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PROPAGATION OF CYLINDRICAL SHEAR WAVES IN NONHOMOGENEOUS ELASTIC MEDIA

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ABSTRACT

The method of characteristics is applied to the set of equations which governs the propagation of axially symmetric torsional shear waves in non-homogeneous elastic media. The wave velocities, characteristic equations, and the equations governing the propagation of abrupt changes (discontinuous wave fronts) are derived in closed form. Numerical integration along the characteristic directions was carried out for several examples on an electronic computer. The solutions of three specific examples calculated show general agreement with existing solutions by other methods. For certain problems, the method of characteristics yield additional results which cannot be obtained by the Laplace transform method.

NOMENCLATURE

c = $(G/\rho)^{1/2}$ = shear (or distorsional) wave velocity

G = shear modulus (function of ı

$\bar{G}$ = G/G_o

r = radial distance

r_o = inner radius of plate

$\bar{r}$ = r/r_o

t = time

$\bar{t}$ = $c_o t/r_o$

v = tangential displacement

v_t = $\partial v/\partial t$ = particle velocity

v_r = $\partial v/\partial r$ = shear strain

$\bar{v}$ = $G_o v/\tau_o r_o$

ρ = density of material (function or r)

$\bar{\rho}$ = ρ/ρ_o

τ = torsional shear stress

$\bar{\tau}$ = τ/τ_o

Subscript

o = properties at r_o

INTRODUCTION

The problem of shear wave propagation due to a suddenly applied rotary disturbance in a homogeneous elastic plate was solved by Goodier and Jahsman[1]. The corresponding problem in a nonhomogeneous plate was solved by Sternberg and Chakravorty[2]. In both these cases the Laplace transform technique was used. Except in a few cases of special radial distributions of the shear modulus, all their solutions are in integral form which can be evaluated only by numerical integration. A solution obtained by the Laplace transform technique is applicable for only one type of initial and boundary conditions. To solve for a different type of condition, the problem must be reinitiated and techniques for inversion developed. In their study of the nonhomogeneous plate problems Sternberg and Chakravorty were mainly interested in the qualitative effect of the variation of shear modulus; therefore, a simple exponential variation was selected. Solutions for other types of radial distribution of the shear modulus are not available. For these reasons there is a need for other methods in treating shear waves in nonhomogeneous plates.

In this paper the propagation of cylindrical shear waves in nonhomogeneous elastic bodies is treated by the method of characteristics. By using this method, the distribution of wave velocity (physical characteristic), the characteristic equations, as well as the equations governing the propagation of abrupt change in stress (step input), may be determined in closed form. Numerical integration for the determination of the stress field behind the wave front may be accomplished readily for any type of input; and for any type of radial distribution of the shear modulus and density. As examples, materials with simple exponential distribution of the shear modulus under step input in stress are presented. For a certain class media, the Laplace transform method yields results only up to a certain critical time; whereas the method of characteristics yield solutions beyond this critical time.

In Ref. 3, the method of characteristics was applied to cylindrical and spherical dilatational waves in a homogeneous elastic material. The present paper is an extension of the method, not only to the case of shear waves, but also to nonhomogeneous media.

GOVERNING EQUATIONS

The governing equations in cylindrical coordinates for elastic torsional shear waves under axisymmetrical loading conditions are,

$$\frac{\partial \tau}{\partial r} + \frac{2\tau}{r} = \rho \frac{\partial^2 v}{\partial t^2} \tag{1}$$

$$\tau = G \left(\frac{\partial v}{\partial r} - \frac{v}{r}\right) \tag{2}$$

where r is the radius; t is the time; ρ is the density; G is the shear modulus; τ is the torsional shear stress; and v is the tangential displacement. The shear modulus and the density are in general arbitrary functions of the radius. Substituting eq. (2) into eq. (1), we obtain a single second order equation

$$\frac{\partial^2 v}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{v}{r}\right) + \frac{1}{G}\frac{\partial G}{\partial r}\left\{\frac{\partial v}{\partial r} - \frac{v}{r}\right\} = \frac{1}{c^2}\frac{\partial^2 v}{\partial t^2} \tag{3}$$

where $c = (G/\rho)^{1/2}$ is the torsional shear wave velocity. It may be noted that since only shear stress is involved, these equations are exact for both a plate (plane stress) and a hollowed infinite body (plane strain).

In the application of the method of characteristics, we may use either eqs. (1) and (2), the stress approach; or eq. (3), the displacement approach. The governing equations for both approaches will be given below; while the numerical procedures for the displacement approach only will be presented.

CHARACTERISTIC EQUATIONS

In the displacement approach, we apply the method of characteristics to the single second order equation, (3), and obtain the following two physical characteristics

$$\frac{dr}{dt} = \pm (G/\rho)^{1/2} \tag{4}$$

which will be called the I^+ and I^- characteristics, respectively. Notice that eqs. (4) are of the same form for both homogeneous and nonhomogeneous materials. For homogeneous materials, the physical characteristics are two families of straight lines of constant slope, whereas for nonhomogeneous materials, they are two families of curved lines in the r,t-plane. In both cases, once the distribution of G and ρ are given, the physical characteristics are determined independent of the loading and solution of the problem.

