACEMENTS - THE NODE NUMBER IS FOLLOWED BY THE X AND Y
C      DISPLACEMENTS.  DISPLACEMENTS ARE PUNCHED FOR ALL NODES, EVEN THE
C      CONSTRAINED ONES.
C
      DO 260 I=1,NO
      J=2*(I-1)
  260 PUNCH 904,I,D(J+1),D(J+2)
C
      IF(SENSE SWITCH 2)280,270
C
  270 CONTROL 102
      PRINT 980
      PRINT 985
      CONTROL 102
      PRINT 990
      PRINT 995
  280 PAUSE
```

```
      GO TO 235
C
  890 STOP
C
  901 FORMAT(11HCONSTRAINTS)
  902 FORMAT(5H NODE,6X,2H X,14X,2H Y)
  903 FORMAT(5HLOADS)
  904 FORMAT(I3,2E16.8,I3)
  905 FORMAT(13HDISPLACEMENTS)
  906 FORMAT(5H NODE,6X,2H U,14X,2H V)
  980 FORMAT(49H FOR A NEW PROBLEM OR FOR A CHANGE OF CONSTRAINTS,1H,)
  985 FORMAT(45H THE ENTIRE SET OF PROGRAMS MUST BE RELOADED.)
  990 FORMAT(47H HOWEVER, ANY NUMBER OF ADDITIONAL LOAD VECTORS)
  995 FORMAT(45H MAY BE LOADED AT THIS TIME.  DEPRESS *START*)
  999 FORMAT(1H )
      END
```

C C

TABLE A-2

DIRECT STIFFNESS ANALYSIS - PANEL CONFIGURATION I

DORA IV - 25 NODES - 2 DEGREES OF FREEDOM

NO OF NODES	YOUNG*S MODULUS	POISSON*S RATIO
12	.10000000E 08	.31600000E 00

NODE	X	Y
1	.00000000E-50	.00000000E-50
2	.40000000E 01	.00000000E-50
3	.60000000E 01	.00000000E-50
4	.00000000E-50	.80000000E 01
5	.40000000E 01	.80000000E 01
6	.60000000E 01	.80000000E 01
7	.00000000E-50	.16000000E 02
8	.40000000E 01	.16000000E 02
9	.60000000E 01	.16000000E 02
10	.00000000E-50	.24000000E 02
11	.40000000E 01	.24000000E 02
12	.60000000E 01	.24000000E 02

BARS

NODES		AREA
1	2	.30100000E 00
2	3	.30100000E 00
4	5	.30100000E 00
5	6	.30100000E 00
7	8	.30100000E 00
8	9	.30100000E 00
10	11	.30100000E 00
11	12	.30100000E 00
1	4	.60200000E 00
2	5	.60200000E 00
4	7	.60200000E 00
5	8	.60200000E 00
	10	.60200000E 00
8	11	.60200000E 00

RECTANGULAR PLATES

NODES				THICKNESS
1	2	4	5	.10000000E 00
2	3	5	6	.10000000E 00
4	5	7	8	.10000000E 00
5	6	8	9	.10000000E 00
7	8	10	11	.10000000E 00
8	9	11	12	.10000000E 00

CONSTRAINTS

NODE	X	Y
3	.10000000E 01	.00000000E-50
6	.10000000E 01	.00000000E-50
9	.10000000E 01	.00000000E-50
12	.10000000E 01	.00000000E-50
2	.00000000E-50	.10000000E 01

LOADS

NODE	X	Y
10	.00000000E-50	.50000000E 00
11	.00000000E-50	.50000000E 00

DISPLACEMENTS

NODE	U	V
1	-.50552647E-07	.75252476E-06
2	-.28441223E-07	.00000000E-50
3	.00000000E-50	.15515326E-06
4	.14233045E-06	.89208273E-06
5	.61209393E-07	.68912770E-06
6	.00000000E-50	.63526471E-06
7	.80316400E-07	.12602640E-05
8	.26406319E-07	.11859289E-05
9	.00000000E-50	.12038422E-05
10	.79833870E-07	.17228336E-05
11	.29270008E-07	.16198911E-05
12	.00000000E-50	.15222091E-05

C C

TABLE A-2 (CONTINUED)

STIFFENER STRAINS

NODES		STRAINS
1	2	.55278560E-08
2	3	.14220611E-07
4	5	-.20280265E-07
5	6	-.30604698E-07
7	8	-.13477520E-07
8	9	-.13203159E-07
10	11	-.12640965E-07
11	12	-.14635004E-07
1	4	.17444747E-07
2	5	.86140962E-07
4	7	.46022662E-07
5	8	.62100150E-07
7	10	.57821200E-07
8	11	.54245275E-07

WEB STRAINS

NODES				EXX	EYY	EXY
1	2	4	5	-.73762040E-08	.51792854E-07	-.10177662E-06
2	3	5	6	-.81920420E-08	.73077446E-07	.30925730E-07
4	5	7	8	-.16878892E-07	.54061404E-07	-.40712350E-07
5	6	8	9	-.21903928E-07	.66586169E-07	-.11162630E-07
7	8	10	11	-.13059243E-07	.56033230E-07	-.22010880E-07
8	9	11	12	-.13919082E-07	.47020569E-07	-.19763200E-07

C C

TABLE A-3

DIRECT STIFFNESS ANALYSIS - PANEL CONFIGURATION II

DORA IV - 25 NODES - 2 DEGREES OF FREEDOM

NO OF NODES	YOUNG*S MODULUS	POISSON*S RATIO
24	.10000000E 08	.31600000E 00

NODE	X	Y
1	.00000000E-50	.00000000E-50
2	.20000000E 01	.00000000E-50
3	.40000000E 01	.00000000E-50
4	.60000000E 01	.00000000E-50
5	.00000000E-50	.30000000E 01
6	.20000000E 01	.30000000E 01
7	.40000000E 01	.30000000E 01
8	.60000000E 01	.30000000E 01
9	.00000000E-50	.50000000E 01
10	.20000000E 01	.50000000E 01
11	.40000000E 01	.50000000E 01
12	.60000000E 01	.50000000E 01
13	.00000000E-50	.80000000E 01
14	.20000000E 01	.80000000E 01
15	.40000000E 01	.80000000E 01
16	.60000000E 01	.80000000E 01
17	.00000000E-50	-.80000000E 01
18	.20000000E 01	-.80000000E 01
19	.40000000E 01	-.80000000E 01
20	.60000000E 01	-.80000000E 01
21	.00000000E-50	-.16000000E 02
22	.20000000E 01	-.16000000E 02
23	.40000000E 01	-.16000000E 02
24	.60000000E 01	-.16000000E 02

C C

TABLE A-3 (CONTINUED)

BARS

NODES		AREA
1	2	.30100000E 00
2	3	.30100000E 00
3	4	.30100000E 00
13	14	.30100000E 00
14	15	.30100000E 00
15	16	.30100000E 00
1	5	.60200000E 00
3	7	.60200000E 00
5	9	.60200000E 00
7	11	.60200000E 00
9	13	.60200000E 00
11	15	.60200000E 00
17	18	.30100000E 00
18	19	.30100000E 00
19	20	.30100000E 00
21	22	.30100000E 00
22	23	.30100000E 00
23	24	.30100000E 00
1	17	.60200000E 00
3	19	.60200000E 00
17	21	.60200000E 00
19	23	.60200000E 00

RECTANGULAR PLATES

NODES				THICKNESS
1	2	5	6	.10000000E 00
2	3	6	7	.10000000E 00
3	4	7	8	.10000000E 00
5	6	9	10	.10000000E 00
6	7	10	11	.10000000E 00
7	8	11	12	.10000000E 00
9	10	13	14	.10000000E 00
10	11	14	15	.10000000E 00
11	12	15	16	.10000000E 00
17	18	1	2	.10000000E 00
18	19	2	3	.10000000E 00
19	20	3	4	.10000000E 00
21	22	17	18	.10000000E 00
22	23	18	19	.10000000E 00
23	24	19	20	.10000000E 00

CONSTRAINTS

NODE	X	Y
24	.10000000E 01	.00000000E-50
20	.10000000E 01	.00000000E-50
4	.10000000E 01	.00000000E-50
8	.10000000E 01	.00000000E-50
12	.10000000E 01	.00000000E-50
16	.10000000E 01	.00000000E-50
23	.00000000E-50	.10000000E 01

C C

TABLE A-3 (CONTINUED)

LOADS

NODE	X	Y
13	.00000000E-50	.50000000E 00
15	.00000000E-50	.50000000E 00

DISPLACEMENTS

NODE	U	V
1	.80047330E-07	.12714139E-05
2	.56581539E-07	.12398273E-05
3	.30157771E-07	.11966348E-05
4	.00000000E-50	.12037975E-05
5	.84571695E-07	.14385289E-05
6	.51000175E-07	.13982069E-05
7	.23830666E-07	.13632036E-05
8	.00000000E-50	.13570573E-05
9	.61306633E-07	.15569601E-05
10	.41834539E-07	.14966693E-05
11	.21936972E-07	.14705235E-05
12	.00000000E-50	.14501316E-05
13	.75304050E-07	.17583445E-05
14	.55530950E-07	.15446214E-05
15	.29327916E-07	.16494427E-05
16	.00000000E-50	.14993756E-05
17	.13537860E-06	.90784096E-06
18	.11062759E-06	.78620528E-06
19	.60166070E-07	.70126854E-06
20	.00000000E-50	.64637866E-06
21	-.48537227E-07	.75408037E-06
22	-.46085294E-07	.46944948E-06
23	-.27554959E-07	.00000000E-50
24	.00000000E-50	.15857147E-06

C C

TABLE A-3 (CONTINUED)

STIFFENER STRAINS

NODES		STRAINS
1	2	-.11732895E-07
2	3	-.13211884E-07
3	4	-.15078885E-07
13	14	-.98865500E-08
14	15	-.13101517E-07
15	16	-.14663958E-07
1	5	.55705000E-07
3	7	.55522933E-07
5	9	.59215600E-07
7	11	.53659950E-07
9	13	.67128133E-07
11	15	.59639733E-07
17	18	-.12375505E-07
18	19	-.25230760E-07
19	20	-.30083035E-07
21	22	.12259665E-08
22	23	.92651675E-08
23	24	.13777479E-07
1	17	.45446612E-07
3	19	.61920787E-07
17	21	.19220073E-07
19	23	.87658567E-07

WEB STRAINS

NODES				EXX	EYY	EXY
1	2	5	6	-.14259328E-07	.54249090E-07	-.18153320E-07
2	3	6	7	-.13398320E-07	.54158050E-07	-.21533710E-07
3	4	7	8	-.13497109E-07	.53304760E-07	-.80041000E-09
5	6	9	10	-.13260903E-07	.54223400E-07	-.33260870E-07
6	7	10	11	-.11766770E-07	.51445580E-07	-.18052120E-07
7	8	11	12	-.11441910E-07	.50098550E-07	-.71079800E-08
9	10	13	14	-.98112980E-08	.41556080E-07	-.63887840E-07
10	11	14	15	-.11525151E-07	.37611890E-07	.23183440E-07
11	12	15	16	-.12816222E-07	.38027200E-07	-.41382930E-07
17	18	1	2	-.12054200E-07	.51074686E-07	-.45141640E-07
18	19	2	3	-.19221322E-07	.59311767E-07	-.37215700E-07
19	20	3	4	-.22580961E-07	.65799069E-07	-.13807310E-07
21	22	17	18	-.55747690E-08	.29407274E-07	-.80277350E-07
22	23	18	19	-.79827960E-08	.63626521E-07	-.12331944E-06
23	24	19	20	-.81527780E-08	.74317233E-07	.31402960E-07

C C

TABLE A-4

DIRECT STIFFNESS ANALYSIS - PANEL CONFIGURATION III

DORA IV - 25 NODES - 2 DEGREES OF FREEDOM

NO OF NODES	YOUNG*S MODULUS	POISSON*S RATIO
24	.10000000E 08	.31600000E 00

NODE	X	Y
1	.00000000E-50	.00000000E-50
2	.20000000E 01	.00000000E-50
3	.40000000E 01	.00000000E-50
4	.60000000E 01	.00000000E-50
5	.00000000E-50	.30000000E 01
6	.20000000E 01	.30000000E 01
7	.40000000E 01	.30000000E 01
8	.60000000E 01	.30000000E 01
9	.00000000E-50	.50000000E 01
10	.20000000E 01	.50000000E 01
11	.40000000E 01	.50000000E 01
12	.60000000E 01	.50000000E 01
13	.00000000E-50	.80000000E 01
14	.20000000E 01	.80000000E 01
15	.40000000E 01	.80000000E 01
16	.60000000E 01	.80000000E 01
17	.00000000E-50	.16000000E 02
18	.20000000E 01	.16000000E 02
19	.40000000E 01	.16000000E 02
20	.60000000E 01	.16000000E 02
21	.00000000E-50	-.80000000E 01
22	.20000000E 01	-.80000000E 01
23	.40000000E 01	-.80000000E 01
24	.60000000E 01	-.80000000E 01

C C

TABLE A-4 (CONTINUED)

BARS

NODES		AREA
1	2	.30100000E 00
2	3	.30100000E 00
3	4	.30100000E 00
13	14	.30100000E 00
14	15	.30100000E 00
15	16	.30100000E 00
1	5	.60200000E 00
3	7	.60200000E 00
5	9	.60200000E 00
7	11	.60200000E 00
9	13	.60200000E 00
11	15	.60200000E 00
17	18	.30100000E 00
18	19	.30100000E 00
19	20	.30100000E 00
21	22	.30100000E 00
22	23	.30100000E 00
23	24	.30100000E 00
1	21	.60200000E 00
3	23	.60200000E 00
13	17	.60200000E 00
15	19	.60200000E 00

RECTANGULAR PLATES

NODES				THICKNESS
1	2	5	6	.10000000E 00
2	3	6	7	.10000000E 00
3	4	7	8	.10000000E 00
5	6	9	10	.10000000E 00
6	7	10	11	.10000000E 00
7	8	11	12	.10000000E 00
9	10	13	14	.10000000E 00
10	11	14	15	.10000000E 00
11	12	15	16	.10000000E 00
21	22	1	2	.10000000E 00
22	23	2	3	.10000000E 00
23	24	3	4	.10000000E 00
13	14	17	18	.10000000E 00
14	15	18	19	.10000000E 00
15	16	19	20	.10000000E 00

CONSTRAINTS

NODE	X	Y
24	.10000000E 01	.00000000E-50
8	.10000000E 01	.00000000E-50
4	.10000000E 01	.00000000E-50
12	.10000000E 01	.00000000E-50
16	.10000000E 01	.00000000E-50
20	.10000000E 01	.00000000E-50
23	.00000000E-50	.10000000E 01

C C

TABLE A-4 (CONTINUED)

LOADS

NODE	X	Y
17	.00000000E-50	.50000000E 00
19	.00000000E-50	.50000000E 00

DISPLACEMENTS

NODE	U	V
1	.13206989E-06	.91577810E-06
2	.11225373E-06	.78321554E-06
3	.61489530E-07	.70207611E-06
4	.00000000E-50	.63645268E-06
5	.14320089E-06	.10336221E-05
6	.10357874E-06	.96561388E-06
7	.57020047E-07	.89863685E-06
8	.00000000E-50	.88768051E-06
9	.12558823E-06	.11247121E-05
10	.87080298E-07	.10823834E-05
11	.44502105E-07	.10241376E-05
12	.00000000E-50	.10237987E-05
13	.62587986E-07	.12747602E-05
14	.43476770E-07	.12606629E-05
15	.22566553E-07	.11967597E-05
16	.00000000E-50	.12130666E-05
17	.83810380E-07	.17546426E-05
18	.59394033E-07	.15908938E-05
19	.30876461E-07	.16437561E-05
20	.00000000E-50	.15422935E-05
21	-.50281472E-07	.75928189E-06
22	-.48551816E-07	.47514069E-06
23	-.29089571E-07	.00000000E-50
24	.00000000E-50	.16121557E-06

C C

STIFFENER STRAINS

NODES		STRAINS
1	2	-.99080800E-08
2	3	-.25382100E-07
3	4	-.30744765E-07
13	14	-.95556080E-08
14	15	-.10455108E-07
15	16	-.11283276E-07
1	5	.39281333E-07
3	7	.65520247E-07
5	9	.45545000E-07
7	11	.62750400E-07
9	13	.50016033E-07
11	15	.57540700E-07
17	18	-.12208173E-07
18	19	-.14258786E-07
19	20	-.15438230E-07
21	22	.86482800E-09
22	23	.97311225E-08
23	24	.14544785E-07
1	21	.19562027E-07
3	23	.87759514E-07
13	17	.59985300E-07
15	19	.55874550E-07

WEB STRAINS

NODES				EXX	EYY	EXY
1	2	5	6	-.14859578E-07	.50040380E-07	-.49733370E-07
2	3	6	7	-.24330723E-07	.63159840E-07	-.39219870E-07
3	4	7	8	-.29627395E-07	.74664760E-07	-.19839860E-07
5	6	9	10	-.19532521E-07	.51964880E-07	-.36112020E-07
6	7	10	11	-.22284222E-07	.60567570E-07	-.38559810E-07
7	8	11	12	-.25380538E-07	.65354740E-07	-.59032800E-08
9	10	13	14	-.14404787E-07	.54721250E-07	-.31873790E-07
10	11	14	15	-.15872104E-07	.58483590E-07	-.41460430E-07
11	12	15	16	-.16767164E-07	.60315000E-07	.33607000E-09
21	22	1	2	-.45216260E-08	.29035691E-07	-.82728630E-07
22	23	2	3	-.78254890E-08	.63134435E-07	-.12335849E-06
23	24	3	4	-.80999900E-08	.73582077E-07	.29559230E-07
13	14	17	18	-.10881891E-07	.50632083E-07	-.42140290E-07
14	15	18	19	-.12356948E-07	.45576710E-07	-.[illegible]2460290E-08
15	16	19	20	-.13360754E-07	.48513964E-07	-.20769560E-07

C C

TABLE A-5

DIRECT STIFFNESS ANALYSIS - PANEL CONFIGURATION IV

DORA IV - 25 NODES - 2 DEGREES OF FREEDOM

NO OF NODES	YOUNG*S MODULUS	POISSON*S RATIO
24	.10000000E 08	.31600000E 00

NODE	X	Y
1	.00000000E-50	.00000000E-50
2	.20000000E 01	.00000000E-50
3	.40000000E 01	.00000000E-50
4	.60000000E 01	.00000000E-50
5	.00000000E-50	.30000000E 01
6	.20000000E 01	.30000000E 01
7	.40000000E 01	.30000000E 01
8	.60000000E 01	.30000000E 01
9	.00000000E-50	.50000000E 01
10	.20000000E 01	.50000000E 01
11	.40000000E 01	.50000000E 01
12	.60000000E 01	.50000000E 01
13	.00000000E-50	.80000000E 01
14	.20000000E 01	.80000000E 01
15	.40000000E 01	.80000000E 01
16	.60000000E 01	.80000000E 01
17	.00000000E-50	.16000000E 02
18	.20000000E 01	.16000000E 02
19	.40000000E 01	.16000000E 02
20	.60000000E 01	.16000000E 02
21	.00000000E-50	.24000000E 02
22	.20000000E 01	.24000000E 02
23	.40000000E 01	.24000000E 02
24	.60000000E 01	.24000000E 02

C C

BARS

NODES		AREA
1	2	.30100000E 00
2	3	.30100000E 00
3	4	.30100000E 00
13	14	.30100000E 00
14	15	.30100000E 00
15	16	.30100000E 00
1	5	.60200000E 00
3	7	.60200000E 00
5	9	.60200000E 00
7	11	.60200000E 00
9	13	.60200000E 00
11	15	.60200000E 00
17	18	.30100000E 00
18	19	.30100000E 00
19	20	.30100000E 00
21	22	.30100000E 00
22	23	.30100000E 00
23	24	.30100000E 00
17	21	.60200000E 00
19	23	.60200000E 00
13	17	.60200000E 00
15	19	.60200000E 00

RECTANGULAR PLATES

NODES				THICKNESS
1	2	5	6	.10000000E 00
2	3	6	7	.10000000E 00
3	4	7	8	.10000000E 00
5	6	9	10	.10000000E 00
6	7	10	11	.10000000E 00
7	8	11	12	.10000000E 00
9	10	13	14	.10000000E 00
10	11	14	15	.10000000E 00
11	12	15	16	.10000000E 00
13	14	17	18	.10000000E 00
14	15	18	19	.10000000E 00
15	16	19	20	.10000000E 00
17	18	21	22	.10000000E 00
18	19	22	23	.10000000E 00
19	20	23	24	.10000000E 00

CONSTRAINTS

NODE	X	Y
20	.10000000E 01	.00000000E-50
8	.10000000E 01	.00000000E-50
4	.10000000E 01	.00000000E-50
12	.10000000E 01	.00000000E-50
16	.10000000E 01	.00000000E-50
24	.10000000E 01	.00000000E-50
3	.00000000E-50	.10000000E 01

C C

TABLE A-5 (CONTINUED)

LOADS

NODE	X	Y
21	.00000000E-50	.50000000E 00
23	.00000000E-50	.50000000E 00

DISPLACEMENTS

NODE	U	V
1	-.10867504E-06	.84300032E-06
2	-.98632255E-07	.55634454E-06
3	-.55491384E-07	.00000000E-50
4	.00000000E-50	.25324771E-06
5	.11097950E-06	.85770273E-06
6	.10819231E-06	.59664253E-06
7	.52147767E-07	.33207062E-06
8	.00000000E-50	.31620945E-06
9	.15500238E-06	.88665446E-06
10	.12517964E-06	.70000530E-06
11	.70666772E-07	.50966276E-06
12	.00000000E-50	.49044809E-06
13	.10986507E-06	.97001706E-06
14	.85647165E-07	.85816786E-06
15	.46712091E-07	.74349165E-06
16	.00000000E-50	.73025990E-06
17	.76403165E-07	.13270019E-05
18	.53168969E-07	.13048632E-05
19	.27581046E-07	.12456152E-05
20	.00000000E-50	.12568298E-05
21	.63139130E-07	.18038613E-05
22	.59234065E-07	.16438256E-05
23	.30847897E-07	.16939021E-05
24	.00000000E-50	.15950362E-05

C C

TABLE A-5 (CONTINUED)

STIFFENER STRAINS

NODES		STRAINS
1	2	.50218950E-08
2	3	.21570435E-07
3	4	.27745693E-07
13	14	-.12108955E-07
14	15	-.19467537E-07
15	16	-.23356045E-07
1	5	.49008033E-08
3	7	.11069020E-06
5	9	.14475865E-07
7	11	.88796070E-07
9	13	.27787533E-07
11	15	.77942963E-07
17	18	-.11617098E-07
18	19	-.12793961E-07
19	20	-.13790523E-07
21	22	-.11952532E-07
22	23	-.14193084E-07
23	24	-.15423948E-07
17	21	.59607425E-07
19	23	.56035862E-07
13	17	.44623100E-07
13	19	.62765450E-07

WEB STRAINS

NODES				EXX	EYY	EXY
1	2	5	6	.18141490E-08	.91667280E-08	-.65848980E-07
2	3	6	7	-.32259180E-08	.62061433E-07	-.15281851E-06
3	4	7	8	.83590400E-09	.65838724E-07	.77286494E-07
5	6	9	10	-.81524820E-08	.33078640E-07	-.96674800E-07
6	7	10	11	-.27639353E-07	.70238730E-07	-.10485204E-06
7	8	11	12	-.30703635E-07	.87957690E-07	-.41392100E-08
9	10	13	14	-.13510162E-07	.40254180E-07	-.88736220E-07
10	11	14	15	-.23361985E-07	.65331900E-07	-.86835890E-07
11	12	15	16	-.29344716E-07	.78940110E-07	-.12104050E-07
13	14	17	18	-.11863026E-07	.50230010E-07	-.37618240E-07
14	15	18	19	-.16130749E-07	.59301180E-07	-.46706650E-07
15	16	19	20	-.18573285E-07	.64293341E-07	-.16999700E-08
17	18	21	22	-.11784816E-07	.50988860E-07	-.44743540E-07
18	19	22	23	-.13493523E-07	.49203080E-07	-.17096300E-08
19	20	23	24	-.14607236E-07	.49155833E-07	-.21708660E-07

```
C  C                                    TABLE A-6

                        STIFFENER AND WEB STRAIN PROGRAM LISTING

C      STIFFENER AND WEB STRAINS FROM NODAL DISPLACEMENTS
C   ****************************************************
C
C      E.L. COOK - JUNE 1966
C
       DIMENSION X(25),Y(25),U(25),V(25),IA(25),IB(25),A(25)
       DIMENSION JA(16),JB(16),JC(16),JD(16),T(16)
       DIMENSION DELTA(8),SIGMA(8)
C
C      READ NUMBER OF NODES, YOUNGS MODULUS, AND POISSONS RATIO
C
  801  READ 901,NO,E,VV
       EP=E/(1.0-VV**2)
       GS=E/(2.0*(1.0+VV))
C
C      READ NODAL COORDINATES
C
       DO 10 I=1,NO
  802  READ 901,K,X(I),Y(I)
       IF(K-I)700,10,700
   10  CONTINUE
C
C      READ STIFFENER DATA
C
       IK=1
  803  READ 902,IA(IK),IB(IK),A(IK),ICODE
       IK=IK+1
       IF(ICODE-1)803,30,700
   30  IK=IK-1
C
C      READ WEB DATA
C
       JK=1
  804  READ 903,JA(JK),JB(JK),JC(JK),JD(JK),T(JK),ICODE
       JK=JK+1
       IF(ICODE-1)804,50,700
   50  JK=JK-1
C
C      READ NODAL DISPLACEMENTS
C
       DO 60 I=1,NO
  805  READ 901,J,U(I),V(I)
       IF(I-J)700,60,700
   60  CONTINUE
```