The characteristic equations of (3), with v_t for $\partial v/\partial t$, v_r for $\partial v/\partial r$, are,

$$d(v_t) \mp c\, d(v_r) = \pm c\left(v_r - \frac{v}{r}\right)\left(\frac{dr}{r} + \frac{dG}{G}\right) \tag{5}$$

along the I^+ and I^- characteristics, respectively. For homogeneous material, the term containing dG vanishes in (5).

In the stress approach, we differentiate eq. (2) with respect to time and rewrite (1) and (2) as

$$\frac{\partial \tau}{\partial r} + 2\frac{\tau}{r} = \rho \frac{\partial v_t}{\partial t} \qquad (6)$$

$$\frac{\partial \tau}{\partial t} = G\left(\frac{\partial v_t}{\partial r} - \frac{v_t}{r}\right) \qquad (7)$$

These may be considered as two first order equations in terms of τ and v_t. Applying the method of characteristics to eqs. (6) and (7), we obtain two physical characteristics identical to (4). The corresponding characteristics equations are

$$d\tau \mp \frac{G}{c}\, d(v_t) = (-2\tau \mp \frac{G}{c} v_t)\, \frac{dr}{r} \qquad (8)$$

along I^+ and I^-, respectively. Unlike eqs. (5), the form of eqs. (8) is the same for both homogeneous and nonhomogeneous materials. It can be shown readily that upon substitution of (2) into (8), eqs. (5) are obtained.

For the present problem, the stress approach and the displacement approach yield identical results. This is different from the problems of spherical and cylindrical dilatational waves[3] and the problem of cylindrical flexural waves in a plate[4]. In both those cases, the stress approach produces one extra physical characteristic, $dr/dt = 0$, which has an associated characteristic equation equivalent to a restatement of the static stress-displacement relations.

PROPAGATION OF DISCONTINUITY

Across the physical characteristics the second derivatives of v (or the first derivatives of τ and v_t) may be discontinuous. Discontinuities of the first derivatives of v (or τ and v_t themselves) may also exist across the physical characteristics, but these will not be governed by eqs. (5) or (8). In Ref. 3, the equations governing the discontinuities in the first derivatives (jump conditions) of the displacement variable in dilatational waves are derived by using the stress approach. In Ref. 4, a similar set of jump conditions are derived for flexural waves by using the displacement approach. In this paper, we shall follow the displacement approach and derive the jump conditions for shear waves in nonhomogeneous materials.

Let A and B be two points on a I^- characteristic as shown in Fig. 1. The two I^+ characteristics passing through A and B are represented by I_1^+ and I_2^+, respectively. If a discontinuity of v_t across I_1^+ exists, then $v_{tA} - v_{tB} = [v_t]$ is finite but different from zero as I_2^+ is allowed to approach I_1^+, or as dr approaches zero. Writing eq. (5) (with the lower sign, for I^-) and integrating from A to B, we have

$$(v_{tA} - v_{tB}) + \int_A^B c\, d(v_r) = -\int_A^B c(v_r - \frac{v}{r})\frac{dr}{r} - \int_A^B c(v_r - \frac{v}{r})\frac{dG}{G} \qquad (9)$$

As B approaches A, the right hand side of (9) vanishes, since the integrands contain bounded values of c, v_r, and v, provided the domain in the physical plane does not include the line r = 0. Integrating the left hand side integral by parts, and keeping in mind that v_r is bounded, as B approaches A eq. (9) becomes

$$[v_t] + c[v_r] = 0 \qquad (10)$$

where brackets are used to designate jumps. The variations of $[v_t]$ and $[v_r]$ as they propagate along the I^+ is obtained by writing (5), with the upper signs, along I_2^+ and I_1^+, and subtracting one from the other. As B approaches A, we have

$$d[v_t] - c\, d[v_r] = c[v_r]\frac{dr}{r} + c[v_r]\frac{dG}{G} \qquad (11)$$

where the condition $[v] = 0$ has been utilized. Eliminating $[v_r]$ from (11) by (10), we obtain

$$\frac{d[v_t]}{[v_t]} = -\frac{1}{2}\left(\frac{dr}{r} + \frac{dc}{c} + \frac{dG}{G}\right) \qquad (12)$$

which may be integrated to give

$$[v_t] = -K\left(\frac{c}{Gr}\right)^{1/2} \qquad (13)$$

Equations (10) and (13) then yield

$$[v_r] = +K \left(\frac{1}{Gcr}\right)^{1/2} \tag{14}$$

The corresponding jump in τ obtained from (2) and (14) is governed by

$$[\tau] = +K \left(\frac{G}{cr}\right)^{1/2} \tag{15}$$

Equations (13) to (15) are for jumps across, and the propagation along, a I^+ characteristic. Following the same procedure, equations for those across and along a I^- characteristic can be shown to be

$$\begin{aligned} [v_t] &= +K \left(\frac{c}{Gr}\right)^{1/2} \\ [v_r] &= +K \left(\frac{1}{Gcr}\right)^{1/2} \\ [\tau] &= +K \left(\frac{G}{cr}\right)^{1/2} \end{aligned} \tag{16}$$

The same set of equations (13) to (16) may also be derived from the stress approach.

INITIAL AND BOUNDARY CONDITIONS

The elastic body under consideration will be either an infinite sheet with a circular hole, or an infinite hollow cylinder. These configurations can be represented by $r_o \leq r \leq \infty$, where r_o is a constant. Initially, the body is not loaded, thus the stress and velocity are zero. For time greater than zero, the input is applied at the boundary $r = r_o$, either suddenly or gradually. This input can be in the form of specified time functions of any one of the three variables, v_t, v_r, or τ.

NUMERICAL PROCEDURE

It is convenient to introduce non-dimensional quantities as follows:

$$\begin{aligned} &\bar{r} = r/r_o, \quad \bar{t} = tc_o/r_o, \quad \bar{G} = G/G_o, \quad \bar{v} = G_o v/\tau_o r_o, \quad \bar{\tau} = \tau/\tau_o, \\ &\bar{\rho} = \rho/\rho_o, \quad \bar{c} = c/c_o \end{aligned} \tag{17}$$

Thus, the results in terms of these quantities are true for materials of any values of G_o, ρ_o, and r_o.

A numerical technique for stepwise integration along the physical characteristics is developed. In the $\bar{r},\bar{t}$-plane, the region between $\bar{r} = 1$ and $\bar{r} = 1 + \bar{c}\bar{t}$ is divided into a grid system by the two families of characteristics. The displacement approach is used, thus at each grid point values of $\bar{v}$, $\bar{v}_t$, and $\bar{v}_r$ are calculated. Since only continuous $\bar{v}$ is considered for all regions in the physical plane, we may write the continuity equation

$$d\bar{v} = \bar{v}_t \, d\bar{t} + \bar{v}_r \, d\bar{r} \tag{18}$$

along any directions. In our numerical work, this continuity equation along the I^- characteristics is used. Values of the three variables $\bar{v}$, $\bar{v}_t$, and $\bar{v}_r$ at a typical interior point 1 of Fig. 2 may be calculated from eqs. (5) and (18) expressed in finite-difference form, if all quantities at the neighboring points 2 and 3 are known. Along the leading I^+ characteristic passing through (1,0), all three variables vanish if the prescribed boundary condition at (1,0) is continuous. For jump input at (1,0) values of $\bar{v}_r$ and $\bar{v}_t$ along the leading characteristic are calculated from eqs. (13) and (14). Along the boundary $\bar{r} = 1$, either $\bar{v}_t$ or $\bar{\tau}$ is specified; correspondingly, the I^+ characteristics to the left are absent leaving two equations for two unknowns.

In the numerical calculation, the characteristic grid system was constructed by choosing points on the leading I^+ characteristic with equal horizontal distance, as shown in Fig. 2. The I^- characteristics are constructed from the reflections of the I^+ characteristics from the boundary $\bar{r} = 1$.

SPECIFIC EXAMPLES

A few specific examples of various inputs at $\bar{r} = 1$ are calculated and the results compared with existing solutions by other methods. Although all the equations and the numerical procedures discussed in this paper are applicable to bodies with arbitrary radial distribution of $\bar{G}$ and $\bar{\rho}$, in the specific examples presented below a special distribution is selected, i.e.,

$$\begin{aligned} \bar{G} &= \bar{r}^{\alpha}, \quad \bar{\rho} = 1 \\ \bar{c} &= (\bar{G}/\bar{\rho})^{1/2} = \bar{r}^{\alpha/2} \end{aligned} \tag{19}$$

where α is a constant. With these functions of $\bar{G}$ and $\bar{\rho}$, our results may be compared directly with those of Refs. 1 and 2. For a unit step stress input, the long time asymptotic solution of stress should approach the corresponding static solution, i.e., $\bar{\tau} = 1/\bar{r}^2$, regardless of the elastic properties of the medium.

Homogeneous Medium

For a homogeneous medium, $\alpha = 0$, results of our calculation are shown in Fig. 3, for a unit step $\bar{\tau}$ input at the hole. On this figure, the results obtained by Goodier and Jahsman[1] is also shown. Figure 3 is a plot of $\bar{\tau}$ against time, at three different radial locations. As far as the arrival time, the magnitude of the peak stress, and the asymptotic static values of the stress are concerned, our results and those of Ref. 1 are in agreement. However, a slight discrepancy in $\bar{\tau}$ exists during a time period after the arrival of the wave front.

It is interesting to note that for homogeneous media, the governing equation in terms of displacement, eq. (3), is of the same form as the corresponding equation for cylindrical dilatational waves, eq. (10) of Ref. 3. If the input at $\bar{r} = 1$ is in terms of prescribed velocity, then the solutions (displacement and velocity) for the dilatational wave can be used as those for the shear wave, if the value of the wave velocity is properly adjusted. The stresses must be calculated from the proper stress-displacement equations for each case separately.

Nonhomogeneous Medium, $\alpha = 1$

With a value of $\alpha = 1$, the physical characteristics are curved lines as shown in Fig. 2. With a constant increment in $\bar{r}$ along the leading I^+ characteristic, the grid segments along the I^- characteristics are not of constant length. The change in length of the segments is not severe, and is believed to be tolerable in the numerical calculation within the region of interest. Our calculated stress distribution is shown in Fig. 4. On this figure, the results obtained by Sternberg and Chakravorty[2] are also given for comparison. Again, correct values of arrival time, peak stress, and long time asymptotic stress are obtained by both methods. A slight discrepancy exists for stress during a time period after the arrival of the wave front.

Nonhomogeneous Medium, $\alpha = 10$

This is a case of particular interest because an exact closed form solution exists.[2] For a unit step stress input, closed form solutions exist for those media with the following values of α,

$$\alpha = \frac{2(2k-1)}{3+2k} \qquad (k = 0, \pm 1, \pm 2, \ldots) \tag{20}$$

where $\alpha = 10$ is a special case corresponding to $k = -2$. As shown in Ref. 2, the Laplace transform of the displacement is a modified Bessel function which degenerates into elementary functions if α is given by (20). These elementary functions can be inverted into closed form functions of $\bar{r}$ and $\bar{t}$.