```
C  C                                 TABLE A-6 (CONTINUED)
C       CALCULATE STIFFENER STRAINS
C
        DO 807 IL=1,IK
        IAI=IA(IL)
        XA=X(IAI)
        YA=Y(IAI)
        UA=U(IAI)
        VA=V(IAI)
        IBI=IB(IL)
        XB=X(IBI)
        YB=Y(IBI)
        UB=U(IBI)
        VB=V(IBI)
        ALX=XB-XA
        ALY=YB-YA
        AL2=ALX**2+ALY**2
        ESTR=(ALX*(UB-UA)+ALY*(VB-VA))/AL2
  807   PUNCH 902,IAI,IBI,ESTR
C
C       CALCULATE WEB STRAINS
C
        DO 806 IL=1,JK
        JAI=JA(IL)
        XA=X(JAI)
        YA=Y(JAI)
        DELTA(1)=U(JAI)
        DELTA(5)=V(JAI)
        JBI=JB(IL)
        XB=X(JBI)
        DELTA(2)=U(JBI)
        DELTA(6)=V(JBI)
        JCI=JC(IL)
        XC=X(JCI)
        YC=Y(JCI)
        DELTA(3)=U(JCI)
        DELTA(7)=V(JCI)
        JDI=JD(IL)
        XD=X(JDI)
        DELTA(4)=U(JDI)
        DELTA(8)=V(JDI)
        XBA=XB-XA
        P=(XC-XA)/XBA
        D=(XD-XA)/XBA
        B=D-1.0
        TR=1.0/(D-P)
        YCA=YC-YA
        ETA=0.5
        XI=0.5
        SIGMA(1)=(ETA-1.0)/XBA
        SIGMA(2)=-SIGMA(1)
        SIGMA(3)=-TR*ETA/XBA
        SIGMA(4)=-SIGMA(3)
```

```
C  C                            TABLE A-6 (CONTINUED)
      EXX=0.0
      DO 70 I=1,4
   70 EXX=EXX+SIGMA(I)*DELTA(I)
      SIGMA(5)=(XI-1.0)/YCA
      SIGMA(6)=-XI/YCA
      SIGMA(7)=TR*(D-XI)/YCA
      SIGMA(8)=TR*(XI-P)/YCA
      EYY=0.0
      EXY=0.0
      DO 80 J=1,4
      EYY=EYY+SIGMA(J+4)*DELTA(J+4)
   80 EXY=EXY+SIGMA(J+4)*DELTA(J)+SIGMA(J)*DELTA(J+4)
  806 PUNCH 904,JAI,JBI,JCI,JDI,EXX,EYY,EXY
      GO TO 801
C
  700 STOP
C
  901 FORMAT(I3,4E16.8)
  902 FORMAT(2I3,E16.8,I3)
  903 FORMAT(4I3,E16.8,I3)
  904 FORMAT(4I3,3E16.8)
C
      END
```

APPENDIX B

THE MATRIX FORCE/DISPLACEMENT PROGRAM

Due to the analogy that exists between the matrix force method and the matrix displacement method of structural analysis, it has been possible to write a program that can be used to analyze structures by either method. The analogy between the two methods was first shown by Argyris and Kelsey (Reference 1). Pestel and Leckie (Reference 5) have expanded on the analogy and have developed an excellent notation which will be used here.

The original program was written by the author for a force analysis. It was intended that the experimental panel would be analyzed using only the force method and the direct stiffness method. However, one of the author's graduate students, Mr. Gordon E. Lambert, elected to write a displacement method program for his masters thesis (Reference 3). A study of the force program and the analogy revealed that the force program could be modified and used for both types of analysis. The modifications consist of one additional digit in a control card to specify the type of analysis, some additional headings, and one program segment which is used only in the displacement analysis.

The equations for both methods are derived in References 1, 3, and 5; therefore, the derivation will not be shown here. The equations which must be solved to perform an analysis will be stated, the analogy will be discussed, and some of the unique features of the program will be explained.

The Force Method

The force equations are:

$$[D_{10}] = [B_1]^T[F_v][B_0] \tag{B-1}$$

$$[D_{11}] = [B_1]^T[F_v][B_1] \tag{B-2}$$

$$[X] = -[D_{11}]^{-1}[D_{10}] \tag{B-3}$$

$$[B] = [B_0] + [B_1][X] \tag{B-4}$$

$$[F_d] = [B]^T[F_v][B] \quad \text{(B-5)}$$

$$\{p\} = [B]\{f\} \quad \text{(B-6)}$$

$$\{d\} = [F_d]\{f\} \quad \text{(B-7)}$$

Three of the matrices in the above equations must be determined from the physical and geometric properties of the structure:

$[B_0]$ = a set of internal forces in the structure due to unit values of the external forces. Each column represents a set of internal forces due to one of the external loads.

$[B_1]$ = a set of internal forces in the structure due to unit values of the redundants. Each column represents a set of internal forces due to one of the redundants. For a statically determinate structure $[B_1] = [0]$ and $[B] = [B_0]$.

$[F_v]$ = the flexibility matrix of the unassembled structure. This is a diagonally partitioned matrix where the submatrices are the element flexibility matrices.

Equations (B-1) through (B-7) can be easily evaluated once these three matrices have been determined. The $[B_0]$, $[B_1]$, and $[F_v]$ matrices for the experimental panel are shown later in this appendix. The physical interpretation of the other matrices in the above equations are as follows:

$[D_{10}]$ = the relative displacements at the redundants due to unit values of the external loads. Each column represents the displacements due to one of the external loads.

$[D_{11}]$ = the relative displacements at the redundants due to unit values of the redundants. Each column represents the displacements due to one of the redundants.

$[X]$ = the values of the redundants due to unit values of the external loads. Each column represents the redundants due to one of the external loads.

$[B]$ = the values of the internal forces due to unit values of the external loads. Each column represents the internal forces due to one of the external forces.

$[F_d]$ = the flexibility matrix, i.e., the values of the external displacements due to unit values of the external loads. Each column represents the displacements due to one of the external loads.

p = the column matrix of internal forces.

f = the column matrix of external forces.

d = the column matrix of external displacements.

The Displacement Method

The displacement equations are:

$$[C_{10}] = [A_1]^T [K_p] [A_0] \tag{B-8}$$

$$[C_{11}] = [A_1]^T [K_p] [A_1] \tag{B-9}$$

$$[Y] = -[C_{11}]^{-1} [C_{10}] \tag{B-10}$$

$$[A] = [A_0] + [A_1][Y] \tag{B-11}$$

$$[K_f] = [A]^T [K_p] [A] \tag{B-12}$$

$$\{v\} = [A]\ d \tag{B-13}$$

$$\{f\} = [K_f]\ d \tag{B-14}$$

The three input matrices required for a displacement analysis are:

$[A_0]$ = the deformations due to unit displacements at the external loads.

$[A_1]$ = the deformations due to unit displacements at the kinematic deficiencies. A kinematic deficiency exists for each degree of freedom for which there is no external load. If there are loads corresponding to all unconstrained external displacements, $[A_1] = [0]$ and $[A] = [A_0]$.

$[K_p]$ = the stiffness matrix for the unassembled structure.

The physical interpretation of the other matrices are as follows:

$[C_{10}]$ = the net forces at the kinematic deficiencies due to unit values of the displacements at the external loads. Each column represents the forces due to one of the external loads.

$[C_{11}]$ = the net forces at the kinematic deficiencies due to unit values of the displacements at the deficiencies. Each column represents the forces due to a displacement at one of the deficiencies.

$[Y]$ = the displacements at the kinematic deficiencies due to unit displacements at the external loads. Each column represents the displacements at the deficiencies due to a displacement at one of the external loads.

$[A]$ = the deformations due to unit values of the displacements at the external loads. Each column represents the deformations due to one external displacement.

$[K_f]$ = the stiffness matrix, i.e., the external forces required to produce unit displacements at these forces. Each column represents the external forces due to a unit displacement at one of the forces.

$\{v\}$ = the column matrix of deformations

$\{f\}$ and $\{d\}$ were previously defined.

Equations (B-13) and (B-14) give the deformations and external forces for prescribed displacements. Since the external forces are usually known, rather than the displacements, and the internal forces are needed, rather than the deformations; two additional equations are necessary to fully implement a displacement analysis. They are

$$[B] = [K_p][A][K_f]^{-1} \qquad \text{(B-15)}$$

$$[F_d] = [K_f]^{-1} \qquad \text{(B-16)}$$

Once these two equations are evaluated, Equations (B-6) and (B-7) can be used to calculate the internal forces and the displacements.

The Analogy Between the Force and Displacement Methods

The analogy between the two methods is evident if Equations (B-1) through (B-7) are compared with Equations (B-8) through (B-14). The analogous matrices are tabulated below

Force	Displacement	Force	Displacement
$\{f\}$	$\{d\}$	$[D_{10}]$	$[C_{10}]$
$\{p\}$	$\{v\}$	$[D_{11}]$	$[C_{11}]$
$[B_0]$	$[A_0]$	$[X]$	$[Y]$
$[B_1]$	$[A_1]$	$[B]$	$[A]$
$[F_v]$	$[K_p]$	$[F_d]$	$[K_f]$

References 1, 3, and 5 all present tabulations of the type shown above with varying degrees of detailed operations and explanation.

The Matrix Force/Displacement Program

The Matrix Force/Displacement Program is divided into six parts:

Part I - Input
Part II - Recursion Analysis
Part III - Calculation of Flexibility or Stiffness Matrix
Part IV - Internal Forces and Displacements-Displacement Method
Part V - Internal Forces and Displacements-Output
Part VI - Initial Forces

A program listing is shown in Table B-1. Parts I through V will be discussed here. Part VI, which is used only for thermal analyses, is discussed in Appendix C. Although the program naturally divides into the six parts listed above, the primary reason for the division is the effective utilization of the computer. The program is written in PDQ Fortran for the IBM 1620 described in Appendix A.

Part I - Input. The inputs to the program consist of a control card followed by the flexibilities, or stiffnesses, of the unassembled structure and the nonzero elements of $[B_0]$ and $[B_1]$, or $[A_0]$ and $[A_1]$. The control card contains the basic parameters of the structure such as the number of elements; internal forces; external forces; redundants, or deficiencies; and nonzero elements in $[B_0]$ and $[B_1]$, or $[A_0]$ and $[A_1]$. It also contains a control digit which specifies the type of analysis to be performed. The format of the control card and the other input cards is described in detail in Table B-1.

The input format of the element flexibilities, or stiffness, requires explanation. Since $[F_v]$, or $[K_p]$, has an order equal to the number of internal forces, it is usually very large. The storage requirements are greatly reduced if the element flexibilities, or stiffness, are stored in a special format and used as needed. The minimum storage requirements would result if each flexibility, or stiffness, matrix was given a variable name and properly dimensioned , or if blocks of storage were set aside for each type of element-bars, rectangular plates, etc. This would, however, reduce the generality of the program. To avoid this, a single array of dimensions ℓx5 is used, where ℓ is the number of internal forces. Five columns are specified because, to date, the largest element flexibility, or stiffness, matrix used has been the 5x5 rectangular or trapeqoidal plate matrix. The element flexibilities, or stiffnesses, are right-justified in the array to permit identification of each type of element flexibilities, or stiffnesses. The identification procedure is described in the next section.

The inputs are also punched into cards, with appropriate headings, for problem identification and debugging.

Part II - Recursion Analysis. The solution of the matrix force or displacement equations previously presented requires the inversion of $[D_{11}]$ or $[C_{11}]$. Since the order of $[D_{11}]$ is equal to the number of redundants and the order of $[C_{11}]$ is equal to the number of kinematic deficiencies, the required inversion may be relatively large. Festel and Leckie (Reference 5) have presented a recurrence method where the redundants, or deficiencies, are divided into a number of groups and an equal number of lower-ordered inversions are performed. During the programming of the recurrence method it was discovered that, not only is the programming simplified, the data storage requirements are minimized if the number of groups is equal to the number of redundants, or devidicncies. Therefore, if there are n redundants, or deficiencies, there are n one-by-one inversions or, in fact, no inversions at all. As is shown in Part IV, a displacement analysis requires the inversion of the stiffness matrix $[K_p]$; however, this matrix is usually not large.

The recurrence equations for the force method are as follows:

$$\{D_{01}^{i}\} = \{B_{1}^{i}\}^{T}[F_{V}][B_{01}^{i}] \tag{B-17}$$

$$\{X^{i}\} = - \{D_{10}^{i}\}/D_{11}^{i} \tag{B-18}$$

$$[B_{01}^{i+1}] = [B_{0}^{i}] + \{B_{1}^{i}\}\{X^{i}\}^{T} \tag{B-19}$$

Equation (B-15) is the recurrence method equivalent of a combination of Equations (B-1) and (B-2). These equations could be written as

$$[D_{01}] = [D_{10}|D_{11}] = [B_{1}]^{T}[F_{V}][B_{0}|B_{1}] \tag{B-20}$$

For each recursion, the last column of the matrix $[B_{01}^{i}]$ is taken as $\{B_{1}^{i}\}$. As a result, both $\{D_{01}^{i}\}$ and $\{X^{i}\}$ are vectors rather than rectangular matrices. The unit redundants $\{X^{i}\}$ are determined by dividing all of the elements of $\{D_{01}^{i}\}$, except the last, by the negative of the last element. The matrix $[B_{01}^{i+1}]$ has one less column than $[B_{01}^{i}]$, and after n recursions, $[B_{01}^{n+1}]$ is the unit internal force matrix $[B]$.

The recurrence equations for the displacement method can be written from Equations (B-17) through (B-19), with the aid of the analogy, as

$$\{C_{01}^{i}\} = \{A_{1}^{i}\}^{T}[K_{p}][A_{01}^{i}] \tag{B-21}$$

$$\{Y^{i}\} = - \{C_{10}^{i}\}/C_{11}^{i} \tag{B-22}$$

$$[A_{01}^{i+1}] = [A_0^i] + \{A_1^i\}\{Y^i\}^T \qquad \text{(B-23)}$$

The triple matrix product in Equation (B-17) and (B-21) involves the matrices $[F_v]$ and $[K_p]$ which are stored in the compressed format described in Part I. The operations required for a force analysis will be illustrated. The product $\{B_1^i\}^T [F_v]$ is performed first. The first row of $[F_v]$ is checked, from the left, for the first nonzero element. If the first element in the row is nonzero, the element flexibility matrix is a 5x5 matrix. This matrix is then premultiplied by the first five elements of $\{B_1^i\}$ to obtain the first five elements of the product. Then the sixth row of $[F_v]$ is checked. If the fourth element were the first nonzero element, the element flexibility would be a 2x2 matrix. The sixth and seventh element of $\{B_1^i\}$ would be used to obtain the next two elements of the product. This procedure is continued until all elements of the product have been calculated. Finally, $[B_{01}^i]$ is premultiplied in the normal manner by the product $\{B_1^i\}^T[F_v]$ to obtain $\{D_{01}^i\}$.

Provision is also made in this part of the program for punching intermediate output which can later be used for a thermal analysis (Appendix C).

Part III - Calculation of Flexibility or Stiffness Matrix. The purpose of this part of the program is to evaluate the flexibility matrix, Equation (B-5), for a force analysis or the stiffness matrix, Equation (B-12), for a displacement analysis. It should be noted that these two equations are not given by Pestel and Leckie. It can be shown, however, that they are equivalent to Pestel and Leckie's equations.

A procedure similar to the one used in Part II for $\{B_1^i\}^T[F_v]$, or $\{A_1^i\}^T[K_p]$, is used to obtain $[F_v][B]$, or $[K_p][A]$. The major difference is that the products are rectangular matrices rather than vectors.

Part IV - Internal Forces and Displacements-Displacement Method. This part is used in a displacement analysis to evaluate Equations (B-15) and (B-16). Since the product $[K_p][A]$ was determined in Part III and stored, it is not necessary to perform this operation again in this part.

Part V - Internal Forces and Displacement-Output. The only purpose of this part of the program is to punch the internal force matrix [B] and the flexibility matrix $[F_d]$ for either type of analysis.

Analysis of the Test Panel

The use of the Matrix Force/Displacement Program is illustrated by analyzing Panel Configuration I (Figure A-1) by both the force method and the displacement method. The other panel configurations could not be analyzed because of the size limitations of the program. The results are compared with those calculated by the direct stiffness method.

An exploded view of the panel is shown in Figure B-1. The independent internal forces selected for the analyses are also shown. There are twenty elements, forty-four internal forces, and five reactions.

Force Analysis. The first requirement of a force analysis is to determine the number of redundants. This number is given by the equation

$$R = P + E - D \tag{B-24}$$

where R = number of redundants.

P = number of independent internal forces.

E = number of external reactions.

D = number of nodal degrees of freedom. For Panel Configuration I, there are twelve nodes, each with two degrees of freedom.

therefore $R = 44 + 5 - 2 \times 12 = 25$

The twenty-five redundants must now be chosen. Rather than attempting to define a statically determinate structure by removing selected internal or external constraints, Argyris' method (Reference 1) of self-equilibrating redundant force systems will be used. Figure C-2 shows the four types of systems selected for this analysis. All possible combinations of these systems must be used to obtain the required twenty-five redundants. The number of each type of system is

Rib-Web Systems	=	12
Stringer-Web Systems	=	9
Four Panel Systems	=	2
Two Panel Systems	=	2
Total Systems	=	25

The unit redundant matrix $[B_1]$ can be determined using Figures B-1 and B-2. Each column of $[B_1]$ consists of the internal forces due to one of the redundant systems. The seventy-six nonzero elements of $[B_1]$ are given in Table B-2 along with their row and column designation.

The internal forces due to unit values of the external forces are shown in Figure B-3. Each of the two systems of forces is statically equivalent to its corresponding unit external load. The nonzero elements of $[B_0]$ are given in Table B-2 with their row and column desigration. Column 1 corresponds to the outboard load and column 2 to the inboard load.

The element flexibilities were determined by inverting the stiffnesses. The calculation of the stiffnesses is discussed in the following section. The flexibilities are shown in Table B-2 in the format in which they are stored in the computer. Although all of the ribs and stringers are followed by the plate elements in this example, any order could have been used.

The results of the analysis are also shown in Table B-2. The two columns labeled "Internal Forces" are the matrix [B] and the "Displacements" are the matrix $[F_d]$. These results are discussed in a later section.

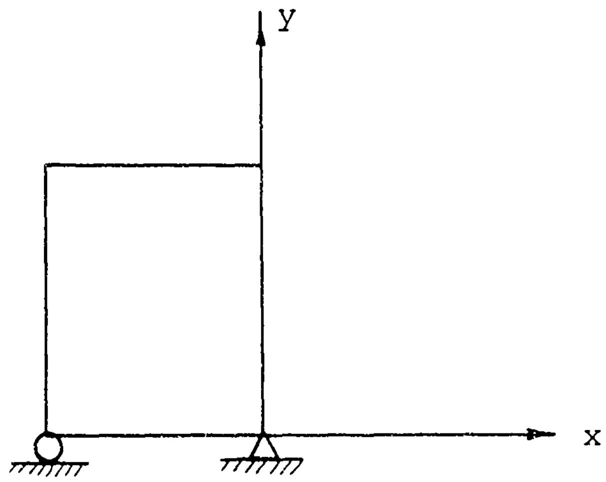
Displacement Analysis. The number of kinematic deficiencies for a displacement analysis is given by the equation

$$K = D - F - E \tag{B-25}$$

where K = number of kinematic deficiencies.
D = number of nodal degrees of freedom.
F = number of external forces.
E = number of external reactions.

therefore $K = 2 \times 12 - 2 - 5 = 17$

Each unconstrained nodal displacement must be given a unit value, and the corresponding element deformations must be obtained to determine the matrix $[A_1]$. Figure B-4 shows the required displacements for the example. The unit displacements at the loads (**) are also shown. The deformations resulting from these displacements are used to determine the matrix [A_0]. The reference system used to determine the plate element deformations is shown below. The X-axis is fixed to the lower edge of the plate and thy Y-axis remains perpendicular to the X-axis. This system is consistant with the independent internal forces



(Figure B-1). The nonzero elements of [A_1] and [A_0] are shown in Table B-3.

The rib and stringer stiffnesses were determined from the equation

$$K_g = AE/L \tag{B-26}$$

The plate element stiffnesses were calculated from the same rectangular plate stiffness matrix used in DORA. The 8x8 singular stiffness matrix was reduced to a 5x5 nonsingular matrix by striking out the rows and columns corresponding to the constraints shown in the sketch above. The plate flexibility matrices used in the force analysis were obtained by inverting the stiffness matrices. The element stiffnesses are tabulated in Table B-3.

The results are tabulated in Table B-3. As in the force analysis, the unit internal force matrix [B] is labeled "Internal Forces" and the flexibility matrix [F_d] is labeled "Displacements".

Comparison of Results

A comparison of both the "Internal Forces" and the "Displacements" calculated by the force and displacement methods shows that the maximum difference is one digit in the seventh significant digit. This difference can be considered to be negligible.

A comparison of the deflections at the loads can also be made between the force/displacement method and the direct stiffness method. Using the force method flexibility matrix, the displacements at the loads are

$$\begin{bmatrix} d_{outboard} \\ d_{inboard} \end{bmatrix} = 10^{-5} \begin{bmatrix} .21905393 & .12551318 \\ .12551320 & .1986546 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

$$= 10^{-5} \begin{bmatrix} .17228355 \\ .16198933 \end{bmatrix}$$

The above results agree with V_{10} and V_{11} of Table A-2 through the sixth significant digit. Although the difference here is greater than the difference between the force and displacement method results, it is still considered negligible.

The Matrix Force/Displacement Program could be easily modified to include three-dimensional structures. The number of columns in $[F_V]$ would be increased and the procedures for the multiplications involving $[F_V]$ would be expanded.

The major disadvantage of the present program is that the three input matrices must be determined by hand. Preliminary investigation indicates that, at least for the displacement method, all of these matrices can be machine generated. This possibility should receive additional investigation.

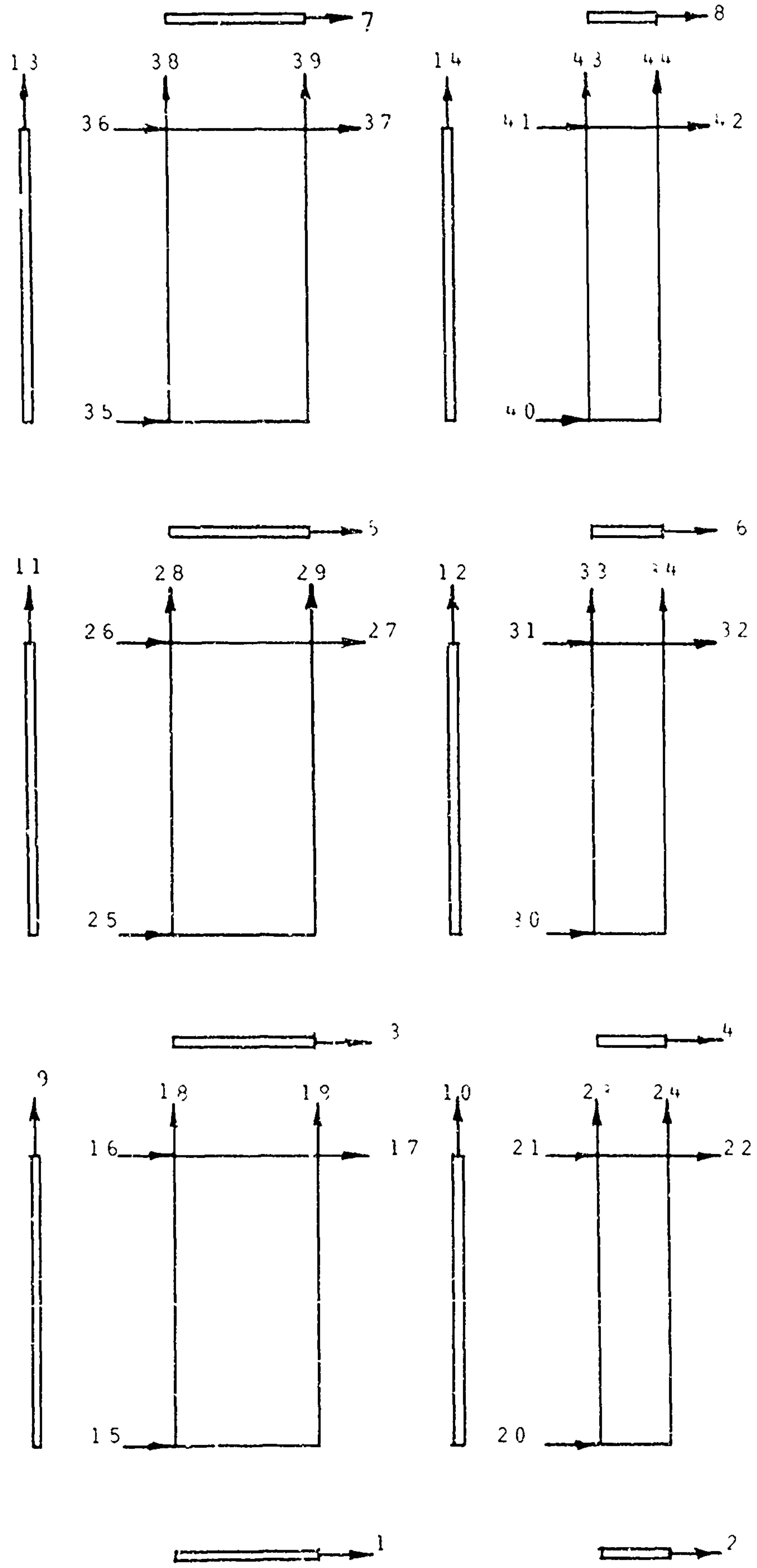


Figure B-1 Panel Elements with Internal Forces

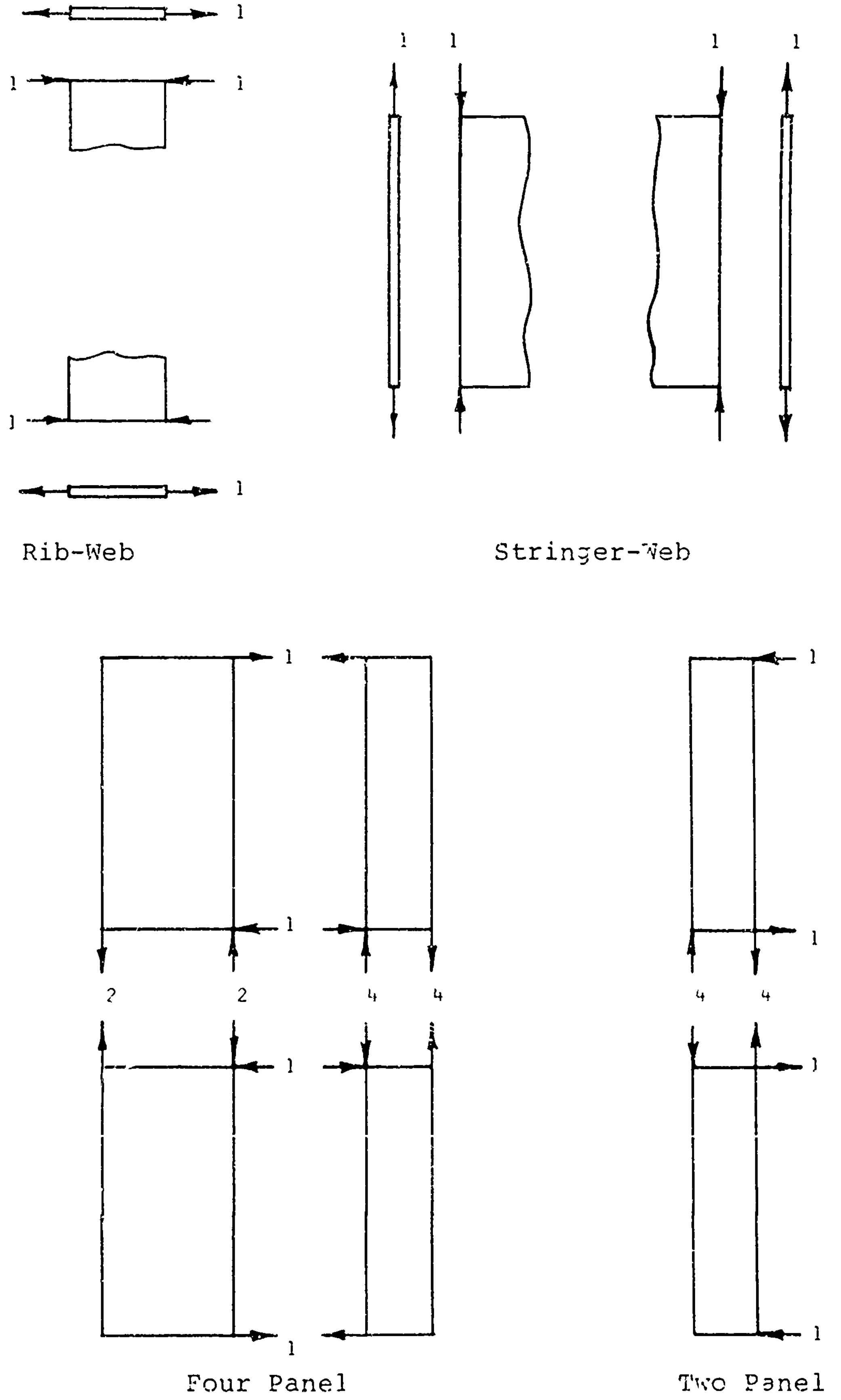


Figure B-2 Systems of Redundants for the Force Analysis

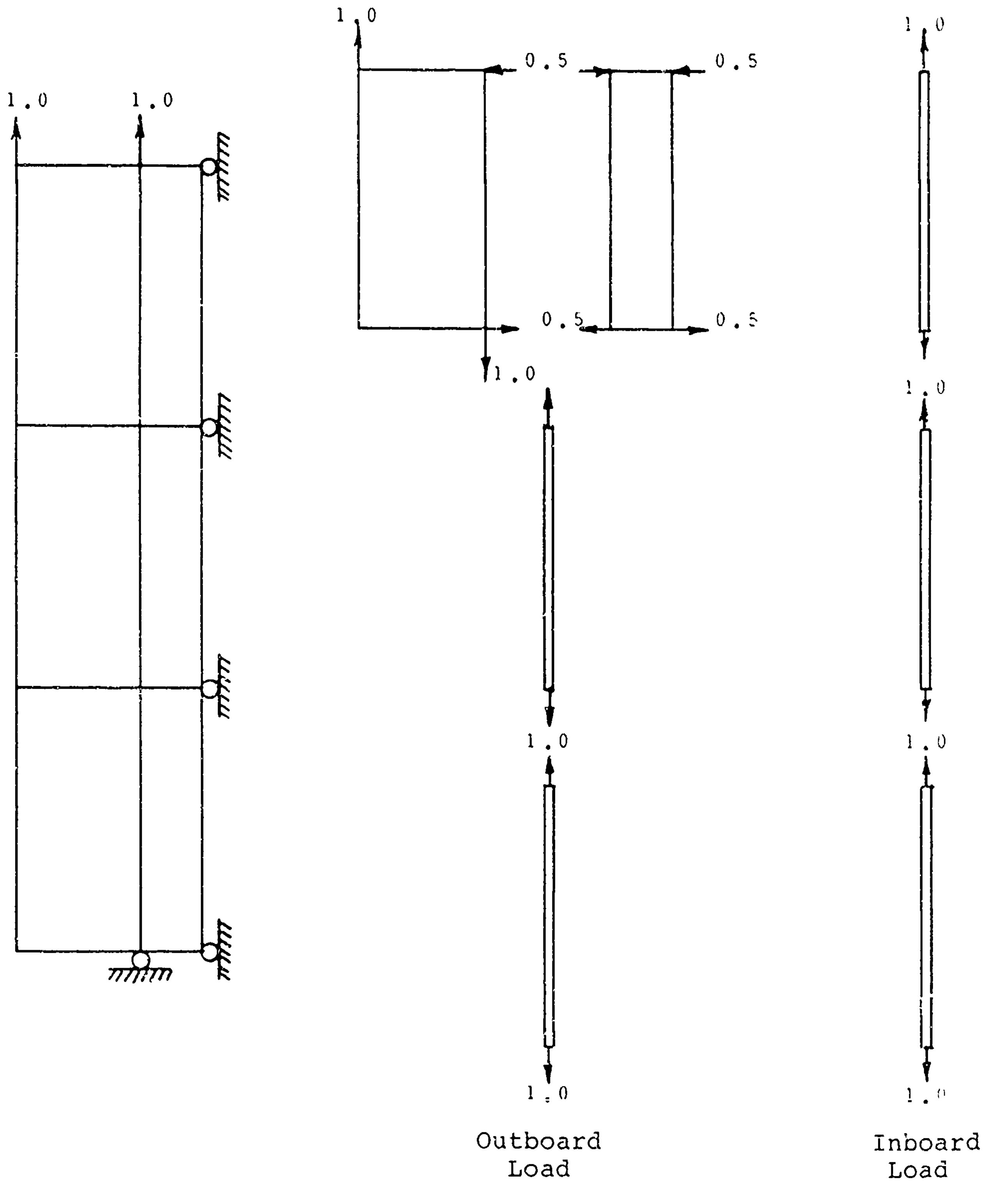


Figure B-3 External Load Systems for the Force Analysis

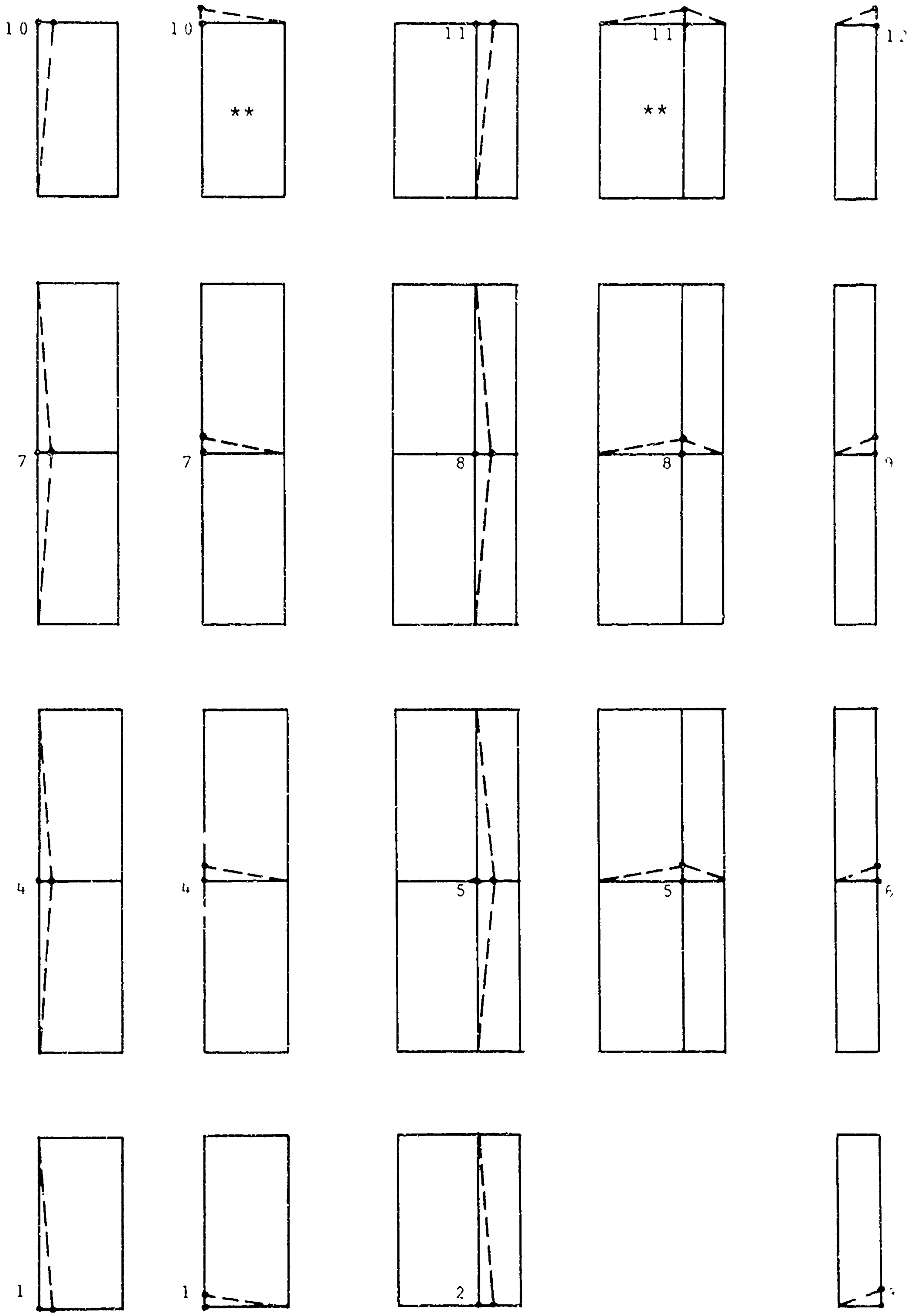


Figure B-4 Unit Displacements for the Displacement Analysis

C C

TABLE B-I

MATRIX FORCE/DISPLACEMENT PROGRAM LISTING

E.L. COOK AND G.E. LAMBERT

REVISED JULY [illegible]

THIS PROGRAM MAY BE USED TO ANALYZE STRUCTUTES BY EITHER THE MATRIX FORCE METHOD OR THE MATRIX DISPLACEMENT METHOD. THE STRUCTURE MAY HAVE ANY COMBINATION OF DIFFERENT TYPES OF ELEMENTS AS LONG AS NO ELEMENT HAS A FLEXIBILITY OR STIFFNESS MATRIX LARGER THAN FIVE-BY-FIVE.
OTHER RESTRICTIONS ARE-

1. MAXIMUM NUMBER OF ELEMENTS = 55
2. MAXIMUM NUMBER OF INTERNAL FORCES = 55
3. MAXIMUM NUMBER OF EXTERNAL FORCES = 10
4. MAXIMUM NUMBER OF EXTERNAL FORCES
   PLUS REDUNDANTS OR KINEMATIC DEFICIENCIES = 45

THE INPUTS TO THE PROGRAM ARE AS FOLLOWS-