For problems with $\alpha > 2$, the Laplace transform method yields a critical time $\bar{t}_\infty$, where solutions exist only in the time interval

$$0 < \bar{t} < \bar{t}_\infty \tag{21}$$

We shall show that solutions above this critical time can be obtained from the method of characteristics.

Integrating the dimensionless physical characteristic with $\bar{c}$ given by (19), we have the equation for the leading I^+ characteristic

$$\bar{t} = -\left(\frac{2}{2-\alpha}\right)\{\bar{r}^{(2-\alpha)/2} - 1\} \tag{22}$$

This characteristic has an asymptote at $\bar{t} = \bar{t}_\infty = 2/(\alpha-2)$, where $\bar{r} \to \infty$. In Fig. 5, this leading I^+ characteristic for $\alpha = 10$ is labeled as curve OA. As $\bar{r} \to \infty$, the wave speed $\bar{c}$ and the stress $\bar{\tau}$ all approach infinity. In the Laplace transform method, the transformed displacement is governed by an ordinary differential equation with $\bar{r}$ as the independent variable, which cannot tolerate unbounded boundary values. Consequently, no solution can be obtained for $\bar{t} > \bar{t}_\infty$ from the Laplace transform method.

From the principle of domain of dependence in the method of characteristics, if proper values of $\bar{v}$ is prescribed along OB (Fig. 5), which is not a characteristic, and proper values of $\bar{v}$ and $\bar{v}_t$ are prescribed on OA, then the solution is uniquely determined in the region OAB, where AB is the I^- characteristic passing through A and B. Notice that point B is at a time larger than $\bar{t}_\infty$, which is 0.25 for $\alpha = 10$. Along the leading I^+ characteristic, values of $\bar{v}$ and $\bar{v}_r$ increase without bound as $\bar{r}$ increases. The domain with which the solution can be obtained is therefore bounded by line CD, the I^- characteristic asymptotic to the line $\bar{t} = \bar{t}_\infty$. In applying the numerical integration along characteristics, accurate solution cannot be obtained for points very close to the line CD; because the characteristic grids are greatly distorted for large $\bar{r}$ and the values of $\bar{v}$ and $\bar{v}_r$ are too large.

Results of our calculation and those of Laplace transform are shown in Fig. 6, in the form of $\bar{\tau}$ against $\bar{t}$ at different radii. For $\bar{t} < \bar{t}_\infty$, the Laplace transform solution indicates $\bar{\tau}$ is constant for fixed radius. The results from method of characteristics are in complete agreement with this for $\bar{r} = 1.1$ and 1.2. The Laplace transform solution stops abruptly at $\bar{t} = 0.25$; whereas the method of characteristics yields results beyond this time, and the stress $\bar{\tau}$

remains constant for $\bar{t} > 0.25$ at $r = 1.1$. For large values of $\bar{r}$, or for values of $\bar{t}$ close to the curve CD, our results show an increase in stress, which is probably due to the inaccuracy introduced by the large distorted grids.

DISCUSSION

For the case of $\alpha = 0$ and $\alpha = 1$, the numerical results from the method of characteristics deviate slightly from the curves presented in Refs. 1 and 2, which were obtained by the Laplace transform technique. The curves presented in Refs. 1 and 2 were in rather small scale without enough resolution for accurate evaluation. Therefore, the discrepancy may be due to either inaccuracy in plotting and reading of curves; or inaccuracy in one or both of the numerical results. The basic procedure of the present calculation remains unchanged for different values of α; and for the case of $\alpha = 10$ the present calculation is very accurate. This seems to give a certain degree of confidence in the accuracy of the present method.

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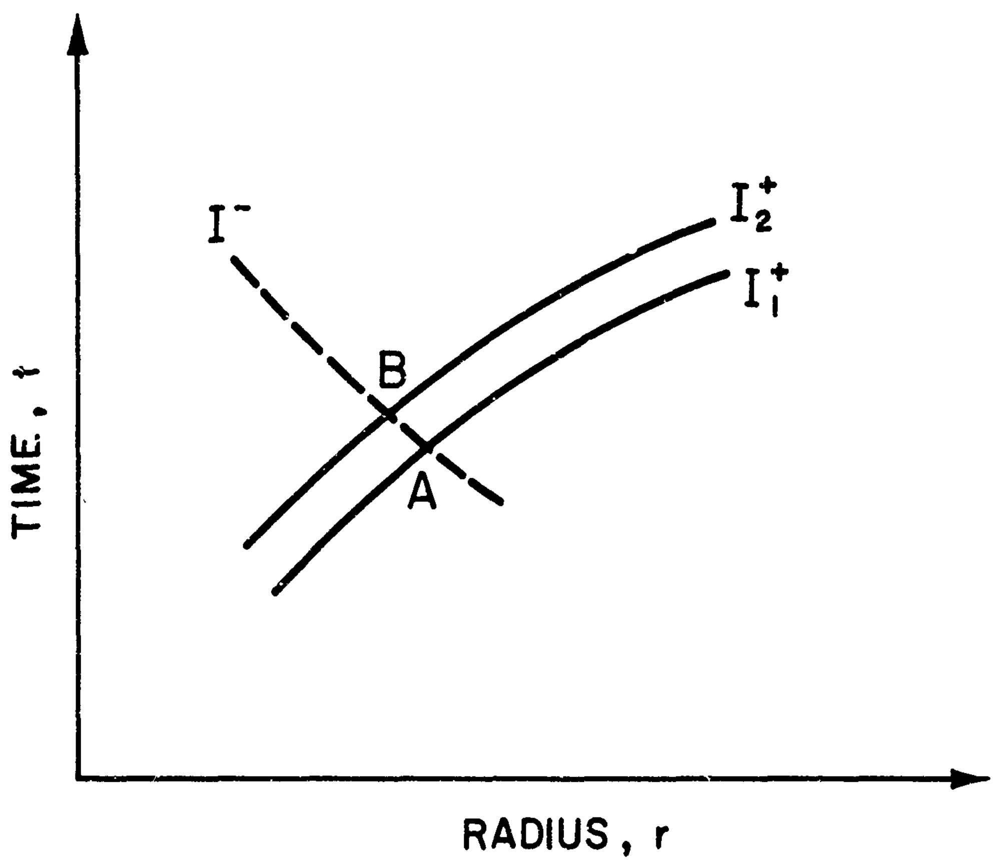

FIGURE 1 PROPAGATION OF DISCONTINUITIES ALONG A I^+ CHARACTERISTIC

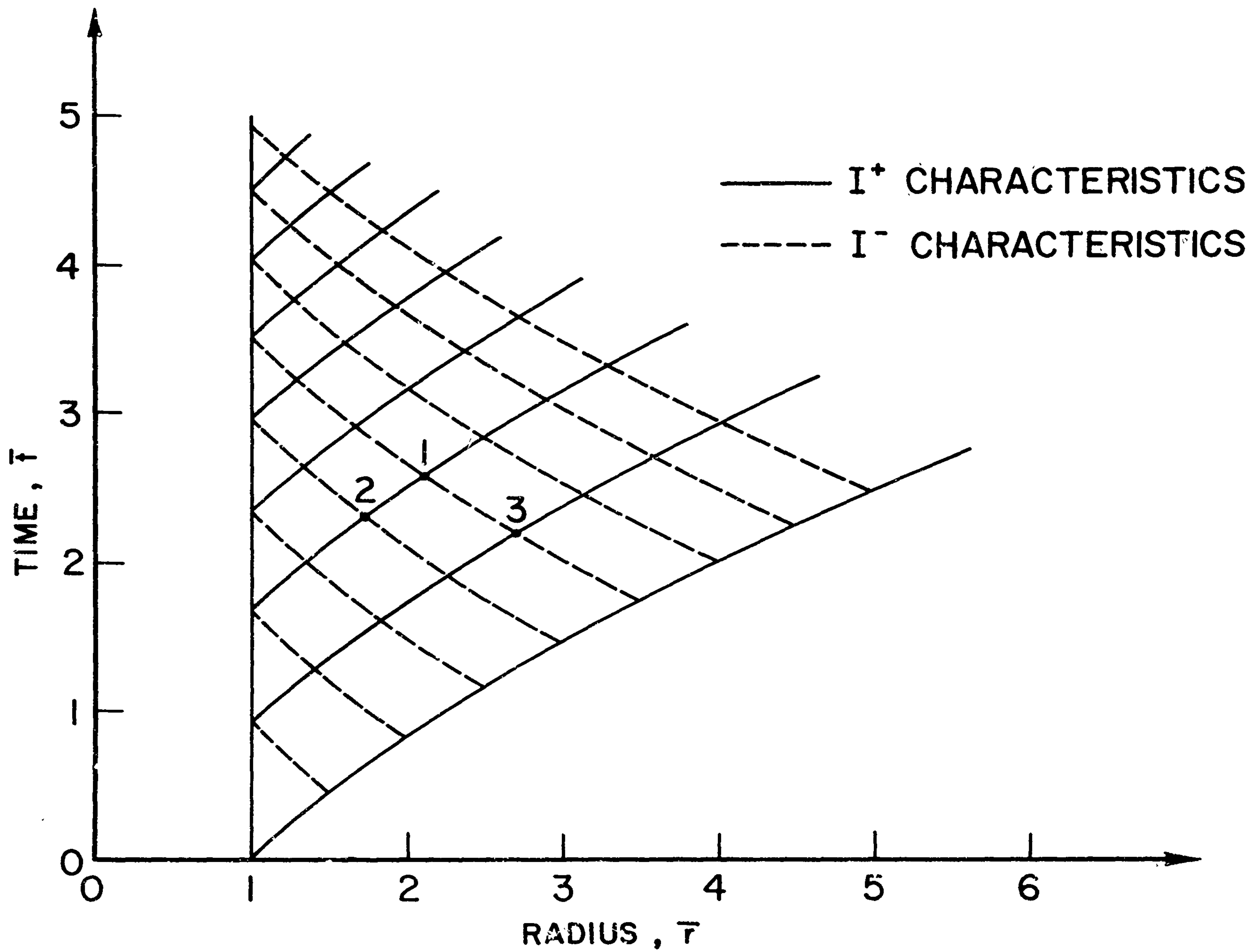

FIGURE 2 CHARACTERISTIC GRID SYSTEM FOR $\alpha = 1$

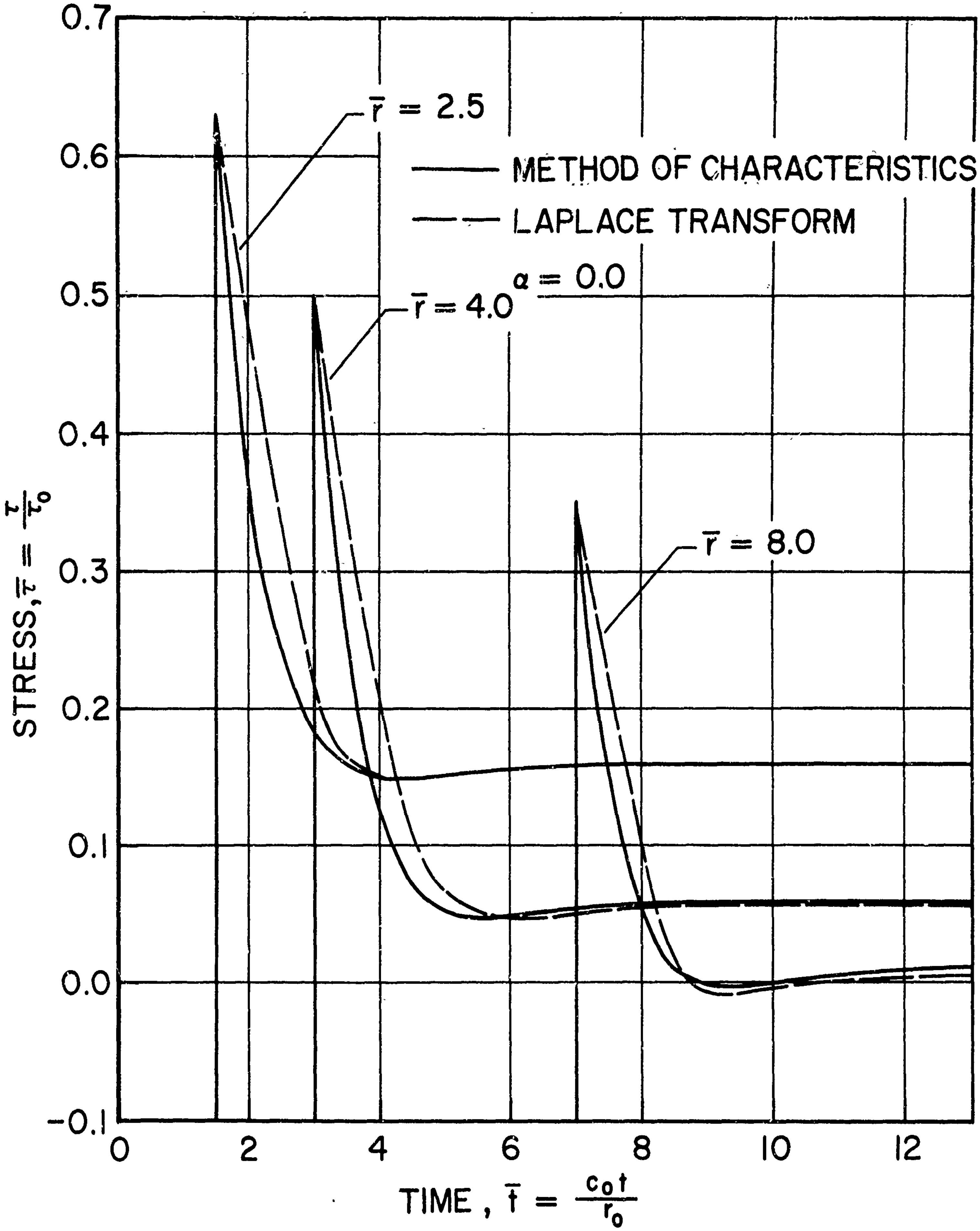

FIGURE 3 STRESS $\bar{\tau}$, VERSUS TIME $\bar{t}$, DUE TO STEP STRESS INPUT AT $\bar{r} = 1$.