```
************  PARAMETER CARD - NUMBER OF ELEMENTS (K), NUMBER OF
*          *  INTERNAL FORCES (L), NUMBER OF EXTERNAL FORCES (M),
*  CARD 1  *  NUMBER OF REDUNDANTS OR DEFICIENCIES (N), NUMBER OF NON-
*          *  ZERO ELEMENTS IN BO OR AO (NZEBO), NUMBER OF NONZERO
************  ELEMENTS IN BI OR AI (NZEBI), AND A DESIGNATION FOR THE
              METHOD BEING USED (KK).  FOR THE FORCE METHOD, KK=1, AND
FOR THE DISPLACEMENT METHOD, KK=2.  THESE PARAMETERS MUST BE IN FIXED
POINT NOTATION (NO DECIMAL POINTS) AND MUST BE SEPARATED BY AT LEAST
ONE BLANK, I.E.,

                         12  24  2  11  7  54  1

************  ELEMENT FLEXIBILITIES OR STIFFNESSES - THESE CARDS
*          *  CONTAIN THE ELEMENT FLEXIBILITIES OR STIFFNESSES IN A
* CARDS 2  *  SPECIAL FORMAT.  EACH CARD MUST CONTAIN FIVE NUMBERS IN
* THRU L+1 *  FLOATING POINT NOTATION (DECIMAL POINTS REQUIRED).  A
*          *  TWO-FORCE MEMBER WILL HAVE A SINGLE CARD WITH FOUR ZEROS
************  FOLLOWED BY THE FLEXIBILITY OR STIFFNESS OF THE ELEMENT-

               0.0 0.0 0.0 0.0 .66666667E-06
```

A BEAM ELEMENT(SAY PINNED AT THE ENDS WITH MOMENTS AT BOTH ENDS) WILL HAVE TWO CARDS WITH THREE ZEROS FOLLOWED BY THE ELEMENT FLEXIBILITY OR STIFFNESS, I.E.,

```
               0.0 0.0 0.0 .11111111E-06 -.05555555E-06
               0.0 0.0 0.0 -.05555555E-06 .11111111E-06
```

AND SO ON UP TO A PLATE ELEMENT WHICH WILL HAVE FIVE CARDS CONTAINING AN ELEMENT FLEXIBILITY OR STIFFNESS MATRIX OF THE FORM -

```
     .40000E-05 .29999E-05 .99999E-06 -.33333E-06 -.33333E-06
     .29999E-05 .86666E-05 .56666E-05 .29999E-05 -.29999E-05
     .99999E-06 .56666E-05 .66666E-05 .26666E-05 -.33333E-05
     -.33333E-06 .29999E-05 .26666E-05 .39999E-05 -.19999E-05
     -.33333E-06 -.29999E-05 -.33333E-05 -.19999E-05 .39999E-05
```

TABLE B-1 (CONTINUED)

```
************  NONZERO ELEMENTS IN B0 OR A0 - EACH OF THESE CARDS CONTAINS
*   NEXT   *  ONE NONZERO ELEMENT OF B0 OR A0 PRECEEDED BY NUMBERS
*  NZEB0   *  INDICATING THE ROW AND COLUMN IN WHICH IT APPEARS.
*  CARDS   *  THE ROW AND COLUMN ARE IN FIXED POINT NOTATION AND THE
************  ELEMENT IS IN FLOATING POINT NOTATION, I.E.,

               10 3 -.70710680E+01

************    NONZERO ELEMENTS IN B1 OR A1 - THESE CARDS ARE REQUIRED
*   NEXT   *  ONLY FOR STATICALLY INDETERMINATE OR KINEMATICALLY
*  NZEB1   *  DEFICIENT STRUCTURES.  THEY CONTAIN THE NONZERO ELEMENTS
*  CARDS   *  OF B1 OR A1 IN THE SAME FORMAT AS THE CARDS IN THE
************  PRECEDING GROUP.
```

TABLE B-1 (CONTINUED)

```
C     MATRIX FORCE/DISPLACEMENT PROGRAM
C
C     PART I - INPUT
C  ****************
C
C     E.L. COOK AND G.E. LAMBEPT - REVISED APRIL 1966
C
      COMMON K,L,M,N,MN,KK,FV(55,5),B01(55,45)
C
C     READ AND PUNCH PARAMETERS
C
    1 READ 100,K,L,M,N,NZEB0,NZEB1,KK
      GO TO(501,502),KK
  501 PUNCH 301
      GO TO 503
  502 PUNCH 302
  503 PUNCH 300,K,L,M,N,NZEB0,NZEB1
C
C     READ AND PUNCH FV OR KP BY ROWS, FIVE VALUES PER CARD
C
      GO TO(511,512),KK
  511 PUNCH 311
      GO TO 513
  512 PUNCH 312
  513 DO 2 I=1,L
      READ 101,FV(I,1),FV(I,2),FV(I,3),FV(I,4),FV(I,5)
    2 PUNCH 101,FV(I,1),FV(I,2),FV(I,3),FV(I,4),FV(I,5)
C
C     CLEAR B01 OR A01
C
      MN=M+N
      DO 3 I=1,L
      DO 3 J=1,MN
    3 B01(I,J)=0.0
C
C     READ AND PUNCH NONZERO ELEMENTS IN B0 OR A0
C
      GO TO(521,522),KK
  521 PUNCH 321
      GO TO 523
  522 PUNCH 322
  523 DO 4 IA=1,NZEB0
      READ 102,I,J,B01(I,J)
    4 PUNCH 102,I,J,B01(I,J)
      IF(N)14,7,530
```

```
C  C                              TABLE B-I (CONTINUED)
C       READ AND PUNCH NONZERO ELEMENTS IN B1 OR A1
C
  530 GO TO(531,532),KK
  531 PUNCH 331
      GO TO 533
  532 PUNCH 332
  533 DO 6 IA=1,NZEBI
      READ 102,I,J,B1IJ
      PUNCH 102,I,J,B1IJ
      JM=J+M
    6 B01(I,JM)=B1IJ
C
    7 ALINK=LINK(I,0)
C
   14 STOP
C
  100 FORMAT( I5)
  101 FORMAT(5E16.8)
  102 FORMAT(2I5,E16.8)
  300 FORMAT(6I5/)
  301 FORMAT(26H FORCE METHOD - PARAMETERS/)
  302 FORMAT(33H DISPLACEMENT METHOD - PARAMETERS/)
  311 FORMAT(22H ELEMENT FLEXIBILITIES/)
  312 FORMAT(20H ELEMENT STIFFNESSES/)
  321 FORMAT(/23H NONZERO ELEMENTS IN B0/)
  322 FORMAT(/23H NONZERO ELEMENTS IN A0/)
  331 FORMAT(/23H NONZERO ELEMENTS IN B1/)
  332 FORMAT(/23H NONZERO ELEMENTS IN A1/)
C
      END
```

```
C  C                                    TABLE B-1 (CONTINUED)
C     MATRIX FORCE/DISPLACEMENT PROGRAM
C
C     PART II - RECURSION ANALYSIS
C  ******************************
C
C     E.L. COOK AND G.E. LAMBERT - REVISED JULY 1966
C
      COMMON K,L,M,N,MN,KK,FV(55,5),B01(55,45)
      DIMENSION B1TFV(55),D01(45)
C
      PRINT 102
      PAUSE
C
      DO 19 IR=1,N
C
C     B1 TRANSPOSE TIMES FV OR A1 TRANSPOSE TIMES KP
C
      DO 2 I=1,L
    2 B1TFV(I)=0.0
      J=1
      DO 15 IS=1,K
      IF(FV(J,1))20,5,3
    3 J4=J+4
      DO 4 IA=1,5
      IAJ=IA+J-1
      DO 4 JA=J,J4
    4 B1TFV(IAJ)=B1TFV(IAJ)+B01(JA,MN)*FV(JA,IA)
      J=J+5
      GO TO 15
    5 IF(FV(J,2))20,8,6
    6 J3=J+3
      DO 7 IA=2,5
      IAJ=IA+J-2
      DO 7 JA=J,J3
    7 B1TFV(IAJ)=B1TFV(IAJ)+B01(JA,MN)*FV(JA,IA)
      J=J+4
      GO TO 15
    8 IF(FV(J,3))20,11,9
    9 J2=J+2
      DO 10 IA=3,5
      IAJ=IA+J-3
      DO 10 JA=J,J2
   10 B1TFV(IAJ)=B1TFV(IAJ)+B01(JA,MN)*FV(JA,IA)
      J=J+3
      GO TO 15
   11 IF(FV(J,4))20,14,12
   12 J1=J+1
      DO 13 IA=4,5
      IAJ=IA+J-4
      DO 13 JA=J,J1
   13 B1TFV(IAJ)=B1TFV(IAJ)+B01(JA,MN)*FV(JA,IA)
      J=J+2
      GO TO 15
   14 B1TFV(J)=B01(J,MN)*FV(J,5)
      J=J+1
   15 CONTINUE
```

```
C     CALCULATION OF DOI(BITFV TIMES BOI OR AITKP TIMES AOI)
      DO 16 I=I,MN
      DOI(I)=0.0
      DO 16 J=I,L
   16 DOI(I)=DOI(I)+BITFV(J)*BOI(J,I)
C
C     PUNCH INTERMEDIATE OUTPUT FOR INITIAL STRAINS
C
      IF(SENSE SWITCH 1)21,23
   21 PUNCH 101,DOI(MN)
      DO 22 I=1,L
   22 PUNCH 101,BOI(I,MN)
   23 CONTINUE
C
C     CALCULATION OF REDUNDANTS OR DEFICIENCIES(STORED IN DOI)
C
      DO 17 I=1,MN
   17 DOI(I)=-DOI(I)/DOI(MN)
C
C     CALCULATION OF NEW BOI OR AOI
C
      DO 18 I=I,L
      DO 18 J=I,MN
   18 BOI(I,J)=BOI(I,J)+BOI(I,MN)*DOI(J)
   19 MN=MN-1
      MN=M+N
C
      ALINK=LINK(1.0)
C
   20 STOP
C
  101 FORMAT(5E16.8)
  102 FORMAT(38H TURN SWITCH I ON FOR THERMAL ANALYSIS/12H PRESS START)
C
      END
```

```
C   C                               TABLE B-1 (CONTINUED)
C       MATRIX FORCE/DISPLACEMENT PROGRAM
C
C       PART III - CALCULATION OF FLEXIBILITY OR STIFFNESS MATRIX
C   *************************************************************
C
C       E.L. COOK AND G.E. LAMBERT - REVISED APRIL 1966
C
        COMMON K,L,M,N,MN,KK,FV(55,5),B(55,45),FD(10,10)
C
C       CALCULATION OR FV TIMES B OR KP TIMES A(STORED IN B)
C
        MN=M+M
        M1=M+1
        DO 25 I=1,L
        DO 25 J=M1,MN
     25 B(I,J)=0.0
        DO 15 IR=1,M
        IRM=IR+M
        J=1
        DO 15 IS=1,K
        IF(FV(J,1))20,5,3
      3 J4=J+4
        DO 4 IA=1,5
        IAJ=IA+J-1
        DO 4 JA=J,J4
      4 B(IAJ,IRM)=B(IAJ,IRM)+B(JA,IR)*FV(JA,IA)
        J=J+5
        GO TO 15
      5 IF(FV(J,2))20,8,6
      6 J4=J+3
        DO 7 IA=2,5
        IAJ=IA+J-2
        DO 7 JA=J,J4
      7 B(IAJ,IRM)=B(IAJ,IRM)+B(JA,IR)*FV(JA,IA)
        J=J+4
        GO TO 15
      8 IF(FV(J,3))20,11,9
      9 J4=J+2
        DO 10 IA=3,5
        IAJ=IA+J-3
        DO 10 JA=J,J4
     10 B(IAJ,IRM)=B(IAJ,IRM)+B(JA,IR)*FV(JA,IA)
        J=J+3
        GO TO 15
     11 IF(FV(J,4))20,14,12
     12 J4=J+1
        DO 13 IA=4,5
        IAJ=IA+J-4
        DO 13 JA=J,J4
     13 B(IAJ,IRM)=B(IAJ,IRM)+B(JA,IR)*FV(JA,IA)
        J=J+2
        GO TO 15
     14 B(J,IRM)=B(J,IR)*FV(J,5)
        J=J+1
     15 CONTINUE
```

```
C   C                                   TABLE B-1 (CONTINUED)
C       CALCULATION OF FD OR KP
C
        DO 18 I=1,M
        IM=I+M
        DO 18 J=1,M
        FD(I,J)=0.0
        DO 18 K=1,L
        FD(I,J)=FD(I,J)+B(K,IM)*B(K,J)
        IF(ABS(FD(I,J))-.10**20)17,17,18
   17   FD(I,J)=0.0
   18   CONTINUE
C
        ALINK=LINK(1.0)
C
   20   STOP
C
        END
```

```
C  C                              TABLE B-I (CONTINUED)
C       MATRIX FORCE/DISPLACEMENT PROGRAM
C
C       PART IV - INTERNAL FORCES AND DISPLACEMENTS - DISPLACEMENT METHOD
C   ****************************************************************************
C
C       E.L. COOK AND G.E. LAMBERT - REVISED APRIL 1966
C
        COMMON K,L,M,N,MN,KK,FV(55,5),B(55,45),FD(10,10)
C
        GO TO(18,600),KK
C
C       INVERSION OF KF TO OBTAIN FD
C
   600 DO 604 I=1,M
        STORE=FD(I,I)
        FD(I,I)=1.0
        DO 601 J=1,M
   601 FD(I,J)=FD(I,J)/STORE
        DO 604 K=1,M
        IF(K-I)602,604,602
   602 STORE=FD(K,I)
        FD(K,I)=0.0
        DO 603 J=1,M
   603 FD(K,J)=FD(K,J)-STORE*FD(I,J)
   604 CONTINUE
C
C       CALCULATION OF B
C
        DO 41 I=1,L
        DO 41 J=1,M
        B(I,J)=0.0
        DO 41 K=1,M
        IM=K+M
    41 B(I,J)=B(I,J)+B(I,IM)*FD(K,J)
C
    18 ALINK=LINK(1,0)
C
        END
```