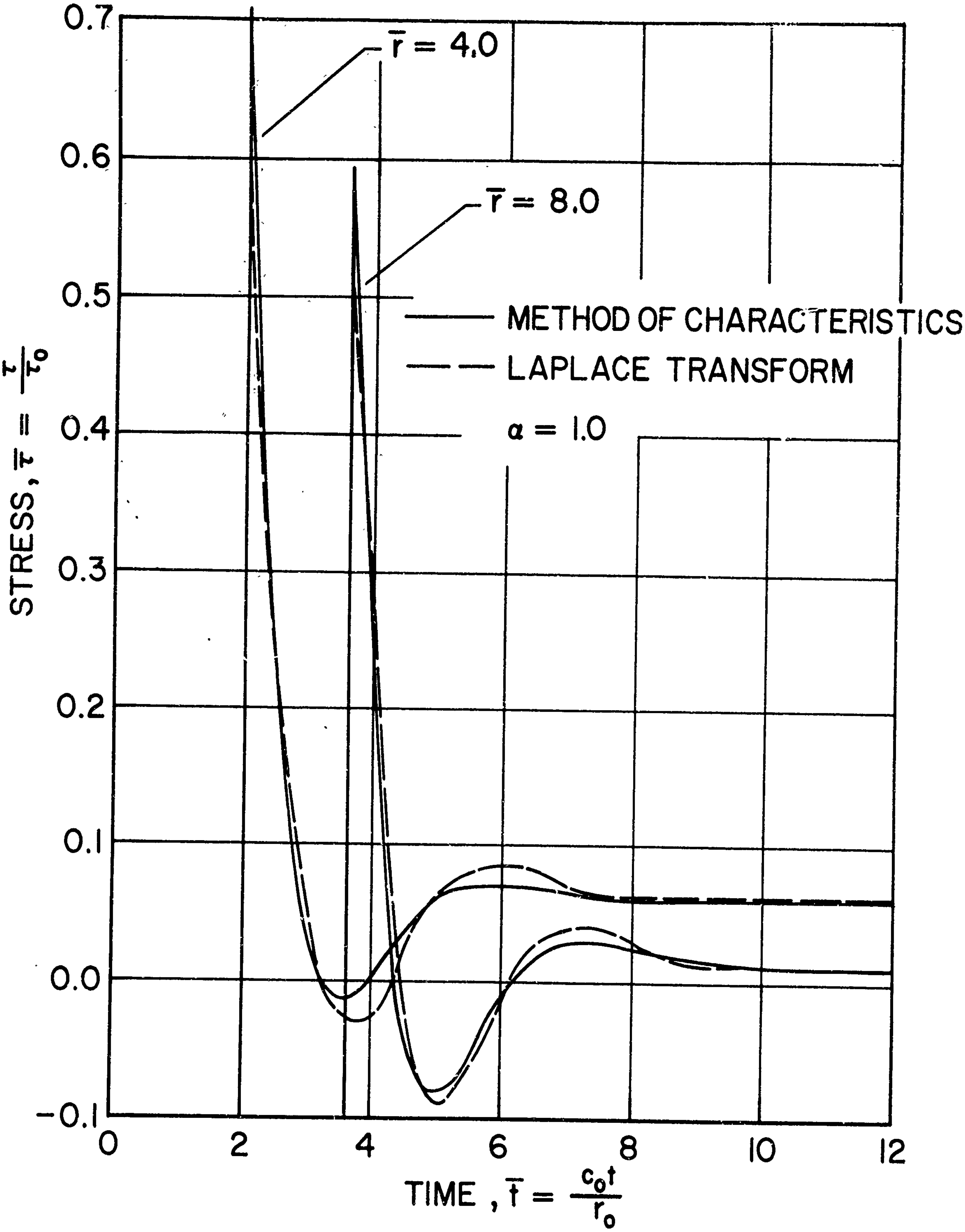

FIGURE 4 STRESS $\bar{\tau}$, VERSUS TIME $\bar{t}$, DUE TO STEP STRESS INPUT AT $\bar{r} = 1$.