```
C  C                                TABLE B-1 (CONTINUED)
C      MATRIX FORCE/DISPLACEMENT PROGRAM
C
C      PART V - INTERNAL FORCES AND DISPLACEMENTS - OUTPUT
C  ***************************************************
C
C      E.L. COOK AND G.E. LAMBERT - REVISED APRIL 1966
C
       COMMON K,L,M,N,MN,KK,FV(55,5),B(55,45),FD(10,10)
C
C      INTERNAL FORCES
C
       PUNCH 401
       GO TO(611,621,631,641,651),M
   611 DO 612 I=1,L
   612 PUNCH 400,B(I,1)
       GO TO 700
   621 DO 622 I=1,L
   622 PUNCH 400,B(I,1),B(I,2)
       GO TO 700
   631 DO 632 I=1,L
   632 PUNCH 400,B(I,1),B(I,2),B(I,3)
       GO TO 700
   641 DO 642 I=1,L
   642 PUNCH 400,B(I,1),B(I,2),B(I,3),B(I,4)
       GO TO 700
   651 DO 652 I=1,L
   652 PUNCH 400,B(I,1),B(I,2),B(I,3),B(I,4),B(I,5)
C
C      DISPLACEMENTS
C
   700 PUNCH 402
       GO TO(711,721,731,741,751),M
   711 DO 712 I=1,M
   712 PUNCH 400,FD(I,1)
       GO TO 800
   721 DO 722 I=1,M
   722 PUNCH 400,FD(I,1),FD(I,2)
       GO TO 800
   731 DO 732 I=1,M
   732 PUNCH 400,FD(I,1),FD(I,2),FD(I,3)
       GO TO 800
   741 DO 742 I=1,M
   742 PUNCH 400,FD(I,1),FD(I,2),FD(I,3),FD(I,4)
       GO TO 800
   751 DO 752 I=1,M
   752 PUNCH 400,FD(I,1),FD(I,2),FD(I,3),FD(I,4),FD(I,5)
C
   800 ALINK=LINK(1.0)
C
   400 FORMAT(5E16.8)
   401 FORMAT(/16H INTERNAL FORCES/)
   402 FORMAT(/14H DISPLACEMENTS/)
C
       END
```

```
C  C                               TABLE B-1 (CONTINUED)
C      MATRIX FORCE/DISPLACEMENT PROGRAM
C
C      PART VI - INITIAL FORCES
C   **************************
C
C      E.L. COOK - JULY 1966
C
       COMMON K,L,M,N
       DIMENSION BH(55,55),BI(55)
C
       DO 10 I=I,L
       DO 10 J=I,L
    10 BH(I,J)=0.0
       DO 14 IA=1,N
       READ 101,D11
       DO 12 J=1,L
    12 READ 101,B1(J)
       DO 14 I=1,L
       DO 14 J=1,L
    14 BH(I,J)=BH(I,J)-B1(I)*B1(J)/D11
       DO 16 I=1,L
       DO 16 J=1,L
    16 PUNCH 101,BH(I,J)
C
   101 FORMAT(5E16.8)
C
       END
```

C C

TABLE B-2

FORCE ANALYSIS - PANEL CONFIGURATION 1

FORCE METHOD - PARAMETERS

20 44 2 25 10 76

ELEMENT FLEXIBILITIES

.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.13289036E-05
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.66445180E-06
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.13289036E-05
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.66445180E-06
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.13289036E-05
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.66445180E-06
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.13289036E-05
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.66445180E-06
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.13289036E-05
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.13289036E-05
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.13289036E-05
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.13289036E-05
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.13289036E-05
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.13289036E-05
.17438655E-05	.50000000E-06	.12438654E-05	.31600000E-06	.31600000E-06
.50000000E-06	.14887080E-04	.14387081E-04	.48775398E-05	-.42455399E-05
.12438654E-05	.14387081E-04	.15630947E-04	.45615403E-05	-.45615405E-05
.31600000E-06	.48775398E-05	.45615403E-05	.42807699E-05	-.28076989E-06
.31600000E-06	-.42455399E-05	-.45615405E-05	-.28076989E-06	.42807699E-05
.91097958E-06	.24999967E-06	.66097930E-06	.31599989E-06	.31599999E-06
.24999967E-06	.37481823E-04	.37231824E-04	.69919585E-05	-.63599639E-05
.66097930E-06	.37231824E-04	.37892806E-04	.66759589E-05	-.66759643E-05
.31599989E-06	.69919585E-05	.66759589E-05	.56689901E-05	.23310084E-05
.31599999E-06	-.63599639E-05	-.66759643E-05	.23310084E-05	.56689920E-05
.17438655E-05	.50000000E-06	.12438654E-05	.31600000E-06	.31600000E-06
.50000000E-06	.14887080E-04	.14387081E-04	.48775398E-05	-.42455399E-05
.12438654E-05	.14387081E-04	.15630947E-04	.45615403E-05	-.45615405E-05
.31600000E-06	.48775398E-05	.45615403E-05	.42807699E-05	-.28076989E-06
.31600000E-06	-.42455399E-05	-.45615405E-05	-.28076989E-06	.42807699E-05
.91097958E-06	.24999967E-06	.66097930E-06	.31599989E-06	.31599999E-06
.24999967E-06	.37481823E-04	.37231824E-04	.69919585E-05	-.63599639E-05
.66097930E-06	.37231824E-04	.37892806E-04	.66759589E-05	-.66759643E-05
.31599989E-06	.69919585E-05	.66759589E-05	.56689901E-05	.23310084E-05
.31599999E-06	-.63599639E-05	-.66759643E-05	.23310084E-05	.56689920E-05
.17438655E-05	.50000000E-06	.12438654E-05	.31600000E-06	.31600000E-06
.50000000E-06	.14887080E-04	.14387081E-04	.48775398E-05	-.42455399E-05
.12438654E-05	.14387081E-04	.15630947E-04	.45615403E-05	-.45615405E-05
.31600000E-06	.48775398E-05	.45615403E-05	.42807699E-05	-.28076989E-06
.31600000E-06	-.42455399E-05	-.45615405E-05	-.28076989E-06	.42807699E-05
.91097958E-06	.24999967E-06	.66097930E-06	.31599989E-06	.31599999E-06
.24999967E-06	.37481823E-04	.37231824E-04	.69919585E-05	-.63599639E-05
.66097930E-06	.37231824E-04	.37892806E-04	.66759589E-05	-.66759643E-05
.31599989E-06	.69919585E-05	.66759589E-05	.56689901E-05	.23310084E-05
.31599999E-06	-.63599639E-05	-.66759643E-05	.23310084E-05	.56689920E-05

C C

TABLE B-? (CONTINUED)

NONZERO ELEMENTS IN B0

10	1	.10000000E 01
12	1	.10000000E 01
37	1	-.50000000E 00
38	1	.10000000E 01
40	1	-.50000000E 00
41	1	.50000000E 00
42	1	-.50000000E 00
10	2	.10000000E 01
12	2	.10000000E 01
14	2	.10000000E 01

NONZERO ELEMENTS IN B1

1	1	.10000000E 01
15	1	.10000000E 01
2	2	.10000000E 01
20	2	.10000000E 01
3	3	.10000000E 01
25	3	.10000000E 01
4	4	.10000000E 01
30	4	.10000000E 01
5	5	.10000000E 01
35	5	.10000000E 01
6	6	.10000000E 01
40	6	.10000000E 01
3	7	.10000000E 01
16	7	.10000000E 01
17	7	-.10000000E 01
4	8	.10000000E 01
21	8	.10000000E 01
22	8	-.10000000E 01
5	9	.10000000E 01
26	9	.10000000E 01
27	9	-.10000000E 01
6	10	.10000000E 01
31	10	.10000000E 01
32	10	-.10000000E 01
7	11	.10000000E 01
36	11	.10000000E 01
37	11	-.10000000E 01
8	12	.10000000E 01
41	12	.10000000E 01
42	12	-.10000000E 01
9	13	.10000000E 01
18	13	-.10000000E 01
10	14	.10000000E 01
23	14	-.10000000E 01
11	15	.10000000E 01
28	15	-.10000000E 01
12	16	.10000000E 01
33	16	-.10000000E 01
13	17	.10000000E 01
38	17	-.10000000E 01
14	18	.10000000E 01
43	18	-.10000000E 01

C C TABLE B-2 (CONTINUED)

10	19	.10000000E 01
19	19	-.10000000E 01
12	20	.10000000E 01
29	20	-.10000000E 01
14	21	.10000000E 01
39	21	-.10000000E 01
17	22	-.10000000E 01
18	22	.20000000E 01
19	22	-.20000000E 01
20	22	-.10000000E 01
21	22	.10000000E 01
23	22	-.40000000E 01
24	22	.40000000E 01
27	22	.10000000E 01
30	22	.10000000E 01
31	22	-.10000000E 01
27	23	-.10000000E 01
28	23	.20000000E 01
29	23	-.20000000E 01
30	23	-.10000000E 01
31	23	.10000000E 01
33	23	-.40000000E 01
34	23	.40000000E 01
37	23	.10000000E 01
40	23	.10000000E 01
41	23	-.10000000E 01
22	24	.10000000E 01
23	24	-.40000000E 01
24	24	.40000000E 01
32	24	-.10000000E 01
32	25	.10000000E 01
33	25	-.40000000E 01
34	25	.40000000E 01
42	25	-.10000000E 01

C C

INTERNAL FORCES

.19722703E-01	.13555068E-01
.48524270E-01	.37084025E-01
-.57857788E-01	-.64229601E-01
-.79863338E-01	-.10437724E 00
-.16905167E-01	-.64229588E-01
.24894030E-01	-.10437719E 00
-.89653643E-01	.13555014E-01
-.12518682E 00	.37084137E-01
.11632817E 00	.93706815E-01
.50709291E 00	.53004589E 00
.34600593E 00	.20810741E 00
.31432608E 00	.43336064E 00
.60246070E 00	.93706822E-01
.12306753E 00	.53004593E 00
.19722703E-01	.13555068E-01
-.10884170E 00	-.84239114E-01
-.61164867E-01	-.55106279E-01
.22368496E 00	.18498399E 00
-.24874727E-02	.33410857E-01
-.12148229E 00	-.10226137E 00
-.33272969E-01	-.78377421E-02
.56951259E-01	.31159175E-01
.60668359E-01	.64566851E-01
.94713037E-01	.93285627E-01
.50983918E-01	.20009516E-01
-.94638121E-01	.20009494E-01
-.29107679E-01	-.20009506E-01
.24149878E 00	.70583396E-01
-.15221922E-01	.13620602E 00
-.32959770E-03	.42805897E-01
-.69947840E-01	.42805857E-01
.52983390E-01	-.42805858E-01
.86535900E-01	.58456955E-01
.26855220E-01	.93285599E-01
.77732955E-01	-.84239085E-01
-.89653643E-01	.13555014E-01
-.11659404E 00	.12579042E 00
.39753934E 00	-.93706822E-01
-.15461664E 00	.31210171E 00
.12340027E-01	-.78376081E-02
.81060831E-01	-.10226128E 00
-.87774645E-01	.78939869E-01
.31549070E-01	.15785242E 00
.00000000E-50	.00000000E-50

DISPLACEMENTS

.21905393E-05	.12551318E-05
.12551320E-05	.19846546E-05

C C

TABLE B-3

DISPLACEMENT ANALYSIS - PANEL CONFIGURATION I

DISPLACEMENT METHOD - PARAMETERS

20 44 2 17 5 84

ELEMENT STIFFNESSES

.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.75250000E 06
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.15050000E 07
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.75250000E 06
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.15050000E 07
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.75250000E 06
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.15050000E 07
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.75250000E 06
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.15050000E 07
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.75250000E 06
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.75250000E 06
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.75250000E 06
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.75250000E 06
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.75250000E 06
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.75250000E 06
.80394546E 06	.30698789E 06	-.40197270E 06	.72210671E 04	-.18274854E 06
.30698789E 06	.80394546E 06	-.70896066E 06	-.18274854E 06	.72210671E 04
-.40197270E 06	-.70896066E 06	.80394546E 06	-.72210671E 04	.18274854E 06
.72210671E 04	-.18274854E 06	-.72210671E 04	.43844839E 06	-.16071504E 06
-.18274854E 06	.72210671E 04	.18274854E 06	-.16071504E 06	.43844839E 06
.15129061E 07	.70896070E 06	-.75645310E 06	.72210671E 04	-.18274854E 06
.70896070E 06	.15129061E 07	-.14654137E 07	-.18274854E 06	.72210671E 04
-.75645310E 06	-.14654137E 07	.15129061E 07	-.72210671E 04	.18274854E 06
.72210671E 04	-.18274854E 06	-.72210671E 04	.59916340E 06	-.46029672E 06
-.18274854E 06	.72210671E 04	.18274854E 06	-.46029672E 06	.59916340E 06
.80394546E 06	.30698789E 06	-.40197270E 06	.72210671E 04	-.18274854E 06
.30698789E 06	.80394546E 06	-.70896066E 06	-.18274854E 06	.72210671E 04
-.40197270E 06	-.70896066E 06	.80394546E 06	-.72210671E 04	.18274854E 06
.72210671E 04	-.18274854E 06	-.72210671E 04	.43844839E 06	-.16071504E 06
-.18274854E 06	.72210671E 04	.18274854E 06	-.16071504E 06	.43844839E 06
.15129061E 07	.70896070E 06	-.75645310E 06	.72210671E 04	-.18274854E 06
.70896070E 06	.15129061E 07	-.14654137E 07	-.18274854E 06	.72210671E 04
-.75645310E 06	-.14654137E 07	.15129061E 07	-.72210671E 04	.18274854E 06
.72210671E 04	-.18274854E 06	-.72210671E 04	.59916340E 06	-.46029672E 06
-.18274854E 06	.72210671E 04	.18274854E 06	-.46029672E 06	.59916340E 06
.80394546E 06	.30698789E 06	-.40197270E 06	.72210671E 04	-.18274854E 06
.30698789E 06	.80394546E 06	-.70896066E 06	-.18274854E 06	.72210671E 04
-.40197270E 06	-.70896066E 06	.80394546E 06	-.72210671E 04	.18274854E 06
.72210671E 04	-.18274854E 06	-.72210671E 04	.43844839E 06	-.16071504E 06
-.18274854E 06	.72210671E 04	.18274854E 06	-.16071504E 06	.43844839E 06
.15129061E 07	.70896070E 06	-.75645310E 06	.72210671E 04	-.18274854E 06
.70896070E 06	.15129061E 07	-.14654137E 07	-.18274854E 06	.72210671E 04
-.75645310E 06	-.14654137E 07	.15129061E 07	-.72210671E 04	.18274854E 06
.72210671E 04	-.18274854E 06	-.72210671E 04	.59916340E 06	-.46029672E 06
-.18274854E 06	.72210671E 04	.18274854E 06	-.46029672E 06	.59916340E 06

C C TABLE B-3 (CONTINUED)

NONZERO ELEMENTS IN A0

13	1	.10000000E 01
38	1	.10000000E 01
14	2	.10000000E 01
39	2	.10000000E 01
43	2	.10000000E 01

NONZERO ELEMENTS IN A1

1	1	-.10000000E 01
15	1	.10000000E 01
9	2	-.10000000E 01
16	2	-.20000000E 01
17	2	-.20000000E 01
18	2	-.10000000E 01
1	3	.10000000E 01
2	3	-.10000000E 01
15	3	-.10000000E 01
16	3	-.10000000E 01
17	3	-.10000000E 01
20	3	.10000000E 01
21	4	.40000000E 01
22	4	.40000000E 01
24	4	-.10000000E 01
3	5	-.10000000E 01
16	5	.10000000E 01
25	5	.10000000E 01
9	6	.10000000E 01
11	6	-.10000000E 01
18	6	.10000000E 01
[illegible]	6	-.20000000E 01
27	6	-.20000000E 01
28	6	-.10000000E 01
3	7	.10000000E 01
4	7	-.10000000E 01
17	7	.10000000E 01
21	7	.10000000E 01
25	7	-.10000000E 01
26	7	-.10000000E 01
27	7	-.10000000E 01
30	7	.10000000E 01
10	8	.10000000E 01
12	8	-.10000000E 01
19	8	.10000000E 01
23	8	.10000000E 01
26	8	.20000000E 01
27	8	.20000000E 01
29	8	-.10000000E 01
31	8	-.40000000E 01
32	8	-.40000000E 01
33	8	-.10000000E 01
24	9	.10000000E 01
31	9	.40000000E 01
32	9	.40000000E 01
34	9	-.10000000E 01

C	C	TABLE B-3 (CONTINUED)
5	10	-.10000000E 01
26	10	.10000000E 01
35	10	.10000000E 01
11	11	.10000000E 01
13	11	-.10000000E 01
28	11	.10000000E 01
36	11	-.20000000E 01
37	11	-.20000000E 01
38	11	-.10000000E 01
5	12	.10000000E 01
6	12	-.10000000E 01
27	12	.10000000E 01
31	12	.10000000E 01
35	12	-.10000000E 01
36	12	-.10000000E 01
37	12	-.10000000E 01
40	12	.10000000E 01
12	13	.10000000E 01
14	13	-.10000000E 01
29	13	.10000000E 01
33	13	.10000000E 01
36	13	.20000000E 01
37	13	.20000000E 01
39	13	-.10000000E 01
41	13	-.40000000E 01
42	13	-.40000000E 01
43	13	-.10000000E 01
34	14	.10000000E 01
41	14	.40000000E 01
42	14	.40000000E 01
44	14	-.10000000E 01
7	15	-.10000000E 01
36	15	.10000000E 01
7	16	.10000000E 01
8	16	-.10000000E 01
37	16	.10000000E 01
41	16	.10000000E 01
44	17	.10000000E 01

C C TABLE B-3 (CONTINUED)

INTERNAL FORCES

.19722669E-01	.13555044E-01
.48524245E-01	.37084015E-01
-.57857780E-01	-.64229596E-01
-.79863380E-01	-.10437723E 00
-.16905155E-01	-.64229580E-01
.24894110E-01	-.10437721E 00
-.89653650E-01	.13555120E-01
-.12518663E 00	.37084060E-01
.11632805E 00	.93706734E-01
.50709297E 00	.53004577E 00
.34600586E 00	.20810740E 00
.31432610E 00	.43336056E 00
.60246065E 00	.93705820E-01
.12306755E 00	.53004587E 00
.19722711E-01	.13555080E-01
-.10884170E 00	-.84239126E-01
-.61164824E-01	-.55106275E-01
.22368487E 00	.18498394E 00
-.24874490E-02	.33410889E-01
-.12148231E 00	-.10226139E 00
-.33273048E-01	-.78378230E-02
.56951298E-01	.31159214E-01
.60668326E-01	.64566807E-01
.94713022E-01	.93285584E-01
.50983897E-01	.20009525E-01
-.94638130E-01	.20009470E-01
-.29107669E-01	-.20009491E-01
.24149880E 00	.70583420E-01
-.15222030E-01	.13620587E 00
-.32953000E-03	.42806090E-01
-.69947790E-01	.42806190E-01
.52983370E-01	-.42806150E-01
.86535826E-01	.58456838E-01
.26855288E-01	.93285660E-01
.77732990E-01	-.84239030E-01
-.89653670E-01	.13555060E-01
-.11659407E 00	.12579024E 00
.39753937E 00	-.93706730E-01
-.15461661E 00	.31210163E 00
.12340156E-01	-.78378750E-02
.81060900E-01	-.10226148E 00
-.87774710E-01	.78940080E-01
.31549050E-01	.15785242E 00
-.19122933E-07	-.23611934E 07

DISPLACEMENTS

.21905389E-05	.12551317E-05
.12551316E-05	.19846539E-05

APPENDIX C

RECURRENCE METHOD FOR INITIAL STRAINS

Pestel and Leckie have developed the matrix force equations for structures with initial strains in Reference 5, Section 9-3. They have not, however, presented a recurrence method for solving these equations. The following pages present the derivation of such a method and an example of its use.

The basic equations for the internal forces and the displacements in the absence of external loads are

$$\{p\} = -[B_1][D_{11}]^{-1}[B_1]^T\{h\} \qquad \text{(C-1)}$$

$$\{d\} = [B]^T\{h\} \qquad \text{(C-2)}$$

where $\{p\}$ = the internal forces due to the initial strains.

$\{d\}$ = the displacements at the external load points due to initial strains.

$\{h\}$ = the deformations in the unassembled structure due to the initial strains.

$[B_1]$ = the internal forces in the structure due to unit values of the redundants. This is the same as the unit redundant matrix used in an isothermal analysis.

$[D_{11}]$ = the matrix of displacements at the redundants for unit values of the redundants. This matrix is also used in the isothermal analysis.

$[B]$ = the internal forces due to unit values of the external forces. This matrix is one of the primary results of an isothermal analysis.

The only quantity in Equations (C-1) and (C-2) that is not available from an isothermal analysis, when the recurrence method is used, is the matrix $[D_{11}]$. This means that $[D_{11}]$ must either be calculated for thermal analyses or that a recurrence method must be derived which obviates the need for $[D_{11}]$. The equations which can be used to derive the recurrence equations are

$$\{p\} = [B_1]\{x\} \qquad \text{(C-3)}$$

$$\{v\} = \{h\} + [F_v]\{p\} \qquad \text{(C-4)}$$

$$[B_1]^T\{v\} = \{0\} \qquad \text{(Compatibility)} \qquad \text{(C-5)}$$

where $\{x\}$ = the redundants due to the initial strains.

$\{v\}$ = the deformations at the internal forces.

$[F_v]$ = the flexibility matrix for the unassembled structure.

As in the isothermal recurrence method, the n redundants are eliminated one at a time with n recursions. Assume that Equation (C-3) can be rewritten as

$$\{p\} = [B_0^i]\{f^i\} + \{B_1^i\}\,x^i + [B_h^i]\{h\}$$

For the first recursion

$$\{p\} = [B_0^I]\{f^I\} + \{B_1^I\}\,x^I + [B_h^I]\{h\} \qquad \text{(C-6)}$$

where $[B_0^I]$ = first n-1 columns of actual $[B_1]$.

$\{B_1^I\}$ = last column of actual $[B_1]$.

$[B_h^I]$ = first approximation of initial stress matrix $[B_h]$.

$\{f^I\}$ = all redundants except the first one to be eliminated.

x^I = the first redundant to be eliminated.

n = the number of redundants.

$[B_h^I] = [0]$ since the structure is originally assumed to be determinate.

Substituting Equation (C-6) in Equation (C-4)

$$\{v^I\} = \{h\} + [F_v](\{[B_0^I]\{f^I\} + \{B_1^I\}\,x^I + [B_h^I]\{h\}) \qquad \text{(C-7)}$$

The relative displacement at the first redundant x^I can be eliminated by using Equation (C-7) in Equation (C-5) giving

$$\{B_1^I\}^T\{h\} + \{D_{10}^I\}^T\{f^I\} + D_{11}^I x^I + \{D_{1h}^I\}^T\{h\} = 0 \qquad \text{(C-8)}$$

where $\{D_{10}^I\}^T = \{B_1^I\}^T[F_v][B_0^I]$

$D_{11}^I = \{B_1^I\}^T[F_v]\{B_1^I\}$

$$\{D_{1h}^{I}\}^T = \{B_1^I\}^T[F_v][B_h^I] = \{0\} \quad \text{since } [B_h^I] = [0]$$

Solving Equation (C-8) for x^I

$$x^I = \{X^I\}^T\{f^I\} - \frac{1}{D_{11}^I}\{B_1^I\}^T\{h\} \tag{C-9}$$

where $\{X^I\}^T = -\frac{1}{D_{11}^I}\{D_{10}^I\}^T$

The first redundant x^I can now be eliminated from Equation (C-6) by using Equation (C-9).

$$\{p\} = ([B_0^I] + \{B_1^I\}\{X^I\}^T)\{f^I\} - (\frac{1}{D_{11}^I}\{B_1^I\}\{B_1^I\}^T)\{h\} \tag{C-10}$$

The force equation can also be written in terms of the second redundant as the unknown, giving

$$\{p\} = [B_0^{II}]\{f^{II}\} + \{B_1^{II}\}x^{II} + [B_h^{II}]\{h\} \tag{C-11}$$

Comparing Equations (C-10) and (C-11)

$[B_0^{II}]$ = the first n-2 columns of $[B_0^I] + \{B_1^I\}\{X^I\}^T$

$\{B_1^{II}\}$ = the last column of $[B_0^I] + \{B_1^I\}\{X^I\}^T$

$[B_h^{II}] = -\frac{1}{D_{11}^I}\{B_1^I\}\{B_1^I\}^T$

$\{f^{II}\}$ = the first n-2 elements of $\{f^I\}$

x^{II} = the last element of $\{f^I\}$

The $[B_0^{II}]$ and $\{B_1^{II}\}$ matrices are defined exactly as in a iso-thermal analysis with zero loads.

The second recursion can be continued by substituting Equation (C-11) in Equation (C-4).

$$\{v^{II}\} = \{h\} + [F_v]([B_0^{II}]\{f^{II}\} + \{B_1^{II}\}x^{II} + [B_h^{II}]\{h\}) \tag{C-12}$$

Now substituting Equation (C-12) in Equation (C-5)

$$\{B_1^{II}\}^T\{h\} + \{D_{10}^{II}\}^T\{f^{II}\} + D_{11}^{II}\,x^{II} + \{D_{1h}^{II}\}^T\{h\} = 0 \tag{C-13}$$

where $\{D_{10}^{II}\}^T = \{B_1^{II}\}^T[F_v][B_0^{II}]$

$D_{11}^{II} = \{B_1^{II}\}^T[F_v]\{B_1^{II}\}$

$\{D_{1h}^{II}\} = \{B_1^{II}\}^T[F_v][B_h^{II}]$

The last expression above can be shown to be zero as follows:

$$[B_0^{II}|B_1^{II}]^T[F_v][B_h^{II}]$$

$$= ([B_0^I] + \{B_1^I\}\{X^I\}^T)^T[F_v](-\frac{1}{D_{11}^I}\{B_1^I\}\{B_1^I\}^T)$$

$$= -[B_0^I]^T[F_v]\{B_1^I\}\frac{1}{D_{11}^I}\{B_1^I\}^T - \{X^I\}\{B_1^I\}^T[F_v]\{B_1^I\}\frac{1}{D_{11}^I}\{B_1^I\}^T$$

$$= -\frac{\{D_{10}^I\}}{D_{11}^I}\cdot\{B_1^I\}^T - \{X^I\}\frac{D_{11}^I}{D_{11}^I}\{B_1^I\}^T = \{X^I\}\{B_1^I\}^T - \{X^I\}\{B_1^I\}^T$$

$$= [0|0]$$

Solving Equation (C-13) for x^{II}

$$x^{II} = \{X^{II}\}^T\{f^{II}\} - \frac{1}{D_{11}^{II}}\{B_1^{II}\}^T\{h\} \tag{C-14}$$

Substituting Equation (C-14) in Equation (C-11)

$$\{p\} = ([B_0^{II}] + \{B_1^{II}\}\{X^{II}\}^T)\{f^{II}\} + ([B_h^{II}] - \frac{1}{D_{11}^I}\{B_1^{II}\}\{B_1^{II}\}^T\{h\} \tag{C-15}$$

Writing the force equation now in terms of the next redundant

$$\{p\} = [B_0^{III}]\{f^{III}\} + \{B_1^{III}\}x^{III} + [B_h^{III}]\{h\} \tag{C-16}$$

Comparing Equations (C-15) and (C-16)

$[B_0^{III}]$ = all but the last column of $[B_0^{II}] + \{B_1^{II}\}\{X^{II}\}^T$

$\{B_1^{III}\}$ = last column of $[B_0^{II}] + \{B_1^{II}\}\{X^{II}\}^T$

$$[B_h^{III}] = [B_h^{II}] - \frac{1}{D_{11}^{II}}\{B_1^{II}\}\{B_1^{II}\}^T$$

$$\{f^{III}\} = \text{all but the last element of } \{f^{II}\}$$

$$x^{III} = \text{the last element of } \{f^{II}\}$$

The third recursion can now be continued by substituting Equation (C-16) in Equation (C-4)

$$\{v^{III}\} = \{h\} + [F_v]([B_o^{III}]\{f^{III}\} + \{B_1^{III}\}x^{III} + [B_h^{III}]\{h\}) \tag{C-17}$$

Substituting Equation (C-17) in Equation (C-5)

$$\{B_1^{III}\}^T\{h\} + \{D_{10}^{III}\}^T\{f^{III}\} + D_{11}^{III}x^{III} + \{D_{1h}^{III}\}^T\{h\} = 0 \tag{C-18}$$

where
$$\{D_{10}^{III}\}^T = \{B_1^{III}\}^T[F_v][B_0^{III}]$$

$$D_{11}^{III} = \{B_1^{III}\}^T[F_v]\{B_1^{III}\}$$

$$\{D_{1h}^{III}\}^T = \{B_1^{III}\}^T[F_v][B_h^{III}]$$

The last equation above can be shown to be zero as follows:

$$[B_0^{III} | B_1^{III}]^T [F_v][B_h^{III}]$$

$$= ([B_0^{III}] + \{B_1^{II}\}\{X^{II}\}^T)^T [F_v]([B_h^{II}] - \frac{1}{D_{11}^{II}}\{B_1^{II}\}\{B_1^{II}\}^T)$$

$$= ([B_0^{II}]^T + \{X^{II}\}\{B_1^{II}\}^T)[F_v]([B_h^{II}] - \frac{1}{D_{11}^{II}}\{B_1^{II}\}\{B_1^{II}\}^T)$$

$$= [B_0^{II}]^T[F_v][B_h^{II}] + \{X^{II}\}\{B_1^{II}\}^T[F_v][B_h^{II}]$$

$$- [B_0^{II}]^T[F_v]\{B_1^{II}\}\frac{1}{D_{11}^{II}}\{B_1^{II}\}^T$$

$$- \{X^{II}\}\{B_1^{II}\}^T[F_v]\{B_1^{II}\}\frac{1}{D_{11}^{II}}\{B_1^{II}\}^T$$

It has been previously shown that

$$[B_0^{II}|B_1^{II}]^T [F_v] [B_h^{II}] = [0|0]$$

therefore, the first two terms in the above equation are zero, giving

$$[B_0^{II}|B_1^{II}]^T [F_v] [B_h^{II}]$$

$$= -[B_0^{II}]^T [F_v] \{B_1^{II}\} \frac{1}{D_{11}^{II}} \{B_1^{II}\}^T$$

$$-\{X^{II}\} \{B_1^{II}\}^T [F_v] \{B_1^{II}\} \frac{1}{D_{11}^{II}} \{B_1^{II}\}^T$$

$$= -\frac{\{D_{10}^{II}\}}{D_{11}^{II}} \{B_1^{II}\}^T - \{X^{II}\} \frac{D_{11}^{II}}{D_{11}^{II}} \{B_1^{II}\}^T$$

$$= \{X^{II}\}\{B_1^{II}\}^T - \{X^{II}\}\{B_1^{II}\}^T = [0|0]$$

Solving Equation (C-18) for x^{III}

$$x^{III} = \{X^{III}\}^T \{f^{III}\} - \frac{1}{D_{11}^{III}} \{B_1^{III}\}^T \{h\} \tag{C-19}$$

Substituting Equation (C-19) in Equation (C-16)

$$\{p\} = ([B_0^{III}] + \{B_1^{III}\}\{X^{III}\}^T)\{f^{III}\} + ([B_h^{III}]$$

$$- \frac{1}{D_{11}^{III}} \{B_1^{III}\}\{B_1^{III}\}^T)\{h\} \tag{C-20}$$

The recursions are continued until all redundants are eliminated. For instance, if there were only three redundants, the next force equation would be

$$\{p\} = [B_0^{IV}]\{f^{IV}\} + [B_1^{IV}]\{X^{IV}\} + [B_h^{IV}]\{h\} \tag{C-21}$$

Since all redundants have been eliminated

$$\{f^{IV}\} = \{X^{IV}\} = 0 \tag{C-22}$$

Therefore

$$\{p\} = [B_h^{IV}]\{h\} = ([B_h^{III}] - \frac{1}{D_{11}^{III}} \{B_1^{III}\}\{B_1^{III}\}^T)\{h\}$$

$$= ([B_h^{II}] - \frac{1}{D_{11}^{II}} \{B_1^{II}\}\{B_1^{II}\}^T - \frac{1}{D_{11}^{III}} \{B_1^{III}\}\{B_1^{III}\}^T)\{h\}$$

$$= (- \frac{1}{D_{11}^{I}}\{B_1^{I}\}\{B_1^{I}\}^T - \frac{1}{D_{11}^{II}} \{B_1^{II}\}\{B_1^{II}\}^T$$

$$- \frac{1}{D_{11}^{III}} \{B_1^{III}\}\{B_1^{III}\}^T)\{h\} \qquad (C-23)$$

It can be concluded that

$$\{p\} = (-\sum_{i=1}^{n} \frac{1}{D_{11}^{i}} \{B_1^{i}\}\{B_1^{i}\}^T)\{h\} = [B_h]\{h\} \qquad (C-24)$$

where

$$[B_h] = -\sum_{i=1}^{n} \frac{1}{D_{11}^{i}} \{B_1^{i}\}\{B_1^{i}\}^T \qquad (C-25)$$

The means for evaluating Equation (C-25) has been incorporated into the Matrix Force/Displacement Program (Appendix B) as follows:

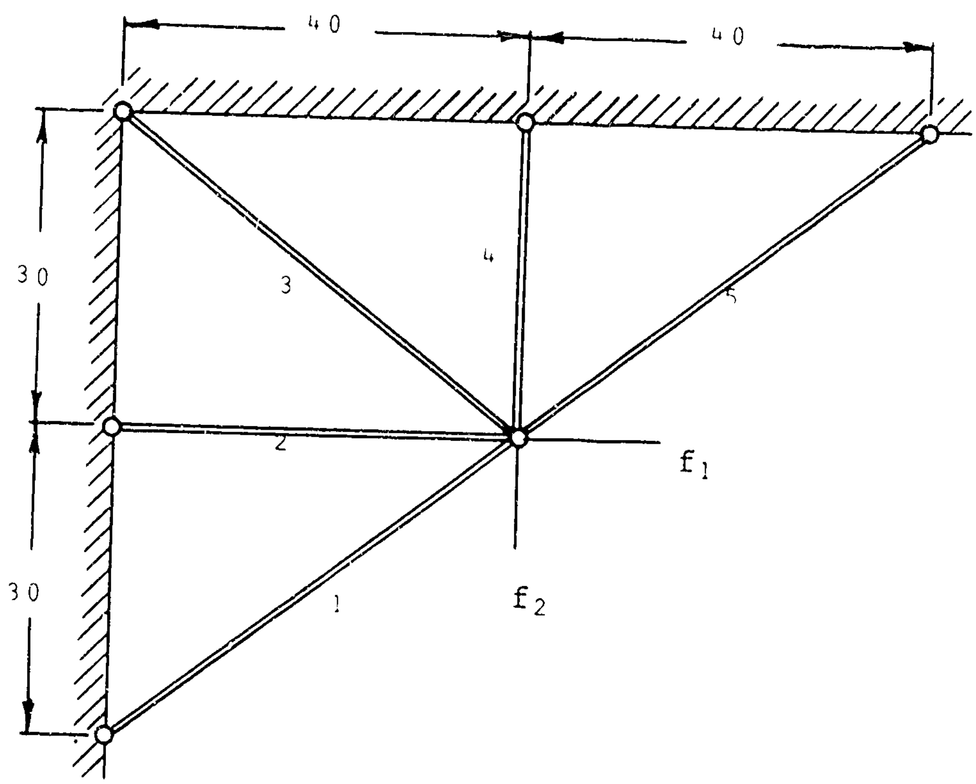
1. In Part II (Recursion Analysis), D_{11}^{i} and $\{B_1^{i}\}$ may be punched into cards during each recursion.
2. In Part VI (Initial Forces), this intermediate output is reread and $[B_h]$ is determined.

The procedure is illustrated for the simple bar structure shown in Figure C-1. The results are shown in Table C-1. The "Internal Forces" in the table are the unit load matrix [B] and can be used in Equation (C-2) to obtain the thermal displacements at the load points. The "Displacements" shown in the table are the isothermal flexibility matrix $[F_d]$ and cannot be used for thermal displacements. The "Initial Forces" are the thermal unit loads $[B_h]$ and can be used in Equation (C-24) to calculate the internal forces due to the initial strains.

As a check on the validity of the recurrance method, the same structure was also analyzed by the direct method. The program written for the check is shown in Table C-3. This program is written in 1620 Fortran II and utilizes the IBM System/360 Scientific Subroutine Package. The two special subroutines NOZERO and MATPCH are listed. The remaining ones were taken directly from the package. The results are shown in Table C-2. All of the forces and displacements agree to six significant figures.

The test panel was not used for this comparison because the dimensions in the Fortran II program could not be made large enough. The Force/Displacement Program can, however, be used for the thermal analysis of the test panel.

An initial stress analysis for the displacement method has not been incorporated into the Force/Displacement Program. Pestel and Leckie have derived an equation for this case (Reference 5, p. 314) and the displacement analogy of Equation (C-25) appears in their equation along with other known quantities. Therefore, the Force/Displacement Program can be extended to include displacement thermal analyses.



$A = 0.10$ in.^{2}

$E = 10 \times 10^6$ psi

$L/AE = L \times 10^{-6}$ in./lb.

$x_1 = p_1$, $x_2 = p_3$, and $x_3 = p_5$

Figure C-1. Structure for Initial Strain Example.

C C

TABLE C-1

EXAMPLE - INITIAL STRAINS

MATRIX FORCE/DISPLACEMENT PROGRAM

FORCE METHOD - PARAMETERS

5	5	2	3	2	9

ELEMENT FLEXIBILITIES

.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.50000000E-04
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.40000000E-04
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.50000000E-04
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.30000000E-04
.00000000E-50	.00000000E-50	.00000000E-50	.00000000E-50	.50000000E-04

NONZERO ELEMENTS IN B0

2	1	.10000000E 01
4	2	-.10000000E 01

NONZERO ELEMENTS IN B1

1	1	.10000000E 01
2	1	-.80000000E 00
4	1	.60000000E 00
2	2	-.80000000E 00
3	2	.10000000E 01
4	2	-.60000000E 00
2	3	.80000000E 00
4	3	-.60000000E 00
5	3	.10000000E 01

INTERNAL FORCES

.22524931E 00	.17908264E 00
.40503980E 00	-.70783641E-01
.29320162E 00	-.26968572E 00
.94378202E-01	-.62328940E 00
-.22524931E 00	-.17908265E 00

DISPLACEMENTS

.16201592E-04	-.28313458E-05
-.28313459E-05	.18698682E-04

INITIAL FORCES

ROW 1

```
-.14247019E 05
 .56312329E 04
 .14549974E 04
-.59694215E 04
-.57529809E 04
```

ROW 2

```
 .56312329E 04
-.14874005E 05
 .73300406E 04
 .23594551E 04
-.56312329E 04
```

ROW 3

```
 .14549974E 04
 .73300406E 04
-.12072546E 05
 .69895242E 04
-.14549973E 04
```

ROW 4

```
-.59694215E 04
 .23594551E 04
 .89895242E 04
-.12557020E 05
 .59694213E 04
```

ROW 5

```
-.57529809E 04
-.56312329E 04
-.14549973E 04
 .59694213E 04
-.14247020E 05
```

C C

TABLE C-2

EXAMPLE - INITIAL STRAINS

GENERAL MATRIX FORCE ANALYSIS PROGRAM

INTERNAL FORCES

1	.22524929E+00	.17908262E+00
2	.40503990E+00	-.70783650E-01
3	.29320158E+00	-.26968568E+00
4	.94378200E-01	-.62328950E+00
5	-.22524929E+00	-.17908263E+00

DISPLACEMENTS

1	.16201592E-04	-.28313465E-05
2	-.28313463E-05	.18698683E-04

INITIAL FORCES

1	-.14247018E+05	.56312330E+04	.14549972E+04	-.59694208E+04	-.57529805E+04
2	.56312330E+04	-.14874003E+05	.73300397E+04	.23594552E+04	-.56312320E+04
3	.14549972E+04	.73300396E+04	-.12072544E+05	.89695231E+04	-.14549974E+04
4	-.59694208E+04	.23594549E+04	.89895230E+04	-.12557018E+05	.59694209E+04
5	-.57529805E+04	-.56312320E+04	-.14549973E+04	.59694208E+04	-.14247018E+05

TABLE C-3

GENERAL MATRIX FORCE ANALYSIS PROGRAM LISTING

```
C C
C     GENERAL MATRIX FORCE ANALYSIS WITHOUT STORAGE COMPRESSION
C
C     E.L. COOK - JULY 1966
C
C     DIMENSIONS FV((LL+L)/2),B0(LM),B1(LN),B1TFV(NL),D10(NM),D11(NN),
C    1LWV(N),MWV(N),X(NM),B1X(LM),B(LM),BTFV(ML),FD(MM),B1D111(LN),
C    2B1T(NL),BH(LL),RWV(L)
C
C     NOTE - FV AND B0 ARE STORED IN BH, THEREFORE, L MUST BE EQUAL TO
C      OR GREATER THAN 1+2M.
C
      DIMENSION FV(325),B0(50),B1(125),B1TFV(125),D10(10),D11(25),
     1LWV(5),MWV(5),X(10),B1X(50),B(50),BTFV(50),FD(4),B1T(125),
     2B1D111(125),BH(625),RWV(25)
C
      EQUIVALENCE (BH(1),FV(1)),(BH(352),B0(1)),(B1TFV,B1T),(B,B0),
     1(B1D111,B1)
C
C     READ NO. OF INTERNAL FORCES, EXTERNAL FORCES, AND REDUNDANTS
    1 READ 900,L,M,N
C
C     READ NONZERO ELEMENTS IN FV (SYMMETRIC FORM)
      CALL NOZERO(FV,L,L,1)
C
C     READ NONZERO ELEMENTS IN B0 (GENERAL FORM)
      CALL NOZERO(B0,L,M,0)
C
C     READ NONZERO ELEMENTS IN B1 (GENERAL FORM)
      CALL NOZERO(B1,L,N,0)
C
C     CALCULATE B1 TRANSPOSE TIMES FV
      CALL TPRD(B1,FV,B1TFV,L,N,0,1,L)
C
C     CALCULATE D10 = B1 TRANSPOSE TIMES FV TIMES B0
      CALL GMPRD(B1TFV,B0,D10,N,L,M)
C
C     CALCULATE D11 = B1 TRANSPOSE TIMES FV TIMES B1
      CALL GMPRD(B1TFV,B1,D11,N,L,N)
C
C     INVERT D11
      CALL MINV(D11,N,DET,LWV,MWV)
C
C     CALCULATE -X = D11 INVERSE TIMES D10
      CALL GMPRD(D11,D10,X,N,N,M)
C
C     CALCULATE B1 TIMES -X
      CALL GMPRD(B1,X,B1X,L,N,M)
C
C     CALCULATE AND PUNCH B =B0 - B1(-X)
      CALL GMSUB(B0,B1X,B,L,M)
      CALL MATPCH(B,L,M,0,RWV)
C
C     CALCULATE B TRANSPOSE TIMES FV
      CALL TPRD(B,FV,BTFV,L,M,0,1,L)
```

```
C   C                           TABLE C-3 (CONTINUED)
C       CALCULATE AND PUNCH FD = B TRANSPOSE TIMES FV TIMES B
        CALL GMPRD(BTFV,B,FD,M,L,M)
        CALL MATPCH(FD,M,M,0,RWV)
C
C       CALCULATE BI TRANSPOSE
        CALL GMTRA( ,,BIT,L,N)
C
C       CALCULATE BI TIMES DII INVERSE
        CALL TPRD(BIT,DII,BIDIII,N,L,0,0,N)
C
C       CALCULATE -BH = BI TIMES DII INVERSE TIMES BI TRANSPOSE
        CALL GMPRD(BIDIII,BIT,BH,L,N,L)
C
C       CALCULATE AND PUNCH BH = -BH(-1.0)
        CALL SMPY(BH,-1.0,BH,L,L,0)
        CALL MATPCH(BH,L,L,0,RWV)
C
        GO TO 1
C
  900 FORMAT(3I3)
C
        END


        SUBROUTINE NOZERO(A,N,M,MS)
C
        DIMENSION A(1)
C
        CALL SCLA(A,0.0,N,M,MS)
        READ 900,NO
        DO 2 IA=1,NO
        READ 900,I,J,AIJ
        CALL LOC(I,J,IR,N,M,MS)
    2   A(IR)=AIJ
C
  900 FORMAT(2I3,E14.8)
C
        RETURN
C
        END
```