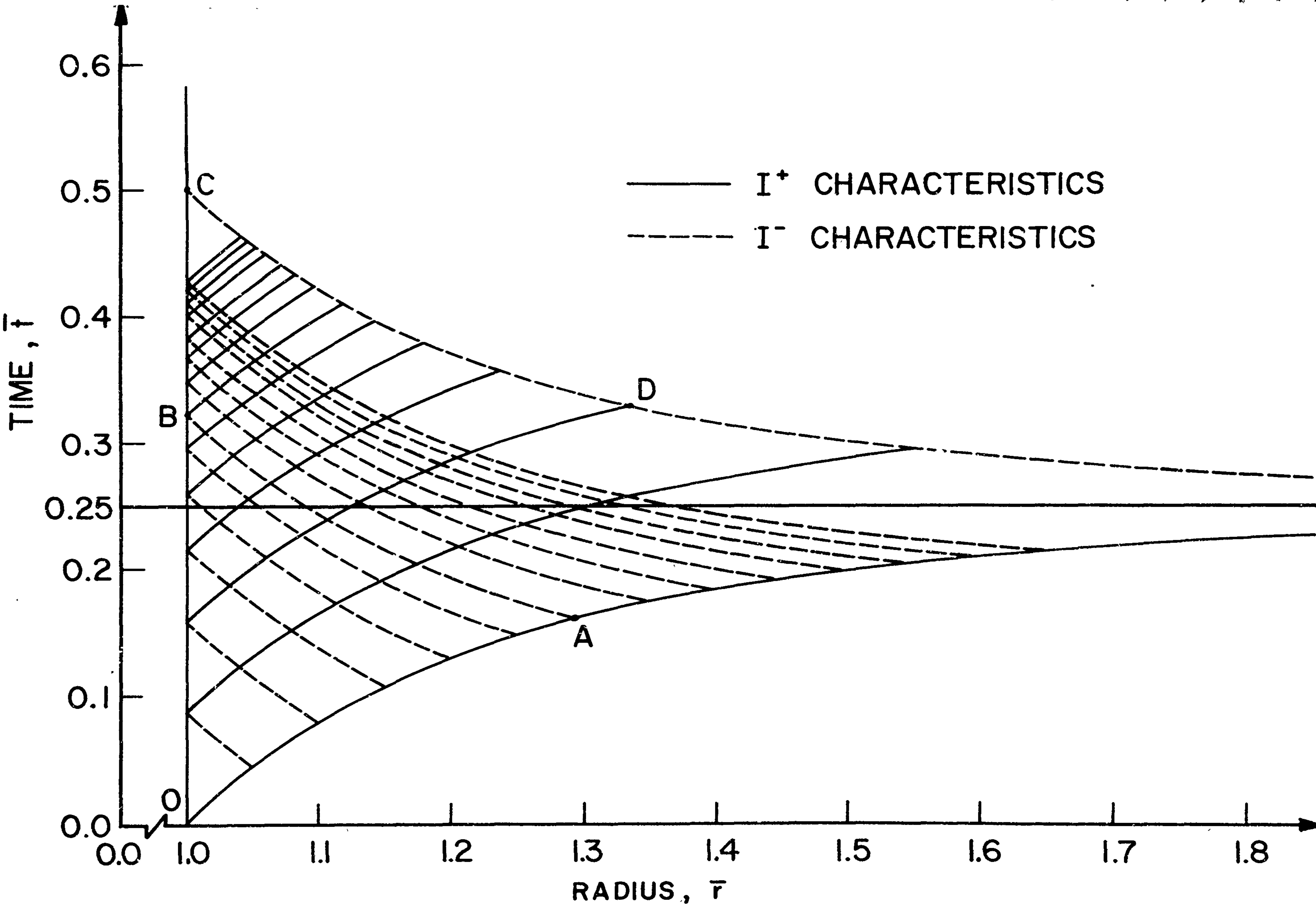

FIGURE 5 CHARACTERISTIC GRID SYSTEM FOR $\alpha = 10$

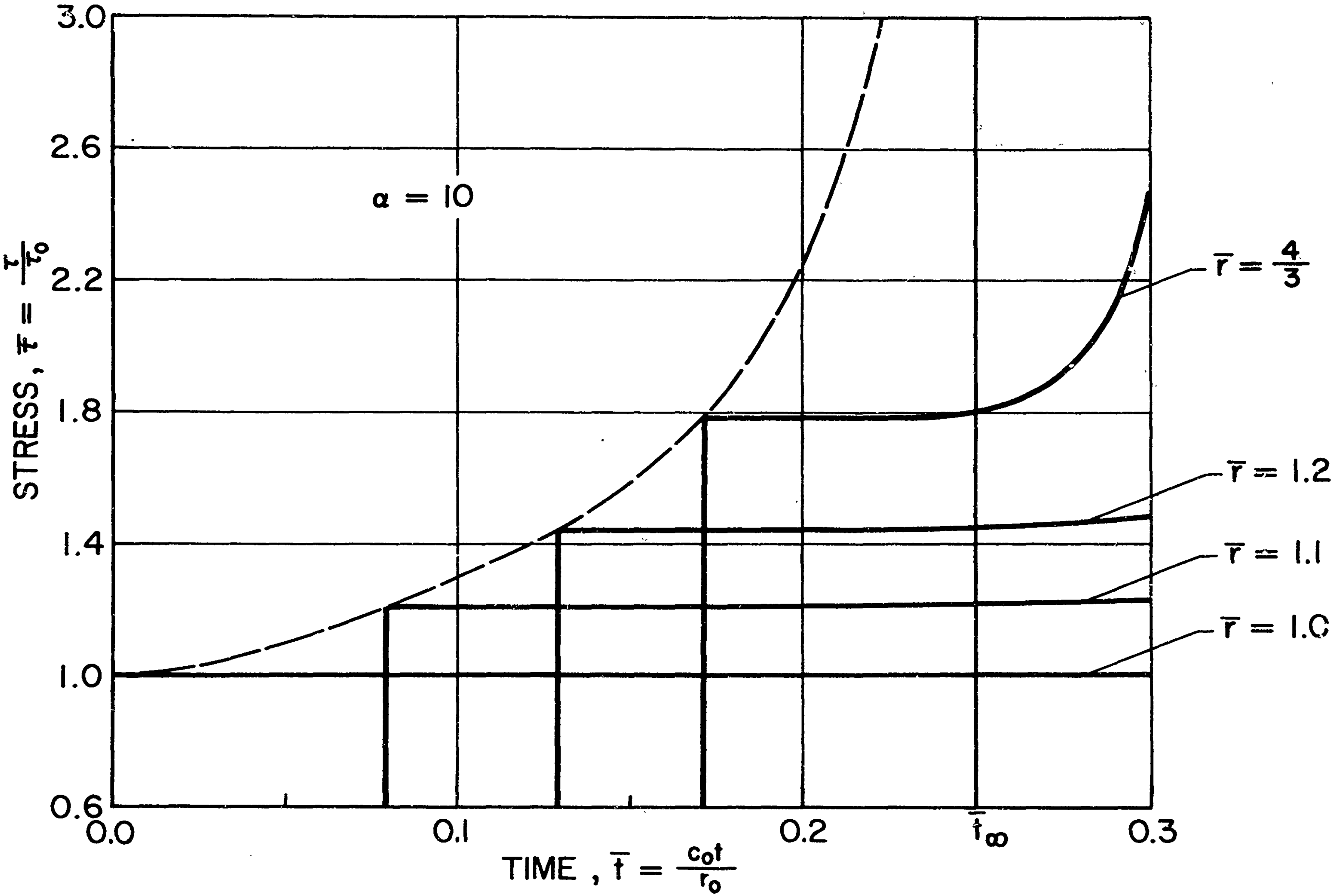

FIGURE 6 STRESS $\bar{\tau}$, VERSUS TIME $\bar{t}$, DUE TO STEP STRESS INPUT AT $\bar{r} = 1$; METHOD OF CHARACTERISTICS

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13. ABSTRACT

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METHOD OF CHARACTERISTICS

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