```
C  C                        TABLE C-3 (CONTINUED)
      SUBROUTINE MATPCH(A,N,M,MS,R)
C
      DIMENSION A(1),R(1)
C
      IF(MS-2)20,10,400
C
C     DIAGONAL FORM
   10 PUNCH 920
      PUNCH 900,(A(I),I=1,N)
      RETURN
C
C     GENERAL OR SYMMETRIC FORM
   20 IF(M-5)30,30,150
C     OUTPUT IN COLUMN FORMAT
   30 PUNCH 920
      DO 130 I=1,N
      CALL LOC(I,1,IR1,N,M,MS)
      IF(M-1)400,80,40
   40 CALL LOC(I,2,IR2,N,M,MS)
      IF(M-2)400,90,50
   50 CALL LOC(I,3,IR3,N,M,MS)
      IF(M-3)400,100,60
   60 CALL LOC(I,4,IR4,N,M,MS)
      IF(M-4)400,110,70
   70 CALL LOC(I,5,IR5,N,M,MS)
      GO TO 120
   80 PUNCH 910,I,A(IR1)
      GO TO 130
   90 PUNCH 910,I,A(IR1),A(IR2)
      GO TO 130
  100 PUNCH 910,I,A(IR1),A(IR2),A(IR3)
      GO TO 130
  110 PUNCH 910,I,A(IR1),A(IR2),A(IR3),A(IR4)
      GO TO 130
  120 PUNCH 910,I,A(IR1),A(IR2),A(IR3),A(IR4),A(IR5)
  130 CONTINUE
      RETURN
C
C     OUTPUT IN ROW FORMAT
  150 DO 170 I=1,N
      DO 160 J=1,M
      CALL LOC(I J,IR,N,M,MS)
  160 R(J)=A(IR)
      PUNCH 920
  170 PUNCH 900,(R(J),J=1,M)
      RETURN
C
  400 STOP
C
  900 FORMAT(5X,5E15.8)
  910 FORMAT(I5,5E15.8)
  920 FORMAT(1H )
C
      END
```

APPENDIX D

ANALYTICAL PREDICTION OF THE TEMPERATURE DISTRIBUTION IN AN INTEGRALLY STIFFENED PANEL

D.L. Hull

This report has to do with one phase of a research project which is to conduct an analytical and experimental analysis of an integrally stiffened panel subjected to combined mechanical and thermal loading. The phase to be studied here will be the analytical prediction of the temperature distribution which would result from heating the edges of a panel which is cooled on the upper and lower surfaces by forced convection. The panel to be studied is shown in Figure D-1.

NUMERICAL ANALYSIS

To obtain a general idea of what the effect would be of changing the geometry of the panel and the values for the coefficients of thermal conductivity and convective heat transfer, a section was taken out of the panel which was one inch in depth with the configuration shown in Figure D-2. This section was assumed to have no heat flowing from the sides or the right end (center line of the plate).

Making use of the symmetry of the panel, one-half of the section was divided into 26 elements and finite difference equations were written for the heat transfer. These equations were programmed for the IBM 1620 digital computer and solved for the temperature distribution along the length of the section. The program used was one which makes use of matrices for the solution of simultaneous equations. The equations are of the form shown below for element number one.

$$hA_i \; (T_\infty - T_i) + \frac{kA_i}{L} (T_{i+1} - T_i) + \frac{kA_i}{L} (T_{i+3} - T_i) + A_i \; (q_R) = 0$$

where: A_i is the area between adjacent elements perpendicular to a line drawn between the elements

L is the distance between the elements

T_i (i = 1 to 26) is the temperature of element i

T_∞ is ambient temperature

k is thermal conductivity BTU/hr-°F-in.

h is convective heat transfer coefficient

BTU/hr-°F-in^2

substituting the geometric variables the equation becomes

$$h \frac{B}{4} (T_\infty - T_1) - k(\frac{H}{6})(\frac{2}{B})(T_2 - T_1) - k(\frac{B}{4})(\frac{3}{H})(T_4 - T_1) - \frac{H}{6} q_R = 0$$

Rearranging, dividing by k and setting $T_\infty = 0$ gives

$$- \frac{hB}{4k} + \frac{H}{3B} + \frac{3B}{4H} T_1 + \frac{H}{3B} T_2 + \frac{3B}{4H} T_4 = - \frac{H}{6k} q_R$$

These equations can be written in the following matrix form:

$$[A]\ [T] = [R]$$

where: A is the temperature coefficients

T is the temperature

R is the applied heat

solving for T

$$[T] = [A]^{-1}[R]$$

The above equations were solved using thirteen different sets of variables. One set of variables was selected as a base and then various variables were changed and the results compared with the results from the original set. This comparison gives an indication of the effects of varying plate geometry and coefficients of thermal conductivity and convective heat transfer. These effects are illustrated in Figures D-3 through D-15. The variables used in the thirteen cases were as follows:

Set I

H = .875 in.
t = .125 in.
B = C = .6 in.
W = 3.4 in.
k = 8.0 BTU/hr-F-in.
h_3 = .167 BTU/hr-F-in.^{2}
$h_1 = h_2 = h_4 = 0$

Set II

H = .875
t = .125
B = C = .6
W = 3.4
k = 8.0
h_3 = .167
*$h_2 = h_4 = .167$
h_1 = 0

Set III

H = .875
t = .125
B = C = .6
W = 3.4
k = 8.0
h_3 = .167
*$h_1 = h_2 = h_4 = .167$

Set IV

H = .875
t = .125
B = C=.6
W = 3.4
*k = 6.0
h_3 = .167
$h_1 = h_2 = h_4 = 0$

Set V

*H = .90
*t = .10
B = C=.6
W = 3.4
k = 8.0
h_3 = .167
$h_1 = h_2 = h_4 = 0$

Set VI

*H = .750
t = .125
B = C=.6
W = 3.4
k = 8.0
h_3 = .167
$h_1 = h_2 = h_4 = 0$

Set VII

H = .875
t = .125
B = C=.6
*W = 3.0
k = 8.0
h_3 = .167
$h_1 = h_2 = h_4 = 0$

Set VIII

H = .875
t = .125
*B = C=.750
W = 3.4
k = 8.0
h_3 = .167
$h_1 = h_2 = h_4 = 0$

Set IX

H = .875
t = .125
*B = C=.50
W = 3.4
k = 8.0
h_2 = .167
$h_1 = h_2 = h_4 = 0$

Set X

H = .875
t = .125
B = C=.60
W = 3.4
k = 8.0
*h_3 = .050
$h_1 = h_2 = h_4 = 0$

Set XI

H = .875
t = .125
B = C=.60
W = 3.4
k = 8.0
*h_3 = .083
$h_1 = h_2 = h_4 = 0$

Set XII

H = .875
t = .125
B = C=.60
W = 3.4
k = 8.0
*h_3 = .100
$h_1 = h_2 = h_4 = 0$

Set XIII

H = .875
t = .125
B = C=.60
W = 3.4
k = 8.0
*h_3 = .200
$h_1 = h_2 = h_4 = 0$

* indicates the variable which has been changed.

The program used for the solution of the above sets of equations made use of matrix inversion and multiplication. This requires a large number of storage locations in the computer and a considerable amount of computer time. The original program written for this purpose had to be divided into four separate programs so as not to exceed the memory storage capacity of the 1620. The first of these programs calculates the constants to be used in the second and third programs which calculate the non-zero elements of the coefficient matrix [A]. The fourth program inverts the matrix [A] and multiplies it by the column matrix [R] (the heat added to the panel) to obtain the temperatures of the twenty-six elements. The approximate computer time for the four programs is shown below.

Program No.	Input Time	Calculation Time	Print Time	Punch Time
1	1.5 min.	.4 min.		.05 min.
2	1.5 min.	.3 min.		.4 min.
3	1.5 min.	.3 min.		.4 min.
4	2.5 min.	15.3 min.	1.1 min.	
	7.0 min.	16.3 min.	1.1 min.	.85 min.

This gives a total time for each run of approximately 25.25 minutes for each set, or a total time of approximately 5.5 hours.

The above two dimensional analysis gives a good indication of what to expect in a three dimensional analysis. Making use of the fact that the panel is symmetric about the center lines it was decided to use only one-fourth of the panel in the three dimensional analysis. As a first attempt at obtaining the temperature distribution for the quarter panel it was decided that the same system would be used that was used for the two dimensional analysis. The quarter panel was divided into 364 elements, these elements were arranged in 14 sections of two rows each with 13 elements in each row. This created 14 sets of simultaneous equations which can be solved for temperature. The temperatures obtained for a given section are dependent upon the temperatures of the adjacent sections therefore, it is necessary to make repeated iterations through all 14 sections in order to obtain the correct temperature distribution. It was found for this system that the temperatures for each section had to be punched out by the computer then used to find 14 new sets of column matrices [R] (the heat supplied to each section) then these column matrices were used to solve for a new temperature distribution in the next iteration. Due to the relative slow input/output procedure on the 1620 it required approximately 35 minutes for a complete iteration of the 364 equations and it was found after 30 iterations that it would take an estimated 100 iterations before the temperature would converge on an equilibrium set of values, the time required for this would be excessive (approximately 50 hours) therefore, it was decided to use another method to solve for the temperature distribution.

For the second attempt at solving for the temperature distribution the quarter panel was divided into 112 elements as shown in Figure D-16, and finite difference equations were written for these elements. Since the solution of a large number of simultaneous equations by the matrix method was found to be impractical on the available 1620, the Gauss-Seidel Iteration method was chosen as a means of solving the equations. Using this method and an IBM 1620 with 40,000 units of storage instead of 20,000 units it was found that it required approximately 21 seconds for each iteration. This allows a large number of iterations in a relatively short time as compared to the method used in the previous attempt. Using this system six sets of variables were chosen to investigate the effects of changing panel geometry and the convective heat transfer coefficient. These six sets are shown below. These six sets of variables required a total of 596 iterations and a total time of 3.6 hours.

Set I	Set II	Set III	Set IV
A = 7.70	*A = 9.70	A = 7.70	A = 7.70
B = 3.40	B = 3.40	B = 3.40	*B = 3.0
C = .30	C = .300	C = .30	C = .30
D = .60	D = .600	D = .60	D = .60
E = 1.0	E = 1.0	E = 1.0	E = 1.0
t = .125	t = .125	*t = .10	t = .125
Q = 100.	Q = 100.	Q = 100.	Q = 100.
k = 8.0	k = 8.0	k = 8.0	k = 8.0
h = .167	h = .167	h = .167	h = .167

Set V	Set VI
A = 7.70	A = 7.70
B = 3.40	B = 3.40
C = .30	C = .30
*D = .750	D = .750
E = 1.0	E = 1.0
t = .125	t = .125
Q = 100.	Q = 100.
k = 8.0	k = 8.0
h = .167	h = .083

The results of the three dimensional analysis for Set I are shown pictorially in Figure D-17 and the results for all cases are tabulated in Table D-1.

Using the temperatures obtained from the numerical analysis the problem of solving for the thermal stresses in the plate due to these temperature distributions will be undertaken. After this problem has been solved an experiment will be set up to obtain the actual stresses due to a temperature gradient in the panel and mechanical loads applied at various points on the panel.

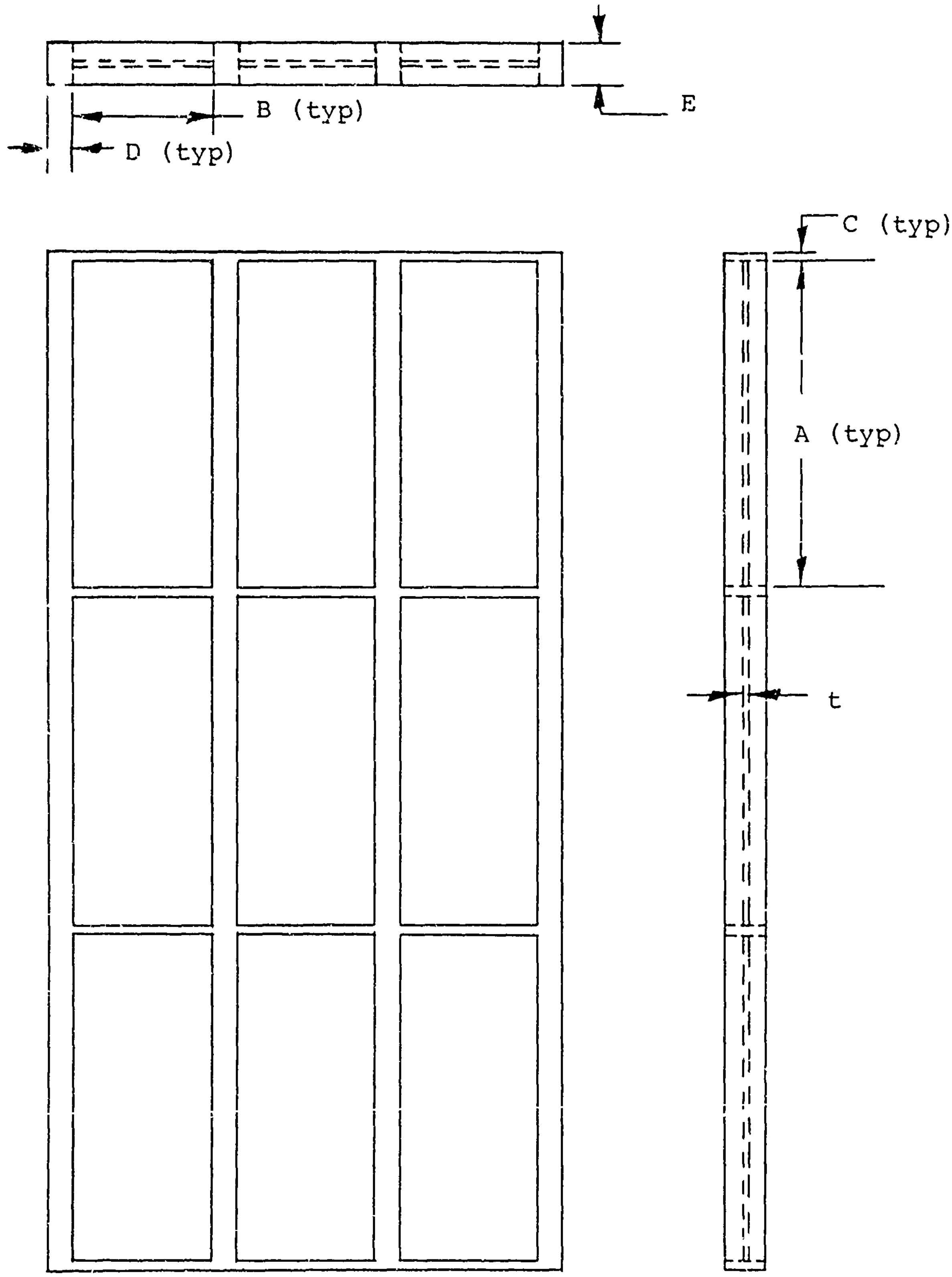


Figure D-1. Panel Geometry

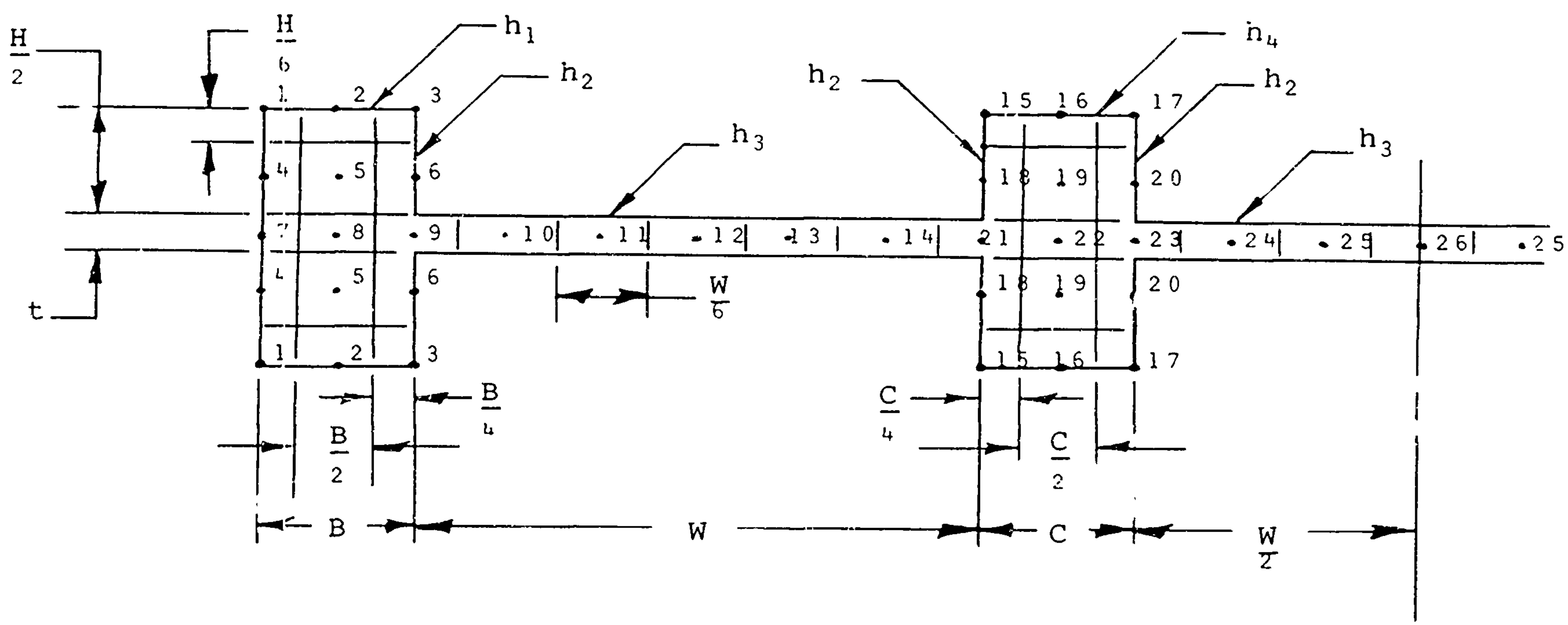


Figure D-2. Elements Used in Two-dimensional Analysis.

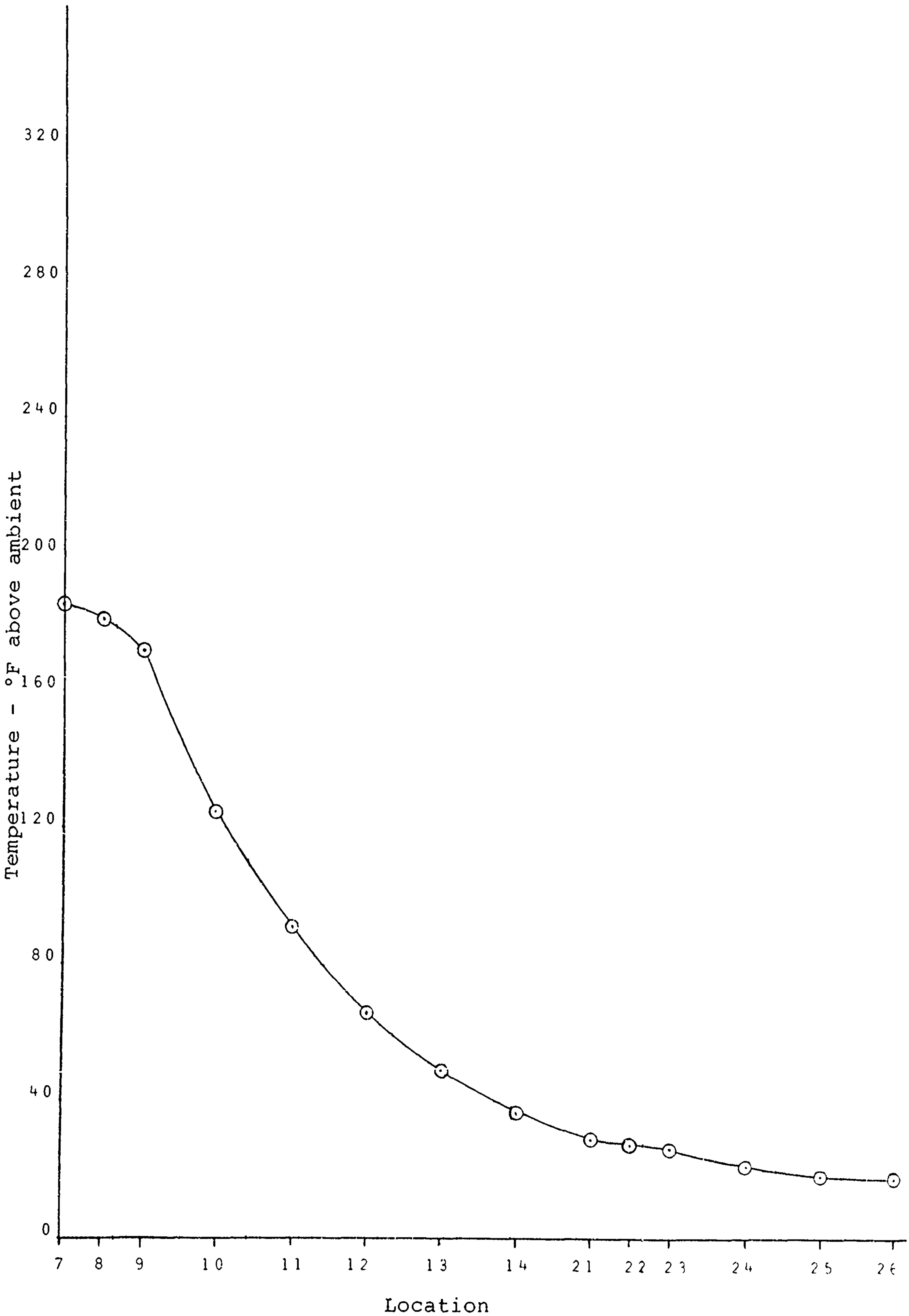


Figure D-3. Temperature Distribution for Variable Set I.

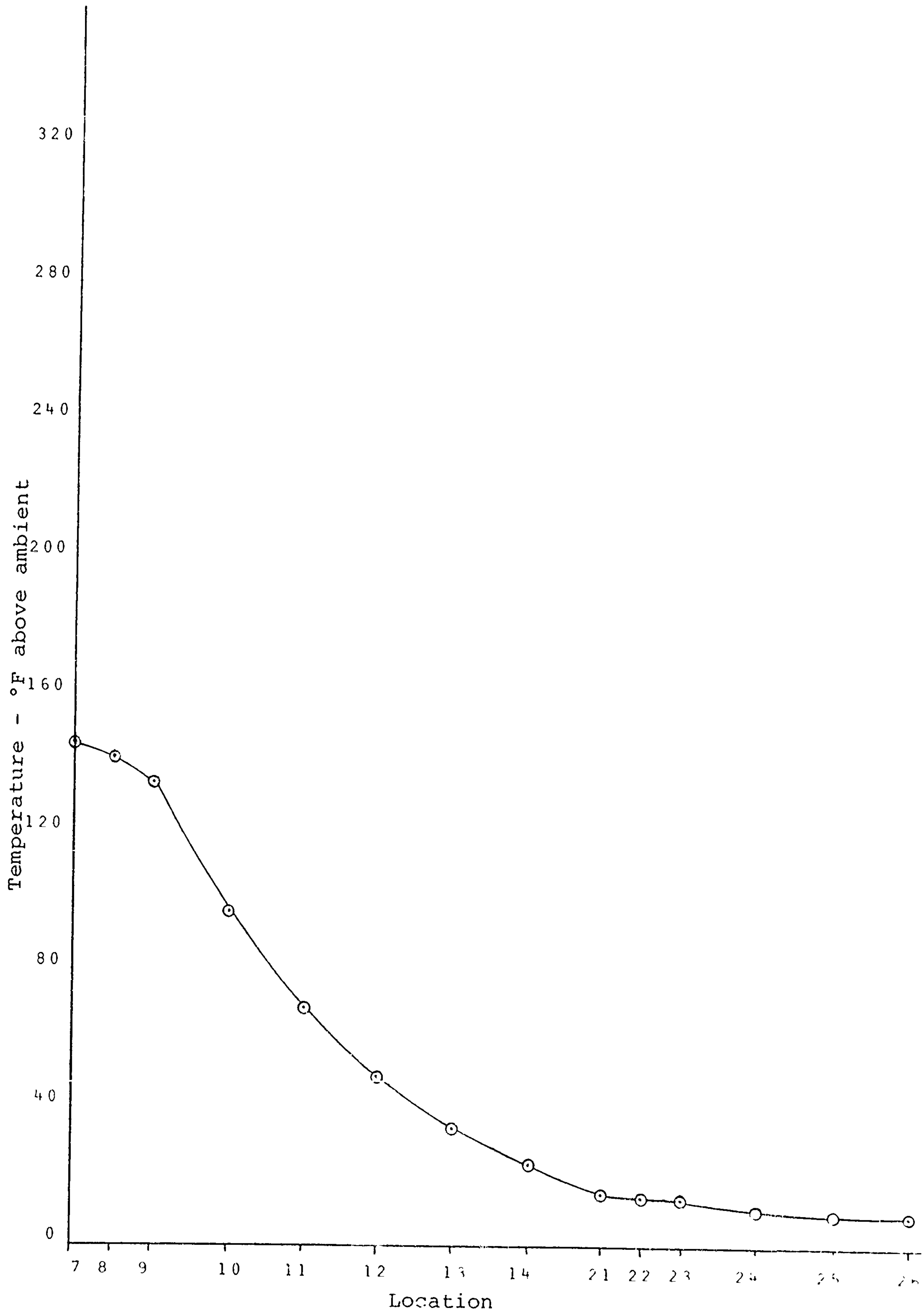


Figure D-4. Temperature Distribution for Variable Set II.

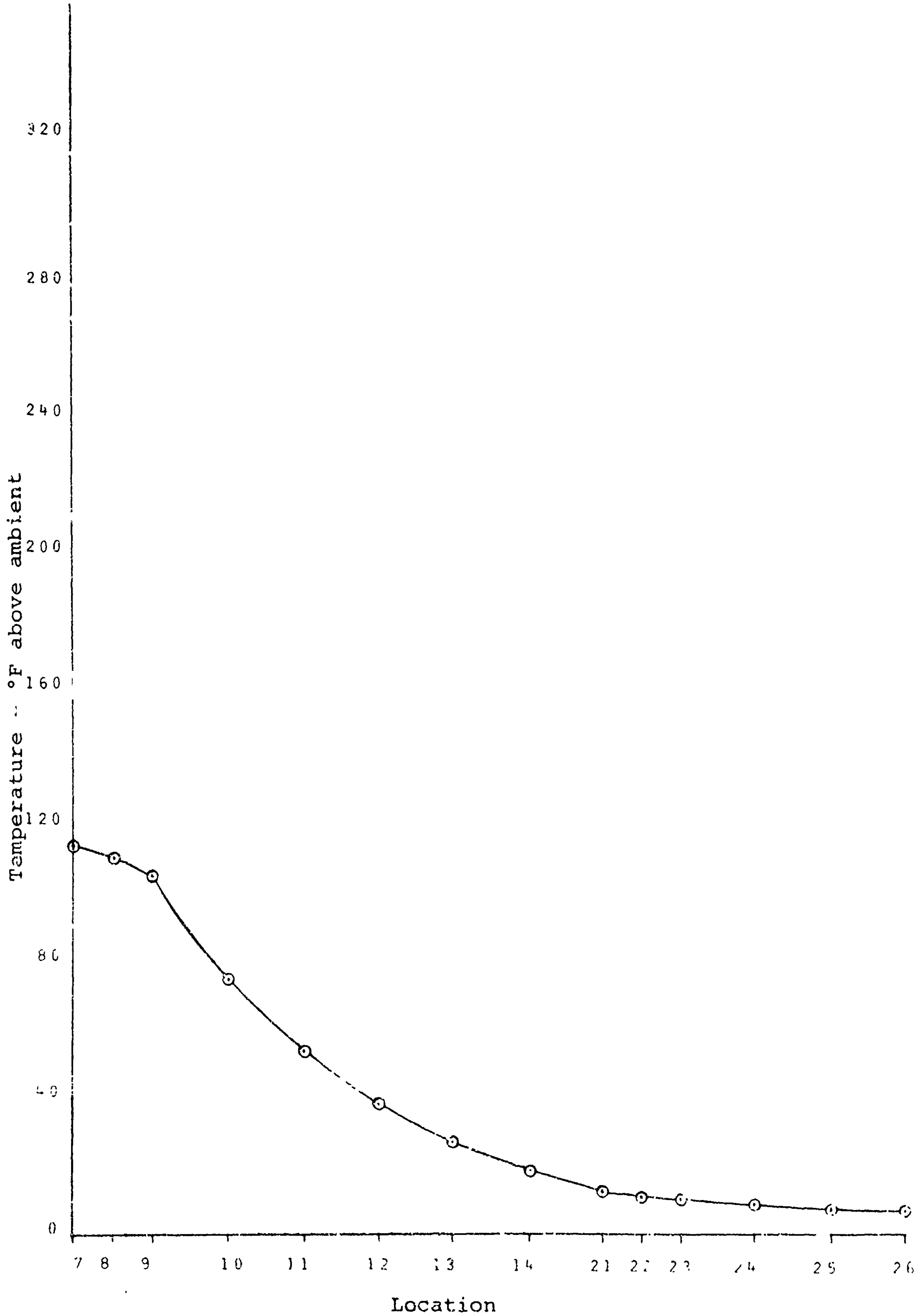


Figure D-5. Temperature Distribution for Variable Set III.

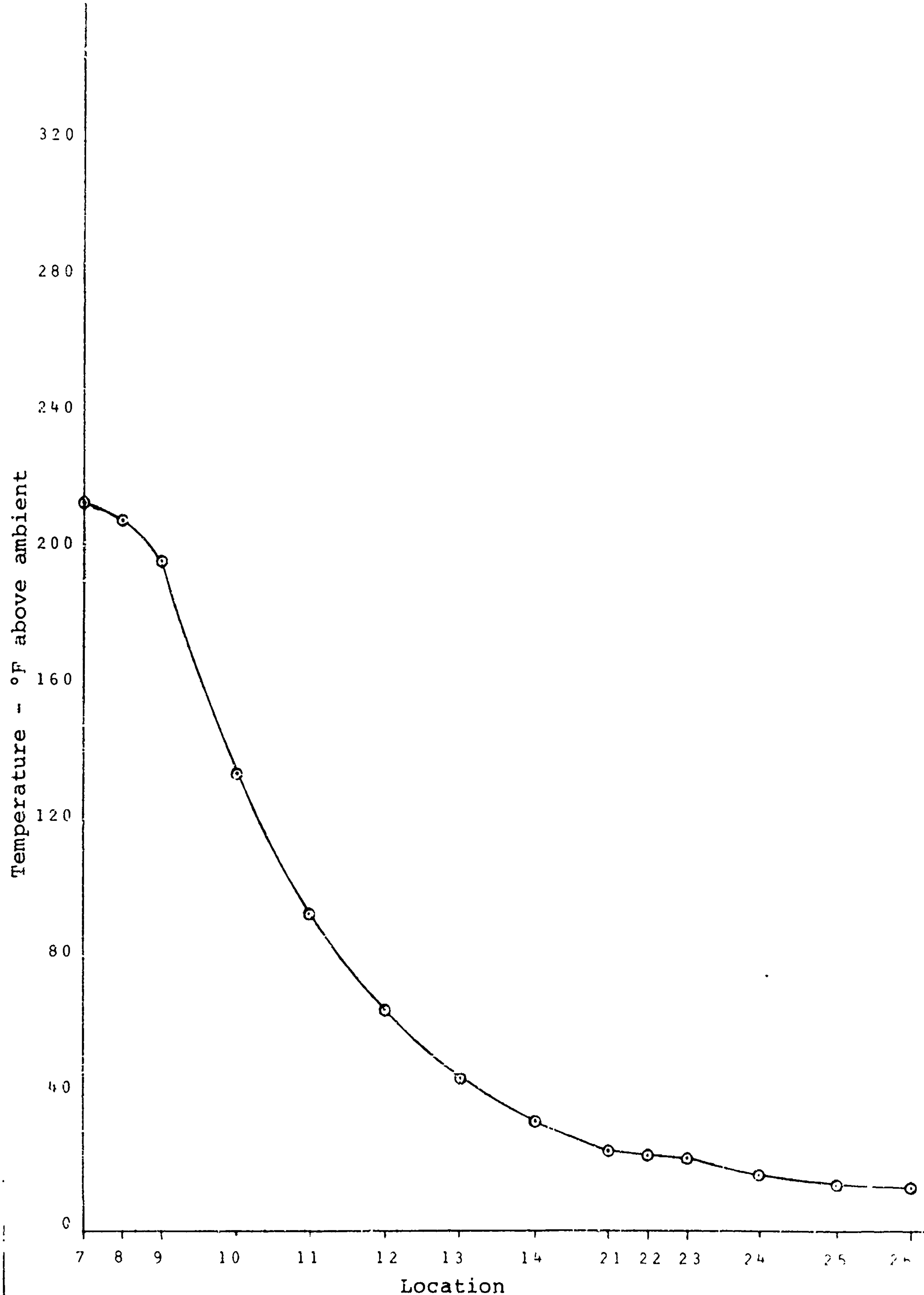


Figure D-6. Temperature Distribution for Variable Set IV.

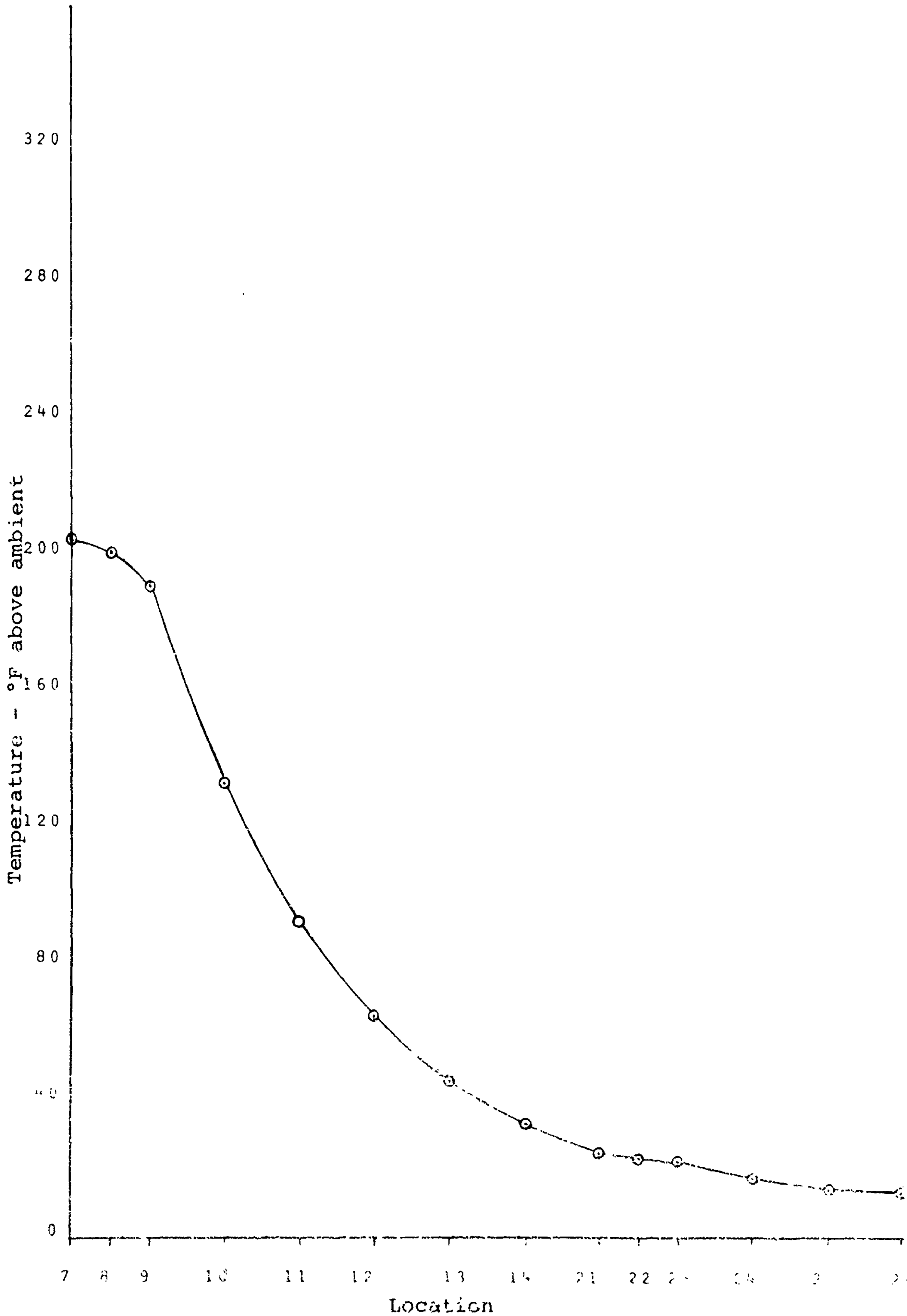


Figure D-7. Temperature Distribution for Variable Set V.

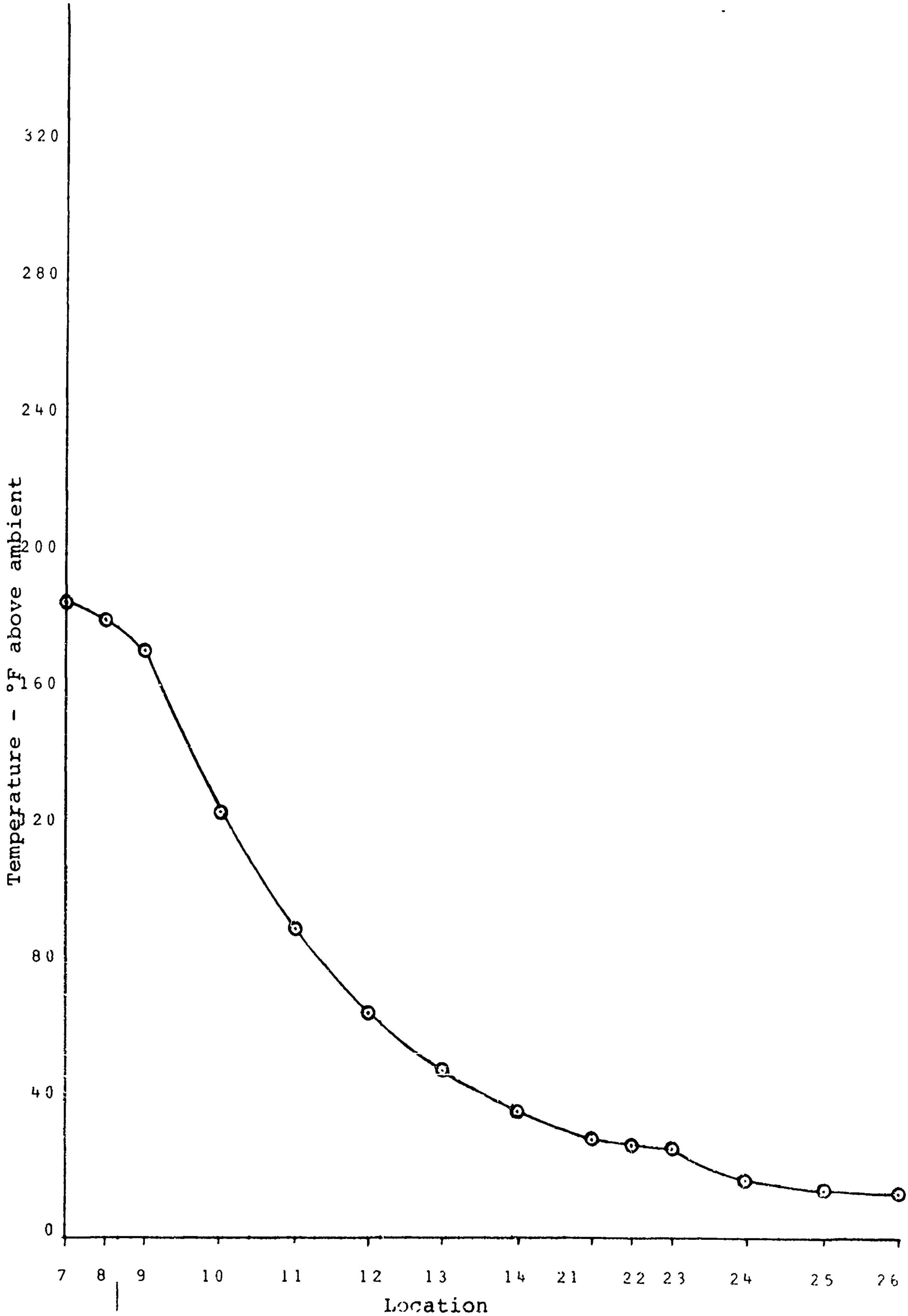


Figure D-8. Temperature Distribution for Variable Set VI.

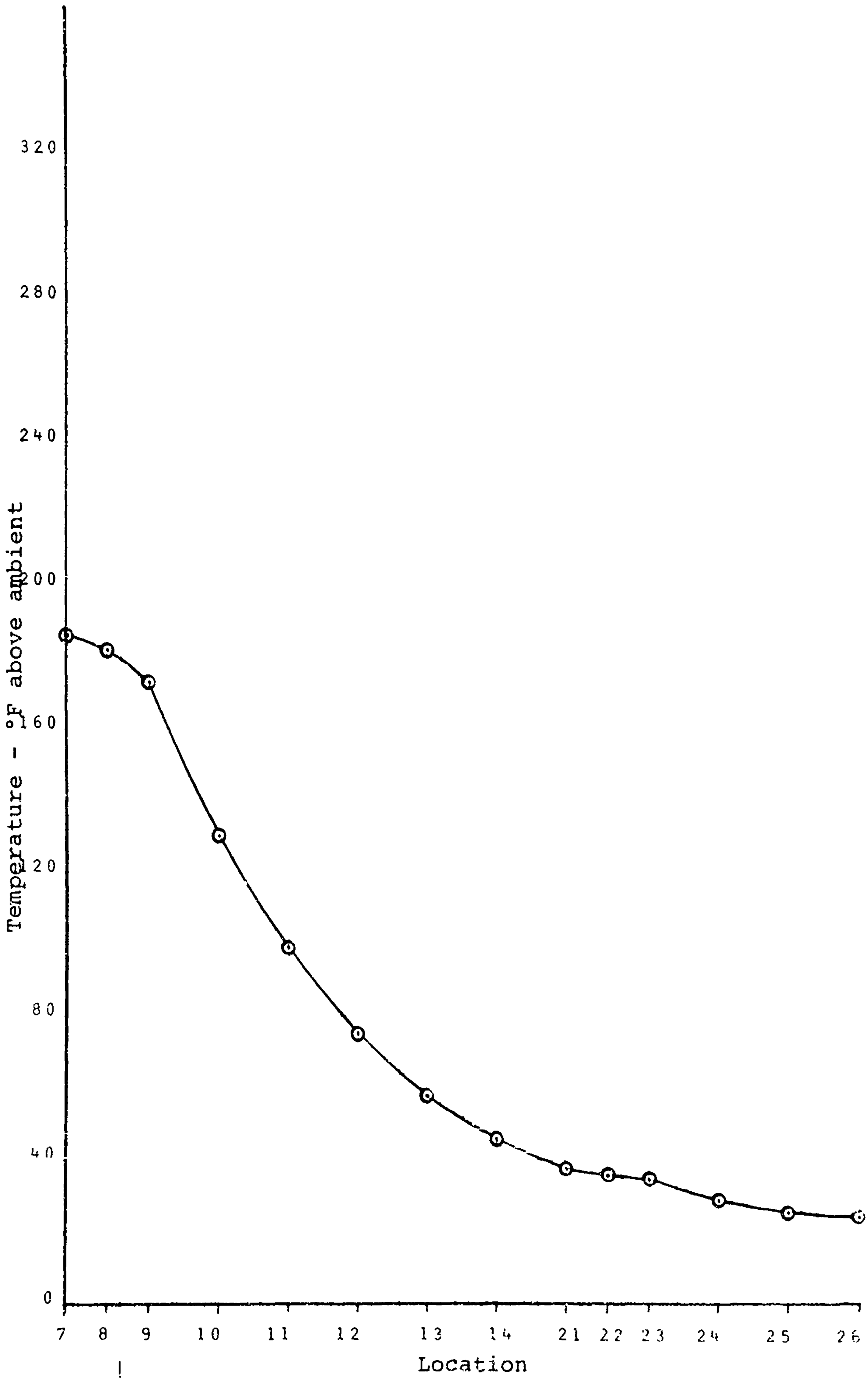


Figure D-9. Temperature Distribution for Variable Set VII.

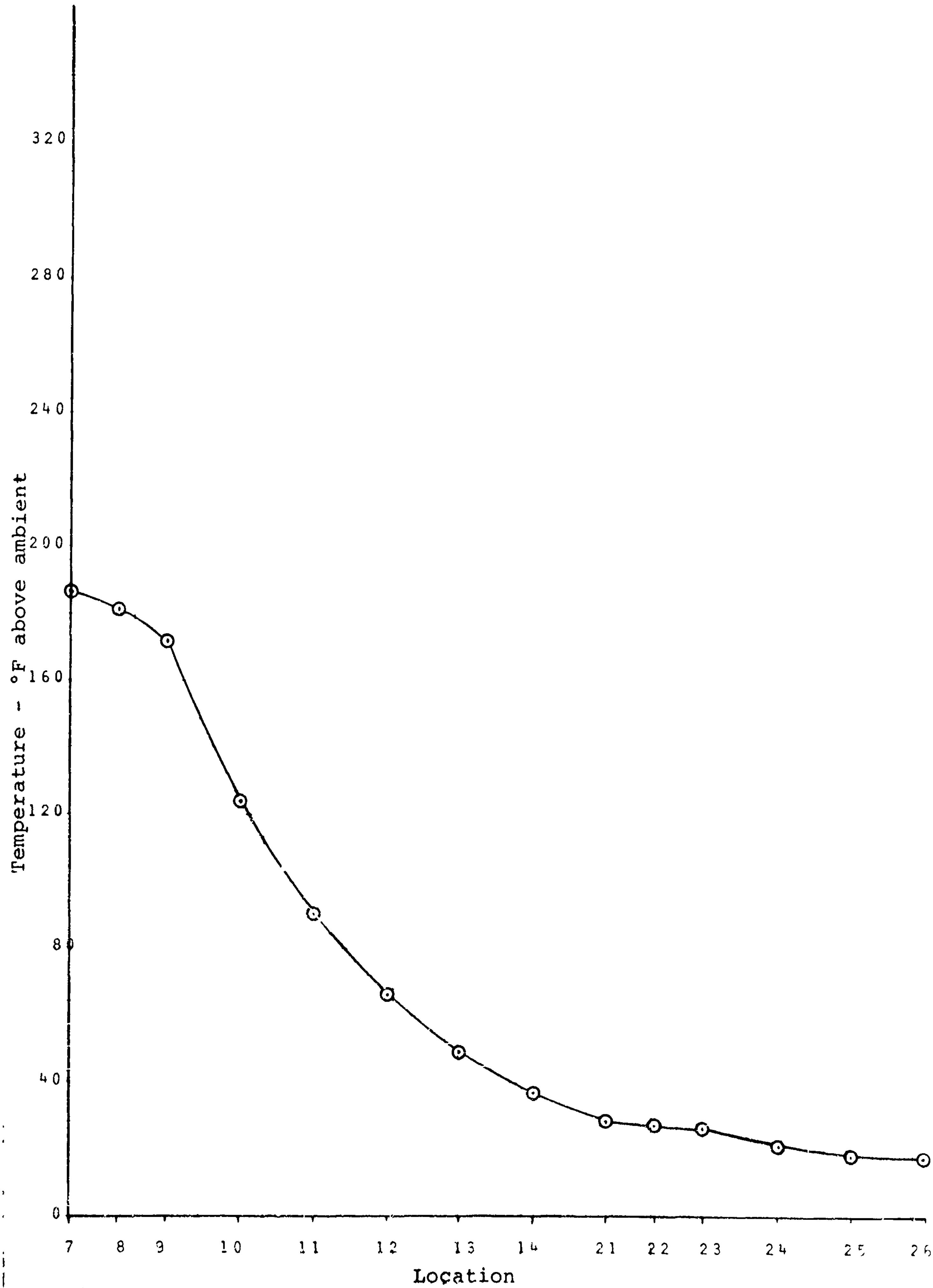


Figure D-10. Temperature Distribution for Variable Set VIII.

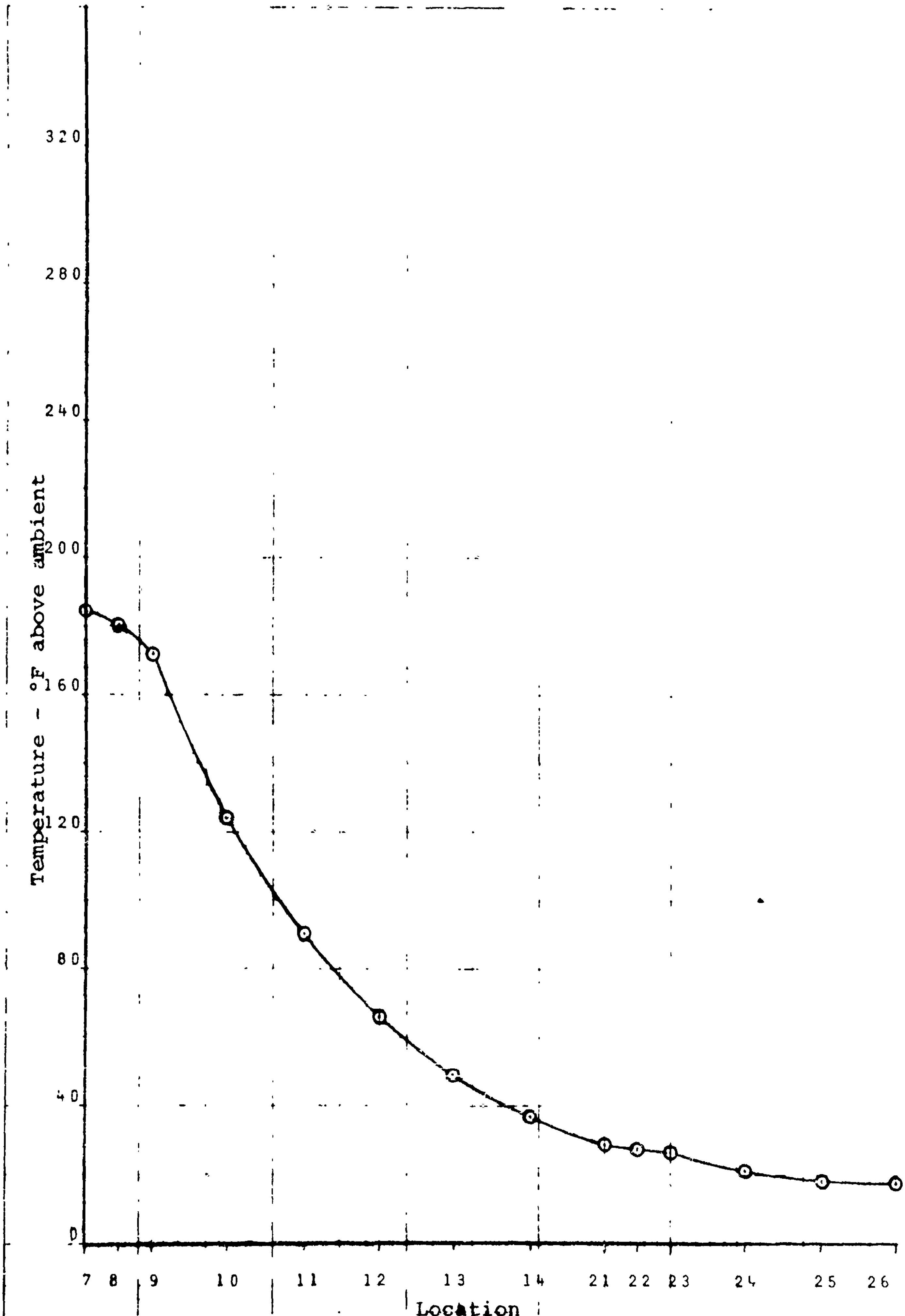


Figure D-11. Temperature Distribution for Variable Set IX.

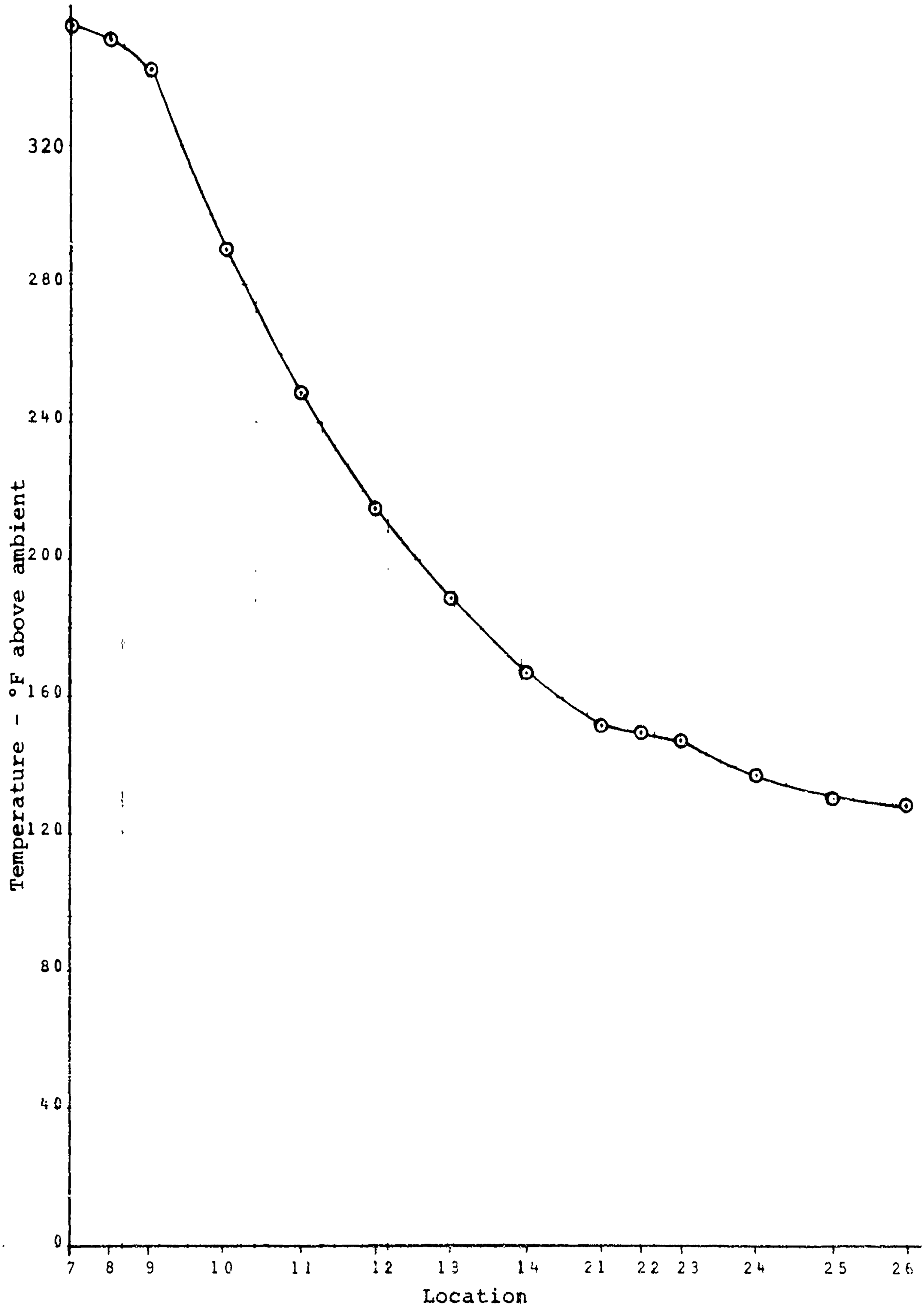


Figure D-12. Temperature Distribution for Variable Set X.

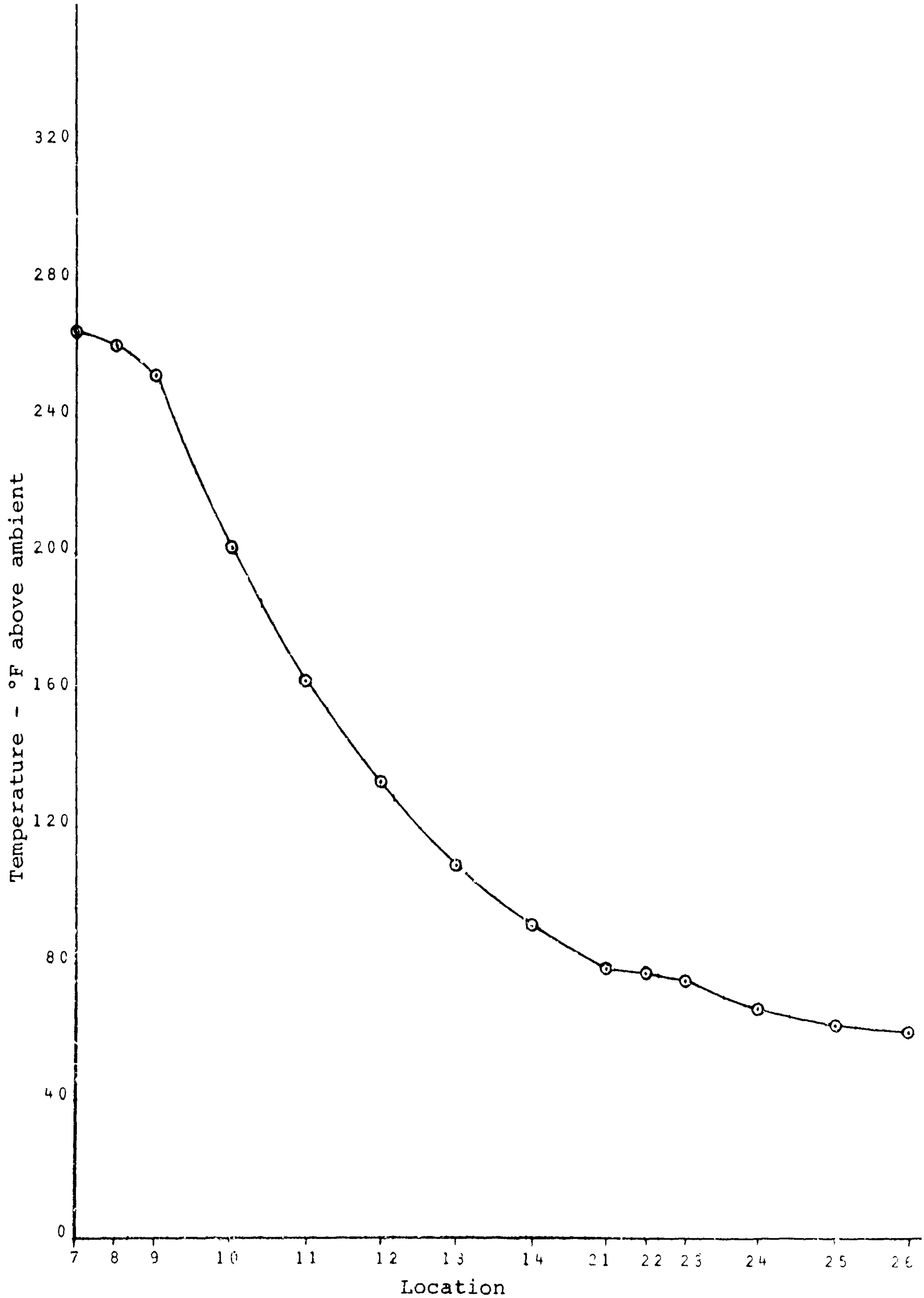


Figure D-13. Temperature Distribution for Variable Set XI.

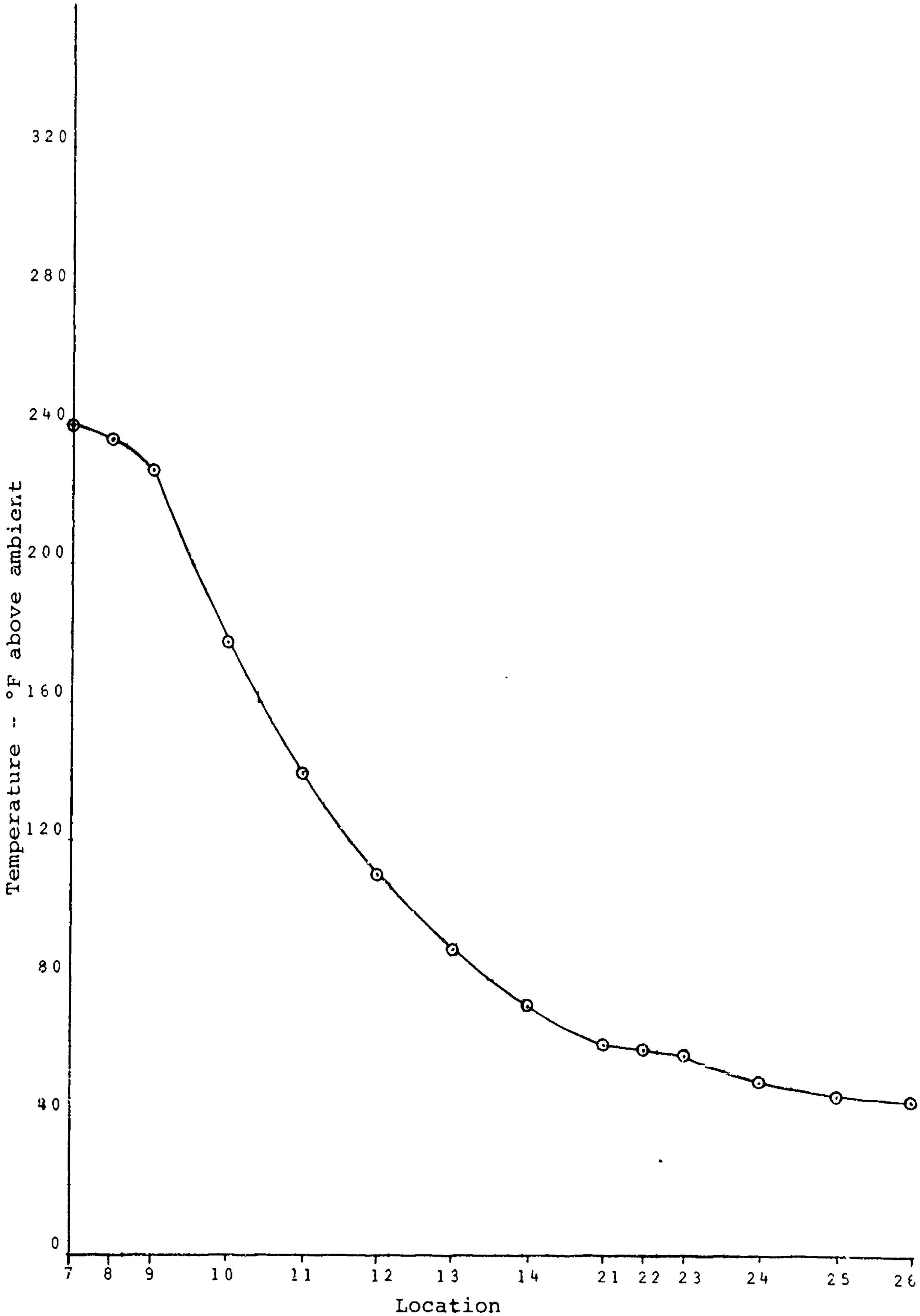


Figure D-14. Temperature Distribution for Variable Set XII.

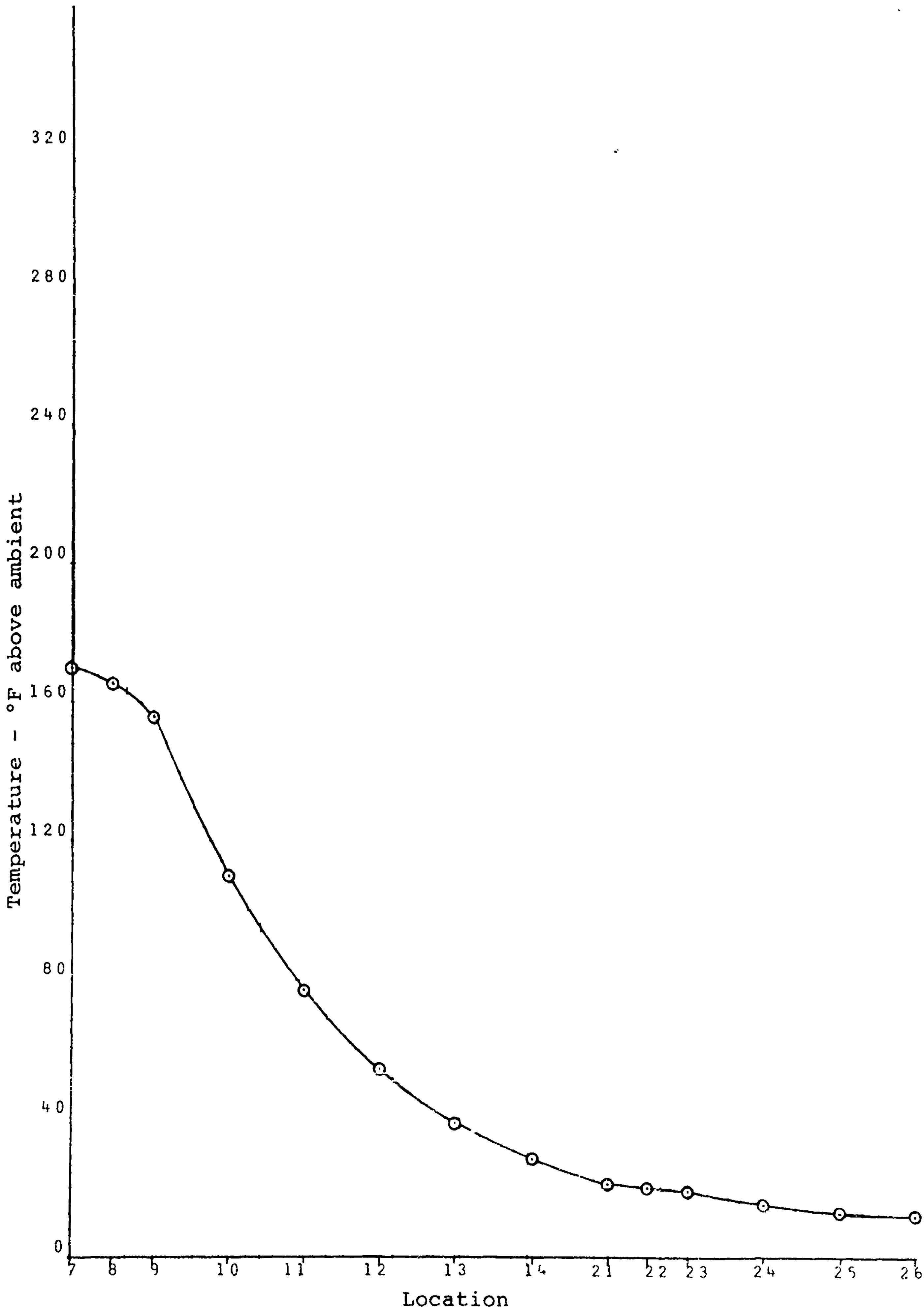


Figure D-15. Temperature Distribution for Variable Set XIII

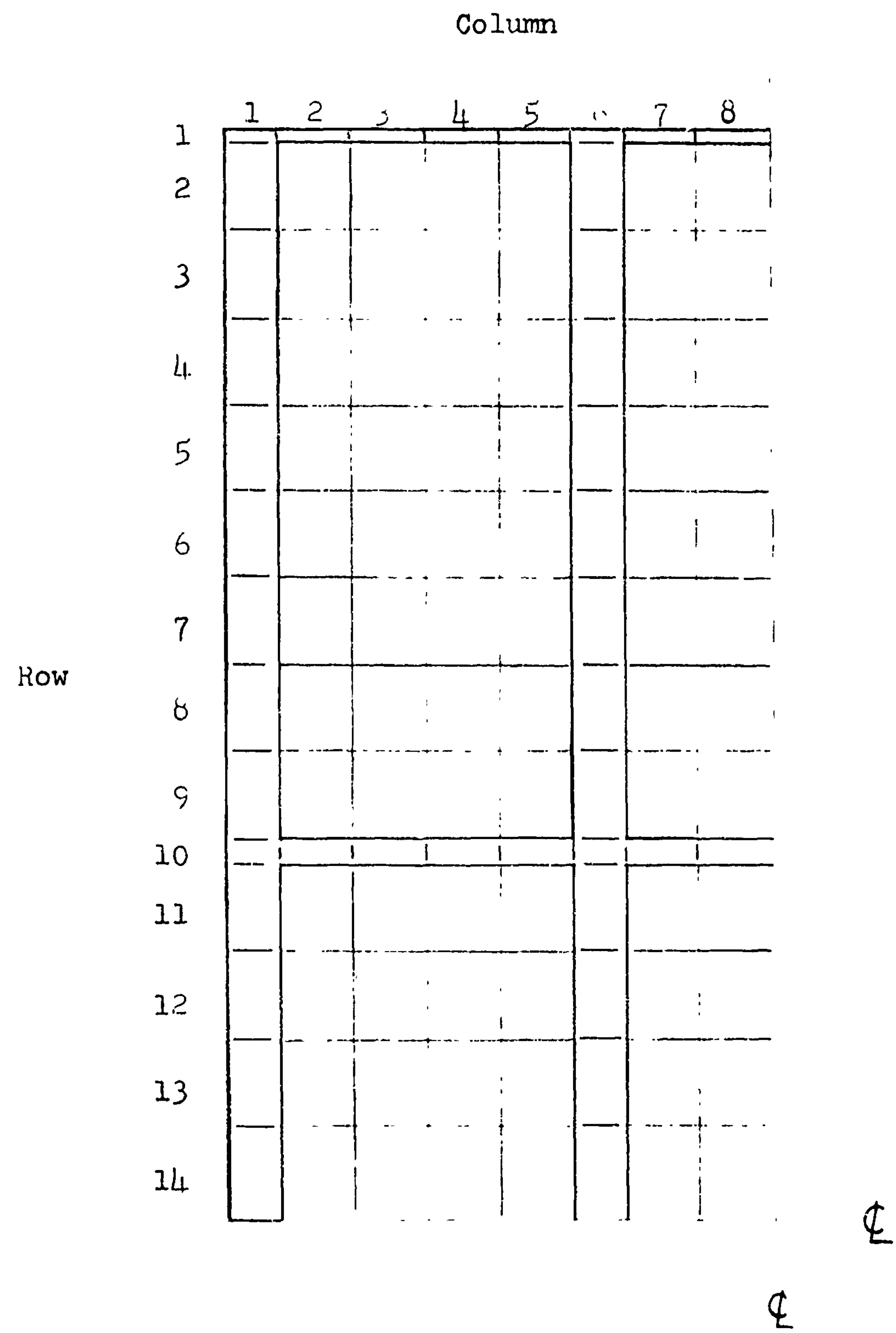


Figure D-16. Elements Used in Three-Dimensional Analysis.

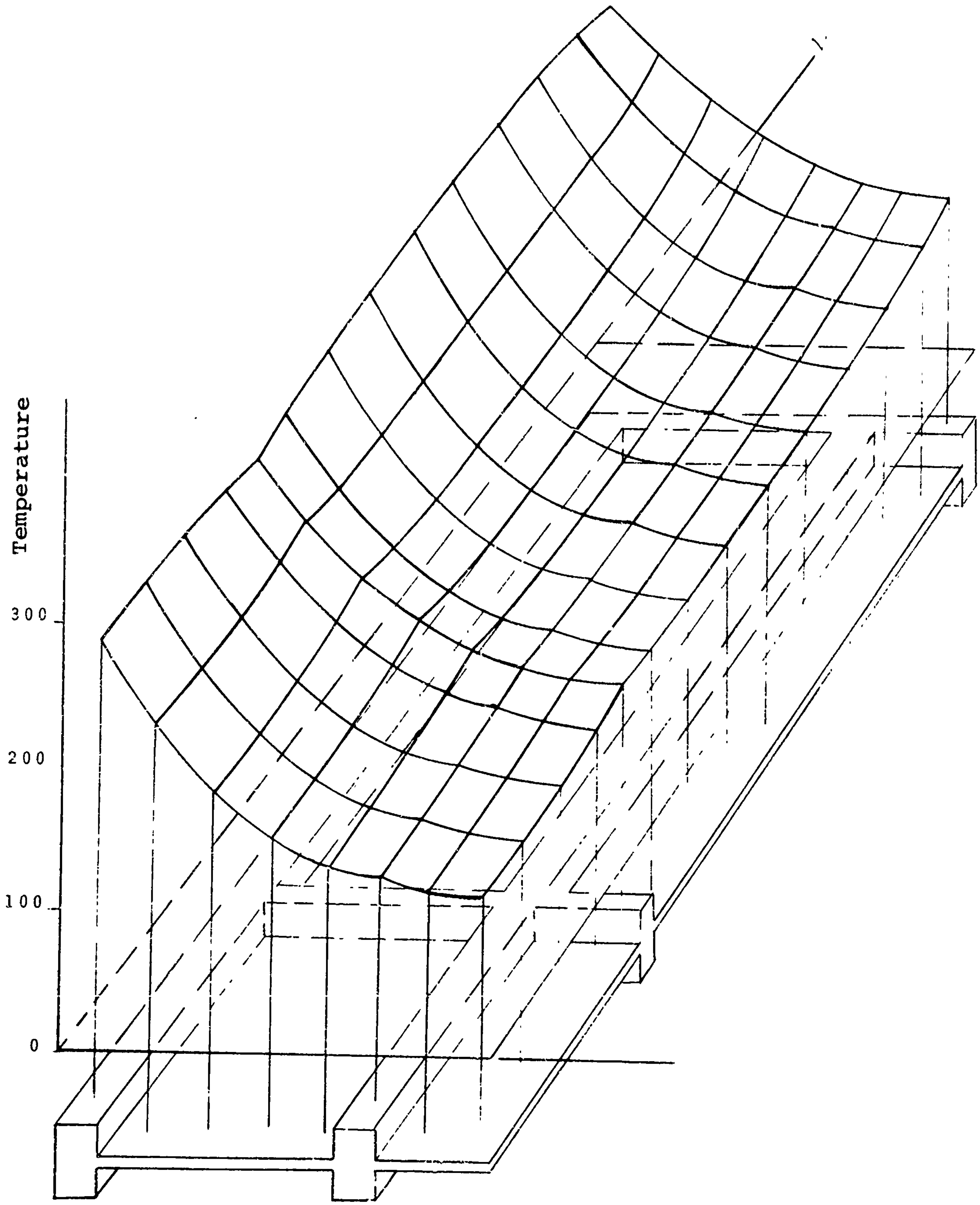


Figure D-17. Temperature Distribution for Variable Set I.

TABLE D-1

Three-Dimensional Temperature Distribution

SET I

Columns

Rows	1	2	3	4	5	6	7	8
1	168.73	137.99	106.33	81.16	61.94	49.20	43.79	40.43
2	178.18	128.80	92.72	68.70	53.01	46.01	37.55	33.38
3	186.44	126.71	85.18	60.29	46.20	41.53	32.30	27.62
4	191.31	127.26	82.70	56.68	42.65	38.28	29.15	24.42
5	193.93	128.06	82.02	55.24	40.90	36.29	27.38	22.72
6	194.80	128.49	82.06	54.96	40.33	35.46	26.67	22.06
7	194.02	128.55	82.69	55.68	40.83	35.87	26.90	22.33
8	191.38	128.79	84.72	58.06	42.69	36.85	28.13	23.65
9	186.11	131.32	90.66	63.94	44.82	38.00	30.[illegible]9	26.55
10	179.56	139.81	101.23	72.70	52.56	40.10	34.09	30.37
11	186.38	131.48	90.72	63.91	46.70	38.59	30.51	26.40
12	192.09	129.21	84.87	57.97	42.35	3[illegible].28	27.65	23.23
13	195.25	129.27	82.92	55.49	40.23	34.[illegible]6	26.05	21.57
14	19[illegible].69	129.56	82.36	54.61	39.35	33.05	25.32	20.85

SET II

Columns

Rows	1	2	3	4	5	6	7	8
1	170.02	139.40	107.58	82.08	[illegible]2.45	49.32	43.95	40.61
2	181.27	129.17	91.70	67.34	51.84	4[illegible].26	36.53	32.21
3	190.69	128.14	84.59	58.77	44.42	39.[illegible]9	30.51	25.81
4	195.91	12[illegible].47	82.93	5[illegible].70	40.94	35.98	27.13	22.51
5	198.58	130.58	82.7	54.64	3[illegible].35	33.93	25.36	20.86
6	199.34	130.99	82.78	54.44	38.88	33.20	24.74	20.31
7	19[illegible].37	130.74	83.10	54.98	3[illegible].[illegible]	[illegible]	25.14	20.69
8	195.33	130.32	84.51	57.[illegible]1	41.29	3[illegible].2[illegible]	2[illegible].64	22.19
9	18[illegible].14	132.00	90.15	[illegible]3.04	45.07	37.[illegible]7	29.73	2[illegible].[illegible]3
10	181.24	1[illegible]1.55	10[illegible].58	73.[illegible]0	[illegible]2.75	39.7[illegible]	[illegible]3.80	30.12
11	189.32	132.10	90.19	[illegible]3.03	4[illegible].01	37.[illegible]	29.[illegible]3	25.45
12	195.81	130.60	84.62	58.97	41.10	34.93	28.36	21.94
13	199.23	131.26	83.29	54.89	39.03	33.01	24.61	20.22
14	200.73	131.80	83.06	54.28	38.23	32.10	23.82	19.48

TABLE D-1 (Continued)

SET III

Columns

Row	1	2	3	4	5	6	7	8
1	184.37	149.98	114.74	86.67	65.04	50.44	44.91	41.47
2	195.41	136.69	95.97	69.80	53.21	46.49	36.96	32.45
3	205.44	133.26	85.79	58.83	44.59	41.08	30.60	25.47
4	211.55	133.76	82.52	54.29	40.26	37.16	26.91	21.75
5	214.93	134.74	81.68	52.55	38.16	34.76	24.87	19.82
6	216.10	135.32	81.76	52.25	37.51	33.74	24.05	19.10
7	215.18	135.49	82.64	53.22	38.15	33.98	24.33	19.45
8	211.93	136.05	85.46	56.37	40.52	35.39	25.79	21.00
9	205.54	140.04	93.77	64.34	45.95	37.76	28.92	24.56
10	197.85	152.53	108.84	76.61	53.76	39.41	33.27	29.46
11	205.95	140.28	93.86	64.31	45.80	37.50	28.71	24.39
12	212.99	136.66	85.69	56.28	40.12	34.67	25.23	20.53
13	216.99	13[illegible].53	83.02	53.05	37.44	32.71	23.34	18.59
14	218.83	136.85	82.27	51.92	36.36	31.72	22.48	17.77

SET IV

Columns

Row	1	2	3	4	5	6	7	8
1	170.72	142.45	114.24	91.09	72.80	59.02	54.32	51.17
2	180.01	133.82	100.63	78.01	63.01	55.93	46.96	42.70
3	188.07	132.24	93.63	69.67	55.01	50.86	40.88	35.97
4	192.79	133.90	91.51	66.23	52.13	47.24	37.29	32.28
5	195.32	133.86	90.99	64.08	50.32	45.06	35.29	30.34
6	196.13	134.26	91.02	64.59	49.74	44.18	34.52	29.61
7	195.33	134.23	91.52	65.24	50.25	44.47	34.84	29.97
8	192.71	134.25	93.28	67.51	52.20	45.87	36.33	31.58
9	187.50	136.33	98.88	73.44	56.67	48.19	39.44	35.14
10	181.02	141.40	109.65	82.88	63.15	49.77	43.62	39.97
11	187.74	136.47	98.92	73.40	56.53	47.97	39.25	34.97
12	193.33	134.61	93.39	67.39	51.84	45.28	35.82	31.11
13	196.42	134.85	91.70	65.02	49.61	43.41	33.92	29.13
14	197.82	135.21	91.26	64.20	40.70	42.49	33.06	28.27

TABLE D-1 (Continued)

SET V

Columns

Row	1	2	3	4	5	6	7	8
1	176.77	141.19	106.50	82.69	63.02	48.61	42.04	39.58
2	184.83	130.45	94.08	69.77	53.84	45.88	36.81	32.74
3	192.18	12[illegible].98	85.73	60.93	46.93	41.93	31.84	27.21
4	196.65	126.85	82.79	57.09	43.36	38.96	28.89	24.16
5	199.16	127.37	81.92	55.55	41.[illegible]1	37.08	27.21	22.34
6	200.07	127.79	81.93	55.25	41.04	36.22	26.50	21.89
7	199.48	128.06	82.69	56.03	41.53	36.30	26.65	22.10
8	197.23	128.81	85.04	58.56	43.39	37.23	27.73	23.31
9	192.74	132.33	91.52	64.71	47.61	38.81	30.11	26.07
10	187.30	141.85	102.54	73.72	53.50	39.82	33.39	29.77
11	193.05	132.51	91.57	64.67	47.46	38.55	29.90	25.89
12	198.06	129.27	85.19	58.45	43.01	36.56	27.19	22.83
13	200.89	128.83	82.93	55.81	40.85	35.12	25.70	21.26
14	202.19	128.91	82.23	54.86	39.97	34.39	25.02	20.59

SET VI

Columns

Row	1	2	3	4	5	6	7	8
1	242.67	210.50	175.93	147.09	124.04	108.25	101.20	96.75
2	252.75	203.90	163.96	134.94	114.55	104.45	93.24	87.42
3	261.81	203.01	156.92	12[illegible].12	106.78	98.95	86.05	79.17
4	267.24	204.04	154.44	122.01	102.41	94.84	81.53	74.29
5	270.20	205.07	153.70	120.21	100.09	92.19	78.88	71.56
6	271.20	205.58	153.70	119.77	99.25	91.02	77.76	70.45
7	270.38	205.60	154.35	120.54	99.77	91.19	78.03	70.82
8	267.51	205.60	15[illegible].37	123.13	101.91	92.58	79.76	72.84
9	261.82	207.29	161.95	129.24	10[illegible].55	94.91	83.37	77.18
10	254.96	213.69	171.32	137.88	112.74	96.43	87.97	82.62
11	262.14	207.48	161.99	129.14	106.30	94.54	83.01	76.84
12	268.33	206.08	156.45	122.85	101.25	91.59	78.82	71.93
13	271.80	206.42	154.50	120.04	98.63	89.47	76.41	69.24
14	273.38	206.82	153.90	118.98	97.50	88.40	75.28	68.05

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13 ABSTRACT

The purpose of the investigation ~~described in this report~~ was: (1) To design and test on integrally stiffened rectangular shear panel, (2) To develop analytical procedures for shear panels subjected to thermomechanical loading, and (3) To compare the analytical and experimental results to evaluate the accuracy of the analysis methods. The body of the report presents a summary description of the methods of analysis which were investigated, the design of the test panel, the experimental program, and the analytical and experimental results. The details of the analysis methods and the digital computer programs developed to implement the use of these methods on the test panel are given in the appendices.

Key Words

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